Public Debt, Interest Rates, and Negative Shocks

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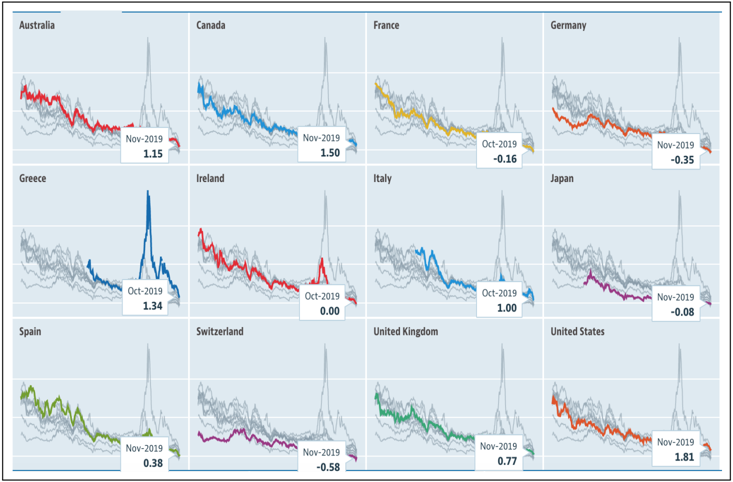
12 OECD Countries, 10-yr govt bond rate, 1981-2018



- Interest rates have broadly declined over last 28 years

Blanchard (2019), Fig. 4, Avg int rate vs. growth, USA

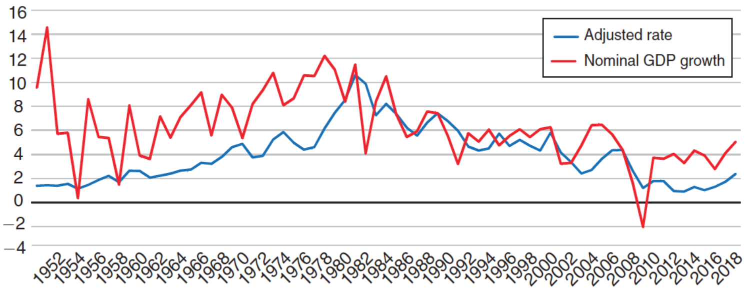
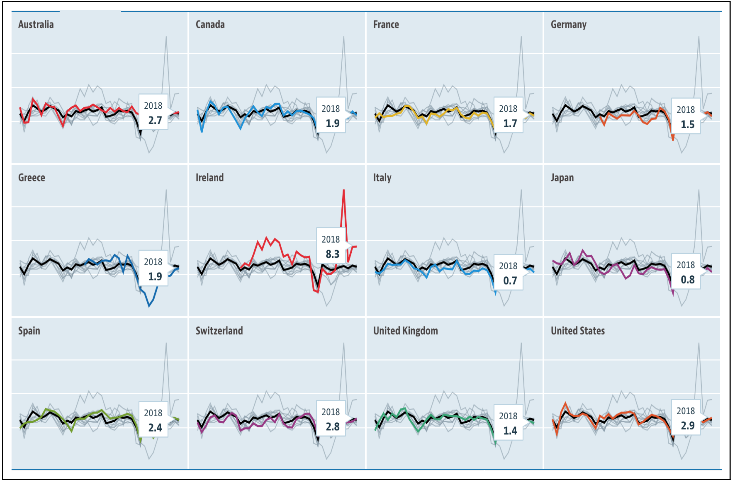


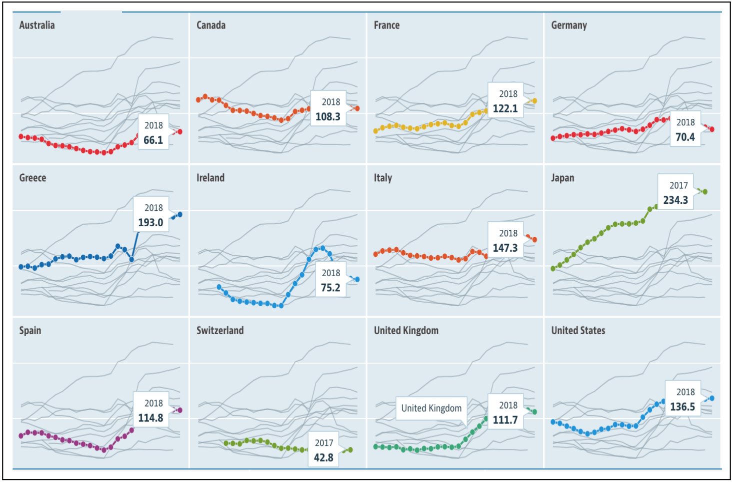
Figure 4. Nominal GDP Growth Rate and Adjusted Rate, 1950–2018

12 OECD Countries, GDP growth rate, 1981-2018



- All current growth rates are higher than 10-year bond rates.

12 OECD Countries, Total Debt/GDP, 1995-2018



- Debt dynamics and responses are varied across countries

My goal and results

Specific question

What are the long-run average welfare costs and risks of increased government debt when interest rates are low?

1. Replication study of Blanchard (2019)
    - Model almost identical to Evans, Kotlikoff, Phillips (2013)
    - **No parameterizations with long-run avg. utility gains**
2. How do results change as more realistic risk added?
    - Reduce safety-net endowment x_1 to young
    - **Long-run average utility losses exacerbated**
3. Calibration using equity prem. rate spread may bias results
    - Rare disaster macro fincancial literature: Rebelo, Wang, Yang (2019), Tsai and Wachter (2015), Evans, Kotlikoff, Phillips (2013), Gourio (2012), Barro (2009)
    - **Higher spreads associated with existence of rare disasters and fiscal stress**

Model summary

- Two-period-lived agent overlapping generations
- Inelastic labor supply: $n_1 = 1$, $n_2 = 0$
- Representative CES production
- 100-percent depreciation
- Aggregate TFP shocks
- Government transfer obligation to old from young

Households

$$\max_{c_{1,t},k_{2,t+1},c_{2,t+1}} (1-\beta)\ln(c_{1,t}) + \beta\frac{1}{1-\gamma}\ln\left(E_t\left[(c_{2,t+1})^{1-\gamma}\right]\right) \quad \forall t \tag{1}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} = w_t + x_1 - H_t \tag{2}$$

$$\text{and} \quad c_{2,t+1} = R_{t+1}k_{2,t+1} + H_{t+1} \tag{3}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} > 0 \tag{4}$$

$$H_t = \min\left(\bar{H}, w_t + x_1 - c_{min} - K_{min}\right) \quad \forall t \tag{5}$$

Young-age endowment x_1 prevents default, violation of (4)

$$\frac{1-\beta}{c_{1,t}} = \beta\frac{E_t\left[R_{t+1}\left(c_{2,t+1}\right)^{-\gamma}\right]}{E_t\left[\left(c_{2,t+1}\right)^{1-\gamma}\right]} \quad \text{and} \quad \bar{R}_t = \left(\frac{1-\beta}{\beta}\right)\frac{E_t\left[\left(c_{2,t+1}\right)^{1-\gamma}\right]}{(c_{1,t})E_t\left[\left(c_{2,t+1}\right)^{-\gamma}\right]} \quad \forall t \tag{6}$$

Firms

$$Y_t = F(K_t, L_t, z_t) = A_t \left[\alpha (K_t)^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \alpha)(L_t)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \quad \forall t \quad (7)$$

$$z_t = \rho z_{t-1} + (1 - \rho)\mu + \epsilon_t$$
$$\text{where } \rho \in [0, 1), \ \mu \geq 0, \ \epsilon_t \sim N(0, \sigma), \text{ and } A_t \equiv e^{z_t} \quad (8)$$

$$\max_{K_t, L_t} Pr_t = F(K_t, L_t, z_t) - w_t L_t - R_t K_t \quad \forall t \quad (9)$$

$$R_t = \alpha (A_t)^{\frac{\varepsilon - 1}{\varepsilon}} \left[\frac{Y_t}{K_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (10)$$

$$w_t = (1 - \alpha)(A_t)^{\frac{\varepsilon - 1}{\varepsilon}} \left[\frac{Y_t}{L_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (11)$$

Government Transfer Program

$$c_{1,t} + k_{2,t+1} = w_t + x_1 - H_t \quad \forall t \tag{2}$$

$$c_{2,t} = R_t k_{2,t} + H_t \quad \forall t \tag{3}$$

$$H_t \equiv \begin{cases} \bar{H} & \text{if} \quad w_t \geq \bar{H} - x_1 + c_{min} + K_{min} \\ w_t + x_1 - c_{min} - K_{min} & \text{if} \quad w_t < \bar{H} - x + c_{min} + K_{min} \end{cases} \quad \forall t$$
$$= \min\left(\bar{H}, w_t + x_1 - c_{min} - K_{min}\right) \quad \forall t \tag{5}$$

- Balanced budget government transfer program
- Is debt because obligation to old

Market clearing and equilibrium

$$L_t = 1 \quad \forall t \tag{12}$$

$$K_t = k_{2,t} \quad \forall t \tag{13}$$

$$0 = B_t = b_{2,t} \quad \forall t \tag{14}$$

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \quad \forall t \tag{15}$$

Eqlb. Def.: stationary price and allocation functions s.t.

- Households optimize in every period (6)
- Firms optimize in every period (10), (11)
- Government transfers (5)
- Markets clear (12), (13), and (14)

Blanchard (2019) calibration

- Annual data avg $r_{t,an}$ in $[0.00, 0.04]$ and avg $\bar{r}_{t,an}$ in $[-0.02, 0.01]$
- $\sigma = 0.2$ to match std. dev of annual log stock returns of 15%
- μ : when $\bar{H} = 0$ and $\varepsilon = \infty \;\Rightarrow\; E_t\left[R_{t+1}\right] = \alpha e^{\mu + \frac{\sigma^2}{2}}$
- γ : when $\bar{H} = 0$ and $\varepsilon = 1$ or $\infty \;\Rightarrow$ $\ln\left(E_t\left[R_{t+1}\right]\right) - \ln\left(\bar{R}_t\right) = \gamma\sigma^2$
- β : some algebra when $\bar{H} = 0$ and $\varepsilon = 1 \;\Rightarrow$ $\beta = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2E[R_{t+1}]}$
- $x_1 = 100\%$ of average wage when $\bar{H} = 0$ and $\varepsilon = 1 \;\Rightarrow$ $x_1 = \left[(1-\alpha)e^{\mu + \frac{\sigma^2}{2}}(2\beta)^{\alpha}\right]^{\frac{1}{1-\alpha}}$

Blanchard (2019), constant mu (Figs. 7, 9)

Percent change in long-run average lifetime utility from increased promised transfer $\bar{H}$

linear prod.		average $\bar{R}$ (annual)		
$\varepsilon = \infty$		-2.0%	-0.5%	1.0%
average	0.0%	3.0%	0.3%	-1.1%
R_t	2.0%	2.8%	0.1%	-1.3%
(annual)	4.0%	2.6%	-0.3%	-1.5%

Cobb-Douglas		average $\bar{R}$ (annual)		
$\varepsilon = 1$		-2.0%	-0.5%	1.0%
average	0.0%	3.0%	0.2%	-0.4%
R_t	2.0%	0.2%	-0.4%	-0.5%
(annual)	4.0%	0.1%	-0.4%	-0.5%

Evans replication of Blanchard (2019), constant mu

Percent change in long-run average lifetime utility from increased promised transfer $\bar{H}$

linear prod. $\varepsilon = \infty$		average $\bar{R}$ (annual) -2.0%	-0.5%	1.0%
average	0.0%	-0.59%	-0.59%	n/a
R_t	2.0%	-0.73%	-0.73%	-0.73%
(annual)	4.0%	-0.86%	-0.86%	-0.86%

Cobb-Douglas $\varepsilon = 1$		average $\bar{R}$ (annual) -2.0%	-0.5%	1.0%
average	0.0%	-0.78%	-0.77%	n/a
R_t	2.0%	-1.62%	-1.58%	-1.54%
(annual)	4.0%	-3.35%	-3.23%	-3.10%

Evans replication of Blanchard (2019), variable mu

Percent change in long-run average lifetime utility from increased promised transfer $\bar{H}$

linear prod.		average $\bar{R}$ (annual)		
$\varepsilon = \infty$		-2.0%	-0.5%	1.0%
average	0.0%	-0.66%	-0.66%	n/a
R_t	2.0%	-0.31%	-0.31%	-0.31%
(annual)	4.0%	-0.16%	-0.16%	-0.16%

Cobb-Douglas		average $\bar{R}$ (annual)		
$\varepsilon = 1$		-2.0%	-0.5%	1.0%
average	0.0%	-1.00%	-0.98%	n/a
R_t	2.0%	-0.52%	-0.51%	-0.49%
(annual)	4.0%	-0.32%	-0.31%	-0.30%

Welfare from increased transfer: 0.5x

Percent change in long-run average lifetime utility from increased promised transfer $\bar{H}$

linear prod.		average $\bar{R}$ (annual)		
$\varepsilon = \infty$		-2.0%	-0.5%	1.0%
average	0.0%	-1.44%	-1.44%	n/a
R_t	2.0%	-0.55%	-0.55%	-0.55%
(annual)	4.0%	-0.27%	-0.27%	-0.27%

Cobb-Douglas		average $\bar{R}$ (annual)		
$\varepsilon = 1$		-2.0%	-0.5%	1.0%
average	0.0%	-3.14%	-3.08%	n/a
R_t	2.0%	-1.28%	-1.23%	-1.19%
(annual)	4.0%	-0.71%	-0.68%	-0.65%

Welfare from increased transfer: x=0

Percent change in long-run average lifetime utility from increased promised transfer $\bar{H}$

linear prod. $\varepsilon = \infty$		average $\bar{R}$ (annual) -2.0%	-0.5%	1.0%
average	0.0%	-20.59%	-20.59%	n/a
R_t	2.0%	-1.83%	-1.83%	-1.83%
(annual)	4.0%	-0.73%	-0.73%	-0.73%

Cobb-Douglas $\varepsilon = 1$		average $\bar{R}$ (annual) -2.0%	-0.5%	1.0%
average	0.0%	-39.87%	-38.30%	n/a
R_t	2.0%	-19.05%	-18.01%	-17.00%
(annual)	4.0%	-5.84%	-5.43%	-5.04%

Summary

1. Replication results: no positive long-run welfare gains in any calibrated parameterization
2. Reducing young-agent endowment x_1 exacerbates long-run welfare losses
3. Calibration using small interest rate spread (low equity premium) likely biases results toward beneficial government debt