Nonparametric Bounds on Treatment Effects with Imperfect Instruments

Kyunghoon Ban and Désiré Kédagni

Department of Economics
Iowa State University

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  - Relevance: $Cov(D, Z) \neq 0$;
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- Parametric Example: $Y = D\theta + U$
  - Relevance: $Cov(D, Z) \neq 0$;
  - Exogeneity: $Cov(Z, U) = 0$ for an instrument Z.
- Exogeneity could be difficult to justify.

Key Contributions

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2. Building bridges between SDC and the literature

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   - **SDC**: Same direction of correlation
     - $Cov(D, U)Cov(Z, U) \geq 0$
   - **LEI**: Less endogenous instrument
     - $|Corr(Z, U)| \leq |Corr(D, U)|$
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   - **LEI**: Less endogenous instrument
     - $|Corr(Z, U)| \leq |Corr(D, U)|$
2. Building bridges between SDC and the literature
   - **MTS-MIV**: Monotone treatment selection and monotone IV (Manski and Pepper, 2000, 2009)
   - **Comonotone IV**: Comonotonicity between D and Z

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Potential Outcome Model (POM)

$$Y = \sum_{d=1}^{T} Y_d \mathbb{1}\{D = d\} \tag{1}$$

- Y: an outcome variable taking values in $\mathcal{Y} \subset \mathbb{R}$
- D: a discrete endogenous treatment variable taking values in $\mathcal{D} = \{1, 2, \ldots, T\}$
- Y_d: a potential outcome that would have been observed if the treatment D had externally been set to d
- Z: an imperfect IV (IIV) in the sense that it may be correlated with the potential outcome Y_d with $Z \in \mathcal{Z} \subseteq \mathbb{R}^+$
  - Leading example: Parental education (Kédagni and Mourifié, 2020; Mourifié et al., 2020)

Potential Outcome Model (POM)

$$Y = \sum_{d=1}^{T} Y_d \mathbb{1}\{D = d\} \tag{1}$$

- The objects of interest:
  - Potential outcome means $\theta_d \equiv \mathbb{E}[Y_d] < \infty$
- Average treatment effects:
  - $ATE(d, d') \equiv \theta_d - \theta_{d'}$, for $d, d' \in \mathcal{D}$
  - $ATE(d, d')$ may vary across (d, d')
- Average treatment effect on the treated:
  - $ATT(d, d') \equiv \mathbb{E}[Y_d - Y_{d'} | D = d]$
- Average treatment effect on the untreated:
  - $ATU(d, d') \equiv \mathbb{E}[Y_d - Y_{d'} | D = d']$

Identifying Assumptions

Assumption 1 (Bounded Support (BoS))

$$Supp(Y_d|D \neq d) = Supp(Y_d|D = d) = \left[\underline{y}_d, \overline{y}_d\right]$$

- The support of the counterfactual outcome is the same as that of the factual
- It is standard and similar to the usual bounded outcome assumption considered in Manski (1990, 1994)
- It does not require the support of the potential outcome Y_d to be uniform across all treatment levels d

Identifying Assumptions

Assumption 2 (Same direction of correlation (SDC))

$$Cov\,(Y_d, D)\, Cov\,(Y_d, Z) \geq 0$$

- It is equivalent to Assumption 3 in Nevo and Rosen (2012)
- The correlation between the imperfect instrument Z and the potential outcome Y_d has weakly the same sign as the correlation between the endogenous treatment D and the potential outcome
- If either the treatment D or the instrument Z is exogenous, then SDC holds
- If $D = Z$, then SDC trivially holds

Bridges to SDC

- SDC is a weaker version of the concepts of MTS-MIV (Manski and Pepper, 2000, 2009)
    - MTS: Monotone treatment selection ($\mathbb{E}[Y_d | D = \ell]$ is monotone in ℓ)
    - MIV: Monotone IV ($\mathbb{E}[Y_d | Z = z]$ is monotone in z)
- SDC is weaker than a comonotonicity between Z and D (CoMIV)
- MTS-MIV and CoMIV are two different sufficient conditions for SDC, but neither implies the other

Binarized MTS-MIV

- In order to establish a connection between MTS-MIV and SDC, we introduce an intermediate concept

Definition 1 (Binarized MTS-MIV)

The variable Z is a binarized MTS-MIV for D if for each $d \in \mathcal{D}$,

$$\left(g_d^+(j) - g_d^-(j)\right)\left(h_d^+(z) - h_d^-(z)\right) \geq 0 \quad \textit{for all } j,\ z.$$

where $g_d^+(j) = \mathbb{E}[Y_d|D \geq j]$, $g_d^-(j) = \mathbb{E}[Y_d|D < j]$, $h_d^+(z) = \mathbb{E}[Y_d|Z \geq z]$, and $h_d^-(z) = \mathbb{E}[Y_d|Z < z]$.

Binarized MTS-MIV

Lemma 1

MTS-MIV in the same direction for D and Z implies that Z is a binarized MTS-MIV for D.

Lemma 2

If Z is a binarized MTS-MIV for D, then Assumption SDC holds.

Binarized MTS-MIV

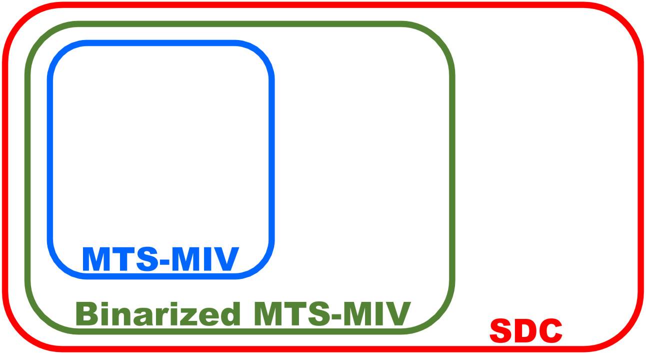

Figure: Illustration of Lemmas 1 and 2

CoMIV

Definition 2 (Comonotonicity)

Let $(\Omega, \mathcal{F})$ be a measurable space. Two random variables X_1 and X_2 defined on Ω are said to be comonotonic if

$$(X_1(\omega) - X_1(\omega'))\,(X_2(\omega) - X_2(\omega')) \geq 0 \quad \textit{for all} \quad \omega, \omega' \in \Omega.$$

Definition 3 (Comonotone instrumental variable (CoMIV))

The variable Z is said to be a comonotone instrumental variable (CoMIV) for the treatment D if Z and D are comonotonic.

CoMIV

Lemma 3

The following results hold.

1. *If D is a deterministic increasing function of Z (or vice versa), then Z is a CoMIV for D.*
2. *Suppose $D = h(Z, V)$, where h is increasing in both of its arguments, and V represents unobserved heterogeneity. If Z and V are comonotonic, then Z is a CoMIV for D.*

- For example, when $D = 2Z + V$ and $Z = e^V$, Z is a CoMIV for D

Lemma 4

If Z is a CoMIV for D, then Assumption SDC holds.

CoMIV

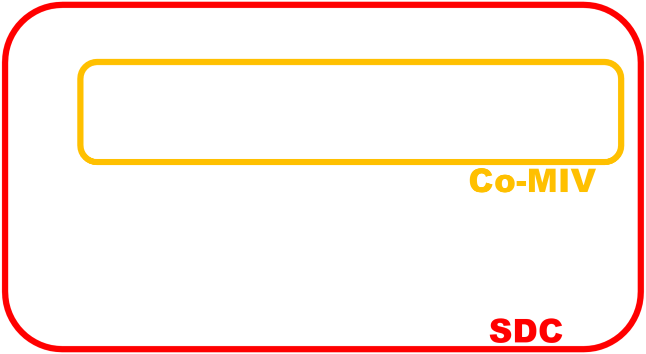

Figure: Illustration of Lemma 4

Example: Not MTS-MIV, but SDC

Example 1

Consider the following data generating process (DGP)

$$\begin{cases} Y & = & 2D + U \\ D & = & 0 \cdot \mathbb{1}\{V \in [0,1]\} + 1 \cdot \mathbb{1}\left\{V \in (1, \frac{3}{2}]\right\} + 2 \cdot \mathbb{1}\left\{V \in (\frac{3}{2}, 5]\right\} \\ Z & = & 2D \\ U & = & 4V\mathbb{1}\{V \in [1,2]\} + V\mathbb{1}\{V \notin [1,2]\} \end{cases}$$

where $V \sim \mathcal{U}_{[0,5]}$.

- The DGP does not satisfy MTS-MIV
- The DGP satisfies binarized MTS-MIV
  - Thus, the DGP satisfies SDC by Lemma 1
- The DGP also satisfies CoMIV
  - Thus, the DGP satisfies SDC by Lemma 4

Example: Not MTS-MIV, but SDC

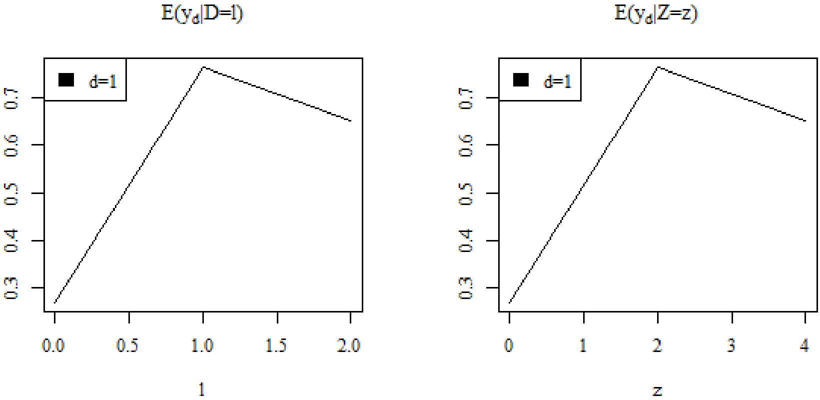

Figure: Numerical illustration of a violation of MTS and MIV

Example: Not MTS-MIV, but SDC

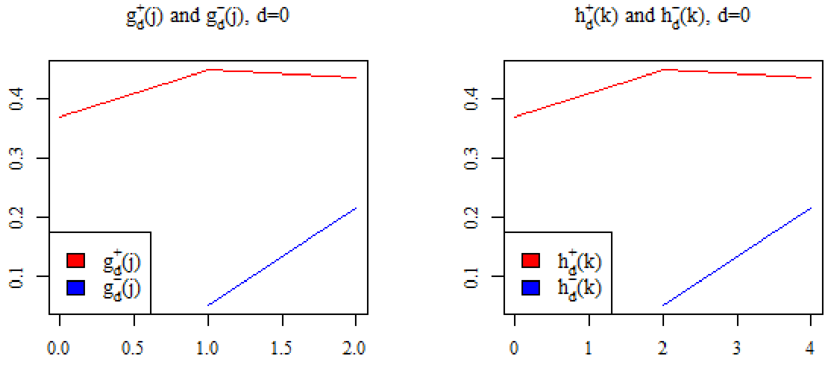

Figure: Numerical illustration of binarized MTS-MIV 1

Example: Not MTS-MIV, but SDC

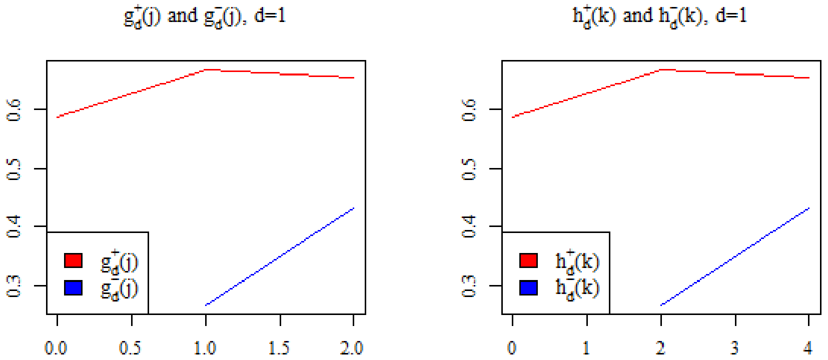

Figure: Numerical illustration of binarized MTS-MIV 2

Example: Not MTS-MIV, but SDC

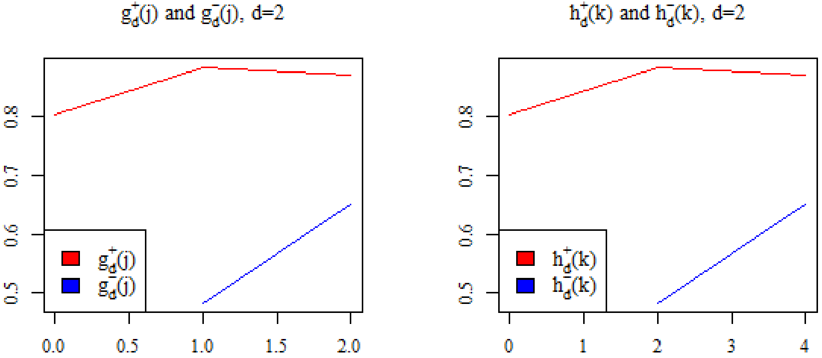

Figure: Numerical illustration of binarized MTS-MIV 3

Relationship between the Assumptions

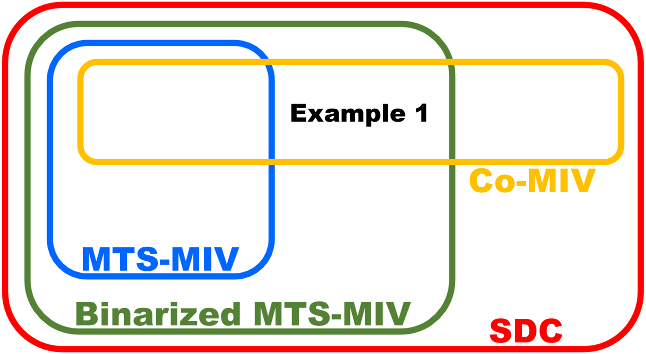

Figure: Illustration of Example 1 and Assumptions

Relationship between the Assumptions

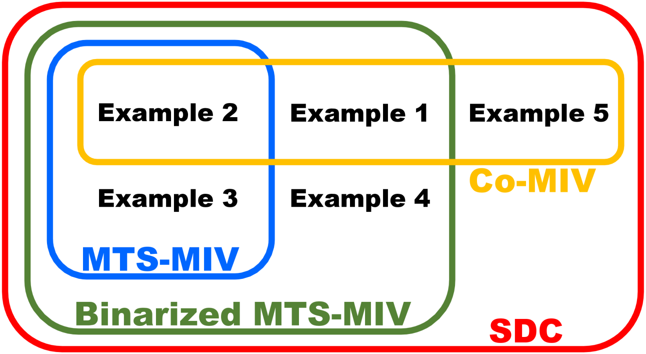

Figure: Illustration of Supplementary Examples

Identifying Assumptions

Assumption 3 (Less endogenous instrument (LEI))

$$| \rho_{Y_d D} | \geq | \rho_{Y_d Z} |$$

where ρ_{UV} denotes the coefficient of correlation between two random variables U and V.

- It is the same assumption as Assumption 4 in Nevo and Rosen (2012)
- The imperfect instrument Z is less correlated with the potential outcome than is the endogenous treatment D
- In the context of our empirical example, it reasonable to assume that parental education is less correlated with the individual's potential wage than is the individual's own education

Identifying Assumptions

Assumption 4 (Monotone treatment response (MTR))

$$Y_d \geq Y_{d'} \quad \textit{for all} \quad d > d'.$$

- The potential outcome weakly increases with the level of the treatment (Manski, 1997)
- In the returns to schooling example, it implies that the wage that a worker earns weakly increases as a function of the worker's years of schooling.

Identifying Assumptions

Assumption 5 (Roy Selection (RS))

$$\{D = d\} \iff \{Y_d > Y_{d'} \quad \textit{for all} \quad d' \neq d\}$$

- Agents choose the level of treatment that maximizes their potential outcome (Roy, 1951)
  - This version implicitly assumes that agents have perfect foresight
- Note that Assumption RS is not compatible with the MTS and MTR assumptions, while Assumption SDC is

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Nevo and Rosen (2012)'s Approach

- Consider the simple linear model:

$$Y = \theta D + U$$

- From the model, we have

$$\theta^{OLS} = \frac{Cov(Y, D)}{Var(D)} = \theta + \frac{Cov(D, U)}{Var(D)}$$
$$\theta^{IV} = \frac{Cov(Y, Z)}{Cov(D, Z)} = \theta + \frac{Cov(Z, U)}{Cov(D, Z)}$$

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- From the model, we have

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$$\theta^{IV} = \frac{Cov(Y,Z)}{Cov(D,Z)} = \theta + \frac{Cov(Z,U)}{Cov(D,Z)}$$

- Under SDC,
$Cov(D,U) \geq 0,\ Cov(Z,U) \geq 0$ **or** $Cov(D,U) \leq 0,\ Cov(Z,U) \leq 0$

$$\Longrightarrow \quad \theta^{IV} \leq \theta \leq \theta^{OLS} \text{ or } \theta^{OLS} \leq \theta \leq \theta^{IV} \quad (\text{if } \rho_{DZ} < 0)$$
$$\theta \leq \min\{\theta^{OLS}, \theta^{IV}\} \text{ or } \max\{\theta^{OLS}, \theta^{IV}\} \leq \theta \quad (\text{if } \rho_{DZ} > 0)$$

Identification under SDC

- Assumption SDC is equivalent to

$$\mathbb{E}\left[Y_d \tilde{D}\right] \mathbb{E}\left[Y_d \tilde{Z}\right] \geq 0$$

where $\tilde{D} \equiv D - \mathbb{E}[D]$ and $\tilde{Z} \equiv Z - \mathbb{E}[Z]$

- That is equivalent to: either

$$\mathbb{E}\left[Y_d \tilde{D}\right] \geq 0, \quad (2)$$

$$\mathbb{E}\left[Y_d \tilde{Z}\right] \geq 0, \quad (3)$$

or

$$\mathbb{E}\left[Y_d \tilde{D}\right] \leq 0, \quad (4)$$

$$\mathbb{E}\left[Y_d \tilde{Z}\right] \leq 0. \quad (5)$$

Identification under SDC

- Inequality (2) implies that, for any $\alpha \in [0, 1)$, we have the following inequalities

$$\mathbb{E}\left[Y_d \alpha \tilde{D}\right] \geq 0 \text{ and } \mathbb{E}\left[-Y_d \alpha \tilde{D}\right] \leq 0,$$

- They are equivalent to

$$\mathbb{E}\left[Y_d\left(1+\alpha\tilde{D}\right)\right] \geq \mathbb{E}[Y_d] \equiv \theta_d \text{ and } \mathbb{E}\left[Y_d\left(1-\alpha\tilde{D}\right)\right] \leq \mathbb{E}[Y_d] \equiv \theta_d,$$

which we rewrite using the identity $\mathbb{1}\{D = d\} + \mathbb{1}\{D \neq d\} = 1$ as

$$\mathbb{E}\left[Y\left(1+\alpha\tilde{D}\right)\mathbb{1}\{D=d\} + Y_d\left(1+\alpha\tilde{D}\right)\mathbb{1}\{D \neq d\}\right] \geq \theta_d, \quad (6)$$

$$\mathbb{E}\left[Y\left(1-\alpha\tilde{D}\right)\mathbb{1}\{D=d\} + Y_d\left(1-\alpha\tilde{D}\right)\mathbb{1}\{D \neq d\}\right] \leq \theta_d, \quad (7)$$

respectively, given that $Y = Y_d$ when $D = d$.

Identification under SDC

- Using Assumption BoS, we can bound the counterfactuals (second terms of (6) and (7)) as follows:

$$
\begin{aligned}
Y_d\left(1+\alpha\tilde{D}\right)\mathbb{1}\{D \neq d\} &\leq \\
&\max\left\{\underline{y}_d\left(1+\alpha\tilde{D}\right), \overline{y}_d\left(1+\alpha\tilde{D}\right)\right\}\mathbb{1}\{D \neq d\} \\
Y_d\left(1-\alpha\tilde{D}\right)\mathbb{1}\{D \neq d\} &\geq \\
&\min\left\{\underline{y}_d\left(1-\alpha\tilde{D}\right), \overline{y}_d\left(1-\alpha\tilde{D}\right)\right\}\mathbb{1}\{D \neq d\}.
\end{aligned}
$$

Identification under SDC

- Therefore, the inequalities (6) and (7) imply that

$$\mathbb{E}\left[\overline{f}_d\left(Y, D, 1+\alpha\tilde{D}\right)\right] \geq \theta_d \text{ and } \mathbb{E}\left[\underline{f}_d\left(Y, D, 1-\alpha\tilde{D}\right)\right] \leq \theta_d$$

for any $\alpha \in [0, 1)$, where we define the function $\underline{f}_d$ and $\overline{f}_d$ as

$$\begin{aligned} \underline{f}_d(Y, D, \delta) &\equiv \mathbb{1}\{D = d\}\delta Y + \mathbb{1}\{D \neq d\}\min\left\{\delta \underline{y}_d, \delta \overline{y}_d\right\} \\ \overline{f}_d(Y, D, \delta) &\equiv \mathbb{1}\{D = d\}\delta Y + \mathbb{1}\{D \neq d\}\max\left\{\delta \underline{y}_d, \delta \overline{y}_d\right\}. \end{aligned}$$

Identification under SDC

- Finally, we can then take the supremum and the infimum of the lower and upper bounds over α, respectively, to obtain the following bounds for θ_d:

$$I^d_{SDC1} \equiv \left[\sup_{\alpha \in [0,1)} \mathbb{E}\left[\underline{f}_d \left(Y, D, 1 - \alpha \tilde{D} \right) \right], \inf_{\alpha \in [0,1)} \mathbb{E}\left[\overline{f}_d \left(Y, D, 1 + \alpha \tilde{D} \right) \right] \right].$$

which is implied by the inequality (2)

Identification under SDC

- In the same manner, from (3), (4), and (5), we have

$$
\begin{aligned}
I_{SDC2}^{d} &\equiv \left[\sup_{\alpha\in[0,1)} \mathbb{E}\left[\underline{f}_d\left(Y,D,1-\alpha\tilde{Z}\right)\right], \inf_{\alpha\in[0,1)} \mathbb{E}\left[\overline{f}_d\left(Y,D,1+\alpha\tilde{Z}\right)\right]\right], \\
I_{SDC3}^{d} &\equiv \left[\sup_{\alpha\in[0,1)} \mathbb{E}\left[\underline{f}_d\left(Y,D,1+\alpha\tilde{D}\right)\right], \inf_{\alpha\in[0,1)} \mathbb{E}\left[\overline{f}_d\left(Y,D,1-\alpha\tilde{D}\right)\right]\right], \\
I_{SDC4}^{d} &\equiv \left[\sup_{\alpha\in[0,1)} \mathbb{E}\left[\underline{f}_d\left(Y,D,1+\alpha\tilde{Z}\right)\right], \inf_{\alpha\in[0,1)} \mathbb{E}\left[\overline{f}_d\left(Y,D,1-\alpha\tilde{Z}\right)\right]\right],
\end{aligned}
$$

Identification under SDC

Proposition 1

Under Assumptions BoS and SDC, the identification region for the parameter θ_d is:

$$I^d_{SDC} \equiv \left(I^d_{SDC1} \cap I^d_{SDC2} \right) \cup \left(I^d_{SDC3} \cap I^d_{SDC4} \right).$$

- We relax the parametric linear assumption at the expense of the bounded support assumption
- The bounds derived in Proposition 1 may not be sharp

Identification under SDC and LEI

Proposition 2

Under Assumptions BoS, SDC and LEI, the identification region for θ_d *is:*

$$I_{LEI}^d \equiv \left(I_{LEI1}^d \cap I_{SDC2}^d\right) \cup \left(I_{LEI2}^d \cap I_{SDC4}^d\right).$$

where

$$I_{LEI1}^d \equiv \Bigg[\sup_{\alpha \in [0,1)} \mathbb{E}\left[\underline{f}_d\left(Y, D, 1 - \alpha\left(\tilde{D}\sigma_Z - \tilde{Z}\sigma_D\right)\right)\right], \inf_{\alpha \in [0,1)} \mathbb{E}\left[\overline{f}_d\left(Y, D, 1 + \alpha\left(\tilde{D}\sigma_Z - \tilde{Z}\sigma_D\right)\right)\right]\Bigg]$$

$$I_{LEI2}^d \equiv \Bigg[\sup_{\alpha \in [0,1)} \mathbb{E}\left[\underline{f}_d\left(Y, D, 1 + \alpha\left(\tilde{D}\sigma_Z - \tilde{Z}\sigma_D\right)\right)\right], \inf_{\alpha \in [0,1)} \mathbb{E}\left[\overline{f}_d\left(Y, D, 1 - \alpha\left(\tilde{D}\sigma_Z - \tilde{Z}\sigma_D\right)\right)\right]\Bigg]$$

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Inference of the set I^d_{SDC}

$$I^d_{SDC} = \left(I^d_{SDC1} \cap I^d_{SDC2}\right) \cup \left(I^d_{SDC3} \cap I^d_{SDC4}\right)$$

- This is an intersection-union test as described in Berger (1982)

1. Construct confidence regions for the sets $I^d_{SDC1} \cap I^d_{SDC2}$ and $I^d_{SDC3} \cap I^d_{SDC4}$ using the intersection bounds framework of Chernozhukov et al. (2013) or Andrews and Shi (2013)
2. Take the union of the two confidence regions, which has at least the same coverage rate as each confidence region (Berger and Hsu, 1996)

Inference of the set I^d_{SDC}

- If we draw U from the uniform distribution over $[0,1)$, independently of the data (Y,D,Z), then we have

$$\mathbb{E}\left[\underline{f}_d\left(Y,D,1-U\tilde{D}\right)\Big|U=\alpha\right]=\mathbb{E}\left[\underline{f}_d\left(Y,D,1-\alpha\tilde{D}\right)\right]$$

- Then, for instance, we have

$$I^d_{SDC1}=\left[\sup_{\alpha\in[0,1)}\mathbb{E}\left[\underline{f}_d\left(Y,D,1-U\tilde{D}\right)\Big|U=\alpha\right],\right.$$
$$\left.\inf_{\alpha\in[0,1)}\mathbb{E}\left[\overline{f}_d\left(Y,D,1+U\tilde{D}\right)\Big|U=\alpha\right]\right]$$

which takes the from of conditional moment inequalities

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Data

- A data set drawn from the National Longitudinal Survey of Young Men (NLSYM)
  - This data includes 3,010 young men who were ages 24-34 in 1976
- The outcome variable (Y) is log hourly wage in cents (*lwage*)
- The treatment variable (D) is education (*educ*) grouped in 4 categories:
  1. less than high school ($educ < 12$ *years*)
  2. high school ($12 \leq educ < 16$)
  3. college degree ($16 \leq educ < 18$)
  4. graduate ($educ \geq 18$)
- Imperfect IV (Z) is parental education
  - An individual's ability can be dependent on her parents' ability, which is correlated with parental education

Estimation

- For practical reasons, we follow Ginther (2000) to trim the log wage
  - In theory, the outcome variable *lwage* is unbounded
  - $Y = \tau$-quantile of *lwage* if *lwage* is less than or equal to its τ-quantile
  - $Y = (1-\tau)$-quantile of *lwage* if *lwage* is greater than or equal to its $(1-\tau)$-quantile
  - $Y =$ *lwage* otherwise
  - We set $\tau = 0.05$
- Two-sided confidence bounds on the potential average log wages using the *clr2bound* command of Chernozhukov et al. (2015) in the Stata software
- The results with mother's education as an IIV are presented

Estimated Confidence Intervals

Table: Confidence sets for parameters under SDC

Parameters	95% conf. LB	95% conf. UB
θ_0 ($<$ high)	5.53	6.86
θ_1 (high)	5.89	6.66
θ_2 (college)	5.65	6.88
θ_3 (graduate)	5.55	6.94
$\theta_0 - \theta_1$	-1.13	0.97
$\theta_2 - \theta_1$	-1.01	0.98
$\theta_3 - \theta_1$	-1.11	1.05

* conf. LB: confidence lower bound; conf. UB: confidence upper bound.

Estimated Confidence Intervals

Table: Confidence sets for parameters under SDC and LEI

Parameters	95% conf. LB	95% conf. UB
θ_0 ($<$ high)	5.53	6.86
θ_1 (high)	5.89	6.66
θ_2 (college)	5.65	6.86
θ_3 (graduate)	5.55	6.94
$\theta_0 - \theta_1$	-1.13	0.97
$\theta_2 - \theta_1$	-1.01	0.97
$\theta_3 - \theta_1$	-1.11	1.05

* conf. LB: confidence lower bound; conf. UB: confidence upper bound.

Estimated Confidence Intervals

Table: Confidence sets for parameters under SDC, LEI, and MTR

Parameters	95% conf. LB	95% conf. UB
θ_0 (< high)	5.53	6.30
θ_1 (high)	6.30	6.46
θ_2 (college)	6.46	6.82
θ_3 (graduate)	6.82	6.94
$\theta_0 - \theta_1$	-0.93	0.00
$\theta_2 - \theta_1$	0.00	0.52
$\theta_3 - \theta_1$	0.36	0.64

* conf. LB: confidence lower bound; conf. UB: confidence upper bound.

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Conclusion

- Non-parametric bounds on the average treatment effect are derived when an imperfect instrument is available
  - Nevo and Rosen (2012)'s identification results are extended
- We show that the MTS-MIV restrictions introduced by Manski and Pepper (2000, 2009), jointly imply the SDC assumption
- We introduce the concept of comonotone IV, which also satisfies the SDC assumption
- The identified set takes the form of intersection bounds, which can be implemented using the Chernozhukov et al. (2013) inferential method
- We illustrate our methodology using the National Longitudinal Survey of Young Men data to estimate returns to schooling

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