Trading Volume Alpha*

Ruslan Goyenko — McGill

Bryan Kelly — Yale, AQR, and NBER

Tobias Moskowitz — Yale, AQR, and NBER

Yinan Su — Johns Hopkins

Chao Zhang — HKUST (GZ)

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Abstract

Portfolio optimization focuses on risk and return prediction, yet implementation costs critically matter. Predicting trading costs is challenging because costs depend on trade size and trader identity, thus impeding a generic solution. We focus on a component of trading costs that applies universally – trading volume. Individual stock trading volume is highly predictable, especially with machine learning. We model the economic benefits of predicting volume through a portfolio framework that trades off tracking error versus net-of-cost performance – translating volume prediction into net-of-cost alpha. The economic benefits of predicting individual stock volume are as large as those from stock return predictability.

Keywords: machine learning, AI, neural networks, portfolio optimization, trading volume, trading costs, investments

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1 Introduction

Research on portfolio optimization chiefly focuses on mean return prediction, and to a lesser extent, variance-covariance prediction.[1] However, real-world implementation costs also play a critical role in the efficacy of portfolios. While the benefits and pitfalls of mean return and volatility forecasts are well-studied in the literature, trading costs have received relatively little attention, and *forecasting* trading costs has received no attention.[2]

Predicting trading costs, particularly at the individual stock level, is challenging since the biggest component of these costs for a large investor is price impact, which depends on the size of the trade and the amount traded by other traders in that security, as well as the identity of the trader (Frazzini, Israel, and Moskowitz, 2018).[3] Since each trader may face their own cost function, finding a generic solution to this piece of the portfolio problem is challenging. Moreover, the size of the trade is endogenously a function of expected trading costs, finding an estimate of expected costs is critical to the portfolio decision.

To circumvent these issues, we take a unique approach to predicting stock-specific expected trading costs by focusing on the component of costs that is neither trader-specific nor endogenous – the level of total trading volume in the stock. Trading volume is driven by other traders in the market for the same security, which is generic to all traders. As Frazzini, Israel, and Moskowitz (2018) show, trade size divided by daily trading volume – termed the market participation rate in a stock – is the key driver of price impact costs. Following Kyle (1985), price impact is an increasing function of participation rate. Holding trade size constant, the less trading volume in the stock the greater the trader's price impact will be.[4] Our empirical strategy is to forecast trading volume for

[1] A very long literature in asset pricing focuses on return prediction, with summaries on the state of this literature, including some of its criticisms, found in Harvey, Liu, and Zhu (2016); McLean and Pontiff (2016); Jensen, Kelly, and Pedersen (2022). A good summary of the literature on volatility prediction can be found in Engle (2004).

[2] Several papers focus on trading costs from a theoretical perspective: Kyle (1985); Gârleanu and Pedersen (2016). Even less work has provided empirical estimates of trading costs for use in a portfolio optimization context: Frazzini, Israel, and Moskowitz (2012, 2018).

[3] While many different trading cost models exist, they universally contain these three elements: trade size, market size, and trader specifics (identify, information, motive, patience, etc.).

[4] Theoretical foundations of trading costs in the classic market microstructure literature include asymmetric information (Kyle, 1985; Glosten and Milgrom, 1985) and inventory costs (Stoll, 1978; Ho and Stoll, 1983; Grossman and Miller, 1988). In either case, volume is the major determinant of liquidity (Benston and Hagerman, 1974; Glosten and Harris, 1988; Brennan and Subrahmanyam, 1995) and is used to proxy for trading costs or liquidity (Campbell, Grossman, and Wang, 1993; Datar, Naik, and Radcliffe, 1998; Amihud, 2002).

each security as a proxy for expected trading costs, and then use this forecast to optimize portfolios net of those costs and quantify its benefits.

Importantly, our aim is to provide a general forecast of costs for an individual stock that abstracts from the need to specify trade size and applies universally across all traders. Our goal is not to provide the "best" or most reliable trading cost model or forecast. Rather, our simple objective is to provide a forecast of trading costs that any trader could use. This aim necessitates a simple, rather than sophisticated, trading cost model. The goal is to focus on one component of a trading cost model that is generic and see how valuable that can be.

To quantify the economic magnitude of volume prediction, we incorporate volume prediction into a portfolio theory problem. We model a portfolio framework that seeks to maximize the net-of-cost performance of the portfolio using a mean-variance utility function, where the cost of transacting scales linearly with participation rate (motivated by theory and empirical work from the literature). The optimization trades off the cost of trading versus the (opportunity) cost of not trading – minimizing trading costs versus minimizing tracking error to the before-cost optimal portfolio – where trading costs and tracking error are endogenously negatively related. In the model, we take the first and second moments of security returns as given and focus solely on the tradeoff between trading costs and tracking error to the pre-cost optimal portfolio.

Using this framework, we can couch the volume prediction problem into a portfolio problem and quantify the economic value of volume prediction in terms of its benefit to after-cost returns or Sharpe ratio.[5] In essence, we translate trading volume predictability into portfolio alpha, which we term "trading volume alpha." This translation opens up after-cost portfolio modeling, which has been restricted due to limited access to trading cost data, to the widely available volume data. This paper demonstrates the economic value of volume prediction with one set of predictors and standard neural network implementation. Yet the framework laid out in the paper illustrates the importance of and enables more work on constructing volume prediction signals and models.[6]

We impose a functional form for trading costs due to price impact motivated by theory (Kyle,

[5] Related, Balduzzi and Lynch (1999) and Çetin, Jarrow, and Protter (2004) model transaction costs in portfolio settings and study the economic value in various transaction cost models.

[6] This research answers the call to integrate market microstructure frictions into asset pricing studies using machine learning tools (Goldstein, Spatt, and Ye, 2021).

1985) and empirical evidence (Frazzini, Israel, and Moskowitz, 2018), where price impact (Kyle's Lambda) is an increasing linear function of the trader's participation rate. All else equal, a higher predicted volume allows the trader to trade more aggressively (larger size) because price impact per dollar traded will be lower. Conversely, a lower predicted volume causes the trader to scale back the trade (even perhaps to zero and not trade) because the price impact per dollar will be higher. By modeling trading costs and benefits as a tracking error problem, we abstract from return or variance prediction and focus exclusively on volume prediction and its economic impact through expected trading costs.

An interesting feature emerges from the model that generates an asymmetry in the economic value of volume prediction. Price impact costs are linear in participation rate, but non-linear in trading volume. Very low trading volume implies exponentially high impact costs, whereas very high volume implies negligible costs. As volume tends to zero, price impact costs approach infinity, whereas when volume becomes large, impact costs are bounded by zero. Hence, predicted changes in volume have much more economic impact when volume is low versus high, thus creating asymmetric costs of volume forecast errors. Conversely, tracking error, or the opportunity cost of not trading, is independent of volume. The combination of these two effects implies the optimization will penalize overestimating volume more than underestimating volume. Trading too much when you overestimate volume is more costly than trading too little when you underestimate it. At lower volume, the cost of trading with respect to volume is very steep, and at high volume the cost-volume relationship is flat. Intuitively, an illiquid stock's price impact is very sensitive to small changes in volume and in a highly non-linear way – participation rates move by orders of magnitude for small changes in volume – resulting in trading costs increasing at a much faster rate when trading too aggressively. Conversely, a very liquid security's price is fairly inelastic to changes in volume, because having more or less liquidity at that point has little impact on costs. As a result, the optimization seeks to be conservative, rather than aggressive.[7]

Since participation rate drives trading costs, it is not only trading volume that matters, but also

[7] Our framework, and its implication that portfolio optimization will seek to trade conservatively rather than aggressively because of the asymmetric cost of volume forecast errors, implies that arbitrage activity may be limited as a consequence. This implication provides a novel and additional source of limits to arbitrage activity in the spirit of Shleifer and Vishny (1997).

the size of the trade. Trade size is determined endogenously and is a function of expected trading volume and aversion to tracking error. In the model, the size (assets under management, AUM) and volatility of the fund (and risk aversion of the investor) also affect the costs and benefits of trading. Because price impact is an increasing function of participation rate, trading costs increase with AUM endogenously, and the relative penalty for tracking error decreases with AUM. The optimal tradeoff between trading costs and tracking error will therefore vary with the size of the portfolio, and so will the economic impact of volume prediction. For small AUM, tracking error considerations likely dominate trading cost considerations, hence the economic benefit to predicting volume may be relatively less valuable. For large AUM, trading cost considerations likely dominate.[8]

Applying the model framework to data, we run a series of trading experiments for optimally designed portfolios that take into account trading costs using stock-level volume prediction as the sole input. Since liquidity is an unknown quantity to the portfolio manager, she uses volume prediction as an input to alter the expected cost and benefit of trading, endogenously responding to her forecast of volume by altering her portfolio. We assess the out-of-sample performance of this portfolio, net of trading costs, to evaluate the economic impact of predicting volume. More accurate volume predictions provide more efficient implementation and hence more efficient portfolios net of costs.

We experiment with various sets of target positions to mimic realistic trading tasks. We start by simulating an extremely profitable (*before-cost*) daily quantitative trading strategy, which represents an unachievable target. This portfolio, which requires high turnover and aggressive trading, is much less profitable after accounting for trading costs. We then consider expected trading costs in the optimization to maximize the after-cost performance of the fund (at various fund sizes). We also target a host of factor portfolios from Jensen, Kelly, and Pedersen (2022) that are based on ex-ante return-predicting characteristics to examine the effectiveness of trading volume alpha across the spectrum of factors. We examine the net of trading cost performance of an investment strategy that seeks to target each of these factor portfolios while taking expected trading costs into account,

[8]From this analysis, it is also possible to assess the optimal dollar size of the portfolio, including the break-even fund size where trading costs exactly offset portfolio returns or the fund size that maximizes after-cost dollars rather than returns (Frazzini, Israel, and Moskowitz, 2018).

using predicted market trading volume as the only input into forecasting expected trading costs. Expected trading costs dictate how frequently to trade, which stocks to trade, and how much to trade, with each of these choices simultaneously affecting tracking error to the portfolio.

We find that volume prediction has a measurable and significant effect on after-cost portfolio performance. To predict volume, we use technical signals, such as lagged returns and lagged trading volume, as well as firm characteristics that the literature finds capture return anomalies, but not necessarily trading volume. We then add indicators for various market-wide or firm-level events associated with volume fluctuation, including upcoming and past earnings releases. We analyze both linear and non-linear prediction methods using various neural networks designed to maximize out-of-sample predictability. Finally, we alter the objective/loss function of the neural network to take into account the portfolio problem's economic objective when predicting volume. Our aim, again, is not to find the best prediction model for stock trading volume, but rather to couch the prediction problem into economic outcomes and measure its costs and benefits accordingly. To that end, we assess a number of different models and variables for predicting volume in order to highlight how different prediction methods lead to different economic consequences.[9]

We find that volume prediction improves significantly over moving averages of lagged trading volume when using technical signals. Adding firm characteristics (such as BE/ME) further improves volume predictability, even though these variables are primarily used for return forecasting and are not necessarily related to trading volume. Information on events such as earnings releases further enhances volume predictability. Non-linear functions from neural network searches provide better predictability over simple linear models, controlling for the same set of variables/information. Recurrent neural networks, which learn dynamic predictive relationships, yield additional improvements.

Finally, we find that imposing an economic loss function consistent with the portfolio problem greatly improves the value of volume predictability from a portfolio performance perspective. Specifically, we fine-tune the neural networks on the economic objective (derived from the portfolio problem) rather than on a statistical objective, such as mean squared errors used to pre-train the

[9] For example, we leave out many potential variables that relate to trading volume, such as other microstructure variables, cross-security lead-lag effects, etc.

volume prediction model. Fine-tuning yields significant gains in out-of-sample portfolio performance compared to the statistically pre-trained model. This result obtains because the portfolio problem recognizes that trading costs are not a linear function of trading volume. The neural network places greater weight on observations that impact trading costs more and worries less about predicting volume where trading costs are less affected, such as recognizing the asymmetric costs of over- versus underestimating volume. While an MSE objective criterion may maximize the out-of-sample R^2 of volume prediction, an economic loss function directly tied to the portfolio problem provides more valuable volume predictability that results in better out-of-sample after-cost portfolio performance.

Since portfolio size affects the tradeoff between the cost and benefit of trading, the resulting portfolio solution also changes with different levels of AUM. In addition, while volume prediction has benefits across all factors, some factors have a greater trading volume alpha than others due to the varying tradeoff between the cost of trading and the opportunity cost of not trading. Intuitively, we find that factors with a higher turnover (e.g., momentum, short-term reversals) benefit more from portfolio optimization that accounts for expected trading costs based on volume forecasts.

In general, we find that trading volume alpha is substantial. The marginal improvement on a portfolio from trading volume alpha is as large as finding return alpha. For example, for a $1 billion fund, the after-cost improvement in portfolio performance due solely to trading volume prediction beyond using lagged volume measures, can be as much as double in terms of expected returns or Sharpe ratio after trading costs. Among popular asset pricing factors, the improvement in after-cost returns ranges from 20 bps to 100 bps above using a moving average of lagged volume to predict future volume. Refining the prediction methods and deepening the prediction models could add substantially to these improvements, generating even larger trading volume alpha.

The rest of the paper is organized as follows. Section 2 covers some preliminaries to the analysis: a motivation for predicting trading volume and a description of our data. Section 3 examines volume prediction from a purely statistical perspective to maximize out-of-sample predictability. Section 4 presents a theoretical optimal portfolio framework for quantifying the economic value of volume prediction in terms of portfolio performance, which we call "trading volume alpha." Section 5 discusses the empirical results of predicting volume through the lens of our theoretical framework using a variety of machine learning methods. Section 6 applies these insights and methods to trading experiments that characterize the net of cost performance improvement of simulated real-world portfolios. Section 7 concludes.

2 Preliminaries: Motivation and Data

We first motivate why predicting volume is interesting and useful and then describe the data.

2.1 Motivation

Trading costs are critical to realizing investment performance, yet insufficient empirical attention has been paid to them regarding optimal portfolio construction. This lack of research is largely due to the challenges imposed by modeling trading costs and finding a generic portfolio solution. As a consequence, and despite its importance, portfolio theory mainly focuses on modeling and forecasting the first and second moment of returns gross of costs.

One of the main challenges in modeling and forecasting trading costs is that the largest component of these costs for a large investor is price impact, which depends on the size of the trade, the amount traded by other traders (in the same and in opposite directions), and the identity of the trader – different traders may face different price impacts. These features frustrate a generic portfolio solution that incorporates trading costs. In particular, since optimal portfolio weights should reflect the expected net of cost return, a solution requires an expected cost function, which varies by investor, and hence so will the portfolio solution.

Following Kyle (1985) and subsequent empirical work (Frazzini, Israel, and Moskowitz, 2018), price impact depends on the trader's participation rate, defined as the dollar amount traded by investor n in security i relative to total dollar volume in stock i (the amount traded in the market by everybody in security i) at the same time t,

$$ParticipationRate_{n,i,t} = \frac{\$Traded_{n,i,t}}{\$Volume_{i,t}}.$$

Price impact is an increasing function of participation rate (modeled linearly in Kyle, 1985 and the positive relation empirically verified in Frazzini, Israel, and Moskowitz, 2018), whose elasticity varies by investor n. The numerator is also endogenous to expected price impact, which itself is a function of the participation rate. The circular nature of trade size, and its variation across investors, makes modeling trading costs particularly challenging. However, the denominator of the participation rate is independent of n and exogenous to the trader's desired trade size (assuming a single investor's trade is too small to materially affect total dollar volume). Thus, trading volume is generic and exogenous to each trader and provides a variable universal to all investors for modeling costs.

From a prediction standpoint, total dollar volume is also easier to forecast than a specific investor's trades. Predicting market-level trading activity (total buys and sells) is an easier task and high frequency data on total trading volume is readily available, while data on individual traders is not. Moreover, as we will find, machine learning techniques can significantly improve our ability to forecast volume, due in part to the non-linear nature of volume and its relation to trading costs. With these insights, we model expected trading costs solely using forecasted total dollar volume for a stock. Although this exercise is only a partial solution to the trading cost problem, it is a general one, and it allows us to showcase the economic value of predicting trading volume in an optimal portfolio framework.[10]

2.2 Data

We compile a data panel of daily stock-level dollar trading volume ($\widetilde{V}_{i,t}$) and 175 predictors ($X_{i,t}$). The unit of observation is stock-day (i, t). We adopt the convention that $X_{i,t}$ is observed by day $t-1$, whereas the associated prediction target $\widetilde{V}_{i,t}$ is observed until the end of day t. We use a tilde to denote a random variable conditional on the information before day t.

The sample period is 2018 to 2022, or 1,258 days. The cross-section covers around 4,700 stocks, with an average of 3,500 stocks per day, or 4,400,000 observations in total. We split the data into

[10] Korajczyk and Sadka (2004), Novy-Marx and Velikov (2016), and DeMiguel et al. (2020) evaluate the after-cost profitability of factor portfolios, rather than actively seeking cost mitigation in portfolio optimization. Jensen et al. (2022) take price impact transaction costs as given in portfolio optimization. We tackle the forecasting problem given uncertain transaction costs and apply it within a portfolio optimization framework.

a 3-year training sample and a 2-year testing sample. All models are trained once in the training sample and evaluated out of the sample. We avoid re-sampling methods such as cross-validation and rolling-window re-estimations.

Our analysis focuses on predicting out-of-sample trading volume. Reasonable in-sample fit, often evaluated in the literature (Chordia, Huh, and Subrahmanyam, 2007; Chordia, Roll, and Subrahmanyam, 2011), does not often lead to good out-of-sample (OOS) performance, which is of primary interest to evaluate the robustness and economic impact of volume predictability.

While it is common in the stock return predictability literature to use large sets of variables (e.g., "factor zoo") to identify the best predictors, it is less common to use large data sets to predict market microstructure variables such as trading volume. We show that using big data improves the precision of volume forecasts significantly.

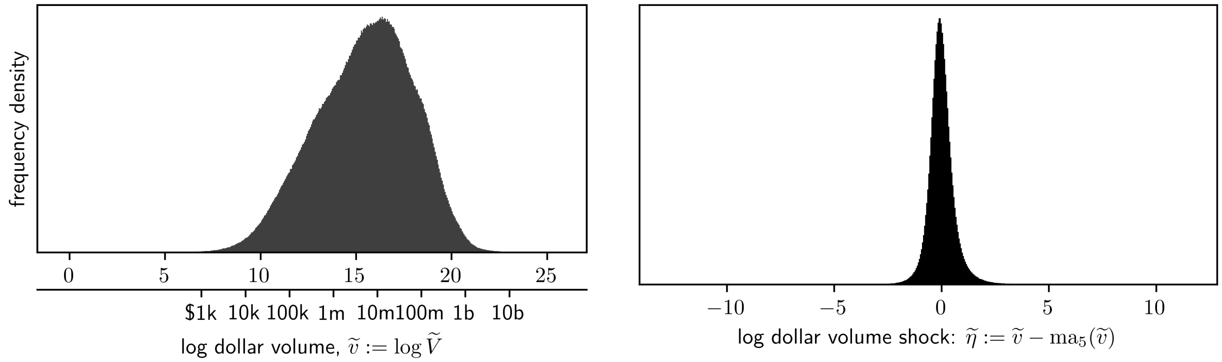
The main variable we aim to predict is dollar trading volume, which we measure as the natural logarithm of end-of-day transacted total dollar trading volume for each stock. This variable is highly persistent. We focus on predicting innovations in trading volume as well, which we show has significant impact on trading costs. When volume is suddenly much lower than expected, an investor will "overpay" in transaction costs. If volume is higher than expected, then there is more liquidity, and an investor incurs opportunity costs from not trading aggressively enough. We examine the predictive content of several sets of predictors, including technical, fundamental, and event-based variables, which we describe below.

2.3 Prediction objects: daily stock trading volume

Daily dollar trading volume $\widetilde{V}_{i,t}$ ranges widely (from thousands to billions of dollars) across stocks and is highly skewed. We take the log of dollar volume, $\widetilde{v} = \log \widetilde{V}$, whose distribution, shown in Figure 1, is relatively well-behaved, being close to normal and symmetrical.

Log dollar volume is highly persistent in the time series and cross-section of stocks, and can be easily predicted by lagged moving averages of various frequencies. The five-day moving average predicts log dollar volume with an R^2 of 93.68%, higher than the one-day lag (92.53%), moving average of 22 days (92.60%), or 252 days (86.12%).

Figure 1: Distributions of daily stock-level log dollar volume ($\widetilde{v}$) and its shock ($\widetilde{\eta}$), panel pooled



Histograms of $\widetilde{v}_{i,t}$ and $\widetilde{\eta}_{i,t}$ in the full sample of around 4,400,000 stock-day observations. The second horizontal axis in the left panel is dollar volume $\widetilde{V}_{i,t}$ in the log scale. Log dollar volume shock $\widetilde{\eta}$ is daily log dollar volume minus the moving average in the past five days: $\widetilde{\eta}_{i,t} := \widetilde{v}_{i,t} - \frac{1}{5}\left(\widetilde{v}_{i,t-1} + \cdots + \widetilde{v}_{i,t-5}\right)$.

We focus on predicting the log dollar volume *shock* defined as daily log dollar volume minus the moving average in the past five days, $\widetilde{\eta}_{i,t} := \widetilde{v}_{i,t} - \frac{1}{5}\left(\widetilde{v}_{i,t-1} + \cdots + \widetilde{v}_{i,t-5}\right) := \widetilde{v}_{i,t} - [\mathrm{ma}_5]_{i,t}$. This exercise is comparable to predicting asset returns (change in log price) instead of price levels. Figure 1 (right panel) shows the pooled distribution of $\widetilde{\eta}$, which is relatively symmetric, centered around zero, and has long tails.

2.4 Predictors

We use a total of 175 predictors from various sources, including technical signals, firm fundamentals, and market and corporate events. We show that the virtue of complexity approach in return prediction (Kelly, Malamud, and Zhou, 2024) is also useful for volume prediction. We find that each subset of variables provides incremental improvement to predicting volume, while using all variables has the greatest OOS predictability. We list the sets of predictors that are cumulatively added to the prediction model.

1. Technical signals ("tech"): lagged moving averages of returns and log dollar volume over the past 1, 5, 22, and 252 days. (8 predictors.)
2. A small set of commonly used fundamental firm characteristics ("fund-1"): market equity, standardized earnings surprise, book leverage, book-to-market equity, Dimson beta, and firm

age. (6 predictors.)[11]

3. The remaining firm characteristics from the JKP dataset ("fund-2"), which are merged and transformed in the same way as fund-1 variables. (147 predictors.)

4. Calendar dates with large effects on trading volume ("calendar"). We hard code four binary features based on the dates of the following four types of events.

   - Early closing days for the exchanges (July 3rd, Black Friday, Christmas Eve, and New Year's Eve).
   - Triple witching days (four times a year when the index futures contract expires at the same time as the index option contract and single stocks options).
   - Double witching days (eight times a year when two of the three events above, the single stock options and index options expiration, coincide).
   - Russell index re-balancing (once a year, the fourth Friday in June).[12]

   Early closing days have substantially less trading volume, while the other three are associated with positive spikes in trading volume.

5. Earnings release schedule ("earnings"): We construct 10 categorical dummy variables (one-hot encoding) indicating whether the firm has an upcoming earnings release or just had one in the past few days. We first construct the number of days until the next known scheduled earnings release event. For example, a value of zero implies the current day is previously known to have a scheduled release. A negative value means there are no known scheduled events in the future and indicates how many days since the last event. We convert this number into 10 dummy variables of categorical bins: $\leq -4, -3, -2, -1, 0, 1, 2, 3, 4, \geq 5$. The data source is the Capital IQ Key Developments dataset. (10 predictors.)

This list of variables for predicting volume is not exhaustive. Other variables that could add

[11] These predictors are from the JKP dataset (Jensen, Kelly, and Pedersen, 2022). We forward fill the monthly firm characteristics in time when merging to the daily panel. Hence, the characteristics are still always ex-ante available. On each day, we rank standardize in the cross-section each characteristic to a uniform distribution from -1 to +1.

[12] Triple witching happens on the third Friday in March, June, September, and December. Double witching is on the third Friday in the other eight months. Russell index re-balancing is on the fourth Friday of June when the Russell 1000, Russell 2000, Russell 3000, and other Russell indexes are reconstituted, and has "often been one of the highest-volume trading days of the year" for the exchange, due to indexes tracking funds adjusting their holdings to reflect the updates. Rerferenes: https://www.nasdaq.com/articles/the-powerful-impact-of-triple-witching-2021-06-10., and https://www.nasdaq.com/articles/2023-russell-rebalancing:-what-you-need-to-know.

predictive power are microstructure variables, intraday observations, and lead-lag relations across stocks in terms of trading (e.g., large to small stocks, within industry, etc.). As stated previously, we do not attempt to provide the best volume prediction model. Rather, we translate the volume prediction problem into an economic problem whose objective is after-cost portolio performance. Using our framework, future work can add further predictors for volume that may provide even larger economic benefits than we show here. However, our framework provides a way to assess those predictive contributions in economic terms.

3 Volume prediction from a statistical perspective

We start with a statistical prediction of daily trading volume using various subsets of predictors and a variety of methods, including machine learning techniques.

3.1 Prediction methods

We run predictive regressions of $\widetilde{\eta}$ (changes in daily dollar volume) on a set of predictors, X, in the training sample panel to estimate the models.[13] We compare linear models (ols), with neural networks (nn) that allow for non-linear transformations and complex interactions, as well as recurrent neural networks (rnn) that, in addition, allow for state variables to incorporate time series dynamics. The simplest baseline is predicting $\widehat{v}_{i,t} = [\mathrm{ma}_5]_{i,t}$, or in other words, $\widehat{\eta}_{i,t} = 0$. (The "hat" denotes predicted values.) Linear regression is also a simple benchmark comparison.

The neural network implementation is kept simple, standard, and fixed throughout the paper in order to facilitate transparency. The network architecture has three fully-connected hidden layers of 32, 16, and 8 ReLU nodes, respectively, and one linear output node. The size of the input layer is the number of predictors supplied.

Recurrent neural network architecture is particularly appealing for this application as it is designed to capture time-series dynamics. An rnn is analogous to state space models like GARCH, in which the forecast $\widehat{\eta}_{i,t}$ is not only a (non-linear) function of the concurrent predictors $X_{i,t}$, but

[13] As explained above, predicting $\widetilde{\eta}$ or $\widetilde{v}$ are essentially the same: predicting $\widetilde{\eta}$ as $nn(X)$ is just predicting $\widetilde{v}$ as $nn(X) + \mathrm{ma}_5$, where ma_5 is one of the predictors. From the machine learning perspective, this is implementing a simple residual connection (ResNet, He et al. 2016) as illustrated in Figure 4.

also of a set of state variables that is the output of the network applied to the previous data point $\{i, t-1\}$. That is, $(\widehat{\eta}_{i,t}, state_{i,t}) = rnn(X_{i,t}, state_{i,t-1})$, where rnn represents the neural network function and *state* are the state variables. The *recurrent* neural network processes data sequentially, and recursively passes the state variables to the next time period. Essentially, rnn extracts predictive information from concurrent and lagged predictors $X_{i,t}, X_{i,t-1}, X_{i,t-2}, \ldots$, in contrast to a nn that uses only the concurrent predictors but nothing from the past. Although $X_{i,t}$ contains moving averages of $\widetilde{v}_{i,t-1}, \widetilde{v}_{i,t-2}, \widetilde{v}_{i,t-3} \ldots$, for example, the way such lagged information enters the model without an rnn architecture is highly restrictive. With rnn, the model can "learn" flexible dynamics, where time-series dependencies are parameterized by trainable network weights. We implement the rnn with the popular and standard lstm (long short-term memory) architecture. The number of layers and neurons are kept the same as nn, but the total number of parameters increases by four times (due to the flow of lagged information).[14]

Appendix A.1 contains other details on implementing the machine learning methods, including the optimizer, training scheme, and infrastructure. We do not tune or optimize the hyperparameters, the architecture, or the training scheme to improve the results.

3.2 Prediction results

Table 1 reports the OOS prediction accuracy of each method. We cumulatively add sets of predictors in the columns from left to right to highlight the prediction improvement from using larger data sets. The rows correspond to different prediction methods. We express the accuracy in two R^2's. Panel A reports the explained percentage of the total sum of squared errors of log dollar volume relative to $\widetilde{\eta}$, the residual after controlling for the five-day moving average. Panel B converts that to the percentage in the total sum of squared of log of dollar volume ($\widetilde{v}$). Lastly, Panel C reports the number of parameters estimated in each model with each set of predictors.

Volume is highly predictable. The most sophisticated model using all predictors can predict

[14] Specifically, the bottom hidden layer in the aforementioned 3-layer network is upgraded to an lstm layer with 32 hidden states and cell states, with the rest of the two layers unchanged. Lstm is a standard and popular type of rnn with four specific internal mechanisms, or gates, that control the flow of information from both the short- and long-term past (Hochreiter and Schmidhuber, 1997). See Kelly and Xiu (2023) for a general reference and Appendix A.1 for our specifications.

Table 1: Prediction accuracy

cumulatively adding predictor sets total number of predictors	tech 8	fund-1 14	fund-2 161	calendar 165	earnings 175
A: R^2 relative to $\widetilde{\eta}$ ($\widetilde{v} - \text{ma}_5$)					
ma_5	0				
ols	12.09	12.26	12.27	14.85	15.99
nn	14.31	14.90	14.42	17.13	18.45
rnn	15.80	16.25	15.47	18.12	19.86
B: R^2 relative to $\widetilde{v}$ (log dollar volume)					
ma_5	93.68				
ols	94.44	94.45	94.45	94.62	94.69
nn	94.58	94.62	94.59	94.76	94.85
rnn	94.68	94.69	94.64	94.86	94.93
C: number of parameters					
ma_5	0				
ols	9	15	162	166	176
nn	961	1,153	5,857	5,985	6,305
rnn	6,049	6,817	25,633	26,145	27,425

Each row represents a prediction model, and each column cumulatively adds to the set of predictors. Panels A and B respectively express the OOS prediction accuracy in two different R^2's. The R^2 relative to $\widetilde{\eta}$ is $1 - \text{MSE}/\text{avg}(\widetilde{v} - \text{ma}_5)^2$; R^2 relative to $\widetilde{v}$ is $1 - \text{MSE}/\text{avg}(\widetilde{v} - \text{avg}(\widetilde{v}))^2$, where $\text{MSE} := \text{avg}(\widetilde{v} - \widehat{v})^2 = \text{avg}(\widetilde{\eta} - \widehat{\eta})^2$ and $\text{avg} := \frac{1}{|\text{OOS}|}\sum_{i,t\in\text{OOS}}$ is the OOS average. Each reported R^2 value is the average across five independent runs initialized with different random seeds to ensure the results' robustness and reproducibility. Panel C reports the number of parameters, for which rnn is about four times of nn due to the four gates in lstm, see exact formulas in Appendix A.1.

nearly 20% of future variation in daily trading volume changes. In comparison, daily stock returns are hardly predictable with a positive OOS R^2, even with state of the art models and predictors, and monthly returns are only slightly predictable (Gu, Kelly, and Xiu, 2020).

Even simple ols models can deliver double-digit R^2, especially when using the largest set of predictors, but the nn and rnn add an additional 3-4% of OOS R^2 predictability.[15]

Adding more predictors improves accuracy in general. All 175 predictors can increase the R^2 by more than three percentage points compared to just the eight technical signals. The exception is that adding the large set of fundamental signals (fund-2) makes the methods (even ML methods)

[15]We experimented with regularization on the linear model (lasso and ridge regressions), and did not find significant improvements.

perform a little worse. This may be due to overfitting when the number of features increases and where we do not use regularization techniques to try to correct for that. The predictors associated with expected returns do not necessarily work for predicting volume. Market-wide calendar events are quite effective in capturing volume changes, however. Scheduled earning announcements also add a sizable gain in prediction accuracy.

The results show that machine learning is critical, and that complexity has its virtue in the context of predicting volume. The prediction accuracy of the rnn is better than the nn, which in turn is better than ols, uniformly across each configuration of included predictors. Panel C shows the improved prediction accuracy is achieved through a significant increase in the number of parameters, a measure of model complexity. Appendix A.1 shows the computational costs of the complex models are higher but manageable.

Panel B reports an alternative R^2 measured as the explained percentage of the total variation of $\widetilde{v}$. Under this metric, the R^2 is always high since the trailing moving average explains $\widetilde{v}$ to a large extent already, and the gain is at most around 1.2 percentage points on top of ma_5, which does not look impressive. This begs the question: which one is the right metric to gauge the economic value of this statistical task? Similarly, why take the log of volume and not predict dollar volume $\widetilde{V}$ or some other transformation of it? In the next section, we will formulate an economic objective as a function of volume by modeling a portfolio problem and show the economic value from the seemingly small gain in R^2 is indeed large and valuable.

Appendix C.1 reports that larger firms have higher prediction accuracy than smaller firms, while the overall patterns of predictability across methods are robust in each firm size sub-sample. The R^2's evaluated in the mega firms are roughly twice those of the nano firms. Smaller firms have a greater magnitude of unexpected trading volume shocks that are hardest to predict. This result makes sense since small firms are volatile and have low trading volume, hence unexpected events that give rise to volume spikes are more likely for these firms. This finding indicates that in addition to small firms being less liquid on average, their liquidity is also less predictable and more volatile. In other words, their trading costs are less predictable. We examine whether firms of different size groups should be modeled differently by implementing a mixture of experts (moe) method, which is shown to be beneficial for the linear model but not for the neural networks.

4 Volume alpha: the economic value of volume forecasting

To quantify the economic value of predicting volume, we set up a portfolio problem that features a tradeoff between tracking a target portfolio versus minimizing trading costs. The key choice is whether to trade aggressively toward the target or passively to avoid trading costs. The optimal balancing point depends on the volume forecast and the trading costs associated with it. We evaluate to what extent more accurate volume forecasts translate to better trading execution.

The incentive to trade is modeled with an objective that penalizes the tracking error toward a target portfolio. The tracking target, potentially informed by the various return forecasting signals, is taken as given since the target itself plays a tangential role in the core tradeoff analysis. Thus, we fix and set aside the return prediction problem to focus on the improvement afforded by the volume prediction problem. Once this problem is solved, we experiment with a range of pre-specified tracking targets to evaluate the economic benefit achieved in different tracking tasks, such as implementing short-term reversal factor portfolios or quantitative strategies.

4.1 Tracking error optimization and its portfolio microfoundation

The tracking error objective is modeled with a simple mean-variance portfolio optimization framework. In this framework, the tracking target is rooted in return forecasts. Regardless of the mean-variance microfoundation, the tracking error objective is also relevant for circumstances such as tracking a benchmark index.

Consider a portfolio manager who chooses dollar portfolio positions $x_{i,t}$ in stock i at the start of day t to maximize a mean-variance certainty equivalence, adjusted for trading costs:

$$A(1 + r_{\mathrm{f},t}) + \sum_i x_{i,t} m_{i,t} - \frac{\gamma}{2A} \sum_{i,j} x_{i,t} x_{j,t} \sigma^2_{ij,t} - \sum_i TradingCost_{i,t}, \tag{1}$$

where A is the manager's assets under management (AUM) or fund size and γ is her risk aversion coefficient.[16] To improve the investment outcome, much empirical work has been devoted to predicting mean excess returns ($m_{i,t}$) and the variance-covariances ($\sigma^2_{ij,t}$). Instead, we assume a simple form for the return moments and take them as given in order to illustrate the economic value solely of the trading cost term. Assuming $\mathbb{V}\mathrm{ar} r_i = \sigma^2$, with zero covariances, the objective function is

$$-\frac{\gamma\sigma^2}{2A}\sum_i\left(x_{i,t}-\frac{A}{\gamma\sigma^2}m_{i,t}\right)^2-\sum_i TradingCost_{i,t}+\left(A(1+r_{\mathrm{f},t})+\frac{A}{2\gamma\sigma^2}\sum_i m_{i,t}^2\right). \quad (2)$$

The task is to balance the tradeoff between the first term, which is the tracking error penalty as the result of the mean-variance optimization, and the second term, the transaction cost, to be detailed below. The third term can be ignored in the optimization since it is irrelevant to the x choices.

The tracking error penalty (first term) is quadratic,

$$TrackingError_{i,t} := \frac{1}{2}\mu(x_{i,t}-x^*_{i,t})^2, \quad (3)$$

where the target x^* is the before-cost mean-variance efficient portfolio position, which increases in the asset's return expectations as well as the total portfolio size A. In implementation, the trading target is formed in a separate process without immediate trading cost consideration. We analyze a general strategy that optimizes the trading rate toward the target based on volume predictions. Parameter $\mu := \frac{\gamma\sigma^2}{A}$ controls the strength of the tracking error penalty. In later empirical analyses, we do not calibrate μ from risk coefficients but instead treat it as a hyperparameter and tune the optimal μ under various AUM levels according to investment performance. Still, the qualitative relationship is preserved – a larger investor penalizes tracking error (measured in dollars) less, meaning they trade less aggressively toward the target in general.

Trading costs are modeled as,

$$TradingCost_{i,t} := \frac{1}{2}\widetilde{\lambda}_{i,t}(x_{i,t}-x^0_{i,t})^2, \quad (4)$$

[16] We consider a simple situation where the AUM is constant due to an immediate payout program, see Eq. 13 for detail. In the second term, the risk aversion coefficient is explicitly adjusted by A such that the before-cost Markowitz optimal dollar position, x^*, scales up with A.

where x^0 is the starting position. We specify $\widetilde{\lambda}$ as a function of volume: $\widetilde{\lambda} = 0.2/\widetilde{V} = 0.2\exp(-\widetilde{v})$, following Frazzini, Israel, and Moskowitz (2018). Underlying the quadratic functional form, it is assumed that the price impact is linear in the trade's size relative to the volume of the day (a.k.a. participation rate): $PriceImpact = 0.1\frac{x-x^0}{\widetilde{V}}$ (Kyle, 1985). For example, buying (or selling) 10% of the daily volume would move the price by 1% (or −1%). And the trading cost is the price impact multiplied by the dollar trade size: $TradingCost = PriceImpact \cdot (x - x^0)$.[17]

The aggregate tracking error optimization problem is,

$$\min_{\{x_{i,t}\}} \sum_{i,t} \left(TrackingError_{i,t} + TradingCost_{i,t}\right). \tag{5}$$

Central to our paper, $\widetilde{v}_{i,t}$, or equivalently $\widetilde{\lambda}_{i,t}$, is not known when choosing $x_{i,t}$ (emphasized by the tilde). One must predict them based on available conditioning information represented by predictors $X_{i,t}$. Taking $x^0_{i,t}$ and $x^*_{i,t}$ as given, the problem becomes $\{i,t\}$-separable.[18] Then, the problem is

$$\min_{x\in\sigma(\mathcal{X})} \mathbb{E}\left[\frac{1}{2}\widetilde{\lambda}(x-x^0)^2 + \frac{1}{2}\mu(x-x^*)^2\right].$$ [19]

4.2 Normalized tracking error and trading rate (z)

To implement this problem empirically, we first normalize the problem by the target trade size $x^* - x^0$ so that we analyze the loss for a one-dollar target trade. The problem scales quadratically: a \$1,000-dollar target trade will incur 10^6 times the loss of a \$1-trade. In detail, let the choice

[17] For simplicity, we assume away cross-impact on $\widetilde{\lambda}$ from related stocks as well as other determinants of $\widetilde{\lambda}$. Also, other functional forms for $\widetilde{\lambda}$, such as the quadratic form often shown empirically, $\widetilde{\lambda} = \frac{0.2}{\sqrt{\widetilde{V}}} = 0.2\exp(-\frac{1}{2}\widetilde{v})$, (Frazzini, Israel, and Moskowitz, 2018) can also be used. In this case, all the analyses carries through but $\widetilde{v}$ will be twice as large. We stick with the linear specification consistent with theory (Kyle, 1985) and empirical evidence on the unexpected component of trading volume (see Frazzini, Israel, and Moskowitz 2018).

[18] Ideally, $x^0_{i,t}$ should not be taken as given, as it is affected by the choice on the previous day, but we are not considering the dynamics of the problem here. By taking $x^0_{i,t}$ and $x^*_{i,t}$ as given, the problem is $\{i,t\}$-separable and easier to solve. In the trading experiments (Section 6), however, we consider the dynamics by evaluating the recursively traded portfolios.

[19] Here "$x \in \sigma(\mathcal{X})$" restricts x as a random variable that depends only on predictive information available at the time of the choice, with $\mathcal{X}_{i,t} := [X_{i,t}, X_{i,t-1}, X_{i,t-2}, \dots]$. This unconditional expectation minimization is equivalent to solving $\min_{x\in\mathbb{R}} \mathbb{E}\left[\frac{1}{2}\widetilde{\lambda}(x-x^0)^2 + \frac{1}{2}\mu(x-x^*)^2 \middle| \mathcal{X}\right]$ for each $\mathcal{X}$ realization.

variable be trading rate $z := \frac{x-x^0}{x^*-x^0}$, then the minimization objective becomes

$$\frac{1}{2}\widetilde{\lambda}(x-x^0)^2 + \frac{1}{2}\mu(x^*-x)^2 = \frac{1}{2}\big(x^*-x^0\big)^2\big(\widetilde{\lambda}z^2 + \mu(1-z)^2\big). \tag{6}$$

Since the factor, $\big(x^*-x^0\big)^2$, does not matter for the choice of z, define the economic loss as

$$loss^{\text{econ}}(\widetilde{v}, z; \mu) := \widetilde{\lambda}z^2 + \mu(1-z)^2, \tag{7}$$

and the normalized problem as

$$\min_{z\in\sigma(\mathcal{X})} \mathbb{E}\left[loss^{\text{econ}}(\widetilde{v}, z; \mu)\right], \tag{8}$$

which is the main focus of economic machine learning. Being able to separate x^*, x^0 affords many conveniences. It means the core problem is independent of the target strategy or fund size. It allows us to look at each i, t observation independently in a volume prediction setting. After the prediction task is done, we evaluate the investment performance under various pre-specified target strategies (x^*) with different AUM levels.

4.3 The optimal policy ignoring forecast error (function s)

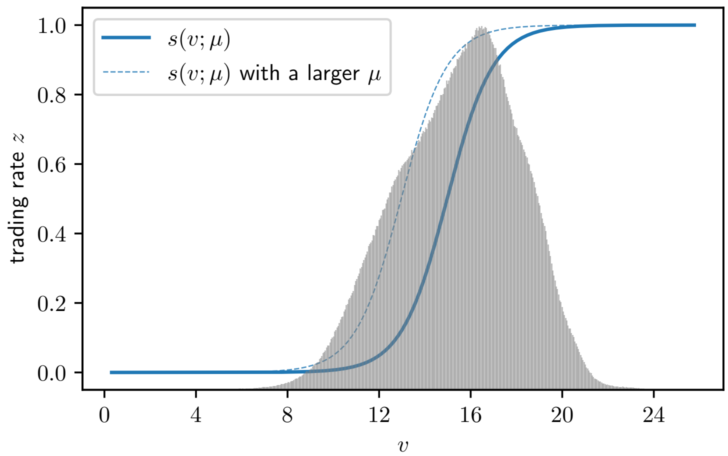
Suppose $\widetilde{v} = v$ and we ignore the inaccuracy in the prediction, the optimal policy is then,

$$s(v;\mu) := \arg\min_z loss^{\text{econ}}(v, z; \mu) = \frac{\mu}{\mu+\lambda} = \frac{1}{1+\exp\left(-v+\log 0.2 - \log\mu\right)}. \tag{9}$$

We plot this function in Figure 2. It is a sigmoid function with a horizontal offset determined by μ. The optimal trading rate z ranges from 0 to 1 as v increases. Then, the optimal dollar position choice is $x = x^0 + s(v;\mu)(x^* - x^0)$. This is the finding of Gârleanu and Pedersen (2013) that the optimal strategy should "trade partially toward the aim."[20] In their setting, the trading rate is a

[20]The other finding in Gârleanu and Pedersen (2013) is that the optimal strategy should also "aim in front of the target." This dynamic effect is abstracted away in our problem since we are considering the $\{i, t\}$-separable optimization. One interpretation is that x^* is already the aim that is in front of the target that implicitly embeds the dynamic effect.

Figure 2: The policy function (s) that maps log volume (v) to trading rate (z)



The solid curve uses a μ value relevant for \$1b AUM. The dashed curve changes to a greater μ (AUM = \$100m), increasing z across the spectrum. The background is the histogram of $\widetilde{v}$ data repeated from Figure 1, to show that a typical $\widetilde{v}$ corresponds to a $\widetilde{z}$ somewhere in the middle between 0 and 1 given the chosen μ's.

fixed constant. Here, it is still irrelevant to either the trading target or the starting point (x^*, x^0), but importantly, it depends on the volume prediction, v. Instead of assuming liquidity as a constant known by the agent, the innovation and emphasis of this paper is that volume prediction alters the tradeoff between the cost and benefit of trading, and hence more accurate predictions lead to more efficient portfolio implementation with a z that varies with forecasted volume. Additionally, the optimal z also does not explicitly depend on the scale of the fund. If the AUM doubles, and both x^* and x^0 double, the optimal z remains the same, while the dollar position choice x doubles. However, a smaller fund will find a larger μ more relevant for their investment performance optimization (μ-tuning detailed further below). In that case, they will trade more aggressively uniformly across v, as illustrated by the upward displacement of the dashed curve in Figure 2.

5 Machine learning for the economic value of volume prediction

We provide empirical methods to construct the policy of choosing z given $\mathcal{X}$.

5.1 The statistical and economic tasks of volume prediction

We consider two ways of approaching the portfolio optimization problem. The first conducts a statistical prediction of volume and then plugs the volume forecasts into the optimal trading policy $s(v;\mu)$ to form a trading plan. This indirect approach we call "statistical" learning. The second is an economic learning approach, which instead learns trading rate z as a function of conditioning information directly to minimize the economic loss. We argue directly choosing z is also "predicting volume," but with a different optimization objective rather than the least squares loss commonly used in statistical predictions. We term this approach "economic" learning. We show their theoretical differences: the economic loss penalizes inaccuracies in volume forecasting asymmetrically, where overestimating volume is more costly. Therefore the model optimized for the economic goal should be more conservative, at the expense of compromising on accuracy measured in terms of squared errors.

- Approach 1, statistical learning:

  Step 1: run machine learning regressions of $\widetilde{v}$ onto $\mathcal{X}$ in the training sample as in Section 3

$$v^*(\cdot) = \arg\min_{v(\cdot)} \sum_{i,t\in\text{train}} (\widetilde{v}_{i,t} - v(\mathcal{X}_{i,t}))^2. \tag{10}$$

  Step 2: plug the OOS predictions $\widehat{v}_{i,t} := v^*(\mathcal{X}_{i,t})$ into policy equation (9) to trade $\widehat{z}_{i,t} = s(\widehat{v}_{i,t};\mu)$.

- Approach 2, economic learning:

  parameterize z as a neural network, optimize the economic objective in the training sample

$$z^*(\cdot) = \arg\min_{z(\cdot)} \sum_{i,t\in\text{train}} loss^{\text{econ}}(\widetilde{v}_{i,t}, z(\mathcal{X}_{i,t});\mu) \tag{11}$$

  and trade $\widehat{z}_{i,t} = z^*(\mathcal{X}_{i,t})$ in the testing sample.[21]

The difference between the two approaches boils down to the different loss functions deployed in penalizing volume prediction errors. For example, a trading action $\widehat{z} = z(\mathcal{X})$ implies an underlying

[21] For both approaches, the machine learning framework can be rnn or nn, in which $\mathcal{X}_{i,t}$ includes concurrent and lagged predictors (X) or only the concurrent.

volume forecast $\widehat{v} = s^{-1}(\widehat{z}; \mu)$, and equivalently the economic loss can be written as a function of the z-implied v instead of z itself: $loss^{\text{econ}}_{\text{vv}}(\widetilde{v}, v; \mu) := loss^{\text{econ}}(\widetilde{v}, s(v; \mu); \mu)$. Hence, the economic learning approach is equivalent to first solving

$$\min_{v(\cdot)} \sum_{i,t \in \text{train}} loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_{i,t}, v(\mathcal{X}_{i,t}); \mu) \tag{12}$$

followed by the $s(\,\cdot\,; \mu)$ transformation, which is also required in the first approach.

Given that the two approaches are only different in the loss functions, we compare $loss^{\text{econ}}_{\text{vv}}$ with the least squares loss function, $loss^{\text{ls}}(\widetilde{v}, v) := \frac{1}{2}(v - \widetilde{v})^2$, used in statistical predictions. To understand how the two approaches behave differently, first note that the two functions are the same for the smallest possible loss being attained, which is when the forecast, v exactly equals the target, $\widetilde{v}$. In empirical experiments, we label the strategy made with perfect foresight "oracle" as the unattainable ideal ($\widehat{v}_{i,t} = \widetilde{v}_{i,t}$), which yields the smallest mean squared error (MSE) and smallest mean economic loss (MEL).

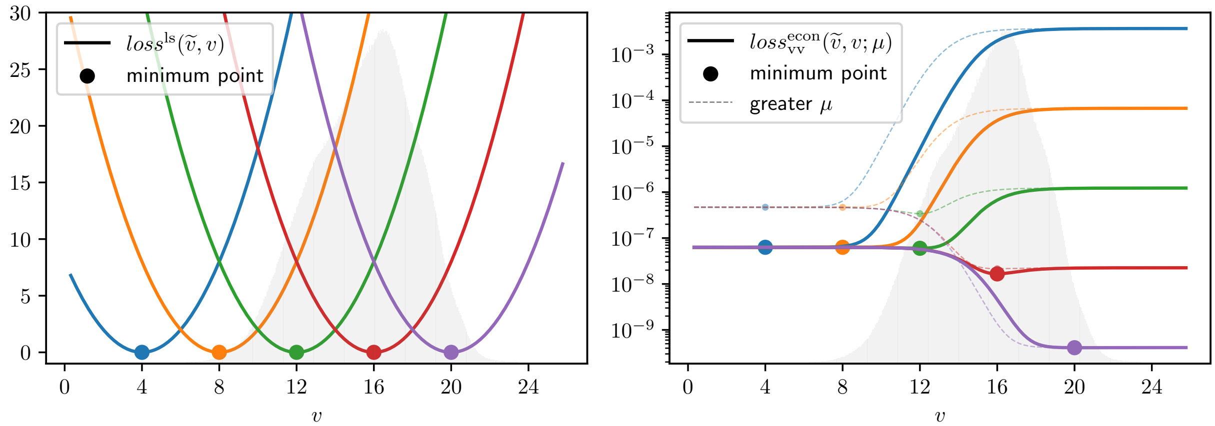
Second, both loss functions monotonically increase as the forecast v deviates away from the true $\widetilde{v}$. Therefore, it is intuitive to think that making forecasts that are close to $\widetilde{v}$ in the least squares sense, will translate to better portfolio implementation as evaluated by the economic loss.[22] However, forecast errors will not guarantee this outcome because of the differences between the two loss functions. From a theoretical perspective, it is well known that the conditional expectation, $\mathbb{E}[\widetilde{v}|\mathcal{X}]$, is the minimizer of the problem $\min_{v \in \sigma(\mathcal{X})} \mathbb{E}\left[loss^{\text{ls}}(\widetilde{v}, v)\right]$, so that the statistical learning method recovers the conditional expectation with the neural network tools. However given a different economic loss function,

Proposition 1. *The least squares minimizer,* $\mathbb{E}[\widetilde{v}|\mathcal{X}] = \arg\min_{v \in \sigma(\mathcal{X})} \mathbb{E}\left[loss^{\text{ls}}(\widetilde{v}, v)\right]$, *does not optimize the economic loss minimization problem* $\min_{v \in \sigma(\mathcal{X})} \mathbb{E}\left[loss^{\text{econ}}_{\text{vv}}(\widetilde{v}, v; \mu)\right]$.

Even with unlimited data, the "perfect" statistical learning would not optimize the economic prob-

[22]The above mentioned properties expressed mathematically are $\arg\min_v loss(v, \widetilde{v}) = \widetilde{v}, \forall \widetilde{v}$; and $loss(v, \widetilde{v})$ is increasing in $|v - \widetilde{v}|, \forall v, \widetilde{v}$. Both $loss^{\text{econ}}_{\text{vv}}$ and $loss^{\text{ls}}$ satisfy these properties. In Figure 3, the dots mark the minimums. In the right panel, the dips around the minimums are too shallow to be noticeable, though analytically they are indeed the minimums. See Appendix B.4 for more details on the local curvature around the minimum points.

Figure 3: The statistical (least squares) and economic loss functions



The two panels visualize $loss^{ls}(\widetilde{v}, v)$ and $loss^{econ}_{vv}(\widetilde{v}, v; \mu)$, respectively. We pick five different true values $\widetilde{v} = 4, 8, 12, 16, 20$ (in five colors), and respectively plot the loss curves for $v \in [0, 24]$. The dots mark the minimums of the loss curves, attained at $v = \widetilde{v}$. The right panel is in the log scale. The solid and dash curves use the μ values in columns 2 and 3 in Table 2 (corresponding to \$1b and \$100m AUM), respectively.

lem. The theoretical foundation of this claim is that $\mathbb{E}[\widetilde{v}|\mathcal{X}] = \arg\min_{v\in\sigma(\mathcal{X})} \mathbb{E}[\phi(\widetilde{v}, v)]$ <u>if and only if</u> the generic loss function ϕ is in the Bregman class (Banerjee, Guo, and Wang, 2005; Patton, 2020). As is well known, the least squares loss belongs to the Bregman class, the economic loss function does not (proofs in Appendix B.3).

More intuitively, we plot and compare the two loss functions in Figure 3. We pick five different true values for $\widetilde{v} = 4, 8, 12, 16, 20$, and respectively plot the loss curves for a range of forecasts $v \in [0, 24]$. The distinguishing feature of the economic loss is its asymmetric penalty for overestimating volume when actual volume is low. For example, the blue loss curve (actual $\widetilde{v} = 4$) is very high if the forecast is mistakenly large ($v > 12$). The economic intuition for why this particular forecasting error is so costly is that it implies trading aggressively (z close to 1) when actual liquidity is low. Analytically, $\lim_{z\uparrow 1} loss^{econ}(\widetilde{v}, z; \mu) = \widetilde{\lambda}$, which increases exponentially if $\widetilde{v}$ is small.

In contrast, errors in the other direction are much more forgiving (e.g., the purple curve with $\widetilde{v} = 20$).[23] Trading too little when actual liquidity turns out to be ample delivers a loss

[23]Notice the vertical axis is in log scale, meaning the purple is much more flat compared to blue in terms of the difference of its right and left ends. The same plot in linear scale is in Appendix Figure B.2.

equal to the opportunity cost of not better tracking the target portfolio, which is fixed at μ ($\lim_{z\downarrow 0} loss^{\text{econ}}(\widetilde{v}, z; \mu) = \mu, \forall \widetilde{v}$). In summary, for low levels of true volume, overestimating volume is very costly, but for high levels of volume, the economic cost of volume forecast error is relatively small.

We state the asymmetric property formally in Proposition 4 in Appendix B.4, and provide further analysis. The analytical results rely on the quadratic functional forms assumed in equations (3) and (4). However, the qualitative points carry over to more general settings.

These findings have important implications. Off-the-shelf machine learning tools are not the most suitable for specific portfolio problems because they minimize statistical error rather than economic error. Hence, an altered financial machine learning design that accounts for economic loss can be more effective. The ranking of forecasts evaluated by the two loss functions can be reversed – a set of forecasts with a smaller sample MSE might have a greater economic loss – something we will see empirically in Table 2. The asymmetry in the economic loss has important implications for implementing a model optimized for the statistical criteria to the trading task. The model should "learn" to be more careful about the potential of a liquidity dry-up and be conservative in avoiding overestimating volume. When necessary, it should compromise least squares accuracy in favor of minimizing economic losses.

Both the statistical and economic learning approaches are implemented with neural networks, given the many benefits of deep learning such as the ability to handle high-dimensional data and non-linear relations. Next, we emphasize one aspect of the implementation that is particularly relevant for transferring statistical learning results to finance applications such as portfolio optimization.

5.2 Transfer learning via pre-training and fine-tuning

We implement a transfer learning paradigm in which the statistical and economic learning models are trained sequentially, as illustrated in Figure 4 right panel.

The statistical volume prediction is an upstream task. It learns valuable but generic information on the predictive relationship and serves as the foundation model (center node in Figure 4 right panel). It enjoys off-the-shelf machine learning programs optimized for this typical task. The background knowledge can be transferred to more specific downstream tasks such as portfolio optimization in our case. The finance-motivated tasks benefit from a good "foundation". Fine-tuning the pre-trained foundation model per the economic loss objective further improves the economic performance. The different specific downstream tasks require separate economic training routines. (The tasks are different because the economic loss function is modulated by μ.) The same pre-trained foundation model serves as the common starting point for different downstream fine-tuning routines.[24]

Pre-training optimizes a nn or rnn from X to $\widehat{\eta}$ (the blue part in Figure 4 left panel) as described in the previous section. Using the pre-trained network as is, flowing the output $\widehat{\eta}$ through additional transformations, "+ma_5" and "sigmoid $s(\,\cdot\,;\mu)$" shown in black, the resulting $\widehat{z}$ implements the plug-in step. Taking this foundation model as the initialization, fine-tuning conducts further stochastic gradient descent in the economic loss function evaluated at the same training sample. The resulting fine-tuned network implements the economic learning approach. Only the trainable part (in blue) is updated in fine-tuning. The economic approach only innovates on the loss function, not the network architecture or data. Neural networks tackle the non-linearity not only in the predictive relationship but also in the (marginal) economic loss function. Experiments show fine-tuning requires only a small number of epochs of training to significantly improve OOS economic performance compared to the pre-trained foundation model.[25]

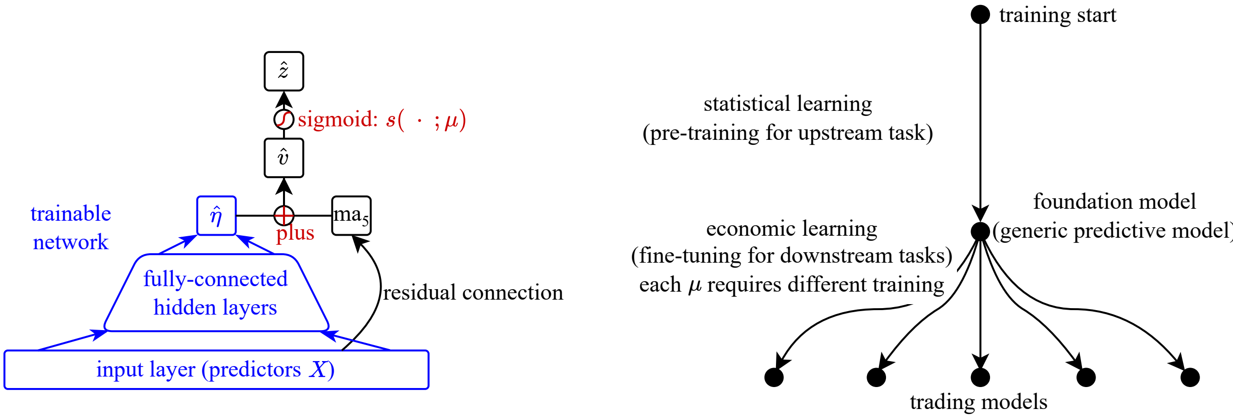
Many other finance problems share a similar structure, in which statistical results are transferred into actionable strategies that are applied towards an economically motivated problem. Markowitz portfolio optimization is a classic example. Another example is financial risk management, which relies on volatility forecasts. The transfer learning procedure adopted here provides a unified framework for applying machine learning techniques in these scenarios.[26] The procedure is also

[24] An even lower-level downstream task is to make decisions given target position x^*. We do not directly train for that, but do evaluate the performance in such trading experiments further below.

[25] Alternatively, side-stepping pre-training but directly training for the economic loss from scratch is generally less robust and takes more time (epochs over the sample) to train, probably because the machine learning program is not optimized for such a loss. Not to mention that each μ would require a separate training routine as they do not share the common pre-trained baseline.

[26] Some financial machine learning studies, including Jensen et al. (2022), Chen et al. (2023), Cong et al. (2021) and Chen, Pelger, and Zhu (2023) involve directly training for the economic target.

Figure 4: Network architecture and transfer learning procedure



The left illustration shows the network architecture. The blue part "trainable network" is a standard feed-forward neural network, specified with three fully connected hidden layers with 32, 16, and 8 neurons, respectively. The black parts are non-trainable transformations from $\widehat{\eta}$ to log volume prediction $\widehat{v}$ and ultimately trading rate $\widehat{z}$. The recurrent connections in rnn are omitted in this illustration. The right figure illustrates the training procedure for transfer learning. Each dot represents a trained model, i.e., a parameterization of the network. The arrows show economic learning as specific fine-tuning steps based on the common foundation model pre-trained statistically.

similar to how GPT models are "P"re-trained on language representations and applications like ChatGPT are fine-tuned for generating conversational responses or other tasks.

5.3 Economic and statistical prediction results

We present a systematic comparison of how different predictor sets and models perform in terms of both statistical and economic performance. The results show the economic performance is indeed optimized by fine-tuning the training process on the same objective, and that more predictors and the network model lead to improved performance.

Table 2 Panel A evaluates the OOS mean economic loss (MEL), and Panel B the mean squared error (MSE). For ease of comparison, Panels A′ and B′ normalize these metrics as a percentage loss reduction such that the "oracle" (which represents the perfect prediction $\widehat{v} = \widetilde{v}$) is at 100% accuracy and the baseline ma_5 at 0%. (The percentage reduction in MSE is then the R^2 predicting $\widetilde{\eta}$ as in Table 1 Panel A.) The four columns correspond to different μ values that modulate the economic loss function, representing four different downstream economic tasks relevant to different

AUM magnitudes. As the AUM decreases, the relevant economic objective is parameterized by a greater μ, such that overall trading becomes more aggressive. The average trading rate ranges from trading conservatively near the starting position (13% for AUM = \$10b) to trading almost all the way to the target position (95% for AUM = \$10m).

The rows include the statistical prediction methods described in Section 3 (ma_5, ols, nn, and rnn). Additionally, the economic learning approaches (those ending with ".econ") conduct fine-tuning on top of the statistical forecasts to optimize the economic loss function under the four different μ's. The lines spanning the columns indicate that the statistical forecasts do not vary with μ, whereas economic learning generates different (z-implied) volume forecasts as μ varies. The predictor sets include the 8 "tech" signals or "all" of the 175 signals.

Looking first at the statistical performances of the forecasts, model complexity and feature richness help reduce MSE. Configuration rnn_{all} yields the highest $R^2 \approx 20\%$. However, smaller MSEs do not necessarily lead to better economic performance. For example, nn_{tech} accrues greater MELs than ols_{tech}, albeit nn is more accurate in terms of R^2 (comparing Panels A and B). For the low AUM task in particular, the various volume forecasts are even worse than the baseline ma_5, likely due to the need to trade aggressively in this task and the disproportional penalty when aggressive trades are the result of overestimating volume. This result is intuitive given the theoretical analysis, and argues why economic fine-tuning produces better OOS portfolio performance.

The economic training methods lead to better economic outcomes. The fine-tuned networks with all the predictors yield the best performance, reaching an OOS economic performance that is about 43%∼70% of the unattainable oracle benchmark at various AUM scales. These improved OOS outcomes are not mechanically guaranteed since the fine-tuning is to minimize in-sample loss. The empirical results demonstrate the validity of the economic learning design. Looking at Panel B′, the statistical accuracy retreats after fine-tuning, often to levels even worse than the ma_5 baseline resulting in negative R^2. This means to achieve improvements in economic outcomes, the models compromise MSE. Optimal trading actions are conservative against volume overestimation given the economic loss function.

Furthermore, for a smaller AUM or, equivalently, a higher μ value, the economic models tend

Table 2: Economic and statistical performance of different methods

	1	2	3	4	1	2	3	4
μ	1.2e-9	6.3e-8	4.7e-7	9.4e-6	1.2e-9	6.3e-8	4.7e-7	9.4e-6
avg $\widetilde{z}$	0.13	0.57	0.78	0.95	0.13	0.57	0.78	0.95
relevant AUM	\$10b	\$1b	\$100m	\$10m	\$10b	\$1b	\$100m	\$10m
A. Mean economic loss (MEL) ($\times 10^{-8}$)					A′. % reduction in mean economic loss			
ma_5	0.1046	3.163	15.41	93.0	0.0	0.0	0.0	0.0
ols_{tech}	0.1041	3.011	14.81	93.2	27.9	29.6	11.1	-0.3
nn_{tech}	0.1043	3.100	14.97	99.7	19.2	12.3	8.0	-12.8
rnn_{tech}	0.1040	2.955	14.37	102.2	33.6	40.8	19.2	-17.7
nn.econ_{tech}	0.1041	2.991	12.35	66.8	31.3	33.6	56.5	50.2
rnn.econ_{tech}	0.1039	2.855	11.78	64.2	39.7	60.4	67.0	55.3
ols_{all}	0.1040	3.024	14.97	94.7	32.4	27.2	8.1	-3.3
nn_{all}	0.1040	3.019	15.07	106.5	33.3	28.2	6.2	-25.9
rnn_{all}	0.1040	3.012	14.78	109.8	34.8	29.5	11.7	-32.1
nn.econ_{all}	0.1039	2.810	11.56	61.9	39.6	69.2	70.9	59.6
rnn.econ_{all}	0.1038	2.812	11.60	66.4	43.7	68.8	70.3	51.0
oracle	0.1029	2.653	9.99	40.8	100	100	100	100
B. Mean squared error (MSE)					B′. R^2 (% reduction in MSE)			
ma_5	——— 0.437 ———				——— 0.0 ———			
ols_{tech}	——— 0.385 ———				——— 12.1 ———			
nn_{tech}	——— 0.375 ———				——— 14.3 ———			
rnn_{tech}	——— 0.368 ———				——— 15.8 ———			
nn.econ_{tech}	0.389	0.449	0.457	0.630	11.2	-2.6	-4.5	-44.1
rnn.econ_{tech}	0.392	0.492	0.487	0.481	10.3	-12.4	-11.3	-10.0
ols_{all}	——— 0.367 ———				——— 16.0 ———			
nn_{all}	——— 0.357 ———				——— 18.4 ———			
rnn_{all}	——— 0.350 ———				——— 19.9 ———			
nn.econ_{all}	0.394	0.555	0.590	1.979	10.0	-26.8	-34.9	-352.5
rnn.econ_{all}	0.377	0.440	0.477	0.785	13.9	-0.6	-9.0	-79.5
oracle	——— 0.00 ———				——— 100 ———			

Panel A: Mean economic loss (MEL) := avg $loss^{\text{econ}}(\widetilde{v}, \widehat{z}; \mu)$; A′: % reduction in mean economic loss := $(\text{MEL}_{\text{ma5}} - \text{MEL}_m)/(\text{MEL}_{\text{ma5}} - \text{MEL}_{\text{oracle}})$; B: MSE := avg $(\widetilde{v} - \widehat{v}_m)^2$; B′: $R^2 := 1 -$ avg $(\widetilde{v} - \widehat{v}_m)^2/$avg $(\widetilde{v} - \widehat{v}_{\text{ma5}})^2 = (\text{MSE}_{\text{ma5}} - \text{MSE}_m)/\text{MSE}_{\text{ma5}}$, for each method m and μ ("avg" is the OOS average over i, t). $\widehat{z}$ varies over each method and μ. For statistical methods, $\widehat{v}$ does not depend on μ, hence the horizontal lines indicate the MSE and R^2 do not depend on μ for these methods. These R^2 numbers repeat those from Table 1 row "$\widetilde{\eta}$" by construction. In the header, the two additional rows help interpret the μ values: avg $\widetilde{z} =$ avg $s(\widetilde{v}; \mu)$ is the average trading rate given true volume; Under each "relevant AUM", the corresponding μ is backed out in portfolio optimization hyperparameter tuning (see Footnote 28 for details on tuning μ).

introduce more statistical bias, as indicated by the increasingly negative R^2 values. This can be explained by the changes in the economic loss functions. With a smaller μ, trading is more intensive in general (s curve in Figure 2 shift to the left). That means the penalty for overestimating low volume is more stringent and takes effect earlier (dashed curves in Figure 3 shift to the left). Thus, for smaller AUM, there is a greater difference between the least squares loss and the economic loss.

6 Investment performance in trading experiments

We now apply the analysis to real-world investment portfolios in a set of trading experiments.

6.1 Trading experiment design

The trading experiment forms a set of trades $x_{i,t}$ by applying the various trading strategies detailed in the previous section to dynamically track a set of given target positions $x^*_{i,t}$. The target positions are not optimized on trading cost considerations, but formed in a separate process focused solely on return prediction. We evaluate the outcome of the implemented trades $x_{i,t}$, including the mean return, Sharpe ratio, and turnover, in the OOS period. We examine whether a volume prediction method brings improved investment performance net of trading costs and tracking error considerations.

Various sets of target positions $\{x^*_{i,t}\}$ are exogenously supplied to mimic realistic trading tasks. The first set of experiments mimic a quantitative strategy. We simulate an extremely profitable before-cost trading strategy assuming the agent can, with some probability, forecast the realized direction of stock price change. We experiment with different AUM levels, in which the dollar positions scale linearly while the trading costs increase quadratically. As a result, the optimal μ, which controls the overall trading aggressiveness, varies. The second set of experiments tracks monthly-rebalanced factor portfolios sorted on firm characteristics from the literature as the trading targets. These experiments reveal the effectiveness of volume prediction across the spectrum of investment styles.

Given the target $\{x^*_{i,t}\}$, the implemented trading outcome $\{x_{i,t}\}$ is constructed following the trading rate strategy, such as $\widehat{z}_{i,t} = \text{rnn.econ}(\mathcal{X}_{i,t};\mu)$, which is formed in the training sample. In particular, $x_{i,t} = x^0_{i,t} + \widehat{z}_{i,t}(x^*_{i,t} - x^0_{i,t})$, where the starting position is formed recursively as $x^0_{i,t} := x_{i,t-1}R^{\text{raw}}_{i,t}$ and $R^{\text{raw}}_{i,t} = 1 + r_{\text{f},t-1} + r_{i,t}$ is the arithmetic raw return accrued on day $t-1$.[27] Note the dynamic effect here: the portfolio choice matters for the starting position on the next day, although the optimization does not explicitly consider it. Additionally, the target amount to trade $x^* - x^0$, varies across i, t, which is another aspect abstracted from in the theoretical analysis.

Given the resulting trades $\{x_{i,t}\}$, the accounting of the investment outcome is standard. The daily dollar payoff is such that the AUM is fixed over time:

$$\text{payoff}_t = A(1 + r_{\text{f},t}) + \sum_i x_{i,t}\widetilde{r}_{i,t} - \sum_i TradingCost_{i,t} - A. \tag{13}$$

We normalize by A to get the net-of-cost excess return of the implementation:

$$\widetilde{r}_{\text{implemented},t} := \frac{\text{payoff}_t}{A} - r_{\text{f},t} = \sum_i w_{i,t}\widetilde{r}_{i,t} - A\sum_i \frac{\widetilde{\lambda}_{i,t}}{2}\left(w_{i,t} - w^0_{i,t}\right)^2, \tag{14}$$

where $w_{i,t} := \frac{x_{i,t}}{A}$ and $w^0_{i,t} := \frac{x^0_{i,t}}{A}$ are portfolio weights as a ratio of AUM. We have the familiar result that the before-cost return (the first term) is scale-invariant while the percentage trading cost due to price impacts (the second term) scales with the AUM linearly. We report the mean and Sharpe ratio of $\widetilde{r}_{\text{implemented},t}$. Additionally, we also evaluate the annualized turnover as

$$\text{Turnover} := \frac{1}{T}\sum_{i,t}\frac{\left|x_{i,t} - x^0_{i,t}\right|}{2A} \times 252 = \frac{1}{2T}\sum_{i,t}\left|w_{i,t} - w^0_{i,t}\right| \times 252. \tag{15}$$

The second equation indicates turnover is scale invariant.

6.2 Implementing a simulated quantitative strategy

We consider a set of trading targets $\{x^*_{i,t}\}$ that simulate a quantitative investment strategy. We simulate a trading signal that, with 1% chance, perfectly forecasts whether a stock goes up or down over the next five days. The signal is independent across i, t. Following the signal, all stocks are

[27] To initiate the recursive calculation, let $x^0_{i,t} = 0$ on the first day a stock appears in the sample.

allocated to either the long or the short group. When no signal is received, the stock position stays the same. We let, $x^*_{i,t}$ be the equal-weighted long-short strategy in which each leg sums up to 50% of the AUM.

This portfolio has an unrealistically high before-cost OOS Sharpe ratio of around 7, which is brought down substantially (to more reasonable levels) after trading costs. We experiment with a sequence of AUM magnitudes up to $10 billion which command varying levels of trading costs. We first evaluate across a grid of μ values that vary the aggressiveness of trades, and then perform the μ-tuning method that allows for the comparison of the highest investment performance attained by each volume prediction method.

We run a trading experiment for every method and each μ value for various AUM levels. Each experiment plots one dot in Figure 5, with OOS turnover versus mean return (or Sharpe ratio) as the coordinates. Each curve corresponds to one method, connecting the dots with varying μ values.

To understand the general shape of the curves, first, note that under different μ values, the strategies vary from always passively holding ($z = 0$) to trading all the way to the target ($z = 1$). As the implemented portfolio becomes more aggressive (μ increase), the turnover increases. On the vertical axis, the mean return (or Sharpe ratio) first increases as a result of active trading for profit and then bends downward due to trading costs, whose effect shows up with high turnover. The two opposing forces result in the inverted U-shaped curves. For higher AUM, the trading costs' effect is stronger, resulting in lower inverted U's that peak at smaller turnover levels (i.e., lower μ values). In the next subsection, We implement the highest attainable investment outcome by selecting the μ value from the peak of the in-sample curves.

The investment gain of a better trading rate strategy $z(\mathcal{X})$ is shown in the vertical displacement of the curves. The improvement comes from two aspects. First, more closely tracking the target portfolio delivers a higher return. Second, reducing trading costs when liquidity is expected to be low, for a given amount of turnover, lowers the trading cost drag on the portfolio. These two objectives are contradictory in our framework, hence the value of predicting volume more accurately is to strike a better balance in the tradeoff between trading costs and tracking error. At the two extremes of the curves ($\mu = 0$ or $+\infty$), there is no room for tradeoff, and hence predicting volume

Figure 5: Trading experiment performance, with only tech predictors

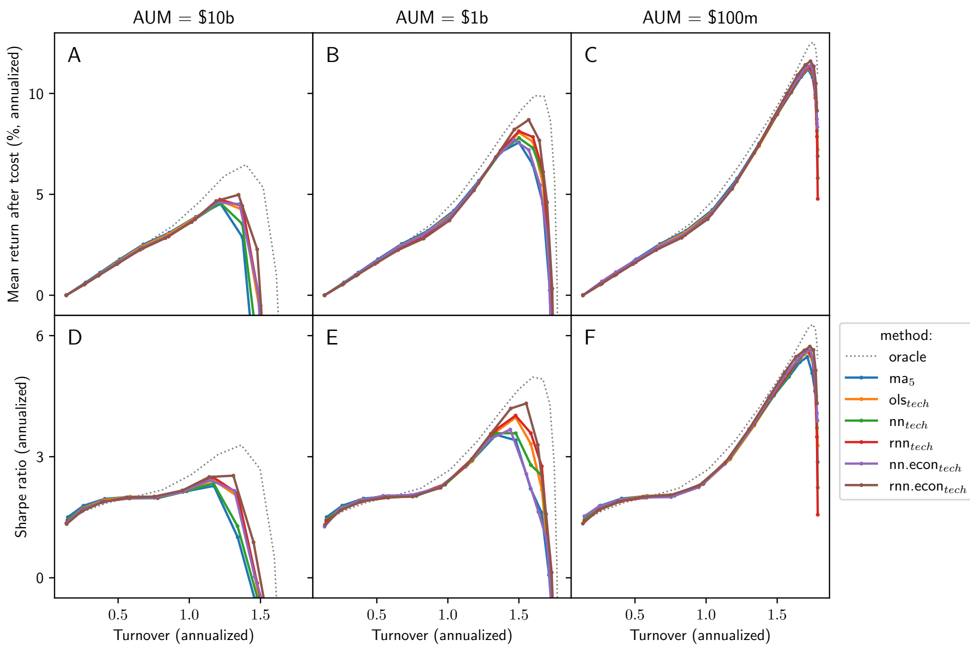
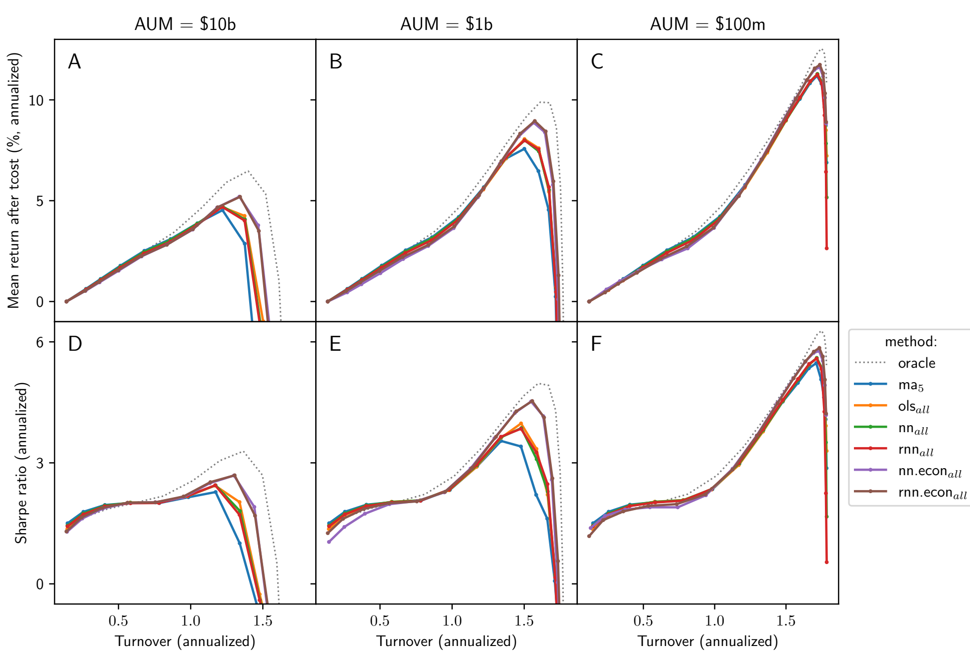


Figure 6: Trading experiment performance, with all predictors



Each dot plots the outcome of one trading experiment: the horizontal coordinate is turnover, vertical is the mean return or Sharpe ratio. Each curve is for a method with varying μ values. Figures 5 and 6 differ in using only the tech or all sets of predictors, while the benchmarks curves "ma_5" and "oracle" are the same.

Table 3: Investment performance in trading experiments

A. Mean return (%, annualized)					B. Sharpe ratio (annualized)			
AUM	\$10b	\$1b	\$100m	\$10m	\$10b	\$1b	\$100m	\$10m
ma_5	3.88	6.47	11.19	13.20	2.00	2.21	5.47	6.55
ols_{tech}	3.82	7.60	11.28	13.14	2.16	3.32	5.59	6.59
nn_{tech}	3.76	7.30	11.32	13.13	2.14	2.79	5.63	6.59
rnn_{tech}	3.74	7.84	11.33	13.13	2.18	3.58	5.64	6.59
nn.econ_{tech}	4.60	7.20	11.29	13.22	2.13	2.57	5.59	6.63
rnn.econ_{tech}	4.67	8.69	11.59	13.30	2.50	4.32	5.73	6.66
ols_{all}	3.82	7.60	11.28	13.14	2.17	3.35	5.60	6.59
nn_{all}	3.86	7.44	11.28	13.13	2.19	3.09	5.60	6.58
rnn_{all}	3.79	7.55	11.25	13.09	2.18	3.26	5.59	6.56
nn.econ_{all}	4.64	8.87	11.61	13.29	2.18	4.24	5.74	6.66
rnn.econ_{all}	4.68	8.95	11.77	13.30	2.50	4.53	5.85	6.68
oracle	6.47	9.89	12.54	13.56	3.05	4.97	6.28	6.80

For each combination of AUM and method, we report the OOS mean return (Panel A) and Sharpe ratio (Panel B) at the tuned μ, which is selected to maximize the in-sample mean return (or Sharpe ratio) over a grid of μ values.[28]

has no value, in which case all curves converge.

Comparing the methods, we can analyze the sources of improvement. First, using the full set of predictors is better than using only the set of technical predictors, which is still much better than no additional information besides the five-day moving average. Second, holding the information set constant, the neural network model predicts volume better than linear regression. Lastly, given the pre-trained neural network, further fine-tuning the trading strategy by directly optimizing the investment performance provides further improvement.

Finally, we tune the hyperparameter μ to report each method's highest attainable investment performance. We choose the μ value that maximizes the in-sample expected return (or Sharpe ratio) and report the out-of-sample performance at the tuned μ. Effectively, μ is selected from the peaks of the in-sample version of the curves (not plotted) and then applied to the OOS curves (as plotted). We expect the in-sample and OOS curves to peak at relatively close μ ranges so that the method attains an OOS performance close to the peak of the OOS curves.

[28] The "relevant AUM" row in Table 2 is calculated according to the μ-tuning result under each AUM level under method ma_5.

The results are reported in Table 3. Applying better prediction methods and using a larger set of predictors improves investment performance uniformly across AUM levels. The economic magnitude of improvement is significant, comparable to if not more than the marginal improvement from innovating on return prediction signals. For \$10b of AUM, the average annual return increases from 3.88% when using the baseline prediction method of only a 5-day moving average volume, to 4.68% when making volume predictions with an economic objective optimization imbedded within a rnn. For \$1b AUM, the magnitude of improvement is even greater, going from 6.47% to 8.98%, and more than doubling the Sharpe ratio from 2.21 to 4.53.

For smaller AUM (\$10m), the improvement is still noticeable but smaller, because price impact shrinks and all methods therefore prescribe trading very aggressively. The investment performance converges to the high before-cost level regardless of the prediction method. For more realistic considerations, future research could consider per-unit trading costs such as bid-ask spread in addition to price impacts, which tend to show up as dollar trade sizes shrink.

6.3 Implementing factor zoo portfolios

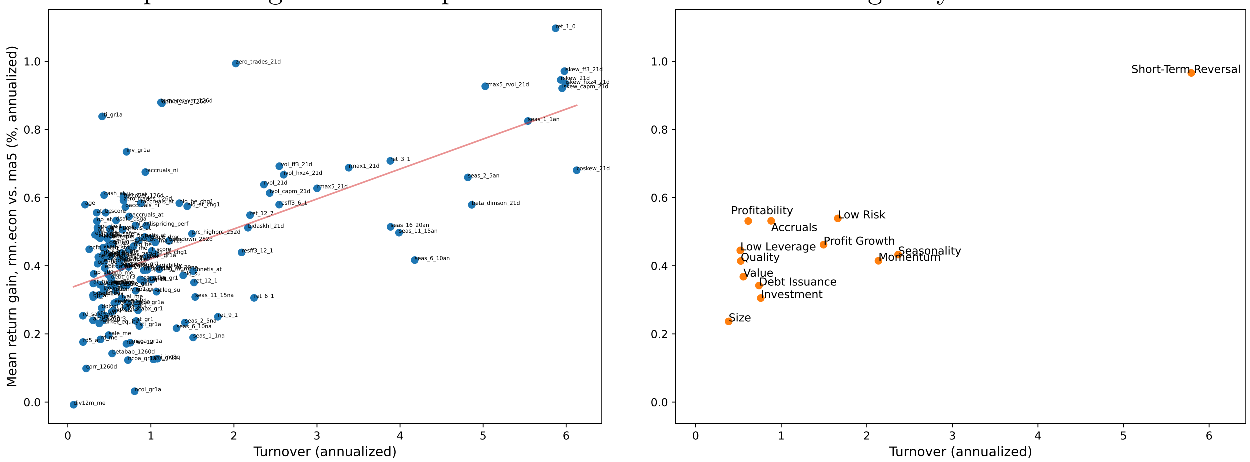
As another set of trading experiments, we use as trading targets the portfolios sorted on characteristics in the JKP dataset that come from the asset pricing literature. The goal is to examine the improvement in implementation outcomes across different investment styles.

For each of the 153 characteristics, the target $\{x^*_{i,t}\}$ is formed in a standard fashion without considering trading costs: at the start of every month, the cross section of stocks is split at the 50th quantile into equal-weighted long-short portfolios based on each characteristic. We fix the AUM at \$10 billion (\$5 billion for each of long and short legs).

We fix μ across the 153 factors for consistent comparison, and because factor-specific μ tuning is likely to be unstable. For example, consider factors that happen to earn negative realized returns during the training sample period, the factor-specific optimal μ would be zero – no trading at all. By evaluating at a fixed positive μ, we can address whether this factor that happens to lose money in the sample, would lose less with a better implementation. We pick the μ that optimizes the average gain across all factors, with results robust to perturbations in μ.

Figure 7: Mean return improvements in implementing each factor portfolio

A. implementing each factor portfolio B. averaged by theme clusters



Each dot implements one JKP factor portfolio. The y-axis is the difference in after-cost mean excess return between implementing with the rnn.econ$_{all}$ and the ma$_5$. The x-axis is the turnover of the factor portfolio target (i.e., Eq. 15 with $x_{i,t} = x^*_{i,t}, x^0_{i,t} = x^*_{i,t-1}$). Panel B averages the points in A by style clusters (from JKP).

Figure 7 plots the gain in mean return when implementing the factor portfolios with the rnn.econ$_{all}$ volume prediction compared with using the ma$_5$ volume prediction. The horizontal axis is the turnover of the target factor portfolio. Panel B averages the points by investment style clusters (from JKP). The plots show volume prediction from the rnn.econ$_{all}$ benefits portfolio implementation across the factors. The average gain in mean after-cost return across factors is 0.44% per year from volume prediction alone using the rnn.econ$_{all}$ model versus the simple moving average. Almost all of the 153 factors have positive gains. With the \$10b AUM scale, this translates into an additional \$44m per year in implementation cost-saving from improved volume prediction.

Across factors, the gain is larger for those factors with higher turnover. In the right region of Figure 7, some raw factors have a turnover approaching six (600% per year or roughly turning over half of the AUM every month). The gains for these factors, including various short-term reversal strategies, are around 0.5% to 1.0%. These factors are constructed with technical signals over a shorter window.[29] In the left part of the figure, even factors with low turnover (those relying on

[29] Examples: `ret_1_0` short term reversal; `iskew_capm_21d` idiosyncratic skewness from the CAPM; `iskew_ff3_21d` idiosyncratic skewness from the FF3F; `rmax5_rvol_21d` highest 5 days of return; `rskew_21d` return skewness 21d; `seas_1_1an` 1 Year Annual Seasonality; and `coskew_21d` coskewness.

quarterly fundamental signals and signals with greater persistency) show gains that range from 0.2% to 0.6% per year from volume prediction.

Appendix Figure C.3 plots the same gains in the vertical axis but changes the horizontal axis to the mean return attained by ma_5, i.e., the baseline level in the gain calculation. The figure shows that, regardless of the baseline, the gain is independently distributed around a positive center. That is, a better volume prediction is uniformly effective, and the improvement is not concentrated on factors that have positive (or negative) realized returns. Appendix Figure C.4 reports similar plots with gains measured in Sharpe ratio space instead of mean returns.

7 Conclusion

We translate volume predictability into net-of-cost portfolio performance by linking it to expected trading costs – a term we call "trading volume alpha." Volume is highly predictable, especially when using machine learning techniques, large data signals, and exploiting the virtue of complexity in prediction. We find that volume prediction can be as valuable as return prediction in achieving optimal mean-variance portfolios net of trading costs.

We find that incorporating an economic objective function directly into machine learning is even more effective for obtaining useful predictions. This feature may be general to many finance applications of machine learning, where incorporating the economic objective directly may dominate a two-step process that first satisfies some statistical objective and then incorporates that statistical object into an economic framework. For volume prediction, the asymmetric cost of overestimating versus underestimating volume is captured (ignored) by an economic (statistical) objective, and delivers sizeable economic impact.

While we find substantial economic benefits from volume predcition using our framework and methods, there is much room for improvement. Our goal is not to develop the best trading cost model or even the best volume prediction model, but rather to translate the prediction problem into economic consequences, which yield interesting insights. A more exhaustive search for prediction variables and models that forecast volume more accurately could translate into even larger economic benefits than we show here. Some promising candidates for additional features and methods are lead-lag volume relations across stocks, more seasonal indicators, other market microstructure variables, and more complex nn and rnn models.

Our simple framework for predicting and characterizing volume alpha also has limitations. For one, we study a very simple functional form for trading costs that maps volume prediction directly into costs. Other functional forms and other determinants of costs beyond volume may lead to novel results. In addition, we separate the volume prediction problem from the expected return and variance/covariance modeling problem. Combining all three could generate further portfolio improvements and the interaction between these three prediction problems could be enlightening. Lastly, our trading experiments are merely a tool to illustrate some possible applications of our insights, but are not designed to optimize any performance outcome. Specifically, two things not considered in our design are: dynamic effects from trading and heterogeneous trade tasks. Adding these more complex features would be an interesting area for future research.

Trading volume prediction in general is an interesting research area worthy of further exploration. While we have couched the volume prediction problem into a portfolio context to translate the problem into economic consequences, understanding the role of volume more generally – its causes and consequences – is interesting. Using some of our techniques may shed light on this question and may help pinpoint what components of volume are most valuable to trading costs and, as a byproduct, portfolio construction. For example, informed versus uninformed volume, volume with temporary versus permanent price impact, and short- versus long-term volume may be interesting research pursuits. Examining various aspects of volume could be very useful for improving portfolio optimization and understanding trading activity more broadly. We leave these issues for future work.

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Internet Appendix

Trading Volume Alpha

Ruslan Goyenko — McGill

Bryan Kelly — Yale, AQR, and NBER

Tobias Moskowitz — Yale, AQR, and NBER

Yinan Su — Johns Hopkins

Chao Zhang — HKUST (GZ)

A Technical details

A.1 Neural network implementation details

The nn architecture consists of three fully-connected hidden layers with 32, 16, and 8 neurons, respectively.

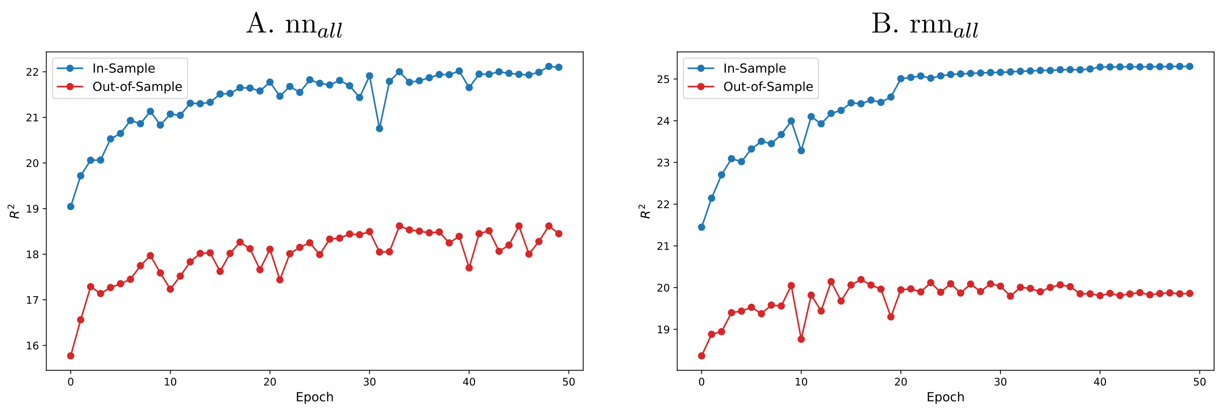
The rnn architecture is similar to that of the nn. The first (bottom) hidden layer in the 3-layer network is upgraded to an lstm layer with 32 hidden states and cell states, respectively. The remaining two layers are unchanged: fully connected with 16 and 8 neurons, respectively.

The formulas for the number of parameters in nn and rnn, as reported in Table 1 Panel C, are stated below. For nn (three fully-connected hidden layers with 32-16-8 neurons), the formula is $(\# \text{ of predictors} + 1) \times 32 + (32 + 1) \times 16 + (16 + 1) \times 8 + (8 + 1)$. In rnn, the first hidden layer has 32 hidden states and 32 cell states with four gates, changing the formula to $(\# \text{ of predictors} + 32 + 1) \times 32 \times 4 + (32 + 1) \times 16 + (16 + 1) \times 8 + (8 + 1)$.

In training the rnn, we implement the "many-to-one" type data pipeline, where the model recursively processes a sequence of 10 inputs, $X_{i,t-9}, \ldots, X_{i,t}$, and produces a single output $\widehat{\eta}_{i,t}$ to calculate the training loss at each data point $\{i, t\}$. (When increasing the sequence length from 10 to 50, the results had minimum improvements but required much larger GPU memory and longer training time.) For data points at the beginning of a stock's observed period where lagged predictors (e.g., $X_{i,t-9}$) are not available, we fill in with zero vectors.

In training both nn and rnn models, we use the Adam optimizer, with default learning rates and other parameters. The batch size is 1024. The $\widetilde{\eta}$ prediction models are trained with 50 epochs. For the sake of clear benchmarking, we do not adopt early stopping, weights dropout, or hyperparameter tuning with cross-validation, though these techniques could further boost the prediction accuracy. The machine learning program is implemented with the PyTorch package.

Figure A.1: Learning curves



R^2 of the $\widetilde{\eta}$ prediction models as training progresses (epochs).

The learning curves show the gap between the IS and OOS R^2 is relatively small, and does not widen with continued training. This indicates limited in-sample overfitting at this neural network configuration. The rnn learning curves show slightly more severe overfitting, though the OOS learning curve is still relatively stable as training continues. The learning curves display fluctuation due to the randomness of the stochastic gradient descent, though the extent to which is not severe. We also note the exact results depend on the inherent randomness of the training program. We find the quantitative results are insensitive to random seeds and report the average of five independent runs to obtain a robust evaluation outcome.

Table A.1 summarizes computational costs in terms of the training times and memory usage for nn and rnn with all predictors for a single random seed. These experiments were conducted on a system equipped with an Nvidia A100 GPU with 40 GB of GPU memory, an AMD EPYC 7713 64-Core Processor @ 1.80GHz with 128 cores, and 1.0TB of RAM, running Ubuntu 20.04.4 LTS.

Table A.1: Training time and memory usage

	Training time (hours)	CPU memory usage (GB)	GPU memory usage (GB)
nn_{all}	0.48	10.87	1.22
rnn_{all}	0.63	144.98	1.48

B Additional theoretical analysis

B.1 Microfoundation of the tracking error part in the portfolio objective

The track error penalty term in the portfolio optimization objective function can be economically founded. We show the quadratic tracking error penalty can be derived from a mean-variance utility function. This analysis connects the target positions x_i^* to the before-cost mean-variance efficient portfolio weight, which increases in the asset's return expectations as well as the total portfolio size (AUM). It also provides a microfoundation of the hyperparameter μ and the quadratic penalty: as the position deviates away from the target, the mean-variance loss increases in a quadratic fashion.

Assume the agent has a mean-variance utility function, $U = \mathbb{E}A' - \frac{\gamma}{2A}\mathbb{V}\text{ar}A' - TCost$, where A' is the before-cost investment outcome, A is the initial wealth (AUM), and γ is the risk aversion coefficient, $TCost$ is the term for the objective of minimizing transaction costs (not the focus here as we are justifying the tracking error part). A' is the outcome of the portfolio strategy: $A' = A(1 + r_{\text{f}}) + \sum_i x_i r_i$, where x_i is the dollar position in risky asset i with excess return r_i. Assume $\mathbb{E}r_i = m_i$, $\mathbb{V}\text{ar}r_i = \sigma^2$, and zero covariances. The agent's portfolio optimization problem is choosing $\{x_i\}$ to maximize U.

Then, the objective function is

$$U = A(1 + r_{\text{f}}) + \sum_i x_i m_i - \frac{\gamma}{2A}\sum_i x_i^2\sigma^2 - TCost \tag{16}$$

$$= \sum_i \left(-\frac{\gamma\sigma^2}{2A}x_i^2 + m_i x_i\right) + A(1 + r_{\text{f}}) - TCost \tag{17}$$

$$= -\frac{\gamma\sigma^2}{2A}\sum_i \left(x_i^2 - \frac{2A}{\gamma\sigma^2}m_i x_i\right) + A(1 + r_{\text{f}}) - TCost \tag{18}$$

$$= -\frac{\gamma\sigma^2}{2A}\sum_i\left(x_i - \frac{A}{\gamma\sigma^2}m_i\right)^2 + \left(\frac{A}{2\gamma\sigma^2}\sum_i m_i^2 + A(1+r_{\mathrm{f}})\right) - TCost \tag{19}$$

The first term matches the tracking error term modeled in Eq. 3. The second term is the before TCost utility at the zero tracking error portfolio ($x = x^*$). This term is constant of the x choice, hence can be ignored in the optimization problem.

Comparing the first term with the tracking error modeled in Eq. 3, we see the target portfolio is $x_i^* = \frac{A}{\gamma\sigma^2}m_i$. As expected, the target positions are the result of Markowitz mean-variance optimization. They are proportional to the return expectations in the cross section. They scale linearly with the AUM (A) and inversely with the risk aversion coefficient and volatility. Additionally, the overall tracking error penalizing coefficient $\mu = \frac{\gamma\sigma^2}{A}$, which is decreasing in AUM. The rationale is that the quadratic penalty stems from the quadratic risk change as the position deviates away from the target, and that the absolute risk aversion coefficient decreases with the wealth level. Although the main analysis takes μ as a hyperparameter ignoring its microfoundation, we still observe the negative relationship between the tuned μ and the economically relevant AUM (e.g., in Section 5.3).

B.2 The economic task as predicting $\widetilde{z}$

We have shown the economic task of choosing the z strategy, $\min_{z(\cdot)}\sum_{i,t\in\text{train}} loss^{\text{econ}}(\widetilde{v}_{i,t}, z(\mathcal{X}_{i,t});\mu)$, can be seen as a prediction task of predicting the $\widetilde{v}$ with the economic loss function: $\min_{v(\cdot)}\sum_{i,t\in\text{train}}$ $loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_{i,t}, v(\mathcal{X}_{i,t});\mu)$. In this appendix, we provide the equivalent representation as a prediction problem of the oracle trading rate $\widetilde{z} := s(\widetilde{v};\mu)$.

Define $loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z;\mu) := loss^{\text{econ}}(s^{-1}(\widetilde{z};\mu), z;\mu)$, where $s^{-1}(\,\cdot\,;\mu)$ is inverse of $s(\,\cdot\,;\mu)$ function. Under this definition, the economic task can be viewed as the problem of looking for a function $z(\cdot)$ that maps $\mathcal{X}$ into z to minimize the training sample average loss:

$$\min_{z(\cdot)}\sum_{i,t\in\text{train}} loss^{\text{econ}}_{\text{zz}}(\widetilde{z}_{i,t}, z(\mathcal{X}_{i,t});\mu) \tag{20}$$

According to the definition, the analytical expression of $loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu)$ is

$$loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu) = \frac{\mu}{\widetilde{z}}(z - \widetilde{z})^2 + \mu(1 - \widetilde{z}). \tag{21}$$

In this expression, the loss can be seen as the squared z prediction error weighted by $\frac{\mu}{\widetilde{z}}$. The last term is constant of choice z so can be ignored in optimization. It equals $loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, \widetilde{z}; \mu)$, the baseline loss incurred even with the perfect prediction.

To derive this expression, notice $\widetilde{z}$ is already defined as the perfect trading given $\widetilde{\lambda}$ or $\widetilde{v}$.

$$\widetilde{z} = \frac{\mu}{\mu + \widetilde{\lambda}} \Longrightarrow \widetilde{\lambda} = \frac{\mu}{\widetilde{z}} - \mu \tag{22}$$

Then, start from Eq. 7, represent $\widetilde{\lambda}$ with $\widetilde{z}$ and then complete the square:

$$\begin{aligned}
loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu) &= \widetilde{\lambda} z^2 + \mu(1 - z)^2 \\
&= \left(\frac{\mu}{\widetilde{z}} - \mu\right) z^2 + \mu(1 - z)^2 \\
&= \frac{\mu}{\widetilde{z}}(z - \widetilde{z})^2 + \mu(1 - \widetilde{z})
\end{aligned}$$

We show $loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu)$ is still meaningfully different from the standard squared error loss function, and that $\mathbb{E}\left[\widetilde{z}|\mathcal{X}\right]$ will not be the optimal choice either further below.

B.3 Economic loss functions are not in Bregman class

We show functions $loss^{\text{econ}}_{\text{vv}}(\widetilde{v}, v; \mu)$ and $loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu)$ are not in the Bregman class, for all μ.

Without loss of generality, any loss function $F(p, q)$ can be normalized as $\bar{F}(p, q) := F(p, q) - F(p, p)$, such that that the function acquires the convenient property that $\bar{F}(p, p) = 0$, and that the solution to the optimization problem $\min_{q \in \sigma\{\mathcal{X}\}} \mathbb{E}\left[F(p, q)\right]$ does not change. Therefore, in the following propositions, we normalize accordingly and consider $\overline{loss}^{\text{econ}}_{\text{vv}}(\widetilde{v}, v; \mu) := loss^{\text{econ}}_{\text{vv}}(\widetilde{v}, v; \mu) - loss^{\text{econ}}_{\text{vv}}(\widetilde{v}, \widetilde{v}; \mu)$ and $\overline{loss}^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu) := loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu) - loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, \widetilde{z}; \mu)$.

The definition of the Bregman loss function is (Banerjee, Guo, and Wang, 2005):

Definition 1. *Let* $\phi : \boldsymbol{R}^d \to \boldsymbol{R}$ *be a strictly convex differentiable function, then, the Bregman loss function* $D_\phi : \boldsymbol{R}^d \times \boldsymbol{R}^d \to \boldsymbol{R}$ *is defined as:*

$$D_\phi(p, q) := \phi(p) - \phi(q) - \langle p - q, \nabla\phi(q) \rangle \tag{23}$$

We consider the simpler case where p, q are scalars. In this case, a Bregman function has the property that its partial second derivative in the first argument is independent of the second argument.

$$\frac{\partial^2 D_\phi(p, q)}{\partial p^2} = \phi''(p) \tag{24}$$

The propositions below rely on this property.

Proposition 2. *Function* $\overline{loss}_{\text{zz}}^{\text{econ}}(\widetilde{z}, z; \mu)$ *is not in the Bregman class, for all* μ*.*

Proof. We verify that $\overline{loss}_{\text{zz}}^{\text{econ}}(\widetilde{z}, z; \mu)$ violates the property in Eq. 24.

$$\overline{loss}_{\text{zz}}^{\text{econ}}(\widetilde{z}, z; \mu) = \frac{\mu}{\widetilde{z}}(z - \widetilde{z})^2 \tag{25}$$

$$\frac{\partial^2 \overline{loss}_{\text{zz}}^{\text{econ}}(\widetilde{z}, z; \mu)}{\partial \widetilde{z}^2} = \frac{2\mu z^2}{\widetilde{z}^3} \tag{26}$$

It is clear that this is not irrelevant to z. □

Proposition 3. *Function* $\overline{loss}_{\text{vv}}^{\text{econ}}(\widetilde{v}, v; \mu)$ *is not in the Bregman class, for all* μ*.*

Proof. Given the property in Eq. 24, a Bregman loss function must be unbounded as $p \to +\infty$. This is because for any fixed q, when $p > q$, $D_\phi(p, q)$ is increasing and convex in p.

However, we verify that $loss_{\text{vv}}^{\text{econ}}(\widetilde{v}, v; \mu)$ is bounded by showing it converges to a finite number as $\widetilde{v} \to +\infty$.

$$\begin{aligned}\overline{loss}_{\text{vv}}^{\text{econ}}(\widetilde{v}, v; \mu) := &\frac{0.2\exp(-\widetilde{v}) + \mu\left(\exp(-v + \log 0.2 - \log\mu)\right)^2}{\left(1 + \exp(-v + \log 0.2 - \log\mu)\right)^2} \\ &- \frac{0.2\exp(-\widetilde{v}) + \mu\left(\exp(-\widetilde{v} + \log 0.2 - \log\mu)\right)^2}{\left(1 + \exp(-\widetilde{v} + \log 0.2 - \log\mu)\right)^2}\end{aligned}$$

$$= \frac{(0.2\mu)^2 \exp(-\widetilde{v}) (\exp(\widetilde{v}) - \exp(v))^2}{(\mu \exp(\widetilde{v}) + 0.2) (\mu \exp(v) + 0.2)^2} \tag{27}$$

By L'Hôpital's rule, we have the limit of it as:

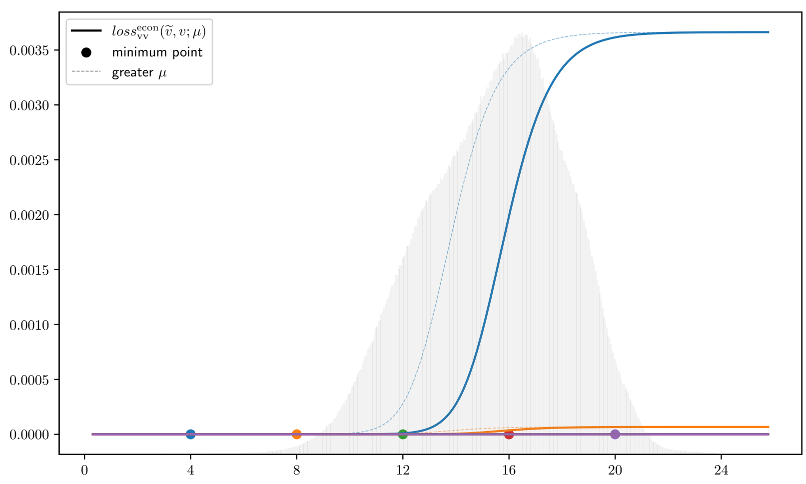
$$\begin{aligned} \lim_{\widetilde{v} \to \infty} \overline{loss}_{\text{vv}}^{\text{econ}}(\widetilde{v}, v; \mu) &= \lim_{\widetilde{v} \to \infty} \frac{(0.2\mu)^2 (\exp(\widetilde{v}) - \exp(2v - \widetilde{v}))}{\mu (\mu \exp(v) + 0.2)^2 \exp(\widetilde{v})} \\ &= \frac{0.04\mu}{(\mu \exp(v) + 0.2)^2} \end{aligned} \tag{28}$$

□

B.4 Further analysis on the loss functions

The following is a visualization of the economic loss function in addition to the one in Figure 3 Panel B. The vertical axis is changed to the linear scale from the log scale. For large $\widetilde{v}$ (12, 16, 20, in green, red, purple), the curves are indistinguishable from a flat line because the blue curve is at a much greater magnitude, which is for the loss of overestimating low actual volume ($\widetilde{v} = 4$).

Figure B.2: Economic loss function in linear scale



Note: same as Figure 3 Panel B but with the vertical axis in linear scale.

The following proposition formally states the asymmetric property of the loss function for over/under-estimating volume.

Proposition 4. *Consider two symmetrical cases with low and high liquidity* $\widetilde{v}_1$ *and* $\widetilde{v}_2$ *such that* $\widetilde{z}_1 = 1 - \widetilde{z}_2 < 0.5$. *Suppose one makes an overestimation in the low-volume case* $\widehat{v}_1 = \widetilde{v}_1 + \varepsilon$, *comparing with an equal amount of underestimation in the high-volume case* $\widehat{v}_2 = \widetilde{v}_2 - \varepsilon$, *the additional loss incurred in the first case is greater than the second:*

$$loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_1, \widehat{v}_1; \mu) - loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_1, \widetilde{v}_1; \mu) > loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_2, \widehat{v}_2; \mu) - loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_2, \widetilde{v}_2; \mu), \quad \forall \mu > 0. \tag{29}$$

Proof. We first show that, given $\widetilde{z}_2 = 1 - \widetilde{z}_1$, as well as $\widehat{v}_1 - \widetilde{v}_1 = \widetilde{v}_2 - \widehat{v}_2 = \varepsilon$, we have $\widehat{z}_2 = 1 - \widehat{z}_1$, and that $\widetilde{z}_1 - \widehat{z}_1 = \widehat{z}_2 - \widetilde{z}_2$.

We know $s(v; \mu) = \frac{1}{1+\exp(-v+\log 0.2-\log \mu)}$. From $\widetilde{z}_1 = 1 - \widetilde{z}_2$, we have:

$$\begin{aligned}
\frac{1}{1 + \exp(-\widetilde{v}_2 + \log 0.2 - \log \mu)} &= 1 - \frac{1}{1 + \exp(-\widetilde{v}_1 + \log 0.2 - \log \mu)} \\
\exp(-\widetilde{v}_1 - \widetilde{v}_2 + 2 \log 0.2 - 2 \log \mu) &= 1 \\
\widetilde{v}_2 &= 2(\log 0.2 - \log \mu) - \widetilde{v}_1
\end{aligned} \tag{30}$$

Then we have:

$$\begin{aligned}
\widehat{z}_1 + \widehat{z}_2 &= \frac{1}{1 + \exp(-\widetilde{v}_1 - \varepsilon + \log 0.2 - \log \mu)} + \frac{1}{1 + \exp(-\widetilde{v}_2 + \varepsilon + \log 0.2 - \log \mu)} \\
&= \frac{1}{1 + \exp(-\widetilde{v}_1 - \varepsilon + \log 0.2 - \log \mu)} + \frac{1}{1 + \exp(\widetilde{v}_1 + \varepsilon - \log 0.2 + \log \mu)} = 1
\end{aligned}$$

The last equation comes from the fact that $\frac{1}{1+\exp(x)} + \frac{1}{1+\exp(-x)} = 1$. Then we make use of the loss function expressed in terms of $\widetilde{z}$ and z, as defined in Eq. 21:

$$loss^{\text{econ}}_{\text{zz}}(\widetilde{z}, z; \mu) = \frac{\mu}{\widetilde{z}}(z - \widetilde{z})^2 + \mu(1 - \widetilde{z}) \tag{31}$$

Since $\widetilde{z}_1 - \widehat{z}_1 = \widehat{z}_2 - \widetilde{z}_2$ and $\widetilde{z}_1 < \widetilde{z}_2$, that is $\frac{\mu}{\widetilde{z}_1} > \frac{\mu}{\widetilde{z}_2}$, we have the required result:

$$loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_1, \widehat{v}_1; \mu) - loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_1, \widetilde{v}_1; \mu) > loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_2, \widehat{v}_2; \mu) - loss^{\text{econ}}_{\text{vv}}(\widetilde{v}_2, \widetilde{v}_2; \mu), \quad \forall \mu > 0. \tag{32}$$

□

C Additional empirical results

C.1 Prediction results in firm size groups and "mixture of experts" forecasts

Table C.2 Panel A provides additional assessments of prediction accuracy by evaluating volume forecasts in different size groups. We use the five groups from the JKP data sorted on the firms' market capitalization.[30]

Table C.2: Prediction accuracy (R^2 in %) in different size groups and "mixture of experts"

size group	jointly	nano	micro	small	large	mega
training obs	2,522,619	300,790	797,880	680,209	479,839	263,901
testing obs	1,893,067	273,792	467,413	552,503	384,819	214,540
A: pooled training evaluated in size groups and jointly (same models as in Table 1)						
ols_{all}	15.99	13.32	12.60	20.90	25.49	26.16
nn_{all}	18.45	15.80	14.86	23.71	27.76	29.12
rnn_{all}	19.86	16.63	16.14	26.00	30.50	32.02
B: size group training evaluated in size groups and jointly (mixture of experts)						
ols+moe_{all}	16.34	13.68	12.73	21.43	25.93	27.47
nn+moe_{all}	17.78	15.29	14.43	22.69	26.57	27.71
rnn+moe_{all}	18.26	15.24	14.71	24.76	29.02	30.99

Panel A evaluates the benchmark models (pooled training) in the five size groups, respectively, in the OOS period. Each model uses "all" 175 predictors. Column "jointly" repeats Table 1, Panel A, last column. Panel B trains "expert" models for each size group separately and evaluates them in their corresponding size groups in the OOS period. Column "jointly" evaluates the mixture of experts (moe) model, which predicts with the corresponding expert model trained on the same size group for each OOS data point. Each R^2 value is the average of five runs, same as Table 1.

The prediction accuracy increases as firm size increases, regardless of the prediction method. The R^2's evaluated in the mega firms are roughly twice those of the nano firms. As explained in the main text, smaller firms have a greater magnitude of unexpected trading volume shocks that are hardest to predict. This result makes sense since small firms are volatile and have low trading

[30]The five groups are defined according to the market capitalization breakpoints of NYSE stock percentiles: mega stocks, greater than the 80th percentile; large, 50–80; small, 20–50; micro, 1-20; and nano, below the 1st percentile.

volume, hence unexpected events that give rise to volume spikes are more likely for these firms. This finding also indicates that in addition to small firms being less liquid on average, their liquidity is also less predictable and more volatile. Hence, tiny firms are not only costly to trade in general, but their costs are less predictable. These results are intuitive and suggest that our prediction models are capturing true variation in volume and not simply noise.

We also examine whether firms of different size groups should be modeled differently. We train a model on each size group separately to attempt to better capture the heterogeneity across the firm size dimension rather than pooling all firms in the same model. We implement a simple mixture of experts (moe) method, where each size group is trained separately to form "expert" models and then compared against the pooled training model in Panel A.

The moe improves performance of the ols method. Comparing the first lines in Panels A and B, linear models catered to different size groups are more accurate than the pooled ols. For nn and rnn, however, the sample size reduction outweighs the potential benefits of separate training, making the mixture of expert models less accurate (either conditioning on size groups or jointly). Since the potential non-linear effects of firm size are already allowed in the neural networks, forcing size groups into different models is less effective. For this reason, we stick with pooled training samples.

C.2 Additional results of implementing factor zoo portfolios

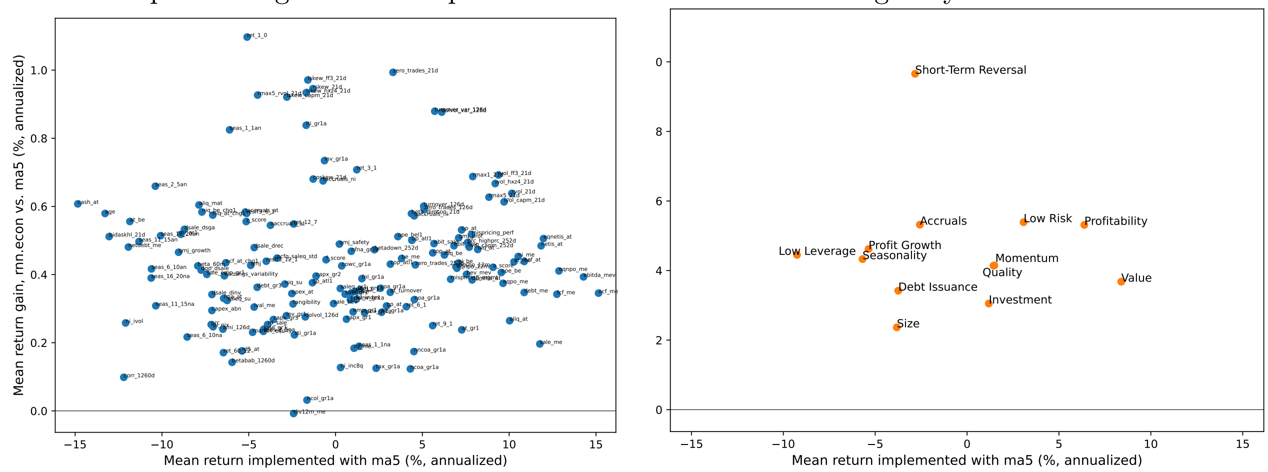
We provide results on the trading experiments that implement the factor zoo portfolios in addition to Subsection 6.3.

Figure C.3 has the same vertical axis as Figure 7, which is the improvement in after-cost mean excess return from ma_5 to rnn.econ_{all}. The horizontal axis changes to the mean excess return achieved with the ma_5 method, i.e., the baseline level of the improvement. The plot shows the gain is distributed around a positive center uncorrelated with the baseline. That is, a better volume prediction is uniformly effective, and the improvement is not concentrated on factors that have positive (or negative) realized returns.

Figure C.4 is the Sharpe ratio version of Figure 7 by showing the gain in Sharpe ratio instead of the mean return. The plot shows a similar pattern to the one reported in the main text. The gains in Sharpe ratios are larger for those factors with higher turnover, reaching around 0.3 to 0.4 per year.

Figure C.3: Mean return improvements in implementing each factor portfolio

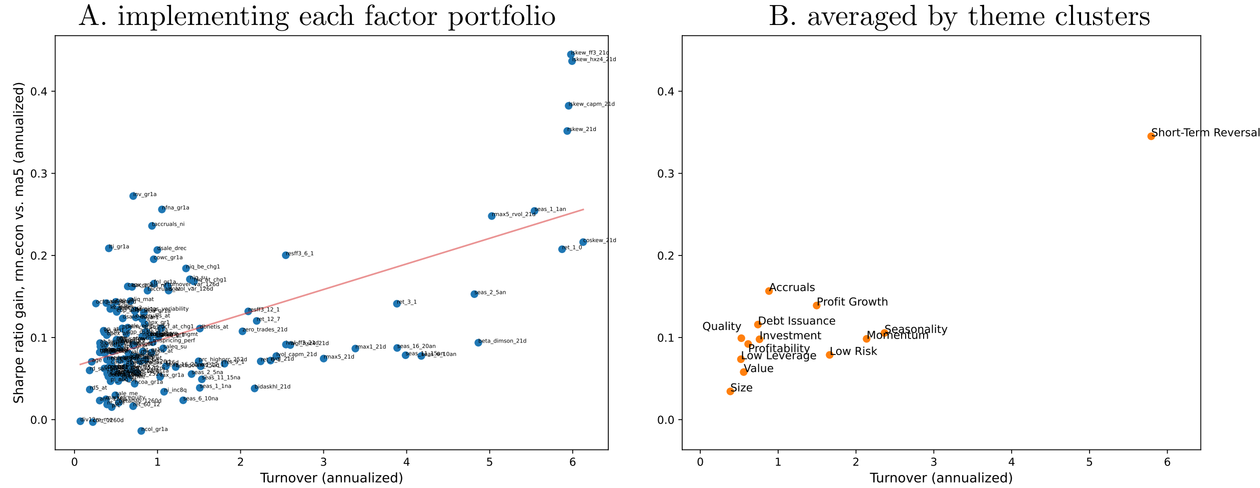
A. implementing each factor portfolio B. averaged by theme clusters



The same plot as Figure 7, but changing the x-axis to mean return achieved with the ma_5 method, i.e., the baseline of the gain.

Figure C.4: Sharpe ratio improvements in implementing each factor portfolio

A. implementing each factor portfolio B. averaged by theme clusters



The same plot as Figure 7, but showing the gain in Sharpe ratio instead of mean return.