Central bank balance sheet size, net interest income, and policy rate amplification*

Joseph Kachovec†

Reserve Bank of Australia

July 16, 2025

Abstract

This paper builds a quantitative New-Keynesian model with a central bank balance sheet, fiscal policy, and a representative financial intermediary facing uncertainty to show how interest rate policy has an additional counter-cyclical fiscal impact via central bank income when reserves are abundant as opposed to scarce. A counterfactual analysis between the US economies with small and large central bank balance sheets of 2005 and 2018 shows that in response to demand, supply, and government spending shocks, the real effects of identical interest rate changes are amplified in the abundant reserves economy. In response to a 1% preference shock, cumulative fluctuations are 4.5% lower in the output gap and 3.4% lower in inflation in the abundant reserves economy.

JEL Classification: E58, E52, E32, E60

*Any opinions expressed are those of the author and do not reflect those of the Reserve Bank of Australia. I am grateful to Gaetano Antinolfi, Miguel Faria-e-Castro, and Ping Wang for primary academic guidance and support. I am grateful to Chacko George, Luke Watson, and all other members of the FDIC Center for Financial Research seminar series. I am thankful for helpful comments from Michele Boldrin, Michael Boutros, Paco Buera, David Cashin, Manu García, Philipp Grübener, Michael Jenuwine, John Kandrac, Rody Manuelli, Filip Milosavljević, and Yongseok Shin throughout various stages of the paper. I additionally thank attendees of the Midwest Macro Meetings (Richmond) and Southern Economic Association Meetings (DC) for valuable feedback.

†Contact email: joseph.kachovec@gmail.com

1 Introduction

From 2010-2022, the Federal Reserve System (Fed) transferred an average of $83 billion per year in net income to the US Treasury Department. As of October 2024, the Fed had a deferred loss to the US Treasury department in excess of $200 billion. Changes in net income of the Fed are determined by the net interest income on the Fed's portfolio of assets.[1] Since the Fed holds interest bearing assets, net income is determined by the size and composition of the Fed's balance sheet as well as the level of interest rates in the economy. When reserves are scarce, and Fed liabilities primarily consist of interest free currency, there is a positive correlation between policy rates and net income. However, when liabilities consist of interest bearing reserves, the correlation between policy rates and central bank income turns negative. Figure 1 displays this correlation inversion between net income of the Fed (orange dashed line) and the policy rate (blue solid line) after the Fed expanded its balance sheet via quantitative easing (QE) in 2009. The policy implication of this correlation inversion is that there exists an additional counter-cyclical fiscal channel of interest rate policy under abundant reserves.

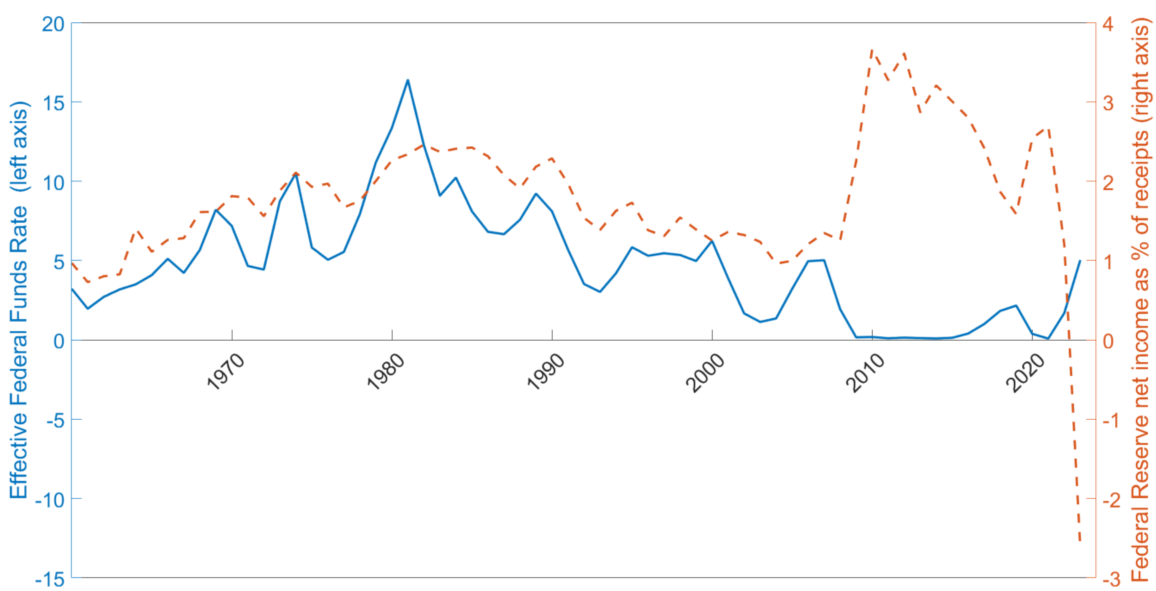

Figure 1: Federal Reserve net income and policy rates

In order to study the central bank net income mechanism, a quantitative New-Keynesian (NK) model is utilized to show how the interest rate policy of central banks with large balance sheets have larger real effects via central bank income. This paper supplements the baseline NK-model with a balance sheet for the central bank, a fiscal authority, and a financial intermediary facing uncertainty. The central bank balance sheet and fiscal authority

[1]The Fed books all assets at par value. Any premiums or discounts on asset purchases are amortized over the life cycle of the bond.

are fundamental to the topic of the paper, and the financial intermediary facing uncertainty allows us to derive interest rate spreads on various financial assets.

Results from the quantitative exercises in the paper show that "large" central bank balance sheet economies experience fewer cumulative fluctuations in response to identical aggregate shocks. Specifically, the calibration of two different economies enables the model to give a quantitative estimate of the real impact of central bank interest rate changes. A counterfactual exercise is conducted between two calibrated steady states for the US economies of 2005 and 2018. The Fed had a balance sheet of 6% of GDP in 2005 and 21% in 2018. The abundant reserves economy of 2018 experiences cumulative fluctuations 4.5% lower in the output gap and 3.5% lower in inflation for identical 1% aggregate preference shocks. Lower fluctuations in the abundant reserves economy are the result of identical interest rate changes having an additional counter-cyclical fiscal impact via central bank income. When reserves are abundant, raising interest rates decreases central bank net income, causing additional fiscal tightening.

Given the New-Keynesian framework this paper uses, the additional counter-cyclical "fiscal" effect makes the central bank's policy rate response to demand shocks more effective, while responses to supply shocks become even less effective. In the benchmark NK model an aggregate supply shock raises potential output more than aggregate demand leading to deflation and a monetary loosening. In response to a 1% supply shock, the model estimates 7.8% lower fluctuations in inflation and 2% higher cumulative deviations in output in the abundant reserves economy. This asymmetry is because the central bank is lowering interest rates while prices are falling and output is expanding. Interest rate policy's pro-cyclical impact on output and counter-cyclical impact on prices are both amplified.

The paper proceeds as follows. Section 2 reviews the relevant literature and contribution of the paper. Central bank interest rate policy has different real impacts depending on whether reserves are scarce or abundant. Holding the size of the central bank balance sheet fixed allows me to make conditional statements about interest rate policy based on an inherited size for the balance sheet. This paper documents how the interest rate policy of central banks with abundant reserves has an additional counter-cyclical impact compared to central banks with scarce reserves.

Section 3 gives a technical accounting of central bank net interest income including an algebraic example to show how changes in central bank income can constrain the budget of the fiscal authority. Section 4 builds the New Keynesian model used for the quantitative exercises in the paper. The model endogenizes the correlation between policy rates and central bank income by modeling a balance sheet for the central bank. The central bank holds a small fraction of GDP in floating rate bonds to replicate the positive correlation of central bank income and interest rates when reserves are scarce. The model then allows for an increase in the size of the central bank balance sheet through asset holdings in fixed interest rate bonds. These fixed rate bond holdings replicate the Fed's balance sheet post-QE where they control the interest rate on their liabilities and their assets pay a fixed rate.

Section 5 uses a baseline model calibration to demonstrate how central bank income feeds through the economy to obtain an additional counter-cyclical impact for interest rate policy. In this model calibration all model parameters are held constant except for the parameter determining the size of fixed rate QE bond holdings. We compare cumulative fluctuations in output and inflation when exclusively changing the central bank balance sheet size parameter. As the size of the central bank balance sheet is increased, output gaps close at a faster pace due to the additional counter-cyclical tightening via central bank income.

Section 6 uses the paper's quantitative New-Keynesian model to conduct a counterfactual exercise between two calibrated steady states of the 2005 and 2018 economies. Assuming that US central bankers have maintained Taylor-implied preferences over time, the real impacts of interest rate policy are greater in the large central bank balance sheet economy of 2018. Cumulative fluctuations in the output and inflation gaps and lower in response to preference and government spending shocks. In response to supply shocks cumulative fluctuations are greater in the output gap and lower in inflation, consistent with amplified effects of interest rate policy in the New-Keynesian model.

Section 7 compares the Federal Reserve deferred asset accounting procedure to the period-to-period transfer scheme used in previous sections of the paper. Instead of receiving a period-to-period transfer to/from the US Treasury department, the Federal Reserve only remits profits to the Treasury. In cases where the Fed experiences negative net income, a negative liability is booked and the Fed prints a corresponding positive reserve balance liability. Over the entirety of the model simulation, a deferred asset accounting exacerbates the counter-cyclical "fiscal effect" documented in this paper when compared to a period to period transfer procedure. Concluding remarks are offered in Section 8.

2 Literature

There is a large literature documenting how an increase in the central bank policy rate can increase government debt service costs. The topic those papers seek to cover is the impact of higher nominal interest rates on government service costs, debt issuance, and government spending. I examine an entirely separate fiscal/monetary interaction channel on the link between higher nominal interest rates, central bank income, and fiscal space.

Central banks now have two primary tools for which to conduct monetary policy, balance sheet and interest rate policy. Understanding the mechanisms in which these two policy levers interact is essential as the literature on central bank balance sheet policy continues to grow. The key contribution of this paper is in identifying, documenting, and estimating such a novel interaction channel. The real effects of interest rate policy are amplified when a central bank has a balance sheet large enough such that its liabilities must pay an interest rate.

The contribution of this paper lies at the intersection of two growing literatures on central bank net income and the effects of unconventional monetary policy. The literature regarding central bank net income is somewhat scarce due to the United States having no legal mandate regarding central bank net income. Many policy makers and academics argue that it should be of no concern, as it does not impact the macroeconomic stabilization mandates. Section 3.2 uses a simple balance sheet policy accounting to show how central bank income can affect fiscal/monetary interactions which has impacts on both fluctuations and policy.

Carpenter et al. 2015 gives the definitive explanation of Federal Reserve balance sheet operations and net income accounting. Auerbach and Obstfeld 2005 first examines the economic stabilization benefits from open market operations at the zero lower bound via increased fiscal seignorage of asset purchases. Bassetto and Messer 2013 examines the potential trade-offs between inflation and fiscal risks via central bank income when reserves are abundant. Berentsen et al. 2014 compares the optimal operating framework for the Federal Reserve by comparing a fed funds targeting channel system to a floor system, specifically focusing on the role of central bank net interest income in a floor system. Empirical evidence in Goncharov et al. 2023 suggests that central banks may be less prone to raise interest rates due to negative profitability concerns, leading to lower interest rates and higher inflation. Cutsinger and Luther 2022 demonstrates the rising importance of central bank income in real terms under the Federal Reserve's abundant reserves operating framework.

The paper most closely related to the approach of this paper is Bhattarai et al. 2022 where central bank net income enters directly into the loss function of the government. The government has preferred level of spending and the cost of collecting taxes is distortionary. Quantitative easing is modeled as a shortening of the maturity structure of government debt. As a result, raising the short term nominal interest rate causes the government to lose revenue from central bank profits. This creates an additional rollover cost to raising interest rates, implying central bank asset purchases at the zero lower bound (ZLB) give forward guidance that interest rates will be kept at lower levels for a longer amount of time due to increased fiscal costs of raising interest rates. While using a similar model of both fiscal and monetary authorities, this paper abstracts from asset purchases at the zero lower bound and focuses on the problem a monetary policy maker inherits at time t with an inherited size of the balance sheet and level of interest rates.

Focusing exclusively on the monetary policy problem still leaves two policy tools to examine; interest rate and balance sheet policy. The literature on the interaction between central bank balance sheet and interest rate policy as operationally independent tools is ambiguous. Sims and Wu 2020 and Debortoli et al. 2019 suggest that interest rate policy and balance sheet policy are substitutable with respect to their impact on the real economy. However Boehl et al. 2022 finds that through supply side channels this is not the case. Their work shows that QE lowers the cost of credit to a point that supply side factors make QE disinflationary, ruining the substitutability of QE and interest rate policy. Sims, Wu, and Zhang 2021 directly implies that both balance sheet and interest rate policy are needed to stabilize fluctuations.

In response to this ambiguity, this paper examines the problem of a central bank policy maker at time t in which they inherit a condition on the balance sheet size and interest rate levels. Given identical aggregate shocks, what real effects do the response of the policy maker have? With an operating framework of administered rates, the Fed can raise or lower interest rates to any level in one meeting. However the size and composition of the balance sheet can only be implemented over longer periods of time given the interaction with market participants. Since interest rate policy is the primary short term countercyclical tool, this paper keeps interest rates flexible and the balance sheet size fixed. The approach of this paper is similar to that of Del-Negro and Sims 2015 in which a central bank implements monetary policy via an interest rate rule unless specific circumstances occur in which central bank income is significantly negative.

3 Central bank income

3.1 Institutional details

As seen in Figure 2, before the global financial crisis in 2008, central bank liabilities were primarily made up of currency in circulation which pays no interest rate. In this time the Fed only expanded the balance sheet with long term purchases to meet demands for physical currency.

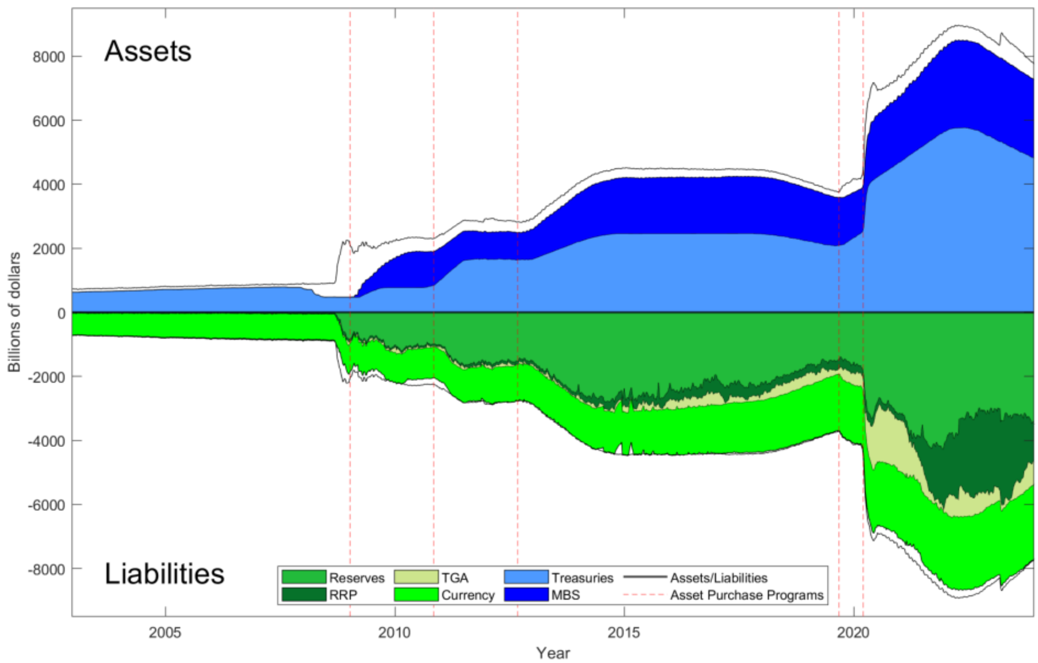

Figure 2: Federal Reserve balance sheet

As a result, central banks earned a small profit[2] from holding positive yielding treasury assets and largely interest free liabilities. The QE programs in which the central banks purchased trillions of dollars in longer-term bonds supplied money into the banking system that exceeded physical currency in the form of reserves. As a result of the QE purchases, the relationship between the size of the central bank balance sheet, short-term interest rates, and central bank profits has fundamentally changed. I proceed with a short summary of these changes, while a more thorough account can be found in Hall and Reis 2015.

In order to maintain control over the overnight policy rate, the Fed must pay an interest rate on these new reserve liabilities. A full accounting of this operating framework switch is documented in Appendix A. Paying an administered rate on liabilities has significant implications for central bank net income. Section (1) of Table 1 shows the central bank return on assets pre-QE programs. Liabilities consisted of currency in circulation which paid an interest rate of zero. On the asset side the Fed held primarily short-term treasury bills which pay a rate close to the policy rate. In this operating framework central bank liabilities will always pay 0 interest regardless of the policy rate. However, central bank assets earn an interest rate correlated to the policy rate as their treasury bill holdings are rolled over at short term market rates. As a result, the correlation between the policy rate and central bank income is positive.

[2]Goodfriend 1994 outlines the actual weekly process by which central bank income is distributed to the US Treasury department.

	Scarce Reserves	Abundant Reserves
Assets: Short Duration	**(1)** $i_t^{bill} - 0$ $Corr(i_t^{policy}, CBI_t) > 0$	**(2)** $i_t^{bill} - i_t^{IOR}$ $Corr(i_t^{policy}, CBI_t) \sim 0$
Assets : Long Duration	**(3)** $i_t^{10yearBond} - 0$ $Corr(i_t^{policy}, CBI_t) >\sim 0$	**(4)** $i_t^{10yearBond} - i_t^{IOR}$ $Corr(i_t^{policy}, CBI_t) < 0$

Table 1: Central Bank return on assets correlation to the policy rate

The post-QE operating framework is seen in Section (4) of Table 1, which is a mathematical representation of Figure 1. Asset purchases of longer term bonds that are in excess of the demand for physical currency are funded by crediting reserve accounts with an overnight deposit. The asset side of the balance sheet pays a positive long term interest rate represented by $i_t^{10yearBond}$, while the liability side pays out roughly the policy rate. As a result, the interest rate on the asset side is somewhat fixed in the short run, as it takes a longer time for assets to roll over into the higher yielding new issues. However the interest rate paid on the liability side increases at almost the same rate as the policy rate. This is why the correlation between the policy rate and central bank income is negative here. While this paper models the impact on central bank income from the Fed switching from section (1) to (4), the analysis is robust to discussions of sections (2)[3] and (3).[4]

When lowering interest rates with abundant reserves, the central bank makes profits on its portfolio of assets which eases the budget constraint of the fiscal authority. Likewise, when raising interest rates with abundant reserves, central bank net income declines leading to a tightening of the balance sheet of the fiscal authority. This is in direct contrast to a "scarce reserves" central bank balance sheet in which interest rates and central bank net income is positively correlated.

[3]A substantive critique of implementing QE in a way that a central bank finds itself in Section (2) would be the liquidity objectives that come with a balance sheet policy of forcing the banking system at large to absorb the Fed's short debt (reserves) over that of the Treasury (bills). Two primary economic reasons given for implementing QE were lowering risk and term premiums, both of which would be unaffected by purchasing shorter dated bonds. Section (2) also comes with feasibility problems because as of February 2024 there were ∼\$6 trillion in outstanding treasury bills, while the size of the Fed's balance sheet was ∼\$7.5 trillion.

[4]In this section the correlation between policy rates and central bank income is still positive as in section (1), just weakly so. As the central bank increases the policy rate, there is no change in its funding costs as reserves are scarce. Over a longer time horizon, central bank assets *slowly* roll over into higher yielding maturities. As a result, the composition of the central bank balance sheet yields the same dynamic as Section (1), just weakly so.

3.2 Algebraic example on central bank net income

This section presents a simple algebraic example of how the size of the central bank balance sheet implies different paths for central bank net income and can cause different impacts of interest rate policy. I make three changes to the canonical set up in Sargent and Wallace 1981 in which B_t is government bonds, G_t is government spending, T_t are taxes which carry an assumed dead-weight loss, and M_t is currency. This algebraic example makes three changes from the cannonical example. First, this example assumes the path of fiscal policy is given and does not depend on future monetary policies. Secondly, the decision making processes and balance sheets of the monetary and fiscal authorities are separated. The third change is seen below in the nominal government budget constraint with a term for central bank income, CBI_t.[5]

$$P_tG_t = B_t - B_{t-1}(1 + i_{t-1}^{bond}) + P_tT_t + (M_t - M_{t-1}) + CBI_t \tag{1}$$

In equation 1 the usual term for seigniorage $(M_t - M_{t-1})$ is separate from the new term for central bank income. This is because the model allows for reserve funded central bank purchases of bonds, B_t, which pay an interest rate i_t^{bond}. The central bank buys some proportion λ_t of nominal government bonds B_t and holds them on the asset side of the balance sheet. The liability side of the central bank balance sheet contains both currency and reserves (RE_t). This is shown mathematically below in equation 2.

$$\lambda_tB_t = M_t + RE_t \tag{2}$$

Though the central bank controls the nominal size of its balance sheet through λ_t policy, it does not directly control the composition of its liabilities. Varying economic agents determine how much physical currency M_t they hold and the amount of reserves in the banking system $\lambda_tB_t - M_t$ can be thought of as a residual. Reserves are modeled as a one-period bond that the Fed is issuing to banks in time t that pays out in time t+1. However the key assumption is that the interest rate on reserves is decided by the central bank in time t+1, not by the market. The formula for central bank income is seen below in equation 3.

$$CBI_t = M_{t-1} * i_{t-1}^{bond} + RE_{t-1} * (i_{t-1}^{bond} - i_t^{IOR}) \tag{3}$$

For any government bonds purchased to absorb physical currency demand M_{t-1}, the central bank earns $M_{t-1} * i_{t-1}^{bond}$, because currency is an interest free liability. However, on any reserve liabilities RE_{t-1}, the central bank earns the same interest rate from government bond holdings, but has to pay out the interest rate on reserves i_t^{IOR}, which it decides in

[5] In this simple algebraic example it is required that $B_t > 0$, which I am defining as the government being a net issuer of debt.

time t. Equation 4 shows the simplified equation for central bank income in time period t by combining equations 2 and 3.

$$CBI_t = \lambda_{t-1} B_{t-1} (i_{t-1}^{bond} - i_t^{IOR}) + M_{t-1} i_t^{IOR} \tag{4}$$

Equation 4 can be interpreted as treating all central bank liabilities as reserves, on which the central bank earns $(i_{t-1}^{bond} - i_t^{IOR})$, and then adding back the interest on reserve payments for asset holdings that correspond to currency in circulation on the liability side of the balance sheet. Taking this simplified equation, we can now see how central bank income changes with respect to changes in both levers of monetary policy, interest rate and balance sheet policy.

$$\frac{\partial CBI_t}{\partial i_t^{IOR}} = M_{t-1} - B_{t-1}\lambda_{t-1} < 0 \tag{5}$$

$$\frac{\partial CBI_t}{\partial \lambda_{t-1}} = B_{t-1}(i_{t-1}^{bond} - i_t^{IOR}) \tag{6}$$

From (5) it can be seen mathematically[6] that an increase in i_t^{IOR} decreases central bank income resulting in either a decrease in government spending, or a mix of increases in distortionary taxes T_t and bond issuance B_t. In either case, equation (6) reveals a situation when there is a nominal interest rate policy $i_t^{IOR} > i_{t-1}^{bond}$. In this case legacy asset purchases λ_{t-1} end up restricting fiscal policy in time t. In equation (1), we see how nominal government spending $P_t G_t$ has a relationship with central bank income CBI_t. We now see how nominal government spending is dependent on both current central bank interest rate policy i_t^{IOR} as well as legacy balance sheet policy λ_{t-1} via central bank income.

4 Model

The problems of the household and firm are standard for the NK-model. The central bank's interest rate rule is standard New-Keynesian, but the central bank now has a balance sheet of assets and liabilities for which it earns net income. The central bank will primarily hold a fixed rate higher yielding bond by construction. This modeling scheme replicates the composition of central bank balance sheets after conducting QE on the longer end of the yield curve. The fiscal authority raises distortionary taxes on the household and issues one period fixed and floating rate bonds to finance a preferred level of spending.

The economy faces three types of aggregate shocks; an aggregate preference shock on the utility function of the household, an aggregate supply shock on each firm's production

[6] $\lambda_{t-1} B_{t-1} \geq M_{t-1}$ by definition.

function, and a government spending shock to the level of preferred government spending. These shocks are used to compare fluctuations between economies with a different size of the central bank balance sheet.

4.1 Households

$$\max_{\{c_t(i),h_t(i),D_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{H_t^{1+\chi}}{1+\chi} \right) \xi_t$$

$$\int_0^1 c_t(i)p_t(i)di + P_tT_t + D_t \leq \int_0^1 w_t(i)h_t(i)di + (1+i_{t-1}^d)D_{t-1} + \int_0^1 \phi_t^{firm}(i)di + \phi_t^{FI}$$

$$log(\xi_t) = \rho_a log(\xi_{t-1}) + \varepsilon_t^{\xi}$$

The representative household maximizes utility subject to the time t flow budget constraint. Households gain utility from personal consumption C_t and have a disutility from labor H_t. The household can save through nominal deposits D_t in a representative financial intermediary. The time t profits of firm i are $\phi_t^{firm}(i)$, the representative financial intermediary's profits are ϕ_t^{FI}, $w_t(i)$[7] is the nominal wage in industry i, and ξ_t is a preference shock. The inverse Frisch elasticity is given by $\chi \geq 0$, the inverse elasticity of intertemporal substitution is given by $\sigma > 0$, and $\psi > 0$ is a scaling parameter.

4.2 Firms

$$\max_{\{p_t(i)\}_{t=0}^{\infty}} \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \left(p_{t+j}(i)y_{t+j}(i) - w_{t+j}(i)\frac{y_{t+j}(i)}{A_{t+j}} - \frac{\varphi}{2}\left(\frac{p_{t+j}(i)}{\pi^* p_{t+j-1}(i)} - 1 \right)^2 Y_{t+j}P_{t+j} \right)$$

$$y_t(i) = A_t h_t(i)$$

$$log(A_t) = \rho_a log(A_{t-1}) + \varepsilon_t^A$$

$$\mathcal{D}_{t,t+j} = \beta^j [U_c(t+j)/U_c(t)]$$

There is a continuum of monopolistically competitive firms, indexed by i, with production functions that are linear in labor. The price adjustment cost parameter φ determines the degree of price stickiness as in Rotemberg 1982. The problem is set up such that the steady state price adjustment costs to output are zero. When solving the profit maximization problem, firms take as given the aggregate demand curve: $y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$ derived

[7] For the industry i nominal wage $w_t(i)$ and price $p_t(i)$ we have : $X_t = \left[\int_0^1 x_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$.

from household optimality conditions and government preferences.

4.3 Financial Intermediaries

$$\max_{\{B_t^{FI}, L_t^{FI}, RE_t\}} \sum_{t=0}^{\infty} \beta^t \left((1+i_{t-1}^{bill})B_{t-1}^{FI} + (1+i_{ss}^{bond})L_{t-1}^{FI} + (1+i_{t-1}^{iorr})\alpha D_{t-1} - (1+i_{t-1}^{d})D_{t-1} + \Xi_{t-1} \right)$$

$$Z_t D_t = B_t^{FI} + RE_t + L_t^{FI}$$
$$RE_t > \alpha D_t$$
$$RE_t + B_t^{FI} \geq \zeta D_t$$
$$i^{iorr} = 0$$

The financial intermediation sector features a representative financial intermediary. The intermediary has liabilities in the form of household deposits D_t and holds one-period assets which pay an interest rate. The three assets the intermediary can invest in are a floating rate government bond (B_t^{FI}), a fixed rate government bond (L_t^{FI}), and reserve balances (RE_t). Uncertainty on the liability side of the balance sheet, Z_t, is introduced into the FI problem to derive interest rate spreads which replicate the negative correlation between policy rates and central bank income under abundant reserves. While no term structure exists as these financial assets are all one period assets, this setting is useful for analyzing the impacts of interest rate risk during short-term fluctuations around a steady state.

As there are no retained earnings in the model and the intermediary faces a balance sheet constraint of $D_t = RE_t + B_t^{FI} + L_t^{FI}$. The intermediary also faces two additional constraints in the form of a reserve requirement, where they must hold some minimum proportion α of deposit liabilities as reserves, and a liquidity coverage ratio (LCR) constraint, where a certain percentage of deposits ζ must be held in either reserves or floating rate bonds. If a bank ends with a negative reserve position they must raise funds to clear their balance sheet.

Choices of RE_t, L_t^{FI}, B_t^{FI} seek to maximize the expected discounted flow of profits accounting for the costs of deposit withdraw uncertainty Ξ_t.[8] This uncertainty creates spreads between the interest rates paid on different financial assets. There still exists a no arbitrage condition on the various financial market assets, but it does not directly relate to the *return* on those assets. These spreads are detailed in section 4.4, and allow for positive steady state central bank income as well as fluctuations in central bank income in response to exogenous shocks.

[8] If a bank has a negative reserve position they must pay an overdraft fee equal to the interest rate on reserves to the central bank.

Intermediaries will hold reserves RE_t in excess of legal requirements as in Poole 1968 as a precaution against a liquidity/payment shock Z_t.[9] We mathematically define the preferences over liquidity uncertainty Ξ_t below:

$$\begin{aligned}\Xi_t =&(1+i_t^{ioer}) \int_{\frac{B_t^{FI}+L_t^{FI}}{(1-\alpha)(B_t^{FI}+L_t^{FI}+RE_t)}}^{Z^U} (RE_t + (D_tZ_t - D_t) - \alpha D_t Z_t) f(Z_t) dZ_t \\ &+ (1+i_t^{ioer}+\Delta_{DW}) \int_0^{\frac{B_t^{FI}+L_t^{FI}}{(1-\alpha)(B_t^{FI}+RE_t+L_t^{FI})}} (RE_t + (D_tZ_t - D_t) - \alpha D_t Z_t) f(Z_t) dZ_t \\ &+ \Delta_{LCR} \int_0^{1+\zeta-\frac{RE_t+B_t^{FI}}{B_t^{FI}+L_t^{FI}+RE_t}} (RE_t + B_t^{FI} + (D_tZ_t - D_t) - \zeta D_t) f(Z_t) dZ_t\end{aligned}$$

Following Bianchi and Bigio 2022, we assume $Z_t \sim lognormal(0, \sigma^z)$. The expected costs and benefits to holding excess reserves $(RE_t - \alpha D_t > 0)$ depend on the expected value of deposit withdraws (Z_t). The intermediary will either hold excess reserves and earn an interest rate (i_t^{ioer}) or have a reserve deficiency and have to borrow at a penalty rate $(i_t^{ioer} + \Delta_{DW}$[10]$)$ in order to fund the reserve gap. As a consequence, banks have an aversion to (in increasing magnitude) holding excess reserves and having to fund a negative reserve position. The probability of each of these scenarios is dependent upon the intermediary's portfolio choice problem. We algebraically derive the bounds of integration below:

$$\underbrace{\alpha D_t Z_t}_{\text{Required reserves after payments shock}} = \underbrace{RE_t + (D_tZ_t - D_t)}_{\text{Total reserves after payments shock}}$$

$$\underbrace{Z_t = \frac{B_t^{FI}+L_t^{FI}}{(1-\alpha)(B_t^{FI}+RE_t+L_t^{FI})}}_{\text{Reserves perfectly satiate the reserve requirement}}$$

An additional cost to deposit withdraw uncertainty comes from a LCR-regulation. Banks must pay some fine Δ_{LCR} on the deficiency in high quality liquid assets (HQLA) if they do not meet the threshold $(\zeta D_t - B_t^{FI} - RE_t > 0)$. Note that this is an *additional* cost to being short on reserves balances. If a bank fails to meet both legal requirements they have to pay discount window costs Δ_{DW} to clear their balance sheet, and then face an additional

[9] An example of which can be found in Appendix B.

[10] Δ_{DW} can be thought of as a loan from the discount window to fill their funding gap.

LCR fine. A bank's LCR constraint binds in expectation when $Z_t = 1 + \zeta - \frac{RE_t + B_t^{FI}}{B_t^{FI} + L_t^{FI} + RE_t}$ as shown below.

$$\underbrace{RE_t + B_t^{FI} + (D_t Z_t - D_t)}_{\text{HQLA after payments shock}} = \underbrace{\zeta D_t}_{\text{HQLA required to avoid a fine}}$$

$$\underbrace{Z_t = 1 + \zeta - \frac{RE_t + B_t^{FI}}{B_t^{FI} + L_t^{FI} + RE_t}}_{RE_t + B_t^{FI} \text{perfectly satiate the LCR requirement}}$$

Assuming intermediaries retain no earnings and pay any profits back to the household as a dividend, gives the term for realized financial intermediary transfers to the household in equation 7:

$$\begin{aligned} \phi_t^{FI} =& (i_{t-1}^{bill} - i_{t-1}^{d}) B_{t-1}^{FI} + (i_{ss}^{bond} - i_{t-1}^{d}) L_{t-1}^{FI} + (i_{t-1}^{iorr} - i_{t-1}^{d}) \alpha D_{t-1} \\ &+ (i_{t-1}^{ioer} - i_{t-1}^{d})(RE_{t-1} - \alpha D_{t-1}) - \Delta_{LCR}(\zeta D_t - RE_t - B_t^{FI}) \end{aligned} \tag{7}$$

4.4 Interest Rate Spreads

The key aim of the model is to show how fluctuations in central bank income can add or remove fiscal space. As a result the model necessarily derives spreads between the interest on reserves and government bonds. The interest rate on the fixed rate bond is derived as an equilibrium derived interest rate, i_{ss}^{bond}. The problem the central bank faces with respect to its net income depends on legacy asset purchases, and not necessarily current interest rate spreads in the economy. For example if central bank assets consist entirely of 10 year treasury bonds purchased 5 years ago, central bank income does not depend on the current interest rate spread between the policy rate and the ten-year treasury. What matters for central bank income is the interest rate on legacy central bank asset holdings relative to the current day policy rate.

The first order conditions for the financial intermediary are used to derive various interest rate spreads in this model. For this section we assume the interest rate on required reserves, i_t^{iorr}, is set equal to zero.[11] The optimality conditions for the modeled fixed and floating rate bonds directly imply that bonds earn a higher yield than bills. This is because in the model floating rate bonds (bills) have an additional liquidity benefit in the form of fulfilling the LCR constraint, so for banks to hold the fixed rate bond in equilibrium, they must pay a higher interest rate.

[11] Modeling the problem with $i_t^{ioer} = i_t^{iorr}$ makes reserves a safe asset and bills a risky asset. As a result the interest rate on reserves is bounded above by the interest rate on deposits.

$$i_{ss}^{bond} = i_t^{bill} + \Delta_{LCR} P\left[Z_t < 1 + \zeta - \frac{RE_t + B_t^{FI}}{B_t^{FI} + L_t^{FI} + RE_t}\right] \tag{8}$$

Further, combining the optimality conditions for floating rate bonds and reserves, 9 shows that $i_t^{ioer} \leq i_t^{bill}$. This is because reserves carry two separate liquidity benefits in fulfilling both the reserve requirement and the LCR, while floating rate bonds only help fulfill the LCR. As a result, for banks to hold floating rate bonds in equilibrium, they must pay a higher interest rate than reserves.

$$i_t^{bill} = i_t^{ioer} + \Delta_{DW} P\left[Z_t < \frac{B_t^{FI}}{(1-\alpha)(B_t^{FI} + RE_t)}\right] \tag{9}$$

One period bonds must also pay a higher interest rate than the rate on deposits, as shown in 10.

$$i_t^{bill} = i_t^{d} + \alpha\left(i^{ioer} + \Delta_{DW} P\left[Z_t < \frac{B_t^{FI}}{(1-\alpha)(B_t^{FI} + RE_t)}\right]\right) \tag{10}$$

Modeling the problem with $i_t^{iorr} = 0$ makes the choice to hold excess reserves riskier, because now holding excess reserves carries a large opportunity cost. As a result, both bills and reserves are treated as risky assets (though reserves less so than bills), and we get the interest rate structure of $i_t^{bond} \geq i_t^{bill} \geq i_t^{d} > i_t^{iorr} = 0$.

$$i_t^{d} = (1-\alpha)\left(i^{ioer} + \Delta_{DW} P\left[Z_t < \frac{B_t^{FI}}{(1-\alpha)(B_t^{FI} + RE_t)}\right]\right) \tag{11}$$

As seen above in 11, under unique circumstances the rate on deposits can exceed the rate on excess reserves. If reserves are sufficiently scarce, banks are willing to have their funding costs exceed the return of the asset due to the additional liquidity preference and aversion to being short reserves. This scenario is avoided in most reasonable model calibrations.

4.5 Government

The government is given simple rules for real spending[12] and taxes in response to real bond issuance rising above its steady state level. The government purchases rule is shown in Equation 12 where the government purchases a constant fraction γ of steady state output. Government purchases being defined as a percentage of steady state output instead of time

[12]The government buys from each firm producing differentiated goods in the same way as the household does, i.e $g_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} G_t$.

t output is done in order to isolate the impacts from a preference shock on government spending. In the case of the government purchasing a constant fraction of time t output, a household preference shock has implications for the path of government spending. This difference will be essential when we analyze the impacts of aggregate shocks in sections 5 and 6.

$$F_t = (1-\rho_F)\gamma Y^{ss} + \rho_F F_{t-1} + \varepsilon_t^F \tag{12}$$

Shown in Equation 13, there exist output costs to taxation as in Barro 1979 and more recently Bhattarai et al. 2022 in which G_t represents the government purchases of the consumption good and F_t represents government spending including a dead-weight loss of raising tax revenue. Convex output costs of raising tax revenue can be interpreted as the costs of targeting individuals who implement tax avoidance measures. Eventually the fixed costs of these avoidance measures will outweigh the potential benefits, and taxes become more costly to collect.

$$F_t = G_t + \phi_T\left(\left((T_t - \tau Y_{ss}) + 1\right)^2 - 1\right) \tag{13}$$

In this model setting, government spending can not be financed exclusively with new bond issuance as in Angeletos et al. 2024. The tax rule in 14 increases taxes in proportion to the level of private marketable government debt held by intermediaries. The model requires $\phi_B > 0$ at a sufficient level such that the model converges back to steady state in response to exogenous shocks. This guarantees the co-movement taxes and nominal bond issuance when financing government spending.

$$T_t = \tau Y_t + \phi_B\left(\frac{(B_t^{FI} + L_t^{FI})/P_t}{Y_t} - \frac{(B_{ss}^{FI} + L_{ss}^{FI})/P}{Y^{ss}}\right) \tag{14}$$

Government bond issuance to financial intermediaries balances the flow budget constraint in 16. The outstanding levels and well as new issuance levels of the two types of bonds, B_t, L_t, is set at an exogenous constant ratio Ω defined below in Equation 15.

$$\Omega = \frac{B_t^{CB} + B_t^{FI}}{L_t^{CB} + L_t^{FI} + B_t^{CB} + B_t^{FI}} \tag{15}$$

The government budget constraint is defined in Equation 16 with expenditures equaling income. The term for central bank income CBI_t is detailed in section 4.6.

$$\begin{aligned} Expenditures :& P_tG_t + (1 + i_{t-1}^{bill})B_{t-1} + (1 + i_{ss}^{bond})L_{t-1} \\ Income :& P_tT_t + B_t + L_t + \Delta_{LCR}(\zeta D_t - RE_t - B_t^{FI}) + CBI_t \end{aligned} \tag{16}$$

4.6 Monetary Policy

Interest rate policy is set following a Taylor type rule in response to fluctuations in the output gap and inflation target. There is a fixed size for the central bank balance sheet which the central bank earns income on. Relaxing this assumption is discussed in more detail in Appendix D.

$$i_t^{ioer} = \mu_i i_{t-1}^{ioer} + (1-\mu_i)\left[(i^{ioer})^{ss} + \mu_\pi(\pi_t - \pi^*) + \mu_y\left(\frac{Y_t - Y^N}{Y^N}\right)\right] \tag{17}$$

The central bank has a baseline balance sheet size of Λ which will be calibrated to 6% of steady state output. This consistently matches the ratio of the size of the Fed's balance sheet to US GDP since 1980. The asset holdings for this baseline balance sheet size are exclusively holdings of marketable treasury bonds B_t^{CB} that pay the market interest rate i_t^{bill}. These bond holdings can be thought of as "minimum size of the central bank balance sheet" bond holdings in the pre-QE period. A central bank needs to supply money into the economy to meet demand for physical currency and maintain some level of reserves in the banking system for payments system purposes. The model uses interest free required reserves that necessitate these bond holdings by the central bank, whereas the application to real world examples is currency in circulation. The difference in modeling here is simply an accounting exercise. The central bank balance sheet is defined below in Equation 18.

$$\underbrace{B_t^{CB} + L_t^{CB}}_{Assets} = \underbrace{\Lambda P_t Y_{ss} + \lambda P_t Y_{ss}}_{Assets} = \underbrace{RE_t}_{Liabilities} \tag{18}$$

In addition, the central bank decides how many longer term bonds L_t^{CB} to hold. These are considered our "QE" bond purchases and vary based on a percentage λ of steady state output. These bond holdings pay the equilibrium derived interest rate i_{ss}^{bond}, which does not fluctuate with respect to the policy rate. This is done to replicate the longer dated nature of QE purchases, and have the model replicate the longer dated maturities of QE purchases, as evidenced in Figure 3. As referenced in Section 4.4, central bank income is not dependent on current interest rate spreads, because a longer dated bond purchased during a QE program does not pay an interest rate that is dependent on the current short rate. As the short term interest rates rise, fixed rate bond holdings not held by the central bank are still absorbed by the financial intermediary. This modeling strategy is consistent with the primary dealer system in the United States, and explains how fiscal authorities are able to issue longer term debt while yield curves are inverted.

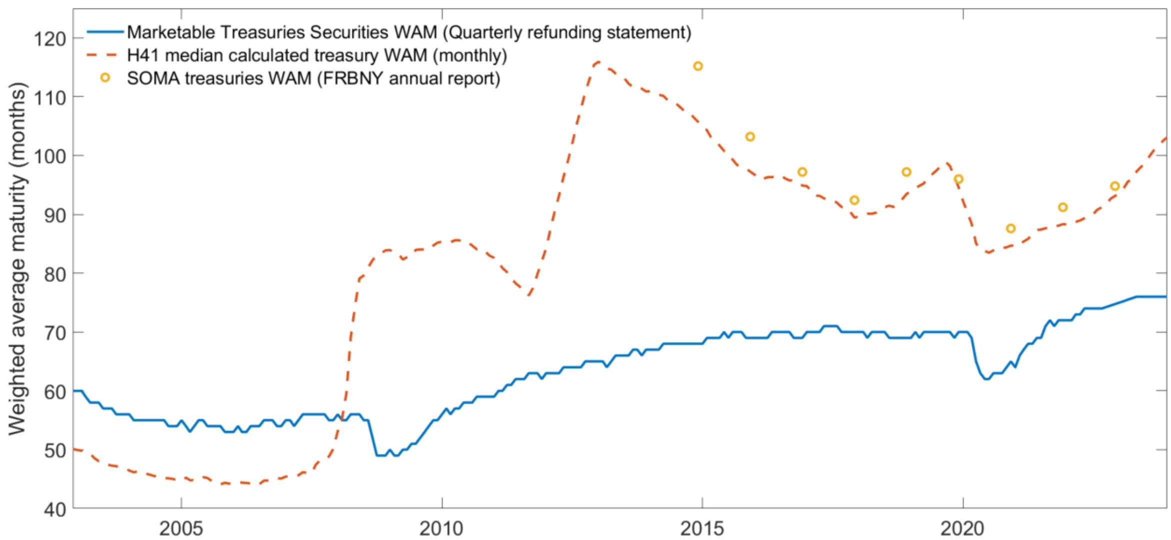

Figure 3: Evidence for the longer term nature of QE purchases

Central bank income is defined below in Equation 19. Physical currency has been removed from the model, but is approximated using αD_t which are interest free required reserves. In the model calibrations of later sections, this parameter is estimated to target the amount of currency in circulation.

$$CBI_t = B^{CB}_{t-1}(1 + i^{bill}_{t-1}) + L^{CB}_{t-1}(1 + i^{bond}_{ss}) - (RE_{t-1} - \alpha D_{t-1})(1 + i^{ioer}_t) - \alpha D_{t-1}(1 + i^{iorr}_{t-1}) \quad (19)$$

4.7 Equilibrium and Market Clearing

In a symmetric equilibrium, all firms choose the same price $p_t(i) = P_t$ and nominal wage $w_t(i) = W_t$. Aggregate income in the economy is listed below in equation 20.

$$Y_t = C_t + G_t + \frac{\varphi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 Y_t \quad (20)$$

The financial market can be thought of as similar to the primary dealer system that exists in the United States, where the central bank and primary dealers support U.S Treasury auctions. Equation 21 is requires that aggregate reserves printed by the central bank equal aggregate reserves held by the banking system.

$$RE_t = B^{CB}_t + L^{CB}_t \quad (21)$$

Using the above market clearing condition and the balance sheet of the intermediary, we derive 22. Though not immediately intuitive, household choices of deposits D_t must match outstanding government bonds. Any deposits that the household has with financial intermediaries is used to buy government bonds, or deposited at the central bank in the form of reserves, which are matched on the asset side of the central bank balance sheet with government bonds.

$$D_t = B_t^{FI} + B_t^{CB} + L_t^{FI} + L_t^{CB} \tag{22}$$

5 Model Mechanism w/ Baseline Calibration

5.1 Calibration

In order to display the basic model mechanism of the paper, this section uses a baseline calibration in which all model parameters outside of λ remain the same. I then compare the impulse response functions of each economy to show how the differing paths for central bank income change aggregate fluctuations in each λ-specific economy. Holding all model parameters constant besides the size of the central bank balance sheet allows us to focus exclusively on central bank balance sheet size and net income. Table 2 below shows the baseline calibration for more standard parameters such as the persistence of shocks.

Table 2: Standard Parameters

Parameter	Value	Description
μ_i	.8	Policy rate persistence
ρ_i	.85	Shock persistence ($i = a, x$)
ρ_g	.15	Government spending persistence
π^*	1.02	Steady state inflation
σ	.5	Inverse I.E.S
θ	11	Elasticity of substitution
ψ	1	Scaling parameter
χ	.5	Inverse Frisch elasticity
φ	98.06	Price adjustment cost

As opposed to more standard model parameters, a more detailed explanation is given for the model specific calibration below in Table 3. While it is important to note the key to this exercise is keeping all model parameters fixed across different $\lambda \in [0, .3]$-economies, I now detail why specific values are chosen for each parameter. The pre-QE balance sheet size as a percentage of GDP, Λ, is set at .06 to match the size of the Fed's balance sheet in proportion to GDP for almost all of it's history before the global financial crisis asset purchase programs.

Table 3: Baseline Calibration

Parameter	Value	Description	Source
λ	[0,.3]	ratio of QE purchases to GDP	comparative statics
Λ	.06	pre-QE (CB assets)/GDP	BEA, H41
β	.992	Discount factor	OECD
γ	.2	SS (gov. spending/GDP)	OMB, BEA
τ	.207	SS (taxes/GDP)	H41, H8
Ω	.18	(Tbills)/(total debt) ratio	MSPD
α	.05	"Reserve Requirement"	excess reserves ~ 0 for $\lambda = 0$
Δ_{DW}	.004/4	DW spread to IOER	full sample average
Δ_{LCR}	.004/4	LCR penalty	$= \Delta_{DW}$
σ_z^2	.12	Deposit Volatility	Bianchi and Bigio 2022
ζ	.2	Liquidity coverage ratio	$\overset{max}{\zeta}$ s.t $\zeta D_t \leq RE_t + B_t^{FI}$
ϕ_B	.18	Tax/bond issuance rule	min s.t $\exists$ convergence
ϕ_T	.2	Output costs to taxation	mechanism >> reserve demand
ϕ_π	1.5	Taylor coefficient on inflation	Taylor Principle
ϕ_y	.125	Taylor coefficient on output	ε_t^F inflationary

The interest rate paid on bank deposits is pinned down by the household euler equation and dependent on the discount rate β. A value of .992 is chosen in order to match the 3 month CD rate in the United States of 3.52% in December 2023 as calculated by the OECD. The ratio of government spending to GDP, γ, was chosen in line with the historical average for the USA, pre-covid pandemic. The steady state taxes to GDP ratio, τ, determines the level of steady state government debt which makes it a key determinant of the composition of bank balance sheets. A level for τ is chosen to match the bank balance sheet moments in Figure 4.

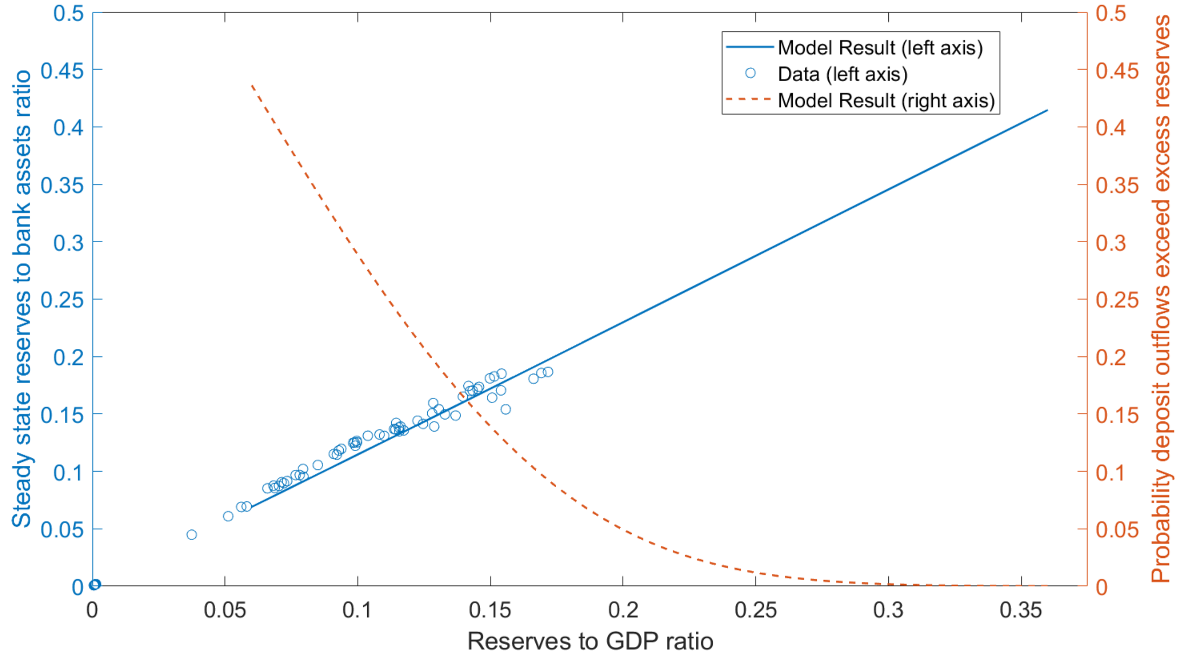

Figure 4: Matched moments of financial intermediary balance sheets

The x-axis in Figure 4 is the ratio of reserves in the banking system to GDP. This axis is directly proportional to the comparative static parameter λ. The left hand y-axis (blue) is the ratio of reserves to bank assets. The blue circles are data points for the reserves to bank assets ratio calculated from the Federal Reserve H.8 release on the assets and liabilities of commercial banks in the United States from 2003-2023. The blue line is the model implied-ratio for the reserves to bank assets ratio under the baseline calibration. The right y-axis (orange) represents the implied probability that a bank expects deposit withdraws to cause a reserve deficiency. Deposit withdraw volatility σ_Z^2 is taken from Bianchi and Bigio 2022. The probability decreases as the amount of reserves in the banking system determined by λ increases.

The ratio of treasury bills to outstanding government debt ratio, Ω, is calculated from the March 2024 monthly statement of public debt (MSPD) by diving total public outstanding treasury bills by total outstanding treasury public debt. A reserve requirement, α, is chosen such that the level of excess reserves in the $\lambda = 0$ (pre-QE) economy is zero. The LCR constraint is set at the minimum level such that in equilibrium it is never violated. Penalty interest rates for violations of the reserve requirement and LCR are set equal to each other($\Delta_{DW} = \Delta_{LCR}$) at the average spread between the discount window primary credit rate and the interest rate on reserves over the entire sample under which reserves have earned an interest rate.

For the coefficients on our policy rules, the fixed Taylor rule coefficients ϕ_π and ϕ_y are set in line with the Taylor principles such that a government spending shock has an immediate inflationary impact. The impact of output costs to taxation ϕ_T are set high enough so that other factors in the model are outweighed by the counter-cyclical fiscal effect of central bank income. In the tax-bond issuance rule for the fiscal authority, it is preferred to have the government finance deficits mostly through bond issuance. Therefore ϕ_B is set at the minimum level such that the model can still converge back to equilibrium in response to the exogenous shocks defined in the model.

5.2 Model Solution and Results

The steady state model solution is derived analytically around an interest rate for the fixed rate bond with the optimality conditions of the household, firms, financial intermediary, as well as policy rules for the fiscal and monetary authority. A comprehensive list of model equations can be found in Appendix C. When the economy is hit with aggregate supply or demand shocks, this interest rate on bonds L_t are unaltered in order to capture the effect of central bank income when holding a portfolio of fixed rate assets. Due to market clearing, these fixed rate bonds are held on the balance sheets of the central bank or financial intermediary. Relative demand for floating rate assets changes based on nominal policy rate responses to aggregate shocks and determines market response of the interest rate, i_t^{bill}, on the floating rate bond B_t.

Figure 5 below shows the impulse response functions of economies with different sizes for the central bank balance sheet. In these impulse response functions we see that in response to an initial 1% preference shock, which raises household demand for both consumption and savings, the central bank raises interest rates almost exactly the same amount. In the abundant reserves economy (dashed orange line) central bank net income declines, while in the scarce reserves economy (blue line) central bank income increases. The shortage of government revenue in the abundant reserves economy implies a higher path for distortionary taxes.[13] As a result of these higher distortionary taxes, the output gap closes more quickly in abundant reserve economy.

[13] In all λ-economies, a positive aggregate shock increases the rate of inflation. As a result, in all economies we see a decline in taxes due to increased seignorage from inflation. While this seignorage channel is not the focus of this paper, Angeletos et al. 2024 find that a boom in real economic activity both expands the tax base and creates a surge in inflation. Both of these factors erodes the real value of nominal government debt and deficits can become "self-financing".

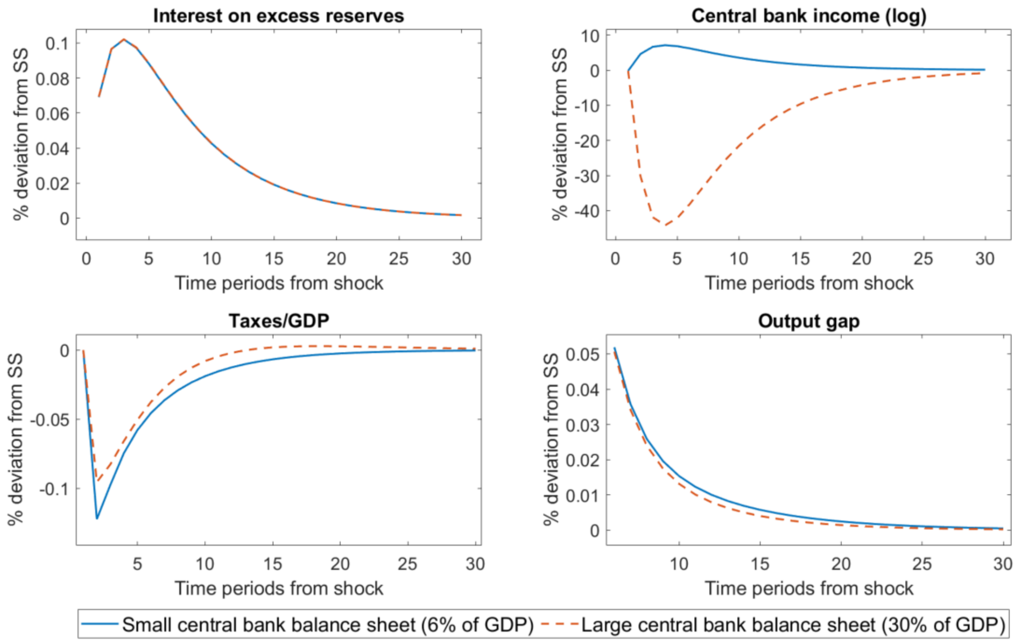

Figure 5: IRFs for aggregate preference shock

I measure economic fluctuations in output and inflation as compared to a baseline where the central bank balance sheet size is 6% of steady state output. In equation 23 the variable $X_{t,i}$ is the value the impulse response function for variables: $(\pi_{t,i} - \pi^*)$ and $(Y_{t,i} - Y_{t,i}^N)/Y_{t,i}^N$, where t is the number of time periods since the shock, and i is the type of shock (aggregate supply shock, preference shock, or government spending shock).

$$Deviations_{\lambda,i} = \frac{\sum_{t=0}^{\infty} |X_{t,i} - X_{ss}|_{\lambda} - \sum_{t=0}^{\infty} |X_{t,i} - X_{ss}|_{\lambda=0}}{\sum_{t=0}^{\infty} |X_{t,i} - X_{ss}|_{\lambda=0}} \tag{23}$$

Figure 6 displays the results from the quantitative exercise described above. Conducting interest rate policy "naively" by ignoring fiscal impacts of central bank net income can either dilute or amplify cumulative economic fluctuations. For a 1% government spending shock, economies that have a central bank balance sheet 30% of GDP experiences cumulative fluctuations that are 1% lower in output, and 1% lower in inflation compared to an economy with a CB BS size of 6% of GDP. These results can be interpreted as recessions/expansions being $\sim$ 1% smaller in economies with the larger central bank balance sheet, meaning instead of a 2% recession we have a 1.99% recession.

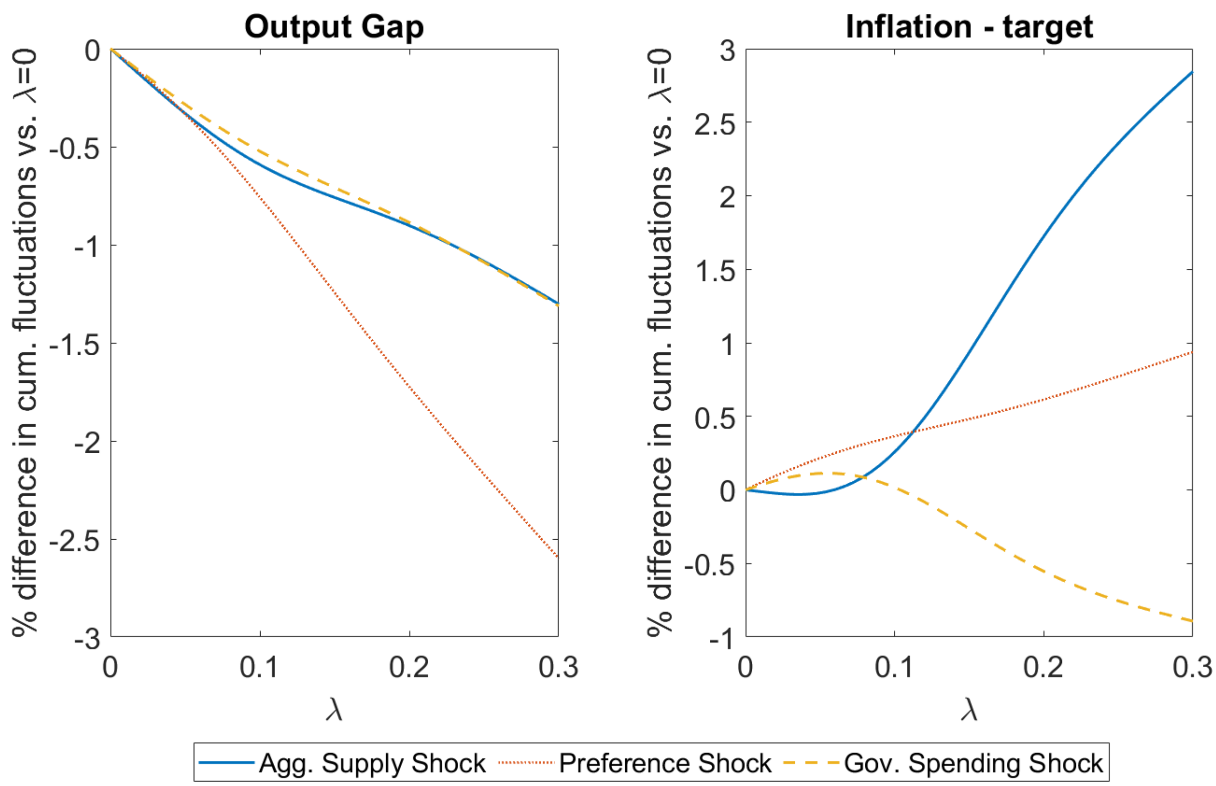

Figure 6: Results for the quantitative model

Results in this section are dependent on changing both the size *and* composition of the central bank balance sheet. The scarce reserves balance sheet has exclusively floating rate bond on the asset side, while the abundant reserves balance sheet has a majority fixed rate, higher yielding bonds. This calibration is chosen to replicate the observable changes in asset composition of global central banks pre and post large scale asset purchase programs. Results in Appendix E show that the results of this section are robust to both isolating exclusively on balance sheet size and exclusively on balance sheet composition. These results demonstrate that with abundant reserves there exists a channel where raising interest rates decreases central bank net income tightening the government budget constraint withdrawing fiscal space. The larger the central bank balance sheet, the larger the additional counter-cyclical "fiscal effect" of interest rate changes.

With a positive preference shock, we get an increase in inflation, increasing government seignorage, and allowing for a decrease in taxes to clear the government budget constraint. As a result we see a reduction in distortionary taxes, increasing government purchases for the consumption good[14], which are exogenous defined preferences. Since the path for taxes

[14] As taxes decrease, the government devotes fewer resources to tax collection, and can purchase more of he consumption good as defined by the government spending rule $G_t = \bar{F} - \phi_T\left(\left((T_t - \tau Y_{ss}) + 1\right)^2 - 1\right)$.

is *lower* the larger the size of the central bank balance sheet, these secondary effects cause higher price fluctuations in response to positive preference shocks. Given these conflicting impacts, we turn to a more data-dependent analysis in the next section to estimate the "fiscal effect" of central bank income in different λ-economies.

6 Linearized 2006 vs. 2018 Steady State comparison

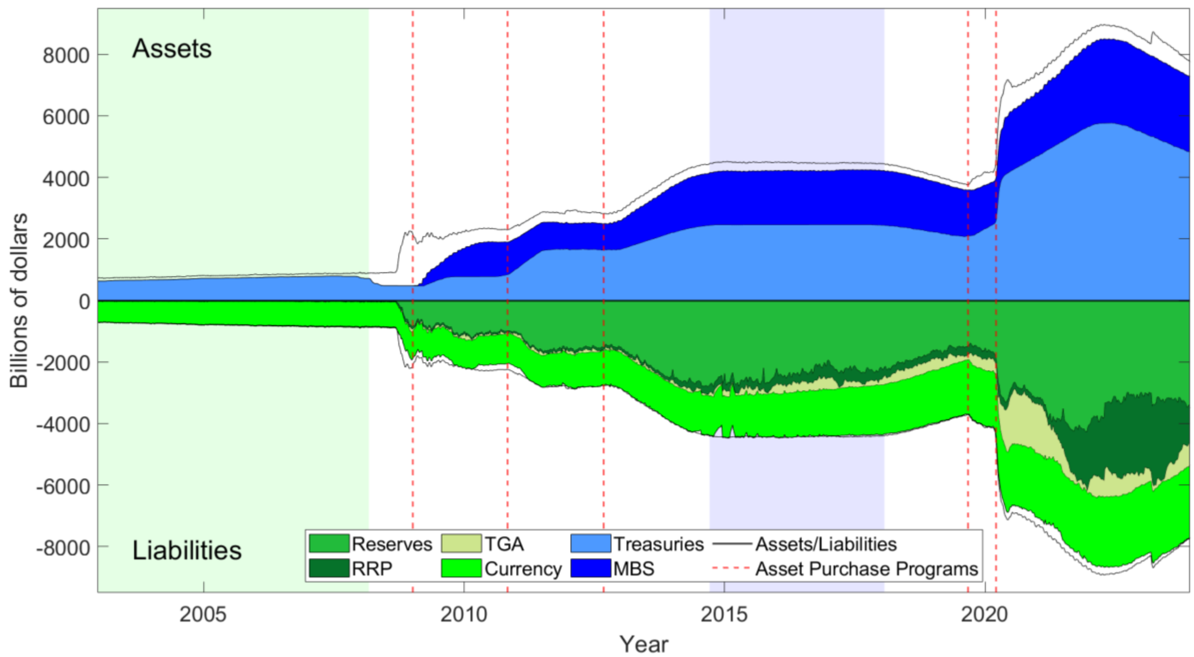

Figure 7: Federal Reserve balance sheet periods of stability

A primary feature of my analysis in section 5 is assuming a fixed size for the central bank balance sheet. This leaves open a criticism that a small central bank balance sheet economy and a large central bank balance sheet economy could have underlying differences. In order to obtain a quantitative answer to how much interest policy is amplified by larger central bank balance sheets, I use two periods of relative stability in the size of the Federal Reserve's balance sheet. These areas are shaded in Figure 7 above. The first period (green shaded area)is pre-2008, when the composition of the the balance sheet was Treasury bills on the asset side and currency on the liability side. The second period of stability in the balance sheet was between 2014-2018 (blue shaded area) when the Fed had a policy of reinvesting principal payments into new issues of Treasuries and MBS in order to maintain the size of the balance sheet. The quantitative exercise in this section measures how each economy responds to identical aggregate shocks.

For the scarce reserves calibration 2006 is used as the base year and for the abundant reserves calibration 2018 is used. In both of these years the Federal Reserve was tightening the policy rate close to a terminal level of a Fed funds rate. In a tightening cycle we can think of the economy being close to the steady state. I avoid using an average of years or the final year of the tightening cycle due to transition dynamics. The final year of the tightening cycle when the terminal policy rate is obtained is avoided due to the yield curve inversions occurring in each of these economies. Interest rate spreads are a key targeted moment in the counterfactual analysis.

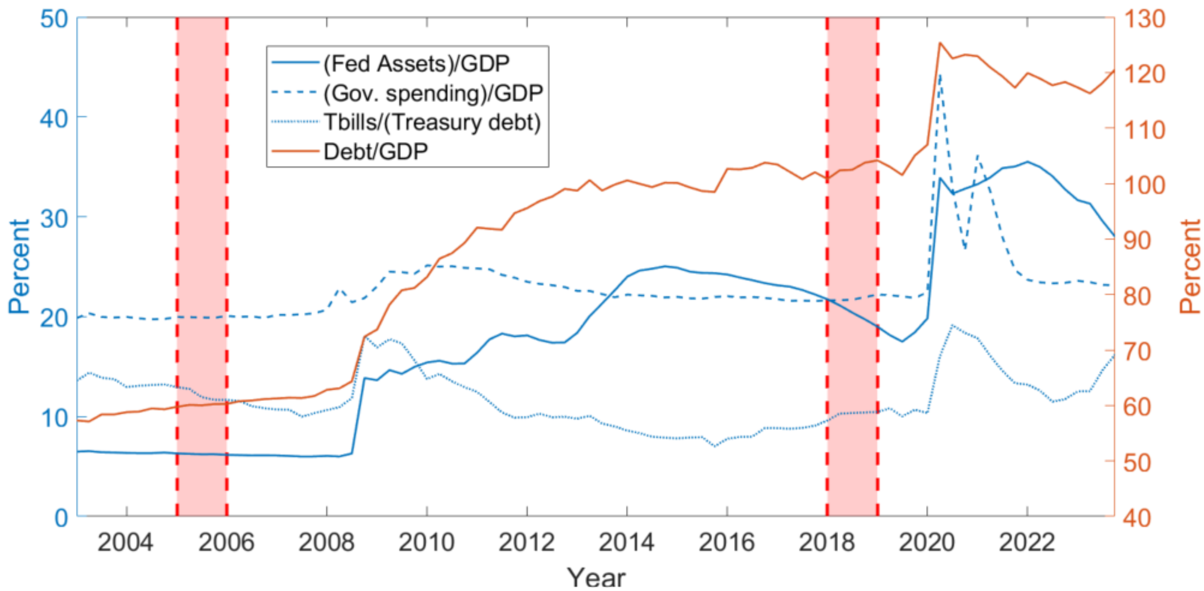

Figure 8: Calibrated steady states

Therefore these two years are chosen for a comparative statics exercise in which the model of section 4 is calibrated to match four ratios plus the slope and level of the yield curve. These six moments are chosen as they are the key identifying ratios of the quantitative model. These ratios are the size of the federal reserve balance sheet to GDP, government spending to GDP, debt to GDP, and the Treasury bills to treasury debt ratio. A time series of each ratio is chosen above in Figure 8. The yield curve is calibrated to three different assets to match the three assets of the financial intermediary portfolio choice problem. The interest rate on reserve balances is calibrated as the federal funds rate (policy rate). The floating rate bond is calibrated as the three month treasury bill, and the fixed rate bond is calibrated as the ten year treasury rate.

6.1 Calibration

The external parameters for the calibration are listed below in Table 4. I externally calibrate three out of the four moments used to define the steady states. The size of the federal reserve balance sheet to GDP is determined exogenously by λ, government spending to GDP by γ, and the Treasury bills to treasury debt ratio by Ω. The forth real moment of the debt to GDP ratio is calibrated endogenously. I detail the calibration process below.

Table 4: Externally calibrated parameters

Parameter	2005	2018	Description	Source
λ	0	.15	ratio of QE asset holdings to SS GDP	H41, BEA
Λ	.06	.06	ratio of pre-QE asset holdings to SS GDP	H41, BEA
γ	.2	.22	SS (gov. spending/GDP)	BEA
Ω	.14	.1	(Tbills)/(total debt) ratio	MSPD
Δ_{DW}	.00975/4	.0053/4	DW spread to FF	H15, NY fed

Tables 5 and 6 detail the process for what parameter values were chosen to match which moments in the data. The discount factor values of β are chosen to match the level of the yield curve. The level of the yield curve is primarily determined in the model based on the household Euler equation and the interest rate on deposits, matched in the data as the 3-month CD rate. After choosing β to pin down the level of the yield curve, the steady state level of government bonds in the economy (and thus steady state debt to GDP) is pinned down by τ. Given values of β and γ we can pick a value for τ, steady state taxes, to match the steady state debt/GDP ratios of 51% and 100% for 2006 and 2018 respectively.[15]

Table 5: 2005 counterfactual calibration

Parameter	Value	Description	Source
β	.99956	discount factor	level of yield curve
τ	.20225	SS (taxes/GDP)	Debt/GDP $\sim$ 51%
α	.112	Reserve Requirement	(Currency/CB Liabilities) $\sim$ 97 %
ζ	.14	LCR	$\overset{max}{\zeta}$ s.t $\zeta D_t \leq RE_t + B_t^{FI}$
Δ_{LCR}	.0212/4	LCR penalty	(10yr - 3 month spread) $\sim$ 1.07%
σ_z^2	.001	Deposit Volatility	3 month - FF spread $\sim$ 0bps

[15]This calibration implies a specific proportion of steady state tax receipts to GDP that are slightly higher than they are in the data. However, the model results avoid this as a problem by setting steady state output costs to taxation equal to zero.

Table 6: 2018 counterfactual calibration

Parameter	Value	Description	Source
β	.9988	discount factor	level of yield curve
τ	.22115	SS (taxes/GDP)	Debt/GDP $\sim$ 100%
α	.0815	Reserve Requirement	(Currency/CB Liabilities) $\sim$ 39 %
ζ	.251	LCR	$\overset{max}{\zeta}$ s.t $\zeta D_t \leq RE_t + B_t^{FI}$
Δ_{LCR}	.018/4	LCR penalty	(10yr - 3 month spread) $\sim$ 98bps
σ_z^2	.26	Deposit Volatility	3 month - FF spread $\sim$ 16 bps

The model parameter α is the reserve requirement, for which the interest rate on required reserves is set to zero. Since the required reserve ratio has been moved to zero in many advanced economies such as the USA and Canada, I use "required reserves" in my model as an approximation for currency in circulation. All domestic currency in circulation in an economy is booked as an interest free liability of the central bank, which matches non interest bearing required reserves in my model. I use the paramater α to calibrate the model to have "required reserves" be the equivalent of currency in circulation in proportion to the central bank balance sheet. This ratio is equal to 97% in the 2005 economy and 39% in the 2018 economy. Since the size of the central bank balance sheet is also proportional to GDP, the level of currency in circulation to GDP is also effectively hit.

The parameter ζ is the liquidity coverage ratio constraint in the model. This constraint means that the intermediary must pay a fine Δ_{LCR} to the government if they hold too few reserves and floating rate bonds. The parameter ζ for the LCR constraint is chosen such the constraint is never violated. The FI problem is a tool to obtain interest rate spreads in the model in order to study central bank income. Therefore I calibrate the model away from these LCR transfer payments to avoid another channel for income to be redistributed to the fiscal authority. Additionally, since the LCR constraint is never violated, we can set the penalty rate Δ_{LCR} to whatever level needed in order to derive the interest rate spread between our fixed rate and floating rate bond. This value is set to match the interest rate spreads between the 10 year treasury and 3 month treasury bill in both 2005 and 2018.

Finally, the σ_z^2 is set to target the interest rate spread between reserves and the floating rate government bond. In the data this is estimated as the spread between the policy rate (federal funds rate) and the 3 year treasury bill rate. In Table 7 the targeted moments from the parameter calibration are shown. Due to the calibration technique used, we can effectively hit most targets.

Table 7: Targeted moments

Moment	Data (2005)	Model (2005)	Data (2018)	Model (2018)
(Fed Assets)/GDP	.06	.06	.21	.21
(Treasury bills)/(Treasury debt)	.14	.14	.1	.1
(Government spending)/GDP	.2	.2	.22	.22
(Government debt)/GDP	.51	0.51	1.02	0.99
10 year - 3 month spread	.0107	.0108	.0094	.0092
3 month - policy rate spread	0	0	.0014	.0015
(Currency)/(Fed liabilities)	.97	.95	.39	.39

Table 8: Non-targeted moments

Moment	Data (2005)	Model (2005)	Data (2018)	Model (2018)
Deposit rate	.035	.038	.022	.025
(Central bank income)/GDP	.0016	.0018	.003	.0031
(Central bank income)/receipts	.01	.009	.019	.014

Non-targeted moments can be seen above in Table 8 and below in Figure 9. We can target the moments on the yield curve very effectively given the calibration strategy in the previous section choosing Δ_{LCR}. This is the penalty interest rate banks must pay if their holdings of floating rate bonds and reserves are below a certain percentage of assets.

In the left panel of Figure 9 we see results for the targeted yield curve moments of the 2005 and 2018 economies. The non-targeted moments for central bank income as a percentage of GDP is seen in the right panel of Figure 9. Central bank income in proportion to GDP is marginally higher in the model under both calibrations due to simplifying the central banks' balance sheets. The moments of central bank income with respect to percentage of government tax receipts is a bit lower due to steady state taxes exceeding steady state government spending. Such an assumption is needed to obtain a positive steady state level for government debt, but makes tax receipts higher than we see in the data. This lowers the model-based estimates of steady state taxes to GDP.

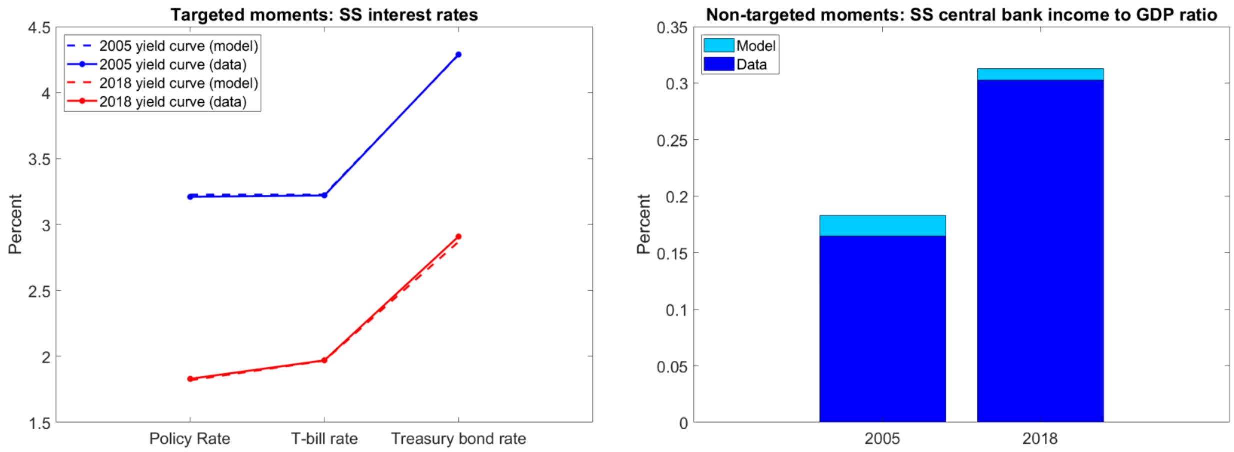

Figure 9: Targeted and non-targeted moments

6.2 Counterfactual exercise results

Table 9 displays the quantitative estimate of the different impacts of interest rate policy in 2018 vs 2005 using the same technique as section 5 and Equation 23. In response to a 1% aggregate preference shock, fluctuations in the large central bank balance sheet economy of 2018 are 4.5% lower in the output gap and 3.4% lower in inflation. This means that in response to an identical shock, the 2018 economy returns to steady state $\sim 4\%$ faster than the 2005 economy.

Table 9: 2018 fluctuations compared to 2005

1% log deviation	Output Gap	Inflation
ξ_t	4.5% lower	3.4 % lower
A_t	2.07 % higher	7.8 % lower

A feature of this new counter-cyclical seignorage channel is that it amplifies the effect of interest rate policy whether or not that policy increases or decreases fluctuations. Take for example the aggregate supply shock in Table 9. In the benchmark NK model an aggregate supply shock raises potential output more than aggregate demand leading to deflation. In response, the central bank conducts a monetary loosening to get inflation back to target. We see more deviations in output with a larger balance sheet because the central bank is loosening while output is expanding. Thus there is an additional "pro-cyclical" impact on output, instead of an additional counter-cyclical impact. The central bank net income mechanism is still adding an additional impact to interest rate policy.

Table 10 shows the results from a government spending shock that is 1% of GDP. We do not analyze a 1% log deviation in government spending because the 2018 shock is roughly

10% larger than the 2005 shock ($\gamma^{2018} = .22 > .2 = \gamma^{2005}$). The 2018 economy experiences lower economic fluctuations in both the output gap and inflation, representative of the additional counter-cyclical impact of central bank income when raising interest rates with abundant reserves.

Table 10: 2018 fluctuations compared to 2005

1% of GDP Shock	Output Gap	Inflation
F_t	0.54% lower	1.08% lower

7 Policy Analysis: deferred asset accounting scheme

To this point in the paper we have shown with a larger balance sheet, interest rate changes of central banks have an additional counter-cyclical effect. Therefore less of an interest rate change is needed to obtain the same real impact. If a central banker "naively" ignores this channel, they are conducting policy under the wrong information set about all the channels of interest rate policy. As the paper does not take a stance on any "joint optimal" policy of the central banks balance sheet and interest rate policy, we now examine central bank accounting procedures designed to minimize the central bank income effect. This allows for the central banker to solve a monetary policy problem under the traditional setting.

Throughout this paper we have assumed there are no retained earnings of the central bank, and that on a period to period basis the central bank either transfers money to the fiscal authority, or receives a transfer if net income turns negative. In reality, while the Federal Reserve remits weekly profits to the fiscal authority, they do not receive a transfer if net income goes negative. Instead, the Federal Reserve system books an IOU to the treasury in the form of a negative liability or "deferred asset". Deferred asset accounting works as follows in Equation 24, where CBI_t is the central banks current net income and DA_{t-1} is the inherited deferred asset. A direct corollary is that the central bank's transfer to treasury is always strictly positive: $CB_t^{transfer} = max\{CBI_t + DA_t^-, 0\}$.

$$DA_t = \begin{cases} 0 & \text{if } DA_{t-1} + CBI_t \geq 0 \\ DA_{t-1} + CBI_t & \text{if } DA_{t-1} + CBI_t < 0 \end{cases} \tag{24}$$

Including this new negative liability, the updated balance sheet for the central bank is show below in Equation 25.

$$\underbrace{B_t^{CB} + L_t^{CB}}_{Assets} = \underbrace{RE_t + DA_t^-}_{Liabilities} \tag{25}$$

The central bank determines how many assets to hold, which pins down the size of the balance sheet. However, the composition of the liabilities of the central bank can change when the Fed is booking the deferred loss. In this case "negative capital" of deferred losses causes the level of reserve balances to become greater than the amount of bonds held as assets. As the Fed is taking losses on its portfolio, commercial banks are on the other side of that trade making money. If the Fed does not take a transfer payment from the fiscal authority, the only way the Fed can pay out its interest expense is to print money in the form of reserves. The central bank balance sheet changes for a non-zero deferred asset are shown in 26.

$$\underbrace{(=)B_t^{CB} + (=)L_t^{CB}}_{(=)Assets} = \underbrace{(\uparrow)RE_t + (\downarrow)DA_t^-}_{(=)Liabilities} \tag{26}$$

Non-linear perfect foresight simulation results for the 2018-calibrated economy are shown below in Figure 10. I compare the period-by-period transfer rule used in sections 5 and 6 of the paper to the deferred asset accounting scheme used by the Federal Reserve and described in this section. Both economies are hit with identical preference shocks, and have the same response in in terms of the nominal policy rate on excess reserves. Four time periods after the shock, the central bank experiences it's first net income loss. In order to fund its losses the period-to-period transfer economy (solid blue line) receives a transfer from the fiscal authority. The central bank in the deferred asset economy (dashed orange line) funds these losses by printing more reserves. These additional reserves being printed into the banking system are not matched with purchases of interest bearing bonds. Rather, they are used to offset the negative IOU liability.

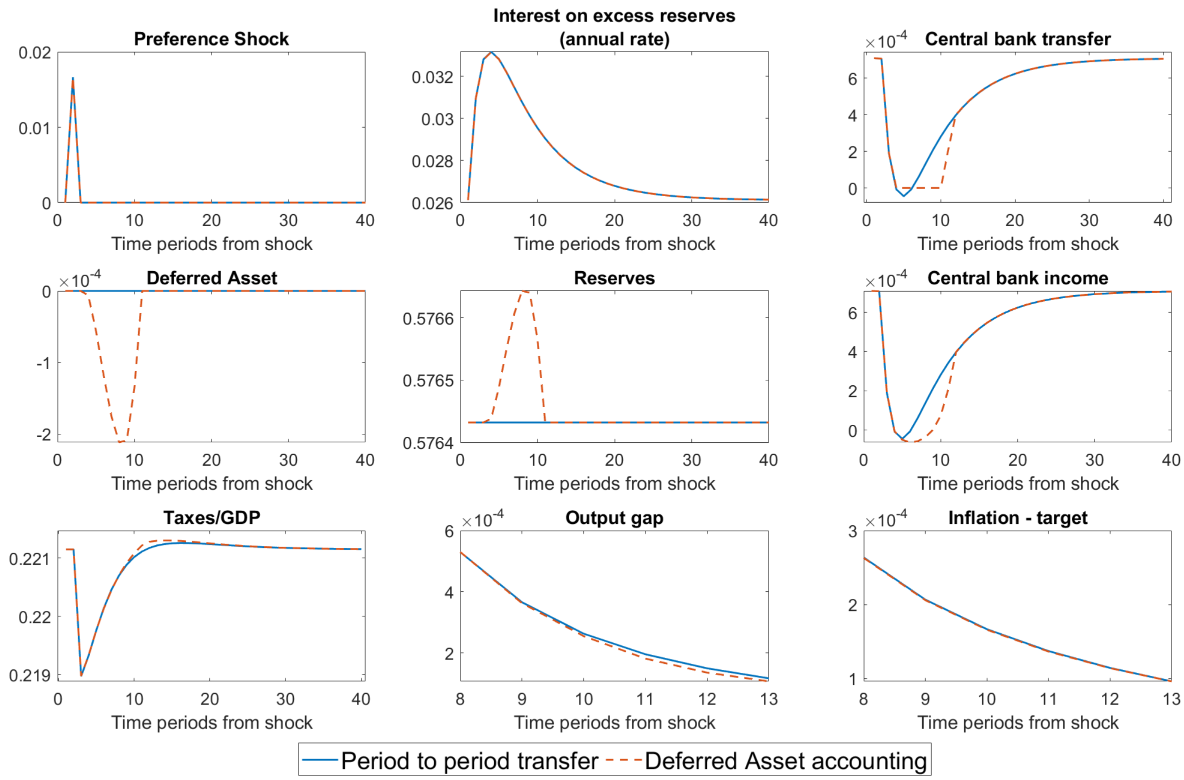

Figure 10: Deferred asset counterfactual comparison

As we see in the model simulation above, this causes aggregate seignorage from central bank income in the deferred asset accounting procedure to be lower. As a result, with deferred asset accounting there is a higher path for taxes, and this exacerbates the central bank income interest rate policy amplification over the entire cycle. By using the deferred asset accounting cumulative fluctuations are 0.5% lower in the output gap and 0.2% lower in inflation.

A first best central bank income accounting procedure would follow from the permanent income hypothesis. If the central bank was allowed to retain capital and smooth dividend payments to the fiscal authority over time, this entire interest rate policy amplification should be mitigated regardless of the size of the balance sheet. However, this type of central bank accounting scheme would come with its own set of restrictions both for implementation, as well as retaining large amounts of earnings. The Federal Reserve earned over $700 billion in profits from 2009-2019. The question of how much net income to retain, and how this impacts the size and composition of liabilities would be an important question for the Fed. Given both the political and legal restrictions on Federal Reserve surplus accounts such as Fessenden and Richardson 2016, a policy of retaining earnings is politically unlikely.

8 Conclusion

A policy setting committee ignoring central bank income would be a mistake, because swings in central bank income have different implications for the fiscal authority's budget constraint, which has real impacts. When central bank liabilities consist predominately of interest free liabilities such as currency in circulation, there exists a positive correlation between central bank income and the central bank policy rate. Large scale asset purchase programs, colloquially referred to as QE, were funded by printing reserves liabilities. In order to maintain control of overnight interest rates, the Federal Reserve began paying an interest rate on these reserve liabilities. As a result the correlation between central bank income and policy rates changed from positive to negative.

The quantitative New-Keynesian model in this paper features a central bank balance sheet, fiscal policy, and a representative financial intermediary facing uncertainty to derive the central bank income mechanism described in the introduction. The traditional interest rate policy mechanisms of the standard NK model are present in this model, and are amplified by the paper's novel seignorage channel. With an abundant reserves balance sheet, lowering (raising) interest rates increases (decreases) central bank net income which loosens (tightens) the government budget constraint providing (withdrawing) fiscal space. This is due primarily to changing the paths for central bank income resulting changes in taxes. This directly implies that identical interest rate changes have a larger impact under abundant reserves because rate changes have an additional counter-cyclical "fiscal" effect.

Under an abundant reserves balance sheet, interest rate policy is amplified by central bank net interest income. This means that, *holding Taylor rule policy constant*, interest rate policy is a more effective tool for demand shocks, and an even more inefficient for supply shocks. This is shown in the paper with a counterfactual exercise comparing the 2005 scarce reserves balance sheet economy against the 2018 abundant reserves balance sheet economy. Results show that for 1% preference shocks, the larger the central bank balance sheet implies there are fewer economic fluctuations by around $\sim 4\%$. This means that *holding Taylor rule policy constant*, instead of 2% recessions/expansions we would have 1.92% recessions/expansions with a larger central bank balance sheet.

Since identically sized interest rate moves can have different impacts depending on the size and composition of a central bank's balance sheet, policy rule coefficients could warrant re-adjustment. With abundant reserve balance sheets identical rate changes have larger real impacts than the scarce reserves counterfactual. While normative interpretations of this result depend on the preferences of the central banker, let us consider the example of the central banker who inherits a large balance sheet and a positive aggregate shock. Amplifying the real effects of interest policy could push a hawk's interest rate response to be overly restrictive.

Regardless of central bank preferences, the Federal Reserve System deferred asset accounting procedure does not mitigate central bank income amplifying the real effects of interest rate policy. To mitigate the impacts of central bank income, a central bank removing accommodation from a ZLB period with abundant reserves should begin balance sheet normalization before interest rate tightening. The smaller the balance sheet size, the smaller the impact this effect will have. The paper does not take a stance on any joint "optimal" policy of the central bank's two levers of balance sheet and interest rate policy, as preferences of the central banker are ambiguous. However, tightening first with the balance sheet before interest rates will mitigate the additional counter-cyclical impact of interest rate policy. Similarly, if the size of the balance sheet was not decreased, rolling over maturing assets into shorter duration holdings whose interest rate is positively correlated with the policy rate would also mitigate this effect.

Works Cited

Angeletos, Lian, and Wolf (2024). "Can Deficits Finance Themselves?" *Econometrica* 92 (5), pp. 1351–1390.

Auerbach and Obstfeld (2005). "The Case for Open-Market Purchases in a Liquidity Trap". *American Economic Review* 95 (1).

Barro (1979). "On the Determination of the Public Debt". *Journal of Political Economy* 87, pp. 940–971.

Bassetto and Messer (2013). "Fiscal Consequences of Paying Interest on Reserves". *Fiscal Studies* 34 (4), pp. 413–436.

Berentsen, Marchesiani, and Waller (2014). "Floor systems for implementing monetary policy: Some unpleasant fiscal arithmetic". *Review of Economic Dynamics* 17 (3), pp. 523–542.

Bhattarai, Eggerston, and Gafarov (2022). "Time Consistency and the Duration of Government Debt: A Model of Quantitative Easing". *Review of Economic Studies* 0, pp. 1–41.

Bianchi and Bigio (2022). "Banks, liquidity management, and monetary policy". *Econometrica* 90 (1), pp. 391–454.

Boehl, Goy, and Strobel (2022). "A structural investigation of quantitative easing". *The Review of Economics and Statistics*.

Carpenter et al. (2015). "The Federal Reserve's balance sheet and earnings: a primer and projections". *International Journal of Central Banking* 11 (2).

Cutsinger and Luther (2022). "Seigniorage payments and the Federal Reserve's new operating regime". *Economics Letters* 220 (110880).

Debortoli, Gali, and Gambetti (2019). "On the empirical (ir)relevance of the zero lower bound constraint". *NBER Macroeconomics Annual* 34 (1), pp. 141–170.

Del-Negro and Sims (2015). "When does a central bank's balance sheet require fiscal support". *Journal of Monetary Economics* 73 (July), pp. 1–19.

Fessenden and Richardson (2016). "The Cost of Fed Membership". *Federal Reserve Bank of Richmond Economic Brief* 16 (2).

Goncharov, Ioannidou, and Schmalz (2023). “(Why) Do central banks care about their profits”. *Journal of Finance*.

Goodfriend (1994). “Why we Need an Accord for Federal Reserve Credit Policy: A Note”. *Journal of Money, Credit and Banking* 26 (3), pp. 572–580.

Hall and Reis (2015). “Maintaining Central-Bank Financial Stability under New-Style Central Banking”. *NBER Working Paper* (21173).

Poole (1968). “Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy”. *The Journal of Finance* 23 (5), pp. 769–791.

Rotemberg (1982). “Sticky Prices in the United States”. *Journal of Political Economy* 90, pp. 1187–1211.

Sargent and Wallace (1981). “Some Unpleasant Monetarist Arithmetic”. *Federal Reserve Bank of Minneapolis Fall 1981 Quarterly Review* 5 (3).

Sims and Wu (2020). “Are qe and conventional monetary policy substitutable?” *International Journal of Central Banking* 16 (1), pp. 195–230.

Sims, Wu, and Zhang (2021). “The Four Equation New Keynsian Model”. *The Review of Economics and Statistics*.

Appendix

A Fed funds targeting vs. administered rates

Pre-QE, the Federal Reserve conducted monetary policy via “fed funds targeting”. Reserves were scarce in this operating regime, and the central bank targeted a short term interest rate by changing the supply of reserves in the banking system. This operating framework is shown in Figure 11, and is common in most money and banking textbooks.

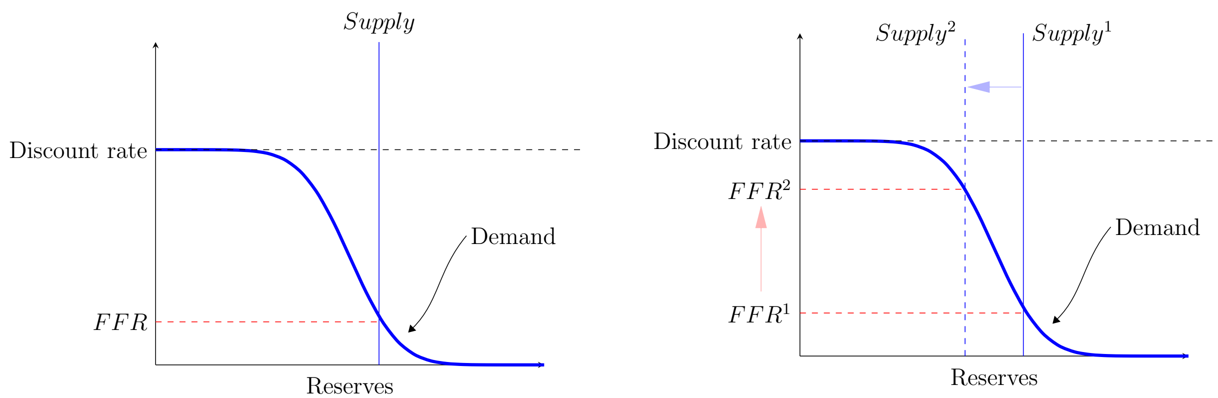

Figure 11: Raising rates with scarce reserves

However, once the supply of reserves is increased via QE, a price on reserves cannot be targeted by changing the supply because reserve demand is so thoroughly satiated. In order to implement interest rate policy by changing the quantity of reserves to target the federal funds rate, the Fed would be required to drain a significant amount of the reserves created by the quantitative easing programs on a daily basis. This would both operationally infeasible and contradictory to the bond buying programs in the first place. Instead of targeting a federal funds rate by changing the supply of reserves, the Fed now pays a variety of administered rates. The Fed switched from changing quantities to target the price for reserves to using administered rates to set the price of reserves directly.

As seen below in Figure 12, paying an administered rate on reserve balances effectively sets a floor on other short term interest rates as banks should not lend at a rate below what they are getting paid on their reserve balances. This is primarily done with two interest rates, interest rate on excess reserves and the interest rate for the Fed’s overnight reverse repurchase facility. The interest rate on reserves acts as a “soft floor” because institutions who do not earn an interest rate on their reserves such as the Federal home loan banks, Fannie, and Freddie are willing to lend at a federal funds rate below the interest rate on reserves. Foreign banks borrow at this rate and earn a risk free arbitrage profit.[16] The

[16] This is not a profitable trade for domestic commercial banks because of deposit insurance premiums.

Federal Reserve can now use balance sheet policy and interest rate policy independently of each other, while pre-financial crisis this was not the case.

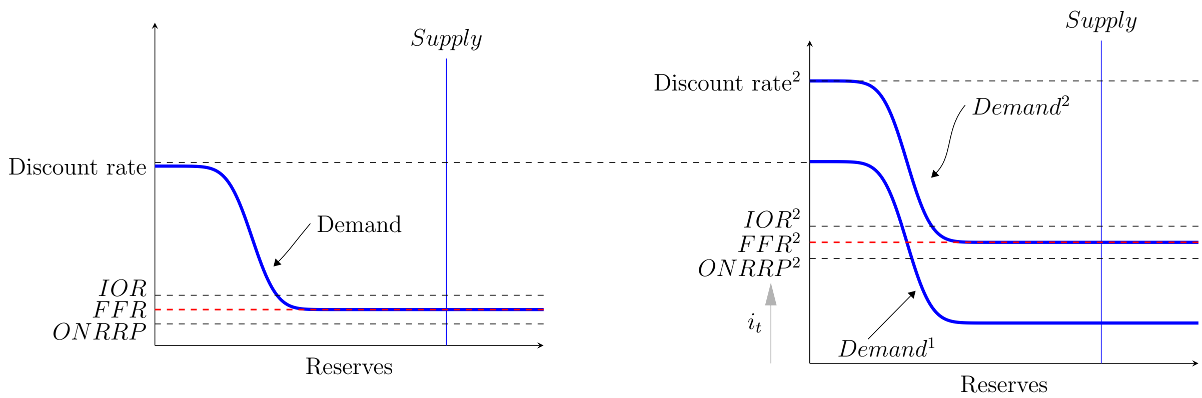

Figure 12: Raising rates with abundant reserves

B Payments system uncertainty example

An example of payments system uncertainty is shown below in Figure 13. In this example, a depositor at Bank A buys an item from a business who banks with Bank B for 10 units. Bank A funds the outflow to Bank B by transferring 10 units of reserve balances.

Bank A balance sheet ($Z_t = .9$)

$$\begin{aligned} Z_t D_t &= B_t^{FI} + RE_t^{FI} \\ .9 * 100 &= B_t^{FI} + 20 \\ -10 + 100 &= B_t^{FI} + 20 - 10 \\ 90 &= B_t^{FI} + 10 \end{aligned}$$

Bank B balance sheet ($Z_t = 1.1$)

$$\begin{aligned} Z_t D_t &= B_t^{FI} + RE_t \\ 1.1 * 100 &= B_t^{FI} + 20 \\ +10 + 100 &= B_t^{FI} + 20 + 10 \\ 110 &= B_t^{FI} + 30 \end{aligned}$$

Figure 13: Example of payments system uncertainty

C Equilibrium Model Conditions

$$Y_t = C_t + F_t + \frac{\varphi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 Y_t \tag{27}$$

$$Y_t = A_t H_t \tag{28}$$

$$C_t + T_t + \frac{D_t}{P_t} = \frac{W_t}{P_t} H_t + (i^d_{t-1} + 1)\frac{D_{t-1}}{P_{t-1}}\pi_t^{-1} + \frac{\phi_t^{firm}}{P_t} + \frac{\phi_t^{FI}}{P_t} \tag{29}$$

$$\pi_{t+1} = \beta(1 + i^d_t)E_t\left[\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right] \tag{30}$$

$$\psi H_t^\chi = C_t^{-\sigma}\frac{W_t}{P_t} \tag{31}$$

$$\begin{aligned}\phi_t^{FI} = (i^{bill}_{t-1} - i^d_{t-1})B^{FI}_{t-1} + (i^{bond}_{t-1} - i^d_{t-1})\left(\frac{1-\zeta}{\zeta}\right)(RE_{t-1} + B^{FI}_{t-1})\\ +(i^{ioer}_{t-1} - i^d_{t-1})(RE_{t-1} - \alpha D_{t-1}) + (i^{iorr}_{t-1} - i^d_{t-1})\alpha D_{t-1}\end{aligned} \tag{32}$$

$$\begin{aligned}0 =&\left(i^{bill}_{t-1} + \alpha(1 + i^{iorr}_{t-1}) - i^d_{t-1} + \left(\frac{1-\zeta}{\zeta}\right)(i^{bond}_{t-1} + \alpha(1 + i^{iorr}_{t-1}) - i^d_{t-1})\right)\\ &- (1 + i^{ioer}_t)\frac{1}{\zeta}\alpha P\left[Z_t > \frac{B^{FI}_t + L^{FI}_t}{(1-\alpha)(B^{FI}_t + L^{FI}_t + RE_t)}\right]\\ &- \alpha\frac{1}{\zeta}(1 + i^{ioer}_t + \Delta_1)P\left[Z_t < \frac{B^{FI}_t + L^{FI}_t}{(1-\alpha)(B^{FI}_t + L^{FI}_t + RE_t)}\right]\end{aligned} \tag{33}$$

$$\begin{aligned}0 =&\left(\alpha(1 + i^{iorr}_{t-1}) - (1 + i^d_{t-1}) + \left(\frac{1-\zeta}{\zeta}\right)(i^{bond}_{t-1} + \alpha(1 + i^{iorr}_{t-1}) - i^d_{t-1})\right)\\ &+ (1 + i^{ioer}_t)(1 - \alpha\frac{1}{\zeta})P\left[Z_t > \frac{B^{FI}_t + L^{FI}_t}{(1-\alpha)(B^{FI}_t + L^{FI}_t + RE_t)}\right]\\ &+ (1 + i^{ioer}_t + \Delta_1)(1 - \alpha\frac{1}{\zeta})P\left[Z_t < \frac{B^{FI}_t + L^{FI}_t}{(1-\alpha)(B^{FI}_t + L^{FI}_t + RE_t)}\right]\end{aligned} \tag{34}$$

$$\frac{D_t}{P_t} = \frac{B_t^{FI}}{P} + \frac{RE_t}{P} + L_t^{FI} \tag{35}$$

$$\varphi\frac{\pi_t}{\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) + \theta\left[\frac{(\theta-1)}{\theta} - \frac{W_t}{P_t A_t}\right] = \beta\varphi E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right)\right] \tag{36}$$

$$F_t = (1-\rho_F)\gamma Y^{ss} + \rho_F F_{t-1} + \varepsilon_t^F \tag{37}$$

$$F_t = G_t + \phi_T\left(\left((T_t - \tau Y_{ss}) + 1\right)^2 - 1\right) \tag{38}$$

$$T_t = \tau Y_t + \phi_B\left(\frac{(B_t^{FI} + L_t^{FI})/P_t}{Y_t} - \frac{(B_{ss}^{FI} + L_{ss}^{FI})/P}{Y^{ss}}\right) \tag{39}$$

$$T_t = F_t + (1 + i_{t-1}^{bill})\frac{B_{t-1}}{P_{t-1}}\pi_t^{-1} - \frac{B_t}{P_t} + (1 + i^{bond})\frac{L_{t-1}}{P_{t-1}}\pi_t^{-1} - \frac{L_t}{P_t} - \frac{CBI_t}{P_t} \tag{40}$$

$$\Omega = \frac{B_t^{CB} + B_t^{FI}}{L_t^{CB} + L_t^{FI} + B_t^{CB} + B_t^{FI}} \tag{41}$$

$$i_t^{ioer} = \mu_i i_{t-1}^{ioer} + (1-\mu_i)\left[(i^{ioer})^{ss} + \mu_\pi(\pi_t - \pi^*) + \mu_y\left(\frac{Y_t - Y^N}{Y^N}\right)\right] \tag{42}$$

$$\frac{B_t^{CB}}{P_t} = \Lambda Y_{ss} \tag{43}$$

$$\frac{L_t^{CB}}{P_t} = \lambda Y_{ss} \tag{44}$$

$$\frac{RE_t}{P_t} = \frac{L_t^{CB}}{P_t} + \frac{B_t^{CB}}{P_t} \tag{45}$$

$$CBI_t = B_{t-1}^{CB}(1 + i_{t-1}^{bill}) + L_{t-1}^{CB}(1 + i_{t-1}^{bond}) - (RE_{t-1} - \alpha D_{t-1})(1 + i_t^{ioer}) - \alpha D_{t-1}(1 + i_{t-1}^{iorr}) \tag{46}$$

D Varying the size of the central bank balance sheet via λ-shocks

The main assumption in the paper is that the size of the central bank balance sheet remains fixed. We examine the implications for relaxing this assumption in Figure 14, which shows the impulse response functions for a 1% increase in the size of the ratio of central bank assets to GDP. As a result of expanding the size of the central bank balance sheet we see an immediate decline in market interest rates, and as a result a booming economy with inflation.

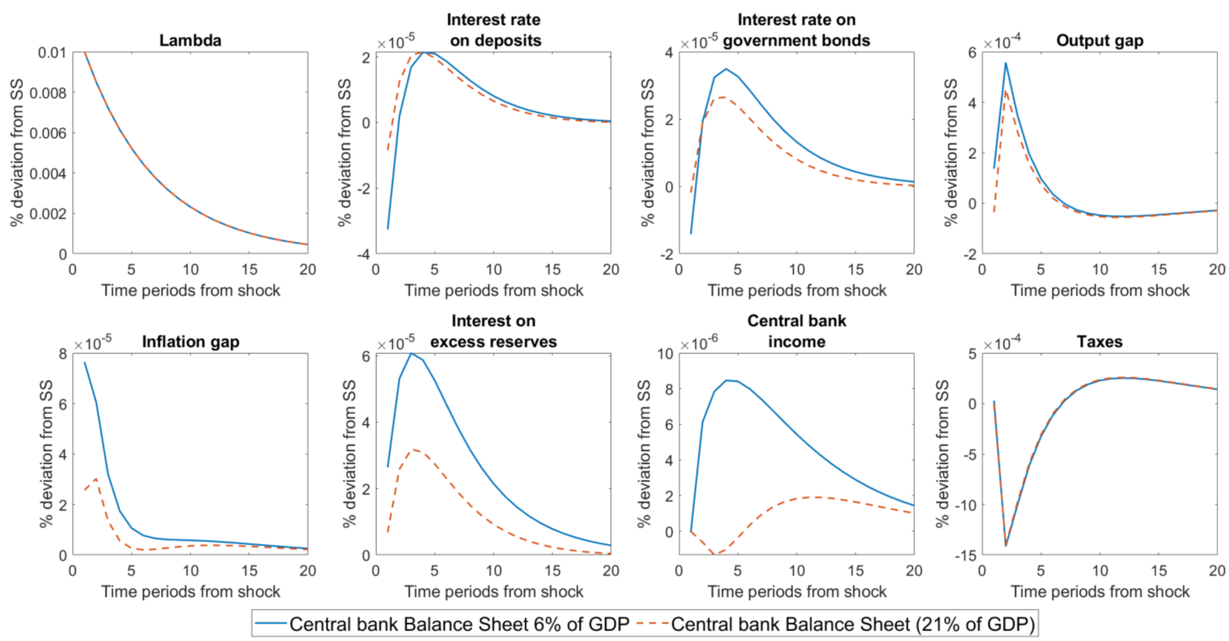

Figure 14: Impulse response functions for a λ shock

The policy rate of the interest rate on excess reserves responds with a monetary tightening due to the inflation and positive output gap. QE stimulates the economy which causes a tightening with interest rate policy. First order effects of an expansionary-QE are present, however the general equilibrium effects of QE require a richer model that considers the joint determination of optimal policy tools.

E Central bank balance sheet size vs. composition

In Section 5 the model counterfactuals are in comparison to a central bank with a balance sheet size of 6% of GDP ($\Lambda = .06$) consisting of the floating rate bond (B_t^{CB}). I then run different model simulations where the central bank balance sheet continues holding the same amount of floating rate bonds, but increases its holdings of fixed rate bonds (L_t^{CB}) via the parameter $\lambda \in [0, .3] \times 100\%$ of GDP. The claim I am making in the paper is that exclusively changing the *size* of the central bank balance sheet changes the real effects of interest rate policy. However the exercise in Section 5 is changing both the size *and* composition of the balance sheet by having the scarce reserves balance sheet have only floating rate bonds on the asset side, and the abundant reserves balance sheet having a majority fixed rate bonds on the asset side. This appendix shows that model results do not significantly change, with the exception that the abundant reserves balance sheet with floating rate assets comes with questions about both feasibility and policy relevance.

Governments primarily issue debt in long term bonds. The Fed's balance sheet reached a height of $\sim$ \$9 trillion, and treasury bills outstanding at their peak are $\sim$\$6 trillion. It would not be feasible for the Fed to have a balance sheet exclusively of short dated Treasury bills. In addition to a feasibility problem there is a policy relevance question about why the central bank would maintain an abundant reserves balance sheet while holding short term assets. In this case the Fed is forcing the banking sector to absorb reserves by removing short duration treasuries from the market. This would go against several reasons for implementing QE such as term premium effects, the risk channel, and signaling impacts.

E.1 Isolating balance sheet size (long duration assets)

Setting the floating rate bond holdings of the central bank to zero allows for the isolation of central bank balance sheet *size* from central bank balance sheet *composition*. This counterfactual can be run by setting parameter values of $\Lambda = 0$ and $\lambda \in [.06, .3]$. This implies the central bank holds 0% of GDP in floating rate bonds and 6% - 30% of GDP in fixed rate bonds. Results comparing cumulative fluctuations in these economies in response to 1% aggregate shocks are shown in Figure 15 and are virtually identical to Figure 6 of Section 5. The similarity in results displays a robustness of the original claim on balance sheet size alone increasing the real effects of interest rate policy. This graph can be effectively used as a comparison to the other sections below as the results are very similar in magnitude to those from Section 5.

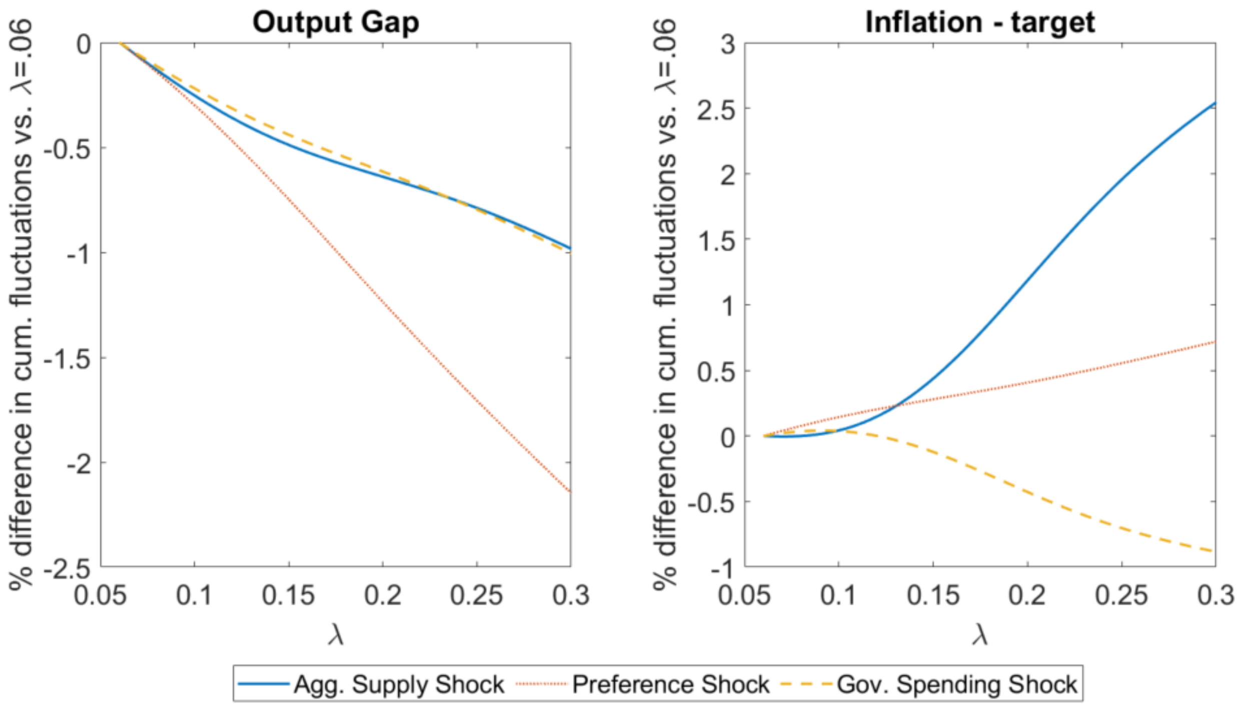

Figure 15: Cumulative fluctuations of large fixed rate balance sheets

Results are very close to those of the text because we are only changing the asset composition of 6% of GDP. Figure 16 shows that in the small central bank balance sheet economy (solid blue line), we now see a small decrease in central bank income, but still very small compared to the significantly negative path for central bank income in the large central bank balance sheet economy. In the text, the path for central bank income in the scarce reserves economy increases slightly, while in this counterfactual the path actually slightly decreases. This is because now when raising interest rates, the floating rate bonds continue to pay the same interest rate instead of paying a higher rate as the floating rate bonds do.

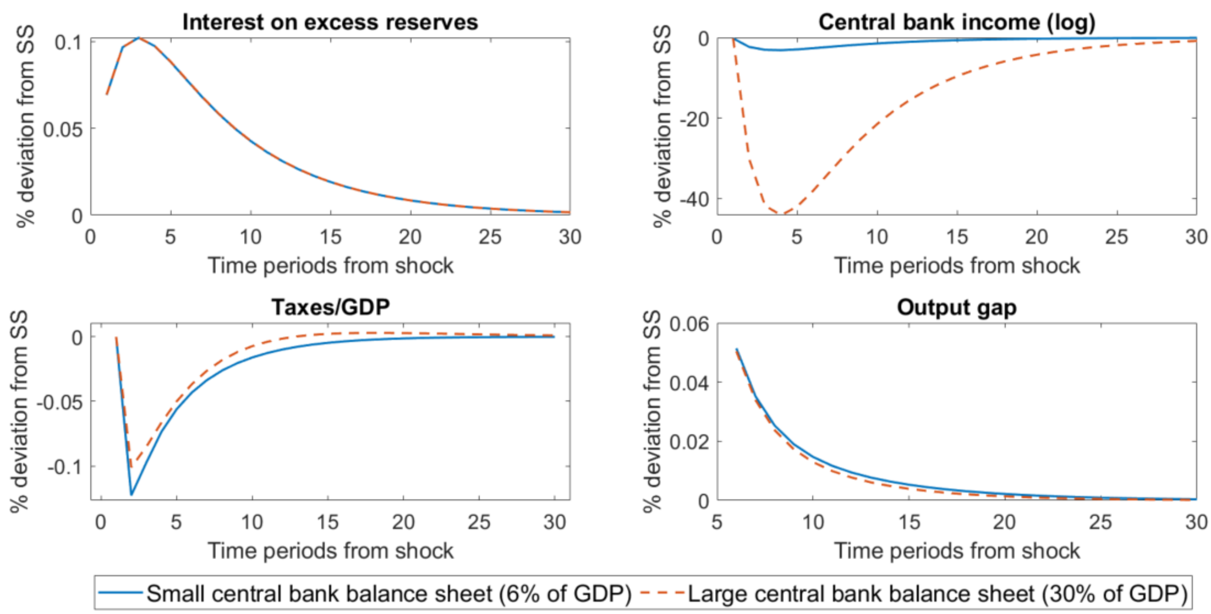

Figure 16: IRFs for aggregate preference shock

E.2 Isolating balance sheet size (short duration assets)

In this subsection we isolate on balance sheet size as well, but this time with short duration assets, represented by the floating rate bond in this model. In this subsection we set the percentage of fixed rate bonds of GDP (λ) equal to zero, and change the percentage of floating rate bonds to GDP (Λ) held by the central bank. Floating rate bonds represent treasury bills in the model, and come with a feasibility constraint where we must only consider values $\Lambda \in [.06, .15]$. In the baseline calibration, steady state debt to GDP is just above 88%. The fixed/floating rate bond issuance is governed by the parameter Ω to match the US ratio of $\sim$ 20%. As a result there is only $\sim$ 18% of GDP in floating rate bonds for the central bank of the model (and in the data) to buy. As of October 2024 there are \$ 6 trillion in outstanding US treasury bills, and the size of the Fed's balance sheet is over \$7 trillion. The Federal Reserve System must have a balance sheet that is composed of *some* longer term assets or the model faces feasibility constraints. As a result, we can only consider $\Lambda \in [.06, .15]$. Figure 17 shows the model results comparing cumulative fluctuations as the central bank purchases a larger share of floating rate bonds.

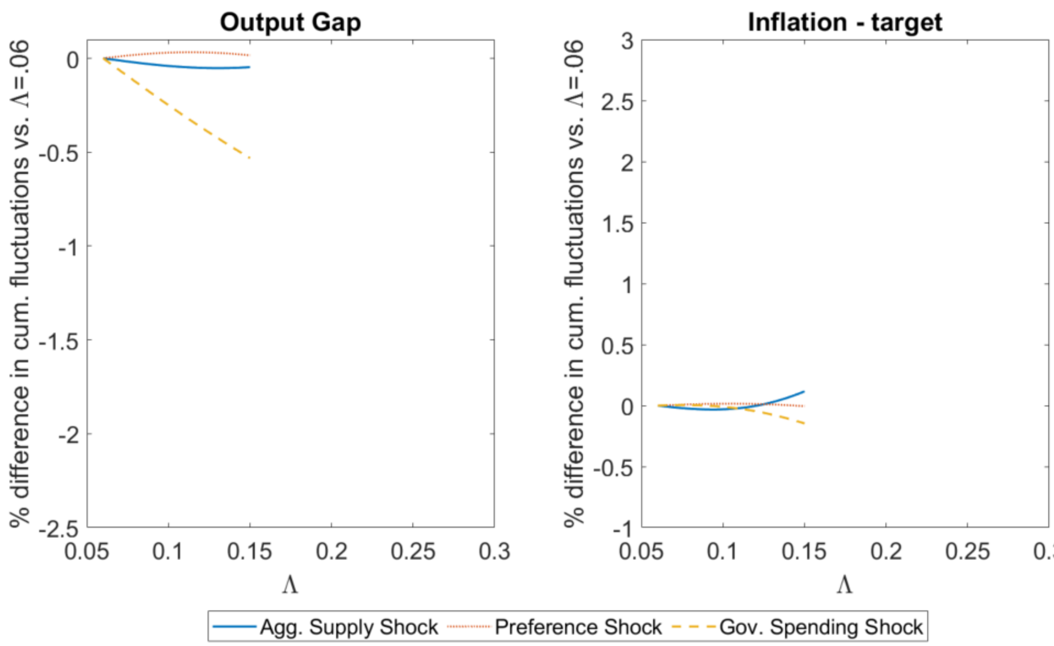

Figure 17: Cumulative fluctuations of large floating rate balance sheets

The axes are kept constant to have a direct comparison to previous exercises. Results are small in magnitude due to the high correlation between policy rates on the liability side and floating rate bonds on the asset side. This exercise is replicating section (2) of Table 1. If a central bank acquires a large amount of floating rate assets, the correlation between central bank income and policy rates remains positive, and the real effects of interest rate policy are relatively similar to the scarce reserves framework.

E.3 Isolating balance sheet composition

In this subsection we isolate on the *composition* of the central bank balance sheet. Here we compare cumulative fluctuations in two different economies where the central bank has an abundant reserves balance sheet, but the composition of assets is either all fixed rate bonds or all floating rate bonds. This exercise comes with the same set of feasibility restrictions described in the previous section about an abundant reserves balance sheet for the central bank which holds exclusively short term government debt. To compare counterfactual scenarios we must maintain $\lambda + \Lambda \in [.06, .15]$.

In the economy with the central bank holding exclusively floating rate bonds we have $\Lambda \in [.06, .15]$ and $\lambda = 0$. In the economy with the central bank holding exclusively fixed rate bonds we have $\Lambda = 0$ and $\lambda \in [.06, .15]$. The results for these model counterfactual scenarios are shown in Figure 18. The x-axis shows that we compare counterfactuals in which the size of the central bank balance sheet is identical, but the composition is different.

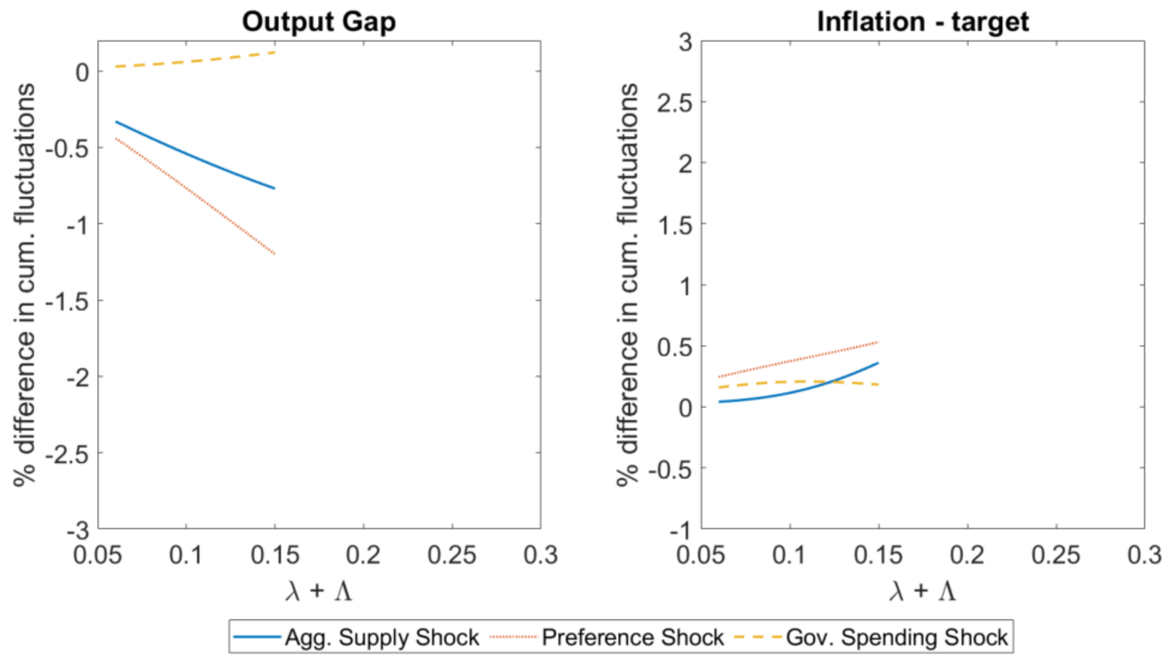

Figure 18: Cumulative fluctuations of large fixed rate vs. large floating rate balance sheets

Model results are similar to the text as we see the economy with the floating rate bonds experiences fewer fluctuations in output due to losses imposed by raising interest rates. The one exception is the government spending shock. In the economy where the central bank holds exclusively the floating rate bond, the interest rate on longer term bonds rises as there are fewer high yielding safe assets for intermediaries to hold. As a result the central bank income mechanism becomes a secondary factor driving the results when comparing these economies.