Firm Revenue Elasticity and Business Cycle Behaviour

Daisoon Kim[a], and Anthony Savagar[b]

[a]North Carolina State University, Email: dkim29@ncsu.edu

[b]University of Kent, Email: A.Savagar@kent.ac.uk

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Motivation

Aggregate measures of price markups are informative for various macro topics.

- Recently, markups rise: De Loecker et al. (2020)

However, recent literature notes the limitations of using revenue elasticities to proxy output elasticities when estimating markups.

- Syverson (2019); Bond et al. (2020)
- revenue elasticities may not unlock markups.

What can revenue elasticities tell us about macroeconomic behaviour?

Intuitions

Revenue elasticity is easy to measure and important in firm behaviour

$$\text{revenue elasticity} = \frac{\text{output elasticity}}{\text{markup}} = \frac{\text{variable costs}}{\text{revneue}}$$

- With the assumption (cost-minimization), the revenue elasticity is easily measured by the ratio of variable costs to sales
- Or we can measure it directly from revenue function estimations.

Why important?

- Supply side: output elasticity $\Rightarrow$ profits $\downarrow$ (e.g., Hopenhayn 1992)
- Demand side: markups $\Rightarrow$ profits $\uparrow$ (e.g., Melitz 2003)
- Their ratio determines a firm's profitability

This paper is

We develop theory to relate output fluctuations to demand and supply shocks, conditional on revenue elasticity and markups. Using U.S. firms data, we test the relationship.

Our simple framework predicts

- Higher revenue elasticity firms generate greater business cycle amplification in reacting to supply shocks.

We document trends of revenue elasticities

- Decreasing revenue elasticity on average but Increasing cross-sectional dispersion
- these facts are arose from small revenue elasticity firms

Our local projections show

- Higher revenue elasticity firms' sales are more procyclical
- and more sensitive to firm- and aggregate-level productivity changes

Related literature

Macroeconomics with markups and returns to scale

- Hall (1986); Hornstein (1993); Rotemberg and Woodford (1993); Devereux et al. (1996); Basu and Fernald (2001)
- Atkeson and Kehoe (2005); Hopenhayn (2014)

Returns to scale, markups, production function estimation

- Hall (1986); De Loecker et al. (2020); Traina (2018)
- Bond et al. (2020); Syverson (2019); Basu (2019)

Systemic differences in firm behaviour to the business cycle.

- Covas and Den Haan (2011); Begenau and Salomao (2018); Crouzet and Mehrotra (2020); Burstein et al. (2020)

Framework with Revenue elasticity (1/2)

Environment

- Production function

$$Q = \mathcal{F}(AX)$$

- Demand and inverse demand functions

$$Q = \mathcal{D}(P) \quad \text{and} \quad P = \mathcal{P}(Q),$$

The output and markups

- Output elasticity

$$\gamma = \frac{\partial \mathcal{F}}{\partial X}\frac{X}{Q} = \mathcal{F}'(AX)\frac{AX}{Q},$$

- Markups

$$\mu = \left(-\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q}\right)\left(-\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q} - 1\right)^{-1}$$

  - from profit maximization of monopolistically competitive firms

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Notation and remarks:

Q and P quantity and price

X and A total factor and productivity

Framework with Revenue elasticity (2/2)

Revenue function

$$PQ = \mathcal{P}(Q)Q = \mathcal{P}(\mathcal{F}(AX))\mathcal{F}(AX) = \mathcal{R}(AX).$$

Revenue elasicity

$$\zeta = \frac{\partial \mathcal{R}}{\partial X}\frac{X}{PQ} = \left[\frac{\partial \mathcal{P}}{\partial Q}\frac{\partial \mathcal{F}}{\partial X} + P\frac{\partial \mathcal{F}}{\partial X}\right]\frac{X}{PQ} = \left[-\left(\frac{\partial \mathcal{D}}{\partial P}\frac{P}{Q}\right)^{-1} + 1\right]\frac{\partial \mathcal{F}}{\partial X}\frac{X}{Q} = \frac{\gamma}{\mu}$$

- Cost-minimization yields

$$\zeta = \frac{WX}{PQ}$$

Notation and remarks:

γ	output elasticity
μ	markups
Q and P	quantity and price
X and W	total factor and price

Propagations with Revenue elasticity (1/2)

Log-lienarization of demand and variable costs

$$\Delta \ln Q \approx = -\left(\frac{\mu}{\mu - 1}\right)\Delta \ln P + \Delta \ln \xi$$

$$\Delta \ln WX \approx= \Delta \ln \frac{W}{A} + \frac{1}{\gamma}\Delta \ln Q$$

Log-lienarization of markups and marginal costs

$$\Delta \ln P = \Delta \ln \mu + \Delta \ln \mathrm{MC}$$

$$\Delta \ln \mathrm{MC} = \Delta \ln WX - \Delta \ln Q - \Delta \ln \gamma$$

Notation and remarks:

ξ demand/preference shock

Propagations with Revenue elasticity (2/2)

Higher revenue elasticity firms generate greater business cycle amplification in reacting to supply shocks.

$$\Delta \ln PQ \approx \frac{\zeta}{1-\zeta}(\Delta \ln A - \Delta \ln W + \Delta \ln \zeta) + \frac{1-1/\mu}{1-\zeta}\Delta \ln \xi,$$

Notation and remarks:

ζ	revenue elasticity
γ	output elasticity
μ	markups
ΔPQ	revenue change
Δ	factor price change
ΔA and $\Delta \xi$	productivity and demand shocks

Data and variables

Compustat Fundamentals Annual: North America

- 22,745 US firms during 1975 (3,726 firms) – 2016 (4,863 firms)

Revenue elasticity: 3-year moving average

- Benchmark: Cost of Goods Sold (COGS) / Sales
- Alternative I: (COGS + capital costs) / sales
- Alternative II: ii) (COGS + SGA) / sales
  - SGA is not variable costs but it would be variable in the long-run

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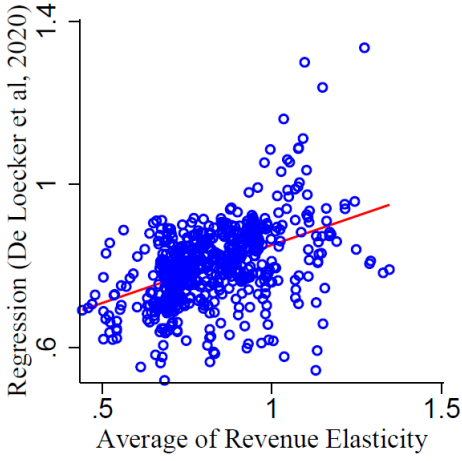
Notation and remarks:

COGS	cost of goods sold
SGA	selling, general, and administrative expenses

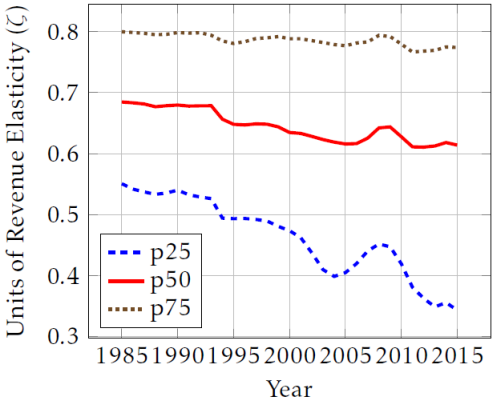
Revenue Elasticity Measurement and Trends (1/4)

Cost-share approach vs Revenue function estimation approach: Two-digit NAICS

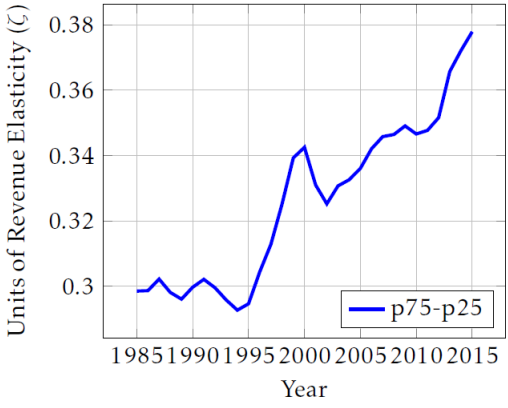


Revenue Elasticity Measurement and Trends (2/4)

Revenue Elasticity Quartile Trends



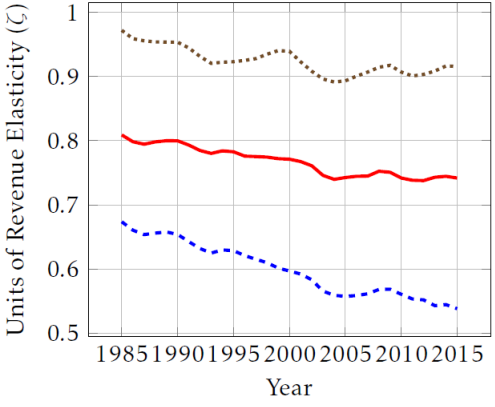
(a) Quartiles



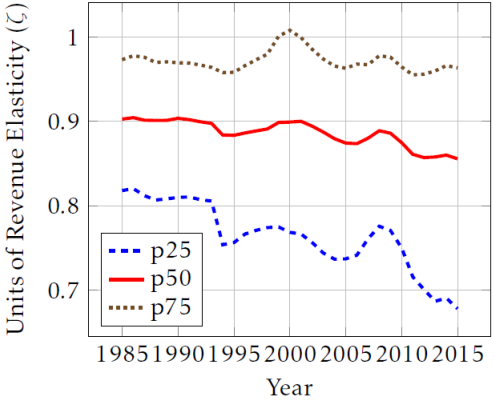
(b) Inter-Quartile Range

Revenue Elasticity Measurement and Trends (3/4)

Alternative Revenue Elasticity Quartile Trends



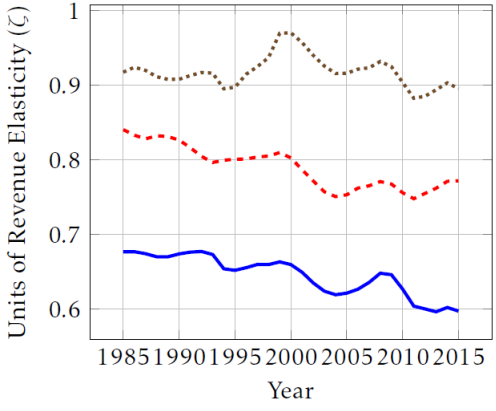
(a) Alternative I



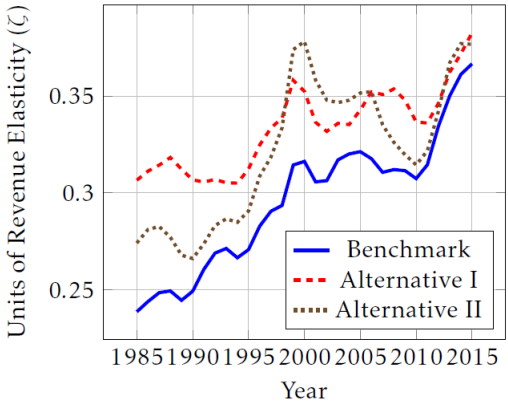
(b) Alternative II

Revenue Elasticity Measurement and Trends 4/4)

Revenue Elasticity Mean and Standard Deviation Trends



(a) Mean



(b) Standard Deviation

Empirical Methodology (1/2)

Quantify the effect of shocks on firm revenues conditional on firm revenue elasticity.

- In order to estimate the dynamics of differential responses across firms, we use local projection estimation following Jorda (2005).

Specification with Continuous Measure of Revenue Elasticity

$$\Delta^h \ln P_{j,t}Q_{j,t} = \beta_0^h \text{shock}_{j,t} + \beta_{1,\zeta}^h(\text{shock}_{j,t} \times \ln \zeta_{j,t}) + (\text{shock}_{j,t} \times \mathbf{traits}_{j,t}^{\intercal})\mathbf{b}_1^h$$
$$+ \beta_2^h \ln \zeta_{j,t} + \mathbf{traits}_{j,t}^{\intercal}\mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h$$

- $\beta_{1,\zeta}^h$: a firm's revenue change following a shock in t relative to a firm with a (log) unit lower revenue elasticity.

Notation and remarks:

- We index a firm with j and $h \geq 1$ represents the forecast horizon.
- The delta operator Δ^h represents the difference between $t+h$ and t, such that $\Delta^h \ln P_{j,t}Q_{j,t} \equiv \ln P_{j,t+h}Q_{j,t+h} - \ln P_{j,t}Q_{j,t}$ for $h = 1, 2, 3, 4$.
- The variable $\text{shock}_{j,t}$' represents a shock.
- The variable $\mathbf{traits}_{j,t}$ is a vector of controls.

Empirical Methodology (2/2)

Specification with Discrete Measure of Revenue Elasticity

$$\Delta^h \ln P_{j,t}Q_{j,t} = \beta_0^h \text{shock}_{j,t} + \beta_{1,UQ}^h(\text{shock}_{j,t} \times UQ_{j,t}) + \beta_{1,LQ}^h(\text{shock}_{j,t} \times LQ_{j,t}) + \beta_{2,UQ}^h UQ_{j,t} + \beta_{2,LQ}^h LQ_{j,t} + \mathbf{traits}_{j,t}\mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h.$$

- The dummy variable $UQ_{j,t}$ is 1 if firm j is in the upper quartile of revenue elasticities.
- The dummy variable $LQ_{j,t}$ is 1 if firm j is in the lower quartile of revenue elasticities.
- $\beta_{1,UQ}^h - \beta_{1,LQ}^h$: the difference in revenue response of high and low revenue elasticity firms to shocks.

Notation and remarks:

- We index a firm with j and $h \geq 1$ represents the forecast horizon.
- The delta operator Δ^h represents the difference between $t+h$ and t, such that $\Delta^h \ln P_{j,t}Q_{j,t} \equiv \ln P_{j,t+h}Q_{j,t+h} - \ln P_{j,t}Q_{j,t}$ for $h = 1, 2, 3, 4$.
- The variable $\text{shock}_{j,t}$' represents a shock.
- The variable $\mathbf{traits}_{j,t}$ is a vector of controls.

Firm-Level Shocks

Firm-level labour productivity

$$\Delta^1 LP_{j,t} \equiv \ln LP_{j,t} - \ln LP_{j,t-1} \quad \approx \Delta PQ - \Delta X$$

- Proxy for $\Delta PQ - \Delta X$: difference between revenue and factor growth rates
- **Note**: The simple difference is systemically biased with non-unit elasticity.

Firm-level labour productivity with correction

$$\Delta PQ \approx \zeta(\Delta A + \Delta X) \quad \Rightarrow \quad \Delta A \approx \Delta PQ - \Delta X + \left(\frac{1}{\zeta} - 1\right) \Delta PQ$$

Notation and remarks:

- labour productivity: $LP_{j,t} = \text{sales}_{j,t}/\text{employees}_{j,t}$

Aggregate-level Shocks

Aggregate-level TFP changes

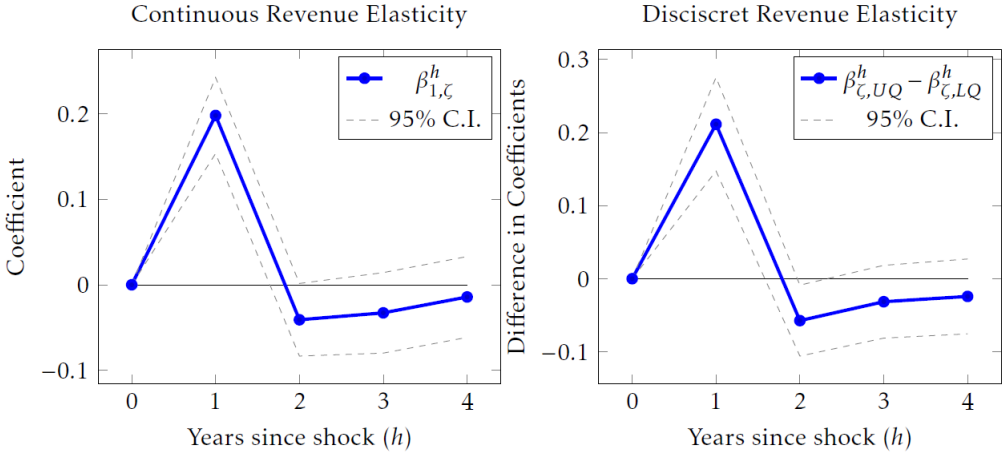
- aggregate total factor productivity growth rates at constant national prices (RTFPNA) from Penn World Table 9.1

Aggregate (real) GDP changes

- a firm's response to the aggregate GDP represents its cyclicality, in other words, cyclical sensitivity to the business cycle.

Cyclical Sensitivity over Revenue Elasiticity (1/4)

Impulse Response Functions to Firm-Level Labor Productivity Shock

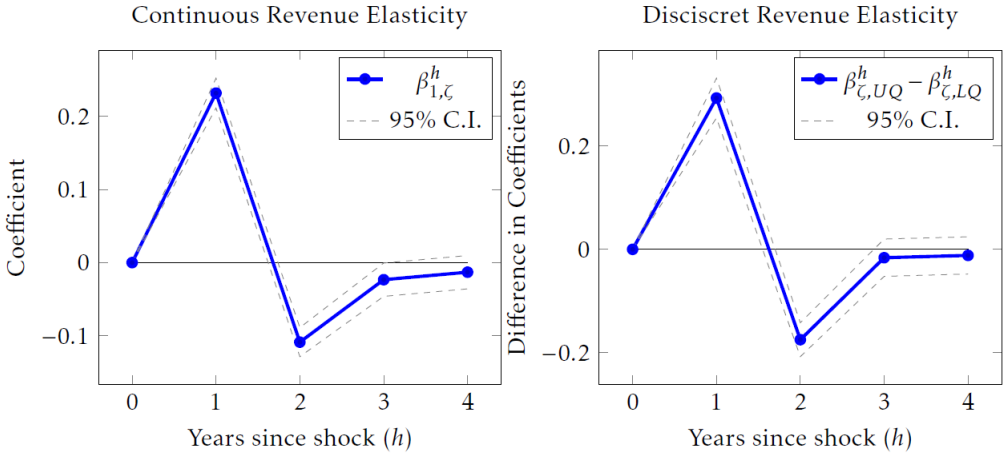


(a) Regression Equation (23)

(b) Regression Equation (24)

Cyclical Sensitivity over Revenue Elasiticity (2/4)

Impulse Response Functions to Corrected Firm-Level Labor Productivity Shock

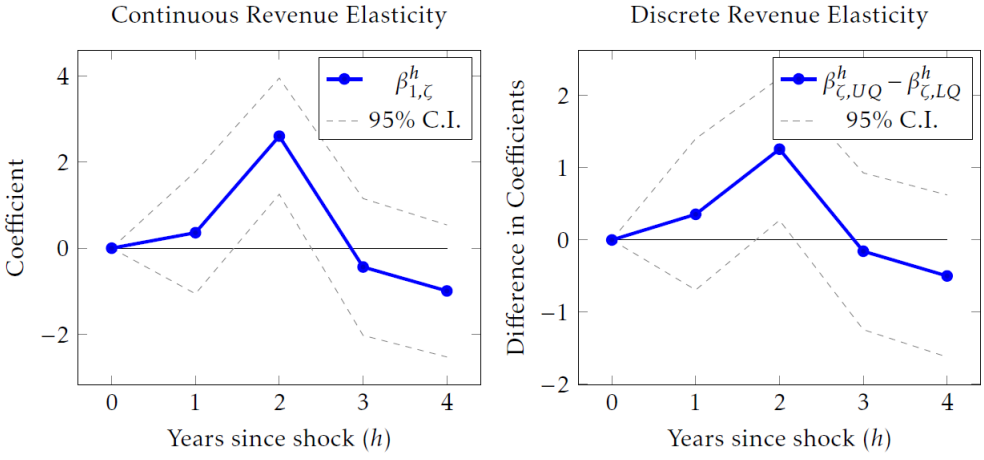


(a) Regression Equation (23)

(b) Regression Equation (24)

Cyclical Sensitivity over Revenue Elasiticity (4/4)

Impulse Response Functions to Corrected Firm-Level Labor Productivity Shock



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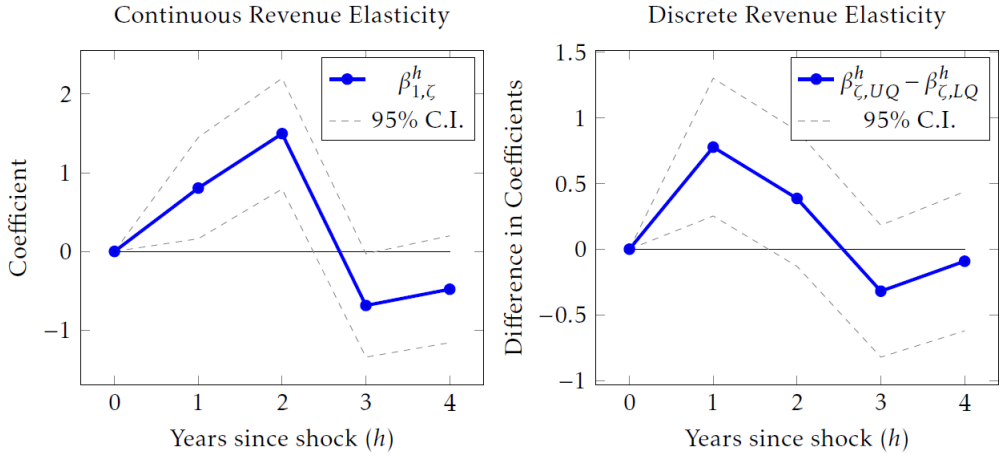

(a) Regression Equation (23)

(b) Regression Equation (24)

Concluding Remarks

We analyse the effect of firm-level revenue elasticities on firm business cycle behaviour.

- We focus on revenue elasticities because they are simple to obtain at the firm level, but are understudied relative to the related concepts of price markups and output elasticities.
- We present empirical results on the behaviour of revenue elasticities of U.S. firms over the last three decades.
- We present theory to show that higher revenue elasticities generate greater business cycle amplification.
- We test this theoretical relationship on U.S. data and find evidence in support of the theory.

Thank you!

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