

# Asset Pricing Under Imperfect Foresight

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## Starting Point

Most of finance theory that we teach our students and encourage them to practice in the field relies on one key assumption:

### Perfect Foresight

Perfect foresight is the ability to predict equilibrium prices in *all* future contingencies.

- Radner (1972) proves existence of **Perfect Foresight Equilibrium** for multi-asset economies with sequential trade
- Kreps (1982) and Duffie and Huang (1985) demonstrate (conditional) price equivalence to Arrow-Debreu equilibrium

## Intuition & Relevance

Consider the following scenario:

- Tomorrow, either the sun shines or it rains
- You may (or may not) know the chances
- Perfect foresight requires you to know the equilibrium prices of, say, ice cream in either case (how?!)

Perfect foresight is of fundamental importance for:

- Corporate finance: absence of arbitrage as in, e.g., Modigliani and Miller (1958)
- Derivatives: option pricing à la Black and Scholes (1973)
- Investments: *any* multi-period investment problem

## Important Distinction

- Note, perfect foresight does **not** imply perfect foresight of the future (i.e., allows for uncertainty)!
- The concept of “perfection” is core to game theory (subgame perfect Nash equilibrium)

## Implications of Perfect Foresight

- There is only *one* source of uncertainty (e.g., sunshine vs. rain)
- Given the state, there is *no* price uncertainty
- Hence, risk premia only depend on fundamental risk (e.g., the weather)

Under perfect foresight, standard theory fails to reconcile the sizable equity premium with historically low consumption risk.

## Perfect Foresight vs. Myopia

### Is Perfect Foresight Realistic?

Clearly, perfect foresight imposes very demanding level of rationality.

We propose a more realistic (?) alternative:

### Myopia

- Do not even try to forecast future prices
- Only take into account what you *do* know:
  - How much of any given asset do you own (without trading) in every contingency
  - What is traded today and at what prices

Remarks:

- Under myopia, everything is as in the standard theory, except that one ignores future (re-)trading opportunities → no need to forecast prices
- Related to *narrow framing* (Barberis et al., 2006), but less extreme
- Under narrow framing, one ignores any holdings in non-traded assets
- Different to “myopia” in dynamic investments under log utility

## Theory

### Theorem: Price Equivalence

Under quadratic utility, myopic prices are exactly the same as if agents had perfect foresight, while their choices (allocations) may be vastly different. Thus, in a **Myopic Equilibrium**, “prices are right,” however, “allocations are wrong.”

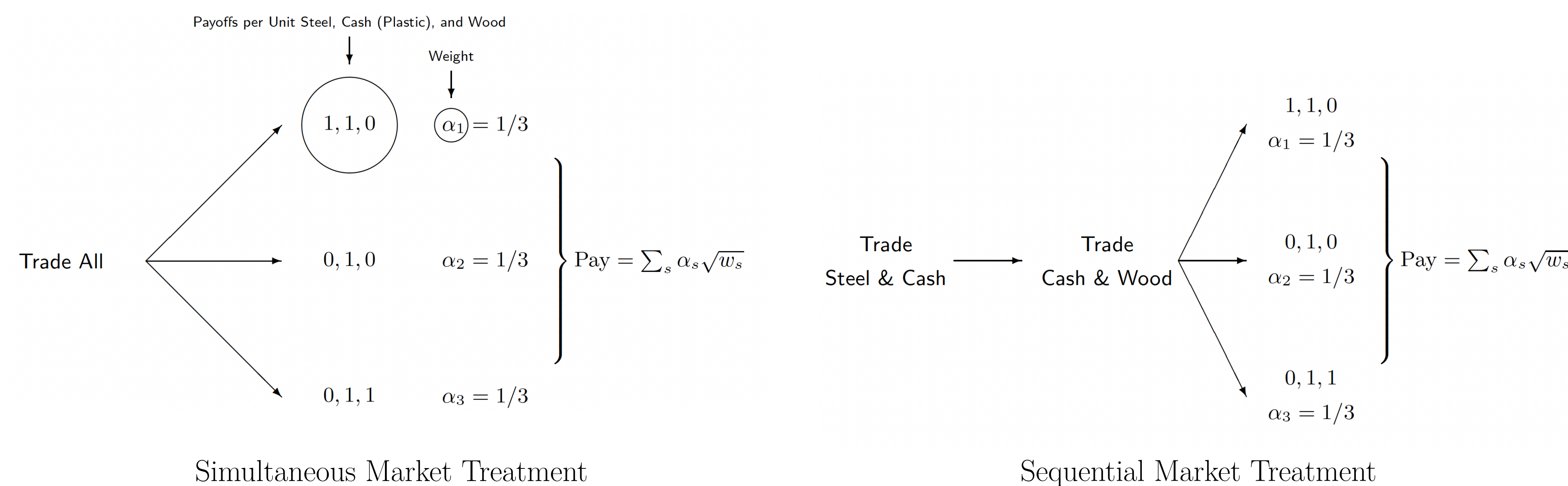
### Intuition

Myopia does not mean that agents are ignoring contingent endowments that they cannot trade away from in the current period. Instead, they do take these endowments into account; only, they assume that these are (permanently) non-tradeable. Therefore, market prices reflect knowledge of the scarcity or abundance of currently non-traded endowments, and as such prices behave as if these endowments had been available for trade.

### Proof

- The proof *assumes* the existence of a Radner equilibrium with *interior* solutions (otherwise only a quasi-equilibrium may exist)
- Two versions, both exhibit intrinsic (price) uncertainty (see Paper Appendix for details, <https://ssrn.com/abstract=3610634>):
  - No extrinsic uncertainty and assets are traded sequentially, or equivalently, extrinsic uncertainty that is not resolved till the end (see experiment)
  - There exists extrinsic uncertainty; uncertainty is resolved gradually as trade happens

## Experiment



Note:  $w_s = \sum_{i \in \{\text{Steel, Cash, Wood}\}} n_i \text{payoff}(i, s)$ , where  $n_i$  is number of asset  $i$  held, and  $\text{payoff}(i, s)$  is payoff of asset  $i$  in state  $s$

**Summary**

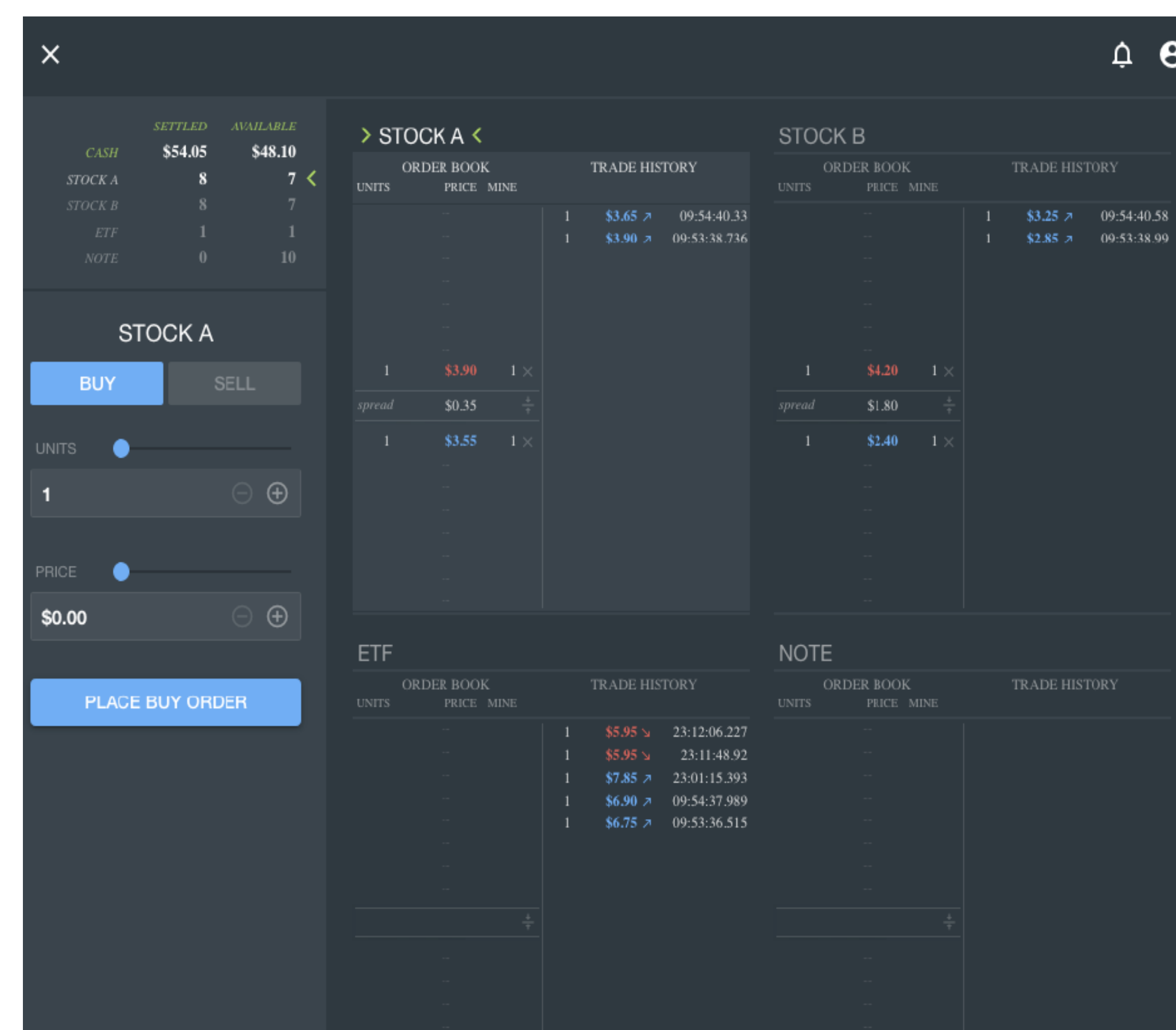
You will trade ‘inputs’ (wood, steel, and plastic) in an online market with other participants. The goal is to collect inputs which will let you produce Widgets. You should try to produce as many Widgets as possible because your earnings will depend on it. You will have access to a spreadsheet which will tell you how many Widgets you can get for a given amount of wood, steel, and plastic.

Plastic is a special input because it can be used to replace wood or steel in the production of Widgets.

Steel can be traded for plastic in the ‘steel market’ and wood can be traded for plastic in the ‘wood market’. You cannot trade steel and wood for one-another directly but must trade through plastic. So, plastic acts as “cash”, and we will often refer to it as cash.

This game will be replicated several times, switching between situations where you can simultaneously trade in the steel and wood markets, and situations where you must first trade in the steel market and then in the wood market.

### Instructions



Multi-Market Trading Interface

- “Expected utility” is induced: production is “as if” there were 3 states with equal probabilities; states are distinguished by which inputs are lost in production (plastic (i.e., cash) is never lost; steel (wood) is lost in the middle and lower (upper) state)
- No gradual revelation of information: in sequential markets, state-dependent assets (steel & wood) are traded *sequentially*
- We pay based on expected production, not by drawing a state → *full* control for confounding effects due to risk aversion
- Participants are *not* given these complicated trees; they have access to a Google spreadsheet that computes expected production changes as a function of input combinations
- Production: decreasing marginal productivity per state
- We use square-root production functions instead of quadratic because *we want to test whether myopia holds and not whether we get the same prices in both treatments*

Initial Holdings		Current Holdings	
Plastic (Cash)	5	Plastic (Cash)	5
Steel	8	Steel	8
Wood	0	Wood	0
<b>Performance</b>	<b>\$4.35</b>	<b>Performance</b>	<b>\$4.35</b>

Possible Future Trades				
	Buy		Sell	
	Quantity	Price (Plastic)	Quantity	Price (Plastic)
Steel				
Wood				

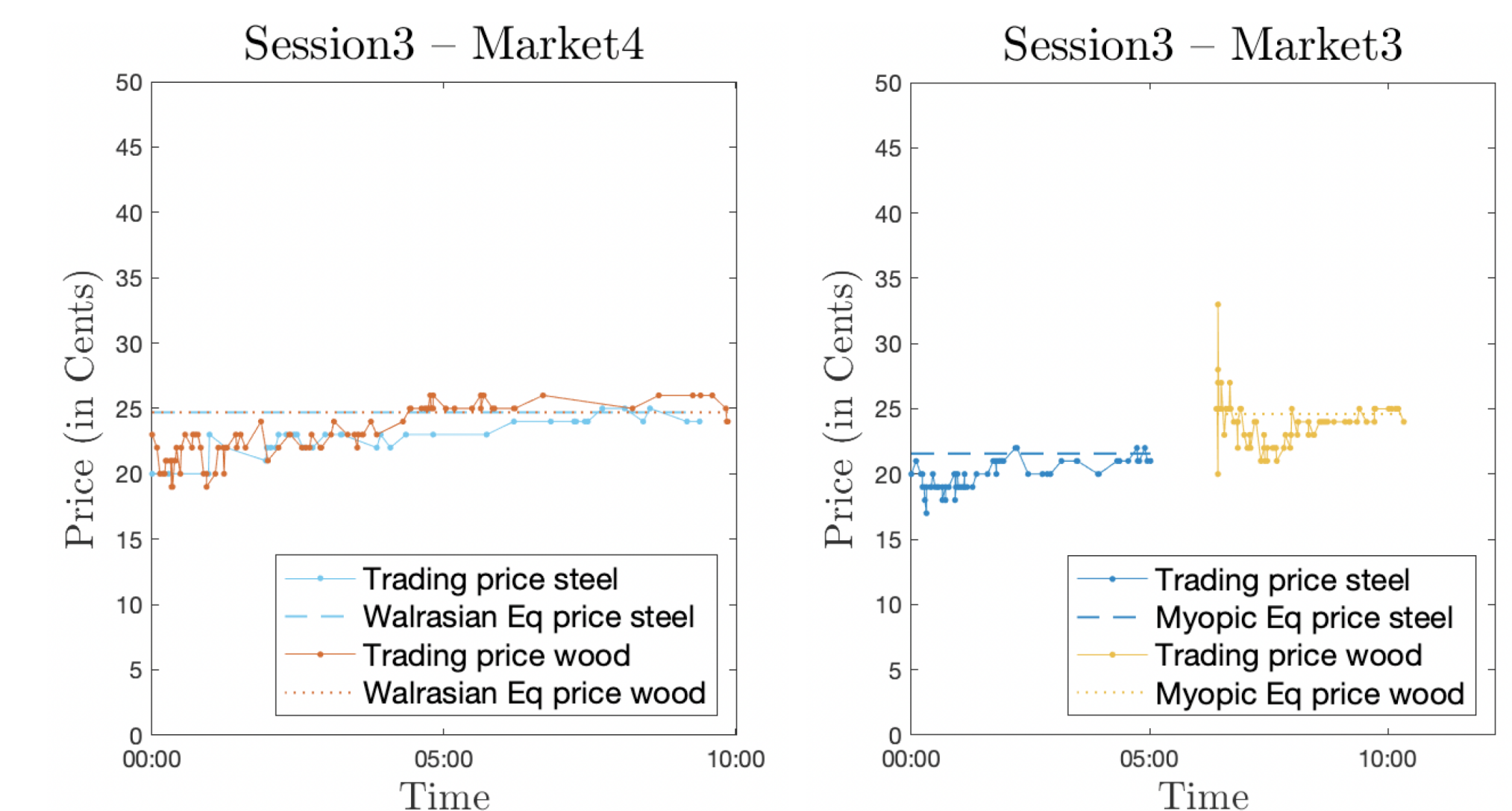
Post-Trade Holdings	
Plastic (Cash)	5
Steel	8
Wood	0
<b>Performance</b>	<b>\$4.35</b>

<b>Perf. Change</b>	<b>\$0.00</b>
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Google Spreadsheet

Screenshots of Experimental Software

## Results



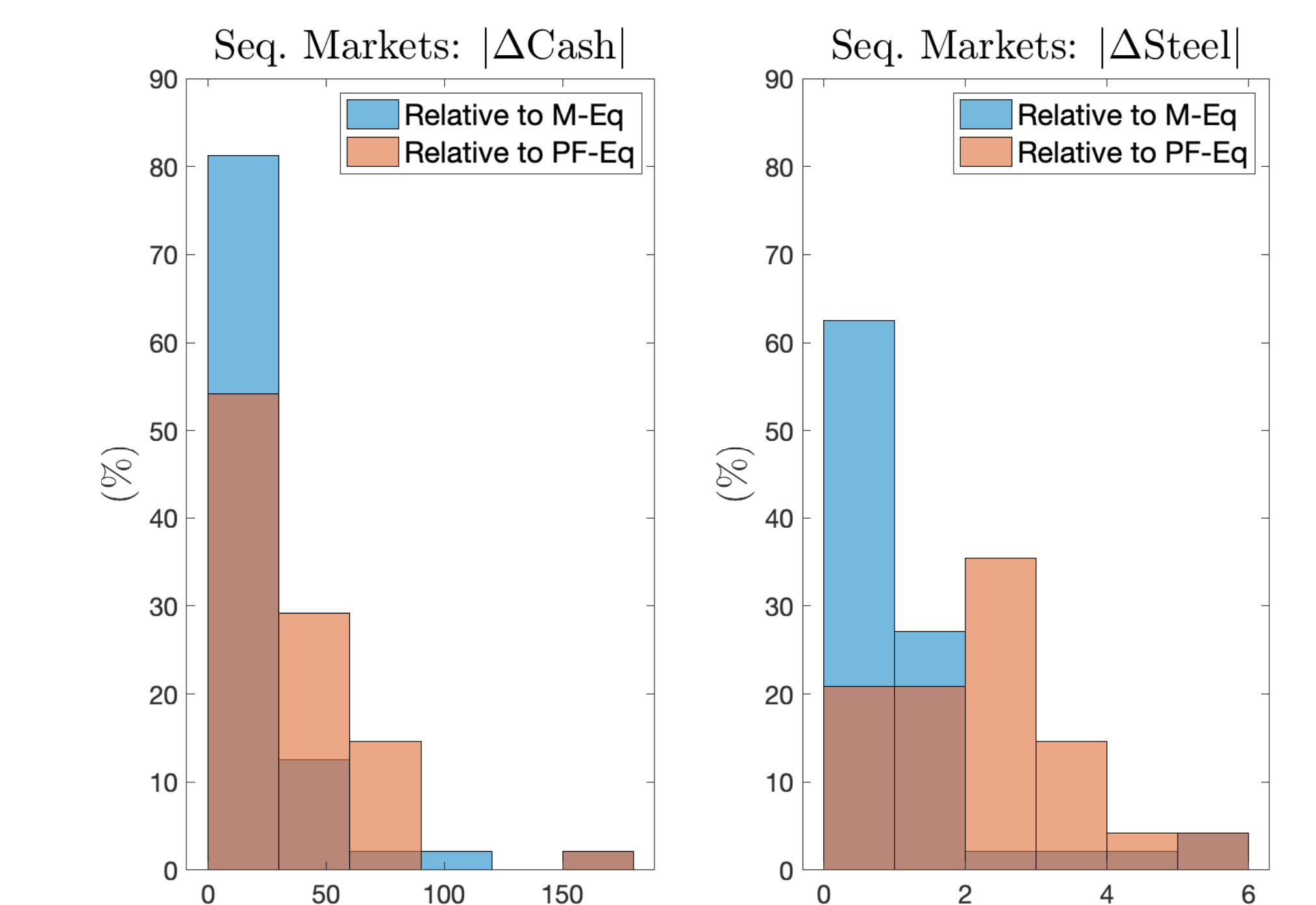
Simultaneous Market Prices

Sequential Market Prices

Note: Walrasian Equilibrium is appropriate notion of equilibrium in simultaneous market treatment

	(1)	(2)
Intercept	1.299 (0.91)	-0.382 (-0.26)
PF-Eq price	0.831 (-22.43) <sup>1</sup>	0.872 (-14.08) <sup>1</sup>
Δ M-Eq price	0.764 (13.48)	
Orth. Δ M-Eq price		0.764 (13.48)
PF-Eq price × $D_{SIM}$	0.073 (14.58)	0.073 (14.58)
Session RE	YES	YES
Market type RE	YES	YES
Replication RE	YES	YES
Participant RE	NO	NO
Observations	4,119	4,119
AIC	18,766	18,766
BIC	18,817	18,817

<sup>1</sup>  $t$ -stats for null hypothesis of unit slope



Sequential Market Allocations

## Summary & Conclusion

- The theory does *not* require strong assumptions about rationality of price forecasts
- Prices will still be the same if agents exhibit a mild form of narrow framing: *myopia*
- Prices and allocations in the experiment prove that deviations from perfect foresight are driven by myopia*

EVEN IN A WORLD WITHOUT PERFECT FORESIGHT PRICING CAN STILL BE “PERFECT.”

When could the theory fail?

- When agents *have* to predict (cannot be myopic), because of, e.g., cash flow smoothing à la Lucas
- When they *want* to speculate (hedge funds, high-beta stocks)

## References

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