COVID-19 epidemic and generational welfare[1]

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Abstract: We study the effects of COVID-19, and the ensuing lockdown and fiscal policies, on the welfare of different age-groups within a life-cycle macroeconomic scheme, adapted from Gertler (1999), where the pandemic is represented as a shock to the mortality rate. We obtain two main results. First, we show that lockdown policies have a negative impact on the dynamics of economic welfare of younger agents relative to that of older agents, thus providing analytical support to the idea that the management of the COVID-19 pandemic through lockdown policies has mainly hit the young generations. Second, we show that expansionary fiscal policies aimed at supporting income after the lockdown affect the relative welfare index of age-groups mainly through the repayment scheme of the consequent public debt; the more such repayment scheme entails a postponement of the debt repayment, the more older agents are favored (in relative terms).

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1 Introduction

The literature, both theoretical and empirical, on the macroeconomic effects of the COVID-19 pandemic has rapidly flourished, in the urgent need to understand the severity of the shock and to identify the optimal policies (including lockdown and social distancing) to adopt in order to limit its consequences on individual incomes and on the real economy.

In spite of the wide spectrum of addressed issues, in our opinion, the theoretical contributions presented so far tend to disregard one clear and statistically significant indicator of the pandemic, i.,e., that it affects in extremely different ways the different age cohorts of the population. To drastically summarize, "young" people experience extremely small mortality rates due to COVID-19 and suffer from zero to low health damages, whereas "old" people are dramatically hit, with heavy health consequences and high to very high mortality rates. From a purely economic viewpoint, this implies that the young generations are asked to bear most of both the present costs (e.g., the reduction of income and employment) and the future costs (e.g., the servicing and repayment of an increased public debt) of the severe containment policies introduced in almost every country, without enjoying at the same time the appropriation of (a significant part of) the economic benefits represented by reduced health damages and saved lives. "Old" agents benefit instead the most from lockdowns or other containment policies, especially in terms of reductions of mortality. For example, to provide a quantitative indication on this issue, Greenstone and Nigam (2020) monetize the impact of moderate social distancing on deaths from COVID-19. Using the projected age-specific reductions in death and age-varying estimates of the United States Government's value of a statistical life, they calculate that around 90% of the monetized mortality benefits of social distancing accrue to people aged 50 or older. These differentiated effects of COVID-19 on different generations suggest that the age dimension cannot be disregarded when studying the effects of the pandemic and evaluating the differentiated costs of the policies to implement in order to limit the contagion.

Although the amount of studies and research devoted to the economic effects of COVID-19 is continuously growing, it seems that the pandemic's demographic features previously described continue to be not adequately addressed; most of the proposed theoretical analyses do not model the COVID-19 pandemic as a shock to the differentiated mortality rates of different generations of agents. Just to provide a few examples, Faria-e-Castro (2021) conceive the pandemic, within a DSGE New Keynesian model with financial frictions, as a large negative shock to the utility of consumption (contact-intensive services); Fornaro and Wolf (2020), within a New Keynesian representative-agent economy, represent the pandemic as a negative shock to the growth rate of productivity that may produce a demand-driven slump, a supply-demand doom loop and open stagnation traps induced by pessimistic animal spirits; Guerrieri et al. (2020) as a temporary shock to the labour supply due to shutdowns which, in their turn, produce short-term effects on demand.

Some studies, more directly related to epidemiological issues, provide a partial account of the demographic features of the COVID-19 pandemic. For example, among the recent contributions based on epidemiological SIR/SIER models of contagion[2], Berger,

[2]See Atkeson (2020) for an overview of these models.

Herkenhoff and Mongey (2020) construct an extended model with immunity and random testing; Eichenbaum, Rebelo and Trabandt (2020) build a real one-sector dynamic model to analyze the effects of the pandemic, taking into account the optimal rational responses by private agent, and the optimal Pigouvian policy capable to internalize the externalities produced by individual actions taking the contagion rate as given. Probably, among the recent contributions, the model which is more directly related to our research topic is that of Gagnon et al. (2020), who study the effects of mortality during the COVID-19 pandemic on the economy's productive capacity in the context of an overlapping-generations model. Nevertheless, they do not focus explicitly on the consequences for the distribution of welfare among different generations and do not include in their analysis the impact of fiscal interventions aimed at providing immediate income support or of other forms of policy. On the empirical side, Jordà et al. (2020) study persistent pandemics, focusing on the longer-term effects of death, to show that, whereas wars increase the real rate of interest, pandemics do the opposite, with long lasting negative effects spanning over decades. Finally, Alvarez et al. (2020) study the optimal dynamic shutdown policy.

These observations motivate our aim to study the differentiated effects of the COVID-19 on different age groups within a life-cycle macroeconomic scheme where the pandemic is represented, per se, as a shock to the mortality rate, to be kept distinct from the other shocks that encapsulate the effects of policy measures. To this aim, we make use of the model originally developed by Gertler (1999), and recently used (among others) by Carvalho et al. (2016), which preserves the main life-cycle properties of the economy in an analytically tractable way, without including the detailed demographic structure of a complete overlapping-generations scheme. The model's analytical tractability allows us to define an explicit index of the relative welfare of the two main age-groups in which the population is partitioned: "young" agents (active workers) and old agents (retirees), and to study its dynamic evolution in response to the pandemic shock and to different policy programs.

This version of Gertler's (1999) model lends itself to a wide number of analytical and numerical applications. By restricting the attention to the research problem previously described, we obtain two main results. First, the model's structure allows us to investigate the impact of a fall in the old agents' life expectancy due to the pandemic on the relative welfare index of the two groups. We use to the results provided by the available statistical and demographic research about the quantitative impact of COVID-19 on the mortality of age-classes (Goldstein and Lee 2020) to calibrate numerically the model's relevant variables and parameters and show that, under a counterfactual scenario under which no lockdown or social restriction policy is undertaken, the burden of welfare losses due to the pandemic falls mainly on the old agents *via* a pure "demographic channel". When instead lockdown policies are active, the fall of old agents' life expectancy is contained, but the economy as a whole is hit by a much more severe recession; in this case the welfare losses are more evenly spread between the two age-groups, but the cost paid in terms of output reduction is substantial. The second result concerns the effects of fiscal policies that could be implemented in the wake of the pandemic and the ensuing lockdown in order to provide a form of immediate income support to households and firms. Expansionary fiscal policies of this kind entail an increase in the level of public debt, which can be repaid according to different schemes and time horizons. These debt repayment schemes imply different time evolutions of the age-groups' relative welfare: the more the repayment scheme entails a postponement of debt repayment, the more the old agents are favoured. Nevertheless, the effects of expansionary fiscal programs and debt repayment schemes on the dynamics of the relative welfare index are comparatively smaller than those of the lockdown/containment policies.

The paper is organized as follows. In Section 2 we present a synthetic description of the analytical framework in order to ease the reading of the subsequent results. In Section 3 the rationale behind the choice of the relative welfare index adopted in the paper is explained and the index is derived from the model's closed-form solutions. Section 4 presents and discusses the results obtained from the model's numerical simulations. Section 5 concludes.

2 The model economy

In this section we describe the main elements of the model, which is solved along the lines of Gertler (1999). The economy consist of three types of economic agents: individual (private) agents, firms and the government. Each individual agent operates within two distinct phases of the life-cycle: when they enter the economy as "young" agents (they are born) they provide labor services used in production, save and consume. They can become "old" with an exogenous probability, and continue to provide labor services (although their productivity is lower) while also collecting revenues from accumulated wealth and from social security payments. Old agents can hence be only partially and somehow inappropriately considered as "retirees". When old, agents face a constant and exogenous probability of surviving into the next time period and a complementary probability of death.

Saving instruments are of two distinct typologies: physical capital and government bonds. Final goods, represented by a single net output used also as a numeraire, are produced by perfectly competitive firms and are used for consumption and investment in physical capital. The government decides the amount of spending and of lump sum taxes, and the one-period debt evolves according to the fiscal budget constraint. Aggregate uncertainty is absent and we only consider (initially) unexpected changes in the relevant demographic parameters, i.e., the probability of survival of old/retirees agents, together with exogenous changes in some of the relevant policy parameters and variables. Even though we are aware that it can only approximate all the relevant features of our research problem, we adopt this scheme for its analytical tractability, which allows us to highlight in a simple way the different economic effects of COVID-19 on large and distinct fractions of the population.

We calibrate the "retirement" probability of young/working agents to match an average age of 65, and the probability of survival of old/retired agents to match an expected life duration of 79 (as a reference value). In the model, the policies implemented to combat the epidemic can act through the probabilities of death and also entail costs, in terms of foregone production (as it is the case for social distancing and generalized quarantine) and greater public debt, that are differently distributed between age groups.

2.1 Life-cycle structure: young/workers and old/"retirees"

At each time period t, each agent belongs to one of two distinct groups indexed by z: young/workers ($z = y$) and old/retirees ($z = o$). The population of young agents N_t^y evolves according to the following dynamic law:

$$N_t^y = (1 - \omega_t + n_t)N_{t-1}^y + \omega_t N_{t-1}^y = (1 + n_t)\, N_{t-1}^y \tag{1}$$

where $1 - \omega_t$ is the probability of becoming old (of retirement) and n_t is the rate at which new young agents are born. The population of old/retired agents N_t^o follows the rule:

$$N_t^o = (1 - \omega_t)\, N_{t-1}^y + \gamma_t N_{t-1}^o \tag{2}$$

where γ_t is the probability to survive into the next period. In the subsequent sections we specify the structure of γ_t, that we assume to depend on two elements: $\gamma_t = \gamma\,(u_t; \vartheta_t)$, where u_t represents the COVID-19 shock and ϑ_t is a parameter related to the containment policies (lockdown) and/or social distancing behaviors (see section 4 below). From (1)-(2), we obtain the dynamic law of the population structure $\psi_t = N_t^o/N_t^y$:

$$(1 + n_t)\,\psi_t = (1 - \omega_t) + \gamma_t \psi_{t-1} \tag{3}$$

The preferences of a typical agent are described by a recursive non-expected utility function of the class proposed by Kreps and Porteus (1978) and by Epstein and Zin (1989):

$$V_t^z = \left\{ \left[(C_t^z)^q \,(1 - l_t^z)^{1-q} \right]^\rho + \beta_{t+1}^z \left[\mathbb{E}_t \left(V_{t+1} | z \right) \right]^\rho \right\}^{\frac{1}{\rho}} \tag{4}$$

where V_t^z represents period-t utility, C_t^z is consumption, the parameter $q \in (0, 1)$ and l_t^z is the fraction of time allocated to work by the agent. Young and old agents have distinct discount factors, as the old agents must also consider the probability of death:

$$\beta_{t+1}^y = \beta; \qquad \beta_{t+1}^o = \gamma_{t+1}\beta \tag{5}$$

The continuation value V_{t+1} in (4), which is different for young and old agents due to the transition in the next phase of the life-cycle, is conditional on the agent remaining young ($z = y$) or becoming old ($z = o$). In particular, if the agent is initially young, he/she must consider the probability of entering the other group, given by $(1 - \omega_{t+1})$, and this explains the expectation operator $\mathbb{E}_t$ in (4). Hence we have:

$$\mathbb{E}_t \left(V_{t+1} | z \right) = \begin{cases} V_{t+1}^o & \text{if } z = o \\ \omega_{t+1} V_{t+1}^y + (1 - \omega_{t+1})\, V_{t+1}^o & \text{if } z = y \end{cases}$$

The functional form of V in (4) is motivated as in Gertler (1999)[3]: it allows to separate risk-aversion from the intertemporal elasticity of substitution, so that the individual can be risk neutral with respect to income fluctuations while displaying a different elasticity of intertemporal substitution $\sigma = (1 - \rho)^{-1}$. In this way, it is possible to have at the same time reasonable saving choices by the young agents (eliminating excessive reactions to random income fluctuations) and a correct response to changes in the interest rate.

[3] See also Farmer (1990).

This life-cycle structure poses some limits to the ability of properly taking into account the effects of COVID-19 on the economic welfare of different age-classes of the population. For example, whereas the increase of death probability due to COVID-19 among age classes between 40 and 65 is not negligible, in the model it can only act indirectly through the increase in γ, as young agents face no risk of death.

Nevertheless, the life-cycle structure of the model has the crucial advantage of preserving analytical tractability, due to the assumption of independence of the probabilities ω and γ from age and time to "retirement". In addition, our assumptions capture some of the main features of the economic impact of the pandemic among different age-classes. Preliminary empirical evidence - as discussed in the introduction - documents in fact a sharp increase in the risk of death related to COVID-19 moving from the age class 40-59 to the age bracket over 60. For example, Goldstein and Lee (2020) estimate that the un-normalised death rates due to COVID-19 (up to may-june 2020) for the United States are: (i) very close to zero among the agents under 40; (ii) below 0.0005 in the age bracket 50-59; iii) around 0.0005 in the age bracket 60-69, iv) close to 0.001 in the age bracket 70-79 and v) close to 0.0025 among the agents over 80.

2.2 Old agents' choices

An agent born at s and entered into old age at τ solves the following dynamic optimization problem:

$$V_t^o(s,\tau) = \max\left\{\left[C_t^o(s,\tau)^q\left(1-l_t^o(s,\tau)\right)^{1-q}\right]^\rho + \beta\gamma_{t+1}\left(V_{t+1}^o(s,\tau)\right)^\rho\right\}^{\frac{1}{\rho}} \tag{6}$$

$$\text{s.t.} : C_t^o(s,\tau) + K_t^o(s,\tau) + B_t^o(s,\tau) = E_t^o + W_t\eta l_t^o(s,\tau) - T_t^o + \frac{1}{\gamma_t}\left[\left(r_t^K+1\right)K_{t-1}^o(s,\tau) + R_{t-1}B_{t-1}^o(s,\tau)\right] \tag{7}$$

where $C_t^o(s,\tau)$ is consumption, $l_t^o(s,\tau)$ is supply of work-time, $K_t^o(s,\tau)$ is investment in new capital stock, $B_t^o(s,\tau)$ is the demand of government's bond, r_t^K is the (net) rate of return on physical capital, W_t is the real wage rate and R_t is the bond's real interest factor. The parameter $\eta \in (0,1)$ measures the productivity of a unit of labor supplied by an older person relative to a younger person. The initial stocks K^o and B^o of the old agent, at the moment in which he/she enters the old-age phase, must coincide with the corresponding values of K and B he/she held as a young worker in that time period; hence it must be:

$$K_{\tau-1}^o(s,\tau) = K_{\tau-1}^y(s); \qquad B_{\tau-1}^o(s,\tau) = B_{\tau-1}^y(s) \tag{8}$$

As in Yaari (1965) and Blanchard (1985), old agents can insure themselves against the possibility of death. They have no bequest motive and sell contingent claims to their wealth to insurance companies operating in a competitive market. At the beginning of each period, insurance companies collect the financial assets from the deceased members of a cohort and make premium payments to survivors. Actuarial fairness results in the insurance market, so that the premium payment per (real) dollar of assets held by survivors is $\frac{1-\gamma}{\gamma}$. Incorporating the return on the insurance contract, $1+\frac{1-\gamma}{\gamma}=\gamma^{-1}$, into the flow budget constraint for survivors gives equation (7).

We include in the model a simple social security system (run by the public authority) based on transfers to old agents (pensions) financed by the taxes (or contributions) paid by all the private agents. Each old agent receives an exogenous lump-sum transfer equal to E_t^o and pays an exogenous lump-sum tax equal to T_t^o.

From the first order conditions of the problem (6)-(7) we obtain the Euler equations for the two assets B^o and K^o, together with the optimal supply of labor:[4]

$$\begin{aligned}
C_{t+1}^o(s,\tau)^{\frac{1}{\sigma}} &= \left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho} \beta\left(r_{t+1}^K+1\right) C_t^o(s,\tau)^{\frac{1}{\sigma}} \\
C_{t+1}^o(s,\tau)^{\frac{1}{\sigma}} &= \left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho} \beta R_t C_t^o(s,\tau)^{\frac{1}{\sigma}} \\
l_t^o(s,\tau) &= 1-\frac{\varsigma}{\eta W_t} C_t^o(s,\tau) \quad \text{with: } \varsigma=\frac{1-q}{q}
\end{aligned} \tag{9}$$

The first two equations determine the equilibrium discount factor for the old agents:

$$\begin{aligned}
1 &= \left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho} \beta\left(r_{t+1}^K+1\right) \frac{C_{t+1}^o(s,\tau)^{\rho-1}}{C_t^o(s,\tau)^{\rho-1}} \\
&= \left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho} \beta R_t \frac{C_{t+1}^o(s,\tau)^{\rho-1}}{C_t^o(s,\tau)^{\rho-1}}
\end{aligned} \tag{10}$$

These equations imply that the rate of change of consumption is equal for all old agents - irrespectively of birth and retirement dates - as they depend on economy-wide variables (in equilibrium). They also imply the no-arbitrage equation equalizing the returns on the two assets:

$$R_t = r_{t+1}^K + 1 \tag{11}$$

The equality between R and r^K in equilibrium allows us to define the total wealth of the old agent:

$$A_t^o(s,\tau) = K_t^o(s,\tau) + B_t^o(s,\tau)$$

so that the budget constraint reduces to:

$$C_t^o(s,\tau) + A_t^o(s,\tau) = \frac{R_{t.-1} A_{t-1}^o(s,\tau)}{\gamma_t} + E_t^o + W_t \eta l_t^o(s,\tau) - T_t^o$$

In order to find the complete solution of the old agents choice problem, we conjecture that the consumption function prescribes a time-varying marginal propensity of consumption (m.p.c.) ξ_t^o out of total wealth/resources:

$$C_t^o(s,\tau) = \xi_t^o\left[\frac{1}{\gamma_t} R_{t-1} A_{t-1}^o(s,\tau) + D_t^o + H_t^o\right] \tag{12}$$

where D_t^o and H_t^o are the discounted values of, respectively, the stream of social security payments and the old agent's (net) *human wealth* (the economic value obtained by employing/exchanging the agent's personal resources different from financial assets):

[4] See the Technical Appendix, available from the authors upon request.

$$D_t^o = \sum_{i=0}^{\infty} \frac{E_{t+i}^o}{\prod_{j=1}^{i} \frac{R_{t+j-1}}{\gamma_{t+j}}}; \qquad D_t^o = E_t^o + \frac{D_{t+1}^o}{R_t/\gamma_{t+1}} \tag{13}$$

$$H_t^o = \sum_{i=0}^{\infty} \frac{W_{t+i}\eta l_{t+i}^o - T_{t+i}^o}{\prod_{j=1}^{i} \frac{R_{t+j-1}}{\gamma_{t+j}}}; \qquad H_t^o = W_t \eta l_t^o (s,\tau) - T_t^o + \frac{H_{t+1}^o}{R_t/\gamma_{t+1}}$$

By substituting the conjecture (12) into the Euler equation: $C_{t+1}^o (s,\tau) = \left\{ \left(\frac{W_t}{W_{t+1}} \right)^{(1-q)\rho} \beta R_t \right\}^{\sigma}$ $C_t^o (s,\tau)$, we can derive the equilibrium dynamic law of the marginal propensity ξ^o:

$$\frac{1}{\xi_t^o} = 1 + \gamma_{t+1} \left[\left(\frac{W_t}{W_{t+1}} \right)^{(1-q)\rho} \beta \right]^{\sigma} R_t^{\sigma-1} \frac{1}{\xi_{t+1}^o} \tag{14}$$

We now conjecture that the solution of the value function is linear in consumption:

$$V_t^o (s,\tau) = \Delta_t^o C_t^o (s,\tau) \left(\frac{\varsigma}{\eta W_t} \right)^{1-q}; \qquad \text{with } \Delta_t^o > 0 \tag{15}$$

and substitute this conjecture into (4). The conjecture allows us to derive the closed-form solution of the old agent's value function:

$$V_t^o (s,\tau) = \left(\frac{\varsigma}{\eta W_t} \right)^{1-q} (\xi_t^o)^{\frac{\sigma}{1-\sigma}} C_t^o (s,\tau) \tag{16}$$

because the optimal solution requires the equality: $\Delta_t^o = (\xi_t^o)^{\frac{\sigma}{1-\sigma}}$.

2.3 Young agents' choices

A typical young agent, born at time s without any bequest left from past generations, chooses to consume, save and provide labor services. Given the no-arbitrage equation (11), both assets pay the return R_{t-1} and we can directly consider his/her total wealth $A_t^y (s) = K_t^y (s) + B_t^y (s)$. Her optimization problem is then:

$$V_t^y (s) = \max \left\{ \begin{array}{c} \left[C_t^y (s)^q \left(1 - l_t^y (s)\right)^{1-q} \right]^{\rho} + \\ \beta \left[\omega_{t+1} V_{t+1}^y (s) + (1-\omega_{t+1}) V_{t+1}^o (s, t+1) \right]^{\rho} \end{array} \right\}^{\frac{1}{\rho}} \tag{17}$$

$$\text{s.t.} : \quad C_t^y (s) + A_t^y (s) = R_{t-1} A_{t-1}^y (s) + W_t l_t^y (s) - T_t^y \tag{18}$$

where l_t^y is the time of work supplied and T_t^y is the lump-sum pax paid by the young.

The optimality conditions of problem (17)-(18) can be combined to write:[5]

$$q C_t^y (s)^{q\rho-1} \left[1 - l_t^y (s)\right]^{(1-q)\rho} = \beta \left[\omega_{t+1} \frac{\partial V_{t+1}^y (s)}{\partial A_t^y (s)} + (1-\omega_{t+1}) \frac{\partial V_{t+1}^o (s,t+1)}{\partial A_t^y (s)} \right] \times \left[\omega_{t+1} V_{t+1}^y (s) + (1-\omega_{t+1}) V_{t+1}^o (s,t+1) \right]^{\rho-1}$$

[5]See the Technical Appendix for derivations.

together with the agent's labour supply:

$$l_t^y(s) = 1 - \frac{\varsigma}{W_t} C_t^y(s) \tag{19}$$

Analogously to the case of the old agents, we formulate the conjecture that the solution of their value function is proportional to consumption:

$$V_t^y = \left(\frac{\varsigma}{W_t}\right)^{1-q} \Delta_t^y C_t^y(s) \tag{20}$$

We substitute this conjecture, together with the equivalent one for the old agents, $V_t^o(s,t) = \Delta_t^o\, C_t^o(s,t)\left(\frac{\varsigma}{W_t \eta}\right)^{1-q}$, into the previous equation to obtain the Euler equation:

$$\begin{aligned}\left[\left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho} \beta R_t \Omega_{t+1}\right]^{\sigma} C_t^y(s) &= \omega_{t+1} C_{t+1}^y(s) + \\ &\quad (1-\omega_{t+1})\,\chi \left(\frac{\Delta_{t+1}^o}{\Delta_{t+1}^y}\right) C_{t+1}^o(s, t+1)\end{aligned} \tag{21}$$

where the adjustment factor Ω_t is defined as:

$$\Omega_t = \omega_t + (1-\omega_t)\,\chi \left(\frac{\Delta_t^o}{\Delta_t^y}\right)^{1-\rho} \quad ; \qquad \chi = \left(\frac{1}{\eta}\right)^{1-q}$$

We then make the following conjecture for the young agent's consumption function:

$$C_t^y(s) = \xi_t^y \left[R_{t-1} A_{t-1}^y(s) + H_t^y + D_t^y\right] \tag{22}$$

where ξ^y is the m.p.c. and H_t^y is a measure of the young worker's human wealth. The term D_t^y is the present value of the stream of social benefits (net of taxes) to be received in case of retirement from t onwards. By substituting (22) into (21), it can be shown that the human wealth evolves according to the dynamic rule:

$$H_t^y = W_t l_t^y(s) - T_t^y + \frac{\omega_{t+1}}{\Omega_{t+1} R_t} H_{t+1}^y + \left(\frac{\Omega_{t+1} - \omega_{t+1}}{\Omega_{t+1} R_t}\right) H_{t+1}^o \tag{23}$$

where H_{t+1}^o is the (expected value of the) human wealth that the young agent would receive were he/she to become old at $t+1$. The dynamic law of the stream of social benefits D_t^y can be defined in a similar way:

$$D_t^y = \frac{\omega_{t+1}}{\Omega_{t+1} R_t} D_{t+1}^y + \left(\frac{\Omega_{t+1} - \omega_{t+1}}{\Omega_{t+1} R_t}\right) D_{t+1}^o \tag{24}$$

By making use of these equations for D_t^y and H_t^y, together with the Euler equation and the budget constraint, the equilibrium evolution of the m.p.c. of the young agent writes:

$$\frac{1}{\xi_t^y} = 1 + \left[\left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho} \beta\right]^{\sigma} (R_t \Omega_{t+1})^{\sigma-1} \frac{1}{\xi_{t+1}^y} \tag{25}$$

Finally, it can be show that the coefficient Δ_t^y in conjecture (20) must satisfy the condition:

$$\Delta_t^y = (\xi_t^y)^{\frac{\sigma}{1-\sigma}}$$

so that the previous conjectures on $V_t^y(s)$ and $C_t^y(s)$ are confirmed.

2.4 Aggregation of consumption, labor and wealth

We now aggregate the equilibrium consumption functions of young and old agents (12) and (22). As shown by (14) and (25), the m.p.c. ξ_t^o and ξ_t^y do not depend on specific individual features and this makes it possible to carry out a straightforward aggregation. Considering, for instance, the old agents, the total consumption of agents born at s and retired at a generic time $\tau \in [s,t]$ is equal to: $C_t^o(s) = \sum_{\tau=s}^{t} \int_0^{N_t^o(s,\tau)} C_t^o(s,\tau)\, di$, where $N_t^o(s,\tau) < N_\tau^o$ is the number of agents born at s and retired (old) at τ. Starting the analysis at time $t_0 = 0$ (at this initial time there must be an initial cohort of old agents coming from an un-modeled past, i.e., born at $t = -1$), the total consumption of all the old agents at t is equal to the sum over all the birth dates s: $C_t^o = \sum_{s=-1}^{t} C_t^o(s)$. Applying an analogous procedure for the old agents' wealth, its total amount at t is equal to: $A_{t-1}^o = \sum_{s=-1}^{t-1} A_{t-1}^o(s)$.

As the variables ξ_t^o and R_{t-1} are independent from individual characteristics, the total amount of consumption of old agents is given by the aggregate function:

$$C_t^o = \xi_t^o \left(R_{t-1}A_{t-1}^o + D_t + H_t\right) \tag{26}$$

where D_t and H_t are, respectively, the discounted value of overall social security payments to the old agents and the discounted values of their human wealth.[6] The aggregate labor supply of the old agents at t is given by the sum of equations (9):

$$L_t^o = N_t^o - \frac{\varsigma}{W_t \eta} C_t^o$$

where L_t^o is the total labor input supplied at t by all the existing old agents.

The aggregate value of the old agents' net human wealth H_t follows the dynamic law:

$$H_t = \left(W_t \eta L_t^o - T_t^{*o}\right) + \frac{\psi_t H_{t+1}}{\psi_{t+1}\left(1+n_{t+1}\right) R_t/\gamma_{t+1}}$$

which includes the effects of possible changes in the demographic evolution of old agents through the factor $\frac{\psi_t}{\psi_{t+1}(1+n_{t+1})}$, together with the aggregate lump-sum payments $T_t^{*,o} = N_t^o T_t^o$. The aggregation of D stems directly from the assumption that current individual payments E_t^o are the same for all old agents, so that the aggregate value of the social benefit (13) is equal to:

$$D_t = E_t + \frac{\psi_t D_{t+1}}{\psi_{t+1}\left(1+n_{t+1}\right) R_t/\gamma_{t+1}}$$

As for the total consumption of young agents, we directly obtain:

$$C_t^y = \xi_t^y \left(R_{t-1}A_{t-1}^y + \bar{H}_t + \bar{D}_t\right) \tag{27}$$

where $\bar{H}_t$ and $\bar{D}_t$ are the aggregate values of (young agents') human wealth and social security payments, and $T_t^{*y} = T_t^y N_t^y$. The aggregate labor supply of the young agents is:

$$L_t^y = N_t^y - \frac{\varsigma}{W_t} C_t^y$$

[6] That is: $D_t^o(s) = \sum_{\tau=s}^{t} \int_0^{N_t^o(s,\tau)} D_t^o(s,\tau)\, di$ and $D_t = \sum_{s=-1}^{t} D_t^o(s)$, with an analogous definition for H_t.

The dynamic equations of the aggregate values of the human wealth $\bar{H}_t$ and of the social security payments $\bar{D}_t$ must take into account the change in the populations of both young and old agents, yielding:

$$\begin{aligned}
\bar{H}_t &= \left(W_t L_t^y - T_t^{*y}\right) + \frac{\omega_{t+1}\bar{H}_{t+1}}{(1+n_{t+1})\,\Omega_{t+1}R_t} + \frac{(\Omega_{t+1}-\omega_{t+1})\,H_{t+1}}{(1+n_{t+1})\,\psi_{t+1}\Omega_{t+1}R_t} \\
\bar{D}_t &= \frac{1}{1+n_{t+1}}\left[\frac{\omega_{t+1}}{\Omega_{t+1}R_t}\bar{D}_{t+1} + \left(\frac{\Omega_{t+1}-\omega_{t+1}}{\psi_{t+1}\Omega_{t+1}R_t}\right)D_{t+1}\right].
\end{aligned}$$

Total consumption is equal to $C_t = C_t^y + C_t^o$. Denoting $A_{t-1} = A_{t-1}^o + A_{t-1}^y$ the overall amount of financial wealth, we can use the expressions $A_{t-1}^o = \lambda_t A_{t-1}$ and $A_{t-1}^y = (1-\lambda_t)\,A_{t-1}$, where $\lambda_t = A_{t-1}^o / A_{t-1}$ is the fraction of total financial wealth held by old agents, to write total consumption as:

$$C_t = \xi_t^y\left[(1-\lambda_{t-1})\,R_{t-1}A_{t-1} + \bar{H}_t + \bar{D}_t\right] + \xi_t^o\left(\lambda_{t-1}R_{t-1}A_{t-1} + H_t + D_t\right)$$

The time evolution of A_t^o and A_t^y depends also on the transition into old age of the new cohorts of young agents. The amount of wealth of old agents A_t^o available at $t+1$ depends on the total savings made in the previous period by this class of agents, plus the amount of savings made by the fraction $1-\omega_{t+1}$ of young agents entering the retirement/old age between t and $t+1$. Hence, from the aggregate budget constraints of the two classes of agents, we can write:

$$\begin{aligned}
A_t^o \;=\;& R_{t-1}A_{t-1}^o + E_t + W_t\eta L_t^o - T_t^{*o} - C_t^o + \\
& (1-\omega_{t+1})\left[R_{t-1}A_{t-1}^y + W_t L_t^y - T_t^{*y} - C_t^y\right]
\end{aligned} \tag{28}$$

This also implies that the amount of A_t^y available at $t+1$ for the young agents depends on the saving made only by the fraction ω_{t+1} of agents remaining young between t and $t+1$:

$$A_t^y = \omega_{t+1}\left[R_{t-1}A_{t-1}^y + W_t L_t^y - T_t^{*y} - C_t^y\right]$$

Hence, by substituting $\frac{A_t^y}{\omega_{t+1}} = R_{t-1}A_{t-1}^y + W_t L_t^y - T_t^{*y} - C_t^y$ and $C_t^o = \xi_t^o\left(R_{t-1}A_{t-1}^o + H_t + D_t\right)$ into (28), and by switching to the share λ, we obtain:

$$A_t = \frac{\omega_{t+1}}{\lambda_t + \omega_{t+1} - 1}\left[(1-\xi_t^o)\,R_{t-1}\lambda_{t-1}A_{t-1} + E_t + W_t\eta L_t^o - T_t^{*o} - \xi_t^o\,(H_t + D_t)\right]$$

Finally, note that, recalling equations $A_t^o(s,\tau) = K_t^o(s,\tau) + B_t^o(s,\tau)$ and $A_t^y(s) = K_t^y(s) + B_t^y(s)$, the aggregate value of wealth A_t is equal to:

$$A_t = K_t + B_t$$

2.5 Firms and production

The representative firm operates in competitive markets for goods and production inputs, and adopts a constant elasticity of substitution (CES) production function:

$$Y_t = \left[(1-\alpha)\,K_{t-1}^{\phi} + \vartheta_t\alpha\,(X_t L_t)^{\phi}\right]^{\frac{1}{\phi}}; \qquad \alpha \in (0;1)$$

where the total real output Y_t is affected by exogenous (Harrod-neutral) technical progress represented by the growth factor X_t applied to the total labour input L_t used by the firms. The parameter α represents the labour's share in income distribution and ϕ determines the (constant) elasticity of substitution between inputs. Technology evolves through time at a constant rate x:

$$X_t = (1 + x) X_{t-1} \tag{29}$$

The variable $\vartheta_t > 0$, which represents the impact of social distancing policies (and/or voluntary behavior induced by the pandemic), is one of the key elements of the policy analysis proposed in this paper. In normal times, when no particular restriction is posed on the persons' movements, it is $\vartheta = 1$. During the COVID-19 pandemic episode, its value changes, to mimic the effect of the social distancing policies and restriction on workers movements (lockdown) that were introduced in several countries and that imposed a severe reduction in production activities.

The firm maximizes its profit Π_t:

$$\max_{K_{t-1};N_t^y} \Pi_t = Y_t - W_t L_t - \left(r_t^K + \delta\right) K_{t-1}$$

where δ is the depreciation rate on capital. Firm's optimization leads to the following equations for the demand of inputs:

$$W_t = \alpha\vartheta_t (X_t)^{\phi} (L_t)^{\phi-1} Y_t^{1-\phi} = \alpha\vartheta_t X_t \left(\frac{Y_t}{X_t L_t}\right)^{1-\phi}; \tag{30}$$

$$r_t^K = (1-\alpha) K_{t-1}^{\phi-1} Y_t^{1-\phi} - \delta = (1-\alpha)\left(\frac{Y_t}{K_{t-1}}\right)^{1-\phi} - \delta. \tag{31}$$

2.6 Fiscal policy and government's budget

Fiscal and social security policies can be financed by lump-sum taxes T and by issuing one-period government's bonds B. The accruals from these sources are used for unproductive expenditures G and the payments of social benefits E, so that the flow budget constraint of the government is:

$$B_t = G_t + E_t - (T_t^{*y} + T_t^{*o}) + R_{t-1} B_{t-1} \tag{32}$$

We assume that the individual lump sum tax T_t^o for the old agent is proportional to the individual tax of the young one:

$$T_t^o = a^o T_t^y \quad \text{with} \quad a^o \geq 0$$

Hence, total fiscal revenues can be written as: $T_t^{*y} + T_t^{*o} = (a^o N_t^o + N_t^y) T_t^y = (1 + a^o \psi_t) T_t^{*y}$.

2.7 Macroeconomic equilibrium

A general equilibrium for the model economy (expressed in aggregate form) can be defined along the lines of Gertler (1999). The goods market equilibrium is given by the economy's resources constraint:

$$Y_t = C_t + I_t + G_t \tag{33}$$

where I_t is the amount of new capital goods (produced by converting consumption goods on a one-to-one basis). The amount of net aggregate investment is coherent with the time evolution of aggregate physical capital:

$$K_t = I_t + (1-\delta) K_{t-1} = Y_t - C_t - G_t + (1-\delta) K_{t-1} \tag{34}$$

The labor market clears according to the following condition:

$$L_t = L_t^y + \eta L_t^o$$

where ηL_t^o accounts for the effective labor time (i.e., weighted by its relative productivity η) supplied by the old agents.

We assume that the (young agents') population growth rate n_t and the probability of remaining young ω_t are constant through time and equal to their long-run (average) values: $n_t = n$ and $\omega_t = \omega$. Given: i) the initial values for the predetermined variables K_{t-1}, λ_{t-1}, X_{t-1}, N_{t-1}^y and ψ_{t-1}; ii) a sequence of exogenous variables $\{X_t; N_t^y\}$; iii) a sequence of exogenous fiscal policy variables $\{G_t; E_t; B_t\}$, a *macroeconomic equilibrium* is a sequence of endogenous variables $\{\ K_t,\ \lambda_t,\ \xi_t^o,\ \xi_t^y,\ \Omega_t,\ H_t,\ \bar{H}_t,\ Y_t,\ C_t,\ W_t,\ R_t,\ A_t,\ \psi_t,\ T_t^{*y},\ D_t,\ \bar{D}_t,\ L_t,\ L_t^o,\ L_t^y\ \}$ such that the equilibrium system detailed in Appendix 1) is satisfied, the exogenous variables follow their dynamic equations (1) and (29), and the variables γ_t and ϑ_t follow the exogenous processes specified below.

Detrended variables and real wage rigidity

Due to the exogenous dynamics of population (1) and of technical progress (29) aggregate variables grow at the compound rate $(1+n)(1+x)$ along the balanced growth path (BGP). We hence express the endogenous variables in detrended values by dividing them by the "effective" amount of young agents $X_t N_t^y$ and indicate with a generic lower-case variable s_t the detrended value $s_t = S_t/(X_t N_t^y)$. The detrending of the labor input L_t, L_t^z only requires to divide the aggregate values by N_t^y:

$$\widetilde{l}_t = \frac{L_t}{N_t^y} = \frac{L_t^y}{N_t^y} + \eta \frac{L_t^o}{N_t^y} = \widetilde{l}_t^y + \eta \widetilde{l}_t^o$$

while the detrended value of the real wage rate is equal to:

$$w_t = \frac{W_t}{X_t} = \alpha \vartheta_t \left[(1-\alpha) \left(\frac{k_{t-1}}{\widetilde{l}_t} \right)^{\phi} + \vartheta_t \alpha \right]^{\frac{1-\phi}{\phi}} = \alpha \vartheta_t \left(\frac{y_t}{\widetilde{l}_t} \right)^{1-\phi}$$

and the detrended labor supply functions are:

$$\widetilde{l}_t^y = 1 - \frac{\varsigma}{w_t} c_t^y; \qquad \widetilde{l}_t^o = \psi_t - \frac{\varsigma}{w_t \eta} c_t^o$$

In order to add an element of realism, we include in our model economy a form of real wage rigidity. Along the lines of the staggered nominal price setting widely adopted in the macroeconomics literature, we assume that the current level of the detrended real wage w_t is determined by a staggered mechanism of adjustment according to which, in each time period, only a random number of young workers can obtain a level of w_t coherent with their utility maximization, while the remaining agents obtain a wage set in previous periods. We also assume, for simplicity, that the objective level of the wage for the young worker j, $w^{ob}_{j,t+i}$, is common to all young workers and coincides with the value deriving from utility maximization and the aggregate labor supply, $\widetilde{l}^y_t = 1-\varsigma c^y_t/w_t$:

$$w^{ob}_{j,t+i} = w_{t+i} = \frac{\varsigma c^y_{t+i}}{1-\widetilde{l}^y_{t+i}} = m^{RS}_{t+i} \tag{35}$$

Under this assumption, and by assuming that the probability of re-setting the wage is $1-\rho_w$, with $\rho_w \in (0;1)$, we can derive[7] the following dynamic equilibrium rule for the real wage:

$$w_t = \frac{(1-\rho_w)(1-\rho_w\beta)}{1+\rho_w^2\beta} m^{RS}_t + \frac{\rho_w\beta}{1+\rho_w^2\beta} E_t w_{t+1} + \frac{\rho_w}{1+\rho_w^2\beta} w_{t-1} \tag{36}$$

After detrending, we focus on the dynamic evolution of the following vector of variables: $\mathbf{v}_t = [k_t,\ \lambda_t,\ \xi^o_t,\ \xi^y_t,\ \Omega_t,\ h_t,\ \bar{h}_t,\ y_t,\ c_t,\ c^y_t,\ c^o_t,\ a_t,\ \psi_t,\ R_t,\ w_t,\ d_t,\ \bar{d}_t,\ \theta_t,\ g_t,\ e_t,\ b_t,\ \widetilde{l}_t,\ \widetilde{l}^o_t,\ \widetilde{l}^y_t]$ according to the equilibrium system:

$$f(\mathbf{v}_{t+1}; \mathbf{v}_t; \mathbf{v}_{t-1}) = \mathbf{0}$$

which is detailed in Appendix 3, where θ_t is defined as $\frac{T^{*y}_t}{X_t N^y_t} = \theta_t$ and the total fiscal revenues are: $\frac{T^{*y}_t + T^{*o}_t}{N^y_t X_t} = (1 + a^o\psi_t)\,\theta_t$.

Fiscal policy is defined by the following equations:

$$\begin{aligned} e_t &= r^e_t y_t; \\ g_t &= \rho_g g_{t-1} + \left(1-\rho_g\right) r^g_t y_t; \\ \theta_t &= \rho_\theta \theta_{t-1} + (1-\rho_\theta)\left[r^\theta_t y_t + \delta_B\left(\frac{b_t}{y_t} - r^b\right)\right]. \end{aligned} \tag{37}$$

The exogenous processes of the ratios r^g_t , r^e_t, r^θ_t are set by the policy makers and r^b is a target value for the debt-to GDP ratio b/y, which we assume to be equal to the long-run stationary value calibrated for the United States economy. As it will be clarified below, the ratios r^g_t and r^θ_t include a stationary, long-run component and a temporary one, so that a short-run change in fiscal policy can be described as a shock to the temporary components of r^g_t and r^θ_t. The last two equations of (37) also include, *via* the coefficients $\rho_{\theta,g} \in (0;1)$, the possibility of a gradual adjustment of public expenditures g_t and fiscal revenues θ_t towards their stationary values after a policy change. We also assume that the government adjusts lump sum taxes θ_t (*via* $\delta_B \in (0;1)$) in response to deviations of

[7] See Appendix 2.

the debt-to-GDP ratio from its stationary value r^b, so as to ensure debt sustainability in the long run (see, e.g., Auray and Eyquem 2020). The variable b_t hence endogenously adjusts to verify the budget equation:

$$(1+n)(1+x)b_t = g_t + e_t - (1 + a^o\psi_t)\theta_t + R_{t-1}b_{t-1} \tag{38}$$

The other exogenous variables γ_t and ϑ_t follow stochastic processes appropriately defined so as to describe, respectively, the impact of the pandemic on the probability of survival of old agents and the effect of lockdown policies on production. Before the occurrence of the pandemic shock and of the subsequent policies, these two variables, together with the ratios r_t^g, r_t^b and r_t^θ, are set equal to their constant long-run values, coherently with the initial position of the economy along the BGP: $r_t^g = r^g$, $r_t^e = r_t^e$, $r_t^\theta = r^\theta$, $\gamma_t = \gamma$ and $\vartheta_t = 1$.

The stationary solution of the model can be carried out by focusing on a restricted set of variables and equations, which are described in Appendix 4. Coherently with the subsequent numerical exercises, we assume that:

$$\rho < -1 \quad \rightarrow \quad \sigma \in (0;1)$$

This implies that in the stationary state (where values are denoted without the time index) it must be $\Omega > 1$ and consequently $\xi^o > \xi^y$.

3 Welfare indicators of the two demographic classes

In the light of our main research interest, we aim at exploring the consequences of both the decreased survival probability produced by the pandemic and of the different policies undertaken in response to the COVID-19 on the different economic and demographic groups which co-exist in the economy. To this aim, it is necessary to define an appropriate *welfare index* for the two main groups $z \in \{y, o\}$. We are aware of the difficulty of carrying out a complete and rigorously founded welfare analysis in the context of agents' heterogeneity and overlapping generations.[8] Nevertheless, the analytical tractability of the model here adopted allows us to define an index for the aggregate welfare of each of the *two groups* in every time period t, which is suitable for our purposes.

We denote V_t^o and V_t^y the aggregate utility indexes at period t of the old agents and of the young agents, respectively. These indexes V_t^z will be used to define a ("relative") between-groups welfare indexe, which must take into account the specific features of the model and satisfy some requirements posed by our research objective. This raises three main issues. First of all, the analysis must be carried out in terms of *commonly detrended* variables. Given this necessity, we can exploit the fact that, along the BGP,

[8] The main issues are, among others, the choice of a sound discount factor for the aggregate welfare index and/or the choice of the correct weight to be assigned to the utility of each heterogenous agent in the aggregate welfare index. See, e.g., the discussion in Fujiwara and Teranishi (2008) and in Baska and Munkacsi (2019).

the total population $N_t = N_t^y + N_t^o$ grows at the rate n to use $X_t N_t^y$ as the common detrending factor in the construction of the welfare indexes.

Second, the overlapping generations structure prevents us from keeping track of the welfare of a specific cohort of agents, as well as measuring through an aggregate index the welfare of an ensemble of cohorts of young or old agents specified at a certain point in time t. We hence include in V_t^o the utility levels of *all the old* agents and in V_t^y the utilities of *all the young* agents who exist at t. This should not be considered a serious limitation of the model because, even though it requires to deal with aggregate indexes whose composition in terms of individual agents constantly changes as time t elapses,[9] in the real world economists and policy makers typically identify and analyze specific groups of agents changing through time, and design/evaluate economic policies addressing, e.g., the younger/older workers, the retirees, the unemployed, etc.

Third, lacking a specific hypothesis on the distribution of material wealth among individuals at a certain point in time (or at the stationary state), the model does not allow us to measure the present value of each individual utility at a certain time t and hence the economic cost of an individual life lost in each period (the life of each heterogeneous old agent perished at each t). This can be grasped by noting the presence of A_{t+i-1}^o in the present value $\sum_{i=0}^{\infty} \beta^i \gamma_{t+i} \left(\frac{\varsigma}{W_{t+i}\eta} \right)^{1-q} \left(\xi_{t+i}^o \right)^{\frac{1}{1-\sigma}} \left[\frac{1}{\gamma_{t+i}} R_{t+i-1} A_{t+i-1}^o (s, \tau) + D_{t+i}^o + H_{t+i}^o \right]$, as implied by equations (12) and (16). Rather, as the loss of economic value due to old agents' death is implicitly contained in the aggregate index of group utility V_t^o, it becomes important to properly take into account the demographic effect of a fall in γ_t on the population structure ψ_t when constructing welfare indexes based on V_t^o and V_t^y.

In the light of these three issues, we exploit the closed-form solutions of (6) and (17) and carry out a straightforward aggregation to define the *group* indexes V_t^z in the following way:

$$
\begin{aligned}
V_t^o &= \sum_{s=-1}^{t} \left[\int_0^{N_t^o(s,\tau)} V_t^o (s, \tau)\, di \right] = \sum_{s=-1}^{t} \left[\int_0^{N_t^o(s,\tau)} \Delta_t^o C_t^o (s, \tau) \left(\frac{\varsigma}{W_t \eta} \right)^{1-q} di \right] \\
&= \left(\frac{\varsigma}{W_t \eta} \right)^{1-q} \Delta_t^o C_t^o; \\
V_t^y &= \sum_{s=-1}^{t} \left[\int_0^{N_t^y(s)} V_t^y (s)\, di \right] = \sum_{s=-1}^{t} \left[\int_0^{N_t^y(s)} \Delta_t^y C_t^y (s) \left(\frac{\varsigma}{W_t} \right)^{1-q} di \right] \\
&= \left(\frac{\varsigma}{W_t} \right)^{1-q} \Delta_t^y C_t^y
\end{aligned}
$$

with $\Delta_t^z = (\xi_t^z)^{\frac{\sigma}{1-\sigma}}$. The two indexes V_t^z hence depend on group-aggregate quantities:

$$
V_t^y = \left(\frac{\varsigma}{W_t} \right)^{1-q} (\xi_t^y)^{\frac{\sigma}{1-\sigma}} C_t^y; \qquad V_t^o = \left(\frac{\varsigma}{W_t \eta} \right)^{1-q} (\xi_t^o)^{\frac{\sigma}{1-\sigma}} C_t^o
$$

[9] In each period some of the old agents contributing to V_t^o exit the economy, while some other (young) agents enter the group, and an analogous observation holds for V_t^y. Furthermore, these inflows and outflows are composed of heterogenous agents, due to their different amounts of individual wealth and resources.

A direct measure of the relative welfare *between groups* is then:

$$\begin{aligned} v_t^{ratio} &= \frac{V_t^y}{V_t^o} = \frac{1}{\chi}\left(\frac{\xi_t^y}{\xi_t^o}\right)^{\frac{\sigma}{1-\sigma}} \frac{C_t^y}{C_t^o} = \frac{1}{\chi}\left(\frac{\xi_t^y}{\xi_t^o}\right)^{\frac{\sigma}{1-\sigma}} \frac{c_t^y}{c_t^o} \\ &= \frac{1}{\chi}\left(\frac{\xi_t^y}{\xi_t^o}\right)^{\frac{1}{1-\sigma}} \frac{(1-\lambda_{t-1}) R_{t-1} a_{t-1} + \bar{h}_t + \bar{d}_t}{\lambda_{t-1} R_{t-1} a_{t-1} + h_t + d_t} \end{aligned} \tag{39}$$

In the model's context, v_t^{ratio} can be conceived as a relative welfare index capable of addressing the three main issues previously discussed and will hence be used in the subsequent analysis.

4 Numerical analysis and policy exercises

Pandemic and social distancing measures

In order to explore the model's prediction on the effects of pandemic-related policies, we specify a numerical version using empirical figures for the U.S. The time period t corresponds to one quarter.

As already pointed out, a special role in our analysis is played by the probability of survival, which can be described, in general terms, as:

$$\gamma_t = \gamma(u_t, \vartheta_t)$$

where u_t is a shock variable representing the impact of the COVID-19 pandemic, with $\frac{d\gamma_t}{du_t} < 0$ (an increase in u , i.e., the presence of COVID-19, reduces the probability of survival). The stationary level of γ_t (when u_t shocks are absent) is set according to long-term demographic data for the U.S. In general, we assume that γ_t is also dependent on the presence of lockdown policies: $\frac{d\gamma_t}{d\vartheta_t} > 0$. When lockdown policies are active, then $\vartheta_t > 1$, so that ϑ_t rises above its stationary value (see the discussion below) and this (partially) offsets the negative effect of u_t on γ_t, as the presence of these policies slows down the diffusion of the pandemic. Temporary deviations of ψ_t from its stationary value $\psi = (1-\omega)/(1+n-\gamma)$ are traced back to the pandemic shock u_t via the dynamic equation (3). Lockdown policies also directly impact on production via ϑ_t as shown by the production function: $y_t = \left[(1-\alpha) k_{t-1}^{\phi} + \vartheta_t \alpha \widetilde{l}_t^{\phi}\right]^{\frac{1}{\phi}}$. We assume that:

$$\vartheta_t = \vartheta(u_t)$$

so that, when lockdown and/or social distancing policies are active it is $\frac{d\vartheta_t}{du_t} > 0$ and these measures can have a direct impact on production activities. We choose to represent the impact of lockdown as an increase in ϑ_t for numerical reasons: in our exercises we will set $\phi < 0$ and, under this assumption, in order to induce a reduction in y_t (and an overall recessionary push), the temporary change in ϑ_t must be positive. The functional forms of γ and ϑ, relating the pandemic shock to the survival probability and the "lockdown effect", will be subsequently specified, in accordance with the specific needs of our numerical applications.

Intuitively, the model predicts that under a pandemic shock ($u_t > 0$) the activation of lockdown/social distancing measures (i.e., an increase in ϑ_t above 1) generates a severe recessionary push. In order to mitigate the economic consequences of lockdown measures - which were often of dramatic proportions - many governments implemented, immediately after the insurgence of the pandemic, a number of expansionary fiscal programs, in particular through the increase of public expenditures frequently coupled with reductions in the tax burden or the postponement of the related payments. These types of fiscal policy can be generically represented in the model by defining the exogenous ratios r_t^g and r_t^θ in the following way:

$$r_t^g = f\left(r^g; u_t^g\right); \qquad r_t^\theta = f\left(r^\theta; u_t^\theta\right).$$

The terms r^g and r^θ represent the (calibrated) stationary values of the ratios $\frac{g}{y}$ and $\frac{\theta}{y}$ respectively, and the exogenous variables u_t^g and u_t^θ describe deviations of expenditures and lump sum taxes from their stationary values and can be related to the policies introduced by the government in the face of the adverse supply effects of lockdown (f is a monotonic increasing function). Finally, we keep the social security payments in line with the stationary ratio, so that: $e_t = r^e y_t$.

4.1 Initial stationary state and parameterization

We compute the stationary state by using data for the U.S. economy. We consider a baseline parameterisation in which the model's main parameters are set according to Table 1.

Table 1 - Baseline parameterisation

$\alpha = 0.67$	$\beta = 0.998$	$\delta = 0.025$	$\sigma = 0.42$
$q = 0.4$	$\phi = -0.16$	$\eta = 0.6$	$\rho_w = 0.85$
$x = 0.02/4$	$n = 0.01/4$	$\gamma = 0.98214$	$\omega = 0.99444$
$r^g = \frac{g}{y} = 0.150$	$r^e = \frac{e}{y} = 0.0405$	$r^b = \frac{b}{y} = 0.983$	$a^o = 1$

The values of the main preference and technological parameters, together with the capital depreciation rate $(\alpha, \beta, \delta, \sigma)$, are commonly adopted in the literature, while η, q are taken from Gertler (1999) and ϕ is set so as to obtain a realistic figure for the long run interest factor R. As for the average retirement rate ω, as well as for γ, we adopt a strategy centered on the main demographic features of the U.S. Our point of departure is the average life expectancy for the U.S. population, which is 78.86 years at birth (OECD 2017 data, see also Goldstein and Lee 2020). We then assume that people, roughly in line with the approach inaugurated by Auerbach and Kotlikoff (1987), on average, enter the labor market at 20 and leave it at 65, for a total of 45 years of productive/active life; after that period, they are left with 14 years to be spent in retirement, so that the total (average) life-span is 79. The implied values of the two parameters are: $\omega = 1 - \frac{1}{45} = 0.9777$ and $\gamma = 1 - \frac{1}{14} = 0.9285$.[10] These values are then converted into quarterly figures: $\omega = 1 - \frac{1}{45\times 4} = 0.99444$ and $\gamma = 1 - \frac{1}{14\times 4} = 0.98214$.

[10] The resulting stationary value of the population structure, $\psi = 0.27362$, is roughly in line with the average ratio of the old age population (65 and more) and the working age population (15-64), which is equal to 0.199 for the period 1977-2018.

As for the fiscal policy ratios, we use data from the FRED database for the U.S. economy at quarterly frequency, in the range 2009-2019,[11] and compute the values in Table 1 as averages of the corresponding ratios. We choose this time range for $r^{g,e,b,\theta}$ in order to have, as a benchmark, a description of the fiscal structure over a relatively short time period, so that these figures can also be applied to the subsequent fiscal policy experiments and scenarios in which the economy is allowed to start from a stationary state characterized by fiscal parameters close to recent estimates. The factor multiplying the old agents' taxes, a^o, is kept equal to 1, while for the persistence parameter ρ_w in the wage equation (36) we choose a value close to the figure frequently adopted for the Calvo rule parameter in models with sticky nominal wages.

Under the parameterization of Table 1, we numerically solve the system of stationary equations specified in Appendix 4 according to the following strategy. We set a target value for the stationary Debt-to-GDP ratio $r^b = \frac{b}{y}$ and require the fiscal variables g and e to adjust according to their stationary ratios: $\frac{g}{y} = 0.150$ and $\frac{e}{y} = 0.0405$.[12] The fiscal revenues θ then adjusts in order to satisfy the budget equation $(1 + a^o\psi)\,\theta = g + e + [R - (1+n)(1+x)]\,b$, and we use the resulting value ($\theta = 0.1756$) to compute the stationary ratio $r^\theta = \frac{\theta}{y} = 0.14997$. This value of r^θ is subsequently inserted in the third equation of (37) to carry out the numerical simulations of the dynamic system described in Appendix 3. By so doing, we obtain values for the main endogenous variables which are roughly in line with some relevant empirical findings for the U.S. economy. For example, the main ratios of aggregate demand components over GDP are: $\frac{c}{y} = 0.6$ and $\frac{y-c-g}{y} = 0.24$, while the real interest factor is: $R = 1.00714$, which corresponds to 2.8% at yearly frequency. Analogously to Gerteler (1999), the m.p.c. of old agents ($\xi^o = 0.021$) is almost double than that of the young agents ($\xi^y = 0.011$), while the amount of labor services supplied by the old agents ($l^o = 0.08$) is significantly lower than that supplied by the young ones ($l^y = 0.48$), who constitute the bulk of the labor force and are relatively more productive.

4.2 Economic dynamics following a pandemic shock

Our first simulation exercise focuses on the economic impact of the pandemic shock. We here wish to first investigate how our model economy reacts to the COVID-19 pandemic when the main fiscal variables, g_t and θ_t, are anchored to the BGP and consequently set $r^g_t = r^g$ and $r^\theta_t = r^\theta$. This allows us to gain a direct insight of the basic reactions of agents and markets to a shock which is demographic and health-related in its nature (within the limits allowed for by the model structure). This experiment requires to pin down the dynamic behavior of γ_t and ϑ_t during the period in which the pandemic unfolds. In order to have a reliable framework, it is necessary to include also ϑ_t because, as discussed above, the causes of economic disruption under the pandemics cannot be uniquely related to measures imposed by governmental authorities (lockdown or similar restrictions), but also to "precautionary" or fear-related behavioral responses by individual agents, so that changes in ϑ_t inevitably include both phenomena.

[11] The ratio of social security expenditure to GDP, r^e, is computed by using data with annual time frequency.

[12] This is also coherent with equations (37) computed at their stationary level.

In shaping the numerical impact of the COVID-19 shock on the dynamic behavior of γ_t, we follow the estimates proposed by Goldstein and Lee (2020). They analyse the effect of the present pandemic on a number of demographic variables and provide a quantitative evaluation of some of these indicators based on three possible scenarios for the U.S.: i) in the "worst" scenario, they assume that during the year 2020 the pandemic causes 2,000,000 deaths; ii) in the "medium" scenario they assume a total death toll of 1,000,000 people; iii) in the last scenario, the one that is more compatible with the adoption of (partial) lockdown and other social distancing policies, they assume 250,000 deaths. For each scenario, Goldstein and Lee (2020) compute the effects of these increases in mortality on the overall life expectancy of the population; in particular, scenario i) should bring life expectancy down by 5.08 years, while scenario iii) should imply a reduction of only 0.84 years in life expectancy.

Starting form the stationary values of Parameterisation A in Table 1, we take advantage of the indications provided by the above scenarios i) and iii), and use Goldstein and Lee's (2020) estimation of the effects on life expectancy to carry out the following experiment. We consider two scenarios: in the first one, labelled *Pure Pandemic* (*PP*), we assume - counterfactually - that COVID-19 is not followed by social distancing policy/behavior, so as to replicate the "worst" scenario ii), in which γ_t falls coherently with a reduction of 6 years in life expectancy.[13] In the second scenario, labelled *Pandemic and Lockdown* (*PL*), we replicate scenario iii), in which the fall of γ_t is coherent with a reduction of only one year in life expectancy,[14] while at the same time ϑ_t increases above its stationary value (equal to 1). To set the numerical values of ϑ_t we follow a direct strategy. According to FRED data, the U.S. real percapita output recorded a fall of 1.26% in the first quarter of 2020 and a reduction of 8.98% in the second quarter of the year. We run a deterministic simulation of the model under the appropriate time-evolution profile of γ_t and then compute the values of ϑ_t that are required to obtain the same reduction of y_t in first two periods (1.2% and 9%).

The exogenous dynamic laws of γ_t and ϑ_t are hence specified in the following way:

$$\gamma_t = \gamma - u_t^{\gamma}; \qquad \vartheta_t = 1 + u_t^{\vartheta} \tag{40}$$

where the shock variables u^{γ} and u^{ϑ} take on different values under the *PP* and *PL* scenarios. More specifically:

- *PP:* the initial impact of the virus brings down life expectancy to 73, so that the implied value of the survival probability in the model is $\gamma_t = 0.96875$ in the

[13] This figure can also be considered as roughly in line with the discussion in Gagnon et al. (2020), in which they assume that a total death toll of 2.5 millions for COVID-19 would be consistent with the economy's population reaching herd immunity. See also Verity (2020).

[14] As of February 2021, the total COVID-19 death toll for the United States amounts to over 400,000 persons. This would suggest to increase the effect on γ_t so as to obtain a sharper fall in life expectancy, but in this case we do not have a numerical value supported by a definite model, such as that provide by Goldstein and Lee (2020). Nevertheless, as robustness check, we run simulations in which γ_t is reduced so as to bring down life expectancy to 1.6 years - i.e., to 77.4 years - and checked that the main results of the model remain qualitatively unchanged. In particular, the deterministic simulations of the *PL* scenario show the same qualitative pattern of that portrayed in Figure 3 below, and the relationship between the v_t^{ratio} in the two scenarios *PP* and *PL* remains substantially unaltered with respect to the one displayed in Figure 4 (with a fall of 1.6 years in life expectancy, the distance of v_t^{ratio} under the *PL* scenario from the stationary value is only slightly more pronounced).

quarters in which the pandemic's lethal effect concentrates; in terms of deviations from the stationary level, we assume the following dynamics: $u^{\gamma}_{t=1,2,3,4}(PP) = [0.0007; 0.0136; 0.0136; 0.0007]$, while we set $u^{\gamma}_{t}(PP) = 0$ in the following ts; (b) as the lockdown effect is absent, it is $u^{\vartheta}_{t}(PP) = 0$ for the whole simulation.

- *PL:* (a) the reduction in life expectancy is of one year only and the impact on the survival probability is $\gamma_t = 0.98077$; we then assume that in this case γ_t falls below its stationary value according to the following process: $u^{\gamma}_{t=1,2,3,4,5}(PL) = [0.0007; 0.00137; 0.001; 0.0005; 0.0001]$, followed by $u^{\gamma}_{t}(PP) = 0$ in the subsequent periods; (b) at the same time, we set the lockdown effect to: $u^{\vartheta}_{t=1,2,3,4}(PL) = [0.0012; 0.00624; 0.0023; 0.001]$ and zero otherwise.

Figure 1 shows the reaction of some of the main economic variables in the *PP* scenario:

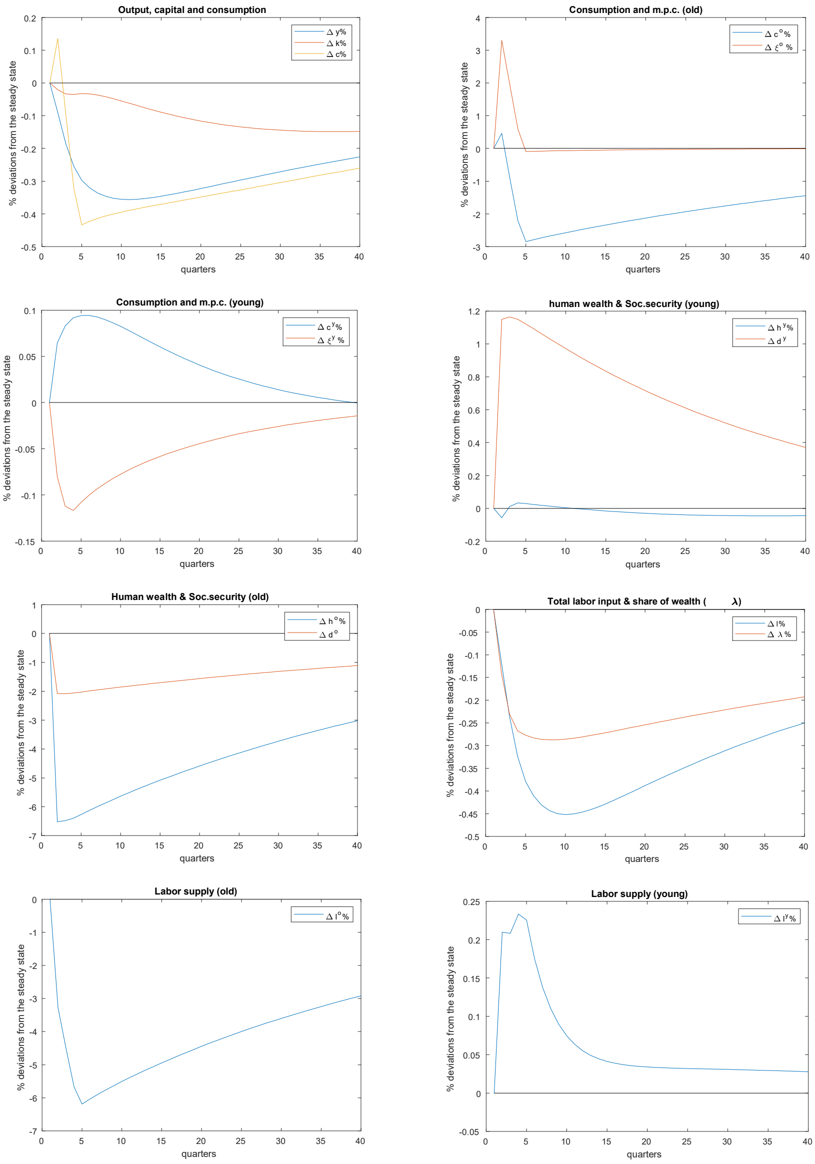

Figure 1 – deterministic simulation of the *PP* scenario: response of y, k, c, c^o, c^y, ξ^o, ξ^y, λ, h, $\bar{h}$, d, $\bar{d}$, $\widetilde{l}$, $\widetilde{l}^o$, $\widetilde{l}^y$. The figure plots the percentage change from the initial

level (i.e., the stationary value according to Table 1) for the first 40 periods (equivalent to 10 years). The parameters of the fiscal rules (37) are: $\rho_g = \rho_\theta = 0$ and $\delta_B = 0.4$.

This scenario is particularly interesting, as it describes the reaction of the model economy to a demographic/pandemic shock only, which makes it possible to isolate and study a mechanism driven by a kind of *"demographic" channel*. We first focus on the consumption levels c_t^y, c_t^o and on the m.p.c.'s, which show an heterogenous behavior. The initial increase in ξ^o can be attributed to the fact that the decrease in life expectancy magnifies the effect produced by each unit of consumption on group welfare, thus fostering asset decumulation (this effect prevails only temporarily, as shown by the short duration of the increase of c^o), whereas the fall in ξ^y is due to the fact that the increase in the wealth of the young group is spread over all the expected infinite life (young agents become old with probability $1-\omega$). As a result, the share of wealth held by the old (young) agents λ $(1-\lambda)$ falls (increases). Asset decumulation by old agents tends to reduce the real rate, which implies lower returns for the old agents and higher discounted flows of future income for the young ones.

The discounted stream of social security payments falls for the old group and increases for the young group: the former group mainly suffers from the reduction in the flow of social expenditure e_t caused, via the public budget constraint, by the lower resources available, while the latter group benefits from the reduction in the real rate.

In the labor market, total labor input $(\widetilde{l}_t = \widetilde{l}_t^y + \eta \widetilde{l}_t^o)$ falls, due to an increase in $\widetilde{l}_t^y$ more than compensated by a sharp fall in $\widetilde{l}_t^o$. Formally, the optimal supply of labor by old agents, $\widetilde{l}_t^o = \psi_t - \frac{\varsigma}{w_t \eta} c_t^o$, shows that the reduction in c_t^o and the increase in w_t (which is sticky and multiplied by the scale parameter η, measuring the lower productivity of old agents) only partly compensate the lethal effect of the pandemic on the old share of the population, that is, the effect of the fall in γ_t on the ratio $\psi_t = N_t^o/N_t^y$. The increase in $\widetilde{l}_t^y = 1 - \frac{\varsigma}{w_t} c_t^y$ is instead the result of the small increase in c_t^y and the increase in w_t: the price (wage) effect induces the young agents to increase their labor supply.

The behavior of the price variables, the interest factor R_t and the real wage w_t, which is shown in Figure 2, is particularly important to test the ability of the model to match the empirical evidence and hence deserves separate comments.

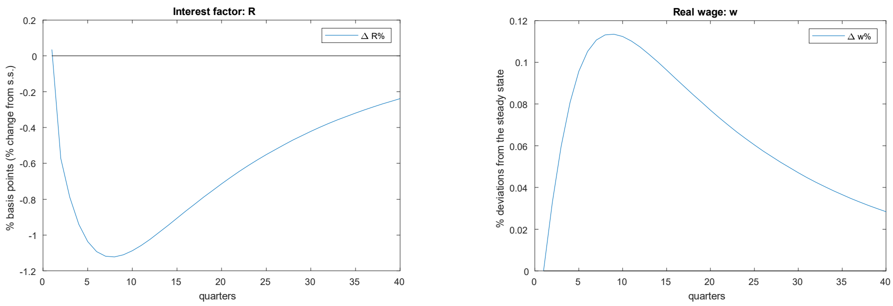

Figure 2 – Evolution of R_t (in basis points change from the s.s. value) and w_t under the deterministic simulation of the *PP* scenario.

The response of R_t and w_t is coherent with the economic mechanism discussed above: DSGE models with a neoclassical structure, such as ours, should actually react to a demographic/health-related shock to γ_t as depicted in Figure 2. The demographic channel reduces the total labor input and pushes the economy into a long-lasting recession: the fall in total labor supply drives the real wage upwards. The increases in the wage signals that: (i) the fall in labor supply is greater than that in labour demand (the effect of the pandemic on the supply side is hence the driver of labor market changes); (ii) the wage elasticity of the labor supply is much greater for the young agents than for the old agents. At the same time, the long-lasting recessionary environment causes a reduction in the demand for capital and in accumulation, inducing a long-term fall in the real interest rate. The behavior of R_t and w_t displayed by our model is also qualitatively in line with the empirical analysis carried out by Jordà et al.(2020), who show that in most historical cases the real "natural" interest rate undergoes a long-term fall following a pandemic, while the real wage index shows a long-lasting increase, which is due to a demographic effect (i.e., the reduction in the labor force) akin to the one depicted in our model.[15]

The outcome of the simulation under the *PL* scenario is shown in Figure 3:

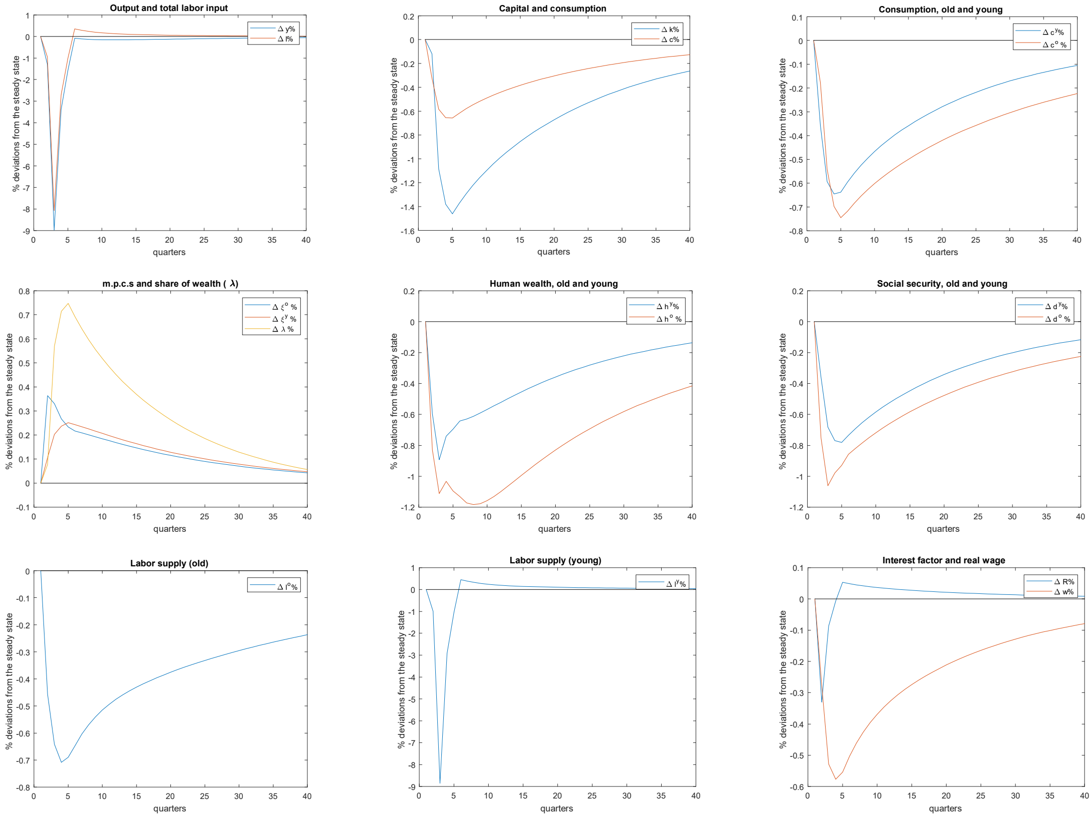

Figure 3 – deterministic simulation of the *PL* scenario: response of y, k, c, c^o, c^y, ξ^o, ξ^y, λ, h, $\bar{h}$, d, $\bar{d}$, $\widetilde{l}$, $\widetilde{l}^o$, $\widetilde{l}^y$. The figure plots the percentage change from the initial level (i.e., the stationary value according to Table 1) for the first 40 periods (10 years). The parameters of the fiscal rules (37) are: $\rho_g = \rho_\theta = 0$ and $\delta_B = 0.4$.

The introduction of a "lockdown" effect – via policies and/or voluntary social distancing behavior – forces the economy to respond in a strongly different way, as com-

[15]In Jordà et al. (2020), the average time period in which the real interest rate lies below its initial level spans over 40 years, which is much longer than the recovery time-span shown in Figure 2. Even though our model, under the baseline paramerisation, cannot replicate with precision this feature, the qualitative similarity of the dynamics of R_t depicted in Figure 2 with the results in Jordà et al. (2020) provides satisfactory empirical support to our model.

pared with the PP scenario. Now, besides the demographic channel previously described, also a *"social restrictions" channel* is present (activated by the change in ϑ_t) which, besides dampening the demographic channel *via* a reduction of the fall in γ_t, heavily impacts on the markets. First of all, the labor inputs, $\widetilde{l}_t^y$ and $\widetilde{l}_t^o$, show substantial reductions due to the direct effect of ϑ_t. Young agents, who are the main component of the labor force are affected the most by these restrictions: the contraction in $\widetilde{l}_t^y$ is ten times greater than that of the old agents $(\widetilde{l}_t^o)$. As a consequence, as long as the lockdown is active, the economy plummets into a deep recession, and the partial containment of the fall of γ_t due the social distancing measures (the reduction in the demographic effect) is completely overwhelmed by the forced restriction on input usage which also causes a wage reduction. Clearly, the dramatic effect of the lockdown is tightly related (in the model) to the duration of the change in ϑ_t: when lockdown restrictions are lifted, and ϑ_t recovers its "normal" value 1, output sharply bounces back to higher values, in line with the recent empirical evidence pointing to a sharp rebound of GDP in the third quarter of 2020[16]. Nevertheless, the recovery in not complete: when the effect of ϑ_t peters out (from $t = 3$ onwards), the highly persistent effect of the (now milder) reduction of γ_t continues to keep output below its stationary level[17]. Contrary to the PP scenario, when lockdown and/or social distancing policies are present, a transfer of wealth from younger agents (whose income loss is the greatest) to old ones takes place (while at same time total wealth a_t falls). The severe recession incentivates old agents to save more, as witnessed by the sharp increase in $1 - \xi_t^o$.

It is particularly interesting for our research objective to compare the reaction of the welfare indicators in the two scenarios. To this aim, Figure 4 depicts the behavior of the level of the relative welfare v_t^{ratio} under PP and PL:

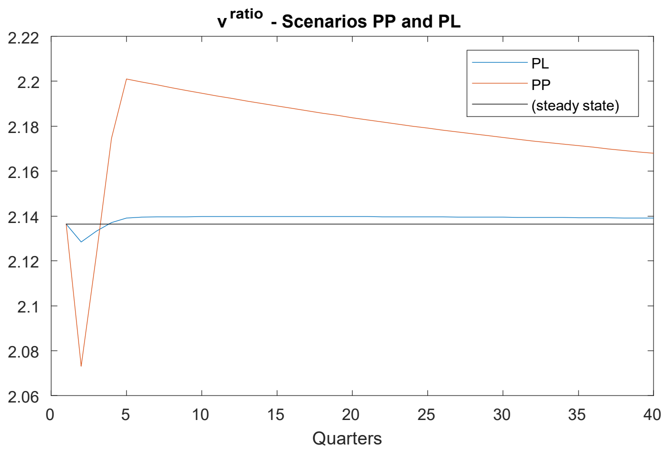

Figure 4 – response of v_t^{ratio} under PP and PL (level).

Under both scenarios, the initial impact of the shock is unfavorable to young agents, but the subsequent dynamics of v_t^{ratio} is quite different: under the PP scenario, the relative welfare index shows a substantial and persistent increase above the initial level,

[16]The strong reaction of y_t in the model is also due to the deterministic nature of the simulation: agents perfectly forecast the end of the lock-down policies and immediately start to provide more labor input, as confirmed by the behavior of $\widetilde{l}_t^y$ shown in Figure 3.

[17]This is however barely visible in the first panel of Figure 3 due to the scale of the lockdown effect.

while in the *PL* scenario this increase is significantly dampened. The difference in the initial response of v_t^{ratio} between the two scenarios can be traced back (via equation 39) to the behavior of the two ratios ξ_t^y/ξ_t^o and c_t^y/c_t^o. Under *PP*, the old agents' variables (ξ_t^o and c_t^o) show, at impact, a behavior markedly different from that of the corresponding variables of young agents: the strong increase the old agents' m.p.c., coupled with a small initial increase in consumption (Figure 1), can be thought of as an anticipation of the adverse consequences of the reduction in life expectancy. On the other hand, under *PL*, the overall recessionary impact of ϑ_t forces both groups to a more uniform behavior, as shown by the dynamics of ξ_t^z and c_t^z in Figure 3: the consequence is a dampening of the dynamic evolution of v_t^{ratio}. The evolution of the relative welfare index confirms this picture: the dynamics of the main endogenous variables depicted in Figure 3 is much more uniform between the two groups than the one shown in Figure 1 for the *PP* scenario.

The main insight offered by this exercise, and portrayed in Figure 4, is that under *PP* the *demographic channel* - the only active one - clearly favours the younger age-group in the transition dynamics, due to the intrinsic nature of the fall in γ_t. Under the *PL* scenario, the sharp prevalence of a *social restrictions channel* (the rise of ϑ_t) over the demographic channel reduces the distance between the two welfare indexes V_t^y and V_t^o, because the strong recessionary drag tends to affect both age groups in a more symmetric way. Under *PL*, while the old agents experience a greater fall in the non-financial components of their wealth (h_t^o and d_t^o), the young agents must bear a stronger decrease in the main source of their income, i.e., the labour one, $w_t\widetilde{l}_t^y$.

Clearly, the *PP* scenario must be considered as a "counterfactual", and also the *PL* scenario cannot be considered as an entirely realistic picture, due to the model's simplified structure. Nevertheless, in this analytical context, it is possible to provide a qualification and a clarification of the claim that the management of the COVID-19 pandemic through containment policies (the only ones at hand before the development of a vaccine) has mainly hit the young generations.[18] Without lockdown policies (under *PP* scenario), the cost of the pandemic is mainly sustained by older agents, in the form of the economic value lost due to the increased mortality of this specific age-group, i.e., to the intrinsic demographic nature of this specific pandemic episode. The introduction of lockdown policies (under the *PL* scenario) shifts a relevant part of the economic cost of the pandemic from older age-groups to younger ones. In other words, lockdown policies entail a form of "re-allocation" of economic welfare among age-groups: social restrictions affects all agents in a more symmetric way and hence contribute to spread the welfare costs more homogeneously among the age-groups. The downside of this type of polices is the ensuing strong recession: the more even sharing of welfare costs due to the implementation of lockdown measures - as depicted by the *PL*-series in Figure 4 - is paid out with a much stronger fall of total output and hence of overall material resources available to all agents in the system.[19]

[18]Even if we abstract, as we do in this model, from some of the most obvious disadvantages suffered from the younger generations during the lockdown, such as the reduction in opportunities for education and human capital accumulation, or for social contacts.

[19]Recall that the difficulties mentioned in Section 3 prevent a general and consistent analysis of the total welfare losses borne by society as a whole in the two scenarios *PP* and *PL*.

4.3 Fiscal policy in the wake of the pandemic shock

In order to study the consequences of different fiscal policy programs on the relative welfare index v_t^{ratio}, we now remove the "conservative" assumption that $r_t^g = r^g$ and $r_t^\theta = r^\theta$ and consider the possibility to increase g_t and θ_t through direct interventions. We hence pose $r_t^g = r^g \exp\left(\vartheta^G u_t^g\right)$ and $r_t^\theta = r^\theta \exp\left(\vartheta^\theta u_t^\theta\right)$, with: $\vartheta^G = 0.8$, $\vartheta^\theta = 0.2$, and assume a (deterministic) process for the "policy shocks" $u_t^{g,\theta}$ in the aftermath of the pandemic episode. While the model is sufficiently flexible to allow us to study different types of fiscal policies, it is also constrained by the chosen analytical structure.

Endogenous variables respond to shocks to the (wasteful) public expenditures g_t, while shocks on θ_t seem to have a minor impact; this is due to equations (37), (38), and is confirmed by a number of simulations.[20] Nevertheless, the impact of g_t on total output (as shown in Figure 5) is relatively small. This is related to the way in which this variable is included in the model: coherently with the standard RBC framework, changes in un-productive public consumption induce a relevant crowding-out of private consumption and investment. Although relevant public investment with a direct impact on production, such as infrastructural programs, have been included in the agenda of many countries hit by the pandemic shock (e.g., the Next Generation EU plan for the European Countries), the immediate fiscal policy reaction to the crisis generated by COVID-19 seems to be more oriented to provide an immediate income support to firms and households. For instance, the America Rescue Plan of the newly elected U.S. administration provides 1.9 \$ trillions to be allocated mainly in direct income support (emergency unemployment insurance, per-person checks, and analogous measures, up to 52% of the planned budget), public health provisions and schools re-opening, small business grants and support to local communities.

This type of intervention can be reasonably interpreted, in the context of our model, as a series of positive shocks u_t^g *(and* u_t^θ*)* taking place in the years immediately following the pandemic shock. In the following numerical experiments, we hence assume, starting from the *PL scenario*, that $u_t^g > 0$ for 3.5 years, coupled with $u_t^\theta > 0$ for 1 year (in order to include the effect of tax payments postponements and money transfers). We focus on this expansionary fiscal policy scenario:

- *PLG:* (a) we assume: $u_{t=1,2...,14}^g = [0.1; 0.12; 0.12; 0.12; 0.5; 0.55; 0.58; 0.55; 0.5; 0.3; 0.25; 0.2; 0.1; 0.1]$. This series generates (in the *PL* scenario) an increase in the ratio g_t/y_t of more than 9% in the quarters 7, 8 and 9; from the FRED data, an increase of 1.9 \$ trl. in public expenditures would bring the g_t/y_t - computed in 2020 - from 14% to 24%. (b) We also assume: $u_{t=1,2,3,4}^\theta = [1.8; 1.6; 0.3; 0.1]$; recall that, in the model, a shock to r_t^θ has a minor impact on the endogenous variables (on output in particular).

Coherently with our main research objective, we are keen to investigate the impact on the relative welfare of age-groups of *Debt Repayment Schemes* (DRS), following the expansionary fiscal policies undertaken during the pandemic. In this perspective, we perform a number of simulations under the *PLG* scheme by changing the parameter δ_B

[20] Available from the authors upon request. Actually, the model's structure forces θ_t and b_t to adjust in order to satisfy the government budget and a stable equilibrium.

so as to generate different DRSs; we then compare these simulations with the benchmark behavior of the system under the PL scenario (in which it is $\delta_B = 0.4$ and $\rho_\theta = 0$), as shown in Figures 3-4. The different DRS are described by the following values: $\delta_B = [0.7; 0.4; 0.2; 0.1; 0.06]$, while the parameter ρ_θ is kept fixed at 0.95.[21]

The main results of this simulation are depicted in Figure 5.

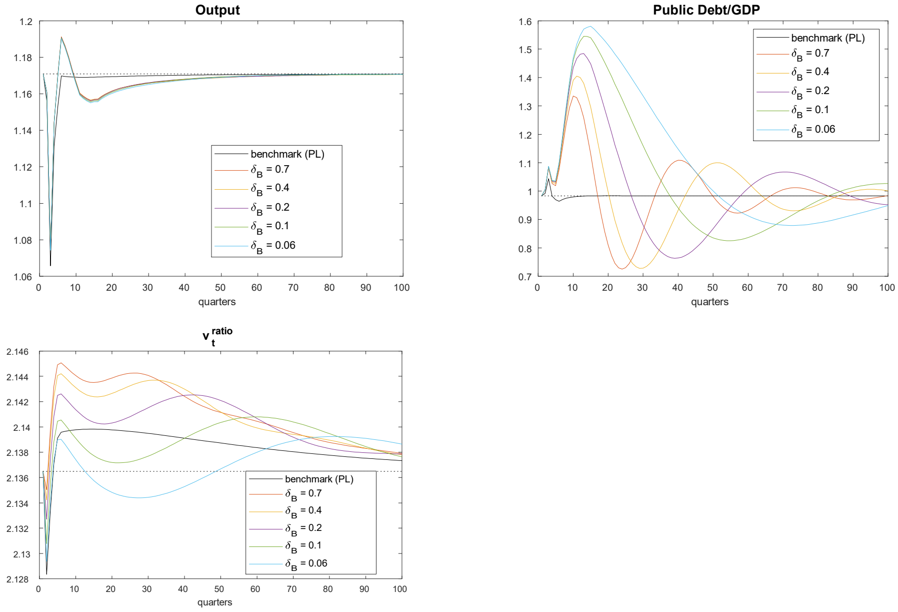

Figure 5 - Behavior of y_t, b_t/y_t and v_t^{ratio} under different DRS in the PLG scenario. The figure depicts the resulting 100 periods (25 years) of a deterministic simulation of the model; the dotted line represents the initial (stationary state) value.

The choice of a particular DRS does not have a noticeable impact on the dynamic evolution of GDP, but things are different for the Debt-to-GDP ratio and, in particular, for the relative welfare index v_t^{ratio}. The time profile of the debt repayment following the expansionary fiscal policy do have some sizeable reallocation effect on the welfare of the age-groups. The more the DRS entails a postponement of the repayment, the more the old agents are favoured; in particular, if we focus on the most rapid DRS ($\delta_B = 0.7$) and on the most delayed one ($\delta_B = 0.06$) and carry out a comparison by taking the evolution of v_t^{ratio} under PL as a benchmark, a reversal of the effect of fiscal policy on the relative welfare appears evident. If the fiscal expansion is shortly covered by the DRS, young agents can benefit in relative terms with respect to old ones, while the opposite is true when the debt repayment is significantly shifted later in time. Clearly, the more the DRS entails a substantial postponement, the more the Debt-to-GDP ratio grows, reaching a peak of almost 160%.

In order to gain an insight on the economic mechanism behind the reallocation effect on the relative welfare index shown in Figure 5, we can inspect the behavior of the main endogenous variables under the different DRS, focusing on those directly relevant for v_t^{ratio} from equation (39).

[21] Robustness check (available upon request) confirms that changing ρ_θ in the interval $[0; 0.99]$ generates only minor changes in the dynamics of v_t^{ratio}, which is our main variable of interest.

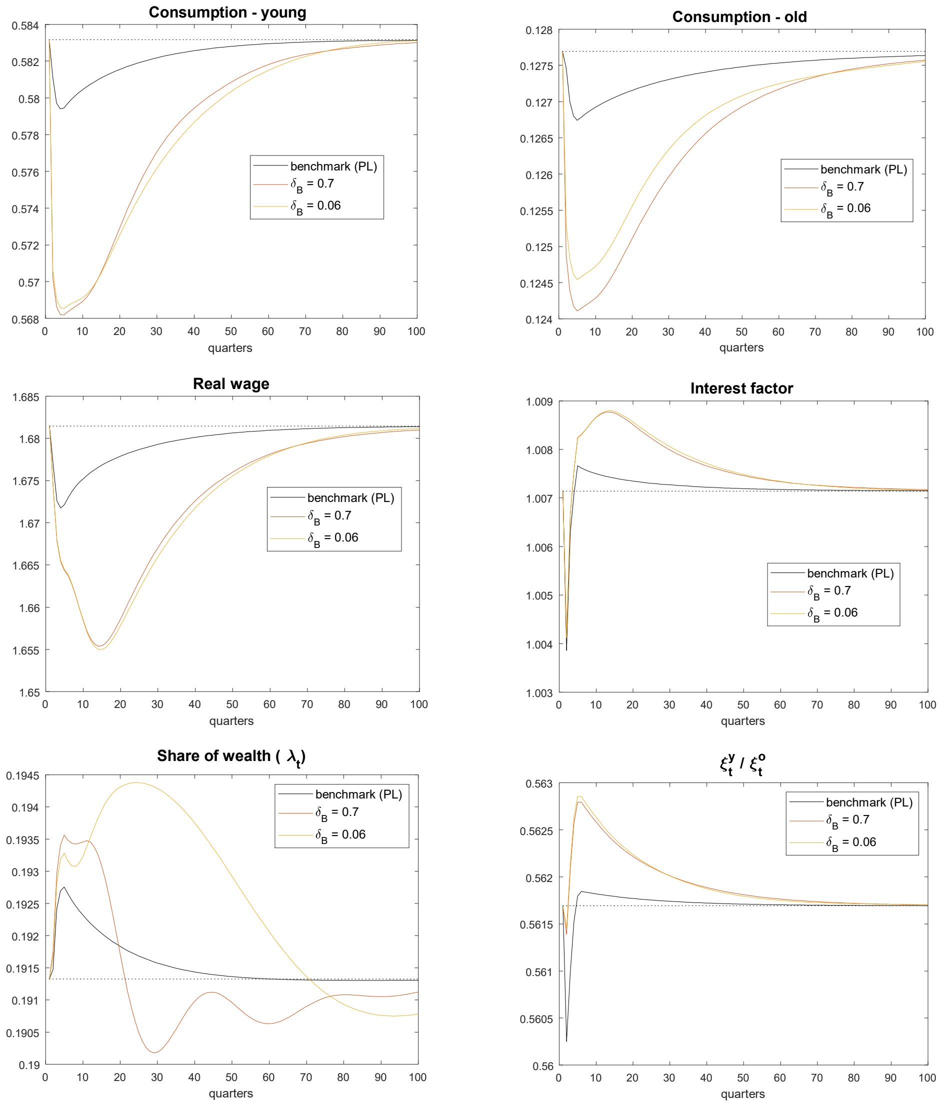

Figure 6 - Behavior of some endogenous variables under the two "extreme" DRS in the *PLG* scenario. The figure depicts the resulting 100 periods (equivalent to 25 years) of a deterministic simulation of the model; the dotted line represents the initial (stationary state) value.

In order to obtain a clearer picture, Figure 6 shows only the two "extreme" DRS in the *PLG* scenario, the most "rapid" and the most "delayed" ones, i.e., respectively $\delta_B = 0.7$ and $\delta_B = 0.06$, . When the DRS is rapid, the burden of the repayment is shared more evenly between the age-groups, while a longer postponement of the repayment puts a greater share of the fiscal adjustment on the shoulders of the young agents. In the DRS $\delta_B = 0.7$ the fraction of total financial wealth held by old agents (λ_t) sharply falls after 10 periods, more or less in synchrony with the fall of the Debt-to-GDP ratio following the peak (see Figure 5). When instead the DRS is substantially delayed ($\delta_B = 0.06$), the share λ_t keeps growing for a longer time - roughly further 20 periods - before starting to converge towards the stationary value; in this case, λ_t follows a transitional dynamics qualitatively similar to that of the Debt-to-GDP ratio. Hence the favorable dynamics of their share of wealth under the "delayed" DRS helps old agents to restrain the fall in consumption, c_t^o, as compared to what happens under the "rapid" DRS. The different dynamic behavior of the share λ_t in the two extreme DRS helps to explain the evolution of v_t^{ratio} in the two scenarios.

5 Conclusions

In this paper, we aimed to study the impact of the COVID-19 epidemics on the relative economic welfare of different age groups of the population, as well as the effects of the "lockdown" measures and fiscal policy programs undertaken in the wake of the health emergency during and immediately after the pandemic. To this aim, we adapted the macroeconomic model with life-cycle by Gertler (1999), which partitions the population into two main groups, "young" agents (active workers) and old agents or "retirees". The possibility of obtaining closed-form solutions allowed us to define an appropriate relative welfare index (the ratio between the aggregate utilities of the two groups at equilibrium) and to study its dynamic evolution. This scheme is particularly useful in the light of our research problem, because it allows us to directly include the impact of the COVID-19 pandemic on the life expectancy of the relevant age-group, i.e., the old agents, who are (accordingly to the available demographic and statistical analysis) the most affected ones.

We obtained two main results. The first one is that, when fiscal policy is relatively "neutral" - in the sense that is does not reacts explicitly and directly to the pandemic shock - we can isolate two main channels through which the fall of life expectancy due to the COVID-19 and the restrictions imposed by lockdown policies impact the relative welfare index. In the absence of lockdown policies, only a "demographic channel" is active (which, in the case of COVID-19, is clearly counterfactual in its nature); this channel is activated by the strong fall of old agents' life expectancy and induces a relevant reduction of the their share in the population. This implies a reduction of the overall labor supply that brings about a rise in the wage rate coupled with a fall in the real interest rate, a pattern in line with that identified by the empirical analysis of historical pandemics of Jordà et al. (2020). This also implies that the "pure" demographic effects of a pandemic such as COVID-19 tend to penalize mainly the welfare of old agents.

When instead lockdown and/or social restriction policies are active, the picture is different and a "social restriction channel" prevails on the demographic one. Lockdown policies can effectively reduce the pandemic-related mortality among the old agents, and hence contain the fall of their life expectancy, but they also induce a sudden and strong recessionary pull hitting the economy as a whole. In this case, the relative welfare index shows substantially smaller changes as compared to the hike (favouring the young agents) shown in the case of the demographic channel alone. Seen under this perspective, the introduction of lockdown policies actually shifts a relevant part of the economic cost of the pandemic from older age-groups to younger ones and hence implicitly implement a more even form of "re-allocation" of welfare losses among age-groups. Nevertheless, the fall in output induced by lockdown policies is consistently greater than the mild reduction that takes place when they are absent and only the demographic channel is active, suggesting that the more even distribution of welfare costs is paid out with a stronger macroeconomic recession.

Our second result concerns the effects of fiscal policies aimed at providing immediate income support to firms and households, in a context in which lockdown and social distancing measures have already been implemented. Within our model, this type of programs takes the form of increases in public expenditures lasting two or three years after the pandemic peak, and we focus on the consequences of different repayment schemes for the resulting public debt. Although this form of fiscal intervention does not produce a particularly strong reaction of total output (due to the RBC nature of the model), the different Debt Repayment Schemes (DRS) have an impact on the time evolution of the relative welfare index. In general, the postponement of the debt repayment tends to favour the old agents; on the opposite, when the fiscal expansion is rapidly covered by the DRS young agents tend to be advantaged in terms of relative welfare. Nevertheless, the impact of lockdown policies on the relative welfare index remains the prevailing one: expansionary fiscal interventions and different DRS can modify the time evolution of the groups' relative welfare index only to a minor extent.

The general picture that can be drawn from these results is sufficiently unequivocal, and the common opinion that the management of the COVID-19 pandemic through containment/lockdown policies has mainly hit the young generations receives a clear support. Given the highly simplified structure of the model, there are however many different routes to pursue in order to deepen and clarify this issue. In particular, the model abstracts from some obvious disadvantages suffered from the younger generations during the lockdown, such as the reduction in opportunities for education and human capital accumulation, and these effects could be included in an extended version of the model with a more accurate description of the long-term process of economic growth. Furthermore, the specific features of the life-cycle scheme adopted in the model prevents a general and consistent analysis of the total welfare losses which are sustained by society as a whole. The relative welfare index derived from the model is adequate only for a preliminary investigation of the differentiated welfare effects of policies on the two age-groups, and a more complete and refined social welfare analysis remains a target for future research.

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Appendix

1) *Macroeconomic equilibrium*
Equilibrium system:

$$
\begin{aligned}
K_t &= Y_t - C_t - G_t + (1-\delta) K_{t-1};\\
A_t &= \frac{\omega\left[\left(1-\xi_t^o\right) R_{t-1}\lambda_{t-1}A_{t-1} + E_t + W_t\eta L_t^o - a^o\psi_t T_t^{*y} - \xi_t^o\left(H_t + D_t\right)\right]}{\omega + \lambda_t - 1};\\
\frac{1}{\xi_t^y} &= 1 + \left[\left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho}\beta\right]^{\sigma}\left(R_t\Omega_{t+1}\right)^{\sigma-1}\frac{1}{\xi_{t+1}^y};\\
\frac{1}{\xi_t^o} &= 1 + \gamma_{t+1}\left[\left(\frac{W_t}{W_{t+1}}\right)^{(1-q)\rho}\beta\right]^{\sigma} R_t^{\sigma-1}\frac{1}{\xi_{t+1}^o};\\
\Omega_t &= \omega + (1-\omega)\chi\left(\frac{\xi_t^o}{\xi_t^y}\right)^{\frac{1}{1-\sigma}}; \qquad (1+n)\psi_t = (1-\omega) + \gamma_t\psi_{t-1};\\
H_t &= W_t\eta L_t^o - a^o\psi_t T_t^{*y} + \frac{\gamma_{t+1}\psi_t H_{t+1}}{\psi_{t+1}(1+n)R_t}; \qquad A_t = K_t + B_t;\\
\bar{H}_t &= W_t L_t^y - T_t^{*y} + \frac{\omega}{(1+n)\Omega_{t+1}R_t}\bar{H}_{t+1} + \left(\frac{\Omega_{t+1}-\omega}{(1+n)\psi_{t+1}\Omega_{t+1}R_t}\right)H_{t+1}\\
L_t^y &= N_t^y - \frac{\varsigma}{W_t}C_t^y; \qquad L_t^o = \psi_t N_t^y - \frac{\varsigma}{W_t\eta}C_t^o; \qquad L_t = L_t^y + \eta L_t^o;\\
Y_t &= \left[(1-\alpha)K_{t-1}^{\phi} + \vartheta_t\alpha\left(X_tL_t\right)^{\phi}\right]^{\frac{1}{\phi}}; \quad W_t = \alpha\vartheta_t X_t\left(\frac{Y_t}{X_tL_t}\right)^{1-\phi};\\
C_t &= \xi_t^y\left[\left(1-\lambda_{t-1}\right)R_{t-1}A_{t-1} + \bar{H}_t + \bar{D}_t\right] + \xi_t^o\left(\lambda_{t-1}R_{t-1}A_{t-1} + H_t + D_t\right);
\end{aligned}
$$

$$
\begin{aligned}
R_t &= r_{t+1}^K + 1 = (1-\alpha)\left(\frac{Y_{t+1}}{K_t}\right)^{1-\phi} + (1-\delta);\\
B_t &= G_t + E_t - \left(1 + a^o\psi_t\right)T_t^{*y} + R_{t-1}B_{t-1}\\
D_t &= E_t + \frac{\psi_t\gamma_{t+1}D_{t+1}}{\psi_{t+1}(1+n)R_t}; \quad (1+n)\bar{D}_t = \frac{\omega}{\Omega_{t+1}R_t}\bar{D}_{t+1} + \left(\frac{\Omega_{t+1}-\omega}{\psi_{t+1}\Omega_{t+1}R_t}\right)D_{t+1}.
\end{aligned}
$$

2) *Staggered real wages*

An individual worker faces the opportunity to re-set the wage in each time period; he/she also faces a quadratic cost when he/she is not allowed to re-set the wage, so that the staggering scheme of Rotemberg (1987) and Blanchard and Galí (2005) can be applied (see also Kilponen et al,. 2006). The opportunities of wage re-setting fall on a worker according to an exogenous Poisson process, so that in each time period the probability for the worker to re-set the wage is $1-\rho_w$ and the expected time of wage re-setting is $\frac{1}{1-\rho_w}$. If the worker j cannot re-set the wage, the cost borne is given by: $\frac{1}{2}\left(w_{j,t}^* - w_{j,t}^{ob}\right)^2$, where $w_{j,t}^*$ is the level of the wage assigned at t to the worker and $w_{j.t}^{ob}$ is the objective level of the wage, as defined in equation (35). Once the wage is set, it can remain fixed at this level for an arbitrary number of periods, according to the Poisson process previously described. Hence, if the worker can set the wage $w^*_{j,t}$, he/she will try to minimize the overall cost function:

$$\frac{1}{2}E_t\sum_{i=0}^{\infty}\rho_w^i\beta^i\left(w^*_{j,t}-w^{ob}_{j,t+i}\right)^2$$

where ρ_w is the probability of not adjusting the wage. From the first order condition for a minimum w.r.t. $w^*_{j,t}$: $w^*_{j,t}=(1-\rho_w\beta)\,E_t\sum_{i=0}^{\infty}\rho_w^i\beta^i w^{ob}_{j,t+i}$ and from equation (35), we obtain the level of the "re-set" wage: $w^*_t=(1-\rho_w\beta)\;E_t\sum_{i=0}^{\infty}\rho_w^i\beta^i m^{RS}_{t+i}$, that is:

$$w^*_t=(1-\rho_w\beta)\,m^{RS}_t+\rho_w\beta E_t w^*_{t+1}$$

which is common to all agents under the assumption that the objective wage $w^{ob}_{j,t+i}=w_{t+i}=m^{RS}_{t+i}$ defined in equation (35) is common to all the young agents. As previously described, in each period a fraction $1-\rho_w$ of workers reset their wage to the level w^*_t, while the remaining fraction ρ_w obtains the previous level w_{t-1}, so that the law of motion for the aggregate real wage is given by: $w_t=(1-\rho_w)\,w^*_t+\rho_w w_{t-1}$. By combining this equation with the previous one, equation (36) is derived.

3) *Detrended equilibrium equations*

The equilibrium system of stationary variables: $f\left(\mathbf{v}_{t+1};\mathbf{v}_t;\mathbf{v}_{t-1}\right)=\mathbf{0}$ includes the following equations:

$$\Omega_t = \omega+(1-\omega)\,\chi\left(\frac{\xi^o_t}{\xi^y_t}\right)^{\frac{1}{1-\sigma}};\quad (1+n)(1+x)\,k_t=y_t-c_t-g_t+(1-\delta)\,k_{t-1};$$

$$\frac{1}{\xi^y_t} = 1+\left[\left(\frac{w_t}{(1+x)\,w_{t+1}}\right)^{(1-q)\rho}\beta\right]^{\sigma}\frac{\left(R_t\Omega_{t+1}\right)^{\sigma-1}}{\xi^y_{t+1}};$$

$$\frac{1}{\xi^o_t} = 1+\gamma_{t+1}\left[\left(\frac{w_t}{(1+x)\,w_{t+1}}\right)^{(1-q)\rho}\beta\right]^{\sigma}\frac{R_t^{\sigma-1}}{\xi^o_{t+1}};$$

$$a_t = \frac{\omega\left[(1-\xi^o_t)\,R_{t-1}\lambda_{t-1}a_{t-1}+e_t+w_t\eta\widetilde{l}^o_t-a^o\psi_t\theta_t-\xi^o_t\,(h_t+d_t)\right]}{(1+n)(1+x)(\omega+\lambda_t-1)};$$

$$h_t = w_t\eta\widetilde{l}^o_t-a^o\psi_t\theta_t+\frac{(1+x)\,\gamma_{t+1}\psi_t}{\psi_{t+1}R_t}h_{t+1};$$

$$\bar{h}_t = w_t\widetilde{l}^y_t-\theta_t+\frac{(1+x)\,\omega}{\Omega_{t+1}R_t}\bar{h}_{t+1}+\frac{(1+x)\,(\Omega_{t+1}-\omega_{t+1})}{\psi_{t+1}\Omega_{t+1}R_t}h_{t+1};$$

$$\widetilde{l}_t = \widetilde{l}^y_t+\eta\widetilde{l}^o_t;\qquad \widetilde{l}^y_t=1-\frac{\varsigma}{w_t}c^y_t;\qquad \widetilde{l}^o_t=\psi_t-\frac{\varsigma}{w_t\eta}c^o_t;$$

$$w_t = \alpha\vartheta_t\left(\frac{y_t}{\widetilde{l}_t}\right)^{1-\phi};\quad m^{RS}_t=\frac{\varsigma c^y_t}{1/\varphi_t-\widetilde{l}^y_t};\quad y_t=\left[(1-\alpha)\,k^{\phi}_{t-1}+\vartheta_t\alpha\widetilde{l}^{\phi}_t\right]^{\frac{1}{\phi}};$$

$$w_t = \frac{(1-\rho_w)(1-\rho_w\beta)}{1+\rho_w^2\beta}m^{RS}_t+\frac{\rho_w\beta}{1+\rho_w^2\beta}E_t w_{t+1}+\frac{\rho_w}{1+\rho_w^2\beta}w_{t-1};$$

$$R_t = (1-\alpha)\left(\frac{y_{t+1}}{k_t}\right)^{1-\phi}+1-\delta;\quad (1+n)\,\psi_t=(1-\omega)+\gamma_t\psi_{t-1};$$

$$
\begin{aligned}
a_t &= k_t + b_t; \qquad (1+n)(1+x)\,b_t = g_t + e_t - (1+a^o\psi_t)\,\theta_t + R_{t-1}b_{t-1};\\
c_t &= \xi_t^y\left[(1-\lambda_{t-1})\,R_{t-1}a_{t-1} + \bar{h}_t + \bar{d}_t\right] + \xi_t^o\left(\lambda_{t-1}R_{t-1}a_{t-1} + h_t + d_t\right);\\
d_t &= e_t + \frac{(1+x)\,\psi_t\gamma_{t+1}}{\psi_{t+1}R_t}d_{t+1}; \qquad \frac{1}{1+x}\bar{d}_t = \frac{\omega}{\Omega_{t+1}R_t}\bar{d}_{t+1} + \left(\frac{\Omega_{t+1}-\omega}{\psi_{t+1}\Omega_{t+1}R_t}\right)d_{t+1}\\
e_t &= r_t^e y_t; \quad g_t = \rho_g g_{t-1} + \left(1-\rho_g\right)r_t^g y_t;\\
\theta_t &= \rho_\theta\theta_{t-1} + (1-\rho_\theta)\left[r_t^\theta y_t + \delta_B\left(\frac{b_t}{y_t} - r^b\right)\right];\\
c_t^y &= \xi_t^y\left[(1-\lambda_{t-1})\,R_{t-1}a_{t-1} + \bar{h}_t + \bar{d}_t\right]; \qquad c_t^o = \xi_t^o\left(\lambda_{t-1}R_{t-1}a_{t-1} + h_t + d_t\right).
\end{aligned}
$$

4) Stationary equations and variables

The main stationary variables are: $\{\, k,\ \lambda,\ \xi^o,\ \xi^y,\ \Omega,\ h,\ \bar{h},\ y,\ c,\ c^y,\ c^o,\ R,\ w,\ d,\ \bar{d},\ \widetilde{l},\ \widetilde{l}^y,\ \widetilde{l}^o, \theta,\ g,\ e,\ b,\ \psi \,\}$ and the stationary equilibrium equations are:

$$
\begin{aligned}
y &= \left[(1+n)(1+x) - 1 + \delta\right]k + c + g; \qquad \Omega = \omega + (1-\omega)\,\chi\left(\frac{\xi^o}{\xi^y}\right)^{\frac{1}{1-\sigma}};\\
\lambda &= \omega\frac{e + w\eta\widetilde{l}^o - a^o\psi\theta - \xi^o(h+d)}{\left[(1+n)(1+x) - \omega(1-\xi^o)R\right](k+b)} + \frac{(1-\omega)(1+n)(1+x)}{(1+n)(1+x) - \omega(1-\xi^o)R};\\
h &= \frac{R\left(w\eta\widetilde{l}^o - a^o\psi\theta\right)}{R-(1+x)\gamma}; \qquad \bar{h} = \frac{\psi\Omega R\left(w\widetilde{l}^y - \theta\right) + (1+x)(\Omega-\omega)\,h}{\psi\left[\Omega R - (1+x)\,\omega\right]};\\
w &= \alpha\left(\frac{y}{\widetilde{l}}\right)^{1-\phi}; \qquad R = (1-\alpha)\left(\frac{y}{k}\right)^{1-\phi} + 1 - \delta; \qquad y = \left[(1-\alpha)\,k^\phi + \vartheta_t\alpha\widetilde{l}^\phi\right]^{\frac{1}{\phi}};\\
c &= c^y + c^o; \qquad (1+a^o\psi)\,\theta = g + e + \left[R - (1+n)(1+x)\right]b;\\
c^y &= \xi^y\left[(1-\lambda)(k+b)\,R + \bar{h} + \bar{d}\right]; \qquad c^o = \xi^o\left[\lambda(k+b)\,R + h + d\right];\\
d &= \frac{R}{R-(1+x)\gamma}e; \qquad \bar{d} = \frac{(1+x)(\Omega-\omega)}{\left[\Omega R - (1+x)\,\omega\right]\psi}d;\\
\xi^y &= 1 - \bar{\beta}^\sigma(R\Omega)^{\sigma-1}; \qquad \xi^o = 1 - \gamma\bar{\beta}^\sigma R^{\sigma-1}; \quad \text{with: } \bar{\beta} = \beta\,(1+x)^{-(1-q)\rho};\\
g &= r^g y; \qquad e = r^e y; \qquad b = r^b y; \qquad \psi = \frac{1-\omega}{1+n-\gamma};\\
\widetilde{l} &= \widetilde{l}^y + \eta\widetilde{l}^o; \qquad \widetilde{l}^y = 1 - \frac{\varsigma}{w}c^y; \qquad \widetilde{l}^o = \psi - \frac{\varsigma}{w\eta}c^o; \qquad m^{RS} = w.
\end{aligned}
$$

Note that the three exogenous fiscal policy variables g, e, b are set according to their (constant) stationary ratios: $\frac{g}{y} = r^g$; $\frac{e}{y} = r^e$; $\frac{b}{y} = r^b$.