The Distributional Impact of the Minimum Wage in the Short and Long Run*

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Abstract

We develop a framework with worker heterogeneity, monopsony power, and putty-clay technology in order to study the distributional impact of the minimum wage in the short and long run. We discipline our model to match the small employment effects of the minimum wage in the short run and the large estimated elasticities of substitution across different workers in the long run. We find that in the short run, both small and large increases in the minimum wage have small impacts on employment and increase the labor income of the workers earning less than the new minimum. In the long run, the effects of the minimum wage greatly differ depending on the size of its increase: a small increase has a beneficial long-run impact on low-income workers in that it increases their employment, income, and welfare, whereas a large increase reduces the employment, income, and welfare of precisely the low-income workers it is meant to support. In either case, these long-run effects take time to fully materialize because firms slowly adjust their mix of inputs. Existing transfer programs, such as the earned-income tax credit (EITC) or a progressive tax system, are more effective than large increases in the minimum wage at improving long-run outcomes for workers at the low end of the wage distribution. But combining existing programs with a small increase in the minimum wage provides even larger welfare gains for those workers.

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1 Introduction

Proposals to increase the federal minimum wage from its current level of $7.25 per hour to at least $15 have recently been advanced in the United States. Their goal is to improve the welfare of workers currently earning less than this new minimum, especially those at the bottom of the wage distribution. In contrast to past changes in the minimum wage, an increase of this magnitude would impact a large fraction of the U.S. workforce. According to data from the American Community Survey (ACS) between 2017 and 2019, about 40% of workers with less than a bachelor's degree have an hourly wage lower than $15.[1] This high degree of wage dispersion suggests that a large change in the minimum wage would have very different effects across workers. For example, given the current wage distribution, a $15 minimum wage would double the wages of workers currently earning $7.50 per hour but would not bind at all on workers in the top 60% of the non-college wage distribution.

The goal of this paper is to develop a framework to assess the distributional impact of both small and large minimum wages in both the short and long run. Given the substantial heterogeneity in wages across workers observed in the data, a key question is: to what extent will firms substitute away from workers for whom a minimum wage binds? The empirical literature offers two sets of estimates of such elasticities of substitution. First, an influential literature estimates that workers with different levels of skills are fairly substitutable with each other in the long run. Prominent studies include Katz and Murphy (1992), which estimates an elasticity of substitution *across* workers of different education groups of about 1.5, and Card and Lemieux (2001), which finds an elasticity of substitution among workers *within* an education group between 4 and 6. Hence, workers within education groups are not perfectly substitutable but much more so than across education groups. Second, a separate and influential literature has documented only small effects of the minimum wage on employment one or two years following a minimum wage increase, implying a low elasticity of substitution across workers in the short run.[2] Taken together, these estimates suggest that a study of the distributional impact of the minimum wage across workers must distinguish between short- and long-run effects.

We distinguish between small and large changes in the minimum wage because the effect of the minimum wage is non-monotonic in its size when firms have monopsony power. The idea that increases in the minimum wage can help alleviate some of the distortions arising from firms' wage-setting power dates back to Robinson (1933). Critically, in such contexts, small minimum wage changes can increase employment and earnings in the long run as they correct monopsony distortions. But once the minimum wage is increased beyond the level needed to correct these distortions, employment in the long run will decline. These insights suggest that the short- and long-run effects of small minimum

[1]In Section 3, we discuss the sample used for these statistics and how hourly wages were measured.
[2]We discuss this literature in more detail later on in the paper.

wage changes cannot simply be extrapolated to learn about the short- and long-run distributional effects of larger minimum wage changes.

Our framework incorporates rich worker heterogeneity, a production technology in line with long-run estimates of input substitutability, a putty-clay input adjustment process consistent with short-run estimates of input substitutability, and monopsony power in labor markets. Specifically, our framework allows for firms facing changes in the relative prices of inputs to (i) substitute among workers of different productivities *within* an education group, (ii) substitute among workers *across* different education groups, and (iii) substitute between labor and capital. We parsimoniously model input adjustment costs by assuming a putty-clay technology.[3] This type of technology captures the idea that when deciding on a production plan, and so on new investments, firms can choose any desired ratio of capital to the labor of workers of each type that lies on the frontier of the underlying production function (putty phase). Once installed, however, each unit of capital requires a fixed amount of each type of labor to operate (clay phase). That is, our production function is effectively CES in the long run but Leontief in the short run. In the short run, then, our model is consistent with the view that the minimum wage can be set quite high without adversely affecting employment, as long as it remains profitable for firms to continue to operate. Over time, though, as new capital goods embodying new ratios of capital to labor are installed, firms' labor-to-capital ratios shift to their preferred input mix governed by the underlying CES production function.

In order to capture monopsony power, and therefore allow for non-monotonic long-run effects of the minimum wage on employment and labor market participation, we embed monopsonistic competition among firms in a directed search setting that features endogenous labor market participation. Our monopsonistically competitive directed search equilibrium extends the traditional Robinson (1933) monopsony model to allow for multiple firms and frictional labor markets, which are central to modern treatments of labor market dynamics.[4] In this framework, firm monopsony power not only distorts the participation margin of workers deciding whether to look for a job, but also the job-creation margin of firms deciding whether to hire more workers. The framework we propose avoids the need to specify ad-hoc rationing rules in the allocation of workers to jobs when the minimum wage induces labor supply to exceed labor demand. In our framework, instead, whenever labor supply exceeds labor demand at a given level of posted vacancies, firms find it optimal to simply decrease vacancies until this excess supply is dissipated.[5]

[3]Our modeling of putty-clay capital builds on work that dates back at least to Johansen (1959) and was extended by Calvo (1976). Our formulation follows that in Atkeson and Kehoe (1999).

[4]Our framework builds on a long line of work that has studied the minimum wage through the lens of frictional models of the labor market. See, in particular, Eckstein and Wolpin (1990), Flinn (2006), Ahn, Arcidiacono and Wessels (2011), and, recently, Engbom and Moser (2021). We add to this literature by augmenting the search framework to allow for monopsonistic competition among firms and by estimating the distributional effects of the minimum wage in both the short and long run.

[5]Kudlyak, Tasci and Tüzemen (2022) exploits cross-state evidence to show that increases in the minimum wage do

The two key parameters that are quantitatively important for determining the long-run distributional effects of minimum wage changes are (i) the extent to which firms can substitute among workers of different productivities within an education group and (ii) the extent of firm monopsony power faced by workers in the labor market. The higher is the elasticity of substitution among workers, or the smaller is the degree of firm monopsony power, the larger the potentially negative long-run consequences of the minimum wage. In our baseline parameterization, the elasticity of substitution among workers of different productivities within an education group is 4 as estimated by Card and Lemieux (2001).[6] We discipline the degree of firms' monopsony power in the labor market so that our model matches recent estimates on wage markdowns from Seegmiller (2021), Lamadon, Mogstad and Setzler (2022), Berger, Herkenhoff and Mongey (2022a), and Yeh, Macaluso and Hershbein (2022), which document that workers are paid, on average, between 15% and 35% less than their marginal products. In our baseline parameterization, we target a 25% wage markdown. We then show how our model's predictions differ for the whole range of estimates of the wage markdown and the elasticity of substitution across workers.

We find that a *small* increase in the minimum wage benefits low-wage workers in both the short and long run. Specifically, we study the transition dynamics following an increase of the minimum wage to \$8.50 (an increase of 17% over the current minimum of \$7.25). In the short run, this small increase raises the wages of workers at the lowest percentiles of the wage distribution and has a negligible effect on their employment. In the long run, employment of these workers actually *increases* because the small minimum wage also partially offsets monopsony markdowns. Hence, small increases in the minimum wage unambiguously benefit low-wage workers.

A *large* increase in the minimum wage to, say, \$15, which more than doubles the current minimum, benefits low-wage workers in the short run but ultimately hurts them in the long run. In the short run, such a large minimum wage has only a small negative effect on employment and, hence, generates a large short-run increase in labor income for the lowest-paid workers. The main force behind this result is our putty-clay technology, which leads to frictions to the adjustment of the mix of any inputs to production. In the long run, however, firms are able to substitute away from the low-wage workers who have become more expensive, which decreases their employment, income, and welfare—adversely affecting precisely the group of workers that the minimum wage is designed to help. As a result, nearly one-fifth of all non-college workers, and 40% of non-college workers initially earning less than \$15, experience a decline in labor income in the long run. These earning losses are concentrated among workers in the bottom 20% of the initial wage distribution initially earning less than about \$10.

in fact reduce firm vacancy positing. These findings motivate our choice to explicitly model the firm vacancy-posting decision.

[6] In the appendix, we show that the Card and Lemieux (2001)' estimates map naturally into our framework even though our model includes monopsony power and frictional labor markets.

The economic mechanisms behind these long-run consequences of a large minimum wage increase are twofold. First, for low-wage workers, their *efficient* wage—the wage they would earn absent firm monopsony power—is much lower than \$15, since their marginal productivity is fairly low. More generally, the heterogeneity in workers' productivity within education groups, as implied by the wage distributions observed in the data, leads to efficient wages that are highly dispersed. Hence, a single minimum wage is too blunt a tool that cannot eliminate monopsony distortions for *all* workers at once. In particular, choosing a high enough minimum to eliminate the monopsony distortion faced by the average worker requires setting it much above the efficient level of wages for low-productivity workers, thereby reducing their employment. Second, given the high long-run elasticity of substitution among non-college workers of differing abilities estimated by Card and Lemieux (2001), eventually firms will sharply substitute away from lower productivity non-college workers towards higher productivity ones in response to a \$15 minimum wage. But firms using a putty-clay technology find it optimal to adjust their workforce only slowly over time. Indeed, even four years after the higher minimum wage is introduced, only about one-quarter of the total long-run employment adjustment has taken place.

The overall impact of the minimum wage on worker welfare results from integrating these short-run and long-run effects along the entire transition of the economy to the new equilibrium with the higher minimum wage. According to our baseline parameterization, essentially all non-college workers weakly benefit from a small increase in the minimum wage, especially those earning less than the new minimum. Even for large minimum wage increases, all but the lowest 16% of current non-college workers initially earning less than \$15 benefit from the introduction of a \$15 minimum wage, once we account for the entire dynamics of the transition. That is, even for large minimum wage changes, the benefits of higher wages and slow employment adjustments in the short run outweigh the larger employment losses in the long run for most low-wage workers.

As noted above, our long-run quantitative results hinge on estimates of the wage markdown and the elasticity of substitution among workers within an education group. We perform an extensive analysis of the robustness of our quantitative results to alternative parameter values. We find that as long as there is at least a modest amount of firm monopsony power, a small minimum wage robustly increases the welfare of low-wage workers. By contrast, if a \$15 minimum wage is to have positive welfare effects on low-wage workers in the long run, one needs to assume either (i) that the long-run elasticity of substitution across workers of differing productivities within an education group is much smaller than the estimates from the current literature (in particular, less than 2) or (ii) that the wage markdown faced by low-wage workers is much larger than the estimates from the current literature (in particular, greater than 40%). Our framework highlights that researchers and policy makers making quantitative claims about the long-run effects of large minimum wage changes are implicitly making assumptions about the extent of monopsony power in labor markets and of long-run input substitutability on the part of the firms.

Although most of our analysis focuses on a permanent increase in the real value of the minimum wage, we also examine the case in which the minimum wage erodes in real terms over time. We interpret this case as capturing in a stylized way what happens between legislated changes of a nominal minimum wage that gradually becomes less binding due to a combination of inflation and growth in labor productivity. Interestingly, even a large, but temporary, minimum wage increase can improve the labor income of the lowest-productivity workers in every period in which it binds. This occurs because firms do not find it optimal to vary much their input mix in response to temporary changes in the minimum wage.

Given the long-run costs associated with large minimum wage changes implied by our counterfactual exercises, we conclude the paper by showing that existing tools in the U.S. tax and transfer system, such as the EITC (earned-income tax credit), are much more effective at raising the employment, income, and welfare of a broader set of lower-wage workers in the long run. We explore these policies in a revenue-neutral way by funding them with a tax on firm profits whose rate we set so that it replicates the loss in firm profits stemming from a $15 minimum wage. These policies dominate the minimum wage because they directly target low-income workers and do not create incentives for firms to adjust their input mix in the long-run. That is, firms are not directly paying the additional cost to hire lower-productivity workers. In contrast, in the long run, a $15 minimum wage—which is motivated by a desire to help more workers than would a smaller minimum wage —is beneficial for workers initially earning between $12 and $15 but is detrimental for workers at the bottom of the wage distribution. In this sense, the EITC is more effectively targeted at reducing the monopsony distortions of these workers without adversely affecting outcomes for even lower-wage workers. We also show that for low-wage workers, the EITC coupled with a modest increase in the minimum wage to about $9 is more beneficial than an EITC policy alone. The reason is that by offsetting the monopsony distortions for such workers, the minimum wage complements the redistributive effects of an EITC policy.

Throughout the paper, we show how our calibrated model generates results in line with the results in the large empirical literature measuring the labor market impact of minimum wage changes.[7] The literature, as a whole, has not yet found a way to isolate the *long-run* labor market impacts of *large* minimum wage changes.[8] The value of our framework is that it allows us to make predictions about the long-run effects of large minimum wage changes given assumptions about firm input substituability

[7]Fully reviewing this literature is beyond the scope of this paper. However, some prominent papers in this extensive literature include Brown (1988), Card and Krueger (2015), Brown (1999), Neumark et al. (2008), and Cengiz et al. (2019). We discuss some recent papers in the empirical minimum wage literature in more detail Appendix B.

[8]The lack of long-run empirical evidence has been pointed out in Brown (1999)'s review article where he states that "[t]here is a simply a stunning absence of credible evidence—indeed, of credible attempts—[to identify] the long run effects [of the minimum wage]."

and the extent of monopsony power in the labor market.

As seen from our model, the effects of the minimum wage on employment and earnings are highly nonlinear, so results from small minimum wage changes cannot simply be extrapolated to learn about the effects of large minimum wage changes. However, we do replicate key patterns from the literature on the short-run responses to small minimum wage changes in order to build confidence in our long-run counterfactuals from large minimum wage changes. To do so, we show that our model replicates the consensus short-run employment elasticities to minimum wage changes documented in Neumark and Shirley (2022).[9] We also show that our model matches the findings in Cengiz et al. (2019), who develop a clever identification strategy to examine how employment responds in both the short- and medium-run to small changes in state-level minimum wage changes. They find that, for small minimum wage increases (about a 10% increase on average), total employment did not change much up to seven years after the minimum wage is enacted. This result is exactly what our model predicts for small minimum wage changes, which help to correct monopsony distortions. Finally, our framework replicates the recent findings in Clemens and Strain (2021) that for medium-sized minimum wage changes, employment does not adjust in the short run but does start to decline four to six years after the minimum wage increase was enacted. We go further and show that the well-identified longer responses to small minimum wage changes can help discipline the set of relevant parameters in our model for our long-run counterfactuals about the effects of larger minimum wage changes.[10]

Our paper builds on the insights found in Sorkin (2015) and Aaronson et al. (2018), who use a variant of the standard putty-clay capital setup to argue that the effect of the minimum wage on employment is smaller in the short run than in the long run. Our paper complements these papers exploring the distributional impact of small and large minimum wage changes over the entire transition path in a model with rich worker heterogeneity and in which firms have monopsony power. Our focus on worker heterogeneity leads us to discipline our model using well-known estimates in the labor economics literature of the long-run elasticities of substitution among workers (specifically Katz and Murphy (1992) and Card and Lemieux (2001)). The values of these elasticities turn out to be crucial for our quantitative results. In addition, since we are interested in large changes in the minimum wage that impact a significant fraction of the economy, we build a general equilibrium model parameterized to be representative of the entire U.S. economy.

[9]Neumark and Shirley (2022) surveys 109 published studies examining the employment effects from state-level minimum wage changes on various sub-populations. In particular, they document that nearly all of the 109 published studies that they review find small short-run employment effects in the two years after a minimum wage increase.

[10]There is a recent literature that provides additional evidence that firm slowly adjust their input mix in response to minimum wage changes. For example, Meer and West (2016) estimate that an increase in the minimum wage reduces employment but such an effect takes several years to materialize. In response to a large and persistent minimum wage increase in Hungary, Lindner and Harasztosi (2019) estimate that firms responded to it by substituting away from labor towards capital. Clemens, Kahn and Meer (2021) exploit cross-regional variation to document that firms substitute away from low-productivity workers towards higher-productivity ones in response to minimum wage increases.

Our model of monopsonistic competition is the natural labor market analogue of the model of monopolistic competition in the goods market, adapted to a search setting. In our model, firms' monopsony power arises from an imperfect substitutability of jobs across firms in workers' preferences, as in Berger, Herkenhoff and Mongey (2022a). In contemporaneous work, Berger, Herkenhoff and Mongey (2022b) adapt this setup, which ports the Atkeson and Burstein (2008)'s model of Cournot competition in the goods market to a labor market setting, and pursue a normative analysis of the long-run optimal level of the minimum wage. Our paper differs from Berger, Herkenhoff and Mongey (2022b) in several ways. Most importantly, we focus on the distributional impact of the minimum wage across workers in an environment with rich worker heterogeneity, whereas Berger, Herkenhoff and Mongey (2022b) focus on the differential effect of the minimum wage across firms in an environment with rich firm heterogeneity. Given the different short-run and long-run elasticity of substitution across workers documented in the literature as discussed above, we heavily focus on the dynamics of the effect of the minimum wage over time. By the same token, our model abstracts from the rich oligopsonistic market structure studied in Berger, Herkenhoff and Mongey (2022b). Therefore, we view the two papers as complementary.

2 Model

We begin by briefly highlighting the main features of our model. First, we incorporate the notion of firm monopsony power in labor markets by allowing workers to view jobs at different firms as imperfectly substitutable with each other. When this is the case, large minimum wage increases can actually increase employment and labor market participation by potentially reducing the resulting monopsony distortions in the labor market. We embed this setup in an economy with labor markets subject to search frictions to be able to distinguish among the employment, unemployment, and non-participation effects of minimum wage policies. In particular, by incorporating a non-participation margin, we capture the idea that a higher minimum wage may incentivize non-participants to enter the labor market and search for jobs. Importantly, in such a framework, monopsony power not only distorts the participation margin of workers who look for jobs, but also an often neglected margin, namely, the vacancy-posting margin of firms deciding to hire more workers. We will show that the effect of tax policies such as the EITC subtly depend on how they affect these two margins.

Second, we include in the model rich worker heterogeneity, both within and across education groups, so as to explore the distributional impact of the minimum wage. We allow for this heterogeneity within a production technology consistent with the long-run evidence on the elasticity of substitution across multiple inputs. For example, we ensure that our model matches the long-run elasticity of substitution between college and non-college workers estimated by Katz and Murphy (1992).

Likewise, we make our model consistent with the estimated elasticity of substitution across workers with different skills *within* an education group, documented in work such as Card and Lemieux (2001). We also let firms substitute between labor and capital.

Finally, to guarantee that our technology is consistent with the evidence of low input substitutability in the short run, we assume that firms operate technologies of the putty-clay type. Specifically, we capture in an intuitive way the notion that adjusting the ratio of any inputs is costly for firms in the short run by assuming that the production technology is embedded in the capital stock so its labor intensity is irreversible once capital is installed. The idea is that a new piece of capital can be built to be used in combination with low- and high-educated workers of any ability in any ratio. These ratios, however, are fixed after the new capital is installed. We view this putty-clay structure as a tractable way to allow for a complex set of adjustment costs for firms when they decide to alter their input mix over time.

In our quantitative exercises, we show how the presence of putty-clay capital slows down the transition of the economy to the new steady state after the introduction of a higher minimum wage. Putty-clay capital thus helps the model reproduce the well-documented feature that employment responses to increases in the minimum wage tend to be muted in the short run, but can be much larger in the long run, as consistent with a large literature on long-run input substitution patterns. We next discuss our model in greater detail.

2.1 Preferences, Production and Matching

We consider an infinite-horizon economy in discrete time populated by consumers and firms. We describe next consumers, firms, production, output and labor markets, and the timing of events in a period.

Consumer Heterogeneity and Preferences. Consumers are heterogeneous in two dimensions. First, they differ in their *education level* $g \in \{\ell, h\}$, where ℓ denotes the group of low-educated consumers (those with less than a bachelor's degree) and h denotes the group of high-educated consumers (those with a bachelor's degree or more). Second, within each group g, consumers are characterized by an *ability level* z drawn from education-specific discrete distributions. Ability differences among consumers allow us to match the observed wage distribution within each education group. We index a consumer by i, which denotes both a consumer's education group and ability level so that $i \in I \equiv I_\ell \cup I_h$, where $I_\ell = \{i|\ z_i \in \{z_{\ell 1}, \ldots, z_{\ell M}\}\}$ represents the set of abilities of low-educated consumers and $I_h = \{i|\ z_i \in \{z_{h1}, \ldots, z_{hM}\}\}$ represents the set of abilities of high-educated consumers. As shorthand, we let $i = (g, z_i)$ denote an education-ability pair.

The economy consists of a measure μ_i of families of each type i.[11] Each type of family is composed of a large number of household members of the same education group and ability level. Risk sharing within such families implies that each member of a household of type i consumes the same amount of goods at date t, regardless of the idiosyncratic shocks that such a member experiences.[12]

The utility function of a family of type i is $\sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it}, s_{it})$, where c_{it} is the consumption of a representative family member, n_{it} is the index of the disutility of work of the family, and $s_{it} = \sum_j s_{ijt}$ are the total searchers, where s_{ijt} denotes the number of family members of type i searching for jobs at firm j in period t. The index of the disutility of work is defined as

$$n_{it} = \left[\sum\nolimits_j n_{ijt}^{\frac{1+\omega}{\omega}} dj\right]^{\frac{\omega}{1+\omega}} \quad \text{with } \omega > 0, \tag{1}$$

where n_{ijt} is the number (measure) of family members who work at firm j in t. The parameter ω measures the imperfect substitutability of employment at different firms in terms of workers' disutility of work at them and can be interpreted as arising from workers' idiosyncratic preferences over different firms, locations, or amenities. The smaller ω is, the less substitutable jobs at a same firm are. Here we adapt the standard way of modeling imperfect substitutability in consumers' preferences across differentiated goods to modeling imperfect substitutability in workers' preferences across differentiated jobs. This imperfect substitutability in preferences for jobs will generate an upward-sloping labor supply curve for each firm's jobs that is analogous to the downward-sloping demand curve for each firm's goods, which arises in standard models of monopolistic competition.[13] As we discuss below, ω is a key parameter that governs the extent of firms' monopsony power in the labor market. In our quantitative analysis, we discipline ω using estimates of the extent to which wages are marked down relative to workers' marginal products.

Production Technology. A large number of identical firms indexed by j produce the same homogeneous final good. Firm j uses capital k_{jt}, an aggregate of efficiency units of low-educated labor $\bar{n}_{\ell jt}$, and an aggregate of efficiency units of high-educated labor $\bar{n}_{hjt}$. The law of motion for capital accumulation is $k_{jt+1} = (1-\delta)k_{jt} + x_{jt}$, where δ is the depreciation rate and x_{jt} is the investment of new capital made by firm j in period t. As noted above, consumers view the labor supplied to these different firms as differentiated. We assume a nested CES production function over capital k_{jt}, the

[11]Letting J denote the (integer) number of firms in the market and assuming that for each J, there is a total measure $\mu_i J$ of consumers of type i, here we focus on an economy with J large enough so that it is well-approximated by $J = \infty$.

[12]This type of risk-sharing arrangement in search models is familiar from the work of Merz (1995) and Andolfatto (1996).

[13]See Berger, Herkenhoff and Mongey (2022a) and Deb, Eeckhout and Warren (2021) for related preferences along with a discussion of various interpretations and alternative microfoundations of them. Intuitively, this specification can be primitively derived from idiosyncratic shocks to the value of working at any firm due to firm's different locations, amenities, and similar.

aggregate of low-educated labor $\bar{n}_{\ell jt}$ and the aggregate of high-educated labor $\bar{n}_{hjt}$ of the form

$$F(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt}) = \left[\psi(k_{jt})^{\frac{\rho-1}{\rho}} + (1-\psi)G(\bar{n}_{\ell jt}, \bar{n}_{hjt})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{2}$$

where

$$G(\bar{n}_{\ell jt}, \bar{n}_{hjt}) = \left[\lambda\left(\bar{n}_{\ell jt}\right)^{\frac{\alpha-1}{\alpha}} + (1-\lambda)(\bar{n}_{hjt})^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}; \tag{3}$$

this production function will underlie both the version of our model with standard capital and that with putty-clay capital. The outer nest in $G(\cdot)$ is a CES production function over capital k_{jt} and an aggregate $G(\bar{n}_{\ell jt}, \bar{n}_{hjt})$ of the efficiency units of low- and high-educated labor, $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$. The inner nest in $G(.)$ is a CES production function over $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$. The parameters ρ and α are key as they capture the degree of substitutability among inputs: the larger ρ is, the more substitutable capital and the labor aggregate $G(\bar{n}_{\ell jt}, \bar{n}_{hjt})$ are, whereas the larger α is, the more substitutable low-educated and high-educated labor are. The parameter α will govern how much firms substitute away from low-educated labor towards high-educated labor when low-educated labor becomes more expensive from the imposition of a large minimum wage. In our quantitative exercise, we make this parameter consistent with the estimates of the elasticity of substitution between high- and low-educated workers from Katz and Murphy (1992). The parameter ρ, instead, will govern how much firms will substitute from the labor aggregate as a whole towards capital as the minimum wage increases the overall cost of labor. We will show the sensitivity of our results to using different estimates of capital-labor subsitutability from the literature.

The labor inputs $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$ used by firm j are themselves CES aggregates of the labor inputs of workers of different abilities within each education group,

$$\bar{n}_{\ell jt} = \left[\sum\nolimits_{i \in I_\ell} z_i(\mu_i n_{ijt})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \text{ and } \bar{n}_{hjt} = \left[\sum\nolimits_{i \in I_h} z_i(\mu_i n_{ijt})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{4}$$

where n_{ijt} is the amount of labor of a family of type i supplied to firm j in t and $\mu_i n_{ijt}$ is the total amount of labor from all families of type i supplied to firm j in t. The parameter ϕ will play an important role in our assessment of the distributional effects of the minimum wage, because it governs the extent to which firms are willing to substitute *across* workers of differing ability, as indexed by z, within an education group. For instance, if some low-ability workers become more expensive due to a higher minimum wage, firms can substitute towards higher-ability workers in production. Our nested CES production structure for labor in (3) and (4) exactly follows the framework in Card and Lemieux (2001). Our setup differs from theirs only because we allow for capital in the overall production function in (2). To discipline ϕ, we rely on estimates from Card and Lemieux (2001), which finds a relatively high elasticity of substitution between workers of differing ability within an education group.

Matching Technology. We consider a directed search setting in which each firm j posts a measure of vacancies $\mu_i a_{ijt}$ directed at consumers of type i searching for jobs at firm j, where a_{ijt} denotes the measure of vacancies posted by firm j aimed at each family of type i and s_{ijt} denotes the number of searchers from family i. The cost of posting a measure $\mu_i a_{ijt}$ of vacancies for type i consumers is $\kappa_i \mu_i a_{ijt}$. The matches created by a measure $\mu_i a_{ijt}$ of vacancies and a measure $\mu_i s_{ijt}$ of searchers of type i are determined by the constant-returns-to-scale Cobb-Douglas matching function

$$m(\mu_i a_{ijt}, \mu_i s_{ijt}) = B_i(\mu_i a_{ijt})^{\eta}(\mu_i s_{ijt})^{1-\eta}. \tag{5}$$

If firm j posts $\mu_i a_{ijt}$ vacancies for type-i consumers and, in total, families of type i send $\mu_i s_{ijt}$ consumers searching for that firm's jobs, then firm j creates a measure $m(\mu_i a_{ijt}, \mu_i s_{ijt}) = \lambda_f(\theta_{ijt})\mu_i a_{ijt}$ of new matches with consumers of type i, where $\lambda_f(\theta_{ijt}) = m(\mu_i a_{ijt}, \mu_i s_{ijt})/\mu_i a_{ijt} = m(a_{ijt}, s_{ijt})/a_{ijt}$ is the probability that a posted vacancy is filled or the *job-filling rate* and $\theta_{ijt} = a_{jit}/s_{ijt}$ denotes market tightness. We can also express these new matches as $m(\mu_i a_{ijt}, \mu_i s_{ijt}) = \lambda_w(\theta_{ijt})\mu_i s_{ijt}$, where $\lambda_w(\theta_{ijt}) = m(\mu_i a_{ijt}, \mu_i s_{ijt})/\mu_i s_{ijt} = m(a_{ijt}, s_{ijt})/s_{ijt}$ is the probability that a consumer of type i finds a job at firm j or the *job-finding rate*. This directed search framework ensures that labor market allocations are pinned down even when the minimum wage binds for some workers. It also provides a tractable way to distinguish among the responses of unemployment and labor force participation to changes in the minimum wage.

Timing. The timing of events within a period is as follows. Each period t consists of two stages. In stage 1, each firm j posts vacancies $\{a_{ijt}\}$ aimed at consumers of type i that determines the tightness $\{\theta_{ijt}\}$ of the markets for such consumers, and commits to a present-value of wages $\{W_{ijt}\}$ for each consumer of type i who is hired in t and begins to work in $t+1$. Each family chooses the total number of its members $\{s_{it}\}$ searching for jobs. In stage 2, after having observed all firms' offers, each family allocates its searching members among the j firms $\{s_{ijt}\}$, where $s_{it} = \sum_j s_{ijt}$. For a family of type i, such a plan specifies the number of consumers s_{ijt} who search for each firm j when confronted with the offers $\{\theta_{ijt}, W_{ijt}\}$. At the end of period t, a proportion σ of matches exogenously terminate. These two stages should be thought of as occurring at the beginning of each period t.[14]

2.2 A Family's Problem

We describe here a family's problem, the implied optimality conditions, and how consumers' preferences over jobs give rise to firms' monopsony power.

[14]Note that we have separated across the two stages a household's decision about the total measure of its members searching for jobs in period t, s_{it}, and about the allocation of this measure across the existing firms, $\{s_{ijt}\}_j$. What matters is that when a firm j decides on the offer $\{\theta_{ijt}, W_{ijt}\}$ for any worker of type i in period t, it takes as given the measure of workers of type i in the market searching for jobs and so the tightness of the corresponding market, θ_{it}.

2.2.1 Setup and Optimality

Consumers of each family i face the risk of not finding a job when looking for one and of losing a job when employed. But since there are no aggregate shocks and families consist a large number of members, there is no aggregate uncertainty at the family level. Thus, our economy is a deterministic one in terms of aggregates. Accordingly, the date-0 budget constraint of family i is

$$\sum_{t=0}^{\infty} Q_{0,t}c_{it} \leq \sum_{t=0}^{\infty} Q_{0,t} \sum_j W_{ijt}\lambda_w(\theta_{ijt-1})s_{ijt-1} + \psi_i\Pi_0, \tag{6}$$

where $Q_{0,t}$ denotes the price of the homogeneous good in period t in units of that good in period 0; W_{ijt} is the present value of wages of newly employed workers; Π_0 is the present value of all firms' profits; and ψ_i is the share of profits of the firms owned by the family. To better understand the first term on the right side of (6), note that if s_{ijt-1} consumers of a family of type i search for jobs at firm j in period $t-1$, then $\lambda_w(\theta_{ijt-1})s_{ijt-1}$ of them find a job, start working in period t, and earn the present value of wages W_{ijt} in units of period-t goods. Since a consumer of type i employed at firm j in t separates from it in $t+1$ with probability σ, the transition law for consumers of type i employed at firm j in t is

$$n_{ijt+1} = (1-\sigma)n_{ijt} + \lambda_w(\theta_{ijt})s_{ijt} \text{ for all } j, \tag{7}$$

where $\lambda_w(\theta_{ijt})$ is the job-finding rate at firm j in t for type-i consumers.

In period 0, a family of type i chooses consumption c_{it}, the number $\{s_{ijt}\}$ of its members looking for jobs across firms, and the number $\{n_{ijt+1}\}$ of its members employed across firms subject to the budget constraint (6), the transition law for employment at each firm (7), and a nonnegativity constraint on the number of searchers $s_{ijt} \geq 0$, with $s_{it} = \sum_j s_{ijt}$ and n_{it} satisfying (1) for all t, in order to maximize the present value of its utility. Dropping the subscript i for simplicity and letting ζ, $\beta^{t+1}\nu_{jt+1}$, and $\beta^t\chi_{jt}$ be, respectively, the multipliers on the budget constraint, the transition law for employed consumers, and the nonnegativity constraint on $\{s_{ijt}\}$, the first-order conditions for the problem of a family of type i with respect to consumption, the number of employed, and the number of searchers imply

$$\beta\frac{u_{ct+1}}{u_{ct}} = Q_{t,t+1}, \tag{8}$$

$$\frac{\nu_{jt+1}}{u_{ct+1}} = \frac{u_{nt+1}}{u_{ct+1}}\left(\frac{n_{jt+1}}{n_{t+1}}\right)^{\frac{1}{\omega}} + \beta(1-\sigma)\frac{\nu_{jt+2}}{u_{ct+2}}\frac{u_{ct+2}}{u_{ct+1}}, \tag{9}$$

$$-\frac{u_{st}}{u_{ct}} = \lambda_w(\theta_{jt})\frac{\beta u_{ct+1}}{u_{ct}}\frac{\nu_{jt+1}}{u_{ct+1}} + \frac{\beta u_{ct+1}}{u_{ct}}\lambda_w(\theta_{jt})W_{jt+1} + \frac{\chi_{jt}}{u_{ct}}, \tag{10}$$

where we have used $\beta^t u_{ct} = \zeta Q_{0,t}$ to derive (8), which is the standard Euler equation for consumption. To understand the next two equations, recall that the family of an employed consumer is paid the present value of wages W_{jt+1} for a match with firm j, which lasts until an exogenous separation occurs. In (9), ν_{jt+1} is the discounted marginal disutility from a marginal increase in the number of the family's members who work at firm j in $t+1$, of whom $(1-\sigma)$, $(1-\sigma)^2$, and so on are still employed in $t+2$, $t+3$, and subsequent periods. To express this disutility in consumption units, define $V_{jt+1} \equiv \nu_{jt+1}/u_{ct+1}$ and substitute $Q_{t+1,t+2} = \beta u_{ct+2}/u_{ct+1}$ into (9) to express V_{jt+1} recursively as

$$V_{jt+1} = \frac{u_{nt+1}}{u_{ct+1}} \left(\frac{n_{jt+1}}{n_{t+1}} \right)^{\frac{1}{\omega}} + Q_{t+1,t+2}(1-\sigma)V_{jt+2}. \tag{11}$$

Further substituting $\nu_{jt+1}/u_{ct+1} = V_{jt+1}$ and $Q_{t,t+1} = \beta u_{ct+1}/u_{ct}$ into (10) gives

$$-\frac{u_{st}}{u_{ct}} = Q_{t,t+1}\lambda_w(\theta_{jt})(W_{jt+1} + V_{jt+1}) + \frac{\chi_{jt}}{u_{ct}} \text{ for all } j. \tag{12}$$

To understand this condition, note that a marginal increase in the number of consumers who search for jobs at firm j in t leads to a corresponding increase in the disutility from searching u_{st}/u_{ct} when expressed in consumption units. This term is the left side of (12). The benefit of incurring this cost is that with probability $\lambda_w(\theta_{jt})$ such consumers find jobs in period $t+1$ and receive the present value of wages W_{jt+1} in units of period- $t+1$ consumption goods, net of the present value of the disutility of work V_{jt+1}. Expressed in period-t consumption units, this expected net benefit is $Q_{t,t+1}\lambda_w(\theta_{jt})(W_{jt+1} + V_{jt+1})$, which corresponds to the first term on the right side of (12). For consumers who actively search in period t in that $s_{jt} > 0$, it follows that $\chi_{jt} = 0$ and so the last term on the right side of (12) is zero. Hence, for consumers who actively search for jobs at firm j in t, (12) implies that the value of doing so must be at least as high as the value of searching for jobs at any other firm j' so that

$$\lambda_w(\theta_{ijt})[W_{ij+1} + V_{ijt+1}] \geq \mathcal{W}_t \equiv \max_{j'}\{\lambda_w(\theta_{ij't})(W_{ij't+1} + V_{ij't+1})\}. \tag{13}$$

In the firm's problem below, when firm j makes employment offers to consumers, it understands that it will attract them only if this constraint is satisfied, which arises from consumers' optimal search behavior. Hence, this constraint is the key one on firms when they make wage and vacancy-posting decisions.

2.2.2 The Participation Constraint and Firm Monopsony Power

We now examine how the constraint (13) simplifies in our symmetric equilibrium and how it encodes firms' monopsony power. In such an equilibrium, each firm needs to anticipate what happens if it deviates from a symmetric allocation. Specifically, consider an allocation in which all firms but one, say firm j, offer the common value $\lambda_w(\theta_{it})\,(W_{it+1} + V_{it+1})$ to type-i consumers and suppose that firm j contemplates offering a potentially different value, $\lambda_w(\theta_{ijt})(W_{ijt+1} + V_{ijt+1})$. Then, for firm j to attract a consumer, it must offer at least that common value. That is, firm j's offer must satisfy

$$\mathcal{W}_t(\theta_{ijt}, W_{ijt}) \equiv \lambda_w(\theta_{ijt})(W_{ijt+1} + V_{ijt+1}) \geq \mathcal{W}_t = \mathcal{W}_t(\theta_{it}, W_{it+1}) = \lambda_w(\theta_{it})\left(W_{it+1} + V_{it+1}\right). \quad (14)$$

We refer to this constraint as the *participation constraint* and note that is the symmetric version of (13). The second stage of an equilibrium is summarized by this constraint (14). Solving forward the recursive expression in (11) for the discounted marginal disutility resulting from a marginal increase in the number of family i's members who work at firm j in $t+1$ yields

$$V_{ijt+1} = \frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + Q_{t+1,t+2}(1-\sigma)\frac{u_{nit+2}}{u_{cit+2}}\left(\frac{n_{ijt+2}}{n_{it+2}}\right)^{\frac{1}{\omega}} + \ldots \quad (15)$$

Monopsony power affects a firm's problem through the derivatives of V_{ijt+1} with respect to vacancies a_{ijt} and market tightness θ_{ijt}. See Appendix A for details.

Although the supply curve of workers for a firm in period t is a dynamic object that depends on wages and market tightness in t as well as the expectations of these variables in all future periods, we can provide some intuition about it in steady state, assuming that preferences $u(c_i, n_i, s_i)$ are of the form $u(c_i - v(n_i) - h(s_i))$ as in Greenwood, Hercowitz and Huffman (1988), which we will use in our quantitative analysis. In this case, the participation constraint in steady state reduces to

$$\frac{\lambda_w(\theta_{ij})}{r+\sigma}\left[w_{ij} - v'(n_i)\left(\frac{n_{ij}}{n_i}\right)^{\frac{1}{\omega}}\right] = \mathcal{W}_i.$$

Holding fixed θ_{ij} and differentiating this constraint with respect to w_{ij} and n_{ij}, we obtain that

$$\frac{dw_{ij}}{dn_{ij}} = \frac{1}{\omega}\frac{v'(n_i)}{n_i}\left(\frac{n_{ij}}{n_i}\right)^{\frac{1}{\omega}-1} > 0. \quad (16)$$

In this sense, the (inverse) labor supply curve for firm j slopes upward in n_{ij}. But as ω becomes arbitrarily large, the slope of this curve converges to zero. This upward-sloping labor supply curve facing firms in our framework is akin to the simple static upward-sloping labor supply curve studied in Robinson (1933).

Firms' monopsony power, as captured by this labor supply curve, affects a firm's first-order conditions for offered wages and vacancies through the derivatives of workers' participation constraint, which capture how the present value of the disutility of work increases when any firm j changes its wages and vacancies, and so market tightness, to attract more workers, holding fixed the value of search in the common market.[15] Put differently, this upward sloping supply curve is the source of firms' monopsony power. Intuitively, firms realize that to attract additional workers of any type i, they need to compensate workers of the same type already employed for their increased disutility

[15]In Appendix A, we discuss how this upward-sloping labor supply curve for a firm's jobs that our model gives rise to is analogous to the downward-sloping demand curve for a firm's goods that arises in models of monopolistic competition.

of work. As a result, they end up hiring fewer workers and paying lower wages in equilibrium. As we show below, an additional distortion arises in our framework, since firms' monopsony power also depresses job creation. In later sections, we discuss the extent to which the alternative policies we consider alleviate or compound all these monopsony distortions.

2.3 A Firm's Problem

We consider two versions of a firm's problem that differ by the type of capital that firms use. In the first version, we assume that firms use a standard type of capital, often referred to as *putty-putty* capital, which is homogeneous and whose substitutability with other inputs is the same before and after it is installed. Intuitively, if we think of each piece of capital as a machine, then this assumption implies that even after a machine is built, it is possible to alter it to use it with different amounts of labor. An issue with this version of the model is that it will have predictions for employment in response to an increase in the minimum wage that are at odds with the data. Motivated by this issue, we consider a second version of a firm's problem with *putty-clay* capital such that after a machine is built, its labor intensity is irreversible. This feature will imply that in this version, employment in the short run reacts much less to increases in the minimum wage than in the putty-putty version and, hence, its response is more in line with the data.

2.3.1 A Firm's Problem with Standard Capital

Consider a firm's problem with standard capital. Given an initial capital stock k_0 and an exogenous sequence of prices of investment goods $\{q_t\}$ expressed in units of consumption goods, each firm chooses sequences of tightnesses $\{\theta_{ijt}\}$ for markets for consumers of type i, measures of vacancies $\{\mu_i a_{ijt}\}$ to post aimed at consumers of type i, the measures of consumers of type i to employ $\{\mu_i n_{ijt+1}\}$, the present values of wages $\{W_{ijt}\}$ for these consumers, and new capital $\{k_{t+1}\}$ in order to maximize

$$\sum_{t=0}^{\infty} Q_{0,t} \left\{ F\left(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt}\right) - q_t x_{jt} - \sum_i W_{ijt} \lambda_f(\theta_{ijt-1}) \mu_i a_{ijt-1} \right\}, \tag{17}$$

subject to the law of motion for capital $k_{jt+1} = (1-\delta)k_{jt} + x_{jt}$, the transition laws for employment for consumers of type i

$$\mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt} \text{ all } i, \tag{18}$$

and the *participation constraints* for attracting consumers of type i

$$\lambda_w(\theta_{ijt})(W_{ijt+1} + V_{ijt+1}) \geq \mathcal{W}_t \equiv \max_{j'} \{\lambda_w(\theta_{ij't})(W_{ij't+1} + V_{ij't+1})\} \tag{19}$$

in each period. For this economy, given an exogenous sequence of investment goods prices $\{q_t\}$, a *monopsonistically competitive search equilibrium* with standard capital and $k_{j0} = k_0$ for all j is a collection of allocations of consumption, employment, searchers, and capital $\{c_{it}, n_{it}, s_{it}, \bar{n}_{it}, k_t\}$, vacancies and market tightnesses $\{a_{it}, \theta_{it}\}$, and prices $\{W_{it+1}, Q_{0t}\}$ such that at these prices and allocations i) consumers' decisions are optimal for each family i, ii) firms' decisions are optimal, and iii) markets clear.

Since families can perfectly insure the idiosyncratic risk of their members and there are no aggregate shocks, it is without loss to adopt the convention that a firm fulfills its present-value wage offer W_{ijt} by offering a constant period wage w_{ijt} over the course of a match that begins at t so that

$$W_{ijt} = w_{ijt} + (1-\sigma)Q_{t,t+1}w_{ijt} + (1-\sigma)^2 Q_{t,t+2}w_{ijt} + \ldots, \tag{20}$$

where $Q_{t,s}$ is the price of goods in $s > t$ in units of goods in t. Since $W_{ijt} = d_t w_{ijt}$ where $d_t \equiv \left[1 + (1-\sigma)Q_{t,t+1} + (1-\sigma)^2 Q_{t,t+2} + \ldots\right]$, we can equivalently think of firms as choosing W_{ijt} or w_{ijt}.

2.3.2 A Firm's Problem with Putty-Clay Capital

Suppose now that capital is of the putty-clay type—we drop the subscript j denoting a firm for notational simplicity. The idea behind this version of the model is most easily understood when all low-educated consumers and all high-educated consumers have the same ability, so that there are only two types of consumers. Ex-ante capital is putty-like in that it is possible to build a machine with any ratio of low-educated and high-educated labor to capital that lies on the frontier of the production function in (2), that is, the output technology is CES ex ante. Once a machine is built, however, it is clay-like in that it requires a fixed amount of low-educated labor and high-educated labor to operate at full capacity, that is, the output technology is Leontief ex-post. Hence, given a stock of machines, demand for low-educated labor and high-educated labor is inelastic in the short run as long as total profits from operating the machines are positive, because a firm cannot substitute between existing capital and either type of labor. Over time, though, new machines embodying new labor-to-capital ratios can be installed. Thus, in the long run, firms can substitute away from the type of labor that becomes more expensive, for instance, low-educated labor when the minimum wage increases, towards both high-educated labor and capital.

More formally, consider the case of interest in which low-educated and high-educated workers differ in their ability level, z. With many types of labor of type i, a capital type $v = \{v_i\}$ denotes the education and ability intensity of capital, that is, how much labor of each education and ability capital needs in order to produce a certain amount of output. Each v_i then specifies the type-i labor-to-capital ratio necessary to run a machine of type v at full utilization. As a result, $k(v)$ units of capital of type v provide $k(v)$ units of capital services only if, for all i, this capital is combined with at least $n_i = k(v)v_i$ units of labor for all i. If $n_i > k(v)v_i$, then the excess workers remain idle whereas if $n_i < k(v)v_i$, then the excess capital remains idle. If $k(v)$ units of capital are combined with $n_i = k(v)v_i$ units of labor for all i, then $f(v)$ units of output are produced according to the constant-returns-to-scale production function $F(\cdot)$ with $f(v)$ defined by

$$F(k, \{n_i\}) = kF(1, \{n_i/k\}) = kF(1, \{v_i\}) = kf(v).$$

More generally, if some arbitrary amount of labor $\{n_i(v)\}$ is combined with $k_i(v)$ units of capital of type v, then the total output produced with type-v capital is $y(v) = \min\left[k(v), \{n_i(v)/v_i\}\right] f(v)$, were $\min\left[k(v), \{n_i(v)/v_i\}\right]/k(v)$ can be thought of as the utilization rate of the $k(v)$ units of capital of type v. The total output of a firm in period t is thus

$$y_t = \int_v \min\left[k_t(v), \{n_{it}(v)/v_i\}\right] f(v)dv.$$

Firms invest $x_t(v)$ units of output to accumulate type-v capital according to the accumulation law

$$k_{t+1}(v) = (1-\delta)k_t(v) + x_t(v) \tag{21}$$

subject to the nonnegativity constraints $x_t(v) \geq 0$.

Given some initial vector of capital $\{k_0(v)\}$ that a firm owns and an exogenous sequence of prices of investment goods $\{q_t\}$ expressed in units of consumption goods, a firm chooses sequences of market tightnesses $\{\theta_{it}\}$, vacancies $\{a_{it}\}$, employed workers $\{n_{it+1}(v)\}$ for each type of capital v, present value of wages $\{W_{it+1}\}$, and investment $\{x_t(v)\}$ for each type of capital in order to maximize

$$\sum_{t=0}^{\infty} Q_{0,t} \left\{ \int_v [F(k_t(v), \{n_{it}(v)\}) - q_t x_t(v)]dv - \sum_i \left[W_{it}\lambda_f(\theta_{it-1})a_{it-1} - \kappa_i a_{it}\right] \right\}, \tag{22}$$

subject to the transition laws for workers (18), the participation constraints for employed workers (19), along with the transition law for each type of capital (21), the adding-up constraints $n_{it} \leq \int n_{it}(v)dv$ for the uses of labor of each type i, the Leontief constraints on labor

$$n_{it}(v) \leq v_i k_t(v), \tag{23}$$

and the nonnegativity constraints on each type of investment $x_t(v) \geq 0$. To understand the constraints in (23), note that with a capital stock $k_t(v)$ of type $v = (v_i)$ that can be used to produce the output $y_t(v) = \min[k_t(v), \{n_{it}(v)/v_i\}]f(v)$, if, say, the firm uses $n_{it}(v)$ units of type-i labor such that $n_{it}(v) > k_t(v)v_i$, then the excess labor $n_{it}(v) - v_i k_t(v)$ is wasted, so this is never optimal. Hence, we can impose the constraints in (23) and drop the Leontief function from the firm's problem. The non-negativity constraint $x_t(v) \geq 0$ implies that firms cannot disassemble their existing types of capital. Without this friction, the firm's problem would reduce to the putty-putty problem previously described.

2.4 Steady-State Properties

Here we discuss the steady state of the economy and discuss some key features of it. It is immediate that the steady state of the model with standard capital and the model with putty-clay capital are identical. Intuitively, once factor prices and the intertemporal prices of consumption that firms face become constant, firms invest in the unique type of capital that is ideally suited to their technologies at those prices and let all past capital depreciate. Eventually, all the old capital stock is replaced. In a steady state with putty-clay capital, firms invest in exactly the same type of capital as they do in a steady state with standard capital. In our baseline, we focus on Greenwood, Hercowitz and Huffman (1988) (GHH) preferences,

$$u(c_i, s_i, n_i) = U\left[c_i - v(n_i) - h(s_i)\right], \tag{24}$$

for each family i. Consider the steady state of this version of the model in which all variables, including the price of capital, are constants. In the steady state, the firm's Euler equation for capital is

$$q\left[\frac{1}{\beta} - (1-\delta)\right] = F_k, \tag{25}$$

the firm's vacancy posting condition is

$$\frac{\kappa}{\lambda_f(\theta_i)} = \frac{F_i - v'(n_i) - v'(n_i)/\omega}{r+\sigma}, \tag{26}$$

a family's first-order condition for the number of searchers is

$$h'(s_i) = \frac{\lambda_w(\theta_i)\left[w_i - v'(n_i)\right]}{r+\sigma}, \tag{27}$$

and equilibrium wages satisfy

$$w_i = \eta\left[F_i - \frac{v'(n_i)}{\omega}\right] + (1-\eta)v'(n_i), \tag{28}$$

where $F_k = F_k(k, \bar{n}_\ell(n_i), \bar{n}_h(n_i))$, $F_i = F_i(k, \bar{n}_\ell(n_i), \bar{n}_h(n_i))$, and $r = 1/\beta - 1$. Finally, the steady-state law of motion for employment reduces to

$$\lambda_w(\theta_i)s_i = \sigma n_i, \tag{29}$$

where $\bar{n}_\ell$ and $\bar{n}_h$ satisfy the symmetric steady-state version of (4). Consumption satisfies a steady-state version of the budget constraint in (6). With GHH preferences, which imply no income effects, the steady state equations then split into two blocks. First, we can solve for the monopsonistically competitive search equilibrium wages $\{w_i\}$ and the associated allocations $\{\theta_i, s_i, n_i\}$ and k from (25)–(29). Then, given these allocations and prices, we can solve for consumption $\{c_i\}$ from the budget constraint.

Notice that firms' monopsony power distorts the wage equation and this distortion is captured by the term $v'(n)/\omega$: for a given marginal product of labor and marginal disutility of work, a firm offers a smaller wage than under the competitive search equilibrium for our economy, in which case $v'(n)/\omega = 0$ since $\omega = \infty$. As apparent from the first-order condition in (27), this inefficiently low level of wages results in consumers searching too little for jobs. Firms' vacancy-posting condition (26) features both the indirect distortion from the inefficient level of wages and the direct distortion due to the term $v'(n)/\omega = 0$. Despite these two distortions have countervailing effects on the marginal benefits of posting vacancies, it turns out that in equilibrium firms post too few vacancies. Hence, firms create too few jobs, consumers search too little for them, and both wages and employment are lower than in the competitive search benchmark—the efficient case for our economy. Since a firm with monopsony power pays its workers only a fraction of their marginal products, a simple measure of firms' monopsony power is then the markdown of wages relative to workers' marginal products, $1 - w_i/F_{ni}$, or, equivalently, the percentage difference between a worker's marginal product and wage, $(F_{ni} - w_i)/F_{ni}$. In a slight abuse of language, we refer to w_i/F_{ni} as the *wage markdown*. Letting $\kappa(\theta_i) = (r+\sigma)\kappa_i/\lambda_f(\theta_i)$, we can combine the equilibrium vacancy-posting condition and the wage equation to show that the implied markdown for workers of a family of type i is

$$\frac{w_i}{F_{ni}} = \left[1 + \underbrace{\frac{\kappa(\theta_i)}{\frac{\eta}{1-\eta}\kappa(\theta_i) + v'(n_i)}}_{\text{efficient component}} + \underbrace{\frac{\frac{1}{\omega}v'(n_i)}{\frac{\eta}{1-\eta}\kappa(\theta_i) + v'(n_i)}}_{\text{monopsony component}} \right]^{-1}. \tag{30}$$

In (30), the *efficient component* of the markdown is defined as the markdown that would arise in the absence of firms' monopsony power. This efficient component corresponds to the level of the wage markdown needed for firms to recoup their vacancy-posting costs and hence earn zero expected profits per vacancy. More interesting is the *monopsony component*, which arises because firms with monopsony power set wages below their competitive search level and so their markdowns are larger than the competitive search ones. As this equation makes clear, for any given size of measured markdowns, the larger the efficient component is, the smaller the monopsony component is. In our quantitative exercises, we find that the overwhelming majority of the markdown in wages is due to the monopsony distortion.

A simple policy implication of this analysis is as follows. Since all distortions emanate from firms paying too low a wage to each consumer type, a policy that mandates that firms must pay a *type-specific* minimum wage $\underline{w}_i$ for each consumer type i, set equal to the wage for that worker type in the competitive search equilibrium, would fix all distortions and lead to efficient allocations. More precisely, define the competitive search equilibrium wages $\{w_i^*\}_i$, associated allocations $\{\theta_i^*, s_i^*, n_i^*, c_i^*\}$, and k^* that satisfy the competitive search equilibrium wage equation

$$w_i^* = \eta F_i^* + (1-\eta)v'(n_i^*) \tag{31}$$

along with the conditions (25)-(27), (29), and a steady-state version of the budget constraint. As we consider a sequence of economies in which firms' monopsony power converges to zero in that ω diverges to infinity, the monopsonistically competitive search equilibrium wages converge to the competitive search equilibrium wages and allocations converge to those of the competitive search equilibrium. We summarize this discussion in the following result.

***Proposition* 1.** *If a minimum wage for each worker type is set equal to the competitive search equilibrium wage for that type and that constraint binds, that is, $\underline{w}_i = w_i^*$, then wages and allocations in the minimum wage economy coincide with those of the competitive search equilibrium. Also, as ω diverges to infinity, wages and allocations in the monopsonistically competitive search economy converge to those of the competitive search equilibrium.*

For the second part of the proposition, note that as ω becomes arbitrarily large, the monopsonistically competitive search equilibrium wage in (28) converges to the competitive search equilibrium wage in (31). Hence, the distortions to wages vanish and, from an inspection of (26) and (27), so do the distortions to the vacancy-posting condition and the first-order condition for search.

The issue we will address in later sections is that although a very rich set of type-specific minimum wages could fix the distortions induced by firms' monopsony power, in practice setting such a complex system of minimum wages is infeasible. We will then analyze the other extreme, which corresponds to the minimum wage policies advocated in practice consisting of only *one* mandated minimum wage for all workers. In such a scenario, if large enough differences in skill and ability exist across workers, then a single minimum wage can have perverse distributional effects.

3 Quantification

We choose the parameters in our model to match key features of the U.S. labor market, which inform the mechanisms described above. We assume that a model period is one month in order to adequately capture worker flows in the labor market. We maintain that the utility function has the Greenwood, Hercowitz and Huffman (1988)' form, namely,

$$u(c_{it}, n_{it}, s_{it}) = \sum_{t=0}^{\infty} \beta^t \log\left(c_{it} - \chi_{g,n}\frac{n_{it}^{1+1/\gamma_n}}{1+1/\gamma_n} - \chi_{g,s}\frac{s_{it}^{1+1/\gamma_s}}{1+1/\gamma_s}\right), \tag{32}$$

where $\chi_{g,n}$ governs the disutility of work for education group $g \in \{\ell, h\}$, $\chi_{g,s}$ governs the disutility of search for each such group, and γ_n and γ_s help control the elasticity of labor supply and job search.[16]

3.1 Disciplining Key Features of the Model

We start by summarizing how we parameterize the key features of our model: the degree of firms' monopsony power, the distribution of worker heterogeneity, the elasticities of substitution across workers and with capital, and the putty-clay technology as summarized by the depreciation rate of physical capital. Table 1 reports this set of parameters. The degree of monopsony power is crucial because it determines the potential for the minimum wage to increase employment and labor market participation. In our model, the degree of monopsony power is controlled by the substitutability across jobs in workers' preferences, ω. We discipline this parameter by targeting existing empirical estimates of the average wage markdown in the data. This target is informative because, as equation (30) shows, ω determines the size of the monopsony component of the wage markdown. As ω becomes large, firms' monopsony power falls to zero and the inefficient component of the wage markdown vanishes. A growing literature has measured wage markdowns in the United States and estimated that, on average, workers are paid between 0.65 and 0.85 of their marginal product.[17] As a baseline, we target the value 0.75, which is midpoint of this estimated range. In Section 5, we show the sensitivity of our results to either higher or lower estimates of wage markdowns.

The distribution of workers' labor market productivity within each education group, captured by $\{\mu_i\}$, governs the degree of wage dispersion within each group and thus the distributional impact of the minimum wage across workers. For both education groups $g \in \{\ell, h\}$, we assume this distribution is lognormal with group-specific standard deviation σ_g, and choose the parameters σ_g to match the dispersion of the distributions of wages from the 2017-2019 American Community Survey.[18] Specifically, we target the ratio of the 50th percentile to the 10th percentile of the wage distribution of each education group in order to precisely match the left tail of each distribution, which is most directly affected by a higher minimum wage.[19]

[16] In principle, the preferences (32) may imply that households violate their time constraint in that $n_{it} + s_{it} > 1$. We ensure that the time constraint is always satisfied by augmenting (32) with a positive utility from leisure $\chi_\ell \log(1 - n_{it} - s_{it})$ and setting χ_ℓ to 0.01. Standard Inada conditions imply that households do not violate their time constraint. Otherwise, this term has minimal effects on our results. We view this specification as simply a technical device to ensure that the time constraint holds in each period without having to deal with an occasionally binding time constraint.

[17] See, for example, Berger, Herkenhoff and Mongey (2022a), Yeh, Macaluso and Hershbein (2022), Lamadon, Mogstad and Setzler (2022), and Seegmiller (2021). Manning (2021) provides a recent survey of this literature.

[18] We restrict the sample to include all individuals aged 16 and above, exclude all individuals residing in group quarters, and those who report themselves as being students, which mirrors the sample restrictions used by the BLS to compute labor market statistics. Given that we exclude students from our analysis, there are very few individuals in our sample between the ages of 16 and 20. As a result, our key findings are essentially unchanged if we calculate the wage distribution using a sample of individuals aged 21 and above. See Appendix B for more details.

[19] We also place a low weight on the 90-50 wage ratios within each education group in our moment-matching procedure to ensure that we do not severely overpredict the dispersion of wages at the top of the distribution.

Table 1: Parameters Governing Key Features of Model

Parameter	Description	Value	Discipline
Monopsony power			
ω	Substitutability across firms	2.74	Match literature's estimates of wage markdowns
Worker heterogeneity			
μ_i	Distribution of productivities z_i	lognormal	Match wage distribution from ACS
Long-run elasticities of substitution			
ρ	Elasticity of substitution between high-educated and low-educated workers	1.40	Fixed: Estimated value from Katz and Murphy (1992)
ϕ	Elasticity of substitution across workers within an education group	4.00	Fixed: Lower bound estimate from Card and Lemieux (2001)
α	Elasticity of substitution between capital and workers	1.00	Fixed: Cobb-Douglas
Putty-clay frictions			
δ	Depreciation rate	10% annual	Fixed: BEA data

Note: Baseline parameters governing key features of the model: monopsony power, worker heterogeneity, elasticities of substitution, and putty-clay frictions.

The production function parameters ρ, α, and ϕ govern the long-run substitutability between different types of workers and between capital and labor. An active debate in the literature concerns the elasticity of substitution between capital and labor, α, which has been found to range from a value suggesting greater complementarity than Cobb-Douglas in Oberfield and Raval (2021) ($\alpha = 0.5$ to 0.7) to a value suggesting greater substitutability than Cobb-Douglas in Karabarbounis and Neiman (2014) ($\alpha = 1.25$). As a compromise between these estimates, we set Cobb-Douglas ($\alpha = 1$) as our baseline, but show in Section 5 that our results are not very sensitive to values of α in this range.[20]

We choose the long-run elasticities of substitution across workers ϕ and ρ in order to match the values in Card and Lemieux (2001), who estimate the same CES production structure as in our model using long-run variation in the relative supplies of workers of different age (which they interpret as corresponding to different levels of ability, as in our model). The within-group elasticity ϕ is particularly important for our analysis because it controls the extent to which firms substitute away from low-wage workers in response to the minimum wage. Card and Lemieux (2001) find estimates for ϕ between 4 and 6. As a benchmark, we set $\phi = 4$ to match the lower end of this range and show in Section 5 that raising ϕ to 6 amplifies the negative effects of the minimum wage in the long run for low-wage workers.[21]

Card and Lemieux (2001) estimate the across-group elasticity ρ using variation in the relative

[20] In Appendix D, we study an alternative version of the model in which capital and non-college labor are substitutes but capital and college labor are complements, as in Krusell et al. (2000). The main results for this alternative specification are very similar to those discussed below.

[21] Card and Lemieux (2001) find similar estimates for both high- and low-education workers, so we set the same value of ϕ for both college and non-college workers.

supply of aggregated labor inputs controlling for trends in labor demand, by an approach similar in spirit to the seminal work of Katz and Murphy (1992). We choose $\rho = 1.4$ to be consistent with the benchmark Katz and Murphy (1992) value, which is within the range of estimates obtained by Card and Lemieux (2001) as well. Taken together, these benchmark estimates from the labor literature imply that the elasticity of substitution among workers *between* education groups is much smaller than the elasticity of substitution among workers *within* an education group, that is, $\rho < \phi$. We show in Section 5 that our key distributional results only change slightly when we use a higher value of the across-group elasticity, such as Bils, Kaymak and Wu (2020)'s recent estimate of $\rho = 4$. These results confirm that the within-education-group elasticity of substitution among workers ϕ is quantitatively more important than the between-education-group elasticity of substitution among workers α.

Appendix C shows that Card and Lemieux (2001)'s estimated parameter elasticities ϕ and ρ map into the true parameter values of our model, despite the fact that Card and Lemieux (2001)'s framework does not include monopsony power or search frictions. The reason is that, in our model, the markdown of wages relative to marginal products is approximately constant across workers (see Figure 2 below). Therefore, ratios of wages between different types of workers approximately equals the ratio of their marginal products, which is the key condition in Card and Lemieux (2001)'s estimation procedure.[22] Appendix C also shows that the approximation error induced by our markdowns not being exactly constant is small in the sense that applying Card and Lemieux (2001)'s estimation strategy to data simulated from our model, recovers the true parameter values almost exactly.

Finally, due to our putty-clay technology, the dynamic effects of the minimum wage depend on the speed at which firms adjust their capital stock. Since firms in our model cannot uninstall existing capital, the speed of this adjustment process is largely determined by the depreciation rate of capital, δ. We set this rate to imply an annual depreciation rate of 10%, which roughly matches the aggregate depreciation rate for the U.S. economy, and later explore the impact of alternative values for it.

3.2 Details of the Quantification Procedure

We now describe the details of our quantification procedure, including how we discipline the remaining parameters which are less important for our main results. We proceed in two steps: we first exogenously fix a subset of additional parameters based on external evidence and then choose the remaining ones in order to match several informative statistics of the data. The parameters governing the search portion of the model—the cost of job posting, the parameters of the matching function, and those governing a household's disutility of labor market search—mainly determine the degree to which a change in employment from an increase in the minimum wage manifests itself as a change in labor

[22] Given that our production structure is weakly separable in capital and labor, these marginal product ratios fall into the class estimated by Card and Lemieux (2001).

force participation rather than in unemployment.

Table 2: Other Fixed Parameters

Parameter	Description	Value
γ_n	Labor supply elasticity	1.00
π_ℓ	Fraction of non-college households	0.69
β	Discount factor	$(1.04)^{-1/12}$
σ	Job destruction rate	2.8%
η	Elasticity of matching function w.r.t. vacancies	0.50
γ_s	Search supply elasticity	5.00
χ_s	Scale of search disutility	3.8×10^6

Note: Other parameters (in addition to ρ, α, ϕ, and δ discussed in Table 1) exogenously fixed in the calibration. A model period is one month.

Table 2 shows the other parameters that we exogenously fix, in addition to ρ, ϕ, α, and δ already discussed in Table 1. We set the parameter γ_n of the utility function, which governs the elasticity of labor supply, to 1, but we show that our results are robust to alternative values of it in Section 5. We fix the share of college-educated households in the population to $1 - \pi_\ell = 31\%$ in order to match their proportion in the ACS data. We choose a value of $(1.04)^{-1/12}$ for households' discount factor β so that the annualized real interest rate r equals 4%. We set the job destruction rate σ to 2.8% per month and the elasticity of the matching function with respect to the measure of searchers to $\eta = 0.5$. Finally, we note that there exists a locus of values for the parameters γ_s and χ_s governing the disutility of search that imply approximately identical steady-state moments but differ in the response of search effort to an increase in the minimum wage. We choose a pair of values on this locus that imply a relatively muted response of search effort to the minimum wage, as in the data; see the discussion in Section 4 below.

Table 3: Fitted Parameters

Parameter	Description	Value
Monopsony power		
ω	Monopsony power	2.74
Worker productivity distribution $\log\mathcal{N}(0, \sigma_b)$		
σ_ℓ	SD of non-college z	0.69
σ_h	SD of college z	0.78
Production function		
ψ	Coefficient on non-college labor n_ℓ	0.42
λ	Coefficient on capital k	0.30
Search frictions		
B	Matching function productivity	0.48
Labor Disutility		
$\chi_{\ell,n}$	Scale of non-college labor disutility	1.82
$\chi_{h,n}$	Scale of college labor disutility	2.43

Note: Parameters endogenously chosen to match the statistics in Table 4.

Table 3 contains the fitted parameters, which we choose to match the statistics in Table 4. We include the monopsony power ω and distribution of productivity z_i in this table, even though we already discussed them in Section 3.1, because the fitted parameters are all jointly determined. As already discussed, the degree of monopsony power ω is primarily determined by the average wage markdown in the data, and the dispersion of worker productivity σ_ℓ and σ_h is primarily determined by the ratio of the 50th percentile to the 10th percentile of the wage distribution of each education group. The scale parameters of the production function, ψ and λ in equations (2) and (3), govern the aggregate labor share and the share of total labor income accruing to college-educated workers. The parameter B governs the efficiency of the matching function, which determines the steady state unemployment rate—we target a steady state unemployment rate of 5.9% to be consistent with the data before the Great Recession. Finally, the parameters $\chi_{g,n}$ of the disutility of work of each group $g \in \{\ell, h\}$ control the steady-state employment rates of each group.[23]

TABLE 4: Targeted Statistics

Moment	Description	Data	Model
Average wage markdown			
$\mathbb{E}[w_{ni}]/\mathbb{E}[F_{ni}]$	Average wage markdown	0.75	0.75
Wage Distribution, ACS 2017-2019			
$w_{\ell 50}/w_{\ell 10}$	Non-college 50-10 ratio	2.04	2.00
w_{h50}/w_{h10}	College 50-10 ratio	2.30	2.06
Income shares			
$\mathbb{E}[w_i n_i]/Y$	Aggregate labor share	57%	57%
$\pi_h \mathbb{E}[w_{hz} n_{hz}]/\mathbb{E}[w_i n_i]$	College income share	55%	56%
Unemployment rate			
$\mathbb{E}[s_i]/(\mathbb{E}[s_i] + \mathbb{E}[n_i])$	Average unemployment rate	5.9%	5.9%
Employment Rates			
$\mathbb{E}_\ell[n_i]$	Non-college employment rate	47%	47%
$\mathbb{E}_h[n_i]$	College employment rate	62%	61%

Note: Statistics targeted using the parameters in Table 3. The average wage markdown is the midpoint of the range of estimated markdowns discussed in the main text. The average labor share is from Karabarbounis and Neiman (2014). The wage distribution targets, college-income share, and employment rates are calculated using the ACS 2017-2019 data described in Appendix B.

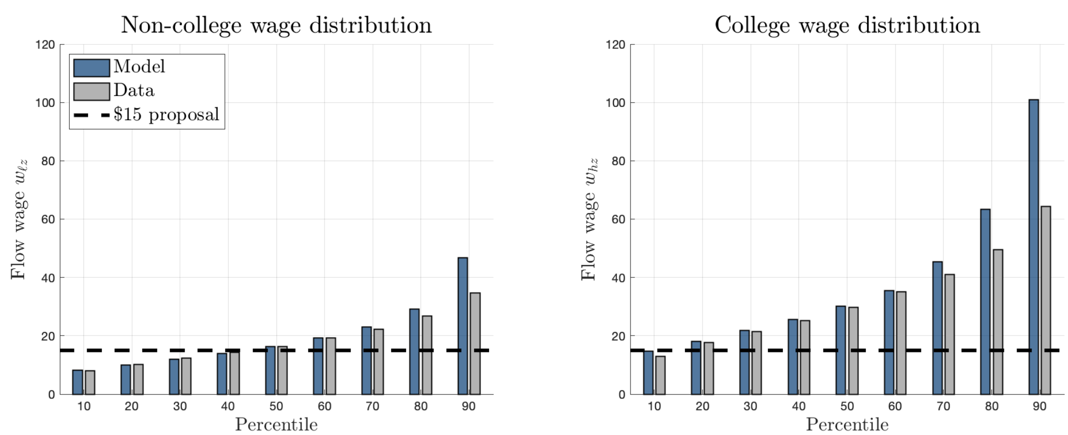
Table 4 shows that the model reproduces these targets extremely well. Importantly, the model matches the average wage markdown, and therefore the degree of firms' monopsony power, almost exactly. The unemployment rate, aggregate and college labor shares, the employment shares by education group, and the non-college 50-10 wage ratio are almost identical in the model and in the data. The average job-finding rate is 0.44 in the model, which is similar to the value of 0.45 in the data (see Shimer (2005)).

[23] As with many search models, there is a set of vacancy-posting costs κ_0 and matching-function productivity B which deliver allocations that are identical except for the average value of market tightness. Following Shimer (2005), we resolve this indeterminacy by normalize the vacancy-posting cost κ_0 such that market tightness is 1 in steady state.

3.3 Implied Wage Distribution

Figure 1 compares the wage distributions in our calibrated model to those from the ACS data. The model matches the left tail of the wage distribution within education groups fairly well by construction, since we target their 50-10 ratio in our calibration. As a point of reference, a $15 minimum wage would bind for approximately 45% of non-college workers in both the model and the data. However, the required dispersion in worker productivity measured by σ_g and the shape of the lognormal distribution imply that the right tail of the two fitted wage distributions is more dispersed in our model than in the data. For example, the interquartile range of non-college wages is 2.1 in the data vs. 2.3 in our model. We are comfortable with this tradeoff between the fit of the model at the low and high end of the wage distributions of the two groups given that the effects of the minimum wage are primarily determined by the left tail of these distributions. Across education groups, the model predicts that the median college wage is 1.82 times larger the median non-college wage, which is in line with a ratio of 1.81 in the data.[24]

Figure 1: Implied Wage Distribution



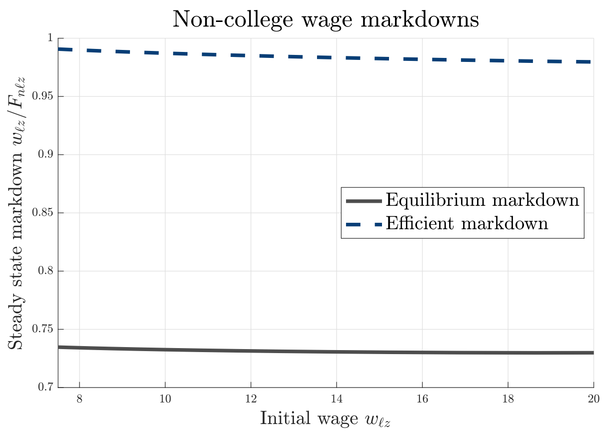
Note: Wage distribution in calibrated model (blue bars) and the data (grey bars). The wage distribution in the data is drawn from the pooled 2017-2019 waves of the ACS, as described in Appendix B.

Figure 2 plots the model's implied wage markdowns as a function of the steady state wage $w_{\ell z}$ by worker education group and ability. Recall from Section 2 that the steady state markdown is given by the expression in (30). The efficient component of the markdown reflects the fact that firms

[24]Curtis et al. (2021) estimates the firm-level response of capital and labor to the Bonus Depreciation Allowance, a temporary tax incentive for investment. They find that capital and labor increase roughly proportionally to this incentive. As a further validation of our model, we replicated the Bonus shock as a change in the after-tax relative price of capital, and found that capital and labor increase roughly proportionally due to our putty-clay technology. The Bonus only changes capital-labor ratios on new investment, which is small relative to the total size of the capital stock. In contrast, the model with standard capital cannot match this finding because firms adjust the capital-labor ratios on their entire stock of capital.

must recoup the annuitized cost of recruiting a worker of any type i, $\kappa_i/\lambda_f(\theta_i)$, regardless of their monopsony power. In our calibration, the efficient component of the markdown is about 1% to 2% and therefore accounts at most for about 4% of the average markdown of 0.75. The remaining portion of the markdown is due to firms' monopsony power, which implies that firms earn substantial monopsony profits in equilibrium.

FIGURE 2: Implied Wage Markdowns



Note: Steady-state wage markdowns $w_{\ell z}/F_{\ell z}$ of selected z-types among non-college workers. "Equilibrium markdown" corresponds to our calibrated model. "Efficient markdown" corresponds to the equilibrium of the model without monopsony power ($\omega \to \infty$). The x-axis corresponds to the initial wage $w_{\ell z}$ earned by a particular z-type.

4 The Short-Run and Long-Run Effects of the Minimum Wage

We now use our quantitative model to study the distributional effects of minimum wage changes of various sizes in both the short- and long-run. Our goal is to highlight how short-run labor market responses of minimum wage changes differ from long-run responses due to adjustment frictions. We further show that the differences between the short- and long-run responses are more pronounced for larger minimum wage changes.

For our counterfactuals, we assume that the minimum wage is unexpectedly introduced starting from the initial steady state without a minimum wage.[25] We begin with the steady-state implications for both small and large minimum wage changes in order to study long-run consequences. We then investigate the dynamic path of the economy from the original steady state to the new one and highlight how our putty-clay technology slows down the transition between them. Finally, we contrast the effects of a permanent increase in the minimum wage with those of a temporary one, which mimics the erosion by inflation and real productivity growth of a permanent increase in the *nominal* minimum

[25]With our calibrated wage distributions, the current national minimum wage of $7.25 would barely bind so that the initial steady state is a useful approximation to the current national policy regime.

wage. Throughout, we highlight how our model's quantitative results match many of the empirical results estimates from the minimum wage literature.

4.1 Long-Run Effects of the Minimum Wage

The minimum wage $\underline{w}$ modifies our steady state equilibrium conditions relative to those described in Section 2. In particular, for a household of type i for which the minimum wage binds, the steady-state wage equation in (28) no longer applies. We also replace w_i with $\underline{w}$ in the vacancy-posting condition in (26) and the first-order condition for search in (27). The remaining conditions are unchanged.

Aggregate Results. We begin our long-run analysis by studying the consequences of the minimum wage on aggregate non-college employment and labor income.[26] The left panel of Figure 3 shows that aggregate employment is a hump-shaped function of the minimum wage, which we refer to as an *employment Laffer curve.* As discussed in Section 2, this shape reflects the fact that a small increase in the minimum wage reduces the average monopsony distortion in the labor market, bringing wages and therefore employment closer to their efficient levels. Therefore, small minimum wage changes — such as a change of a \$1 or so that is often studied in the empirical literature — have a small, and positive, effect on employment in the long run. A large minimum wage increase, however, pushes the affected workers' wages above their efficient level reducing employment in the long run. For example, a \$15 minimum wage is well beyond the peak of the Laffer curve and reduces non-college employment in the long-run by about 12%.

The left panel of the Figure 3 also shows that the majority of these changes in employment from large minimum wage changes are due to changes in the labor force participation rate rather than changes in the unemployment rate. To see how these rates are linked, note that

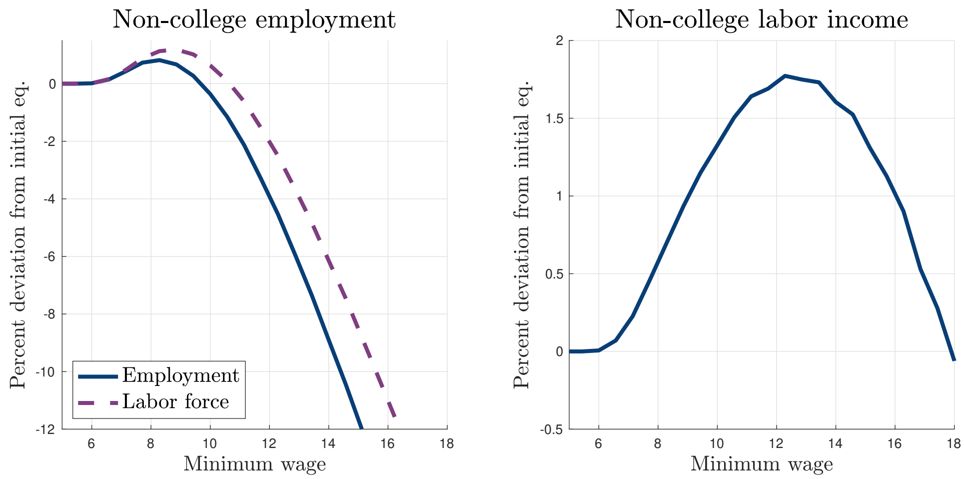
$$\underbrace{\Delta \log n_g}_{\text{employment}} \approx \underbrace{\Delta \log(s_g + n_g)}_{\text{labor force}} - \underbrace{\Delta \left(\frac{s_g}{s_g + n_g} \right)}_{\text{unemployment}}, \tag{33}$$

where n_g is the aggregate employment rate of education group $g \in \{\ell, h\}$, s_g is aggregate search effort of the group, $s_g + n_g$ is the labor force participation rate of the group, and $s_g/(s_g + n_g)$ is the unemployment rate of the group.[27] Across the range of minimum wages considered, the total change in non-college employment $\Delta \log n_\ell$ is primarily driven by changes in the participation rate $\Delta \log(s_\ell + n_\ell)$ rather than changes in the unemployment rate $\Delta(s_\ell/s_\ell + n_\ell)$. For example, a \$15 minimum wage

[26] We focus on non-college workers because the new minimum wage is barely binding for college workers. Appendix D shows how college employment and labor income vary with the minimum wage at both the aggregate and micro level.

[27] To derive this decomposition, observe that $n_g = \frac{n_g}{s_g + n_g} \times (s_g + n_g) = \left(1 - \frac{s_g}{s_g + n_g}\right)(s_g + n_g)$. Taking logs of both sides, it follows that $\log n_g = \log\left(1 - \frac{s_g}{s_g + n_g}\right) + \log(s_g + n_g)$. Since $\log x \approx x - 1$ for small x, this latter expression becomes $\log n_g \approx -\frac{s_g}{s_g + n_g} + \log(s_g + n_g)$. Finally, by taking differences, we obtain (33).

Figure 3: Long-Run Aggregate Minimum Wage Laffer Curves



Note: Steady-state outcomes as a function of the minimum wage $\underline{w}$. The left panel plots the percentage change in aggregate non-college employment and the percentage change in of aggregate non-college labor force as a function of the minimum wage (see the decomposition (33)). The right panel plots the aggregate labor income of non-college workers as a function of the minimum wage.

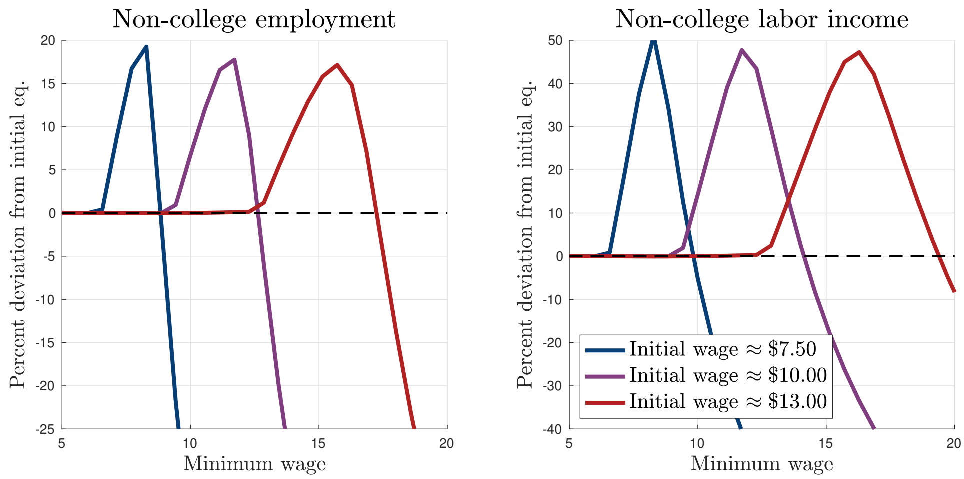
reduces non-college employment by 11.7%, decreases the non-college labor force participation rate by 8.5%, and increases the non-college unemployment rate by 3.2 percentage points.[28]

The right panel of Figure 3 shows that aggregate labor income as a function of the minimum wage also has a Laffer curve shape. This *labor income Laffer curve* peaks at a higher level of the minimum wage than the employment Laffer curve (at about $13 rather than about $9). This result occurs because the minimum wage has two effects on labor earnings. First, a higher minimum wage directly *increases* the wages of those individuals who remain working and who were initially earning a wage lower than the new minimum wage. For example, if a worker was initially earning $7.50 and that worker remained employed after the imposition of a $15 minimum wage, that worker would directly receive a 100% increase in their hourly wage. But second, as noted in the left panel, a large minimum wage also *reduces* employment of some workers, reducing their total labor income. On net, a $1.25 increase in the minimum wage (from $7.25 to $8.50) will increase the aggregate labor income of non-college workers by a little over 0.5%. Likewise, on net, a $15 minimum wage will increase aggregate non-college labor income by about 1.4% in the long run even though overall employment falls for this group; the direct effect of increased wages for those who remain employed due to the imposition of a large minimum wage change offsets the fact that employment will fall for some workers.[29]

[28] The fact that the long-run disemployment effect of the minimum wage manifests itself through lower labor force participation rather than higher unemployment is consistent with findings in the literature. For example, Adams, Meer and Sloan (2022) document that search effort does not significantly increase after an increase in the minimum wage.

[29] In the appendix, we highlight how aggregate income, firm profits, and the capital stock all monotonically decline when the minimum wage increases beyond the peak of the employment Laffer curve. We also discuss how when the

Figure 4: Disaggregated Long-Run Minimum Wage Laffer Curves



Note: Changes in steady-state employment (left panel) and labor income (right panel) of particular z-types among non-college workers as a function of the minimum wage. The x-axis is the level of the minimum wage. Initial wages rounded to the nearest half dollar.

Micro-Level Results. How do changes in the minimum wage affect individual workers of differing productivity within the non-college group? Figure 4 plots type-specific long-run employment and labor income Laffer curves for three types of non-college workers with different levels of latent ability z such that their initial wages are \$7.50, \$10, and \$13. These type-specific Laffer curves have different peaks corresponding to their different efficient level of their wage. For example, consider a worker who is initially earning \$7.50 (the blue line). The existence of monopsony power in our calibrated model implies that such workers will have their wages marked down by 25% relative to their marginal product. As a result, the peak of that worker's Laffer curve would be roughly \$9.40 (e.g., \$7.50(1 + 0.25)). Therefore, as the minimum wage increases up to \$9.40, these workers will see their employment increase; however, above this efficient level, employment will fall as firms will substitute away from these low-productivity levels (governed by the long-run substitutability as estimated by Card and Lemieux (2001), which our model replicates).

Now contrast the blue line with the red line in Figure 4, which shows the Laffer curves for a worker initially earning \$13.00 (red lines). The level of the wage which maximizes these workers' Laffer curves would be about \$16.25, so as the minimum wage increases up to \$16.25 these workers will see employment gains but a minimum wage above \$16.25 will induce employment declines.

Combining this logic across the different types of workers reveals the distributional makeup of our

minimum wage gets sufficiently high, the labor earnings of non-college workers remain relatively constant. As low wage workers become more expensive after the imposition of a sufficiently high minimum wage, firms find it optimal to switch towards higher wage workers. This force leads to an increase in the wages of college workers on average. However, at the same time, a higher minimum wage reduces the aggregate capital stock which reduces the wage of college workers.

aggregate Laffer Curves. For example, as the minimum wage wage increases by about \$1.25 (from \$7.25 to \$8.50), the employment of low productivity workers will increase while the employment of higher productivity workers would be unaffected; hence aggregate non-college employment can increase after the imposition of a small minimum wage change, as in Figure 3. However, for large minimum wage changes—for example a change in the minimum wage to \$15—the employment of some non-college workers will rise (such as those initially earning \$13 per hour) while the employment of other lower productivity workers will fall (such as those initially earning \$7.50 or \$10.00 per hour). Quantitatively, aggregate employment for non-college workers falls with a \$15 minimum wage as the employment losses from lower productivity non-college workers swamp the employment gains from higher productivity non-college workers.

The key takeaway from Figure 4 is that there is no level of the minimum wage that simultaneously increases the employment or labor income of workers initially earning \$7.50, \$10.00, and \$13.00 per hour. The reason is simple: the minimum wage is too blunt an instrument to do so.

Figure 5 contrasts the the long run distributional impact of a small and a large minimum wage change across workers. Specifically, our small minimum wage is \$8.50 (a 17% increase over \$7.25), which is around the size of minimum wage commonly explored in the empirical minimum wage literature (see, for example, Cengiz et al. (2019)). Our large minimum wage is \$15, which is the most common proposal in the United States. Figure 5 plots the long-run effects of these changes on employment, labor income, and welfare across workers in the economy. We define the *long-run welfare change* as the value of Δ_i which solves

$$u\left((1+\Delta_i)c_i^* - v(n_i^*) - h(s_i^*)\right) = u\left(\widetilde{c}_i - v(\widetilde{n}_i) - h(\widetilde{s}_i)\right), \tag{34}$$

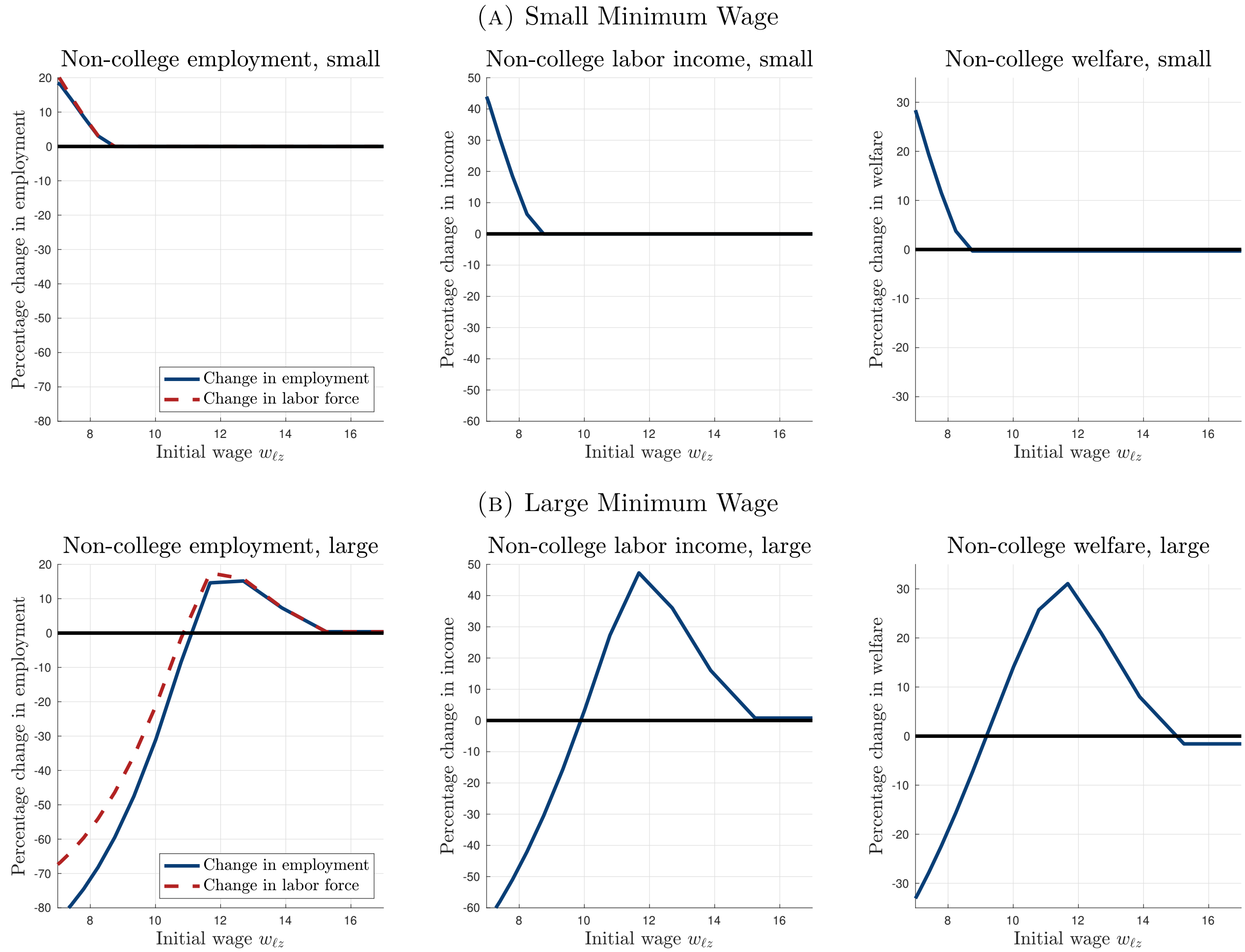
where x^* denotes the value of variable x in the old steady state and $\widetilde{x}$ denotes its value in the new steady state with the higher minimum wage. Here, Δ_i measures the percentage change in steady-state consumption that would make a household indifferent between the old and new equilibrium. Hence, Δ_i is positive if the policy makes the household better off and negative if the policy makes the household worse off.[30]

Panel A of Figure 5 shows that the small minimum wage does not negatively impact any worker in the economy, and increases the employment, labor income and welfare of all workers initially earning less than \$8.50. For example, for those workers initially earning \$7.25, an increase in the minimum wage of a \$1.25 increases employment by about 15%, increases labor income by about 30%, and increases welfare by about 20%.

In contrast, Panel B shows that the large minimum wage of \$15 has substantially negative long-run

[30] Our welfare analysis depends on the precise rule for distributing profits across households. We assume that profits are distributed in proportion to each households' share of total labor income.

Figure 5: Distributional Effects of an \$8.50 and \$15 Minimum Wage in the Long Run



Note: Steady-state outcomes for selected z-types among non-college workers for a \$8.50 minimum wage (Panel A) and \$15 minimum wage (Panel B). The left panel plots the percentage change in employment $n_{\ell z}$ relative to the initial steady state, the middle panel plots the percentage change in labor income $w_{\ell z} n_{\ell z}$ relative to the initial steady state, and the right panel plots the change in welfare for non-college workers $\Delta_{\ell z}$ from (34). The x-axis corresponds to the initial wage $w_{\ell z}$ of a type-z worker in the initial steady state.

effects for the lowest-wage workers in the economy. For example, employment falls for all non-college worker types who were initially earning less than \$11 per hour (who account for 26% of all non-college workers and 62% of the workers on whom the minimum wage binds). The decline is largest among lowest-earning workers because their efficient wages are substantially below the \$15 minimum. To the extent that employment gains occur, they are concentrated among workers whose initial wage was already close to \$15 - those workers whose initial wage was between \$11 and \$15. The middle panel shows that the same broad pattern holds in terms of labor income, although the set of workers whose labor income falls is smaller because wages rise for those workers who remain employed. Indeed, we find that 17% of all non-college workers and 41% of non-college workers initially earning less than

$15 experience declines in labor income. Consistent with these patterns, the right panel shows that in the long run, workers initially earning less than $9.00 experience a decline in welfare after the imposition of a $15 minimum wage, accounting for one-third of households initially earning less than $15 experience a decline in welfare after such a large minimum wage change.

Taken together, these results highlight the blunt nature of the minimum wage for reducing monopsony distortions: a small minimum wage can increase the employment, income and welfare of initially low-wage workers, but those benefits are exclusively concentrated among those workers; a large minimum wage can successfully reduce the monopsony distortion for higher-wage workers, but at the expense of lowering income, employment and welfare for the lowest earning workers. Table 5 previews the results of our various counterfactuals explored in the paper. The first three columns summarize results from a $1.25 minimum wage change whereas the last three columns summarize results from a $15 minimum wage. In particular, we summarize the total change in employment (columns 1 and 4), the total change in labor income (columns 2 and 5), and the fraction of individuals who experience a decline in labor earnings (columns 3 and 6). Panel A summarizes results for all non-college workers whereas Panel B summarizes results for non-college workers with wages initially lower than the new-minimum wage. For example, with the $15 minimum wage counterfactual this panel summarizes how employment and labor income evolve for those workers initially earning less than $15. The long-run results from Figure 5 are summarized in the top row of each panel. This table will provide a way to easily compare our results from the various counterfactuals we explore over differing time horizons throughout the paper. The table shows that no worker is made worse from a $1.25 change in the minimum wage in the long-run. However, labor income falls in the long-run for 41% of non-college workers earning less than $15 per hour in response to a $15 minimum wage.

Additional Distributional Effects. Appendix D.5 uses our framework to explore the long-run distributional impact of the minimum wage across different U.S. states or sectors of the U.S. economy, given that different states and sectors occupy different regions of the aggregate wage distribution. We show that states or sectors with lower average wages, such as the low-wage states of Mississippi or West Virginia or the low-wage sectors of Retail Trade and Personal Services, would be substantially worse off in the long run than high-wage states or sectors after an increase in the minimum wage to $15. However, we emphasize that these results should only be viewed as illustrative because we do not formally model linkages or reallocation frictions across states or sectors.

4.2 Short Run vs. Long Run

Having explored the long-run consequences of the minimum wage, we now turn to studying the transition path of the economy to the new steady state. Along this transition path, the minimum

Table 5: Summary of Counterfactual Minimum Wage Changes in Short and Long Run

Time Period	\$8.50 Min Wage			\$15 Min Wage		
	Δn_ℓ	$\Delta w_\ell n_\ell$	Share $\Delta w_{\ell z} n_{\ell z} < 0$	Δn_ℓ	$\Delta w_\ell n_\ell$	Share $\Delta w_{\ell z} n_{\ell z} < 0$
Panel A: All Non-College Workers						
Long run (new steady state)	0.9%	0.9%	0%	-11.7%	1.4%	17%
Short run (first 5 years)	0.7%	4.1%	0%	-3.5%	10.5%	0%
Total effect: current cohorts	0.66%	2.7%	0%	-6.6%	7.6%	6%
Panel B: Workers With Wages Initially Lower Than New Minimum Wage						
Long run (new steady state)	7.4%	25.2%	0%	-28.0%	4.8%	41%
Short run (first 5 years)	9.0%	39.9%	0%	-8.6%	40.7%	0%
Total effect: current cohorts	10.0%	39.4%	0%	-16.9%	26.5%	16%

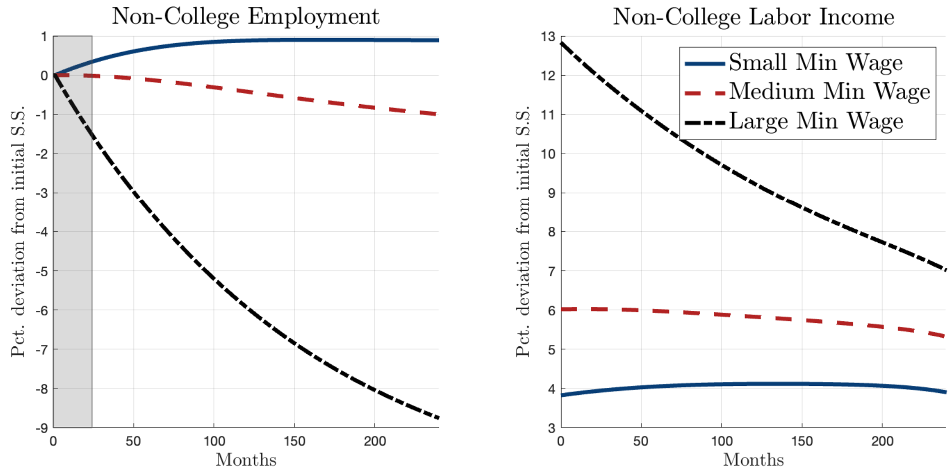
Note: Table shows a summary of our counterfactual analysis of a \$1.25 change in minimum wage (first three columns) and a \$15 minimum wage (last three columns). For each counterfactual change, we show the change in total non-college employment (columns 1 and 4), total non-college labor income (columns 2 and 5), and the fraction of workers who experience a decline in labor income (columns 3 and 6). Panel A computes statistics for all non-college workers, whereas Panel B computes statistics for non-college workers with an initial wage lower than the new counterfactual minimum wage. "Long run (new steady state)" entries compare the final steady state with the counterfactual minimum wage change to the initial steady state. "Short run (first 5 years)" entries compute analogous statistics along the transition path up to 5 years after the introduction of the higher minimum wage. "Total effect: current cohorts" entries compute the present values of changes over the entire transition path. In this case, Δn_ℓ is the change in the present value of non-college employment, $\Delta w_\ell n_\ell$ is the change in the present value of aggregate non-college labor income, and Share($\Delta w_{\ell z} n_{\ell z} < 0$) is the share of initial workers whose present value of labor income declines relative to the initial steady state. "Long run: temporary change" shows the long run results where the minimum wage changes in nominal terms and then a combination of productivity growth and inflation equals 5% per year.

wage imposes a lower bound $w_{ijt} \geq \underline{w}$ on the period wage that a firm can offer. Given a sequence of intertemporal prices $\{Q_{t,s}\}$, this constraint on period wages implies a constraint on the present value of wages of the form

$$W_{ijt} \geq \underline{W}_t \equiv \underline{w} + (1-\sigma)Q_{t,t+1}\underline{w} + (1-\sigma)^2 Q_{t,t+2}\underline{w} + \dots. \tag{35}$$

Note that $\underline{W}_t$ is the smallest present value of wages consistent with the minimum-wage constraint $w_{ijt} \geq \underline{w}$ in each period of a match, and depends on time because the intertemporal prices do. We add the constraint in (35) to a firm's problem. If for a consumer of type i the minimum wage constraint does not bind, then the first-order conditions are the same as before. When the minimum wage constraint binds, we set $W_{ijt} = \underline{W}_t$ so that a firm's wage offer satisfies (35). Upon the introduction of the new minimum wage, firms can fire existing workers or increase an existing worker's wage if desirable. All new hires must be paid at least the minimum wage and all workers retained by a firm

FIGURE 6: Transition Dynamics Following Permanent Increase in Minimum Wage



Note: Transition paths for non-college employment (left panel) and non-college labor income (right panel) following an unexpected imposition of a \$8.50 (blue solid line), \$11 (red dashed line), and \$15 (black dot-dash line) minimum wage.

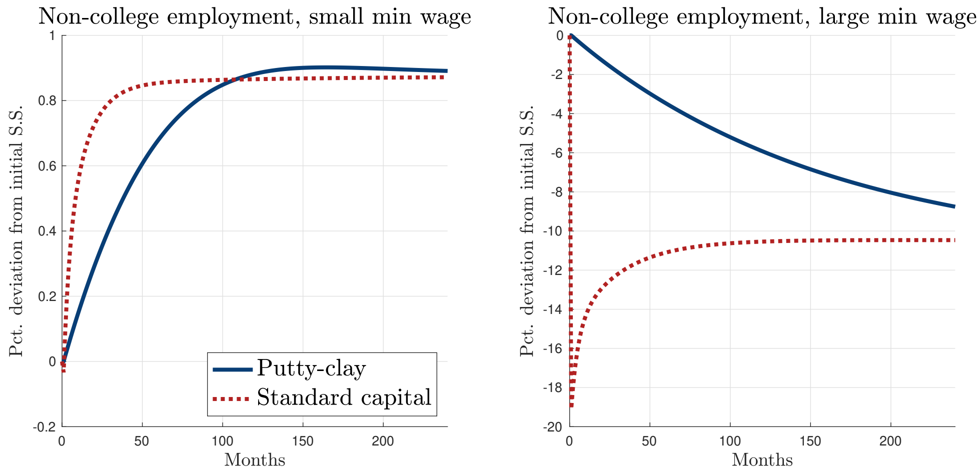
must be paid the larger of their existing wage and the minimum wage.[31] We turn to examine the dynamics of employment, income, and welfare.

Employment Dynamics. The left panel of Figure 6 plots the transition path of employment for all non-college workers in response to three different minimum wages: \$8.50 (red line), \$11.00 (blue line), and \$15.00 (black line). In all three cases, the change in employment is roughly similar in the first two years of the transition path (the shaded area of the figure): employment increases by 0.2% for the \$8.50 minimum wage, doesn't change for the \$11 minimum wage, and falls by 1% for the \$15 minimum wage. However, strong differences emerge over time for the larger changes in the minimum wage. Specifically, for the \$15 minimum wage, only about 10% of the eventual employment decline occurs in those first two years, while most of the effect of the \$1.25 minimum wage occur over the same period. Hence, it is inappropriate to use results from small minimum wage changes in either the short-run or the long-run to learn about the long-run effects of large minimum wage changes.

The long-run response to the \$15 minimum wage is very different from the short-run response

[31] If, instead, we allow a firm to lower the wage of an existing worker for whom the minimum wage does not bind, then a firm has an incentive to lower the wage until such a worker is just indifferent between quitting or remaining with the firm. We assume that the original contract the worker signed contains a clause that specifies that the firm cannot lower the wage in the event that a minimum wage is introduced. We note that absent such a clause, in our baseline there is a small group of workers for whom the firm would like to lower wages—workers with slightly higher productivity than those for whom the new minimum wage binds, who are the closest substitutes of the workers for whom the new minimum binds. We rule out this possibility because, without such a clause, the unexpected introduction of a minimum wage allows firms to renege on an existing wage contracts. More technically, we imagine that all agents believe that with probability ε a minimum wage will be introduced in the next period and the economy we consider is the limit of such an economy when ε converges to zero.

Figure 7: Role of Putty-Clay Frictions in Transition Dynamics



Note: Transition path following an unexpected imposition of a \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel) starting from the initial equilibrium without the minimum wage with putty-clay capital (blue solid line) and standard putty-putty capital (red dashed line). "Putty-clay" refers to the baseline model. "Standard capital" refers to the version of the model with standard capital described in Section 2.3.1.

because it takes time for firms to adjust their input mix. After the imposition of the new minimum wage, firms immediately reduce the non-college labor intensity of newly installed capital. However, their existing capital still requires low-productivity workers, so firms retain these workers in order to use the capital that they have already installed. Over time, this installed capital depreciates away and is continually replaced with less labor intensive capital, causing employment to fall. Appendix Figures D.6 and D.7 illustrate these mechanism by plotting various features of the distribution of capital types over the transition path.

Consistent with this discussion, Figure 7 shows that the model with standard capital converges much more quickly than our model with putty-clay frictions. The left panel shows that, in the model with standard capital and the small minimum wage change, the economy roughly converges to the new steady state in about 4 years instead of 8 years. The right panel shows that the differences between putty-clay and standard capital are much more pronounced with large minimum wage changes. In the model with standard capital, total non-college employment immediately falls *below* its new steady-state level upon the introduction of the new minimum wage. This abrupt decline occurs because firms immediately fire the lowest-ability workers because they are not needed to operate the existing capital as in the putty-clay model. Over time, firms hire new workers of intermediate ability to replace the low-ability ones, but this hiring process takes time due to the search frictions in the labor market. These hiring dynamics are fairly rapid in the sense that non-college employment converges to its new steady-state level only after a few years.

The depreciation rate δ is crucial in determining the speed of transition in our putty-clay model because it determines the rate at which the old, labor intensive capital is replaced by new, less labor-intensive capital. Our baseline analysis assumes a depreciation rate of 10% to be consistent with aggregate NIPA data, but we show in the appendix that our results are robust to larger depreciation rates. Ultimately, we view the putty-clay frictions as a parsimonious way to capture the rich set of frictions to firms adjusting their input mix. As long as firms face some reasonable adjustment costs to altering their production mix, the short-run employment response to large minimum wage changes will markedly differ from the long-run response.

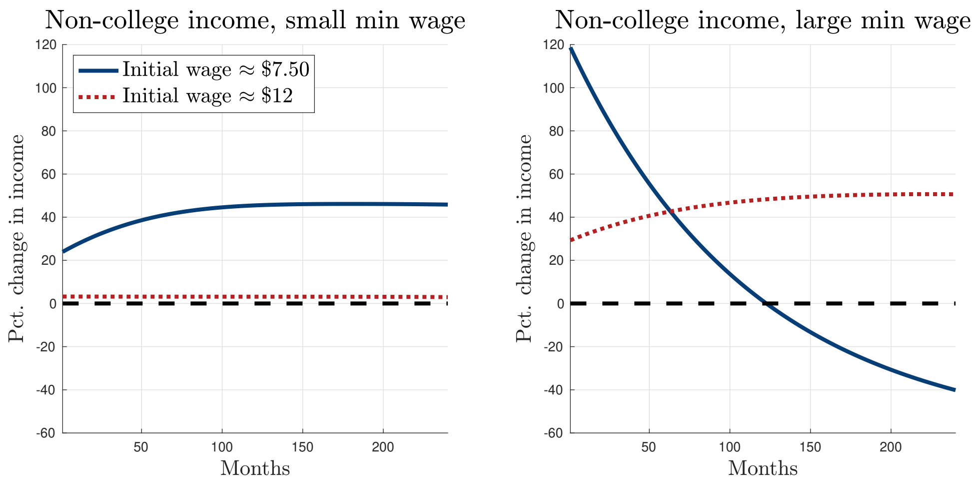
Income and Welfare Dynamics. The right panel of Figure 6 shows the transition dynamics for total labor income for non-college workers when the minimum wage is raised to either \$8.50, \$11.00, or \$15.00. In all cases, labor income immediately increases because the minimum wage raises wages and employment has not yet changed. For the small- and medium-sized minimum wages, these income gains largely persist over the transition path because employment does not substantially over time. However, for the large minimum wage, employment gradually falls over time, causing the increase in labor income to dissipate as well.

Figure 8 delves deeper into the micro-level dynamics of labor income in response to either a small (\$8.50) or a large (\$15) minimum wage change. Specifically, we contrast the time paths of labor income for a worker type who was earning \$7.50 in the initial equilibrium to another type who was earning \$12 in the initial equilibrium. In response to the small minimum wage change, the worker initially earning \$7.50 experiences an increase in earnings of about 20%; most of this increase is due to the direct effect of their wage being increased from \$7.50 to the new minimum wage of \$8.50, but some of the increase is due to the fact that employment increases over time as well. In contrast, there is no effect of a small minimum wage increase on workers initially earning about \$12 per hour because they are far above the new minimum.

The \$15 minimum wage generates more dramatic changes in labor income both over time and across the two types of workers. Initially, labor income increases for both types, but much more so for the type initially earning \$7.50 than worker initially earning \$12. This result occurs because \$15 minimum wage immediately doubles low-ability workers' wages and their employment has not yet significantly changed due to the putty-clay dynamics frictions. Over time, however, their labor income falls as firms substitute away from these workers; after about ten years, a low-ability worker's labor income falls below its initial level and remains below forever after. In contrast, labor income further rises for the type initially earning \$12 because the minimum wage reduces the monopsony distortion for these workers, inducing firms to hire more of them.

These results highlight how the effects of a large minimum wage substantially change over time

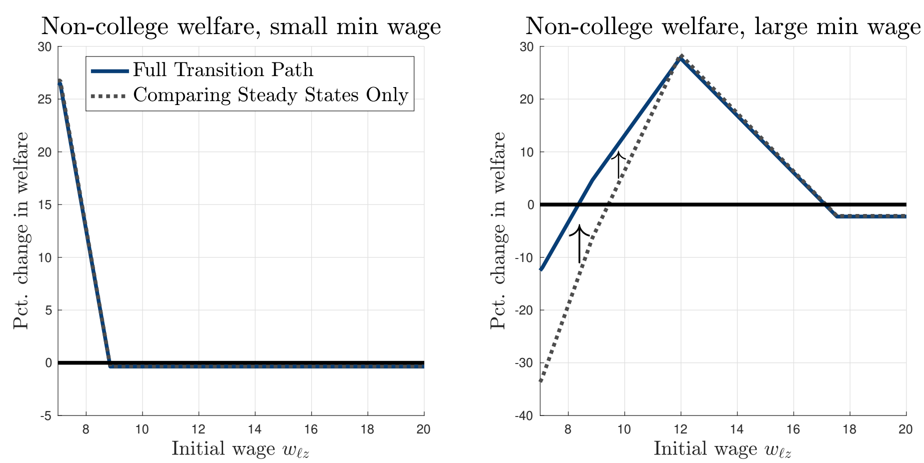
Figure 8: Labor Income for Two Workers in Response to Minimum Wage



Note: Flow labor income $w_{\ell z,t} n_{\ell z,t}$ for two types of non-college workers along the transition path following the unexpected imposition of an $8.25 minimum wage (left panel) and a $15 minimum wage (right panel) starting from the initial equilibrium without the minimum wage.

due to our putty-clay frictions (as previewed in Table 5). For low-wage workers, a large minimum wage change has sizable short-term benefits because employment has not decline, but eventually generates large long-term costs as the employment declines are realized. Cohorts of the lowest paid workers who start their careers after the introduction of the new minimum wage enjoy less of the short-term benefits and bear more of the long-term costs—the more so, the later these cohorts enter.

Figure 9: Welfare Effects of the Minimum Wage



Note: Change in welfare due to an $8.50 minimum wage (left panel) and a $15 minimum wage (right panel). The solid blue line plots welfare along the transition path computed as $\widetilde{\Delta}_i$ from (36). The dotted gray line plots welfare changes comparing only the new to the initial steady state computed as Δ_i from (34). The x-axis corresponds to the initial wage $w_{\ell z}$ of a type-z worker in the initial equilibrium.

Since the current cohort of lower-wage workers earn a higher income in the early phase of the transition to the new steady state, the steady state-to-steady state welfare comparisons in Section 4.1 overstate their decline in welfare along the transition path. To assess by how much, we calculate the consumption-equivalent *dynamic welfare change* taking into account the entire transition dynamics, $\widetilde{\Delta}_i$, from

$$\sum_{t=0}^{\infty} \beta^t u((1+\widetilde{\Delta}_i)c_i^*, n_i^*, s_i^*) = \sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it}, s_{it}), \tag{36}$$

where x^* denotes the value of variable x in the old steady state and the right hand side is evaluated along the entire transition path. As before, the minimum wage decreases welfare if $\widetilde{\Delta}_i < 0$ but increases welfare if $\widetilde{\Delta}_i > 0$.

Figure 9 plots both the steady state and dynamic welfare effects of the small minimum wage (left panel) and the large minimum wage (right panel). In response to the small minimum wage, the static and dynamic welfare changes are very similar because the economy transitions fairly quickly to the new steady state (recall Figure 6). However, the two measures differ in response to the large minimum wage; the set of workers whose welfare falls is much smaller once we account for the transition dynamics of the economy because of their short-term gains in terms of higher labor income described above. In fact, only 6% percent of the current cohort of non-college workers are made worse off after the introduction of a $15 minimum wage. In this sense, the steady-state welfare comparison overstates the true welfare change for the large minimum wage.

4.3 Model Predictions vs. Existing Empirical Estimates

In this subsection, we discuss how our framework is able to replicate many of the key patterns from the empirical labor literature. While this literature has primarily focused on the short-run labor market responses of small minimum wage changes, there are a few recent papers examining the longer-run labor market response of small and medium minimum wage changes. The fact that our model can replicate these findings gives us confidence that it is a useful laboratory to explore the long-run effects of large minimum wages changes, which the empirical literature has yet to convincingly tackle.

Short-Run Responses to Minimum Wage Changes. Neumark and Shirley (2022) provides a recent review of the empirical literature, which estimates the short-run employment effects of (mainly) small minimum wage changes. Specifically, they conduct a meta-analysis of 109 published papers based on cross-state variation in the minimum wage and compute the implied elasticity of employment with respect to the minimum wage.[32] As Neumark and Shirley (2022) highlight, essentially all of

[32]Given the typical multiplicity of findings in this literature even within a given paper, the authors compute the elasticities of interest by identifying the core estimates that support the conclusions from each study, in most cases, as they explain, by relying on responses from the authors of the studies.

Table 6: Counterfactual Short-Run Employment Elasticities to Minimum Wage Change

Minimum Wage Change	All Non-College Workers	Non-College Workers Initially Earning < \$15	Non-College Workers Initially Earning < \$10
\$1.00 Increase (13.8%)	0.03	0.07	0.15
\$1.50 Increase (20.7%)	0.01	0.03	0.06
\$2.00 Increase (27.6%)	0.02	0.04	0.07
\$2.50 Increase (34.5%)	0.01	0.03	0.07

Note: Table shows employment elasticities to various changes in the minimum wage for different groups of non-college workers. Column 1 shows the employment elasticities for a group including all non-college workers. Column 2 shows results for non-college workers initially earning less than \$15. Column 3 shows results for non-college workers initially earning less than \$10. Employment elasticities are computed as the change as the model's predicted percentage change in employment two years after the minimum wage change relative to the percentage change in the minimum wage.

the papers examine the employment effects of relatively small minimum wage changes (\$3 or less) and over relatively short time horizons (one or two years). Specifically, the vast majority of papers examine employment changes up to two years following an increase in the minimum wage. Also, all these papers tend to focus on the employment effects of lower-earning workers, such as teenagers and young adults, who are most likely to be effected by changes in the minimum wage. Neumark and Shirley (2022) document that roughly 80% of the studies they review find zero to small short-run employment effects in the two years after a minimum wage increase.

Table 6 shows that our model yields small short-run employment elasticities that are consistent with these estimates. Specifically, we compute two-year employment elasticities for various groups of workers in response to various small- and medium-sized changes in the minimum wage.[33] Two years after the imposition of a minimum wage change, employment does not change much for most sizes of the minimum wage changes and for most groups of workers, resulting in employment elasticities that are close to zero. We find it reassuring that our model is able to replicate the range of findings on short-run employment responses to small and medium sized minimum wage changes typically found in the empirical minimum wage literature.

Long-Run Responses to Small Minimum Wage Changes. In a recent paper, Cengiz et al. (2019) develop a novel empirical strategy to estimate both the short- and long-run responses to small minimum wage changes. There are two important points about the analysis in Cengiz et al. (2019) in the context of our analysis. First, the minimum wage changes they explore are small; averaging across all their minimum wage changes, the minimum wage increased by about 10%. Second, Cengiz

[33]For direct comparability to the empirical literature, we study a "small open economy" version of the model in that we hold consumption prices $Q_{t,t+1} = \beta$ fixed at each horizon along the transition path, in order to capture the idea that the minimum wage changes considered in Neumark and Shirley (2022) exploit state, as opposed to national, minimum wage increases. We do this for direct comparability but if we do not impose such a restriction the results in Table 6 are relatively unchanged.

et al. (2019) are able to use their methodology to explore changes in employment up to seven years after the minimum wage change. We interpret these results as estimating the long-run effect of small minimum wage changes because our Figure 6 shows that the economy has essentially converged to the new steady state seven years after a small minimum wage change. Cengiz et al. (2019) find that there were small positive employment effects in the few years after the minimum wage change which persist through this seven-year horizon (although, the employment increases were not statistically different from zero). They conclude that there does not seem to be any significant labor-labor substitution for small minimum wage changes up to seven years after the minimum wage change.

The left panel of Figure 6 shows that our model replicates the spirit of the findings in Cengiz et al. (2019). Specifically, in response to our small minimum wage change (which was a 17% increase in the minimum wage — slightly larger than the average size minimum wage change studied in Cengiz et al. (2019)), non-college employment increased slightly after the imposition of the minimum wage and remained roughly constant seven years after the change. In our model, small minimum wage changes do not lead to labor-labor substitution but instead encourages firms to hire more low-wage workers by lowering the monopsony distortion they face. The input substitution on the part of firms primarily occurs when the minimum wage change is sufficiently large so that it forces firms to pay workers in excess of their marginal product.

Long-Run Responses to Larger Minimum Wage Changes. Recent work in Clemens and Strain (2021) estimate the employment effects of smaller (less than $2.50) and larger (more than $2.50) state-level changes in the minimum wage in both the short-run (up to four years after the minimum wage change) and the medium-run (from four to six years after the minimum wage change). They find that in both the short and medium run, small and large minimum wage changes have insignificant effects on employment, and also that in the medium run, only large minimum wage changes have negative and statistically significant effects on employment. Our model replicates these findings these findings as well: the five year employment response to a large minimum wage change in Figure 6 is much larger than our one-year one. Likewise, we find very small elasticities in both the short and medium run when we consider minimum wage increases of only $1.25, that is, an increase from $7.25 to $8.50, as typically studied in the applied literature.[34]

[34]Clemens and Strain (2021) refer to small minimum wage changes as changes of about $1 and to large minimum wage changes as minimum wage changes of about $3 and focus on low-skill workers (aged between 16 and 25 years without a high-school diploma) and young workers (all those aged between 16 and 21). These authors document that in the short run, small minimum wage increases have effectively no impact on employment whereas large minimum wage increases have a small but insignificant effect on employment. In the medium run (4+ years ahead), small minimum wage increases have increased the employment rate in the medium run by 0.57 percentage points but this effect is statistically insignificant and not robust across specifications. Large minimum wage increases, instead, have decreased the employment rate by 2.65 percentage points, which is a robust and statistically significant effect.

Summary. Overall, we view it as a strength that our model can broadly match the short-run employment responses to minimum wage changes surveyed in Neumark and Shirley (2022), the long-run employment responses to small minimum wage changes documented in Cengiz et al. (2019), and the difference between short- and medium-run findings to small and larger minimum wage changes as documented in Clemens and Strain (2021). Importantly, because of putty-clay frictions, our model implies that the long-run responses of employment to larger increases in the minimum wage are likely to be much larger than the estimated short- and medium-run ones.

4.4 Temporary Changes in the Minimum Wage

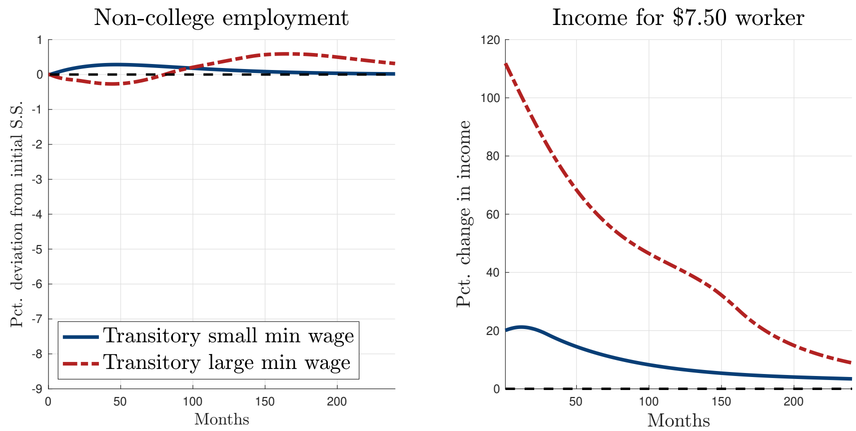
Although our analysis so far has considered only permanent increases in the real minimum wage, actual legislation in the United States has changed the nominal minimum wage, so the real minimum has declined in between legislated changes due to general price inflation. In addition, productivity growth implies that the real value of the minimum wage may decrease relative to average wages in the economy over time.

Here we consider a simple example in order to examine the implications of these dynamics. Specifically, we model a temporary increase in the real minimum wage through a path for it that decays at constant rate g, such that: $\underline{w}_t = \underline{w}_0(1+g)^t$. This process for $\underline{w}_t$ is a simple way to capture the periods between legislated changes in the nominal minimum wage. The decay rate g corresponds to the sum of inflation and real productivity growth over time. For illustration, we consider a value of g equal to 5% annually, which we interpret as corresponding to 2% productivity growth and an inflation rate of 3% (which can be thought of as the sum of 2% baseline inflation plus any additional inflation induced by the minimum wage).

The left panel of Figure 10 shows the time path of employment for non-college workers in response to small ($8.50) and large ($15) temporary minimum wage changes. The response of employment to the temporary minimum wage change displays two important differences relative to the permanent real minimum wage change. First, the overall response of employment to the transitory changes is an order of magnitude smaller than the response to the permanent change. This difference is partly due to the fact that the present value of the transitory minimum wage increase is smaller than the present value of the permanent minimum wage increase. But even in the early phase of the transition, when these present values are similar, the irreversibility of capital implied by the putty-clay technology induces firms to reduce employment by much less in the transitory case than in the permanent case. This result occurs because firms anticipate that any new capital they install will be in use later on in the transition, when the decaying minimum wage implies that lower-productivity workers will once again be relatively inexpensive to hire.

The second difference between the permanent and transitory paths for large minimum wage

Figure 10: Transitory Minimum Wage Increases



Note: Transition paths following an unexpected imposition of a time-varying minimum wage $\underline{w}_t$ that starts at \$15 and then decays at 5% per year. The left panel plots the time path of employment for all non-college workers for a \$8.50 (small) and \$15.00 (large) temporary minimum wage changes. The right panel plots the time path of employment for a worker initially earning \$7.50 in response to small and large temporary minimum wage changes.

changes is that the temporary large minimum wage path feature non-monotonic dynamics: employment declines below its initial level immediately after the introduction of the minimum wage, but then increases above the initial level before reverting back to its steady-state level. This occurs because the real value of the minimum wage is initially relatively high—above the efficient level of wages for sufficiently many workers that it decreases employment—but later declines to a relatively low level—close to the efficient level of wages for a large enough set of workers that it actually increases employment. For a 5% decay rate, the real value of the minimum wage eventually falls enough that it becomes nonbinding for both small and large minimum wage changes, at which point the economy returns to the initial steady state.

The right panel of Figure 10 compares the paths of labor income for a low-wage worker initially earning around \$7.50 in response to small and large temporary increases in the minimum wage. After the introduction of a \$15 minimum wage, which doubles the wage of workers initially earning \$7.50, the labor income of such workers increases by 100%. This change in labor income is completely driven by the increasing wage because, as seen in the left panel, employment barely changes for these workers after temporary changes in the minimum wage. As the real minimum wage increase dissipates over time, so do the labor income gains. Interestingly, the labor income for such workers is strictly higher in every period under a large temporary minimum wage increase than under a small one. Thus, for such low-income workers, a large temporary minimum wage increase strictly dominates a small one. Or, in other words, a one-time unanticipated introduction of a decaying large minimum wage allows low-wage workers to reap the short-run benefits of a large minimum wage without incurring its long-run costs. Note that we use this temporary case to illustrate the mechanisms of our model rather than to suggest that this is a policy that policymakers could repeatedly exploit—a full analysis of such a policy would require a carefully modeling of its impact on expectations.

5 The Role of Key Parameters and Extensions

Our analysis so far has used our particular calibration of the model to show how large changes in the minimum wage affect the path of employment and labor income over time. In this section, we show that two specific parameter values—the extent of firm monopsony power (ω) and the extent to which firms are willing to substitute among workers of differing productivities within an education group (ϕ)—are critical for determining the extent of employment declines for low-productivity workers in response to large minimum wage changes. While we have drawn on the existing literature discipline these parameters — the wage markdown literature for monopsony power ω and Card and Lemieux (2001) for the elasticity of substitution ϕ — in this section we explore how our key results change to alternate values. We then discuss the combinations of ϕ and ω that are consistent with a no long-run employment response to small minimum wage changes, as in the data, in order to disciple the long-run employment response of larger minimum wage changes. We briefly discuss the robustness of our key results to changes in other model parameters which are much less important for our results. Overall, these results highlight the assumptions needed on wage markdowns and the extent to which firms substitute among workers of differing productivities to justify the prediction that there will be no long-run employment declines in response to a $15 minimum wage. Finally, we discuss possible extensions of our framework.

5.1 The Importance of Firm Monopsony Power and Labor-Labor Substitutability

Table 7 shows how the long-run effect of a $15 minimum wage on non-college employment depends on the degree of monopsony power and the elasticity of substitution across workers ϕ.[35] Our baseline parameterization features a value of ϕ of 4 and targets a wage markdown of 0.75, in which case the $15 minimum wage leads to a 11.6% decline in non-college employment. The size of this employment decline is larger if monopsony power in the economy is weaker; for example, if the targeted wage markdown is small at 0.9, non-college employment declines regardless of whether the labor-labor substitution parameter ϕ, but if the markdown is larger at 0.6, then the minimum wage leads to either a smaller employment decline or even an increase in employment. These results occur because more monopsony power, as implied by the larger markdown, implies a larger gap between market wages and efficient wages that the minimum wage can fill. Hence, a belief that a permanent $15 real

[35] For this table, we recalibrate the model for each of the different values for ϕ and targets for the wage markdown.

Table 7: Robustness of Long-Run Non-College Employment Changes to a $15 Minimum Wage

	Targeted Wage Markdown		
ϕ	0.6	0.75	0.9
2	9.1%	-1.4%	-9.6%
4	-1.2%	-11.6%	-18.3%
6	-1.6%	-14.1%	-20.8%

Note: Table shows the total employment decline for non-college workers in response to a $15 minimum wage (permanent) for various values of ϕ and our targeted wage markdown. As a reminder, our baseline choice of ϕ is 4 and our baseline targeted wage markdown is 0.75.

minimum wage will have no adverse employment effects on non-college workers could be justified by a belief that firm monopsony power is quite large.

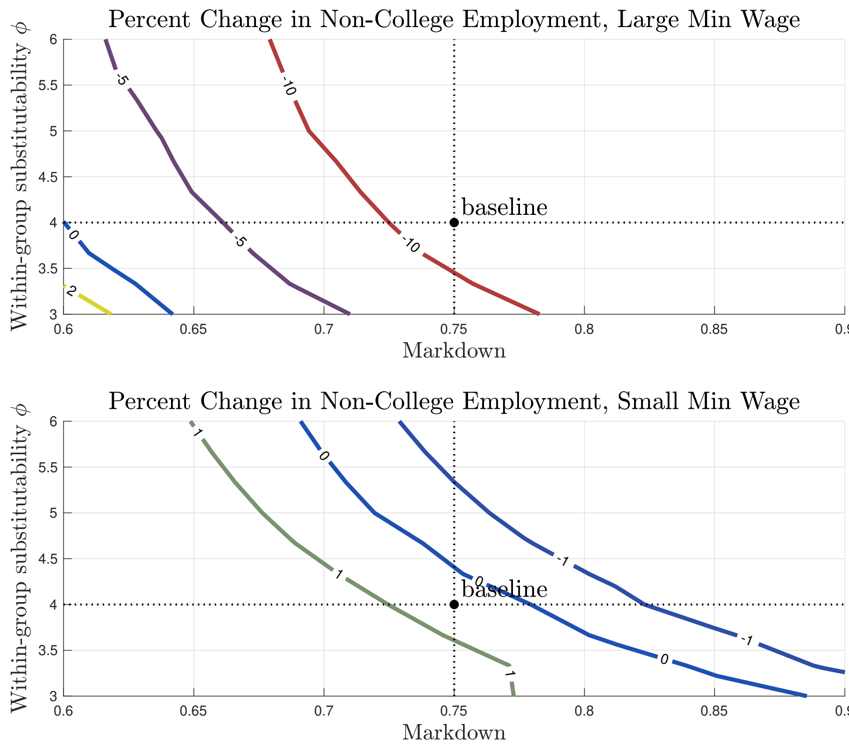
Likewise, the long-run employment response for non-college workers to a $15 minimum wage also hinges on the elasticity of substitution ϕ. All else equal, lower levels of ϕ lead to smaller employment declines for non-college workers. This result is not surprising; if workers of differing productivities are not very substitutable with one another, then firms are less inclined to adjust their input mix as lower productivity workers become more expensive. For example, if one believes the wage markdown is 0.75, then a $15 minimum wage will have only small long-run employment effects for non-college workers when $\phi = 2$. Therefore, one could also justify a belief that a permanent $15 real minimum wage will have no adverse employment effects on non-college workers with a belief that workers are not very substitutable with each other.

Figure 11 illustrates these points in a different way.[36] The top panel plots iso-employment lines for the values of ϕ and the targeted wage markdown that lead to the same long-run decline in non-college employment from a $15 minimum wage. For example, the red line plots the set of (ϕ, ω) that generate a 10% decline in non-college employment; pairs of parameters to the northeast of this line imply larger declines in employment. Similarly, the blue line corresponds to the set of parameter values that generate no change in non-college employment, and pairs of parameter values to the southwest of this line lead to increases in employment. Hence, in order for the $15 minimum wage to not decrease non-college employment in the long run, it must be that the markdown is below 0.65 and the within-group substitutability of workers is below 4, both of which are outside the consensus range of estimates in the literature.

The bottom panel of Figure 11 plots analogous iso-employment lines for a small minimum wage. In contrast to a $15 minimum wage, there is a wide range of parameter estimates—again, all values to the southwest of the blue line—that imply an increase in employment in response to a small minimum wage. This result highlights the nonlinear response of employment to increases in the minimum wage in

[36]For this analysis, for simplicity, we hold the other parameters fixed at their baseline values.

Figure 11: Sensitivity Analysis to Monopsony Power and Within-Group Substitutability

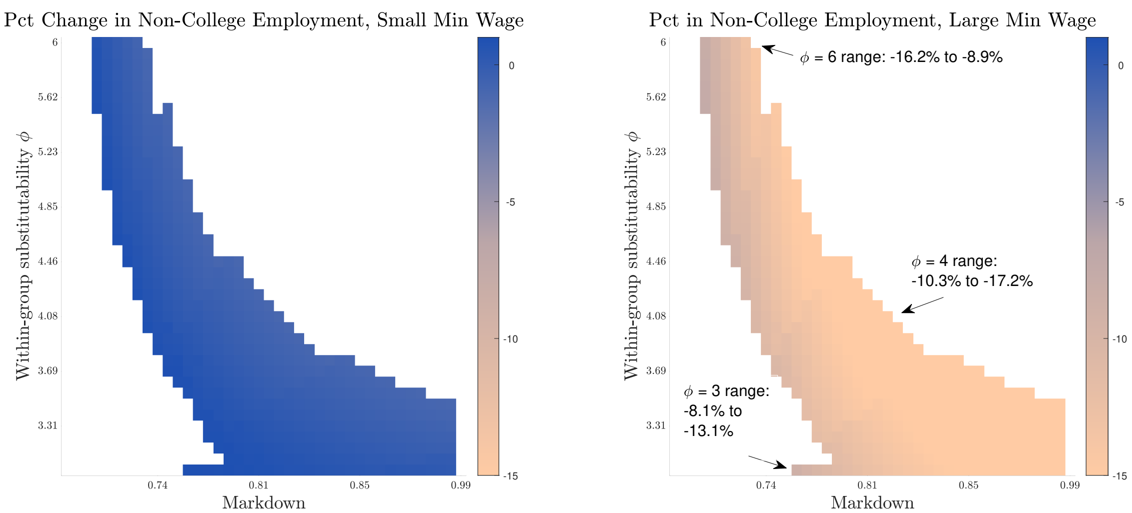


Note: Level curves of the percent change in non-college employment following the minimum wage as a function of the degree of monopsony power ω and worker substitutability within education group ϕ. Top panel plots level curves for large minimum wage (\$15) and bottom panel plots curve for smaller minimum wage (\$9). Each line corresponds to sets of parameters which deliver the same percentage change in non-college employment.

our model. The empirical literature finds that small minimum wage changes do not have a significant effect on the employment of non-college workers in the long run. Through the lens of our model, this evidence suggests that we are close to the blue iso-employment line, suggesting that ϕ is not too small, that the markdown is not too large, or both.

Figure 12 highlights that all of the parameter values generating a *small* long-run effect of small minimum wage changes imply a *large* long-run effect of large minimum wage changes. We search over all parameters such that a small change in the minimum wage yields between a -1% and +1% change in employment in the long run for non-college workers and plot the resulting heatmap of those changes in the left panel. This interval of employment responses is consistent with the estimates of the long-run employment effect of small minimum wage changes found in Cengiz et al. (2019) and Clemens and Strain (2021). Within this set of parameter values, we then ask what the long-run employment response for non-college workers would be to a \$15 minimum wage. The right panel of Figure 12 shows that these parameter values imply that a \$15 minimum wage would reduce non-college employment by 7% to 14% in the long run.

Figure 12: Effects of Small and Large Minimum Wage for Select Parameterizations



Note: Heatmaps of the effects of small and large minimum wage change on non-college employment for model parameterizations such that the small minimum wage generates a change in non-college employment between -1% and +1%. The left panel plots the effect of a small minimum wage ($10) and the right panel plots the effect of a large one ($15).

5.2 Robustness of Findings to Other Parameters

Table 8 shows the robustness of our results on the effects of a $15 minimun wage to the values of production elasticities ρ and α, the value of the labor supply elasticity as governed by the parameter γ_n, and the degree of search frictions as governed by κ_0, η, and σ.

First, consider the production elasticities. We set a higher value of the across-education-group elasticity $\rho = 4$ from recent work by Bils, Kaymak and Wu (2020), as opposed to $\rho = 1.4$ in our baseline from Katz and Murphy (1992), and find that our main results are not very sensitive to even large changes in the value of this parameter.[37] We then consider two different values of the elasticity of substitution between capital and labor α, corresponding to greater complementarity than Cobb-Douglas ($\alpha = 0.7$ from Oberfield and Raval (2021)) and greater substitutability than Cobb-Douglas ($\alpha = 1.25$ from Karabarbounis and Neiman (2014)). In either case, the results are fairly similar to those from our baseline. Overall, these results underscore that the most important elasticity of response on the part of firms following a $15 minimum wage stems from the within-education-group substitutability as highlighted above, as opposed to either the across-education-group substitutability or the substitutability between capital and labor.

The next entry of Table 8 shows that our results do not vary much with the parameter γ_n for the

[37]The findings in Bowlus et al. (2021) suggest an even greater substitutability between these workers than reported in previous studies. They document an elasticity of substitution between high-school- and college-educated workers that ranges between 3 and 8, depending on the years considered and the modelling of skill-biased technical change.

Table 8: Robustness of Long-Run Results to Other Parameter Values for a \$15 Minimum Wage

Parameterization	Δn_ℓ	Share ($\Delta n_{\ell z} < 0$)	Share ($\Delta w_{\ell z} n_{\ell z} < 0$)
Baseline	-11.7%	26%	17%
Other Production elasticities			
Cross-group $\rho = 4$ (Bils, Kaymak and Wu, 2020)	-11.0%	22%	18%
Capital-labor $\alpha = 0.7$ (Oberfield and Raval, 2021)	-10.0%	26%	17%
Capital-labor $\alpha = 1.25$ (Karabarbounis and Neiman, 2014)	-10.5%	22%	18%
Labor supply elasticity			
Labor supply elasticity $\gamma_n = 1/2$ (Chetty et al., 2011)	-13.0%	28%	19%
Search frictions (sensitivity to 50% above baseline)			
Vacancy posting costs κ_0	-12.4%	25%	21%
Matching function elasticity η	-11.9%	26%	17%
Job separation rate σ	-12.6%	26%	21%

Note: Change in steady state outcomes due to \$15 minimum wage; Δn_ℓ is the change in non-college employment, Share($\Delta n_{\ell z} < 0$) is the share of initial non-college workers who experience an employment decline, and Share($\Delta w_{\ell z} n_{\ell z} < 0$) is the share of initial non-college workers who experience an income decline. Rows correspond to different parameterizations of the model. "Baseline" refers to baseline calibration from Section 3. "Monopsony power" entries correspond to targeting different values of the markdown in the calibration (and simultaneously recalibrating all other fitted parameters as well). "Production elasticities" entries correspond to different values of the production elasticities (recalibrating all other fitted parameters as well). "Labor supply elasticity" corresponds to a different value of γ_n (recalibrating all other fitted parameters as well). "Search frictions" corresponds to setting a parameter to 50% above its baseline value, without recalibrating the other fitted parameters.

elasticity of labor supply. We set this elasticity to 0.5 as a value representative of the values in the *micro range* from Chetty et al. (2011), as opposed to our benchmark value of 1 from the *macro range*, and recalibrate our model given this lower elasticity. Clearly, our results are not sensitive to this change. The bottom rows of Table 8 show that different values of the search frictions have minimal impact on our results. Since there is little external evidence on the magnitudes of these parameters, we simply perform a sensitivity analysis in which we increase their size by 50%.

Finally, Appendix D shows that our main results are very similar in a version of our model with a slightly different production function featuring in which capital and non-college labor are substitutes but capital and college labor are complements (as in Krusell et al. (2000)).

5.3 Extensions

Here we discuss three fruitful extensions of our framework: incorporating imperfect risk sharing within a family, modeling the entry and exit of firms, and endogenizing skill acquisition by workers.

Imperfect Risk Sharing. Our model abstracts from imperfect risk sharing *within* a family despite allowing for imperfect risk sharing *across* families. Indeed, our model uses a representative family construct, which implies perfect risk-sharing against the income losses arising from both unsuccessful searches and job loss within worker types z. We conjecture that in such an environment if we break this perfect risk sharing by adding incomplete markets, the resulting model would imply that the welfare losses from the large minimum wage displayed in Figure 5 would be even larger because workers who lose their jobs would experience a larger decline in consumption. One way to accommodate a type of imperfect risk sharing within a family is by adding to the model home production, which represents any combination of home-produced goods and transfers from the government to non-employed family members with value smaller than the average value of the output of employed family members. Imperfect risk sharing then arises if all non-employed members of a family are *hand-to-mouth consumers* in that they simply consume their home-produced goods and all employed members share risk with each other as before. In our baseline model with perfect risk sharing within a family, when the minimum wage rises, total employment in a low-skilled family falls but total labor income falls much less. The reason is that total labor income, which includes the higher labor income of employed family members due to the increase in the minimum wage, is shared evenly across all family members, thus cushioning the impact of any decline in income on any one member. With imperfect risk-sharing of the type described, however, the same fall in total labor income leads to a worsening of the distribution of consumption within a family, since the newly non-employed members of a family bear much more drastic falls in consumption than the employed members.

Firm Entry and Exit. In our model, which assumes a fixed number of firms in the market, a large increase in the minimum wage decreases firms' profits, as Appendix Figure D.5 illustrates. We conjecture that allowing for the entry and exit of firms would increase the adverse effects of a large minimum wage increase, because it would imply that many firms eventually exit and are not replaced even in the long run. Specifically, imagine, in the spirit of Hopenhayn (1992), that there exists a large number of potential entrants that must pay a fixed cost to enter the market and that newly entered firms optimally choose their input mix. Entry occurs until the cost of entry for the marginal firm is exactly balanced by the expected present discounted value of profits after entry. Exit occurs through firms exogenously dying. In this setting, an unexpected increase in the minimum wage decreases the profits of any firm after entry. Because the entry cost is sunk for existing firms, immediately after the minimum wage increase, existing firms continue to produce as long as their flow profits from doing so are positive. Once this initial cohort of firms dies out, however, they will not all be replaced because the costs of entry are the same as before but profits are smaller. Hence, the total number of firms in the market and job varieties will decrease. This force reinforces the welfare losses from a minimum wage increase relative to our baseline model.

Skill Upgrading. Our model abstracts from endogenous skill acquisition. An interesting extension of our framework would be to endogenize the distribution of worker skills in response to permanent changes in the minimum wage. It is a priori ambiguous, though, if adding this feature would reinforce or diminish the detrimental effects of a large minimum wage increase on workers with low productivity.

On the one hand, since the minimum wage equalizes wages for all workers at the low end of the skill distribution, the minimum wage may disincentivize these workers from investing in new skills. On the other hand, since these workers also face lower job-finding rates, they may have an incentive to acquire more skills in order to increase their probability of finding a job. Hence, exploring the role of skill acquisition requires carefully modeling and pinning down how this process responds to these incentives, in particular its benefits and costs. Such a model would then have to confront the fact that workers who can in principle benefit the most from accumulating more skills—those at the bottom of the skill distribution—may also face the lowest returns to doing so. For example, existing work has found that low-wage workers tend to have lower levels of human capital as they face higher monetary or opportunity costs to acquiring skills (see, for instance, the papers reviewed by French and Taber (2011)). Such an analysis would be an ambitious and worthwhile endeavor not just for studying the minimum wage but also many other labor market policies.

6 Alternative Policies Part of the Tax and Transfer System

The previous section showed that although the minimum wage may benefit low-wage workers in the short run, it imposes significant costs on such workers in the long run unless it is set to a very low level. More generally, as Figure 4 showed, because it is such a blunt instrument, there is no minimum wage that simultaneously increases the labor income of workers for a range of low-wage workers, such as those initially earning \$7.50, \$10.00, and \$13.00 per hour. In contrast, existing policies which are part of the U.S. tax and transfer system can increase labor income for a whole range of low-wage workers. Section 6.1 describes how we model a tax and transfer system and ensure that the alternative policies we consider are quantitatively comparable to the minimum wage. Section 6.2 examines the effects of the EITC, a particularly important component of the transfers to low-income households in the data, and Section 6.3 explores its interaction with the minimum wage. Section 6.4 evaluates the minimum wage in the context of an approximation to the entire U.S. tax and transfer system.

6.1 Modeling Tax and Transfer Programs

Consider a general tax and transfer system $T(w_i)$, where $T(w_i)$ denotes labor income taxes so negative taxes $T(w_i) < 0$ indicate transfers. Let $A(w_i) = w_i - T(w_i)$ denote after-tax labor income, which may be greater or smaller than pre-tax income. The tax and transfer system affects both the incentives for firms to hire workers and for households to search for jobs in the labor market. Consider first how the system affects firms' labor demand, as summarized by the steady state vacancy posting condition

$$\frac{\kappa_i}{\lambda_f(\theta_i)} = \frac{1}{r+\sigma}\left[F_{ni} - w_i - \frac{1}{\omega}\frac{v'(n_i)}{A'(w_i)}\right], \tag{37}$$

where $A'(w_i)$ is marginal after-tax income. Equation (37) shows that a positive marginal tax rate, which implies $A'(w_i) < 1$, exacerbates monopsony distortions relative to our baseline model with $A'(w_i) = 1$, by increasing the magnitude of the last term in (37).

To understand this mechanism, recall that a monopsony distortion arises because hiring a marginal worker increases the marginal disutility of work for all inframarginal workers, so a firm needs to compensate inframarginal workers with a higher wage thereby increasing its cost of hiring. A positive marginal tax rate reduces the after-tax wage that inframaginal workers receive and therefore increases the required before-tax wage that a firm must offer, further increasing firms' cost of hiring. Conversely, a negative marginal tax rate, namely, a tax credit which results in $A'(w_i) > 1$, alleviates monopsony distortions by reducing firms' costs of hiring and so bringing them closer to the planner's costs—that is, they reduce the last term on the right side of (37).

Whereas *marginal* tax rates affect monopsony distortions on labor demand, *average* tax rates influence households' labor supply by affecting their search decisions. The optimal search decision satisfies

$$h'(s_i) = \frac{\lambda_w(\theta_i)}{r+\sigma}\left[A(w_i) - v'(n_i)\right], \tag{38}$$

where $r = 1/\beta - 1$. A positive average tax rate, which implies $A(w_i) < w_i$, reduces the after-tax wage relative to our baseline model, thus depressing the incentive to search, as equation (38) shows. By contrast, negative average tax rates imply $A(w_i) > w_i$ and so increase the incentive to search. The marginal tax rate is irrelevant for households' search decisions because they involve the extensive margin of whether or not to participate in the labor market. This discussion suggests that policies within the existing tax and transfer system, such as the EITC, that more directly target average and marginal wages may be more effective at alleviating monopsony distortions than the minimum wage.

We ensure that the policies we consider are comparable to a \$15 minimum wage as follows. First, note that a \$15 minimum wage reduces firms' flow profits by an amount $\Delta\pi^*$, so we can think of the minimum wage as corresponding to an implicit tax on profits. For each of our alternative policies, we then assume that there is no minimum wage but that the government levies an explicit linear corporate income tax on firms' profits of rate τ_f, which raises the same amount of revenues $\Delta\pi^*$. We assume that both investment and vacancy posting costs are fully deducted from corporate taxes, which implies that this tax does not distort any of the firms' marginal decisions, since it is a pure profit tax.[38] We then use these tax revenues to fund each of our alternative policies, that is, set $\tau_f\pi = \Delta\pi^* = -\sum_i T(w_i)$ where π denotes flow profits in the new equilibrium. Hence, each policy transfers on net $\Delta\pi^*$ from firms to households. Policies only differ in the schedule $T(w_i)$ that

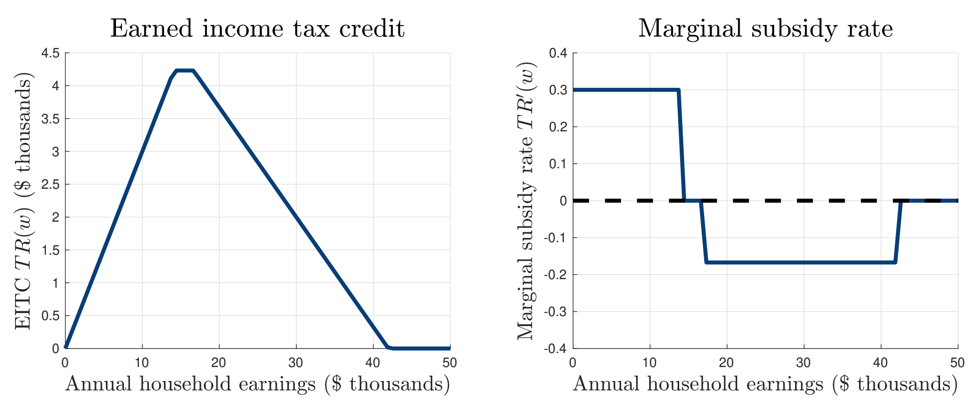
[38] Our pure profit tax is nondistortionary because our model features a fixed number of active firms. If we allowed for the free entry of firms, then reducing steady-state profits may reduce entry and so the number of firms. Our profit tax ensures that the entry margin would be distorted by the same amount across all alternative policies we consider.

determines the distribution of these transfers across households.

6.2 Earned Income Tax Credit

The earned income tax credit (EITC) is one of the largest components of transfer payments in the United States. The empirical EITC schedule has several kinks due to the implied subsidy being phased in and out at different income levels, which lead to three regions: (i) a *phase-in* region in which the subsidy (the tax credit) is proportional to household income, (ii) a *plateau* region in which the subsidy is capped at its maximum benefit, and (iii) a *phase out* region in which the subsidy is phased out. We choose our tax and transfer schedule $T(w_i)$ to mimic this empirical one adjusting magnitudes to ensure that it is budget-equivalent to a $15 minimum wage. Figure 13 plots it. We set the phase-in rate to 30%, as in the data. In the phase-in region, households face both a positive average subsidy rate, since the total tax credit is positive, and a positive marginal subsidy rate, since the credit is being phased in. In the plateau region, households face a zero marginal subsidy rate but still a positive average subsidy rate, since they are still receiving the benefit. Finally, we set the phase-out rate to approximately 18%. In the phase-out region, households face a positive marginal tax rate, that is, a negative marginal subsidy rate, because each dollar of earnings reduces their transfer payments.

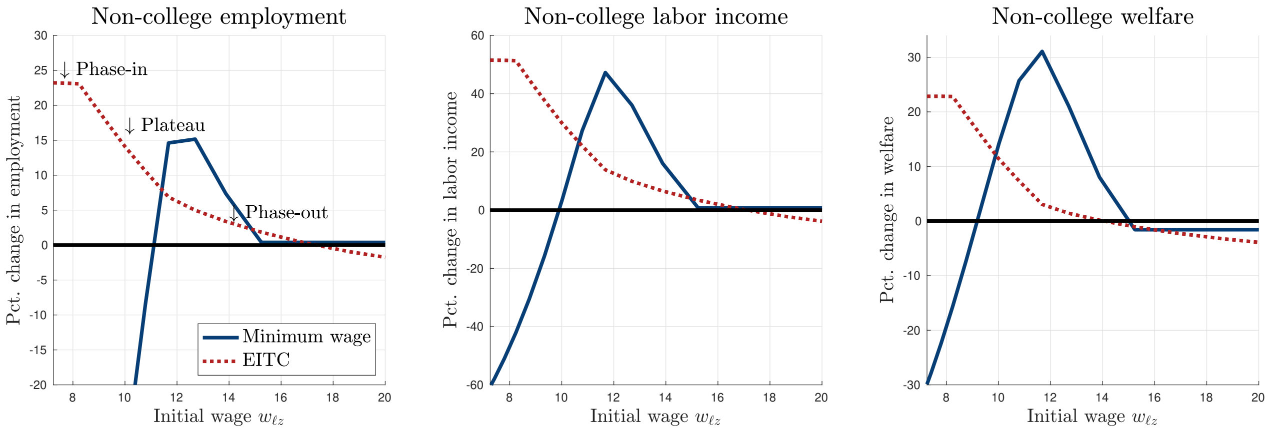
Figure 13: Earned Income Tax Schedule



Note: The left panel plots an EITC schedule that is budget-equivalent to a $15 minimum wage. The right panel plots the implied marginal tax rate. In each panel, the x-axis rescales steady state labor income to annual earnings assuming each household works 1800 hours per year.

Figure 14 shows how the EITC affects employment, income, and welfare in the long run for non-college workers. The lowest-wage workers are in the phase-in region and so enjoy both the positive effect of the positive average subsidy on their marginal benefit of searching for jobs and of the positive marginal subsidy on firm's marginal benefit of posting vacancies. Workers with somewhat higher wages are in the plateau region, where the effect of the EITC is still positive but decreasing in initial wages because the credit becomes a smaller share of total wages. Finally, workers in the phase-out region experience an even smaller increase, or perhaps even a decrease, in their employment, as the policy implies a positive marginal tax rate for them, as discussed.

Figure 14: Effect of Earned Income Tax Credit on Employment



Note: Steady-state employment (left panel), labor income (middle panel), and welfare (right panel) of selected z-types under the EITC. The y-axis is normalized relative to employment in the initial equilibrium (without any policies). The x-axis is the wage $w_{\ell z}$ of a z-type worker in the initial equilibrium without either policy. Steady-state welfare change Δ_i from $u\left((1+\Delta_i)c_i^* - v(n_i^*) - h(s_i^*)\right) = u\left(\widetilde{c}_i - v(\widetilde{n}_i) - h(\widetilde{s}_i)\right)$, where the superscript * denotes steady-state values in the initial equilibrium and the superscript $\widetilde{}$ denotes steady-state values in the new equilibrium.

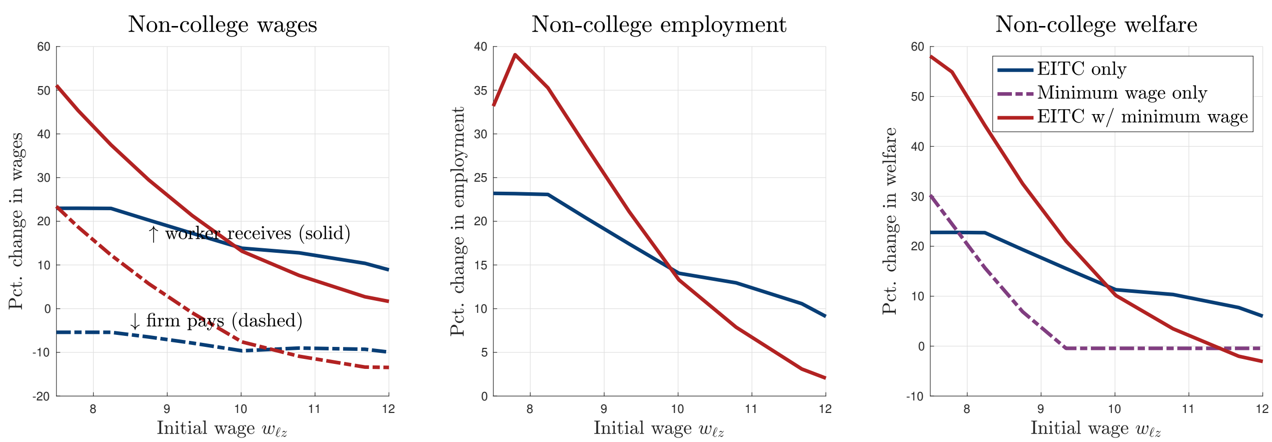
The key implication of Figure 14 is that, because it is a much richer policy instrument than the minimum wage, the EITC is able to increase the employment, labor income, and welfare for a wide range of low-wage workers—an outcome than any single minimum wage is unable to achieve. The EITC directly subsidizes low-wage workers when they work and, hence, this policy increases low-wage workers' incentives to look for jobs. From the firm's point of view, the EITC makes it cheaper to hire the now-subsidized low-wage workers relative to hiring the unsubsidized higher-paid workers. Hence, firms naturally hire more rather than fewer low-wage workers.

6.3 Interaction Between Transfer Policies and the Minimum Wage

A long-standing issue with the EITC and similar programs is that they also benefit firms because given the increase in labor supply they stimulate, firms can lower the wages they offer and so appropriate part of the benefits. This issue has led some authors to suggest that a modest minimum wage may complement transfer programs like the EITC because it prevents firms from reducing the wages they pay (see Neumark and Wascher (2011), Lee and Saez (2012), and Vergara (2022), for example.) Here, we reassess this argument in the context of our model. We confirm the intuition that a modest minimum wage is helpful in preventing firms from lowering pre-transfer wages. But, in our monopsony context, its main benefit is that it locally forces firms to pay workers more than the monopsony wage, thus reducing monopsony power and increasing employment.

Figure 15 compares the effects of our original budget-equivalent EITC program above to a version of that program together with a modest minimum wage of $\overline{w} = .25$, which is near the peak of the employment Laffer curve. We choose the EITC schedule in this latter case to ensure that the tax credits and the implicit tax on corporate profits from the minimum wage are together budget-equivalent to the \$15 minimum wage. The blue lines in left panel of Figure 15 show that without a minimum wage, firms optimally lower the wages of workers to partially appropriate the subsidy to workers. The red lines in the figure show that pairing the EITC with a modest minimum wage of $\underline{w}$ = \$9.25 leads to much larger increase in employment, labor income, and welfare than the EITC program alone does. One reason is that this minimum wage prevents firms from lowering wages in response to the transfer to low-income workers. But the much more quantitatively important reason is that the minimum wage alleviates the monopsony distortions these workers face.

FIGURE 15: Effect of EITC with and without $\underline{w}$ = \$9.25 Minimum Wage



Note: Steady-state wages (left panel), employment (middle panel), and labor income (right panel) in response to an EITC system that is budget-equivalent to a \$15 minimum wage. The y-axis is the log-change relative to the initial equilibrium (without any policies) and the x-axis is the wage $w_{\ell z}$ of a z-type worker in this initial equilibrium. Dashed lines denote the wages that firms offer, as opposed to those that workers receive, under the two cases.

We conclude from this analysis that the minimum wage can have a valuable role in supporting transfer programs like the EITC. The size of the minimum wage, however, must be chosen carefully. If the minimum wage is set too high, it may end up hurting low-income workers for the reasons described throughout the paper. As an illustrative example, Appendix E shows that a \$12 minimum wage substantially *reduces* the benefit of the EITC for low-income non-college workers, since it significantly reduces their employment.

6.4 Progressive Tax and Transfer System and Minimum Wage

As discussed, we model the entire U.S. progressive tax and transfer system using the parametric tax function $T(w_i) = w_i - \lambda w_i^{1-\tau}$, where $T(w_i)$ is the labor income tax schedule, λ and τ are parameters that govern the level and progressivity of the system, and negative taxes $T(w_i) < 0$ indicate transfers. Heathcote, Storesletten and Violante (2017) show that $\tau = 0.181$ provides a good description of the overall progressivity of the U.S. system, except at the very bottom of the income distribution because transfers are phased in and out at various income levels, which introduces kinks in the implied tax rates.[39] We follow Heathcote, Storesletten and Violante (2017) in excluding the EITC and setting the progressivity parameter τ at 0.181. We then choose the scale parameter λ to ensure that the aggregate net transfer payment is budget-equivalent to a \$15 minimum wage.

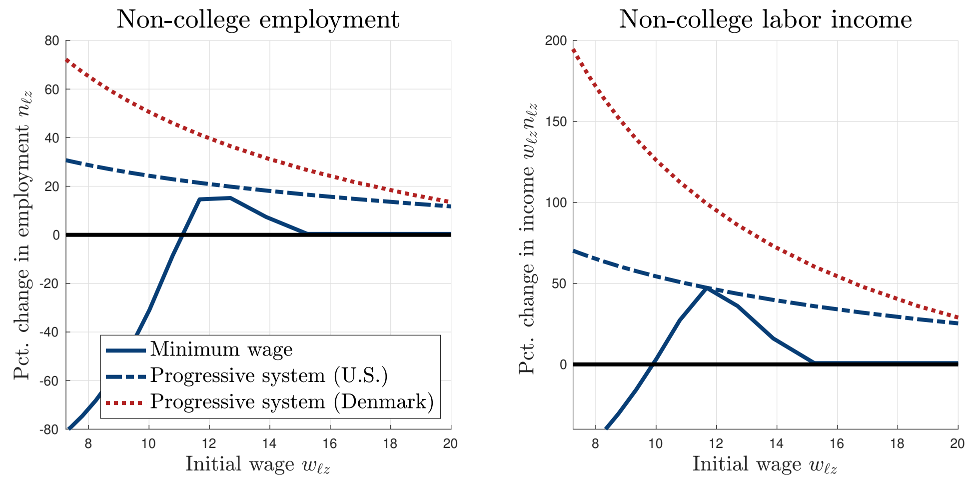
In the Appendix in Figure E.2 plots the average tax rate $T(w_i)/w_i$ and marginal tax rate $T'(w_i)$ of our budget-equivalent system as a function of labor income. The fact that the system is progressive ($\tau > 0$) implies that marginal tax rates are higher than average tax rates throughout. The lowest-income households face both a negative average tax rate, so they receive transfers ($A(w_i) > w_i$), and a negative marginal tax rate, so transfers are phased in ($A'(w_i) > 1$). For these households, the tax and transfer system reduces the monopsony distortion by (37) and increases search effort by (38). Middle-income households continue to receive transfers, which encourages their search effort, but face positive marginal tax rates, which exacerbates monopsony distortions. Finally, high-income households face both positive average and marginal tax rates, which reduces their search effort and exacerbates monopsony distortions. In this sense, a progressive tax and transfer system differentially affects the monopsony distortions faced by different workers.

The left panel of Figure 16 shows that this progressive system succeeds in substantially increasing the employment of non-college workers, especially for the low-wage workers whom the system especially targets. The right panel shows that similar results hold for after-tax labor income. Labor income increases for all the z-type in the figure, although it falls for some of the higher-z types not pictured who earn higher wages, especially among college workers. For sake of comparison, Figure 16 also shows the effect of an even more progressive tax and transfer system with $\tau = 0.463$ meant to capture the degree of progressivity in Denmark.[40] This more progressive system further increases the employment of low-z types but decreases that of college workers more than the U.S. system does (not

[39]See page 1700 of their paper: "[O]ur tax/transfer scheme tends to underestimate marginal tax rates at low income levels ... marginal rates vary substantially across households, and some households simultaneously enrolled in multiple welfare programs face high marginal tax rates where benefits are phased out. Although our parametric functional form cannot capture this variation in tax rates at low income levels ..."

[40]Heathcote, Storesletten and Violante (2020) estimate the progressivity of the tax system in Denmark and a number of other countries. Unfortunately, due to data limitations, these estimates only include taxes excluding transfers. We impute a value of the progressivity of the tax and transfer system by scaling our baseline progressivity parameter for the U.S. tax and transfer system by the ratio of the progressivity of the Danish tax system to the U.S. one.

Figure 16: Effect of Progressive Tax System on Non-College Workers



Note: Steady-state employment (left panel) and labor income (right panel) of selected z types for three different policies: a \$15 minimum wage (blue line), the budget-equivalent progressive tax system with the U.S. level of $\tau = 0.181$ (red line), and the budget-equivalent tax system with the Danish level of $\tau = 0.463$ (purple line). The y-axis is normalized relative to employment in the initial equilibrium (without any policies). The x-axis is log individual productivity $\log z$ relative to its mean value, expressed in standard deviations from the mean.

pictured), which underscores the policy trade-offs associated with different levels of progressivity.

7 Conclusion

Recently, many policy proposals to increase the federal minimum wage to \$15 per hour or more have been introduced in the United States. What would be the distributional impact of such a large national increase? How would its long-run effects differ from its short-run effects? The existing empirical literature on the minimum wage has not been able to answer such questions yet because it focuses on short-run labor market responses to small minimum wage changes. For example, the typical minimum wage study tends to examine employment and labor income responses over the one to three years after a minimum wage change. The studies that do estimate longer-run impacts of the minimum wage examine much smaller changes than the \$15 proposal, which have been implemented at a much narrower geographical level (city or state) than the national one.

Given this paucity of evidence, we develop a general equilibrium framework with rich worker heterogeneity, firm monopsony power in the labor market, and frictions to adjusting firms' input mix in order to study the short-run and long-run effects of increasing the minimum wage by the magnitudes being proposed. To build confidence in our model's dynamic predictions, we ensure that it matches both the literature that has estimated the short-run effects of small minimum wage changes and the literature that has estimated long-run elasticities of substitution among workers. Our model's ability to replicate all this evidence implies that short-run employment responses to minimum wage changes are uninformative about potential long-run responses, because firms only slowly adjust their use of inputs over time.

We incorporate monopsony power in our framework in order to capture the standard argument in favor of the minimum wage: as the minimum wage increases, employment can increase since the minimum wage offsets monopsony distortions. As the minimum wage rises above the efficient level, however, firms start substituting away from the workers whose marginal product is smaller than the minimum wage. This result then implies that even long-run estimates of the employment effect of small minimum wage changes cannot be linearly extrapolated to learn about the effects of larger ones.

We find that in the short run, even a large increase in the minimum wage has a small effect on employment so it works as its proponents hope: it leads to a sizable increase in the labor income and welfare of workers at the low end of the wage distribution. Over time, however, firms will substitute away from low-productivity workers towards higher-productivity ones, as implied by the elasticities estimated by Card and Lemieux (2001). Hence, in the long run, a high minimum wage has perverse distributional implications in that it reduces the employment, income, and welfare of precisely the lowest-income workers it is designed to support. Taking into account the entire time path of these effects, though, we find that large short-run welfare gains help to offset the large long-run costs for most of the current generation of workers experiencing such a change in the minimum wage.

Our results are primarily driven by the elasticity of substitution among workers of differing productivities and by the extent of firms' monopsony power. According to our model, proponents of a large minimum wage in the long run must believe that monopsony power is much larger than estimated by the literature and/or that the elasticity of substitution among workers within an education group is much smaller than estimated by the literature. Furthermore, we show that the parameter values that generate a small long-run effect of *small* minimum wage increases all imply that a *large* minimum wage substantially decreases long-run employment.

We end by illustrating that a more effective way to increase the welfare of low-productivity workers is through a progressive tax and transfer scheme, such as the EITC. In our model, the EITC, unlike the minimum wage, reduces the monopsony distortions of targeted workers without adversely affecting the wages of other lower-wage workers. As a result, we find that a modest minimum wage complements the EITC and is more effective at raising the welfare of the lowest-income workers than either a minimum wage policy on its own or an EITC on its own. In this sense, a modest minimum wage is a valuable tool that enhances the efficacy of existing tax and transfer policies.

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A Omitted Model Details

We present here details omitted from the main text.

A.1 Analogy Between Monopolistic and Monopsonistic Competition

Here we discuss how the upward-sloping labor supply curve for each firm's jobs that our model gives rise to is analogous to the downward-sloping demand curve for each firm's goods that arises in models of monopolistic competition. In these latter models, consumers view each firm's good as imperfectly substitutable with any other. Then, the downward-sloping demand curve of consumers for a firm's good as a function of all firms' prices is the constraint that captures monopoly power in a firm's problem. Analogously, in our setup, workers view each firm's job as imperfectly substitutable with any other. Thus, the upward-sloping supply curve of searchers for a firm's jobs as a function of all firms' wages is the constraint that captures monopsony power in a firm's problem.

To elaborate, recall that standard analyses of monopolistic competition derive the static demand curve of consumers for goods of each type j and then impose this demand curve as a constraint on firm j's problem. Equivalently, one could derive the first-order conditions for a consumer's problem and impose the constraint that the marginal utility from buying good j is at least as high as that from buying any other good. More formally, the static part of a consumer's dynamic problem with a standard utility function over differentiated goods, given total expenditure pc, is to choose $\{c_j\}$ to maximize $u(c)$ subject to $\sum_j p_j c_j \leq pc$ with multiplier λ, where $c = \left(\sum_j c_j^{\frac{\omega-1}{\omega}} dj\right)^{\frac{\omega}{\omega-1}}$ is the consumption aggregate and p is the associated price index. The first-order condition for buying good j implies that

$$u'(c)\left(\frac{c_j}{c}\right)^{-\frac{1}{\omega}} - \lambda p_j = \max_{j'}\left\{u'(c)\left(\frac{c_{j'}}{c}\right)^{-\frac{1}{\omega}} - \lambda p_{j'}\right\}. \tag{39}$$

In a symmetric allocation with $c_{j'} = c$ and $p_{j'} = p$, (39) reduces to

$$u'(c)\left(\frac{c_j}{c}\right)^{-\frac{1}{\omega}} - \lambda p_j \geq u'(c) - \lambda p, \tag{40}$$

which is the participation constraint under monopolistic competition. Notice the similarity of this participation constraint for attracting a consumer to buy from firm j and the participation constraint for attracting a searching consumer to the labor market (θ_j, w_j) created by firm j, namely, (14). The main difference between the two constraints is that choosing which good to buy given a level of expenditure is a static decision whereas searching for a firm offering a long-term labor contract is a dynamic one.

B Data and Empirical Literature

This appendix contains details about our data sources and construction referenced throughout the main text as well as additional details on the empirical literature that has estimated the employment effects of the minimum wage. We use data from the pooled 2017-2019 American Community Survey (ACS). All observations are weighted using the weights provided by the ACS.

Share of College Workers. We define two education groups within the paper: "college" and "non-college." We define college individuals as those individuals who report having a bachelor's degree or higher. During the 2017-2019 period, 31.3% of our sample had at least a bachelor's degree.

Employment Rates. As part of our calibration, we match "full-time" employment rates by education group. By focusing on "full-time" employment, we measure workers with a strong attachment to the labor force. We define an individual as being "full-time" employed if (1) they are currently working at least 30 hours per week, (2) they reported working at least 29 weeks during the prior year, and (3) they reported positive labor earnings during the prior 12 month period. For our 2017-2019 sample, 46.8% of non-college individuals and 62.4% of college individuals worked full-time.

Share of Income Earned by College Workers. For the 2017-2019 period, 37.8% of individuals working full-time were college educated. Conditional on being a full-time worker, mean annual earnings for college individuals totaled $91,706 while mean annual earnings for non-college individuals total $44,871. Given these numbers, we compute that 55.5% of all earnings of full-time workers accrued to workers with at least a bachelor's degree.

Wage Distribution. We compute hourly wages for our sample of full-time workers by dividing annual labor earning by annual hours worked. We compute annual hours worked as the multiple of weeks worked last year and reported usual hours worked. We make two other sample restrictions when computing the wage distribution. First, we restrict the sample to only those workers who report at least $5,000 of labor earnings during the prior year. Second, we then truncate the distribution at the top and both 1% of the wage distribution. All wages are converted to 2019 dollars using the June CPI-U. From this data, we compute the median wage as well as the ratios of wages between the 10th percentile and the median and the ratio of wages between the 90th percentile and the median separately for each of the education groups. These moments are used as part of our calibration. We also show that even though only those three moments are targeted for each education group, our model matches the full distribution of wages for each education group quite closely.

Estimates of the Employment Effects of the Minimum Wage. The recent survey by Neumark and Shirley (2022), which we reference in the main text, takes stock of the large body of work on the employment effects of the minimum wage in the United States and illustrates how, despite a lack of agreement in the literature on how to interpret the existing evidence, most studies report negative employment effects of minimum wage increases. Specifically, the authors argue that negative estimates are pervasive in the literature, that this evidence is stronger for teens, young adults, and less-educated workers, that the evidence from studies of directly-affected workers points even more strongly to negative effects, and that the evidence from studies of low-wage industries is less clear cut. These authors focus on the evidence that has relied on geographical (subnational) variation in the minimum wage within the United States assembling the entire set of published studies and identify the core estimates (more than a hundred) supporting the conclusions from each study. According to their preferred estimate from each study, a 1% increase in the minimum wage reduces employment by 0.151%, based on the mean such estimate across studies, or by 0.116%, based on the median such estimate across studies—–estimates range from -1% to 1.7% (see their Figure 1). Instead, according to the median estimates from each study, the authors calculate that a 1% increase in minimum wage reduces employment by 0.133%, based on the mean such estimate across studies, or by 0.110%, based on the median such estimate across studies—–estimates range from -1% to 1.7% (see their Figure 3).[41]

[41] Numerous papers have pointed out the challenges of measuring the effects of the minimum wage, which include isolating reliable identifying information, constructing accurate control groups, controlling for confounding factors, and accounting for potential issues of endogeneity in controls and selection in the sample examined, as well as the lack of robustness of results across alternative estimated specifications of the effects of interest. For instance, Neumark, Salas and Wascher (2014) revisit studies that claim that panel-data estimates commonly used are flawed, as they fail to account for spatial heterogeneity, and doing so supports the notion that minimum wages in the United States have not reduced employment. The authors' results, though, confirm the evidence of negative employment effects and so the

Positive effects of the minimum wage on employment have been detected for other countries, though. Dustmann et al. (2022), for example, study the impact of the introduction of a nationwide minimum wage in Germany that affected 15% of all employees and find that the minimum wage raised wages, did not not lower employment, and led to a reallocation of low-wage workers from smaller to larger, from lower- to higher-paying, and from less to more productive establishments.

We note that most of the empirical literature evaluating the minimum wage exploits local changes in it, which are quite small relative to the current proposed changes in the national minimum wage, and examines short-run employment responses among narrow groups of workers. Cengiz et al. (2019), for instance, which is one of the most recent papers reviewed by Neumark and Shirley (2022), rely on state-level minimum wage changes to examine the extent to which the minimum wage affects the employment of workers who were initially below the new minimum and find that the employment of low-wage workers declines only slightly in the few years following a minimum wage increases.[42] Similar impacts have been documented in other studies; when positive employment effects are detected, they are relatively small and concentrated among high-wage workers. See Dube and Lindner (2021) for analogous results.

Our results are consistent with this evidence for two reasons. First, we find that small changes in the minimum wage have only small effects on the employment rates of all workers, including low-productivity ones. Second, and more importantly, we find that even large changes have only small short-run employment effects. In line with these results, by exploiting state-level changes in the minimum wage between 1975 and 2012, Meer and West (2016) estimate that an increase in the minimum wage reduces employment but such an effect takes several years to materialize. When the authors regress annual changes in (log) employment of all non-agricultural employees on changes in (log) minimum wage, they find that an increase in the minimum wage has no contemporaneous effect on employment but it has a negative effect after one year. Specifically, a 1% increase in the minimum wage reduces employment by 0.03% after one year, by 0.06% after two years, and by 0.07% after three years (see their Table 4). The authors also regress long differences (from one to eight years) in (log) employment on long differences in (log) minimum wage and report that an increase in the minimum wage has no contemporaneous effect on employment but it has negative effects after one year. In particular, a 1% increase in the minimum wage reduces employment by 0.0387% after one year and by approximately 0.05% after two years—–the effect is constant thereafter.[43]

As we emphasize, allowing for reasonable frictions to input adjustment in the form of a putty-technology implies that employment effects are small in the first few years after a minimum wage increase even though long-run responses are large. These implications of our model are consistent with the evidence in Lindner and Harasztosi (2019). In response to a large and persistent minimum wage increase in Hungary, these authors find that employment elasticities are negative but small even four years after the reform and that firms reacted by substituting labor with capital.[44]

trade-off at the heart of minimum wage policies between higher wages for some workers and job losses for others, which our distributional analysis highlights.

[42]Clemens, Kahn and Meer (2021) also exploit cross-regional variation to document that firms switch away from low-productivity workers towards higher-productivity workers in response to minimum wage increases. This finding is consistent with the key adjustment mechanism in our model.

[43]Jardim et al. (2022) find similar non-linear effects of city-level changes in the minimum wage on hours worked. Namely, an increase in the minimum wage in Seattle from \$9.5 to \$11 did not impact hours worked but a further increase to \$13 reduced hours worked and employment for low-wage workers, that is, workers earning less than \$19 per hour across all industries. In particular, the authors study two successive increments in the minimum wage in the city between 2015 and 2016 and find that the first increase from \$9.5 to \$11 is associated with an increase in average wages by 1.7% and no statistically significant effect on hours worked (see their Table 6). They then document that the second increase from \$11 to \$13 is associated with an increase in average wages by 3.2% (see their Table 5) and a reduction in hours worked by 6.9% and in the number of jobs by 5.9% (see their Table 6).

[44]Thee authors propose a model of monopolistic competition in the output market to address these findings as well

C Mapping Card and Lemieux (2001) Estimates to Our Model

This appendix shows that Card and Lemieux (2001)'s estimated elasticities of substitution across workers, ϕ and ρ, map into the same elasticities in our model despite the fact that Card and Lemieux (2001) estimation framework does not allow for monopsony power or search frictions. First, we show that their procedure works arbitrarily well as the size of the search frictions shrinks to zero (in which case markdowns become constant). Second, we show that our calibrated model is quantitatively close to that limit in the sense that Card and Lemieux (2001)'s estimation procedure, when applied to data simulated from our model, recovers the true parameter values almost exactly.

C.1 Limit with Small Search Frictions

We begin by showing that Card and Lemieux (2001)'s procedure recovers the true parameter values in our model as the search frictions shrink to zero. The following proposition relates steady state wage markdowns to the search frictions and the degree of monopsony power:

***Proposition* 2.** *Consider the steady state of our model and let i denote a worker type. Then:*

$$\frac{w_i}{F_{ni}} = \frac{1}{1+\frac{1}{\omega}} - (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}\frac{\omega(1-\eta)-\eta}{(1+\omega)(1-\eta)}\frac{1}{F_{ni}}. \tag{41}$$

where $r = \frac{1}{\beta} - 1$.

Proof. Consider the steady state conditions for vacancy posting and the wage equation:

$$F_{ni} = (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)} + w_i + \frac{1}{\omega}v'(n_i) \tag{42}$$

$$w_i = \eta(F_{ni} - \frac{1}{\omega}v'(n_i)) + (1-\eta)v'(n_i) \tag{43}$$

Rearrange the wage equation (43) to get an expression for the marginal disutility of labor:

$$v'(n_i) = \frac{w_i - \eta F_{ni}}{1-\eta-\frac{\eta}{\omega}}. \tag{44}$$

Now plug this expression into the first order condition for vacancy posting and collect terms to get

$$F_{ni}\left(1+\frac{\eta}{\omega(1-\eta-\frac{\eta}{\omega})}\right) = (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)} + w_i\left(1+\frac{1}{\omega(1-\eta-\frac{\eta}{\omega})}\right). \tag{45}$$

Simplify the expression to get

$$F_{ni}\frac{\omega}{1+\omega} = (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}\frac{\omega(1-\eta)-\eta}{(1+\omega)(1-\eta)} + w_i. \tag{46}$$

Finally, divide by F_{ni} and rearrange to get the expression (41) in the proposition. ∎

Proposition 2 shows that search frictions are the only reason that markdowns are not constant in our model; monopsony power in isolation would lead to constant markdowns because the disutility of labor supply (1) has a CES form. In the calibrated model, the search frictions are small in the

as the fact they document that negative employment effects are greater in industries where the pass-through of wage costs to output is likely lower, like in the tradable, manufacturing, and exporting sectors.

sense that the second term in (41) accounts for less than 1% of the overall markdown for the typical worker. Intuitively, the second term depends on the size of the search frictions annuitized over the entire life of the match, which is small relative to flow output.

In the limit where search frictions shrink in the sense that the second term in equation (41) goes to zero, Card and Lemieux (2001)'s estimated values for the elasticities of substitution $\widehat{\phi}$ and $\widehat{\rho}$ would exactly equal their true values, i.e. $\widehat{\phi} = \phi$ and $\widehat{\rho} = \rho$. To see this, note that in this limit we would have that wages are proportional to marginal products, $w_i = \frac{1}{1+\frac{1}{\omega}} F_{ni}$. Therefore, ratios of wages across workers would equal the wages of their marginal products, $\frac{w_i}{w_j} = \frac{F_{ni}}{F_{nj}}$. Since our production function is weakly separable in capital, the ratios of marginal products would fall within the same class estimated by Card and Lemieux (2001).

To elaborate, let us now show how Card and Lemieux (2001)'s estimates identify the true parameter values in this limiting case of our model. First consider how Card and Lemieux (2001) estimate the within-group elasticity ϕ. Under either their model without monopsony power or search, or in the limit of our model without search frictions, the ratio of wages between two worker types within education group is, in logs,

$$\log\left(\frac{w_i}{w_j}\right) = \log\left(\frac{z_i}{z_j}\right) - \frac{1}{\phi}\log\left(\frac{N_i}{N_j}\right), \tag{47}$$

where N_i and N_j are the measures of workers of two arbitrary types i and j. Hence, wage ratios are related to the ratio of worker productivities and their relative supply. Card and Lemieux (2001) identify ϕ from equation (47) by assuming that the relative supply of workers $\log \frac{N_i}{N_j}$ changes over time but their productivities $\log \frac{z_i}{z_j}$ do not. Below, we conceptualize this variation by assuming that we observe at least two steady state which differ in their initial distributions of types, generating differences in the relative supplies of workers.[45] Taking differences of (47) across the two steady states gives

$$\Delta \log\left(\frac{w_i}{w_j}\right) = -\frac{1}{\phi}\Delta\log\left(\frac{N_i}{N_j}\right). \tag{48}$$

Card and Lemieux (2001) estimate this regression equation and recover $\widehat{\phi}$ from the regression coefficient, which corresponds exactly to the true elasticity ϕ. The equation (48) does not precisely hold in our full model with strictly positive search frictions, but we show below that it approximately holds in the sense that running the mis-specified regression (48) would recover the true value of ϕ almost exactly.

Now consider how Card and Lemieux (2001) estimate the across-group elasticity ρ. In either Card and Lemieux (2001)'s model without monopsony or search or in the limit of our model described above, the ratio of wages of workers across educations groups can be written as

$$\log\left(\frac{w_{hi}}{w_{\ell j}}\right) = \log\left(\frac{1-\lambda}{\lambda}\right) + \left(\frac{1}{\phi} - \frac{1}{\rho}\right)\log\left(\frac{\overline{n}_h}{\overline{n}_\ell}\right) + \log\left(\frac{z_{hi}}{z_{\ell j}}\right) - \frac{1}{\phi}\log\left(\frac{N_{hi}}{N_{\ell j}}\right), \tag{49}$$

where $\overline{n}_h$ and $\overline{n}_\ell$ are the aggregated labor inputs defined in (4) and λ is a scale parameter in the production function (so that $\frac{1-\lambda}{\lambda}$ captures the degree of "skill bias" in production). In an intermediate step, Card and Lemieux (2001) construct estimates z_{hi} and $z_{\ell j}$ and, using their estimated value for

[45] We assume Card and Lemieux (2001)'s empirical variation corresponds to steady state changes in our model because their sample covers a forty year period (by which point any transition path would converge to steady state in our model). To the extent that their variation does not exactly correspond to steady state changes, their estimated value $\widehat{\phi}$ would be *lower* than the true value ϕ because the putty-clay frictions would reduce the observed short-run substitutability. In this case, the long-run effects of the minimum wage would be even worse for low-wage workers than in our baseline results presented in the main text.

ϕ, construct the aggregate labor inputs $\overline{n}_h$ and $\overline{n}_\ell$. In that case, the only unknowns in the equation (49) is the degree of skill bias $\frac{1-\lambda}{\lambda}$ and the elasticity of interest ρ. Collect these unknowns on the right-hand side of the regression and difference (49) across steady states to get

$$\underbrace{\Delta \log\left(\frac{w_{hi}}{w_{\ell j}}\right) - \frac{1}{\phi}\Delta \log\left(\frac{\overline{n}_h}{\overline{n}_\ell}\right) + \Delta\frac{1}{\phi}\log\left(\frac{N_{hi}}{N_{\ell j}}\right)}_{\Delta y_i} = \Delta \log\left(\frac{1-\lambda}{\lambda}\right) - \frac{1}{\rho}\Delta \log\left(\frac{\overline{n}_h}{\overline{n}_\ell}\right). \tag{50}$$

Card and Lemieux (2001) assume that the degree of skill bias $\log \frac{1-\lambda}{\lambda}$ follows a linear time trend. In this case, its time-difference is constant, so this term becomes a constant in the regression of Δy_i on $-\frac{1}{\rho}\Delta \log \frac{\overline{n}_h}{\overline{n}_\ell}$. The elasticity of interest ρ can therefore be exactly recovered from the regression coefficient on the change in the aggregate labor inputs, $\Delta \log \frac{\overline{n}_h}{\overline{n}_\ell}$. Again, this regression (48) is not exactly correct in our full model, but below we will show that it holds approximately.

To summarize the discussion so far, we have shown that in the limit of our model with vanishingly small search frictions, Card and Lemieux (2001)'s estimation strategy exactly recovers the true elasticities of substitution ρ and ϕ. In this case, it is valid for us to use their estimates for the parameter values of our model, despite the fact that our model has monopsony power while Card and Lemieux (2001)'s does not. We now turn to showing that this result holds approximately holds in our calibrated model with non-vanishing search frictions as well.

C.2 Quantitative Results in Calibrated Model

In order to replicate Card and Lemieux (2001)'s estimates in our calibrated model, we assume that we observe three steady states corresponding to different distributions of workers and degrees of skill bias, which mirrors the empirical variation used by Card and Lemieux (2001). Specifically, we assume the third and final steady state corresponds to our calibrated model (which is based on recent data). We then assume that the dispersion of worker abilities z within the two education groups we consider, $\sigma_{\ell z}$ and σ_{hz}, are 10% and 30% lower in the second and first steady states to capture the lower degree of wage dispersion in previous decades. We also assume that the fraction of college-educated workers is 10% and 40% lower in the second and first steady states to reflect the lower rates of educational attainment in the past. Finally, we assume that the degree of skill bias in production $(1-\lambda)/\lambda$ is 15% and 30% lower in the second and first steady states, which captures the pattern of the college wage premium over time. Although these parameter changes are somewhat arbitrary, we show below that our results are extremely robust to other changes in the distributions or the degree of skill bias.

We construct Card and Lemieux (2001)'s estimators of our elasticities, $\widehat{\phi}$ and $\widehat{\rho}$, using the procedure outlined above. Specifically, to estimate the within-group elasticity ϕ, we run the regression implied by the first-differenced version of equation (48). We label the resulting estimate $\widehat{\phi}$ because that equation will not literally hold in our model and therefore represents an estimate of the true parameter ϕ. We then use the estimated value of $\widehat{\phi}$ to construct estimates of the ratio of type-specific productivities, $\frac{z_i}{z_j}$ from the non-differenced version of the equation, (47). We then construct the aggregate labor inputs by education group $\overline{n}_h$ and $\overline{n_\ell}$, therefore giving us the left-hand side variable Δy_i in equation (49). Due to Card and Lemieux (2001)'s assumption that skill-biased technical change follows a linear time trend, the term $\Delta \log[(1-\lambda)/\lambda]$ is constant across the two pairs of steady state. Therefore, to estimate the across-group elasticity of substitution ρ, we double difference this equation to obtain

$$\Delta^2 y_i = -\frac{1}{\rho}\Delta^2 \log\left(\frac{\overline{n}_h}{\overline{n}_\ell}\right). \tag{51}$$

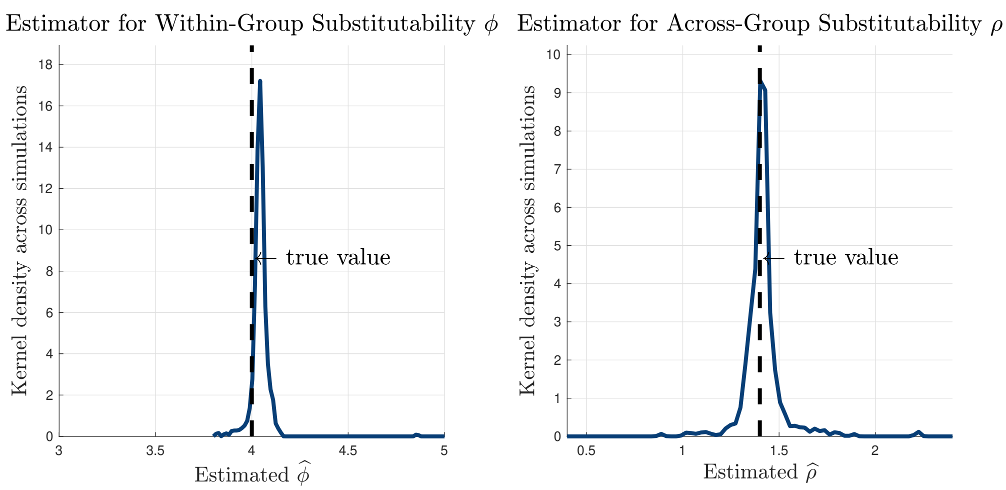
Again, the regression equation in (51) does not hold exactly in our model, so we denote the resulting

Table C.1: Relationship Between Card and Lemieux (2001) Estimates and True Parameters

Card and Lemieux (2001)'s Estimates		
	True value	Estimated value
Within-group elasticity ϕ	4.000	4.036
Across-group elasticity ρ	1.400	1.388
Targeting Card and Lemieux (2001)'s Estimates		
	Calibration target	Calibrated parameter
Within-group elasticity ϕ	4.000	4.065
Across-group elasticity ρ	1.400	1.383

Note: Top panel reports the Card and Lemieux (2001) estimates of the within-group elasticity of substitution ϕ and across-group elasticity ρ following the procedure described in the text. Bottom panel reports the calibrated values of the elasticities ϕ and ρ if we instead include the Card and Lemieux (2001) estimators $\widehat{\phi}$ and $\widehat{\rho}$ as additional targets in our calibration procedure (all other parameters are recalibrated following the procedure described in Section 3).

Figure C.1: Kernel Densities of Card and Lemieux (2001) Estimates on Model-Simulated Data



Note: Kernel density of distribution of Card and Lemieux (2001) estimates across different simulations of the model. Different simulations correspond to different parameter values in the first two steady states, as described in the text. The estimation procedure is also described in the text.

estimate $\widehat{\rho}$ to distinguish it from the true elasticity ρ.

The top panel of Table C.1 shows that Card and Lemieux (2001)'s estimation procedure uncovers the true parameter values nearly exactly; the estimate of $\widehat{\phi} = 4.036$ and $\widehat{\rho} = 1.388$ are both within 1% of the true values $\phi = 4$ and $\rho = 1.4$. Given this extreme similarity, we conclude that Card and Lemieux (2001)'s estimation procedure approximately applies in our model, allowing us to directly use their parameter estimates in our calibration, i.e. to assume $\phi \approx \widehat{\phi}$ and $\rho \approx \widehat{\rho}$. An alternative approach would have been to treat the Card and Lemieux (2001) estimators $\widehat{\phi}$ and $\widehat{\rho}$ as moments to be matched in our calibration procedure (along with all the other parameters already described in Section 3). The bottom panel of Table C.1 pursues this approach and shows that the calibrated values of ϕ and ρ are also nearly identical to our baseline choices. Given this similarity, we conclude that the parametrization strategy we follow in our baseline entails no obvious loss.

Finally, Figure C.1 shows that our conclusions in this subsection are robust to using a wide range of different distributions and skill-biased technical change to generate the time variation in our model. Specifically, we construct the Card and Lemieux (2001)'s estimates 500 times, each of which correspond to random draws of parameters:

- Dispersion of college z's $\sigma_{zh} \sim U[0.6, 0.9]\times$ baseline for initial steady state; second steady state 1/3 between initial and final steady state.
- Dispersion of non-college z's $\sigma_{z\ell} \sim U[0.6, 0.9]\times$ baseline; second steady state 1/3 between initial and final steady state.
- Fraction of college-education people $\pi_s \sim U[0.6, 0.9]\times$ baseline; second steady state 1/3 between initial and final steady state.
- Production function parameter $\lambda \sim U[1.1, 1.3]\times$ baseline; second steady state is equidistant between initial and final steady state.

The two panels of Figure C.1 plot the kernel density of the distribution of estimates $\widehat{\phi}$ and $\widehat{\rho}$ across these 500 simulations. The distribution of estimated within-group elasticities $\widehat{\phi}$ are tightly bunched around the true value $\phi = 4$; over 99% of the estimates are within 5% of the true value. The distribution of estimated across-group elasticities $\widehat{\rho}$ are somewhat more dispersed, but still bunched around the true value. Furthermore, Appendix D shows that our results do not strongly depend on the value of ρ within this range. Hence, our conclusion that Card and Lemieux (2001)'s procedure identifies our elasticities of interest nearly exactly is robust to different degrees of, and sources of, variation used in their estimation procedure.

D Additional Results About the Minimum Wage

This appendix contains a number of additional results about the minimum wage.

D.1 Results with Capital-Skill Complementarity

In the main text, we assumed that capital is equally substitutable with college and non-college workers (see equation (2)). In very influential work, Krusell et al. (2000) argue that, instead, capital is more substitutable with non-college workers than it is with college workers, a configuration they refer to as "capital-skill complementarity." In this appendix, we show that our main results are robust to using this alternative specification. This robustness occurs because the distributional impact of the minimum wage is primarily determined by the substitutability of workers within an education group, not the substitutability of the two education groups with capital.

This alternative version of the model replaces the long-run production structure (2) and (3) with

$$F(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt}) = \left[\psi(\bar{n}_{\ell jt})^{\frac{\rho-1}{\rho}} + (1-\psi)G(k_{jt}, \bar{n}_{hjt})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{52}$$

$$G(k_{jt}, \bar{n}_{hjt}) = \left[\lambda\,(k_{jt})^{\frac{\alpha-1}{\alpha}} + (1-\lambda)(\bar{n}_{hjt})^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}. \tag{53}$$

In this formulation, the parameter α is the elasticity of substitution between capital and college labor and the parameter ρ is the elasticity of substitution between non-college labor and the capital-college labor bundle $G(k_{jt}, \bar{n}_{hjt})$. Krusell et al. (2000) estimate that $\rho > \alpha$, which implies that non-college labor is more substitutable with capital than is college labor. The aggregated labor inputs $\bar{n}_{\ell jt}$ and $\bar{n}_{\ell jt}$ depend on type-level productivity z_i with the within-group elasticity ϕ as in the main text.

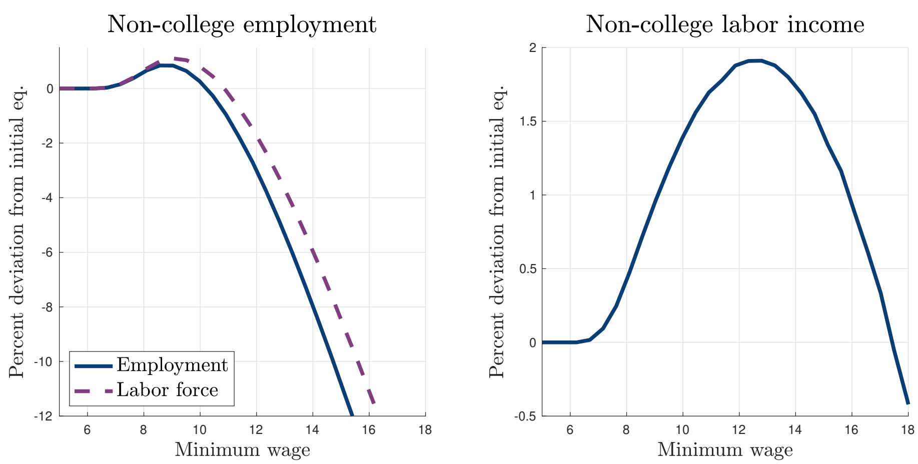
We recalibrate the model with this alternative production structure to target the same statistics as the baseline model in the main text. Table D.1 shows that the alternative model matches the targets

Table D.1: Targeted Statistics, Baseline vs. Capital-Skill Complementarity

Moment	Description	Data	Baseline Model	KORV Model
Average wage markdown				
$\mathbb{E}[w_{ni}]/\mathbb{E}[F_{ni}]$	Average wage markdown	0.75	0.75	0.75
Wage Distribution, ACS 2017-2019				
$w_{\ell 50}/w_{\ell 10}$	Non-college 50-10 ratio	2.04	2.00	1.90
w_{h50}/w_{h10}	College 50-10 ratio	2.30	2.06	2.06
Income shares				
$\mathbb{E}[w_i n_i]/Y$	Aggregate labor share	0.57	0.57	0.57
$\pi_h \mathbb{E}[w_{hz} n_{hz}]/\mathbb{E}[w_i n_i]$	College income share	0.55	0.56	0.55
Unemployment rate				
$\mathbb{E}[s_i]/(\mathbb{E}[s_i]+\mathbb{E}[n_i])$	Average unemployment rate	5.9%	5.9%	5.9%
Employment Rates				
$\mathbb{E}_\ell[n_i]$	Non-college employment rate	0.47	0.47	0.47
$\mathbb{E}_h[n_i]$	College employment rate	0.62	0.61	0.61

Note: Statistics targeted using parameters in Table 3. "Baseline model" corresponds to the model in the main text. "KORV model" refers to model with capital-skill complementarity described in the appendix text. The average wage markdown is the midpoint of the range of estimated markdowns discussed in the main text. The average labor share is from Karabarbounis and Neiman (2014).

Figure D.1: Long-Run Aggregate Minimum Wage Laffer Curves w/ Capital-Skill Complementarity

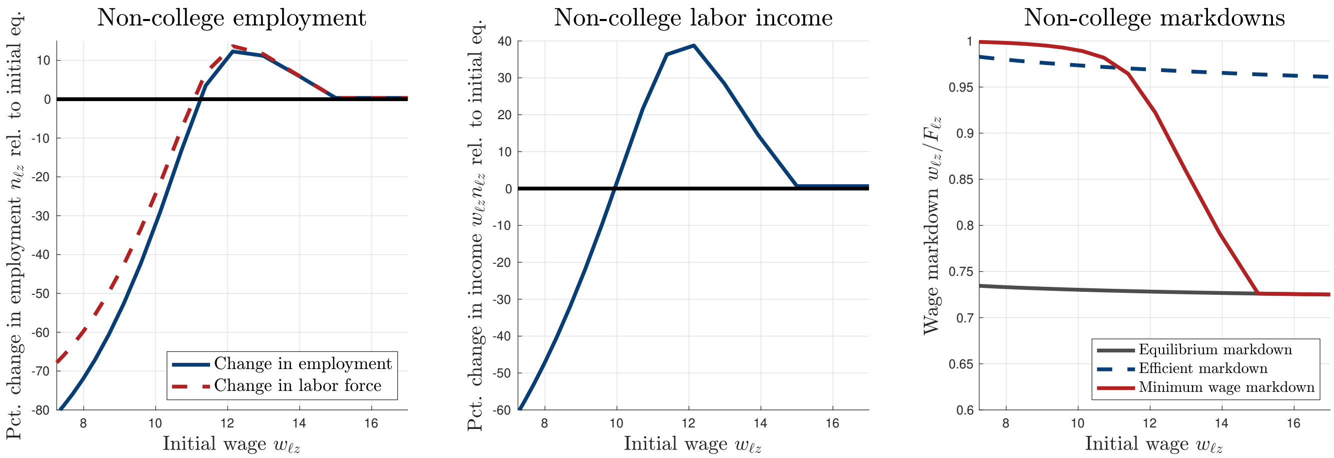


Note: Steady-state outcomes as a function of the minimum wage $\underline{w}$ in the model with capital-skill complementarity. The left panel plots the log-change of aggregate non-college employment, the log-change of aggregate non-college labor force, and the change in the unemployment rate relative to their levels in the initial steady state with $\underline{w} = 0$ (see the decomposition (33)). The right panel plots aggregate labor income of non-college workers. The x-axis is the level of the minimum wage $\underline{w}$ that relative to the median non-college wage in the initial equilibrium is the same as in the data.

about as well as our baseline model. While the parameter values are also similar to the baseline model, we make one important change: we assume that capital depreciates at $\delta = 15\%$ annually rather than $\delta = 10\%$ annually as in the main text. We make this change in order to exclude structures from the specification of capital-skill complementarity, following Krusell et al. (2000).

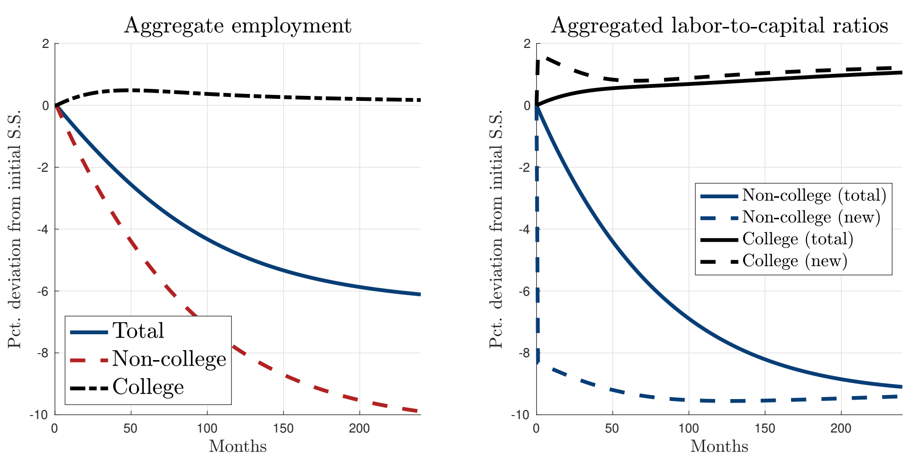
The long-run effects of the minimum wage in this alternative model are extremely similar to the baseline model in the main text. Figure D.1 plot the employment and labor income Laffer Curves for non-college workers; they are nearly identical to Figure 3 in the main text. For example, the $15 minimum wage decreases non-college employment by about 12% in both cases. Figure D.2 shows that

FIGURE D.2: Distributional Effects of $15 Minimum Wage with Capital-Skill Complementarity



Note: Steady-state outcomes for selected z-types among non-college workers for a $15 minimum wage in the model with capital-skill complementarity. The left panel plots the percentage change in employment n_i relative to the initial steady state without the minimum wage, the middle panel plots the percentage change in labor income $w_i n_i$ relative to the initial steady state without the minimum wage, and the right panel plots the levels of the markdowns w_i/F_{ni} for three different parameterizations: (i) $\underline{w} = 0$ ("equilibrium markdown"), (ii) $\underline{w} = 0$ and $\omega \to \infty$ ("efficient markdown"), and (iii) $\underline{w} = \$15$ ("minimum wage markdown"). The x-axis corresponds to the initial wage $w_{\ell z}$ of a type-z worker in the initial equilibrium.

FIGURE D.3: Employment Dynamics Along the Transition Path with Capital-Skill Complementarity

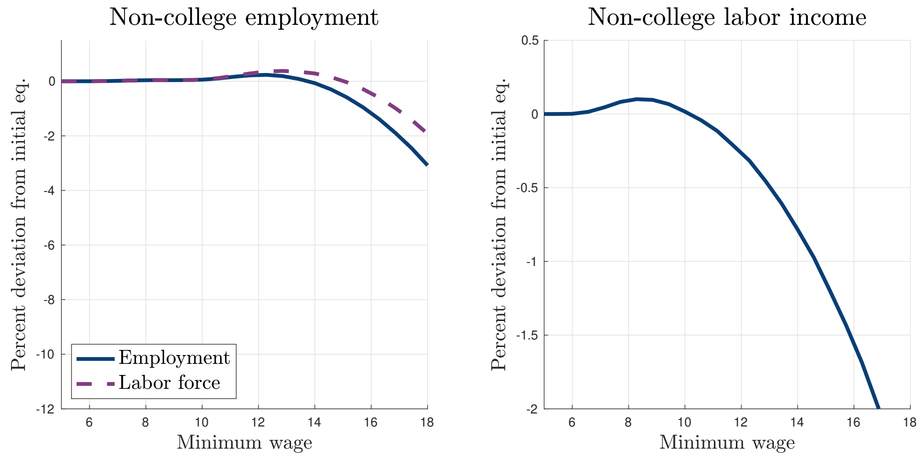


Note: Transition paths following an unexpected imposition of the minimum wage $\underline{w}$, starting from the initial equilibrium with $\underline{w} = 0$, in the model with capital-skill complementarity. The left panel plots the aggregated employment of non-college workers, college workers, and total employment. The right panel plots the associated labor-to-capital ratios.

the distributional impact of the $15 minimum wage across individual non-college types is also nearly identical to its counterpart, Figure 5, in the main text.

Figure D.3 shows that the transition path in this alternative model is extremely similar to the baseline model in the main text as well. As in our baseline model, it takes employment more than

Figure D.4: Long-Run Aggregate Minimum Wage Laffer Curves for College Workers



Note: Steady-state outcomes as a function of the minimum wage $\underline{w}$ in the model with capital-skill complementarity. The left panel plots the log-change of aggregate college employment, the log-change of aggregate college labor force, and the change in the unemployment rate relative to their levels in the initial steady state with $\underline{w} = 0$ (see the decomposition (33)). The right panel plots aggregate labor income of college workers. The x-axis is the level of the minimum wage $\underline{w}$ that relative to the median college wage in the initial equilibrium is the same as in the data.

twenty year to converge to its new steady state value due to the putty-clay technology. While the transition dynamics are somewhat faster in the alternative model than in the main text, that difference is due to the higher depreciation rate rather than the alternative production structure (recall that we are using $\delta = 15\%$ annually in this alternative model). As described in the main text, the depreciation rate controls the speed of transition because it determines how quickly the old, labor-intensive capital exits production.

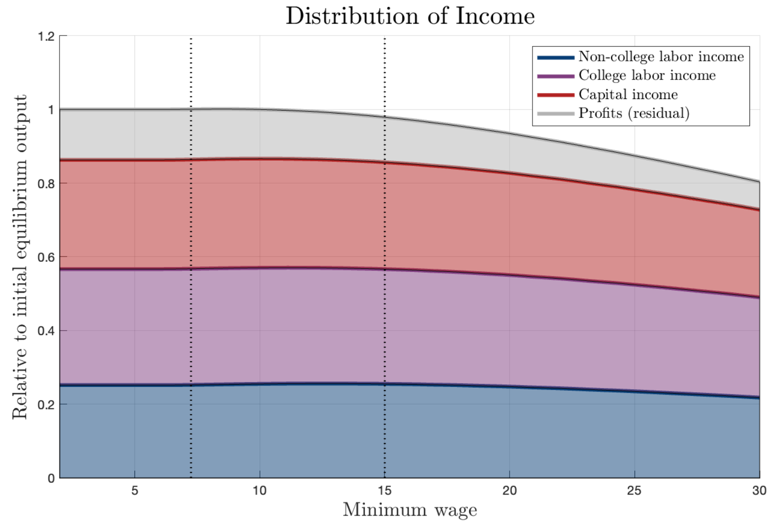
D.2 Long-Run Effects of the Minimum Wage on College Workers

The analysis in the main text focuses on non-college workers because the minimum wage primarily binds among those workers; relatively few college-educated workers earn below a \$15 minimum wage for example. However, for the sake of completeness, Figure D.4, plots the aggregated employment and income Laffer curves for college workers. The left panel shows that college employment has the similar shape, but that it only starts to decline for higher levels of the minimum wage than for the non-college workers. In contrast, the right panel shows that labor income declines by more for college workers than non-college workers. This occurs because the lower non-college employment and lower capital stock reduce the marginal product, and therefore the wages, of the college workers.

D.3 Composition of Aggregate Income For Different Minimum Wages

In order to illustrate how the minimum wage redistributes aggregate income across different groups, Figure D.5 plots how the minimum wage affects the four components of aggregate income in the long run: non-college labor income, college labor income, capital income, and firms' profits, which, as discussed, primarily reflect firms' monopsony power. Consistent with the labor income Laffer curve in Figure 3, a \$15 minimum wage raises non-college labor income by 1.4%. Firm profits fall substantially, suggesting that the minimum wage successfully redistributes resources from firms to workers. This redistribution, however, has a cost: the minimum wage reduces total employment, the aggregate

Figure D.5: Distribution of Aggregate Income



Note: Steady-state output and shares accounted for by non-college labor income, college labor income, capital income, and profits as a function of the minimum wage. The y-axis is normalized such that aggregate income equals 1 without the minimum wage. The x-axis is the level of the minimum wage that binds on the same amount of initial workers as in the data.

capital stock, and ultimately lowers total output by 2.2%. We turn next to investigate which workers pay the price for this redistribution.

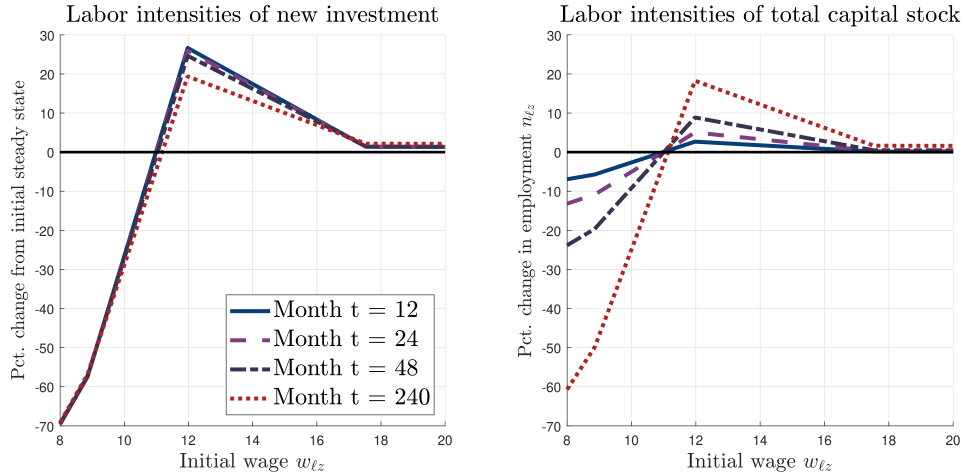
D.4 Distribution of Capital Types Over Time

In order to understand the role of the putty-clay technology in driving this dynamics, Appendix Figure D.6 plots the labor intensities of new and existing capital over the transition path for individual non-college workers in response to a \$15 minimum wage.[46] The left panel plots the labor-to-capital ratios $v_{\ell z}$ of newly installed capital goods for each worker type (y-axis) against that worker type's initial wage $w_{\ell z}$ (x-axis). In response to a \$15 minimum wage, firms immediately substitute away from low-ability workers towards higher-ability ones for the newly installed capital. The magnitude of such a substitution is fairly stable at various horizons along the transition. The right panel of the figure, though, shows that the implied dynamics for the labor intensity of the *total* stock of capital is much slower. The reason is that firms continue to operate their existing capital stock along the transition path, which requires the old steady-state mix of worker types chosen before the introduction of the \$15 minimum wage. This old capital accounts for the majority of the aggregate capital stock early on in the transition, but as it depreciates away, it is replaced by new, less labor intensive capital. Hence, the depreciation rate δ is crucial in determining the speed of transition, as the distribution of capital types along the transition path in Appendix D highlights.

Figure D.7 plots the distribution of capital types along the transition path in response to the permanent \$15 minimum wage. Before the introduction of the minimum wage, firms hold only one type of capital, namely the type that is optimal at the original steady state prices. The minimum wage induces firms to invest in less labor-intensive types of capital, but in the early stages of the

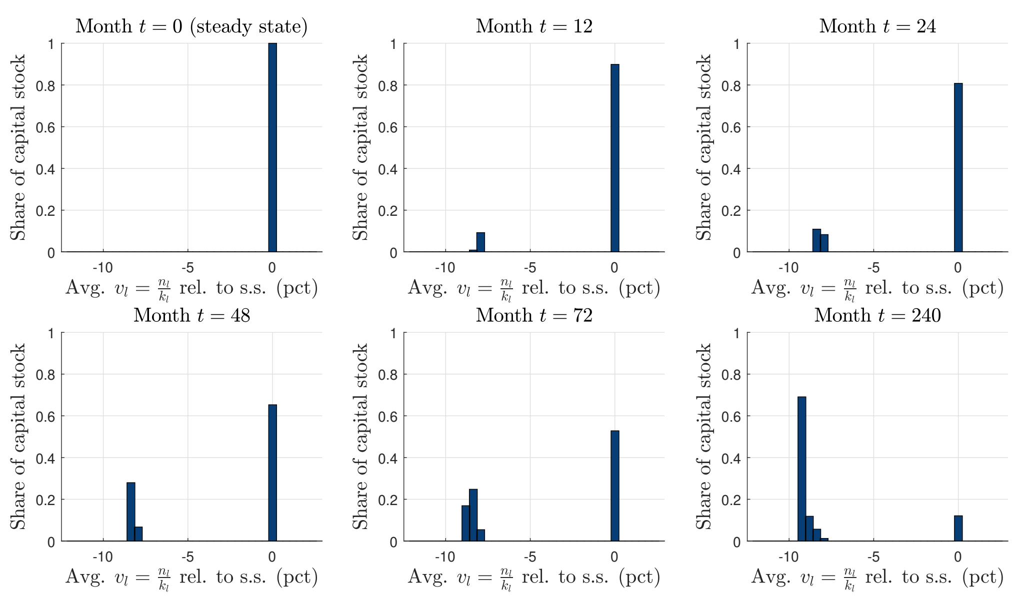
[46] The curves are less smooth along the transition than in steady state because we use a coarser grid of z-types when computing the transition paths.

Figure D.6: Labor Intensities in Response to a $15 Minimum Wage at Various Time Horizons



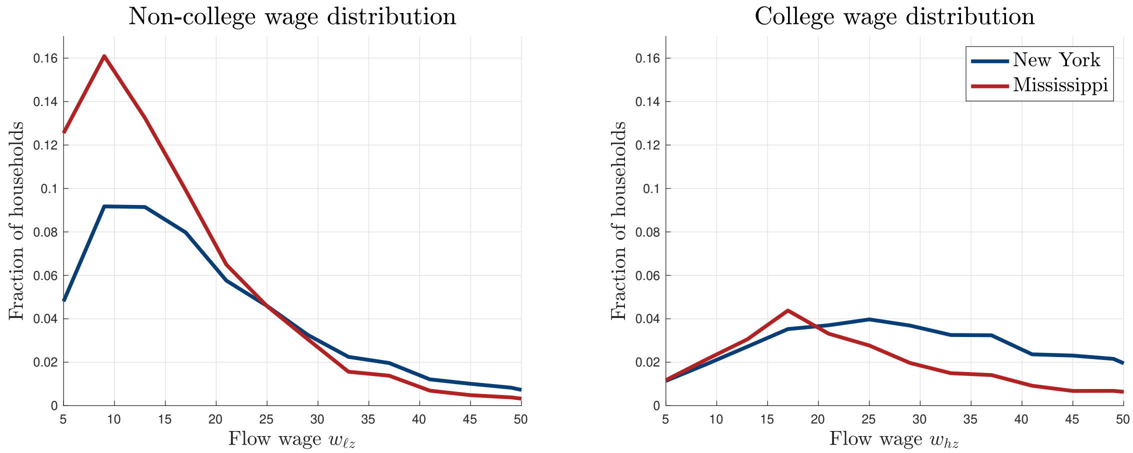
Note: Labor intensities along the transition path following an unexpected imposition of a $15 minimum wage starting from the initial equilibrium, as a function of the initial wage $w_{\ell z}$ earned by a non-college worker with ability z. The left panel plots non-college labor intensities on new capital goods as a function of worker ability, $v_{\ell z}$. The right panel plots the corresponding non-college labor intensities on the entire capital stock.

Figure D.7: Distribution of Capital Types Along the Transition Path



Note: Distribution of capital types along the transition path. The x-axis indexes the type of capital by its average non-college labor-to-capital ratio.

Figure D.8: Distribution of Wages in Mississippi vs. New York



Note: Distribution of wages in Mississippi (red line) and New York (blue line). The left panel plots the distribution for workers without a college degree and the right panel plots the distribution for workers with college degree. The wage grid has been aggregated to $4 bins. Distributions across non-college and college workers sum to one within each state.

transition that new capital accounts for a small share of the total capital stock. The employment dynamics are so prolonged because it takes time for the old capital stock to depreciate away; the bottom right panel shows that, even twenty years after the introduction of the minimum wage, more than 10% of the initial stock still exists.

D.5 Additional Distributional Effects

In this appendix, we highlight the potential for a $15 minimum wage to differential impact different states or sectors to the extent that these different states/sectors populate different portions of the aggregate wage distribution. Our model provides a mapping from an individual types' initial wage and their long-run change in employment or labor income in response to the minimum wage. The results in the main text aggregate up these changes using the aggregate wage distribution; now, we re-weight those individual-level changes by the distribution of wages in a given state or sector from the ACS. These results should only be viewed illustratively; in order to formally explore this heterogeneity, we would need to formally model linkages and reallocation frictions across states or sectors.

In order to illustrate the differences in wage distributions across states, Figure D.8 plots the distribution of wages in Mississippi and New York from our pooled 2017-2019 ACS data. The left panel plots the distributions for non-college workers and the right panel for college workers. Note that the distributions in these two panels have not been normalized, e.g. the entire wage distribution across both types of workers together sums to one. There are two important differences between these two states. First, New York has a much higher share of of college-educated workers (45%) compared to Mississippi (27%). Second, even within the two education groups, the distribution of wages in Mississippi is shifted left compared to New York. Both facts contribute to the stronger effects of the minimum wage in Mississippi in Table D.2.

The top panel of Table D.2 illustrates the distributional effects of the $15 minimum wage for selected states. The top three states correspond to those with the highest average hourly wage per worker in the ACS during 2017-2019 (Massachusetts, Connecticut, and New Jersey), while the bottom three states have the lowest average hourly wage per worker in the ACS during 2017-2019 (West Virginia, Arkansas, and Mississippi). According to this table, the $15 minimum wage will decrease

TABLE D.2: Heterogeneous Long-Run Effects Across States and Sectors

State	Employment change	Share($w_i < \overline{w}$)	Share($\Delta n_i < 0$)	Share($\Delta w_i n_i < 0$)
Massachusetts	-3.9%	0.17	0.09	0.06
Connecticut	-4.4%	0.18	0.11	0.07
New Jersey	-6.0%	0.21	0.13	0.09
⋮				
West Virginia	-10.8%	0.37	0.23	0.15
Arkansas	-11.2%	0.19	0.39	0.16
Mississippi	-13.6%	0.40	0.27	0.19

Sector	Employment change	Share($w_i < \overline{w}$)	Share($\Delta n_i < 0$)	Share($\Delta w_i n_i < 0$)
FIRE	-3.4%	0.15	0.08	0.05
Manufacturing	-5.2%	0.25	0.16	0.13
Professional services	-6.2%	0.24	0.17	0.09
⋮				
Entertainment	-11.4%	0.38	0.50	0.39
Retail Trade	-17.2%	0.51	0.34	0.23
Personal Services	-18.2%	0.51	0.35	0.25

Note: Long-run effects of the \$15 minimum wage across U.S. states and sectors. We compute the state-level outcome as the weighted value of that outcome across z_i types, using the distribution of z_i implied by that state/sector's wage distribution in the ACS. "Employment change" is change in total employment. "Share($w_i < \overline{w}$)" is the share of initial workers whose wage is below the \$15 minimum wage. "Share($\Delta n_i < 0$)" is the share of initial workers whose employment declines due to the \$15 minimum wage. "Share($\Delta w_i n_i < 0$)" is the share of initial workers whose labor income declines due to the \$15 minimum wage.

employment by four times as much in Mississippi as in Massachusetts. This occurs because the distribution of wages in Mississippi is shifted left compared to Massachusetts; for example, the \$15 minimum wage would bind on nearly 40% of workers in Mississippi but only 17% of workers in Massachusetts (Appendix D plots state-level wage distributions for comparison). Consistent with that, 27% of *all* workers (including college and non-college) in Mississippi would experience an employment decline vs. only 9% in Massachusetts. The other states line up in between these two extremes.

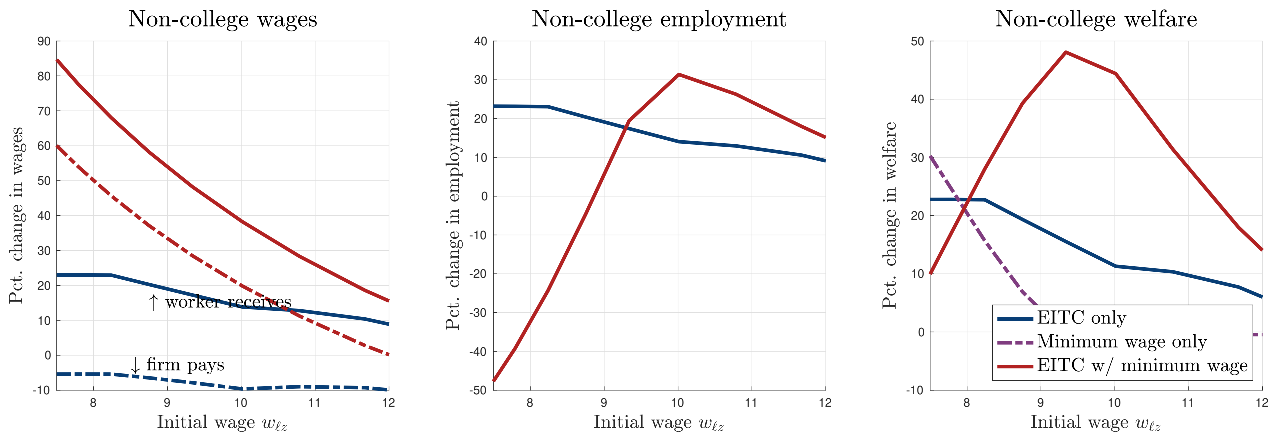
The bottom panel of Table D.2 shows the long-run results of a \$15 minimum wage for the three sectors with the highest average hourly wage per worker (Finance/Insurance/Real Estate, Manufacturing, and Professional Services) and the three sectors with the lowest average hourly wage per worker (Entertainment, Retail Trade, and Personal Services). A \$15 minimum wage will have a relatively small effect on the FIRE sectors because there are few workers (15%) in that sector who currently earn less than \$15 per hour. Conversely, a \$15 minimum wage will have much more dramatic effects on workers in the Retail Trade and Personal Services sectors—more than 50% of workers currently each less than \$15.

Again, these results should be only considered illustrative; in order to assess fully the effects of a large change in the minimum wage across locations and sectors, one would need to have a model allowing for linkages across space and sectors. Additionally, one would potentially want to allow for the production technologies to differ across sectors. However, the back-of-the-envelope findings we highlight here suggest that a large change in the minimum wage will disproportionately affect some states and sectors relative to others because of differences in the underlying wage distribution.

E Additional Results About Alternative Policies

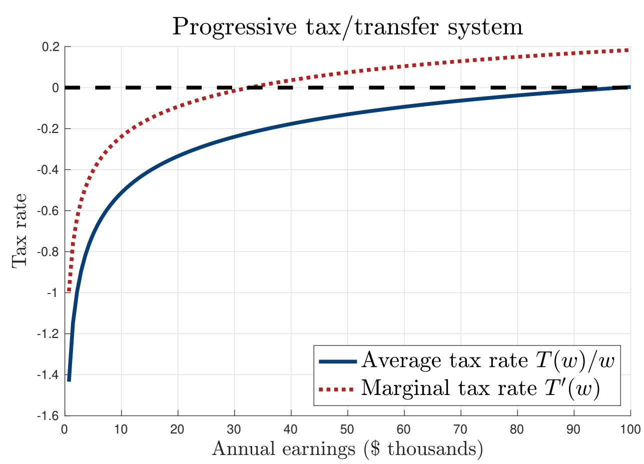
This appendix shows two additional results. First, Figure E.1 shows how a \$12 minimum wage interacts with our budget-equivalent EITC from Section 6. Second, Figure E.2 plots our budget-equivalent tax and transfer system described in the main text.

FIGURE E.1: Effect of EITC with and without $\underline{w}$ = \$12 Minimum Wage



Note: Steady-state wages (left panel), employment (middle panel), and labor income (right panel) in response to our budget-equivalent EITC system from Section 6. The y-axis is the log-change relative to the initial equilibrium (without any policies). The x-axis corresponds to the initial wage $w_{\ell z}$ of a particular type-z worker in the initial equilibrium.

FIGURE E.2: Progressive Tax and Transfer System



Note: Average tax rates $T(w)/w$ and marginal tax rates $T'(w)$ from the budget-equivalent tax and transfer system described in the main text (with the U.S. level of progressivity $\tau = 0.181$). In each panel, the x-axis rescales steady-state labor income to annual earnings assuming each household works 1800 hours per year.