

Overview

Introduction:

- Consumers have preferences for certain products, but markets may fail to supply adequate quantities due to issues such as imperfect competition or fixed production costs. Consequently, the socially optimal level of welfare might not be achieved.
- Activist funds can potentially address this issue by holding pivotal shares in firms, aiming to attract investment flows.
- Investors, who are in the same household as the consumers, care about their consumption utilities, and trade-off between index and activist fund when investing.

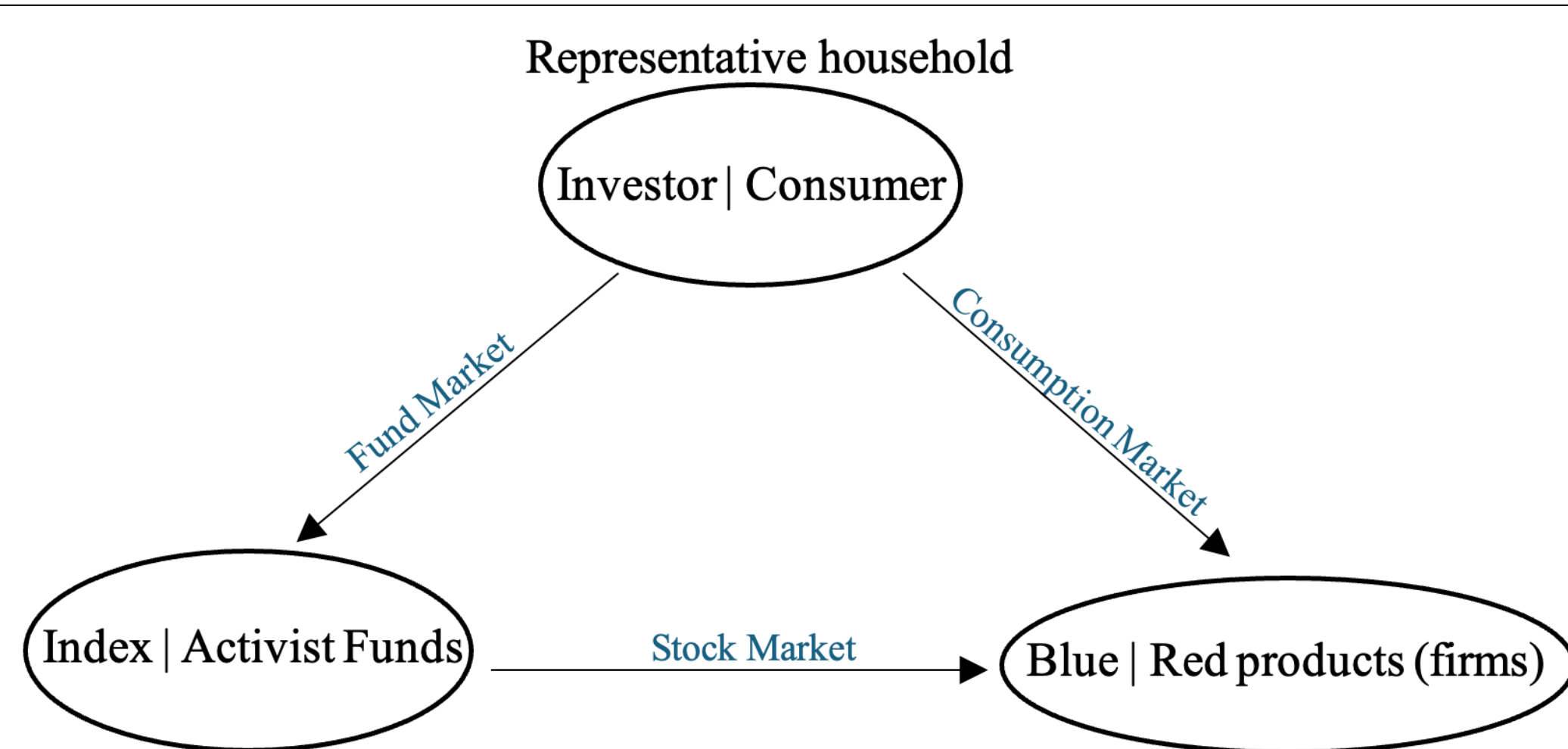
Research Questions:

- Can the financial market help improve welfare? If so, when?

Main Results:

- The socially optimal level of welfare can only be restored under a moderate range of consumption preferences and fixed production costs.

Model Setup



Consumption market:

- The representative household utility function:

$$U_c = \left[\int_0^\gamma b \bar{y}_i^{\frac{\epsilon-1}{\epsilon}} di + \int_\gamma^1 \bar{y}_i^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $b > 1$ is the preference for blue varieties relative to red; γ is the fraction of blue varieties; $\epsilon \in (1, \infty)$ is the elasticity of substitution. Budget constraint follows:

$$\int_0^\gamma y_i p_i di + \int_\gamma^1 \bar{y}_i \bar{p}_i di \leq I$$

Where I is the consumption endowment.

- 1 continuum of monopolistic firms choose to produce blue ($\theta_i = 1$) or red ($\theta_i = 0$) varieties, with marginal cost φ . If it produces blue variety, it incur a fixed cost κ .
- $\hat{\pi}_i = \pi_i - \kappa = (p_i - \varphi)y_i$, $\bar{\pi}_i = (\bar{p}_i - \varphi)\bar{y}_i$ are the profits for blue and red firms.

Fund market:

- Tow funds: 1 activist fund and 1 index fund. Household has 1 endowment for investment, invest α in activist fund, and $1 - \alpha$ in index fund.
- Index fund has no strategy, while activist fund chooses portfolio size χ and per unit management fee R . Investor has utility function:

$$U_I(\alpha, \gamma) = \Pi(\alpha) + U_c(\gamma)$$

where $\Pi(\alpha) = \pi_{active} - \alpha R + \pi_{index}$.

- If activist fund holds more than τ fraction of its portfolio firms, it can have control over the firms and take over the fixed cost χ . It has payoff function:

$$\Omega(\chi, R) = \max\{\alpha R - \kappa \chi, 0\}$$

- $\alpha < 1$ number of passive investors with 1 endowment each, only in index fund.

Stock market:

- For every dollar index fund receives, it invests equally in all firms; for every dollar activist fund receives, it invests equally in χ portfolio firms
- Portfolio firms have stock price s_1 , while the rest of the firms have stock price s_2 .
- Stock market follows cash-in-the-market pricing. Number of shares for firms are normalised for 1.

Timing:

- T=1: Activist fund picks portfolio size χ and fee R ;
- T=2: Household makes investment decision α ;
- T=3: Influence of firms is realised; firms decide to produce blue ($\theta_i = 1$) or red ($\theta_i = 0$) varieties and set prices;
- T=4: Household consumes accordingly, all payoffs realise.

Subgame perfect equilibrium:

- Activist fund picks χ and R : $\max_{\{\chi, R\}} \Omega(\chi, R)$;
- Investor picks α : $\max_\alpha U_I(\alpha, \gamma)$;
- Firm decides to produce blue $\theta_i = 1$ or red $\theta_i = 0$, and picks $p_i = \operatorname{argmax} \hat{\pi}_i$ and $\bar{p}_i = \operatorname{argmax} \bar{\pi}_i$;
- Consumer picks y_i and \bar{y}_i : $\max_{\{y_i, \bar{y}_i\}} U_c(\gamma, y_i, \bar{y}_i)$

Benchmark and Constrained Optimum

Benchmark:

- Suppose only consumption market exists.
- Firms choose to be blue or red, equilibrium arrives when $\pi_i(\gamma_{BM}^*) - \kappa = \bar{\pi}_i(\gamma_{BM}^*)$

which gives:

$$\frac{I(g-1)}{\epsilon} [1 + (g-1)\gamma_{BM}^*]^{-1} = \kappa$$

Constrained Optimum:

- Social planner cares about both the utility of consumers and the profit of firms. Define the social planner's welfare function as:

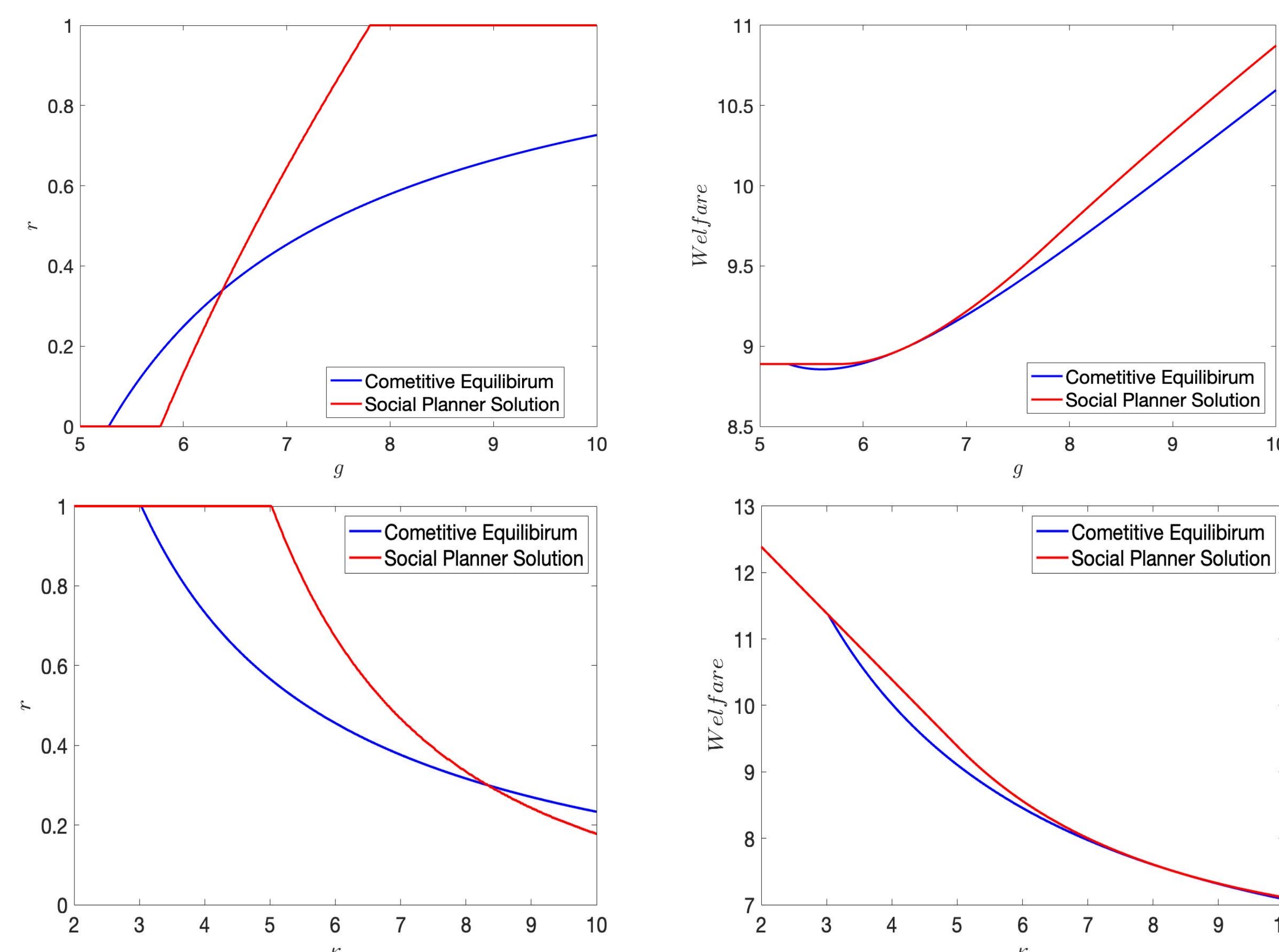
$$W(\gamma) = U_c(\gamma) + \int_0^\gamma (\pi_i - \kappa) di + \int_\gamma^1 \bar{\pi}_i di$$

- Suppose social planner takes the market structure as given and respects the monopolistic competition among firms. She maximises welfare by picking γ , which gives:

$$\frac{I(g-1)}{\epsilon\varphi} [1 + (g-1)\gamma_{SO}^*]^{-1} = \kappa$$

Key intuitions:

- In the benchmark equilibrium, a firm decides to be blue or red by considering which side of the profit is larger, given a certain γ .
- It does not consider the impact of its decision on γ , nor the impact on other firm's profit.
- In the social planner's case, γ is chosen to maximise the total welfare in the economy.
- It considers the externalities of one firm being blue or red on other firms, as well as the impact of γ on consumer's utility gain.



Key intuitions:

- Increasing g represents the increase of consumer's relative preference between blue and red products.
- This leads to more varieties in the market in both benchmark and social planner's outcome, and thus an increase in welfare in both cases.
- Similarly, an increase of κ decreases the profit of becoming a green firm and hence, reduces the equilibrium γ .
- As the fixed cost increases, the total welfare decreases.

Financial Market Equilibrium

- At T=4, consumer consumes given consumption choices γ^* .
- At T=3, changes from benchmark equilibrium only happens when $\text{if } \chi \geq \gamma_{BM}^*$ and activist fund holds enough shares:

$$\gamma^* = \begin{cases} \chi, & \text{if } \chi \geq \gamma_{BM}^* \text{ and } \alpha \geq T \\ \gamma_{BM}^*, & \text{otherwise} \end{cases}$$

where $T = \frac{\tau\chi(1+\alpha)}{(1-\tau)+\tau\chi}$.

- At T=2, investor observes the activist fund choice of χ and R , and faces a **trade-off**:

- $\pi_i > \bar{\pi}_i \Rightarrow$ payoff of activist fund $>$ index fund \Rightarrow activist fund more attractive $\Rightarrow \alpha \uparrow$
- $\alpha \uparrow \Rightarrow s_1 \uparrow, s_2 \downarrow \Rightarrow$ payoff of activist fund $<$ index fund \Rightarrow index fund more attractive $\Rightarrow \alpha \downarrow$

Depending on χ , the equilibrium of α is different:

- $\chi < \gamma_{BM}^*$: investor always prefers influence in place, and choose $\alpha_1^* = \operatorname{argmax} U_I(\alpha, \gamma_{BM}^*)$;
- $\chi \geq \gamma_{BM}^*$: investor might prefer to invest all in index fund, and picks $\alpha = 0$ or $\alpha_2^* = \operatorname{argmax} U_I(\alpha, \chi)$;

$$\alpha = \begin{cases} 0, & U_I(0, \chi) > U_I(\alpha_2^*, \chi) \\ \alpha_2^*, & U_I(0, \chi) \leq U_I(\alpha_2^*, \chi) \end{cases}$$

- At T=1, activist fund faces a **trade-off**:

- $\chi \downarrow \Rightarrow U_c(\chi) - U_c(\gamma_{BM}^*) \downarrow \Rightarrow \alpha \downarrow \Rightarrow \alpha R \downarrow \Rightarrow \Omega(\chi, R) \downarrow$
- $\chi \uparrow \Rightarrow \kappa \chi \uparrow \Rightarrow \Omega(\chi, R) \downarrow$

In equilibrium, activist fund only prefers to enter the market when $\chi \geq \gamma_{BM}^*$, or only under this situation consumption surplus is positive.

- Activist fund picks χ to maximise investor's surplus: $\chi^* = \operatorname{argmax} U_I(\alpha_2^*, \chi) - U_I(0, \chi)$, or chooses to stay outside the market:

$$\chi = \begin{cases} 0, & U_I(\alpha_2^*, \chi^*) - U_I(0, \chi^*) < 0 \\ \chi^*, & U_I(\alpha_2^*, \chi^*) - U_I(0, \chi^*) \geq 0 \end{cases}$$

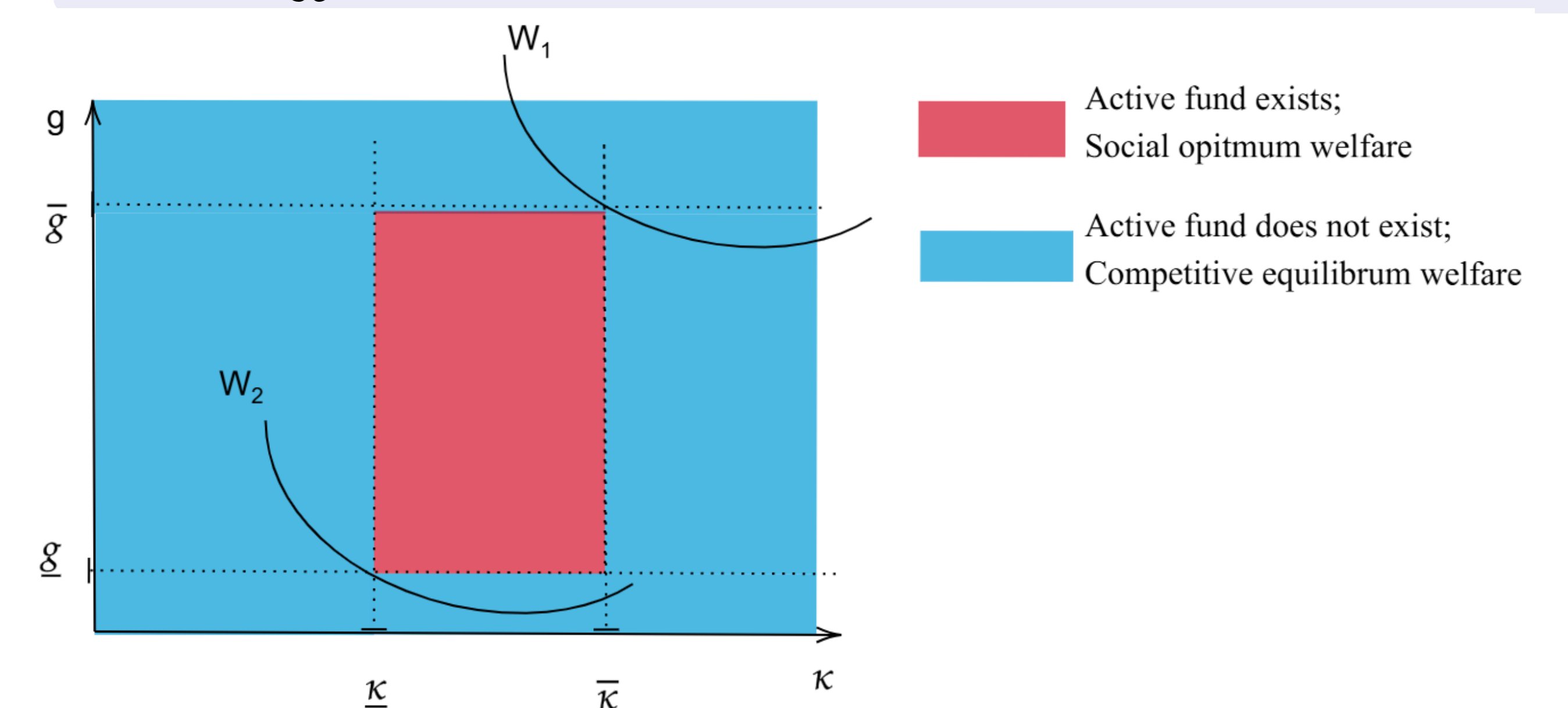
- R is chosen to extract all the surplus: $R = U_I(\alpha_2^*, \chi^*) - U_I(0, \chi^*)$.

Key intuitions:

- As it is possible that $\alpha_2^* > T$, investor might not just invest enough to have control over the portfolio firms. In other words, having influence does not necessarily mean sacrificing financial payoffs.
- Activist fund has the incentive to maximise profit and thus maximises investor's surplus. This maximisation incentive guarantees social optimum level of welfare is achieved.

Proposition

For $\kappa \in [\underline{\kappa}, \bar{\kappa}]$ and $g \in [g, \bar{g}]$, active fund exists in the market and social optimum level of welfare and γ_{SO}^* can be restored.



Key Intuitions:

- When κ is large, the cost of monitoring deters activist fund to enter the market.
- When κ is small, γ_{BM}^* is already large without activist fund. In this case, it is difficult for activist fund to induce utility surplus from consumption utility.
- When g is large, investor cares more about green consumption. In this case, γ_{BM}^* is already large without activist fund and activist fund finds it difficult to enter the market.
- When g is small, to increase consumption utility, activist fund must pick a very large χ , which might not be profitable as the cost $\chi\kappa$ might deter activist fund to enter.

