Foundations for the New Keynesian Model II

AEA Continuing Education, 2022, DSGE Modeling

January 9, 2022

Standard New Keynesian Model

- Discuss log linearized version of the and some of its key properties.
  - Phillips curve, natural rate of output, effect of price rigidities on response of model economy to shocks.
- Taylor rule: designed, so that in steady state, inflation is zero ($\bar{\pi} = 1$).
  - $\bar{\pi} = 1$: simplifies algebra
- Employment subsidy extinguishes monopoly power in steady state

$$(1 - v)\frac{\varepsilon}{\varepsilon - 1} = 1$$

Equations of the NK Model

- Assume $G_t = 0$,so $C_t = Y_t$.

$$\frac{1}{C_t} - \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} = 0$$

$$1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} - F_t = 0$$

$$C_t - p_t^* e^{a_t} N_t = 0$$

$$F_t \left[\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} - K_t = 0$$

$$\frac{1}{p_t^*} - \left[(1-\theta) \left(\frac{1 - \theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right] = 0$$

$$\frac{\varepsilon}{\varepsilon-1} \frac{(1-\nu)\, e^{\tau_t} C_t N_t^{\varphi}}{e^{a_t}} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t = 0$$

In steady state

Natural Equilibrium

- Natural equilibrium: NK model under optimal policy achieves flexible price equilibrium.

- Output and employment (in logs)

$$\overbrace{e^{\tau_t} C_t N_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

$$y_t^* = a_t - \frac{1}{1+\varphi}\tau_t,$$
$$n_t^* = \frac{1}{1+\varphi}\tau_t,$$

- Intertemporal Euler equation after taking logs, and ignoring V adjustment term

$$\overbrace{\qquad}^{0}$$

The Natural Rate of Interest

- Intertemporal euler equation in natural equilibrium, denoted with superscript *.

$$\overbrace{a_t - \frac{1}{1+\varphi}\tau_t}^{y_t^*} = -[r_t^* - rr] + E_t\,\overbrace{a_{t+1} - \frac{1}{1+\varphi}\tau_t}^{y_{t+1}^*}$$

- Back out the natural rate

$$r_t^* = rr + \rho\Delta a_t + \frac{1}{1+\varphi}(1-\lambda)\tau_t$$

- Shocks

$$\tau_t = \lambda\tau_{t-1} + \varepsilon_t^{\tau},\ \ \Delta a_t = \rho\Delta a_{t-1} + \varepsilon_t$$

NK IS Curve

- Euler equation in Taylor and natural two equilibria
  - Taylor rule equilibrium

$$y_t = -[r_t - E_t\pi_{t+1} - rr] + E_t y_{t+1}$$

Natural equilibrium

$$y_t^* = -[r_t^* - rr] + E_t y_{t+1}^*$$

- Output gap: $x_t = y_t - y_t^*$

$$x_t = -[r_t - E_t\pi_{t+1} - r_t^*] + E_t x_{t+1}$$

Output in the NK equilibrium

- Aggregate output relation

$$y_t = ln(p_t^*) + n_t + a_t$$

$$ln(p_t^*) = \left\{ \begin{array}{c} = 0 \text{ if } P_{it} = P_{jt} \text{ for all } i, j \\ \leq 1 \text{ otherwise} \end{array} \right\}$$

- Cool fact: since steady state inflation is 1, to a first order approximation

$$p_t^* \approx 1$$

The NK Phillips Curve

- Log-linearly expand the price setting equations about steady state.

$$1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} - F_t = 0$$

$$F_t \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} - K_t = 0.$$

$$\frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu)\, e^{\tau_t} C_t N_t^{\varphi}}{e^{a_t}} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t = 0$$

- Log-linearly expanding about steady state we obtain the NK Phillips curve

$$\hat{\bar{\pi}}_t = \frac{(1-\beta\theta)(1-\theta)}{\theta}(1+\varphi)x_t + \beta \hat{\bar{\pi}}_{t+1}$$

- Iterating forward, we see that inflation depends on current and all future output gaps, with the rate of decay depending on the Calvo parameter, θ.

- See [illegible] 5 of Derivation for a detailed derivation of these results

Equations of Equilibrium Closed by Adding Policy Rule

- Taylor rule

$$r_t = r + \alpha(r_{t-1} - r) + (1-\alpha)[rr + \phi_\pi \pi_t + \phi_x x_t]$$

- Phillips curve

$$\beta E_t \pi_{t+1} + \kappa x_t - \pi_t = 0$$

- IS equation

$$-[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} - x_t = 0$$

- Definition of the natural rate

$$r_t^* = \rho \Delta a_t + \frac{1}{1+\varphi}(1-\lambda)\tau_t$$

- All variables are deviations from steady state

Solving the model

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^{\tau} \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{bmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -1 & 1 \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ r_t \\ r^*_t \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} s_{t+1} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\rho & \frac{-1}{1+\varphi}(1-\lambda) \end{bmatrix} s_t$$

$$E_t \left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0$$

Solving the Model,

$$E_t \left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0$$

- Use method of undetermined coefficients Solving
- Solution

$$z_t = A z_{t-1} + B s_t$$

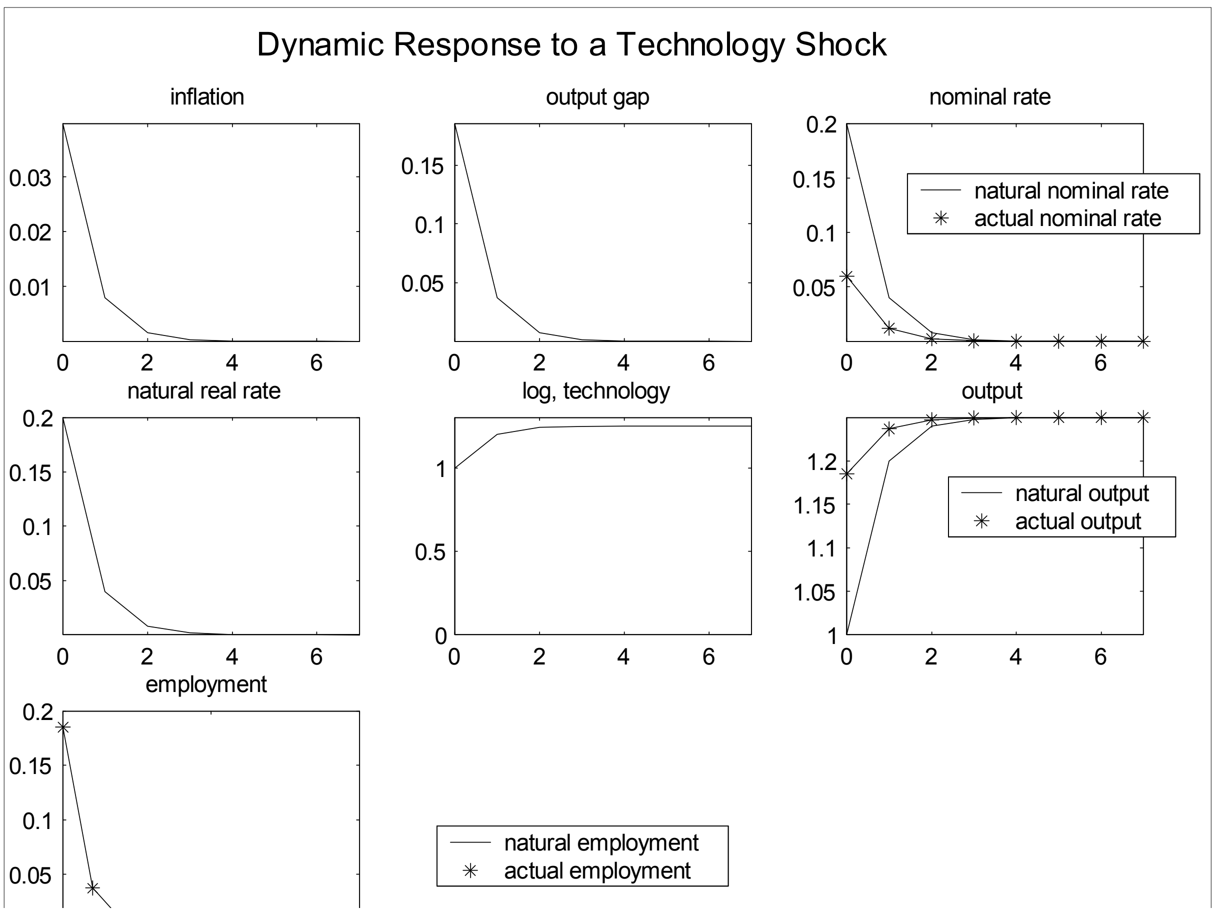
where

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0,$$

$$F = \left(\beta_0 + \alpha_0 B \right) P + \left[\beta_1 + \left(\alpha_0 A + \alpha_1 \right) B \right] = 0$$

Technology Shock

$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$



Technology Shock

- Since technology is an AR in growth rates, a shock today means that technology will be even higher in the future.
- Consumption smoothing leads households to want to increase consumption today by a lot.
- So they work really hard and the output gap and inflation are positive.
- The nominal interest rate rises but by less than the natural rate
  - In the efficient allocation, the natural rate has to rise by enough to keep hours worked and consumption demand at the pre-shock level.

What if technology was an AR(1) in log levels?

- Hours worked and inflation fall, so the output gap will be negative.
- Technology will be lower in the future than today.
- Demand goes up but not by as much as when technology is AR(1) in growth rates.
- So now firms can meet demand with less workers who are more productive.

What if technology was an AR(1) in ln levels?

- Relative to the natural equilibrium, price rigidity effectively limits the demand increase

- If prices were flexible, in the period of the shock, P_t would immediately fall
  - So expected inflation would rise.
  - Other things equal, this means the real interest rate would fall exerting upwards pressure on demand.

- With price stickiness, inflation falls, and stays *persistently* low
  - (waves of firms come each period and cut their prices, so inflation stays low for a while).
  - So expected inflation falls, rather than rise as it would if prices were flexible.
  - So the real interest rate rises, which works to lower demand.

Preference shock: acts like a negative shock to labor supply