Risk taking under assimilation and contrast

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Motivation: risk is not evaluated in isolation

- Classical economic models postulate that risky prospects are evaluated in isolation
- Evaluation of future risky prospects path dependent
  - One type of path dependence: boiling frog effect Da et al. (2014), Grinblatt and Moskowitz (2004), Krugman (2009)
- Kahneman and Miller (1986): evaluation is anchored to a history dependent reference point
  - This work: risky prospects are either assimilated or contrasted away from the ones we are accustomed to

Contrast and Assimilation

- Contrast: overestimation of large changes. E.g.: estimated weight of an object is inflated, if a very light object is lifted before.
- Assimilation: underestimation of small changes. Physical judgements (brightness, length, weight). E.g.: people underestimate weight of an object if they have just lifted a moderately lighter object

This Work:

- Decision makers **assimilate** prospects that change by a little relative to what they remember, and **underreact** to changes. They **contrast** risks that change by a lot, and **overreact**.

$$U(A_t|A_t^m) = \underbrace{\mathbb{E}_{A_t^m}[u]}_{\textit{memory anchor}} + \underbrace{g(A_t, A_t^m)}_{\textit{adjustment}} \underbrace{\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^m}[u]\}}_{\textit{deviation}}$$

This work

1. A model of decision making under memory based distortion of probability distributions

- A lottery can look similar or very different from lotteries in memory: past distributions distort the current distribution
- **Underreaction** to small changes in distribution and **overreaction** to large changes. Existing models only capture one of the two.
- Tractable representation for applications. Example: make sense of (i) anomalies in asset pricing (ii) anomalies in the (dis) amenity premia estimated empirically.

This work

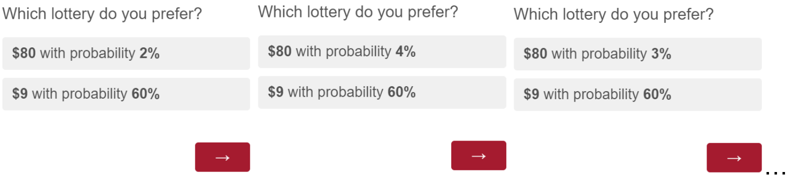
2. Novel experimental evidence. Choice **underreacts** to small changes in probability of payments (**assimilation**), **overreacts** to large changes (**contrast**)

- Lottery chosen after drastic improvement, not chosen after gradual one (Boiling Frog).
- Lottery chosen if it comes after a much worse one, not chosen if it comes after a much better one (Contrast)
- Lottery chosen if it comes after a slightly better one, not chosen if it comes after a slightly worse one (Assimilation)
- Hard to jointly explain with existing models

	KR (2006)	BRS (2021)	KLW (2020)	EU
Boiling Frog	no	yes	no	no
Contrast (upward vs downward changes)	no	no	no	no
Assimilation (upward vs downward changes)	yes	no	yes	no

The Boiling Frog

Subjects make sequences of choices between two binary lotteries that resolve at final period T

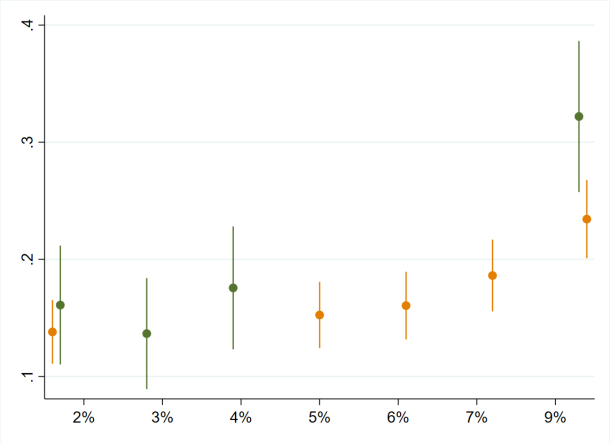


- **Drastic** ascending treatment:

(\$80, **2**%) vs(\$9, 60%), (\$80, **4**%) vs(\$9, 60%), (\$80, **3**%) vs(\$9, 60%), (\$80, **9**%) vs(\$9, 60%).

- **Gradual** ascending treatment:

(\$80, **2**%) vs(\$9, 60%), (\$80, **5**%) vs(\$9, 60%),
(\$80, **6**%) vs(\$9, 60%),(\$80, **7**%) vs(\$9, 60%), (\$80, **9**%) vs(\$9, 60%).



Fact:($80, 9%) chosen almost 10% more frequently after a drastic probability increase relative to a graual $\implies$ cumulation of small changes appreciated less than one big change

Related Literature

1. Theory.
   1.1 Preferences. Bell (1985), Koszegi Rabin (2007, 2009), Khaw et al. (2021), Bushiong et al. (2020), Bordalo et al. (2020), Gabaix (2014), Gul (1991), Loomes and Sugden (1986); Rubinstein (1988)
   1.2 Beliefs. Mullainathan (2002), Rabin and Vayanos (2009), Bordalo et al (2016)
2. Experiments: Freeman, Halevy, and Kneeland (2019), Frydman and Jin (2020), Shram and Sonnemans (2011), Sprenger (2015)
3. Facts from financial and housing markets: Chay and Greenstone (2005) Davis (2004), Gallagher (2014), Gayer et al. (2002), Greenstone and Gallagher (2008). Della Vigna and Pollet (2007), Da et al. (2014), Giglio and Shue (2014), Huang et al. (2021),

Outline

Evaluation of a probability distribution

- At every period t, choice set

$$C_t = \{A_t, \delta\}$$

- A_t, δ are lotteries which pay at T. δ is deterministic
- A_t is distorted by a function of remembered lotteries, A_t^m

Distortion of A_t

$$\pi\left(A_t(x)\,|A_t^m\right) = \underbrace{A_t^m(x)}_{\textit{memory anchor}} + \underbrace{g\left(F_{A_t}, F_{A_t^m}\right)}_{\textit{adjustment}} \underbrace{\left\{A_t(x) - A_t^m(x)\right\}}_{\textit{deviation}} \quad x \in \mathbb{R}$$

- distorted lottery is anchored to remembered lottery A_t^m, plus a weighted deviation with weight $g(\cdot,\cdot)$ that depends on cdf of A_t and A_t^m.

Representation in the spirit of BGS (2021) on consumption choice. Difference: present paper is on choice under risk

Distortion weight $g(\cdot,\cdot)$

Monotonicity wrt First order Stochastic Dominance

$$F >_{FOSD} G >_{FOSD} H \implies g(F,H) > g(G,H).$$

Moreover, for every H there exists $\hat{H}$ such that $g\left(\hat{H},H\right) = 1$

Continuity

$g(\cdot,\cdot)$ is continuous in both arguments in the L^2 norm. Moreover, $g(H,H) = 0$ for every H

Positivity and simmetry

$g(\cdot,\cdot) \geq 0.$
$g(F,G) = g(G,F)$. This can be relaxed

Example

Evaluation of A_t

- Expected utility with respect to *distorted* lottery $\pi(A_t | A_t^m)$

$$U(A_t | A_t^m) = \int u(x)\, d\pi(A_t(x) | A_t^m)$$

$$U(A_t | A_t^m) = \underbrace{\mathbb{E}_{A_t^m}[u]}_{\text{memory}} + \underbrace{g(A_t, A_t^m)}_{\text{adjustment}} \underbrace{\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^m}[u]\}}_{\text{deviation}}$$

Definition (Assimilation and Contrast)

A_t is assimilated to A_t^m if $g(A_t, A_t^m) < 1$

A_t is contrasted away from A_t^m if $g(A_t, A_t^m) > 1$

Example

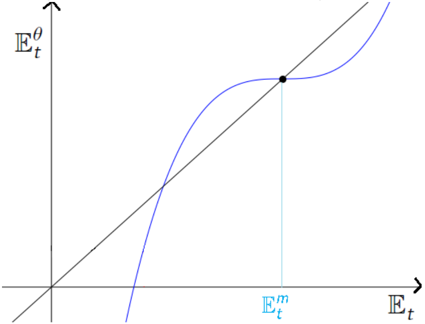
Simple case $g(A_t, A_t^m) = \theta|\mathbb{E}_t - \mathbb{E}_t^m|$

Example

Assume u linear

$$\mathbb{E}_t^\theta[X] = U(A_t|A_t^m) = \mathbb{E}_t^m + \begin{cases} \theta(\mathbb{E}_t - \mathbb{E}_t^m)^2 & \mathbb{E}_t > \mathbb{E}_t^m \\ -\theta(\mathbb{E}_t - \mathbb{E}_t^m)^2 & \mathbb{E}_t < \mathbb{E}_t^m \end{cases}$$

where $\mathbb{E}_t^m = \int x \mathrm{d}A^m(x)$ and $\mathbb{E}_t = \int x \mathrm{d}A_t(x)$



Remembered lottery A_t^m

- DM remembers recent lotteries better than past ones, and only retrieves one lottery per past period
- Memory is backward looking average of past similar lotteries, discounted by factor ρ

$$A_{t-1}^m(x) = (1-\rho)\sum_{j=1}^{t}\rho^{j-1}A_{t-j}(x) + \rho^t A_t(x)$$

DM retrieves A_t if she has no memory history
If δ is not deterministic, A_{t-1}^m constructed following a similar criterion. memory

- Evaluation of a risky prospect is history dependent
    1. similar past lotteries attract evaluation towards them (assimilation)
    2. very different past lotteries repel evaluation away (contrast)
    3. less reaction to gradual changes in risks (boiling frog)
    4. Contrast increases with size of stakes

Results are presented for simple binary lotteries and $u(x) = x$. More general results are in the links.

Assimilation&Contrast

Proposition 1

▸ Prop 1

Let $\delta = (Kp, 1)$ and $A_t = (K, p; 0, 1 - p)$. Assume a decision maker has seen choice sets

$$C_s = \{(K, p_s; 0, 1 - p_s), \delta\}$$

at every period $s < t$. If t is large, or ρ is low, there are boundaries $\bar{p} > p > \underline{p}$, such that

1. Assimilation: if $p < p_s < \bar{p} \, \forall s$ then

   $$A_t \succ \delta$$

   if $\underline{p} < p_s < p \, \forall s$ then

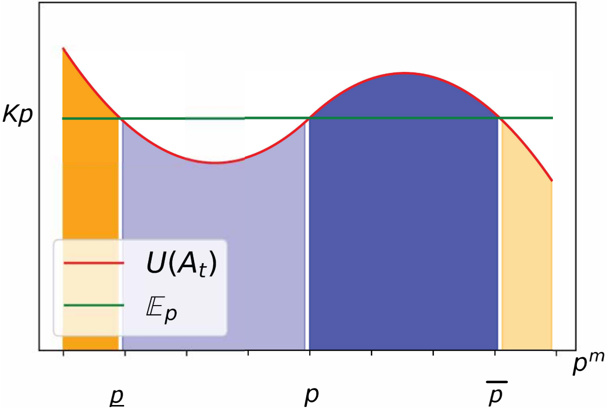
   $$A_t \prec \delta$$

2. Contrast: if $p_s > \bar{p} \, \forall s$ then

   $$A_t \prec \delta$$

   if $p_s < \underline{p} \, \forall s$ then

   $$A_t \succ \delta$$



Fix p and vary remembered probability p^m

$$p^m = \sum \rho^{t-s+1} p_s$$

- Assimilation
  - light blue area: $\underline{p} < p^m < p$, : A_t **assimilated** downwards to $A_t^m \implies$ downward bias in evaluation
  - dark blue area: $p < p^m < \bar{p}$, : A_t **assimilated** upwards to $A_t^m \implies$ upward bias in evaluation
- Contrast
  - dark orange area: $p > p^m$, $g > 1$: A_t **contrasted** away upwards from $A_t^m \implies$ upward bias in evaluation
  - light orange area: $p < p^m$,$g > 1$: A_t**contrasted** away downwards from $A_t^m \implies$ downward bias in evaluation

Boiling frog

Proposition 2

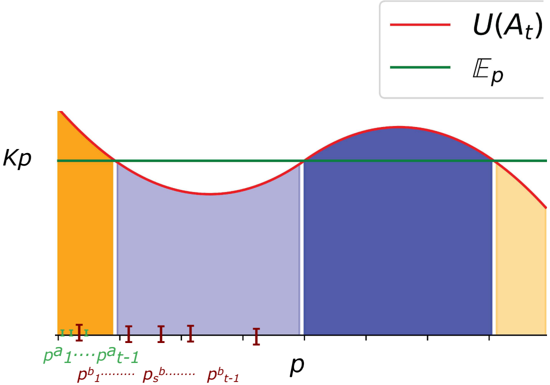
Take two decision makers a and b facing the same choice set at time t $C_t = \{A_t, \delta\}$, where $A_t = (K, p; 0, 1-p)$ and $\delta = (x, 1)$. Assume a faces a drastic path leading to time t, while b a gradual one. That is

$$C_s = \left\{ \left(K, p_s^i; 0, 1 - p_s^i \right), \delta \right\} \; i = a, b$$

at every period $s < t$, such that $p > p_s^i > p_{s-1}^i$. with $p_0^i = p_0 < \bar{p}$ for $i = a, b$. $p_s^a \leq \underline{p}$ for all $s < t$, and $p_{t-1}^a < p_s^b < p$ for some s, then

$$A_t \succ_b \delta \implies A_t \succ_a \delta$$

Boiling frog



- Path of b more gradual than $a \implies p^{m,a} < p^{m,b} \implies$ less contrast (more assimilation) in b than in a

Other predictions from contrast

- Payoffs at stake determine the strength of contrast versus assimilation ▸ stake
  - a 5% change in the probability of receiving \$50 is neglected, while a 5% change in receiving $\$50,000$ is overreacted to
- First order risk aversion needs not imply unreasonable rejection of good risky prospects
- Endowment effect for risk (Sprenger 2015): contrast increases risk aversion if one is accustomed to riskless lottery.

Other predictions from assimilation

- Evidence of noisy encoding can be recast as evidence of assimilation
  - Frydman and Jin (2022) finds larger risk aversion when decision makers are accustomed to relatively worse lotteries
  - Assimilation predicts that evaluation is anchored to past lotteries, *if they are close enough*. ▸ FJ

Multi-period lotteries

Model can be extended to the domain of multi period lotteries

$$\boldsymbol{A}_t = \left(A_{t,t}, A_{t,t+1}, ..., A_{t,T}\right)$$

$$V\left(\boldsymbol{A}_t|\boldsymbol{A}_t^m\right) = \sum_{j=1}^{T-t} \delta^j U\left(A_{t,t+j}|A_{t,t+j}^m\right)$$

- This model nests the one period model as a special case. Useful for the application in the next subsection.
- Assume linear consumption utility, and suppose $\boldsymbol{A}_t$ is the marginal distribution of a stochastic process $\boldsymbol{X}_t$ (the alternative choice is a constant)

$$V\left(\boldsymbol{A}_t|\boldsymbol{A}_t^m\right) = \sum_{j=1}^{T-t} \delta^j \mathbb{E}_t^\theta\left[X_{t+j}\right]$$

Where

$$\mathbb{E}_t^\theta\left[X_{t+j}\right] = \mathbb{E}_t^m\left[X_{t+j}\right] + g\left(A_{t,t+j}, A_{t,t+j}^m\right)\left\{\mathbb{E}_t\left[X_{t+j}\right] - \mathbb{E}_t^m\left[X_{t+j}\right]\right\}$$

Setup

- Investors decide whether to hold cash or a **risky** asset in finite supply which pays D_T at final period T. In period t it produces u_t so that

$$D_T = \bar{D} + \sum_{j=1}^{T} u_j$$

where $u_j \sim_{i.i.d} \mathcal{N}\left(0, \sigma^2\right)$ and realize at time j

- Investor has linear consumption utility (can relax this with mean variance), distorted by assimilation and contrast

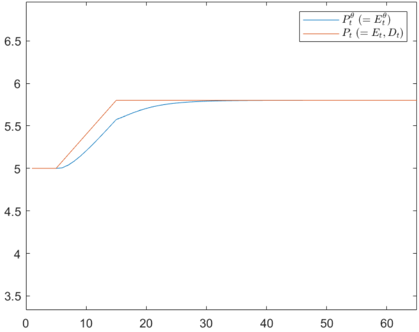
  meanvariance

- In equilibrium the price of the asset is

$$P_t^\theta = \mathbb{E}_t^\theta \left[D_T\right]$$

$\mathbb{E}_t^\theta$

Underreaction to sequence of small news (Giglio Shue 2014)



- suppose a **sequence of small surprises** starts arriving at t_0

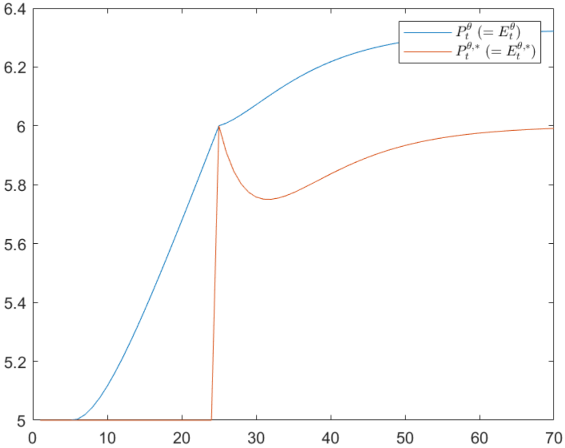
$$u_{t_0+j} = u$$

for $j = 1, .., J$

$$\mathbb{E}_t^\theta [D_T] = \mathbb{E}_t [D_T] + \left(\sum_{j=0}^{t-t_0-1} \rho^j u_{t-j}\right) \left\{\theta | \sum_{j=0}^{t-t_0-1} \rho^j u_{t-j} | \theta - 1\right\}$$

- When $t = t_0$, u_{t_0} is small $\implies$ P_t^θ underreacts
- As the surprises cumulate, $\mathbb{E}_t$ depart further from $\mathbb{E}_{t-1}^m$ but never far enough to be contrasted against $\implies$ P_t^θ stays below P_t
- As the surprises stop, $\mathbb{E}_{t-1}^m$ generating momentum after the surprise.

Boiling frog (Da et al. 2014, Grinblatt Moskowitz 2004)



- Conditional on the same past price increase, smooth price increase ΔP_t^θ predicts higher future returns than discrete price increase $\Delta P_t^{\theta,*}$ ▸ statement
- Intuition:
  - smooth ΔP_t^θ after small bits of surprises $\Longrightarrow$ underreaction
  - Discrete $\Delta P_t^{\theta,*}$ after large surprise $\Longrightarrow$ overreaction
- Different from predictions of other behavioral models ▸ comparison

Other predictions

- Overreaction to extreme earnings surprises, underreaction to small ones ▸ figure
- A very long sequence of small surprises of the same sign is associated with initial underreaction and delayed overreaction ▸ figure
- Surprises lead to more overreactions when occurring in increasing order, rather than in an inconsistent one ▸ figure
- Low market volatility $\implies$ momentum, high volatility $\implies$ reversal

Amenity premia in house prices

- Two cities A and B. Workers can move in either city frictionless. Wages equal in both cities for simplicity.
  - City A: amenities evolve as a random walk (pollution is persistent) $a_{t+1} = a_t + \epsilon_{t+1}$
  - City B: rent is fixed at $\bar{R}$ and amenities are fixed at 0.
  - Equilibrium rent in city A is $R_t = \bar{R} + a_t$
- Landlords biased by assimilation and contrast, and linear consumption utility, can either own a house paying price P_t or keep cash ▸ value
  - In equilibrium, they must be indifferent between holding a house and not holding it

$$P_t = R_t + \delta \mathbb{E}_t^{\theta} [P_{t+1}]$$

- Assume Landlords are naive about the future distortions, that is, they believe future price to be set rationally

$$P_{t+1} = P_{t+1}^{*} = R_{t+1} + \delta \mathbb{E}_{t+1} \left[P_{t+1}^{*} \right]$$

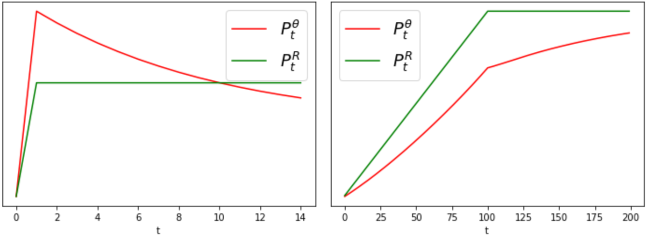
Pollution premia (Currie et al. 2015, Chay and Greenstone 2004, Greenstone and Gallagher 2008)

- Denote P_t^θ the price prevailing in the behavioral economy at time t

$$P_t^\theta = \bar{P} + a_t + \beta \mathbb{E}_t^\theta \left[\frac{1}{1-\beta} a_{t+1} \right]$$

- Compare against the rational price

$$P_t^R = \bar{P} + \frac{1}{1-\beta} a_t$$



- Amenity premium estimated using the drastic variation in the shock is larger than rational, and larger than the premium during the slow moving shock ▸ result

Goal: detect history dependent risk aversion

- Vary history of past choice sets so to change choice at t.
  - subjects make a sequence of choices (4 or 5) among binary lotteries. Only one gets implemented with some chance.
  - design close to Bordalo et al. (2022) on intertemporal choice.
- 2010 subjects on prolific are randomly assigned to 8 treatments. Pre registered, incentivized experiment on Prolific.

Experiment

Which lottery do you prefer?

$80 with probability **22%**

$9 with probability **60%**

→

Which lottery do you prefer?

$80 with probability **21%**

$9 with probability **60%**

→

Which lottery do you prefer

$80 with probability **23%**

$9 with probability **60%**

Treatment summary

	T1	T2	T3	T4
Stage 1	\$80, 9% vs \$9, 60%	\$80, 2% vs \$9, 60%	\$80, 21% vs \$9, 60%	\$80, 2% vs \$9, 60%
Stage 2	\$80, 7% vs \$9, 60%	\$80, 5% vs \$9, 60%	\$80, 23% vs \$9, 60%	\$80, 4% vs \$9, 60%
Stage 3	\$80, 6% vs \$9, 60%	\$80, 6% vs \$9, 60%	\$80, 22% vs \$9, 60%	\$80, 3% vs \$9, 60%
Stage 4	\$80, 5% vs \$9, 60%	\$80, 7% vs \$9, 60%	\$80, 9% vs \$9, 60%	\$80, 9% vs \$9, 60%
Stage 5	\$80, 2% vs \$9, 60%	\$80, 9% vs \$9, 60%		

	T5	T6	T7	T8
Stage 1	\$80, 9% vs \$5, 60%	\$80, 9% vs \$1, 60%	\$80, 9% vs \$9, 60%	\$80, 9% vs \$1, 60%
Stage 2	\$80, 9% vs \$4, 60%	\$80, 9% vs \$2, 60%	\$80, 9% vs \$8, 60%	\$80, 9% vs \$2, 60%
Stage 3	\$80, 9% vs \$2, 60%	\$80, 9% vs \$4, 60%	\$80, 9% vs \$8.5, 60%	\$80, 9% vs \$1.5, 60%
Stage 4	\$80, 9% vs \$1, 60%	\$80, 9% vs \$5, 60%	\$80, 9% vs \$5, 60%	\$80, 9% vs \$5, 60%

- **T1-T4**. $\$80, p$ vs $\$9, 60\%$. **Probability** p changes across stages
- **T5-T8**. $\$80, 9\%$ vs $\$k, 60\%$. **Payoff** k changes across stages

Predictions summary - probabilities

	T1	T2	T3	T4
Stage 1	\$80, 9% vs \$9, 60%	\$80, 2% vs \$9, 60%	\$80, 21% vs \$9, 60%	\$80, 2% vs \$9, 60%
Stage 2	\$80, 7% **vs** \$9, 60%	\$80, 5% **vs** \$9, 60%	\$80, 23% vs \$9, 60%	\$80, 4% vs \$9, 60%
Stage 3	\$80, 6% **vs** \$9, 60%	\$80, 6% **vs** \$9, 60%	\$80, 22% vs \$9, 60%	\$80, 3% vs \$9, 60%
Stage 4	\$80, 5% **vs** \$9, 60%	\$80, 7% **vs** \$9, 60%	\$80, 9% **vs** \$9, 60%	\$80, 9% **vs** \$9, 60%
Stage 5	\$80, 2% vs \$9, 60%	\$80, 9% vs \$9, 60%		

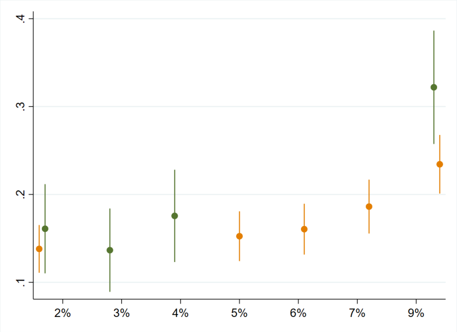
- **T1vsT2**. \$80, p **assimilated to FOS dominant** options in **T1**. \$80, p **assimilated to FOS dominated** options in **T2**. $P(\$80, p|T1) > P(\$80, p|T2)$
- **T3vsT4**. \$80, 9% **contrasted away from FOS dominant** options in **T3**. \$80, 9% **contrasted away from FOS dominated** options in **T4**. $P(\$80, 9\%|T3) < P(\$80, 9\%|T4)$
- **T2vsT4**.Boiling Frog \$80, 9% in T2 is assimilated more to (or contrasted less from) lower paying options compared to T4. $P(\$80, 9\%|T2) < P(\$80, 9\%|T4)$

Predictions summary - payoffs

	T5	T6	T7	T8
Stage 1	\$80, 9% vs \$5, 60%	\$80, 9% vs \$1, 60%	\$80, 9% vs \$9, 60%	\$80, 9% vs \$1, 60%
Stage 2	\$80, 9% **vs** \$4, 60%	\$80, 9% **vs** \$2, 60%	\$80, 9% vs \$8, 60%	\$80, 9% vs \$2, 60%
Stage 3	\$80, 9% **vs** \$2, 60%	\$80, 9% **vs** \$4, 60%	\$80, 9% vs \$8.5, 60%	\$80, 9% vs \$1.5, 60%
Stage 4	\$80, 9% vs \$1, 60%	\$80, 9% vs \$5, 60%	\$80, 9% vs \$5, 60%	\$80, 9% **vs** \$5, 60%

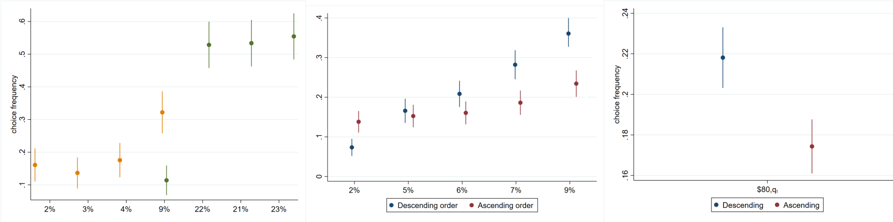
- **T5vsT6**. $\$k, 60\%$ **assimilated to FOS dominant** options in **T5**. $\$k, 60\%$ **assimilated to FOS dominated** options in **T6**.
  $P(\$k, 60\% | T5) > P(\$k, 60\% | T6)$
- **T7vsT8**. $\$5, 60\%$ **contrasted away from FOS dominant** options in **T7**. $\$5, 60\%$ **contrasted away from FOS dominated** options in **T8**.
  $P(\$5, 60\% | T7) < P(\$5, 60\% | T8)$
- **T6vsT8**.Boiling Frog $\$5, 60\%$ in T6 is assimilated more to (or contrasted less from) lower paying options compared to T8. $P(\$5, 60\% | T6) < P(\$5, 60\% | T8)$

Boiling Frog



- ($\$80, 9\%$) chosen almost 10% more frequently after a drastic probability increase relative to a graual

Figure: Contrast, Assimilation. Changes in probability



Yellow dots: T4 (drastic upward change) in green. Green dots: T3 (drastic downward change). Red dots: T2 (gradual upward). Blue dots: T1 (gradual downward).

table

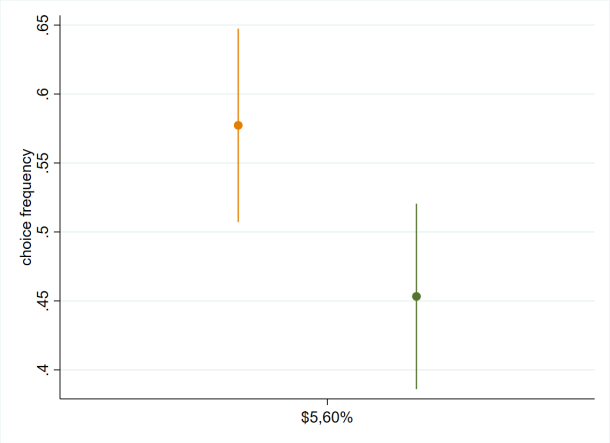
Assimilation and contrast - probability changes

- Contrast effect: choice frequency of $(\$80, 9\%)$is 20% less when it comes after a drastic downward change relative to a drastic upward one
- Pooled data reveals overall assimilation, $(\$80, q)$ chosen 5% more frequently in T1 than in T2. Sizeable effect: baseline frequency is 17%
- Asymmetric effects:
  - Effect comes all from larger probabilities (6%,7% and 9%).
  - $(\$80, 2\%)$ chosen **less** after a gradual downward change than when seen in isolation $\implies$ contrasted away from preceding options
  - Model can fit this effect heterogeneity with an asymmetric g, but it is not the focus of the theory.

Switching point between assimilation and contrast

- Suppose subjects with a remembered lottery $(\$80, 5.5\%)$ evaluate a new lottery $(\$80, p)$: for which p does the data predict assimilation, and when contrast?
- Answer: infer the "switching points" $\underline{p}$, $\bar{p}$ between assimilation and contrast
  - Impose a functional form assumption and estimate preference parameters ▸ estimation
  - 3 parameters estimated in the data: curvature of consumption utility γ, contrast parameter θ, memory discount factor ρ
- Given $(\$80, 5.5\%)$, $\bar{p} = 12.6\%$, $\underline{p} = 0$. $\bar{p}$ shrinks with payoff at stake ▸ figure

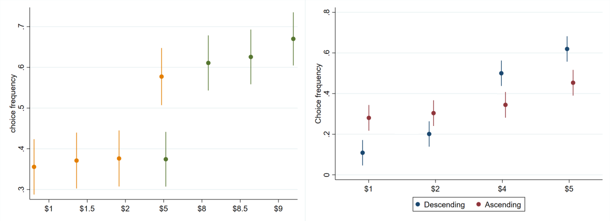
Boiling frog - changes in payoffs



Green dots: T6 (gradual change in payoffs). Yellow dots: T8 (drastic change in payoffs)

- Choice frequency of ($5, 60%) after a gradual payoff increase is lower than after a drastic increase by 10%

Figure: Contrast, Assimilation



Yellow dots: T8 (drastic upward change) in green. Green dots: T7 (drastic downward change). Red dots: T6 (gradual upward). Blue dots: T5 (gradual downward).

▸ table

- Contrast effect: choice frequency of ($5, 60%)is 20% less when it comes after a drastic downward change relative to a drastic upward one
- Asymmetric assimilation (pooled data reveals **no** assimilation overall):
    - ($5, 60%) chosen less after a gradual upward change than when seen in isolation $\Longrightarrow$ assimilated to preceding options
    - ($1, 60%) chosen less after a gradual downward change than when seen in isolation $\Longrightarrow$ contrasted away from preceding options
    - Model can fit that with an asymmetric g, but it is not the focus of the theory.

Summary

- Contrast: $(\$80, 9\%)$ chosen more frequently after $(\$80, 3\%)$ than after $(\$80, 22\%)$
- Assimilation: $(\$80, q)$ chosen more often when q is in descending order than ascending
- Assimilation: $(\$80, 9\%)$ chosen more frequently after a gradual ascending order of options than a drastic one

Conclusion

- Path dependent distortion of risky prospects
  - boiling frog effect with novel predictions about assimilation and contrast,
  - connects empirical facts across different domains
- Experiment:
  - evidence of the boiling frog effect
  - test for contrast and assimilation

Comparison

- Diagnostic expectations (Bordalo et al. 2018): $\mathbb{E}_t^{Diag}$ always overreacts if u_t is normal
- Gambler's and hot hand fallacy (Rabin and Vayanos (2009)). Commonality: $\mathbb{E}_t^{GF}$ initial underreaction followed by overreaction. Difference: in steady state underreaction to u_t. My model: size of u_t matters.
- Categorization (Mullainathan (2002)). Commonality: initial underreaction followed by overreaction. Difference: does not predict Frog in the pan.

back

Assimilation and Frydman and Jin (2021)

- Model can fit finding of experiment (2) in Frydman and Jin (2021).
- two decision makers a, b face choice sets $\left\{\left(X_t^i, p\right), C_t^i\right\}$. $i = a, b$. The empirical frequency oof X_t^i is cdf F^i on $[k, K]$. $F^a >_{FOSD} F^b$. C_t^i is distributed according to pX_t^i Payments occur at time T.

Proposition 4

Assume u is linear and $g(F, G) = |\mathbb{E}_F[X] - \mathbb{E}_G[X]|$. If $|K - k| \leq \frac{1}{2\theta p}$, then for any $x \in [k, K]$

$$Pr\left(U^b(x, p; 0, 1-p) > U^b(xp, 1)\right) < Pr\left(U^a(x, p; 0, 1-p) > U^a(xp, 1\right.$$

back

Example of g

$$g(F, G) = |\int x \mathrm{d}F - \int x \mathrm{d}G|$$

back

Example: A_t binary

A decision maker remembers a lottery paying K with probability p^m that is

$$A_t^m(x) = \begin{cases} p^m & \text{if } x = \$K \\ 1 - p^m & \text{if } x = \$0 \end{cases}$$

She evaluates lottery

$$A_t(x) = \begin{cases} p & \text{if } x = \$K \\ 1 - p & \text{if } x = \$0 \end{cases}$$

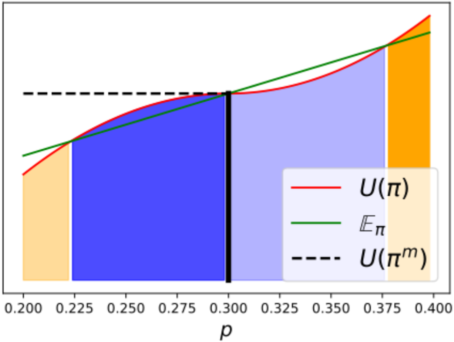
Evaluation distorted by g

The decision maker evaluates A_t as

$$U(A_t | A_t^m) = p^m K + g((K,p),(K,p^m))\{pK - p^m K\}$$

Note: monotonicity implies g increases both in K and in p if $p > p^m$

$$U\left(A_t | A_t^m\right) = p^m K + g\left(\left(K, p\right), \left(K, p^m\right)\right)\left\{pK - p^m K\right\}$$



light blue area: $p > p^m$, $g < 1$: A_t assimilated downwards to $A_t^m \implies$ downward bias in evaluation
dark orange area: $p > p^m$, $g > 1$: A_t contrasted away upwards from $A_t^m \implies$ upward bias in evaluation
dark blue area: $p < p^m$, $g < 1$: A_t assimilated upwards to $A_t^m \implies$ upward bias in evaluation

light orange area: $p < p^m$, $g > 1$: A_t contrasted away downwards from $A_t^m \implies$ downward bias in evaluation

back

Assimilation

Proposition 1

Let $k = u^{-1}\left(\mathbb{E}_{A_t}[u]\right)$. Let δ^k assign probability 1 to k. Consider choice sets of the form $C_s = \left\{\lambda_s, \delta^k\right\}$, $s < t$. There exist

$$\bar{\mu} >_{FOSD} A_t >_{FOSD} \underline{\mu}$$

such that if $\bar{\mu} >_{FOSD} \lambda_s >_{FOSD} A_t$ then

$$A_t \succ \delta^k,$$

and if $\underline{\mu} <_{FOSD} \lambda_s <_{FOSD} A_t$

$$A_t \prec \delta^k$$

Assimilation&Contrast

Proposition 2

Let $k = u^{-1}\left(\mathbb{E}_{A_t}[u]\right)$. Let δ^k assign probability 1 to k. Consider choice sets of the form $C_s = \left\{\lambda_s, \delta^k\right\}$, $s < t$. If either t is large enough or ρ is small enough. There exist

$$\bar{\mu} >_{FOSD} A_t >_{FOSD} \underline{\mu}$$

such that

1. Assimilation: if $\bar{\mu} >_{FOSD} \lambda_s >_{FOSD} A_t$ then
$$A_t \succ \delta^k,$$
and if $\underline{\mu} <_{FOSD} \lambda_s <_{FOSD} A_t$
$$A_t \prec \delta^k$$
2. Contrast: if $\lambda_s >_{FOSD} \bar{\mu}$ then
$$A_t \prec \delta^k,$$
and if $\lambda_s <_{FOSD} \underline{\mu}$
$$A_t \succ \delta^k$$

Stake dependent contrast vs assimilation

Proposition 3

Assume $u(x) = \frac{1}{1-\sigma} x^{1-\sigma}$ and $g(F, G) = |\mathbb{E}_F[u] - \mathbb{E}_G[u]|^{\beta}$.
Assume $C_s = \{\underline{\mu}^{\alpha}, \delta\}$ for $s < t$ and $C_t = \{A_t^{\alpha}, \delta^{\alpha}\}$, with t large.
Define

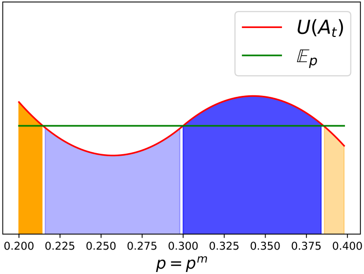
$$\underline{\mu}^{\alpha}(x) = \underline{\mu}(\alpha x)$$
$$A_t^{\alpha}(x) = A_t(\alpha x)$$
$$\delta^{\alpha}(x) = \delta(\alpha x)$$

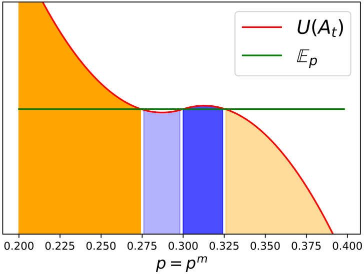
with A_t, δ and $\underline{\mu}$ constructed as in Proposition 1. Then,

$$A_t \succ \delta \iff \alpha > 1$$

Figure back

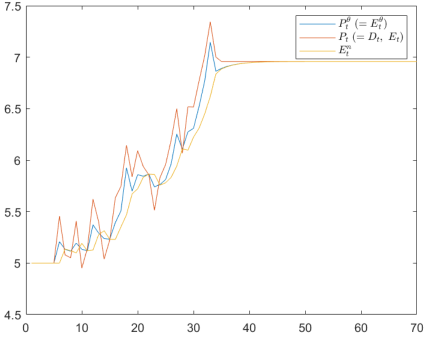
Stake dependent contrast vs assimilation

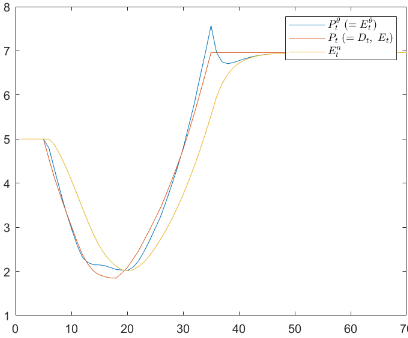




back

Permutation of surprises leads to different price overvaluation





▸ Statement ▸ back

Consistent vs alternating surprises

Proposition Assume the market at t_0 is in steady state. Consider a stream of shocks $\{u_t\}_{t_0}^{t_1}$ such that $u_t \geq u_{t-1}$ $\forall t$ (at least one of the inequalities being strict), and such that $\sum_t u_t > \kappa$. Consider another stream of news $\{u_t^*\}_{t_0}^{t_1}$ which is a permutation of $\{u_t\}_{t_0}^{t_1}$. Then,
- the expected price change at t_1 after $\{u_t\}_{t_0}^{t_1}$ will be lower than after $\{u_t^*\}_{t_0}^{t_1}$, that is

$$\mathbb{E}_{t_1}\left[\Delta P_{t+1}\right] < \mathbb{E}_{t_1}\left[\Delta P^*_{t_1+1}\right]$$

- the expected $t+h$ return under $\{u_t\}_{t_0}^{t_1}$ is lower than under $\{u_t^*\}_{t_0}^{t_1}$ for h large
- $\lim_{h\to+\infty} \mathbb{E}_{t_1}\left[P_{t_1+h}\right] = \lim_{h\to+\infty} \mathbb{E}_{t_1}\left[P^*_{t+h}\right] = P_t = P^*_t$

Figure back

Remembered lottery A_t^m

$$A_t^m = (1-\rho)\sum_{j=1}^{t} \rho^{j-1} \hat{A}_{t-j}(x) + \rho^t A_t(x)$$

where

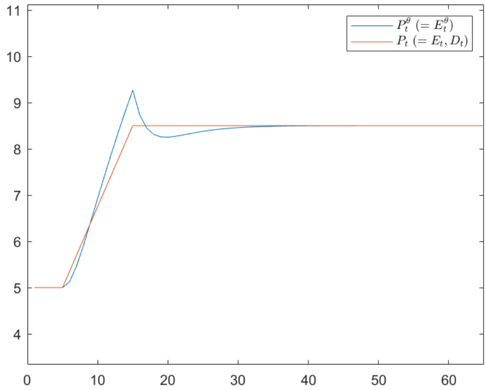
$$\hat{A}_{t-j} = \begin{cases} \arg\min_{\nu_{t-j} \in C_{t-j}} s(\nu_{t-j}, A_t) & \textit{if } A_t \textit{ non degenerate} \\ A_t & \textit{else} \end{cases}$$

where

$$s(\nu, \mu) = \begin{cases} \int (F_\mu(x) - F_\nu(x))^2 \, \mathrm{d}x & \#\mathrm{supp}\mu, \mathrm{supp}\nu > 1 \\ +\infty & \textit{else} \end{cases}$$

back

Under and overreaction to a long sequence of small surprises



- When $t = t_0$, u_{t_0} is small $\implies$ P_t^θ underreacts
- As the news cumulates, $\mathbb{E}_t$ gets saliently far from $\mathbb{E}^m_{t-1}$ $\implies$ P_t^θ overreacts
- The news stops coming, $\mathbb{E}^m_{t-1}$ catches up $\implies$ P_t^θ reverts back

statement back

Under and overreaction to a long sequence of small news

Proposition

If there is sequence of dividend surprises $u_t = u > 0$ for $t \in \{t_0, .., t_1\}$, there exist constants $\underline{u}, \bar{u}$ such that
- If $u < \underline{u}$, the price is below rational and $\mathbb{E}_{t_1}\left[R^{\theta}_{t_1,t_1+h}\right] > 0$ for any h above some constant.
- If $u \in [\underline{u}, \bar{u}]$, the price is below rational for some $t < t_1$ and above rational afterwards, so that $\mathbb{E}_{t_1}\left[R^{\theta}_{t_1,t_1+h}\right] < 0$ for any h above some constant.
- If $u > \bar{u}$, the price is above rational and $\mathbb{E}_{t_1}\left[R^{\theta}_{t_1,t_1+h}\right] < 0$ for any h above some constant.

back to Fig back

d

	T1	T2	T3	T4
Stage 1	\$80, 9% vs \$9, 60%	\$80, 2% vs \$9, 60%	\$80, 21% vs \$9, 60%	\$80, 2% vs \$9, 60%
Stage 2	\$80, 7% **vs** \$9, 60%	\$80, 5% **vs** \$9, 60%	\$80, 23% vs \$9, 60%	\$80, 4% vs \$9, 60%
Stage 3	\$80, 6% **vs** \$9, 60%	\$80, 6% **vs** \$9, 60%	\$80, 22% vs \$9, 60%	\$80, 3% vs \$9, 60%
Stage 4	\$80, 5% **vs** \$9, 60%	\$80, 7% **vs** \$9, 60%	\$80, 9% **vs** \$9, 60%	\$80, 9% **vs** \$9, 60%
Stage 5	\$80, 2% vs \$9, 60%	\$80, 9% vs \$9, 60%		

	T5	T6	T7	T8
Stage 1	\$80, 9% vs \$5, 60%	\$80, 9% vs \$1, 60%	\$80, 9% vs \$9, 60%	\$80, 9% vs \$1, 60%
Stage 2	\$80, 9% **vs** \$4, 60%	\$80, 9% **vs** \$2, 60%	\$80, 9% vs \$8, 60%	\$80, 9% vs \$2, 60%
Stage 3	\$80, 9% **vs** \$2, 60%	\$80, 9% **vs** \$4, 60%	\$80, 9% vs \$8.5, 60%	\$80, 9% vs \$1.5, 60%
Stage 4	\$80, 9% vs \$1, 60%	\$80, 9% vs \$5, 60%	\$80, 9% **vs** \$5, 60%	\$80, 9% **vs** \$5, 60%

Test 2: Assimilation

T2: gradual **ascending**	T4: drastic **ascending**
$80, 2% **vs** $9, 60%	
$80, 5% **vs** $9, 60%	$80, 2% **vs** $9, 60%
$80, 6% **vs** $9, 60%	$80, 4% **vs** $9, 60%
$80, 7% **vs** $9, 60%	$80, 3% **vs** $9, 60%
$80, 9% **vs** $9, 60%	$80, 9% **vs** $9, 60%

- As in the video, small changes are unnoticed in a gradually moving environment
- T2: $80, 9% **assimilated** to preceding ones. T4: $80, 9% **contrasted** away. $C(\$80, 9\%|T4) > C(\$80, 9\%|T2)$

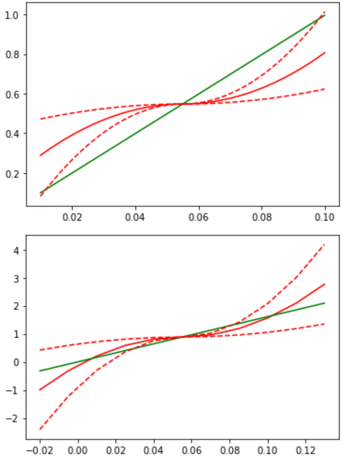
Test 2: Assimilation

T1: gradual descending	T2: gradual ascending
$80, 9% **vs** $9, 60%	$80, 2% **vs** $9, 60%
$80, 7% **vs** $9, 60%	$80, 5% **vs** $9, 60%
$80, 6% **vs** $9, 60%	$80, 6% **vs** $9, 60%
$80, 5% **vs** $9, 60%	$80, 7% **vs** $9, 60%
$80, 2% **vs** $9, 60%	$80, 9% **vs** $9, 60%

- T1: riskier option looks similar to the preceding one, which is slightly better
- T2: riskier option looks similar to the preceding one, which is slightly worse
- Prediction: $C(\$80, q|T1) > C(\$80, q|T2)$

Switching point between assimilation and contrast

Figure: Estimated preferences



red the distorted expected utility of a lottery $U((\$K, p, \$0, 1-p) \mid (\$K, 5.5\%, \$0, 94.5\%))$. *Bottom panel:* $K = 80$, *while in the top panel* $K = 200$.

back

Switching point between assimilation and contrast

- Estimate probit model

$$U\left(\left(x_{i,t},p_{i,t}\right)|\mu_{i,t}^{m}\right)=\left(\frac{1-\rho}{2}\right)\sum_{j=1}^{t-1}\rho^{j-1}\left(p_{i,t-j}u\left(x_{i,t-j}\right)\right)+\left(\frac{\rho^{t-1}}{2}\right)p_{it}u\left(x_{i,t}\right)+ \qquad (1)$$

$$+\begin{cases}\theta\frac{1}{4}\left\{\left(1-\rho^{t-1}\right)u\left(x_{i,t}\right)p_{i,t}-(1-\rho)\sum_{j=1}^{t-1}\rho^{j-1}\left(p_{i,t-j}u\left(x_{i,t-j}\right)\right)\right\}^{2} & if\ \left(\frac{1-\rho^{t-1}}{1-\rho}\right)p_{i,t}u\left(x_{i,t}\right)>\sum_{j=}^{t} \\ -\theta\frac{1}{4}\left\{\left(1-\rho^{t-1}\right)p_{i,t-j}u\left(x_{i,t}\right)-(1-\rho)\sum_{j=1}^{t-1}\rho^{j-1}\left(p_{i,t-j}u\left(x_{i,t-j}\right)\right)\right\}^{2} & if\ \left(\frac{1-\rho^{t-1}}{1-\rho}\right)u\left(x_{i,t}\right)<\sum_{j=1}^{t-1} \end{cases}$$

- Assume $u\left(x\right)=x^{\beta}$
- Recover via MLE

$$\left(\hat{\beta},\hat{\rho},\hat{\theta},\hat{\sigma}\right)=\begin{pmatrix}0.28\\0.02\\1.28\\1.53\end{pmatrix}$$

- Standard errors obtained via block bootstrap where each individual i is a block to take into account individual level serial correlation.

back

Amenity and health risk premia in house prices

- $$V_t(y_t) = \begin{cases} R_t + \delta \mathbb{E}_t^\theta [P_{t+1}] - \frac{1+r}{\delta} P_t + \beta E_t V_{t+1}^\theta (y_{t+1}) & \text{if } y_t = own \\ 0 + E_t V_{t+1}^\theta (y_{t+1}) & \text{if } y_t = not \end{cases}$$

$$E_t V_{t+j}^\theta (y_{t+1}) = \begin{cases} \mathbb{E}_t^\theta [R_{t+j}] + \delta \mathbb{E}_t^\theta [P_{t+j+1}] - \delta \mathbb{E}_t^\theta [P_{t+j}(1+r)] + \delta E \\ 0 + \delta E_t V_{t+j+1}^\theta (y_{t+j+1}) \end{cases}$$

back

Amenity premia in house prices

Proposition 5

Suppose to run the following regression

$$\Delta P_t = \alpha + \beta \Delta a_t + u_t$$

Let σ^2 be the variance of the amenity shock. Then $\frac{\partial}{\partial \sigma}\beta > 0$
Moreover

$$cov\left(\Delta P_t, \Delta P_{t-1}\right) \geq 0 \iff \sigma^2 \leq k$$

- Amenity premium estimated using the drastic variation in the shock is larger than rational, and larger than the premium during the slow moving shock back

Boiling frog

Consider two different price paths such that $P_0 = P_0^*$ and $P_t = P_t^*$, where $\{P_j\}_{j=t_0}^{t}$ is generated by a stream of positive news $\{u_j\}_j$ with $u_j > 0$, while $\left\{P_j^*\right\}_{j=t_0}^{t}$ is generated by $\left\{u_j^*\right\}_j$ with $u_j^* = 0$ for $j < t$ and $u_t > 0$. If ΔP_t^* is large enough, then

-

$$\mathbb{E}_t\left[R_{t,t+h}^*\right] < \mathbb{E}_t\left[R_{t,t+h}\right]$$

for h above some constant k

- Moreover $\lim_{h\to+\infty} \mathbb{E}_t\left[P_{t+h}\right] > \lim_{h\to+\infty} \mathbb{E}_t\left[P_{t+h}^*\right]$

back

Portfolio choice: mean variance preferences with **distorted mean** $\mathbb{E}_t^\theta[D_T]$

$$\max_{X_t} \left\{ X_t \left(\mathbb{E}_t^\theta[D_T] - P_t \right) - \frac{\gamma}{2} X_t^2 \mathbb{V}[D_T] \right\}$$

Solution

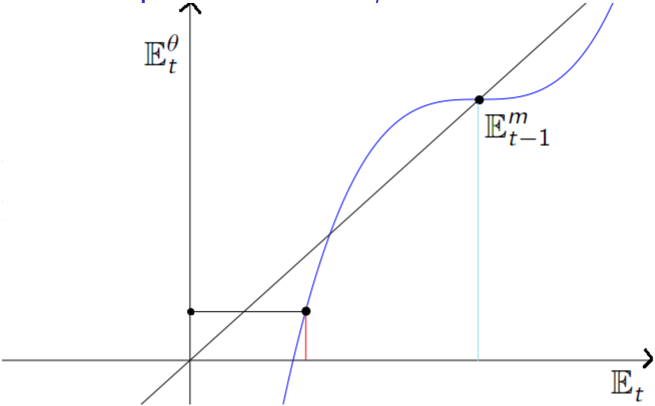
$$X_t = \frac{\mathbb{E}_t^\theta[D_T] - P_t}{\gamma \mathbb{V}[D_T]}$$

Assume supply of 1. The equilibrium price is thus

$$P_t = \mathbb{E}_t^\theta[D_T] - \gamma \mathbb{V}[D_T]$$

back

Distorted expected value $\mathbb{E}_t^{\theta}$



$$\mathbb{E}_t^{\theta} = \mathbb{E}_{t-1}^{m} + \begin{cases} \left(\mathbb{E}_t - \mathbb{E}_{t-1}^{m}\right)^2 & \mathbb{E}_t > \mathbb{E}_{t-1}^{m} \\ -\left(\mathbb{E}_t - \mathbb{E}_{t-1}^{m}\right)^2 & \mathbb{E}_t < \mathbb{E}_{t-1}^{m} \end{cases}$$

$$\mathbb{E}_{t-1}^{m} := (1-\rho)\sum_{j=1}^{\infty} \rho^{j-1}\mathbb{E}_{t-j} = (1-\rho)\sum_{j=1}^{\infty} \rho^{j-1} D_{t-j}$$

Can express $\mathbb{E}_t^{\theta}[D_T]$ as a functon of the rational expectation and the surprises until t

$$\mathbb{E}_t^{\theta}[D_T] = \mathbb{E}_t[D_T] + \left(\sum_{j=0}^{t-t_0-1} \rho^j u_{t-j}\right) \underbrace{\left\{\theta \left|\sum_{j=0}^{t-t_0-1} \rho^j u_{t-j}\right| - 1\right\}}_{\substack{<0 \text{ small surprises} \\ >0 \text{ if large surprises}}}$$

back

Assimilation: underreaction

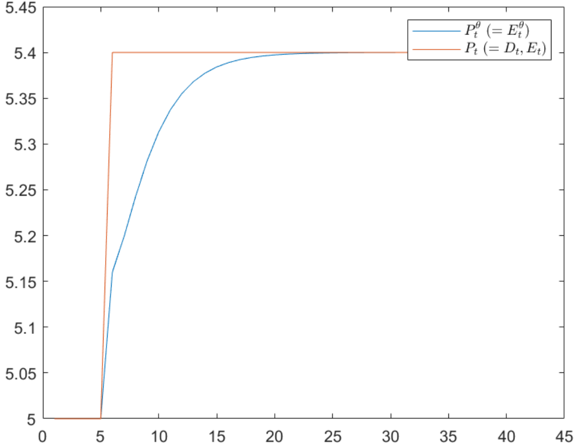
- Suppose no news occurred before t_0, and **positive surprise** $u_{t_0} > 0$ **arrives at** t_0:

$$\mathbb{E}_{t_0}^{\theta}\left[D_T\right] = \mathbb{E}_{t_0}\left[D_T\right] + u_{t_0}\left\{\theta|u_{t_0}| - 1\right\}$$

- Agent underreacts if

$$u_{t_0} < \frac{1}{\theta}$$

- Interpretation: the posterior on D_T is assimilated to the prior because it is close to it. The expected value is shrunk towards the memorized prior.

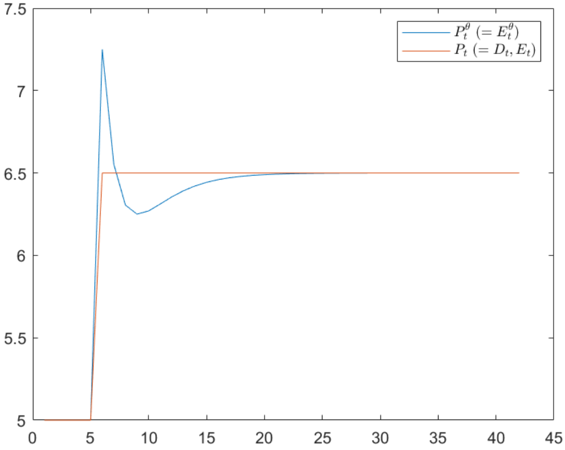


P_t^θ **underreacts** on impact to a permanent change in D_T. When $t > t_0$, memorized $\mathbb{E}_{t-1}^m$ slowly adjusts upwards, hence P_t^θ converges upwards.

Contrast: overreaction to large shock

- Agent overreacts if
$$u_{t_0} > \frac{1}{\theta}$$
- Interpretation: the posterior on D_T is contrasted away from the prior. The expected value is exaggerated away from the memorized prior in the direction of the surprise.

Contrast: overreaction to large shock



P_t^θ **overreacts** on impact to a permanent change in D_T. When $t > t_0$, the memorized $\mathbb{E}_{t-1}^m$ **slowly adapts upwards**, hence $\theta \left| \sum_{j=0}^{t-t_0-1} \rho^j u_{t-j} \right|$ decreases

▸ back

Table: Assimilation, Contrast and Boiling Frog

	(1) ($80,9%)	(2) ($80,9%)
D Drastic effect	-0.208***	
	(0.0404)	
A Drastic	0.322***	
	(0.0281)	
A Drastic effect		0.0876**
		(0.0351)
A Gradual		0.234***
		(0.0174)
Observations	398	828
R-squared	0.063	0.008

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) 9%	(2) 7%	(3) 6%	(4) 5%	(5) 2%
D Drastic effect	0.126***	0.0959***	0.0480**	0.0133	-0.0645***
	(0.0260)	(0.0242)	(0.0223)	(0.0211)	(0.0177)
A Gradual	0.234***	0.186***	0.161***	0.152***	0.138***
	(0.0181)	(0.0168)	(0.0155)	(0.0147)	(0.0123)
Observations	1,208	1,208	1,208	1,208	1,208
R-squared	0.019	0.013	0.004	0.000	0.011

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

back

Table: Contrast, Assimilation and Boiling frog

	(1) ($5,60%)	(2) ($5,60%)
D Drastic effect	-0.203***	
	(0.0492)	
A Drastic	0.577***	
	(0.0352)	
A Drastic effect		0.124**
		(0.0493)
A Gradual		0.453***
		(0.0340)
Observations	397	408
R-squared	0.041	0.015

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) $5	(2) $4	(3) $2	(4) $1
D Gradual effect	0.166***	0.154***	-0.103**	-0.172***
	(0.0496)	(0.0491)	(0.0437)	(0.0395)
A Gradual	0.453***	0.346***	0.304***	0.280***
	(0.0337)	(0.0334)	(0.0297)	(0.0268)
Observations	398	398	398	398
R-squared	0.028	0.024	0.014	0.046

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

back