Asset Pricing with Option-implied Consumption Growth

Jinji Hao
Victoria University of Wellington

AEA 2022, Virtual
January 1, 2022

Motivation

- Asset pricing puzzles
  - equity premium puzzle (Mehra and Prescott, 1985)
  - risk-free rate puzzle (Weil, 1989)
- The consumption-based asset pricing literature has been
  - in search of more appropriate measures or models of consumption growth
  - to explain the puzzles with reasonable preference parameters

Motivation

- For example
  - more appropriate measures of consumption growth
    - the luxury goods consumption (Aït-Sahalia, Parker, and Yogo, 2004), the ultimate consumption (Parker and Julliard, 2005), the garbage (Savov, 2011), the unfiltered NIPA consumption (Kroencke, 2017), or the fourth quarter year-on-year growth (Jagannathan and Wang, 2007), among others
  - more appropriate models of consumption growth
    - unprecedented rare disasters (e.g., Rietz, 1988) or hard-to-detect low-frequency consumption volatility (e.g., Bansal and Yaron, 2004).
  - subjective distributions of consumption growth
    - based on learning models (e.g., Weitzman, 2007) or data in other countries (Barro, 2006)

Contributions

- Strong restrictions on the specification of consumption growth risk by option prices
  - The model gives the data of option prices and stock returns enough flexibility in picking either a combination of low consumption risk and high risk aversion or a combination of high consumption risk and low risk aversion.
  - The data indeed picks the latter with a risk aversion of 4.72
  - The market index option prices largely reveal the market subjective beliefs about the distribution of consumption growth
- The asset pricing with this option-implied consumption growth explains the data well
  - equity premium puzzle/return predictability; risk-free rate puzzle; conditional volatility/predictability; puzzles in the options market

The Model

Assumption

There is a representative agent with an Epstein-Zin preference over consumption flows. The time preference is ρ, the risk aversion is γ, and the elasticity of intertemporal substitution is ψ. Define $\vartheta \equiv (1-\gamma)/(1-1/\psi)$.

Assumption

Although updating their beliefs over time, when making decisions at each time t, the investors always approximate their subjective beliefs about the future consumption growth rate as an i.i.d. process. Define the CGF of the consumption growth rate under the subjective beliefs at time t as $c_t(\theta) \equiv \log \mathbb{E}_t[e^{\theta G_{t+1}}]$.

The Model

Result

Martin (2013a) implies that the risk-free rate, the dividend-price ratio, and the risk premium of the λ-asset at time t are given by

$$r_{f,t} = \rho - c_t(-\gamma) + c_t(1-\gamma)(1-1/\vartheta), \tag{1}$$

$$(d/p)_t = \rho - c_t(\lambda-\gamma) + c_t(1-\gamma)(1-1/\vartheta), \tag{2}$$

$$rp_t = c_t(\lambda) + c_t(-\gamma) - c_t(\lambda-\gamma). \tag{3}$$

The Model

- Define the risk-neutral CGF of the log return on the asset price as

$$m_t(\eta) \equiv \log \mathbb{E}_t^Q[e^{\eta z_{t+1}}] = \log \mathbb{E}_t^Q\left[\left(\frac{P_{t+1}}{P_t}\right)^{\eta}\right]. \tag{4}$$

- It can be replicated by European put and call prices

Proposition

The CGF of consumption growth under the investors' subjective beliefs is connected to the risk-neutral CGF of log asset price changes via

$$c_t(\theta) = m_t\left(\frac{\theta+\gamma}{\lambda}\right) - m_t\left(\frac{\gamma}{\lambda}\right). \tag{5}$$

The Model

Proposition

Within any consumption-based asset pricing models with an Epstein-Zin preference and with the subjective beliefs about consumption growth being approximated as i.i.d., the risk-free rate, the dividend-price ratio, and the equity premium predicted by the option-implied consumption growth at each time t are, respectively,

$$r_{f,t} = \rho + \frac{1}{\vartheta} m_t\left(\frac{\gamma}{\lambda}\right) + \left(1 - \frac{1}{\vartheta}\right) m_t\left(\frac{1}{\lambda}\right), \tag{6}$$

$$(d/p)_t = \rho - m_t(1) + \frac{1}{\vartheta} m_t\left(\frac{\gamma}{\lambda}\right) + \left(1 - \frac{1}{\vartheta}\right) m_t\left(\frac{1}{\lambda}\right), \tag{7}$$

$$rp_t = m_t\left(1 + \frac{\gamma}{\lambda}\right) - m_t\left(\frac{\gamma}{\lambda}\right) - m_t(1). \tag{8}$$

The Model

Proposition

The option-implied conditional variance of the λ-asset is

$$Var_t\left(\log \frac{P_{t+1}}{P_t}\right) = \lambda^2 c_t''(0) = m_t''\left(\frac{\gamma}{\lambda}\right). \tag{9}$$

Asset Pricing: Equity Premium Puzzle

- Equity Premium Puzzle
  - a calibrated risk aversion of 4.72
- The option-implied equity premium in real time predicts the future market returns better than the historical mean forecast as well as the popular predictors (Goyal and Welch, 2008; Campbell and Thompson, 2008)

Asset Pricing: Equity Premium Puzzle

Table: Summary Statistics of Option-implied Equity Premium: Annual Horizon

γ / λ	mean	median	1st pct.	99th pct.	s.d.
0.20	0.97	0.85	0.36	3.18	0.53
0.60	2.74	2.42	1.05	8.83	1.48
1.00 (Martin's LB)	4.31	3.85	1.69	13.75	2.29
1.40	5.73	5.14	2.29	18.07	3.01
1.70	6.70	6.05	2.71	20.91	3.49
1.75	6.86	6.18	2.78	21.36	3.57
1.80	7.02	6.32	2.84	21.81	3.65
1.85	7.17	6.46	2.91	22.25	3.72
1.90	7.32	6.59	2.97	22.68	3.79
1.95	7.47	6.73	3.04	23.12	3.87
2.00	7.62	6.87	3.10	23.55	3.94
2.05	7.76	7.00	3.17	23.97	4.01
2.10	7.91	7.13	3.23	24.39	4.08
2.15	8.05	7.27	3.30	24.80	4.15
2.60	9.29	8.42	3.84	28.64	4.75
2.80	9.80	8.89	4.07	30.11	4.99
3.00	10.30	9.34	4.30	31.53	5.23
4.00	12.57	11.37	5.30	38.11	6.33
5.00	14.55	13.13	6.13	44.18	7.32
Data	8.10	11.52	-40.50	44.59	17.81
Rolling Mean forecast	6.24	6.24	5.80	6.60	0.18

Asset Pricing: Equity Premium Puzzle

Table: Out-of-Sample R^2, 1996–2017

	Full Sample 1996–2017					Excluding 2008:01–2009:06				
γ/λ	$h=1$	$h=2$	$h=3$	$h=6$	$h=12$	$h=1$	$h=2$	$h=3$	$h=6$	$h=12$
0.20	-1.40	-2.60	-3.96	-6.83	-13.04	-2.62	-5.47	-8.64	-16.03	-25.89
0.60	-0.57	-0.98	-1.64	-1.76	-5.21	-0.97	-2.25	-3.55	-6.41	-12.69
1.00	-0.12	-0.11	-0.42	1.26	-0.52	0.34	0.21	0.27	0.56	-3.82
1.40	0.00	0.15	-0.09	2.75	1.93	1.36	2.03	3.07	5.49	1.93
1.70	-0.10	0.00	-0.31	3.08	2.65	1.95	3.02	4.59	8.12	4.68
1.75	-0.13	-0.05	-0.38	3.08	2.70	2.04	3.15	4.80	8.48	5.03
1.80	-0.16	-0.11	-0.47	3.06	2.73	2.12	3.28	5.00	8.82	5.35
1.85	-0.20	-0.17	-0.56	3.03	2.73	2.20	3.41	5.19	9.14	5.65
1.90	-0.24	-0.24	-0.65	2.99	2.72	2.27	3.52	5.36	9.44	5.92
1.95	-0.29	-0.32	-0.76	2.93	2.69	2.34	3.63	5.53	9.72	6.16
2.00	-0.34	-0.40	-0.88	2.86	2.64	2.41	3.73	5.68	9.98	6.38
2.05	-0.39	-0.49	-1.00	2.77	2.58	2.48	3.82	5.83	10.23	6.57
2.10	-0.45	-0.59	-1.14	2.68	2.50	2.54	3.91	5.96	10.46	6.74
2.15	-0.51	-0.69	-1.28	2.57	2.41	2.59	3.99	6.09	10.67	6.89
2.60	-1.19	-1.85	-2.87	1.09	0.94	2.97	4.43	6.79	11.85	7.31
2.80	-1.58	-2.50	-3.74	0.18	-0.02	3.06	4.47	6.88	12.02	7.05
3.00	-2.01	-3.22	-4.71	-0.87	-1.13	3.11	4.43	6.84	11.98	6.55
4.00	-4.77	-7.76	-10.75	-7.78	-8.58	2.74	3.09	5.08	9.35	1.21
5.00	-8.38	-13.56	-18.32	-16.84	-18.24	1.52	0.23	1.20	3.56	-7.57

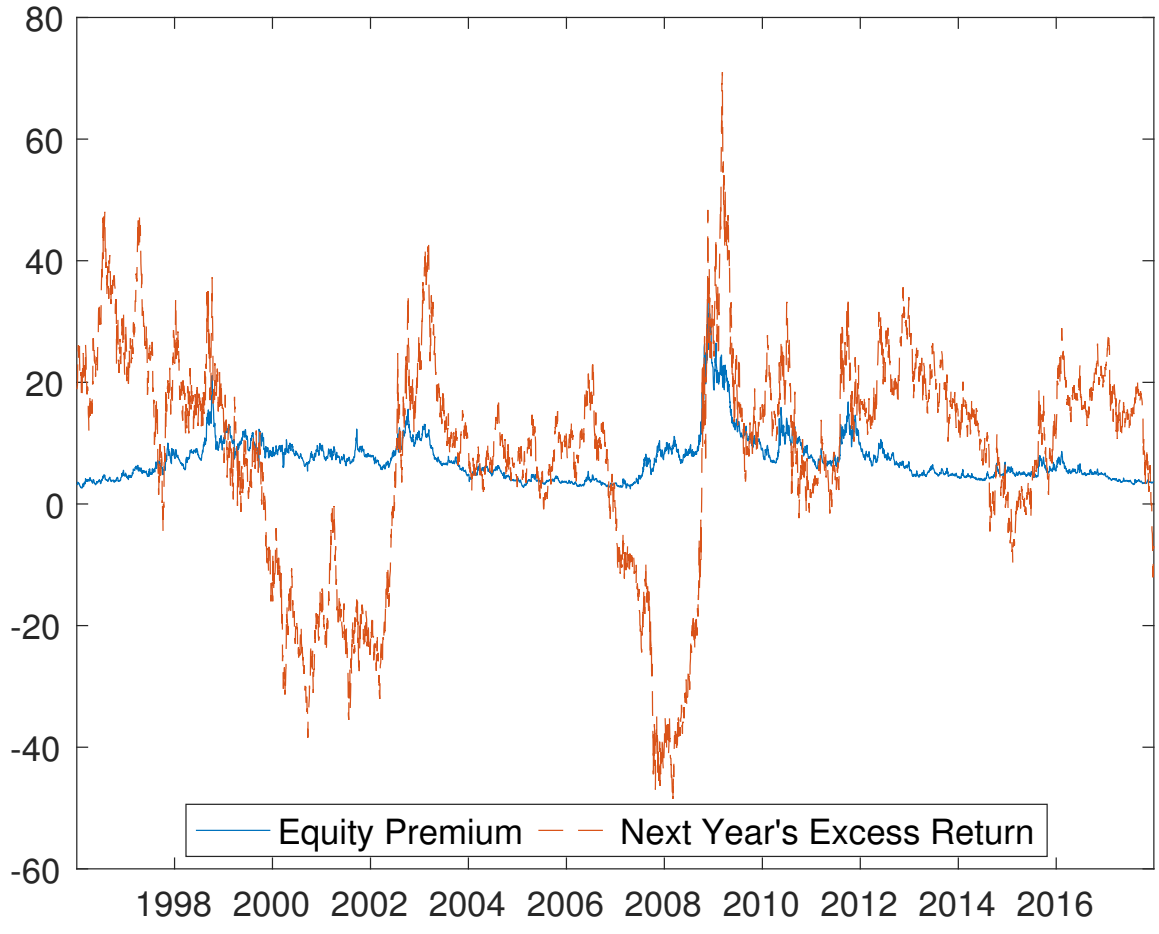
Asset Pricing: Equity Premium Puzzle

$$r_{t,t+h} = RP_{t,h}(\gamma/\lambda) + \epsilon_{t,t+h}$$

Table: Estimates of the Delevered Risk Aversion

Methods	$h = 1$	$h = 2$	$h = 3$	$h = 6$	$h = 12$	Average
MM	1.99	1.89	1.88	1.95	2.16	1.97
	[0.76]	[0.77]	[0.80]	[0.91]	[1.17]	
LS	1.36	1.39	1.37	1.72	1.84	1.54
	[0.82]	[0.81]	[0.88]	[0.84]	[0.94]	
WLS	2.01	1.82	1.76	1.74	1.94	1.85
	[0.78]	[0.78]	[0.82]	[0.94]	[1.23]	

Asset Pricing: Equity Premium Puzzle



(a) Equity Premium and Next Year's Excess Return (%)

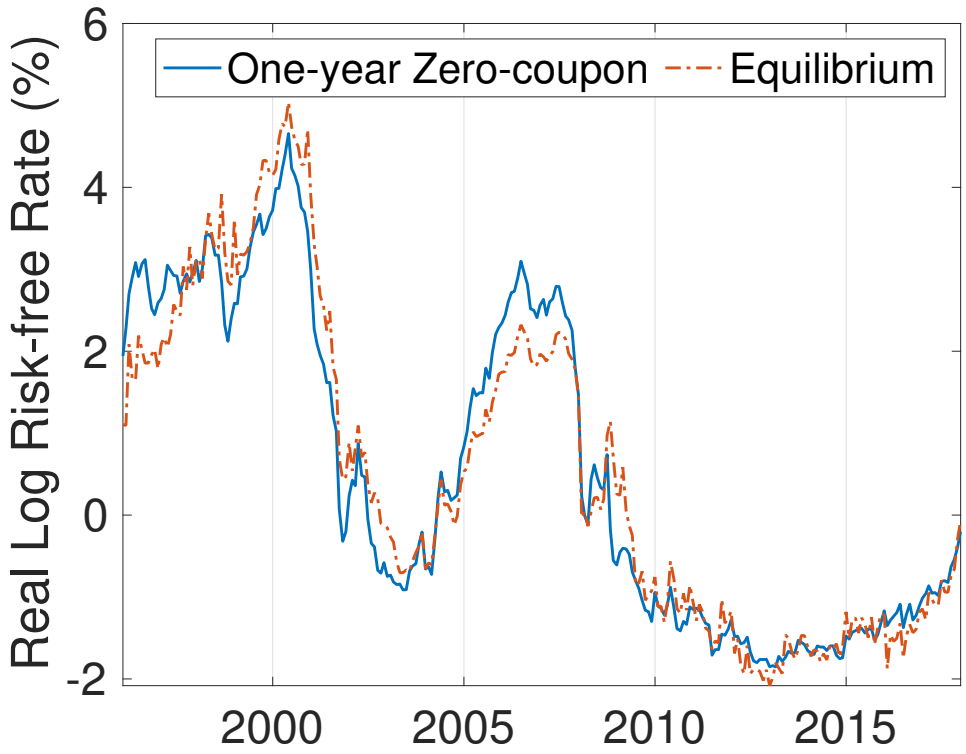
Asset Pricing: Return Predictability

Table: Out-of-Sample R^2 of Return Predictors, 1996–2017

Predictors	$h = 1$	$h = 2$	$h = 3$	$h = 6$	$h = 12$
DP	-1.07	-3.06	-4.85	-6.91	-21.68
DY	-1.98	-4.08	-4.87	-7.61	-23.69
EP	-1.30	-3.59	-6.06	-10.86	-19.89
DE	-0.48	-0.92	-1.36	-3.06	-3.25
RVOL	0.08	0.30	0.49	0.74	0.80
BM	-1.98	-5.85	-8.16	-7.72	-16.73
NTIS	-3.16	-6.61	-10.44	-19.36	-34.34
TBL	0.00	0.00	0.01	0.93	2.83
LTY	0.36	0.58	0.75	1.36	1.59
LTR	-0.29	-0.60	-1.03	-1.47	-1.46
TMS	-0.95	-1.71	-2.45	-2.21	1.10
DFY	-0.75	-1.74	-1.71	0.20	1.70
DFR	-0.10	-0.93	-0.32	-0.37	-0.86
INFL	-0.49	-0.63	0.29	1.61	1.97
RP6	-0.09	0.47	0.63	3.58	2.69
RP12	-0.08	0.32	0.49	3.28	2.89

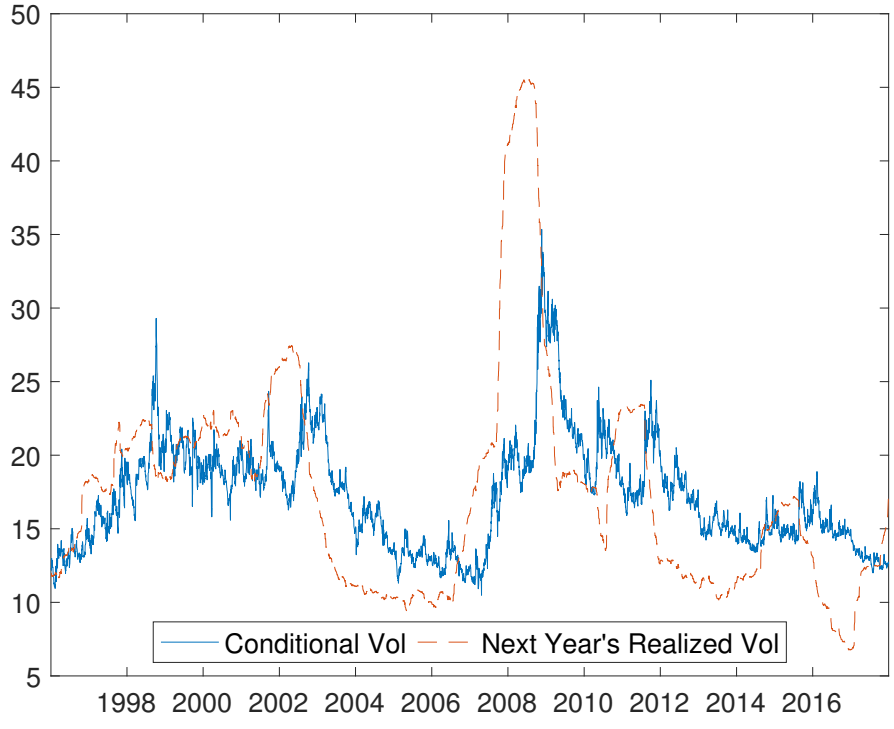
Asset Pricing: Risk-free Rate Puzzle

- The equilibrium risk-free rate matches the sample moments
- The equilibrium risk-free rate is highly correlated with the data



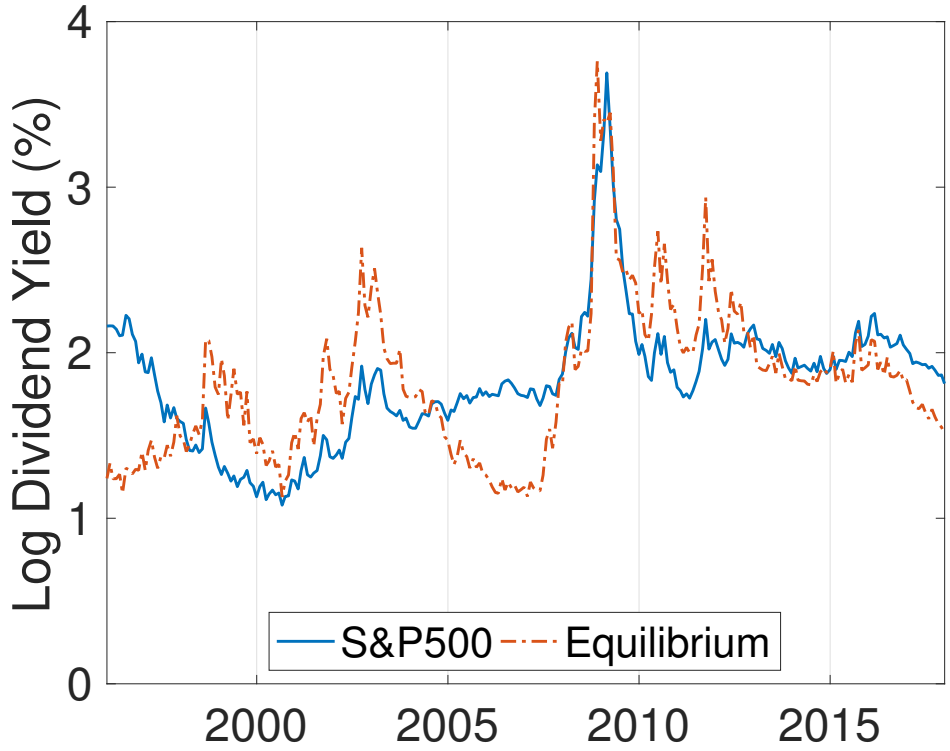
Asset Pricing: Conditional Volatility

- The model-implied market conditional volatility in real time matches the sample volatility on average
- Predicts the future volatility much better than the lagged realized volatility or the option-implied risk-neutral volatility



Asset Pricing: Dividend-price Ratio

- The model captures the dynamics of dividend-price ratio



Asset Pricing: Puzzles in Options Market

- Asset pricing puzzles in the options market (e.g., the implied volatility skew)
  - option prices are taken as inputs

Option-implied Consumption Growth

Table: Summary Statistics of Growth Rates

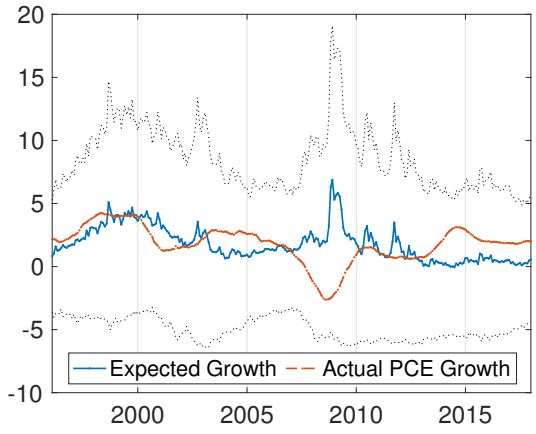
Panel A: Average Moments of Option-implied Consumption Growth

λ	mean	s.d.	skew.	kurt.
1	5.43	17.25	-1.56	7.52
2	2.37	8.62	-1.56	7.52
2.55	1.80	6.76	-1.56	7.52
3	1.50	5.75	-1.56	7.52
4	1.10	4.31	-1.56	7.52
5	0.86	3.45	-1.56	7.52

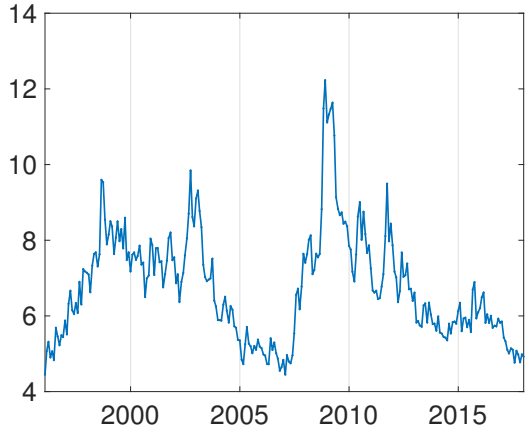
Panel B: Per Capita Real Macroeconomic Variables

	Sample: 1889–2017				Sample: 1996–2017			
Variable	mean	s.d.	skew.	kurt.	mean	s.d.	skew.	kurt.
Consumption	1.97	3.41	-0.34	4.31	1.80	1.47	-0.86	4.26
GNP	1.95	4.90	-0.13	4.83	1.54	1.56	-1.59	6.26
Dividend	0.21	11.37	-0.66	7.09	2.72	8.63	-1.86	7.61

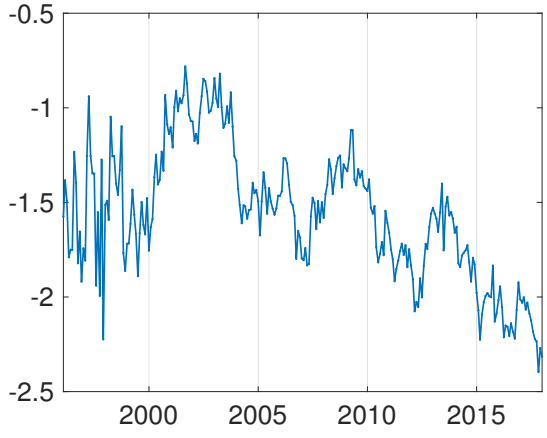
Moments of Option-implied Distributions of Annual Consumption Growth



(b) Mean (%)



(c) Standard Deviation (%)



(d) Skewness

Conclusion

- The option prices reveal the expected consumption growth
  - Given enough flexibility, the stock returns and options data pick a combination of a high consumption risk and a low risk aversion
- The asset pricing with the option-implied consumption growth solves many asset pricing puzzles