Signing-out Confounding Shocks in Variance-Maximizing Identification Methods*

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Economists have long debated the driver(s) of business cycles, attempting to answer this question through the lens of structural vector autoregressions (SVARs) and estimated DSGE models (Galí, 1999; Smets and Wouters, 2007). More recently, a range of variance-maximizing SVAR estimators have been used to identify some of these potential drivers, including technology shocks (Francis et al., 2014), "news" shocks (Barsky and Sims, 2011), and other attempts to dissect the business-cycle anatomy (Angeletos, Collard and Dellas, 2020). However, identification performance is poor when shocks other than the target of interest also play a nontrivial role in driving volatility at the targeted horizon or frequency, thus confounding the estimation. The result is that these identifications capture a *hybrid* shock rather than a dominant shock (Dieppe, Francis and Kindberg-Hanlon, 2021).

We suggest a simple enhancement to sharpen the variance-maximizing identification procedure that reduces the influence of confounding shocks. This enhancement is to include theoretically-informed sign and elasticity restrictions in the identification stage of the VAR.

When applying our solution of combining sign and magnitude restrictions in the frequency domain, we establish the relevant importance of different classes of shocks in driving the U.S. business cycle. We find that "demand"-type shocks, which drive up inflation and output, explain a roughly similar proportion of business-cycle variation in GDP as "supply"-type drivers of output, which lower inflation.

* The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

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I. An example of overlapping shocks in a variance-maximizing identification

Variance-maximizing SVARs identify shocks as those which dominate the variance of a particular variable of interest. However, the objective variance of interest can take several forms; for example, it can reflect the forecast error variance at a specific horizon (Max-Share approach), or it can reflect the variance within a particular frequency domain (Spectral Max-Share approach), reflecting business-cycle or longer-term variance. A reduced-form VAR can be used to compute the objective variance, V (as a function of the variance-covariance matrix of residuals Σ_u and MA coefficient matrix D), modified appropriately to reflect either the forecast error variance at a targeted horizon, k.

$$V = \left(\sum_{\tau=0}^{k-1} D^{\tau} \Sigma_u D^{\tau\prime} \right)$$

Identifying the shock of interest involves the Lagrangian for V:

$$L(\alpha) = \alpha'(V)\alpha - \lambda(\alpha'\alpha - 1)$$

whose first order conditions reduce to solving for the eigenvector associated with the largest eigenvalue of V.

The identified vector α, is then used to generate a single structural shock, $\tilde{A}\alpha$, where $\tilde{A}$ is the Cholesky decomposition of Σ_u. Other structural shocks are left undetermined.

While this approach seeks to identify a dominant structural driver, it is a linear *combination* of structural shocks that often accounts for the largest share of variance. A simple New Keynesian model is used to demonstrate how the variance maximizing methodology can erroneously produce results combining the effects of a demand shock and a supply-side shock, even when the demand shock drives majority of the variance of output:

$$\tilde{y}_t = \frac{-1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - R_t^{NR}\right) + E_t[\tilde{y}_{t+1}] + \eta_t$$
$$\pi_t = \kappa MC_t + \beta E_t[\pi_{t+1}]$$
$$i_t = \phi_y \tilde{y}_t + \phi_\pi \pi_t$$
$$MC_t = (\sigma + \chi) y_t - (1+\chi)\vartheta_t$$

Where $\tilde{y}_t$ is the output gap, i_t is the nominal interest rate, π_t is inflation, MC_t is marginal costs, and R_t^{NR} is the natural rate of interest. η_t represents a demand (preferences) shock, while ϑ_t reflects a supply-side shock, such as technology.[1]

The solution to the model can be written as:

$$\begin{bmatrix} \tilde{y} \\ \pi \end{bmatrix} = \begin{bmatrix} \Psi_{y\eta} & \Psi_{y\vartheta} \\ \Psi_{\pi\eta} & \Psi_{\pi\vartheta} \end{bmatrix} \begin{bmatrix} \eta_t \\ \vartheta_t \end{bmatrix}$$

Take the highly-simplified example in which both η and ϑ have unit variance and in which the empiricist is searching for the shock which maximizes the initial impact variance of the output gap ($\tilde{y}$). In the case of a standard parameterization, where $\Psi_{y\eta} > \Psi_{y\vartheta}$, the demand shock η drives the largest share of the variance of the output gap. However, an eigenvalue-eigenvector decomposition shows that the variance of $\tilde{y}$ on impact is actually maximized by a combination of η and ϑ (Appendix A.A1). More specifically, the "dominant" shock's impact on $\tilde{y}$ will be $\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}$, while the shock's impact on π will be $(\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta})/(\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2})$, rather than the true impact of $\Psi_{y\eta}$ and $\Psi_{\pi\eta}$ respectively.

Note that this identification is equivalent to the standard Cholesky identification solution in this basic case, although maximization over longer periods will deviate from this solution. The smaller supply shock ϑ may exert considerable influence on the properties of the identified shock. Notice for example, that even in cases where the impact of ϑ on the output gap, $\Psi_{y\vartheta}$, is small, the bias to the inflation IRF can still be large if the supply shock's effect on inflation, $\Psi_{\pi\vartheta}$, is large.[2]

In general, the researcher will not restrict her search for the dominant driver of the initial impact variance of the endogenous variables, but rather the forecast error variance at longer horizons, or the variance within a particular frequency band. However, we argue that the same principles shown above still apply; the identified shock will capture a range of influences, in proportion to their impacts at the chosen horizon or frequency band.

In summary, without further identifying restrictions, the search for a dominant driver of a variable of interest will be confounded by other shocks.

II. Methodology

To sharpen identification, we propose a maximization procedure that imposes additional restrictions to reduce the influence of shocks that are of less interest to the researcher. Our estimation procedure is to maximize,

$$V(\alpha) = \alpha' V \alpha$$

subject to

$$\alpha'\alpha = 1$$

$$C_R^{L\prime}\alpha \geq a$$

$$\frac{C_R^{NL1\prime}\alpha}{C_R^{NL2\prime}\alpha} \geq b$$

Here, α is chosen as a linear combination of the reduced-form innovations to the variance-covariance matrix of the target variable of interest (V). It also satisfies the unit-length constraint, and is subject to the restriction that it satisfies a set of linear inequality restrictions (C_R^L) and nonlinear inequality restrictions that can be used to regulate the elasticity of the response of variables relative to one another (C_R^{NL}). With the inequality constraint, the problem is solved using a constrained maximization algorithm.[3] Our approach differs from standard sign-

[1] σ is the inter-temporal elasticity of substitution, χ is the Frisch elasticity of labor supply. κ is the slope of the Phillips curve and is a function of the probability of not being able to reset prices each period (θ) and the discount rate (β) : $\kappa = (1-\theta)(1-\beta\theta)/\theta$.

[2] Our concept of "confounding" shocks in variance-maximizing restrictions has many parallels with the issue of "masquerading" shocks that can lead to misleading results when applying pure sign restrictions (Wolf, 2020).

[3] Additional iterative procedures have been identified which solve constrained eigenvector-eigenvalue decomposition where the linear constraints hold with equality at a. However, even in

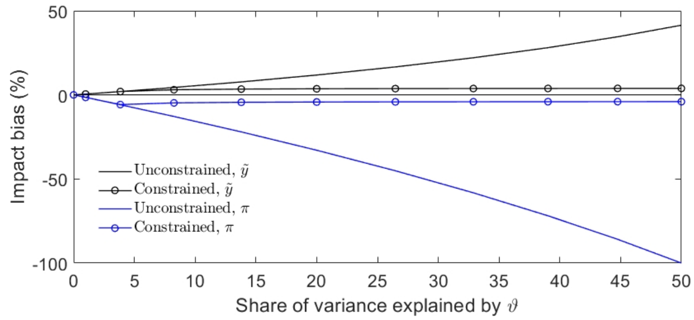

Figure 1. : **Bias of the output gap and inflation impact response to the identified dominant driver of the output gap, with and without restrictions**

restricted identifications in that the draws that satisfy the sign restrictions are not from a uniformly random distribution (Haar prior). Instead, the draws that are kept satisfy the sign restriction constraints and dominate the objective variance function of interest.

III. Constrained Maximization: Applied to the Simple New Keynesian Model

Taking the above New Keynesian model as an example, it is possible to compare the IRF bias that would result from an unconstrained variance maximization identification with a constrained maximization procedure that imposed sign and elasticity restrictions. The unconstrained maximization procedure used to capture the dominant driver of the output gap, η, is increasingly biased for both the impact on $\tilde{y}$ and π as the standard deviation of ϑ increases (Figure 1). Even when the variance of ϑ is low, the inflation response is substantially negatively biased. This bias could lead to erroneous conclusions that the slope of the Phillips curve is flat, or even nonexistent, in response to the main business-cycle driver of the model, a key finding of (Angeletos, Collard and Dellas, 2020).

If the researcher instead imposes *a priori* knowledge of the Philips curve relationship then the bias is substantially lowered for all except the smallest levels of interference from ϑ. The restriction imposed is that inflation increases at least one-third (i.e., $b = \frac{1}{3}$) as much as the increase of the output gap, consistent with standard model parameters. Applying elasticity restrictions that are too low or too high are also found to reduce IRF biases for a wide range of tolerances.[4] In addition, the application of elasticity restrictions is also found to sharpen identification in larger and more complex models and when the objective variance is expressed in frequency-domain form (Appendix A.A2).

IV. Constrained maximization: What drives the U.S. business cycle?

We now apply this methodology to identify the dominant driver of the variance of U.S. GDP at business-cycle frequencies. V now takes a more complicated form based on a transformation of the MA-coefficient matrix D to capture business-cycle frequencies (ω):

$$V = \left(\sum_{\tau=0}^{k-1} D^{\tau}(e^{-i\tau\omega})\Sigma_u D^{\tau}(e^{i\tau\omega})' \right)$$

Here, k is set to 40 such that D admits a long-term, but finite series with which to assess the spectral density of the endogenous variables (Dieppe, Francis and Kindberg-Hanlon, 2021).

We estimate a quarterly VAR over the period 1953-2018 containing: log real GDP levels per capita, the cumulative utilization-adjusted TFP log difference series of Fernald (2014), total hours worked, the unemployment rate, the share of investment in GDP, the share of consumption in GDP, the consumption deflator, and the Federal Funds rate of interest (Appendix

this case a search algorithm is employed and the solution may have multiple roots (Gander, Golub and von Matt, 1989).

[4] Applying elasticity restraints that are too low perform at least as well as the unrestricted case. Applying elasticity restrictions of up to 0.5 reduce bias in all cases for inflation, and reduce the bias of the output gap impact in cases where ϑ explains at least 20 percent of the variance of the output gap.

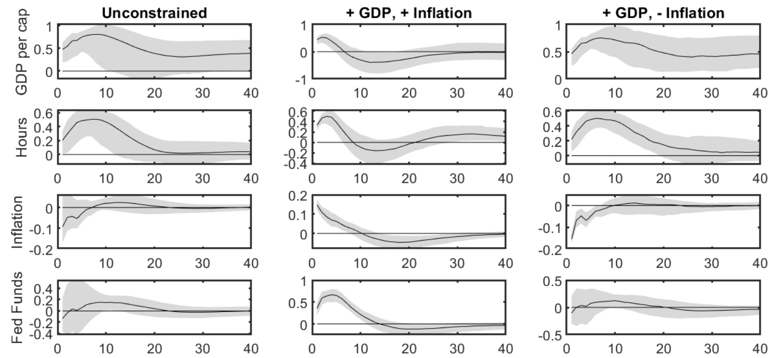

Figure 2. : **Targeting output at business-cycle frequencies, constrained and unconstrained: U.S Data**

Note: 16th and 84th percentile error bands. Columns reflect the unconstrained shock which maximizes business-cycle frequency variation of GDP; the shock where the impact on GDP is constrained to be positive, and the impact on inflation is at least 0.3 times the GDP impact; and, the shock where the impact is constrained to be positive for GDP, but at least -0.3 times the GDP impact for inflation.

A.A3). The shock which maximizes GDP variation at business-cycle frequencies (6-32 quarters) is identified, in some cases subject to constraints on the response of variables.

The results vary significantly when constraints on the response of inflation are introduced (Figure 2). In the case of a constraint that the initial impact on inflation is at least one-third of the size of the impact on GDP combined with a requirement that the GDP response is positive, there is a less persistent response of GDP relative to the unconstrained case. Furthermore, the response of interest rates is also positive in the demand case, while TFP falls, in contrast to the persistent rise in the unconstrained case (Appendix A.A3). In the case of a constraint that inflation *falls* by at least one-third of the increase in GDP, the identified shock takes on the properties of a positive supply-side innovation: the GDP response is still positive at the 10-year horizon, as is the response of TFP and consumption, while interest rates rise very little.

The unconstrained identification produces a hybrid of these two restricted identifications. Even the restricted identifications may continue to be subject to interference, and the imposition of the one-third restriction on the reaction of the inflation rate relative to output is still subject to much debate in the literature. Nonetheless, we argue that the restriction can be varied and still be informative about the contributions of different subsets of shocks to the business cycle.

V. How important are different drivers of the business cycle?

Natural questions that arise from the restricted maximizing shocks are: what proportion of business-cycle variation in output do the newly identified shocks explain relative to the unconstrained case? And, how sensitive are these identified shocks to the elasticity restriction on the response of inflation relative to output?

Both restricted shocks explain around half of the business-cycle variance of GDP. That suggests that both classes of shocks are broadly similar in importance in driving the business cycle (Table 1). As the elasticity restriction is increased, the share of explained variance gradually falls, although only slightly; in the case of the restriction requiring a positive response of inflation, the share falls from 54 to 50 percent as the elasticity is increased from 0.05 to 0.4, while in the case of the negative restriction, it falls from 53 to 50. Clearly, there is likely to be continued overlap between the shocks contained in either identification; it cannot be the case that two independent shocks explain 50 percent or more of the total business-cycle variation of output. Nonetheless, the elasticity restrictions go some way to reducing the degree to which different classes of shocks are included. For example, the unrestricted shock explains about 60 percent of the business cycle variation of output.[5]

[5]Furthermore, we find that supply-side drivers of the business-cycle are similarly important in driving long-run variation for GDP (Appendix A.A5).

Table 1—: **Contribution of identified shocks to business-cycle variation of GDP**

Scale of restriction	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**
Positive response of inflation	54	54	52	51	50
	(48, 61)	(48, 61)	(46, 59)	(46, 57)	(46, 57)
Negative response of inflation	53	54	52	51	50
	(47, 60)	(48, 61)	(45, 60)	(45, 60)	(45, 56)

Note: The median percent contribution of the SVAR-identified shock to business cycle frequency variation in GDP as the scale of the inflation response to the identified shock relative to the GDP response is altered. 16th and 84th percentiles shown in brackets.

VI. Conclusion

This paper has highlighted potential shortfalls of employing variance-maximizing SVAR identifications to identify dominant structural drivers. It shows that the identified shock will likely be a composite of shocks that can contain very different properties. Even in cases where a single shock dominates the variance of the target variable of interest, the impulse responses for other variables can be significantly biased. We propose additional restrictions that can be employed to sharpen variance-maximizing identifications. Sign and elasticity restrictions are shown in examples of model-generated data to generate IRFs that are closer to the true dominant structural shocks. In addition, they can also be used to establish the properties of different categories of shock, for example, those with "demand"-type properties and those resembling supply shocks. However, these restrictions rely on *a priori* knowledge of the structure of the economy. When applied to a VAR estimated on U.S. data, demand shocks, which raise output and inflation, and supply shocks, which raise output but lower inflation, account for a similar proportion of the variance of GDP at business-cycle frequency.

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APPENDIX

A1. Confounding nature of shocks in a simple New Keynesian model

In the New Keynesian model outlined in section I, the solution to the path of the endogenous variables can be written as a function of the structural shocks, η and ϑ. Ψ is a 2×2 matrix which reflects the impact coefficients on the endogenous variables, $\tilde{y}$ and π.

$$\begin{bmatrix}\tilde{y}\\ \pi\end{bmatrix} = \begin{bmatrix}\Psi_{y\eta} & \Psi_{y\vartheta}\\ \Psi_{\pi\eta} & \Psi_{\pi\vartheta}\end{bmatrix}\begin{bmatrix}\eta_t\\ \vartheta_t\end{bmatrix}$$

Where through the method of undetermined coefficients,

$$\Psi_{\pi\eta} = \frac{-\sigma\tilde{\kappa}}{(1-\beta\rho_\eta)(\sigma(1-\rho_\eta)+\phi_y)+\tilde{\kappa}(\phi_\pi-\rho_\eta)},$$
$$\Psi_{y\eta} = \frac{-\sigma(1-\beta\rho_\eta)}{(1-\beta\rho_\eta)(\sigma(1-\rho_\eta)+\phi_y)+\tilde{\kappa}(\phi_\pi-\rho_\eta)},$$
$$\Psi_{\pi\vartheta} = \frac{-\Psi_{NR}\tilde{\kappa}}{(1-\beta\rho_\vartheta)(\sigma(1-\rho_\vartheta)+\phi_y)+\tilde{\kappa}(\phi_\pi-\rho_\vartheta)},$$
$$\Psi_{y\vartheta} = \frac{-(1-\beta\rho_\vartheta)}{(1-\beta\rho_\vartheta)(\sigma(1-\rho_\vartheta)+\phi_y)+\tilde{\kappa}(\phi_\pi-\rho_\vartheta)},$$

In the simulated IRF biases in section III, the following parameter values are used: $\beta = 0.99$, $\sigma = 1$, $\chi = 1$, $\theta = 0.66$, $\phi_\pi = 1.5$, $\phi_{\tilde{y}} = 0.125$, $\rho_\eta = 0$, and $\rho_\vartheta = 0$. Ψ_{NR} is equal to $-\sigma\frac{1+\chi}{\sigma+\chi}$ and $\tilde{\kappa}$ is equal to $(\sigma+\chi)(1-\theta)(1-\beta\theta)/\theta$.

The structural shocks can be mapped to the reduced form impacts that would be observed by the practitioner, and the variance maximizing shock determined as a function of these true underlying impulses. We focus on the initial impact period in order to minimize the complexity of the algebra. The reduced-form residuals are

$$\varepsilon_{\mathbf{t}} = \begin{bmatrix}\varepsilon_t^{\tilde{y}}\\ \varepsilon_t^{\pi}\end{bmatrix} = \begin{bmatrix}\Psi_{y\eta} & \Psi_{y\vartheta}\\ \Psi_{\pi\eta} & \Psi_{\pi\vartheta}\end{bmatrix}\begin{bmatrix}\eta_t\\ \vartheta_t\end{bmatrix}$$

Assuming uncorrelated structural shocks with unit variance, and the fact that $E[\eta_t\vartheta_t] = 0$ the variance covariance matrix of residuals is

$$\Sigma = \begin{bmatrix}\Psi_{y\eta}^2+\Psi_{y\vartheta}^2 & \Psi_{y\eta}\Psi_{\pi\eta}+\Psi_{y\vartheta}\Psi_{\pi\vartheta}\\ \Psi_{y\eta}\Psi_{\pi\eta}+\Psi_{y\vartheta}\Psi_{\pi\vartheta} & \Psi_{\pi\eta}^2+\Psi_{\pi\vartheta}^2\end{bmatrix}$$

Let $\tilde{A}$ be the Cholesky decomposition of Σ, using the fact that:

$$\Sigma = \begin{bmatrix}a & 0\\ b & c\end{bmatrix}\begin{bmatrix}a & b\\ 0 & c\end{bmatrix} = \begin{bmatrix}a^2 & ab\\ ab & b^2+c^2\end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix}\sqrt{\Psi_{y\eta}^2+\Psi_{y\vartheta}^2} & 0\\ \frac{\Psi_{y\eta}\Psi_{\pi\eta}+\Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2+\Psi_{y\vartheta}^2}} & \frac{\Psi_{\pi\eta}\Psi_{y\vartheta}-\Psi_{\pi\vartheta}\Psi_{y\eta}}{\sqrt{\Psi_{y\eta}^2+\Psi_{y\vartheta}^2}}\end{bmatrix}$$

$\tilde{A}$, can be combined with the selection matrix ($s = \begin{bmatrix}1 & 0\end{bmatrix}$) to target the output gap, $\tilde{y}$, in order to form the matrix that is used to identify the dominant shock using the eigenvalue-eigenvector approach of Faust (1998).

$$V = \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2} & 0 \\ \frac{\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} & \frac{\Psi_{\pi\eta}\Psi_{y\vartheta} - \Psi_{\pi\vartheta}\Psi_{y\eta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} \end{bmatrix} \right]' \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2} & 0 \\ \frac{\Psi_{y\eta}\Psi_{\pi\eta} + \Psi_{y\vartheta}\Psi_{\pi\vartheta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} & \frac{\Psi_{\pi\eta}\Psi_{y\vartheta} - \Psi_{\pi a}\Psi_{y\eta}}{\sqrt{\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2}} \end{bmatrix} \right]$$

The eigenvalues of V are the vector $[\Psi_{y\eta}^2 + \Psi_{y\vartheta}^2, 0]$, while the eigenvector corresponding to the largest eigenvalue is $\Gamma_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Notice, that the coefficients from both shocks appear in the eigenvalue, and therefore, the structural rotation in the variance-maximizing approach to identification. The final identification matrix, A^{-1}, is equal to $\tilde{A}\Gamma$, where Γ is a matrix of eigenvectors in descending order following the first column. The first column of Γ is the only shock of interest in this approach. The relative impact of both shocks will determine how close we get to the true structural space (given we have normalized both structural shocks to have unit variance). Notice also that the impact of the dominant shock on inflation will also be increasingly biased in proportion to the size of $\Psi_{y\vartheta}\Psi_{\pi\vartheta}$ relative to $\Psi_{y\eta}\Psi_{\pi\eta}$.

The maximization problem will take a more complex form where the maximization targets dominant shocks over an extended horizon (k) or frequency, in which case:

$$V = \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \sum_{\tau=0}^{k} D^{\tau}\tilde{A} \right]' \left[\begin{bmatrix} 1 & 0 \end{bmatrix} \sum_{\tau=0}^{k} D^{\tau}\tilde{A} \right]$$

The principle is the same however, with the final identified structural shock vulnerable to bias the larger the share of variance driven by the shock that is not of interest.

A2. Constrained variance maximization in larger models

Sign and elasticity constraints can sharpen identification in larger and more complex models than the simple two-variable model given in the main text, and when using a more complicated form for V in the maximization problem. For example, the model of Smets and Wouters (2007) contains 7 shocks. Three of the shocks have characteristics of a typical "demand" shock, in the sense that they generate a positive co-movement between output, prices, and interest rates, and three have "supply" shock characteristics, generating a negative relationship between output relative to inflation and interest rates. The model also contains an additional monetary policy shock.[6]

Seven variables are included in the VAR, based on 1000 periods of simulated data from the Smets-Wouters model: output, hours, wages, investment, consumption, interest rates, and inflation. In the unconstrained case, the shock that dominates business-cycle variation in output is highly persistent; the positive boost of output is accompanied by initial reductions in prices and interest rates (Figure A1). It is also clearly a hybrid shock, as in the previous example, capturing elements of both the demand and supply shocks shown in the blue and red lines respectively, although the latter appear dominant. In the Smets-Wouters model, the supply-side shocks marginally dominate the business-cycle frequency variance of GDP, accounting for around 60 percent.

Sign and elasticity constraints are then applied using our knowledge of the model. The Phillips curve is very flat in the model and the output-inflation elasticity is estimated to be just 0.05. This minimum elasticity is imposed on impact to isolate demand-drivers of output. To isolate the supply-type drivers, a negative inflation-to-output elasticity is imposed. As some of the supply-type drivers do not affect output on impact, the positive output restriction is imposed after one year. The smallest inflation to output elasticity of the supply drivers in the model is 0.1, so this is imposed as a minimum constraint.

[6]The "demand"-type shocks are an exogenous spending shock, a risk premia shock, and an investment-specific technology shock. The "supply" shocks reflect neutral technology shocks, a price markup, and a wage markup shock.

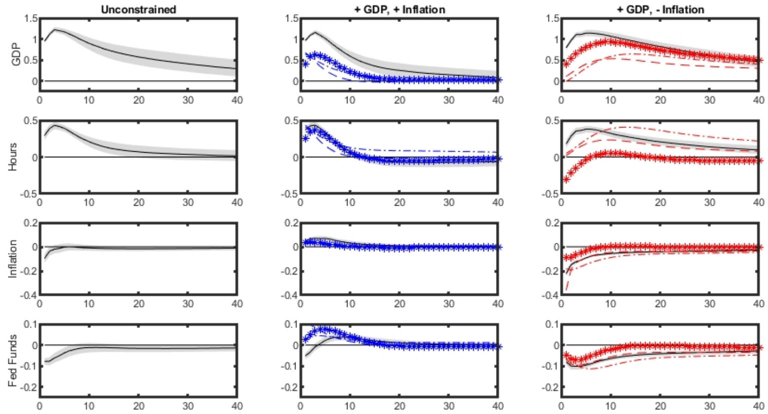

Figure A1. : **Dominant driver of output in Medium-Scale New Keynesian model**

Note: Note: Identified dominant driver of GDP in simulated data produced by the Smets-Wouters model. Blue lines show IRFs from "demand" type shocks in the model, red lines show shocks from supply-type shocks in the model.

In each restricted identification, the impact response of output is larger than any one of the model's demand (blue) or supply (red) shocks (Figure A1). This reflects the identification capturing a composite combination of the shocks of interest. For the positive and negative elasticity restricted cases, the impact on GDP is smaller than in the unconstrained case, capturing a smaller subset of shocks, and either less (demand) or more persistent (supply) IRFs than the unconstrained case. Second, the restricted shocks more closely match the shocks of interest than in the unrestricted case, even where no direct elasticity restriction is applied. For example, the response of interest rates is more positive in the positive inflation restriction case, and more negative in the negative inflation restriction case than in the unrestricted case. The response of hours is less persistent in the positive inflation restricted case than the unrestricted case, more closely matching the underlying demand shocks.

A3. Data and complete IRFs in US VAR

The VAR estimated on U.S. data contains 8 variables, constructed from the Reserve Bank of St. Louis's FRED database (Table A1):

Table A1—: **US data and FRED codes or source**

Variable	Code and transformation
GDP per capita	log(A939RX0Q048SBEA)*100
Utilization-adj. TFP	log(cumsum(Util-adj TFP/4))*100 Fernald (2014)
Hours per capita	log(PRS85006023*(CE16OV/CivPop))*100
Unemployment rate	UNRATE
Investment share of GDP	log(100*((PCDG+GPDI)./GDP))*100
Consumption share of GDP	log(100*((PCND+PCESV)./GDP))*100
Inflation	(log(DPCERD3Q086SBEA)-log(lag(DPCERD3Q086SBEA))*100;
Interest rates	FEDFUNDS

Only a subset of IRFs are shown in Figure 2 of the main text to conserve space. All IRFs under the three identifications are shown below in Figure A2

A4. Employment-targeting shock

In Angeletos, Collard and Dellas (2020), the dominant driver of the unemployment rate at business cycle frequencies is the focus of much of the paper, rather than GDP. Targeting unemployment rather

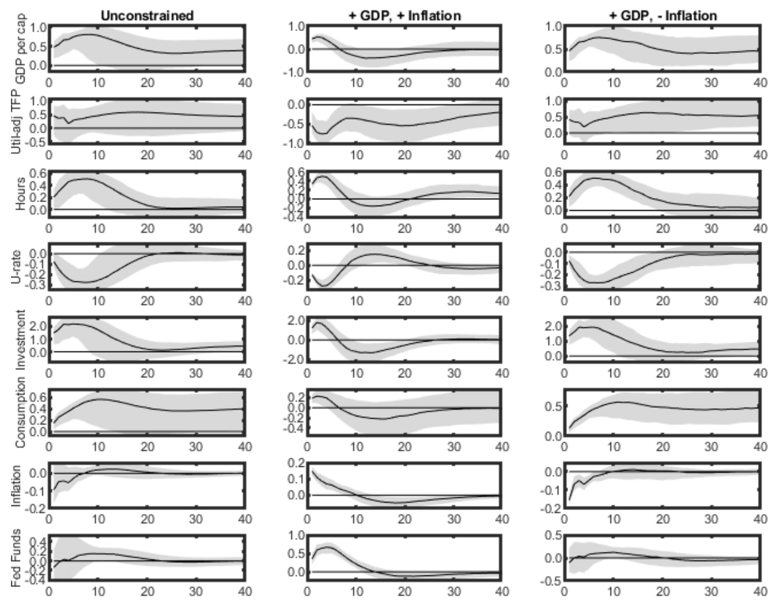

Figure A2. : **Targeting output at business-cycle frequencies, constrained and unconstrained: U.S. Data**

Note: 16th and 84th percentile error bands. Columns reflect the unconstrained eigenvalue-eigenvector solution to the shock which maximizes business-cycle frequency variation of real GDP; the maximizing shock where the impact on GDP is constrained to be positive, and the impact on inflation is at least 0.3 times the GDP impact; and, the maximizing shock where the impact is constrained to be positive for GDP, but at least -0.3 times the GDP impact for inflation.

than GDP yields IRFs which are more consistent with a "demand"-type shock. This may be because demand shocks drive a larger proportion of business-cycle frequency variation in unemployment than GDP. For example, in the Smets-Wouters model, the three demand shocks in the model account for about 60 percent of the business cycle variation of unemployment, but just 40 percent of the variation of GDP.

When targeting the shock that maximizes the business-cycle frequency variation of unemployment, some differences and similarities emerge relative to the shock targeting GDP (Figure A3). First, the unrestricted shock is inflationary, rather than initially deflationary, likely reflecting the increased importance of demand-drivers in unemployment. Second, the shock where the ratio of the response of inflation to GDP is constrained to be positive is also less persistent for GDP than in the unrestricted case, while the negative restriction results in a more persistent impact. This second result is also common to the GDP-targeting maximization.

A5. Do dominant business-cycle shocks also explain low-frequency variation of macroeconomic variables?

Angeletos, Collard and Dellas (2020), using a main-business cycle shock targeting unemployment (6-32q), find a disconnect between the dominant business-cycle and long-run drivers of the macroeconomy. Using the same VAR provided in the main text, it is also found that the unconstrained unemployment-targeting business-cycle shock explains 42 percent of business-cycle variation of GDP per capita, but just 10 percent of long-run variation (40+ quarters). However, we find this to be largely the result of the contamination of the main business-cycle driver of unemployment with both demand and supply side drivers.

In contrast, when we apply our methodology of including additional constraints in the maximization problem, we find that supply-side components of the main business cycle shock driving unemployment explain over 25 percent of both the business-cycle and long run variation of GDP for a sufficiently large restriction on the inflation response relative to GDP. It is the demand components of the shock which do not explain low frequency variation in GDP per capita, while driving a large

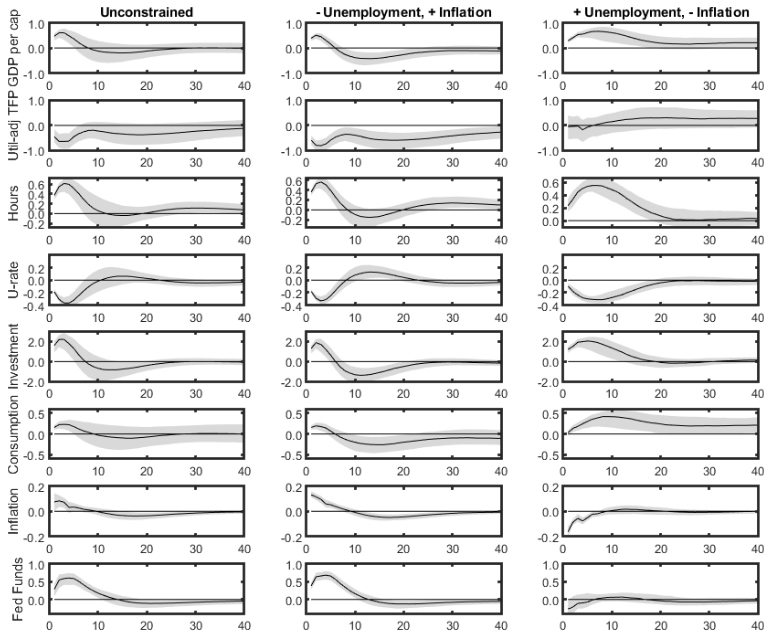

Figure A3. : **Targeting unemployment at business-cycle frequencies, constrained and unconstrained: U.S. Data**

Note: 16th and 84th percentile error bands. Columns reflect the unconstrained eigenvalue-eigenvector solution to the shock which maximizes business-cycle frequency variation of unemployment; the maximizing shock for unemployment where the impact on GDP is constrained to be positive, and the impact on inflation is at least 0.3 times the GDP impact; and, the maximizing shock for unemployment where the impact is constrained to be positive for GDP, but at least -0.3 times the GDP impact for inflation.

share of business-cycle variation (Table A2).

A6. Sign and magnitude restrictions without variance maximization

As we note in the main text, simple sign and magnitude restrictions without the conditioning assumption of an objective maximization problem are simply drawn from a uniform distribution (Haar prior). They therefore can provide very different IRFs than our constrained maximization identification. Figure A4 shows the results of applying the same magnitude restrictions (demand: inflation must rise by 0.3 times the impact on output, supply: inflation must fall by 0.3 times the impact on output). The sign restrictions-only estimations mainly deliver uninformative impulse responses that are indistinguishable from zero.

Table A2—: **Maximizing business cycle variation in unemployment: share of variance of GDP explained at business and long-run frequencies.**

Business cycle (6-32q)					
Scale of restriction	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**
Positive inflation	41	41	40	39	39
	(32, 50)	(32, 50)	(32, 49)	(30, 48)	(31, 48)
Negative inflation	36	37	35	37	38
	(28, 43)	(29, 43)	(25, 43)	(25, 44)	(28, 45)
Long-run (40+q)					
Scale of restriction	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**
Positive inflation	10	9	8	10	11
	(4, 22)	(4, 21)	(3, 21)	(3, 22)	(4, 24)
Negative inflation	16	18	23	25	28
	(8, 26)	(8, 29)	(8, 33)	(11, 37)	(12, 41)

Note: The median percent contribution of the dominant driver of business-cycle frequency variation in unemployment to business-cycle and low-frequency variation in GDP as the restriction on the scale of the inflation response to the identified shock relative to the GDP response is altered. 16th and 84th percentiles shown in brackets.

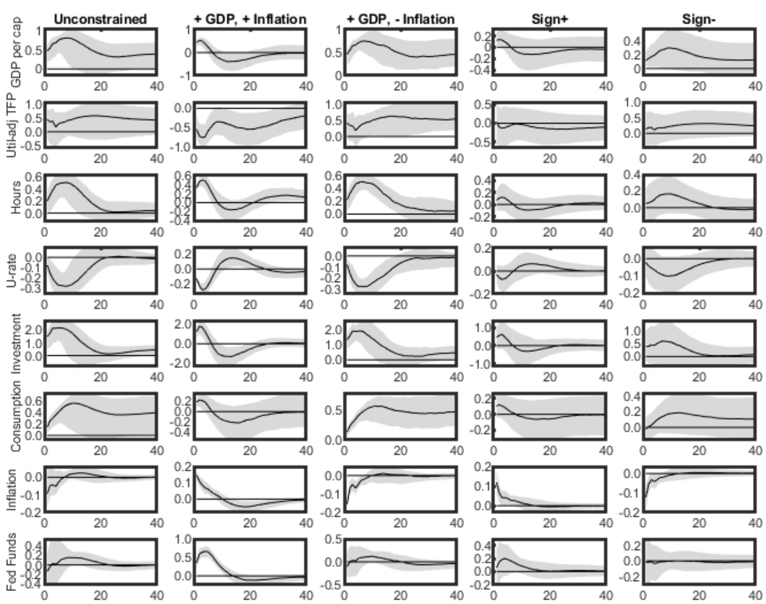

Figure A4. : **Targeting output at business-cycle frequencies, constrained and unconstrained maximization compared to sign and magnitude restrictions: U.S Data**

Note: 16th and 84th percentile error bands. Columns reflect the unconstrained eigenvalue-eigenvector solution to the shock which maximizes business-cycle frequency variation of real GDP; the maximizing shock where the impact on GDP is constrained to be positive, and the impact on inflation is at least 0.3 times the GDP impact; and, the maximizing shock where the impact is constrained to be positive for GDP, but at least -0.3 times the GDP impact for inflation. Final two columns reflect only sign and magnitude restrictions that constrain the inflation response to be +0.3 and -0.3 of the GDP impact, respectively.