Adaptive Importance Sampling for DSGE Models

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Motivation

- **Dynamic Stochastic General Equilibrium** (DSGE) models have been widely adopted to study the business cycle, policy analysis and forecasting.
- Over time, DSGE models have increased their level of complexity. **This implies that their estimation has become a more challenging task.**
- Starting with Schorfheide (2000) and Otrok (2001), **MCMC algorithms, and more specifically the RWMH, have been the cornerstones for DSGE estimation**.
- Recently Herbst and Schorfheide (2014) and Cai et al. (2020) proposed the application of the **Sequential Monte Carlo (SMC) algorithm for DSGE estimation**.

Our Contributions

- We propose a new methodology that is based on the Mixture of (Student's) *t* by Importance Sampling weighted Expectation-Maximization (MitISEM).
- Our adaptive scheme provides large benefits.
  - The algorithm is "**embarrassingly parallelizable**" on multiple processors or graphics processing units. **It does not require the parameters to be tuned for the user**. It can easily handle the **asymmetry, non-normality and multi-modality** of the posteriors. The adaptive EM step improves and speeds up **posterior convergence** in the case of **parameter identification**.
- We apply the MitISEM methodology to a two-country DSGE model that aims to analyse the effects of government spending shocks on the economy.
- Our DSGE model presents new Keynesian features and considers two types of government expenditures, namely, productive and unproductive spending.

MitISEM in a nutshell

(1) **Initialization**: Simulate draws $\boldsymbol{\theta}^1, \ldots, \boldsymbol{\theta}^N$ from a 'naive' candidate distribution.

(2) **Adaptation**: Estimate the target distribution's mean and covariance matrix using IS with the draws $\boldsymbol{\theta}^1, \ldots, \boldsymbol{\theta}^N$ from g_{naive}. Use these estimates as the mode and scale matrix of Student-t density $g_{adaptive}$. Draw a sample $\boldsymbol{\theta}^1, \ldots, \boldsymbol{\theta}^N$ from this adaptive Student-t distribution with density $g_0 = g_{adaptive}$, and compute the IS weights for this sample.

(3) Apply the **IS-weighted EM algorithm** given the latest IS weights and the drawn sample of step (2). The output consists of the new candidate density g with optimized ζ, the set of $\mu_h, \Sigma_h, \nu_h, \eta_h$ $(h = 1, \ldots, H)$. Draw a new sample $\boldsymbol{\theta}^1, \ldots, \boldsymbol{\theta}^N$ from the distribution that corresponds with this proposal density and compute corresponding IS weights.

MitISEM in a nutshell (cont.)

(4) **Iterate on the number of mixture components.** Given the current mixture of H components, take a percentage (%) of the sample $\boldsymbol{\theta}^{(1)}_{h,Adap}, \ldots, \boldsymbol{\theta}^{(N)}_{h,Adap}$ that corresponds to the highest IS weights. Construct a new mode μ_{H+1} and scale matrix Σ_{H+1} with these draws and IS weights, which are the starting values for the additional component in the mixture candidate density. This step ensures that the new component covers a region of the parameter space in which the previous candidate mixture had a relatively low probability mass.

(5) **Assess convergence of the candidate density's quality by inspecting the IS weights** and return to step (3) unless the algorithm has converged.

Estimate the model parameters with the IS using the constructed density.

Summary of results

1. **Simulation results** show how the MitISEM achieves identification of the model parameters and how it can estimate complex features, such as parameter bimodality.
2. We use the MitISEM to estimate **two workhorse DSGE models**: the small **new Keynesian (NK) model** and the **Smets and Wouters (SW) model**.
   - We compare the **estimates of the NK and SW models** using MitISEM with those of the standard MCMC: differences are negligible but the MitISEM presents an enormous advantage in terms of computing time.
3. We estimate a **new Keynesian DSGE model with a two-country framework** that is based on 164 equations with 86 parameters to estimate.

Summary of results (cont.)

- **Estimated results for the open-economy model.**
- In the presence of **nominal rigidities**, an increase in **productive spending generates a crowding-in effect on domestic private consumption**.
- An increase in **unproductive government spending induces a fall in domestic private consumption** even in the presence of nominal rigidities.
- Irrespective of the type of government expenditure, an increase in **public spending** for the domestic economy induces **an exchange rate appreciation** and an improvement in the **trade balance**.
- **Output multipliers** for the domestic economy are **larger in the presence of nominal rigidities**.
- **Government spending shocks have different effects** on output and consumption multipliers **depending on the degree of trade openness of the economy**.

Thank you!