

# The Pricing of Continuous and Discontinuous Factor Risks

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## Abstract

- We consider a continuous-time version of the **Fama-French (2015) five-factor model**, allowing stocks' **exposures** on the factors' **continuous, jump, and overnight** movements to be **different**.
- Our results show that stocks' **continuous, jump, and overnight betas** with respect to a given factor can be **very different** and are only **weakly related**.
- Strong evidence for **positive pricing of continuous** and **negative pricing of overnight** market betas, **contradicting** the findings of **Bollerslev et al. (2016)** that indicate positive pricing of jump and overnight but no pricing of continuous market betas.
- Continuous size, value, profitability, and investment factor betas** mostly **negatively priced** while **overnight betas positively priced**.
- Our results give an indication on the **source** of the factors' documented **return premia**.

## Motivation

- Factor pricing models predict a **positive linear relation** between expected (excess) **returns** and **factor exposures**.
- Empirically, this relation is much **weaker than predicted** or **does not hold** for the factors of the Fama-French (2015) five-factor model (e.g. Jegadeesh et al. (2019)).
- We investigate the following explanation:
  - Stocks may have **different exposures** to **continuous, jump, and overnight factor movements**.
  - Exposure to different types of factor risks may carry **different risk premia**.
  - Monthly/daily factor betas** do not differentiate between stocks' exposures to continuous, jump, and overnight movements in the factors and may thus be **unable to accurately reflect the exposure to the priced type of factor risk**.

## Data

- Sample period: **1993 - 2019**
- Monthly and daily stock data on **all common US stocks** from **CRSP**
- High-Frequency stock data from **TAQ**
- Firm fundamentals data from **Compustat**
- Construction of **high-frequency and overnight versions** of the five Fama-French factors (market (MP), size (SMB), value (HML), profitability (RMW), investment (CMA))

## Beta Estimation Methodology

- Estimation of individual stocks' betas:**
  - At the end of each month from June 1993 to December 2019
  - Six-month estimation window
  - Individual sampling frequencies (15-, 30-, or 75-minutes) depending on stocks' microstructure noise
- Continuous beta** of stock  $i$  on factor  $k$  ( $\beta_{i,t}^{k,C}$ ): from a multivariate regression of the stock's high-frequency returns on the five factors' high-frequency returns, using only observations for which none of the factors exhibits a jump.
- Jump beta** of stock  $i$  on factor  $k$  ( $\beta_{i,t}^{k,J}$ ): from a univariate regression of the stock's adjusted high-frequency returns on factor  $k$ 's high-frequency returns, using only observations for which factor  $k$  but none of the other factors exhibits a jump (the stock's returns are adjusted for its continuous exposures to the other factors).
- Overnight beta** of stock  $i$  on factor  $k$  ( $\beta_{i,t}^{k,N}$ ): from a multivariate regression of the stock's overnight returns on the five factors' overnight returns.
- Identification of jumps based on the TOD estimator of Bollerslev et al. (2013):** high-frequency returns that exceed, in absolute terms, three times a local volatility estimate are classified as jumps (accounting for the time-of-day volatility pattern).
- Cross-Sectional correlations** between betas (averaged across the five factors):
  - Corr(continuous, jump): ca. 0.35
  - Corr(continuous, overnight): ca. 0.25
  - Corr(jump, overnight): ca. 0.15
 Only weak positive relation between betas

## Decomposition of Factor Returns

- Market premium** earned in **overnight** returns; average **continuous and jump** returns **close to zero**.
- Size, value, profitability, and investment premia** earned in **continuous** returns; average **overnight** returns strongly **negative**, average **jump** returns **close to zero**.
- Consistent with results of Lou et al. (2019).

## Risk Premia

- Factor pricing models imply a contemporaneous expected return-beta relation. Thus, we investigate the **pricing of the factor betas in contemporaneous returns**.
- Estimation of the following **cross-sectional Fama-MacBeth regression** in each month from June 1993 to December 2019:

$$r_{i,t}^e = \gamma_{0,t} + \sum_{k=1}^5 \gamma_{k,t}^C \cdot \hat{\beta}_{i,t}^{k,C} + \sum_{k=1}^5 \gamma_{k,t}^J \cdot \hat{\beta}_{i,t}^{k,J} + \sum_{k=1}^5 \gamma_{k,t}^N \cdot \hat{\beta}_{i,t}^{k,N} + \sum_{c=1}^C \gamma_{c,t}^X \cdot X_{i,t}^c + \varepsilon_{i,t}$$

- $r_{i,t}^e$  is stock  $i$ 's average excess return from month  $t-5$  to month  $t$
- $\hat{\beta}_{i,t}^z$  are estimated from month  $t-5$  to month  $t$
- $X_{i,t}^c$  are stock characteristics as measured at the end of month  $t-6$
- Regressions are **estimated with weighted least squares** (weights are stocks' market capitalizations)

- Risk Premium Estimates:**  $\hat{\gamma}_k^z = \frac{1}{T} \cdot \sum_{t=1}^T \hat{\gamma}_{k,t}^z$  for  $z \in \{C, J, N\}$

	Coefficients			t-statistics		
	(1)	(2)	(3)	(1)	(2)	(3)
Const	0.56***	1.28***	1.72***	(3.58)	(6.16)	(6.59)
MP Cont	0.84***	1.12***	1.09***	(2.84)	(3.76)	(3.53)
MP Jump	0.04	0.02	-0.01	(0.56)	(0.23)	(-0.15)
MP ON	-0.54***	-0.47***	-0.55***	(-3.61)	(-3.14)	(-3.67)
SMB Cont	-0.09	-0.72***	-0.74***	(-0.59)	(-4.27)	(-4.22)
SMB Jump	-0.09*	-0.15***	-0.15***	(-1.71)	(-2.65)	(-2.73)
SMB ON	0.16***	0.12**	0.13**	(2.78)	(2.13)	(2.24)
HML Cont	-0.30*	-0.47***	-0.55***	(-1.77)	(-2.68)	(-3.05)
HML Jump	0.05	0.05	0.04	(1.25)	(1.17)	(1.08)
HML ON	0.15*	0.14*	0.12	(1.90)	(1.77)	(1.60)
RMW Cont	0.03	0.11	0.07	(0.24)	(0.83)	(0.48)
RMW Jump	-0.02	-0.01	0.00	(-0.71)	(-0.20)	(-0.10)
RMW ON	0.07	0.08	0.10**	(1.37)	(1.64)	(2.03)
CMA Cont	-0.19*	-0.24**	-0.24**	(-1.86)	(-2.30)	(-2.23)
CMA Jump	-0.02	-0.03	-0.03	(-0.92)	(-1.52)	(-1.30)
CMA ON	0.12*	0.10	0.10	(1.86)	(1.58)	(1.61)
Controls	No	Yes	Yes			
$\bar{R}^2$	0.317	0.342	0.367			

Table: Risk premium estimates in percent per month.

(1): No additional controls

(2): Controls: size, book-to-market, operating profitability, investment

(3): Controls: size, book-to-market, operating profitability, investment, momentum, short-term reversal, idiosyncratic volatility, illiquidity, coskewness, cokrutosis, realized skewness, realized kurtosis

- Risk premia display the opposite pattern of factor returns' realizations:**
  - Market premium is **earned overnight**, but overnight beta is **negatively priced**.
  - Market return is **zero intraday**, but continuous beta is **positively priced**.
  - Size, value, profitability, and investment premia are **earned intraday**, but their continuous betas are mostly **negatively priced**.
  - Size, value, profitability, and investment factor returns are **negative overnight**, but their overnight betas are **positively priced**.
- Results are robust** to sampling frequencies, estimation window lengths, jump beta estimation methodology, restriction to S&P500 stocks, and errors-in-variables correction following Jegadeesh et al. (2019).
- Reasons for **differences to the results of Bollerslev et al. (2016)**: we investigate a **contemporaneous** relation (rather than a predictive), employ **all common US stocks** (rather than only S&P500), and use **value-weights** (rather than equal-weights).

## Conclusion

- Market betas:** Continuous beta positively priced, overnight beta negatively priced, jump beta not priced.
- Size, value, profitability, and investment betas:** Continuous betas mostly negatively priced, overnight betas positively priced, jump betas hardly priced.
- Overall, we **cannot document a clearly upward sloping (multivariate) security market line**: negative risk premia for continuous factor exposures mostly overcompensate positive risk premia for overnight factor exposures.

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