Tail Risk and Expectations

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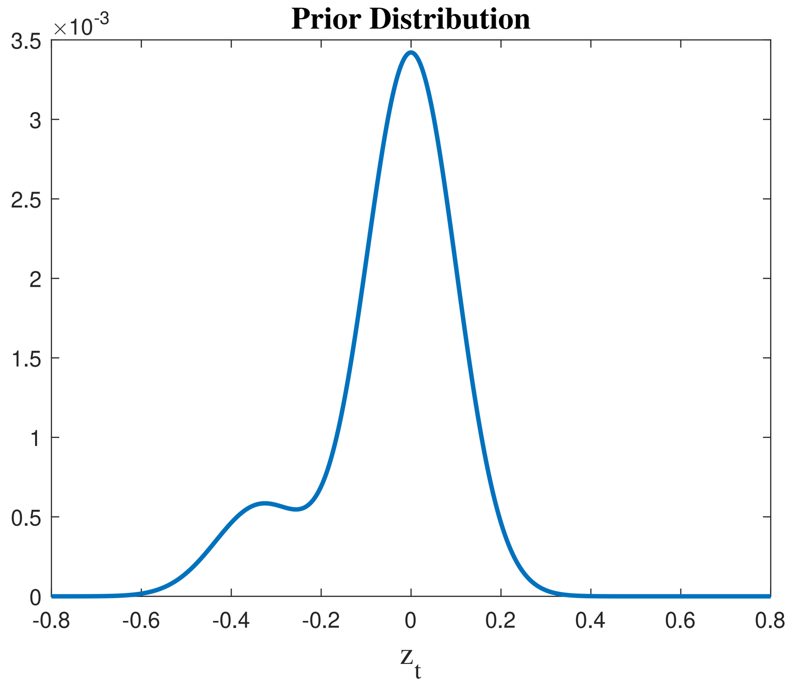
Motivation and Research Question

- Tail Risk refers to the probability of extreme negative losses.
  - Examples of Tail Risk events: 2008 Financial Crisis, Covid-19 Recessions.
  - Relationship to Expectations: Endogenously generate persistent changes in beliefs (Kozlowski et al. (2019)).

Figure: Distribution with Tail Risk



Research Question:

- We incorporate tail risk in a Bayesian learning framework and study how first and second moment shocks affect expectations:

Our Findings:

- **Result 1:** Individuals overreact under tail risk as compared to framework without tail risk.
  - Individuals are excessively optimistic and pessimistic compared to a Bayesian learning framework without tail risk.
- **Result 2:** An increase in uncertainty $\implies$ more pessimistic forecasts.
- **Result 3:** Magnitude of overreaction depends on uncertainty.
  - Higher uncertainty $\implies$ individuals overreact more.

Model Set-up

1. *Period 0.* Each forecaster starts with a belief about the hidden state Z_t. This is denoted by the prior distribution of Z_t: $f(z_t)$.
2. *Period 1.* Each forecaster then observes a signal $s^z_{i,t}$. The signal $s^z_{i,t}$ is given by:

$$s^z_{i,t} = z_t + e_{i,t} \tag{1}$$

where $e_{i,t} \sim N(0, \sigma^{e2}_t)$.
Based on their signals received, they update their beliefs about the hidden state Z_t. This is denoted by the posterior distribution of Z_t: $f(z_t|s^z_{i,t})$.

A. The Benchmark Model.

In the benchmark model, Z_t follows the following process:

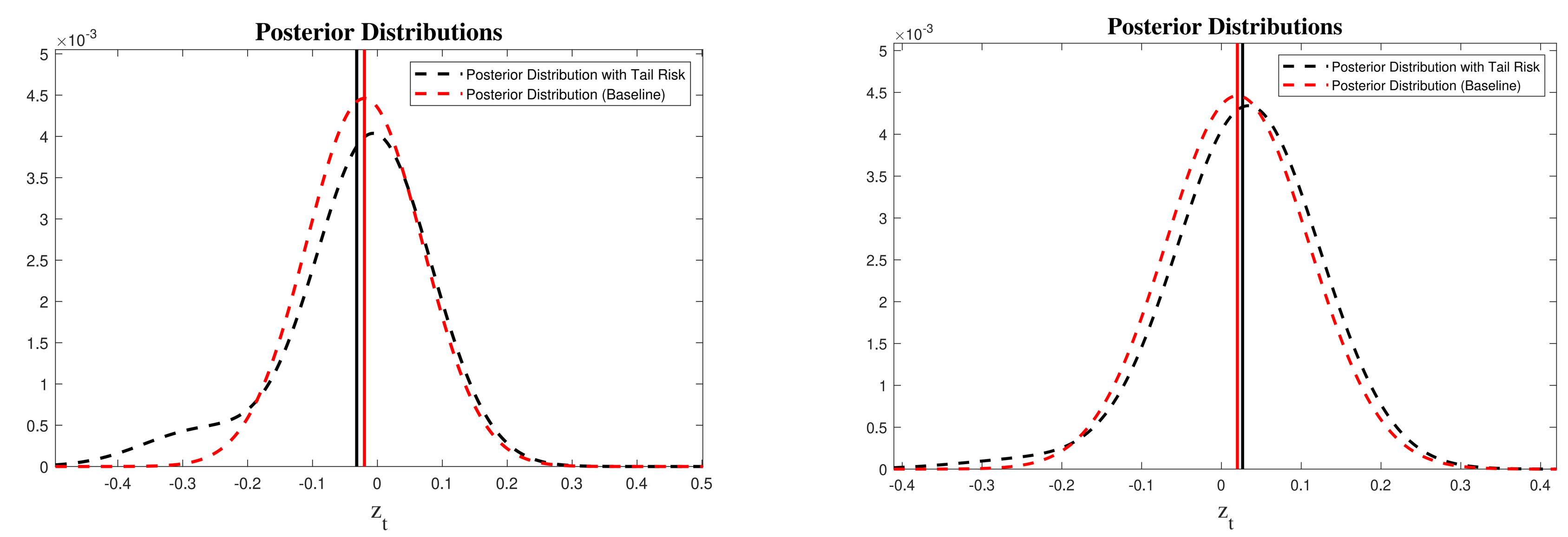
$$\log Z_t = u_t \tag{2}$$

B. The Tail Risk Model.

$$\log Z_t = -I_t\gamma + u_t \tag{3}$$

where I_t is an indicator that equals 1 if the economy is in a disaster state, and equals 0 if the economy is not in a disaster state. Hence, I_t follows a Bernoulli distribution with $Pr(I_t = 1) = p$.

Result 1: Forecasters Overreact to First Moment Shocks



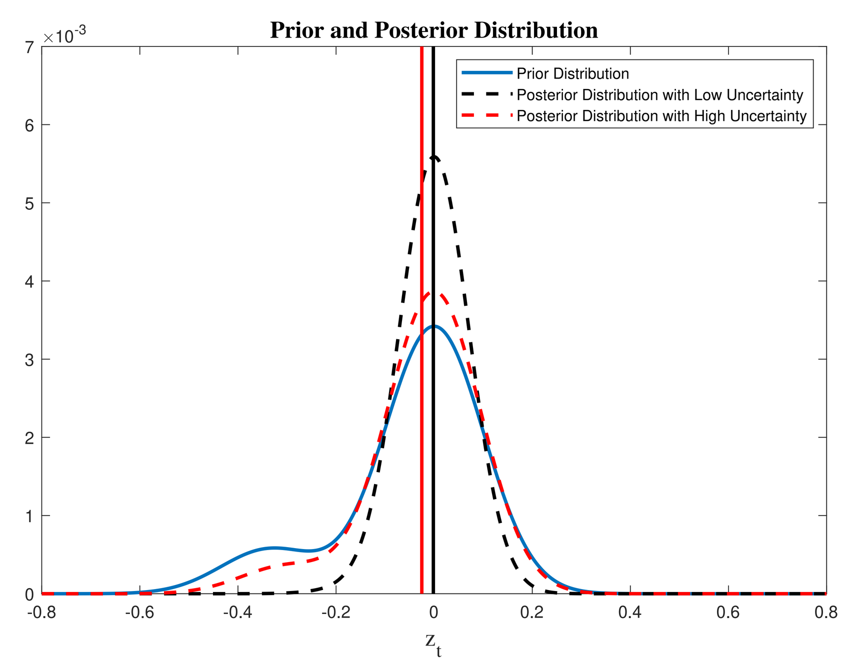
(a) Negative TFP shock (b) Positive TFP shock

Notes: This figure presents the impact of a negative and positive hidden state shock to posterior expectations in panels (a) and (b) respectively. Vertical lines are posterior expectations.

Individuals overreact when there is a first moment shock, that is, individuals are excessively optimistic and pessimistic compared to a Bayesian learning framework without tail risk. When forecasters receive a bad signal, they cannot tell if the bad signal is due to the disaster shock. As such, they overweigh the bad signal to the disaster shock and become overly pessimistic. On the other hand, when forecasters receive a favorable signal, they are confident of the absence of disaster risk. As such, they underweigh the disaster shock and become overly optimistic.

Result 2: Second Moment Shocks Lead To More Pessimism

Figure: Relationship between uncertainty shocks and posterior expectations for median forecaster



Notes: This figure presents the posterior distribution of the forecaster's beliefs about the hidden state with low and high uncertainty by decreasing (increasing) the standard deviation by half its original value. Vertical lines are posterior expectations.

A positive shock to uncertainty (defined as the variance of noise in the signal) leads to a decrease in posterior expectations (defined as expectations of the hidden state conditional on signals received) in a model with tail risk. This implies more pessimistic forecasts in an environment with higher uncertainty. When there is an increase in the variance of noise in the signal, due to downside risk, forecasters perceive an increase in the probability of a low hidden state, even though they receive a neutral signal. Consequently, forecasters attribute a higher weight to disaster risk. Hence, this lowers their expectations of the hidden state. In comparison, in the absence of tail risks, uncertainty shocks do not influence posterior expectations as there is no disaster risk.

Result 3: First and Second Moment Shocks: Larger Overreaction

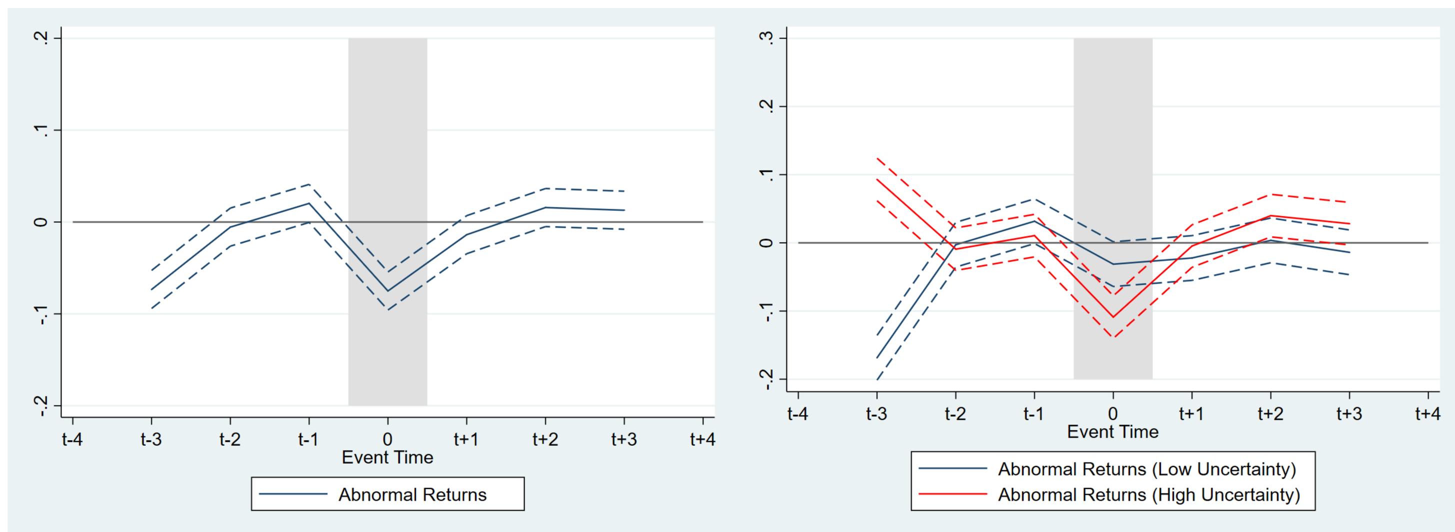
- Combination of results 1 and 2.

Empirical Validation

- Time period: 1978Q1 to 2016Q4

  Data:
- VIX index: To measure uncertainty.
- University of Michigan's Consumer Sentiment Index: To measure expectations.
- Survey of Professional Forecasters: To measure expectations and overreaction behavior (Following Bordalo et al. (2020))
  - Negative coefficient $\implies$ overreaction behavior.
- US Bureau of Economic Analysis: To measure tail risk and GDP growth distribution (Adrian et al. (2019)).
  - Tail risk measure defined as difference between median and 5th percentile.
  - Tail risk episodes: Measure of tail risk exceeding 75th percentile.

Validation of Result 1 and 3: Overreaction in Tail Risk Episodes



(a) Overreaction in Tail Risk Episodes (b) Larger Overreaction with Higher Uncertainty

- Result 1: Coefficient in Bordalo et al. (2020) is more negative in a tail risk episode (shaded grey area) $\implies$ larger overreaction in a tail risk episode.
- Result 3: Coefficient in Bordalo et al. (2020) is more negative with higher uncertainty in a tail risk episode. Larger overreaction when uncertainty is higher.

Validation of Result 2: Uncertainty Shocks Decrease Expectations

Table: Regression Results between Expectations and Uncertainty

Dependent Variable:	$\log S_t$ (1)	$\log S_t$ (2)	$Y_{t+1\|t}$ (3)	$Y_{t+1\|t}$ (4)
$\log VIX_t$	-0.106**	0.034	-5.30***	-0.923
	(0.045)	(0.041)	(1.71)	(1.13)
TR_t		0.305***		12.38***
		(0.071)		(3.85)
$\log VIX_t \times TR_t$		-0.279***		-10.19***
		(0.058)		(2.96)
Observations	156	156	156	156
R^2	0.04	0.25	0.08	0.17

Notes: Newey-West standard errors with a lag length of 4 quarters are in parenthesis. *, ** and *** denotes significance level at 10%, 5% and 1% respectively.

- Result 2: A rise in uncertainty decreases expectations in a tail risk episode.

Conclusion

This paper studies how individuals react under tail risk. First, we show that individuals overreact under tail risk. Second, under tail risk, uncertainty shocks lead to more pessimistic expectations. Third, we find that the magnitude of overreaction under tail risk depends on the level of uncertainty in the economy. Our findings shed light on factors driving overreaction in expectations and highlight the importance of uncertainty shocks in propagating macroeconomic stability.