Revisiting the Optimal Inflation Rate with Downward Nominal Wage Rigidity: The Role of Heterogeneity

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October 30, 2020

Abstract

In this paper, I study the optimal inflation rate in a sticky price economy in which workers are heterogeneous in labor productivity and wage changes are subject to asymmetric adjustment costs. The model calibrated to U.S. micro wage data implies downward nominal wage rigidity (DNWR). The optimal inflation rate is substantially higher than stated in the literature in the presence of worker heterogeneity. A key to understanding the result is that DNWR causes an inefficient cross-sectional allocation of labor as well as inefficient aggregate dynamics, enlarging the "grease the wheels" effect of inflation.

JEL classification: E24; E31; E52.

Keywords: Optimal inflation rate; Downward nominal wage rigidity; Heterogeneous agent model.

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This is a revised version of a chapter of my dissertation at Boston College. I am deeply grateful to Susanto Basu, Pablo A. Guerrón-Quintana, and Dongho Song for their invaluable guidance and support. I thank Kosuke Aoki, Anthony M. Diercks, Nils Gornemann, Adam Guren, Peter Ireland, Pierre De Leo, Kazushige Matsuda, Shin-Ichi Nishiyama, and seminar and conference participants at Boston College, Johns Hopkins Carey Business School, and 25th International Conference CEF at Carleton University for their helpful comments and suggestions. Previous versions of this paper were circulated as "Optimal Monetary Policy Rule in a Heterogeneous Agent Model with Nominal Rigidities" and "Optimal Monetary Policy Rule with Downward Nominal Wage Rigidity: The Role of Heterogeneity." The views expressed herein are those of the author and should not be attributed to the IMF, its Executive Board, or its management.

1 Introduction

There has been a long-lasting debate regarding the optimal inflation rate. It is one of the most fundamental questions in monetary economics in that it involves the costs and benefits of inflation. In the literature, one of the first studies to address the question is Friedman (1969), who argues that the inflation rate should be negative so as to minimize the opportunity cost of holding money. In the wake of the New Keynesian theory around the 1990s, a widely accepted view is that zero inflation is optimal in a cashless economy because both inflation and deflation generate welfare losses through relative price dispersion (e.g., King and Wolman 1999).

In contrast to the early literature that supports the inflation rate at or below zero, many central banks in both advanced and emerging economies have adopted positive inflation targets. Schmitt-Grohé and Uribe (2010) point out that the observed positive inflation targets are puzzling in the light of the conventional wisdom of monetary theories.

One potential justification for positive inflation targets is downward nominal wage rigidity (DNWR).[1] Numerous studies report that nominal wages are more downwardly rigid than upwardly (e.g., Card and Hyslop 1997, Bewley 1999, and Grigsby et al. 2019 more recently). In this regard, Tobin (1972) claims that positive inflation facilitates real wage declines upon a contractionary shock even if nominal wages are downwardly rigid, mitigating the adverse effect of DNWR. However, the magnitude of such "grease the wheels" effect is still controversial. For example, Kim and Ruge-Murcia (2009, 2019) (KRM henceforth)build a representative agent New Keynesian model with DNWR and find that the optimal inflation

[1] Other candidates include the zero lower bound on nominal interest rates (ZLB) and trends in relative prices as well as biases in measuring the inflation rate. More discussion is provided shortly.

rate in the model economy is positive but close to zero.

In this paper, I demonstrate that the implications of DNWR for the optimal inflation rate are substantially altered once worker heterogeneity is taken into account. Specifically, I develop a sticky price model in which workers are heterogeneous in labor productivity. To capture the lumpy and asymmetric wage adjustments observed in the data, I introduce asymmetric wage adjustment costs. The specification nests the cases of absolute DNWR and fully flexible wages. When calibrated to U.S. micro wage data, the model suggests a much larger cost for downward wage adjustments than upward ones, which implies the presence of DNWR.

In the baseline heterogeneous agent (HA) model, I find that the optimal inflation rate is around 2 percent per year. Notably, the optimal inflation rate is substantially higher than in a representative agent (RA) model that is often used by previous studies.[2] The main reason behind the higher optimal inflation rate in the HA model is that individual workers seek to adjust their wages in response to idiosyncratic shocks as well as to aggregate conditions. Consequently, the lack of wage adjustments due to DNWR leads to a sizable welfare loss through an inefficient cross-sectional allocation of labor, whereas such cross-sectional implications of DNWR are abstracted in a RA model. In other words, the benefits of holding positive inflation become larger in the presence of worker heterogeneity.

Regarding the level of the optimal inflation rate obtained in this paper, it is worth noting that the specification of nominal price rigidity is conservative for generating a positive optimal inflation rate. Specifically, I adopt the staggered price setting of Calvo (1983) with

[2] While I consider a simple monetary policy rule as in Taylor (1993), several previous studies including KRM2009 investigate the Ramsey policy, which can potentially lower the optimal inflation rate. However, I verify in the later sections that the difference in terms of the optimal inflation rate between the HA model and the RA model remains under the same monetary policy.

non-zero steady-state inflation as a form of nominal price rigidity. It is known that the welfare cost of inflation is larger in the Calvo (1983) model than in other popular specifications, though there is a substantial debate on the empirical validity of each specification.[3]

This paper is related to a long line of literature on the optimal inflation rate.[4] In particular, a number of studies explore the benefits of positive inflation after the Global Financial Crisis when many advanced economies suffered a severe downturn in a low-inflation environment. Some of these studies focus on DNWR (e.g., KRM2009, KRM2019, Abo-Zaid 2013, Carlsson and Westermark 2016); the ZLB (e.g., Coibion et al. 2012, Andrade et al. 2019); trends in relative prices (e.g., Wolman 2011, Ikeda 2015); and biases in measuring the inflation rate (e.g., Schmitt-Grohé and Uribe 2012). On the other hand, previous studies point out that shifting the steady-state inflation rate into a positive territory generates a considerable welfare loss through nominal price rigidity (e.g., Coibion et al. 2012). In fact, most of the studies mentioned above conclude that a moderate inflation rate or an inflation rate close to zero is optimal as a consequence of the trade-off between the costs and benefits of inflation. The optimal inflation rate in a heterogeneous agent model is a relatively new research field. Previous studies address various dimensions of heterogeneity including firms' productivity growth (Adam and Weber 2019), firms' price setting (Blanco forthcoming), and houholds' asset holdings (Menna and Tirelli 2017). I contribute to this line of the literature by investigating the role of worker heterogeneity in the labor market.

As for the literature on DNWR, several studies including Akerlof et al. (1996), Benigno

[3]For example, Burstein and Hellwig (2008) study the menu cost model, Lombardo and Vestin (2008) the quadratic price adjustment cost model of Rotemberg (1982), and Coibion et al. (2012) the the fixed-duration pricing model of Taylor (1980). All of these studies find that the welfare cost of inflation in the Calvo model is larger than that in other models.

[4]A comprehensive survey on the optimal inflation rate is conducted by Diercks (2017).

and Ricci (2011), and Daly and Hobijn (2014) consider some sort of worker heterogeneity in an economy with DNWR, though quantitative investigation for the optimal inflation rate is not provided.[5] An exception is Fagan and Messina (2009), who study the optimal inflation rate by in a stationary environment with worker heterogeneity. Built upon these previous studies on DNWR, the novelty of this paper is threefold. First, I explicitly compare the optimal inflation rate in a HA model and a RA model to focus on the role of worker heterogeneity. Second, I develop a general setting for individual workers' productivity process and wage adjustment costs, and exploit micro wage data to discipline the model. Indeed, the model setting of this paper is more general than the studies mentioned above. Third, I take the aggregate dynamics into account, and thereby investigate both cross-sectional and time-series dynamics as well as their potential interactions. These features allow the model to quantitatively evaluate the costs and benefits of inflation and therefore the optimal inflation in a rigorous manner.[6]

The remainder of the paper is organized as follows. Section 2 develops the model, and Section 3 describes the computation method and calibration procedure. Section 4 investigates the optimal inflation rate in the calibrated model. Section 5 offers various sensitivity analyses. Section 6 concludes.

[5] For example, Benigno and Ricci (2011) mention the possibility that the optimal inflation becomes positive in the presence of DNWR, though quantitative analysis is not conducted. Wagner (2018), using a model developed by Daly and Hobijn (2014), investigate the transition dynamics under different levels of the steady-state inflation rates.

[6] Compared with the model of Fagan and Messina (2009), I develop a more general setting that accommodates job-changes and different trend productivity growth across workers in order capture the individual workers' wage dynamics observed in the data. Regarding nominal price rigidity, the quadratic price adjustment cost, which Fagan and Messina (2009) employ, has little micro-founded interpretation in a stationary environment. On the other hand, I employ the Calvo-type nominal price rigidity and evaluate the welfare loss arising from the inflation dynamics as well as the steady-state allocation.

2 Model

2.1 Stylized partial equilibrium model

I start by building a stylized partial equilibrium model that embeds DNWR to explore its welfare implications. Specifically, I consider a wage-setting problem in an environment in which (i) each worker stays in the same job forever, (ii) nominal wages are subject to absolute DNWR, and (iii) there are no aggregate shocks. Each of these assumptions is relaxed in a quantitative model developed in Section 2.2. Note that, though the model developed in this section shares many features with that of my companion paper, Mineyama (2020), I add several modifications in Section 2.2 according to the purpose of this paper.

2.1.1 Wage setting with DNWR

There is a continuum of households indexed by j on the unit interval, each of which supplies a differentiated labor service to the production sector:

$$l_t(j) = z_t(j)h_t(j), \tag{1}$$

where $l_t(j)$ is the effective unit of labor and $h_t(j)$ is hours worked. $z_t(j)$ denotes worker-specific labor productivity. The aggregate labor supply takes the Dixit-Stiglitz form $L_t = (\int_0^1 l_t(j)^{(\theta_w-1)/\theta_w} dj)^{\theta_w/(\theta_w-1)}$ where θ_w represents labor demand elasticity. The user of the labor service minimizes the cost of using a certain amount of composite labor inputs, taking each worker's wage as given. The first-order condition (FOC) for the cost minimization problem yields the following individual labor demand function:

$$l_t(j) = \left(\frac{w_t(j)/z_t(j)}{W_t} \right)^{-\theta_w} L_t, \tag{2}$$

where $w_t(j)$ is the wage rate of worker j, and the aggregate wage index W_t satisfies $W_t = \{\int_0^1 (w_t(j)/z_t(j))^{1-\theta_w} dj\}^{1/(1-\theta_w)}$

Each household receives utility from consumption $c_t(j)$ and disutility from hours worked $h_t(j)$. Expected lifetime utility is given by

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s u\big(c_{t+s}(j), h_{t+s}(j)\big) \right], \tag{3}$$

where β is the subjective discount factor. Budget constraint is given by

$$c_t(j) + \frac{b_t(j)}{P_t} \leq \frac{w_t(j)}{P_t} h_t(j) + R_{t-1} \frac{b_{t-1}(j)}{P_t} + \frac{\tau_t(j)}{P_t} + \Phi_t(j), \tag{4}$$

where $b_t(j)$, $\tau_t(j)$, and $\Phi_t(j)$ are the amount of nominal bond holdings, lump-sum transfer, and share of the producer's real profits distributed to household j. P_t is the aggregate price index and R_t is the gross nominal interest rate.

In this stylized model, nominal wages are subject to absolute DNWR:

$$w_t(j) \geq w_{t-1}(j)\ . \tag{5}$$

The assumption is a parsimonious way to represent the empirical fact that nominal wage cuts are rare to occur (e.g., Barattieri et al. 2014); similar assumptions are often used in previous studies (e.g., Benigno and Ricci 2011, Daly and Hobijn 2014). Note, however, that the assumption is relaxed in Section 2.2 by allowing for a more general form of adjustment cost and estimating the relevant parameters according to U.S. micro data.

Each household j maximizes expected lifetime utility (3) by choosing $c_t(j)$, $b_t(j)$, and $w_t(j)$ subject to (4), (2), and (5).[7] The FOC for $w_t(j)$ along with the complementary slackness conditions for (5) yields the following wage rule:

$$\frac{w_t(j)}{P_t} = \max\left\{ \frac{w_t^d(j)}{P_t} \ , \ \frac{1}{\Pi_t^p}\frac{w_{t-1}(j)}{P_{t-1}} \right\}, \tag{6}$$

$$\text{where} \quad \frac{w_t^d(j)}{P_t} = \mu_w z_t(j) \underbrace{\frac{u_{h,t}(j)}{u_{c,t}(j)}}_{mrs_t(j)} - \beta \mathbb{E}_t[\psi_{t+1}(j)] \left(u_{c,t}(j) \frac{\theta_w h_t(j)}{w_t^d(j)} \right)^{-1}. \tag{7}$$

$u_{c,t}(j) = \partial u(c_t(j), h_t(j))/\partial c_t(j)$ and $u_{h,t}(j) = \partial u(c_t(j), h_t(j))/\partial h_t(j)$. $\Pi_t^p = P_t/P_{t-1}$ is the gross price inflation rate, $\mu_w = \theta_w/(\theta_w - 1)$ is the steady-state wage markup that arises from the workers' monopolistic power over their labor service, $mrs_t(j)$ is the marginal rate of substitution (MRS) of hours worked for consumption, and $\psi_t(j) \geq 0$ denotes the Lagrange multiplier for (5). $w_t^d(j)$ denotes the desired wages, which are the wages chosen by workers when DNWR does not bind in the current period. According to (6), the wages take a max function, with the first element corresponding to the case in which DNWR does not bind in the current period and the second element to the case in which it binds.

On the other hand, without the DNWR constraint, $\psi_t(j) = 0$ holds for all j and t. In

[7] Wage is a workers' choice variable under monopolistic competition of labor supply. In other words, wages are posted by firms along with the corresponding labor demand and workers choose the most preferable wage available. In this regard, the literature provides ample evidence on firms' reluctance to offer nominal wage cuts (e.g., Bewley 1999).

that case, the optimality conditions are reduced to

$$\frac{w_t^f(j)}{P_t^f} = \mu_w z_t(j) mrs_t^f(j), \tag{8}$$

where x_t^f denotes the variable x_t under flexible prices and wages. Equation (8) suggests that the flexible wages are determined by labor productivity $z_t(j)$ and the MRS $mrs_t^f(j)$, as well as the steady-state wage markup μ_w.

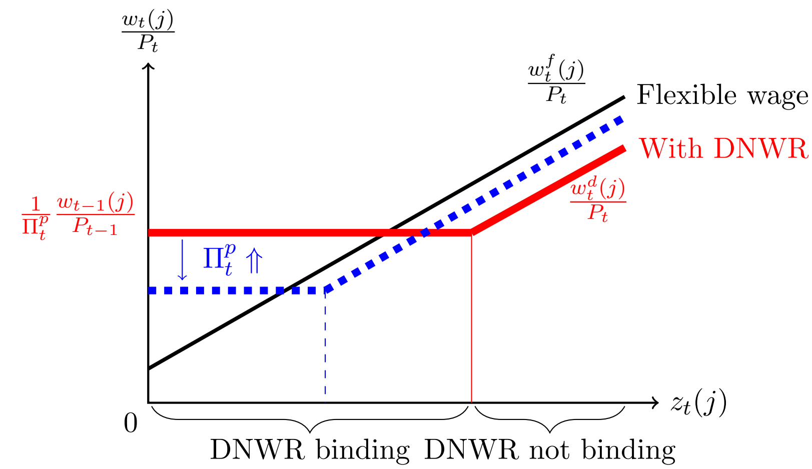
2.1.2 Welfare implications of DNWR

Wage rules (6) and (7) imply that the wages in this economy can deviate from the flexible ones both upwardly and downwardly, as illustrated in Figure 1. The upward deviations are straightforward to see in (6). When DNWR binds in the current period, the actual wages tend to be higher than the flexible wages.[8] On the other hand, downward deviations occur due to a forward-looking effect of DNWR. Specifically, (7) suggests that the desired wages are weakly lower than the flexible wages due to the Lagrange multiplier included in the second term of the right-hand side. Intuitively, a worker internalizes the possibility that the DNWR constraint might bind in future periods, and therefore desires to hold a buffer to prevent future constraints from binding. In other words, the actual wages can be lower than the flexible wages due to the precautionary motives for a future downturn.[9] It is also noteworthy that positive inflation mitigates these effects of DNWR by ensuring room for

[8] Strictly speaking, the actual wages can be lower than the flexible ones even when DNWR binds if the forward-looking effect is strong enough.

[9] It should be noted that the flexible wages are still distorted by the workers' monopolistic power over their labor service, which is represented by the steady-state wage markup μ_w. Hence, the precautionary motives to keep a lower wage provided for a future downturn compress wage markup, which can be welfare-improving. However, in the numerical analysis in the following sections, I find that the welfare-deteriorating effect of DNWR by generating deviations from the flexible wages is quantitatively dominant.

Figure 1: Wage setting with DNWR



real wage declines upon an adverse shock according to (7).

Importantly, the presence of heterogeneity in workers' labor productivity $z_t(j)$ implies that the desired wages are more dispersed than that for a representative worker, leading to larger deviations from the flexible wages at the individual level. This is a potential source of welfare loss. In this regard, the aggregate welfare consequences of DNWR depend on the state of the distribution of individual workers' lagged wages as well as the current labor productivity. Moreover, there can be aggregate feedback through the rest of the economy though this stylized partial equilibrium model is agnostic about them. In the next subsection, therefore, I develop a quantitative model that will be used for assessing welfare consequences of DNWR in the presence of worker heterogeneity.

2.2 Quantitative general equilibrium model

In this subsection, I develop a quantitative general equilibrium model by generalizing the stylized model in Section 2.1. The economy consists of households, monopolistically competitive firms, and a central bank. Households supply differentiated labor service, earn labor income, and make saving-consumption decisions. Importantly, each worker receives individual labor productivity shocks, and wage changes are subject to asymmetric adjustment costs. Firms produce differentiated goods and set prices under the staggered contract á la Calvo (1983). The central bank follows an interest-rate feedback rule of Taylor (1993). Compared to the model of Mineyama (2020), this paper's model employs the Calvo-type nominal price rigidity with non-zero steady-state inflation rates to rigorously evaluate the cost of inflation. It also considers time-variations of job-changes.

2.2.1 Households

Nominal wage changes are subject to asymmetric adjustment costs for positive and negative wage changes. The adjustment costs are composed of a fixed cost and a linear cost that is proportional to the size of wage changes.[10] Note that the specification nests the absolute DNWR in Section 2.1 when the fixed cost for downward wage adjustments is infinitely large and that the costs for upward changes are zero. It is also more general than the one employed by previous studies.[11] In addition, job changes are introduced to capture potential heterogeneity in the degree of DNWR depending on the job-change status. For simplicity, job changes are assumed to occur randomly with a time-varying probability $\delta_t \in [0, 1]$ and job-changers—workers who switch jobs—are free from wage adjustment costs. This assumption

[10]In Online Appendix C, alternative specifications, such as a fixed cost only and a combination of fixed and quadratic costs, are assessed. It is verified that the baseline specification of a combination of fixed and linear costs fits micro evidence on wage adjustments better than alternative ones.

[11]For example, Benigno and Ricci (2011) and Daly and Hobijn (2014) consider absolute DNWR with a random fraction of being free from the constraint, which corresponds to ζ in my model. Fagan and Messina (2009) use a fixed adjustment cost, while Elsby (2009) employs a linear one. Jo (2020) considers five different cases; flexible wage, Calvo wage rigidity, long-term contract, symmetric menu cost, and absolute DNWR. Though my model does not take into account the long-term contract, it nests the other four specifications.

reflects the empirical findings of previous studies, by which wages change frequently when workers move to another job (e.g., Barattieri et al. 2014). Moreover, a random fraction $\zeta \in [0, 1]$ of job-stayers—workers who remain in the same job—are also assumed to be free from wage adjustment costs to capture small wage changes observed in the data.[12] In sum, the wage adjustment costs $m_t(j)$ are given below.

$$m_t(j) = \begin{cases} (m_0^+ + m_1^+ \ln \Pi_t^w(j)) \mathbf{1}_{\{w_t(j) > w_{t-1}(j)\}} + (m_0^- + m_1^- \ln \Pi_t^w(j)) \mathbf{1}_{\{w_t(j) < w_{t-1}(j)\}} \\ \qquad \text{if } s_t(j) = s^1 \qquad \ldots \text{job-stayers with adjustment costs} \\ 0 \qquad \text{if } s_t(j) = s^2 \qquad \ldots \text{job-stayers without adjustment costs} \\ 0 \qquad \text{if } s_t(j) = s^3 \qquad \ldots \text{job-changers} \end{cases}, \quad (9)$$

where $\Pi_t^w(j) = w_t(j)/w_{t-1}(j)$, and m_0^+, m_1^+, m_0^-, and m_1^- are parameters for the adjustment costs. $s_t(j)$ denotes the status regarding job-change and adjustment costs; $s_t(j) = s^1$ for job-stayers with adjustment costs, $s_t(j) = s^2$ for job-stayers without adjustment costs, and $s_t(j) = s^3$ for job-changers. The fraction of workers in each status is $(1-\delta_t)(1-\zeta)$, $(1-\delta_t)\zeta$, and δ_t, respectively. The adjustment costs captures various factors that potentially prevent wage adjustments, including psychological costs and effects on workers' morale for negative ones. Though the adjustment costs enter the households' budget constraint in the baseline model, I consider an alternative setting in which they are rebated back to households so as to eliminate the direct effects of paying these costs in Section 5.

Labor productivity $z_t(j)$ consists of a deterministic growth component $z_{1,t}(j)$ and a

[12] This specification is often used for the same purpose in the price-setting literature (e.g., Vavra 2014).

stochastic one $z_{2,t}(j)$:

$$\ln z_t(j) = \ln z_{1,t}(j) + \ln z_{2,t}(j). \tag{10}$$

Different labor productivity processes are considered for job-stayers and job-changers. For job-stayers, a labor productivity shock is assumed to hit infrequently with a probability $(1-\gamma) \in (0,1)$ along with a deterministic trend growth rate $\mu_z^{st} > 0$:

$$\ln z_{1,t}(j) = \mu_z^{st} + \ln z_{1,t-1}(j), \tag{11}$$

$$\ln z_{2,t}(j) = \begin{cases} \rho_z^{st} \ln z_{2,t-1}(j) + \varepsilon_{z,t}(j),\ \ \varepsilon_{z,t}(j) \sim i.i.d.N(0, \sigma_z^{st2}) & \text{with prob. } 1-\gamma \\ \ln z_{2,t-1}(j) & \text{with prob. } \gamma \end{cases}. \tag{12}$$

The infrequent productivity shock accommodates promotion, performance evaluation conducted periodically, and other occasional events that affect labor productivity. It could replicate fat tails of wage change distribution observed in the data. Similar specifications are often employed in the literature on income risk (e.g., Guvenen et al. 2019) and price setting (e.g. Vavra 2014).

For job-changers, on the other hand, labor productivity follows a random walk with a deterministic growth rate $\mu_z^{ch} > 0$. μ_z^{ch} can differ from μ_z^{st}, and the average growth rate among workers in each period is defined as $g_t = (1-\delta_t)\mu_z^{st} + \delta_t \mu_z^{ch}$. To capture the larger variations of job-changers' wage changes observed in the data, a shock for job-changers is assumed to be drawn from a uniform distribution with a support $U_z^{ch} > 0$. The shock captures a match-specific labor productivity. The process is written as

$$\ln z_{1,t}(j) = \mu_z^{ch} + \ln z_{1,t-1}(j), \tag{13}$$

$$\ln z_{2,t}(j) = \ln z_{2,t-1}(j) + \varepsilon_{z,t}(j),\ \ \varepsilon_{z,t}(j) \sim i.i.d.U[-U_z^{ch}, U_z^{ch}]. \tag{14}$$

The period utility takes the following form:

$$u\left(c_t(j), h_t(j)\right) = \ln c_t(j) - \frac{h_t(j)^{1+1/\eta}}{1+1/\eta}, \tag{15}$$

where η is the Frisch labor supply elasticity. Notice that the log-utility of consumption is consistent with balanced growth. Reflecting the generalization mentioned above, the period budget constraint (4) is modified to

$$c_t(j) + m_t(j)c_t(j) + \frac{b_t(j)}{P_t} \leq \frac{w_t(j)}{P_t}h_t(j) + Q_{t-1}R_{t-1}\frac{b_{t-1}(j)}{P_t} + \frac{\tau_t(j)}{P_t} + \Phi_t(j), \tag{16}$$

where wage adjustment costs are proportional to consumption. Q_t is an exogenous risk premium, which is described shortly.

I assume that households have access to a complete contingent claim market for consumption so that they consume the same amount, although they are still subject to an uninsurable idiosyncratic labor productivity shock.[13] This assumption allows one to focus on the heterogeneity of individual wages and the cross-sectional allocation in the labor market. The

[13]Note that the setting is crucially different from many recent papers that deal with heterogeneity in consumption and asset holdings in an incomplete market, the so-called "HANK" literature (e.g., Kaplan et al. 2018). The assumption is largely due to computational burden to deal with wage and consumption heterogeneity simultaneously, and is commonly used by previous studies that study DNWR with heterogeneous workers (e.g., Fagan and Messina 2009, Benigno and Ricci 2011, Daly and Hobijn 2014, Wagner 2018, Jo 2020). I discuss the welfare implications of relaxing this assumption in Online Appendix C.

FOCs for consumption and nominal bond holdings yield the consumption Euler equation:

$$\beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{Q_t R_t}{\Pi_{t+1}} \right] = 1. \tag{17}$$

The individual wage setting problem is written in a recursive representation:

$$\begin{aligned} V\left(\frac{w_{t-1}(j)}{P_{t-1}}, z_t(j), s_t(j)\right) = \max_{w_t(j)} - \frac{h_t(j)^{1+1/\eta}}{1+1/\eta} + \underbrace{C_t^{-1}}_{u_{c,t}} \left(\frac{w_t(j)}{P_t} h_t(j) - m_t(j) C_t\right) \\ + \frac{\beta}{g} \mathbb{E}_t \left[V\left(\frac{w_t(j)}{P_t}, z_{t+1}(j), s_{t+1}(j)\right)\right], \end{aligned} \tag{18}$$

subject to (1), (2), (9), and (10)–(14), given all of the aggregate variables. Note that the wage setting problem is separable from consumption choice conditional on $u_{c,t}$ due to additive separability in preference.

2.2.2 Firms

There is a continuum of firms indexed by i on the unit interval. Firm i produces a differentiated good using a linear production technology:

$$y_t(i) = l_t(i), \tag{19}$$

using composite labor inputs $l_t(i) = (\int_0^1 l_t(i,j)^{(\theta_w - 1)/\theta_w} dj)^{\theta_w/(\theta_w - 1)}$ where $l(i,j)$ denotes the labor service supplied by household j and used by firm i.

The aggregate output Y_t is given by the CES aggregator:

$$Y_t = \left(\int_0^1 y_t(i)^{\frac{\theta_p - 1}{\theta_p}} di \right)^{\frac{\theta_p}{\theta_p - 1}}, \tag{20}$$

where θ_p is the goods demand elasticity. Firms face individual demand:

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t, \tag{21}$$

where the price index is given by $P_t = (\int_0^1 p_t(i)^{1-\theta_p} di)^{1/(1-\theta_p)}$.

Firms set their prices under the staggered contract á la Calvo (1983). In each period, a fraction $\xi \in (0, 1)$ of firms keeps their prices unchanged, whereas the remaining fraction $(1 - \xi)$ of firms resets their prices. The reset price P_t^* maximizes the expected real profits:

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \xi^s \underbrace{\beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-1}}_{\Lambda_{t,t+s}} \underbrace{\left(\frac{P_t^*}{P_{t+s}} y_{t+s|t} - \frac{W_{t+s}}{P_{t+s}} y_{t+s|t} \right)}_{\Phi_{t+s|t}} \right], \tag{22}$$

subject to the individual good demand $y_{t+s|t} = (P_t^*/P_{t+s})^{-\theta_p} Y_{t+s}$, where $\Lambda_{t,t+s}$ is the stochastic discount factor between time t and $t + s$ and $\Phi_{t+s|t}$ is the real profit at $t + s$ of the firms that reset their prices at t. Notice that the firm index i is dropped because the optimization problem is identical across the firms that reset their prices at t. The FOC is derived as below.

$$\frac{P_t^*}{P_t} = \frac{\Omega_{1,t}}{\Omega_{2,t}}, \tag{23}$$

$$\text{where} \quad \Omega_{1,t} = \mu_p \frac{W_t}{P_t} C_t^{-1} Y_t + \xi\beta\mathbb{E}_t\left[(\Pi_{t+1}^p)^{\theta_p}\Omega_{1,t+1}\right],$$

$$\Omega_{2,t} = C_t^{-1} Y_t + \xi\beta\mathbb{E}_t\left[(\Pi_{t+1}^p)^{\theta_p-1}\Omega_{2,t+1}\right].$$

$\mu_p = \theta_p/(\theta_p - 1)$ is the steady-state price markup that arises from the firms' monopolistic power over their products. The price index is rearranged as[14]

$$1 = (1-\xi)\left(\frac{P_t^*}{P_t}\right)^{1-\theta_p} + \xi(\Pi_t^p)^{\theta_p-1}. \tag{25}$$

Integrating the individual production function (19) over firms yields

$$Y_t = \frac{L_t}{D_t}, \tag{26}$$

where $D_t = \int_0^1 (p_t(i)/P_t)^{-\theta_p} di$ and $L_t = \int_0^1 l_t(i) di$. D_t represents the misallocation associated with relative price dispersion. It evolves according to a recursive formula:

$$D_t = \xi(\Pi_t^p)^{\theta_p} D_{t-1} + (1-\xi)\left(\frac{P_t^*}{P_t}\right)^{-\theta_p}. \tag{27}$$

[14]Taking the first-order approximation of (23) and (25) around the zero-inflation steady state yields the well-known linearized New Keynesian Phillips curve:

$$\pi_t^p = \beta\mathbb{E}_t\left[\pi_{t+1}^p\right] + \kappa\widehat{mc}_t, \tag{24}$$

where $\kappa = (1-\xi)(1-\beta\xi)/\xi$, $\pi_t^p = \ln \Pi_t^p$, $MC_t = W_t/P_t$, and variables with hats denote the log-deviations from the steady-state values.

The goods market clearing condition is given by

$$Y_t = C_t\left(1 + M_t\right), \tag{28}$$
$$\text{where} \quad M_t = \int_0^1 m_t(j)dj.$$

2.2.3 Central bank

The central bank follows the Taylor (1993) rule in which it sets the gross nominal interest rate R_t to stabilize the gross inflation rate Π_t around its target rate Π^* and the output gap $Y_t^{gap} = Y_t/Y_t^f$:[15]

$$R_t = R^*\left(\frac{\Pi_t^p}{\Pi^*}\right)^{\phi_\pi}\left(\frac{Y_t^{gap}}{Y^*}\right)^{\phi_y}, \tag{29}$$

where ϕ_π and ϕ_y are the responsiveness to inflation and the output gap, R^* is the steady-state nominal interest rate, and Y_t^f is the output under flexible prices and wages.[16]

2.2.4 Aggregate shock

Regarding an aggregate shock, I focus on exogenous fluctuations in the risk premium on the nominal interest rate.[17] Following a convention of the literature, such as Krusell and Smith

[15] In this specification, the target inflation rate Π^* is achieved in the deterministic steady state in which all exogenous shocks are muted. As is pointed out by Coeurdacier et al. 2011, however, the deterministic steady state does not necessarily coincide with the stochastic mean or the risky steady state in a non-linear environment. Since Π^* is used as an instrument for the optimal inflation analysis, I adjust Y^* so that Π^* coincides with the stochastic mean of the inflation rate under calibrated parameters.

[16] Throughout this paper, I focus on a simple monetary policy rule rather than the Ramsey policy. The choice is partly due to theoretical and computational challenges of implementing the Ramsey policy in my heterogeneous agent setting. However, as Schmitt-Grohé and Uríbe (2007) argue, a simple policy rule can be justifiable as a feasible solution to an economic problem that central banks face in reality. I also conduct sensitivity analyses with respect to the monetary policy rule in Section 5.

[17] A risk premium shock is a parsimonious way to capture the fluctuations in the aggregate demand. Coibion et al. (2012) argue that the fluctuations in risk premium have similar effects to net-worth shocks in

(1998), I assume that the aggregate state follows a two-state Markov chain:

$$\ln Q_t = \begin{cases} -\Delta & \text{if} \quad a_t = h \\ \\ \Delta & \text{if} \quad a_t = l \end{cases} , \tag{30}$$

where a_t represents the aggregate state. The transition probabilities are given by $P(a_t = j | a_{t-1} = i) = p_{ij}$ with $\sum_j p_{ij} = 1$ for $i, j = h, l$. $\Delta > 0$ is the size of the risk premium shock. A higher risk premium indicates a contractionary state in which households lose their desire to consume in the current period. In addition, I assume that the probability of job changes evolves according to the aggregate state, i.e., $\delta_t = \delta_i$ if $a_t = i$ for $i = \{h, l\}$ with $\delta_h > \delta_l$.

2.2.5 Equilibrium

A recursive competitive equilibrium consists of a households' policy function for real wage $\tilde{w}_t(j) \equiv w_t(j)/P_t = h(\tilde{w}_{t-1}(j), z_t(j), s_t(j); \chi_{t-1}, D_{t-1}, a_t)$, a policy function for a set of aggregate variables $\mathbf{X}_t \equiv \{Y_t, L_t, C_t, \Pi_t^p, R_t, D_t, Y_t^f\} = f(\chi_{t-1}, D_{t-1}, a_t)$, and a law of motion Γ for the cross-sectional density of real wages g_t, given exogenous processes $\{z_t(j), s_t(j)\}_{j \in [0,1]}$, and a_t, such that (i) a households' policy function h solves the individual wage setting problem; (ii) an aggregate policy function f satisfies the aggregate conditions; (iii) markets clear; and (iv) the cross-sectional density χ satisfies a recursive rule:

$$\chi_t = \Gamma(\chi_{t-1}, D_{t-1}, a_t).$$

a model with financial frictions.

2.2.6 Social welfare

I define social welfare as the unconditional expectation of household life-time utility:

$$SW \equiv \frac{1}{1-\beta}\mathbb{E}\left[\ln C_t - \int_0^1 \frac{h_t(j)^{1+1/\eta}}{1+1/\eta}dj\right]. \tag{31}$$

In what follows, I measure welfare losses as the deviations of the social welfare in the model economy from that in the flexible price and wage economy.

2.2.7 Numerical method

I solve the model by using the modified Krusell-Smith algorithm developed by Mineyama (2020). The details are provided in Online Appendix A.

3 Calibration

I follow a two-step procedure to set parameter values of the model. In the first step, I calibrate several parameters according to external evidence. In the second step, I estimate the parameters for cross-sectional wage distribution using the simulated method of moments (SMM).

3.1 Externally fixed parameters

I calibrate the model to U.S. macro and micro data. The time frequency is quarterly. The externally fixed parameters are listed in Table 1. For preference, the subjective discount factor β is set to 0.995. The Frisch labor supply elasticity η is set to 0.5 according to the value that Chetty et al. (2011) report as the mean of the estimates in the existing studies using micro data. The elasticity of substitution across individual goods θ_p is set equal to 7 following Coibion et al. (2012). The value implies that the steady-state price markup is around 17 percent, which is consistent with empirical estimates in the literature such as Basu and Fernald (1997). The elasticity of substitution across individual labor service θ_w is set to 3.5 following KRM2009. The degree of price stickiness ξ is set to 0.63 based on the frequency of price changes reported by Nakamura and Steinsson (2008).[18] The average productivity growth rate $g = \mathbb{E}[g_t]$ is set equal to 1.4 percent per year. The value corresponds to the mean growth rate of real GDP per capita in the past few decades. For the monetary policy rule, I set the responsiveness to inflation ϕ_π equal to 1.50 and that to the output gap ϕ_π to 0.25. Regarding exogenous processes, the remaining probability in each state $p_{hh} = p_{ll} = 0.94$ and the size of risk premium shocks $\Delta = 0.0054$ are set to match the persistence and variance of real GDP per capita after the Greenspan era. Following Grigsby et al. (2019), the probability of job changes δ is set according to the Job-to-Job Flow Data from the Longitudinal Employer-Household Dynamics (LEHD) program. I split the available sample depending on whether the unemployment rate is above or below its median, which corresponds to the low and high states in the model, and use the average of each period for δ_l and δ_h.

[18] Nakamura and Steinsson (2008) report that the median duration of price changes is 7.4–8.7 months during different time periods. I use the mean of these values, implying $\xi = 1 - 1/(0.5 \times (7.4 + 8.7)/3) \approx 0.63$.

Table 1: Externally fixed parameters

Description	Symbol	Value	Source
Subjective discount factor	β	0.995	—
Frisch labor supply elasticity	η	0.50	CGMW (2011)
Goods demand elasticity	θ_p	7.00	CGW (2012)
Labor demand elasticity	θ_w	3.50	KRM (2009)
Price stickiness	ξ	0.63	NS (2008)
Average productivity growth (% per year)	g	1.40	GDP growth per capita
Responsiveness to inflation	ϕ_π	1.50	—
Responsiveness to the output gap	ϕ_y	0.25	—
Remaining prob. of aggregate state	p_{hh}, p_{ll}	0.94	Persistence and Std.
Size of aggregate shock	Δ	0.0054	of GDP per capita
Prob. of job changes in low state	δ_l	0.045	J2J Flow Data
Prob. of job changes in high state	δ_h	0.056	J2J Flow Data
Support of labor productivity	U_z^{ch}	1.000	—

Notes: The abbreviations are as follows; CGMW (2011): Chetty et al. (2011), CGW (2012): Coibion et al. (2012), KRM (2009): Kim and Ruge-Murcia (2009), and NS (2008): Nakamura and Steinsson (2008).

3.2 Internally estimated parameters

3.2.1 Simulated method of moments (SMM)

I use the SMM to estimate the parameters for cross-sectional wage distribution. To be precise, I choose the values of a set of parameters $\hat{\boldsymbol{\Theta}}$ to minimize the distance between the target and model moments:[19]

$$\hat{\boldsymbol{\Theta}} = \arg\min_{\Theta} : \ \big(\boldsymbol{d} - \boldsymbol{m}(\Theta)\big)' \boldsymbol{W_\Theta} \big(\boldsymbol{d} - \boldsymbol{m}(\Theta)\big), \tag{32}$$

where $\boldsymbol{d}$ is a vector of the target moments, $\boldsymbol{m}(\Theta)$ is a vector that collects the corresponding model moments, and $\boldsymbol{W_\Theta}$ is a weighting matrix. For the target moments, I use the moments of individual workers' wage changes reported by Grigsby et al. (2019) (GHY henceforth). They collect administrative payroll data from a large U.S. payroll processing company, and report detailed facts regarding nominal wage adjustments, such as the

[19]To generate the model moments, I solve for the stationary equilibrium of the model due to the computational burden of repeatedly deriving the recursive competitive equilibrium.

size and frequency of wage changes for both job-stayers and job-changers.[20] I use the probability, median size, and mean size of positive and negative wage changes, as well as the median, mean, and standard deviation of unconditional wage changes. These moments are available for different job statuses (job-stayers, job-changers, and all workers) and different time frequencies (quarterly and yearly). The total number of target moments included in $\boldsymbol{d}$ is 46.[21] On the other hand, the number of model parameters to estimate is 9, i.e., $\boldsymbol{\Theta} = \{m_0^+, m_0^-, m_1^+, m_1^-, \zeta, \gamma, \mu_z^{st}, \rho_z^{st}, \sigma_z^{st}\}$.[22] Lastly, I use a diagonal matrix for the weighting matrix $\boldsymbol{W_\Theta}$ and weight the moments for job-stayers, job-changers, and all workers with their unconditional fractions in the LEHD.

3.2.2 Choice of target moments

I next make a heuristic identification argument that justifies the choice of target moments. Each parameter is disciplined by empirical moments as follows.

First, regarding the adjustment cost parameters $m_0^+, m_0^-, m_1^+, m_1^-$, fixed and linear costs are guided by the frequency and size of wage changes in the data given productivity processes. For example, a larger fixed cost lowers the frequency of wage changes, whereas a larger linear cost reduces the size of wage changes. Moreover, positive and negative costs are disciplined

[20]Their data record administrative measures of hourly wage for hourly paid workers. For salaried workers, data contain the employee's contracted earnings per pay period, such as weekly or monthly. Since the data is according to payroll record, it is supposed to be free from measurement errors that are present in survey based wage measures. GHY argue that the empirical moments reported by them are consistent with other empirical studies after conducting necessary adjustments, though they provide more comprehensive information to discipline the model.

[21]The median and mean size of positive and negative wage changes for all workers are not available in GHY. Consequently, the total number of target moments available is 9(moments) $\times$ 3(job-status) $\times$ 2(frequency) $-$ 4(moments) $\times$ 2(frequency) = 46.

[22]μ_z^{ch} is computed consistently with g once the estimate for μ_z^{st} is obtained. Moreover, I fix $U_z^{ch} = 1$ due to computational difficulty in searching for an equilibrium under a larger value of U_z^{ch}. Though this is a parsimonious approach, the empirical moments are closely matched by the model under the fixed value of U_z^{ch} as is shown in Table 3

by the empirical moments for positive and negative wage changes, respectively.

Second, the probability of not being subject to adjustment costs ζ governs the overall degree of the connection between the moments of productivity changes and those of wage changes.

Third, the probability of not receiving a productivity shock γ is related to the kurtosis of wage change distribution. A higher γ implies that the distribution has a fatter tail as well as a large mass around zero. These features are captured by the difference between the mean and median of wage changes and the frequency of them in the target moments. Note that this parameter can be separately estimated from m_0^+, m_0^- because remaining at the same productivity level still leads to positive nominal wage changes under positive trend inflation whereas a large m_0^+, m_0^- implies zero nominal wage changes.

Fourth, the mean and standard deviation of innovations of productivity μ_z^{st} and σ_z^{st} govern the mean and overall dispersion of wage changes.

Fifth, the persistence of a productivity shock ρ_z^{st} governs the differences in desires of wage adjustments at each time horizon. Note that the literature on price rigidity often employs the hazard function to identify this parameter (e.g., Nakamura and Steinsson 2008). Analogously, the differences between the empirical moments of quarterly and annual wage changes play a corresponding role in the estimation here.

3.2.3 Estimation result

The estimated parameters are listed in Table 2 whereas the target and model moments are reported in Table 3.[23] As GHY discuss in detail, nominal wage adjustments in the data

[23] The standard deviation of the estimates is not obtained because the variance-covariance matrix of the target moments is not available.

Table 2: Internally estimated parameters

Description	Symbol	Value
For job-stayers		
Fixed cost for positive wage changes	m_0^+	0.012
Fixed cost for negative wage changes	m_0^-	0.097
Linear cost for positive wage changes	m_1^+	0.675
Linear cost for negative wage changes	m_1^-	3.626
Prob. of not subject to adjustment cost	ζ	0.011
Prob. of not receiving labor productivity shock	γ	0.848
Mean growth of labor productivity (% per year)	μ_z^{st}	1.692
Persistence of labor productivity	ρ_z^{st}	0.698
S.D. of innovations to labor productivity	σ_z^{st}	0.097
For job-changers		
Mean growth of labor productivity (%)	μ_z^{ch}	8.085

have the following features: (i) negative wage changes are quite rare for job-stayers whereas job-changers often receive them; (ii) the size of wage changes is larger for job-changers than for job-stayers; (iii) the mean wage change is higher for job-changers than for job-stayers. Consistent with these features, the estimated parameters have the following features.

First, the estimated fixed and linear costs are much larger for negative wage adjustments than for positive ones ($m_0^- > m_0^+, m_1^- > m_1^+$). In particular, a large fixed cost is essential for replicating the infrequent nominal wage adjustments observed in the data. The estimated values imply the adjustment cost for a median size wage increase is 1.0% of annual consumption whereas that for a wage cut is 9.0%.[24] The result suggests the presence of DNWR. Since only some fraction of workers change their nominal wages in each period, the total resource cost associates with nominal wage adjustments is around 0.35% of annual consumption.

Second, the probability of not being subject to adjustment costs for job-stayers (ζ) is quite low. This also reflects infrequent wage changes in the data.

[24] The larger cost for negative wage changes partly reflect a large size of wage cuts. The cost for a 1% wage increase is around 0.5% and that for a 1% wage decrease is 3.3%. The estimates are close to those of Fagan and Messina (2009), who estimated the menu cost for negative wage changes to be 37% of wage rate (roughly equivalent to 9% of annual consumption) according to micro wage data in the PSID after correcting measurement errors.

Table 3: Targeted moments

Moment	Quarterly changes		Yearly changes	
	Data	Model	Data	Model
Job-stayers				
Probability of positive wage changes	0.185	0.187	0.639	0.638
Probability of negative wage changes	0.009	0.011	0.024	0.035
Median size of positive wage changes	0.033	0.041	0.035	0.046
Median size of negative wage changes	–0.077	–0.074	–0.066	–0.072
Mean size of positive wage changes	0.057	0.055	0.063	0.067
Mean size of negative wage changes	–0.087	–0.080	–0.073	–0.080
Median of unconditional wage changes	0.000	0.000	0.024	0.035
Mean of unconditional wage changes	0.010	0.010	0.039	0.040
S.D. of unconditional wage changes	0.037	0.029	0.065	0.056
Job-changers				
Probability of positive wage changes	0.527	0.589	0.568	0.610
Probability of negative wage changes	0.374	0.402	0.380	0.371
Median size of positive wage changes	0.167	0.191	0.185	0.202
Median size of negative wage changes	–0.136	–0.173	–0.158	–0.161
Mean size of positive wage changes	0.235	0.209	0.261	0.223
Mean size of negative wage changes	–0.165	–0.187	–0.185	–0.178
Median of unconditional wage changes	0.023	0.043	0.046	0.065
Mean of unconditional wage changes	0.063	0.048	0.080	0.070
S.D. of unconditional wage changes	0.259	0.238	0.293	0.238
All workers				
Probability of positive wage changes	0.206	0.207	0.627	0.634
Probability of negative wage changes	0.032	0.030	0.087	0.095
Median of unconditional wage changes	0.000	0.000	0.025	0.036
Mean of unconditional wage changes	0.012	0.012	0.044	0.046
S.D. of unconditional wage changes	0.067	0.060	0.120	0.114

Notes: Data moments are those reported by GHY, who use administrative payroll data from a large payroll-processing company in the U.S. The sample period is from 2008 to 2016. Model moments are obtained from the stationary equilibrium. In the estimation, the average productivity growth g, the target inflation rate Π^*, and the probability of job changes δ are set consistent with those in the dataset of GHY. Specifically, Π^* is set to 1.5% per year according to the average inflation rate during 2008–2016 in the GDP deflator. g is set to 3.1% per year consistent with the average nominal wage growth rate in the dataset of GHY (4.6%), which correspond to $g\Pi^*$ in the model. δ is set to 4.8% according to the average job-change rate during 2008–2016 in the Job-to-Job Flow Data from the LEHD.

Third, the implied standard deviation of a labor productivity shock is larger for job-changers than for job-stayers ($U_z^{ch}/\sqrt{6} > \sigma_z^{st}$). This is consistent with the larger dispersion of wage changes for job-changers in the data.

Fourth, the mean growth rate of labor productivity is higher for job-changers than for job-stayers ($\mu_z^{ch} > \mu_z^{st}$). Notice that though the estimated mean and standard deviation of labor productivity change for job-changers are substantially large, the realized wage changes are compressed due to precautionary motives, i.e., each worker will be subject to wage adjustment costs as a job-stayer from the next period.

Lastly, the estimated parameters for labor productivity process ρ_z^{st} and γ imply that a labor productivity shock for job-stayers arrives on average once in 1.6 years and the half-life is 2 quarters. These values are broadly consistent with the estimates of previous studies.[25]

3.3 Validation analysis

To check the validity of the model, Table 4 reports the untargeted moments in the estimation procedure. The table compares the cross-sectional moments under high and low wage-growth periods.[26] As GHY emphasize, the data indicates state-dependency of wage changes; there are fewer positive changes and slightly more negative ones, as a consequence of which the fraction of workers with wage freezes becomes larger, under a low wage-growth rate. The model is successful in replicating these patterns. This is because under a lower wage-growth rate more workers stay close to their DNWR constraint—i.e., while more workers' desired wages decline, they keep their wages unchanged due to the large adjustment cost for downward wage changes. Consistently, the the standard deviation of wage changes is slightly lower in the low wage growth period as in the data.

[25]For example, Kaplan et al. (2018) estimate earning process using Social Security Administration data and find that a transitory shock arrives on average once 3 years and has the half-life of around 1 quarter.

[26]GHY split the sample period of 2008-2016 into two sub-periods: from March 2009 to December 2010 and January 2012 to December 2016. I refer to the former as "low wage-growth period" and the latter as "high wage-growth period."

Table 4: Untargeted moments

Moment	Low wage growth		High wage growth	
	Data	Model	Data	Model
Job-stayers				
Probability of positive wage changes	0.525	0.539	0.674	0.755
Probability of negative wage changes	0.042	0.079	0.020	0.021
Probability of zero wage changes	0.433	0.382	0.306	0.224
S.D. of unconditional wage changes	0.063	0.055	0.070	0.060
Job-changers				
Probability of positive wage changes	0.505	0.579	0.581	0.631
Probability of negative wage changes	0.440	0.400	0.364	0.351
Probability of zero wage changes	0.055	0.021	0.055	0.018
S.D. of unconditional wage changes	0.314	0.239	0.289	0.245
All workers				
Probability of positive wage changes	0.520	0.546	0.660	0.733
Probability of negative wage changes	0.106	0.137	0.084	0.080
Probability of zero wage changes	0.374	0.317	0.256	0.177
S.D. of unconditional wage changes	0.121	0.114	0.124	0.118

Notes: The listed moments are those of yearly wage changes. The data moments are those reported by GHY. The low wage-growth period in the data is from May 2009 to December 2010, whereas the high wage-growth period is from January 2012 to December 2016. The average nominal wage growth rate in each period is 2.8% and 6.4% per year, respectively.

4 Welfare analysis

4.1 Welfare losses under calibrated Taylor rule

Table 5 shows the welfare losses and relevant moments of the baseline HA model under the calibrated Taylor rule (column 1).[27] The target inflation rate Π^* is set to 2.1 percent in the annual rate according the mean inflation rate after the Greenspan era. For comparison purposes, the table also reports the values of the RA model with asymmetric smooth wage adjustment cost (column 2)[28] and those of both models under flexible wages (columns 3

[27]In Online Appendix B, I derive the second-order approximation of social welfare and show that welfare losses arise from four sources: (i) aggregate mean, (ii) aggregate variance, (iii) cross-sectional variance, and (iv) cross-sectional covariance. Table 5 reports the relevant moments of each component.

[28]In the representative agent model with asymmetric smooth wage adjustment cost, the aggregate wage growth $\Pi_t^w = W_t/W_{t-1}$ is governed by the following wage Phillips curve:

$$\Psi_w^{'}(\Pi_t^w)\Pi_t^w = \beta\mathbb{E}_t\left[\Psi_w^{'}(\Pi_{t+1}^w)\Pi_{t+1}^w\right] + \theta_w\left(H_t^{1/\eta}C_t - \frac{1}{\mu_w}\frac{W_t}{P_t}\right)\frac{H_t}{C_t}, \tag{33}$$

$$\text{where}\quad \Psi_w(\Pi_t^w) = \phi_w\left(\frac{exp(-\psi_w(\Pi_t^w - 1)) + \psi_w(\Pi_t^w - 1) - 1}{\psi_w^2}\right).$$

Table 5: Welfare losses under a calibrated Taylor rule

	(1)	(2)	(3)	(4)
	With wage rigidity		Without wage rigidity	
	HA model (baseline)	RA model	HA model	RA model
Welfare loss (CE, %)	–0.97	–0.20	–0.27	–0.22
$\sigma_j\big(\ln w_t(j)\big)$ (%)	17.21	—	21.05	—
$\rho_j\big(\ln w_t(j), \ln z_t(j)\big)$	0.98	—	1.00	—

Notes: HA denotes heterogeneous agent and RA representative agent. All the specifications embed nominal price rigidity. The target inflation rate Π^* is set to 2.1% per year. The value corresponds to the mean of the inflation rate after the Greenspan era. Welfare losses are in terms of the consumption equivalent (CE) loss compared with the economy under flexible prices and wages. $\sigma_j(\cdot)$, and $\rho_j(\cdot,\cdot)$ denote the standard deviation and correlation of cross-sectional distribution.

and 4).[29] All the specifications embed nominal price rigidity. Note that the asymmetric smooth wage adjustment cost is often used in the literature to approximate DNWR in a representative agent setting (e.g., KRM2009, Aruoba et al. 2017).

Several points are noteworthy in the table. First, the welfare loss of the baseline HA model in column (1) is around five times larger than that of the RA model in column (2). The welfare loss arising from wage rigidity can be measured by the size of the wage markup, i.e., the deviations of wages from the socially optimal levels. In this regard, individual workers face idiosyncratic fluctuations in their optimal wages due to individual labor productivity shocks in the presence of worker heterogeneity, even when the aggregate wage is stable. Consequently, the lack of wage adjustments due to adjustment costs results in a larger welfare loss in the HA model than in the RA model.

Second, the point above is also verified by comparing the cases with and without wage rigidity in columns (1) and (3). Regarding the cross-sectional moments, the standard de-

$\Psi_w(\Pi_t^w)$ is wage adjustment cost. The parameter ψ_w governs the degree of asymmetry, whereas ϕ_w determines the overall degree of wage rigidity. The parameter values are calibrated according to the estimates by KRM2019. The details are presented in Online Appendix D. Note that though the model considers a continuum of workers, I label it as a representative agent model in the sense that equilibrium is characterized only by the relationships among aggregate variables.

[29]The allocations under flexible wages are described in Online Appendix B.

viation of wages is smaller with wage rigidity, and the correlation between wage and labor productivity is slightly lower. These features indicate that wage rigidity impedes wage adjustments upon idiosyncratic shocks. As for the mean of the aggregate variables, the output is inefficiently high due to the wage adjustment cost represented by M_t. Consequently, the presence of wage rigidity leads to a sizable welfare loss in the HA model.

Third, it is also notable that the welfare implication of wage rigidity is flipped once worker heterogeneity is taken into account. In the RA models in columns (2) and (4), adding wage rigidity to a sticky price setting does not enlarge the welfare loss, consistent with the findings of previous studies (e.g., Galí and Monacelli 2016). This is because wage rigidity makes real wages and therefore marginal cost sticky. This reduces the fluctuations in the inflation rate, leading to a smaller welfare loss arising from nominal price rigidity. On the other hand, in the HA models in columns (1) and (3), the welfare deterioration arising from the increased labor market inefficiency due to wage rigidity overwhelms the benefits of reducing the inflation variations.

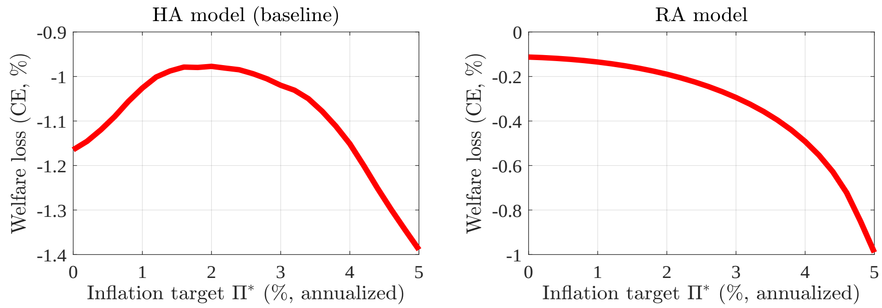
4.2 Optimal inflation rate

The welfare analysis in Section 4.1 indicates that the presence of worker heterogeneity substantially increases the welfare loss arising from wage rigidity. Next, I investigate the consequences of these welfare differences for the optimal inflation rate. Specifically, I search for the level of inflation target Π^* that maximizes social welfare.[30]

Figure 2 displays the welfare losses under different levels of Π^*. Social welfare is maxi-

[30] Notice that maximizing social welfare by changing Π^* is isomorphic to minimizing the welfare loss from the flexible wage and price economy, because monetary policy does not affect allocations in the flexible price and wage economy.

Figure 2: Welfare losses under different levels of Π^*



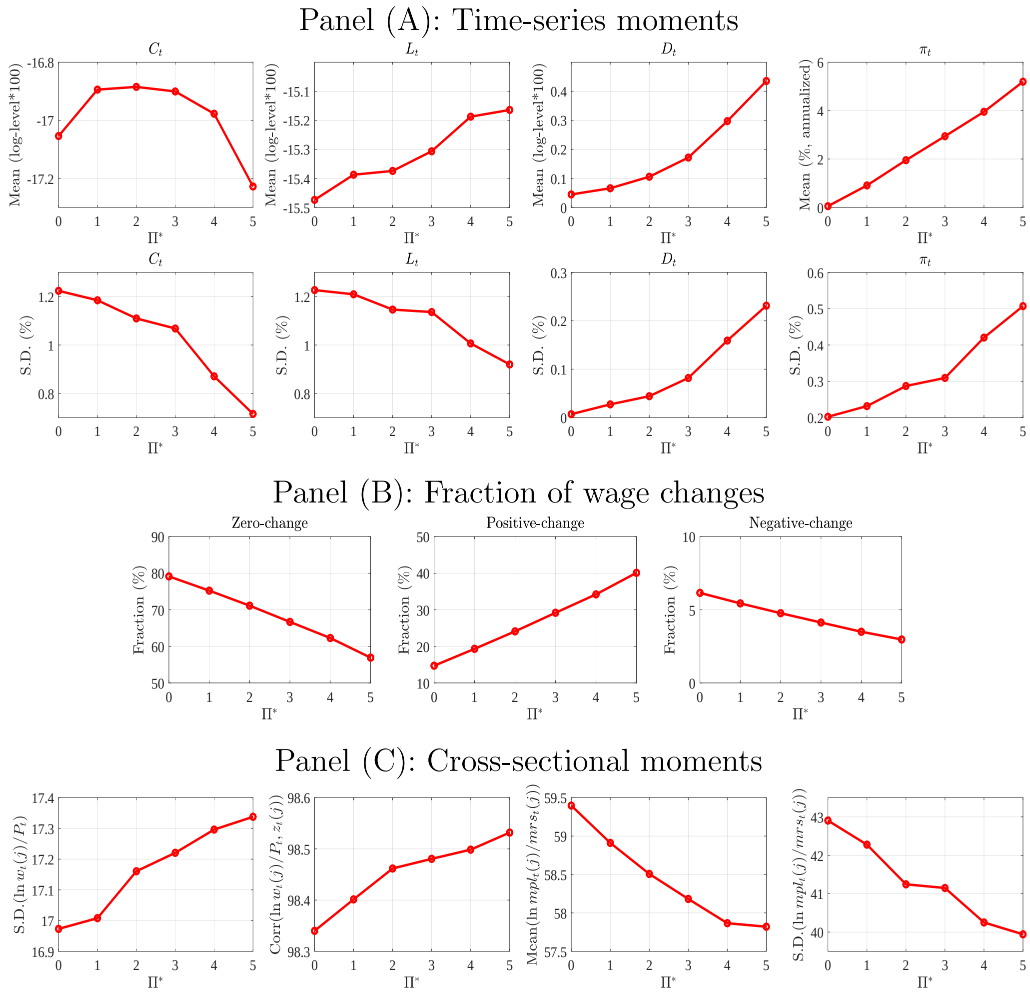
Notes: The y-axis represents the consumption equivalent welfare loss compared with the economy under flexible prices and wages. For the ease of presentation, the welfare losses are shown in negative values. They are computed at different levels of the inflation target Π^* with an interval of 0.2 percentage points. In the heterogeneous agent model in the left panel, the moving average for the five points centering each point is taken to smooth out small deviations.

mized when Π^* is around 2 percent in the baseline HA model shown in the left panel. Positive inflation can benefit the economy because fewer workers suffer DNWR at a higher inflation rate. On the other hand, a higher inflation rate enlarges the distortion due to nominal price rigidity as relative prices become more dispersed. The optimal inflation rate is determined by the trade-off between the benefits and costs of inflation.

It is also notable that the curvature of welfare losses around the optimum is considerably flat. For example, the range of Π^* from 1.0 to 3.0 percent delivers welfare differences from the optimum within 0.05 percent of consumption, and the range of Π^* from 0.8 to 3.6 percent within 0.1 percent of consumption.

The right panel of Figure 2 shows the case of the RA model with asymmetric smooth wage adjustment cost. The result is striking; social welfare is monotonically decreasing in Π^* in a positive territory although the model embeds asymmetric wage adjustment cost. Precisely speaking, I find that the optimal inflation rate is –0.4 percent in the RA model. The reason behind the negative optimal inflation rate is the presence of positive trend productivity growth. Due to a positive trend growth rate in productivity, real wages have a upward

Figure 3: Selected moments under different levels of Π^*



Notes: The x-axis is the target inflation rate (%, annualized).

trend. Hence, a negative inflation rate brings the nominal wage growth down close to zero, reducing the welfare loss associated with wage adjustments. This result is also obtained by Amano et al. (2009).[31] Overall, the analysis here suggests that the benefit of positive inflation through DNWR, or the "grease the wheels" effect, becomes substantially larger in the presence of worker heterogeneity.

[31]Consistently, the previous studies that abstract trend productivity growth report non-negative values for the optimal inflation rate in an economy with DNWR (e.g., KRM2009).

4.3 Determinants of optimal inflation rate

To examine the mechanism that determines the optimal Π^*, Figure 3 displays selected moments of the baseline HA model under different levels of Π^*.

Panel (A) reports the stochastic mean and standard deviation of each aggregate variable. Notably, the mean consumption C_t is hump-shaped; it increases in Π^* at low inflation rates, and it starts to decrease when Π^* exceeds a certain point. There are two offsetting forces behind this property. On the one hand, the distortion arising from nominal price rigidity increases as Π^* rises. More specifically, the increase in the relative price dispersion D_t generates inefficiency in production. Indeed, the labor input L_t monotonically increases in Π^* while C_t is hump-shaped due to the inefficiency. On the other hand, the misallocations in the labor market are lessened as Π^* rises as fewer workers suffer DNWR at a higher Π^*. Regarding the standard deviation, that of C_t decreases, whereas that of π_t^p increases, in Π^*. This is because wage changes become more frequent at a higher Π^*, which makes marginal cost and thereby inflation more flexible.

Panel (B) indicates that fewer workers experience wage freezes at a higher Π^*. Consequently, the standard deviation of wages and the correlation between wages and labor productivity shown in Panel (C) become higher, getting closer to the allocations under flexible wage. These lead to a lower cross-sectional mean and standard deviation of labor wedge, defined as the difference between the marginal product of labor (MPL) and the MRS.

5 Sensitivity analysis

In this section, I conduct various sensitivity analyses regarding the baseline quantitative results presented in Section 4. I focus on alternative settings in terms of (i) trend productivity growth, (ii) wage adjustment cost, (iii) labor market parameters such as idiosyncratic variations of productivity and labor supply and demand elasticities, and (vi) monetary policy rule.[32] The results are summarized in Table 6. Essentially, the difference between the HA and RA models remains considerable in these alternative settings though the impact of each factor is in a broad range.

5.1 Trend productivity growth

Trend productivity growth is a crucial factor for the optimal inflation rate because it promotes real wage growth, making DNWR less binding given the level of Π^*. Row (2) and (3) of Table 6 show the optimal inflation rates when the productivity growth rate g becomes lower or higher by 0.5 percentage points from the baseline calibration ($g = 0.9\%$ in Row 2 and $g = 1.9\%$ in Row 3). These results indicate that the optimal Π^* is negatively related to the level of g as is expected. Moreover, the relationship is almost one-to-one; the lower and higher g respectively lead to the optimal Π^* of 2.6 percent and 1.6 percent, with the changes in the optimal Π^* close to those of g.

[32] Sensitivity to other calibrated parameters is investigated in Online Appendix C. The appendix also discusses potential effects of consumption heterogeneity, which is abstracted in the baseline model.

Table 6: Optimal inflation rates in alternative settings

		Opt. Π^* in HA model	Wel. diff. <0.05% in HA model		Opt. Π^* in RA model
(1)	Baseline	2.0	[1.0	3.0]	–0.4
	Trend productivity growth g				
(2)	Lower g by 0.5%p	2.6	[1.4	3.4]	–0.4
(3)	Higher g by 0.5%p	1.6	[0.4	2.8]	–0.6
	Degree of wage adjustment cost m $(m_0^+, m_0^-, m_1^+, m_1^-)$				
(4)	Lower m by 50%	1.8	[1.0	2.4]	–0.7
(5)	Higher m by 50%	2.2	[1.6	2.6]	–0.4
(6)	Symmetric m	–1.4	[–1.8	–1.0]	–0.5
	Rebating adjustment costs				
(7)	Rebating adjustment costs	1.4	[1.2	3.2]	0.0
	Idiosyncratic labor productivity shock σ_z^{st}				
(8)	Lower σ_z^{st} by 50%	1.2	[0.8	3.0]	—
(9)	Higher σ_z^{st} by 50%	2.2	[1.4	3.4]	—
	Frisch labor supply elasticity η				
(10)	Lower η ($\eta = 0.3$)	2.2	[1.4	3.0]	–1.0
(11)	Higher η ($\eta = 1.0$)	1.2	[0.8	2.0]	–0.1
	Labor demand elasticity θ_w				
(12)	Lower θ_w (θ_w=2)	1.0	[0.4	1.4]	0.0
(13)	Higher θ_w (θ_w=5)	2.4	[1.2	3.6]	–1.0
	Monetary policy rule				
(14)	Higher δ_π ($\delta_\pi = 3$)	2.0	[1.0	2.8]	–0.1
(15)	Higher δ_y ($\delta_y = 0.5$)	2.4	[0.4	3.2]	–0.1
(16)	Wage growth rule	1.6	[0.4	2.4]	–0.5
	Aggregate volatility σ_q				
(17)	Lower σ_q by 50%	2.4	[0.8	3.2]	0.0
(18)	Higher σ_q by 50%	1.8	[1.6	2.2]	–1.0

Notes: The HA model is solved at different levels of Π^* with an interval of 0.2 percentage points due to the computational burden of deriving equilibrium repeatedly at different Π^*. The RA model is solved with an interval of 0.1 percent points, and the range of Π^* is truncated at –1.0 percent because equilibrium tends to be unstable at a lower Π^*. The second and third columns report the range of Π^* within which the differences in welfare loss from the optimum are within 0.05 percent of consumption.

5.2 Wage adjustment cost

Degree of wage adjustment cost. The degree of wage rigidity is apparently one of the key factors for the optimal Π^*. Row (4) and (5) consider the cases in which the parameters for wage adjustment cost are decreased or increased by 50 percent from the baseline values ($m_0^+ = 0.006, m_0^- = 0.0485, m_1^+ = 0.3375, m_1^- = 1.813$ in Row 4 and $m_0^+ = 0.0218, m_0^- = 0.1455, m_1^+ = 1.0125, m_1^- = 5.439$ in Row 5). As is expected, the lower (higher) degree of wage rigidity tends to lead to a lower (higher) optimal Π^*. However, it is worth noting that the sensitivity of the optimal Π^* is low. One reason behind the low sensitivity is that idiosyncratic labor productivity shocks and wage adjustment costs are quite large compared to the aggregate fluctuations. Hence, the changes in the degree of wage adjustment costs do not drastically alter the responsiveness to the aggregate conditions that are caused by the changes in Π^*.

Asymmetry of wage adjustment cost. Row (6) considers the case in which wage adjustment cost is symmetric. For this analysis, I set the wage adjustment costs at the mean of positive and negative adjustment costs in the baseline calibration ($m_0^+ = m_0^- = 0.0545, m_1^+ = m_1^- = 2.1505$). In this case, the optimal Π^* becomes negative as in the RA model. In other words, asymmetry in wage adjustment cost is essential to capture the benefit of holding positive inflation.

Eliminating resource cost. In Row (7), I eliminate the effect of wage adjustment cost on household budget by rebating them through lump-sum transfer to households, so as to focus on the effect on the misallocation in the labor market. Note that the entire output is consumed in this case, i.e., $C_t = Y_t$. The optimal Π^* in the HA model remains higher than in RA model, implying that the main driver of the higher optimal Π^* in the HA model is not the resource cost of wage adjustment itself but the resulting misallocation.

5.3 Labor market parameters

Idiosyncratic variations of productivity. Row (8) and (9) consider the cases in which the standard deviation of labor productivity shocks for job-stayers σ_z^{st} becomes lower or higher by 50 percent of the baseline value ($\sigma_z^{st} = 0.0485$ in Row 8 and $\sigma_z^{st} = 0.1455$ in

Row 9). The lower (higher) σ_z^{st} leads to a lower (higher) optimal Π^* because the desired wages of individual workers become less (more) dispersed, reducing (enlarging) the effects of wage rigidity. Interestingly, the sensitivity is asymmetric; the decline in the optimal Π^* corresponding to the smaller value of σ_z^{st} is relatively large whereas the optimal Π^* is insensitive to the larger value of σ_z^{st}. This is presumably because idiosyncratic shocks are much larger than aggregate fluctuations, and therefore the effects of the further increase of σ_z^{st} cannot be mitigated by the changes in Π^*.

Labor supply elasticity. Row (10) and (11) investigate the sensitivity with respect to the Frisch labor supply elasticity η. Note that the higher value ($\eta = 1.0$) is often used in the macro literature (e.g., Coibion et al. 2012) though it is above the range of the most estimates based on micro data (e.g., Chetty et al. 2011). The higher (lower) η results in a lower (higher) optimal Π^*. This is because a higher η implies that disutility from labor supply is less convex, which causes smaller welfare losses due to insufficient wage adjustments. However, the difference of the optimal Π^* between the HA and RA models remains considerable.

Labor demand elasticity. Row (12) and (13) turn to the assessment of sensitivity regarding the labor demand elasticity θ_w. The lower and higher values of θ_w analyzed here ($\theta_w = 2$ in Row 12 and $\theta_w = 5$ in Row 13) imply the steady-state wage markup μ^w is 100% and 25% respectively, whereas the baseline value of $\theta_w = 3.5$ implies that of 40%. A higher θ_w means that labor demand is more elastic to wages, enlarging the labor market misallocations arising from wage rigidity. Consequently, the optimal Π^* is lower under a lower θ_w, though it is still substantially higher than that in the RA model.

5.4 Monetary policy rule

Alternative monetary policy rules are analyzed. Row (14) and (15) consider stronger responsiveness to inflation and the output gap in the Taylor rule, respectively ($\delta_\pi = 3.0$ in Row 14 and $\delta_y = 0.5$ in Row 15). Row (16) considers a monetary policy rule that responds to wage growth instead of inflation.[33]

The results suggest that the high responsiveness to the output gap has strong stabilization power, especially mitigating the adverse effects of at a high Π^* associated with price rigidity. This leads to a slightly higher optimal Π^* than the baseline calibration. On the other hand, the wage growth rule addresses the distortion arising from wage rigidity, which results in a slightly lower optimal Π^*. Still, the difference between the HA and RA models remains considerable under alternative monetary policy rules at least within the moderate degree of modifications that are considered here.

5.5 Aggregate volatility

One of key distinctions of this paper from previous studies such as Fagan and Messina (2009) is the presence of an aggregate shock. To see its consequence, Row (17) and (18) consider the cases in which the standard deviation of aggregate risk premium shock σ_q becomes lower or higher by 50 percent ($\sigma_q = 0.0027$ in Row 17 and $\sigma_q = 0.0081$ in Row 18). Since aggregate uncertainty affects both the costs and benefits of inflation, the effect on the optimal Π^* is unclear a priori. It turns out that lower aggregate uncertainty leads to a higher optimal Π^*. There are two reasons behind the result. First, since the welfare loss arising from price

[33] The alternative monetary policy rule is given by $R_t = R^*(\Pi_t^w/\Pi^{w*})^{\phi_{\pi^w}}(Y_t^{gap}/Y^*)^{\phi_y}$, where the responsiveness ϕ_{π^w} is set to 1.5 consistent with the baseline calibration.

rigidity is highly convex in terms of the fluctuations in inflation, it increases as aggregate uncertainty is enlarged. Second, higher aggregate uncertainty leads to stronger precautionary behavior in wage setting as is discussed in Section 2.1, and therefore the benefit of a higher inflation rate is weakened.[34] Both contribute to the higher Π^* under the lower σ_q.

6 Conclusion

In this paper, I develop a heterogeneous agent model in which heterogeneity stems from worker productivity and wage changes are subject to asymmetric adjustment costs. Worker heterogeneity enlarges the welfare loss that arises from wage rigidity by generating an inefficient cross-sectional allocation of labor. Reflecting the larger "grease the wheels" effect, the optimal inflation rate obtained in the heterogeneous agent model is substantially higher than in a representative agent model that is often used in the literature.

A number of extensions are possible for future research. First, elaborating the source of welfare loss associated with wage rigidity and exploring endogenous mechanism behind wage rigidity, reflecting developments of recent theoretical and empirical literature (e.g., Dupraz et al. 2020), would be interesting. This point is also related to the allocativeness of wages (e.g., Basu and House 2016). Second, though this paper focuses on the consequences of DNWR on the optimal inflation rate, interactions of DNWR with other factors affecting the costs and benefits of inflation, such as the ZLB, money holdings, etc., would be worth investigating. Third, international comparison would be an important policy question. Although the inflation targets adopted by central banks in advanced economies are somewhat

[34]The point is consistent with Wagner (2018), who claims that the effects of different levels of Π^* are partly mitigated by the precautionary behavior.

concentrated around 2 percent, different degrees of DNWR across economies can lead to divergent implications on the optimal inflation rate.

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Online Appendix for "Revisiting the Optimal Inflation Rate with Downward Nominal Wage Rigidity: The Role of Heterogeneity"

October 30, 2020

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This Appendix is structured as follows:

- Appendix A provides the details of computation methods. It explains the modified Krusell-Smith algorithm and other elements for the equilibrium computation.
- Appendix B provides miscellaneous elements of the baseline heterogeneous agent model; decomposition of social welfare and allocations under flexible wages.
- Appendix C provides sensitivity analyses. It assesses alternative specification of wage adjustment costs and sensitivity to calibrated parameters that supplement the analyses in the main article. It also discusses potential effects of consumption heterogeneity, which is abstracted in the baseline model.
- Appendix D describes a representative agent model with asymmetric smooth wage adjustment cost.

A Computation

In this appendix, I present the equilibrium computation method. Though the method largely follows that developed in my companion paper, Mineyama (2020), I add several modifications according to the model setting of this paper.

A.1 Modified Krusell-Smith algorithm

Approximated equilibrium. To deal with the infinite dimensionality of the cross-sectional distribution, Krusell and Smith (1998) propose an approximated equilibrium in which each agent perceives the evolution of aggregate state variables as a function of a small number of moments of the cross-sectional distribution. One of their key findings is that using a very small set of moments, usually the mean of distribution, is sufficient to achieve a good approximation. Adopting their insight, I assume that the aggregate endogenous state variable, aggregate real wage $\tilde{W}_t$, follows an aggregate law of motion (ALM):

$$\tilde{W}_t = \Gamma_{\tilde{W}}(\tilde{W}_{t-1}, D_{t-1}, a_t), \tag{A.1}$$

where the variables with tilde denote those detrended by the deterministic real growth rate g. Other notations follow those in the main article.

Specification of ALM. To parameterize the ALM $\Gamma_{\tilde{W}}$, I first forecast the wage growth rate Π_t^w using a log-linear form of lagged real wage $\tilde{W}_{t-1}$ and relative price dispersion D_{t-1}.[1]

[1] I find that this specification delivers higher accuracy than the ALM that directly forecasts the current real wage $\tilde{W}_t$.

Specifically, I consider a semiparametric specification that allows for different coefficients of the ALM for each aggregate exogenous state s_t. The ALM is given by

$$\ln \Pi_t^w = \ B_0^a + B_1^a \ln \tilde{W}_{t-1} + B_2^a \ln D_{t-1} \quad \text{for} \quad a_t = a_i, \tag{A.2}$$

where $i = \{l, h\}$. The coefficients of the ALM $\mathbf{B} = \{B_0^i, B_1^i, B_2^i\}_{i \in \{l,h\}}$ govern the dynamics of the aggregate state variable. It should be noted that, although the ALM takes a simple functional form in terms of $\tilde{W}_{t-1}$, it can capture rich nonlinear dynamics because of the semiparametric specification of the coefficients $\mathbf{B}$.[2]

Once the forecast of the wage growth rate is obtained, the current real wage $\tilde{W}_t$ can be recovered according to the following equation:

$$\tilde{W}_t = \frac{\Pi_t^w}{\Pi_t^p} \tilde{W}_{t-1}, \tag{A.3}$$

where the inflation rate Π_t^p is obtained when solving the aggregate part of the economy.

Algorithm. The algorithm takes the following steps for each iteration $m = 1, 2, 3, ...$

1. (Forecasting) Given the aggregate state variables $\mathbf{S}_t = \{a_t, D_{t-1}, \tilde{W}_{t-1}\}$, each agent uses the ALM (A.2) with the coefficients $\mathbf{B}^{(m)}$ to forecast the current period aggregate state variable $\tilde{W}_t^{fore}$.

2. (Aggregate problem) Given the forecast variable $\tilde{W}_t^{fore}$, the policy function $f^{(m)}$ for

[2] The log-linear specification is often used in the literature to approximate the ALM (e.g., Krusell and Smith (1998), Krueger et al. (2016)). Though my companion paper, Mineyama (2020) uses a quadratic ALM for the model with the ZLB and AR(1) shocks, I find that a log-linear ALM delivers enough explanatory power for the specification of this paper.

aggregate jump variables $\mathbf{X}_t \equiv \{Y_t, L_t, C_t, \Pi_t^p, R_t, D_t, Y_t^f\}$ is obtained by solving the aggregate part of the economy. The aggregate part is a New Keynesian system that consists of the Euler equation (A.4), pricing equations (A.5)–(A.8), and the Taylor rule (A.9), along with the aggregate production function (A.10), the law of motion of price dispersion (A.11), and the resource constraint (A.12). This step is feasible because the aggregate part of the economy does not depend on the cross-sectional distribution once being conditional on $\tilde{W}_t^{fore}$. Note that, when computing the aggregation of adjustment costs $M_t = \int_0^1 m_t(j)dj$ in (A.12), I approximate the fraction of wage changes to its stochastic mean, i.e., $M_t \approx \mathbb{E}[\int_0^1 m_t(j)dj]$. The approximation is due to the computation burden to obtain the fraction of wage changes for each point of aggregate state variables. However, the approximation errors are considered to be quantitatively small because the changes in the fraction of workers who pay menu costs has up to the second order effects.

$$\beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{Q_t R_t}{\Pi^p_{t+1}} \right] = 1, \tag{A.4}$$

$$\frac{P^*_t}{P_t} = \frac{\Omega_{1,t}}{\Omega_{2,t}}, \tag{A.5}$$

$$where \quad \Omega_{1,t} = \mu_p \frac{W_t}{P_t} C_t^{-1} Y_t + \xi \beta \mathbb{E}_t \left[(\Pi^p_{t+1})^{\theta_p} \Omega_{1,t+1} \right], \tag{A.6}$$

$$\Omega_{2,t} = C_t^{-1} Y_t + \xi \beta \mathbb{E}_t \left[(\Pi^p_{t+1})^{\theta_p - 1} \Omega_{2,t+1} \right], \tag{A.7}$$

$$1 = (1-\xi) \left(\frac{P^*_t}{P_t} \right)^{1-\theta_p} + \xi (\Pi^p_t)^{\theta_p - 1}, \tag{A.8}$$

$$R_t = R^* \left(\frac{\Pi^p_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{Y^{gap}_t}{Y^*} \right)^{\phi_y}, \tag{A.9}$$

$$Y_t = \frac{L_t}{D_t}, \tag{A.10}$$

$$D_t = \xi (\Pi^p_t)^{\theta_p} D_{t-1} + (1-\xi) \left(\frac{P^*_t}{P_t} \right)^{-\theta_p}, \tag{A.11}$$

$$Y_t = C_t \left(1 + M_t \right). \tag{A.12}$$

3. (Individual problem) Given the aggregate state and jump variables $\mathbf{S}_t$ and $\mathbf{X}_t$, along with the individual state variables $\mathbf{s}_t(j) \equiv \{\tilde{w}_{t-1}(j), z_t(j), s_t(j)\}$, individual households solve the following wage-setting problem to derive their policy function $h^{(m)}$.

$$\begin{aligned} V\Big(\tilde{w}_{t-1}(j), z_t(j), s_t(j) \Big) = \max_{\tilde{w}_t(j)} & - \frac{h_t(j)^{1+1/\eta}}{1+1/\eta} + C_t^{-1} \Big(\tilde{w}_t(j) h_t(j) - m_t(j) C_t \Big) \\ & + \frac{\beta}{g} \mathbb{E}_t \left[V\Big(\tilde{w}_t(j), z_{t+1}(j), s_{t+1}(j) \Big) \right], \end{aligned} \tag{A.13}$$

subject to

$$l_t(j) = z_t(j)h_t(j), \tag{A.14}$$

$$l_t(j) = \left(\frac{\tilde{w}_t(j)/z_t(j)}{\tilde{W}_t}\right)^{-\theta_w} L_t, \tag{A.15}$$

where

$$\ln z_t(j) = \ln z_{1,t}(j) + \ln z_{2,t}(j), \tag{A.16}$$

for job-stayers ($s_t(j) = 1, 2$),

$$\ln z_{1,t}(j) = \mu_z^{st} + \ln z_{1,t-1}(j), \tag{A.17}$$

$$\ln z_{2,t}(j) = \begin{cases} \rho_z^{st} \ln z_{2,t-1}(j) + \epsilon_{z,t}(j),\ \ \epsilon_{zt}(j) \sim i.i.d.N(0, \sigma_z^{st2}) & w.pr.\ 1-\gamma \\ \ln z_{2,t-1}(j) & w.pr.\ \gamma \end{cases} \tag{A.18}$$

and for job-changers ($s_t(j) = 3$),

$$\ln z_{1,t}(j) = \mu_z^{ch} + \ln z_{1,t-1}(j), \tag{A.19}$$

$$\ln z_{2,t}(j) = \ln z_{2,t-1}(j) + \epsilon_{z,t}(j),\ \ \epsilon_{z,t}(j) \sim i.i.d.U[-U_z^{ch}, U_z^{ch}]. \tag{A.20}$$

4. (Stochastic aggregation) Given the aggregate policy function $f^{(m)}$ and the individual policy function $h^{(m)}$, the model economy is simulated with N households for T periods. I numerically integrate individual real wages according to the definition (A.21) to

recover the aggregate real wage:

$$\tilde{W}_t = \left\{ \int_0^1 \left(\frac{\tilde{w}_t(j)}{z_t(j)} \right)^{1-\theta_w} dj \right\}^{\frac{1}{1-\theta_w}}. \tag{A.21}$$

The simulation delivers the series of aggregate variables $\{\mathbf{S}_t^{(m)}, \mathbf{X}_t^{(m)}\}_{t=T_0+1}^{T}$. The initial cross-sectional wage distribution is set at one in the stationary equilibrium, and the initial T_0 periods are discarded. I set $N = 10,000$, $T = 2,200$, and $T_0 = 200$.[3] When random shocks are drawn, I adjust the number of workers in each state so that it is equal to the respective number in the ergodic distribution following the method proposed by Heer and Maussner (2009).

5. (Updating) Using the simulated variables $\{\mathbf{S}_t^{(m)}, \mathbf{X}_t^{(m)}\}_{t=T_0+1}^{T}$, the suggested coefficients $\hat{\mathbf{B}}$ are obtained by running the OLS of the ALM (A.2). The coefficients $\mathbf{B}^{(m+1)}$ are updated according to the rule:

$$\mathbf{B}^{(m+1)} = \lambda \hat{\mathbf{B}} + (1-\lambda)\mathbf{B}^{(m)}, \tag{A.22}$$

where λ is the weight for updating. I set λ to 0.1.

6. Repeat from step 1 to step 5 until convergence criteria for the coefficients $\mathbf{B}$ are attained.

Convergence criteria of ALM. I use two convergence criteria for the ALM coefficients $\mathbf{B}$. First, I repeat iterations until the maximum quadratic distance between the original and updated coefficients becomes smaller than 10^{-5}. In addition, to guarantee the accuracy of the

[3] I confirm that the computation results do not change even if I further increase N or T.

ALM, I check whether the model's dynamics do not change over iterations. More specifically, I verify that the changes in the mean, the standard deviation, and the first-order autocorrelation of each aggregate variable from those in the previous iteration stay within $0.5 \times 10^{-2}\%$.

Accuracy of ALM. To assess the accuracy of the ALM, I use the maximum distance statistic proposed by Den Haan (2010) as well as standard measures such as the R-squared statistics (R2) and forecast error. The statistic measures the maximum distance between the aggregate state variables computed according only to the ALM for the entire time period $\{\tilde{W}_t^{alm}\}_{t=T_0+1}^T$, and those derived from equilibrium conditions in the simulation $\{\tilde{W}_t\}_{t=T_0+1}^T$:

$$maxDH = \max_{t \in [T_0+1, T]} |\ln \tilde{W}_t - \ln \tilde{W}_t^{alm}|. \tag{A.23}$$

Den Haan (2010) proposes using the statistic rather than R^2 to check the accuracy of the ALM because R^2 only measures the average error in the one-period-ahead forecast.

The accuracy of the converged ALM under the calibrated Taylor rule is reported in Table A.1. The R^2 is slightly lower than that in previous studies that report the values such as above 0.99 by forecasting the aggregate capital (e.g., Krusell and Smith 1998). One reason for the slightly lower R^2 in my model is that the autocorrelation of real wage is lower than that of capital, and therefore the explanatory power of the ALM tends to be lower. However, it does not necessarily indicate a low forecast accuracy. In fact, the mean absolute forecast error (MAFE) is smaller than 0.05%, verifying the accuracy of the forecasting rule. The Den Haan (2010) statistics is around 0.47%, which indicates that the cumulative error of agents' prediction of the aggregate real wage is smaller than 0.5% over 2,000 periods. The value is

Table A.1: Accuracy of forecasting rule

	Forecast variable	R^2	MAFE (%)	maxDH (%)	aveDH (%)
Baseline	Π_t^w	0.9375	0.0449	0.4660	0.1033

Notes: The R^2 is computed as the average across exogenous states as follows: $R^2 = 1 - \sum_{t=T_0+1}^{T}(\ln \tilde{W}_t - \ln \tilde{W}_t^{fore})^2 / \sum_{t=T_0+1}^{T}(\ln \tilde{W}_t - \overline{\ln \tilde{W}})^2$ with $\overline{\ln \tilde{W}} = \sum_{t=T_0+1}^{T} \ln \tilde{W}_t/(T-T_0)$. The mean absolute forecast error $MAFE$ is defined as $MAFE = \sum_{t=T_0+1}^{T} \; |\ln \tilde{W}_t - \ln \tilde{W}_t^{fore}|/(T-T_0)$. Notice that both statistics measure the accuracy of the one-period-ahead forecast $\tilde{W}_t^{fore}$. On the other hand, $maxDH$ refers to the maximum Den Haan (2010) statistic defined in (A.23), whereas the average Den Haan (2010) statistics $aveDH$ is defined as $aveDH = \sum_{t=T_0+1}^{T} \; |\ln \tilde{W}_t - \ln \tilde{W}_t^{alm}|/(T-T_0)$.

on the same order as those of previous studies.[4]

A.2 Global solution method

Global solution method. I use a global solution method to solve each of the aggregate and individual parts of the model. In this regard, a local solution method, such as the perturbation method, cannot be applied due to the nonlinearity of my model. More precisely, since the fixed cost for wage adjustments causes the individual policy function to be kinked, the function is not differentiable.

For that reason, I apply the policy function iteration method of Coleman (1990) to the aggregate problem. The version I use is the time iteration method. To this end, Richter et al. (2014), who compare several variations of the policy function iteration method, argue that the time iteration method performs in a balanced way in terms of accuracy, speed, and robustness. To solve the individual problem, I employ the value function iteration method. In both methods, I discretize the state spaces and numerically search for the functions that

[4]For example, Den Haan (2010) compares several computation algorithms to solve a heterogeneous agent model, and finds that the Krusell-Smith algorithm with stochastic aggregation, which is the most accurate one, gives around 0.2% as the maximum Den Haan (2010) statistic and 0.05% as the average one over 10,000 periods.

Table A.2: Time-series moments

	σ (%)		ρ		ρ_Y	
	Data	Model	Data	Model	Data	Model
Y	1.13	1.13	0.87	0.87	1.00	1.00
L	1.35	1.17	0.94	0.88	0.86	1.00
C	0.95	1.13	0.89	0.87	0.91	1.00
π^p	0.22	0.28	0.68	0.95	0.34	0.84
π^w	0.20	0.29	0.55	0.91	0.55	0.88
i	0.55	0.64	0.94	0.93	0.49	0.94

Notes: The standard deviation σ, first-order autocorrelation ρ, and correlation with output ρ_Y are reported. In the data series, Y is the real GDP per capita, L is the total hours worked per capita, C is the real personal consumption expenditure per capita, π^p is the GDP deflator, π^w is the earnings per hour in the non-farm business sector, and i is the effective federal funds rate. Y, L, and C are taken as log and detrended by the HP-filter. π^p, π^w, and i are the quarterly rates. The sample spans from 1987Q4 to 2008Q4. The start point of the sample corresponds to the Greenspan Era whereas the end point is determined to exclude the ZLB periods. To compute the moments of the model, the inflation target Π^* is set equal to the mean of the inflation rates during the sample periods.

satisfy the equilibrium conditions. To evaluate expectations, I use linear interpolation and numerical integration over the discretized grid points.

A.3 Welfare evaluation

Once I solve for equilibrium, I conduct the stochastic simulations to evaluate welfare losses. In this step, I approximate the unconditional expectation operator in social welfare function by taking the mean of the simulated series.

A.4 Time-series moments

Table A.2 compares the time-series moments of the data and the model while cross-sectional moments are reported in the mail article. Though the model lacks some of the ingredients and sources of shocks that are often introduced in a medium-scale DSGE model to match the data, it captures salient features of the data. These includes: (i) low standard deviation of inflation and wage growth relative to that of output and labor input, (ii) moderate persistence of each variable, and (iii) comovements among variables.[5]

B Details of baseline heterogeneous agent model

B.1 Decomposition of welfare loss

In this appendix, I derive analytical expression for the source of welfare loss in the model. The approach adopted here follows Rotmberg and Woodford (1997) and Erceg et al. (2000). I take the second order Taylor expansion of the current social welfare SW_t around the deterministic steady state:

$$
\begin{aligned}
SW_t &\equiv \ln C_t - \frac{1}{1+1/\eta}\int_0^1 h_t(j)^{1+1/\eta} dj \qquad \text{(A.24)}\\
&\approx S\bar{W} + \left(\frac{dC_t}{\bar{C}}\right) - \frac{1}{2}\left(\frac{dC_t}{\bar{C}}\right)^2 \\
&\quad - \bar{h}^{1+1/\eta}\left\{\int_0^1 \left(\frac{dh_t(j)}{\bar{h}}\right) dj - \frac{1}{2\eta}\int_0^1 \left(\frac{dh_t(j)}{\bar{h}}\right)^2 dj\right\}, \qquad \text{(A.25)}\\
&\text{where} \quad \frac{da_t}{\bar{a}} \equiv \frac{a_t - \bar{a}}{\bar{a}}.
\end{aligned}
$$

$\bar{a}$ is the value of variable a_t in the deterministic steady state where prices and wages are flexible and any exogenous shocks are muted.

[5] There are several dimensions of the data that the model cannot precisely match. First, the model cannot match the different dynamics of output, labor input, and consumption, presumably because capital investment and productivity shocks are abstracted. Second, the model generates a slightly higher standard deviation of inflation and wage growth than that of the data.

Following Erceg et al. (2000), two approximations are used:

$$\frac{da_t}{\bar{a}} \equiv \frac{a_t - \bar{a}}{\bar{a}} \approx \hat{a}_t + \frac{1}{2}\hat{a}_t^2, \tag{A.26}$$
$$\text{where} \quad \hat{a}_t \equiv \ln a_t - \ln \bar{a}$$

and, if $a_t = \left(\int_0^1 a_t(j)^{\varphi} dj\right)^{1/\varphi}$, then

$$\hat{a}_t \approx \mathbb{E}_j[\hat{a}_t(j)] + \frac{1}{2}\varphi \mathbb{V}ar_j(\hat{a}_t(j)) \tag{A.27}$$

where $\mathbb{E}_j[\cdot]$ and $\mathbb{V}ar_j(\cdot)$ are the expectation and the variance across j.

Substituting (A.26) into (A.25) yields

$$\begin{aligned}
SW_t \approx\; & S\bar{W} + \left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) - \frac{1}{2}\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right)^2 \\
& - \bar{h}^{1+1/\eta}\left\{\int_0^1 \left(\hat{h}_t(j) + \frac{1}{2}\hat{h}_t(j)^2\right) dj + \frac{1}{2\eta}\int_0^1 \left(\hat{h}_t(j) + \frac{1}{2}\hat{h}_t(j)^2\right)^2 dj\right\} \\
\approx\; & S\bar{W} + \hat{C}_t - \bar{h}^{1+1/\eta}\left\{\int_0^1 \left(\hat{h}_t(j) + \frac{1}{2}\left(1 + \frac{1}{\eta}\right)\hat{h}_t(j)^2\right) dj\right\} \\
=\; & S\bar{W} + \hat{C}_t - \bar{h}^{1+1/\eta}\left\{\int_0^1 \left((\hat{l}_t(j) - \hat{z}_t(j)) + \frac{1}{2}\left(1 + \frac{1}{\eta}\right)(\hat{l}_t(j) - \hat{z}_t(j))^2\right) dj\right\},
\end{aligned} \tag{A.28}$$

where the last equality comes from $l_t(j) = z_t(j)h_t(j)$. Notice that the third and higher order terms are ignored since I focus on the second-order approximation.

By (A.26) and (A.27), cross-sectional moments of labor service are rearranged to

$$\begin{aligned}
\int_0^1 \hat{l}_t(j)dj &= \mathbb{E}_j[\hat{l}_t(j)] \\
&\approx \hat{L}_t - \frac{1}{2}\frac{\theta_w - 1}{\theta_w}\mathbb{V}ar_j[\hat{l}_t(j)], && \text{(A.29)} \\
\int_0^1 \hat{l}_t(j)^2 dj &= \mathbb{E}_j[\hat{l}_t(j)^2] \\
&= \mathbb{V}ar_j[\hat{l}_t(j)] - \mathbb{E}_j[\hat{l}_t(j)]^2 \\
&\approx \mathbb{V}ar_j[\hat{l}_t(j)] - \left(\hat{L}_t - \frac{1}{2}\frac{\theta_w - 1}{\theta_w}\mathbb{V}ar_j[\hat{l}_t(j)]\right)^2 \\
&\approx \mathbb{V}ar_j[\hat{l}_t(j)] - \hat{L}_t^2, && \text{(A.30)} \\
\int_0^1 \hat{l}_t(j)\hat{z}_t(j)dj &= \mathbb{E}_j[\hat{l}_t(j)\hat{z}_t(j)] \\
&= \mathbb{C}ov_j[\hat{l}_t(j)\hat{z}_t(j)] - \mathbb{E}_j[\hat{l}_t(j)]\mathbb{E}_j[\hat{z}_t(j)] \\
&= \mathbb{C}ov_j[\hat{l}_t(j)\hat{z}_t(j)], && \text{(A.31)}
\end{aligned}$$

where the last equality holds because $\mathbb{E}_j[\hat{z}_t(j)] = 0$.

Using (A.29)–(A.31), (A.28) is rearranged to

$$\begin{aligned}
SW_t \approx S\bar{W} + \hat{C}_t - \bar{h}^{1+1/\eta}\Bigg\{&\left(\hat{L}_t - \frac{1}{2}\frac{\theta_w - 1}{\theta_w}\mathbb{V}ar_j[\hat{l}_t(j)]\right) + \frac{1}{2}\left(1 + \frac{1}{\eta}\right)\left(\mathbb{V}ar_j[\hat{l}_t(j)] + \hat{L}_t^2\right) \\
&- \left(1 + \frac{1}{\eta}\right)\mathbb{C}ov_j[\hat{l}_t(j), \hat{z}_t(j)] + \frac{1}{2}\left(1 + \frac{1}{\eta}\right)\mathbb{E}_j[\hat{z}_t(j)^2]\Bigg\} \\
= S\bar{W} + \Bigg\{&\hat{C}_t - \bar{h}^{1+1/\eta}\hat{L}_t - \bar{h}^{1+1/\eta}\frac{1}{2}\left(1 + \frac{1}{\eta}\right)\hat{L}_t^2\Bigg\} - \bar{h}^{1+1/\eta}\frac{1}{2}\left(\frac{1}{\theta_w} + \frac{1}{\eta}\right)\mathbb{V}ar_j[\hat{l}_t(j)] \\
&+ \bar{h}^{1+1/\eta}\left(1 + \frac{1}{\eta}\right)\mathbb{C}ov_j[\hat{l}_t(j), \hat{z}_t(j)] + \bar{h}^{1+1/\eta}\frac{1}{2}\left(1 + \frac{1}{\eta}\right)\mathbb{E}_j[\hat{z}_t(j)^2], \qquad \text{(A.32)}
\end{aligned}$$

On the firm side, the aggregation of L_t and Y_t implies

$$\hat{L}_t \approx \mathbb{E}_i[\hat{l}_t(i)] + \frac{1}{2}\mathbb{V}ar_i[\hat{l}_t(i)], \tag{A.33}$$

$$\hat{Y}_t \approx \mathbb{E}_i[\hat{y}_t(i)] + \frac{1}{2}\frac{\theta_p - 1}{\theta_p}\mathbb{V}ar_i[\hat{y}_t(i)], \tag{A.34}$$

where i denotes the index for firm. From $y_t(i) = l_t(i)$, (A.33) and (A.34) lead to

$$\hat{L}_t \approx \hat{Y}_t + \frac{1}{2\theta_p}\mathbb{V}ar_i[\hat{y}_t(i)], \tag{A.35}$$

$$\hat{L}_t^2 \approx \hat{Y}_t^2. \tag{A.36}$$

Moreover, the individual labor demand implies

$$\mathbb{V}ar_i[\hat{y}_t(i)] = \theta_p^2 \mathbb{V}ar_i[\hat{p}_t(i)]. \tag{A.37}$$

In the Calvo model, it is shown that

$$\begin{aligned} \mathbb{V}ar_i[\hat{p}_t(i)] &= \mathbb{E}_i\left[(\hat{p}_t(i) - \mathbb{E}_i[\hat{p}_t(i)])^2\right] \\ &= \xi\mathbb{E}_i\left[(\hat{p}_{t-1}(i) - \mathbb{E}_i[\hat{p}_t(i)])^2\right] + (1-\xi)\mathbb{E}_i\left[(\hat{p}_t^*(i) - \mathbb{E}_i[\hat{p}_t(i)])^2\right] \\ &\approx \xi\left(\mathbb{V}ar_i[\hat{p}_{t-1}(i)] + (\hat{\Pi}_t^p)^2\right) + (1-\xi)\left(\frac{\xi(\Pi^*)^{\theta_p - 1}}{1 - \xi(\Pi^*)^{\theta_p - 1}}\right)^2 (\hat{\Pi}_t^p)^2. \end{aligned} \tag{A.38}$$

The derivation follows Rotmberg and Woodford (1997) and Erceg et al. (2000). Notice that the second term in the right-hand side of (A.38) is adjusted for the non-zero trend inflation

Π^*. Taking the unconditional expectation of (A.38) leads to,

$$\mathbb{E}\left[\mathbb{V}ar_i[\hat{p}_t(i)]\right] \approx \underbrace{\frac{\xi\left\{1-\xi(\Pi^*)^{\theta_p-1}+\xi(\Pi^*)^{\theta_p-1}((\Pi^*)^{\theta_p-1}-1)\right\}}{(1-\xi)(1-\xi(\Pi^*)^{\theta_p-1})^2}}_{c_\Pi}\mathbb{E}[(\hat{\Pi}_t^p)^2]. \tag{A.39}$$

Notice that $c_\Pi = \xi/(1-\xi)^2$ if $\Pi^* = 1$. Then, (A.35), (A.37), and (A.39) yield

$$\mathbb{E}[\hat{L}_t] \approx \mathbb{E}[\hat{Y}_t] + \frac{\theta_p}{2}c_\Pi\mathbb{E}[(\hat{\Pi}_t^p)^2]. \tag{A.40}$$

The resource constraint implies

$$\hat{C}_t = \hat{Y}_t - \ln(1+M_t), \tag{A.41}$$

Using (A.36), (A.40), and (A.41), the unconditional expectation of the second term in (A.32) is rearranged to

$$\begin{aligned}
&\mathbb{E}\left[\hat{C}_t - \bar{h}^{1+1/\eta}\hat{L}_t - \bar{h}^{1+1/\eta}\frac{1}{2}\left(1+\frac{1}{\eta}\right)\hat{L}_t^2\right] \\
&\approx \mathbb{E}[\hat{Y}_t] - \mathbb{E}[\ln(1+M_t)] - \bar{h}^{1+1/\eta}\left(\mathbb{E}[\hat{Y}_t] + \frac{\theta_p}{2}c_\Pi\mathbb{E}[(\hat{\Pi}_t^p)^2]\right) - \bar{h}^{1+1/\eta}\frac{1}{2}\left(1+\frac{1}{\eta}\right)\mathbb{E}[\hat{Y}_t^2] \\
&= \left(1-\bar{h}^{1+1/\eta}\right)\mathbb{E}[\hat{Y}_t] - \mathbb{E}[\ln(1+M_t)] - \bar{h}^{1+1/\eta}\frac{\theta_p}{2}c_\Pi\mathbb{E}[(\hat{\Pi}_t^p)^2] - \bar{h}^{1+1/\eta}\frac{1}{2}\left(1+\frac{1}{\eta}\right)\mathbb{E}[\hat{Y}_t^2].
\end{aligned} \tag{A.42}$$

Taking the difference from the social welfare under flexible prices and wages, welfare loss is written as

$$
\begin{aligned}
SW - SW^f &= \frac{1}{1-\beta}\mathbb{E}[SW_t] - \frac{1}{1-\beta}\mathbb{E}[SW_t^f] \\
&\approx \underbrace{\frac{1-\bar{h}^{1+1/\eta}}{1-\beta}}_{\omega_{gap}} \left(\mathbb{E}[\hat{Y}_t] - \mathbb{E}[\hat{Y}_t^f]\right) - \underbrace{\frac{1}{1-\beta}}_{\omega_m} \mathbb{E}[\ln(1+M_t)] \\
&\quad - \underbrace{\frac{\bar{h}^{1+1/\eta}}{1-\beta}\frac{\theta_p}{2}c_\Pi}_{\omega_\pi} \mathbb{E}[(\hat{\Pi}_t^p)^2] - \underbrace{\frac{\bar{h}^{1+1/\eta}}{1-\beta}\frac{1}{2}\left(1+\frac{1}{\eta}\right)}_{\omega_y} \left(\mathbb{E}[\hat{Y}_t^2] - \mathbb{E}[(\hat{Y}_t^f)^2]\right) \\
&\quad - \underbrace{\frac{\bar{h}^{1+1/\eta}}{1-\beta}\frac{1}{2}\left(\frac{1}{\theta_w}+\frac{1}{\eta}\right)}_{\omega_l} \left(\mathbb{V}ar_j[\hat{l}_t(j)] - \mathbb{V}ar_j[\hat{l}_t^f(j)]\right) \\
&\quad + \underbrace{\frac{\bar{h}^{1+1/\eta}}{1-\beta}\left(1+\frac{1}{\eta}\right)}_{\omega_{lz}} \left(\mathbb{C}ov_j[\hat{l}_t(j), \hat{z}_t(j)] - \mathbb{C}ov_j[\hat{l}_t^f(j), \hat{z}_t(j)]\right). \qquad \text{(A.43)}
\end{aligned}
$$

Note that I abbreviate the unconditional expectation operator $\mathbb{E}[\cdot]$ in front of $\mathbb{V}ar_j[\cdot]$ and $\mathbb{C}ov_j[\cdot]$ for the ease of notation.

Here,

$$
\begin{aligned}
\mathbb{E}[\hat{Y}_t] - \mathbb{E}[\hat{Y}_t^f] &= \mathbb{E}[\ln Y_t - \ln Y_t^f] \\
&= \mathbb{E}[\ln Y_t^{gap}], \qquad \text{(A.44)} \\
\mathbb{E}[\hat{Y}_t^2] - \mathbb{E}[(\hat{Y}_t^f)^2] &= \mathbb{E}[(\hat{Y}_t - \hat{Y}_t^f)^2 + 2\hat{Y}_t\hat{Y}_t^f] \\
&= \mathbb{V}ar[\ln Y_t^{gap}] + \mathbb{E}[\ln Y_t^{gap}]^2 + 2\mathbb{E}[\hat{Y}_t\hat{Y}_t^f] \\
&\approx \mathbb{V}ar\left[\ln Y_t^{gap}\right] \qquad \text{(A.45)} \\
\mathbb{E}[(\hat{\Pi}_t^p)^2] &= \mathbb{V}ar[\ln \Pi_t^p] + \mathbb{E}[\hat{\Pi}_t^p]^2 \\
&\approx \mathbb{V}ar[\ln \Pi_t^p], \qquad \text{(A.46)}
\end{aligned}
$$

I assume that $\mathbb{E}[\ln Y_t^{gap}]$ and $\mathbb{E}[\hat{\Pi}_t^p]$ are of second-order according to the insights of Erceg et al. (2000), and the higher order terms are neglected in the second-order approximation. Moreover,

$$
\begin{aligned}
\mathbb{V}ar_j[\hat{l}_t(j)] &= \mathbb{E}_j\left[(\hat{l}_t(j) - \mathbb{E}_j[\hat{l}_t(j)])^2\right] \\
&= \mathbb{E}_j\left[(\ln l_t(j) - \mathbb{E}_j[\ln l_t(j)])^2\right] \\
&= \mathbb{V}ar_j[\ln l_t(j)], \qquad \text{(A.47)} \\
\mathbb{C}ov_j[\hat{l}_t(j), \hat{z}_t(j)] &= \mathbb{E}_j\left[(\hat{l}_t(j) - \mathbb{E}_j[\hat{l}_t(j)])(\hat{z}_t(j) - \mathbb{E}_j[\hat{z}_t(j)])\right] \\
&= \mathbb{E}_j\left[(\ln l_t(j) - \mathbb{E}_j[\ln l_t(j)])(\ln z_t(j) - \mathbb{E}_j[\ln z_t(j)])\right] \\
&= \mathbb{C}ov_j[\ln l_t(j), \ln z_t(j)]. \qquad \text{(A.48)}
\end{aligned}
$$

Using these equations, the welfare is expressed in the following equation:

$$
\begin{aligned}
SW - SW^f = & \underbrace{\omega_{gap}\mathbb{E}[\ln Y_t^{gap}] - \omega_m \mathbb{E}[\ln(1 + M_t)]}_{\text{aggregate mean}} \\
& - \underbrace{\omega_\pi \mathbb{V}ar[\ln \Pi_t] - \omega_y \mathbb{V}ar[\ln Y_t^{gap}]}_{\text{aggregate variance}} \\
& - \underbrace{\omega_l \left(\mathbb{V}ar_j[\ln l_t(j)] - \mathbb{V}ar_j[\ln l_t^f(j)]\right)}_{\text{cross-sectional variance}} \\
& + \underbrace{\omega_{lz}\left(\mathbb{C}ov_j[\ln l_t(j), \ln z_t(j)] - \mathbb{C}ov_j[\ln l_t^f(j), \ln z_t(j)]\right)}_{\text{cross-sectional covariance}} \\
& + (\text{higher order terms}). \qquad \text{(A.49)}
\end{aligned}
$$

(A.49) indicates four main sources of welfare loss in this economy: (i) aggregate mean; (ii) aggregate variance; (iii) cross-sectional variance; and (iv) cross-sectional covariance.

B.2 Allocations under flexible wages

Under flexible wages, real wage is equated to the marginal rate of substitution (MRS) along with individual labor productivity and the steady-state markup:

$$\frac{w_t(j)}{P_t} = \mu_w z_t(j) \underbrace{h_t^{1/\eta} C_t}_{mrs_t(j)}. \tag{A.50}$$

Individual labor demand along with $l_t(j) = z_t(j)h_t(j)$ implies

$$\begin{aligned} l_t(j) &= \left(\frac{\mu_w z_t(j) h_t^{1/\eta} C_t / z_t(j)}{W_t/P_t}\right)^{-\theta_w} L_t \\ \Leftrightarrow \quad l_t(j) &= z_t(j)^{\frac{\theta_w/\eta}{1+\theta_w/\eta}} \left(\frac{L_t^{1/\theta_w} W_t/P_t}{\mu_w C_t}\right)^{\frac{\theta_w}{1+\theta_w/\eta}}. \end{aligned} \tag{A.51}$$

Substituting (A.51) into (A.50), real wage is written as

$$\begin{aligned} \frac{w_t(j)}{P_t} &= \mu_w z_t(j) \left\{ z_t(j)^{\frac{\theta_w/\eta}{1+\theta_w/\eta}} \left(\frac{L_t^{1/\theta_w} W_t/P_t}{\mu_w C_t}\right)^{\frac{\theta_w}{1+\theta_w/\eta}} \frac{1}{z_t(j)} \right\}^{1/\eta} C_t \\ &= \left\{ z_t(j)^{1+\theta_w/\eta - 1/\eta} \left(\frac{W_t}{P_t}\right)^{\theta_w/\eta} \mu_w L_t^{1/\eta} C_t \right\}^{\frac{1}{1+\theta_w/\eta}}. \end{aligned} \tag{A.52}$$

Using the definition of the aggregate wage index, (A.52) implies

$$
\begin{aligned}
\frac{W_t}{P_t} &= \left[\int_0^1 \left\{ z_t(j)^{-1/\eta} \left(\frac{W_t}{P_t}\right)^{\theta_w/\eta} \mu_w L_t^{1/\eta} C_t \right\}^{\frac{1-\theta_w}{1+\theta_w/\eta}} dj \right]^{\frac{1}{1-\theta_w}} \\
&= \left(\int_0^1 z_t(j)^{-\frac{1-\theta_w}{\eta(1+\theta_w/\eta)}} dj \right)^{\frac{1}{1-\theta_w}} \left\{ \left(\frac{W_t}{P_t}\right)^{\theta_w/\eta} \mu_w L_t^{1/\eta} C_t \right\}^{\frac{1}{1+\theta_w/\eta}} \\
\Leftrightarrow \quad \frac{W_t}{P_t} &= \underbrace{\left(\int_0^1 z_t(j)^{-\frac{1-\theta_w}{\eta(1+\theta_w/\eta)}} dj \right)^{\frac{1+\theta/\eta}{1-\theta_w}}}_{c_z} \mu_w L_t^{1/\eta} C_t. \qquad \text{(A.53)}
\end{aligned}
$$

Notice that the coefficient of the right-hand side c_z is constant over time. In other words, under flexible wages, the aggregate dynamics are summarized into the relation among the aggregate variables through worker heterogeneity is present. Substituting (A.53) into (A.52), the individual wages have an analytical expression:

$$
\frac{w_t(j)}{P_t} = z_t(j)^{\frac{1+\theta_w/\eta - 1/\eta}{1+\theta_w/\eta}} c_z^{\frac{\theta_w/\eta}{1+\theta_w/\eta}} \mu_w L_t^{1/\eta} C_t. \qquad \text{(A.54)}
$$

Furthermore, under flexible prices, the price markup remains constant:

$$
\frac{1}{\mu_p} = \frac{W_t}{P_t}. \qquad \text{(A.55)}
$$

Along with the market clearing conditions $Y_t = L_t = C_t$, it can be verified that

$$
l_t(j) = \left\{ (\mu_w \mu_p)^{-\theta_w} z_t(j)^{\theta_w/\eta} L_t^{1-\theta_w} \right\}^{\frac{1}{1+\theta_w/\eta}}, \qquad \text{(A.56)}
$$

Finally, the output under flexible prices and wages is given by

$$Y_t = (c_z \mu_w \mu_p)^{-\frac{1}{1+1/\eta}}, \tag{A.57}$$

C Sensitivity analyses

C.1 Sensitivity to specifications of wage adjustment cost

I investigate alternative specifications of wage adjustment cost. In the baseline case in the main article, the wage adjustment cost is assumed to be the sum of a fixed cost and a linear cost proportional to the size of nominal wage changes:

$$m_t(j) = (m_0^+ + m_1^+ \ln \Pi_t^w(j)) \mathbf{1}_{\{w_t(j) > w_{t-1}(j)\}} + (m_0^- + m_1^- \ln \Pi_t^w(j)) \mathbf{1}_{\{w_t(j) < w_{t-1}(j)\}}. \tag{A.58}$$

As alternative cases, I consider a fixed cost only:

$$m_t(j) = m_0^+ \mathbf{1}_{\{w_t(j) > w_{t-1}(j)\}} + m_0^- \mathbf{1}_{\{w_t(j) < w_{t-1}(j)\}}, \tag{A.59}$$

and the sum of fixed and quadratic costs:

$$m_t(j) = \left(m_0^+ + \frac{m_2^+}{2} \ln \Pi_t^w(j)^2 \right) \mathbf{1}_{\{w_t(j) > w_{t-1}(j)\}} + \left(m_0^- + \frac{m_2^-}{2} \ln \Pi_t^w(j)^2 \right) \mathbf{1}_{\{w_t(j) < w_{t-1}(j)\}}. \tag{A.60}$$

Several points should be noted regarding the alternative specifications. First, I allow for asymmetry for positive and negative wage changes in each specification. The setting

Table A.3: Estimated parameters in different specifications of wage adjustment cost

Description	Symbol	Baseline (fixed+linear)	Alternative 1 (fixed only)	Alternative 2 (fixed+quad.)
For job-stayers				
Fixed cost for positive wage changes	m_0^+	0.012	0.011	0.015
Fixed cost for negative wage changes	m_0^-	0.097	0.155	0.247
Linear cost for positive wage changes	m_1^+	0.675	—	—
Linear cost for negative wage changes	m_1^-	3.626	—	—
Quadratic cost for positive wage changes	m_2^+	—	—	3.464
Quadratic cost for negative wage changes	m_2^-	—	—	48.347
Prob. of not subject to adjustment cost	ζ	0.011	0.010	0.008
Prob. of not receiving productivity shock	γ	0.848	0.710	0.814
Persistence of productivity	ρ_z^{st}	0.698	0.894	0.801
Trend growth of productivity (% per year)	μ_z^{st}	1.692	0.444	1.250
S.D. of innovations to productivity	σ_z^{st}	0.097	0.050	0.096
For job-changers				
Trend growth of productivity (% per year)	μ_z^{ch}	8.085	13.945	8.985

nests symmetric costs when the parameter values for positive and negative wage changes are identical. Second, I include a fixed cost in each specification. This is because a fixed cost is essential to replicate the infrequent wage changes observed in the data. It is also worth noting that the setting nests the case without a fixed cost when the corresponding parameters are zero. Third, fixed, linear, and quadratic costs are widely used in the literature on price- and wage-setting.[6]

I estimate the parameters for wage adjustment costs along with those for cross-sectional wage distribution in each specification adopting the same procedure as the baseline case in the main article.[7] The estimated parameter values are listed in Table A.3, whereas the target and model moments are reported in Table A.4 and A.5.

The first thing to note in these tables is that alternative specifications can replicate major features of nominal wage adjustments observed in the data. Quantitatively, however, the

[6] Although some studies use a linex function to approximate adjustment cost, especially in a representative agent model (e.g., Kim and Ruge-Murcia 2009, Aruoba et al. 2017), I find the function becomes highly convex and unstable in practice when applied to dispersed individual wage changes in the data.

[7] As in the baseline case, I restrict $U_z^{ch} = 1$ because equilibrium tends to be unstable under a large U_z^{ch}.

Table A.4: Targeted moments in different specifications of wage adjustment cost

Moment	Quarterly changes			
	Data	Baseline (fixed+linear)	Alternative 1 (fixed only)	Alternative 2 (fixed+quad.)
Job-stayers				
Probability of positive wage changes	0.185	0.187	0.191	0.188
Probability of negative wage changes	0.009	0.011	0.011	0.009
Median size of positive wage changes	0.033	0.041	0.045	0.044
Median size of negative wage changes	–0.077	–0.074	–0.083	–0.076
Mean size of positive wage changes	0.057	0.055	0.052	0.056
Mean size of negative wage changes	–0.087	–0.080	–0.080	–0.077
Median of unconditional wage changes	0.000	0.000	0.000	0.000
Mean of unconditional wage changes	0.010	0.010	0.009	0.010
S.D. of unconditional wage changes	0.037	0.029	0.025	0.027
Job-changers				
Probability of positive wage changes	0.527	0.589	0.598	0.582
Probability of negative wage changes	0.374	0.402	0.392	0.410
Median size of positive wage changes	0.167	0.191	0.193	0.181
Median size of negative wage changes	–0.136	–0.173	–0.167	–0.175
Mean size of positive wage changes	0.235	0.209	0.218	0.204
Mean size of negative wage changes	–0.165	–0.87	–0.187	–0.194
Median of unconditional wage changes	0.023	0.043	0.043	0.032
Mean of unconditional wage changes	0.063	0.048	0.057	0.040
S.D. of unconditional wage changes	0.259	0.238	0.246	0.242
All workers				
Probability of positive wage changes	0.206	0.207	0.210	0.207
Probability of negative wage changes	0.032	0.030	0.029	0.028
Median of unconditional wage changes	0.000	0.000	0.000	0.000
Mean of unconditional wage changes	0.012	0.012	0.012	0.012
S.D. of unconditional wage changes	0.067	0.060	0.060	0.060
Loss in moment-matching method	—	1.645×10^{-3}	2.266×10^{-3}	1.821×10^{-3}

Notes: The data moments and the moments of the baseline specification are identical to those reported in the main article.

baseline case with a combination of fixed and linear costs delivers the best fit to the data, as shown in the loss in the moment-matching method, i.e., the sum of quadratic distances between the data and model moments.

When it comes to the performance of each alternative specification, the specification with a fixed cost only (Alternative 1) fails to simultaneously match the frequency and size of wage changes observed in the data. In particular, Table A.4 and A.5 indicate that the median wage changes for job-stayers tends to be larger than those in the data. This result can be understood by a standard Ss interpretation, i.e., only workers who have strong desires for

Table A.5: Targeted moments in different specifications of wage adjustment cost (cont.)

Moment	Yearly changes			
	Data	Baseline (fixed+linear)	Alternative 1 (fixed only)	Alternative 2 (fixed+quad.)
Job-stayers				
Probability of positive wage changes	0.639	0.638	0.635	0.635
Probability of negative wage changes	0.024	0.035	0.036	0.029
Median size of positive wage changes	0.035	0.046	0.046	0.046
Median size of negative wage changes	–0.066	–0.072	–0.077	–0.074
Mean size of positive wage changes	0.063	0.067	0.063	0.068
Mean size of negative wage changes	–0.073	–0.080	–0.076	–0.077
Median of unconditional wage changes	0.024	0.035	0.036	0.038
Mean of unconditional wage changes	0.039	0.040	0.038	0.041
S.D. of unconditional wage changes	0.065	0.056	0.048	0.054
Job-changers				
Probability of positive wage changes	0.568	0.610	0.626	0.608
Probability of negative wage changes	0.380	0.371	0.353	0.372
Median size of positive wage changes	0.185	0.202	0.205	0.195
Median size of negative wage changes	–0.158	–0.161	–0.159	–0.166
Mean size of positive wage changes	0.261	0.223	0.233	0.222
Mean size of negative wage changes	–0.185	–0.178	–0.180	–0.186
Median of unconditional wage changes	0.046	0.065	0.068	0.058
Mean of unconditional wage changes	0.080	0.070	0.082	0.066
S.D. of unconditional wage changes	0.293	0.238	0.246	0.243
All workers				
Probability of positive wage changes	0.627	0.634	0.634	0.630
Probability of negative wage changes	0.087	0.095	0.092	0.090
Median of unconditional wage changes	0.025	0.036	0.037	0.039
Mean of unconditional wage changes	0.044	0.046	0.046	0.046
S.D. of unconditional wage changes	0.120	0.114	0.115	0.115
Loss in moment-matching method (redisplaying)	—	1.645×10^{-3}	2.266×10^{-3}	1.821×10^{-3}

Notes: The data moments and the moments of the baseline specification are identical to those reported in the main article.

wage adjustments pay a fixed cost, and therefore small wage changes do not occur. This interpretation is consistent with the large estimated values for fixed costs in the specification shown in Table A.3.[8] Regarding the specification with a combination of fixed and quadratic costs (Alternative 2), the mean size of wage changes and the standard deviation of unconditional wage changes are relatively small whereas the median size of wage changes is slightly high. This is because of the convexity of wage adjustment costs in this specification; a large

[8]In this specification, the estimated probability of not receiving idiosyncratic shocks ζ is somewhat low whereas the estimated persistence of productivity ρ_z^{st} is high, compared with the estimates of previous studies (e.g., Kaplan et al. 2018).

Table A.6: Optimal inflation rates in alternative settings

		Optimal Π^* in HA model	Wel. diff. <0.05% in HA model		Optimal Π^* in RA model
(1)	Baseline	2.0	[1.0	3.0]	–0.4
	Steady-state wage and price markups				
(2)	Labor subsidy $\tau_w = \mu_w$	1.8	[1.2	2.6]	–1.0
(3)	Production subsidy $\tau_p = \mu_p$	2.0	[1.0	3.0]	–1.0
(4)	$\tau_w = \mu_w$ and $\tau_p = \mu_p$	1.8	[1.2	2.4]	–1.0
	Production function				
(5)	Decreasing returns $\alpha = 0.66$	2.0	[0.6	3.2]	–0.5
	Heterogeneity in trend productivity growth				
(6)	$\mu_z^{st} = \mu_z^{ch}$	2.4	[1.2	3.8]	—

Notes: The HA model is solved at different levels of Π^* with an interval of 0.2 percentage points due to the computational burden of deriving equilibrium repeatedly at different Π^*. The RA model is solved with an interval of 0.1 percent points, and the range of Π^* is truncated at –1.0 percent because equilibrium tends to be unstable at a lower Π^*. The second and third columns report the range of Π^* within which the differences in welfare loss from the optimum are within 0.05 percent of consumption.

wage change becomes increasingly costly due to the quadratic costs. Consequently, relatively small wage changes are more likely to occur, reducing the mean size and the standard deviation of wage changes. In sum, the baseline specification of a combination of fixed and linear costs does better than alternative ones in matching the size and frequency of wage changes in a balanced manner.

C.2 Sensitivity to calibrated parameters

I provide sensitivity analyses with respect to several calibrated parameters. Specifically, I assess (i) steady-state wage and price markups, (ii) returns to scale in production, and (iii) heterogeneity in trend productivity growth. The results are summarized in Table A.6. Note that other dimensions of sensitivity are investigated in the main article.

C.2.1 Steady-state wage and price markups

Row (2)–(4) assess the effects of steady-state wage and price markups by eliminating them through labor and production subsidies. The presence of steady-state markups leads to inefficiently low output, and therefore the marginal benefits of reducing inefficiency would become larger. Notice, however, that this is the case for both reducing the distortion of wage rigidity and that of price rigidity, and therefore the consequence for the optimal Π^* is ambiguous. Quantitatively, the optimal Π^* is found to be almost unaffected, implying that the marginal effects on wage and price rigidity are almost offset each other.

C.2.2 Returns to scale in production

Row (5) investigates the decreasing returns to scale in production whereas the baseline model assumes the linear production technology.[9] It is confirmed that the baseline result is not drastically changed.

C.2.3 Heterogeneity in trend productivity growth

Row (6) reports the case in which there is no heterogeneity in trend productivity growth for job-stayers and job-changers. Since $\mu_z^{st} < \mu_z^{ch}$ in the baseline calibration, eliminating the heterogeneity means a higher μ_z^{st} and a lower μ_z^{ch} than the baseline calibration. The result indicates a higher optimal inflation in this case, implying that job changes are essential opportunities to adjust wages not only because job-changers are free from wage adjustment costs but also because they receive a higher productivity growth on average.

[9] The production function is modified to $y_t(i) = l_t(i)^\alpha$ where α is calibrated to 0.66.

C.3 Discussion on consumption heterogeneity

The baseline model focuses on the misallocations in the labor market induced by wage rigidity, whereas consumption dynamics are assumed to be summarized in the aggregate Euler equation. Though the assumption is largely due to computational burden of keeping track of the joint distribution of wages and asset holdings, one concern is that adding consumption heterogeneity might change the welfare consequences of wage rigidity.

In this regard, there are two potential interactions between wage rigidity and consumption heterogeneity. On the one hand, consumption heterogeneity would make the MRS more dispersed across households, which causes larger desires for individual wage adjustments. This channel would increase the welfare loss of wage rigidity, leading to a higher optimal inflation rate. On the other hand, wage rigidity renders wages less volatile, and if it reduces income volatility the welfare loss associated with consumption heterogeneity can be mitigated, adding downward pressure on the optimal inflation rate.[10]

Regarding the latter channel, however, I compare income volatility with and without wage rigidity, and find that it is much more volatile with wage rigidity.[11] This is because, although individual wages become less dispersed in the presence of wage rigidity, the imperfect wage adjustments lead to a larger dispersion of labor demand. In other words, wage rigidity would amplify the welfare loss associated with consumption heterogeneity. Therefore, the optimal inflation rate is conjectured to become higher if consumption heterogeneity is present, though a rigorous analysis is left for future research.

[10] In a standard incomplete market setting, welfare loss arises from the lack of consumption smoothing due to borrowing constraint.

[11] To be precise, the standard deviation of labor income $w_t(j)h_t(j)$ is 28.8% with wage rigidity at $\Pi^* = 2.1$ (%, annualized), whereas it is 17.5% without wage rigidity.

D Representative agent model with asymmetric smooth wage adjustment cost

In this appendix, I present the representative agent model with asymmetric smooth wage adjustment cost, with which I compare the baseline heterogeneous agent model in the main article. The specification largely follows that of Kim and Ruge-Murcia (2019).

Households' problem. I consider the following household problem:

$$\max_{w_t(j),c_t(j),b_t(j)} \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s\left(\ln c_{t+s}(j) - \frac{h_{t+s}(j)^{1+1/\eta}}{1+1/\eta}\right)\right], \tag{A.61}$$

$$s.t. \quad c_t(j) + \Psi_w(\Pi^w_t(j))C_t + \frac{b_t(j)}{P_t} \leq \frac{w_t(j)}{P_t}h_t(j) + Q_{t-1}R_{t-1}\frac{b_{t-1}(j)}{P_t} + \frac{\tau_t(j)}{P_t} + \Phi_t(j), \tag{A.62}$$

$$l_t(j) = z_t(j)h_t(j), \tag{A.63}$$

$$l_t(j) = \left(\frac{w_t(j)/z_t(j)}{W_t}\right)^{-\theta_w} L_t, \tag{A.64}$$

$$where \quad \Psi_w(\Pi^w_t(j)) = \phi_w\left(\frac{exp(-\psi_w(\Pi^w_t(j)-1)) + \psi_w(\Pi^w_t(j)-1) - 1}{\psi_w^2}\right). \tag{A.65}$$

$\Psi_w(\Pi^w_t(j))$ is the adjustment cost of nominal wages, which is proportional to the aggregate consumption. The notations of the other variables follow those in the baseline model. The first order condition for $w_t(j)$ takes the following form:

$$\begin{aligned} &- h_t(j)^{1/\eta}\frac{\partial h_t(j)}{\partial w_t(j)} + \lambda_t(j)\left\{\left(\frac{1}{P_t}h_t(j) + \frac{w_t(j)}{P_t}\frac{\partial h_t(j)}{\partial w_t(j)}\right) - \Psi'_w(\Pi^w_t(j))\frac{1}{w_{t-1}(j)}C_t\right\} \\ &- \beta\mathbb{E}_t\left[\lambda_{t+1}(j)\Psi'_w(\Pi^w_{t+1}(j))\left(-\frac{w_{t+1}(j)}{w_t(j)^2}\right)C_{t+1}\right] = 0, \end{aligned} \tag{A.66}$$

where $\lambda_t(j) = 1/c_t(j)$ is the Lagrangian multiplier of the budget constraint (A.62). (A.63) and (A.64) yield

$$\begin{aligned}
\frac{\partial h_t(j)}{\partial w_t(j)} &= \frac{\partial h_t(j)}{\partial l_t(j)} \frac{\partial l_t(j)}{\partial w_t(j)} \\
&= \frac{1}{z_t(j)} \left(-\theta_w \frac{l_t(j)}{w_t(j)} \right) \\
&= -\theta_w \frac{h_t(j)}{w_t(j)}.
\end{aligned} \tag{A.67}$$

Using (A.67), (A.66) is rearranged to

$$\Psi_w^{'}(\Pi_t^w(j))\Pi_t^w(j) = \beta \mathbb{E}\left[\Psi_w^{'}(\Pi_{t+1}^w(j))\Pi_{t+1}^w(j)\right] + \theta_w \left(h_t(j)^{1/\eta} c_t(j) - \frac{1}{\mu_w} \frac{w_t(j)}{P_t} \right) \frac{h_t(j)}{c_t(j)}. \tag{A.68}$$

In the symmetric equilibrium, (A.68) yields the aggregate wage Phillips curve:

$$\Psi_w^{'}(\Pi_t^w)\Pi_t^w = \beta \mathbb{E}\left[\Psi_w^{'}(\Pi_{t+1}^w)\Pi_{t+1}^w\right] + \theta_w \left(H_t^{1/\eta} C_t - \frac{1}{\mu_w} \frac{W_t}{P_t} \right) \frac{H_t}{C_t}. \tag{A.69}$$

Other parts of the model is identical to those of the baseline model.

Resource constraint. The resource constraint of the economy is modified as follows.

$$Y_t = C_t \left(1 + \Psi_w(\Pi_t^w)\right). \tag{A.70}$$

Calibration. For the parameter ϕ_w and ψ_w, I use the estimated values of Kim and Ruge-

Murcia (2019).[12] They report $\phi_w = 33.85$ and $\psi_w = 602.48$ when they estimate the model with normally distributed shocks using U.S. data from 1964Q2 to 2015Q4 (Table 1 of Kim and Ruge-Murcia 2019).

Computation. I use the policy function iteration method of Coleman (1990) to compute equilibrium. However, I find that the wage adjustment cost $\Psi'_w(\Pi^w_t)$ is highly convex under the calibrated parameter values and tend to be unstable in the non-linear solution when the wage inflation rate deviate from the steady-state value.[13] Therefore, I apply the second-order Taylor expansion for $\Psi'_w(\Pi^w_t)$ while keeping the fully non-linear setting for the remaining parts of the model.

[12] I find that the main results are robust to alternative parameter values such as those estimated by Kim and Ruge-Murcia (2009) and Aruoba et al. (2017). I also verify that the quantitative results do not significantly change when the overall degree of rigidity ϕ_w is calibrated according to the frequency of wage changes reported by Grigsby et al. (2019) though a direct measure of the degree of asymmetry ψ_w is not available in Grigsby et al. (2019).

[13] A similar issue is pointed out by Aruoba et al. (2017).

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