Exploring the Variance Risk Premium Across Assets[1]

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Abstract

This paper explores the variance risk premium in option returns across twenty different futures, including equities, bonds, currencies, and commodities (energy, metals, and grains). We implement a novel model-free methodology that constructs tradable option portfolios, which replicate realized variance. In the period 2006–2020, most assets had significant variance risk premiums, but the realized S&P 500 variance risk premium was not significantly different from zero. Within a particular asset, option prices across different strikes are related to the level of volatility and the correlation of volatility with futures returns. Returns to variance are not associated with systematic risk, but are related to fat tails, consistent with option dealers demanding a premium for holding idiosyncratic volatility risk. Contrary to Bollerslev et al. (2009), we find that option-implied variance does not positively predict underlying futures returns for the majority of assets. However, implied variance does predict returns to variance-sensitive option portfolios.

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1. Introduction

The negative variance risk premium on options is a well-documented asset pricing phenomenon. Most of the evidence in the literature to date is based on options on U.S. stocks and stock indices. However, this limited evidence does not indicate whether the properties of the variance risk premium for U.S. equity options extend to other assets. Consequently, the evidence cannot distinguish competing economic explanations for the existence and magnitude of the variance risk premium. For example, the variance risk premium might be due to systematic risk associated with innovations in the level of risk. Alternatively, it might be due to illiquidity and hedging risk in options. Finally, the unconditional variance risk premium might be caused by fluctuating conditional variance risk premiums, which are predictable by observable variables.

This paper explores option returns across twenty different futures, including equities, bonds, currencies, and commodities (energy, metals, and grains). This rich cross-section of assets allows us to analyze whether the results for U.S. stocks and indices are typical, or unusual. We measure the co-movement of option returns across assets and explore the relation between variance risk premiums and measures of risk and liquidity. In addition, we implement a novel model-free methodology that does not require interpolation across strikes and is based on tradable option portfolios.

The literature to date has extensively documented the existence of a variance risk premium in options on U.S. stocks and indices. For example, Fleming (1998) shows that implied volatility of S&P 100 options exceeds realized volatility. Bakshi and Kapadia (2003) study delta-hedged option portfolios on the S&P 500 and find a negative variance risk premium. Carr and Wu (2009) and Goyal and Saretto (2009) also find a negative variance risk premium on portfolios of options on individual stocks. Other papers confirm negative variance risk premiums in stocks, commodities, and Treasuries. Choi et al. (2017) find that the variance risk premium in Treasuries is negative and economically large. Prokopczuk and Chardin (2014) find negative variance risk premiums in commodity markets, although Trolle and Schwartz (2010) show that the negative variance risk premiums in gas and oil have smaller Sharpe ratios than simply shorting the S&P 500. Dew-Becker et al. (2017) study the returns of short-term and long-term straddles across commodities, currencies, equities and Treasuries and also find that realized volatility has a negative risk premium.

These studies of diverse markets are difficult to compare because they employ different methodologies to measure the variance risk premium. Some papers rely on models to construct option portfolios and to infer the variance risk premium. Other studies use interpolated prices or implied volatilities, and their results lack interpretation as returns on tradable strategies. In this paper, we measure the variance risk premium consistently across assets by constructing novel model-independent option portfolios. This approach is easily implementable in practice, and does not rely on interpolation nor any assumptions about the underlying model. In addition, our methodology overcomes some of the drawbacks of the Chicago Board of Options Exchange VIX formula (CBOE (2019)). For example, the CBOE VIX formula occasionally produces negative values for implied variance, but our methodology does not. In addition, our model-free returns produce higher correlations with realized variance than the CBOE VIX formula. Our returns are unambiguous measures of the variance risk premium because they are highly correlated with realized variance, with a median correlation across assets above 99%.

We find that some characteristics of S&P 500 options are not representative for all assets. All assets exhibit a volatility smile across different strike prices. However, S&P 500 options have the most pronounced smirk, where out-of-the-money (OTM) put options have higher implied volatilities than OTM calls. To the contrary, many assets including agricultural commodities, the Japanese yen, and the Swiss franc, have an inverse smirk with higher prices for OTM calls than for OTM puts. The negative (positive) slope of the smirk is associated with negative (positive) correlation between futures returns and option returns across assets.

Inspired by the fact that S&P 500 options exhibit different characteristics than options on other assets, we next measure the variance risk premium across assets. We find that the large and negative variance risk premium of S&P 500 options is largely a feature of the pre-COVID sample. During the March 2020 COVID month, the returns on options reached levels above *2000% per month*, bringing the average S&P 500 variance risk premium to only -11.9%, or roughly half the pre-COVID level of -22.9%. Other assets also experienced large and positive variance returns during COVID, but these effects were much smaller than for the S&P 500. As a result, the S&P 500 has a smaller negative variance risk premium over the whole sample than most other assets. Agricultural commodities and 10 year Treasuries have the most negative variance risk premium. While 19 out of 20 assets have negative realized variance risk premium in our sample, the Swiss franc is the only asset with a positive realized variance risk premium.

After documenting the significant heterogeneity in variance risk premiums across assets, we next turn to studying which factors are related to fluctuations in the premium. We find that the variance risk premium is not associated with the return on the underlying futures. The variance risk premium also does not appear related to measures of systematic risk. Specifically, options on assets with large systematic volatility have smaller (negative) variance risk premiums. Contrary to Bollerslev et al. (2009), we do not find that the ex-ante variance risk premium predicts realized futures returns for the majority of assets. However, we do find that the premium predicts returns on our variance portfolios. These findings suggest an important role for theory to model conditional risk premiums in options.

The rest of the paper is organized as follows. Section 2 describes the data and methodology, and presents basic facts on options for different assets. Section 3 describes the variance risk premium. Section 4 considers three explanations for the variance risk premium: systematic risk, liquidity, and conditional risk. A final section concludes.

2. Data and Methodology

2.1. Data and Futures Statistics

We study options on twenty different futures. The data comes from IVolatility based on options traded on three major exchanges: CME, NYMEX and ICE. We focus on currencies (Australian dollar, British pound, Canadian dollar, Swiss franc, Japanese yen), equities (S&P 500 Index), fixed income (10 year and 30 year US Treasury Notes) and commodities (crude oil, natural gas, gold, silver, copper, live cattle, soybean meal, soybean oil, soybeans, wheat, coffee C, corn). The earliest starting date is January 2006, and the end dates are generally in July-October 2020 (see the last three columns of Table 1). We apply standard filters outlined in the methodology of the CBOE VIX calculation (see CBOE (2019)) to filter out erroneous data points. For example, we exclude option prices with zero bids and zero open interest.

To construct the option portfolios, we use options with 28 days to expiration.[2] Given that various underlying commodities have different options and futures expiration dates, this approach ensures that the results are comparable across assets. We use options on the most liquid futures contracts for each asset.

Fig. 1 shows the average implied volatility curves for each asset, fitted to quadratic regressions across log-moneyness, and scaled by the average at-the-money volatility, for comparison across assets. While the level of implied volatility varies over time, these curves faithfully represent the general shape of the implied volatility curve across strike prices. The curves are all convex and display a "smile" or "smirk". However, there is significant heterogeneity as the slope of the smirk is different across assets. The S&P 500 displays the strongest negative slope, followed by crude oil. By contrast, coffee and the Japanese yen have the most positive slopes. We shall shortly relate the shape of these smirks to the statistical characteristics of futures returns.

Table 1 shows summary statistics on futures returns across the twenty assets. From a U.S. dollar perspective, the Australian dollar futures appreciated, while all other currency futures (British Pound, Canadian dollar, Swiss Franc, and Japanese Yen) lost money over our sample period. Natural gas and wheat lost more than 2% per month, while the S&P 500 gained 0.84%. The risk of the assets also varies significantly. Oil and natural gas futures have the highest risks, with standard

[2] Some assets have missing data on options with 28 days to maturity in some months, and then we use 29 or 30 days. The results are similar if we use only 28 days for the whole analysis.

deviations of 11% and 11.4%, respectively, while 10-Year Treasury Notes futures have the lowest risk, with 1.4% standard deviation. S&P 500 futures have the most prominent skewness at -1.16 and also a high kurtosis of 12.4. Oil has the highest positive skewness, and the largest kurtosis of 14.5. One must be cautious about exact comparison across commodities because they have somewhat different sample periods. However, the table illustrates that futures exhibit diverse behavior, and that the S&P 500 is atypical because it has the second largest mean (after silver), the largest negative skewness, and the second-largest kurtosis. The diverse behavior of futures across assets and the differences with the commonly studied S&P 500 index motivate our curiosity about the behavior of non-S&P-500 assets and their variance risk premiums.

2.2. Variance Portfolio Methodology

To explore the variance risk premium in option returns across assets, we aggregate options of different strike prices to form a representative variance portfolio for each asset. It is important to use a methodology that consistently accommodates assets with different volatilities and uses options with different strike prices. We do this in a model-free way, constructing option portfolios with payoffs that are highly correlated with actual realized variance, with a median correlation above 99% across assets. In this sense, these are truly "variance portfolios". By comparison, Table A.1 in the appendix shows that another commonly used approach to measure variance risk premiums based on at-the-money straddle returns, has relatively low correlation with realized variance.

The dominant benchmarks for option volatility use the CBOE VIX formula for S&P 500 Index options (CBOE (2019)). This formula has also been applied to options on Treasury bonds (TYVIX), currency (EYZ), and individual stocks (VXAPL, VXAZN, VXGOG, VXGS, VXIBM). The formula works well for S&P 500 options with many finely spaced strike prices. But the CBOE formula is inaccurate for some of our futures data, and occasionally even yields negative values (e.g., for Australian dollar and British pound on more than 10 days in our sample). To avoid these limitations of the CBOE formula, we construct variance portfolios using the more accurate Simpson's rule of Heston et al.

(2022).[3]

Let VIX^2 denote the variance index for horizon T (which equals four weeks in our analysis). Then $e^{-rT}VIX^2$ is the price of an option portfolio at time $t = 0$, where r is the risk-free interest rate. The Simpson's formula is:

$$VIX^2 = \frac{2}{T}e^{rT}\sum_i \frac{\Delta_i}{K_i^2}\min(Put(K_i), Call(K_i)) + \frac{\Delta}{3T}\left(\frac{F-K_0}{K_0^2}+\frac{K_1-F}{K_1^2}\right) + \frac{e^{rT}}{3T}\left(\frac{\Delta-\Delta_0}{K_0^2}Put(K_0)+\frac{\Delta-\Delta_1}{K_1^2}Call(K_1)\right)+\frac{1}{T}\left(\log(\frac{F^2}{K_0K_1})-\frac{F}{K_0}-\frac{F}{K_1}+2\right), \tag{1}$$

where $Put(K_i)$ and $Call(K_i)$ represent time $t = 0$ prices of put and call options with strike price K_i and expiration T. K_0 and K_1 denote strike prices just below and just above the forward price F, respectively, and the strike price intervals are $\Delta = K_1 - K_0$ and $\Delta_i = (K_{i+1} - K_{i-1})/2$. Note that this variance index is a portfolio of out-of-the-money options, the underlying futures contract, and the risk-free asset.

Our benchmark portfolio has the advantage of incorporating options of all available out-of-the-money strike prices, thereby diversifying any quotation noise in individual option prices. This benchmark maintains consistent sensitivity to variance because it always includes at-the-money options. In a certain sense, the VIX^2 portfolio is always at-the-money. In contrast, the Bakshi and Kapadia (2003), Goyal and Saretto (2009), and Dew-Becker et al. (2017) approaches of delta-hedging a single option or straddle generally have varying vega sensitivity as the underlying price drifts away from the option strike price.

The prices of our VIX^2 option portfolios measure the variance of options over a 28-day "month" before expiration. The corresponding annualized VIX level is $\sqrt{VIX^2 \times 365/28}$. Table 2 shows summary statistics of the VIX level across assets. Energy commodities (natural gas and oil) have the largest and one of the most volatile VIX, whereas currencies and 10 year Treasuries have the lowest

[3] In addition to superior accuracy, Simpson's rule is symmetric - it gives identical answers for foreign currency options from either currency's perspective.

and least volatile VIX. The S&P 500 VIX is ranked 11th across assets but its standard deviation is the third-largest, which illustrates the volatile nature of equity implied variance.

In addition to the level of variance measured by our VIX^2 option portfolios, we want to measure returns. Following Carr and Madan (2001), the payoff on our VIX^2 portfolio of options is approximately $2(S_T/F_0 - 1) - 2\log(S_T/F_0)$, where S_T denotes the spot price at expiration and F_0 denotes the initial forward price. In words, the VIX^2 portfolio effectively replicates two forward contracts minus two units of the log-contract. This replication is exact in the limit with a continuum of options across different strike prices. Following Bondarenko (2014) and Heston et al. (2022), the delta-hedge of our ideal VIX^2 portfolio at time t sells $2/F_0 - 2/F_t$ shares of the underlying asset, where F_t denotes the forward price at time t. Importantly, both the value of the VIX^2 index and the hedge are model-free, because they do not require estimation of any parameters. Carr and Wu (2009) show that the resulting hedged portfolio return replicates the realized variance in continuous time, and Heston et al. (2023) derive it as an approximation in discrete-time. We shall empirically demonstrate this with our daily-hedged VIX^2 benchmarks. Specifically, if r_u^F is the daily futures (excess) return on day u, then the hedged VIX^2 return from time t to time T is approximately determined by the realized daily variance of r_u^F

$$r_{t\to T}^{hedged\ VIX} \approx \frac{RV_{t,T}}{VIX_t^2} - 1, \tag{2}$$

where the realized variance is the sum of squared daily returns over holding period from time t until the option expiration T.

$$RV_{t,T} = \sum_{u>t}^{u\leq T} \left(r_u^F\right)^2.$$

To measure returns consistently across assets, we construct daily-rebalanced delta-hedged VIX^2 portfolio returns using non-overlapping 28-day periods for every asset.

It is important to note that we construct a tradable strategy and do not interpolate the volatility surface, unlike many papers studying the variance risk premium (e.g., Choi et al. (2017), Prokopczuk and Chardin (2014), Dew-Becker et al. (2017)). Strategies that interpolate the volatility surface rely on non-tradable prices which could be unrepresentative of market-observed prices (there are no actual quotes). Instead, our methodology is based on observed prices, and is implementable in practice. By holding options to expiration, our strategy also avoids paying multiple bid-ask spreads when rolling the option position. This reduces the impact of transaction costs on our results.

3. Variance Risk Premium Across Assets

3.1. Returns to Variance

The first column of numbers in Table 3 presents the Carr and Wu (2009) variance swap approximation (2) to the average returns on our model-free variance portfolios. This variance swap approximation to a monthly return is just the realized variance over the month divided by the VIX^2 at the beginning of the month, minus 1. The second column presents the actual realized returns on our model-free option portfolios. While these returns often differ by a few percent, the strong correspondence of the numbers across assets supports the usefulness of the approximation.

The Swiss Franc is the only asset with positive average returns, whereas all the other assets have negative variance returns.[4] Because they involve buying OTM options, our model-free variance portfolios have high leverage, and are quite risky as reflected in their standard deviations. 10 Year Treasury Notes have the lowest average return to variance portfolios at -25.8% per month followed by corn and live cattle. The S&P 500 has also a large negative average return, but this is highly sensitive to an outlying return during the March 2020 COVID month, when the option portfolio had an extremely large positive return above 2000% per month (see Fig. A.1). Before this month, the S&P 500 would have had the second largest (negative) variance risk premium of -22.9%. However, with the COVID month, the sample average variance risk premium is only -11.9%, giving the S&P 500 a smaller (negative) variance risk premium than most of the other assets. This shows

[4]The Swiss Franc had a large positive outlier return in January 2015 after the peg to the euro was removed.

that historical evidence about the variance risk premium based on S&P 500 returns is sensitive. The other assets also experienced large positive variance returns during the COVID month, but these were several orders of magnitude smaller than for the S&P 500, and we only report means over the full sample for these assets.

Fig. 2 shows that the S&P 500 is also atypical in the sense that it has the most negative volatility smile slope, and the largest negative correlation between variance returns and futures returns. The figure also shows that the slope of the volatility smile is increasing in the correlation between variance and futures returns, in line with Heston (1993).

3.2. *Correlations of Returns Across Assets*

Table 4, Panel A, reports the correlations of futures returns across assets and shows that these are mostly positive.[5] The correlation between Australian and Canadian dollar is 72%, and the correlation between gold and silver is 50%. Soybeans and soybean oil and soybean meal, as well as other agricultural commodities like corn and wheat, are also highly correlated. In contrast, the U.S. Treasury futures and Japanese Yen are negatively correlated with about half of the other assets. Natural gas futures is negatively correlated with oil and gold futures and with currencies.

Table 4, Panel B, reports correlations of variance returns across assets. Most variance returns are positively correlated with each other, including U.S. Treasuries. Unreported diagnostics show that the equally weighted average of variance returns for all assets is highly correlated with the first principal component, and serves a good proxy for systematic variance risk. Oil, S&P 500, and silver variance returns are highly correlated with this average, at 84%, 81%, and 75%, respectively (last row).[6] The correlations of oil, S&P 500, and silver variance returns with each other range from 66% to 75%. Hence, they are all similar proxies for systematic variance risk. The correlations among other variance returns are smaller.

[5]Note that one must take care when comparing correlations across commodities when they do not have identical sample periods.

[6]These correlations were somewhat lower if we remove the COVID month of March 2020.

Table 4, Panel C reports the correlation of futures returns with variance returns. Futures returns on a given asset are negatively correlated with variance returns on the same asset in 11 out of 20 cases. The most negative correlation is for the S&P 500, at -63%, followed by oil, at -59%. In contrast, the Swiss Franc, Japanese Yen, U.S. Treasury futures, and Natural Gas returns are positively correlated with their own variance returns and with the equally weighted average variance return across assets.

4. Comparing Theories of the variance risk premium

We consider three theoretical explanations of the variance risk premium based on: (i) systematic risk, (ii) hedging and liquidity risk, and (iii) conditional risk. We study how realized variance risk premiums across our assets relate to empirical proxies for each of these three explanations.

Perhaps the most established theories in asset pricing are based on systematic risk. In the Merton (1973) ICAPM, risk premiums are associated with intertemporal hedging demand for risks that affect investment opportunities. The level of systematic, marketwide variance is a natural candidate for a variable that affects the investment opportunity set.

An alternative theory is that options are difficult and costly to hedge. In a Black and Scholes (1973) world, options are dynamically spanned by underlying futures returns. However, jumps and fat tails in futures prices make perfect hedging impossible (Cochrane and Saa-Requejo (2000)). In addition, market illiquidity makes it difficult to manage risk quickly and efficiently. These microstructural frictions affect option dealers who supply options. Illiquid option markets require more capital to manage positions because they carry more risk. Instead of perfectly hedging risk in illiquid markets, dealers may choose to charge higher prices on options that are hard to hedge, which would make the variance risk premium more negative. This theory predicts that jumps and illiquidity are associated with larger negative variance risk premiums.

The theories of systematic risk and liquidity apply to static models and unconditional variance risk premiums. Bollerslev et al. (2009) show that the level of implied volatility can predict equity market returns, and reflects a changing conditional risk premium. They suggest that the equity risk premium varies due to changes in the level of risk, or changes in aggregate risk aversion. We apply their framework to our futures returns, and then extend it to successfully find predictability in variance returns.

4.1. Systematic Risk

We begin our theoretical exploration by examining whether the variance risk premium is related to systematic risk. Note that Table 2 and Table 3 show a strong association between the average *level* of VIX, and the standard deviation of hedged VIX^2 returns across assets. The cross-sectional correlation between the average VIX return and its standard deviation is also quite high, at .77 (an R^2 of .60) as illustrated by Fig. A.2 in the appendix. If this risk is systematic, then it might explain the returns on our VIX^2 portfolios.

In the context of option returns across many different markets, the meaning of "systematic risk" is ambiguous. Some options might inherit systematic risk from their correlation with the underlying commodity future returns. In the Capital Asset Pricing Model, systematic risk is measured by covariance with the market portfolio. In more general models, systematic risk might include a pervasive variance factor that affects option returns across assets.

Table 5 explores whether risk in the underlying futures returns can explain returns on the associated VIX^2 option portfolios. Note that our VIX^2 option portfolios are already delta-hedged every day. Nevertheless, returns on delta-hedged options can be correlated with returns on futures, due to stochastic volatility. The first column regresses the monthly hedged VIX^2 returns on contemporaneous futures (excess) returns for each commodity.[7] Under the null hypothesis that futures risk explains options risk, the regression intercept should be zero. The intercepts represent "beta-hedged" returns on our VIX^2 option portfolios. The intercepts for the S&P 500 and Swiss franc are insignificantly positive. For the rest of the assets, the intercepts are negative, just like

[7] Percentage changes in futures returns prices are excess returns because they represent the profit from an uncollateralized futures position.

the mean VIX^2 returns in Table 1, and many of them are significant. Hence, futures risk does not seem to explain options risk.[8]

If futures risk is idiosyncratic, then it would not explain option risk. Instead, it is possible that option returns have exposure to risk from the aggregate equity market portfolio, or from a systematic variance factor. Unreported diagnostics indicate that the first principal component of the VIX^2 returns across assets is highly correlated with an equally weighted average of these returns. Moreover, the bottom row of Table 4, Panel B shows that the S&P 500 VIX^2 return has 81% correlation with the equally weighted portfolio of all assets' VIX^2 returns. In other words, this systematic variance factor is highly correlated with the S&P 500 VIX^2 return, a proxy for the variance of the equity market portfolio.[9]

The right side of Table 5 (columns 6-10) uses the average VIX^2 returns across all assets as a systematic risk factor, by making it the independent variable in regressions of individual asset VIX^2 returns. Again, this risk adjustment does not explain the mean VIX^2 returns for the majority of assets. The intercepts remain largely negative and often statistically significant at conventional levels.

Fig. 3 and Fig. 4 illustrate the failure of futures returns and average VIX^2 betas to explain VIX^2 returns across assets. Fig. 3 shows a positive cross-sectional association between the underlying futures return and the average realized VIX^2 return. Recall that the summary statistics in Table 1 showed 11 out of 20 assets have a positive futures risk premium, whereas Table 3 reports that only 1 out of 20 has a positive variance risk premium. The positive cross-sectional relationship means that futures with a larger risk premium tend to have options with a smaller absolute variance risk premium. Similarly, Fig. 4 shows the relationship between average realized VIX^2 returns, and the VIX^2 betas with respect to the average VIX^2 portfolio. Just as in Fig. 3, the rela-

[8] Dew-Becker and Giglio (2022) construct beta-hedged option replication strategies with Black-Scholes hedges and reach similar conclusions about the variance risk premium on the S&P 500.

[9] The last row of Table 4, Panel B also shows a high correlation of the equally weighted VIX^2 portfolio with oil (84%) and silver (75%).

tionship is positive. This means that assets with higher systematic covariance risk tend to have a less negative variance risk premium.[10]

4.2. Liquidity and Hedging

In a Black and Scholes (1973) world, dealers can perfectly hedge option risk by continuously trading the underlying futures. In practice, short-sale constraints can create a demand for options (Bakshi et al. (2010), Hong and Stein (2003)), leaving option dealers with net short positions. This exposes dealers to volatility risk (Hull and White (1987), Heston (1993)). In addition, OTM options can be particularly difficult for dealers to hedge (Cochrane and Saa-Requejo (2000)). If hedging is difficult or costly, then option dealers need more capital to support their option positions. Under such circumstances, it is natural for dealers to increase option prices to compensate for the risk and cost of their positions. According to this theory, we expect to see larger, more negative variance risk premiums in option markets with illiquidity or fat-tailed jumps in prices.

To test this theory, we consider two simple measures of illiquidity. The first is the log of the volume ratio of futures contracts to options contracts. A high ratio indicates low trading in options markets, which should be associated with a more negative variance risk premium. Table 3 shows that the log volume ratio varies greatly across different futures, from 5.7 in Japanese yen, to 16.6 in soybean meal.

The second measure of illiquidity is the average deviation of option quotations from put-call parity, as a percentage of the futures price.[11] If markets are active and liquid, then these apparent arbitrage opportunities should be small and fleeting. Table 1 shows that the measure is a fraction of a percent, but varies across futures markets.[12]

[10] There could other systematic factors that describe variance risk. However, our intuition is that the average VIX^2 return across assets should be a decent proxy for such risk. It is also possible that systematic risk changes over time, but our inspection of subsample returns suggests this is not a promising explanation.

[11] Unfortunately, the data on bid-ask spreads is of poor quality for many assets and days. This is why we did not use bid-ask spreads as measures of liquidity.

[12] A possible caveat is that option dealers might quote prices of less actively traded put (call) options by applying put-call parity to the more actively traded call (put) option, in which case put-call deviations should be small.

Finally, we measure jumps or fatness of tails by the skewness and kurtosis of daily futures returns in the previous month. These statistics measure departures from a Gaussian distribution that create jumps or outliers, which are difficult to hedge. Table 1 shows that many assets display large kurtosis, e.g., oil, the S&P 500, corn and the Swiss franc. If dealers are short volatility, then high kurtosis in futures returns could increase risk for dealers due to jumps. Liquidity preference theories suggest that skewness or kurtosis might be associated with increased magnitude of the variance risk premium.

Table 6 presents cross-sectional Fama and MacBeth (1973) regressions to measure the effect of illiquidity and fat-tails on the variance risk premium.[13] We also include the level and standard deviation of VIX returns to avoid confounding the effects of liquidity and jump risk with those of the general level and risk of implied variance. The table shows that skewness and put-call parity deviations are not significant at conventional levels, whereas the cross-sectional effect of volume is only significant at the 5% (two-tailed) level in the univariate specification but not in the multivariate one. In contrast, kurtosis and the level of VIX are both significant at the 1% level. Note that the intercepts largely remain highly significant compared to the slopes of the cross-sectional regressions. These facts indicate that liquidity characteristics explain little cross-sectional risk in VIX^2 returns, and that the unconditional variance risk premium is large compared to the variation explained by liquidity. These results provide mild support for the theory that the variance risk premium is larger in illiquid option markets.

4.3. Conditional Returns and Predictive Regressions

The two previous subsections used exposure to systematic risk and liquidity to explain average returns on hedged VIX^2 portfolios, i.e., the realized unconditional variance risk premium. This section investigates the possibility of a conditional variance risk premium. Bollerslev et al. (2009) suggest that the premium on implied variance minus historical variance provides an ex-

[13]Unreported diagnostics show low correlation among the explanatory variables in Table 6.

ante measure of biased expectations. The authors then use this measure to forecast the S&P 500 equity premium. We follow a similar procedure, and extend it to additionally forecast the returns on our VIX^2 portfolios.

Bollerslev et al. (2009) run a time series regression to predict the underlying asset's returns based on the premium of implied variance over historical variance:

$$r^F_{t\to T} = \alpha + \beta(VIX^2_t - RV_{t-1,t}) + \epsilon_T, \tag{3}$$

where $r^F_{t\to T}$ is the S&P 500 (futures) return from month t to T, and $RV_{t-1,t}$ is realized variance of daily returns measured by the sum of daily squared S&P 500 returns over the previous month. Bollerslev et al. (2009) interpret a positive value of the slope coefficient β to mean that a high level of option implied variance (relative to historical realized variance) signifies a high level of systematic risk and a correspondingly high conditional equity premium on the underlying S&P 500.

Although Bollerslev et al. (2009) used a measure of implied variance in excess of historical variance to forecast equity returns, this measure is even more relevant for forecasting the variance risk premium. Therefore, we derive a forecasting equation for the variance risk premium analogous to the equity premium regression (3). The variance expectations hypothesis states that the risk-neutral option variance is the expectation of future realized variance:

$$VIX^2_t = E_t[RV_{t,t+1}]. \tag{4}$$

We can nest the expectations hypothesis (4) in a linear regression

$$RV_{t,t+1} - VIX^2_t = \alpha + \beta(VIX^2_t - RV_{t-1,t}) + \epsilon_{t+1}. \tag{5}$$

By law of iterated expectations, the expectations hypothesis (4) predicts that past realized variance has no predictive power for future realized variance in excess of VIX^2, so the slope coefficient β in regression (5) should be zero. The left side of the expectations regression (5) is approximately the VIX^2 return. Substituting the approximation from equation (2) shows a forecasting equation for the variance premium that parallels Bollerslev et al. (2009) equation (3)

$$r_{t\to t+1}^{hedged\ VIX} \times VIX_t^2 \approx \alpha + \beta(VIX_t^2 - RV_{t-1,t}) + \epsilon_{t+1}. \tag{6}$$

We interpret a significant slope coefficient β as evidence of predictable variation in the conditional risk premium of VIX^2 portfolio returns.

Columns 1-4 of Table 7 show the results of the Bollerslev et al. (2009) time-series regression (3) for horizons ranging from one month (T=1) to one year (T=12). The table shows that most of the estimates are statistically insignificant at the 5% level since only 4-5 out of 20 futures have significant estimates across maturities.[14] Our estimate for the 3-month horizon for the S&P 500 is positive and close to the estimate in Bollerslev et al. (2009), but the estimates for 6-month and 12-month horizons change to negative sign, contrary to the positive and significant coefficients found in Bollerslev et al. (2009). In addition, some currencies (British pound, Australian dollar) and 30 year Treasuries have negative estimates, contrary to the positive estimates found for the S&P 500 in Bollerslev et al. (2009).

Column 5 of Table 7 shows the estimates of the variance expectations regression (6) and illustrates that the results for VIX^2 returns are more significant than for futures returns. For VIX^2 returns, most of the assets have significant estimates at the 5% level.[15] Except for currencies, the majority of assets have positive regression slope estimates, meaning that larger realized implied variance (in excess of historical variance) is associated with larger future variance returns. In other

[14]We report t-statistics in the table instead of standard errors, in line with the format used in the original Bollerslev et al. (2009) paper.

[15]In unreported results, we also run the variance expectations regressions for horizons of 3, 6, and 12 months, similar to the maturities for Panel A. The results were unchanged: most of the assets had significant estimates at the 5% level.

words, the time series of implied variance positively forecasts the future variance premium. These results show that the returns on VIX^2 portfolios are predictable and reject the variance expectations hypothesis. This finding is consistent with the empirical results of Giglio and Kelly (2018) on variance swaps.

Bollerslev et al. (2009) suggest that a conditional variance risk premium could reflect changing levels of risk, or changing risk aversion. Table 7 shows that the evidence for conditional variability in the variance risk premium is much stronger than the evidence for predictability of the equity premium, and that it extends to more assets. These conditional variance risk premiums augment our previous evidence on the diverse behavior of unconditional variance risk premiums across assets. Future research can develop theoretical asset pricing models to simultaneously explain the variance risk premiums across assets and the predictable conditional variance risk premiums within assets.

5. Conclusions

This paper explores the variance risk premium across twenty different assets using a novel model-free methodology that measures returns on option portfolios, using a model-free hedge. We find that to some extent, the commonly studied S&P 500 options are representative of options in other assets. Specifically, options on all assets exhibit a volatility smile, and all assets but the Swiss Franc have a negative realized variance risk premium. However, there are significant differences between S&P 500 options and options on other assets. For example, S&P 500 options have the largest negative option smirk, while many commodity options have a positive smirk. The S&P 500 has the second-largest negative variance risk premium excluding the March 2020 COVID crisis, and experienced the largest positive variance return (above *2000%*) across all assets during the COVID crisis.

We find little evidence that the variance risk premium across assets is associated with systematic variance risk. Instead, assets that are positively correlated with systematic variance tend to have a smaller (negative) variance risk premium. However, we find mild evidence that the variance risk premium is associated with hedging constraints as proxied by illiquidity measures (ratio of option volume to futures volume) and measures of jump risk (kurtosis).

Finally, we test whether the Bollerslev et al. (2009) measure of implied variance in excess of historical variance can predict variance and futures returns. In contrast to Bollerslev et al. (2009)'s findings, we find that this measure *negatively* predicts futures returns in our sample for several assets and is insignificant for the majority of assets. The measure does not significantly predict S&P 500 returns except at the 3-months horizon. We also find that the measure successfully predicts returns on VIX^2 option portfolios for most of the assets. These findings suggest that implied variance provides a useful measure of the conditional market variance risk premium. Our results indicate that variance risk premium is not well explained by existing theories and that future research could develop better explanations for unconditional and conditional variance risk premiums across assets.

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Tables and Figures

Table 1
Summary statistics of futures returns
The Table shows the mean, standard deviation, Sharpe ratio, skewness, kurtosis and number of observations for 28-day futures returns as well as the start and end dates and the exchange on which options are traded. Means and standard deviations are in %, Sharpe ratio is annualized. The sample period is generally from January 2006 to October 2020; some assets have missing data on options with 28 days to maturity in some months. *,**, and *** indicate statistical significance of the t-statistics for the mean return at the 10%, 5%, and 1% levels, respectively.

	Futures returns								
	Mean	Std	Sharpe	Skew	Kurt	Obs.	Start date	End date	Exchange
Australian Dollar	0.17	3.30	0.18	0.07	4.12	168	2007-02	2020-10	CME
British Pound	-0.13	2.59	−0.18	-0.60	3.96	166	2006-01	2020-10	CME
Canadian Dollar	-0.07	2.30	−0.11	0.02	3.47	152	2006-01	2020-09	CME
Swiss Franc	-0.07	2.94	−0.11	-0.03	5.22	140	2007-01	2020-08	CME
Japanese Yen	-0.07	2.57	−0.11	-0.24	3.64	167	2006-01	2020-09	CME
S&P 500	0.84	5.33	0.58^{**}	-1.16	12.36	167	2006-01	2020-06	CME
10 Year UST	0.26	1.37	0.69^{**}	0.21	4.11	120	2006-02	2020-04	CBOT
30 Year UST	0.35	2.74	0.47	0.07	3.83	127	2006-02	2020-03	CBOT
Brent Crude Oil	0.46	10.97	0.14	0.61	14.49	94	2011-08	2020-09	NYMEX
Natural Gas	-2.61	11.41	-0.83^{***}	0.26	4.34	162	2006-01	2020-10	NYMEX
Gold	0.79	4.52	0.61^{**}	0.15	2.64	160	2006-01	2020-09	COMEX
Silver	1.01	8.68	0.43	0.30	3.22	152	2006-08	2020-09	COMEX
Copper	0.36	6.48	0.18	0.30	3.95	155	2006-01	2020-07	COMEX
Live Cattle	-0.29	4.74	−0.22	-0.30	4.55	127	2006-01	2020-08	CME
Soybean Meal	0.07	7.38	0.04	0.37	3.26	145	2006-02	2020-08	CBOT
Soybean Oil	0.03	5.56	0.04	0.51	3.60	143	2007-01	2020-08	CBOT
Soybeans	0.56	6.59	0.29	-0.50	4.30	144	2006-02	2020-08	CBOT
Wheat	-2.19	8.18	-0.97^{***}	0.05	3.37	131	2006-02	2020-07	CBOT
Coffee C	-0.69	8.24	−0.29	0.42	3.00	110	2007-01	2020-09	ICE
Corn	-1.49	8.02	-0.69^{**}	0.25	5.98	126	2006-02	2020-07	CBOT

Table 2
VIX summary statistics
The table shows summary statistics on VIX levels. These are computed as $\sqrt{VIX^2 \times 365/28}$, where VIX^2 is the monthly implied variance. "Mean RVol" is the average realized volatility (also annualized). The sample period is from January 2006 to October 2020.

	Mean	Median	Std (%)	Min	Max	Skew	Kurt	Mean RVol
Australian Dollar	12.4	11.2	5.2	6.4	39.6	2.4	10.9	10.6
British Pound	10.3	9.4	4.0	4.9	29.7	2.3	9.9	8.1
Canadian Dollar	9.7	8.7	3.8	4.6	27.5	1.9	8	7.7
Swiss Franc	10.2	9.8	3.6	3.9	24.2	1.2	5.1	9.4
Japanese Yen	10.9	10.2	3.8	4.9	30.1	1.6	7.8	8.1
S&P 500	22.2	17.6	12.8	9.7	78.0	1.8	6.2	17.3
10 Year UST	6.3	5.8	2.0	3.4	14.7	1.3	5.1	4.2
30 Year UST	11.1	10.5	3.4	6.5	24.0	1.3	4.9	8.2
Brent Crude Oil	38.5	31.0	24.8	17.3	155.0	3.0	13.1	30.1
Natural Gas	50.6	45.7	17.8	21.9	111.7	1.0	3.7	37.7
Gold	19.0	18.0	6.5	9.5	51.2	1.6	7.3	14.5
Silver	33.7	32.7	12.4	15.6	78.6	1.2	4.7	26.4
Copper	29.4	26.9	11.3	13.5	76.6	1.6	6.3	21.2
Live Cattle	19.1	18.2	7.2	8.1	68.3	3.0	19.6	13.7
Soybean Meal	29.3	28.4	8.2	14.4	53.5	0.7	3.4	20.9
Soybean Oil	24.7	23.3	6.8	15.2	70.5	2.7	16.8	18.1
Soybeans	25.2	24.1	8.1	12.1	56.1	1.4	5.0	18.6
Wheat	33.5	30.4	10.8	19.1	71.9	1.2	4.2	25.1
Coffee C	35.6	34.2	7.4	22.0	56.5	0.4	2.6	25.1
Corn	32.4	31.3	9.3	17.2	64.9	0.7	3.3	22.9

Table 3

Summary statistics of hedged VIX^2 returns

The table shows the mean variance swap return as well as the mean, standard deviation, Sharpe ratio, skewness, kurtosis for hedged VIX^2 returns, the ratio of trading volume, put-call deviations and correlations. Hedged VIX^2 returns are the returns on a dynamically hedged VIX^2 portfolios formed 28 days before options' maturity. Means and standard deviations are in %, Sharpe ratio is annualized. "Mean VSR" is the Carr and Wu (2009) average variance swap return approximation, "Volume ratio" is log ratio of the average futures' trading volume to that of options. "Put-call" is the average absolute put-call-parity deviation computed as the difference between call price and the price of the replicating portfolio (put plus spot price minus the present value of the strike price) as percentage of the spot price across all options and days. $\rho(r_T^F, r_T^{VIX})$ is the correlation coefficient between futures returns and hedged VIX^2 returns. The sample period is from January 2006 to October 2020; some assets have missing data on options with 28 days to maturity in some months. *,**, and *** indicate statistical significance of the t-statistics for the mean VIX^2 return at the 10%, 5%, and 1% levels, respectively.

	Hedged VIX^2 returns and option measures								
	Mean VSR (%)	Mean (%)	Std (%)	Sharpe	Skew	Kurt	Volume ratio	Put-call (%)	$\rho(r_T^F, r_T^{VIX})$
Australian Dollar	-6.4	-2.1	57.4	-0.14	4.0	27.4	10.4	0.08	-0.36
British Pound	-11.8	-9.4	75.2	-0.46	8.44	94.31	10.7	0.08	-0.22
Canadian Dollar	-13.5	-8.8	61.2	-0.52^{*}	4.6	35.0	10.6	0.05	-0.20
Swiss Franc	14.2	14.8	274.3	0.19	11.2	130.5	12.7	0.08	0.30
Japanese Yen	-19.6	-16.6	61.4	-0.98^{***}	2.7	13.1	5.7	0.08	0.27
S&P 500	-13.1	-11.9	192.0	-0.23	8.6	90.7	14.5	0.05	-0.63
S&P 500 pre-Covid	-24.4	-22.9	100.0	-0.86^{***}	4.56	26.01	14.6	0.05	-0.65
10 Year UST	-31.9	-25.8	31.5	-2.97^{***}	1.4	6.2	12.7	0.09	0.01
30 Year UST	-18.8	-14.4	37.4	-1.41^{***}	1.6	5.9	13.4	0.09	0.16
Brent Crude Oil	-17.2	-12.2	96.2	-0.46	5.4	39.7	16.3	0.05	-0.59
Natural Gas	-15.3	-14.1	62.2	-0.80^{***}	6.4	61.3	10.7	0.09	0.20
Gold	-12.7	-11.7	101.0	-0.42	8.4	88.5	15.9	0.08	-0.08
Silver	-3.7	-5.5	144.1	-0.11	3.5	48.0	12.9	0.33	-0.25
Copper	-18.8	-17.6	45.5	-1.41^{***}	1.8	6.8	14.0	0.12	-0.28
Live Cattle	-25.2	-22.0	52.5	-1.52^{***}	2.8	14.0	13.4	0.12	-0.38
Soybean Meal	-23.2	-15.7	40.9	-1.40^{***}	1.5	5.8	16.6	0.11	0.16
Soybean Oil	-19.2	-16.7	43.3	-1.40^{***}	1.6	7.1	14.0	0.09	-0.04
Soybeans	-24.5	-21.4	42.4	-1.95^{***}	1.5	6.0	15.8	0.08	-0.19
Wheat	-15.2	-13.4	39.6	-1.30^{***}	1.1	4.6	15.7	0.08	0.23
Coffee C	-23.3	-18.5	42.7	-1.57^{***}	1.7	7.9	13.3	0.08	0.38
Corn	-22.8	-22.9	50.3	-1.71^{***}	1.0	6.6	15.4	0.09	0.19

Table 4

Correlations of futures returns and hedged VIX^2 returns

The table shows monthly correlations of futures returns (Panel A), hedged VIX^2 returns (Panel B), hedged VIX^2 returns and futures returns (Panel C). The last line "Eq. Weighted" shows the correlations of the equally weighted index with the particular asset. In Panel C, "F" stands for futures return, "V" for VIX^2 return.

Panel A: Futures correlations

	Australian Dollar	British Pound	Canadian Dollar	Swiss Franc	Japanese Yen	S&P 500	10 Year UST	30 Year UST	Brent Oil	Natural Gas	Gold	Silver	Copper	Live Cattle	Soybean Meal	Soybean Oil	Soybeans	Wheat	Coffee C	Corn
British Pound	0.45																			
Canadian Dollar	0.72	0.44																		
Swiss Franc	0.39	0.53	0.35																	
Japanese Yen	0.05	-0.02	-0.07	0.30																
S&P 500	0.07	-0.06	0.09	-0.13	-0.20															
10 Year UST	-0.01	-0.15	-0.25	-0.12	0.16	-0.47														
30 Year UST	-0.10	-0.21	-0.30	-0.32	0.10	-0.26	0.94													
Brent Crude Oil	0.23	0.24	0.27	-0.03	-0.14	0.27	-0.17	-0.23												
Natural Gas	-0.10	-0.01	-0.10	-0.01	-0.11	0.15	-0.10	-0.04	-0.17											
Gold	0.03	-0.06	0.04	-0.03	0.20	-0.12	0.26	0.18	0.16	-0.08										
Silver	0.24	0.11	0.13	-0.01	0.11	0.07	0.10	0.05	0.20	0.10	0.50									
Copper	0.20	0.19	0.29	0.29	-0.10	0.31	-0.34	-0.42	0.31	0.07	0.09	0.24								
Live Cattle	0.17	0.18	0.08	-0.09	-0.22	-0.01	0.02	0.03	0.26	0.04	-0.01	0.03	-0.07							
Soybean Meal	0.20	0.27	0.20	0.32	0.00	0.10	-0.05	-0.15	0.08	0.19	-0.07	0.04	0.24	-0.04						
Soybean Oil	0.27	0.22	0.27	0.25	0.09	0.11	-0.09	-0.19	0.21	0.05	0.06	0.12	0.25	0.05	0.39					
Soybeans	0.22	0.22	0.19	0.21	-0.07	0.28	-0.12	-0.14	0.09	0.24	-0.12	0.08	0.14	-0.02	0.74	0.6				
Wheat	-0.12	0.00	-0.18	-0.10	0.04	0.03	0.09	0.08	-0.10	0.08	-0.06	0.08	0.06	0.10	0.39	0.14	0.43			
Coffee C	0.17	-0.01	0.22	-0.14	-0.12	0.08	0.03	0.02	0.01	0.25	0.26	0.16	0.05	0.10	0.07	0.22	0.26	0.22		
Corn	-0.01	0.08	-0.10	0.01	0.04	0.15	0.09	0.10	0.01	0.33	-0.05	0.12	-0.04	0.11	0.52	0.34	0.68	0.66	0.23	
Eq. Weighted	0.37	0.33	0.31	0.23	0.02	0.36	-0.01	-0.08	0.51	0.48	0.18	0.49	0.39	0.24	0.66	0.57	0.74	0.52	0.43	0.73

Panel B: Hedged VIX^2 returns correlations

	Australian Dollar	British Pound	Canadian Dollar	Swiss Franc	Japanese Yen	S&P 500	10 Year UST	30 Year UST	Brent Oil	Natural Gas	Gold	Silver	Copper	Live Cattle	Soybean Meal	Soybean Oil	Soybeans	Wheat	Coffee C	Corn
British Pound	0.64																			
Canadian Dollar	0.77	0.68																		
Swiss Franc	0.13	0.44	0.27																	
Japanese Yen	0.32	0.17	0.29	0.03																
S&P 500	0.15	0.10	0.10	0.03	0.47															
10 Year UST	0.08	-0.03	0.07	0.00	0.23	0.11														
30 Year UST	0.08	-0.06	0.06	-0.04	0.28	0.33	0.81													
Brent Crude Oil	0.06	0.14	0.01	0.23	0.52	0.75	0.19	0.18												
Natural Gas	0.01	0.09	0.00	-0.05	0.10	0.08	-0.07	-0.09	0.25											
Gold	-0.01	0.06	0.02	0.00	0.20	0.26	0.02	0.17	0.11	0.03										
Silver	0.00	0.06	-0.05	0.04	0.34	0.66	0.08	0.12	0.66	0.11	0.53									
Copper	0.11	-0.06	0.06	-0.06	0.26	0.33	0.06	0.18	0.27	0.04	0.27	0.37								
Live Cattle	0.35	0.41	0.35	0.02	0.27	0.22	-0.15	-0.07	0.47	0.16	0.05	0.24	0.10							
Soybean Meal	-0.11	-0.06	-0.04	0.00	0.05	-0.01	-0.04	-0.05	-0.05	0.02	0.01	0.09	0.13	0.04						
Soybean Oil	0.01	0.00	-0.07	0.08	0.09	0.13	0.16	0.23	0.11	-0.04	0.24	0.11	0.10	-0.02	0.33					
Soybeans	0.06	-0.04	0.06	-0.02	0.13	0.19	0.01	0.06	0.07	-0.02	0.04	0.12	0.21	0.13	0.57	0.25				
Wheat	-0.06	-0.02	0.00	0.06	0.08	0.08	0.05	0.08	0.10	-0.09	0.13	0.16	0.19	-0.02	0.19	0.10	0.33			
Coffee C	0.18	0.14	0.16	-0.01	0.14	0.03	0.15	0.15	0.26	0.19	0.08	0.13	-0.09	0.27	0.11	0.14	0.21	0.20		
Corn	-0.07	-0.09	-0.10	-0.01	-0.01	-0.22	0.25	0.12	0.11	-0.12	0.15	0.22	0.23	-0.09	0.28	0.04	0.33	0.47	0.04	
Eq. Weighted	0.32	0.32	0.34	0.37	0.55	0.81	0.25	0.31	0.84	0.17	0.45	0.75	0.39	0.39	0.2	0.31	0.32	0.28	0.28	0.21

Table 4
Correlations of futures returns and hedged VIX^2 returns
(Continued from previous page)

Panel C: Futures-Hedged VIX^2 returns cross-correlations

	Australian Dollar, F	British Pound, F	Canadian Dollar, F	Swiss Franc, F	Japanese Yen, F	S&P 500, F	10 Year UST, F	30 Year UST, F	Brent Oil, F	Natural Gas, F	Gold, F	Silver, F	Copper, F	Live Cattle, F	Soybean Meal, F	Soybean Oil, F	Soybeans, F	Wheat, F	Coffee C, F	Corn,F	Eq. Weighted, F
Australian Dollar, V	-0.36	-0.11	-0.25	-0.04	0.17	0.03	0.11	0.03	-0.25	0.03	0.16	0.03	0.03	-0.21	-0.07	-0.03	-0.13	0.06	0.09	-0.03	-0.06
British Pound, V	-0.25	-0.22	-0.21	-0.12	0.04	0.26	0.03	0.04	-0.3	0.04	0.11	-0.02	0.06	-0.26	-0.12	-0.11	-0.12	-0.06	0.01	-0.12	-0.15
Canadian Dollar, V	-0.29	-0.17	-0.20	-0.09	0.07	0.09	0.23	0.15	-0.18	-0.03	0.12	-0.03	0.01	-0.30	-0.10	-0.11	-0.11	-0.10	0.03	-0.14	-0.16
Swiss Franc, V	-0.11	0.13	-0.17	0.30	0.10	0.03	-0.16	-0.18	-0.28	0.00	-0.16	-0.12	0.10	-0.10	0.05	0.00	0.04	-0.02	-0.13	0.01	-0.05
Japanese Yen, V	-0.17	-0.06	-0.11	0.13	0.27	-0.35	0.26	0.25	-0.37	0.02	0.08	-0.08	-0.16	-0.21	0.09	0.09	0.05	0.14	-0.06	0.12	-0.10
S&P 500, V	-0.06	-0.01	-0.09	0.12	0.21	-0.63	0.30	0.20	-0.49	-0.07	0.11	-0.19	-0.22	-0.22	0.08	-0.06	-0.15	0.04	-0.08	-0.07	-0.36
10 Year UST, V	-0.19	-0.05	-0.01	0.09	0.11	-0.20	0.01	0.04	-0.13	-0.01	0.09	-0.03	0.09	-0.08	-0.15	-0.07	-0.08	-0.05	0.08	-0.09	-0.07
30 Year UST, V	-0.15	-0.03	-0.07	0.03	0.18	-0.38	0.13	0.16	-0.17	-0.09	0.08	-0.03	-0.10	-0.06	-0.14	-0.12	-0.14	-0.03	0.03	-0.10	-0.17
Brent Crude Oil, V	-0.12	-0.12	-0.14	0.16	0.23	-0.46	0.15	0.16	-0.59	0.07	-0.04	-0.21	-0.29	-0.36	-0.01	-0.01	-0.09	0.09	0.06	0.04	-0.35
Natural Gas, V	0.04	-0.07	-0.04	-0.03	0.01	-0.04	-0.04	-0.02	-0.23	0.20	0.04	0.01	-0.01	-0.19	-0.01	-0.03	-0.04	-0.02	0.03	-0.01	-0.01
Gold, V	-0.08	0.01	0.02	0.01	0.00	-0.21	0.11	0.15	-0.13	0.01	-0.08	-0.18	-0.14	-0.08	0.00	-0.12	-0.06	0.01	-0.08	-0.11	-0.13
Silver, V	-0.12	0.01	-0.06	0.09	0.09	-0.35	0.05	0.07	-0.42	-0.01	-0.22	-0.25	-0.17	-0.26	-0.09	-0.16	-0.16	-0.13	-0.08	-0.10	-0.35
Copper, V	-0.13	-0.08	-0.14	-0.17	0.11	-0.22	0.18	0.19	-0.23	0.12	-0.09	-0.06	-0.28	0.00	-0.07	0.02	-0.01	0.15	-0.06	0.08	-0.09
Live Cattle, V	-0.20	-0.11	-0.14	-0.06	0.08	0.04	-0.08	-0.03	-0.25	0.10	0.08	0.06	-0.11	-0.38	-0.18	-0.01	-0.23	-0.15	0.13	-0.16	-0.2
Soybean Meal, V	0.06	-0.10	-0.05	-0.07	-0.15	-0.03	-0.03	0.01	0.00	0.15	-0.01	0.15	-0.14	0.06	0.16	-0.11	0.09	0.10	0.07	0.07	0.09
Soybean Oil, V	-0.04	-0.05	-0.12	-0.03	0.02	-0.16	-0.03	0.04	-0.24	0.08	-0.01	0.05	-0.08	0.08	-0.06	-0.04	-0.13	-0.12	-0.02	-0.02	-0.08
Soybeans, V	-0.02	-0.15	-0.14	-0.11	-0.04	-0.27	0.26	0.22	-0.13	0.06	0.12	0.16	-0.15	-0.02	-0.03	-0.17	-0.19	0.06	0.03	0.02	-0.01
Wheat, V	-0.02	0.02	-0.05	0.03	0.03	-0.12	0.08	0.09	-0.16	0.02	-0.08	-0.03	-0.12	0.09	0.12	-0.07	0.00	0.23	0.08	0.07	0.05
Coffee C, V	-0.07	-0.07	0.00	0.04	-0.03	-0.11	-0.07	-0.04	-0.19	0.16	0.13	0.04	-0.03	-0.20	0.13	0.00	0.12	0.07	0.38	0.11	0.12
Corn, V	-0.02	-0.07	-0.03	0.02	-0.03	0.08	0.12	0.17	-0.16	0.20	-0.18	0.02	-0.02	0.06	-0.04	-0.22	0.06	0.21	0.21	0.19	0.2
Eq. Weighted, V	-0.21	-0.08	-0.20	0.15	0.20	-0.43	0.13	0.12	-0.60	0.05	-0.07	-0.20	-0.19	-0.32	-0.03	-0.13	-0.12	0.00	-0.02	-0.02	-0.29

Table 5
Regressions of hedged VIX^2 returns
Hedged VIX^2 returns are the returns on a dynamically hedged VIX^2 portfolios formed 28 days before options' maturity. α and β are the estimates of a regression of hedged VIX^2 returns on the futures excess return: $r_t^{VIX} = \alpha + \beta r_t^F + \epsilon_t$. α_{EW} and β_{EW} are the estimates of a regression of hedged VIX^2 returns on the equally weighted VIX^2 return across assets: $r_t^{VIX} = \alpha_{EW} + \beta_{EW} r_t^{VIX_{EW}} + \epsilon_t$. "se" stands for standard error, "IR" for annualized information ratio (the ratio of α to the standard deviation of ϵ_t). The sample period is from January 2006 to October 2020; some assets have missing data on options with 28 days to maturity in some months. *,**, and *** indicate statistical significance of the t-statistics for α and α_{EW} at the 10%, 5%, and 1% levels, respectively.

	α (1)	α se (2)	β (3)	β se (4)	IR (5)	α_{EW} (6)	α_{EW} se (7)	β_{EW} (8)	β_{EW} se (9)	IR_{EW} (10)
Australian Dollar	-0.01	0.04	-6.25	1.26	−0.07	0.03	0.04	0.41	0.09	0.20
British Pound	-0.10	0.06	-6.49	2.21	−0.49*	-0.02	0.06	0.56	0.13	−0.10
Canadian Dollar	-0.09	0.05	-5.25	2.13	−0.54*	-0.04	0.05	0.45	0.10	−0.25
Swiss Franc	0.17	0.22	28.00	7.58	0.23	0.37	0.22	2.11	0.45	0.52*
Japanese Yen	-0.16	0.05	6.49	1.79	−0.97***	-0.07	0.04	0.77	0.09	−0.49*
S&P 500	0.07	0.12	-22.67	2.18	0.17	0.31	0.09	3.47	0.19	1.00***
10 Year UST	-0.26	0.03	0.22	2.11	−2.97***	-0.22	0.03	0.27	0.10	−2.59***
30 Year UST	-0.15	0.03	2.21	1.20	−1.46***	-0.10	0.03	0.36	0.10	−1.01***
Brent Crude Oil	-0.10	0.08	-5.14	0.74	−0.46	0.10	0.06	1.54	0.10	0.68*
Natural Gas	-0.11	0.05	1.09	0.42	−0.65**	-0.11	0.05	0.23	0.11	−0.65**
Gold	-0.10	0.08	-1.82	1.77	−0.36	0.10	0.08	1.53	0.24	0.40
Silver	-0.01	0.11	-4.13	1.31	−0.03	0.20	0.08	2.30	0.17	0.75***
Copper	-0.17	0.04	-1.97	0.54	−1.40***	-0.13	0.03	0.39	0.07	−1.12***
Live Cattle	-0.23	0.04	-4.22	0.92	−1.71***	-0.18	0.04	0.42	0.09	−1.34***
Soybean Meal	-0.16	0.03	0.87	0.46	−1.42***	-0.12	0.04	0.27	0.11	−1.08***
Soybean Oil	-0.17	0.04	-0.30	0.66	−1.41***	-0.09	0.04	0.47	0.12	−0.79***
Soybeans	-0.21	0.03	-1.22	0.53	−1.82***	-0.15	0.04	0.45	0.11	−1.34***
Wheat	-0.11	0.03	1.11	0.41	−1.03***	-0.08	0.04	0.36	0.11	−0.76**
Coffee C	-0.17	0.04	1.99	0.46	−1.55***	-0.14	0.04	0.37	0.12	−1.23***
Corn	-0.21	0.04	1.17	0.55	−1.53***	-0.18	0.05	0.33	0.14	−1.31***

Table 6

Fama-MacBeth cross-sectional regressions

The table shows estimates of Fama-MacBeth regressions. "Volume ratio" is the log-ratio of futures' trading volume to that of options. "Put-call deviations" is the put-call-parity deviation computed as the absolute difference between call price and the price of the replicating portfolio (put plus spot price minus the present value of the strike price) as percentage of the spot price across all options and days. "Skewness" and "Kurtosis" are monthly skewness and kurtosis computed from daily futures returns within the month. "VIX level" is the value of VIX and "VIX standard deviation" is its daily time-series standard deviation. The sample period is from January 2006 to October 2020. *,**, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Dependent variable: hedged VIX^2 return						
Volume ratio	-0.04**						-0.02
	(0.02)						(0.02)
Put-call deviations		4.68					21.18
		(25.78)					(32.52)
Skewness			-0.02				-0.003
			(0.03)				(0.03)
Kurtosis				0.07***			0.07***
				(0.02)			(0.02)
VIX level					-0.01***		-0.01***
					(0.002)		(0.004)
VIX standard deviation						0.01	0.02**
						(0.02)	(0.01)
Intercept	-0.10	-0.15***	-0.15***	-0.37***	0.01	-0.14***	-0.13*
	(0.08)	(0.03)	(0.02)	(0.06)	(0.03)	(0.02)	(0.08)
Observations	2,804	2,804	2,804	2,804	2,804	2,804	2,804

Table 7
Bollerslev-Tauchen-Zhou predictive regressions
Panel A shows β from the regression $r^F_{t\to T} = \alpha + \beta(VIX^2_t - RV_{t-1,t}) + \epsilon_t$. Panel B shows β from the regression $r^{hedged\ VIX}_{t\to t+1} \times VIX^2_t = \alpha + \beta(VIX^2_t - RV_{t-1,t}) + \epsilon_t$. RV is realized variance measured by the sum of daily squared returns over a given month. $r^{hedged\ VIX}_{t\to T}$ is the return on the hedged VIX^2 portfolio over horizon T months, $r^F_{t\to T}$ is the return on the futures over horizon T months. t-statistics accounting for the overlap following Hodrick (1992) are reported in parentheses below each estimate in brackets.

	Panel A: $r^F_{t\to T}$				Panel B: $r^{hedged\ VIX}_{t\to T}$
	T=$t+1m$	T=$t+3m$	T=$t+6m$	T=$t+12m$	T=$t+1m$
	(1)	(2)	(3)	(4)	(5)
Australian Dollar	-5.44	-3.41	-4.98	-2.07	0.49
	(-0.92)	(-1.03)	(-1.95)	(-1.03)	(2.71)
British Pound	-7.30	-1.11	-1.32	-0.13	0.11
	(-3.39)	(-0.88)	(-1.42)	(-0.26)	(1.58)
Canadian Dollar	-6.53	-0.32	2.64	2.92	0.16
	(-1.73)	(-0.13)	(0.92)	(2.48)	(1.50)
Swiss Franc	1.74	0.11	0.46	0.30	0.07
	(3.18)	(0.67)	(3.86)	(3.55)	(2.35)
Japanese Yen	1.62	-0.28	-1.60	-0.37	-0.30
	(0.97)	(-0.13)	(-1.02)	(-0.37)	(-1.77)
S&P 500	0.12	0.48	-0.07	-0.08	-0.63
	(0.35)	(2.57)	(-0.32)	(-0.68)	(-2.51)
10 Year UST	26.79	5.25	2.51	5.68	-0.56
	(3.91)	(1.23)	(0.64)	(1.94)	(-13.41)
30 Year UST	-4.89	-7.40	-7.14	-1.18	-0.33
	(-0.79)	(-2.24)	(-1.89)	(-0.48)	(-2.34)
Brent Crude Oil	1.03	1.02	1.29	0.16	-1.02
	(1.32)	(6.29)	(1.98)	(0.42)	(-10.53)
Natural Gas	0.07	0.19	0.08	0.05	-0.96
	(0.67)	(3.99)	(4.36)	(2.54)	(-22.07)
Gold	0.72	0.79	0.22	0.18	-0.41
	(0.27)	(0.67)	(0.25)	(0.27)	(-1.99)
Silver	0.69	1.07	-0.10	-0.34	0.04
	(0.69)	(1.68)	(-0.19)	(-1.01)	(0.13)
Copper	2.12	1.63	1.37	1.07	-0.17
	(1.75)	(1.74)	(2.22)	(3.11)	(-0.53)
Live Cattle	-0.33	-0.39	1.02	-0.17	-1.04
	(-0.12)	(-0.2)	(0.81)	(-0.08)	(-2.80)
Soybean Meal	0.00	0.95	0.22	0.05	-0.15
	(0.00)	(0.93)	(0.34)	(0.12)	(-1.09)
Soybean Oil	1.26	-0.16	0.21	0.37	-0.51
	(1.38)	(-0.44)	(0.77)	(1.16)	(-4.72)
Soybeans	3.12	2.89	0.40	0.14	-0.45
	(2.45)	(5.91)	(0.56)	(0.21)	(-5.79)
Wheat	1.06	-1.80	-1.24	-0.96	-0.39
	(3.30)	(-1.69)	(-1.26)	(-1.44)	(-2.85)
Coffee C	0.94	0.15	0.32	-0.11	-0.13
	(0.40)	(0.18)	(0.65)	(-0.26)	(-0.74)
Corn	1.99	0.74	-0.55	-0.42	-0.37
	(0.92)	(0.96)	(-0.70)	(-0.58)	(-1.94)

Fig. 1. Implied volatility smiles

The figure shows scaled implied volatility (scaled by the at-the-money (ATM) implied volatility σ_{ATM} for each asset) as a function of scaled moneyness $m = \log(K/F)/\sigma^{ATM}$, where F is the futures price, K is the strike price of the option, and σ is volatility. The lines plot the function $\sigma(m)/\sigma^{ATM} = \bar{\alpha} + \bar{\beta} m + \bar{\gamma} m^2$ for each asset, where $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are the time-averaged estimates of α, β, γ from a series of cross-sectional (across strikes) regressions $\sigma_t(m_t)/\sigma_t^{ATM} = \alpha + \beta m_t + \gamma m_t^2 + \epsilon_t$ for each asset at each time t.

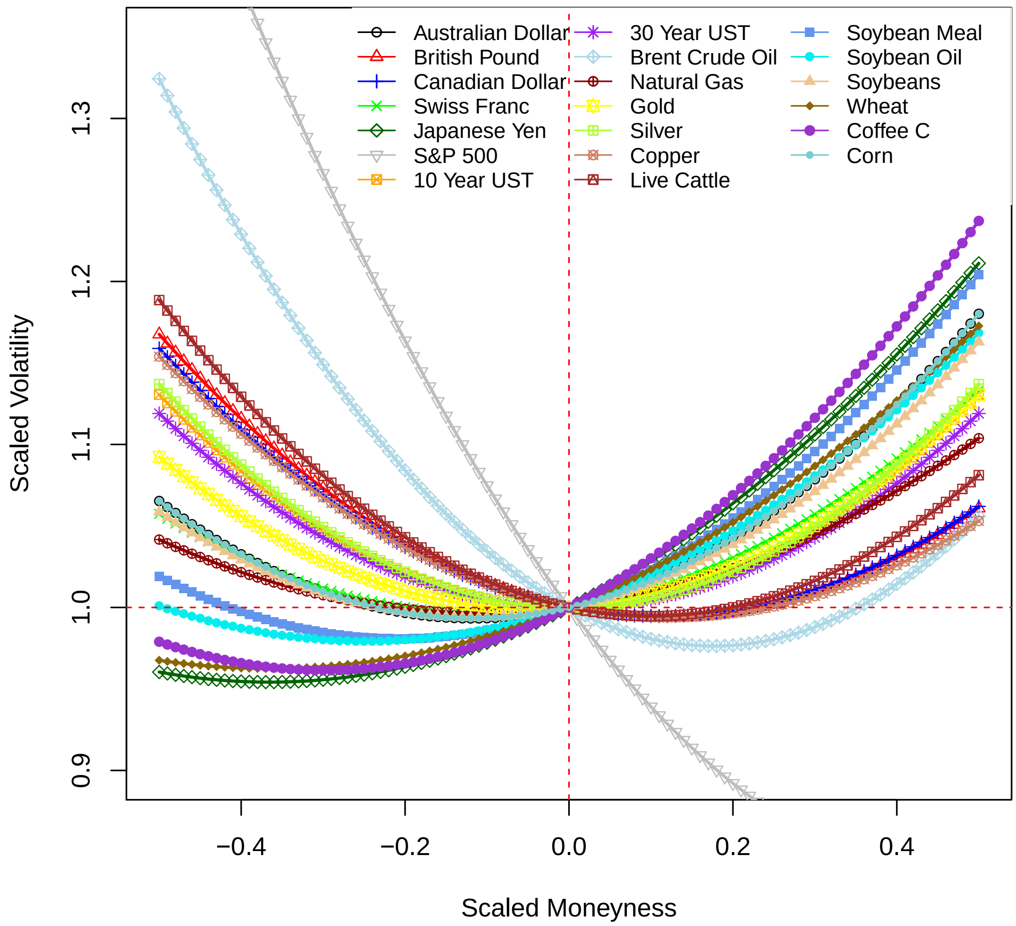


Fig. 2. Variance risk premium and the slope of the volatility smile
The figure shows the correlation between the variance risk premium (VRP), measured by the return on the hedged VIX^2 portfolio and the futures risk premium (FRP), measured by the excess return on the futures contract, and the average slope of the implied volatility smile for each asset.

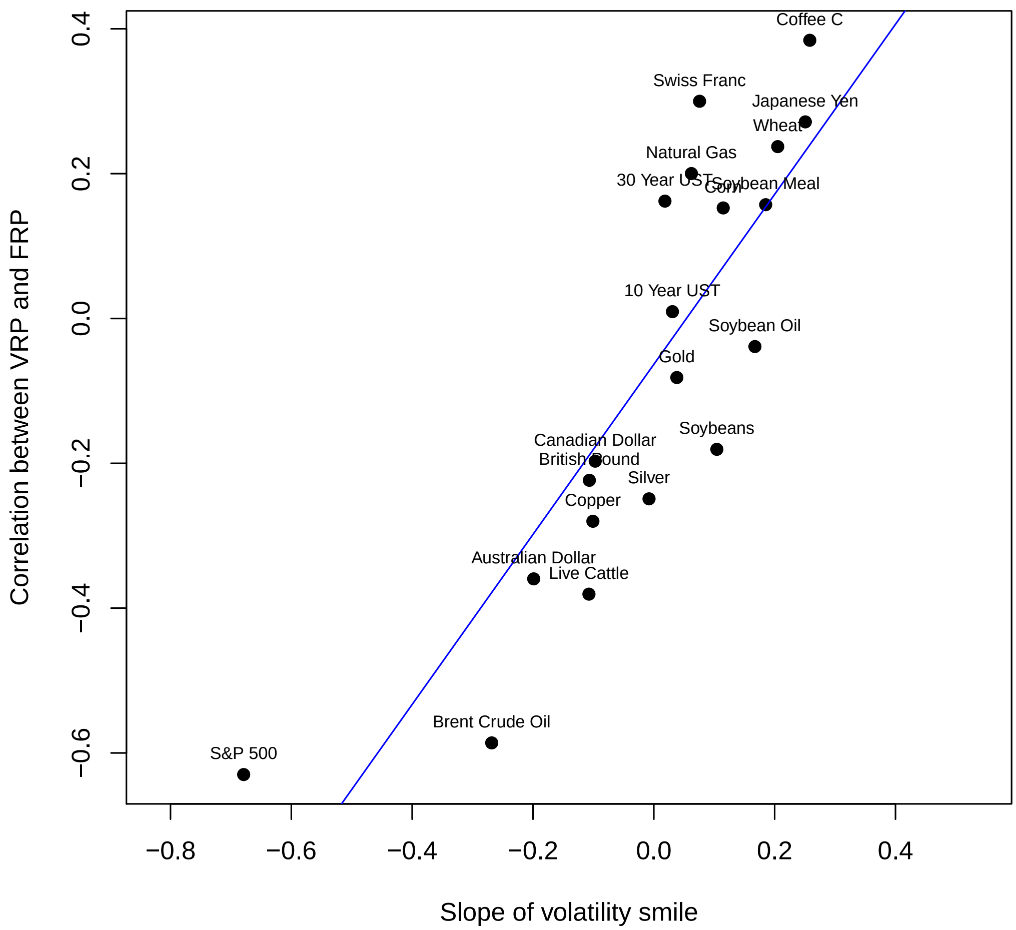


Fig. 3. Variance risk premium and futures risk premium
The figure shows the annualized variance risk premium (measured by the average return on the hedged VIX^2 portfolio) and the futures risk premium (measured by the average excess return on the futures contract).

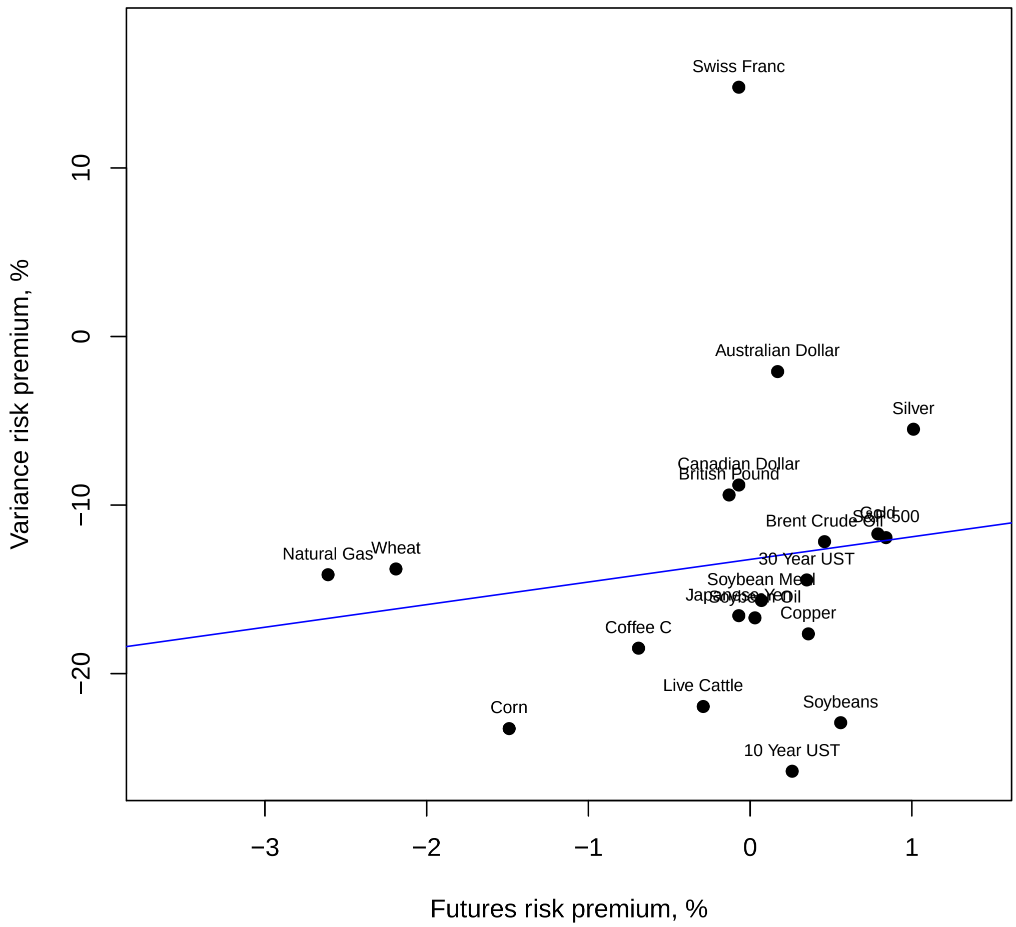
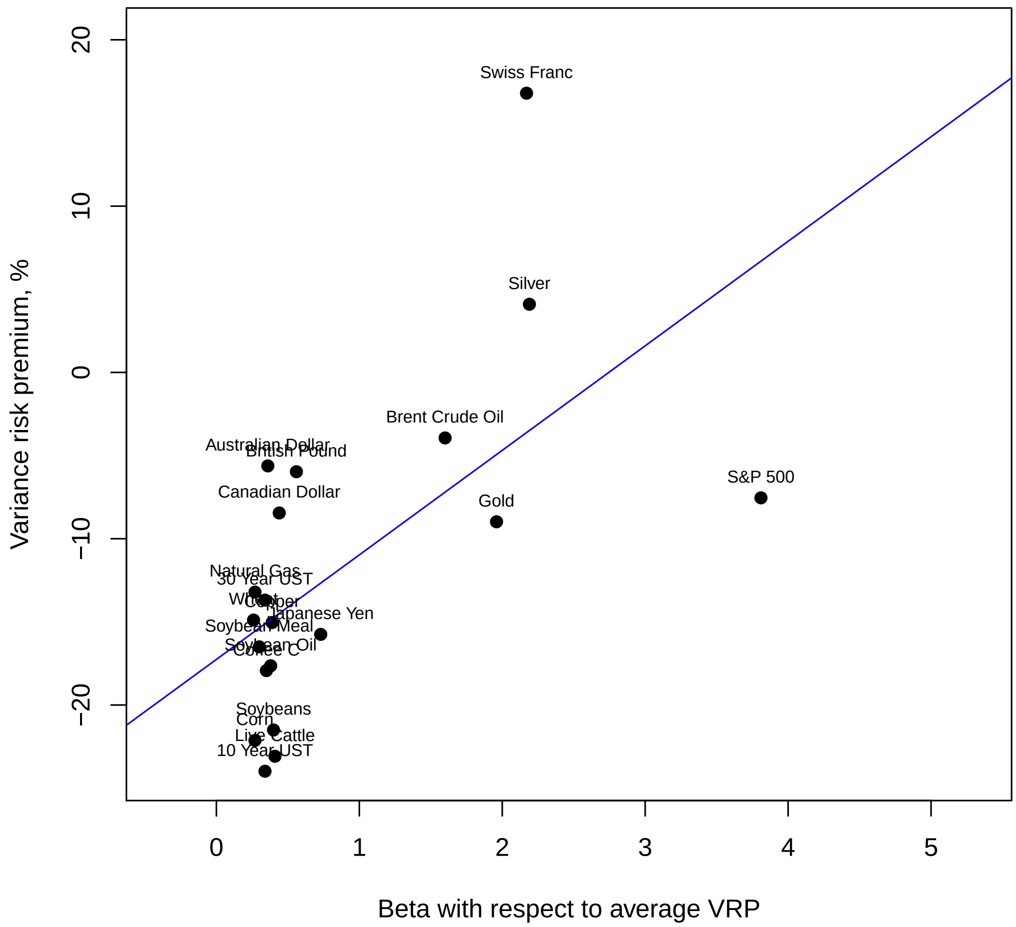


Fig. 4. Variance risk premium and β of systematic variance.
The figure shows the size of the variance risk premium (VRP) and the β from a regression of individual-asset VRP on the average VRP across assets. VRP is measured by the average return on the corresponding hedged VIX^2 portfolio.



6. Appendix A: Additional tables and figures

Table A.1
Summary statistics comparing hedged VIX^2 returns r^{VIX}, Carr and Wu (2009) approximation of variance swap returns r^{VSR}, and at-the-money straddle returns r^{str}. "SR" stands for Sharpe ratio, ρ for time-series correlation coefficient.

	mean r^{VIX}	SR r^{VIX}	mean r^{VSR}	SR r^{VSR}	mean r^{str}	SR r^{str}	$\rho(r^{VSR}, r^{VIX})$	$\rho(r^{VSR}, r^{str})$
Australian Dollar	-2.08	-0.14	-6.37	-0.41	0.05	0.00	0.99	0.41
British Pound	-9.40	-0.46	-11.82	-0.58	-0.51	-0.03	0.96	0.31
Canadian Dollar	-8.81	-0.52	-13.46	-0.82	-3.91	-0.21	0.99	0.35
Swiss Franc	14.79	0.19	14.20	0.17	6.44	0.27	1.00	0.39
Japanese Yen	-16.56	-0.98	-19.55	-1.20	-3.93	-0.17	0.97	0.42
S&P 500	-11.93	-0.23	-13.08	-0.24	-0.75	-0.04	1.00	0.51
10 Year UST	-25.79	-2.97	-31.94	-3.68	-6.14	-0.29	0.97	0.53
30 Year UST	-14.44	-1.41	-18.77	-1.76	5.18	0.21	0.99	0.60
Brent Crude Oil	-12.17	-0.46	-17.18	-0.61	-8.98	-0.59	0.99	0.30
Natural Gas	-14.13	-0.80	-15.31	-0.86	-13.45	-0.68	1.00	0.44
Gold	-11.71	-0.42	-12.70	-0.47	7.45	0.30	1.00	0.41
Silver	-5.50	-0.11	-3.69	-0.07	6.43	0.32	0.89	0.49
Copper	-17.64	-1.41	-18.73	-1.35	-13.30	-0.65	0.92	0.45
Live Cattle	-21.95	-1.52	-25.17	-1.77	-5.93	-0.27	1.00	0.60
Soybean Meal	-15.66	-1.40	-23.22	-2.07	-0.65	-0.03	0.99	0.16
Soybean Oil	-16.69	-1.40	-19.15	-1.63	-10.00	-0.42	0.95	0.17
Soybeans	-22.91	-1.95	-24.51	-2.14	4.82	0.20	0.99	0.35
Wheat	-13.79	-1.30	-15.16	-1.41	-5.83	-0.32	0.99	0.18
Coffee C	-18.49	-1.57	-23.26	-2.00	-7.23	-0.34	0.98	0.54
Corn	-23.26	-1.71	-22.79	-1.67	-8.51	-0.37	0.76	0.50

Fig. A.1. VSR and hedged VIX^2 returns. The graphs show variance swap returns (in blue) and hedged VIX^2 returns (in black) for Australian dollar, British pound, S&P 500, 10 year Treasuries, gold and natural gas.

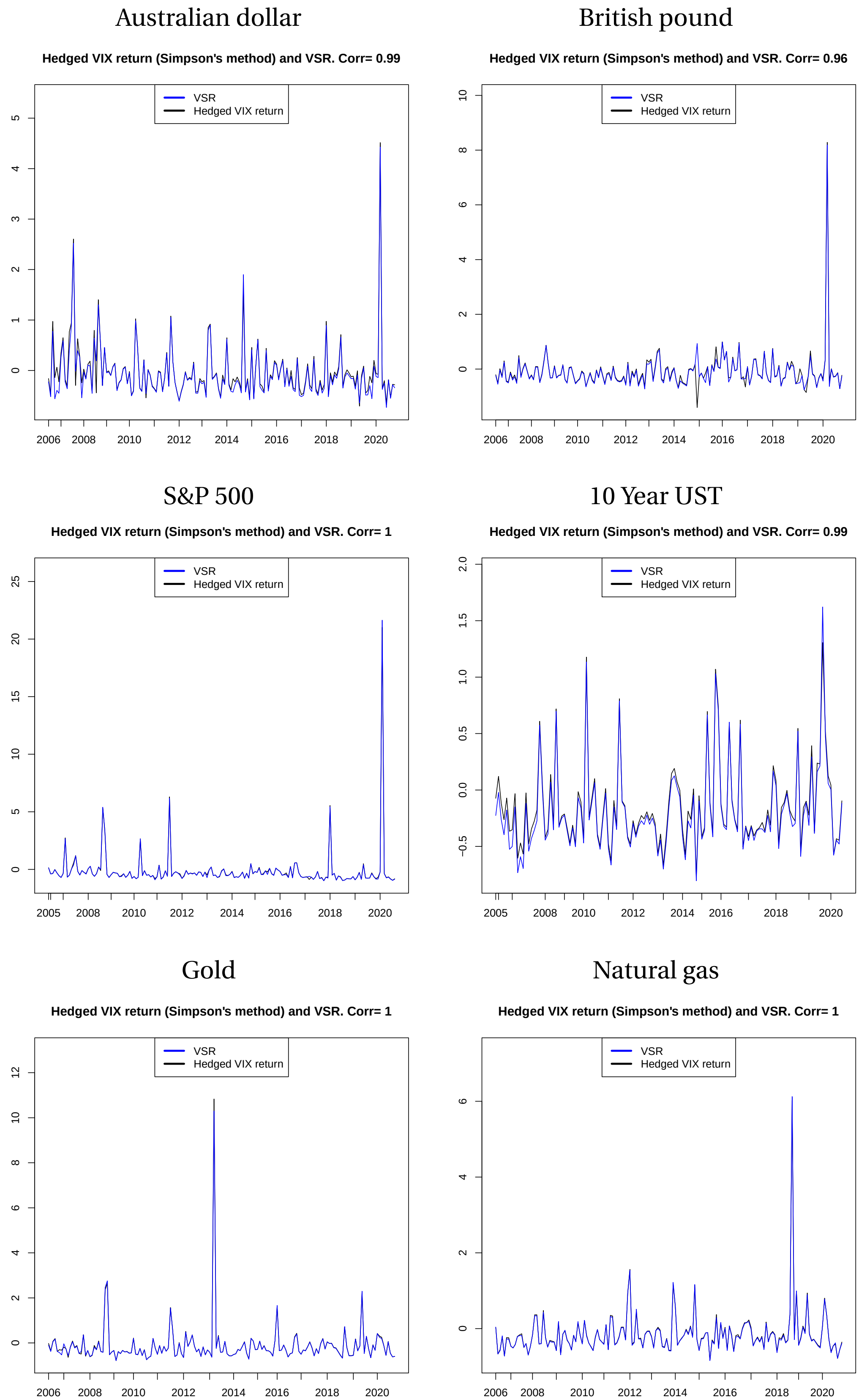
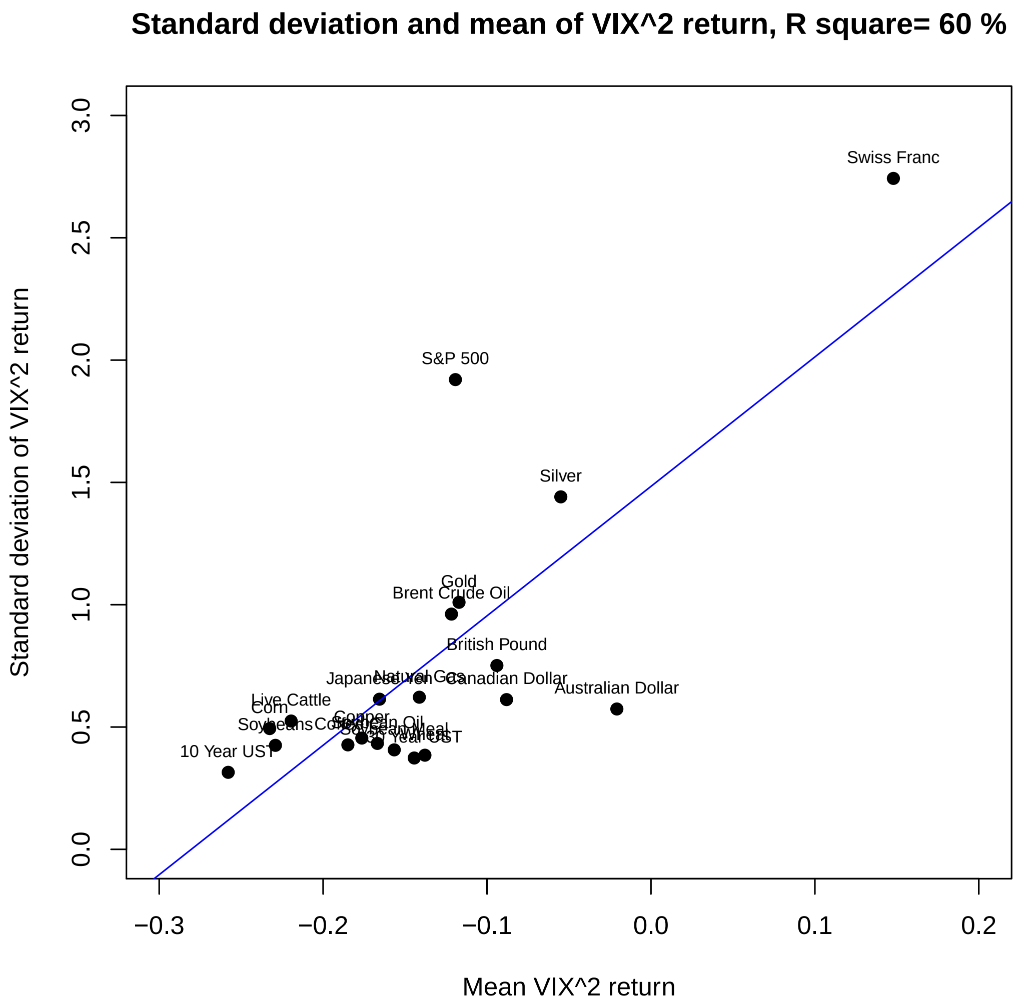


Fig. A.2. Scatterplot of the mean VIX^2 return and its standard deviation. The figure shows the average VIX^2 return over time and its standard deviation.



7. Appendix B: Some details about the data and robustness checks

American vs European. Most of the options we analyze are American, while the VIX CBOE formula assumes European options. For robustness, we recalculated the VIX^2 returns using European option prices (based on the implied volatilities of the American ones) and found that the results are nearly identical. The fact that using American options instead of European ones does not have a substantial impact on the VRP returns is consistent with other studies: e.g., Choi et al. (2017), Dew-Becker et al. (2017).

Convenience yields. The convenience yield of futures cancels out in the dynamic hedge term and the static position. Hence, the existence of convenience yields does not change the results.

Futures vs forwards. We expect the difference due to using futures vs forwards to be small. For example, in the Eurodollar futures market, Flesaker (1993) and Cakici and Zhu (2001) show that the impact of using futures as the underlying as opposed to forwards is very small particularly for options with short maturity.