The circular relationship between productivity and hours worked: A long-term analysis

Gilbert Cette* Simon Drapala** Jimmy Lopez***

Abstract

We analyze the circular relationship between productivity (or wages) and hours worked. We begin by developing a simple theoretical model showing the different channels in this circular relationship: productivity (or wages) impacts hours worked through either a income channel or a substitution channel, while returns to scale of hours worked depend on a fixed-cost channel or a fatigue channel. Then we estimate the two equations of this circular relationship, using the IV estimation method, on two separate datasets for advanced countries: a long-term (1890–2019) country panel database, and a country-industry panel for a shorter, more recent period (1995–2019). The three main results are: (i) the income channel outweighs the substitution channel in the long term: increased productivity or higher wages reduce the number of hours worked; (ii) in the short term, there is no clear order of precedence between the income and the substitution channels; (iii) the fatigue channel outweighs the fixed-cost channel: a reduction in hours worked raises productivity (or hourly wages). According to our results, a productivity revival brought about by the digital revolution and resulting in the same productivity growth as was observed in the US from 1900 to 1975 would reduce hours worked to 25 hours per week by the end of this century.

JEL classification: E24; J22; O33; O47
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*: Neoma Business School; gilbert.cette@neoma-bs.fr
**: Paris School of Economics and Université Paris 1 Panthéon – Sorbonne; simon.drapala@etu.univ-paris1.fr
***: Université de Bourgogne, Laboratoire d'Economie de Dijon; jimmy.lopez@u-bourgogne.fr

1. Introduction

Over the last century and a half, total factor productivity and labor productivity have both greatly improved in all advanced countries (e.g., Bergeaud *et al.*, 2016). During the same period, average annual hours worked per employee have fallen dramatically while average annual wages have risen dramatically. This means that, over this long period, the large productivity gains have been enough to finance an improvement in living conditions in two respects: increased purchasing power due to higher wages and more leisure time due to shorter working hours.

As described by Boppart and Krusell (2020), a productivity increase financing a higher hourly wage may impact hours worked through two channels: an income channel (hours worked decrease when productivity increases) and a substitution channel (hours worked increase). In the long run, the income channel seems to have prevailed over the substitution channel. Part of the literature, for instance Li (2022), confirms this order of precedence (see the literature review in the following section). However, other estimation results, for instance Reif *et al.* (2021), find that the substitution channel could prevail over the income channel, which means that a higher hourly wage or increased productivity would mean longer hours worked. From our point of view, these last estimates may hold for short-term relations between productivity and hours worked.

Some empirical analyses of the impact of productivity (or wages) also take into account that the order of precedence between the income and the substitution channels could depend on the level of development, as Boppart and Krussel (2020) point out. Working on country panel data, Bick *et al.* (2018 and 2022b) claim that the relation between average hours worked and GDP per capita takes the form of an inverted U curve. Hours worked per worker first rise with GDP per capita before falling off beyond a certain level of development. An abundant literature has also shown that hours worked decrease when the income tax rate, and more specifically the labor income tax rate, rises, which corresponds to a substitution effect (e.g., Prescott, 2004; Ohanian *et al.* 2008; or more recently Reif *et al.* 2021).

The number of hours worked per worker could itself impact productivity per hour. Here again, two channels are involved: a fatigue effect implying decreasing returns of working time, and a fixed-cost or learning-by-doing effect implying increasing returns of working time. Whilst the literature is consensual in admitting non-constant returns to scale of hours worked, it is not consensual in concluding as to whether those returns are increasing or decreasing (see the literature review in the following section). Using individual US data, Bick *et al.* (2022a) estimate increasing returns to hours worked when workers are on short hours but decreasing returns for long working hours. With industry or country level data, estimation results for advanced country data lean mainly toward decreasing returns of hours worked (e.g., Bourlès and Cette, 2007; Aghion *et al.*, 2009; Cette, Chang, and Konte, 2011; Bourlès, Cette, and Cozarenco, 2012; or Pencavel, 2015).

The aim of the paper is to estimate this circular relationship from productivity (or wages) to hours worked and from hours worked to productivity. We begin by proposing a simple theoretical model, showing the different channels that operate in this circular relationship. Then we estimate the two relations of this circular relationship, the relation from productivity (or wages) to hours worked, distinguishing between short-term and long-term mechanisms. This empirical investigation draws on Bergeaud *et al.*'s (2016) very long-term productivity panel database: our country-level estimation sample covers the period from 1891 to 2019 for 21 advanced countries. We supplement our analysis using the EUKLEMS & INTANProd industry level panel database. The corresponding estimation sample is unbalanced and covers 22 advanced countries and 23 business sectors for the more recent period 1995–2019. Estimation results obtained with these two datasets through OLS or IV methods are consistent. The main original contributions of our paper are: (i) the link proposed between theoretical and empirical approaches; (ii) the possible coexistence of both income and substitution channels in the estimated relation; (iii) estimations are made using both OLS and IV approaches to avoid endogeneity problems related, for instance, to the circular link between productivity (or wages) and hours worked; (iv) the estimates made on two separate datasets for advanced countries, the first one being a long-term (1890–2019) country*year dataset, and the second one a country*industry*year dataset for a shorter, more recent period (1995–2019).

The three main results of our estimations are the following. First, the income channel takes precedence over the substitution channel in the long term: increased productivity (or wages) reduce the labor supply through shorter hours worked, the long-term elasticity being about -0.1 to -0.2 depending on the sub-period and the dataset. Second, in the short term, there is no clear order of precedence between the income and the substitution channels: the first prevailing over the second for certain sub-periods and sets of countries, and vice-versa for other sub-periods and sets of countries. This suggests that the short-term impact of changes in productivity (or in wages) on the number of hours worked involves other variables that differ among countries and sub-periods. Third, the fatigue channel outweighs the fixed-cost channel: a reduction in hours worked raises productivity (or wages per hour), the elasticity being about -0.4 to -0.6 depending on the sub-period and the dataset.

The paper is organized as follows. Section 2 describes a simple theoretical model showing the circular relationship between productivity (or hourly wages) and hours worked per worker, and presents the estimated models. Section 3 describes the two sets of data used for the estimations. Sections 4 and 5 comment on the estimation results for the relations from productivity (or wages) to hours worked, and from hours worked to productivity (or wages), respectively. Section 6 concludes.

2. Theoretical approach and estimated models

We begin by presenting a simple theoretical model of the circular relationship between productivity and hours worked, and the related literature, before specifying the relations estimated in the following sections of the paper.

2.1. A simple theoretical framework

Consider a standard CES function to represent the utility level U for a representative agent, depending on leisure L and labor income R:

(1) $U = [(1-\lambda)L^{\frac{\rho-1}{\rho}} + \lambda R^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}$. with $0 < \lambda < 1$ and $\rho > 0$

The parameter ρ corresponds to the elasticity of substitution between leisure L and labor income R. The leisure time L is equal to the global time budget D, which is constant, minus the number of hours worked H:

(2) $L = D - H$

The labor income R depends on productivity per hour with a scale effect:

(3) $R = A\, e^{\gamma t} H^{\beta}$ with $\gamma, \beta > 0$

Where γt represents the productivity per hour impact of technical progress, which increases with γ and with t, and β the returns to scale of hours worked H. If $\beta > 1$ ($\beta < 1$), returns to scale of hours worked are increasing (decreasing). As described for instance by Bourlès and Cette (2007), Aghion *et al.* (2009), Cette, Chang, and Konte (2011), Bourlès, Cette, and Cozarenco (2012), Pencavel (2015), Eden (2021), Bick *et al.* (2022a) or Del Rey *et al.* (2022) among others, different types of effects may contribute to non-constant returns to scale of hours worked. A fatigue effect could explain decreasing returns to scale of hours worked whereas fixed-cost and learning-by-doing effects could explain increasing returns of hours worked. As detailed by Eden (2021), this impact of hours worked on productivity per hour depends also on many things, such as the length of the working day, the number and the position of the vacation days in the week and the year, etc. On different country level datasets of advanced countries over the last decades, Bourlès and Cette (2007), Aghion *et al.* (2009), Cette, Chang, and Konte (2011), and Bourlès, Cette, and Cozarenco (2012) estimate on average decreasing returns to scale of hours worked of about 50 percent ($\beta \approx 0.5$). Using different datasets for the UK for the early twentieth century, Pencavel (2015) estimates a fatigue effect and consequently decreasing returns of hours worked. These decreasing returns reportedly become large on exceeding 49 hours' work a week. Using individual US data, Bick *et al.* (2022a) estimate increasing returns to hours worked when workers are on short hours (below 40 hours per week) and decreasing returns for long hours (above 48 hours a week). Del Rey *et al.* (2022) propose a model explaining the results from Bick *et al.* (2022a). As we can see, the literature is consensual in admitting non-constant returns to scale of hours worked. Nevertheless, it is not totally consensual concerning the thresholds from which these decreasing returns may be significant. In the following developments of our analysis, we will consider that returns to scale of hours worked are not necessarily constant.

The first order condition to maximize the utility level U leads to equation (4):

$$(4) \qquad H^{\beta(\rho-1)-\rho}(D-H) = \left(\frac{1-\lambda}{\lambda\beta}\right)^{\rho} Ae^{(1-\rho)\gamma t}$$

The left hand side of this equality is decreasing with H. The right hand side is increasing with γt if $\rho < 1$ and decreasing with γt if $\rho > 1$. This means that the level of working time H decreases with productivity gains γ if the substitution elasticity between leisure and labor income is less than 1 ($\rho < 1$) and on the contrary increases with productivity gains if the substitution elasticity between leisure and labor income is greater than 1 ($\rho > 1$).

Two channels come into play for hours worked preferences in the event of a productivity change: an income channel (negative relation between productivity and hours worked) and a substitution channel (positive relation). The first channel reportedly prevails when the elasticity of substitution between leisure and labor income is less than 1, and on the contrary the second channel prevails when the substitution elasticity between leisure and labor income is greater than 1. This point is raised by Boppart and Krusell (2020) based on a theoretical model. They stress that the value of the substitution elasticity between leisure and labor income could depend on the development level of countries. These opposing effects of the income and substitution channels cannot arise if the substitution elasticity between leisure and work is equal to 1, as assumed by Duernecker and Herrendorf (2018).[1] Using country level data from a set of 15 OECD advanced countries over the period 1963–2006, Reif *et al.* (2021) estimate a positive impact of productivity growth (the parameter γ in our previous equations) on hours worked. This means that for these specific data, the substitution elasticity between leisure and labor income is greater than 1, and that the substitution channel prevails over the income channel. Another strand of literature estimates through VAR or SVAR models the impact of TFP shocks on hours worked. For instance, Li (2022) finds, by such an approach on US data

[1] The main goal of Duernecker and Herrendorf (2018) is to analyze, among working activities, substitution between home and market production. They assume a CES relation between these two types of work. This is also the case in an abundant literature, see, for instance, Ngai and Pissarides (2008).

from 1948 to 2017, a significant and negative impact, which means also that the income channel outweighs the substitution channel.

Bick *et al.* (2018) and Bick *et al.* (2022b) claim, using country panel data, that the relation between the hours worked on average per worker and the GDP per capita corresponds to an inverted U curve. The average number of hours worked per worker first rises with the GDP per capita and then decreases beyond a certain development level. This result suggests that the substitution elasticity between leisure and labor income is greater than 1 for low-income countries and less than 1 for high-income countries. At the same time, Bick *et al.* (2018) and Bick *et al.* (2022b) argue that, contrariwise, the relation between the employment rate and GDP per capita corresponds to a U curve: the employment rate first decreases with the GDP per capita and then increases beyond a certain development level. These two results correspond to intensive and extensive margin behavior in the labor supply depending on development. And they report that the number of hours worked on average per adult, which takes into account both the intensive and the extensive margin labor supply behavior, decreases slightly with development.

In the particular situation of a unitary substitution elasticity between leisure and labor income ($\rho = 1$), the level of working time H corresponds to equation (5):

(5) $$H = \frac{\lambda\beta}{\lambda\beta+(1-\lambda)} D$$

In this situation, the optimal level of hours worked H is constant and does not depend on productivity gains γ. It depends on returns to scale of hours worked β. And in the more specific case of constant returns of hours worked ($\beta = 1$), the level of hours worked is a simple share of the time budget D:

(6) $H = \lambda D$

If the utility function corresponds to a Cobb-Douglas relation, which means a unitary substitution elasticity between leisure and labor income, then:

(1') $U = L^{1-\alpha} R^{\alpha}$ with $0 < \alpha < 1$

Under the same hypotheses as above concerning hours worked H (equation (2)) and labor income R (equation (3)), the first order condition to maximize the utility level U leads to equation (5'):

(5') $$H = \frac{\alpha\beta}{\alpha\beta+(1-\alpha)} D$$

This equation is similar to equation (5) obtained with a CES utility function in the particular case of a unitary substitution elasticity between leisure and labor income. The optimal level of hours worked H is constant and does not depend on productivity gains γ. It depends on returns to scale of hours worked β. And in the specific case of constant returns of hours worked ($\beta = 1$), the level of hours worked corresponds to equation (6'), which is similar to the previous equation (6):

(6') $H = \alpha D$

2.2. Estimated models

Two types of relations are estimated. The first one explains the average working time by productivity level and growth plus other variables, and the second one explains productivity by hours worked plus other variables.

Estimated relations of the first type correspond to equation (7):

$$(7)\quad \log(H) = a1.\log(LP) + a2.\Delta\log(LP) + a3.\log(LP).\Delta\log(LP) + \Sigma_i(c_i.CV_i) + \Sigma_j(d_j.FE_j) + u$$

Where *H* is the average working time per worker, *LP* is labor productivity per hour worked, CV_i are control variables, FE_j are fixed effects, and *u* is the error term. In estimates on the industry*country*year dataset, we replace *LP* by the average wage (noted *w*). To make the notations leaner, we omit country and time indices. Two types of control variables will be considered in the estimations. First, the average employment rate, as it is shown in an abundant literature that productivity decreases (increases) when the employment rate increases (decreases), for the most productive workers are the first employed (see, for instance, Bourlès and Cette, 2007; Aghion *et al.*, 2009; Cette, Chang, and Konte, 2011; and Bourlès, Cette, and Cozarenco, 2012). And at the same time, the employment rate may influence hours worked, as its increase (decrease) often means more (fewer) part-time jobs, mainly among women entering (leaving) the labor market. Second, tax rates, more precisely social and income tax rates, with an abundant literature showing, following Prescott (2004), that taxes are important determinants of hours worked (see for instance Ohanian *et al*., 2008, or more recently Reif *et al*. 2021). Fixed effects will be country and year effects on the country*year dataset, and also industry and crossed fixed effects on the country*industry*year dataset.

Concerning the key variables, related to the productivity impact on hours, the coefficient *a1* corresponds to a long-term effect and coefficients *a2* and *a3* to a short-term effect. If $a1 > 0$ ($a1 < 0$), then the substitution effect between leisure and labor income outweighs (is outweighed by) the income effect. From our point of view, the relation estimated by Reif *et al.* (2021) assumes that $a1 = a3 = 0$, and corresponds to the estimation of a short-term relation.

Estimated relations of the second type correspond to equation (8):

$$(8)\quad \log(LP) = b1.\log(H) + \Sigma_i(c_i.CV_i) + \Sigma_j(d_j.FE_j) + u$$

Using the same notation conventions as for equation (7). Here again, in estimates on the industry*country*year dataset, we replace *LP* by the average wage (noted *w*). Two types of control variables will also be considered in the estimations. First, the average employment rate, for reasons mentioned before. Second, the logarithm of the capital to labor ratio, more precisely the logarithm of the capital (in constant prices) per hour worked, as is usually done in production function estimates.

Concerning the key variables, related to the impact of hours on productivity, the coefficient *b1* corresponds to a long-term effect.

3. The data

Our empirical investigation of the relationship between hours worked and productivity or labor income benefits from two level of analysis. The first is a country level analysis. This allows this relationship to be investigated for a very long period from 1891 to 2019. The second is an industry level analysis over a much shorter period, from 1995 to 2019. This cross-country-industry panel allows us to use a wide set of fixed effects to prevent omission bias. This section introduces first the country level database, then the industry level database.

3.1. Country level database description

The Country Level Database relies on the latest version (version 2.5, 2021) of the Long-Term Productivity Database (Bergeaud *et al.*, 2016[2]). This database is exceptional by its temporal extent and provides Labor Productivity per hour (*LP*) and average working time (*H*) over the period 1890–2019 for 21 countries. These 21 countries correspond to 14 European countries (Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom), 5 other developed countries (Australia, Canada, Japan, New Zealand, and the USA), and 2 less developed countries (Chile and Mexico). Other variables are also provided by the database and used in our study, including education, capital stock, total employment, and total population.

Labor Productivity per hour (*LP*) is simply computed over the period 1890–2019 by using three basic series available in the Long-Term Productivity Database: GDP (*Y*), total employment (*N*), and average working time (*H*). It is the ratio of GDP (*Y*) to Labor ($N * H$); $LP = \frac{Y}{N*H}$. We then extract the low frequency component of Labor Productivity and average working time series using the Hodrick-Prescott (HP) filter with a smoothing parameter of 6.25 (common value for yearly data). We also remove the years related to World War 1 and World War 2 (1914–1918 and 1939–1949) for reasons of representativeness. To prevent these removed years from creating disturbances for the HP filter, we linearly interpolate them before applying the HP filter to our entire dataset.

The variable *Education* corresponds to the average number of years of schooling of the population aged 25 and more. We use for that series of educational attainment updated and completed by Cette *et al.* (2022) from data provided by van Leeuwen and van Leeuwen-Li (2014) updated by Bergeaud *et al.* (2018), with data from the UNESCO and the Penn World Table 9.1. We also build the variable *Labor Income Tax Rate*, which corresponds to the average tax rate on labor income from McDaniel's (2007) updated series. We use the sum of the average tax rate on household income and average payroll tax rate (it includes taxes paid by both employer and employee).

The starting database was that built by Cette *et al.* (2009) for France, Japan, the United Kingdom, and the United States over the 1890–2006 period. Bergeaud *et al.* (2016) have updated and considerably enlarged this first database. They used estimates of long aggregate historical data series (e.g., Maddison, 2001, 2003; Barro and Ursua, 2010; Madsen and Ang, 2016) on GDP, employment, working time, and investment (in two products, equipment, and buildings). For the most recent decades of the analysis, the Long-Term Productivity Database is built from national accounts where available. For others, the database is constructed from data estimated and gathered by economists and historians on consistent assumptions. The data are built at the country level under the hypothesis of constant borders, in their last state. It should be noted that however talented economists and historians are, strong assumptions are required to reconstitute some countries and many of these data are subject to uncertainty and inaccuracy. We may nevertheless consider that the orders of magnitude of our estimates are fairly reliable and meaningful. Series for GDP and capital are given in 2010 constant

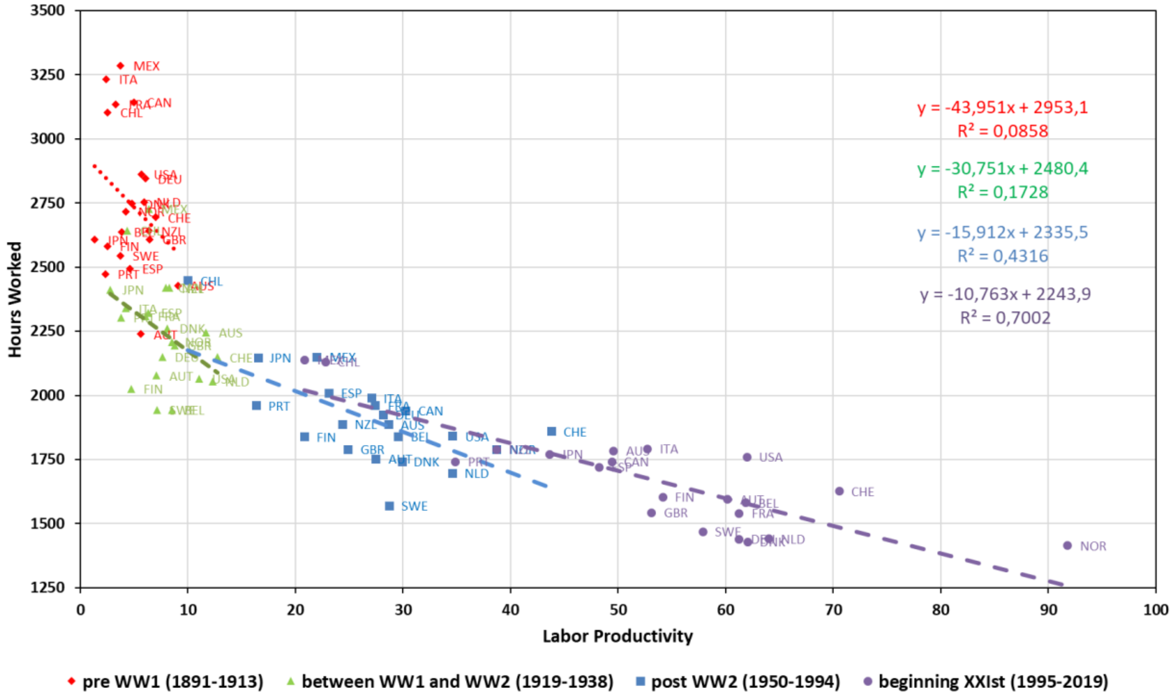
[2] See that paper for a complete description of the methodology used to construct the database. This database can be freely accessed, see http://www.longtermproductivity.com/.

national currencies, and converted to US dollars at purchasing power parity (PPP) with a conversion rate from the Penn World Tables.

Looking at the data, Figure 1 plots the long-run relationships between hours worked and labor productivity at a country average level. The average values for our 21 countries are displayed for our four main sub-periods: before WW1 (1891–1913), between the two World Wars (1919–1938), after WW2 (1950–1994), and around the turn of the twenty-first century (1995–2019). The relationships obtained are always negative and level off for more recent sub-periods. These relationships are consistent with the existing literature: hours worked tend to decrease and labor productivity to increase over time. In a long-run framework and considering only variations between countries, the income channel seems to outweigh the substitution channel.

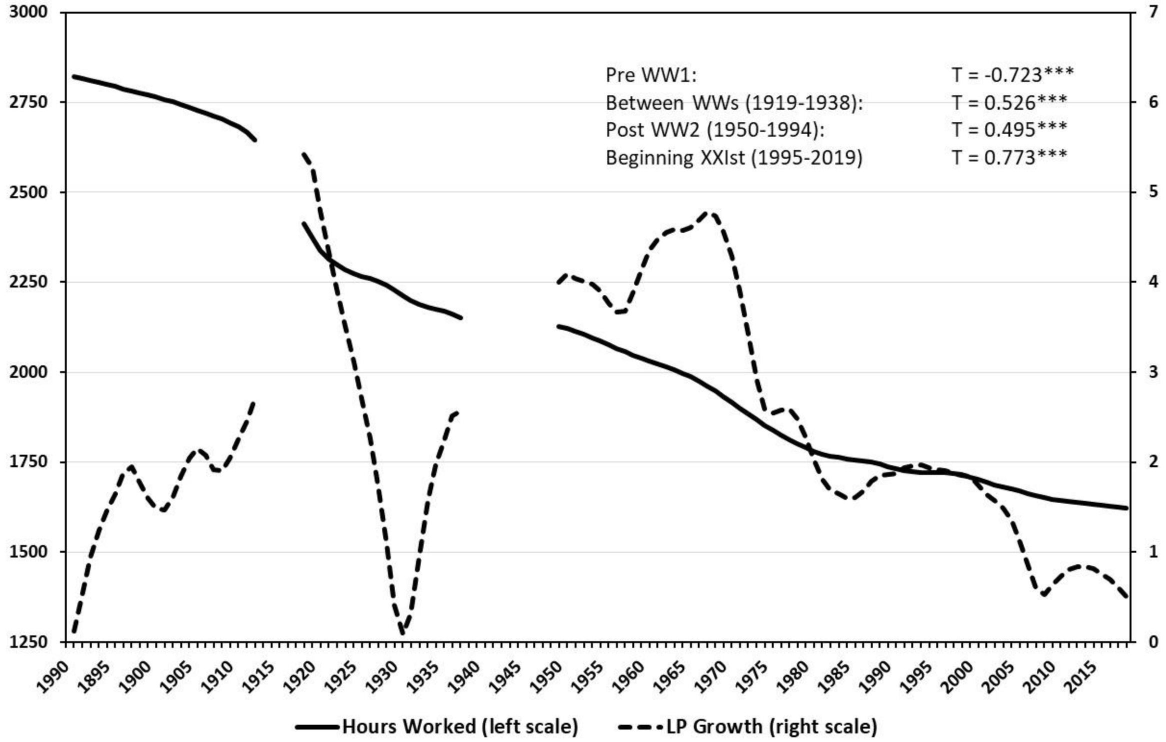
Figure 2 plots the yearly averages of hours worked and labor productivity growth computed for 21 countries. It provides some initial leads to see how hours worked evolve in terms of productivity growth (and conversely) over our sub-periods. This relation can be seen as the short-run relation between hours worked and labor productivity. In a short-run framework and considering only yearly averages over countries, the substitution channel seems to outweigh the income channel, which is consistent with Reif *et al*.'s (2021) results. This relation is not clear for all sub-periods, in particular for the "pre WW1 (1891–1913)" sub-period. This is a first indication that the short-term effect is somewhat volatile.

Figure 1
Hours Worked and Labor Productivity – Country averages (21) over four sub-periods
Country level database



Each point represents a specific country for one sub-period. For this specific country, we represent the average value of labor productivity and hours worked over this sub-period. In the top right corner, linear equation regressions and R^2 values are given for each sub-period (earliest sub-period at the top). Series used for GDP and capital are given in 2010 constant national currencies, and converted to US dollars at purchasing power parity (PPP).

Figure 2
Hours Worked and Labor Productivity Growth – Yearly averages
Country level database



We represent the average value taken by the labor productivity and hours worked computed over 21 countries for each year. The solid lines represent hours worked (left axis) and the dashed lines represent labor productivity growth (right axis). In the top right corner, correlations between the two variables are given (earliest sub-period at the bottom). The Kendall method is used to compute these correlations.

3.2. Industry level database description

Like the country level analysis, our industry level analysis uses data on hours worked and value added, but it benefits also from data on wages, capital stock, and intermediate inputs. As was the case with the country level database, series are given in 2010 constant prices and converted into US dollars at purchasing power parity (PPP). These data are provided by the EU KLEMS Database for 27 EU Member States, Japan, the United Kingdom, and the USA over the period 1995–2019 at the 2 digit ISIC Rev. 4 industry level.[3] However, the capital stock data is available for 22 countries only.[4] This set of countries differs from the country level sample, with 13 countries common to both samples. Our analysis focuses on 23 industries covering the business sector.[5] After cleaning, our main estimation sample is an unbalanced panel of 8116 observations (Appendix A presents the sample in more detail).

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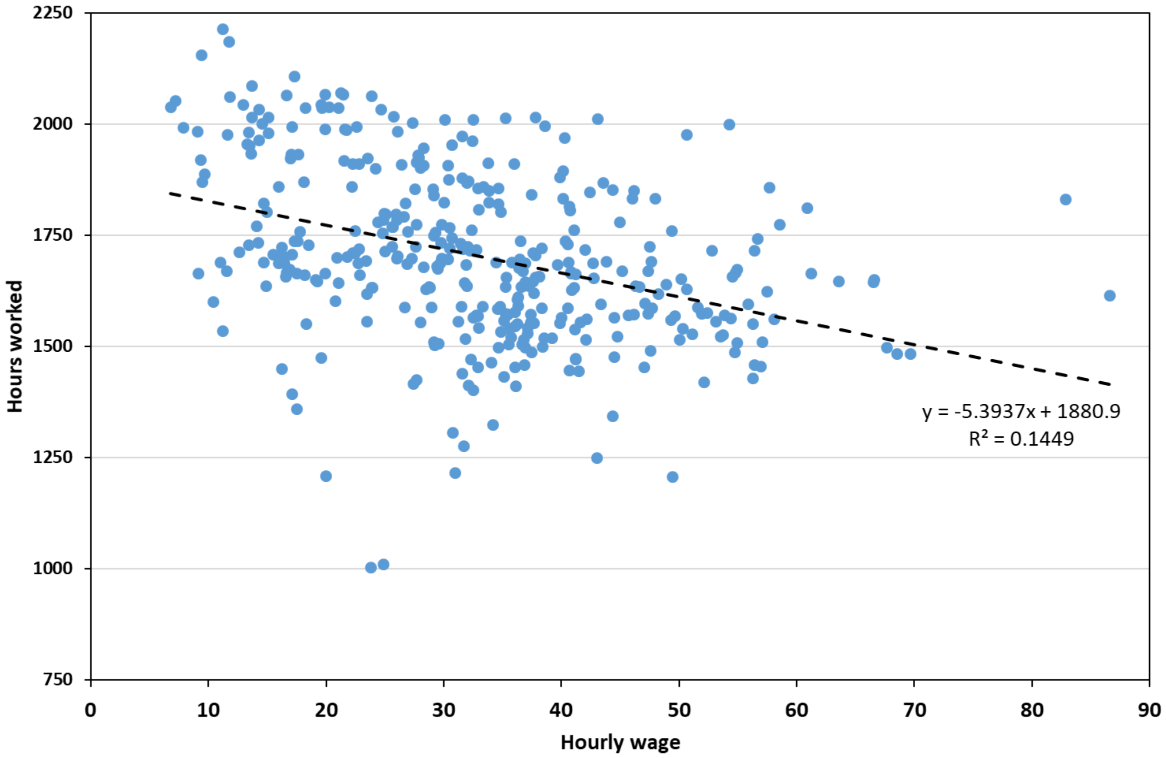
3 We use the Release 2021 of the integrated EUKLEMS & INTANProd database run by the Luiss Lab of European Economics.

4 The countries in our main estimation sample are: Austria, Belgium, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Japan, Lithuania, the Netherlands, Poland, Romania, Slovakia, Slovenia, Spain, Sweden, the United Kingdom, and the United States.

5 Four industries of the business sector are excluded from our estimation sample because the productivity measurement is particularly difficult concerning these industries: “Agriculture, forestry and fishing”, “Mining and quarrying”, “Manufacture of coke and refined petroleum products”, and “Real estate activities”.

The main estimations at the industry level use real wages per hour worked to measure labor income, by contrast with the long-term country level analysis that must use labor productivity as a real wage proxy. However, it is interesting to note that labor productivity and wages are strongly correlated, with a correlation coefficient of 0.70 between the country*industry averages of both variables for instance. Figure 3 plots the relationship between real wages and hours per worker using country-industry averages. It suggests a negative relationship between hours worked and wages, which is consistent with the relation between hours worked and labor productivity shown in Figure 1 in the previous section on country level averages (Figure A in Appendix A shows also the strong relation between productivity and wages). However, these results must be confirmed by using the within-individual data variability in the next section on the estimation results.

Figure 3
Hours Worked and Hourly Wage, country*industry average, period 1995–2019
Industry level database



Each point represents a country-industry specific average for the period 1995–2019

4. From productivity (or wages) to hours worked

Table 1 presents estimation results of the impact of labor productivity on hours worked per worker on our country level database. Columns (5) and (6) correspond to equation (7), while columns (1) to (4) allow us to investigate the sensitivity of the estimation results to the specification choices as well as a comparison with the results of Reif *et al.* (2021). Column (1) shows estimation results using labor productivity growth as in Reif *et al.* (2021), for the same set of 15 advanced countries[6] and the same 1963–2006 period. Our results are similar to their results and remain positive when we extend the estimation period to 1950–2019 (column 2) and the 21-country sample (column 3). This suggests that the impact of the substitution

[6] These 15 countries are: Australia, Austria, Belgium, Canada, Finland, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

channel prevails over the impact of the income channel in the short term. However, Reif *et al.* (2021) assume implicitly that labor productivity growth alone impacts hours worked. On the contrary, the column (4) specification includes only the labor productivity level and shows a negative effect of productivity, suggesting that the income channel dominates. The specification of equation (7) introduces both labor productivity growth and level. The column (5) positive effect of growth and negative effect of level suggest that the substitution channel dominates in the short term whereas the income effect dominates in the long term. A productivity increase of 1 percentage point would increase working time by 0.82 percent the first year, but would reduce working time by 0.16 percent in the long run.

Another interesting result of Table 1 columns (1) to (3) is that the effect of the labor productivity growth declines markedly when we extend the country sample to 21 countries (column 3) relatively to the Reif *et al.* (2021) 15-country sample (columns 1 and 2).[7] Equation (7) introduces an interaction term between growth and level of productivity to test whether the impact of productivity growth depends on the level of productivity reached. Column (5) shows a positive coefficient for this interaction term, suggesting that the substitution effect dominates in the short term when the productivity level is high. On the contrary, when the productivity level is low, the income effect dominates in both the short and long run.[8] Finally, column (6) shows that the previous estimation results are strongly robust when introducing the labor income tax rate and the employment rate. Labor income tax would have a detrimental impact on hours worked: a 1 percentage point increase in the labor income tax rate would decrease working time by 0.29 percent. This is consistent with the literature, see, for instance, Prescott (2004), Ohanian *et al*. (2008), or Reif *et al*. (2021). The employment rate also seems to have a detrimental impact on hours: a 1 percentage point increase in the employment rate would reduce the hours by about 0.41 percent.[9] Our results are also robust to the use of various HP filter parameters.[10]

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7 If we limit the estimation sample to the six new countries (Chile, Denmark, Mexico, New Zealand, Norway, and Portugal), productivity growth has a negative but not significant effect on hours worked.

8 A 1 percentage point productivity increase would reduce the working time in the first year by 0.5 percent for observations for the first decile of labor productivity but would increase the working time in the first year by 1.9 percent for the last decile.

9 The employment rate on our country level database is calculated on the whole population, not only on the working age population as it should be. Therefore, our estimates may overestimate the impact of the employment rate.

10 Estimation results are available from the authors on request.

Table 1
Impact of productivity on hours worked, OLS estimates
Country level database
Dependent variable: Hours worked per worker (in log)

	(1)	(2)	(3)	(4)	(5)	(6)
Period	1963-2006	1950-2019	1950-2019	1950-2019	1950-2019	1950-2018
Country Sample	15	15	21	21	21	15
$\log(LP)$				-0.138***	-0.157***	-0.152***
				(0.008)	(0.007)	(0.022)
$\Delta\log(LP)$	1.284***	1.145***	0.581***		0.823***	1.074***
	(0.163)	(0.185)	(0.120)		(0.113)	(0.169)
$\Delta\log(LP)*\log(LP)$					1.449***	1.575***
					(0.139)	(0.218)
Labor Income Tax Rate						-0.292***
						(0.041)
Employment Rate						-0.412***
						(0.084)
R^2	0.899	0.882	0.889	0.920	0.928	0.908
Observations	660	1050	1470	1449	1449	1010

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.
Country and year fixed effects are included in all the estimated specifications.

Table 2 also presents estimation results of equation (7), but over various periods. Columns (1) to (4) show estimation results on our country level database for four separate periods: before the First World-War (1891–1913), between the two World Wars (1919–1938), and for two sub-periods after the Second World-War, 1950–1994 and 1995–2019. This breakdown of the post WW2 period allows us to compare the results with the industry level database estimations for the same 1995–2019 period, column (7). Columns (5) and (6) cover respectively the two post-WW2 periods (1950–2019) and the whole set of periods (1891–2019).[11] Another important difference with the previous Table 1 estimations is the use of the IV method. Indeed, we may expect three usual sources of endogeneity: (i) hours worked may also have an impact on labor productivity; (ii) omitted factors may have an impact on both productivity and hours worked, for instance labor market regulations; and (iii) labor productivity may be difficult to measure. We use education level and capital stock as external instruments for productivity for the country level analysis, capital stock and intermediate inputs for wages for the industry level analysis, assuming that these variables have a strong impact on productivity and are exogenous. Our identification assumptions are confirmed by the test results: the weak instrument hypothesis is always rejected, the Sargan test does not allow the over-identification restriction to be rejected, and the Durbin-Wu-Hausman test consistently rejects the exogeneity assumption of labor productivity, except for column (6). According to the Table 2 estimation results, the long-run negative impact of labor productivity is quite stable over time, which is consistent with the relation observed using the country averages in Figure 1.

Country level estimation results are confirmed on industry level data, column 7, even though real wages rather than labor productivity and a full set of fixed effects are used: country*industry, country*year, and industry*year fixed effects. On the contrary, the short-term

[11] The set of fixed effects are the same in columns (5) and (6) as in columns (1) to (4), so there are respectively two and four fixed effects per country. This is particularly important for the whole period 1891–2019 as the years of WWs and part of the rebuilding period are excluded from the estimation sample.

effects of labor productivity appear to change between periods as does its relation with the productivity level of the country. Finally, a 1 percent increase in labor productivity would increase the hours worked by about 0.15 percent in the long run whatever the period, but we cannot draw conclusions concerning the short-run impact. This short-term impact could depend on many other factors omitted from our estimates.

Table 2
Impact of productivity on hours worked, IV estimates
Dependent variable: Hours worked per worker (log)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sample	Country level						Industry level(a)
Period	1891-1913	1919-1938	1950-1994	1995-2019	1950-2019	1891-2019	1995-2019
$\log(LP)$	-0.231***	-0.342***	-0.157***	-0.167	-0.178***	-0.152***	-0.107***
	(0.044)	(0.079)	(0.042)	(0.146)	(0.049)	(0.036)	(0.0141)
$\Delta\log(LP)$	-0.707	0.221	1.319**	-0.424	1.458**	0.261	-0.401***
	(0.442)	(0.345)	(0.635)	(0.430)	(0.692)	(0.314)	(0.0623)
$\Delta\log(LP) * \log(LP)$	0.522	-3.082***	2.874*	-4.559**	2.382*	1.172	-0.318***
	(0.872)	(1.116)	(1.602)	(2.010)	(1.364)	(1.037)	(0.123)
R^2	0.986	0.870	0.922	0.982	0.952	0.982	0.325
Observations	420	357	882	462	1407	2184	8116
IV tests, p-values							
Sargan test	0.08	0.44	0.08	0.97	0.11	0.90	0.18
DWH test	0.00	0.00	0.00	0.00	0.1	0.62	0.00

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.
Country and year fixed effects are included in the estimated specifications columns (1) to (6). For columns (5) and (6), two or four sets of country fixed effects are introduced respectively. Country*year and industry*year fixed effects are included in column (7) specifications.
a: Column (7) is estimated on the sectoral database, wages are used instead of Labor Productivity.
Instruments: education and capital stock, in levels and first difference, for columns (1) to (6); capital stock and intermediate inputs for column (7).
All the weak instrument tests are significant at the 1 percent level.

Appendix B provides a sensitivity analysis of the estimation results from productivity or wages to hours worked. Table B1 replicates Table 1 specifications and sample but using the IV estimation method rather than the OLS estimator. Similarly, Table B2 replicates Table 2 but using the OLS estimator rather than the IV method. In both cases, estimation results are similar but with higher values for the impact of labor productivity growth and the interaction terms when using the IV estimator.

5. From hours worked to productivity (or wages)

The estimates in Section 4 have shown the impact of labor productivity on hours worked. Vice-versa, the hours worked may have an effect on productivity. Table 3 estimation results investigate this effect over the same periods as Table 2, using the equation (8) specification and the IV method. Our external instruments are the hours worked in the two closest countries for the country level analysis and of the two closest industries in the same country for the industry level analysis.[12] Our identification assumptions are confirmed by the tests: the weak instrument hypothesis is consistently rejected and the Sargan test does not allow the over-identification restriction to be rejected. The result of the Durbin-Wu-Hausman test depends on the estimation period, which is surprising but may be explained by the low power of this test. However, the Table 2 results have confirmed an impact of labor productivity on hours worked, underlining the endogeneity of the explanatory variable hours worked in equation (8).

Table 3 shows a strong negative impact of hours worked for each period. This suggests decreasing returns to the number of hours worked per worker, with the fatigue effect outweighing the learning effect. According to column (6) estimation results for the whole period 1891–2019, a 1 percent increase in hours worked per worker would decrease labor productivity by 0.62 percent in the long run. This result is similar to those of Bourlès and Cette (2007), Aghion *et al.* (2009), Cette, Chang, and Konte (2011) and Bourlès, Cette, and Cozarenco (2012).

Appendix C provides a sensitivity analysis of Table 3 estimation results. Table C1 replicates Table 3 estimations but using the OLS estimator. As for the impact of productivity or wages on hours worked, the estimated effects are similar but smaller when using the OLS estimator. The estimation results are very similar to Table 3 estimates. This is also the case if we use the three closest countries or industries rather than the two closest or a closeness measure based on the previous period rather than the current period.[13]

12 The two closest countries are those with the highest correlation in terms of hours worked (without year fixed effects) with the country under consideration. The two closest industries are the ones with the highest correlation within country on average across the whole set of countries.

13 Estimation results are available from the authors on request.

Table 3
Impact of hours worked on productivity, IV estimates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sample	Country level						Industry level[(a)]
Dep. var.	log(*LP*)						log(*w*)
Period	1891-1913	1919-1938	1950-1994	1995-2019	1950-2019	1891-2019	1995-2019
$\log(H)$	-0.625*	-0.366**	-0.654***	-0.384***	-0.640***	-0.624***	-0.690***
	(0.276)	(0.152)	(0.084)	(0.126)	(0.080)	(0.074)	(0.180)
$\log(KI)$	0.212***	0.299***	0.635***	0.233***	0.613***	0.564***	0.110***
	(0.033)	(0.055)	(0.013)	(0.028)	(0.013)	(0.013)	(0.00547)
R^2	0.993	0.976	0.985	0.995	0.991	0.997	0.718
Observations	462	399	924	504	1449	2310	8116
IV tests, p-values							
Sargan test	0.86	0.43	0.22	0.68	0.83	0.85	0.13
DWH test	0.12	0.00	0.47	1.00	0 .02	0.02	0.00

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.
Country and year fixed effects are included in the estimated specifications columns (1) to (6). For columns (5) and (6), two or four sets of country fixed effects are introduced, respectively. Country*year and industry*year fixed effects included in column (7) specifications.
a: Column (7) is estimated on the sectoral database, wages are used instead of Labor Productivity.
Instruments: hours worked in the two closest countries for columns (1) to (6) and of the two closest industries in the same country for column (7).
All the weak instrument tests are significant at the 1 percent level.

6. Conclusion

In this paper, we have proposed a theoretical model showing a circular relationship between hours worked per worker and productivity or wages per hour. In the first relationship from productivity (or wages) to hours worked, two channels are at play. The first one is a income channel: hours worked decrease with productivity or wages. The second one is a substitution channel: hours worked increase with productivity or wages. Similarly, two channels operate in the second relationship from hours worked to productivity or wages. The first one is a fatigue effect: productivity or wages per hour increase with hours worked. The second one is a fixed-cost effect: productivity or wages per hour decrease with hours worked.

We have estimated this circular relationship using two different datasets: a country*year dataset made up of 21 countries over the long period 1890–2019, and a country*industry*year dataset covering 30 countries (27 from the EU plus Japan, the UK, and the US) over the short 1995–2019 period. Estimation results obtained on these two datasets through OLS or IV methods are consistent. The three main results are the following. First, the income channel outweighs the substitution channel in the long term: an increase in productivity or wages reduces the labor supply through a reduction in hours worked, the long-term elasticity being about -0.1 to -0.2 depending on the sub-period and the dataset. Second, in the short term, there is no clear hierarchy between the income and the substitution channels: the first outweighing the second over certain sub-periods and sets of countries and on the contrary being outweighed by it over other sub-periods and sets of countries. This suggests that the short-term impact of productivity or wage changes on hours worked depends on other variables that differ among countries and sub-periods. Third, the fatigue channel outweighs the fixed-cost channel: a reduction in hours worked increases productivity or wages per hour, the elasticity being about -0.4 to -0.6 depending on the sub-period and the dataset.

What might be the impact of hours worked on the productivity revival brought about by the digital revolution? To give some order of magnitude, let's assume the very simple hypothesis that this technological revolution could have the same impact on productivity as the previous revolution observed in the US from 1900 to 1975. Over this period, the average growth rate of labor productivity per hour was about 2.6 percent. Using estimation results from Table 2, column 4, hours worked could decrease, during the next three-quarters of the century by about 0.45 percent per year, meaning that, starting at about 1840 hours in the current period, by the end of the century hours worked per worker could average about 1335 hours per year. This averages out at about 25 hours per week. This workweek length is still above the 15 hours predicted by Keynes (1930) for 2030. But as explained by Crafts (2022), Keynes (1930) had not anticipated that a large part of the increased leisure financed by productivity gains also contributes to a longer retirement period and not just a shorter workweek during one's working life. The relationship between hours worked and productivity being circular, part of the observed productivity growth could be explained by the decrease in hours worked, 0.17 percentage points more exactly. Of course, in a secular stagnation scenario with a stable productivity level, hours worked could on the contrary remain stable.

Over the next three-quarters of the century, several types of headwind will have to be financed by productivity gains. The three main headwinds are of course climate policies, ageing population and the reduction of the public debt. This means that in all likelihood a big part of the future productivity gains will not finance the reduction of hours worked, during the working week or during one's working life. The twentieth century was probably an exceptional and specific period, during which high productivity gains financed the improving quality of life both through an increase in household purchasing power and in leisure time. Productivity gains now have to finance the long-term sustainability of our current quality of life. A problem would, of course, occur without substantial productivity gains in the future: we would perhaps have to consider the necessity of longer workweeks to help finance policies designed to face the main headwinds in front of us.

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APPENDIX

Appendix A : Descriptive analysis of the industry level database

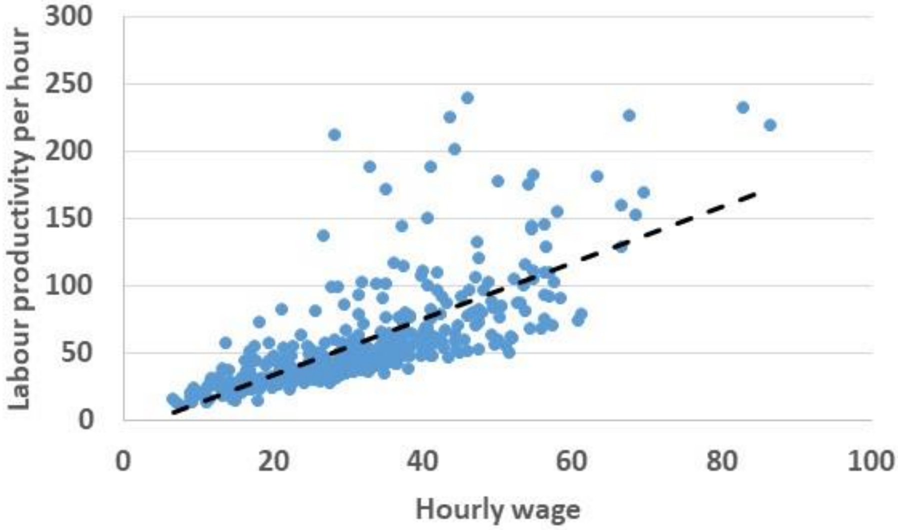
At the industry level, our estimation sample is an unbalanced panel of 22 countries and 23 market sectors for the period 1995–2019. Thirteen of these countries are common to the country level estimation sample. Table A provides more information on the industry level estimation sample.

Table A

Country name	Country code	Nbr of sectors	Period
Austria	AT	23	1995-2018
Belgium	BE	21	1995-2019
Czech Republic	CZ	23	1995-2019
Denmark	DK	23	1995-2019
Estonia	EE	7	2000-2018
Finland	FI	23	1995-2019
France	FR	23	1995-2018
Greece	EL	19	1995-2018
Germany	DE	23	1995-2018
Hungary	HU	7	2010-2018
Italy	IT	22	1995-2017
Japan	JP	21	1995-2018
Lithuania	LT	8	2000-2017
Netherlands	NL	23	1995-2018
Poland	PL	8	2000-2018
Romania	RO	7	2000-2017
Slovakia	SK	16	2000-2017
Slovenia	SI	8	2000-2018
Spain	ES	18	2000-2017
Sweden	SE	19	1995-2017
United Kingdom	UK	22	2000-2017
United States	US	18	1997-2019

The main estimations on the industry level database use the wage per hour rather than the productivity level, as the workers' choices between leisure and consumption of goods depend on their wages. The information on the wages is not available for the very long period of the country level database. However, Figure A shows the strong relation between productivity and wages, with a linear correlation coefficient of 0.70 between their country*industry averages.

Figure A: Wage per hour and labor productivity country*industry averages, period 1995–2019



Each point represents a country-industry specific average over the 1995–2019 period

Appendix B: Sensitivity analysis of the impact of productivity or wages on hours worked

In this Appendix, we investigate how the estimated impact of productivity or wages on hours worked depends on the use of the IV estimator. We find qualitatively similar results for the IV and OLS estimates, but with higher estimated values for the productivity growth and interaction coefficients with the former.

Table B1
Replication of Table 1 but using the IV estimation method

	(1)	(2)	(3)	(4)	(5)	(6)
Period	1963-2006	1950-2019	1950-2019	1950-2019	1950-2019	1950-2018
Country Sample	15	15	21	21	21	15
$\log(LP)$				-0.137***	-0.230***	-0.361***
				(0.011)	(0.034)	(0.077)
$\Delta\log(LP)$	2.240***	2.286***	1.994***		2.032***	1.661***
	(0.244)	(0.217)	(0.190)		(0.555)	(0.513)
$\Delta\log(LP)*\log(LP)$					4.519***	5.474***
					(1.350)	(1.246)
Labor Income Tax Rate						-0.114*
						(0.060)
Employment Rate						-0.652***
						(0.166)
R^2	0.903	0.890	0.888	0.924	0.893	0.859
Observations	615	1005	1407	1407	1407	980
IV test, p-values :						
Sargan test	0.00	0.00	0.00	0.02	0.59	0.43
DWH test	0.00	0.00	0.00	0.00	0.02	0.00

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.
Country and year fixed effects are included in the estimated specifications.
Instruments: education and capital stock, in levels and first difference.
All the weak instrument tests are significant at least at the 5 percent level.

Table B2
Replication of Table 2 but using the OLS estimator

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Period	1891-1913	1919-1938	1950-1994	1995-2019	1950-2019	1891-2019	1995-2019
Sample	Country level						Industry level
$\log(LP)$	-0.089***	-0.158***	-0.147***	-0.172***	-0.141***	-0.114***	-0.0591***
	(0.013)	(0.027)	(0.011)	(0.031)	(0.010)	(0.010)	[0.00452]
$\Delta\log(LP)$	-0.281***	-0.213***	0.404***	-0.194	0.392***	-0.120	-0.102***
	(0.068)	(0.073)	(0.134)	(0.119)	(0.121)	(0.073)	[0.00780]
$\Delta\log(LP) * \log(LP)$	-0.453***	-1.055***	1.650***	-0.982***	1.079***	0.122	0.000810
	(0.064)	(0.189)	(0.210)	(0.186)	(0.155)	(0.082)	[0.0101]
R^2	0.990	0.924	0.924	0.988	0.954	0.983	0.969
Observations	462	399	924	504	1449	2310	8 116

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.
Country and year fixed effects are included. For columns (5) and (6), 2 or 4 sets of country fixed effects are introduced, respectively.
Instruments: education and capital stock, in levels and first difference.
All the weak instrument tests are significant at the 1 percent level.

Appendix C: Sensitivity analysis of the impact of hours worked on productivity or wages

In this Appendix, we investigate the robustness of the estimated impact of hours worked on wages depending of the use of OLS or IV estimator. We find again that estimation results are similar but higher when using the IV estimator.

Table C
Table 3 replication using the OLS estimator
Dependent variable: log(*LP*)
Sample: country level

	(1)	(2)	(3)	(4)	(5)	(6)
Period	1891-1913	1919-1938	1950-1994	1995-2019	1950-2019	1891-2019
$\log(H)$	-0.425	-0.720***	-0.624***	-0.384***	-0.557***	-0.544***
	(0.273)	(0.130)	(0.077)	(0.095)	(0.074)	(0.068)
$\log(KI)$	0.220***	0.225***	0.636***	0.233***	0.618***	0.569***
	(0.034)	(0.049)	(0.013)	(0.025)	(0.012)	(0.013)
R^2	0.993	0.977	0.985	0.995	0.991	0.997
Observations	462	399	924	504	1449	2310

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.
Country and year fixed effects are included.
For columns (5) and (6), two or four sets of country fixed effects are introduced, respectively.