Foreign Reserves, Fiscal Capacity, and Lender of Last Resort

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Abstract

Why do emerging markets accumulate foreign reserves for precautionary purposes while advanced economies do not? In this paper, I argue that, in contrast to advanced economies, developing countries accumulate reserves because they lack the sufficient fiscal capacity - ability to extract resources from its citizens - to provide liquidity during crises successfully. By accumulating reserves, developing countries emulates the liquidity provision capabilities of advanced economies. To show this argument, I develop a three period model of small open economy whose funding costs are driven by a global financial cycle. Moreover, I present empirical evidence for a sample of 100 countries between 1990 and 2018 that countries with lower fiscal capacity tend to have larger stocks of foreign reserves. In terms of policy, it shows that overcoming currency mismatch, without improving fiscal capacity, might not be sufficient to eliminate the need for foreign reserves.

Keywords: Foreign reserves, Fiscal capacity, Liquidity crises
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*Email: hm409economics.rutgers.edu. Preliminary - Please do not cite. I am thankful to the valuable comments received at Money Macro Finance Society Annual Conference (2021), and at two brownbag seminars at the Banco Central de Chile; in particular, I am grateful to Andrés Fernández, Bernabé Celis, and Carlos Rondón for their time. This paper is the first outcome of a never ending exploration around the concept of liquidity and how do governments produce it. I am extremely indebted to Roberto Chang and Michael D. Bordo for their continuous support and guidance, specially, for the time they have spent discussing and revising the ideas in this paper. Finally, this paper is due greatly to the conversations about foreign reserves with Julio Carrillo during my internship at BANXICO. Follow this link for more information about my work

1 Introduction

Official holdings of foreign reserves worldwide in 2020 were more than three times the levels registered at the end of Bretton Woods. Emerging and developing countries, where stocks of reserves quadrupled, have lead this build up. Meanwhile, reserves holdings only doubled in advanced economies during the same period (Figure 1).

Accumulating reserves is not cost free. As argued since Heller (1966), a country is foregoing a greater social return by investing resources in low-yield highly liquid instruments instead of capital.

Monetary authorities justify this cost given that reserves are considered to play a key self-insurance role against balance of payments financing needs.[1] Paraphrasing Rodrik (2006), the cost of foreign reserves accumulation is the price of admission to participate safely in (financial) globalization. Yet, as shown by the trend in reserves accumulation, not everybody is paying the admission fee.

This paper tackles two questions: why do some countries accumulate foreign reserves to act as lenders of last resort while others do not? And how is this related to the level of development? I argue that the answer to both of these questions can be traced down to a government's ability to extract resources from its citizens - which I define as fiscal capacity.[2]

The argument is simple. A lender of last resort's fiscal capacity determines the effectiveness of its liquidity provision policies. However, building this capacity requires making important investments in enforcement, information, and compliance, among others. Additionally, it is a key feature in a country's development path.[3] As a result, not every country has this ability sufficiently developed.

Therefore, if a lender of last resort wants to be able to provide liquidity during a crisis, it can compensate for the lack of fiscal capacity by accumulating reserves ex-ante. By doing so, it emulates mature countries and it can successfully provide liquidity ex-post.

[1]See Chamon et al. (2019) for a recent discussion of the precautionary motive
[2]This definition follows Besley and Persson (2014)
[3]See Besley et al. (2013), Besley and Persson (2014)

In this paper I developed this idea both theoretically as well as empirically. In the following section, I provide quantitative evidence consistent with the hypothesis that countries with lower fiscal capacity tend to have greater stocks of foreign reserves. More importantly, I show that this argument is empirically relevant even in the presence of other accumulation motives considered in the literature.

Then, I develop a three period model à la Holmström and Tirole (1998) and Farhi and Tirole (2012) to depict the main argument of this paper.[4]

I study a small open economy inhabited by a continuum of banking entrepreneurs who have exclusive access to high return projects. These projects require an initial investment and, then, a mandatory reinvestment during the intermediate period. If no reinvestment is made, projects are shutdown and don't produce any return at all in the last period.

This economy has access to international financial markets, and, entrepreneurs finance their projects by issuing claims to foreign investors. Importantly, funding costs in international markets are driven by a global financial cycle which determines, whether markets are in a boom or under stress. Naturally, funding costs are low in a boom whereas they are high under a market stress event.

There is only one final good in this world economy that is used to invest and consume. This implies that any positive demand for reserves in equilibrium is not a result of concerns over balance sheet exposure to real exchange rate movements.

I depart from Arrow and Debreu's complete markets by assuming that entrepreneurs can abscond with a share of the project's total return and default on its liabilities. Foreign lenders, to prevent default, only lend up to the point where entrepreneurs are indifferent between absconding and following through with liabilities. As a result, the value of the claims that can be issued - what can be credibly *pledged or promised* to foreign investors - is less than the value of a project's expected total return.

[4]The literature that emerged from Holmstrom and Tirole's work on liquidity provision is a natural place to start to formally study why emerging countries accumulate reserves while advanced economies do not, since reserves are accumulated to address, precisely, liquidity shocks

This wedge between pledgeable and total return implies that projects reinvestment cannot be financed-as-they-go during market stress episodes. Thus, if an entrepreneur wishes to avoid its project to shutdown, it must preemptively hoard market liquidity.

In a laissez faire environment, entrepreneurs do not hoard liquidity when the probability of a market stress is sufficiently low.[5] In such scenario, a market stress turns into a sudden stop type event since no lending occurs between this small economy and world markets. This sudden stop doesn't happen because international markets exclude this small economy but, instead, because entrepreneurs do not provide sufficient pledgeable income nor market liquidity to attract investors.

Can a lender of last resort eliminate sudden stop type episodes? I show that it depends on the lender's fiscal capacity. And if that is not enough, on whether it accumulates reserves or not.

I device a lender of last resort intervention as a *brokerage role* between foreign investors and entrepreneurs. That is, in the intermediate period, it offers a loan to entrepreneurs and it borrows from international markets. In the final period, it collects payments from projects and entrepreneurs to pay back foreign lenders.

A lender of last resort is characterized by a level of fiscal capacity which determines the maximum share that is owed by entrepreneurs that can be directly collected from them and not through the project. The key assumption is that entrepreneurs cannot default on what is collected directly from them. Thus, this share is not subject to projects limited pledgeability.

First, I find that sufficient fiscal capacity allows the lender of last resort to eliminate sudden stop type episodes from this economy, regardless of the probability of the market stress. This is because with that level of fiscal capacity, public intermediation *completes* markets.

Now, a lender of last resort with low fiscal capacity can still eliminate sudden stop like episodes by accumulating reserves. Reserves accumulation allows them to emulate advanced

[5] This result is consistent with Holmström and Tirole (1998) who show that partial insurance is optimal since there is a trade-off between accumulating liquidity and initial investment scale

fiscal capacity.

However, the lower the fiscal capacity, more reserves are needed. Thus, it is also possible that, in equilibrium, a lender of last resort doesn't accumulate reserves. This is more likely when the probability of a market stress is relatively low. As a result, some low fiscal capacity levels lead to an equilibrium where the economy remains exposed to sudden stop type event.

Moreover, I find multiple equilibria for some parameter values. Strategic complementarities emerge between entrepreneurs under a low fiscal capacity lender of last resort and a relatively high probability of a market stress. While in both equilibria, sudden stops are prevented, the difference lies on who hoards liquidity: in one equilibrium it is banking entrepreneurs while in the other equilibrium it is solely the lender of last resort.

As in Farhi and Tirole (2012), strategic complementarities emerged because implementing liquidity provision policies is costly. When it is not costly, as it is the case for mature lenders of last resort, liquidity can be offered to one entrepreneur or to the continuum of entrepreneurs. Thus, the hoarding decisions of others don't affect the hoarding decisions of the rest.

Lastly, let me finish this introduction explaining why reserves are useful to compensate for lower fiscal capacity. The need for liquidity management comes from the wedge between what can be promised to others and what projects generate. Sufficient fiscal capacity eliminates this wedge.

At the micro level, reserves allow the lender of last resort to offer its economy a cheaper financing source. This lower cost of liquidity increases the value of projects pledgeable income just enough to allow for a full-scale continuation. Reserves do this because they reduce the amount that needs to be collected from the economy to pay back foreign lenders. Meanwhile, at the macro level, reserves become part of the economy's balance sheet on the assets side. Thus, it increases the available market liquidity that this economy can used to attract foreign lending. In this model, reserves are no different from private liquidity holdings which are also part of the economy's balance sheet. This equivalence is underscored by the fact that for equilibria that eliminates sudden stops either entrepreneurs hold liquidity or the lender of last resort does it but not both.

Contribution to the literature. This paper provides a novel rationale to why emerging economies hoard reserves for precautionary purposes while advance economies do not. And, as such, it is related to previous work on why countries accumulate reserves for precautionary reasons. This strand of the literature goes back, at least, to Heller (1966) who views reserves as a useful instrument to cover adjustments in the balance of payments, in particular, coming from trade shocks. More recently, Rodrik (2006) argues that the rise in reserves buffers in developing countries since 1990 is a consequence of financial liberalization and globalization. Aizenman and Lee (2007) provides quantitative evidence of a relationship between capital mobility and reserves accumulation which they interpret as evidence for accumulation due to precautionary purposes. Obstfeld et al. (2010) view reserves as a necessary tool to protect domestic credit markets while limiting external currency depreciation from *internal drains*.[6] Meanwhile, Ghosh et al. (2017) provides evidence that motives behind reserves accumulation have shifted from concerns over shocks of the current account to concerns over shocks to the capital and financial account.

My paper is more related to recent work that merges reserves accumulation with financial frictions. Dominguez (2009) provides empirical evidence that part of the surge in foreign reserves holdings is motivated by the goal to compensate for financial underdevelopment. Similarly, Céspedes and Chang (2019) study the optimal level of reserves when a government uses this instrument to alleviate financial frictions. My paper contributes by highlighting that reserves accumulation are useful to overcome financial frictions only for governments that lack the fiscal capacity to overcome them without reserves in the first place. Thus, reserves emerge when there is fiscal underdevelopment as well.

International Monetary Fund (2011) policy report summarizes the existing predominant

[6] The event where domestic residents withdraw their resources from the economy and buying assets abroad (capital outflows)

view on why advanced economies don't accumulate reserves for precautionary reasons in two main ideas: i) not exposed to sudden stops, and ii) borrow in their own currency. Consistent with this first idea, there are several papers that require a positive probability of a sudden stop to generate a positive demand for reserves. For example, Jeanne and Rancière (2011) estimate the optimal level of reserves to insure an economy against financial dry-ups in international markets while Calvo et al. (2013) due the same in a statistical model. Similarly, in Aizenman and Lee (2007), the demand for reserves comes from a government's objective to stabilize consumption during sudden stops. In contrast, in this paper, sudden stops are a possible outcome of the model and, as such, they are not assumed to be specific to developing countries. Additionally, in this model, reserves are required to attract foreign lending, so there are complements rather than substitutes.

The inability to borrow in its own currency is what Eichengreen et al. (2003) refer to as *Original Sin*. This concept is linked to financial fragility because it exposes countries balance sheet to currency mismatch. For example, Chang and Velasco (2001) place international illiquidity (currency mismatch) at the center of the financial fragility of emerging economies in the 1990s. In principle, thus, a lender of last resort that provides liquidity in foreign currency could alleviate financial instability. But to do so, it is said that it needs to accumulate dollars ex-ante to provide dollars ex-post.

However, following Fischer (1999), a lender of last resort doesn't need to accumulate reserves ex-ante as long as it has the ability to make available those resources to illiquid agents when needed. This has been evident more recently where the use of central bank swap lines and derivatives settled in local currency have allowed central banks in emerging markets to intervene foreign exchange markets without tapping into their stock of reserves. The contribution of this paper is to underscore that the ability to attract resources depends on a country's fiscal capacity. And that the lack of fiscal capacity creates a demand for foreign reserves even in scenarios where there is no currency mismatch.

Recently, some interesting work has emerged on analyzing the optimal reserves holdings when governments can default on their debt. Alfaro and Kanczuk (2009) find that under this context the optimal level of reserves is zero since reserves are a tool to smooth consumption during defaults, and, as such, reduces the opportunity cost of defaulting. As a result, the cost of debt is higher with positive levels of reserves, and the government chooses optimally to not accumulate. In contrast, Bianchi et al. (2018) find that the optimal level of reserves is positive since reserves can be used as a hedging instrument, and, as such, provide an insurance against rollover risk. In my model I abstract from incentives to default. However, consistent with Bianchi et al. (2018), I observe the hedging property of reserves since they are accumulated to be used in periods of high funding costs which is particularly important for countries with low fiscal capacity.

As discussed briefly previously, the model of this paper directly emanates from the work of Holmström and Tirole (1998). I contribute to this literature by highlighting that the effectiveness of government ex-post interventions doesn't rest solely on having the power to tax agents and levy non-pecuniary penalties but on how is the economy being taxed. And the *how* depends on the level of development of the power of taxation. This is, to the best of my knowledge, the formalization of Tirole (2002) idea that increases in public debt may fail to increase aggregate liquidity if the expected tax incidence crowds out pledgeable income.

Lastly, my paper is closely related to Bocola and Lorenzoni (2020) who develop a model of a small open economy to show that liability dollarization emerges in equilibrium when domestic savers have concerns over local financial stability. More importantly, they show that the effectiveness to eliminate bad equilibria depends on a government's fiscal capacity. Similar to my paper, Bocola and Lorenzoni (2020) find that reserves can compensate when there is insufficient fiscal capacity.

However, our papers, besides of their aim and research question, differ importantly on how reserves compensate for the lack of fiscal capacity. In their paper, reserves are a hedging instrument due to being denominated in foreign currency. Thus, for reserves to have any role, the economy's balance sheet needs to be experiencing a currency mismatch. Meanwhile, in my paper, reserves are a instrument to increase the economy's market liquidity. In this case, reserves have a role because the economy's balance sheet is exposed to a maturity mismatch that fiscal capacity cannot overcome. From a policy perspective, the comparison of these two roles would suggest that eliminating currency mismatch is not sufficient to eliminate the need for reserves, you cannot shy away from improving fiscal capacity.

2 Foreign Reserves and Fiscal Capacity in the data

I collect data of foreign reserves, Gross Domestic Product (GDP), and tax revenue, total and income tax, between 1990 and 2018 for 206 countries from the World Development Indicators dataset to take the main argument of this paper to the data.

Following Besley and Persson (2014), I measure a country's fiscal capacity using income tax revenue data as a share of total tax revenue. These authors argue that collecting income tax requires major investments in enforcement and compliance mechanisms compared to other taxes such as tariffs. Both enforcement and compliance mechanisms ultimately improve a government's ability to extract resources from its citizens making income tax revenue an adequate proxy for fiscal capacity.

Figure 2 plots country averages between 1990 and 2018 of foreign reserves holdings against total tax revenue (Panel 2a) and against fiscal capacity (Panel 2b).[7] While there is a weak positive correlation between foreign reserves and total tax revenue, Panel 2b shows that countries with a greater share of taxes collected through income tax tend to have lower levels of foreign reserves in line with the prediction of the model.

The naive approach depicted in Figure 2 provides some initial evidence. However, for a more rigorous exercise, I build on the previous work that empirically estimates the motives behind the accumulation of foreign reserves by emerging markets since 1990.[8]

The predominant empirical approach of this literature assumes that the ratio of foreign

[7] First, I take logs on foreign reserves (% of GDP), tax revenue (% of GDP), and income tax revenue (% of tax revenue), then I calculate the country average between 1990-2018 for each variable

[8] See, for example, Aizenman and Lee (2007), Obstfeld et al. (2010) and Ghosh et al. (2017)

reserves to GDP in year t held by country j ($y_{j,t}$) is a function of the exchange rate regime, the potential financing needs coming from the balance of payments (precautionary motives) and the mercantalist motive.[9] Additionally, to limit the potential endogeneity problems, most regressors are usually lagged one year except those that capture the exchange rate regime.

$$log\ y_{j,t} = \beta X_{j,t-1} + \alpha_0 log(\frac{TR_{j,t-1}}{GDP_{j,t-1}}) + \alpha_1 log(\frac{ITR_{j,t-1}}{TR_{j,t-1}}) + \varphi^t + \epsilon_{j,t} \tag{1}$$

To test whether fiscal capacity has any empirical power, I add to the predominant view model - captured by matrix $X_{j,t-1}$ in Equation 1, two additional regressors: i) the measure of fiscal capacity discussed previously (income tax revenue as share of total revenue - $\frac{ITR_{j,t-1}}{TR_{j,t-1}}$); ii) and total tax revenue ($\frac{TR_{j,t-1}}{GDP_{j,t-1}}$) to control for scale effects of tax collection that I don't want them to be captured by our measure of fiscal capacity.

Any support for the hypothesis in this paper depends on obtaining a negative estimate for α_1. At this point, it is worth mentioning that the chosen approach doesn't have the power to establish causality, however, I use the theoretical model to provide an explanation of why such causality can potentially exist. Moreover, note that Equation 1 includes time fixed effects but not country fixed effects. The reason is that the empirical prediction of the model refers to a between countries and not within countries comparison since the level of fiscal capacity is exogenous.

I follow Obstfeld et al. (2010) to determine the variables that are part of matrix $X_{j,t-1}$. These authors divide the different motives behind reserves hoarding considered by the literature into *models*. The *traditional model* comprises variables that capture risks emanating from the current account. In this exercise,[10] I include the ratio imports of goods and services to GDP, in log units, and the three-year standard deviation of exports over GDP as a measure of volatility in receipts from the world. Additionally, I include the annual standard

[9] This motive understands foreign reserves accumulation as the by-product of a development strategy to explicitly undervalue the currency

[10] Same as Ghosh et al. (2017)

deviation of monthly exchange rate variation to capture the risk of what Heller (1966) called the expenditure-switching adjustment. Lastly, the traditional model includes each country's GDP, in log units, to control for scale effects.

The second model captures financial stability risks by including the share of broad money in the economy and the external short term debt, again relative to the economy, to capture, as argued by Obstfeld et al. (2010), an internal (from deposits to currency) as well as an external drain (from domestic to foreign assets), respectively. Additionally, I include the Chinn-Ito Index (normalized) as a measure of a country's financial openness since where capital moves more freely it is more likely that a financial crisis turns into a balance of payment crisis. This group also includes dummy variables for a country being an high income as defined by the World Bank, and whether the country was implementing during the respective year a hard peg or a soft peg according to Ilzetzki et al. (2019) exchange rate regime index.[11]

The third group is the mercantalist model. Aizenman and Lee (2007) were one of the first to empirically consider the mercantalist strategy as a explanatory variable behind foreign reserves accumulation. Due to data availability, I follow Dominguez (2009) by measuring currency over-valuation equal to the ratio between the Parity Purchasing Power (PPP) conversion factor and the market exchange rate minus one. Hence, if this index is positive than the currency is considered to be over-valued.

The traditional model, the financial stability model and the mercantalist model comprise the existing predominant view in the literature on what drives foreign reserves accumulation, specially in emerging markets.

I include a model that captures the level of development of the financial sector. This is motivated by Dominguez (2009) who provides empirical evidence that countries with underdeveloped capital markets tend to accumulate more reserves. This model includes the sum of domestic private credit creation and stock market capitalization (Domestic Financial

[11]Hard pegs are countries whose Ilzetzki et al. (2019) Fine index was less or equal to 9 or equal to 11, while soft pegs corresponds to categories 10 or 12.

Liabilities as % of GDP),[12] as a measure for financial development, together with the size of the external balance sheet broken down by private and public sectors.

Including the *financial development model* is key because a lender of last resort is by definition only necessary when there are no other lenders. Thus, in the reasoning presented in this paper, fiscal capacity becomes a relevant feature behind foreign reserves accumulation when financial markets are underdeveloped. Excluding the financial development model from this empirical exercise would cause the approach to suffer from endogeneity due to an omitted variable.

Lastly, one of the main arguments in this paper is that fiscal capacity is a relevant motive for foreign reserves hoarding even in an economy with no currency mismatch. I test this argument by including an proxy for *original sin* in the empirical exercise.

I collected annual data for both advanced economies and low income and emerging economies between 1990-2018.[13] I exclude countries that had less than five observations available for the period of analysis as well as economies with a population lower than one million people. The final dataset is an unbalanced panel for 100 countries, of which 29 are advanced economies according to the IMF. Table 1 presents the summary statistics of the different variables.

First, I run Equation 1 without the Original Sin. I do this because the original sin index is only available starting 2000. Table 2 presents the results for 6 different sample groups. The first column of each sample presents the results when I exclude the Fiscal Capacity model. The second column are the results when I include this model to the regression.

Columns I and II present the results for the whole sample. Note that the estimate for fiscal capacity (Income Tax revenue as share of total revenue) is significant with a negative sign as expected. Thus, countries with lower fiscal capacity tended to accumulate more reserves.

[12] Dominguez (2009) consider two additional proxies for financial development. However their results remain the same regardless of the measure

[13] Appendix describes the data, how variables were constructed and the original sources.

Columns III and IV are the results when only considering the emerging and developing countries, as classified by the IMF, in the dataset. Once again, the estimate of fiscal capacity is as expected.

Ghosh et al. (2017) point out that the motives behind foreign reserve accumulation could be shifting through time. To see this with regard to fiscal capacity, I chose as a breaking point the Global Financial Crisis (GFC).[14] Columns V and VI present the results for the period 1990-2007 while columns VII and VIII for 2010-2018. During both periods, the sign of fiscal capacity is negative but the estimate is only significant post GFC.

The dataset is an unbalanced panel due to the lack of data for some variables for some years. If there is an element in each country (i.e. quality of institutions) that explains this lack of data as well as fiscal capacity and foreign reserves accumulation, then the estimate of interest is biased. I identify 17 countries that have data for every year and I run Equation 1. The results (Columns IX and X) not only show that the estimate for fiscal capacity is negative and significant, but also that it is the double in magnitude.

Countries in the euro zone are part of a monetary union but not a fiscal union. That is, they share the same monetary authority but their fiscal capacity is idiosyncratic. In addition, most of the external debt of these countries is in euros which suggest that they don't experience currency mismatch, and, the national central banks of the system still have the prerogative to act as lenders of last resort. This context closely follows the assumptions of the model. Hence, it is an adequate sample to test whether fiscal capacity can explain the variance in levels of foreign reserves between countries.

Columns XI and XII present the results for nine euro zone countries between 1999 and 2018. These countries are part of the first group to put in motion the euro as a currency back in 1999.[15] For this sub-sample, estimates show that countries with lower fiscal capacity

[14]I select this point first because it put to the test the accumulation of foreign reserves as a self-insurance mechanism. And, second, because it is possible to consider that the policy response, such as the implementation of swap lines between reserve central banks and other central banks, could have modify the effect of fiscal capacity on reserves accumulation.

[15]This initial group consisted of eleven countries, however, Spain is not part due to data availability and Luxembourg has a population lower than one million.

tended to have a greater stock of reserves in line with the results of the model developed in this paper.

Table 2 results support that fiscal capacity matters for foreign reserves accumulation. I now move to including original sin in the exercise Table 3 presents the estimates of these regressions.

Fiscal capacity is negatively correlated in most sub-samples with foreign reserves. Unlike the results that excluded original sin, fiscal capacity estimates are significant both before and after the GFC. However, the estimate in the regression with the balanced panel lost significance but the sign remains negative. Overall, the results still support that countries with lower fiscal capacity hold more foreign reserves even when controlling by original sin.

Note, as well, that the estimate for the original sin index is positive and significant in most regressions. Hence, countries with greater original sin have larger stocks of foreign reserves.

In terms of other variables, the results are robust regardless of including, or not, original sin. The results show a robust positive relationship between hard and soft pegs with foreign reserves stock. This is not surprising since reserves are necessary to implement less flexible exchange rate regimes.

Additionally, consistent with Obstfeld et al. (2010), there is evidence that financial stability concerns guide a share of the build up in foreign reserves as indicated by the positive and robust relationship between the dependent variable and broad money. Also consistent with these authors, my results show a negative sign for external short-term debt. This is opposite to the Guidotti-Greenspan rule that suggests that country should accumulate reserves to cover any potential demand for repayment derived from a country's short term debt. Obstfeld et al. (2010) explain this finding to be consistent with the fact that countries accumulate reserves far in excess than short-term debt obligations.

Moreover, similar to Aizenman and Lee (2007) and Ghosh et al. (2017), I find support for the mercantalist motives. Countries with undervalued currency tend to have bigger stocks of reserves.

Lastly, I run a robustness check of Equation 1. Table 4 presents four different specifications for the model excluding original sin, and the same four for the model with original sin. Columns I and V is the main specification which includes year fixed effects (YE), columns II and VI includes country fixed effects (CFE), columns IV and VII includes both year and country fixed effects while columns IV and VIII is a cross-section regression where the observations are the panel average for each country.

Results show that the statistical significance of the correlation between fiscal capacity and foreign reserves is lost in the specifications with country fixed effects. This suggests that the within variation in foreign reserves is driven mainly by the predominant view variables. In addition, in line with Dominguez (2009), financial development appears significant and negatively correlated with foreign reserves with country fixed effects.

Interestingly, the estimate for original sin index changes sign with country fixed effects. This result is surprising since it is believed that lower currency mismatch leads to a lower need for foreign reserves. Taking stock, this section provides both naive and more formal evidence that countries with lower fiscal capacity accumulate more foreign reserves. This empirical evidence is robust to including both measures of original sin and other motives considered in the literature.

3 Model

3.1 Environment

I study the role of foreign reserves in ex-post liquidity provision in an environment similar to **?** and Farhi and Tirole (2012).

The economy Consider a three period economy ($t = 0, 1, 2$) inhabited by two types of agents: banking entrepreneurs and a lender of last resort. There is a continuum of banking entrepreneurs with population normalized to 1. Agents trade, consume and invest the only perishable final good existing in this economy. Moreover, this economy is *open* in the sense that agents have access to international capital markets where they can lend or issue claims, either at period 0 or at period 1.

Foreign lenders are risk neutral and deep pocket. They are willing to lend resources to this economy as long as they obtain, at least, the same expected return that they would get from lending at international financial markets. I denote this marginal opportunity cost between period t and period $t + 1$ with γ_t. I assume that this economy is *small* such that equilibrium returns in international capital markets are not affected by decisions made by either entrepreneurs or the lender of last resort.

Banking Entrepreneurs are risk neutral agents ($U(c) = c_0 + c_1 + c_2$) that receive an initial endowment A of the only good in the economy at the initial period. These agents do not receive further endowments and are protected by limited liability.[16] Entrepreneurs can consume the initial endowment at $t = 0$, they can lend it in international capital markets at the given rate, or they can use it to invest in a project.

Project Technology. Banking entrepreneurs have access to a constant return to scale investment technology where long-term returns require occasional reinvestments (Figure 3). When i units of the perishable good are invested in the initial period, it generates a safe cash flow of πi at $t = 1$. A reinvestment is required at $t = 1$ to generate any return at $t = 2$.

[16] Consumption levels cannot be negative in any period

Thus, if the reinvestment, denoted by j, is positive, the project produces a total return of $\rho_1 j$ at $t = 2$.[17] Whereas, if j is zero, the project is shutdown and doesn't generate any return beyond the safe cash flow.

As stated by Tirole (2011), the demand for liquidity in this type of models comes from the lack of synchronicity between revenues ($t = 1$ and $t = 2$) and outlays ($t = 0$ and $t = 1$). Hence, it is a natural setting for the surge of liquidity demand.[18]

Banking entrepreneurs cover liquidity needs (initial investment and reinvestments) either by using the liability side of their balance sheet (funding liquidity), or by using the asset side (market liquidity).[19] In this model, entrepreneurs tap on their funding liquidity by issuing short-term and long-term claims at international capital markets (private funding liquidity) or by borrowing from the lender of last resort (public liquidity).[20] And for market liquidity, they can use their initial endowment at $t = 0$, and, plausibly, any return they receive from the project or world capital markets at $t = 1$.

Moral Hazard. I introduce a friction to a project's funding liquidity by assuming that banking entrepreneurs are subject to moral hazard.[21] At the start of $t = 2$, an entrepreneur can abscond with a fraction θ of the project's total output. If this happens, the remaining fraction $1 - \theta$ is lost.[22]

Aggregate Shock. As mentioned before, this economy is small and as such, it's funding cost is subject to the opportunity cost in world capital markets which is random. At $t = 1$, the state of the world could either be a *boom* where γ_1 is equal to γ_1^L with probability α or it could be experiencing a stress event with γ_1 equal to γ_1^H with probability $1 - \alpha$.

The funding cost shock at $t = 1$ captures states of the world where funding for small

[17]Reinvestment (j) cannot be greater than the initial investment scale i, thus, the project's size is set at $t = 0$

[18] Moreover, a key assumption is that only banking entrepreneurs, who are credit constrained, have access to this investment technology. Thus, from their perspective, this is a liquidity management problem.

[19]See Tirole (2011) for a further discussion on market and funding liquidity

[20]Short-term claims are backed up by projects safe cash flow while the rest of liabilities are are backed up by projects date-2 pledgeable return - More on this below

[21]See Holmström and Tirole (2011) or Tirole (2011) for different ways to model an agency wedge between total and pledgeable return

[22]The $1 - \theta$ loss can be interpret as the cost that a banking entrepreneur needs to successfully abscond

open economies is potentially either relatively expensive (stress) or relatively cheap (boom). Naturally, this interpretation is consistent with $\gamma_1^L < \gamma_0 \leq \gamma_1^H$.

Additionally, note that the opportunity cost and their probabilities are exogenous to any idiosyncrasies of the small economy. Both are modeling choices that capture a world where financial costs for small economies are driven by a Global Financial Cycle as in Rey (2015).

Assumption 1 (Project's High Return)

1.1 $\frac{\rho_1}{\gamma_1^H} + \pi > 1 + \alpha\gamma_1^L + (1-\alpha)\gamma_1^H$

1.2 $\alpha(\rho_1 - \gamma_1^L) + \pi > 1$

Assumption 1 guarantees that projects have a return attractive enough for entrepreneurs to invest all their net worth even when compared to high funding costs. This assumption is straight forward: a banking entrepreneur needs to invest one unit at $t = 0$ and a second unit additional investment at $t = 1$ with an expected net cost of $\alpha\gamma_1^L + (1-\alpha)\gamma_1^H - \pi$ which is reflected on the right hand side of Numeral 1. The left hand side states project's total return relatively to the high funding cost is sufficient $\frac{\rho_1}{\gamma_1^H}$ to cover for the investment cost. Numeral 2, in turn, states that the expected net return of the project if no reinvestment is donde under market stress is still positive at date-0.

Lender of last Resort (LOLR) is a key agent in this small economy. Following Holmström and Tirole (1998), it is the only player in this economy that has the power to audit incomes and impose non-financial penalties to banking entrepreneurs in order to collect payments.[23] As it will be clearer below, this unique ability provides a potentially welfare improving role for a LOLR when there are financial frictions between agents that demand liquidity (banking entrepreneurs) and those that supply liquidity (foreign lenders).

Following Bagehot's rule,[24] I assume that the LOLR implements an ex-post liquidity provision program to lend freely to illiquid but solvent agents with good collateral to guarantee the continuation of projects even during stress states of the world. Thus, resources are

[23]Note that LOLR only has this power over banking entrepreneurs and not over foreign lenders. That is, its actions are limited by the space of the domestic economy

[24]See Bordo (1990) and Fischer (1999) for a discussion about Bagehot's rule

collected to make available another source of finance to entrepreneurs, specifically, during periods where financing costs are relatively high in world markets.

Lending Scheme At $t = 1$, a banking entrepreneur can ask the LOLR for a loan, τ, to finance reinvestment. In return, the LOLR collects $\hat{R}\tau$ at $t = 2$ where $\mu\hat{R}\tau$ comes directly from entrepreneurs and $(1 - \mu)\hat{R}\tau$ comes from projects. To cover this potential demand for loans at $t = 1$, the LOLR can:

- transfer f_1 from its market liquidity which results from collecting F_0 resources from banking entrepreneurs at $t = 0$. Thus, f_1 is less or equal to F_0. Naturally, each unit collected at $t = 0$ by the LOLR is not invested in the project, and, thus, incurs in a opportunity cost ψ
- Issue bonds, denoted by B_1, at international markets that need to be fully redeemed at $t = 2$

Fiscal Capacity Parameter μ lies between 0 and $\bar{\mu}$, where $\bar{\mu}$ can take any value between zero and 1. In this model, $\bar{\mu}$ captures the level of development of the LOLR's fiscal capacity. I assume that a LOLR with greater fiscal capacity has made the necessary investments in enforcement and compliance such that it can collect a greater share of $\hat{R}\tau i$ directly from entrepreneurs. This assumption is consistent with Besley et al. (2013) who use the share of tax revenue that is collected through income tax as a proxy for a country's fiscal capacity.

Policy Instruments comprise set $\Gamma(\bar{\mu})^s$ and depend on the LOLR's fiscal capacity. These instruments are the amount of reserves accumulated at $t = 0$ (F_0), the cost of public liquidity ($\hat{R}^s$), the depletion of reserves (f_1^s), and bond issuance (B_1^s) at $t = 1$ in every state of the world.

As it will be clear below, when required, a greater stock of F_0 potentially increases reinvestment levels during market stress by reducing the cost of public liquidity ($\hat{R}$) faced by entrepreneurs. This is because, with reserves, a lower share of the repayment of bonds falls to entrepreneurs, specifically, to projects balance sheet. However, a greater F_0 also implies a deviation of resources from projects which are, by Assumption (1), the most productive investment option in the economy.

Policy Objective This trade-off faced by an LOLR is captured by its Policy Objective Function (2). Deviating a unit of initial endowment from projects implies giving up a marginal net return, ψ,[25] times a scale effect that arises in projects due to the equity multiplier (κ).[26]

As explained by Holmström and Tirole (2011), the equity multiplier determines the maximum leverage per unit of own net worth. Thus, I include the equity multiplier to internalize the cost on leverage of reducing entrepreneurs disposable endowment.

$$\psi F_0 \kappa + E_s\big[L(j^s)\big] \tag{2}$$

The second term reflects the expected welfare costs of partial liquidation. Loss Function $L(j^s)$ depicts, in a reduced form, this loss.[27] This approach follows Farhi and Tirole (2012) with the purpose to underscore that a LOLR dislikes the negative spillover effects of downsizing on the economy that, individually, entrepreneurs might fail to do identify.[28]

Assumption 2 (Welfare Loss Function)
Define function $L : [0, i] \to R^+$ with the following characteristics;

1. *Continuous and convex function*
2. *Non-increasing*
3. *Bounded from below by zero when $L(i) = 0$*
4. *bounded from above by a positive constant $L(0) = K$*

[25] I interpret ψ as equivalent to the difference between a project's expected return and the return from lending such unit at the international markets is equal to $\rho_1 + \pi - 1 - (1-\alpha)\gamma_1^H - \alpha\gamma_1^L$ which, by Assumption 1, is strictly positive

[26] Function *kappa* is endogenous determined by the level of investment and liquidity hoarding chosen by entrepreneurs

[27] Function L plausibly reflects, for example, losses due to rises in unemployment or increases in financial fragility, for example

[28] Naturally the inclusion of this loss function is key to our results, otherwise, the agent with the fiscal capacity wouldn't have the motivation to provide liquidity ex-post

I assume that $L(j)$ is bounded from below at zero when full-scale reinvestment is reached ($j = i$). That is, there are no welfare gains for feasible reinvestment levels beyond initial scale. Additionally, $L(j^s)$ is convex to reflect that small levels of downsizing produce lower marginal losses than larger magnitudes.

Moreover, I assume that the loss function is bounded from above by a very large positive constant when projects shutdown. If this were not the case, a LOLR would always do whatever is necessary to prevent a complete shutdown, no matter the cost, thus eliminating interesting equilibrium results. This might be a plausible description for some economies, yet developing countries find costly to insure against all crises since opportunity costs are relatively higher. This upper bound reflects the inability to do "Whatever it Takes".

3.2 Timeline and Optimal Decision Problems

The LOLR and banking entrepreneurs are the only active decision makers in the model. Foreign lenders do not have a maximization problem, but, as described before, they are willing to lend to any entrepreneur as long as, the expected return is, at least, equal to opportunity cost at international markets. I describe the decision process starting from the final period up on to the initial period (Figure 4).

3.2.1 Period 2 - Limited Pledgeability, Limited Liability, and Fiscal Capacity

At this point in time, the state of the economy is described by the policy set $\Gamma(\bar{\mu})$, entrepreneurs balance sheet,[29] and the realized state of the world at $t = 1$ (*boom or stress*).

At the beginning of $t = 2$, banking entrepreneurs decide whether to abscond with share θ of project's total return or not. Following this decision, the LOLR collects $\mu\hat{R}\tau$ directly from entrepreneurs and $(1-\mu)\hat{R}\tau$ from projects whose banking entrepreneur didn't abscond. These resources are then used, together with $\gamma_1^s(F_0 - f_1)$, to redeem any bonds issued at $t = 1$.

[29] As it will be clear below, this is determined by contingent contracts K_1, K_0, and by any investments in foreign markets x_2

Lastly, banking entrepreneurs consume any disposable income after paying $\mu\hat{R}\tau$. At $t = 2$, entrepreneurs gross income consists of the project's net worth (n_2),[30] any additional return from international markets (x_2), and any resources rebated back by the LOLR after redeeming bonds (T_2).

I follow the usual definition of net worth where n_2 is equal to the difference between a project's assets ($\rho_1 j$) and its liabilities. These liabilities, at this point, consist of long term claims owed to foreign investors (l_f) and, potentially, any debt to the LOLR ($(1-\mu)\hat{R}^s\tau^s$).[31] Notice that a project's assets depend exclusively on the level of reinvestment made in the previous period (j).

$$n_2 = \rho_1 j - l_f + (1-\mu)\hat{R}\tau$$

Limited Pledgeability Foreign investors buy long term claims from entrepreneurs up to a project's pledgeable return.[32] That is, foreign lenders do not lend more than what is incentive compatible with abiding (not absconding).

The credibility of this decision rests on satisfying (3). When this incentive compatibility constraint holds, project's net worth (n_2) is sufficiently high such that it is in the benefit of entrepreneurs to follow through with claims.

$$n_2 \geq \theta\rho_1 j \tag{3}$$

Given projects balance sheet, satisfying (3) implies that pledgeable income at $t = 2$ is equal to $\rho_0 j = \rho_1(1-\theta)j$, and, more importantly, that it bounds both the value of long term claims sold in international markets and the payment to the LOLR that is collected directly from the project (4).

[30] which is contingent on absconding or not

[31] Total long term claims l_f is equal to the sum of long term claims sold at international markets at $t = 0$ (l_0) and $t = 1$ (l_1)

[32] Period 2 pledgeable return, denoted by ρ_0, is the maximum share per unit of reinvestment that an entrepreneur can credibly promise outside investors

$$\rho_0 j \geq l_f + (1-\mu)\hat{R}\tau \tag{4}$$

Assumption 3 establishes that projects are liquidity constrained in some states of the world.[33] Numeral 1 says that the marginal expected pledgeable income of the project with positive reinvestment only in boom states is not sufficient to cover the opportunity cost of foreign lenders at the initial period ($\gamma_0 = 1$).[34]

Additionally, Numeral 1 also implies that safe cash flow is not enough to finance solely full-scale reinvestment at $t = 1$ ($1 > \pi$). However, it is enough when paired with pledgeable income (Numeral 2). Thus, full-scale reinvestment is feasible during stress periods thru a combination of market and funding liquidity.

Assumption 3 (Liquidity Constrained Projects)

1. $1 > \pi + \alpha(\rho_0 - \gamma_1^L)$
2. $\frac{\rho_0}{1-\pi} \geq \gamma_1^H$
3. $\gamma_1^H \geq 1 > \rho_0$
4. $min\{\pi, \rho_0\} \geq \gamma_1^L$

Assumption 1 paired with numeral 1 of Assumption 3 implies that projects are socially valuable even in a stress episode ($\frac{\rho_1}{\gamma_1^H} > 1$). Thus, their continuation is warranted at full scale. However, I assume that projects are liquidity constrained at the initial period and during market stress events (Numeral 3).

In contrast, I assume that the funding cost during a boom state is relatively small ($min\{\pi, \rho_0\} \geq \gamma_1^L$). In fact, so small that, at in this state of the world, projects can self-finance any reinvestment ($\rho_0 > \gamma_1^L$).

Hence, with financial frictions, even if projects are capable of generating sufficient return to cover financing costs (Assumption 1), they can potentially shutdown because they don't

[33]If this were not the case, there is no need for liquidity management by entrepreneurs since it could always *finance-as-you-go* any reinvestment - See Tirole (2011)

[34]This is a necessary assumption so the initial investment scale is determined

produce enough *pledgeable* liquidity. In this environment, limited pledgeability is the symptom but moral hazard is the culprit for a project's inability to be liquid in all states of the world.

Limited Liability Entrepreneurs consumption at $t = 2$ is equal to the sum of projects net worth, any additional income derived from lending at world markets (x_2^s),[35] any transfers from LOLR (T_2) net of what the LOLR collects directly from entrepreneurs.

$$c_2^s = n_2^s + x_2 + T_2 - \mu \hat{R}^s \tau^s \tag{5}$$

Since c_2^s cannot be negative, limited liability sets an additional limit on the total amount of resources that an LOLR can extract form its economy in the last period.[36] Note that $n_2+x_2+T_2$ is the upper bound to what the LOLR can collect directly from each entrepreneur $(\mu\hat{R}\tau)$. Thus, the total return of a project plus total aggregate savings have to be, at least, enough to redeem long term claims sold to foreign lenders and to pay back fully the LOLR as well - (6). Unlike (4), this result is independent of an LOLR's fiscal capacity (μ).

$$x_2 + T_2 + \rho_1 j \geq l_f + \hat{R}\tau \tag{6}$$

Fiscal Capacity is a key feature for whether public liquidity provision alleviates moral hazard or not. To see this, note that the share of what an LOLR charges directly to a banking entrepreneur is collected even if it decides to abscond.[37] As a result, limited pledgeability sets a limit on what an LOLR collects from projects $((1-\mu)\hat{R}\tau)$ but not on what it can collect directly from entrepreneurs $(\mu\hat{R}\tau)$. Clearly, if LOLR can choose μ, then it is weakly optimal to set μ equal to $\bar{\mu}$ since it maximizes the share of the revenue that is not limited by pledgeable income. I assume that this holds hereafter.

A better way to see the importance of $\bar{\mu}$ is by comparing extreme values: for a LOLR

[35]Which is equal to $\gamma_1^s(x_1^s - M_1^s - c_1^s)$
[36]The first limit is (4)
[37]See Incentive Compatibility Constraint - Equation 3

with fully developed fiscal capacity ($\bar{\mu} = 1$), $\hat{R}\tau$ is bounded by Equation 6 while, for a LOLR with $\bar{\mu}$ equal to zero, $\hat{R}\tau$ is bounded by Equation 4, which, since $\rho_1 > \rho_0$, is strictly lower.

Upper bounds on $\hat{R}\tau$ matter because these resources are collected to redeem bonds issued at $t = 1$, and, as a result, set a limit on the amount that can be issued. However, recall that the LOLR also collects F_0 at $t = 0$ which can be used to cover some share of the demand for loans reducing the amount of bonds that need to be issued in the first place. This idea is depicted in the model through foreign lenders participation constraint (7).

$$\gamma_1^s \tau - \gamma_1^s F_0 \leq \hat{R}\tau \tag{7}$$

Foreign lenders buy bonds from LOLR as long as the share of liquidity demand not covered with reserves valued at $t = 2$ ($\gamma_1^s \tau - \gamma_1^s F_0$) is less or equal to $\hat{R}\tau$. It is worth mentioning that to derive this condition it is not necessary to assume that reserves are used as collateral to these bonds. In fact, it is sufficient to assume that reserves can be used to cover a share of τ and that an LOLR can only use up to F_0 to do so.[38]

$$\bar{R}(\tau, F_0) \geq \begin{cases} \gamma_1^s & \text{if } \tau = 0 \\ max\{0, \gamma_1^s \left[1 - \frac{F_0}{\tau}\right]\} & \text{if } \tau > 0 \end{cases} \tag{8}$$

I define function $\bar{R}(\tau, F_0)$ as the minimum cost of public liquidity for bonds to be redeemable. As long as $\hat{R}$ is equal or greater to $\bar{R}(\tau, F_0)$, (7) is satisfied and LOLR's bonds are bought by foreign lenders.

For positive values of τ, $\bar{R}(\tau, F_0)$ is a non-increasing function with respect to F_0 with an upper bound equal to the return observed in international markets when F_0 are zero. Accumulating reserves, then, allows an LOLR to offer its domestic economy a financing source that is less expensive than foreign lenders opportunity cost. At this point, it should

[38]This result is relevant because it implies even if I assume that a central bank controls reserves and that bonds are issued by a central government, fiscal capacity affects the decision to accumulate reserves as long as the central bank dislikes liquidation of projects

be clear the relationship between fiscal capacity and reserves. That is, reserves alleviate the constraint on bond issuance by reducing the necessary amount to issue. A feature that is more valuable for an LOLR with lower fiscal capacity.

3.2.2 Period 1 - Boom or Market Stress

At the onset of $t = 1$, projects produce a safe cash flow return πi which is allocated between entrepreneurs and foreign investors as determined by K_0.[39] Once πi is distributed, the aggregate shock is realized. The amount of market liquidity in hands of entrepreneurs (x_1^s) as well as the savings in hands of the LOLR (F_0), and LOLR's fiscal capacity ($\bar{\mu}$) completes the description of the state of the economy at this moment in time.

Banking entrepreneurs, as (9) shows, can use x_1^s to consume immediately, to invest in the project M_1^s, and/or to lend at international markets to obtain x_2 at $t = 2$. Naturally, x_2 is positive only when the following inequality doesn't bind.

$$c_1^s + M_1^s \leq x_1^s \tag{9}$$

Moreover, entrepreneurs have the option to return to world markets a second time to sell additional long-term claims and acquire the necessary funds to reinvest.[40] They offer foreign investors a contract $K_1^s = \{j^s,\ M_1^s,\ \phi_1^s,\ \tau^s,\ l_1^s\}$.

Reinvestment (j^s) is financed using entrepreneur's market liquidity (M_1^s), the transfer from the LOLR (τ^s), and with foreign funds ($\phi_1^s j$). Moreover, I assume that j^s cannot be greater than the initial investment scale to capture that the scale of the model is set at $t = 0$ and it cannot be changed at $t = 1$.

$$j^s = min\{\frac{M_1^s + \tau^s}{1 - \phi_1^s}, i\} \tag{10}$$

[39] A contingent contract between an entrepreneur and foreign lenders determined at $t = 0$

[40] I assume that, even under a stress period, this small open economy keeps her access to international markets

Contract K_1^s has to be attractive enough for foreign lenders. To do so, the date-2 value of claims sold at $t = 1$, denoted by l_1^s, is at least equal to the expected return in international markets times the amount borrowed (11).

$$\gamma_1^s \phi_1^s j^s \leq l_1^s \tag{11}$$

Additionally, K_1^s must also be incentive compatible such that the entrepreneur doesn't abscond. Therefore, l_1^s and τ^s are constrained by (4).[41]

$$C_{1,2}^s = c_1^s + c_2^s \tag{12}$$

At $t = 1$, either under a boom or under a stress state, entrepreneurs want to maximize their consumption at periods 1 and 2 ($C_{1,2}^s$). To do so, an entrepreneur chooses non-negative set $\{c_1^s, K_1^s\}$ subject to (4), (5), (9), (10), (11), and policy set $\Gamma(\bar{\mu})^s$.

The LOLR, simultaneously, establishes a liquidity provision program, as described above, with the objective to minimize the potential welfare losses due to partial liquidation of projects.

At $t = 1$, any demand for public liquidity by entrepreneurs is covered by either issuing bonds or by depleting reserves. Naturally, f_1 is limited by the amount of reserves that were collected at $t = 0$.

$$B_1 + f_1 = \tau^s \tag{13}$$

$$f_1 \leq F_0 \tag{14}$$

Given that the LOLR is only concern about reinvestment scale, there is no reason to set $\hat{R}$ above $\bar{R}(\tau, F_0)$. Therefore, at t=1, a LOLR observes τ and, given F_0, sets $\hat{R}$ equal to $\bar{R}(\tau, F_0)$. This function guarantees satisfying (13), (14), and (7).

[41] Since pledgeable return is contingent on reinvesting, l_1^s has seniority over any previous long term liabilities l_0^s.

Note that what ultimately determines what happens during $t = 1$ are the state variables $\{F_0,\ x_1^s, \bar{\mu},\ \gamma_1^s\}$, where the first two are chosen in the initial period.

3.2.3 Period 0 - Project's initial scale and Reserves Accumulation

At $t = 0$, the LOLR collects F_0 from the domestic economy while banking entrepreneurs consume C_0 and invest in their project. To do so, the latter offer contract K_0 to foreign investors that stipulates the initial investment scale i, the amount of entrepreneur's market liquidity to be invested M_0, and the total amount to borrow ϕ_0 from investors which is collected by issuing short-term debt $(d_f^L i, d_f^H i)$ and long-term debt (l_0^L) contingent on the state of world. That is, K_0 is equal to set $\{i,\ M_0,\ \phi_0,\ d_f^L i,\ d_f^H i,\ l_0^L\}$

A project's initial investment is covered with market liquidity and borrowing from foreign lenders. In turn, market liquidity at the initial period is bounded by an entrepreneurs disposable endowment.[42] However, this endowment can also be used to consume (c_0) and to lend in international markets at the initial period (x_A), thus, the possible uses for $A - F_0$ are described by (15).

$$c_0 + M_0 + x_A = A - F_0 \tag{15}$$

Similarly to $t = 1$, attracting foreign lenders requires that entrepreneurs offer the same expected return than international markets.[43] The opportunity cost of a foreign lender at $t = 1$ is $\bar{\gamma}_0$ which is normalized to one. Thus, a project's borrowing capacity at the initial period is given by (16).

$$(i - M_0) = E_s\left[l_0^s + d_f^s i\right] \tag{16}$$

Foreign lenders expected return depends on the return offered through short-term and

[42] The initial endowment minus the amount collected by the LOLR $(A - F_0)$

[43] Entrepreneurs could offer an expected return higher than international markets but, under this setting, it is not optimal because it doesn't increase investment while it does reduces entrepreneurs expected payoff

long-term claims. Since the aggregate shock is observable to all,[44] I focus on contingent debt. That is, payoffs depend on the realized shock.

Short-term claims are backed up by the safe-cash flow produced by projects at $t = 1$.[45] By offering a higher payoff d^s_f, an entrepreneur increase the amount it can borrow abroad but, by doing so, it reduces the amount of resources available to reinvest through market liquidity at $t = 1$ as shown by (18). As it is shown below, these resources are key to determine whether a project continues o gets shutdown during episodes of market stress.

$$d^s_f i + d^s_e i = \pi i \tag{17}$$

$$d^s_e i + x_A = x^s_1 \tag{18}$$

In turn, long-term claims issued at the initial period are bounded by available pledgeable net worth valued at $t = 0$ (19).[46] Consequently, depending on the state of the world and decisions made at $t = 1$, there could be positive pledgeable income left to back up l^L_0.

Similar to Farhi and Tirole (2012), I focus on a contract where long-term claims are not available for stress periods. This assumption is justifiable if foreign lenders do not observe x^H_1 before buying long-term claims and, thus, are reluctant to buy claims for states of the world where projects need market liquidity to achieve any continuation.[47]

$$l^L_0 \ \epsilon \big[0,\ \rho_0 j^L - l^L_1 - (1 - \bar{\mu})\hat{R}\tau^L\big] \tag{19}$$

At the initial period, entrepreneurs want to maximize their expected consumption (20). To do so, it chooses non-negative set $\{c_0, x_a, K_0\}$ subject to (15), (16), (17), (18), (19) and

[44] After all, it is the realized return at international markets

[45] See (17)

[46] Pledgeable return depends on reinvestment, then, it is plausible to assume that liabilities that finance this reinvestment, such as l^L_1 and $(1 - \bar{\mu})\hat{R}\tau^L$, have seniority over l^L_0

[47] Although it makes the model more tractable, this assumption does eliminate potentially interesting outcomes. For example, in principle, foreign lenders should be more willing to buy long-term claims for states that they anticipate that an LOLR's will provide liquidity assistance. This would increase the amount that projects could borrow and increase initial investment levels

a given collection of F_0.

$$c_0 + E_s\left[C^s_{1,2} - l^s_0\right] \tag{20}$$

As said previously, the only action from the LOLR during the initial period is to collect F_0 from its domestic economy. Given that entrepreneurs are protected by limited liability, LOLR's minimize their policy objective (2) subject to $F_0 \leq A$.

3.3 Brief Discussion of modeling choices

The basic setup of this model has a similar flavor to Farhi and Tirole (2012). However, I model a small open economy to study why countries accumulate reserves which requires minor but yet important modifications.

First, I introduce an LOLR that can have different levels of fiscal capacity. This is intended to capture both developing countries where an important share of economic activity is out of the grasp of taxes and non-financial penalties,[48] as well as advanced economies where this is not the case. Thus, this modeling approach is more general to what is usually observed in the liquidity literature for the public liquidity provider.[49]

Ultimately, fiscal capacity determines to what extent is the LOLR subject to the same financial frictions as private agents. Hence, in contrast to Farhi and Tirole (2012), I explicitly model a financial friction that creates a wedge between total and pledgeable income.

To do so, I assume that entrepreneurs can abscond with a share of the total return of the project. Meanwhile, Holmström and Tirole (1998) model this gap as the result of entrepreneurs choosing different effort levels. At the end, both wedges come from the possibility of capturing a private benefit. What is key for my model is that such private benefit exists and that the level of fiscal capacity determines how much of it can be collected by the

[48] Consider the case of the informal economy

[49] Besides Holmström and Tirole (1998) and Farhi and Tirole (2012), see Holmström and Tirole (2011) for example where governments have a fully developed fiscal capacity which is the case when $\bar{\mu}$ is equal to 1 in my model

LOLR.

Second, Farhi and Tirole (2012) mainly study a liquidity provision program involving reductions in the economy's interest rate.[50] However, following Rey (2015), monetary independence is off the table for small open economies when financing costs are driven by a global financial cycle.[51] Thus, instead, I assume that the liquidity program consists of transfers between a LOLR and its domestic agents at a cost. This type of policy has the added advantage, given my research question, that it is an instrument available in both advanced and developing economies, and, thus, suited for comparability.

Third, as it is a more natural setting for a small open economy, the liquidity shock comes from random funding costs[52] whereas, in Farhi and Tirole (2012), the liquidity shock is modeled as a potential need for reinvestment. Although, at first, these might seem as two different modeling choices, they are not. As discussed by Tirole (2011), what creates the demand for liquidity is the inability to *finance as you go* outlays in some states of the world. Thus, from the perspective of liquidity management, the *no crisis state* in Farhi and Tirole (2012) is equivalent to the *boom state* in my model.

Some comments about reserves. I interpret accumulating F_0 in the initial period by a LOLR equivalent to accumulating foreign reserves. The parallel is straight forward with the International Monetary Fund's definition of such assets.[53] The stock of F_0 is in control of the LOLR and available for its immediate use. Additionally, these resources are assets (claims on foreign lenders) of the small open economy that imply a carry cost since these resources are invested in the technology with lower return (Assumption 1). Finally, as in

[50] In a section of their paper, these authors compare changes in interest rates with direct transfers. They argue that, as long as the liquidity provider cannot perfectly identify firms that are in distressed from those that are not, then interest rates is a preferable policy tool

[51] Rey (2015) shows that the existence of this global force transformed the open macroeconomic trilemma into a "irreconcilable duo" where national monetary independence is only possible when a managed capital account; regardless of the exchange rate regime

[52] This is similar to Holmström and Tirole (1998) with the exception that in their model the cost is continuous while in this model, for tractability, there are only two possible states

[53] See Chapter 6 of the IMF's Balance of Payments and International Investment Position Manual - Sixth Edition

this model, foreign reserves are accumulated in part for precautionary purposes.[54]

Having said that, note that reserves are in control of a lender of last resort. I interpret the LOLR in this economy more as a crisis lender/manager which, as Fischer (1999) discusses, doesn't necessarily need to be a central bank or monetary authority. Hence, the LOLR is closer to a general government than to a central government or a central bank. This wider interpretation implies, for example, that this model's definition of reserves include external assets that are not in direct control of central banks (i.e. sovereign wealth funds).

Lastly, in this environment, an LOLR's is indifferent between issuing debt or depleting its stock of reserves to cover τ. This might seem as a strong assumption. One could add a dead-weight cost to bond issuance which would push LOLR's to fully deplete its stock of reserves before considering issuing any new debt. However, some countries have been reluctant to use their reserves as the primary tool to provide liquidity, even during severe crisis.[55] Basu et al. (2018) argues that this reluctance can be explained because reserves are an instrument with a zero lower bound.[56] In turn, Chamon et al. (2019) suggest that most of the benefit of reserves comes from their role *off equilibrium*, and, as a result, they are almost never used. I abstract, for now, from the analysis of reserves management and fiscal capacity and leave this question open for future work.

4 Laissez Faire Equilibrium

Throughout this paper, I focus on finding Subgame Perfect Nash Equilibria where entrepreneurs don't abscond and the LOLR issues safe bonds to provide liquidity ex-post successfully. This type of equilibrium has the advantage that agents strategies are *time-consistent*. I start with the equilibrium where there is no LOLR liquidity provision policy.[57]

[54] See empirical evidence of this motive in Aizenman and Lee (2007)

[55] See International Monetary Fund (2011)

[56] A country could run out of reserves

[57] Even if an LPP is available, this equilibrium would prevail when $\hat{R}$ is too expensive relative to international markets funding costs such that the LPP is never attractive for entrepreneurs

Definition 1 (Laissez Faire Equilibrium (LFE))
A Laissez Faire Subgame Perfect Nash Equilibrium where banking entrepreneurs' don't abscond is characterized by the following strategy profile

- *Date-2: Entrepreneurs' don't abscond*
- *Date-1:* $\{c_1^s, K_1^s\}_{L,H}$ *solve entrepreneurs date-1 problem*
- *Date-0:* $\{c_0, x_A, K_0\}$ *solve entrepreneurs problem at the initial period*

I present the full derivation of entrepreneurs optimal behavior in a laissez faire environment in the Appendix (A). However, at this point, it is worth highlighting that the driving force behind credit rationed agents is the trade-off between initial investment scale and insurance. To see this, note that optimal investment is given by

$$i = A\kappa(\bar{x}_1^L, \bar{x}_1^H)$$

where $\kappa(\bar{x}_1^L, \bar{x}_1^H)$ is project's equity multiplier as a function of how much entrepreneurs choose to hold liquidity for each state.[58] In turn, the equity multiplier is equal to

$$\kappa(\bar{x}_1^L, \bar{x}_1^H) = \frac{1}{1 - \pi - \alpha(\rho_1 - \gamma_1^L) + \alpha\bar{x}_1^L + (1-\alpha)\bar{x}_1^H}$$

which is a decreasing function with respect to $\{x_1^s\}_{L,H}$, always positive due to Assumption 3, and greater than 1.

The equity multiplier reflects directly the trade-off between insurance and investment scale. By hoarding greater levels of x_1^S, an entrepreneur increases the continuation level of projects at $t = 1$ but, in turn, it sacrifices initial investment scale.

As a result of this trade-off, there are two types of Laissez Faire Equilibria in this model depending on the value of some parameters.[59]

In the *No Crisis Equilibrium*, entrepreneurs optimally hoard liquidity to guarantee the continuation of projects during every state of the world. More specifically, entrepreneurs

[58] As is discussed in Appendix A, $\bar{x}_1^S$ denotes the amount of liquidity holdings per unit of investment (x_1^s/i)
[59] The full description and proof of the LFE can be found in the appendix

choose x_1^H to be $i\big[1-\frac{\rho_0}{\gamma_1^H}\big]$ which is the minimum amount of market liquidity to complement with the maximum possible funding liquidity $(i\frac{\rho_0}{\gamma_1^H})$ to reach full-scale reinvestment.

Proposition 1 (No Crisis - LFE)
In a No Crisis Equilibrium, an entrepreneur loads up in liquidity up to $i\big[1-\frac{\rho_0}{\gamma_1^H}\big]$ *which allows it to complement with funding liquidity and continue at full-scale when a stress episode is realized*

In contrast, in the *Sudden Stop Equilibrium*, entrepreneurs don't hoard liquidity at all ($x_1^H = 0$) which means that, if a stress episode happens, projects are forced to shutdown.

I characterize this last equilibrium as *Sudden Stop* because the small open economy is unable to attract foreign lending when funding costs are high. However, this is not because international markets are unwilling to lend to entrepreneurs but, instead, it is due to its inability to provide both sufficient pledgeable income and own market liquidity to attract expensive resources from abroad.

Proposition 2 (Subject to Sudden Stops - LFE)
Define ω *as follows*

$$\omega = \frac{\pi + \rho_0\frac{\gamma_1^H - 1}{\gamma_1^H} - \gamma_1^L}{1 + \rho_0\frac{\gamma_1^H - 1}{\gamma_1^H} - \gamma_1^L}$$

In a Laissez Faire environment, a market stress event turns into a Sudden Stop if $(1-\alpha) \leq \omega$

Proposition 2 states that the existence of one equilibrium or the other depends on whether the probability of the stress period is higher or lower than a threshold. Entrepreneurs', then, hoard liquidity when the stress event is not *rare*. Once again, this result highlights that partial insurance is optimal in these type of models as argued by Holmström and Tirole (1998).

To highlight the working forces in this model that drive existence of equilibria, I define set $\Omega(\omega) : \{z|z \leq \omega\}$ and present some comparative statics (Corollary 1). Note that, by definition, stress episodes with a probability that is part of set Ω turn into a sudden stop if realized.

Let me start with what happens when γ_1^H increases. In such scenario, more liquidity hoarding is required to insure projects during stress periods which implies a greater sacrifice in investment scale. Naturally, entrepreneurs will require higher probabilities of occurrence to compensate for this cost. Now, greater γ_1^L has the opposite effect. A boom with greater funding costs actually reduces investment scale lowering the cost of insurance.

Corollary 1 (Comparative Statics - Ω)
Set Ω is expanding in the funding cost during stress periods (γ_1^H), and the safe cash flow (π) while it is contracting in the funding cost during booms (γ_1^L), and the level of moral hazard (θ)

Proof Choose an initial value for ω and define the appropriate set $\Omega(\omega)$. Choose any z that belongs to $\Omega(\omega)$. Define ω' such that $\gamma_1^H < \gamma_1^{H\prime}$ or $\pi < \pi'$. Note that z also belongs to $\Omega(\omega')$. Now, choose h equal to ω. By construction, h belongs to $\Omega(\omega)$. Define $\hat{\omega}$ such that $\gamma_1^L < \gamma_1^{L\prime}$ or $\theta < \theta'$. Note that h doesn't belong to $\Omega(\hat{\omega})$. ∎

Since the decision to hoard depends on the trade-off between investment scale and continuation, then, not surprisingly, banking entrepreneurs don't hoard any liquidity for booms in both type of equilibria. This decision follows from modelling booms as states where projects can finance as they go their reinvestments ($\rho_0 > \gamma_1^L$). As such, there is no point in incurring in costly liquidity hoarding.

5 Equilibria with LOLR - No Reserves

Results of the LFE suggest that an LOLR's welfare improving role is warranted for *rare* stress periods.

Naturally, the results of the model when an LOLR is present hinges on how banking entrepreneurs respond. In Appendix B I derive the optimal behavior of banking entrepreneurs, let me briefly present the intuition behind this behavior.

Replacing (5) and (11) with equality, an entrepreneur's objective function at $t = 1$ can be written as follows

$$\left[1 - \gamma_1^s\right]c_1^s + \left[\rho_1 - \gamma_1^s\right]j^s + \gamma_1^s x_1^s + \left[\gamma_1^s - \hat{R}\right]\tau$$

Hence, for all feasible $\hat{R}$ considered, demanding a loan directly from the LOLR has a direct and an indirect benefit. If $\hat{R} < \gamma_1^H$, τ is marginally less expensive than ϕ^1 and M_1 which increases entrepreneurs marginal payoff (*price benefit*). This effect is reflected directly in the payoff function.

Moreover, even when $\hat{R} = \gamma_1^H$, a unit of τ loosens (4) relative to a unit of ϕ_1 as long as $\bar{\mu}$ is not zero. This allows projects to attract more liquidity through their liabilities *(indirect effect)*.

However, this indirect effect is only valuable when a binding (4) prevents greater reinvestment.[60] If not, public liquidity looses a comparative advantage.

Consequently, in case of a boom, banking entrepreneurs are indifferent between private or public funding when $\hat{R}$ is equal to γ_L. This holds because projects reach full-scale reinvestment borrowing from foreign lenders. Thus, without loss of generality, I focus in an equilibrium where τ^L is equal to zero.

This is not true during a market stress. To illustrate the role that fiscal capacity has through a LOLR's *indirect effect* in providing liquidity, I abstract from reserves accumulation for the moment. Thus, for now, $\hat{R}$ is equal to γ_1^H.

The indirect effect of LOLR liquidity provision is positive for $\bar{\mu}$ such that

$$(1-\bar{\mu})\gamma_1^H \leq \rho_0$$

That is, fiscal capacity is sufficiently developed to overcome moral hazard to some degree by having access to some share of entrepreneurs' private benefit.

$$(1-\mu_A)\gamma_1^H = \rho_0 \tag{21}$$

The threshold at which fiscal capacity is *sufficient*, denoted as μ_A, is given by (21). This happens when the marginal cost of public liquidity on total pledgeable income is equal to

[60] States of the world when γ_1^s is greater than ρ_0

pledgeable income per unit of reinvestment. Thus, one unit of τ increases, at least, as much total pledgeable income as it increases its cost.

I denote the group of LOLR that belong to interval $\left[\mu_A, 1\right]$ as Mature. Interestingly, there is no need to have a fully developed fiscal capacity to be mature in this environment. Instead, it suffice to have just enough to compensate for the wedge between the demand for liquidity and pledgeable income.[61]

For a LOLR that is mature, I find the Mature Fiscal Capacity Equilibrium. Proposition 3 summarizes the main characteristics of this equilibrium.[62]

Proposition 3 (Mature Fiscal Capacity Equilibrium - ME)
Whenever $\bar{\mu} \epsilon \left[\mu_A, 1\right]$, and Assumptions 1 and 3 hold, the following characterizes the Mature Fiscal Capacity Equilibrium (ME)

- *Entrepreneurs expect $\hat{R}$ equal to γ_1^S in each state and don't hoard liquidity for neither $\{x_1^s = 0\}_{L,H}$*
- *The LOLR doesn't accumulate reserves ($F_0 = 0$) at $t = 0$*
- *If a boom materializes, entrepreneurs reinvest at full-scale using funding liquidity*
- *If a stress event materializes, banking entrepreneurs demand i to the LOLR who issues i abroad, entrepreneurs reinvest at full-scale*
- *At $t = 2$, after a market stress, LOLR collects $\gamma_1^H i$ to redeem bonds while entrepreneurs finally consume $(\rho_1 - \gamma_1^H)i$*

Similar to the No Crisis LFE, the small open economy achieves full-scale reinvestment and eliminates the possibility of a Sudden Stop. However, in contrast to the No Crisis LFE, there is no holding of private liquidity. Therefore, in an episode of market stress, any borrowing by this economy is done through the intermediation of the LOLR.

Note that the existence of the ME is independent of the probability of a crisis. That is, when a mature LOLR provides assistance, it eliminates the need to hoard liquidity even for probabilities that are relatively high or which in a LFE would be a NO Crisis Equilibrium. The reason is that a Mature LOLR intervention, in practice, *completes* markets by overriding

[61] In fact, note that μ_A is exactly equal to the amount entrepreneurs hoard per unit of investment in a NO Crisis LFE

[62] I refer the reader to the appendix for the proof and complete set of strategies

financial frictions. There is no point to insure against a stress event at the cost of investment scale if a LOLR can provide liquidity at any time.

For LOLR that are not mature ($\bar{\mu} \leq \mu_A$), there exist, once again, two equilibria whose existence depends on the probability of a market stress. Proposition 4 summarizes the main characteristics of this case.

Proposition 4 (No Reserves Equilibria)
In a environment with LOLR intervention but no reserves accumulation, as long as $\bar{\mu} \,\epsilon\, \big[0, \mu_A\big[$, Assumptions 1, and 3 hold, and entrepreneurs expect $\hat{R}$ equal to γ_1^H

1. *if $(1-\alpha) \leq \omega(\bar{\mu}, \gamma_1^H)$ there is a* ***Sudden Stop Equilibrium - No Reserves*** *where*
   - *Entrepreneurs expect $\hat{R}$ equal to γ_1^S in each state and don't hoard liquidity for neither $\{x_1^s = 0\}_{L,H}$*
   - *If a stress event materializes, banking entrepreneurs don't have enough market liquidity so the LOLR can issue bonds*
   - *The economy can't borrow and projects shutdown*

   if $(1-\alpha) > \omega(\bar{\mu}, \gamma_1^H)$ there is a ***No Crisis - Private Hoarding Equilibrium*** *where*
   - *Entrepreneurs expect $\hat{R}$ equal to γ_1^S in each state and hold liquidity only for stress episodes equal to $x_1^H = i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]$*
   - *If a stress event materializes, banking entrepreneurs borrow $\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H} i$*
   - *The economy borrows through LOLR who issues B_1*
   - *Banking Entrepreneurs reinvest at full-scale*

Consequently, unlike Mature LOLR, liquidity provision backed up by lower fiscal capacity fails to eliminate the existence of a Sudden Stop.

Having said that, I define function $\omega(\bar{\mu}, \hat{R})$ in (22). When evaluated at γ_1^H, it determines the existence of which equilibrium presented by Proposition 4. Hence, this function establishes the probability threshold at which the economy shifts from a No Crisis to a Sudden Stop Equilibrium for policy pairs $\{\hat{R}, \bar{\mu}\}$ such that $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$.

$$\omega(\bar{\mu}, \hat{R}) = \frac{(\rho_1 - \rho_0)\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] + \big[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\big]\big[(\rho_0 - \gamma_1^L) - (1-\pi)\big]}{(\rho_1 - \rho_0)\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] + \big[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\big](\rho_0 - \gamma_1^L)} \tag{22}$$

Note that threshold ω from Proposition 2 is equal to $\omega(\bar{\mu}, \hat{R})$ when evaluated at $\{0, \gamma_1^H\}$. This highlights that the intermediation of a LOLR who has no fiscal capacity nor the ability to accumulate reserves cannot play a welfare improving role by intermediation between lenders and borrowers.

In contrast, the limit of $\omega(\bar{\mu}, \hat{R})$ as $\bar{\mu}$ tends to μ_A converges to

$$\frac{(\rho_0 - \gamma_1^L) - (1 - \pi)}{(\rho_0 - \gamma_1^L)}$$

which is the minimum probability of a crisis that is consistent with Numeral 1 of Assumption 3. Therefore, as fiscal capacity gets closer to μ_A, the smaller is the set of market stress probabilities with Sudden Stop equilibria.

Corollary 2 (Role of Fiscal Capacity)
For any LOLR with fiscal capacity strictly above zero, there is at least one probability of market stress, that with public liquidity provision, shifted from a Sudden Stop to a No Crisis.

Proof As discussed previously, Mature LOLR eliminate sudden stop equilibria. Now, I consider the case for $\bar{\mu}$ strictly between $]0, \mu_A[$. Choose pair $\{\gamma_1^H, 0\}$ and define set $\Omega(\gamma_1^H, 0) = \{z \mid z \leq \omega(\gamma_1^H, 0)\}$. By construction, note that $\omega(\gamma_1^H, 0) = \omega$, thus $\Omega(\gamma_1^H, 0)$ is contained in Ω. Select z equal to ω. Note that $z \epsilon\ \Omega$. Choose any $\bar{\mu}'$ strictly between $]0, \mu_A[$ and define set $\Omega(\gamma_1^H, \bar{\mu}')$ Since $\bar{\mu}' > 0$ and $\omega(\hat{R}, \bar{\mu})$ is strictly decreasing $(\rho_1 > \hat{R})$ with respect to $\bar{u}$ for any $\hat{R}$ including γ_1^H, then $z > \omega(\hat{R}, \bar{\mu}')$ and, thus, z doesn't belong to $\Omega(\gamma_1^H, \bar{\mu}')$ ∎

Corollary 2 states that in economies with an LOLR with some fiscal capacity ($\bar{\mu} > 0$) provision of public liquidity shifts at least one probability from a Sudden Stop in LFE to a No Reserves - No Crisis Equilibrium.

As argued previously, banking entrepreneurs face a trade-off between investment scale and insurance. Since public liquidity provision with some fiscal capacity ($\bar{\mu} > 0$) expands to some degree pledgeable income, it reduces the amount of liquidity that needs to be hoarded at $t = 0$.[63] Thus, in return, banking entrepreneurs are willing to insure against a market stress event with lower probability.

[63]From $1 - \frac{\rho_0}{\gamma_1^H}$ to $1 - \frac{\rho_0}{(1-\bar{u})\gamma_1^H}$

Thus, LOLR intermediation decreases the amount of private liquidity holdings but, at the same time, it increases the set of probabilities of full-scale reinvestment.

However, this effect is not true when a LOLR has no fiscal capacity ($\bar{\mu} = 0$). In this case, the LOLR has to rely in other instruments to close the gap between pledgeable and total return. One possibility is to accumulate foreign reserves which I analyze below.

6 LOLR Equilibrium - With Reserves

Accumulating reserves allows an LOLR to offer a cheaper funding source. Thus, public liquidity provision increases pledgeability, relative to the laissez faire scenario, through this price effect. Higher pledgeability potentially provides room for more reinvestment.

However, accumulating reserves can end up being wasteful since they are not state-contingent, and, when a boom materializes, an LOLR deviates resources from more productive investments without reaping any of the benefit.[64] Clearly, similar to entrepreneurs, LOLR face a trade-off between insurance and scale.

I derive the optimal behavior of an LOLR in Appendix D. I show that, when there is a positive τ at $t = 1$, a LOLR guarantees that bonds issued at international markets are redeemable at $t = 2$ by setting $\hat{R}$ equal to (8). Moreover, without loss of generality, I assume that any stock of reserves that goes unused is rebated to entrepreneurs at the end of $t = 2$.[65]

At $t = 0$, the LOLR faces the trade-off between insurance and scale. Accumulating reserves is not optimal when the economy already has other sources to compensate for moral hazard. Two states of the economy fall within this realm. The first is when the LOLR has sufficient fiscal capacity ($\bar{\mu} \geq \mu_A$) while the second state is when the private sector holds enough liquidity already $x_1^H \geq i(1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}$.[66] The intuition for this optimal behavior is

[64] Rodrik (2006) estimated this cost close to an annual premium of 1% of GDP for emerging economies

[65] This assumption is possible since LOLR only cares about reinvestment scale, and not the utility of entrepreneurs. If it did, then, an LOLR would increase utility by rebating reserves at $t = 1$ when a boom is realized. This wouldn't change any results since entrepreneurs would consume any extra market liquidity since $\gamma_1^L < 1$

[66] Note that having a higher $\bar{\mu}$ implies that entrepreneurs need to hold a lower amount of liquidity to reach

simple: no need to incur in the opportunity cost when reserves don't provide additional reinvestment.

Corollary 3 (LOLR - Mature Fiscal Capacity Equilibrium)
Whenever $\bar{\mu} \geq \mu_A$, an LOLR would optimally choose F_0 equal to zero in the Mature Fiscal Capacity Equilibrium described in Proposition 3

Proof Choose $\bar{\mu} \geq \mu_A$. Proposition 3 shows that full-scale reinvestment is reached for any x_1^A when $F_0 = 0$. Choosing no stock of reserves, then, generates an expected welfare cost of zero. Suppose that there exists a positive F_0 that creates a lower expected welfare costs. This is not possible since reinvestment cannot be greater than initial investment. Thus, for any $F_0 > 0$, $\psi F_0 \kappa(x_1^H)$ is strictly greater than zero. ∎

Corollary 4 (LOLR - No Crisis Private Hoarding Equilibrium)
*Define set $\Omega(\bar{\mu}, \hat{R}) = \{\ z \mid z \leq \omega(\bar{\mu}, \hat{R})$ Whenever $\bar{\mu} < \mu_A$, and $(1-\alpha) \ \epsilon\ \Omega^C(\bar{\mu}, \gamma_1^H)$, an LOLR would optimally choose F_0 equal to zero in the **No Crisis Equilibrium - Private Hoarding** described in Proposition 4*

Proof Choose $\bar{\mu} < \mu_A$. Proposition 4 shows accumulating $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]$ by entrepreneurs is a best response to F_0 equal to zero when $(1-\alpha) \ \epsilon\ \Omega^C(\bar{\mu}, \gamma_1^H)$. Given that $x_1^H = i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]$, would an LOLR choose a F_0 equal to zero? The answer is yes. The reason is that with at this level, entrepreneurs reach full-scale reinvestment during market stress with $\hat{R}$ at γ_1^H, thus it generates an expected welfare cost of zero. Suppose that there exists a positive F_0 that creates a lower expected welfare costs. This is not possible since reinvestment cannot be greater than initial investment. Thus, for any $F_0 > 0$, $\psi F_0 \kappa(x_1^H)$ is strictly greater than zero. ∎

Corollary 3 states that when a Mature LOLR has the possibility to accumulate reserves, it chooses not to. So, in effect, F_0 equal to zero is optimal in a Mature Fiscal Capacity Equilibrium (Proposition 3).[67] Likewise, if banking entrepreneurs hold sufficient liquidity to reinvest at full-scale as in the **No Crisis Equilibrium - Private Hoarding**, then an LOLR with $\bar{\mu} < \mu_A$ would choose F_0 equal to zero as well (Corollary 4).

What is the case for a LOLR with low fiscal capacity when private liquidity holdings lie strictly between 0 and $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})}\big]$? The actual optimal response depend on the specifics

this state.

[67]This result, of course, could be different if an LOLR incurred in some dead weight loss when issuing bonds. However, this dead weight loss has to be sufficiently high in expected terms to compensate for the opportunity cost of accumulating reserves. Moreover, it is plausible to assume that any dead weight loss of issuing bonds is lower in economies with greater than with lower fiscal capacity. Hence, it the qualitative implications of the model would remain the same

of $L(J^H)$ and parameter values. Yet, still something can be said about the general lines of this behavior. If x_1^H is close enough to $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})}\big]$, the marginal benefit of increasing reinvestment is around zero since already j^H is close to i while the marginal opportunity cost of F_0 is positive. Thus, it is possible that this LOLR accepts some partial liquidation of projects before accumulating reserves. Now, as X_1^H tends to zero, the marginal benefit of higher reinvestment due accumulating reserves increases while its marginal opportunity cost remains constant. Thus, it is possible to reach an interior solution where both entrepreneurs and the LOLR hoard liquidity.

In turn, if x_1^H is zero, the small economy finds itself at the doors of complete shutdown. LOLR intervention, characterized by pair $\{\bar{\mu}, \hat{R}\}$, is attractive for banking entrepreneurs as long as (23) holds.

$$(1 - \bar{\mu})\hat{R} \leq \rho_0 \tag{23}$$

Condition 23, by definition, doesn't hold for pairs $\{\bar{\mu}, \gamma_1^H\}$ when $\bar{\mu} < \mu_A$. Thus, in this scenario, an LOLR needs to accumulate reserves to observe some reinvestment. I define function $\bar{F}(\bar{\mu})$ as the minimum amount of reserves such that (23) holds with equality.

$$\bar{F}(\bar{\mu}) = A \frac{\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]\kappa(0)}{1 + \big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]\kappa(0)}$$

Amount $\bar{F}(\bar{\mu})$ shows that reserves need to compensate for the wedge between liquidity demand and pledgeable income valued with fiscal capacity $(1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H})$. Hence, a lower fiscal capacity implies a greater $\bar{F}(\bar{\mu})$. Likewise, the denominator captures the fact that by reducing banking entrepreneur's disposable endowment at $t = 0$, investment scale is smaller and as such the amount of reserves required is smaller.

Additionally, note that $\bar{F}(\bar{\mu})$ is feasible for any LOLR since it is strictly less than A precisely due to this effect on lower investment scale.

Proposition 5 (LOLR Optimal Response to $x_1^H = 0$)

When $\bar{\mu} < \mu_A$ and x_1^H is zero, define set $\Lambda(\bar{\mu}) = \{z \mid z \leq \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})\}$. The optimal response of a LOLR at $t = 0$ is

$$F_0 = \begin{cases} 0 & \textit{if } (1-\alpha) \in \Lambda(\bar{\mu}) \\ \bar{F}(\bar{\mu}) & \textit{if } (1-\alpha) \in \Lambda^c(\bar{\mu}) \end{cases}$$

Proposition 5 depicts LOLR's optimal response to $x_1^H = 0$.[68] Just like banking entrepreneurs, LOLR dont' hoard liquidity in the form of reserves when the probability of a market stress is *relatively low.* The threshold is determined by the ratio between the cost of accumulating the minimum necessary amount of reserves and the welfare losses of a complete shutdown.[69]

Proposition 6 (No Crisis - Reserves Equilibrium)
For $\bar{\mu} < \mu_A$, $(1-\alpha) \in \Lambda^c(\bar{\mu})$, and Assumptions 1, and 3 hold, in this small open economy

- *Date-0: banking entrepreneurs invest $i = (A - F_0)\kappa(O)$ and do not hoard liquidity ($X_1^L = 0,\ x_1^H = 0$) while LOLR's accumulate $F_0 = \bar{F}(\bar{\mu})$*
- *Date-1: In both states, reinvestment is done at full-scale ($j = i$). In a market stress, entrepreneurs demand i of public liquidity while the LOLR sets $\hat{R}$ equal to $\frac{\rho_0}{1-\bar{\mu}}$ and issues B_1 for a value of $A\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\frac{\kappa(0)}{1+\left[1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\right]\kappa(0)}$*
- *Date-2: Entrepreneurs do not abscond, LOLR collects $\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}i$ and redeems fully $\gamma_1^H B_1$*

Proof The proof consists on showing that at $t = 0$, optimal response functions are consistent. Behavior for $t = 1$ and $t = 2$ follows from decisions on x_1^H and F_0 and can be found in the Appendix. Choose Suppose that F_0 is equal to $\bar{F}(\bar{\mu})$. Therefore, entrepreneurs expect pair $\{\bar{\mu}, \hat{R}\}$ to be equal to ρ_0 which implies an optimal choice of x_1^H equal to zero. Now, suppose that x_1^H is equal to zero. Since the LOLR is $\bar{\mu} < \mu_A$, and $(1-\alpha) \in \Lambda^c(\bar{\mu})$, then it is optimal to choose F_0 equal to $\bar{F}(\bar{\mu})$. ■

The comparison between the Mature Fiscal Capacity equilibrium and the No Crisis - Reserves Equilibrium (Proposition 6) underscores the main result of this paper. In both equilibria, the small open economy avoids the market stress to turn into a sudden stop, and, reinvestment manages to reach full-scale despite the existence of moral hazard. Moreover, in both equilibria, the private sector doesn't hoard liquidity. The main difference lies in that while Mature LOLR don't need to hoard reserves, the rest of LOLR need to hoard a *sufficient*

[68] You can find the derivation in Appendix D

[69] IF $L(0)$ is high enough, then it is possible for set $\Lambda(\bar{\mu})$ to be empty for any feasible $(1-\alpha)$. The range of probabilities is determined by Assumption 3. In such case, an LOLR will always accumulate reserves.

amount of reserves to provide liquidity ex-post successfully. In other words, a LOLR with a fiscal capacity below μ_A accumulates $\bar{F}(\bar{\mu})$ ex-ante to emulate what a mature LOLR can do ex-post.

An important question is whether LOLR intervention is welfare improving relative to the Laissez Faire Case. Although there are many ways to measure this, I restrict my analysis to whether LOLR eliminates the Sudden Stop episode or not. Not surprisingly, Mature LOLR eliminate sudden stops for all feasible probabilities of a market stress.

LOLR with lower fiscal capacity eliminate the Sudden Stop for $(1-\alpha)\ \epsilon\ \Lambda^c(\bar{\mu})$. Hence, for these economies, only probabilities that are part of Ω and part of $\Lambda^c(\bar{\mu})$ shift from a Sudden Stop to a No Crisis equilibrium with LOLR intervention. Once again, the cost of this shift is for LOLR's to accumulate *sufficient* reserves (Corollary 5).

Corollary 5 (LOLR Intervention - Sudden Stop Elimination)
A LOLR intermediation eliminates the Sudden Stop - LFE if

1. $\bar{\mu} \geq \mu_A$ *(Mature LOLR)*
2. $\bar{\mu} < \mu_A$, *when* $(1-\alpha)\ \epsilon\ \Omega \cap \Lambda^c(\bar{\mu})$

Proof For any feasible $(1-\alpha)$, the Sudden Stop - LFE exists when $(1-\alpha)\ \epsilon\ \Omega$. With $\bar{\mu} \geq \mu_A$, the economy reaches a Mature Fiscal Capacity equilibrium where there is no sudden stop. While when $\bar{\mu} < \mu_A$, the economy emulates a Mature Fiscal Capacity equilibrium with a No Crisis - Reserves Equilibrium when $(1-\alpha)\ \epsilon\ \Lambda^c(\bar{\mu})$. Thus, consequently, LOLR intervention eliminates Sudden Stop for $(1-\alpha)$ that are part of the intersection between Ω and $\Lambda^c(\bar{\mu})$ $\blacksquare$

Given that accumulating reserves is costly, there is the possibility of a Sudden Stop type equilibrium even when a LOLR is present. That is, an equilibrium where the economy fails to borrow from abroad since neither the LOLR nor entrepreneurs chose to accumulate the sufficient amount of liquidity at $t = 0$ to attract funding from foreign lenders during stress event at $t = 1$.

Proposition 7 (Sudden Stop - Reserves Equilibrium)
For $\bar{\mu} < \mu_A$, $(1-\alpha)\ \epsilon\ \Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$, *and Assumptions 1, and 3 hold, in this small open economy*

- *Date-0: banking entrepreneurs invest $i = A\kappa(O)$ and do not hoard liquidity ($x_1^L = 0,\ x_1^H = 0$) while the LOLR doesn't accumulate reserves $\{F_0 = 0\}$*

- *Date-1: Reinvestment only occurs during booms. The LOLR sets $\hat{R}$ equal to γ_1^S. In a market stress, entrepreneurs don't demand public liquidity, and, as a result, the LOLR doesn't issue bonds.*

- *Date-2: Entrepreneurs do not abscond following a boom and pay back foreign lenders. Following a stress event, nothing occurs.*

Proof Suppose that F_0 is equal to zero. Therefore, entrepreneurs expect $\hat{R}\}$ to be equal to γ_1^H which, together with a $\bar{\mu} < \mu_A$ and $(1-\alpha)\ \epsilon\Omega(\bar{\mu}, \gamma_1^H)$, imply an optimal choice of x_1^H equal to zero. Now, suppose that x_1^H is equal to zero. Since the LOLR is $\bar{\mu} < \mu_A$, and $(1-\alpha)\ \epsilon\ \Lambda(\bar{\mu})$, then it is optimal to choose F_0 equal to zero. ∎

Lastly, a direct consequence of the possible existence of a sudden stop type equilibrium is that some economies in the world economy cannot afford to insure against market stress. Lower fiscal capacity implies they require more reserves. Thus, within the spectre of low fiscal capacities, it is possible to observe an economy with lower fiscal capacity in a Sudden Stop - Reserves Equilibrium while another economy with greater fiscal capacity in a No Crisis -Reserves Equilibrium.

Corollary 6 (Comparative Statics - $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$**)**

Set $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$ is contracting with respect to $\bar{\mu}$

Proof Choose $\bar{\mu}$ such that $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$ is none empty. Select z equal to the minimum between $\omega(\bar{\mu})$ and $\frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})\}$. Note that z belongs to set $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$. Select $\bar{\mu'}$ greater than $\bar{\mu'}$. Since both $\omega(\bar{\mu})$ and $\bar{F}(\bar{\mu})$ are strictly decreasing with respect to $\bar{\mu}$, then z doesn't belong to set $\Lambda(\bar{\mu'}) \cap \Omega(\bar{\mu'}, \gamma_1^H)$ ∎

Corollary 6 shows that set $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$ is contracting with respect to fiscal capacity. Thus, the possibility of observing an economy with lower fiscal capacity to be exposed to a sudden stop hinges on that, at least, the set $\Lambda(0) \cap \Omega(0, \gamma_1^H)$ is non-empty. Whether this set is empty or not, ultimately, depends on how big are the welfare costs perceived by the LOLR in the case of a complete shutdown.

In this section, I have shown the possibility of three different equilibria depending on the level of fiscal capacity and the probability of a market stress.

Countries with low fiscal capacity can emulate the ability to provide liquidity ex-post that mature countries have by accumulating reserves. However, low fiscal capacity itself can deter countries from choosing to emulate.

7 Multiple Equilibria

When a market stress event is not *rare*, there is multiple equilibria in environments with a LOLR whose fiscal capacity is below the maturity threshold: one with private liquidity hoarding, and the other with public liquidity hoarding through the accumulation of reserves.

Proposition 8 (Multiple Equilibria)
As long as $\bar{\mu} < \mu_A$, Assumptions 1 and 3 hold, and $(1-\alpha)\ \epsilon\Lambda^C(\bar{\mu}) \cap \Omega^C(\bar{\mu}, \gamma_1^H)$, then, at least two equilibria co-exist:

- *No Crisis - Private Hoarding Equilibrium*
- *No Crisis - Reserves Equilibrium*

Proof Note that I assume that $\bar{\mu} < \mu_A$, and Assumptions 1 and 3 hold. By Definition, any $(1-\alpha)$ that belongs to set $\Lambda^C(\bar{\mu}) \cap \Omega^C(\bar{\mu}, \gamma_1^H)$ is part of $\Omega^C(\bar{\mu}, \gamma_1^H)$. Thus, No Crisis - Private Hoarding Equilibrium exists according to Proposition 6 and Corollary 4. Likewise, any $(1-\alpha)$ that belongs to set $\Lambda^C(\bar{\mu}) \cap \Omega^C(\bar{\mu}, \gamma_1^H)$ is part of $\Lambda^C(\bar{\mu})$. Thus, No Crisis - Reserves Equilibrium exists according to Proposition 6. Now, I show that this intersection is non-empty. First, suppose that $\omega(\bar{\mu}, \gamma_1^H) \geq \frac{\psi\kappa(0)}{L(0)}$. Choose $z\ \epsilon\Omega^C(\bar{\mu}, \gamma_1^H)$ then $z\ \epsilon\Lambda^C(\bar{\mu})$. Suppose that $\omega(\bar{\mu}, \gamma_1^H) < \frac{\psi\kappa(0)}{L(0)}$. Choose $z\ \epsilon\Lambda^C(\bar{\mu})$ then $z\ \epsilon\Omega^C(\bar{\mu}, \gamma_1^H)$ ∎

Yet, multiple equilibria is not a feature of environments under Mature LOLR. As argued by Farhi and Tirole (2012), multiple equilibria occurs in these type of models because strategic complementarities appear between entrepreneurs self-insurance choices due to a costly untargeted policy instrument. In this setting, liquidity provision policies are only costly for LOLR that require foreign reserves to implement them.

To see this, consider if, under a LOLR with $\bar{\mu} < \mu_A$, an entrepreneur would benefit from hoarding liquidity when the rest of entrepreneurs do not. The answer is no, and the reason is that, as long as the probability of the market stress is high enough, the LOLR would optimally choose to accumulate reserves which allows it to implement a $\hat{R}$ sufficiently low that every entrepreneurs reaches full-scale reinvestment, even with no market liquidity. Similarly, would an entrepreneur benefit from choosing $x_1^H = 0$ if all others choose to hoard liquidity? Again, the answer is no. This time, the LOLR would not accumulate reserves, and, therefore, it would not be able to provide an LLP with $\hat{R}$ lower than γ_1^H forcing the deviating entrepreneur to shutdown while others would reinvest at full-scale using their market liquidity.

In contrast, a Mature LOLR allows for full-scale reinvestment without the need to reduce the cost of liquidity. Thus, regardless of what other entrepreneurs do, an entrepreneur can always ask for a transfer at date-1 if necessary, and the Mature LOLR has the capacity to provide it.

The coexistence of this two equilibria underscores another important feature of this model. There are, under the environment with a low fiscal capacity LOLR, two ways to circumvent moral hazard: private liquidity hoarding or accumulation of reserves.

When the probability of a stress event is relatively high, both entrepreneurs and the LOLR are willing to hoard liquidity ex-ante. However, it is not optimal for either to hold liquidity if it expects the other to do the hoarding. This shows that private and public hoarding are substitute instruments to solve the same problem.

8 Final Remarks

In this paper, I provide a novel rationale for why emerging economies accumulate foreign reserves for liquidity provision purposes. I show that reserves accumulation is a potential equilibrium outcome in countries that lack the fiscal capacity to overcome existing financial frictions. To do so, I built a three period theoretical model to underscore the channels through which different levels of fiscal capacity affect incentives to accumulate foreign reserves.

Throughout the paper I argue that currency mismatch is not a necessary condition for LOLRs to accumulate reserves, and that, fiscal capacity can explain a share of why countries do so. Therefore, an obvious extension to this model would be to include two different goods (tradable and non-tradable) to study how real exchange rate movements and fiscal capacity interact. Plausibly, the existence of different levels of fiscal capacity could even justify the surge of currency mismatches.

Moreover, I have shown that reserves are an instrument that can prevent sudden stop type episodes. This is consistent with previous work that has shown empirically and also theoretically that the level of reserves diminishes the incidence and the likelihood of a crisis.[70] The reason they are effective is that reserves increase an economy's market liquidity when fiscal capacity fails to increase funding liquidity capabilities. However, I find also that since reserves is costly, low fiscal capacity countries might be more reluctant to accumulate them

Finally, foreign reserves, more than a liquidity provider instrument, are a policy tool that enhances liquidity capabilities of the economy. In fact, in this model, foreign reserves do not need to be liquid assets nor do they need to be at the LOLR's immediate disposal. The model would achieve same results if I had assumed that reserves could only be used at $t = 2$ to redeem bonds.

This idea that reserves are more than direct liquidity providers could be further explored in a dynamic model where a sequence of liquidity distress episodes are possible. In principle, when reserves compensate for lack of fiscal capacity, they are most useful when they are depleted in episodes with higher funding costs. Thus, if during a market stress, there is the belief that things could get worse, then it would be optimal to issue bonds instead of depleting the stock of reserves. This could provide a rationale of why countries accumulate reserves for liquidity purposes but are reluctant to use them during distressed episodes.

[70] See Frankel and Saravelos (2010) and Céspedes and Chang (2019)

References

Aizenman, J. and Lee, J. (2007). International Reserves: Precautionary Versus Mercantilist Views, Theory and Evidence. *Open Economies Review*, 18(2):191–214.

Alfaro, L. and Kanczuk, F. (2009). Optimal reserve management and sovereign debt. *Journal of International Economics*, 77(1):23–36.

Basu, S. S., Ghosh, A. R., Ostry, J. D., and Winant, P. E. (2018). Managing capital outflows with limited reserves. *IMF Economic Review*, 66(2):333–374.

Besley, T., Ilzetzki, E., and Persson, T. (2013). Weak States and Steady States: The Dynamics of Fiscal Capacity. *American Economic Journal: Macroeconomics*, 5(4):205–235.

Besley, T. and Persson, T. (2014). Why Do Developing Countries Tax So Little? *Journal of Economic Perspectives*, 28(4):99–120.

Bianchi, J., Hatchondo, J. C., and Martinez, L. (2018). International Reserves and Rollover Risk. *American Economic Review*, 108(9):2629–2670.

Bocola, L. and Lorenzoni, G. (2020). Financial crises, dollarization, and lending of last resort in open economies. *American Economic Review*, 110(8):2524–57.

Bordo, M. (1990). The lender of last resort: alternative views and historical experience. *Economic Review*, 76(Jan):18–29.

Calvo, G., Izquierdo, A., and Loo-Kung, R. (2013). Optimal Holdings of International Reserves: Self-insurance against Sudden Stops. *Monetaria*, 0(1):1–35.

Céspedes, L. F. and Chang, R. (2019). Optimal foreign reserves and central bank policy under financial stress. *Working in progress*.

Chamon, M., Hofman, D., Magud, N. E., and Werner, A. M., editors (2019). *Foreign exchange intervention in inflation targeters in Latin America*. International Monetary Fund, Washington, D.C. OCLC: on1089006580.

Chang, R. and Velasco, A. (2001). A Model of Financial Crises in Emerging Markets. *The Quarterly Journal of Economics*, 116(2):489–517. Publisher: Oxford University Press.

Dominguez, K. M. (2009). International reserves and underdeveloped capital markets. In *NBER International Seminar on Macroeconomics*, volume 6, pages 193–221. JSTOR.

Eichengreen, B., Hausmann, R., and Panizza, U. (2003). Currency Mismatches, Debt Intolerance and Original Sin: Why They Are Not the Same and Why it Matters. Technical Report w10036, National Bureau of Economic Research, Cambridge, MA.

Farhi, E. and Tirole, J. (2012). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1):60–93.

Fischer, S. (1999). On the need for an international lender of last resort. *Journal of economic perspectives*, 13(4):85–104.

Frankel, J. A. and Saravelos, G. (2010). Are Leading Indicators of Financial Crises Useful for Assessing Country Vulnerability? Evidence from the 2008-09 Global Crisis. Working Paper 16047, National Bureau of Economic Research. Series: Working Paper Series.

Ghosh, A. R., Ostry, J. D., and Tsangarides, C. G. (2017). Shifting motives: Explaining the buildup in official reserves in emerging markets since the 1980s. *IMF Economic Review*, 65(2):308–364.

Heller, H. R. (1966). Optimal International Reserves. *The Economic Journal*, 76(302):296.

Holmström, B. and Tirole, J. (1998). Private and public supply of liquidity. *Journal of Political Economy*, 106(1):1–40.

Holmström, B. and Tirole, J. (2011). *Inside and Outside Liquidity*, volume 1. The MIT Press, 1 edition.

Ilzetzki, E., Reinhart, C. M., and Rogoff, K. S. (2019). Exchange arrangements entering the 21st century: Which anchor will hold? *Quarterly Journal of Economics*, 134(2):599–646.

International Monetary Fund (2011). Assessing Reserve Adequacy. *Policy Papers*, 2011(8).

Jeanne, O. and Rancière, R. (2011). The Optimal Level of International Reserves For Emerging Market Countries: A New Formula and Some Applications. *Economic Journal*, 121(555):905–930.

Obstfeld, M., Shambaugh, J. C., and Taylor, A. M. (2010). Financial Stability, the Trilemma, and International Reserves. *American Economic Journal: Macroeconomics*, 2(2):57–94.

Rey, H. (2015). Dilemma not trilemma: The global financial cycle and monetary policy independence. Working Paper 21162, National Bureau of Economic Research.

Rodrik, D. (2006). The social cost of foreign exchange reserves. *International Economic Journal*, 20(3):253–266.

Tirole, J. (2002). *Financial Crises, Liquidity and the International Monetary System*. Princeton University Press.

Tirole, J. (2011). Illiquidity and All Its Friends. *Journal of Economic Literature*, 49(2):287–325.

Tables and Figures

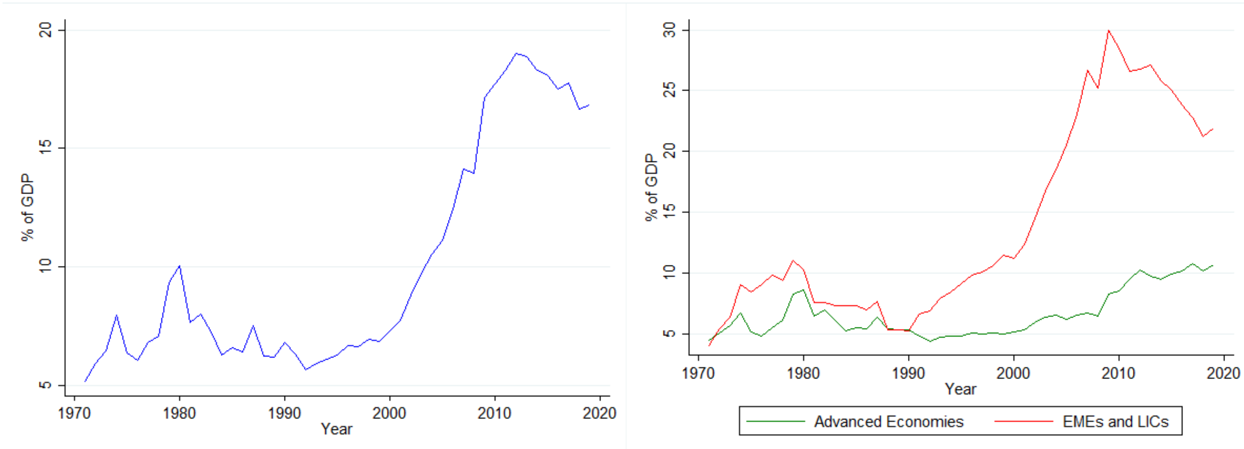
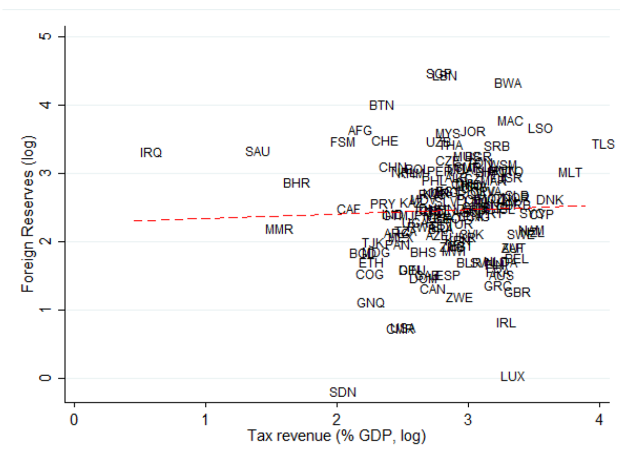
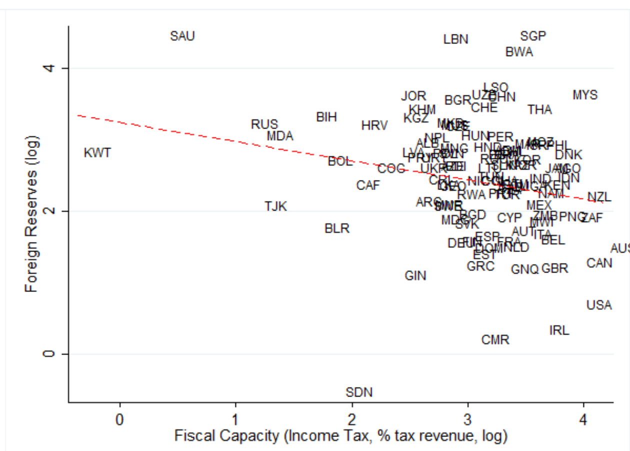


Figure 1: Foreign Reserves Official Holdings - WDI
% of GDP (1970 - 2020)



(a) Total Tax Revenue



(b) Income Tax Revenue

Figure 2: Foreign Reserves and Fiscal Capacity
Country Average (1990 - 2018)

Table 1: Summary statistics

	Mean	SD	Min	Max	N
Dependent Variable					
Foreign Reserves (% GDP, log)	2.6	0.8	0.2	4.6	674
Fiscal Capacity Model					
Tax Revenue (% GDP, log)	2.7	0.5	-2.5	3.5	674
Income Tax Revenue (% TR, log)	3.2	0.7	-0.9	4.2	674
Traditional Model					
Trade (% GDP, log)	4.2	0.6	2.7	6.1	674
Monthly ER Vol. (Annual sd)	0.0	0.0	0.0	0.4	674
GDP (log)	25.9	1.7	21.7	30.5	674
Financial Sta. Model					
Broad Money (% GDP, log)	4.1	0.5	2.1	5.3	674
Chinn Ito Index (0-1)	0.6	0.3	0.0	1.0	674
High Income dummy	0.5	0.5	0.0	1.0	674
Soft Peg dummy	0.4	0.5	0.0	1.0	674
Short Term Debt (% GDP, log)	-2.3	1.1	-5.3	1.3	674
Mercantalist Model					
Currency Overvaluation	-0.4	0.3	-0.9	0.7	674
Financial Dev. Model					
Domestic Financial Liab. (% GDP, log)	4.5	0.8	1.6	6.1	674
Private Foreign Liabilities (% GDP, log)	-2.1	1.1	-8.2	1.3	674
Public Foreign Liabilities (% GDP, log)	-3.9	1.3	-14.3	-1.6	674

Table 2: Foreign Reserves and Fiscal Capacity - OLS Regression

	Whole Sample		EME		Pre-GFC		Post GFC		Balanced Panel		Euro Area	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Imports (% GDP, log)	0.232	0.244	0.230*	0.201	0.201*	0.172	0.365	0.419	0.644**	0.792***	0.524	1.180***
	(0.157)	(0.175)	(0.115)	(0.126)	(0.111)	(0.129)	(0.305)	(0.294)	(0.253)	(0.231)	(0.408)	(0.333)
Exports Vol. (log, 3-year sd)	-0.197	-0.255	-0.117	-0.116	0.117	0.227	-0.164	-0.266	0.159	0.023	1.139	4.253
	(0.196)	(0.186)	(0.170)	(0.168)	(0.369)	(0.346)	(0.164)	(0.202)	(0.510)	(0.392)	(2.785)	(2.826)
Monthly ER Vol. (Annual sd)	-0.000***	-0.000***	-0.000***	-0.000***	-0.000***	-0.000***	-1.181	-0.765	0.597**	0.256	198.433	-234.636
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(2.379)	(2.137)	(0.233)	(0.283)	(565.300)	(542.560)
GDP (log)	-0.031	-0.019	0.017	0.033	-0.066*	-0.034	0.022	-0.009	0.187**	0.324***	-0.023	0.237***
	(0.045)	(0.045)	(0.038)	(0.040)	(0.040)	(0.050)	(0.073)	(0.074)	(0.071)	(0.094)	(0.079)	(0.046)
Broad Money (% GDP, log)	0.569***	0.569***	0.774***	0.777***	0.555***	0.560***	0.511	0.491	0.115	-0.020	-0.421***	-0.450***
	(0.189)	(0.183)	(0.165)	(0.162)	(0.168)	(0.163)	(0.314)	(0.301)	(0.232)	(0.209)	(0.124)	(0.108)
Chinn Ito Index (0-1)	0.133	0.146	0.311*	0.330*	0.063	0.096	0.313	0.365	0.698*	0.771**	-0.199	1.366
	(0.171)	(0.178)	(0.174)	(0.180)	(0.187)	(0.199)	(0.225)	(0.239)	(0.339)	(0.334)	(2.108)	(2.074)
High Income dummy	0.009	-0.061	0.194	0.114	0.333**	0.259	-0.507	-0.685**	0.477**	0.363**		
	(0.189)	(0.183)	(0.153)	(0.132)	(0.161)	(0.170)	(0.372)	(0.330)	(0.181)	(0.145)		
Hard Peg dummy	0.526***	0.464***	0.321***	0.322***	0.508***	0.460***	0.544***	0.494**	0.484*	0.306		
	(0.125)	(0.127)	(0.115)	(0.101)	(0.135)	(0.140)	(0.205)	(0.202)	(0.247)	(0.227)		
Soft Peg dummy	0.670***	0.643***	0.310***	0.338***	0.461***	0.446***	0.816***	0.791***	0.524*	0.421*		
	(0.131)	(0.136)	(0.112)	(0.107)	(0.129)	(0.130)	(0.216)	(0.224)	(0.253)	(0.215)		
Short Term Debt (% GDP, log)	-0.324**	-0.337**	-0.283	-0.281	-0.171	-0.163	-0.566**	-0.606**	-0.106	-0.157	0.170	0.331
	(0.152)	(0.151)	(0.187)	(0.194)	(0.107)	(0.102)	(0.245)	(0.252)	(0.121)	(0.111)	(0.375)	(0.344)
Currency Overvaluation	-1.205***	-1.089**	-0.457	-0.477	-1.661***	-1.707***	-0.468	-0.038	-1.739***	-1.848***	-0.623	-1.778***
	(0.456)	(0.475)	(0.403)	(0.401)	(0.426)	(0.444)	(0.606)	(0.556)	(0.493)	(0.520)	(0.336)	(0.323)
Domestic Financial Liab. (% GDP, log)	0.073	0.097	-0.097	-0.095	0.058	0.055	0.185	0.290	0.041	0.048	0.654*	0.383
	(0.106)	(0.105)	(0.086)	(0.083)	(0.089)	(0.090)	(0.208)	(0.212)	(0.161)	(0.128)	(0.300)	(0.319)
Private Foreign Liabilities (% GDP, log)	0.302**	0.314**	0.294	0.291	0.264***	0.267***	0.370*	0.416*	0.051	0.131	-0.829	-0.933**
	(0.123)	(0.123)	(0.202)	(0.212)	(0.088)	(0.087)	(0.210)	(0.223)	(0.134)	(0.117)	(0.507)	(0.334)
Public Foreign Liabilities (% GDP, log)	-0.041	-0.041	-0.018	-0.019	-0.078	-0.088	0.036	0.059	0.122*	0.085	0.532***	0.404***
	(0.054)	(0.053)	(0.032)	(0.033)	(0.058)	(0.061)	(0.057)	(0.058)	(0.062)	(0.052)	(0.139)	(0.057)
Tax Revenue (% GDP, log)		-0.004		0.131		0.205		-0.428		0.413		2.081***
		(0.202)		(0.151)		(0.180)		(0.295)		(0.251)		(0.548)
Income Tax Revenue (% TR, log)		-0.161**		-0.146**		-0.175		-0.169**		-0.477***		-1.588**
		(0.081)		(0.064)		(0.108)		(0.081)		(0.114)		(0.503)
Observations	1681	1681	1162	1162	915	915	605	605	507	507	152	152
R^2	0.39	0.40	0.51	0.52	0.46	0.47	0.35	0.39	0.62	0.66	0.73	0.82
Countries	98	98	69	69	93	93	92	92	20	20	9	9

Note: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parenthesis. Observations clustered by country. Time fixed effects are not reported but are included in every regression.

Table 3: Foreign Reserves, Fiscal Capacity and Original Sin - OLS Regression

	Whole Sample		EME		Pre-GFC		Post GFC		Balanced Panel		Euro Area	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Imports (% GDP, log)	0.252	0.279	0.340**	0.348**	0.091	0.129	0.203	0.250	0.582**	0.711***	0.546	1.332**
	(0.221)	(0.218)	(0.137)	(0.150)	(0.224)	(0.212)	(0.254)	(0.246)	(0.245)	(0.240)	(0.496)	(0.417)
Exports Vol. (log, 3-year sd)	0.507	0.354	0.544	0.445	0.959*	1.025**	0.820	0.037	0.225	0.087	0.134	2.944
	(0.626)	(0.624)	(0.444)	(0.480)	(0.497)	(0.514)	(1.197)	(1.145)	(0.618)	(0.631)	(2.073)	(2.540)
Monthly ER Vol. (Annual sd)	-0.783	-1.027	-1.393	-1.474*	-1.799	-1.842	0.733	1.081	-1.742	-1.811		
	(1.336)	(1.163)	(0.880)	(0.867)	(1.378)	(1.296)	(2.197)	(1.618)	(1.461)	(1.728)		
GDP (log)	0.013	-0.010	0.064*	0.048	-0.120*	-0.116	0.047	-0.013	0.219***	0.407***	-0.011	0.255***
	(0.051)	(0.050)	(0.036)	(0.036)	(0.064)	(0.074)	(0.059)	(0.062)	(0.068)	(0.113)	(0.091)	(0.055)
Broad Money (% GDP, log)	0.452**	0.445**	0.611***	0.595***	0.368*	0.360*	0.516*	0.513**	-0.018	-0.055	-0.413**	-0.555***
	(0.214)	(0.198)	(0.185)	(0.173)	(0.196)	(0.185)	(0.262)	(0.246)	(0.268)	(0.230)	(0.129)	(0.137)
Chinn Ito Index (0-1)	0.031	0.011	0.233	0.203	-0.294	-0.280	0.420*	0.383*	0.785**	0.845**	-0.596	1.100
	(0.187)	(0.194)	(0.140)	(0.141)	(0.238)	(0.256)	(0.213)	(0.229)	(0.344)	(0.310)	(1.777)	(1.706)
High Income dummy	0.056	-0.031	0.099	-0.000	0.387*	0.339	-0.239	-0.414	0.182	-0.017		
	(0.238)	(0.211)	(0.179)	(0.154)	(0.217)	(0.226)	(0.362)	(0.302)	(0.241)	(0.257)		
Hard Peg dummy	0.685***	0.565***	0.430**	0.401**	0.920***	0.766***	0.543**	0.437*	0.537*	0.462*		
	(0.176)	(0.174)	(0.187)	(0.154)	(0.177)	(0.181)	(0.240)	(0.227)	(0.286)	(0.259)		
Soft Peg dummy	0.750***	0.682***	0.329*	0.332**	0.772***	0.670***	0.657***	0.596**	0.699**	0.682**		
	(0.178)	(0.174)	(0.186)	(0.162)	(0.206)	(0.208)	(0.248)	(0.236)	(0.302)	(0.319)		
Short Term Debt (% GDP, log)	-0.114	-0.144	0.058	0.038	-0.258	-0.317**	0.034	0.018	-0.189	-0.197	0.076	0.382
	(0.145)	(0.138)	(0.112)	(0.101)	(0.158)	(0.143)	(0.186)	(0.185)	(0.168)	(0.164)	(0.317)	(0.242)
Currency Overvaluation	-0.931**	-0.656	-0.254	-0.168	-1.230**	-1.043**	-0.792	-0.317	-1.454***	-1.782***	-0.666*	-2.244***
	(0.413)	(0.418)	(0.522)	(0.565)	(0.497)	(0.497)	(0.536)	(0.474)	(0.449)	(0.459)	(0.303)	(0.443)
Domestic Financial Liab. (% GDP, log)	0.169	0.236*	-0.078	-0.025	0.308**	0.360**	0.215	0.299	0.199	0.062	0.683*	0.253
	(0.140)	(0.141)	(0.108)	(0.103)	(0.129)	(0.140)	(0.190)	(0.194)	(0.230)	(0.235)	(0.298)	(0.238)
Private Foreign Liabilities (% GDP, log)	0.038	0.057	-0.012	-0.001	0.238	0.295*	-0.145	-0.152	0.131	0.292*	-0.792	-1.056**
	(0.148)	(0.138)	(0.135)	(0.115)	(0.180)	(0.154)	(0.178)	(0.178)	(0.184)	(0.166)	(0.500)	(0.399)
Public Foreign Liabilities (% GDP, log)	0.056	0.073	0.033	0.057	-0.085	-0.086	0.176**	0.210**	0.093	0.048	0.506***	0.264*
	(0.081)	(0.081)	(0.047)	(0.047)	(0.112)	(0.113)	(0.087)	(0.089)	(0.067)	(0.060)	(0.127)	(0.133)
Original Sin Index (0-1)	1.674***	1.617***	-1.197**	-1.150***	1.071**	1.069**	2.203***	2.076***	0.386	0.737	0.251	0.006
	(0.348)	(0.339)	(0.566)	(0.427)	(0.457)	(0.456)	(0.356)	(0.356)	(0.432)	(0.468)	(0.763)	(1.109)
Tax Revenue (% GDP, log)		-0.215		-0.108		-0.162		-0.415		0.810**		2.354***
		(0.238)		(0.213)		(0.257)		(0.276)		(0.384)		(0.544)
Income Tax Revenue (% TR, log)		-0.227***		-0.166**		-0.222**		-0.222**		-0.372***		-1.744***
		(0.085)		(0.074)		(0.103)		(0.087)		(0.129)		(0.396)
Observations	1029	1029	606	606	397	397	505	505	312	312	144	144
R^2	0.51	0.54	0.53	0.56	0.61	0.62	0.48	0.53	0.64	0.67	0.74	0.84
Countries	84	84	55	55	69	69	80	80	20	20	9	9

Note: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parenthesis. Observations clustered by country. Time fixed effects are not reported but are included in every regression.

Table 4: Foreign Reserves and Fiscal Capacity - Robustness Check

	Main Specification (MS)				MS with Original Sin			
	(1) YE	(2) CFE	(3) YE+CFE	(4) Between	(5) YE	(6) CFE	(7) YE+CFE	(8) Between
Imports (% GDP, log)	0.244	0.211***	0.142**	0.523*	0.279	0.123	-0.080	0.549*
	(0.175)	(0.054)	(0.056)	(0.276)	(0.218)	(0.098)	(0.119)	(0.283)
Exports Vol. (log, 3-year sd)	-0.255	-0.269*	-0.278**	2.614	0.354	-0.047	-0.265	6.915***
	(0.186)	(0.137)	(0.138)	(1.662)	(0.624)	(0.260)	(0.265)	(2.574)
Monthly ER Vol. (Annual sd)	-0.000***	-0.000	-0.000	-0.006***	-1.027	1.681***	0.641	-0.007***
	(0.000)	(0.000)	(0.000)	(0.002)	(1.163)	(0.614)	(0.649)	(0.002)
GDP (log)	-0.019	0.329***	0.081	0.084	-0.010	0.154***	-0.138	0.103
	(0.045)	(0.038)	(0.071)	(0.063)	(0.050)	(0.050)	(0.089)	(0.076)
Broad Money (% GDP, log)	0.569***	0.294***	0.235***	0.666**	0.445**	0.058	-0.051	0.738***
	(0.183)	(0.064)	(0.067)	(0.259)	(0.198)	(0.095)	(0.102)	(0.190)
Chinn Ito Index (0-1)	0.146	-0.090	-0.178***	0.511*	0.011	0.195*	0.187*	0.490
	(0.178)	(0.066)	(0.069)	(0.299)	(0.194)	(0.104)	(0.103)	(0.336)
High Income dummy	-0.061			-0.452*	-0.031			-0.169
	(0.183)			(0.238)	(0.211)			(0.191)
Hard Peg dummy	0.464***	0.232***	0.205***	0.523*	0.565***	-0.004	-0.042	0.425
	(0.127)	(0.060)	(0.059)	(0.313)	(0.174)	(0.090)	(0.088)	(0.265)
Soft Peg dummy	0.643***	0.223***	0.189***	0.941***	0.682***	0.020	-0.011	0.703***
	(0.136)	(0.065)	(0.065)	(0.315)	(0.174)	(0.102)	(0.101)	(0.245)
Short Term Debt (% GDP, log)	-0.337**	-0.100***	-0.108***	-0.864***	-0.144	-0.035	-0.042	-0.516*
	(0.151)	(0.035)	(0.035)	(0.296)	(0.138)	(0.049)	(0.049)	(0.287)
Currency Overvaluation	-1.089**	-1.146***	-0.892***	-0.336	-0.656	-0.832***	-0.766***	-0.311
	(0.475)	(0.152)	(0.193)	(0.608)	(0.418)	(0.165)	(0.196)	(0.582)
Domestic Financial Liab. (% GDP, log)	0.097	-0.117***	-0.084*	0.124	0.236*	-0.087	-0.010	0.112
	(0.105)	(0.041)	(0.043)	(0.170)	(0.141)	(0.058)	(0.060)	(0.137)
Private Foreign Liabilities (% GDP, log)	0.314**	0.130***	0.123***	0.621**	0.057	0.144***	0.172***	0.338
	(0.123)	(0.031)	(0.031)	(0.245)	(0.138)	(0.054)	(0.055)	(0.275)
Public Foreign Liabilities (% GDP, log)	-0.041	-0.045***	-0.048***	-0.005	0.073	0.051*	0.050*	0.037
	(0.053)	(0.013)	(0.014)	(0.083)	(0.081)	(0.028)	(0.028)	(0.105)
Original Sin Index (0-1)					1.617***	-0.986***	-0.778***	1.487***
					(0.339)	(0.216)	(0.217)	(0.365)
Tax Revenue (% GDP, log)	-0.004	0.245***	0.274***	0.066	-0.215	0.491***	0.588***	-0.030
	(0.202)	(0.084)	(0.084)	(0.274)	(0.238)	(0.134)	(0.134)	(0.300)
Income Tax Revenue (% TR, log)	-0.161**	0.025	-0.005	-0.322***	-0.227***	0.091	0.068	-0.414***
	(0.081)	(0.045)	(0.044)	(0.118)	(0.085)	(0.071)	(0.070)	(0.106)
Observations	1681	1681	1681	98	1029	1029	1029	84
R^2	0.40	0.20	0.23	0.55	0.54	0.10	0.16	0.66

Note: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parenthesis. YE=Year Fixed Effects, CFE=Country Fixed Effects, and Between refers to results of running regressions across panel averages for each country.

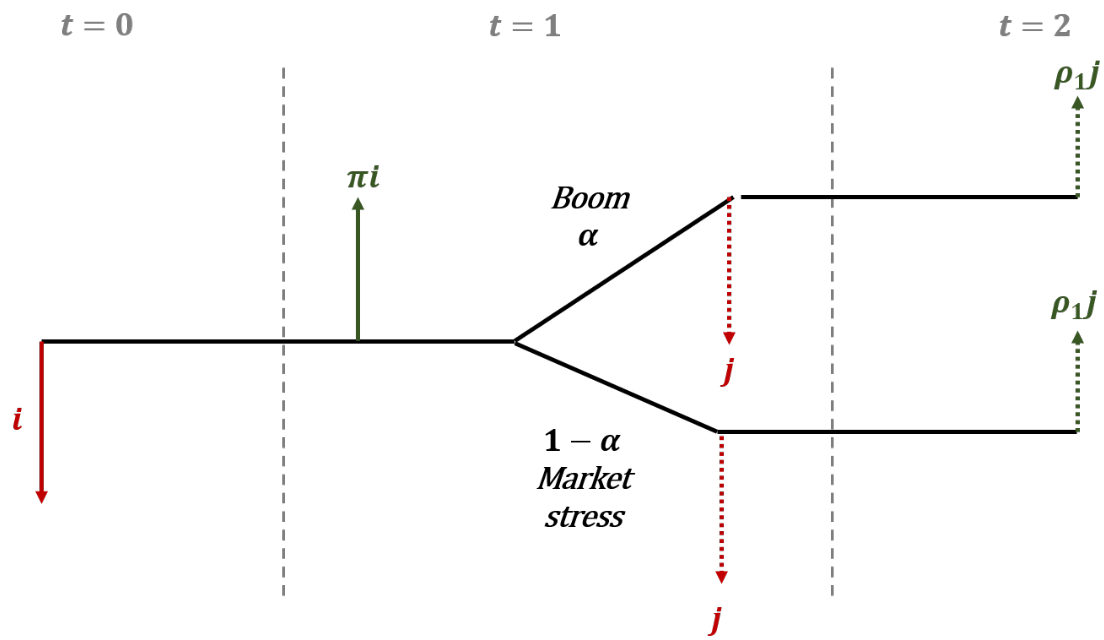


Figure 3: Project Technology - Timeline

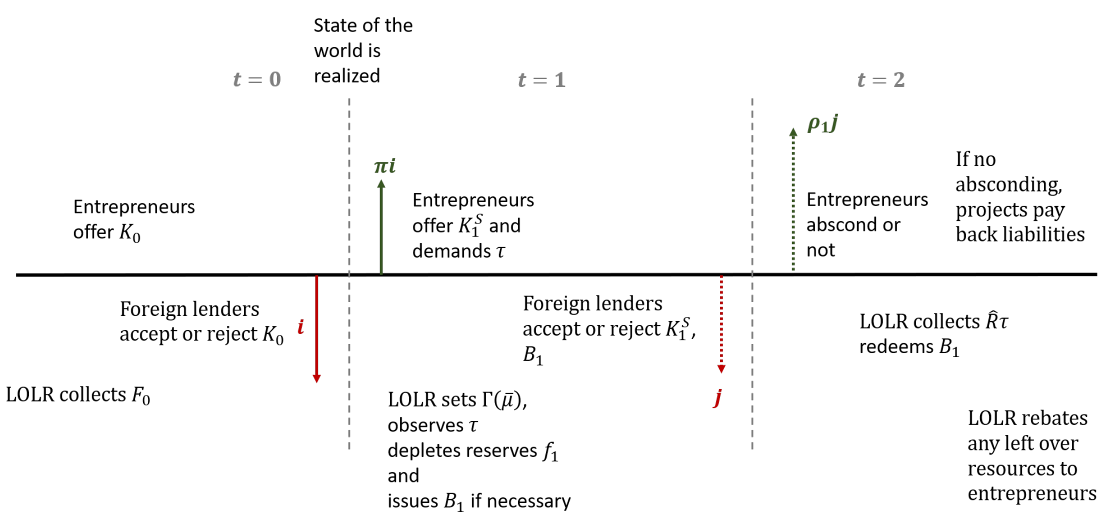


Figure 4: Model Timeline

Appendices

A Laissez Faire Equilibria

I find the set of Laissez Faire Equilibria through backward induction. I start at period $t = 1$ since contracts are designed for banking entrepreneurs to abide by requiring satisfying (4).

A given entrepreneur's period 1 optimization problem can be rewritten as follows:

$$\begin{aligned}
\underset{\{c_1^s, M_1^s, \phi_1^s\}}{\text{Maximize}}\ & \big[1-\gamma_1^s\big]c_1^s + \big[\rho_1 - \gamma_1^s\big]j^s + \gamma_1^s x_1^s \\
\text{subject to: } & j^s = min\{\frac{M_1^s}{1-\phi_1^s}, i\} \\
& \big[\gamma_1^s \phi_1^s - \rho_0\big]j^s \leq 0 \\
& c_1^s + M_1^s \leq x_1^s \\
& \{c_1^s, M_1^s, \phi_1^s\}\ non-negative, \text{ given } \{x_1^s, i\}
\end{aligned}$$

I start by solving this problem for a boom For a given $\{x_1^L, i\}$, recall that $\gamma_1^L < \rho_0$ which implies that $\gamma_1^L < 1$. I argue that, in this scenario, $\{j^L = i, c_1^L = x_1^L, M_1^L = 0, \phi_1^L = 1\}$ is a optimal answer with payoff $\big[\rho_1 - \gamma_1^L\big]i + x_1^L$. To see this, first, now that this solution is feasible since $\gamma_1^L i < \rho_0 i$. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^s}, \hat{M_1^s}, \hat{\phi_1^s}\}$ that is feasible and produces a strictly greater payoff. This is not possible because $1 > \gamma_1^L$ in this scenario, $\hat{j^L} \leq i$ and $\hat{C_1^L} \leq x_1^L$ due to feasibility, and $\gamma_1^L < \rho_1$ as an assumption. Below the optimal strategies for an entrepreneur during a boom.

$$j^L = \begin{cases} i & \text{for all } x_1^L \geq 0 \end{cases}$$

$$c_1^L = \begin{cases} x_1^L & \text{for all } x_1^L \geq 0 \end{cases}$$

$$M_1^L = \begin{cases} 0 & \text{for all } x_1^L \geq 0 \end{cases}$$

$$l_1^L = \begin{cases} \gamma_1^L i & \text{for all } x_1^L \geq 0 \end{cases}$$

$$\rho_0 j^L - l_1^L = \begin{cases} \big[\rho_0 - \gamma_1^L\big]i & \text{for all } x_1^L \geq 0 \end{cases}$$

$$C_{1,2}^L = \begin{cases} \big[\rho_1 - \gamma_1^L\big]i + x_1^L & \text{for all } x_1^L \geq 0 \end{cases}$$

I continue by solving this problem for a stress period For a given $\{x_1^H, i\}$, recall that $\gamma_1^H > \rho_0$ which implies that $\phi_1^H = 1$ is no longer possible (no *finance as you go*). Moreover, $\gamma_1^H > 1$ which implies that it is better to lend at international markets than to consume, thus, c_1^H is equal to zero. **Consider first the scenario where $x_1^H \geq i\big[1 - \frac{\rho_0}{\gamma_1^H}\big]$.** I show that, in this scenario, $\{c_1^H = 0, M_1^H = i\big[1 - \frac{\rho_0}{\gamma_1^H}\big], \phi_1^H = \frac{\rho_0}{\gamma_1^H}\}$ is an optimal answer with payoff $\big[\rho_1 - \gamma_1^H\big]i + \gamma_1^H x_1^H$. To see this, first, i show feasibility where $j^H = \phi_1^H j^H + M_1^H = \frac{\rho_0}{\gamma_1^L} i + i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] = i$ which is less or equal to i. Additionally, $\big[\gamma_1^H \phi_1^H - \rho_0\big] j^s = \big[\gamma_1^H \frac{\rho_0}{\gamma_1^H} - \rho_0\big] i$ which is equal to zero. Lastly, $c_1^H + M_1^H = 0 + i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \le x_1^H$ by assumption for this scenario. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^H}, \hat{M_1^H}, \hat{\phi_1^H}, \hat{j^H}\}$ that is feasible and produces a strictly greater payoff. This is not possible because $1 < \gamma_1^H$ in this scenario, $\hat{j^L} \le i$ due to feasibility. **Consider now subgames when** $x_1^H < i\big[1 - \frac{\rho_0}{\gamma_1^H}\big]$. I show that $\{c_1^H = 0, M_1^H = x_1^H, \phi_1^H = \frac{\rho_0}{\gamma_1^H},\ j^H = \frac{x_1^H}{1 - \frac{\rho_0}{\gamma_1^H}}\}$ is an optimal answer with payoff $\frac{\rho_1 - \rho_0}{\gamma_1^H - \rho_0} \gamma_1^H x_1^H$. To see this, first, I show feasibility where $j^H = \phi_1^H j^H + M_1^H = \frac{\rho_0}{\gamma_1^L} j^H + x_1^H \to j^H = \frac{x_1^H}{1 - \frac{\rho_0}{\gamma_1^H}}$ which is less or equal to i because $x_1^H < i\big[1 - \frac{\rho_0}{\gamma_1^H}\big]$. Additionally, $\big[\gamma_1^H \phi_1^H - \rho_0\big] j^H = \big[\gamma_1^H \frac{\rho_0}{\gamma_1^H} - \rho_0\big] j^H$ which is equal to zero. Lastly, $c_1^H + M_1^H = 0 + x_1^H \le x_1^H$. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^H}, \hat{M_1^H}, \hat{\phi_1^H},\ \hat{j^H}\}$ that is feasible and produces a strictly greater payoff. First, note that since this candidate is feasible then $\hat{j^H} = \hat{\phi_1^H} \hat{j^H} + \hat{M_1^H} \le \frac{\rho_0}{\gamma_1^L} j^H + x_1^H \to \hat{j^H} \le \frac{x_1^H}{1 - \frac{\rho_0}{\gamma_1^H}}$. Thus, any feasible solution cannot generate a reinvestment scale greater than our optimal choice. This finding together with $1 < \gamma_1^H$ and $\hat{c_1^H}$ non-negative imply that no feasible solution produces a greater payoff.

Below the optimal strategies for an entrepreneur during a stress period.

$$j^H = \begin{cases} \frac{x_1^H}{1 - \frac{\rho_0}{\gamma_1^H}} & \text{when } x_1^H < i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \\ i & \text{when } x_1^H \ge i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

$$c_1^H = \begin{cases} 0 & \text{for all } x_1^L \ge 0 \end{cases}$$

$$M_1^H = \begin{cases} x_1^H & \text{when } x_1^H < i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \\ i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] & \text{when } x_1^H \ge i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

$$l_1^H = \begin{cases} \rho_0 \frac{x_1^H}{1 - \frac{\rho_0}{\gamma_1^H}} & \text{when } x_1^H < i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \\ \rho_0 i & \text{when } x_1^H \ge i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

$$C_{1,2}^H = \begin{cases} \frac{\rho_1 - \rho_0}{\gamma_1^H - \rho_0} \gamma_1^H x_1^H & \text{when } x_1^H < i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \\ \big[\rho_1 - \gamma_1^H\big] i + \gamma_1^H x_1^H & \text{when } x_1^H \ge i\big[1 - \frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

Recall that a given entrepreneur's period 0 optimization problem is as follows:

$$
\begin{aligned}
\underset{\{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\}}{\text{Maximize}} \quad & c_0 + \alpha\big[C_{1,2}^L - l_0^L\big] + (1-\alpha)\big[C_{1,2}^H\big] \\
\text{subject to: } & \alpha\big[l_0^L + d_f^L i\big](1-\alpha)\big[d_f^H i\big] = i - M_0 \\
& d_f^s i + d_e^s i = \pi i \\
& l_0^L \ \epsilon \big[0, \rho_0 j^L - l_1^L \gamma_1^L\big] \\
& A - c_0 - M_0 = x_A \\
& d_e^s i + x_A = x_1^s \\
& \{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\} \text{ non-negative, given the optimal} \\
& \text{functions at date-1 that depend on } \{x_1^L, x_1^H\}
\end{aligned}
$$

Could c_0 be positive? No, it can not. Since entrepreneurs are risk neutral, they are always better off to postpone consumption until $t = 1$ after they observe the aggregate shock. If it is a boom, they can consume, while if there is market *stress*, they can lend with a higher return. Either option is at least as good as consuming that unit at $t = 0$ **Could x_A be positive?** A positive x_A increases insurance x_1^s for both states while it sacrifices investment. However, entrepreneurs have another way to accumulate liquidity. That is, by allocating to the project a share of safe cash flow. Since $\pi > 1 - \frac{\rho_0}{\gamma_H^L}$, this cash flow together with the maximum amount of funding liquidity is enough to reach full-scale reinvestment even in stress periods. Additionally, x_A is not state-contingent so it is even more expensive in terms of investment scale. Thus, even in scenarios where x_1^s is positive, there is no need for x_A to be positive. Additionally, following Assumption 1, projects generate have a positive net present value. This confirms that entrepreneurs invest all their net worth in the project. Therefore, M_0 is equal to A. **Should l_0^L be the maximum possible?** Consider the case where reinvestment is only possible in booms since these are long-term claims contigent on a realized boom. If you take the derivative of the objective function relative to l_0^L, the sign will depend on the term of $\alpha(\rho_1 - \gamma_1^L) - (1 - \pi)$. By Assumption (1), this term is positive. Then, contracts at period-0 load up in long-term claims for booms. This result is due to the fact that even if no reinvestment is done during stress periods, projects generate a sufficiently high return that it is worth to load up in long-term contingent debt. I have shown that $\{c_0 = 0,\ x_A = 0,\ M_0 = A,\ l_0^L = \rho_0 j^L - l_1^L\}$. What is left to determine is set $\{d_f^L i,\ d_f^H i\}$ which in turn defines the initial investment scale. Note that with x_A equal to zero, then $x_1^s = d_e^s i$. I define $\bar{x_1}^s$ equal to $\frac{x_1^s}{i} = \bar{x_1}^s$ which is the amount of liquidity hoarding by unit of initial investment. Rewriting foreign lenders participation constraint I get that the initial scale is given by

$$
i(\bar{x_1}^L, \bar{x_1}^H) = \frac{A}{1 - \pi - \alpha(\rho_0 - \gamma_1^L) + \alpha\bar{x_1}^L + (1-\alpha)\bar{x_1}^H}
$$

Recall that $\bar{x_1}^S$ lies between zero and π. Note that the initial investment falls when insurance for in any state increases. The objective function of an entrepreneur at period 0 with the previous findings and the optimal behavior from date-1 onward is given by

$$\left[\alpha(\rho_1 - \rho_0 + \bar{x_1}^L) + (1-\alpha)C^H_{1,2}(\bar{x_1}^H)\right] i(\bar{x_1}^L, \bar{x_1}^H)$$

The sign of the first order condition of this objective function relative to $\bar{x_1}^L$ depends on the term $1 + \alpha\gamma_1^L - \alpha\pi - \alpha\rho_1$ which by Assumption 1 is negative. Thus, an entrepreneur never chooses to hoard liquidity for a boom period. This result is consistent with the fact that hoarding sacrifices investment scale and, with a boom, it provides no insurance since projects can be financed as they go. Note that even if any liquidity is hoarded, it is not used to reinvest but instead it is consumed. **What about $\bar{x_1}^H$?** During stress periods, a project cannot finance reinvestment as it goes. Thus, if it wants to survive, the banking entrepreneur needs to accumulate some liquidity. The F.O.C. of the payoff function with $\bar{x_1}^L = 0$ relative to $\bar{x_1}^H$ is given by

$$\left[\alpha(\rho_1 - \rho_0) + (1-\alpha)C^H_{1,2}(\bar{x_1}^H)\right]\frac{\partial}{\partial \bar{x_1}^H} i(\bar{x_1}^H) + (1-\alpha) i(\bar{x_1}^H)\frac{\partial}{\partial \bar{x_1}^H} C^H_{1,2}(\bar{x_1}^H)$$

The first term of this first order condition captures the cost of insuring as a fall in investment scale times the expected payoff while the second term captures the benefit from continuation. Not that this benefit is weighted by the probability that in fact a stress period happens given that any hoarding, even contingent, is wasted when the aggregate shock is a boom. I evaluate this F.O.C at two points given the discontinuity in $C^H_{1,2}(\bar{x_1}^H)$ (See above).

First, is it optimal to hoard any liquidity? To see this, I evaluate the F.O.C with respect to x_1^H at values close to zero. The sign of the derivative is given by the following term

$$1 - \pi - \alpha\left[(\rho_0 - \gamma_1^L) + (1 - \frac{\rho_0}{\gamma_1^H})\right]$$

First note that it doesn't depend on x_1^H since it is a linear function. Therefore, the optimal choice is a corner solution. Now, when this term is negative, entrepreneurs do not hoard positive levels of liquidity.

A.1 Proposition 2 - Proof

To prove this proposition. I do so by contradiction. Define ω as stated, and the probability of a market stress is equal to z where $z \leq \omega$ by assumption. Suppose, now, that the economy is in a No Crisis Equilibrium, thus, by optimality, the FOC evaluated close to zero must be positive, Thus, the following relationship must hold.

$$0 < 1 - \pi - (1-z)\left[(\rho_0 - \gamma_1^L) + (1 - \frac{\rho_0}{\gamma_1^H})\right]$$

Using the fact that $1 - z$ is greater or equal to $1 - \omega$ which, by definition, is equal to $\frac{1-\pi}{(1-\frac{\rho_0}{\gamma_1^H}) + (\rho_0 - \gamma_1^L)}$, then

$$0 < 1 - \pi - (1-z)\left[(\rho_0 - \gamma_1^L) + (1 - \frac{\rho_0}{\gamma_1^H})\right] \leq 1 - \pi - \frac{1-\pi}{(1-\frac{\rho_0}{\gamma_1^H}) + (\rho_0 - \gamma_1^L)}\left[(\rho_0 - \gamma_1^L) + (1 - \frac{\rho_0}{\gamma_1^H})\right]$$

which is equal to zero. Then I found a contradiction.

A.2 Proposition 1 - Proof

In the case they choose to hoard liquidity, the question is how much. For this to happen, it must be true that $1-\pi \geq \alpha\big[(\rho_0-\gamma_1^L)+(1-\frac{\rho_0}{\gamma_1^H})\big]$. I now evaluate the F.O.C between $\bar{x_1}^H$ is between $1-\frac{\rho_0}{\gamma_1^H}$ and π, the sign of this F.O.C is given by

$$\gamma_1^H\big[1-\pi+\alpha\gamma_1^L+(1-\alpha)\big]-\rho_1-\alpha\rho_0\big[\gamma_1^H-1\big]$$

Note that, once again, this derivative is linear as it does not depend on x_1^H. Additionally, by Assumption 1, $\rho_1>\gamma_1^H\big[1-\pi+\gamma_L^L+(1-\alpha)\big]$, thus this derivative is negative. Consequently, an entrepreneur, if it chooses to hoard, it hoards $i[1-\frac{\rho_0}{\gamma_1^H}]$ which is the minimum amount necessary to continue at full-scale by complementing this liquidity with funding liquidity.

A.3 No Crisis Equilibrium - LFE

Whenever $1-\alpha$ is greater than ω and Assumptions 1 and 3 hold, the following characterizes the NO Crisis Equilibrium - LFE

- Date-2: Entrepreneurs' don't abscond
- Date-1: $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i\}$ and determined by strategy profile functions $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L=\frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H=\frac{l_1^H}{\gamma_1^H}$
- Date-0: $\{c_0=0, x_A=0\}$ and $K_0=\{i=\frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)+(1-\alpha)(1-\frac{\rho_0}{\gamma_1^H})},\ M_0=A,\ \phi_0=i-A,\ d_f^L i=\pi i,\ d_f^H i=i\big[\pi-(1-\frac{\rho_0}{\gamma_1^H})\big],\ l_0^L=(\rho_0-\gamma_1^L)i,\ x_1^L=0, x_1^H=i(1-\frac{\rho_0}{\gamma_1^H})\}$ solve entrepreneurs problem at the initial period

Proof Optimal behavior at date-1 and date-2 was derived through backward induction. Given strategy profiles, $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L=\frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H=\frac{l_1^H}{\gamma_1^H}$ and the previous discussion, since $1-\alpha$ is greater than ω, then the first order condition of the objective function is positive thus optimally choose a positive $x_1^H=$. Proposition 1 shows that it is optimal to select $x_1^H=i(1-\frac{\rho_0}{\gamma_1^H})$. ∎

A.4 Sudden Stop Equilibrium - LFE

Whenever $1-\alpha$ is less or equal than ω and Assumptions 1 and 3 hold, the following characterizes the Sudden Stop Equilibrium - LFE

- Date-2: Entrepreneurs' don't abscond
- Date-1: $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i\}$ and determined by strategy profile functions $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L=\frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H=\frac{l_1^H}{\gamma_1^H}$

- Date-0: $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)},\ M_0 = A,\ \phi_0 = i - A,\ d_f^L i = \pi i,\ d_f^H i = \pi i,\ l_0^L = (\rho_0 - \gamma_1^L)i,\ x_1^L = 0, x_1^H = 0\ \}$ solve entrepreneurs problem at the initial period

Proof Optimal behavior at date-1 and date-2 was derived through backward induction. Given strategy profiles, $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L = \frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H = \frac{l_1^H}{\gamma_1^H}$ and the previous discussion, since $1-\alpha$ is less or equal to ω, then the first order condition of the objective function relative to x_1^H is negative thus an entrepreneur optimally choose $x_1^H = 0$. ∎

B Banking Entrepreneurs Optimal Behavior with a LOLR

B.1 Period 1 - Optimal Behavior

In this section, I derive the best response functions when a LPP with funding cost $\hat{R}$ and fiscal capacity $\bar{\mu}$ is present.

I start at period $t = 1$ since contracts are designed for banking entrepreneurs to abide by requiring satisfying (4).

A given entrepreneur's period 1 optimization problem can be rewritten as follows:

$$\begin{aligned}
&\underset{\{c_1^s, M_1^s, \phi_1^s, \tau\}}{\text{Maximize}} \ \big[1-\gamma_1^s\big]c_1^s + \big[\rho_1 - \gamma_1^s\big]j^s + \gamma_1^s x_1^s + \big[\gamma_1^s - \hat{R}\big]\tau \\
&\text{subject to: } j^s = min\{\frac{M_1^s + \tau^s}{1-\phi_1^s}, i\} \\
&\qquad \hat{R}(1-\bar{\mu})\tau + \big[\gamma_1^s \phi_1^s - \rho_0\big]j^s \leq 0 \\
&\qquad c_1^s + M_1^s \leq x_1^s \\
&\qquad \{c_1^s, M_1^s, \phi_1^s, \tau\} \ non-negative, \text{ given } \{x_1^s,\ i,\ \hat{R},\ \bar{\mu}\}
\end{aligned}$$

I start by solving this problem for a boom First, consider when $\hat{R} \geq \gamma_1^L$. For a given $\{x_1^s,\ i,\ \hat{R} \geq \gamma_1^L,\ \bar{\mu}\}$, recall that $\gamma_1^L < \rho_0$ which implies that $\gamma_1^L < 1$. Moreover, note, from the objective function, that τ should be as small as possible since $\hat{R} \geq \gamma_1^L$. I argue that, in this scenario, $\{j^L = i, c_1^L = x_1^L, M_1^L = 0, \phi_1^L = 1, \tau = 0\}$ is a optimal answer with payoff $\big[\rho_1 - \gamma_1^L\big]i + x_1^L$. To see this, first, note that this solution is feasible since $\gamma_1^L i < \rho_0 i$. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^s}, \hat{M_1^s}, \hat{\phi_1^s}\}$ that is feasible and produces a strictly greater payoff. This is not possible because $1 > \gamma_1^L$ in this scenario, $\hat{j^L} \leq i$ and $\hat{C_1^L} \leq x_1^L$ due to feasibility, $\gamma_1^L < \rho_1$, and $\gamma_1^L < \hat{R}$ as an assumption for this scenario. **Now, consider when $\hat{R} < \gamma_1^L$.** For a given $\{x_1^s,\ i,\ \hat{R} < \gamma_1^L,\ \bar{\mu}\}$, recall that $\gamma_1^L < \rho_0$ which implies that $\gamma_1^L < 1$. Moreover, note, from the objective function, that τ should be as large as possible since $\hat{R} \geq \gamma_1^L$. I argue that, in this scenario, $\{j^L = i, c_1^L = x_1^L, M_1^L = 0, \phi_1^L = 0, \tau = i\}$ is a optimal answer with payoff $\big[\rho_1 - \hat{R}\big]i + x_1^L$. Note that this candidate solution is feasible for any $\bar{\mu}$ since $\rho_0 > \gamma_1^L > \hat{R} \rightarrow \rho_0 > \hat{R}(1-\bar{\mu})$. I prove optimality by contradiction. I assume that there exists another feasible candidate that generates a greater payoff than $\big[\rho_1 - \hat{R}\big]i + x_1^L$. This is not possible because any feasible τ is bounded from above by i, the same as any feasible j^L, while any feasible c_1^L is bounded by x_1^L. Below the optimal strategies for an entrepreneur during a boom. To differentiate from optimal behavior in LFE, I denote these functions with a tilde.

For all $x_1^L \geq 0$

$$\tilde{j}^L = \Big\{ i \qquad \text{for all } \hat{R}$$

$$\tilde{c}_1^L = \Big\{ x_1^L \qquad \text{for all } \hat{R}$$

$$\tilde{M}_1^L = \begin{cases} 0 & \text{for all } \hat{R} \end{cases}$$

$$\tilde{l}_1^L = \begin{cases} \gamma_1^L i & \text{if } \hat{R} \geq \gamma_1^L \\ 0 & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

$$\rho_0 \tilde{j}^L - \tilde{l}_1^L - (1-\bar{\mu})\hat{R}\tau = \begin{cases} \left[\rho_0 - \gamma_1^L\right] i & \text{if } \hat{R} \geq \gamma_1^L \\ \left[\rho_0 - (1-\bar{\mu})\hat{R}\right] i & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

$$\tilde{\tau}^L = \begin{cases} 0 & \text{if } \hat{R} \geq \gamma_1^L \\ i & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

$$\tilde{C}_{1,2}^L = \begin{cases} \left[\rho_1 - \gamma_1^L\right] i + x_1^L & \text{if } \hat{R} \geq \gamma_1^L \\ \left[\rho_1 - \hat{R}\right] i + x_1^L & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

I continue by solving this problem for a stress period For a given $\{x_1^H, i, \hat{R}, \bar{\mu}\}$, recall that $\gamma_1^H > \rho_0$ which implies that $\phi_1^H = 1$ is no longer possible (no *finance as you go*). Moreover, $\gamma_1^H > 1$ which implies that it is better to lend at international markets than to consume, thus, c_1^H is equal to zero.

By assumptions, I only consider scenarios where $0 < \hat{R} \leq \gamma_1^H$. In this case, τ has to be the largest possible given that it is not, in any case, more expensive than market and funding liquidity. This is reflected in the objective function. Simultaneously, τ's cost on pledgeable income is *smoother* due to fiscal capacity $(1-\bar{\mu})$, so, even if $\hat{R} = \gamma_1^H$ and for any given level of x_1^H, an entrepreneur can borrow from the LOLR as a minimum (if $\bar{u} = 0$) the same amount as in international markets. This suggests that is weakly optimal to exhaust pledgeable income with τ. To see this more formally, **I first consider when** $(1-\bar{\mu})\hat{R} \leq \rho_0$ That is, for a given $\hat{R}$, the level of fiscal capacity is such that a project can finance as it goes using public funding liquidity. I argue that $\{j^H = i, c_1^H = 0, M_1^H = 0, \phi_1^H = 0, \tau = i\}$ is optimal with payoff $\left[\rho_1 - \hat{R}\right] i + \gamma_1^H x_1^H$. Note that this candidate is feasible since j^H is equal to i and $\tau = i$ is possible since $(1-\bar{\mu})\hat{R} \leq \rho_0$. I prove optimality by contradiction. I suppose that there is another feasible solution with a payoff greater than $\left[\rho_1 - \hat{R}\right] i + \gamma_1^H x_1^H$. However, this is not possible because γ_1^H is greater than 1, and any feasible τ and j are bounded by i. Note that, since there is an upper limit on $\hat{R}$ equal to γ_1^H, then the case of $(1-\bar{\mu})\hat{R} \leq \rho_0$ is the only possible scenario for when $\bar{\mu} \geq 1 - \frac{\rho_0}{\gamma_1^H}$. Next, consider **when** $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$. In this scenario, projects can no longer finance as they go using public funding, they need to complement with market liquidity. I argue that when $x_1^H \geq i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right]$, $\{j^H = i, c_1^H = 0, M_1^H = i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right], \phi_1^H = 0, \tau = \frac{\rho_0}{(1-\bar{\mu})\hat{R}} i\}$ is optimal with payoff $\left[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\right] i + \gamma_1^H \left[x_1^H - i(1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}})\right]$. First, note that this candidate is feasible precisely because $\rho_0 < (1-\bar{\mu})\hat{R} < \hat{R} \leq \gamma_1^H$ and $x_1^H \geq i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right]$. I show optimality by contradiction. There is no other feasible candidate that creates a greater payoff since any j^H is bounded by i, and, in this scenario, any feasible τ is bounded by $\frac{\rho_0}{(1-\bar{\mu})\hat{R}} i$. **Now, I turn to when** $x_1^H < i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right]$, where I argue that $\{j^H = \frac{x_1^H}{1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}}, c_1^H = 0, M_1^H = x_1^H, \phi_1^H = 0, \tau = \frac{\rho_0}{(1-\bar{\mu})\hat{R}} \frac{x_1^H}{1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}}\}$ is optimal with payoff

$\left[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\right]\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}$. This candidate is feasible. To see this, $j^s < i$ since $x_1^H < i\left[1 - \frac{\rho}{(1-\bar{\mu})\hat{R}}\right]$ Also, the incentive compatibility constraint is binding in this scenario, so it is feasible, and, finally $c_1 + M_1$ is equal to x_1^H which, by definition, is less or equal to x_1^H. Once again, I show optimality by contradiction. There is another solution that generates a greater payoff. This is not possible since: i) any feasible j^H is bounded by $\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}$ since $x_1\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right]$, $\hat{R}(1-\bar{\mu}) \leq \gamma_1^H$, and any ϕ_1^H, τ are non-negative, ii) any feasible τ are argued earlier is bounded by $\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}$ and $\gamma_1^H \geq \hat{R}$.

Below the optimal strategies for an entrepreneur during a stress episode. To differentiate from optimal behavior in LFE, I denote these functions with a tilde. I also define $\bar{\mu}_{AE}$ equal to $1 - \frac{\rho_0}{\gamma_1^H}$

$$\tilde{j}^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ i & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \;\&\; \bar{\mu} < \bar{\mu}_{AE} \\ \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ i & \quad \text{for all } x_1^H \geq i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

$$\tilde{c}_1^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ 0 & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \;\&\; \bar{\mu} < \bar{\mu}_{AE} \\ 0 & \quad \text{for all } x_1^H < i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ 0 & \quad \text{for all } x_1^H \geq i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

$$\tilde{M}_1^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ 0 & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \;\&\; \bar{\mu} < \bar{\mu}_{AE} \\ x_1^H & \quad \text{for all } x_1^H < i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] & \quad \text{for all } x_1^H \geq i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

$$\tilde{l}_1^H + \hat{R}(1-\bar{\mu})\tau = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ (1-\bar{\mu})\hat{R}i & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \;\&\; \bar{\mu} < \bar{\mu}_{AE} \\ \rho_0\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ \rho_0 i & \quad \text{for all } x_1^H \geq i\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

$$\tilde{\tau}^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ i & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ \frac{\rho_0}{(1-\bar{\mu})\hat{R}} \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \\ \frac{\rho_0}{(1-\bar{\mu})\hat{R}} i & \quad \text{for all } x_1^H \geq i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \end{cases}$$

$$\tilde{C}_{1,2}^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ \big[\rho_1 - \hat{R}\big] i + \gamma_1^H x_1^H & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ \big[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\big] \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \\ \big[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\big] i + \gamma_1^H \big[x_1^H - i(1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}})\big] & \quad \text{for all } x_1^H \geq i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \end{cases}$$

B.2 Period 0 - Optimal Behavior

Recall that a given entrepreneur's period 0 optimization problem is as follows:

$$\begin{aligned} \underset{\{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\}}{\text{Maximize}} \quad & c_0 + \alpha\big[C_{1,2}^L - l_0^L\big] + (1-\alpha)\big[C_{1,2}^H\big] \\ \text{subject to: } & \alpha\big[l_0^L + d_f^L i\big](1-\alpha)\big[d_f^H i\big] = i - M_0 \\ & d_f^s i + d_e^s i = \pi i \\ & l_0^L \ \epsilon \big[0, \rho_0 j^L - l_1^L\big] \\ & A - F_0 - c_0 - M_0 = x_A \\ & d_e^s i + x_A = x_1^s \\ & \{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\} \text{ non-negative, given the optimal} \\ & \text{functions at date-1 that depend on } \{x_1^L, x_1^H\} \end{aligned}$$

The proof that any entrepreneur chooses $C_0 = 0, x_1^L = 0, l_0^L = \big[\rho_0 - \gamma_1^L\big] i$ follows the same steps as in the Laissez Faire Equilibrium. If interested, I refer the reader to it. Similarly, regarding M_0, incentives for entrepreneurs are to invest in the project all of their net worth as in the LFE. However, with an LOLR, entrepreneurs can only invest their disposable net worth $A - F_0 = M_0$.

What about $\bar{x_1}^H$? The decision on how much to hoard at $t = 0$ will depend on the expectation of $\hat{R}$ and the LOLR's fiscal capacity. **First, I consider a pair $\{\hat{R}, \bar{\mu}\}$ such that** $\hat{R}(1-\bar{\mu}) = \rho_0$. In this scenario, the objective function is given by

$$\Big[\alpha\big[\rho_1 - \rho_0\big] + (1-\alpha)\big[\rho_1 - \hat{R} + \gamma_1^H \bar{x}_1^H\big]\Big] \frac{A - F_0}{1 - \pi - \alpha(\rho_0 - \gamma_1^L) + (1-\alpha)\bar{x}_1^H}$$

where the sign of the derivative is determined by

$$\gamma_1^H\big[1-\pi-\alpha(\rho_0-\gamma_1^1)\big]-\alpha(\rho_1-\rho_0)-(1-\alpha)(\rho_1-\hat{R})$$

which, because $\gamma_1^H \geq \hat{R}$, is less or equal to

$$\gamma_1^H\big[1-\pi+\alpha\gamma_1^L+(1-\alpha)\big]-\rho_1-\alpha\rho_0(\gamma_1^H-1)$$

This last term is strictly negative because of Assumption 1. Note that this is true independently of probabilities of events. **Next, I consider a pair $\{\hat{R},\bar{\mu}\}$ such that** $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$ If an entrepreneur's expectations are within this environment, then the sign of derivative of the objective function with respect to x_1^H evaluated close to zero depends on the sign of

$$\Big[(\rho_1-\frac{\rho_0}{1-\bar{\mu}})(1-\pi-\alpha(\rho_0-\gamma_L^L))-\alpha\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big](\rho_1-\rho_0)\Big]$$

This term can be negative or positive depending on α. I define $\omega(\bar{\mu},\hat{R})$ equal to

$$\frac{(\rho_1-\rho_0)\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]+\big[\rho_1-\frac{\rho_0}{1-\bar{\mu}}\big]\big[(\rho_0-\gamma_1^L)-(1-\pi)\big]}{(\rho_1-\rho_0)\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]+\big[\rho_1-\frac{\rho_0}{1-\bar{\mu}}\big](\rho_0-\gamma_1^L)} \tag{24}$$

It is quite straight forward to show that if $(1-\alpha) \leq \omega(\bar{\mu},\hat{R})$, and expectations over pair $\{\hat{R},\bar{\mu}\}$ are such that $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$, then an entrepreneur's optimal answer is $x_1^H = 0$. Likewise, with same expectations but when $(1-\alpha) > \omega(\bar{\mu},\hat{R})$ then x_1^H is equal to $i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]$.

C No Reserves Equilibria

C.1 Mature Fiscal Capacity Equilibrium

Whenever $\bar{\mu} \,\epsilon\, \big[\mu_A, 1\big]$, and Assumptions 1 and 3 hold, the following Mature Fiscal Capacity Equilibrium Exists -

- Date-2: Entrepreneurs' don't abscond, and consume according to $\{\tilde{c}_2^L, \tilde{H}_2^L\}$. After a boom event, an LOLR has no action. Following a market stress, LOLR's collects $\gamma_1^H i$ and uses it to redeem bonds to foreign lenders.
- Date-1: Given the realized shock, the LOLR sets $\hat{R}$ equal to γ_1^S accordingly. $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i, \hat{R}\}$ and determined by strategy profile functions $\tilde{j}^L$, $\tilde{c}_1^L$, $\tilde{M}_1^L$, $\tilde{l}_1^L$, $\tilde{\phi}_1^L = \frac{l_1^L}{\gamma_1^L}$, τ^L, $\tilde{j}^H$, $\tilde{c}_1^H$, $\tilde{M}_1^H$, $\tilde{l}_1^H$, $\tilde{\phi}_1^H = \frac{l_1^H}{\gamma_1^H}$, τ^H. Given τ^L and τ^L, the LOLR issues $B_1 = i$ during market stress and zero during a boom. At this point, $\{j^L, j^H\}$ are equal to $\{i, i\}$
- Date-0: Given $\bar{\mu}, \gamma_1^S\}$, $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0 - \gamma_1^L)}$, $M_0 = A$, $\phi_0 = i - A$, $d_f^L i = \pi i$, $d_f^H i = \pi i$, $l_0^L = (\rho_0 - \gamma_1^L)i$, $x_1^L = 0, x_1^H = 0\}$ solve entrepreneurs problem at the initial period. Given $\{x_1^L, x_1^H\}$, the LOLR chooses optimally $F_0 = 0$

Proof Choose $\bar{\mu} \geq \mu_A$. Let start by showing that bonds are redeemable after a market stress with no reserves. Suppose they are not, then $0 < \gamma_1^H B_1 - \hat{R}\tau \rightarrow \gamma_1^H i - \gamma_1^H i = 0 \rightarrow 0 < 0$ which is a contradiction. This only holds if an LOLR can collect fully $\hat{R}\tau$. Again, suppose that is not possible. Then, it must be true that $0 < (1-\bar{\mu})\hat{R}\tau - \rho_0 i = (1-\bar{\mu})\gamma_1^H i - \rho_0 i$ which is strictly less than zero because $\bar{\mu} \geq \mu_A$ and hence a contradiction . This result is also consistent with entrepreneurs not absconding. At date-1, given $\hat{R} = \gamma_1^S$ and $\bar{\mu} \geq \mu_A$, then $\gamma_1^H(1-\bar{\mu}) \leq \rho_0$, so, entrepreneurs demand $\tau^L = 0$ and $\tau^H = i$. Given that $F_0 = 0$, then $\hat{R}$ is set equal to γ_1^S for any demand τ, including $\tau^L = 0$ and $\tau^H = i$ of course. Given $\hat{R} = \gamma_1^H$ and $\bar{\mu} \geq \mu_A$, it is optimal for entrepreneurs to choose $x_1^H = 0$ by Assumption 1. Now, is it optimal for the LOLR given $x_1^H = 0$ and $\bar{\mu} \geq \mu_A$ to choose F_0 equal to zero. Suppose it is not. Then there is a feasible $\hat{F}_0$ greater than zero that generates a lower welfare cost, Then it must hold that $0 > \psi\big[(\hat{F}_0 - F_0)\big]\kappa(0) + (1-\alpha)(L(i) - L(i)) = \psi\big[(\hat{F}_0)\big]$ which is strictly greater than zero since any feasible $\hat{F}_0$ is non negative and by assumption $\hat{F}_0 > 0$ ∎

C.2 Sudden Stop Equilibrium - No Reserves

Whenever $\bar{\mu} < \mu_A$, $(1-\alpha) < \omega(\gamma_1 H, bar\mu)$, and Assumptions 1 and 3 hold, the following following Sudden Stop Equilibrium exists

- Date-2: Entrepreneurs' don't abscond, and consume according to $\{\tilde{c}_2^L, \tilde{H}_2^L\}$. After a boom event, an LOLR has no action. Following a market stress, LOLR doesn't collect since there are no outstanding bonds. Note that, after a stress, entrepreneurs don't consume either because projects were shutdown.

- Date-1: Given the realized shock, the LOLR sets $\hat{R}$ equal to γ_1^S accordingly. $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i, \hat{R}\}$ and determined by strategy profile functions $\tilde{j}^L,\ \tilde{c}_1^L,\ \tilde{M}_1^L,\ \tilde{l}_1^L,\ \tilde{\phi}_1^L = \frac{l_1^L}{\gamma_1^L},\ \tau^L,\ \tilde{j}^H,\ \tilde{c}_1^H,\ \tilde{M}_1^H,\ \tilde{l}_1^H,\ \tilde{\phi}_1^H = \frac{l_1^H}{\gamma_1^H},\ \tau^H$. $\{\tau^L, \tau^H\}$ are equal to $\{0, 0\}$ and doesn't need to issue any bonds in either state. At this point, $\{j^L, j^H\}$ are equal to $\{i, 0\}$

- Date-0: Given $\bar{\mu}, \gamma_1^S\}$, $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)},\ M_0 = A,\ \phi_0 = i - A,\ d_f^L i = \pi i,\ d_f^H i = \pi i,\ l_0^L = (\rho_0 - \gamma_1^L)i,\ x_1^L = 0, x_1^H = 0\}$ solve entrepreneurs problem at the initial period. I assume that the LOLR cannot accumulate reserves.

Proof Choose $\bar{\mu} < \mu_A$. Let start by showing that bonds are redeemable after a market stress with no reserves. This is obvious since there are no outstanding. Since projects shutdown, entrepreneurs don't have any incentives to abscond after a stress period. At date-1, given $\hat{R} = \gamma_1^S$, $\bar{\mu} < \mu_A$, then $\gamma_1^H(1-\bar{\mu}) > \rho_0$, so, entrepreneurs demand $\tau^L = 0$ and $\tau^H = \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H} \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}}$ which is equal to zero since $x_1^H = 0$. Given that $F_0 = 0$, then $\hat{R}$ is set equal to γ_1^S for any demand τ, including $\tau^L = 0$ and $\tau^H = 0$ of course. Given $\hat{R} = \gamma_1^H$ and $\bar{\mu} < \mu_A$, and, that $(1-\alpha) < \omega(\gamma_1 H, bar\mu)$ it is optimal for entrepreneurs to choose $x_1^H = 0$ since the probability of a market stress is lower than the threshold at which the FOC of entrepreneurs shifts to positive. Now, is it optimal for the LOLR given $x_1^H = 0$ and $\bar{\mu} < \mu_A$ to choose F_0 equal to zero? Probably not Suppose it is not. Then there is a feasible $\hat{F}_0$ greater than zero that generates a lower welfare cost, Then it must hold that $0 > \psi\big[(\hat{F}_0 - F_0)\big]\kappa(0) + (1-\alpha)(L(i) - L(i)) = \psi\big[(\hat{F}_0)\big]$ which is strictly greater than zero since any feasible $\hat{F}_0$ is non negative and by assumption $\hat{F}_0 > 0$ ∎

C.3 No Crisis Equilibrium - No Reserves

Whenever $\bar{\mu} < \mu_A$, $(1-\alpha) > \omega(\gamma_1 H, bar\mu)$, and Assumptions 1 and 3 hold, the following No Crisis Equilibrium - No Reserves Exists -

- Date-2: Entrepreneurs' don't abscond, and consume according to $\{\tilde{c}_2^L, \tilde{H}_2^L\}$. After a boom event, an LOLR has no action. Following a market stress, LOLR's collects $\gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i$ and uses it to redeem bonds to foreign lenders.

- Date-1: Given the realized shock, the LOLR sets $\hat{R}$ equal to γ_1^S accordingly. $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i, \hat{R}\}$ and determined by strategy profile functions $\tilde{j}^L,\ \tilde{c}_1^L,\ \tilde{M}_1^L,\ \tilde{l}_1^L,\ \tilde{\phi}_1^L = \frac{l_1^L}{\gamma_1^L},\ \tau^L,\ \tilde{j}^H,\ \tilde{c}_1^H,\ \tilde{M}_1^H,\ \tilde{l}_1^H,\ \tilde{\phi}_1^H = \frac{l_1^H}{\gamma_1^H},\ \tau^H$. Given τ^L and τ^L, the LOLR issues $B_1 = \big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i$ during market stress and zero during a boom. At this point, $\{j^L, j^H\}$ are equal to $\{i, i\}$

- Date-0: Given $\bar{\mu}, \gamma_1^S\}$, $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)+(1-\alpha\bar{x}_1^H}$, $M_0 = A,\ \phi_0 = i - A,\ d_f^L i = \pi i,\ d_f^H i = \pi i - \big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i,\ l_0^L = (\rho_0 - \gamma_1^L)i,\ x_1^L = 0, x_1^H = i[1 - \gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]]\}$ solve entrepreneurs problem at the initial period. I assume that the LOLR cannot collect Reserves.

Proof Choose $\bar{\mu} < \mu_A$. Let start by showing that bonds are redeemable after a market stress with no reserves. Suppose they are not, then $0 < \gamma_1^H B_1 - \hat{R}\tau \rightarrow \gamma_1^H \left[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\right] i - \gamma_1^H \left[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\right] i = 0 \rightarrow 0 < 0$ which is a contradiction. This only holds if an LOLR can collect fully $\hat{R}\tau$. Again, suppose that is not possible. Then, it must be true that $0 < (1-\bar{\mu})\hat{R}\tau - \rho_0 i = (1-\bar{\mu})\gamma_1^H \left[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\right] i - \rho_0 i$ which is strictly less than zero by simplification and hence a contradiction . This result is also consistent with entrepreneurs not absconding. At date-1, given $\hat{R} = \gamma_1^S$ and $\bar{\mu} < \mu_A$, then $\gamma_1^H(1-\bar{\mu}) > \rho_0$, so, entrepreneurs demand $\tau^L = 0$ and $\tau^H = \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H} i$. Given that $F_0 = 0$, then $\hat{R}$ is set equal to γ_1^S for any demand τ, including $\tau^L = 0$ and $\tau^H = \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H} i$ of course. Given $\hat{R} = \gamma_1^H$, $\bar{\mu} < \mu_A$, and that $(1-\alpha) \geq \omega(\gamma_1^H, \bar{\mu})$ it is optimal for entrepreneurs to choose $x_1^H = i[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}]$ since the probability of a market stress is more than enough to compensate for the sacrifice in investment scale. Note that when $(1-\alpha) \geq \omega(\gamma_1^H, \bar{\mu})$ ∎

D Lender of Last Resort Optimal Behavior

At $t = 2$, the LOLR collects any claims on entrepreneurs to redeem bonds potentially issued at $t = 1$. The LOLR collects a total of $\hat{R}\tau$ where a share $\bar{\mu}$ comes from entrepreneurs directly and the remainder from projects as long as entrepreneurs don't abscond. If entrepreneurs abscond, then an LOLR cannot claim $(1 - \bar{\mu})\hat{R}\tau$. As a result, total revenue is limited to $\bar{\mu}\hat{R}\tau$. However, by design, Contracts K_1^S are such that entrepreneurs don't abscond so it is fair to say that total revenue can be collected. Thus, for bonds to be redeemable, the following condition must hold

$$\gamma_1^S B_1 \leq \hat{R}\tau$$

At $t = 1$, the LOLR defines $\hat{R}$, while it issues B_1, and depletes f_1 of their reserves stock (F_0) to cover any demand for public liquidity. So, given τ, B_1 is equal to the $max\{0, \tau - f_1\}$. By replacing this in the previous condition and the fact that $f_1 \leq F_0$, then

$$\gamma_1^s \tau \leq \hat{R}\tau + F_0$$

Then, this condition with equality and knowing that $\hat{R}$ is non-negative, it is straight forwad to derive (8). Note that as long as is greater or equal than (8), (7) is satisfied. To see this, assume that there is exists a R such that is greater or equal to $\bar{R}(\tau, F_0)$ and it doesn't satisfy (7). If this is true then the following must hold, for positive τ,

$$0 < \gamma_1^S \tau - R\tau - \gamma_1^S F_0 \leq \gamma_1^S \tau - \bar{R}\tau - \gamma_1^S F_0 = \gamma_1^S \tau - \gamma_1^S \big[1 - \frac{F_0}{\tau}\big]\tau - \gamma_1^S F_0$$

However, this is a contradiction since the last term is equal to zero. Thus, there is no such R. This condition is also satisfied for τ equal to zero since zero is equal to the product of γ_1^s times zero. This proves that setting $\hat{R}$ equal to $\bar{R}(\tau, F_0)$ guarantees that (7) is satisfied. Thus, at $t = 1$, the LOLR with a given stock of reserves F_0 observes τ and sets $\hat{R}$ accordingly. One important clarification is what should a LOLR do with F_0 is τ is equal to zero. Since there is no demand, then f_1 and B_1 is zero by definition. The LOLR has two options with F_0, either lend it at international markets and rebate the return to entrepreneurs at $t = 2$, or rebate it immediately to entrepreneurs at $t = 1$. Since the LOLR has no preference over entrepreneurs consumption, I establish that it rebates everything at the end of $t = 2$ which is consistent with what determines ψ.

The optimal behavior at $t = 0$ is, given x_1^H, to choose $F_0 \leq A$ to minimize

$$\psi F_0 \kappa(x_1^H) + (1 - \alpha)L(\tilde{j}^H)$$

Note that this version of the objective function already includes that entrepreneurs don't hoard liquidity for booms and that they are able to reinvest at full-scale, even without LOLR assistance. The expected welfare cost objective function has a lower bound at zero when F_0 is equal to zero and $j^H = 1$. Additionally, $\bar{R}(\tau, 0)$ is equal to γ_1^H for any non-negative τ. **First, I consider the case of Mature LOLR** Choose $\bar{\mu} \geq \mu_A$. I argue that for any γ_1^H, the optimal response is F_0 equal to zero. The reason is that when $\bar{\mu} \geq \mu_A$, then $(1 - \bar{\mu})\gamma_1^H \leq \rho_0$. In this scenario, as determined by $\tilde{j}^H$, projects continue at full scale regardless of x_1^H. To prove this, suppose that there exists a positive $\tilde{F}_0$ such that is generates a lower payoff than F_0. This is not possible because $\tilde{F}_0$ incurs in the opportunity cost $\psi\tilde{F}_0\kappa(x_1^H)$ while not reducing the welfare losses due to partial liquidation since, even with F_0 equal to zero, $\tilde{j}^H$ is equal to i. **Now, I consider the case of an economy with a LOLR with $\bar{\mu} < \mu_A$.** I argue that if $x_1^H \geq i[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}$ then the optimal F_0 is equal to zero. Similar to the previous argument, with this amount of liquidity, entrepreneurs are able to continue at full scale at a cost of public liquidity equal to γ_1^H which results from $F_0 = 0$. Thus, F_0 is optimal because it reaches the lower bound of the expected welfare function. Any positive F_0 incurs in cost $\psi\kappa(x_1^H)$ but cannot reduce the welfare losses beyond zero. Next, I consider the case when x_1^H is equal to zero. With no market liquidity, projects are forced to shutdown ($\tilde{j}^H = 0$) unless the marginal cost of public liquidity is at the most such that $(1-\bar{\mu})R = \rho_0$. This lowers the level of $\hat{R}$ sufficiently such that any unit borrowed from LOLR increases pledgeable income in the same magnitude. This is the condition for example, for a cost of funding liquidity to be such that projects can be self-financed. By rearranging (8), you find F_0 as a function of $\hat{R}$.

$$F(\hat{R}, \bar{\mu}) = \big[1 - \frac{\hat{R}}{\gamma_1^H}\big]\tau(\hat{R}, \bar{\mu})$$

Function $(\hat{R}, \bar{\mu})$ is not completely determined since the demand of public liquidity is a function of $\hat{R}$ and F_0 it self. However, this relationship can be used to find $\bar{F}(\bar{\mu})$ which is the level of reserves such that $(1-\bar{\mu})\hat{R} = \rho_0$ when $x_1^H = 0$

$$\bar{F}(\bar{\mu}, x_1^H) = A\kappa(0)\frac{1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}}{1 + \big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]\kappa(0)}$$

It is worth pointing out that $\bar{F}(\bar{\mu})$ is decreasing and strictly concave with respect to $\bar{\mu}$. Thus if a LOLR accumulates $\bar{F}(\bar{\mu})$, then it incurs in a welfare cost equal to $\psi\bar{F}(\bar{\mu})\kappa(0)$. I argue that this is optimal if the expected welfare cost of shutdown is too large. To see this, first, consider a F_0 that is greater than $\bar{F}(\bar{\mu})$ but generates a lower welfare cost. This is not possible since $\bar{F}(\bar{\mu})$ is by definition the minimum amount of reserves that achieve full scale reinvestment when x_1^H is zero. Now consider an F_0 that is lower than $\bar{F}(\bar{\mu})$. Recall that with $x_1^H = 0$, entrepreneurs cannot reinvest at all, and, thus, shutdown their projects. So, $j^H = 0$ for all $F_0 < \bar{F}(\bar{\mu})$ Among those F_0 lower than $\bar{F}(\bar{\mu})$, F_0 equal to zero generates the lower welfare costs since projects shutdown but it doesn't incur in the opportunity cost of deviating resources. Thus, I define set $\Lambda(\bar{\mu}) = \{z \mid z \leq \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})\}$ I argue that when $1-\alpha$ belongs to $\Lambda(\bar{\mu})$, then $F_0 = 0$ is optimal. To see this, suppose that it is not. Therefore, I assume that $\bar{F}(\bar{\mu})$ is optimal then $0 > \psi\kappa(0)(\bar{F}(\bar{\mu}) - 0) + (1-\alpha)(0 - L(0))$ which simplified is $0 > \psi\kappa(0)\bar{F}(\bar{\mu}) - (1-\alpha)L(0)) \geq \psi\kappa(0)\bar{F}(\bar{\mu}) - \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})L(0))$ since $(1-\alpha)$ belongs to $\Lambda(\bar{\mu})$. Note that the last term is equal to zero, thus, I get a contradiction and $F_0 = 0$ is optimal. Similarly, when $1-\alpha$ doesn't belong to $\Lambda(\bar{\mu})$, then $\bar{F}(\bar{\mu})$ is optimal. To see this suppose that it is not. Then, if F_0 is optimal then it mus be true that $0 > -\psi\kappa(0)\bar{F}(\bar{\mu}) + (1-\alpha)L(0)) \geq \psi\kappa(0)\bar{F}(\bar{\mu}) + \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})L(0))$ since $(1-\alpha)$ doesn't belong to $\Lambda(\bar{\mu})$. Again, note that the last term is equal to zero, thus, I get a contradiction and $F_0 = \bar{F}(\bar{\mu})$ is optimal. A comment is relevant. There is the possibility that set $\Lambda(\bar{\mu})$ is empty for feasible probability of market stress if $L(0)$ is too high. Finally, I consider the case when x_1^H is strictly between zero and

$i\big[1 - \frac{\rho}{(1\bar{\mu})\gamma_1^H}\big]$. In this scenario, entrepreneurs have positive levels of liquidity such that the optimal F_0 is an solution determined by the first order condition

$$\psi\kappa(x_1^H) + (1-\alpha)\frac{\partial L(j^s)}{\partial j^s}\frac{\partial j^s}{\partial \hat{R}}\frac{\partial \hat{R}}{\partial F_0}$$

In this scenario, given the continuity of functions, it is possible that the LOLR will accept some partial liquidation in order to reduce the cost of hoarding reserves.