Inflation Target Uncertainty and Monetary Policy

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Abstract

I develop an extension of the standard New Keynesian model to monetary policy regime switching to study the impact of uncertainty around the future inflation target. First, I fully characterize how the responses of inflation and output to inflation target uncertainty depend on the monetary policy rule. If monetary policy is passive, inflation may increase far beyond the anticipated increase in the inflation target, while a strong monetary response to expected inflation results in an immediate drop in the inflation rate. Next, I derive the optimal response of the central bank, which can be achieved by adjusting the current inflation target. A central bank unwilling to adjust the inflation target can optimally adjust other policy rule parameters and can often obtain comparatively similar welfare benefits. Finally, I examine the implications of a perfectly anticipated change in the inflation target and find it is likely to generate cyclical dynamics for inflation and output under a constant policy rule. An optimal time varying policy rule or uncertainty in the period of the inflation target change eliminates cyclical fluctuations and improves welfare.

JEL Classification: E52, E61
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1 Introduction

Since the financial crisis, a growing debate has centered on whether the natural interest rate has permanently declined. A decline in the natural interest rate limits monetary policy's ability to stabilize the economy in a recession, as nominal interest rates are closer to the zero lower bound. In response to this, both academics and members of the Federal Open Market Committee have discussed the possibility of raising the inflation target as a means of restoring pre-crisis flexibility.[1] In 2017, Janet Yellen, chair of the Federal Reserve, addressing these concerns stated,

> So it's that recognition that causes people to think we might be better off with a higher inflation objective... And it's important for our decisions to be informed by a wide range of views and research, which is ongoing inside and outside the fed... But I would say that this is one of the most important questions facing monetary policy around the world in the future.

Despite the attention given to this discussion, the consequences of merely having the discussion are not typically addressed. Yet if the inflation target helps determine inflation, discussions that raise inflation target expectations should have immediate consequences regardless of whether the inflation target is eventually changed.[2] In this paper, I characterize the response of macro aggregates to inflation target uncertainty, how the response depends on current policy, and how a central bank should adjust its policy in response to inflation target uncertainty.

To model inflation target uncertainty, I develop an otherwise standard small scale New Keynesian model that incorporates multiple policy regimes. Modeling inflation target uncertainty in a regime shift framework captures the discrete and long term nature of inflation target switches. Monetary policy is assumed to follow a regime specific policy rule with the regimes determined by a Markov process. Analytical solutions of the model allow me to fully characterize how the responses of inflation and the output gap depend on other policy rule parameters, the probability of a change in the inflation target, and other model parameters.

To my knowledge, Foerster (2016) is the only other paper that studies the response of macro aggregates to inflation target uncertainty. He finds that an expectation of a future increase in the inflation target results in an increase in current inflation and a decrease in current output. However his analysis relies on numerical estimations for a small set of policy parameters and does not address how the response depends on the full monetary policy profile, which determines the qualitative results. Furthermore, by allowing interest rates to respond to expected inflation, I find that an expectation of a potential increase

[1] See Williams (2016), Blanchard et al. (2010), and Ball (2014).

[2] See Mavroeidis et al. (2014) for an overview of the empirical evidence for a Phillips curve relationship linking expected inflation to current inflation and output.

in the inflation target may have the counter intuitive effect of reducing current inflation if output stabilizing policy is not too strong. Alternatively if the current regime is passive, but the inflation target change is accompanied by a shift to active monetary policy, then current inflation will increase by more than the potential future increase.

Having characterized the response of macro aggregates under a constant monetary policy, the natural question is: should the central bank change its policy and how? In the main specification, I prove that by changing the current inflation target the central bank may achieve any feasible outcome conditional on the expectations for the future inflation targets without affecting the volatility of inflation or the output gap. Therefore the optimal response to inflation target uncertainty is a change in the inflation target, while other policy parameters remain at their optimal values in the absence of inflation target uncertainty. Depending on the loss function weights, the optimal response can alleviate up to 95% of the losses generated from inflation target uncertainty. A central bank may be unwilling to change the current inflation target due to the concern that it may destabilize inflation. The concern of destabilizing inflation is incremented since the optimal inflation target would be set at a value that is different from both the desired level of inflation and the mean level of inflation it will generate. A central bank that is unwilling to change the current inflation target may optimally adjust other policy parameters and obtain most of the benefits of the optimal policy response.

While inflation target uncertainty reduces welfare, it may be preferred to a perfectly anticipated change in the inflation target. A plausible alternative to a potential future change in the inflation target is a central bank that knows that in the future the inflation target will change perhaps because of a legislative mandate with a delayed implementation period. For standard calibrations of the policy rule, a perfectly anticipated change in the inflation target generates cyclical movements in inflation and the output gap along the transition path. These cyclical movements can be eliminated, and losses can be reduced if the anticipated change in the inflation target is accompanied by a time varying path for the intercept in the policy rule. However, introducing uncertainty in the period of the inflation target change can also eliminate the cyclical movements and generate a similar reduction in losses.

Finally, I find that the effect of uncertainty in the future monetary policy regime on current outcomes depends primarily on how the expected future policies affect inflation volatility. Expectations of potential regime shifts that are expected to increase inflation stability improve current outcomes, and vice versa. Even if the current regime implements an optimal policy rule, the anticipation of a regime shift to a worse regime but with greater inflation stability results in improved outcomes prior to the regime shift. This is consistent with findings by Davig and Leeper (2007) and Foerster (2016) that an expectations of a regime switch to a passive regime substantially increases inflation volatility but have a small, ambiguous impact on output volatility that depends on the calibration.

This paper relates to several strands of the literature. The model builds on the regime switch framework and its application to monetary policy. A large part of the literature focuses on developing solution methods and applying them to examine under what conditions passive policy can be sustained as part of a determinate equilibrium.[3] I expand upon this approach by studying the implications of inflation target uncertainty under passive policy. A separate branch of the literature focuses on estimating DSGE regime switching models to identify past policy.[4] Particularly relevant is Schorfheide (2005) who finds that monetary policy in the 1970s shifted to a high inflation target regime that lasted through the end of the decade and provides evidence that an inflation target regime switching model is consistent with historical data. My paper complements their work by exploring the implications of this type of regime shifting model on monetary policy. A final branch considers the implications of expected regime shifts on current outcomes and their policy implications.[5] However, Foerster (2016) alone applies this approach to inflation target regime switches. I expand upon his numerical findings by deriving analytical results and allowing for more general policy rules. Additionally, I am able to solve for the optimal policy response to inflation target uncertainty.

The impact of uncertainty shocks has also been addressed outside the regime switch framework. Recent empirical studies by Bloom (2009), Baker et al. (2016), and Creal and Wu (2014) provide evidence that both uncertainty shocks in general and monetary policy uncertainty shocks have detrimental effects on macroeconomic aggregates. Theoretical models such as Ulrich (2012) estimate the effect of monetary policy uncertainty on financial volatility. While this literature considers interest rate uncertainty, it does not distinguish between an increase in variance around the mean from systemic changes in the way future policy will be conducted, as I do in this paper.

Finally, this paper relates to older work on the dynamics of disinflation. New classical papers such as Sargent (1982) argued that disinflation is costless, while Keynesians papers such as Taylor (1983) argued that disinflation is costly unless it is done slowly. Ball (1994) showed that Keynesian models imply that a quick disinflation causes a boom by distinguishing between changes in the growth of money versus changes in the level of money. I expand upon these findings by looking at the effects of an anticipated change in the inflation target and how it depends on the certainty that it will occur at a specific time.

The rest of the paper is organized as follows. In section 2, I present the model. In section 3, I exclude all shocks not related to inflation target uncertainty, characterize

[3]See Leeper and Zha (2003), Davig and Leeper (2007), ,Farmer et al. (2009),Farmer et al. (2011), and Foerster et al. (2016) for some of the approaches to solving regime shift models and their implications for determinacy.

[4]These include Liu et al. (2011), Bianchi and Melosi (2016), Bianchi (2013), Bianchi (2013), and Davig and Doh (2014).

[5]Foerster and Choi (2016) and Foerster (2016).

the impact of inflation target uncertainty, and derive the optimal response to inflation target uncertainty. In section 4, I consider welfare and optimal policy in the full stochastic model with monetary policy regime uncertainty and extend the analysis to inflation target uncertainty in section 5. In section 6, I derive the transition path for a fully anticipated change in the inflation target, solve for the optimal time varying policy rule during a fully anticipated change in the inflation target, and compare the outcomes to the outcomes with uncertainty in the period of the inflation target change. In section 7, I conclude.

2 Modeling Monetary Policy Regime Uncertainty

To evaluate the impact of uncertainty in the future inflation target, I develop an otherwise standard small scale, forward looking New Keynesian model that incorporates multiple policy regimes in the style of Davig and Leeper (2007) and has analytical solutions. The baseline model is a simple staggered price setting model as in Walsh (2010) and Woodford (2003) with an extension where firms which do not get to set the optimal price index their previous period's price as a robustness check. The model consists of households, firms, and a central bank.

The representative household purchases goods for consumption, supplies labor, holds money and bonds, and has preferences over a composite consumption good C_t, real money balances $\frac{M_t}{P_t}$, and time devoted to market employment N_t represented by the utility function:

$$u(C_t, N_t, \frac{M_t}{P_t}) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{\frac{M_t}{P_t}^{1-v}}{1-v} \tag{1}$$

The composite consumption good is defined over a continuum of varieties as

$$C_t = (\int_0^1 C_{it}^{\frac{e_t-1}{e_t}} di)^{\frac{e_t}{e_t-1}} \tag{2}$$

The time varying elasticity of substitution among goods varies over time according to the stationary stochastic process e_t around $\bar{e}$ to generates cost push shocks. The household budget constraint is

$$\int_0^1 P_{it}C_{it}di + M_t + \frac{1}{1+i_t}B_t \leq M_{t-1} + B_{t-1} + W_tN_t + D_t, \tag{3}$$

where P_{it} is the price of variety i, B_t are the bond holdings, W_t is the wage, and D_t are the dividends from the firms. The households problem is to maximize the expected present discount value of utility subject to the budget constraint.

A continuum of monopolistically competitive firms produce the differentiated goods and maximize profits given the technology, price stickiness, and demand. Technology is

summarized by the diminishing returns to scale production function

$$Y_{it} = A_t N_{it}^{1-\alpha} \tag{4}$$

In the main specification, firms face Calvo (1983) price stickiness. Each period a random $1-\omega$ fraction of the firms are selected to optimally adjust prices, while the remaining ω fraction retain their previous period's price. However, this leads to a long run positive relationship between inflation and output. The standard modifications to the Calvo model that eliminates this relationship is the assumption that firms which do not get to set their price optimally index their last period's price by inflation last period. While this is a common assumption in the literature which eliminates the long run relationship between inflation and output and improves inflation dynamics, as shown in Chari et al. (2009) backward indexation of prices conflicts with the microeconomic evidence on price setting. A more analytically tractable price indexing assumption is that firms which do not get to optimize index their previous period's prices to the inflation target as in Woodford (2003) and Yun (1996), and I use this as a robustness check. Finally, the demand function for variety i can be derived from the household problem and is given by

$$C_{it} = (\frac{P_{it}}{P_t})^{e_t} C_t \tag{5}$$

Solving the household utility maximization and firm profit maximization problem and linearizing around the zero inflation steady state results in the canonical New Keynesian Phillips and Euler equations. The log linearized Euler equation is

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + \mu_t^D, \tag{6}$$

where x is the output gap, π is inflation, i is the interest rate, μ_t^D is an aggregate demand shock (productivity shock), and σ is the coefficient of relative risk aversion. The aggregate supply relationship is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + (1-\beta)\bar{\pi} + \mu_t^S, \tag{7}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega} \frac{1-\alpha}{1-\alpha+\alpha\bar{e}} \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha}$, μ_t^S is an aggregate supply shock (cost push shock), and $\bar{\pi}$ is the value to which firms that do not get to optimally set their price index their previous prices by (in the main specification $\bar{\pi} = 0$, in the robustness check it equals the current inflation target). The shock processes are autoregressive of the form

$$\mu_t^j = \rho_j \mu_{t-1}^j + \epsilon^j \quad \forall j, \tag{8}$$

where the ϵ^j are iid exogenous shocks.

The central bank implements monetary policy by setting interest rates according to a policy rule which is a linear function of the inflation target, current and future inflation, and the output gap. Two forms of uncertainty over monetary policy are incorporated into the interest rate rule. An additive, auto regressive shock captures short term non-fundamental deviations from the rule. The key addition to the standard New Keynesian model is uncertainty over the monetary policy regime. That is, uncertainty over how the interest rate in the future will respond to the variables in the policy rule. I model this as uncertainty over a finite number of different monetary policy rules. Formally, the interest rate rule in regime s at time t is:

$$i(s_t) = \phi_{\pi,s}\pi_t + \phi_{\pi',s}E_t\pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1)\pi^\star_s + \phi_{x,s}x_t + \mu^I_t, \tag{9}$$

with the realized regime governed by a time invariant Markov process the with transition matrix

$$\Pi = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix},$$

where k is the number of different monetary policy regimes.

The solution methodology relies on two main features of the model: variables that respond to the realization of the regime do not have any backward looking aspects and the Markov process for the regimes is independent of the rest of the model. This allows a reformulation of equations 6, 7, and 9 in terms of regime conditional variables and expectations. First, re-express $E_t x_{t+1}$ and $E_t\pi_{t+1}$ as

$$E_t\pi_{t+1} = E[\pi_{t+1}|s_t = i, \Omega_t^{-s}] = \sum_{j=1}^{k} p_{ij}E[\pi_{jt+1}|\Omega_t^{-s}] \tag{10}$$

$$E_t x_{t+1} = E[x_{t+1}|s_t = i, \Omega_t^{-s}] = \sum_{j=1}^{k} p_{ij}E[x_{jt+1}|\Omega_t^{-s}] \tag{11}$$

where Ω_t is the full information set and Ω_t^{-s} is the information set excluding the current

regime. Then equations 6, 7, and 9 are each replaced with k state contingent equations:

$$x_{s,t} = \sum_{j=1}^{k} p_{sj} E_t x_{j,t+1} - \sigma^{-1}(i_{s,t} - \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1}) + \mu_t^D \tag{12}$$

$$\pi_{s,t} = \beta \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1} + \kappa x_{s,t} + (1-\beta)\bar{\pi}_s + \mu_t^S \tag{13}$$

$$i_{s,t} = \phi_{\pi,s}\pi_t + \phi_{\pi',s} \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1)\pi_s^\star + \phi_{x,s} x_t + \mu_t^I \tag{14}$$

Thus rewritten, the model can be solved for state contingent variables by standard methods for forward looking models, and simulating the Markov process Π determines which of the state contingent variables are realized in each period.[6]

Rotemberg and Woodford (1996) and Woodford (2002) showed that the period losses equal to the weighted sum of squared deviations of inflation and an output gap from their optimal values can approximate expected utility, but this relies on the assumption that there are subsidies that ensure a steady state output level of zero. Benigno and Woodford (2005) relax this assumption by making second order approximations of the structural equations to eliminate the first order terms in the quadratic approximation of expected utility. This approximation results in period losses equal to the weighted sum of squares of inflation and a welfare relevant output gap. Additionally, they show that by redefining the cost push shock, the aggregate supply relationship is unchanged from 7 except that x_t is a welfare relevant output gap rather than the deviation from the flexible price output level. This provides a micro foundation for using the loss function,

$$L_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2) \tag{15}$$

in the presence of variables with nonzero first moments. As I take the loss function as given rather than deriving it from the model fundamentals, I use robustness checks for the choices of θ_x and θ_i rather than relying on a single model specific value.

The main calibration is shown in table 1 and uses quarterly time periods and standard parameter values. The discount factor (δ) is .99, the coefficient of risk aversion (σ) is 2, the demand elasticity (η) is 6, and φ is 1. The share of firms able to set prices each period, $1-\omega$, is .34 implying prices are on average adjusted every 9 month, which is on the upper limit of empirical estimates but is consistent with the regime switching literature calibrations including Davig and Leeper (2007), Foerster (2016), and Schorfheide (2005). This calibration implies $\kappa = .1786$. For all shocks, persistence is set to .5, and the

[6] In appendix A, I derive the analytic solution for the two regime model. I use Sims (2002) algorithm for solving linear rational expectations models for much of the numerical analysis.

Table 1: Model Calibration

Parameter	Value	Parameter	Value
β	.99	ρ_I, ρ_S, ρ_D	.5
σ	2	σ_S	1.5
φ	1	σ_D	2
$\bar{e}$	6	σ_I	2
α	.33	θ_x	.0298
ω	.66	θ_i	.25

standard deviations of demand, supply, and interest rate shocks are set to 2, 1.5, and 2 respectively.[7] In robustness checks, firms are assumed to index prices to the regime specific inflation target $\bar{\pi}_s = \pi_s^\star$. The main specification for numerical analysis sets $\theta_x = .0298$ and $\theta_i = .2500$, but both are varied for robustness and for analytical results $\theta_i = 0$. For the theoretical analysis, all parameters are restricted to be non-negative and $0 < \beta < 1$.

3 Theoretical Analysis

While the preceding model can be solved analytically for the regime specific values, and in appendix A I show the general form of the solution and a couple of key features derived from it, the solution is too complex to be useful for deriving most of its properties. To analytically examine the implications of an expected increase in the inflation target, I use a simplified model which removes all shocks not related to the expectations of the inflation target change. For most of the analysis, I also remove price indexing by firms that do not get to optimally set their price in a given period. With these assumptions, the model can be rewritten as:

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}), \tag{16}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{17}$$

$$i_{s,t} = \phi_{\pi,s}\pi_t + \phi_{\pi',s} E_t \pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1)\pi_s^\star + \phi_{x,s} x_t, \tag{18}$$

where each equation can be rewritten in the state contingent notation of (12) - (14). All parameters are assumed to be nonnegative, and I allow for the policy parameters to change at the same time as the inflation target. Conceptually this is a reasonable assumption since a central bank may wish to adjust the rest of its policy at the same time as the inflation target. Analytically, this assumption only affects the dynamics prior to the implementation of the new inflation target by changing the eventual steady state

[7] The persistence is reduced from standard values to ensure that there exists an optimal policy rule with finite coefficients.

values after the inflation target changes, and it extends the range of policy parameters that results in a unique solutions.

To examine the implications of the inflation target uncertainty in an analytically tractable setting, I use a two regime model. Prior to period zero the model is in the single regime, zero inflation target steady state. In period zero, new information is revealed that causes everyone to rationally expect that the central bank may raise the inflation target to $\pi^\star$ with probability λ in all future periods. Once the inflation target is raised, it remain at the new level in perpetuity.[8] This formulation is not only consistent with a formal announcement of following a stochastic policy rule as a means of implementing a higher inflation target, but it is also consistent with (stated) uncertainty over what the future inflation target will be.[9]

3.1 Regime Switches Dynamics

In the absence of additional shocks present in the computational model, each period in a given regime is identical. Therefore, I need only solve for three outcomes; the outcome prior to the revelation of a potential regime shift (denoted x_0 and π_0), the outcome in the current regime prior to the change in the inflation target (denoted x_1 and π_1), and the outcome after the regime shifts to a higher inflation regime (denoted x_2 and π_2).

Prior to the revelation of a potential regime shift, the model is identical to a single regime model, with expectations that the inflation target will remain zero forever and no shocks. Therefore,

$$x_0 = \pi_0 = 0. \tag{19}$$

Since I assume that the second regime is absorbing, once the inflation target changes it is expected to remain the same forever. Therefore, x_2 and π_2 are at the steady state values for a single regime model with an inflation target $\pi^\star$, which are

$$x_2 = \pi_2 \frac{1-\beta}{\kappa} \text{ and } \pi_2 = \pi^\star \frac{1-\phi_\pi - \phi_{\pi'}}{1-\phi_\pi - \phi_{\pi'} - \phi_x \frac{1-\beta}{\kappa}}. \tag{20}$$

The output gap in regime two is positive because of the long run relationship between inflation and output embedded in the Phillips curve. Responding to the positive output

[8] In the regime switching notation, the model starts in regime one and the Markov process, Π, unexpectedly switches from $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to $\begin{bmatrix} 1-\lambda & \lambda \\ 0 & 1 \end{bmatrix}$ at time zero, with $\pi_1^\star = 0$ and $\pi_2^\star = \pi^\star$

[9] The analysis presented in this section extends to variations without an absorbing regime, but these variations introduces a new effect where regime one policy parameters have similar effects on regime two outcomes as the effects of regime two parameters on regime one outcomes that are discussed in this section. As an expectation of a change in the inflation target is not generally associated with a significant probability that after the regime change the inflation target may be returned to its old value, an absorbing regime two is a more natural assumption for this application and allows for closer parallels to the perfect foresight case. I present the solution with a nonabsorbing regime two and some brief implications thereof in the appendix.

gap will raise nominal and real interest rates pushing down inflation and therefore the output gap.

Since monetary policy in regime two is independent of monetary policy in regime one, π_2 is exogenous from the perspective of a central bank in regime one. To simplify the notation I solve for the outcomes in regime one as a function of π_2 rather than $\pi^\star$, but they can all be expressed in terms of $\pi^\star$ by (20). In regime one, the expected output gap next period is the probability of remaining in the same regime times the output gap in the current regime next period plus the probability of a regime shift times the output gap next period if the regime shift occurs. But since each regime is in a steady state, $E_t x_{t+1} = (1-\lambda)x_1 + \lambda x_2$. Using this, I can rewrite equations 16 - 18 as

$$\pi_1 = \beta((1-\lambda)\pi_1 + \lambda\pi_2) + \kappa x_1 \tag{21}$$

$$x_1 = (1-\lambda)x_1 + \lambda x_2 - \sigma^{-1}(i_1 - (1-\lambda)\pi_1 - \lambda\pi_2) \tag{22}$$

$$i_1 = \phi_\pi \pi_1 + \phi_{\pi'}((1-\lambda)\pi_1 + \lambda\pi_2) + \phi_x x_1 + \bar{i}_1, \tag{23}$$

where $\bar{i}_1 = (1 - \phi_\pi + \phi_{\pi'})\pi_1^\star = 0$ if the inflation target in regime one is zero. Solving these equations for the output gap and inflation in regime one,

$$x_1 = \frac{(\lambda\kappa\sigma^{-1}(1-\beta\phi_\pi - \phi_{\pi'}) + \lambda(1-\beta)(1-\beta(1-\lambda)))\pi_2 - \kappa\sigma^{-1}(1-\beta(1-\lambda))\bar{i}_1}{\kappa(\kappa\sigma^{-1}(\phi_\pi + (1-\lambda)(\phi_{\pi'}-1)) + (\lambda + \sigma^{-1}\phi_x)(1-\beta(1-\lambda)))} \tag{24}$$

$$\pi_1 = \frac{\lambda(1-\beta+\beta\lambda+\sigma^{-1}\kappa+\beta\sigma^{-1}\phi_x - \sigma^{-1}\kappa\phi_{\pi'})\pi_2 - \sigma^{-1}\kappa\bar{i}_1}{\kappa\sigma^{-1}(\phi_\pi + (1-\lambda)(\phi_{\pi'}-1)) + (\lambda+\sigma^{-1}\phi_x)(1-\beta(1-\lambda))}. \tag{25}$$

Using the preceding two equations, we can characterize the equilibrium and how it depends on monetary policy.

Proposition 1 *If monetary policy is active ($\phi_\pi + \phi_{\pi'} \geq 1$) then*

1. *π_1 is increasing in $\pi^\star$ if $1 + \frac{1-\beta(1-\lambda)}{\kappa\sigma^{-1}} + \frac{\beta\phi_x}{\kappa} > \phi_{\pi'}$*
2. *x_1 is increasing in $\pi^\star$ if $1 + \frac{(1-\beta)(1-\beta(1-\lambda))}{\kappa\sigma^{-1}} > \beta\phi_\pi + \phi_{\pi'}$*
3. *$\frac{\partial^2\pi_1}{\partial\pi^\star\partial\phi_\pi} < 0$ if $1 + \frac{1-\beta(1-\lambda)}{\kappa\sigma^{-1}} + \frac{\beta\phi_x}{\kappa} > \phi_{\pi'}$*
4. *$\frac{\partial^2\pi_1}{\partial\pi^\star\partial\phi_x} < 0$ if $1 + \frac{(1-\beta)(1-\beta(1-\lambda))}{\kappa\sigma^{-1}} > \beta\phi_\pi + \phi_{\pi'} + \frac{\beta(1-\beta(1-\lambda)^2)}{\kappa}\phi_x$*
5. *$\frac{\partial^2\pi_1}{\partial\pi^\star\partial\phi_{\pi'}} < 0$*
6. *If monetary policy parameters aside for the inflation target are the same in both regimes, then $\pi_1 < \pi_2 \leq \pi^\star$ and $x_1 < x_2$.*

Lets first consider the case where $\phi_{\pi'} = \phi_x = 0$. An expectation of a future regime shift to a higher inflation target implies higher expected inflation which creates an incentive

for firms to set higher prices as they may not be able to reset their prices when the regime shift occurs and the optimal price rises. With active monetary policy, nominal interest rates will rise by more than expected inflation and, therefore, real interest rates will also rise. Higher real interest rates create an incentive to save, but if ϕ_π is small this effect may be weaker than the consumption smoothing motive combined with an anticipation of higher consumption when the regime shifts. Depending on the magnitude of ϕ_π, marginal costs will either be slightly positive but lower then in regime two or negative. However, the reduced marginal costs will never cause inflation to fall below zero because interest rates only rise in response to positive inflation. Therefore at time zero inflation shifts from zero to a positive value proportional to but less than the inflation target in regime two. Inflation remains at this level until the regime shift, at which point inflation increases to its steady state value at the new inflation target. At time zero, output shifts to a new level which is below its previous level unless the monetary policy response to inflation is sufficiently small. When the regime shift occurs, the output gap increases to the new steady state level.

Nominal interest rates responding to expected inflation with active monetary policy also generate higher real interest rates and cause a recessionary force that pushes down inflation. However, as the interest rate is responding to expected inflation, which with regime shifts can be positive even if current inflation is zero, a large enough coefficient on expected inflation can generate a recession large enough to force deflation. If in response to monetary policy inflation in regime one falls when the inflation target in regime two is raised, further increasing the responsiveness to expected inflation magnifies the deflation, while increasing the responsiveness to current inflation will reduce the deflation by lowering interest rates.

The effect of interest rates responding to the output gap on inflation in regime one depends on the response of output. If output falls because monetary policy causes a recession and reduces marginal costs, then responding more to the output gap will lower interest rates and result in higher inflation. If real interest rates barely rise and the output gap remains positive, a stronger response to the output gap will raise interest rates which implies a reduction in output, marginal costs, and therefore inflation. However if ϕ_x is already large, further increasing it may result in higher inflation through an equilibrium effect.

The effect of inflation target uncertainty is very different if monetary policy is passive. To emphasize the key aspects of the response, I assume interest rates do not respond to expected inflation or the output gap.[10]

Proposition 2 *If* $\phi_{\pi'} = \phi_x = 0$ *and* $\phi_\pi \leq 1$*, then*

1. *If* $\phi_\pi = 1$*, then* $\pi_1 = \pi_2$ *and* $x_1 = \frac{1-\beta}{\kappa}\pi_2 = x_2$

[10]This assumption is relaxed in appendix A.

2. If $1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})<\phi_\pi<1$*, then* $\pi_1>\pi_2$ *and* $x_1>\frac{1-\beta}{\kappa}\pi_2$ *and*

$$\lim_{\phi_\pi\to[1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})]+}\pi_1=\lim_{\phi_\pi\to[1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})]+}x_1=\infty$$

3. If $1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})>\phi_\pi$*, then* $\pi_1<0$ *and* $x_1<0$

If interest rates respond one to one with inflation, inflation and output instantly adjust to the future steady state value. As the monetary policy response becomes weaker than one for one, inflation and the output gap begin to explode towards infinity. Real interest rates fall because nominal interest rates do not keep up with the rise in expected inflation. This pushes up marginal costs, leading firms to set higher prices not only because they anticipate future higher prices from a higher inflation target but also face higher marginal costs. This is further amplified because the rise in the inflation target is not anticipate for multiple periods during which inflation and the output gap are anticipated to go up for the same reasons. The threshold $\phi_\pi=1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})$ corresponds to the threshold for determinacy in the stochastic model. If ϕ_π is below this threshold, the stochastic model does not have a unique solution, while in the simplified model of this section it results in inflation and the output gap becoming negative.

3.2 Monetary Policy Response

Thus far we have looked at the equilibrium under monetary policy that is set without consideration for the inflation target uncertainty. However if expectations over the future inflation target are formed exogenously from current monetary policy decisions such as from uncertainty over what will be the best estimate of the optimal inflation level in the future, the dynamics generated by such expectations are likely to be undesirable for the central bank. Therefore, it is natural to explore how the central bank can adjust monetary policy to minimize the losses from inflation target uncertainty.

The equilibrium in this model is determined by the intersection of the Philips curve with the IS curve combined with a policy rule. A change in the inflation target in regime two shifts both curves. Changing the current inflation target, shifts the IS curve allowing the central bank to achieve any outcome on the new Phillips curve.

Proposition 3 *By changing the inflation target in the current regime, the central bank can achieve any outcome where*

$$\pi_1=\frac{\pi^\star\beta\lambda+\kappa x_1}{1-\beta(1-\lambda)} \tag{26}$$

without affecting the volatility of inflation or output in the full stochastic model. Other policy parameters have no effect on the set of possible outcomes.

Other policy parameters only matter for the equilibrium in determining how much the inflation target has to be adjusted to achieve a particular outcome or if the central bank is unwilling to change the current inflation target, and for the volatility of the variables in the full stochastic model.

Proposition 4 *Reducing the constant in the policy rule for the current regime, $\bar{i}_1$, will raise inflation and output in the current regime if*

$$\phi_\pi + (1-\lambda)\phi_{\pi'} + \phi_x \frac{1-\beta(1-\lambda)}{\kappa} > 1 - \lambda(1 + \frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa}) \tag{27}$$

Proposition 4 states that if monetary policy is close enough to being active, then reducing the constant in the policy rule results in a lower real interest rate, which incentivizes consumption leading to higher output, marginal costs, and inflation. With active monetary policy, lowering the constant in the policy rule is equivalent to raising the inflation target. Therefore the central bank can raise the inflation target in the current regime to increase output at the cost of higher inflation or lower it to reduce inflation at the cost of lower output.

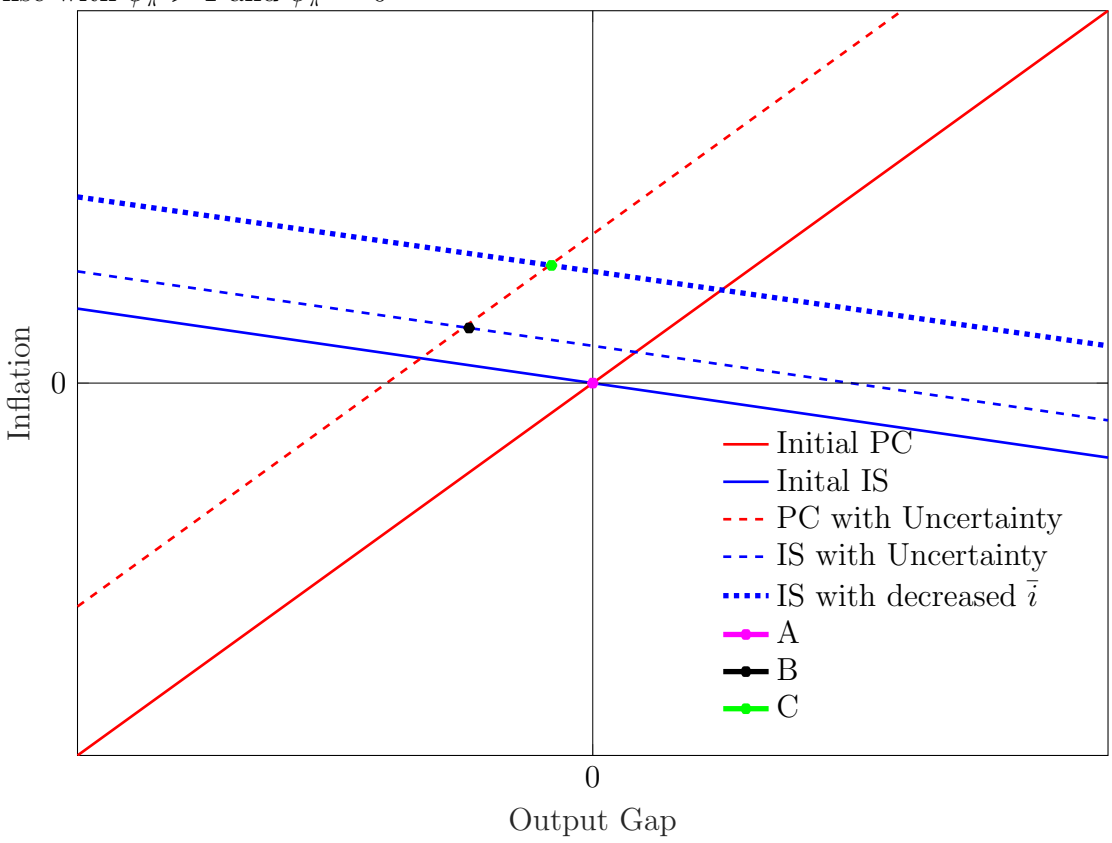
Figure 1 illustrates the preceding results for the case when $\phi_\pi > 1$ and $\phi_{\pi'} = 0$. In the absence of inflation target uncertainty the Phillips and IS curves intercept at point A, which corresponds to zero inflation and no output gap. Once uncertainty about a potential future increase in the inflation target is introduced, the Phillips Curve and the IS curve both shift up and the regime one equilibrium is point B.[11] However, by adjusting the constant in the policy rule the central bank can choose any point on the new Phillips Curve. For example, by reducing the constant the central bank reduces nominal and real rates, incentivizes consumption, and increases output and marginal costs which which pushes up inflation (point C). If the policy rule responds to expected inflation, the results are very similar except that if $\phi_{\pi'}$ is large enough the effect of uncertainty will be to shift the IS curve down, but this does not change the set of feasible outcomes the central bank can achieve by adjusting the intercept of the policy rule.

If a central bank is committed to maintaining its current inflation target and conducts an active monetary policy that does not respond too much to expected inflation, then it can maintain the initial inflation target by creating a recession. To accomplish this, the central bank must set the inflation target in the current regime's policy rule, $\pi_1^\star$, below the actual inflation target the central bank wishes to maintain.

Proposition 5 *To achieve a zero inflation rate in regime one, the central bank needs to*

[11] If ϕ_π is small enough the IS curve shifts up enough that point B corresponds to a positive output gap.

Figure 1: The Effects of Inflation Target Uncertainty and the Central Bank Policy Response with $\phi_\pi > 1$ and $\phi_{\pi'} = 0$



have the output gap in regime one equal to $\frac{-\beta\lambda\pi_2}{\kappa}$ *which can be accomplished by setting*

$$\pi_1^\star = \underbrace{\lambda\pi_2}_{\text{Impact of } \pi^\star \text{ on } E_t\pi_{t+1}} \underbrace{\frac{1}{1-\phi_\pi-\phi_{\pi'}}}_{\text{Direct effect}} \underbrace{(1-\phi_{\pi'}+\frac{\beta\phi_x}{\kappa}+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})}_{\text{Equilibrium effect}}. \tag{28}$$

How much $\pi_1^\star$ needs to be lowered depends on how much the other regimes inflation target raises expected inflation, a direct effect, and an indirect effect.[12] The direct effect is that if ϕ_π or $\phi_{\pi'}$ are larger, then a change in the inflation target causes a greater change in the nominal interest rate. The equilibrium effect captures how much inflation rises from higher expected inflation and falls from a larger intercept in the policy rule. If the equilibrium effect is positive and policy is active, then by proposition 1 inflation in regime one is positive and raising the responsiveness of nominal interest rates to expected inflation reduces the necessary decrease in $\pi_1^\star$ necessary to achieve zero inflation. If $\phi_{\pi'}$ is set such that the equilibrium effect is zero, then changes in the other regime's inflation target does not affect inflation in the current regime. Furthermore since ϕ_π is unrestricted, a continuum of policy rules and inflation and output volatility mixes are possible while

[12] If $\phi_{\pi'}$ is large enough for inflation to fall, then $\pi_1^\star$ will need to be increased to increase the output gap to $\frac{-\beta\lambda\pi_2}{\kappa}$.

keeping the regime one expected inflation rate at zero.

If a central bank cares about both inflation and output stability, then the central bank will not wish to set inflation to zero. Assume that the central bank does not care about interest rate volatility then the central bank loss function (15) becomes

$$E_t \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \theta_x x_t^2). \tag{29}$$

Proposition 6 *If the central bank loss function is given by equation 29 and monetary policy in regime two is exogenous, then the optimal commitment policy in regime one will set the inflation target in regime one such that*

$$x_1 = \frac{-\pi_2 \lambda \beta}{\kappa + \kappa^{-1}(1-\beta(1-\lambda))^2 \theta_x} \text{ and } \pi_1 = \frac{\pi_2 \lambda \beta}{1-\beta(1-\lambda)} \frac{\kappa^{-1}(1-\beta(1-\lambda))^2 \theta_x}{\kappa + \kappa^{-1}(1-\beta(1-\lambda))^2 \theta_x}. \tag{30}$$

As θ_x increases from zero to infinity, the optimal allocation shits up the Philips curve (21) from $x_1 = -\frac{\beta \lambda \pi_2}{\kappa}$ and $\pi_1 = 0$ to $x_1 = 0$ and $\pi_1 = \frac{\pi^\star \lambda \beta}{1-\beta(1-\lambda)}$. As these allocation can be achieved just by adjusting the inflation target, they have no impact on the volatility of output and inflation in a stochastic model, and the trade off between inflation and output in levels can be completely separated from the trade off in variances.

While the separation of the trade offs in levels and volatilities is possible, it requires that the current inflation target be adjusted to any changes in the expected inflation target. Constantly adjusting the monetary policy rule to changes in expectations may be destabilizing. A natural question is what is the optimal policy if the central bank is unwilling to adjust the inflation target and/or the policy rule coefficients. Additionally, how effective is the optimal policy? Both of these questions are not analytically tractable and will be addressed numerically in section 5.

3.3 Indexing

A feature of the preceding analysis is that once the inflation target is increased, the level of the output gap also increase. In this section I show the implications of eliminating the long run relationship between inflation and the output gap by assuming that firms which do not get to optimally set their price in a given period will index their previous period's prices to the inflation target. With this assumption the Phillips curve (17) becomes

$$\pi_t = \beta E_t \pi_{t+1} + (1-\beta)\pi_s^\star + \kappa x_t. \tag{31}$$

Prior to the announcement, the zero inflation and zero output gap steady state is unchanged. After the inflation target changes, there will be a new steady state with

inflation at the inflation target and a zero output gap. Since the output gap in regime two is zero, the expectation of output and consumption growth disappears. Therefore if real interest rates are positive prior to the regime shift then the output gap must be negative. For the same reasons as without price indexing, a rise in the expected inflation target will cause real interest rates to rise if monetary policy is active. Therefore, at the time of of the announcement inflation increases and the output gap decreases and then they both remain constant until the regime shift.

Adding price indexing to the current inflation target has a more substantive effect on the optimal policy response to an expectation of a shift to a regime with a higher inflation target, as it adds a new effect of adjusting the inflation target. Without price indexing, raising the inflation target lowers the nominal and real interest rate and results in a higher output gap. With price indexing, raising the inflation target also increases the price level set by firms directly, implying that the Phillips curve also adjusts. Hence the set of feasible outcomes a central bank can achieve is no longer described by the Phillips curve, and therefore simply adjusting the current policy rule's intercept and holding the other coefficients constant is insufficient to achieve the optimal policy response. However, as I show numerically in section 5, most of the benefits of optimal policy can still be achieved by only changing the policy rule's intercept.

4 Optimal Policy & Welfare with Constant Inflation Target

In section 5, I quantify the effects of inflation target uncertainty and the monetary policy response to it in the stochastic model. Before allowing for inflation target uncertainty, it is useful to understand the implications of regime switches in the stochastic model for optimal policy. In this section, I first show the optimal policy under a single regime and then show the implications of introducing regime switches on optimal policy. In section 5, this will allow us to distinguish how monetary policy is responding to inflation target uncertainty verses responding to the absorbing regime structure. Monetary policy regime switch analysis uniformly uses policy response functions as the monetary policy mechanism. Therefore, I continue to consider constrained optimal policies restricted to the policy response function (9) rather than the unconstrained optimal policy. This necessitates including a positive weight on interest rate stability, as without it the optimal policy rule involves setting arbitrarily large coefficients. As observed policy is inconsistent with very large coefficients in the policy rule, adding an interest rate stabilizing motive allows for more reasonable optimal policy rules. The constrained optimal policy response

Table 2: Optimal Policy Rule Under a Single Regime

Loss Function Weights		Optimal Policy Rule			Optimal Outcomes			
θ_x	θ_i	ϕ_π	$\phi_{\pi'}$	ϕ_x	$E\pi^2$	Ex^2	Ei^2	Losses
0.0298	0.2500	1.5180	1.0090	0.4307	7.0707	13.9107	23.3509	13.3224
0.0000	0.2500	1.5180	1.0090	0.2500	6.7899	16.8648	24.3068	12.8666
0.5000	0.2500	1.5180	1.0090	3.2868	9.4284	2.3165	21.6203	15.9918
1.0000	0.2500	1.5180	1.0090	6.3238	10.2618	0.8725	22.2733	16.7027
2.0000	0.2500	1.5181	1.0091	12.3995	10.8895	0.2777	23.0003	17.1949
0.0298	0.0500	8.3898	4.4449	1.1537	1.7640	37.4617	60.6236	5.9100
0.0298	0.5000	0.6590	0.5795	0.3404	10.8887	12.9213	12.4987	17.5226

function solves

$$\min_{\phi_{\pi,s},\phi_{\pi',s},\phi_{x,s},\pi_s^\star,\Pi} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2) \tag{32}$$

or from a timeless perspective

$$\min_{\phi_{\pi,s},\phi_{\pi',s},\phi_{x,s},\pi_s^\star,\Pi} E(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2) \tag{33}$$

subject to the structural equations 6-9. Since the inflation target will be optimally chosen to be zero, price indexing to the inflation target does not play a role in this section.

Table 2 shows the optimal policy response functions without monetary policy regime uncertainty for various weights on output and interest rate stability. The main specification sets the weights near the model consistent values under a single regime, but I allow for variations in the weights as the model based values are generally inconsistent with the emphasis central bankers place on output stability. The optimal policy rule responds to both inflation and expected inflation as well as the output gap. For the main specification, the optimal policy rule sets $\phi_\pi = 1.5180$, $\phi_{\pi'} = 1.0090$, and $\phi_x = .4307$.[13] The optimal policy rule results in inflation volatility of 7.0707, output gap volatility of 13.9107, interest rate volatility of 23.3509, and expected losses of 13.3224. Changing the weights on inflation and output stability has significant effects on the rules and the outcomes they generate, but will not qualitatively affect the optimal response to inflation target uncertainty.

As shown in Davig and Leeper (2007), Foerster (2016), and other papers, the introduction of regime shifts affects the volatilities of inflation in both regimes. Table 3 shows how inflation, output, and interest rates in each regime respond to a 10% chance of a regime shift for seven alternative regimes, while table 4 shows the expected outcomes across regimes.[14] A chance of a regime shift to a regime with greater inflation stability

[13] While it matters whether interest rates respond to inflation or expected inflation, near the optimum there exist alternative combinations of ϕ_π and $\phi_{\pi'}$ along with $\phi_x = .4307$ that results in computationally identical losses and volatilities.

[14] The numerical algorithm provides results in solutions for the regime contingent variables in the form

increases inflation stability in regime one and overall. Similarly, a chance of a regime shift to a regime with lower inflation stability reduces inflation stability in regime one and overall. As there is a tradeoff between inflation and output stability, a transition to a more inflation stabilizing regime implies implies a transition to a regime with greater output volatility; therefore expected volatility of output also increase. However, the change of output in regime one is ambiguous. In the main specification, the anticipation of a transition to a more inflation stabilizing regime reduces output stability in regime one. However, alternative calibrations of the model may result in output stability also increasing in regime one, but regardless of the direction of the change in output stability, it is always much smaller than the change in inflation stability.

Additionally, since expectations of a regime shift to a more inflation stabilizing policy raise inflation and interest rate stability and only minimally reduce output stability, expectations of a regime shift to a more inflation stabilizing policy also reduce losses in regime one. This can be seen in table 3 from the EL_1 column that shows the expected per period losses while in regime one. This is a robust result and remains true even if the weight on output stabilization is double the weight on inflation stability and for a variety of alternative calibrations of the model. Hence expectations of a regime shift to a worse regime can reduces losses in the short run if the alternative regime has greater inflation stability.

With regime shifts there are three relevant variations of the welfare optimization problem depending on what the central bank controls. One possibility is the central bank sets the optimal regime switching Markov process rule by optimizing over the policy parameters in all regimes and the Markov process itself. This is the exactly the optimization problem from (33). However, the optimal regime switching Markov process rule without absorbing regimes is to have no regime shifts and always implement the single regime optimal policy response function.

of (97) and (98), which allows for an exact estimation for first and second moments for the variables. For the realized outcome, I take expectations across the two regimes outcomes to get exact solutions without simulations. For example, $Ex_t^2 = P(regime = 1) * E(x_{1,t}^2) + P(regime = 1) * E(x_{2,t}^2)$.

Table 3: Regime One and Two Outcomes with $i_1 = 1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4978x_t$ and $p_{11} = p_{22} = .9$

i_2	$E\pi_1^2$	Ex_1^2	Ei_1^2	EL_1	$E\pi_2^2$	Ex_2^2	Ei_2^2	EL_2
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.0707	13.9107	23.3509	13.3224	7.0707	13.9107	23.3509	13.3224
$1.9180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	6.9714	13.9537	22.9117	13.1146	6.2418	15.7066	27.6400	13.6192
$1.1180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.1844	13.8624	23.8569	13.5612	8.0764	12.4612	19.0063	13.1988
$1.5180\pi_t + 1.4090E\pi_{t+1} + 0.4307x_t$	7.0193	13.9329	23.1232	13.2147	6.6357	14.7783	25.5135	13.4539
$1.5180\pi_t + 0.6090E\pi_{t+1} + 0.4307x_t$	7.1254	13.8874	23.5940	13.4372	7.5470	13.1333	21.1860	13.2344
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.8307x_t$	7.1235	13.7846	23.5307	13.4164	7.5595	9.6974	22.0742	13.3667
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.0307x_t$	6.9939	14.1001	23.0976	13.1879	6.4902	21.6111	26.5270	13.7651

Table 4: Expected Outcomes with $i_1 = 1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4978x_t$ and $p_{11} = p_{22} = .9$

i_2	$E\pi^2$	Ex^2	Ei^2	Losses
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.0707	13.9107	23.3509	13.3224
$1.9180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	6.6066	14.8301	25.2758	13.3669
$1.1180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.6304	13.1618	21.4316	13.3800
$1.5180\pi_t + 1.4090E\pi_{t+1} + 0.4307x_t$	6.8275	14.3556	24.3184	13.3343
$1.5180\pi_t + 0.6090E\pi_{t+1} + 0.4307x_t$	7.3362	13.5103	22.3900	13.3358
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.8307x_t$	7.3415	11.7410	22.8024	13.3915
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.0307x_t$	6.7420	17.8556	24.8123	13.4765

Table 5: Optimal Regime 1 Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

i_1^*	i_2	$E\pi^2$	Ex^2	Ei^2	Losses
$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t^\star$	$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.0707	13.9107	23.3509	13.3224
$1.5435\pi_t + 1.0102E\pi_{t+1} + 0.4307x_t$	$1.9180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	6.5745	14.8860	25.3970	13.3667
$1.5025\pi_t + 0.9944E\pi_{t+1} + 0.4307x_t$	$1.1180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.6604	13.1140	21.3167	13.3798
$1.5328\pi_t + 1.0060E\pi_{t+1} + 0.4307x_t$	$1.5180\pi_t + 1.4090E\pi_{t+1} + 0.4307x_t$	6.8109	14.3839	24.3812	13.3342
$1.5122\pi_t + 0.9980E\pi_{t+1} + 0.4307x_t$	$1.5180\pi_t + 0.6090E\pi_{t+1} + 0.4307x_t$	7.3509	13.4864	22.3341	13.3357
$1.5210\pi_t + 1.0029E\pi_{t+1} + 0.4533x_t$	$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.8307x_t$	7.3588	11.5872	22.7505	13.3912
$1.5227\pi_t + 0.9995E\pi_{t+1} + 0.4038x_t$	$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.0307x_t$	6.7226	18.0553	24.8644	13.4760

*There are alternative policy rules for regime one that result in the same losses in the two regime model.

Table 6: Expected Per Period Regime 1 Loss Minimizing Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

i_1^*	i_2	$E\pi_1^2$	Ex_1^2	Ei_1^2	EL_1
$1.1817\pi_t + 0.8324E\pi_{t+1} + 0.4073x_t$	$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	8.1211	12.7033	18.7952	13.1979
$1.1856\pi_t + 0.8330E\pi_{t+1} + 0.4071x_t$	$1.9180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	7.9924	12.7558	18.4916	12.9949
$1.1773\pi_t + 0.8317E\pi_{t+1} + 0.4075x_t$	$1.1180\pi_t + 1.0090E\pi_{t+1} + 0.4307x_t$	8.2685	12.6454	19.1453	13.4312
$1.1829\pi_t + 0.8323E\pi_{t+1} + 0.4071x_t$	$1.5180\pi_t + 1.4090E\pi_{t+1} + 0.4307x_t$	8.0569	12.7281	18.6256	13.0921
$1.1804\pi_t + 0.8325E\pi_{t+1} + 0.4074x_t$	$1.5180\pi_t + 0.6090E\pi_{t+1} + 0.4307x_t$	8.1894	12.6775	18.9764	13.3107
$1.1786\pi_t + 0.8324E\pi_{t+1} + 0.4136x_t$	$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.8307x_t$	8.1964	12.4459	18.8841	13.2878
$1.1862\pi_t + 0.8323E\pi_{t+1} + 0.3980x_t$	$1.5180\pi_t + 1.0090E\pi_{t+1} + 0.0307x_t$	8.0120	13.0921	18.6695	13.0690

An alternative perspective is that there is uncertainty over future monetary policy, but the central bank can only control policy in the current regime. For example, an expectation that future policy will normalize after the unconventional monetary policy of the financial crises, but with the Federal Reserve not having control over the specific form of the anticipated policy. Table 5 shows the optimal response for the same seven alternative regimes as in tables 3 and 4. If under the alternative regime the central bank implements the optimal policy response function, then the regime one optimal policy remains unchanged.[15] However, if the expected future policy response functions are not all the optimal policy response functions, then the variances of inflation and output in both regimes change and minor alternations to policy in regime one will be preferred. There are two key features of the optimal policy response. First, if regime two has higher than optimal output volatility, the optimal policy in regime one may also place a smaller emphasis on output stabilization. Secondly, the optimal policy reduces losses by less than a percent of the increase in losses from introducing a sub-optimal regime two into the model. Given the difficulty of identifying expectations over alternative regimes, the single regime optimal policy provides a good approximation to the optimal policy regardless of expectations for future monetary policy.

A final possibility is that the central bank only cares about welfare in the current regime. If regime one cannot be entered from the other regimes, then the optimal policy problem for a central bank that only controls policy in regime one reduces to minimizing losses in regime one.[16] Intuitively, policy in one regime affects policy in the other regimes through its effect on expected inflation and output. If a regime cannot be transitioned into, then nothing in that regime can affect the other regimes including the policy response function. Therefore conditional on the policy in the other regimes, minimizing losses over the policy response function in regime one is equivalent to minimizing expected losses across regimes from a timeless perspective. Alternatively, this may be the relevant metric if there are deviations from rational expectations, where the central bank is committed to remaining in a regime but cannot convince the public of this, or if changes in regimes are associated with changes in central bankers, and the central bankers only care about losses while they are in power.

Table 6 shows the regime one policy rules that minimize expected per period losses while in regime one. Relative to the benefit of implementing the optimal rule on expected losses, switching to the policy rules that minimize expected per period losses while in regime one has a more substantive affect on expected per period losses in regime one. Furthermore, even if the policy rule in regime two is the optimal rule, expected per period losses in regime one can be reduced by switching to an alternative policy rule for regime one. This will be important in the next section, as the optimal policy parame-

[15] There are alternative policy rules for regime one that generate the same losses.

[16] For two regimes this is shown in the proof of proposition 6.

ters in regime one will change when a second regime with a different inflation target is introduced. However, the change in the regime one loss minimizing policy occurs simply from introducing regime switching and need not be related to an optimal response to uncertainty over the future inflation target.

5 Optimal Policy with Exogenous Inflation Target Uncertainty

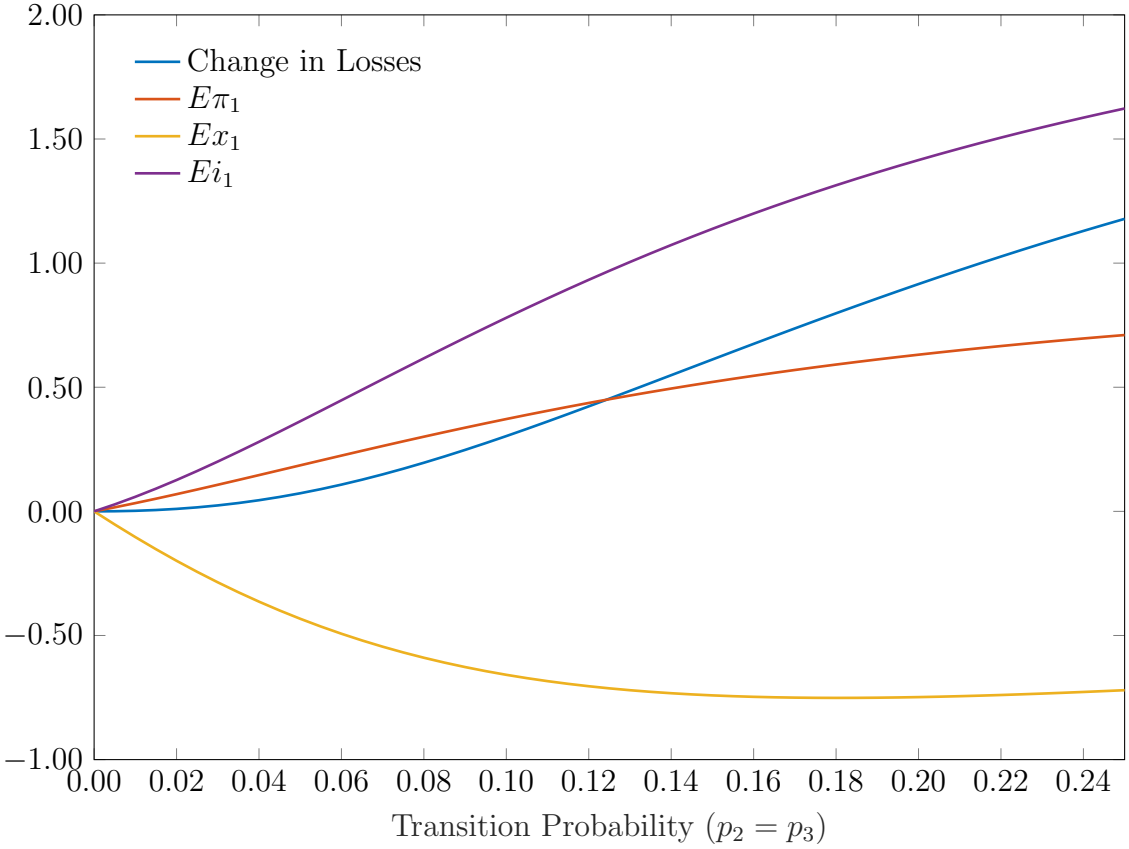
In the preceding sections, I solved the two regime model without any shocks not related to the change in the inflation target and explored the policy response possibilities of the central bank. Separately, I looked at the implications of regime switches and stochastic shocks on optimal policy. Here, I combine inflation target uncertainty with the stochastic shocks. I quantify the effects of inflation target uncertainty and the possible central bank responses to it in the full stochastic model, where I can evaluate the impact of policy parameters on the expected levels of inflation and output jointly with their effects on the volatilities from the other shocks.

Expanding on the analytic two regime model, I use a three regime stochastic model which has uncertainty in both if the inflation target will change and when it will change. The model initiates in regime one at a zero inflation target. For exogenous reasons, such as the current policy discussions of the lower real interest rate reducing the central bank's ability to respond to future crises given the zero lower bound for interest rates, there is uncertainty over the future inflation target. Each period there is a p_2 chance that the central bank will raise the inflation target to 2%. Alternatively, the policy debate may be resolved and the current inflation target may be kept permanently, which is modeled by a p_3 chance to transition to a third regime with a zero inflation target. Both the second and third regimes are absorbing. Therefore once a regime shift occurs, the model reduces to the single regime framework.

As a baseline, policy in all three regimes is assumed to take the single regime optimal policy given the loss function weights, and the transition probabilities p_2 and p_3 are the same and equal to .05. Since regime one is purely transitory, the outcomes after the regime shift are exogenous to policy prior to the regime shift, and expected losses in regime one are the only part of losses that the central bank can affect. Except for the optimal policy analysis, price indexing by firms only has minor quantitative effects, and I continue with no indexing as the main specification. Furthermore, I focus on two cases: $\theta_x = .0298$ and $\theta_x = 1$, since the effects differ substantially based on the policy rule of the central bank.

First lets consider the $\theta_x = .0298$ case. By introducing inflation target uncertainty in this manner, the expected inflation level in regime one jumps to 0.19 while the expected

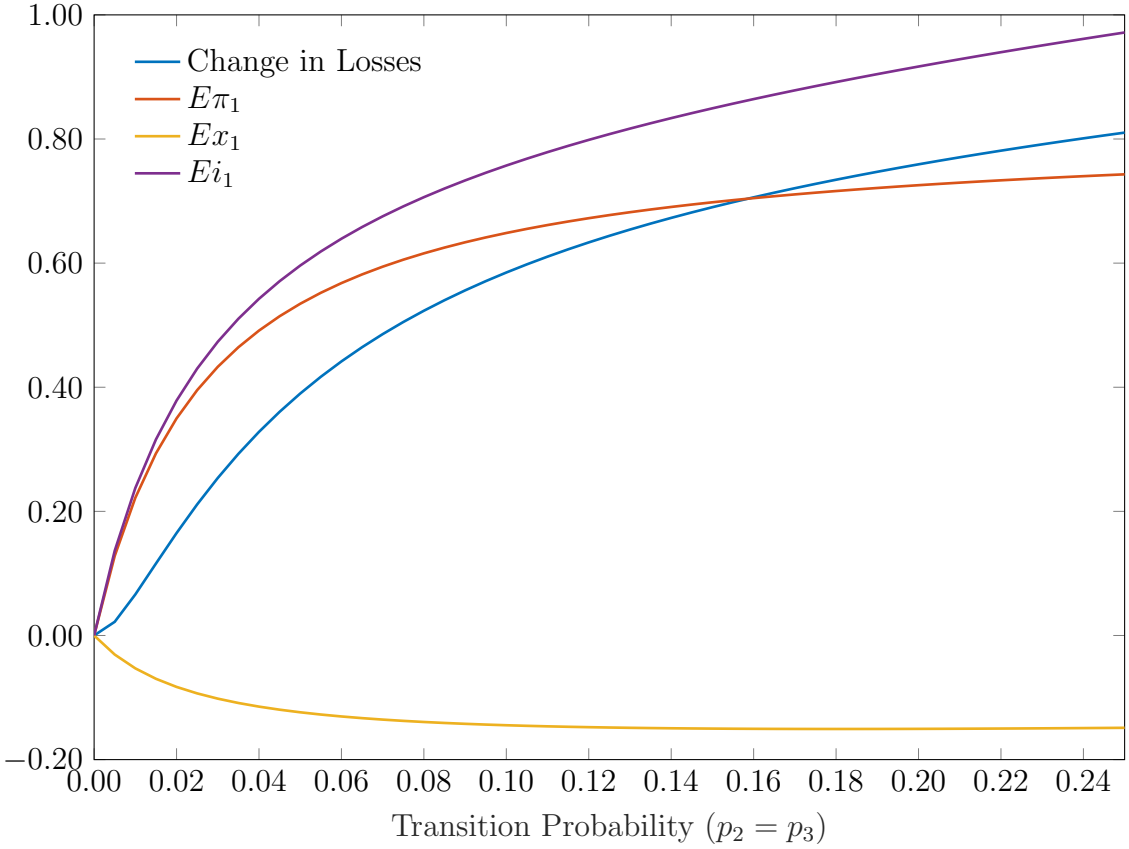
Figure 2: Expected Outcomes in Regime 1 for Different Transition Probabilities with Monetary Policy Focused on Inflation Stability



output gap falls to -0.43, the variances remain the same, and expected losses increase by 0.0735. Furthermore, since $p_1 + p_2 = .1$, the expected duration of regime one is 2.5 years, implying the monetary policy response to the higher expected inflation results in a strong, prolonged recessionary force. Figure 2 shows the results for a range of transition probabilities. For transition probabilities between 0% to 15%, a z% chance of transition to a 2% higher inflation target, results in roughly $4.5 * z$% increase in inflation and an over $5 * z$% decrease in the output gap. Despite sizable movements in the first moments, expected losses exhibit only a minor increase relative to an expected change to a more or less inflation stabilizing policy around a constant inflation target. If $\theta_x = 1$, monetary policy places a greater emphasis on output stability, and inflation goes up 0.53% from just a 5% chance of a shift to the higher inflation target, while the output gap falls to -0.12.

In the previous exercise, the long run probability the model ends up in the 2% inflation target regime was fixed at 50%. Next I hold the probability of resolving the inflation target uncertainty and moving to either regime 2 or 3 fixed at 10%, but vary the probability of transitioning into regime two vs regime three. Figure 4 shows the outcomes in regime one. As the probability of eventually transitioning to the 2% inflation target regime increases from 0 to 1, p_1 increases from 0 to .1 and p_3 decreases from .1 to 0. A .1% increase in

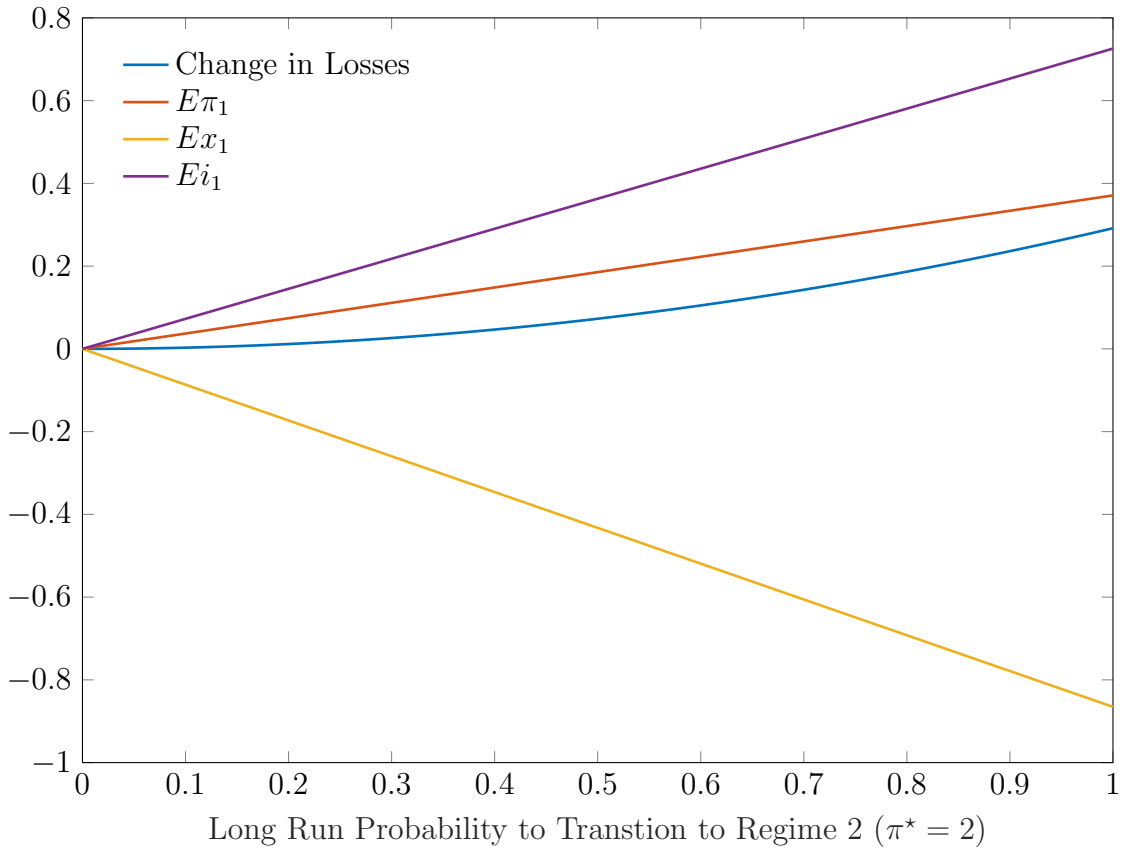
Figure 3: Expected Outcomes in Regime 1 for Different Transition Probabilities with Monetary Policy Focused on Output Stability



the per period probability to transition to regime two, increases the long run probability of ending up in regime 2 by 1%. For the main specification, it also increases the regime one inflation rate by .004% and interest rates by .007%, while reducing the regime one output gap by .009. However, the specific values are highly dependent on the emphasis the central bank places on inflation versus output stability.

If a central bank is fully committed to maintaining the expected level of inflation at zero, it can achieve this by lowering its current inflation target. If the central bank is focusing on inflation stability ($\theta_x = .0298$) and the transition probabilities to the other regimes are .05, then achieving $E\pi_1 = 0$ requires setting $\pi_1^\star = -.23\%$. Reducing the current inflation target will further reduce the expected output gap to -.55 for a net effect of a slight lower losses than with no policy response. Figure 5 shows the results for a range of transition probabilities. If policy is less focused on inflation stabilization, then achieving $E\pi_1 = 0$ requires a greater reduction in the current inflation target both because inflation increases more and reducing the same amount of inflation requires a larger decrease in the inflation target. For example if the central bank is pursuing a policy focused on output stability ($\theta_x = 1$), then achieving zero inflation requires reducing the current inflation target to -1.93% and for higher probabilities of a transition the required reduction in $\pi_1^\star$ exceeds well over the 2% potential future increase in the inflation target.

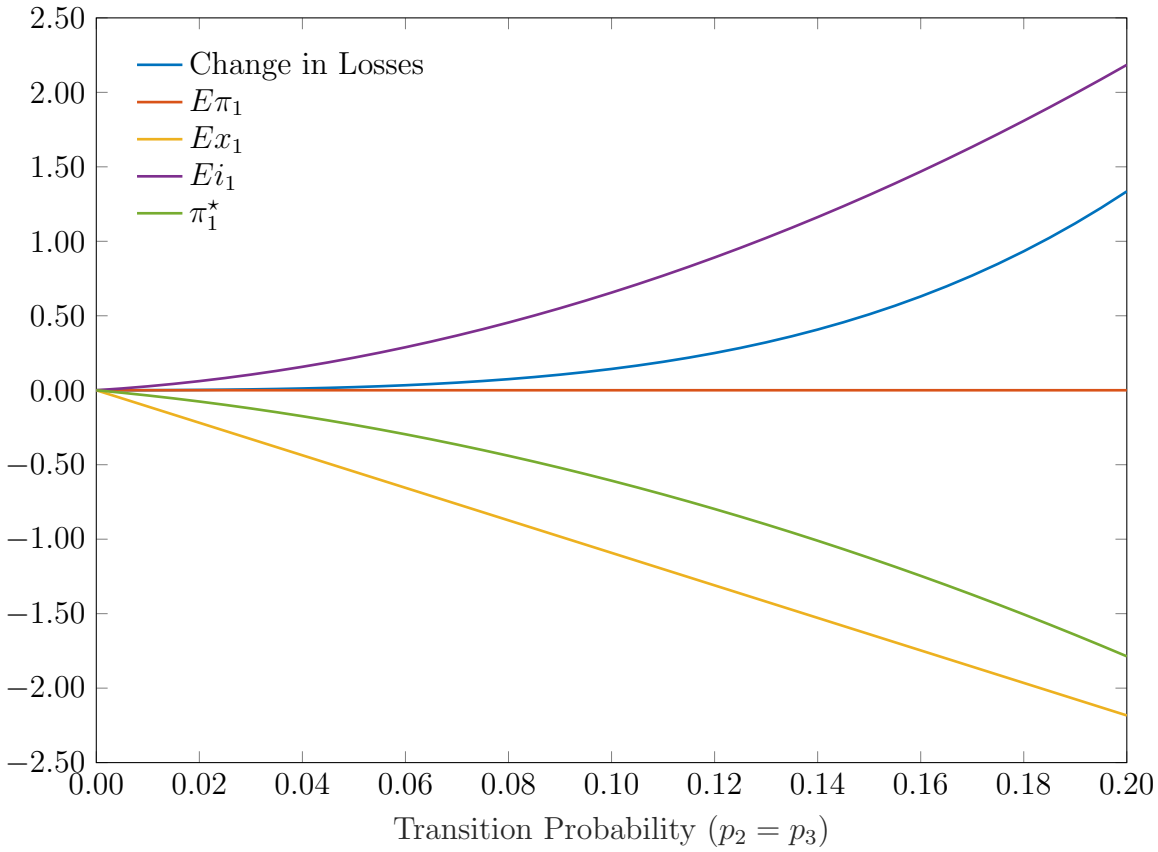
Figure 4: Expected Outcomes in Regime 1 for Different Long Run Transition Probabilities with Monetary Policy Focused on Inflation Stability



Even with equal weights on inflation and output stability, implementing a policy that achieves zero inflation in regime one is welfare improving.

While a central bank can achieve a zero expected level of inflation, the optimal response of the bank could be either to increase or decrease the current inflation target. Additionally, the optimal response involves different policy parameters on the responsiveness of interest rates to inflation and the output gap due to the presence of regime shifts. If the weight on inflation stability, θ_x, is .0298; then the optimal policy response to the regime shifts but without inflation target uncertainty sets $\phi_{\pi,1} = 1.1599$, $\phi_{\pi',1} = 0.8206$, $\phi_{x,1} = 0.4065$, and $\pi_1^\star = 0$ and reduces losses in regime one to 13.1796. Under this policy losses rise to 13.2979 from inflation target uncertainty. The optimal response to the inflation target uncertainty sets $\phi_{\pi,1} = 1.0096$, $\phi_{\pi',1} = 1.1234$, $\phi_{x,1} = 0.4066$, and $\pi_1^\star = -0.3282$. This policy results in an expected inflation of -0.0280%, an expected output gap of -0.5630, expected nominal interest rates of 0.1968%, and losses of 13.1995. Notably while the policy coefficients on inflation and expected inflation change, this does not effect the variances of any variable, and an equally effective policy is to only change the inflation target.Figure 6 shows the optimal policy outcome for a range of transition probabilities. For the main specification, the optimal response eliminates 83% of the losses from inflation target uncertainty.

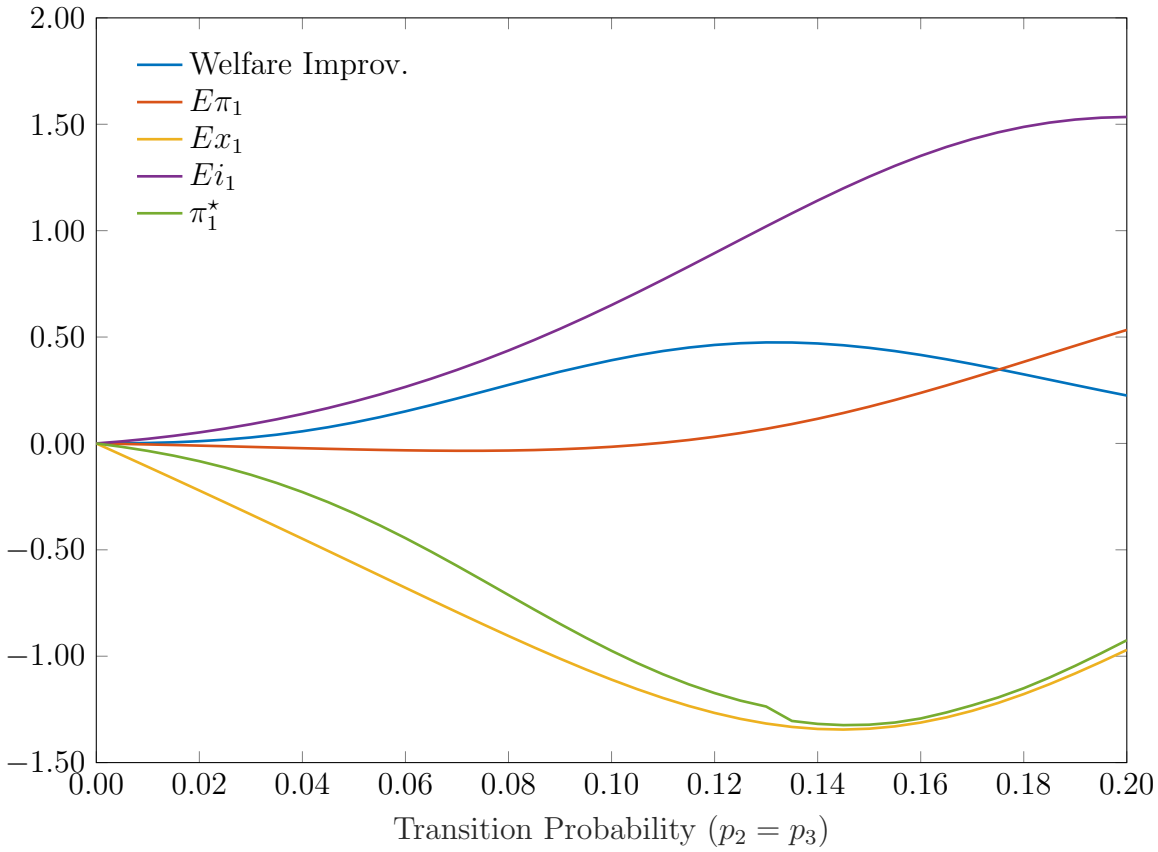
Figure 5: Expected Outcomes in Regime 1 if Central Bank Sets $\pi_1^\star$ such that $E\pi_1 = 0$ with Monetary Policy Focused on Inflation Stability



Price indexing plays a major role in determining the optimal policy response, as introducing it results in the Phillips curve shifting in response to changes in the inflation target and implies that changes in the rest of the response function change the set of feasible allocations the central bank is choosing among. The optimal policy depends on whether interest rates may respond negatively to expected inflation. If they can, then for the main specification the optimal response sets $\phi_{\pi,1} = 2.1141$, $\phi_{\pi',1} = -1.1019$, $\phi_{x,1} = 0.4061$, and $\pi_1^\star = -12.0067$. Such a policy eliminates over 98.7% of the losses from inflation target uncertainty. If coefficients are assumed to be nonnegative, then the optimal policy response and its benefits are qualitatively similar to those without price indexing except for the volatilities which slightly adjust.

The preceding results assume that the central bank may adjust the policy rule in order to exploit the transitory nature of regime one. As previously stated, all the benefits from optimal policy can be obtained from changes in the regime one inflation target, but the rest of the policy profile still matters. Is the optimal adjustment in the inflation target different if all other regime one policy parameters are set at their single regime optimal values? Figure 7 shows the results. It features similar trends to those in figure 6, but the inflation target adjusts less and the welfare benefit of the policy relative to the losses from inflation target uncertainty are smaller.

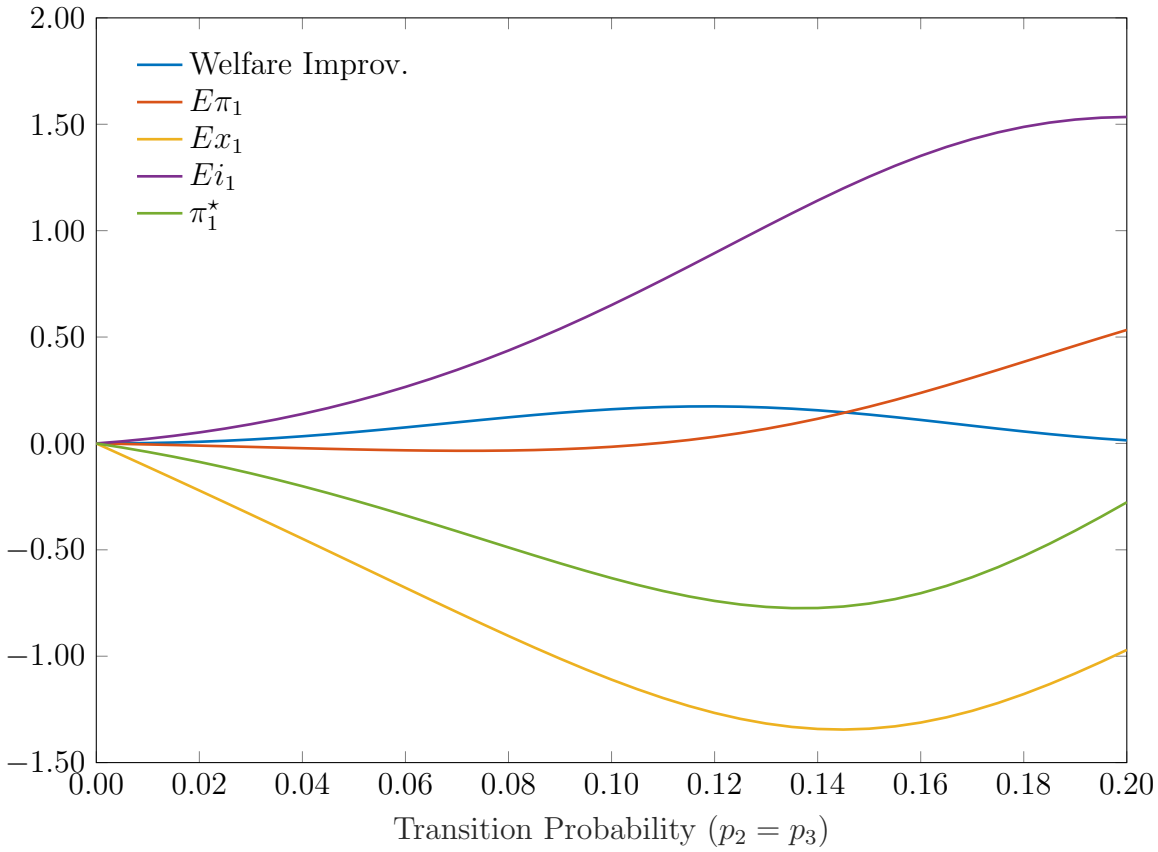
Figure 6: Expected Outcomes Under Optimal Policy with Monetary Policy Focused on Inflation Stability



To address how effective the optimal policy is, I compare the % of the losses generated by inflation target uncertainty that can be eliminated by adjusting monetary policy. Table 7 shows the results of this exercise for the optimal policy response and three constrained optimal policies. The optimal policy eliminates anywhere between 15% and 99% of the losses and is most powerful when the central bank values inflation and interest rate stability more. However, a central bank may be unwilling to change its inflation target in response to changes in expectations over the future inflation target, as such changes may themselves reduce confidence in the central bank's commitment to an inflation target. As discussed in the theoretical analysis, changes in policy responsiveness to inflation, expected inflation, and the output gap can affect the effects of inflation target uncertainty, but also change the inflation-output volatility trade off. Despite this, unless the central bank values output stability substantially more than inflation stability, the constrained optimal policy that keeps the inflation target unchanged is able to achieve at least half of the benefits of the optimal policy. Analytically, I showed that in the absence of the autoregressive shocks any feasible outcome of inflation and output gap levels is achievable just by adjusting the inflation target. Consistent with this, the optimal outcome can also be achieved by only adjusting the inflation target.

To measure the effects of optimal policy without exploiting the transitory nature of

Figure 7: Expected Outcomes at the Optimal $\pi_1^\star$ with Monetary Policy Focused on Inflation Stability



the current regime, consider the exercise in figure 7. Since adjusting the inflation target is sufficient to achieve the optimal outcome, the alternative policy experiment of optimally adjusting the inflation target but keeping all other parameters at their single regime optima provides an alternative measure of the effectiveness of the optimal policy. This effect is shown in the last column of table 7, and depending on the loss function specification it achieves somewhere between 50% and 98% of optimal policy's loss reduction.

The effectiveness of optimal policy in the model extension where there is price indexing by firms is broadly similar, and table 8 replicates the results from table 7 for this extension. The key difference is that simply adjusting the inflation target is no longer sufficient to achieve the optimal outcome. However, if the policy rule coefficients are restricted to be nonnegative then changing the inflation target is still sufficient to achieve at least 95% of the benefits from the optimal policy.

Throughout the optimal policy analysis, I've assumed the central bank may respond to either current or expected inflation as happens to be optimal given the shocks. However the literature often assumes the central bank will only respond to one of the two. In my model with a single regime, responding to either measure of inflation exclusively does not substantially affect losses. However, the response to inflation target uncertainty depends crucially on which measure of inflation the central bank responds to. Figure 8 shows

Table 7: Welfare Benefits of Optimal and Constrained Optimal Policies

Welfare Weights		% of Inflation Target Uncertainty Losses Eliminated			
θ_x	θ_i	Optimal Policy	Optimizing over		
			$\phi_\pi, \phi_{\pi'}, \phi_x$	$\pi_1^\star$	$\pi_1^\star$ and Base Policy[1]
0.0298	0.25	83.16%	76.52%	83.16%	72.69%
0.0000	0.25	87.60%	83.35%	87.60%	78.03%
0.5000	0.25	69.37%	48.11%	69.37%	62.43%
1.0000	0.25	61.29%	36.80%	61.29%	55.63%
2.0000	0.25	49.74%	25.15%	49.74%	45.18%
3.0000	0.25	41.93%	19.03%	41.93%	37.91%
4.0000	0.25	36.30%	15.31%	36.30%	32.64%
0.0298	0.05	15.87%	15.87%	15.87%	8.66%
0.0298	0.10	41.24%	41.24%	41.24%	28.56%
0.0298	0.40	95.22%	83.24%	95.22%	89.71%
0.0298	0.60	99.49%	93.25%	99.49%	97.51%

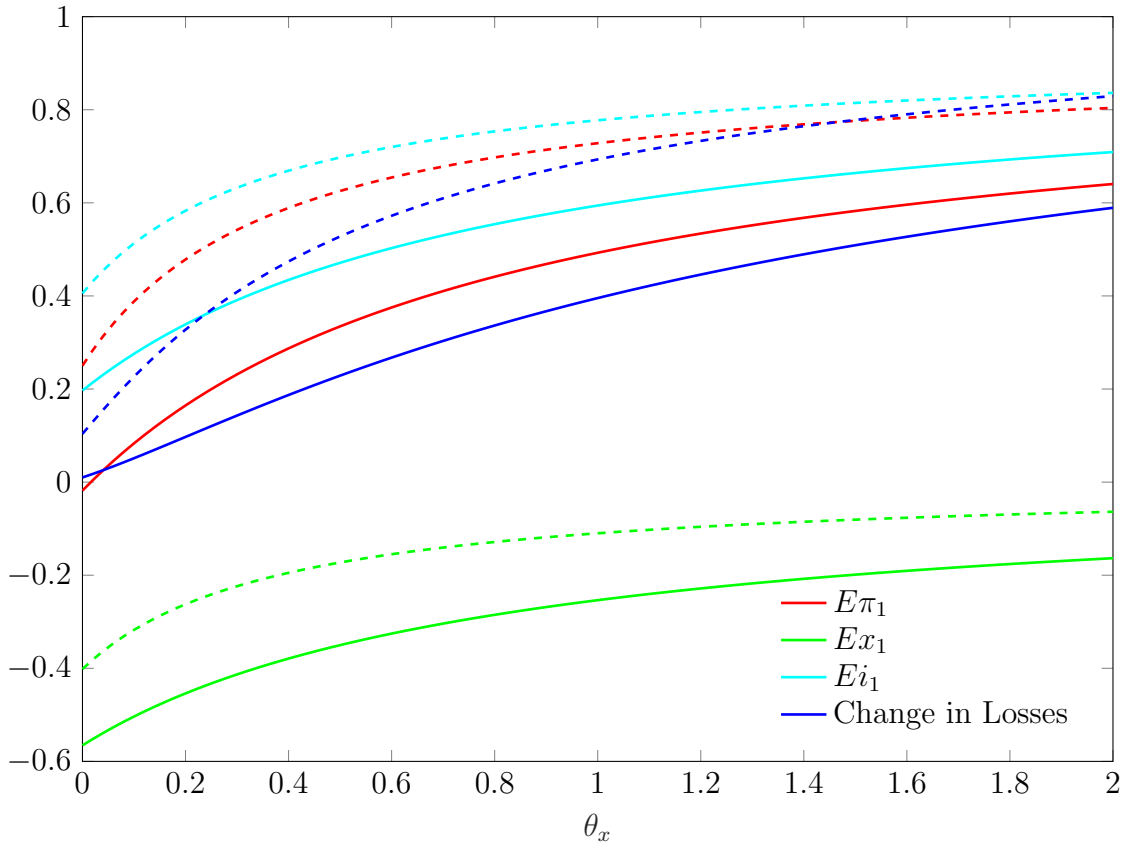
[1] Base policy: All other parameters in regime one are at their single regime optima rather than the regime one optimal policy in the three regime model without $\pi^\star$ uncertainty.

Table 8: Welfare Benefits of Optimal and Constrained Optimal Policies with Price Indexing

Welfare Weights		% of Inflation Target Uncertainty Losses Eliminated			
θ_x	θ_i	Optimal Policy	Optimizing over		
			$\phi_\pi, \phi_{\pi'}, \phi_x$	$\pi_1^\star$	$\pi_1^\star$ and Base Policy[1]
0.0298	0.25	84.90%	76.93%	83.94%	73.40%
0.0000	0.25	88.62%	84.21%	88.19%	78.70%
0.5000	0.25	81.95%	48.67%	76.55%	68.86%
1.0000	0.25	88.20%	38.17%	75.07%	67.51%
2.0000	0.25	94.25%	27.64%	75.17%	66.45%
3.0000	0.25	96.18%	22.04%	76.82%	66.73%
4.0000	0.25	97.12%	18.50%	78.92%	67.57%
0.0298	0.05	16.84%	15.22%	15.95%	8.74%
0.0298	0.10	42.26%	40.36%	41.40%	28.62%
0.0298	0.40	99.58%	83.74%	96.16%	90.59%
0.0298	0.60	99.96%	95.11%	99.34%	99.24%

[1] Base policy: All other parameters in regime one are at their single regime optima rather than the regime one optimal policy in the three regime model without $\pi^\star$ uncertainty.

Figure 8: Inflation Target Uncertainty Effects with Policy Responding to Current Inflation (dashed) and Expected Inflation (solid)



the effect of inflation target uncertainty when the central bank uses the optimal policy that only responds to either current inflation (dashed) or expected inflation (solid) for different loss function weights on output stability. Optimally responding to expected inflation results in lower values of expected inflation, output gap, and interest rates in regime one than responding to current inflation. The net effect of these differences is that expected losses rise more from inflation target uncertainty if interest rates are responding to current inflation instead of expected inflation.

6 Anticipated Change in the Inflation Target

In the preceding analysis, I have assumed that the central bank cannot commit to an inflation target and faces uncertainty over the future level of the inflation target. Here, I consider the alternative setting where the central bank knows that the inflation target will change at a specific point in the future. A future anticipated change in the inflation target may occur if the central bank knows that future government policy will mandate a different inflation target, anticipates new central bank governors pursuing an alternative inflation target, or simply wishes to implement an anticipated increase in the inflation target.

To model this, I use the single regime version of the model described by equations 16-18 to study an anticipated T periods in the future change in the inflation target from 0 to $\pi^\star$. I first solve the model to show the implications of an anticipated increase in the inflation target if the central bank does not act on this information, and I find that it is likely to generate cyclical fluctuations in inflation and output. Next, I allow the central bank to optimally adjust its policy given the anticipated change in the inflation target. Under optimal policy inflation monotonically increases to its new target along the transition path while output monotonically decreases. Finally, I compare the optimal transition path from an anticipated change in the inflation target to the transition path in the regime switching model. Since optimal policy can be implemented by changing the intercept in the policy rule which does not affect the inflation-output volatility trade off, I can study the effects of an anticipated change in the inflation target in a model without shocks. Throughout, I set $\phi_{\pi'}$ and ϕ_x to zero for a clearer narrative.

6.1 Perfect Foresight Dynamics

Let us first consider a one period in advance announcement of the inflation target change, which illuminates much of the analysis of the general case. If at $t = 0$, the central bank announces that at $t = 1$ the inflation target will be permanently raised from zero to $\pi^\star$, then it is straightforward to show that

$$x_t = \begin{cases} 0, & \text{if } t < 0 \\ \frac{1-\beta-\sigma^{-1}\kappa(\beta\phi_\pi-1)}{\kappa(1+\kappa\sigma^{-1}\phi_\pi)}\pi^\star, & \text{if } t = 0 \\ \frac{1-\beta}{\kappa}\pi^\star, & \text{if } t > 0 \end{cases} \quad \text{and} \quad \pi_t = \begin{cases} 0, & \text{if } t < 0 \\ \frac{1+\kappa\sigma^{-1}}{1+\kappa\sigma^{-1}\phi_\pi}\pi^\star, & \text{if } t = 0 \\ \pi^\star, & \text{if } t > 0 \end{cases}. \tag{34}$$

Before period zero, the economy is at the zero inflation steady state. Since the inflation target is always zero and there are no shocks, expected inflation is zero and firms will only adjust prices in response to marginal costs. Additionally, real interest rates are also zero at zero inflation. Hence, there is no incentive to deviate from the steady state savings and consumption allocations. Therefore the output gap and marginal costs are zero and firms optimally choose to keep prices constant. After period zero, the model is similarly in a steady state, but inflation is at the new inflation target, and the output gap is positive. To characterize the outcome at time zero, it is useful to derive a few more results.

Proposition 7 *If equation 34 holds and $\pi^\star > 0$, then*

1. $\pi_0 \in (0, \pi^\star)$ *if* $\phi_\pi > 1$ *and* $\pi_0 \geq \pi^\star$ *otherwise*
2. $x_0 < 0$ *if* $\phi_\pi > \frac{\kappa\sigma^{-1}+1-\beta}{\beta\kappa\sigma^{-1}}$ *and* $x_0 \geq 0$ *otherwise*
3. $\frac{\partial x_0}{\partial \phi_\pi} < 0$ *and* $\frac{\partial \pi_0}{\partial \phi_\pi} < 0$

After the announcement, inflation and output adjust to their new levels which are proportionally to the future inflation target and decreasing in the responsiveness of monetary policy to inflation. If monetary policy does not respond to inflation, then real interest rates fall, agents are incentivized to consume, and the output gap increases. Firms expecting higher prices in the future and facing sticky prices set higher prices. Additionally the higher output gap implies higher marginal costs which further push prices up and lead to inflation overshooting its future steady state. When monetary policy becomes responsive to inflation, real interest rates fall by less and cause a smaller increase in output and marginal costs. Facing lower marginal costs, firms do not increase prices as much. If nominal rates respond at least one for one to inflation, then the real interest rates rises, and marginal costs do not rise enough for inflation to overshoot its future steady state value. As the responsiveness of nominal interest rates to inflation further increases past $\frac{\kappa\sigma^{-1}+1-\beta}{\beta\kappa\sigma^{-1}}$, real rates are high enough to incentivize sufficient savings for the output gap to become negative.[17] A negative output gap implies firms have lower marginal costs and puts downwards pressure on inflation. Further increasing the responsiveness of nominal interest rates to inflation increases real interest rates and savings, while reducing output, marginal costs, and inflation.

While the preceding example is simple, the intuition applies to the general T period in advance announcement. For the general case, the periods prior to the announcement, after the inflation target has changed, and one period prior to the change in the inflation target remain identical. To examine the full dynamics, I first characterize the solution. Since there are no shocks in the model aside for the inflation target, the model can be solved forward with inflation and output being weighed sums of future expected inflation targets. Solving for inflation,

$$\pi_t = \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_t(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\bar{i}_{t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\bar{i}_{t+j}), \tag{35}$$

with $\lambda = \frac{1+\beta+\sigma^{-1}\kappa\pm\sqrt{(1+\beta+\sigma^{-1}\kappa)^2-4\beta(1+\sigma^{-1}\kappa\phi_\pi)}}{2(1+\sigma^{-1}\kappa\phi_\pi)}$.

The dynamics of inflation and output depend on whether λ is real or complex.

Proposition 8 *If*

$$\frac{(1-\beta)^2}{\sigma^{-1}\kappa}+\sigma^{-1}\kappa \geq 2(\beta\phi_\pi-1)+2\beta(\phi_\pi-1), \tag{36}$$

[17] When $1<\phi_\pi<\frac{\kappa\sigma^{-1}+1-\beta}{\beta\kappa\sigma^{-1}}$, real interest rates rise thereby increasing savings, but the output gap remains positive because this effect is dominated by the consumption smoothing motive combined with expectations of higher consumption in the future.

then λ is a real number, and for $t \in [0, T]$

$$\pi_{T-t} = \pi^\star \frac{\lambda_2^{t+1}(1-\lambda_1) - \lambda_1^{t+1}(1-\lambda_2))}{\lambda_2 - \lambda_1} \tag{37}$$

$$x_{T-t} = \pi^\star \frac{\lambda_2^{t+1}(1-\lambda_1)(\lambda_2-\beta) - \lambda_1^{t+1}(1-\lambda_2))(\lambda_1-\beta)}{\kappa(\lambda_2-\lambda_1)}. \tag{38}$$

Otherwise λ is a complex number, and for $t \in [0, T]$

$$\pi_{T-t} = \frac{\pi^\star(\phi_\pi - 1)\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi} r^t \sum_{j=0}^{\infty} r^j \frac{\sin(\omega(t+1+j))}{\sin(\omega)}, \tag{39}$$

where $r = \sqrt{\frac{\beta}{1+\sigma^{-1}\kappa\phi_\pi}}$, $\omega = \cos^{-1}(\frac{1+\beta+\sigma^{-1}\kappa}{2\sqrt{\beta(1+\sigma^{-1}\kappa\phi_\pi)}})$, and $x_{T-t} = \frac{\pi_{T-t} - \beta\pi_{T-t+1}}{\kappa}$.

If the monetary policy response to inflation is sufficiently strong for equation 36 to be violated ($\phi_\pi \geq 1.05$ in the main calibration), then inflation and the output gap exhibit cyclical dynamics that are magnified as the period with the expected change in the inflation target approaches. If the monetary policy response to inflation is sufficiently weak, then equation 36 holds, and inflation and the output gap exhibit non-cyclical dynamics that are either monotonic or can be split into two subsets on which they are monotonic. To analyze the dynamics when λ is real, it useful to look at the recursive formulation of inflation and output for $0 \leq t < T$

$$\pi_{t-1} = \frac{\beta + \sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\pi_t + \frac{\kappa}{1+\sigma^{-1}\kappa\phi_\pi}x_t \tag{40}$$

$$x_{t-1} = \frac{-\sigma^{-1}(\beta\phi_\pi - 1)}{1+\sigma^{-1}\kappa\phi_\pi}\pi_t + \frac{1}{1+\sigma^{-1}\kappa\phi_\pi}x_t, \tag{41}$$

using which I can prove proposition 9.

Proposition 9 *If λ is real and $\pi^\star > 0$, then*

1. *If $\phi_\pi < 1$, then $\pi_{T-t} > \pi^\star$, $x_{T-t} > \frac{1-\beta}{\kappa}\pi^\star$, and both are monotonically decreasing.*
2. *If $\phi_\pi = 1$, then $\pi_{T-t} = \pi^\star$, $x_{T-t} = \frac{1-\beta}{\kappa}\pi^\star$*
3. *If $\frac{1}{\beta} \geq \phi_\pi > 1$, then $0 < \pi_{T-t} < \pi^\star$, $0 < x_{T-t} < \frac{1-\beta}{\kappa}\pi^\star$, and both are monotonically increasing.*
4. *If $\phi_\pi > \frac{1}{\beta}$, then for small t; $\pi_{T-t} < \pi^\star$, $x_{T-t} < \frac{1-\beta}{\kappa}\pi^\star$, and both are monotonically increasing. For large enough t, either or both π_{T-t} and x_{T-t} may be monotonically decreasing.*

The final case need not exist, but for any numerically reasonable calibration it exists, inflation is nonnengative and monotonically increasing, and output is declining for large

t and monotonically increasing for smaller t. Combining proposition 9 with the cyclical case when λ is complex, there are five possible cases for the dynamics, which are all shown in Figure 9.

If interest rates are independent of inflation, then in the period prior to the inflation target change real interest rates fall, agents are incentivized to consume, and the output gap increases, while firms set higher prices in response to the expected inflation and marginal costs. Anticipating this in the earlier periods, inflation is expected to be even higher. Therefore real interest rates are even lower, creating a stronger incentive for consumption and therefore higher output and marginal costs. Firms anticipate higher expected inflation and face even higher marginal costs and set even higher prices. Hence at the time of the announcement, inflation and the output gap jump above their future steady state values and monotonically decline to approach the future steady state values as time moves forward. If monetary policy responds to inflation less than one for one, then the same things happen, but real interest rates fall by less. Therefore there is a smaller incentive to raise consumption, which reduces the rise in output, marginal costs, and inflation.

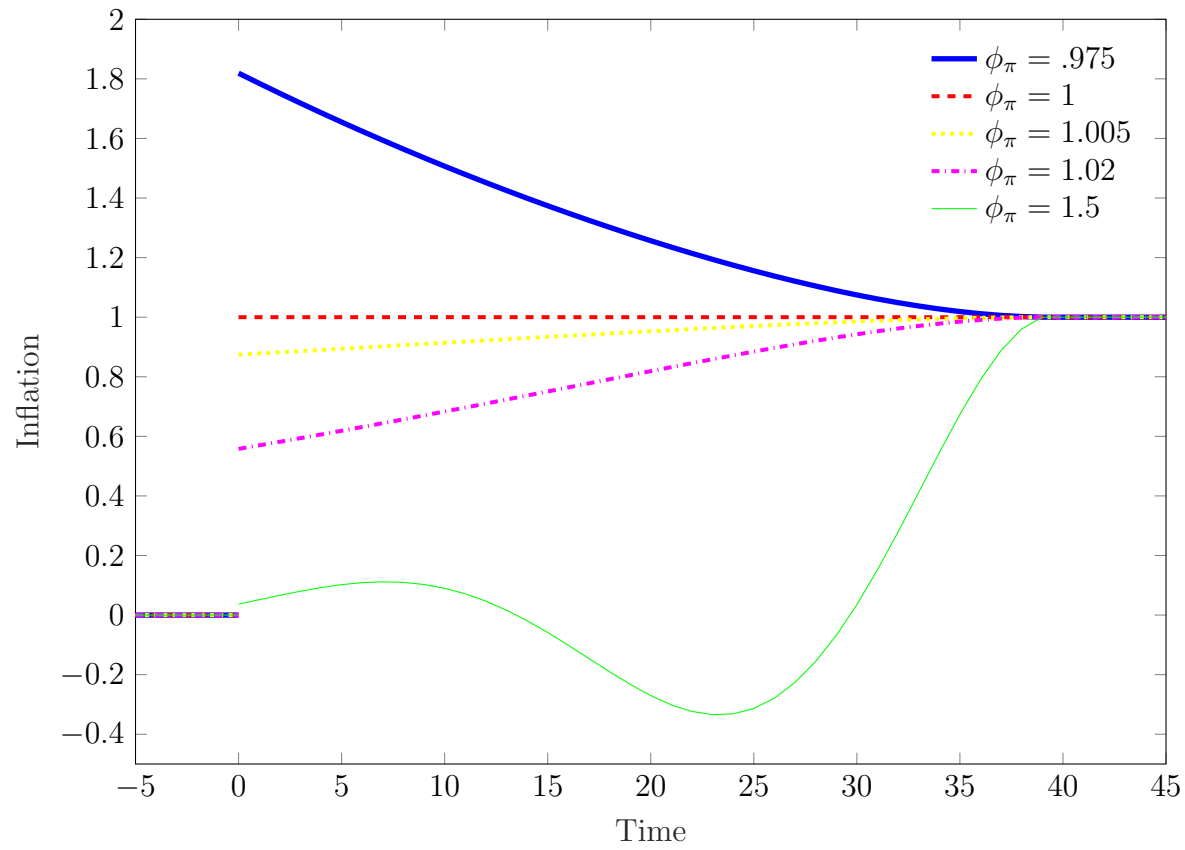
If nominal interest rates respond exactly one for one with inflation, then at the time of the announcement both inflation and output shift up to their future steady states and are constant in the following periods. At $\pi^\star$ inflation, real interest rates will be constant in the period prior to the inflation target change and therefore output will be also be constant. Facing the future steady state output and expected inflation rates, firms will set prices consistent with the future steady state inflation rate of $\pi^\star$. Anticipating this, each prior period will be identical for the same reasons.

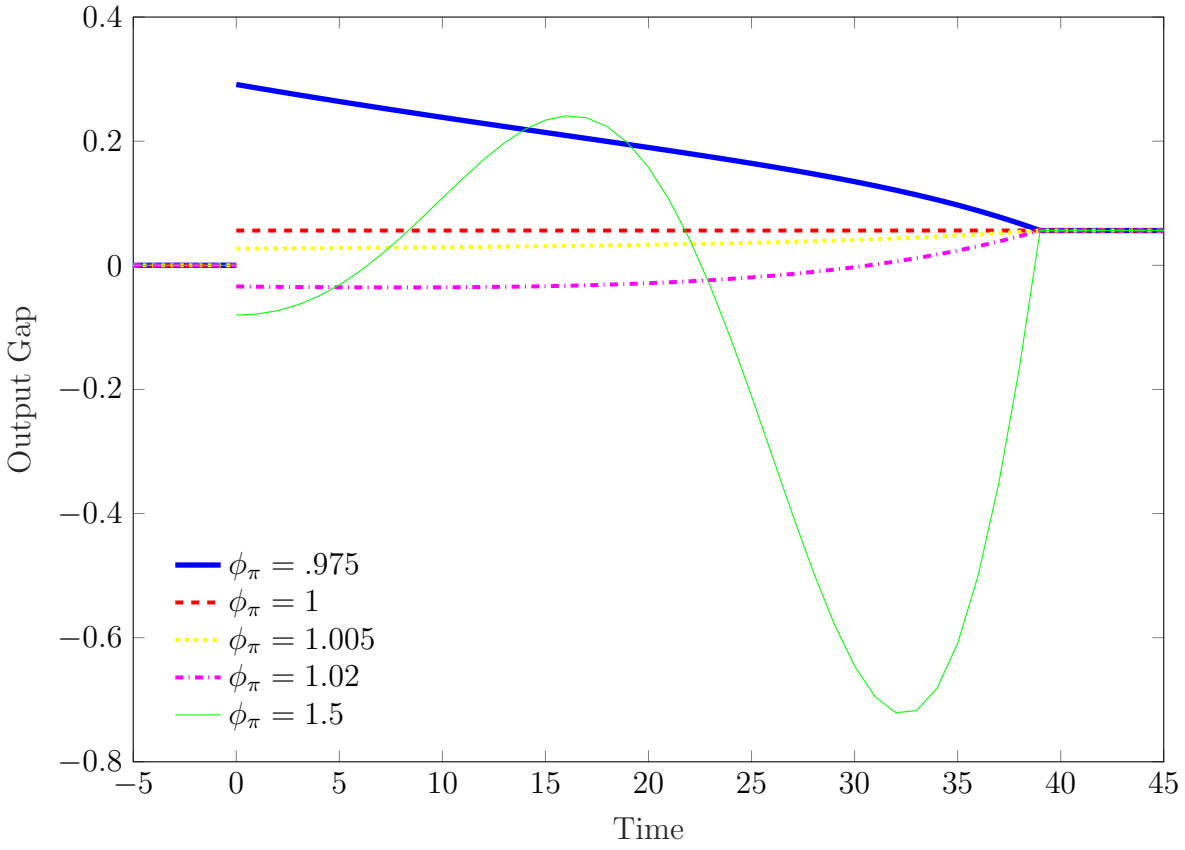
If the monetary policy response to inflation is slightly greater than one ($1 < \phi_\pi < \frac{1}{\beta}$), then real interest rates rise slightly. With positive real interest rates agents face an incentive to consume less thereby reducing output. Lower output implies lower marginal costs, which lead firms to set lower prices. However, the monetary policy response in this range is weak, and the incentive to save does not push output below zero, which implies that marginal costs remain positive and inflation barely falls. So at the time of the announcement inflation and output both jump up, and then they monotonically increase to their future steady state values.

If the monetary policy response is further strengthened ($\frac{1}{\beta} < \phi_\pi$ and equation 36 holds), then real interest rates rise by more. This leads to a greater incentive to save and results in the output gap falling below zero, which pushes marginal costs below zero and further reduces inflation but not below zero. So at the announcement inflation shifts up and monotonically increases to the future steady state value, while output is also monotonically increasing but at the announcement may jump in either direction depending on how far in the future the inflation target change is.

The final case occurs if the response of interest rates to inflation is sufficiently large

Figure 9: Potential Inflation and Output Dynamics of a Period Zero Announcement that the Inflation Target in Period 40 Will be Raised from Zero to One





for equation 36 to be violated. In standard calibrations this implies that $\phi_\pi \geq 1.05$ and is therefore the most relevant case for policy analysis. As ϕ_π is further increased, the direct effect in the periods prior to the change in the inflation target is the expected interest rate rises even more. This amplifies the incentive to save, leading to a larger drop in the output gap. While firms still retain an incentive to set higher prices from the expected higher inflation, the large negative marginal costs reduces prices further. The anticipation of a strong monetary policy response that will cause a large recession prior to the inflation target change, leads firms to anticipate lower future prices, and combined with lower marginal costs reduces inflation below zero. However if the announcement is made further away from the actual change, this will all be anticipated as will the very low nominal interest rates that will accompany inflation below the current inflation target. The very low nominal rates will lead to negative real interest rates and a growth of consumption, output, and marginal costs. Further back from the inflation target change, the monetary response to the low inflation leading to low real interest is anticipated. So output and marginal costs are high, and firms while anticipating low expected inflation in a few periods, increases prices in response to high marginal costs both in the current and future periods and inflation is again positive. Even further back the cycle repeats, with an anticipation of the monetary policy response to this inflationary period causing a recession and so on.

The preceding analysis largely extends to price indexing by the firms which do not get to optimally set their prices each period. Four of the five possibilities previously characterized are possible. If $1 \leq \phi_\pi < \frac{1}{\beta}$, then without price indexation real interest rates rise and encouraged savings. Higher savings result in lower output and lower marginal costs. However as the expected output gap in the future steady state is positive, the small contractionary force is not enough to cause a negative output gap. With price indexing to the inflation target, the future steady state has a zero output gap and a small contractionary force causes the output gap to decrease. The other cases remain qualitatively unchanged.

6.2 Optimal Response to an Anticipated Change in $\pi^\star$

The previous section shows that if the central bank anticipates a change in the inflation target, then for reasonable calibrations of a time invariant policy rule both inflation and output exhibit cyclical dynamics along the transition path. By sticking to the same policy rule, a central bank is therefore destabilizing inflation and output through its response to current inflation. Weakening the response of interest rates to inflation can result in a more monotonic transition path but comes at the cost of impacting the inflation-output volatility trade off. Alternatively by choosing the announcement length the central bank may also eliminate the cyclical elements. For example, in figure 9 if the anticipated

change in the inflation target will occur within six periods, then the transition path is monotonic for $\phi_\pi = 1.5$. However, a central bank may face a situation when the timing of the change in the inflation target is outside of the current central bankers' control.

A natural alternative option is for the central bank to optimal respond to the anticipated change in the inflation target. In particular the central bank may commit to a path for the policy rule intercept along the transition path which minimizes the central bank's loss function. Formally, let the time varying policy rule be

$$i_t = \phi_\pi \pi_t + (1 - \phi_\pi)\pi_t^\star = \phi_\pi \pi_t + \bar{i}_t \tag{42}$$

Then let the $t = 0$ announcement that the inflation target for $t \geq T$ will increase from 0 to $\bar{\pi}$ be accompanied by announced path for $\pi_t^\star$ for $0 \leq t < T$. Since the only thing that changes is $\pi_t^\star$ this has no impact on the inflation-output volatility trade off, and if the central bank sets $\pi_t^\star$ to the optimal path that minimizes losses from the anticipated change in the inflation target, this does not generate sufficient freedom to implement discretionary or time inconsistent policy.

To determine the optimal path of $\pi_t^\star$, the central bank solves the loss minimization problem

$$\min_{\pi_t, x_t} \sum_{t=0}^{T} \beta^t (\pi_t^2 + \theta_x x_t^2) \text{ s.t. } \pi_t = \kappa x_t + \beta \pi_{t+1}, \pi_T = \bar{\pi} \tag{43}$$

for the optimal transition path. Given the transition path of inflation and the output gap, the interest rate that implement the transition path is determined by the linearized Euler equation (16), while the path for $\pi_t^\star$ and $\bar{i}_t$ are determined by equation 42.

Solving the loss minimization problem results in

$$\pi_t = A_1 m_1^t + A_2 m_2^t \tag{44}$$

$$x_t = \frac{m_1^t A_1 (1 - \beta m_1) + m_2^t A_2 (1 - \beta m_2)}{\kappa} \tag{45}$$

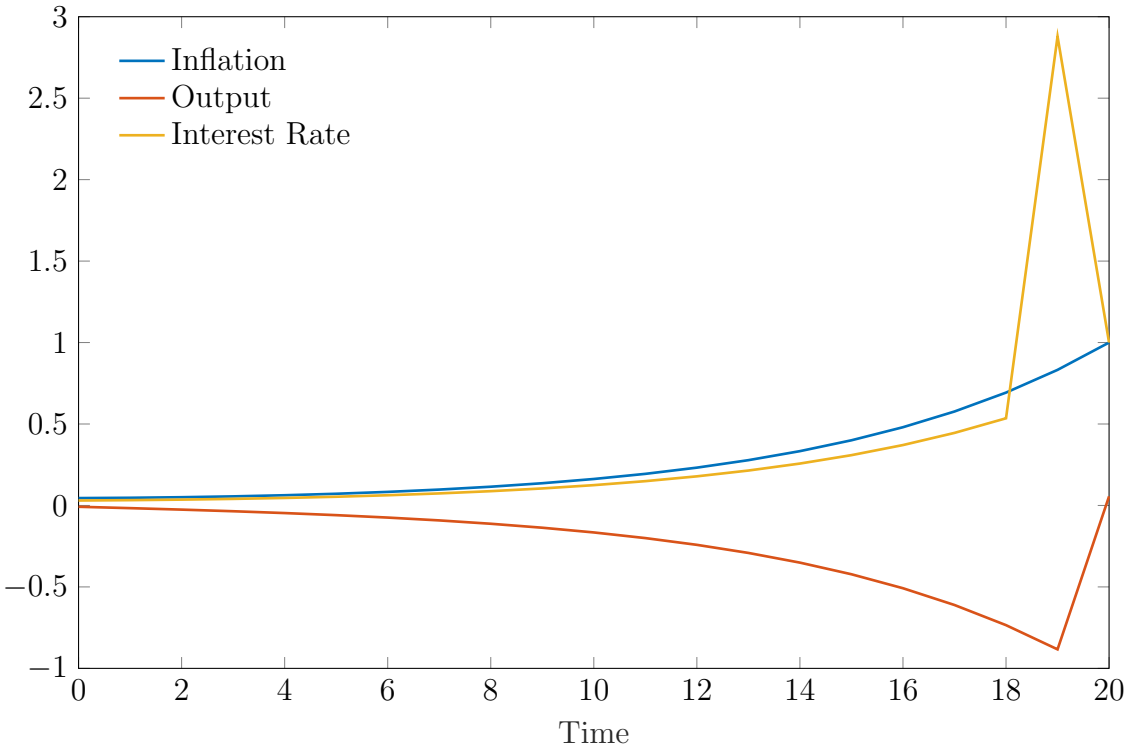
where the parameters A_1, A_2, m_1, and m_2 are given in the appendix.

Proposition 10 *Under the optimal transition path for an anticipate future change in the inflation target from 0 to $\bar{\pi}$, inflation at the time of the announcement jumps in the direction of $\bar{\pi}$ and then monotonically adjusts to $\bar{\pi}$.*

A key implication of the solution's functional form is that cyclical fluctuations never occur under optimal policy. Therefore the optimal policy outcome in the presence of an anticipated future inflation target change is qualitatively different from the outcome under a time unvarying policy rule if equation 36 is violated. Furthermore, inflation monotonically adjusts from the old target to the new target with a jump at the time of the announcement. To characterize the optimal policy and the implied transition path

for the other variables, I use numeric simulations of equations 44 and 45 along with the implied path for i_t, $\bar{i}_t$, and $\pi_t^\star$.

Figure 10: Optimal Transition Path for a Perfectly Anticipated Increase in $\pi^\star$

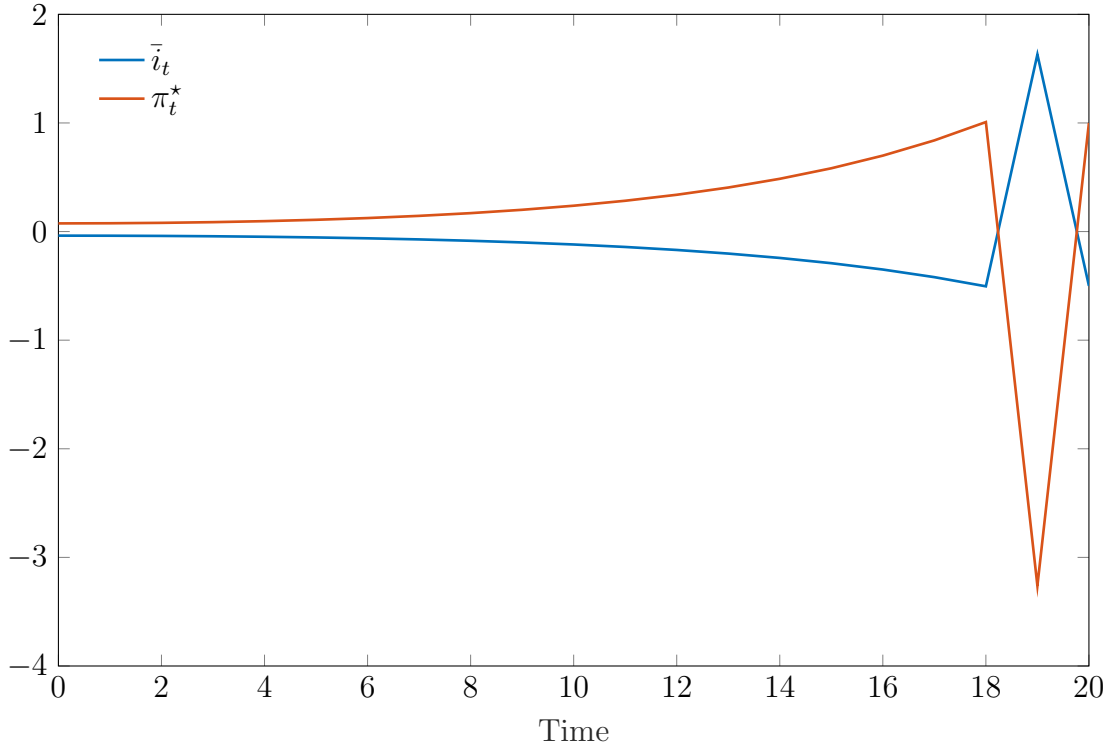


Note: $T = 20$, $\bar{\pi} = 1$, $\theta_x = 1$ and $\theta_i = 0$

Figure 10 shows the optimal transition path for a 20 period in advance anticipated increase in the inflation target from 0 to 1. Along the path inflation is monotonically increasing at an increasing rate, while output is monotonically decreasing along the transition path before increasing to its new steady state value once the inflation target change occurs. As with a time invariant policy rule with $\phi_\pi > 1$, interest rates rise to cause a recession along the transition path, which results in lower marginal costs and lower inflation. The key difference is that the time varying optimal policy rule can increases interest rates arbitrarily in the period right before the change in the inflation target. This allows an arbitrarily large drop in output right before the inflation target changes. Therefore marginal costs are lowest when firms face the largest incentive to raise prices from expectations of higher future inflation. Further away from the inflation target change, firms have a smaller incentive to increase prices from expectations of future inflation. They therefore need less incentive to reduce prices from low marginal costs and the optimal policy is able to generate such a path for marginal costs.

The optimal transition path does not depend on ϕ_π or σ and is qualitatively similar for any θ_x, κ, and T, and for any $\beta > .5$. A smaller θ_x results in a larger drop in output, while inflation remain near zero for longer. A smaller κ results in a more linear transition path, while a smaller β requires a smaller decrease in the output gap to achieve a similar

Figure 11: Optimal Policy for a Perfectly Anticipated Increase in $\pi^\star$



Note: $T = 20$, $\bar{\pi} = 1$, $\theta_x = 1$ and $\theta_i = 0, \phi_\pi = 1.5$

path for inflation.

Figure 11 shows the optimal policy that implements the outcome from figure 10 when $\phi_\pi = 1.5$. Consistent with the spike in interest rates the period before the inflation target change, the intercept of the policy rule increases substantially right before the inflation target change. However, in all prior periods the intercept is slightly negative and is used to stimulate the economy relative to the constant intercept case. Since $\phi_\pi > 1$, increasing the intercept in the policy rule is equivalent to decreasing the inflation target in the policy rule. Hence, $\pi_t^\star$ is inversely proportional to $\bar{i}_t$. The specifics of the optimal policy depend on all the parameters, but remain qualitatively similar across calibrations.

6.3 Perfect Foresight vs Regime Shift Transitions

In the preceding sections, I have characterized the transition path from a perfectly anticipated change in the inflation target both under a constant policy rule and the optimal time varying policy rule. Similarly, I have characterized transition path for an anticipated change in the inflation target at unanticipated date as modeled by a regime switching framework of section 3. In this section, I quantitatively compare the optimal policy transition paths under under the perfect foresight and the regime switching frameworks, and ask which transition path is preferred.

Two compare the two different frameworks, I consider an anticipated increase in the

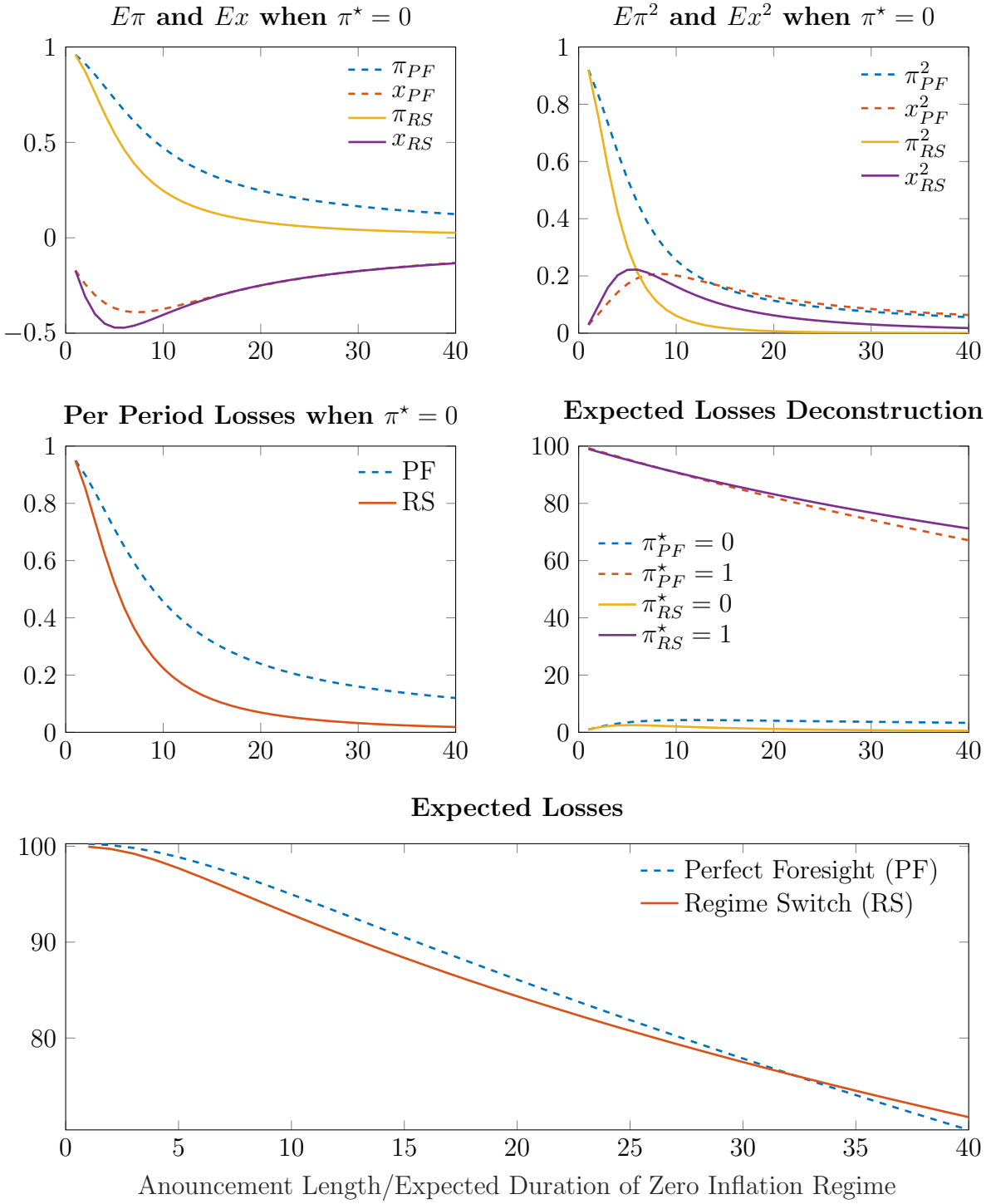
inflation target in T periods and the expectation that in each future period there is a $1/T$ chance the inflation target will be raised to the new value. Under both methods, the expected duration until the adjustment in the inflation target is T periods, but under the regime shift mechanism the actual duration could be much longer or shorter. As in the analytical analysis, the second regime is absorbing to make the two frameworks comparable, and there are no shocks to the structural equations. In both cases policy prior to the inflation target change is set to the optimal policy for that framework. The perfect foresight optimal policy is a time varying interest rate rule as in section 6.2, while the regime switch optimal policy is as described in proposition 6.

Figure 12 shows the losses, a decomposition of the losses, and the expected values of the output gap and inflation at the initial inflation target for announcements of a future inflation target change that are expected to occur between one quarter and 10 years in the future. For illustration purposes $\theta_x = 1$, but alternative values are not qualitatively different. As the optimal inflation rate is zero, this captures the idea that the central bank wishes to maintain its initial target prior to the inflation target change, and policy is valued on its ability to stabilize inflation around the current inflation target. Expected values at the new inflation targets are not shown, because after the inflation target changes the regime shift model and the perfect foresight model are identical.

The average levels of the output gap on the transition path are very similar, while the average inflation rate is higher under perfect foresight although for small θ_x the difference is minimal. However the loss function penalizes squared deviations which are higher under prefect foresight because the transition path is more variable. This results in average per period losses prior to the change in the inflation target to be higher for the perfect foresight transition path. However, the total expected losses need not be higher because with regime shifts the higher loss periods after the inflation target change are more likely to occur earlier when losses are discounted less. There is more room for this effect if the expected change in the inflation target is further in the future. Hence, expected losses are lower under perfect foresight if the inflation target change is expected to occur far in the future but higher for more immediate anticipated changes in the inflation target.

This exercise suggests that a central bank that optimally responds to an anticipated change in the inflation target can achieve roughly similar welfare outcomes regardless of whether the specific period of the inflation target change is known or not. Perfect foresight of the inflation target change is preferred if the anticipated change is known far in advance or if the central bank cares about inflation smoothly adjusting. Furthermore, there may be additional costs to the uncertainty not captured in the log linearized model that may make the perfect foresight outcomes strictly preferred.

Figure 12: Anticipated Increase in the Inflation Target from 0% to 1% Under Optimal Policy



Note: Under perfect foresight (PF) the central bank announces that the inflation target will be raised in T periods, while under regime switching (RS) there is a $1/T$ chance of a regime shift to the absorbing, higher inflation regime.

7 Conclusion

In models with rational expectations, expectations over future monetary policy play a significant role on current outcomes. A growing literature explores how expectations of an alternative policy can result in determinancy even under normally indeterminate monetary policy. However expectations of alternative monetary policy regimes in the future can have detrimental affects on current outcomes. Monetary policy in the future may change with new developments in monetary policy theory, as monetary policy addresses new challenges, and as the decision makers change. For example, the academic and policy discussions on the (potentially) long term lower real interest rate climate has spurred discussions on whether increasing the inflation target will be beneficial. If these discussions generate uncertainty over the future inflation target, then they will have an immediate, unintended affect on inflation and the output gap.

In this paper I use an extension of the standard New Keynesian model to monetary policy regime switching to study the effects of future inflation target uncertainty, how the effects depends on monetary policy, and the policy responses to it. While more complicated models will be helpful to assess specific quantitative policy implications, the model captures the fundamental aspects of a New Keynesian model and has a tractable analytical analog in which the results can be developed. Furthermore, while some recent research suggests that the role of expectations is overstated in the standard New Keynesian model; to the extent this is true, this will weaken the quantitative effects of inflation target uncertainty, but the qualitative results should remain the same.

An expectation that the inflation target may increase in the future raises expected inflation thereby increasing the optimal price firms set in the presence of sticky prices. With active monetary policy, the nominal interest rates rise by more than expected inflation and real interest rates also increase causing increased savings and lower output. Depending on the price indexing of the firms not getting to optimally set their price, a higher future inflation target may lead to higher output in the future, and therefore introduce a consumption smoothing motive that increases current output. The net effect is either slightly higher output or a decline in output and a corresponding change in the marginal cost that may reduce the firms incentive to raise prices. The qualitative and quantitative results depend on the complete monetary policy profile as it determines whether real interest rates rise, and if interest rates respond to expected inflation then the policy response can cause a recession severe enough to reduce marginal costs sufficiently for firms to lower prices.

The central bank can respond to inflation target uncertainty and choose any point along the aggregate supply relationship by changing the current inflation target or equivalently a constant in a policy rule, without effecting the inflation–output volatility tradeoff. By adjusting the current inflation target a central bank can achieve it's initial inflation

target. Since this will also affect the output gap, the optimal level will likely be a different inflation and output gap combination. However, such a policy in the standard framework implies setting an inflation target that is different from the intended mean level of inflation, which may destabilize inflation. Alternatively by adjusting the rest of the policy profile a central bank can minimize the effect of inflation target uncertainty on inflation such as by responding to expected inflation instead of contemporaneous inflation, but this will also impact the volatilities.

Inflation target uncertainty may also be resolved by an announced future change in the inflation target. I show that under standard calibrations of the policy rule, this will introduce cyclical forces to the transition path of inflation and the output gap. However, if the central bank adjusts the policy rule to allow for an optimally set, time varying intercept the cyclical fluctuations can be eliminated. The key feature of the time varying policy rule is a huge spike in interest rates the period before the inflation target changes, which insures that monetary policy creates the largest deflationary force in the same periods as expected inflation creates the strongest inflationary force. Additionally, introducing uncertainty in the period when the inflation target changes also eliminates the cyclical dynamics and under the optimal policy rule generates roughly similar losses as with the optimal time varying perfect foresight transition.

Motivated by proposals to increase the inflation target, this paper focuses on analyzing the effects of expectations that in the future the inflation target might be endogenously changed, with numerical examples focusing on a 2% increase in the inflation target. Consistent with Janet Yelen's comments there is uncertainty as to what is the specific model that describes the economy and how it should be calibrated. As new information and research rolls in, beliefs about the best model and the parameter estimates of the model evolve and a different inflation target may become optimal. Without knowing what the true model is, I evaluate the effects of the uncertainty in the future inflation target in a standard, stylized model. However, within the context of the model, the optimal policy is to credibly commit to maintain inflation at zero percent forever. To generate model consistent changes in the unconstrained optimal inflation target requires changes in the model fundamentals. One possibility, is that shocks hitting the economy change over time and they generate different probabilities the zero lower bound on interest rates. Dordal i Carreras et al. (2016) show that if there is a regime switching risk premium shock that changes the expected duration the zero lower bound will bind the optimal inflation target. For example, they find that if the probability of being at the zero lower bound is 10% and the expected duration of a zero lower bound episode increase from 9 quarters to 18 quarters, the optimal inflation target increase from 2 to 4 percent. Thus one model consistent explanation for why the inflation target will optimally follow a regime switching process is that there is an underlying regime switching process for the risk premiums shocks. However, this approach is somewhat unsatisfying since current proposals to

increase the U.S. inflation target are primarily based on the economic experiences of the past decade and concerns that the real interest rate has already declined rather than beliefs that future shocks to the economy will be more persistent.

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A Proofs and Derivations

A.1 Regime Shift Solution

In this section, I derive the regime shift solution and prove proposition 1. Prior to period zero and after the regime switch, there are no expectations of regime shifts and there are no shocks; therefore, the economy is in a steady state. Solving for the steady state, the Phillips curve implies

$$x = \frac{(1-\beta)\pi}{\kappa}, \tag{46}$$

and the Euler equation implies $i = \pi$. Plugging these results into the policy rule and solving for inflation,

$$\pi = \frac{\pi^\star(1-\phi_\pi-\phi_{\pi'})}{1-\phi_\pi-\phi_{\pi'}-\phi_x(1-\beta)/\kappa}. \tag{47}$$

Equations (46) and (47) describe the regime two outcome, and setting $\pi^\star = 0$ provides the outcomes prior to period zero. Since monetary policy in regime two is independent of monetary policy in regime one, π_2 is exogenous from the perspective of a central bank in regime one, and I solve for the outcomes in regime one as a function of π_2 rather than $\pi^\star$. In regime one, the expected output gap next period is the probability of remaining in the same regime times the output gap in the current regime tomorrow plus the probability of the regime shift times the output gap next period if the regime shift occurs. But since each regime is in a steady state $E_t x_{t+1} = (1-\lambda)x_1 + \lambda x_2$. Using this, I can rewrite equations (16) - (18) as

$$\pi_1 = \beta((1-\lambda)\pi_1 + \lambda\pi_2) + \kappa x_1 \tag{48}$$

$$x_1 = (1-\lambda)x_1 + \lambda x_2 - \sigma^{-1}(i_1 - (1-\lambda)\pi_1 - \lambda\pi_2) \tag{49}$$

$$i_1 = \phi_\pi \pi_1 + \phi_{\pi'}((1-\lambda)\pi_1 + \lambda\pi_2) + \phi_x x_1 + \bar{i}_1, \tag{50}$$

where $\bar{i}_1 = (1 - \phi_\pi + \phi_{\pi'})\pi_1^\star$. Solving for π_1 in equation (48),

$$\pi_1 = \frac{\beta\pi_2\lambda + \kappa x_1}{1 - \beta(1-\lambda)} \tag{51}$$

and combining (49) with (50)

$$\pi_1 = -\frac{\lambda + \sigma^{-1}\phi_x}{\sigma^{-1}(\phi_\pi + (1-\lambda)(\phi_{\pi'} - 1))} x_1 + \frac{\lambda(1 - \beta + \sigma^{-1}\kappa(1-\phi_{\pi'}))\pi_2 - \sigma^{-1}\kappa(1 - \phi_\pi - \phi_{\pi'})\pi_1^\star}{\kappa\sigma^{-1}(\phi_\pi + (1-\lambda)(\phi_{\pi'} - 1))}. \tag{52}$$

Solving these equations for π_1 and x_1 results in equations (24) and (25).

I can now prove proposition 1. First taking the partial derivative of π_2 with respect to $\pi^\star$ (using equation (47)),

$$\frac{\partial \pi_2}{\partial \pi^\star} = \frac{(1 - \phi_\pi - \phi_{\pi'})}{1 - \phi_\pi - \phi_{\pi'} - \phi_x(1-\beta)/\kappa} \equiv c_2, \tag{53}$$

which is positive if $\phi_\pi + \phi_{\pi'} > 1$.[18] Hence if a variable is increasing in π_2 it is increasing in $\pi^\star$. Define $f(\phi_{\pi'}, \phi_x) \equiv \lambda(1 - \beta + \beta\lambda + \sigma^{-1}\kappa + \beta\sigma^{-1}\phi_x - \sigma^{-1}\kappa\phi_{\pi'})$, $g(\phi_\pi, \phi_{\pi'}, \phi_x) \equiv \kappa\sigma^{-1}(\phi_\pi + (1-\lambda)(\phi_{\pi'} - 1)) + (\lambda + \sigma^{-1}\phi_x)(1 - \beta(1-\lambda))$, $h(\phi_\pi, \phi_{\pi'}) \equiv \lambda\kappa\sigma^{-1}(1 - \beta\phi_\pi - \phi_{\pi'}) + \lambda(1-\beta)(1 - \beta(1-\lambda))$, and $c = \kappa\sigma^{-1}(1 - \beta(1-\lambda))$. Then,

$$\pi_1 = \frac{f(\phi_{\pi'}, \phi_x)c_2\pi^\star - \sigma^{-1}\kappa\bar{i}_1}{g(\phi_\pi, \phi_{\pi'}, \phi_x)} \text{ and } x_1 = \frac{h(\phi_\pi, \phi_{\pi'})c_2\pi^\star - c\bar{i}_1}{\kappa g(\phi_\pi, \phi_{\pi'}, \phi_x)}. \tag{54}$$

Taking partial derivatives,

$$\frac{\partial \pi_1}{\partial \pi^\star} = \frac{f(\phi_{\pi'}, \phi_x)c_2}{g(\phi_\pi, \phi_{\pi'}, \phi_x)} \text{ and } \frac{\partial x_1}{\partial \pi^\star} = \frac{h(\phi_\pi, \phi_{\pi'})c_2}{\kappa g(\phi_\pi, \phi_{\pi'}, \phi_x)}. \tag{55}$$

Furthermore,

$$g(\phi_\pi, \phi_{\pi'}, \phi_x) > 0 \Rightarrow \phi_\pi + (1-\lambda)\phi_{\pi'} + \phi_x\frac{1 - \beta(1-\lambda)}{\kappa} > 1 - \lambda(1 + \frac{1 - \beta(1-\lambda)}{\sigma^{-1}\kappa}) > 1 - \lambda \tag{56}$$

[18] Recall, monetary policy in regime two is assumed to be independent of monetary policy in regime one. Formally there are separate policy parameters in each regime and the partial derivatives of c_2 with respect to monetary policy parameters in regime one are all zero.

$$f(\phi_{\pi'}, \phi_x) > 0 \Rightarrow 1 + \frac{1-\beta(1-\lambda)}{\kappa\sigma^{-1}} + \frac{\beta\phi_x}{\kappa} > \phi_{\pi'} \tag{57}$$

$$h(\phi_\pi, \phi_{\pi'}) > 0 \Rightarrow 1 + \frac{(1-\beta)(1-\beta(1-\lambda))}{\kappa\sigma^{-1}} > \beta\phi_\pi + \phi_{\pi'} \tag{58}$$

If $\phi_\pi + \phi_{\pi'} \geq 1$, then $g(\phi_\pi, \phi_{\pi'}, \phi_x) > 0$, $\frac{\partial x_1}{\partial \pi^\star} > 0 \Leftrightarrow h(\phi_\pi, \phi_{\pi'}) > 0$, and $\frac{\partial \pi_1}{\partial \pi^\star} > 0 \Leftrightarrow f(\phi_{\pi'}, \phi_x) > 0$. Taking second derivatives,

$$\frac{\partial^2 \pi_1}{\partial \pi^\star \partial \phi_\pi} = \frac{-\sigma^{-1}\kappa f(\phi_{\pi'}, \phi_x)}{g(\phi_\pi, \phi_{\pi'}, \phi_x)^2} < 0 \Leftrightarrow f(\phi_{\pi'}, \phi_x) > 0, \tag{59}$$

$$\begin{aligned}\frac{\partial^2 \pi_1}{\partial \pi^\star \partial \phi_x} = \frac{\lambda\beta\sigma^{-1} g(\phi_\pi, \phi_{\pi'}, \phi_x) - \sigma^{-1}(1-\beta(1-\lambda)) f(\phi_{\pi'}, \phi_x)}{g(\phi_\pi, \phi_{\pi'}, \phi_x)^2} < 0 \\ \Leftrightarrow \lambda\beta\sigma^{-1} g(\phi_\pi, \phi_{\pi'}, \phi_x) < \sigma^{-1}(1-\beta(1-\lambda)) f(\phi_{\pi'}, \phi_x) > 0 \\ \Leftrightarrow 1 + \frac{(1-\beta)(1-\beta(1-\lambda))}{\kappa\sigma^{-1}} > \beta\phi_\pi + \phi_{\pi'} + \frac{\beta(1-\beta(1-\lambda)^2)}{\kappa}\phi_x,\end{aligned} \tag{60}$$

and

$$\begin{aligned}\frac{\partial^2 \pi_1}{\partial \pi^\star \partial \phi_{\pi'}} = \frac{-\lambda\sigma^{-1}\kappa g(\phi_\pi, \phi_{\pi'}, \phi_x) - \sigma^{-1}\kappa(1-\lambda) f(\phi_{\pi'}, \phi_x)}{g(\phi_\pi, \phi_{\pi'}, \phi_x)^2} < 0 \\ \Leftrightarrow \lambda\sigma^{-1}\kappa g(\phi_\pi, \phi_{\pi'}, \phi_x) + \sigma^{-1}\kappa(1-\lambda) f(\phi_{\pi'}, \phi_x) > 0 \\ \Leftrightarrow \kappa\phi_\pi + \phi_x > -\frac{1-\beta(1-\lambda)}{\sigma^{-1}},\end{aligned} \tag{61}$$

which is always true. This completes the comparative statics portion of the proposition, but that $\pi_1 < \pi_2 \leq \pi^\star$ and $x_1 < x_2$ if monetary policy parameters aside for the inflation target are the same in both regimes still needs to be shown. $\pi_2 \leq \pi^\star$ follows from equation (47) for active monetary policy. To verify that $\pi_1 < \pi_2$,

$$\begin{aligned}\pi_1 = \frac{\lambda(1-\beta+\beta\lambda+\sigma^{-1}\kappa+\beta\sigma^{-1}\phi_x-\sigma^{-1}\kappa\phi_{\pi'})\pi_2}{\kappa\sigma^{-1}(\phi_\pi+(1-\lambda)(\phi_{\pi'}-1))+(\lambda+\sigma^{-1}\phi_x)(1-\beta(1-\lambda))} < \pi_2 \\ \Leftrightarrow \lambda(1-\beta+\beta\lambda+\sigma^{-1}\kappa+\beta\sigma^{-1}\phi_x-\sigma^{-1}\kappa\phi_{\pi'}) < \\ \kappa\sigma^{-1}(\phi_\pi+(1-\lambda)(\phi_{\pi'}-1))+(\lambda+\sigma^{-1}\phi_x)(1-\beta(1-\lambda)) \\ \Leftrightarrow (\phi_\pi+\phi_{\pi'}-1)\sigma^{-1}\kappa+\sigma^{-1}\phi_x(1-\beta) > 0,\end{aligned} \tag{62}$$

but this always holds if $\phi_\pi + \phi_{\pi'} > 1$. Similarly $x_1 < x_2$ implies

$$\begin{aligned}\frac{\lambda\kappa\sigma^{-1}(1-\beta\phi_\pi-\phi_{\pi'})+\lambda(1-\beta)(1-\beta(1-\lambda))\pi_2}{\kappa(\kappa\sigma^{-1}(\phi_\pi+(1-\lambda)(\phi_{\pi'}-1))+(\lambda+\sigma^{-1}\phi_x)(1-\beta(1-\lambda)))} < \frac{1-\beta}{\kappa}\pi_2 \\ \Leftrightarrow \lambda\kappa\sigma^{-1}(1-\beta\phi_\pi-\phi_{\pi'})+\lambda(1-\beta)(1-\beta(1-\lambda)) < \\ (1-\beta)(\kappa\sigma^{-1}(\phi_\pi+(1-\lambda)(\phi_{\pi'}-1))+(\lambda+\sigma^{-1}\phi_x)(1-\beta(1-\lambda))),\end{aligned} \tag{63}$$

which simplifies to

$$\kappa\sigma^{-1}(1-\beta(1-\lambda))(\phi_\pi+\phi_{\pi'}-1)+\sigma^{-1}\phi_x((1-\beta)^2+\lambda(\beta-\beta^2))>0 \tag{64}$$

and always holds if $\phi_\pi+\phi_{\pi'}>1$. This completes the proof of proposition 1.

Before proving proposition 2, first I extend it to the case where monetary policy may respond to both the output gap and expected inflation. It is assumed that monetary policy in regime two is active and therefore has a unique solution for π_2 and x_2 given by (46) and (47). The modified version states:

1. If $f(\phi_{\pi'},\phi_x)=g(\phi_\pi,\phi_{\pi'},\phi_x)$, then $\pi_1=\pi_2$ and $x_1=\frac{1-\beta}{\kappa}\pi_2$
2. If $f(\phi_{\pi'},\phi_x)>g(\phi_\pi,\phi_{\pi'},\phi_x)$ and $g(\phi_\pi,\phi_{\pi'},\phi_x)>0$ then $\pi_1>0$ and $x_1>\frac{1-\beta}{\kappa}\pi_2$ and

$$\lim_{g(\phi_\pi,\phi_{\pi'},\phi_x)\to 0^+}\pi_1=\lim_{g(\phi_\pi,\phi_{\pi'},\phi_x)\to 0^+}x_1=\infty$$

3. If $g(\phi_\pi,\phi_{\pi'},\phi_x)<0$, then $\pi_1<0$ and $x_1<0$

To get to the simplified version in the main text, substitute in $\phi_{\pi'}=\phi_x=0$ into f and g. For the first part, $f(0,0)=g(\phi_\pi,0,0)$ implies $\phi_\pi=1$. For the second part note that $f(0,0)>g(\phi_\pi,0,0)$ implies $\phi_\pi<1$ and $g(\phi_\pi,0,0)>0$ implies $\phi_\pi>1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})$. For part three, note that $g(\phi_\pi,0,0)<0$ implies $\phi_\pi<1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})$

Now for the proof, first express π_1 in terms of f and g notation.

$$\pi_1=\frac{f(\phi_{\pi'},\phi_x)\pi_2-\sigma^{-1}\kappa\bar{i}_1}{g(\phi_\pi,\phi_{\pi'},\phi_x)}=\frac{f(\phi_{\pi'},\phi_x)\pi_2}{g(\phi_\pi,\phi_{\pi'},\phi_x)}, \tag{65}$$

where the second equality holds since $\pi_1^\star=0$ and therefore $\bar{i}_1=0$. From (65), it is evident that setting $\pi_2=\pi_1\Rightarrow f(\phi_{\pi'},\phi_x)=g(\phi_\pi,\phi_{\pi'},\phi_x)$. Additionally since $\pi_2=\pi_1$, (48) simplifies to $x_1=\frac{1-\beta}{\kappa}\pi_2$, which completes the first part of the proposition.

For the second part, under passive policy

$$\begin{aligned}\min_{\phi_{\pi'},\phi_x} f(\phi_{\pi'},\phi_x)&=\min_{\phi_{\pi'},\phi_x}\lambda(1-\beta+\beta\lambda+\sigma^{-1}\kappa+\beta\sigma^{-1}\phi_x-\sigma^{-1}\kappa\phi_{\pi'})\\&=\lambda(1-\beta+\beta\lambda+\sigma^{-1}\kappa-\sigma^{-1}\kappa)=\lambda(1-\beta+\beta\lambda),\end{aligned} \tag{66}$$

which is a positive constant. Therefore as $g(\phi_\pi,\phi_{\pi'},\phi_x)$ approaches 0 from above, the numerator of π_1 is at least $\lambda(1-\beta+\beta\lambda)$, while the denominator approaches zero. Hence $\pi_1\to\infty$. Furthermore since, $f(\phi_{\pi'},\phi_x)$ and $g(\phi_\pi,\phi_{\pi'},\phi_x)$ are positive in this range and by assumption $f(\phi_{\pi'},\phi_x)>g(\phi_\pi,\phi_{\pi'},\phi_x)$, then $\pi_1=\frac{f(\phi_{\pi'},\phi_x)\pi_2}{g(\phi_\pi,\phi_{\pi'},\phi_x)}>\pi_2$. For the output gap, solve (51) for x_1,

$$x_1=\frac{\pi_1(1-\beta(1-\lambda)-\beta\lambda\pi_2}{\kappa} \tag{67}$$

Since in this range $\pi_1 > \pi_2$,

$$x_1 = \frac{\pi_1(1-\beta(1-\lambda)-\beta\lambda\pi_2}{\kappa} > \frac{\pi_2(1-\beta(1-\lambda)-\beta\lambda\pi_2}{\kappa} > \frac{1-\beta}{\kappa}\pi_2 \tag{68}$$

Finally, since $\lim_{g(\phi_\pi,\phi_{\pi'},\phi_x)\to 0^+} \pi_1 = \infty$, then by (67) the limit of x_1 also foes to infinity.

For the final part of the proposition, since $\pi_1 = \frac{f(\phi_{\pi'},\phi_x)\pi_2}{g(\phi_\pi,\phi_{\pi'},\phi_x)}$, $f(\phi_{\pi'},\phi_x)$ is always positive under passive policy, and by assumption $g(\phi_\pi,\phi_{\pi'},\phi_x) < 0$, then $\pi_1 < 0$. For output, since

$$x_1 = \frac{h(\phi_\pi,\phi_{\pi'})\pi_2 - c\bar{i}_1}{\kappa g(\phi_\pi,\phi_{\pi'},\phi_x)} = \frac{h(\phi_\pi,\phi_{\pi'})\pi_2}{\kappa g(\phi_\pi,\phi_{\pi'},\phi_x)}, \tag{69}$$

$h(\phi_\pi,\phi_{\pi'})$ is always positive under passive policy (see (58)), and by assumption $g(\phi_\pi,\phi_{\pi'},\phi_x) < 0$, then $x_1 < 0$.

A.2 Monetary Policy Response Dynamics

Proposition 3 has two components, the set of outcomes the central bank can achieve and the property that any such outcome can be achieved without affecting the volatility of output or inflation in the stochastic model. The outcome in regime one is determined by the intersection of the two linear equations (51)

$$\pi_1 = \frac{\beta\pi_2\lambda + \kappa x_1}{1-\beta(1-\lambda)}$$

and (52) which can be expressed as

$$\pi_1 = c_3 x_1 + c_4\pi_2 + c_5\pi_1^\star. \tag{70}$$

Since $c_3 \neq \kappa/(1-\beta(1-\lambda))$, the two equations will intersect. By changing $\pi_1^\star$, the second equation can be shifted up or down along equation (51). Hence by changing $\pi_1^\star$, any outcome consistent with equation (51) can be achieved. The second part of this proof is to show that changing $\pi_1^\star$ does not affect the volatility of output or inflation, which I do at the end of this subsection as it involves finding an analytical solution to the stochastic model.

Proposition 4 states that reducing the constant in the policy rule for the current regime, $\bar{i}_1$, will raise inflation and output in the current regime if $\phi_\pi + (1-\lambda)\phi_{\pi'} + \phi_x\frac{1-\beta(1-\lambda)}{\kappa} > 1-\lambda(1+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})$. To see this, take partial derivatives of (54) with respect to $\bar{i}_1$,

$$\frac{\partial\pi_1}{\partial\bar{i}_1} = \frac{-\sigma^{-1}\kappa}{g(\phi_\pi,\phi_{\pi'},\phi_x)} \text{ and } \frac{\partial x_1}{\partial\bar{i}_1} = \frac{-c}{\kappa g(\phi_\pi,\phi_{\pi'},\phi_x)}. \tag{71}$$

Both of the above are less than zero if $g(\phi_\pi,\phi_{\pi'},\phi_x) > 0$, which by equation 56 is the condition in the proposition.

Proposition 5 claims that to achieve a zero inflation in regime one, the central bank needs to generate a recession of magnitude $\frac{\beta\lambda\pi_2}{\kappa}$ which can be accomplished by setting $\pi_1^\star = \lambda\pi_2\frac{1}{1-\phi_\pi-\phi_{\pi'}}(1-\phi_{\pi'}+\frac{\beta\phi_x}{\kappa}+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa})$. The Phillips Curve (51) determines the set of possible outcomes in regime one. Setting inflation in regime one to zero and solving for the output gap results in $x_1 = -\frac{\beta\lambda\pi_2}{\kappa}$. Plugging the values for inflation and output in regime one into the Euler equation for regime one merged with the interest rate rule (52), I can solve for the regime one inflation target consistent with a zero inflation outcome.

$$
\begin{aligned}
0 = -\frac{\lambda+\sigma^{-1}\phi_x}{\sigma^{-1}(\phi_\pi+(1-\lambda)(\phi_{\pi'}-1))}(-\frac{\beta\lambda\pi_2}{\kappa})+ \\
\frac{\lambda(1-\beta+\sigma^{-1}\kappa(1-\phi_{\pi'}))\pi_2-\sigma^{-1}\kappa(1-\phi_\pi-\phi_{\pi'})\pi_1^\star}{\kappa\sigma^{-1}(\phi_\pi+(1-\lambda)(\phi_{\pi'}-1))} \\
\Rightarrow (1-\phi_\pi-\phi_{\pi'})\pi_1^\star = \frac{\lambda\pi_2}{\sigma^{-1}\kappa}(1-\beta(1-\lambda)+\sigma^{-1}\kappa(1-\phi_{\pi'})+\sigma^{-1}\beta\phi_x) \\
\Rightarrow \pi_1^\star = \pi_2\frac{\lambda}{1-\phi_\pi-\phi_{\pi'}}(1-\phi_{\pi'}+\frac{\beta\phi_x}{\kappa}+\frac{1-\beta(1-\lambda)}{\sigma^{-1}\kappa}) \quad (72)
\end{aligned}
$$

Proposition 6 describes the optimal allocation a central bank chooses with future inflation target uncertainty. The central bank's problem is to minimize losses as given by (29), but since regime two is an absorbing regime this can be simplified. Starting from (29),

$$
\begin{aligned}
&E_t\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\theta_x x_t^2) \\
&\quad = E_t\sum_{t=0}^{\infty}(\beta(1-\lambda))^t(\pi_{1,t}^2+\theta_x x_{1,t}^2)+\underbrace{\sum_{t=0}^{\infty}(\beta^t\sum_{j=0}^{t-1}\lambda(1-\lambda)^j(\pi_{2,t}^2+\theta_x x_{2,t}^2))}_{\equiv C} \\
&\quad = E_t\sum_{t=0}^{\infty}(\beta(1-\lambda))^t(\pi_{1,t}^2+\theta_x x_{1,t}^2)+C \\
&\quad = \frac{1}{1-\beta(1-\lambda)}(\pi_1^2+\theta_x x_1^2)+C \quad (73)
\end{aligned}
$$

From the solution for $x_{2,t}$ and $\pi_{2,t}$, we know that they do not depend on policy parameters for regime one, and therefore C is exogenous from the perspective of a central banker that is only optimizing regime one policy. Therefore for policy optimization problems over policy parameters for regimes which cannot be transitioned into as part of the Markov process, minimizing losses is equivalent to minimizing losses while the current regime lasts, and from a timeless perspective is equivalent to minimizing $E_t\pi_{1,t}^2+\theta x_{1,t}^2$. To prove proposition 6, I solve for the loss minimizing allocation on the Phillips Curve (which

relates π_1 to x_1 for any π_2):

$$\min_{x_1,\pi_1} \frac{1}{1-\beta(1-\lambda)}(\pi_1^2+\theta x_1^2)+C \text{ such that } \pi_1 = \frac{\beta\pi_2\lambda+\kappa x_1}{1-\beta(1-\lambda)} \tag{74}$$

Substituting in the constraint and taking the first order condition:

$$\frac{2(\pi_2\beta\lambda+\kappa x_1)\kappa}{(1-\beta(1-\lambda))^3}+\frac{2\theta x_1}{1-\beta(1-\lambda)}=0 \Rightarrow x_1 = \frac{-\pi_2\lambda\beta}{\kappa+\kappa^{-1}(1-\beta(1-\lambda))^2\theta} \tag{75}$$

Substituting in x_1 from (75) into (51) and solving for π_1:

$$\begin{aligned}\pi_1 = \frac{\beta\pi_2\lambda}{1-\beta(1-\lambda)}+\frac{\kappa}{1-\beta(1-\lambda)}\frac{-\pi_2\lambda\beta}{\kappa+\kappa^{-1}(1-\beta(1-\lambda))^2\theta}\\ \Rightarrow \pi_1 = \frac{\pi_2\lambda\beta}{1-\beta(1-\lambda)}\frac{\kappa^{-1}(1-\beta(1-\lambda))^2\theta}{\kappa+\kappa^{-1}(1-\beta(1-\lambda))^2\theta}\end{aligned} \tag{76}$$

Finally, substitute in (75) and (76) into (52) to solve for the inflation target in regime one that results in the optimal allocation. Doing so,

$$\begin{aligned}\pi_1^\star = \frac{1}{1-\phi_\pi-\phi_{\pi'}}\frac{\lambda\sigma\pi_2}{\kappa^2+z_1^2\theta}(\beta\lambda\kappa+(1-\beta)(\kappa+\kappa^{-1}z_1^2\theta)+\\ \sigma^{-1}\beta\kappa\phi_x-\lambda\beta z_1\theta\sigma^{-1}\phi_\pi-(\phi_{\pi'}-1)\sigma^{-1}(\kappa^2+z_1^2\theta+\lambda(1-\lambda)\beta z_1\theta)),\end{aligned} \tag{77}$$

where $z_1 \equiv 1-\beta(1-\lambda)$.

A.2.1 Stochastic Two Regime Model Solution

To complete the proof of proposition 3, I need to derive the solution for the stochastic model. That is, I need an analytical solution to the two regime model described by (6) - (9). To simplify the notation denote the Markov process

$$\Pi = \begin{bmatrix} \gamma_1 & 1-\gamma_1 \\ 1-\gamma_2 & \gamma_2 \end{bmatrix}.$$

and define $\gamma_{11} \equiv \gamma_1^2+(1-\gamma_1)(1-\gamma_2)$, $\gamma_{22} \equiv \gamma_2^2+(1-\gamma_1)(1-\gamma_2)$, $\gamma_{12} \equiv \gamma_1(1-\gamma_1)+(1-\gamma_1)\gamma_2$, and $\gamma_{21} \equiv \gamma_2(1-\gamma_2)+(1-\gamma_2)\gamma_1$. The first step is to combine the equations for each regime into a single expression that eliminates interest rate and output gap terms. Starting with (7),

$$\begin{aligned}\pi_{1,t} = \beta E_t(\gamma_1\pi_{1,t+1}+(1-\gamma_1)\pi_{2,t+1})+\kappa x_{1,t}+(1-\beta)\bar{\pi}_1+\mu_t^S\\ \Rightarrow \kappa x_{1,t} = \pi_{1,t}-\beta\gamma_1 E_t\pi_{1,t+1}-\beta(1-\gamma_1)E_t\pi_{2,t+1}-(1-\beta)\bar{\pi}_1-\mu_t^S\end{aligned} \tag{78}$$

Similarly,

$$\kappa x_{2,t} = \pi_{2,t} - \beta(1-\gamma_2)E_t\pi_{1,t+1} - \beta\gamma_2 E_t\pi_{2,t+1} - (1-\beta)\bar{\pi}_2 - \mu_t^S \tag{79}$$

Now multiplying (6) though by κ, simplifying, and plugging in $\kappa x_{1,t}$ and $\kappa x_{2,t}$,

$$\begin{aligned}
\kappa x_{1,t} &= \kappa E_t(\gamma_1 x_{1,t+1} + (1-\gamma_1)x_{2,t+1}) - \kappa\sigma^{-1}(\phi_{1,\pi}\pi_{1,t} + \phi_{1,\pi'}E_t(\gamma_1\pi_{1,t+1} + (1-\gamma_1)\pi_{2,t+1}) \\
&\quad - (\phi_{1,\pi} + \phi_{1,\pi'} - 1)\pi_1^\star + \phi_{1,x}x_{1,t} + \mu_t^I - E_t(\gamma_1\pi_{1,t+1} + (1-\gamma_1)\pi_{2,t+1})) + \kappa\mu_t^D \\
\Rightarrow \kappa x_{1,t}(1+\sigma^{-1}\phi_{1,x}) &= \gamma_1 E_t\kappa x_{1,t+1} + (1-\gamma_1)E_t\kappa x_{2,t+1} - \kappa\sigma^{-1}\phi_{1,\pi}\pi_{1,t} - \kappa\sigma^{-1}\mu_t^I + \kappa\mu_t^D \\
&+ \kappa\sigma^{-1}(\phi_{1,\pi} + \phi_{1,\pi'} - 1)\pi_1^\star + (1-\phi_{1,\pi'})\kappa\sigma^{-1}\gamma_1 E_t\pi_{1,t+1} + (1-\phi_{1,\pi'})\kappa\sigma^{-1}(1-\gamma_1)E_t\pi_{2,t+1} \\
&\Rightarrow (1+\sigma^{-1}\phi_{1,x})(\pi_{1,t} - \beta\gamma_1 E_t\pi_{1,t+1} - \beta(1-\gamma_1)E_t\pi_{2,t+1} - (1-\beta)\bar{\pi}_1 - \mu_t^S) = \\
&\quad \gamma_1 E_t(\pi_{1,t+1} - \beta\gamma_1 E_t\pi_{1,t+2} - \beta(1-\gamma_1)E_t\pi_{2,t+2} - (1-\beta)\bar{\pi}_1 - \mu_{t+1}^S) \\
&\quad + (1-\gamma_1)E_t(\pi_{2,t+1} - \beta(1-\gamma_2)E_t\pi_{1,t+2} - \beta\gamma_2 E_t\pi_{2,t+2} - (1-\beta)\bar{\pi}_2 - \mu_{t+1}^S) \\
&\quad - \kappa\sigma^{-1}\phi_{1,\pi}\pi_{1,t} + (1-\phi_{1,\pi'})\kappa\sigma^{-1}\gamma_1 E_t\pi_{1,t+1} + (1-\phi_{1,\pi'})\kappa\sigma^{-1}(1-\gamma_1)E_t\pi_{2,t+1} \\
&\quad + \kappa\sigma^{-1}(\phi_{1,\pi} + \phi_{1,\pi'} - 1)\pi_1^\star - \kappa\sigma^{-1}\mu_t^I + \kappa\mu_t^D \\
&\Rightarrow \pi_{1,t}(1+\sigma^{-1}\phi_{1,x} + \kappa\sigma^{-1}\phi_{1,\pi}) - E_t\pi_{1,t+1}\gamma_1(\beta(1+\sigma^{-1}\phi_{1,x}) + 1 + (1-\phi_{1,\pi'})\sigma^{-1}\kappa) \\
&+ E_t\pi_{1,t+2}\beta\gamma_{11} - E_t\pi_{2,t+1}(1-\gamma_1)(\beta(1+\sigma^{-1}\phi_{1,x}) + 1 + (1-\phi_{1,\pi'})\sigma^{-1}\kappa) + E_t\pi_{2,t+2}\beta\gamma_{12} \\
&= \underbrace{(1+\sigma^{-1}\phi_{1,x})\mu_t^S - E_t\mu_{t+1}^S - \kappa\sigma^{-1}\mu_t^I + \kappa\mu_t^D}_{\equiv\mu_{1,t}} \\
&+ \underbrace{(1-\beta)((1+\sigma^{-1}\phi_{1,x} - \gamma_1)\bar{\pi}_1 - (1-\gamma_1)\bar{\pi}_2) + \kappa\sigma^{-1}(\phi_{1,\pi} + \phi_{1,\pi'} - 1)\pi_1^\star}_{\equiv a_1} = \mu_{1,t} + a_1 \equiv z_{1,t}
\end{aligned} \tag{80}$$

Similarly,

$$\begin{aligned}
&\pi_{2,t}(1+\sigma^{-1}\phi_{2,x} + \kappa\sigma^{-1}\phi_{2,\pi}) - E_t\pi_{2,t+1}\gamma_2(\beta(1+\sigma^{-1}\phi_{2,x}) + 1 + (1-\phi_{2,\pi'})\sigma^{-1}\kappa) \\
&+ E_t\pi_{2,t+2}\beta\gamma_{22} - E_t\pi_{1,t+1}(1-\gamma_2)(\beta(1+\sigma^{-1}\phi_{2,x}) + 1 + (1-\phi_{2,\pi'})\sigma^{-1}\kappa) + E_t\pi_{1,t+2}\beta\gamma_{21} \\
&= \underbrace{(1+\sigma^{-1}\phi_{2,x})\mu_t^S - E_t\mu_{t+1}^S - \kappa\sigma^{-1}\mu_t^I + \kappa\mu_t^D}_{\equiv\mu_{2,t}} \\
&+ \underbrace{(1-\beta)((1+\sigma^{-1}\phi_{2,x} - \gamma_2)\bar{\pi}_2 - (1-\gamma_2)\bar{\pi}_1) + \kappa\sigma^{-1}(\phi_{1,\pi} + \phi_{1,\pi'} - 1)\pi_1^\star}_{\equiv a_2} = \mu_{2,t} + a_2 \equiv z_{2,t}
\end{aligned} \tag{81}$$

Note that the inflation targets and the indexing parameters based on the inflation targets only occur in the a_1 and a_2 terms. Next define the polynomials,

$$Q_j(y) = \beta\gamma_{jj}y^2 - \gamma_j(\beta(1+\sigma^{-1}\phi_{j,x}) + 1 + (1-\phi_{j,\pi'})\sigma^{-1}\kappa)y + (1+\sigma^{-1}\phi_{j,x} + \kappa\sigma^{-1}\phi_{j,\pi}) \\ \equiv c_{j,1}y^2 - c_{j,2}y + c_{j,3} \quad (82)$$

$$P_j(y) = y(\beta\gamma_{j,-j}y - (1-\gamma_j)(\beta(1+\sigma^{-1}\phi_{j,x}) + 1 + (1-\phi_{j,\pi'})\sigma^{-1}\kappa) \equiv y(d_{j,1}y - d_{j,2}) \quad (83)$$

Rewriting (80) and (81) using lag notation,

$$E_t(Q_1(L^{-1})\pi_{1,t} + P_1(L^{-1})\pi_{2,t}) = z_{1,t} \quad (84)$$

$$E_t(Q_2(L^{-1})\pi_{2,t} + P_2(L^{-1})\pi_{1,t}) = z_{2,t} \quad (85)$$

Next combine (84) and (85) to get one equation in one variable. To eliminate $\pi_{2,t}$, multiply (84) through by $Q_2(L^{-1})$, (85) through by $P_1(L^{-1})$, and subtract them to get

$$E_t(Q_1(L^{-1})Q_2(L^{-1}) - P_2(L^{-1})P_1(L^{-1}))\pi_{1,t} \\ = E_t(Q_2(L^{-1})z_{1,t} - P_1(L^{-1})z_{2,t}) \equiv \tilde{Y}_{1,t} \quad (86)$$

Similarly,

$$E_t(Q_1(L^{-1})Q_2(L^{-1}) - P_2(L^{-1})P_1(L^{-1}))\pi_{2,t} \\ = E_t(Q_1(L^{-1})z_{2,t} - P_2(L^{-1})z_{1,t}) \equiv \tilde{Y}_{2,t} \quad (87)$$

Next, define $\tilde{C(y)} \equiv Q_1(y)Q_2(y) - P_2(y)P_1(y)$, then

$$\tilde{C(y)} = (c_{1,1}c_{2,1} - d_{1,1}d_{2,1})y^4 - (c_{1,2}c_{2,1} + c_{2,2}c_{1,1} + d_{22,1}d_{1,2} + d_{1,1}d_{2,2})y^3 \\ + (c_{1,3}c_{2,1} + c_{1,2}c_{2,2} + c_{1,1}c_{2,3} - d_{2,2}d_{1,2})y^2 - (c_{2,2}c_{1,3} + c_{1,1}c_{2,3})y + c_{2,3}c_{1,3} \quad (88)$$

In this notation, (86) and (87) become

$$E_t(C(\tilde{L}^{-1})\pi_{j,t}) = E_t\tilde{Y}_{j,t} \quad (89)$$

Define $C(y) \equiv \frac{\tilde{C(y)}}{c_{1,1}c_{2,1}-d_{1,1}d_{2,1}}$ and $Y_{j,t} \equiv \frac{\tilde{Y}_{j,t}}{c_{1,1}c_{2,1}-d_{1,1}d_{2,1}}$, then

$$E_t(C(L^{-1})\pi_{j,t}) = E_tY_{j,t} \quad (90)$$

Let λ_1, λ_2, λ_3, and λ_4 be the roots of $C(y)$. For a unique forward solution all the roots must be greater than one in absolute value. Continuing,

$$
\begin{aligned}
&E_t((L^{-1}-\lambda_1)(L^{-1}-\lambda_2)(L^{-1}-\lambda_3)(L^{-1}-\lambda_4)\pi_{j,t}) = E_tY_{j,t} \Rightarrow \\
&\quad E_t(\lambda_1\lambda_2\lambda_3\lambda_4(1-\frac{1}{\lambda_1}L^{-1})(1-\frac{1}{\lambda_2}L^{-1})(1-\frac{1}{\lambda_3}L^{-1})(1-\frac{1}{\lambda_4}L^{-1})\pi_{j,t}) = E_tY_{j,t} \\
&\quad \Rightarrow \pi_{j,t} = E_t[\frac{1}{(1-\frac{1}{\lambda_1}L^{-1})(1-\frac{1}{\lambda_2}L^{-1})(1-\frac{1}{\lambda_3}L^{-1})(1-\frac{1}{\lambda_4}L^{-1})}\frac{1}{\lambda_1\lambda_2\lambda_3\lambda_4}Y_{j,t}] \\
&\qquad = E_t[(\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\frac{1}{\lambda_1}\frac{1}{1-\frac{1}{\lambda_1}L^{-1}} - \frac{1}{\lambda_2}\frac{1}{1-\frac{1}{\lambda_2}L^{-1}})+ \\
&\qquad\quad \frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\frac{1}{\lambda_3}\frac{1}{1-\frac{1}{\lambda_3}L^{-1}} - \frac{1}{\lambda_4}\frac{1}{1-\frac{1}{\lambda_4}L^{-1}}))Y_{j,t}] \\
&\qquad = (\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}^{k+1}E_tY_{j,t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}^{k+1}E_tY_{j,t+k})+ \\
&\qquad\quad \frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}^{k+1}E_tY_{j,t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}^{k+1}E_tY_{j,t+k}) \quad (91)
\end{aligned}
$$

Next simplifying $E_tY_{j,t+k}$, first note

$$E_tz_{j,t+k} = (1+\sigma^{-1}\phi_{j,x}-\rho_S)\rho_S^k\mu_t^S - \kappa\sigma^{-1}\rho_I^k\mu_t^I + \kappa\rho_D^k\mu_t^D + a_j \quad (92)$$

Then,

$$
\begin{aligned}
E_tY_{1,t+k} &= \frac{1}{c_{1,1}c_{2,1}-d_{1,1}d_{2,1}}E_t(Q_2(L^{-1})z_{1,t} - P_1(L^{-1})z_{2,t}) \\
&= \frac{1}{c_{1,1}c_{2,1}-d_{1,1}d_{2,1}}E_t((c_{2,1}L^{-2}-c_{2,2}L^{-1}+c_{2,3})z_{1,t} - (d_{1,1}L^{-2}-d_{1,2}L^{-1})z_{2,t}) \\
&= \frac{1}{c_{1,1}c_{2,1}-d_{1,1}d_{2,1}}[((1+\sigma^{-1}\phi_{1,x}-\rho_S)(c_{2,1}\rho_S^2-c_{2,2}\rho_S+c_{2,3}) + (1+\sigma^{-1}\phi_{2,x}-\rho_S) \\
&\qquad *(-d_{1,1}\rho_S^2-d_{1,2}\rho_S))\mu_t^S - \kappa\sigma^{-1}((c_{2,1}-d_{1,1})\rho_I^2 + (d_{1,2}-c_{2,2})\rho_I + c_{2,3})\mu_t^I \\
&\quad + \kappa((c_{2,1}-d_{1,1})\rho_D^2 + (d_{1,2}-c_{2,2})\rho_D + c_{2,3})\mu_t^D + (c_{2,1}-d_{1,1}+d_{1,2}-c_{2,2}+c_{2,3})a_1] \\
&\qquad \equiv e_{1,1}\mu_t^S + e_{1,2}\mu_t^I + e_{1,3}\mu_t^D + e_{1,4}a_1 \quad (93)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
E_tY_{2,t+k} &= \frac{1}{c_{1,1}c_{2,1}-d_{1,1}d_{2,1}}[((1+\sigma^{-1}\phi_{2,x}-\rho_S)(c_{1,1}\rho_S^2-c_{1,2}\rho_S+c_{1,3}) + (1+\sigma^{-1}\phi_{1,x}-\rho_S) \\
&\qquad (-d_{2,1}\rho_S^2-d_{2,2}\rho_S))\mu_t^S - \kappa\sigma^{-1}((c_{1,1}-d_{2,1})\rho_I^2 + (d_{2,2}-c_{1,2})\rho_I + c_{1,3})\mu_t^I \\
&\quad + \kappa((c_{1,1}-d_{2,1})\rho_D^2 + (d_{2,2}-c_{1,2})\rho_D + c_{1,3})\mu_t^D + (c_{1,1}-d_{2,1}+d_{2,2}-c_{1,2}+c_{1,3})a_2] \\
&\qquad \equiv e_{2,1}\mu_t^S + e_{2,2}\mu_t^I + e_{2,3}\mu_t^D + e_{2,4}a_2 \quad (94)
\end{aligned}
$$

Plugging this into (91) and rearranging the sums such that each element of $E_t Y_{j,t+k}$ is in a separate sum

$$\begin{aligned}
\pi_{j,t} = &(\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}^{k+1}E_t e_{j,1}\mu^S_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}^{k+1}E_t e_{j,1}\mu^S_{t+k})+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}^{k+1}E_t e_{j,1}\mu^S_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}^{k+1}E_t e_{j,1}\mu^S_{t+k})\\
&+(\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}^{k+1}E_t e_{j,2}\mu^I_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}^{k+1}E_t e_{j,2}\mu^I_{t+k})+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}^{k+1}E_t e_{j,2}\mu^I_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}^{k+1}E_t e_{j,2}\mu^I_{t+k})\\
&+(\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}^{k+1}E_t e_{j,3}\mu^D_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}^{k+1}E_t e_{j,3}\mu^D_{t+k})+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}^{k+1}E_t e_{j,3}\mu^D_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}^{k+1}E_t e_{j,3}\mu^D_{t+k})\\
&+(\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}^{k+1}E_t e_{j,4}a_j - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}^{k+1}E_t e_{j,4}a_j)+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}^{k+1}E_t e_{j,4}a_j - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}^{k+1}E_t e_{j,4}a_j)
\end{aligned} \tag{95}$$

Next consider the sums over any one of the four expected shock terms, for example μ^S,

$$\begin{aligned}
&\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}^{k+1}E_t e_{j,1}\mu^S_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}^{k+1}E_t e_{j,1}\mu^S_{t+k})+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}^{k+1}E_t e_{j,1}\mu^S_{t+k} - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}^{k+1}E_t e_{j,1}\mu^S_{t+k})\\
&= \frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_1}(\frac{\rho_S}{\lambda_1})^k e_{j,1}\mu^S_t - \sum_{k=0}^{\infty}\frac{1}{\lambda_2}(\frac{\rho_S}{\lambda_2})^k e_{j,1}\mu^S_t)+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_1)}(\sum_{k=0}^{\infty}\frac{1}{\lambda_3}(\frac{\rho_S}{\lambda_3})^k e_{j,1}\mu^S_t - \sum_{k=0}^{\infty}\frac{1}{\lambda_4}(\frac{\rho_S}{\lambda_4})^{k+1} e_{j,1}\mu^S_t)\\
&= [\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)(\lambda_2-\lambda_1)}(\frac{1}{\lambda_1-\rho_S}-\frac{1}{\lambda_2-\rho_S})+\\
&\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)}(\frac{1}{\lambda_3-\rho_S}-\frac{1}{\lambda_4-\rho_S})]e_{j,1}\mu^S_t\\
&= [\frac{1}{(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)}\frac{1}{(\lambda_1-\rho_S)(\lambda_2-\rho_S)}+\frac{1}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)}\frac{1}{(\lambda_3-\rho_S)(\lambda_4-\rho_S)}]e_{j,1}\mu^S_t\\
&\equiv g_S e_{j,1}\mu^S_t
\end{aligned} \tag{96}$$

Similarly, define g_D, g_I, g_1, where ρ_S is replaced by ρ_D, ρ_I, and 1. Then using these definitions, (95) simplifies to

$$\pi_{j,t} = g_S e_{j,1}\mu_t^S + g_I e_{j,2}\mu_t^I + g_D e_{j,3}\mu_t^D + g_1 e_{j,4} a_j \tag{97}$$

Using (78) and (79), I find the solution for $x_{1,t}$ and $x_{2,t}$ are

$$\begin{aligned}
x_{1,t} &= \kappa^{-1}(\pi_{1,t} - \beta\gamma_1 E_t\pi_{1,t+1} - \beta(1-\gamma_1)E_t\pi_{2,t+1} - (1-\beta)\bar{\pi}_1 + \mu_t^S \\
&= \kappa^{-1}(g_S(e_{1,1}(1-\beta\gamma_1\rho_S)-\beta(1-\gamma_1)\rho_S e_{2,1})+1)\mu_t^S + \kappa^{-1}g_I(e_{1,2}(1-\beta\gamma_1\rho_I)-\beta(1-\gamma_1)\rho_I e_{2,2})\mu_t^I \\
&+\kappa^{-1}g_D(e_{1,3}(1-\beta\gamma_1\rho_D)-\beta(1-\gamma_1)\rho_D e_{2,3})\mu_t^D + \kappa^{-1}g_1(1-\beta\gamma_1)e_{1,4}a_1 - \kappa^{-1}g_1\beta(1-\gamma_1)e_{2,4}a_2) - (1-\beta)\bar{\pi}_1 \\
&\equiv f_{1,S}\mu_t^S + f_{2,I}\mu_t^I + f_{1,D}\, u_t^D + f_{1,1}a_1 + f_{1,2}a_2 - (1-\beta)\kappa^{-1}\bar{\pi}_1
\end{aligned} \tag{98}$$

$$\begin{aligned}
x_{2,t} &= \kappa^{-1}(\pi_{2,t} - \beta\gamma_2 E_t\pi_{2,t+1} - \beta(1-\gamma_2)E_t\pi_{1,t+1} - (1-\beta)\bar{\pi}_2 + \mu_t^S \\
&= \kappa^{-1}(g_S(e_{2,1}(1-\beta\gamma_2\rho_S)-\beta(1-\gamma_2)\rho_S e_{1,1})+1)\mu_t^S + \kappa^{-1}g_I(e_{2,2}(1-\beta\gamma_2\rho_I)-\beta(1-\gamma_2)\rho_I e_{1,2})\mu_t^I \\
&+\kappa^{-1}g_D(e_{2,3}(1-\beta\gamma_2\rho_D)-\beta(1-\gamma_2)\rho_D e_{1,3})\mu_t^D + \kappa^{-1}g_1(1-\beta\gamma_2)e_{2,4}a_2 - \kappa^{-1}g_1\beta(1-\gamma_2)e_{1,4}a_1) - (1-\beta)\bar{\pi}_2 \\
&\equiv f_{2,S}\mu_t^S + f_{2,I}\mu_t^I + f_{2,D}\, u_t^D + f_{2,1}a_1 + f_{2,2}a_2 - (1-\beta)\kappa^{-1}\bar{\pi}_2
\end{aligned} \tag{99}$$

Equations (93), (98), and (99) are the analytic solutions to the stochastic two regime model. Finally, calculating the first and second moments,

$$E\pi_{j,t} = E(g_S e_{j,1}\mu_t^S + g_I e_{j,2}\mu_t^I + g_D e_{j,3}\mu_t^D + g_1 e_{j,4}a_j) = g_1 e_{j,4}a_j \tag{100}$$

$$\begin{aligned}
Ex_{j,t} &= E(f_{j,S}\mu_t^S + f_{j,I}\mu_t^I + f_{j,D}\, u_t^D + f_{j,1}a_1 + f_{j,2}a_2 - (1-\beta)\kappa^{-1}\bar{\pi}_j) \\
&= f_{j,1}a_1 + f_{j,2}a_2 - (1-\beta)\kappa^{-1}\bar{\pi}_j
\end{aligned} \tag{101}$$

$$\begin{aligned}
Var(\pi_{j,t}) &= Var(g_S e_{j,1}\mu_t^S + g_I e_{j,2}\mu_t^I + g_D e_{j,3}\mu_t^D + g_1 e_{j,4}a_j) \\
&= g_S^2 e_{j,1}^2 Var(\mu_t^S) + g_I^2 e_{j,2}^2 Var(\mu_t^I) + g_D^2 e_{j,3}^2 Var(\mu_t^D) \\
&= g_S^2 e_{j,1}^2 \frac{Var(\epsilon_t^S)}{1-\rho_S^2} + g_I^2 e_{j,2}^2 \frac{Var(\epsilon_t^I)}{1-\rho_I^2} + g_D^2 e_{j,3}^2 \frac{Var(\epsilon_t^D)}{1-\rho_D^2}
\end{aligned} \tag{102}$$

$$\begin{aligned}
Var(x_{j,t}) &= Var(f_{j,S}\mu_t^S + f_{j,I}\mu_t^I + f_{j,D}\mu_t^D + f_{j,1}a_1 + f_{j,2}a_2 - (1-\beta)\kappa^{-1}\bar{\pi}_j) \\
&= f_{j,S}^2 Var(\mu_t^S) + f_{j,I}^2 Var(\mu_t^I) + f_{j,D}^2 Var(\mu_t^D) \\
&= f_{j,S}^2 \frac{Var(\epsilon_t^S)}{1-\rho_S^2} + f_{j,I}^2 \frac{Var(\epsilon_t^I)}{1-\rho_I^2} + f_{j,D}^2 \frac{Var(\epsilon_t^D)}{1-\rho_D^2}
\end{aligned} \tag{103}$$

Recall that the regime specific inflation target and the indexing based on them only appear in a_1 and a_2. Therefore the variances of inflation and output do not depend on the inflation targets, completing the proof of proposition 6.

A.3 Perfect Foresight Solution

The model is described by three equations

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) \tag{104}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \text{ and} \tag{105}$$

$$i_t = \phi_\pi \pi_t - (\phi_\pi - 1)\pi_t^\star \tag{106}$$

Furthermore,

$$\pi_t^\star = \begin{cases} 0, & \text{if } t < T \\ \pi^\star, & \text{if } t \geq T \end{cases} \quad \text{and} \quad E_j \pi_t^\star = \begin{cases} 0, & \text{if } j < 0 \\ 0, & \text{if } 0 \leq j < T \text{ and } 0 \leq t < T \\ \pi^\star, & \text{if } j \geq 0 \text{ and } t \geq T \end{cases}.$$

A.3.1 Recursive Formulation

To get the recursive formulation for $0 \leq t < T$, first note that $x_t = E_{t-1} x_t$ and $\pi_t = E_{t-1}\pi_t$ as this is a perfect foresight model and $\pi_t^\star = 0$. Then we can rewrite equations (104), (105), and (106) as

$$\begin{bmatrix} 1 & -\kappa \\ \sigma^{-1}\phi_\pi & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} \beta & 0 \\ \sigma^{-1} & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}. \tag{107}$$

Solving for x_{t-1} and π_{t-1},

$$\begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ \sigma^{-1}\phi_\pi & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta & 0 \\ \sigma^{-1} & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \frac{1}{1 + \kappa\sigma^{-1}\phi_\pi} \begin{bmatrix} \beta + \kappa + \sigma^{-1} & \kappa \\ -\beta\sigma^{-1}\phi_\pi + \sigma^{-1} & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}. \tag{108}$$

Equation (108) is equivalent to equations (40) and (41) in the main text.

A.3.2 One Period in Advance Announcement

Let us first verify the dynamics in equation (34). Prior to the announcement and after the inflation target changes, expectations for the inflation target are constant and there are no other shocks; therefore, the model is in two steady states. Combining equations

104 and 106 and using the steady state property,

$$x = x - \sigma^{-1}(\phi_\pi \pi - (\phi_\pi - 1)\pi^\star - \pi) \Rightarrow \pi = \pi^\star. \tag{109}$$

Then by the Philips curve,

$$x = \frac{1-\beta}{\kappa}\pi^\star. \tag{110}$$

Therefore,

$$x_t = \begin{cases} 0, & \text{if } t < 0 \\ \frac{1-\beta}{\kappa}\pi^\star, & \text{if } t > 0 \end{cases} \quad \text{and} \quad \pi_t = \begin{cases} 0, & \text{if } t < 0 \\ \pi^\star, & \text{if } t > 0 \end{cases}. \tag{111}$$

The $t = 0$ allocation can be obtained by combining equations (108) and (111),

$$\begin{bmatrix} \pi_0 \\ x_0 \end{bmatrix} = \frac{1}{1+\kappa\sigma^{-1}\phi_\pi} \begin{bmatrix} \beta + \kappa\sigma^{-1} & \kappa \\ -\beta\sigma^{-1}\phi_\pi + \sigma^{-1} & 1 \end{bmatrix} \begin{bmatrix} \pi^\star \\ \frac{1-\beta}{\kappa}\pi^\star \end{bmatrix} = \begin{bmatrix} \frac{1+\kappa\sigma^{-1}}{1+\kappa\sigma^{-1}\phi_\pi}\pi^\star \\ \frac{1-\beta-\sigma^{-1}\kappa(\beta\phi_\pi-1)}{\kappa(1+\kappa\sigma^{-1}\phi_\pi)}\pi^\star \end{bmatrix}. \tag{112}$$

Next, I prove proposition 7. Since

$$\pi_0 = \frac{1+\kappa\sigma^{-1}}{1+\kappa\sigma^{-1}\phi_\pi}\pi^\star, \tag{113}$$

if $\phi_\pi = 1$, then $\pi_0 = \pi_\star$. If $\phi_\pi < 1$, then the denominator is less than the numerator and $\pi_0 > \pi^\star$. If $\phi_\pi > 1$, then $\pi_0 < \pi^\star$ and $\lim_{\phi_\pi \to \infty} \pi_0 = 0$. Since

$$x_0 = \frac{1-\beta-\sigma^{-1}\kappa(\beta\phi_\pi - 1)}{\kappa(1+\kappa\sigma^{-1}\phi_\pi)}\pi^\star, \tag{114}$$

$$x_0 < 0 \Leftrightarrow 1-\beta-\sigma^{-1}\kappa(\beta\phi_\pi - 1) < 0 \Leftrightarrow \frac{\sigma^{-1}\kappa + 1 - \beta}{\beta\sigma^{-1}\kappa} > \phi_\pi. \tag{115}$$

Furthermore,

$$\frac{\partial \pi_0}{\partial \phi_\pi} = \frac{-\kappa\sigma^{-1}(1+\kappa\sigma^{-1})\pi^\star}{(1+\kappa\sigma^{-1}\phi_\pi)^2} < 0 \tag{116}$$

and

$$\frac{\partial x_0}{\partial \phi_\pi} = \frac{-\sigma^{-1}(1+\kappa\sigma^{-1})\pi^\star}{(1+\kappa\sigma^{-1}\phi_\pi)^2} < 0. \tag{117}$$

A.3.3 T Period in Advance Announcement

I solve for the outcomes with a T period in advance announcement by rewriting the model as a single difference equation in inflation. I then solve the model using the properties of lag operators. Let $\hat{i}_t = -(\phi_\pi - 1)\pi^\star_t$, then we can rewrite equation (106) as

$$i_t = \phi_\pi \pi_t + \hat{i}_t. \tag{118}$$

Then, I can rewrite the model in lag operator notation and combine equations (118) and (104),

$$E_t(1 - L^{-1})x_t = \sigma^{-1}E_t(L^{-1} - \phi_\pi)\pi_t - \sigma^{-1}\hat{i}_t \tag{119}$$

$$E_t(1 - \beta L^{-1})\pi_t = \kappa x_t \tag{120}$$

Multiplying equation (119) through by κ and equation (120) through by $(1 - L^{-1})$ and then equating them,

$$E_t(1 - L^{-1})(1 - \beta L^{-1})\pi_t = \kappa\sigma^{-1}E_t(L^{-1} - \phi_\pi)\pi_t - \sigma^{-1}\kappa\hat{i}_t. \tag{121}$$

Rearranging,

$$E_t(1 - \frac{1 + \beta + \sigma^{-1}\kappa}{1 + \sigma^{-1}\kappa\phi_\pi}L^{-1} + \frac{\beta}{1 + \sigma^{-1}\kappa\phi_\pi}L^{-2})\pi_t = \frac{-\sigma^{-1}\kappa}{1 + \sigma^{-1}\kappa\phi_\pi}\hat{i}_t. \tag{122}$$

I can rewrite this as

$$E_t(1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})\pi_t = \frac{-\sigma^{-1}\kappa}{1 + \sigma^{-1}\kappa\phi_\pi}\hat{i}_t, \tag{123}$$

where

$$\lambda = \frac{1 + \beta + \sigma^{-1}\kappa \pm \sqrt{(1 + \beta + \sigma^{-1}\kappa)^2 - 4\beta(1 + \sigma^{-1}\kappa\phi_\pi)}}{2(1 + \sigma^{-1}\kappa\phi_\pi)}. \tag{124}$$

If $\phi_\pi > 1$, then $|\lambda_1|$ and $|\lambda_1|$ are less then one and the model can be solved forward. Doing so

$$\begin{aligned}\pi_t &= -E_t\frac{1}{(1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})}\frac{\sigma^{-1}\kappa}{1 + \sigma^{-1}\kappa\phi_\pi}\hat{i}_t \\ &= \frac{\sigma^{-1}\kappa}{1 + \sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1 - \lambda_2}E_t(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\hat{i}_{t+j} + \lambda_2\sum_{j=0}^{\infty}\lambda_2^j\hat{i}_{t+j}),\end{aligned} \tag{125}$$

which is the same as equation (35). If $E_t\hat{i}_{t+j} = 0\ \forall j$, then $\pi_t = 0$ and $x_t = \frac{\pi_t - \beta E_t\pi_{t+1}}{\kappa} = 0$, which is the pre-announcement steady state. Simplifications of equation (125) depend on whether λ_1 and λ_2 are real or complex numbers. They are real if

$$\begin{aligned}&(1 + \beta + \sigma^{-1}\kappa)^2 - 4\beta(1 + \sigma^{-1}\kappa\phi_\pi) \geq 0 \\ &\Leftrightarrow \frac{(1 - \beta)^2}{\kappa\sigma^{-1}} + \kappa\sigma^{-1} \geq 2(\beta\phi_\pi - 1) + 2\beta(\phi_\pi - 1)\end{aligned} \tag{126}$$

Assuming equation (126) holds, I can rewrite equation (125) for period $T-t$ and then simplify to get

$$\begin{aligned}\pi_{T-t} &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_{T-t}(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\hat{i}_{T-t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\hat{i}_{T-t+j})\\ &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}(-\lambda_1\sum_{j=t}^{\infty}\lambda_1^j\hat{i}+\lambda_2\sum_{j=t}^{\infty}\lambda_2^j\hat{i})\\ &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}(-\lambda_1^{t+1}\sum_{j=0}^{\infty}\lambda_1^j+\lambda_2^{t+1}\sum_{j=0}^{\infty}\lambda_2^j)\hat{i}\\ &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}\frac{-\lambda_1^{t+1}(1-\lambda_2)+\lambda_2^{t+1}(1-\lambda_1)}{(1-\lambda_1)(1-\lambda_2)}\pi^\star(1-\phi_\pi)\\ &= \pi^\star\frac{(1-\phi_\pi)\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1+\sigma^{-1}\kappa\phi_\pi}{(\phi_\pi-1)\sigma^{-1}\kappa}\frac{-\lambda_1^{t+1}(1-\lambda_2)+\lambda_2^{t+1}(1-\lambda_1)}{-1(\lambda_2-\lambda_1)}\\ &= \pi^\star\frac{-\lambda_1^{t+1}(1-\lambda_2)+\lambda_2^{t+1}(1-\lambda_1)}{\lambda_2-\lambda_1}, \end{aligned} \tag{127}$$

where $\hat{i}=-(\phi_\pi-1)\pi^\star$. Solving for x_{T-t},

$$x_{T-t}=\frac{\pi_{T-t}-\beta E_{T-t}\pi_{T-t+1}}{\kappa}=\frac{(1-\lambda_1)\lambda_2^t(\lambda_2-\beta)-(1-\lambda_2)\lambda_1^t(\lambda_1-\beta)}{\kappa(\lambda_2-\lambda_1)} \tag{128}$$

Equations (126) though (128) verify the first half of proposition 8. To verify the rest of it, I solve for π_{T-t} when equation (126) is violated. First note we can express $\lambda_{1,2}$ as

$$\begin{aligned}\lambda_{1,2} &= \frac{1+\beta+\sigma^{-1}\kappa\pm i\sqrt{-((1+\beta+\sigma^{-1}\kappa)^2-4\beta(1+\sigma^{-1}\kappa\phi_\pi)}}{2(1+\sigma^{-1}\kappa\phi_\pi))}\\ &\equiv a\pm bi=re^{\pm i\omega}=r(\cos\omega\pm i\sin\omega), \end{aligned} \tag{129}$$

where

$$\begin{aligned}r=\sqrt{a^2+b^2} &= \sqrt{\Big(\frac{1+\beta+\sigma^{-1}\kappa}{2(1+\sigma^{-1}\kappa\phi_\pi)}\Big)^2+\frac{-((1+\beta+\sigma^{-1}\kappa)^2-4\beta(1+\sigma^{-1}\kappa\phi_\pi)}{4(1+\sigma^{-1}\kappa\phi_\pi)^2}}\\ &= \sqrt{\frac{\beta}{1+\sigma^{-1}\kappa\phi_\pi}} \end{aligned} \tag{130}$$

and

$$\begin{aligned}\omega=\cos^{-1}(\frac{a}{r}) &= \cos^{-1}(\frac{1+\beta+\sigma^{-1}\kappa}{2(1+\sigma^{-1}\kappa\phi_\pi)}\Big/\sqrt{\frac{\beta}{1+\sigma^{-1}\kappa\phi_\pi}})\\ &= \cos^{-1}(\frac{1+\beta+\sigma^{-1}\kappa}{2\sqrt{\beta(1+\sigma^{-1}\kappa\phi_\pi)}}). \end{aligned} \tag{131}$$

Now to simplify equation (125),

$$\begin{aligned}\pi_t &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_t(\sum_{j=0}^{\infty}(\lambda_2^{j+1}-\lambda_1^{j+1})\hat{i}_{t+j}) \\ &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{re^{i\omega}-re^{-i\omega}}E_t(\sum_{j=0}^{\infty}((re^{-i\omega})^{j+1}-(re^{i\omega})^{j+1})\hat{i}_{t+j}) \\ &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{2ri\sin\omega}E_t(\sum_{j=0}^{\infty}-2r^{j+1}i\sin(\omega(j+1))\hat{i}_{t+j}) \\ &= \frac{-\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}E_t(\sum_{j=0}^{\infty}r^j\frac{\sin(\omega(j+1))}{\sin\omega}\hat{i}_{t+j}). \quad (132)\end{aligned}$$

Rewriting equation (132) for period $T-t$,

$$\begin{aligned}\pi_{T-t} &= \frac{-\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}E_{T-t}(\sum_{j=0}^{\infty}r^j\frac{\sin(\omega(j+1))}{\sin\omega}\hat{i}_{T-t+j}) \\ &= \frac{-\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\sum_{j=t}^{\infty}r^j\frac{\sin(\omega(j+1))}{\sin\omega}\hat{i} \\ &= \frac{-\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\sum_{j=0}^{\infty}r^{j+t}\frac{\sin(\omega(j+1+t))}{\sin\omega}\pi^\star(1-\phi_\pi) \\ &= \pi^\star\frac{\sigma^{-1}\kappa(\phi_\pi-1)}{1+\sigma^{-1}\kappa\phi_\pi}r^t\sum_{j=0}^{\infty}r^j\frac{\sin(\omega(j+1+t))}{\sin\omega}, \quad (133)\end{aligned}$$

which completes the proof of proposition 8.

Next, I prove proposition 9. Recall equations (40) and (41) and define a, b, c, and d as

$$\pi_{t-1} = \underbrace{\frac{\beta+\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}}_{a}\pi_t + \underbrace{\frac{\kappa}{1+\sigma^{-1}\kappa\phi_\pi}}_{b}x_t \quad (134)$$

$$x_{t-1} = \underbrace{\frac{-\sigma^{-1}(\beta\phi_\pi-1)}{1+\sigma^{-1}\kappa\phi_\pi}}_{c}\pi_t + \underbrace{\frac{1}{1+\sigma^{-1}\kappa\phi_\pi}}_{d}x_t. \quad (135)$$

Let us first consider the $\phi_\pi < 1$ case. First note that $\pi_{T-1} = a\pi_T + bx_T > a\pi_T > \pi_T = \pi^\star > 0$, because $a > 1$, $b > 0$, and $x_T > 0$. Similarly,

$$x_{T-1} > x_T \Leftrightarrow \frac{1-\beta-\sigma^{-1}\kappa(\beta\phi_\pi-1)\pi^\star}{\kappa(1+\kappa\sigma^{-1}\phi_\pi)} > \frac{1-\beta}{\kappa}\pi^\star \Leftrightarrow \phi_\pi < 1. \quad (136)$$

Hence $x_{T-1} > x_T = \frac{1-\beta}{\kappa}\pi^\star > 0$. Now I prove by induction that $0 < \pi_{T-n} < \pi_{T-n-1}$ and $0 < x_{T-n} < x_{T-n-1}$ for all n, which implies the first part of proposition 2. I have just shown that this holds for $n = 0$. Now assume this is true for $n = k-1$, that is

$0 < \pi_{T-k+1} < \pi_{T-k}$ and $0 < x_{T-k+1} < x_{T-k}$. Then for $n = k$,

$$\pi_{T-k-1} = a\pi_{T-k} + bx_{T-k} > a\pi_{T-k+1} + bx_{T-k+1} = \pi_{T-k} > 0$$

$$x_{T-k-1} = cx_{T-k} + dx_{T-k} > c\pi_{T-k+1} + dx_{T-k+1} = x_{T-k} > 0,$$

which completes the proof by induction.

Next, let us consider the $\phi_\pi = 1$ case. From equations (134) and (135), $\pi_{T-1} = \pi^\star = \pi_T$ and $x_{T-1} = \frac{1-\beta}{\kappa}\pi^\star = x_T$. Since π_{T-t} and x_{T-t} are recursively defined and constant between $t = 0$ and $t = 1$, they will be constant for any t.[19]

Next, let us consider the $1 < \phi_\pi \leq \frac{1}{\beta}$ case. Note that, $\pi_{T-1} = \frac{1+\kappa\sigma^{-1}}{1+\kappa\sigma^{-1}\phi_\pi}\pi^\star < \pi^\star = \pi_T$ and $x_{T-1} < x_T$ as this is the opposite case of (136). Additionally, a, b, c, d, x_{T-1}, and π_{T-1} are all positive. Now I prove by induction that $0 \leq \pi_{T-n-1} < \pi_{T-n}$ and $0 \leq x_{T-n-1} < x_{T-n}$ for all n, which implies the third part of proposition 9. I have just shown that this holds for $n = 0$. Now assume this is true for $n = k - 1$, that is $0 < \pi_{T-k} < \pi_{T-k+1}$ and $0 < x_{T-k} < x_{T-k+1}$. Then for $n = k$,

$$\pi_{T-k-1} = a\pi_{T-k} + bx_{T-k} < a\pi_{T-k+1} + bx_{T-k+1} = \pi_{T-k}$$

$$x_{T-k-1} = cx_{T-k} + dx_{T-k} < c\pi_{T-k+1} + dx_{T-k+1} = x_{T-k}.$$

Furthermore since a, b, c, d, x_{T-k}, and π_{T-k} are all positive, then so are π_{T-k-1} and x_{T-k-1}, which completes the proof by induction.

The final case is $\phi_\pi > \frac{1}{\beta}$, and as argued before $x_{T-1} < x_T$ and $\pi_{T-1} < \pi_T$. Furthermore as long as π_t and x_t are both greater than zero, $x_{t-1} < x_t$ and $\pi_{t-1} < \pi_t$. For the output gap, $x_{t-1} = c\pi_t + dx_t < dx_t < x_t$ because $c < 0$ and $d < 1$. For inflation the previous induction argument still holds. So at time $T - 1$ and as long as both inflation and output are positive, they are both monotonically increasing. If t is large enough given the calibration that π_t and x_t are not both greater than zero, then may exist some perhaps different t and t' for which π_t and $x_{t'}$ are monotonically decreasing as a consequence of the functional form of the solutions given in equations (127) and (128) (the solutions' functional form $a\lambda_1^t + b\lambda_2^t$ is where $0 < \lambda_1 < 1$ and $0 < \lambda_2 < 1$ and a and b can be either positive or negative implies there can be at most one change in the sign of the first derivative).

A.4 Time Varying Optimal Policy Rule

In this section I solve for the optimal time varying policy rule and outcome for a T period in advance change in the inflation target described in section 6.2. Let the central bank

[19] This can be shown formally by an induction argument similar to the one used to prove the first part of the proposition.

declare at time $t = 0$ that $t = T$ the inflation target will be raised from 0 to $\bar{\pi}$. The central bank optimization problem is

$$\min_{x_t, \pi_t} \sum_{t=0}^{T} \beta^t (\pi_t^2 + \theta_x x_t^2) \text{ s.t. } \pi_t = \kappa x_t + \beta \pi_{t+1}, \pi_T = \bar{\pi} \tag{137}$$

The Lagrangian is

$$\mathcal{L} = \sum_{t=1}^{T} \beta^t (\pi_t^2 + \theta_x x_t^2 + 2\gamma_t (\pi_t - \kappa x_t - \beta \pi_{t+1})) \tag{138}$$

The FOC for $t = 0, ..., T$ are

$$\theta_x x_t = \gamma_t \kappa \tag{139}$$

$$\pi_t = \gamma_{t-1} - \gamma_t \tag{140}$$

$$\pi_t = \kappa x_t + \beta \pi_{t+1} \tag{141}$$

$$\pi_T = \bar{\pi} \text{ and } \gamma_{-1} = 0 \tag{142}$$

I can rewrite these as

$$\pi_t = \frac{\theta_x}{\kappa}(x_{t-1} - x_t) \tag{143}$$

$$\pi_t = \kappa x_t + \beta \pi_{t+1} \tag{144}$$

for $t = 0, ..., T$ along with $\gamma_{-1} = 0$ and end point $(\pi_{T+1}, x_{T+1}) = (\bar{\pi}, \frac{1-\beta}{\kappa}\bar{\pi})$.

To solve these equations, I turn this into a second order difference equation in the inflation rate and solve for the transition path of inflation. Once the path of inflation is solved for I can back out the path of the output gap and interest rates (and the components thereof). Solving (144) for output:

$$x_t = \frac{\pi_t - \beta \pi_{t+1}}{\kappa} \tag{145}$$

Plugging this into (143) and simplifying

$$\begin{aligned} \pi_t = \frac{\theta_x}{\kappa}\left(\frac{\pi_{t-1} - \beta \pi_t}{\kappa} - \frac{\pi_t - \beta \pi_{t+1}}{\kappa}\right) \\ \Rightarrow \pi_{t+1} - \frac{\kappa^2 + \theta_x(1+\beta)}{\beta \theta_x} \pi_t + \frac{1}{\beta} \pi_{t-1} = 0 \\ \Rightarrow \pi_{t+1} - a\pi_t + b\pi_{t-1} = 0, \end{aligned} \tag{146}$$

where $a = \frac{\kappa^2 + \theta_x(1+\beta)}{\beta \theta_x}$ and $b = \frac{1}{\beta}$.

Guess $\pi_t = Aw^t$ so

$$Aw^{t+1} - aAw^t + bAw^{t-1} = 0 \Rightarrow w^2 - aw + b = 0 \Rightarrow (w - m_1)(w - m_2) = 0, \tag{147}$$

where

$$m_{1,2} = \frac{a \pm \sqrt{a^2 - 4b}}{2} \tag{148}$$

$m_{1,2}$ are real and distinct if

$$\begin{aligned} a^2 - 4b = (\frac{\kappa^2 + \theta_x(1+\beta)}{\beta\theta_x}) - \frac{4}{\beta} > 0 \\ (\kappa^2 + \theta_x(1+\beta))^2 > 4\beta\theta_x^2 \\ \kappa^4 + 2\kappa^2\theta_x + 2\kappa^2\beta\theta_x + \theta_x(1-\beta)^2 > 0 \end{aligned} \tag{149}$$

Since the last line will hold for any set of reasonable parameters (non negative and $\beta < 1$), the general solution will be of the form

$$\pi_t = A_1 m_1^t + A_2 m_2^t \tag{150}$$

Notably, cyclical dynamics such as those exhibited under a constant policy rule will never be optimal.

To solve for the specific solution, I use $\pi_T = \bar{\pi}$ and $\gamma_{-1} = 0$ which in turn implies

$$\pi_0 = -\frac{\theta_x}{\kappa} x_0 \tag{151}$$

Solving this for x_0 and plugging into (144) for $t = 0$ to get

$$\frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x})\pi_0 = \pi_1 \tag{152}$$

Define $z_1 = \frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x})$, then to solve for the specific solution, I solve

$$\bar{\pi} = A_1 m_1^T + A_2 m_2^T \tag{153}$$

$$z_1(A_1 + A_2) = A_1 m_1 + A_2 m_2 \tag{154}$$

Doing so

$$A_2 = \frac{\bar{\pi}}{-m_1^T(\frac{m_2 - z_1}{m_1 - z_1}) + m_2^T} \equiv \frac{\bar{\pi}}{m_1^T z_2 + m_2^T} \tag{155}$$

$$A_1 = -A_2 \frac{m_2 - z_1}{m_1 - z_1} \equiv A_2 z_2, \tag{156}$$

where $z_2 = -\frac{m_2 - z_1}{m_1 - z_1}$. Given A_1 and A_2,

$$\pi_t = A_1 m_1^t + A_2 m_2^t \tag{157}$$

$$x_t = \frac{\pi_t - \beta \pi_{t+1}}{\kappa} = \frac{m_1^t A_1 (1 - \beta m_1) + m_2^t A_2 (1 - \beta m_2)}{\kappa} \tag{158}$$

Furthermore since $i_t = \phi_\pi \pi_t + (1 - \phi_\pi)\pi_t^\star \equiv \phi_\pi \pi_t + \bar{i}_t$, I can back out the values for i_t, $\bar{i}_t$ and $\pi_t^\star$ using the IS equation;

$$x_t = x_{t+1} - \sigma^{-1}(i_t - \pi_{t+1}) \tag{159}$$

$$i_t = \sigma(x_{t+1} - x_t) + \pi_{t+1} \tag{160}$$

$$\bar{i}_t = \sigma(x_{t+1} - x_t) + \pi_{t+1} - \phi_\pi \pi_t \tag{161}$$

$$\pi_t^\star = \frac{\sigma(x_{t+1} - x_t) + \pi_{t+1} - \phi_\pi \pi_t}{1 - \phi_\pi} \tag{162}$$

Next I turn to a proof of proposition 10. As preliminaries to the proof, I first show that $m_1 > z_1 > m_2$, $m_1 > 1 > m_2 > 0$, $z_2 > 0$, and that $A_1, A_2 > 0$ if $\bar{\pi} > 0$ and $A_1, A_2 < 0$ if $\bar{\pi} < 0$.

First to show $m_1 > 0$, recall that $m_1 = \frac{a + \sqrt{a^2 - 4b}}{2}$, but $a = \frac{\kappa^2 + \theta_x(1+\beta)}{\beta \theta_x} > 0$ and we have already shown that $a^2 - 4b > 0$. Hence, $m_1 > 0$.

Next to check that $m_2 > 0$:

$$m_2 = \frac{a - \sqrt{a^2 - 4b}}{2} > 0 \Leftrightarrow a > \sqrt{a^2 - 4b} \tag{163}$$

$$\Leftrightarrow a^2 > a^2 - 4b \tag{164}$$

$$\Leftrightarrow 0 > -4b \Leftrightarrow 0 > -\frac{4}{\beta}, \tag{165}$$

which always holds.[20]

Next to verify that $m_1 > 1$:

$$m_1 = \frac{a + \sqrt{a^2 - 4b}}{2} > 1 \Leftrightarrow \sqrt{a^2 - 4b} > 2 - a, \tag{166}$$

but

$$2 - a = 2 - (\frac{\kappa^2}{\beta \theta_x} + \frac{1}{\beta} + 1) < -\frac{\kappa^2}{\beta \theta_x} < 0 \tag{167}$$

Hence, $\sqrt{a^2 - 4b} > 0 > 2 - a$.

[20] Equation (164) holds because both a and $\sqrt{a^2 - 4b}$ are both always positive. Similarly, equations (169) and (173) hold because the preceding inequalities are between positive numbers.

Now to verify that $m_2 < 1$:

$$m_2 = \frac{a - \sqrt{a^2 - 4b}}{2} < 1 \Leftrightarrow a - 2 > \sqrt{a^2 - 4b} \tag{168}$$

$$\Leftrightarrow a^2 - 4a + 4 = (a-2)^2 > \sqrt{a^2 - 4b}^2 = a^2 - 4b \tag{169}$$

$$\Leftrightarrow 1 < a - b = \frac{\kappa^2 + \theta_x(1+\beta)}{\beta\theta_x} - \frac{1}{\beta} \Leftrightarrow \beta\theta_x < \kappa^2 + \theta_x(1+\beta) - \theta_x \Leftrightarrow 0 < \kappa^2, \tag{170}$$

which always holds.

Next to verify that $m_1 > z_1$:

$$m_1 = \frac{a + \sqrt{a^2 - 4b}}{2} > \frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x}) = z_1 \tag{171}$$

$$\Leftrightarrow \sqrt{a^2 - 4b} > \frac{2}{\beta}(\frac{\theta_x + \kappa^2}{\theta_x}) - a = \frac{2}{\beta}(\frac{\theta_x + \kappa^2}{\theta_x}) + \frac{\kappa^2 + \theta_x(1+\beta)}{\beta\theta_x} = \frac{(1-\beta)\theta_x + \kappa^2}{\beta\theta_x} \tag{172}$$

$$\Leftrightarrow a^2 - 4b > (\frac{(1-\beta)\theta_x + \kappa^2}{\beta\theta_x})^2 \tag{173}$$

$$\Leftrightarrow (\frac{\kappa^2 + \theta_x(1+\beta)}{\beta\theta_x})^2 - \frac{4}{\beta} > (\frac{(1-\beta)\theta_x + \kappa^2}{\beta\theta_x})^2 \Leftrightarrow (\frac{\kappa^2 + \theta_x(1+\beta)}{\beta\theta_x})^2 - (\frac{(1-\beta)\theta_x + \kappa^2}{\beta\theta_x})^2 > \frac{4}{\beta} \tag{174}$$

$$\Leftrightarrow 2\theta_x(1+\beta)\kappa^2 - 2\theta_x(1-\beta)\kappa^2 + \theta_x^2(1+\beta)^2 - \theta_x^2(1-\beta)^2 > 4\beta\theta_x^2 \Leftrightarrow \theta_x\kappa^2 > 0, \tag{175}$$

which always holds.

Now to verify that $m_2 < z_1$:

$$m_2 = \frac{a - \sqrt{a^2 - 4b}}{2} < \frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x}) = z_1 \tag{176}$$

$$\Leftrightarrow a - \frac{2}{\beta}(\frac{\theta_x + \kappa^2}{\theta_x}) < \sqrt{a^2 - 4b} \tag{177}$$

$$\Leftrightarrow \frac{\kappa^2 + \theta_x(1+\beta)}{\beta\theta_x} - \frac{2}{\beta}(\frac{\theta_x + \kappa^2}{\theta_x}) = \frac{-\kappa^2 - \theta_x(1-\beta)}{\beta\theta_x} < 0 < \sqrt{a^2 - 4b}, \tag{178}$$

which always holds.

To verify that $z_2 > 0$:

$$z_2 = -\frac{m_2 - z_1}{m_1 - z_1} > 0 \Leftrightarrow 0 > m_2 - z_1 \Leftrightarrow z_1 > m_2, \tag{179}$$

which we have already confirmed.

Finally, I need to show that A_1 and A_2 are both positive if $\bar{\pi}$ is positive, and they are both negative if $\bar{\pi}$ is negative. From 155 and 156:

$$A_2 = \frac{\bar{\pi}}{m_1^T z_2 + m_2^T} \text{ and } A_1 = A_2 z_2,$$

and I have shown that m_1, m_2, and z_2 are all positive. Therefore A_1 and A_2 have the same sign and are both positive if $\bar{\pi} > 0$ and negative otherwise.

Since $\pi_0 = A_1 + A_2$ and A_1 and A_2 are of the same sign as $\bar{\pi}$:

$$\pi_0 = \begin{cases} > 0, & \text{if } \bar{\pi} > 0 \\ < 0, & \text{if } \bar{\pi} < 0 \end{cases} \tag{180}$$

So inflation at the time of the announcement jumps in the direction of $\bar{\pi}$. To show that it then monotonically adjusts increases (decreases) to approach a positive (negative) $\bar{\pi}$, I use a proof by induction for each case.

Case 1: $\bar{\pi} > 0$. Claim $\pi_t < \pi_{t+1}$. For $t = 1$, from (152):

$$\pi_1 = \frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x})\pi_0 > \pi_0, \tag{181}$$

since $\frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x}) > 1$. Now I need to show that if $\pi_t < \pi_{t+1}$ for $t = k$, then it also holds for $t = k + 1$. Hence, I need to show that:

$$\pi_{k+1} < \pi_{k+2} \Leftrightarrow A_1 m_1^{k+1} + A_2 m_2^{k+1} < A_1 m_1^{k+2} + A_2 m_2^{k+2} \tag{182}$$

$$\Leftrightarrow A_1 m_1^{k+1}(1 - m_1) < A_2 m_2^{k+1}(m_2 - 1) \tag{183}$$

$$\Leftrightarrow \frac{A_1}{A_2} > \frac{m_2^{k+1}}{m_1^{k+1}} \frac{m_2 - 1}{1 - m_1}, \tag{184}$$

but by assumption:

$$\pi_k < \pi_{k+1} \Leftrightarrow A_1 m_1^k + A_2 m_2^k < A_1 m_1^{k+1} + A_2 m_2^{k+1} \tag{185}$$

$$\Leftrightarrow A_1 m_1^k(1 - m_1) < A_2 m_2^{k+1}(m_2 - 1) \tag{186}$$

$$\Leftrightarrow \frac{A_1}{A_2} > \frac{m_2^k}{m_1^k} \frac{m_2 - 1}{1 - m_1} > \frac{m_2^k}{m_1^k} \frac{m_2 - 1}{1 - m_1} \frac{m_2}{m_1} = \frac{m_2^{k+1}}{m_1^{k+1}} \frac{m_2 - 1}{1 - m_1}, \tag{187}$$

where the second inequality in (187) holds because $\frac{m_2^k}{m_1^k} \frac{m_2 - 1}{1 - m_1} > 0$ and $m_2 < m_1$.

Case 2: $\bar{\pi} < 0$. Claim $\pi_t > \pi_{t+1}$. For $t = 1$, from (152):

$$\pi_1 = \frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x})\pi_0 < \pi_0, \tag{188}$$

since $\frac{1}{\beta}(1 + \frac{\kappa^2}{\theta_x}) > 1$ and $\pi_0 < 0$. Now I need to show that if $\pi_t > \pi_{t+1}$ for $t = k$, then it

also holds for $t = k + 1$. Hence, I need to show that:

$$\pi_{k+1} > \pi_{k+2} \Leftrightarrow A_1 m_1^{k+1} + A_2 m_2^{k+1} > A_1 m_1^{k+2} + A_2 m_2^{k+2} \tag{189}$$

$$\Leftrightarrow A_1 m_1^{k+1}(1 - m_1) > A_2 m_2^{k+1}(m_2 - 1) \tag{190}$$

$$\Leftrightarrow \frac{A_1}{A_2} > \frac{m_2^{k+1}}{m_1^{k+1}} \frac{m_2 - 1}{1 - m_1}, \tag{191}$$

but by assumption,

$$\pi_k > \pi_{k+1} \Leftrightarrow A_1 m_1^k + A_2 m_2^k > A_1 m_1^{k+1} + A_2 m_2^{k+1} \tag{192}$$

$$\Leftrightarrow A_1 m_1^k(1 - m_1) > A_2 m_2^{k+1}(m_2 - 1) \tag{193}$$

$$\Leftrightarrow \frac{A_1}{A_2} > \frac{m_2^k}{m_1^k} \frac{m_2 - 1}{1 - m_1} > \frac{m_2^k}{m_1^k} \frac{m_2 - 1}{1 - m_1} \frac{m_2}{m_1} = \frac{m_2^{k+1}}{m_1^{k+1}} \frac{m_2 - 1}{1 - m_1}, \tag{194}$$

where the second inequality in (194) holds because $\frac{m_2^k}{m_1^k} \frac{m_2 - 1}{1 - m_1} > 0$ and and $m_2 < m_1$.

A.5 Price Indexing Solution

In this section I provide a sketch of the preceding analysis where the assumption that firms do not index prices is relaxed. Both the approach and the results largely mimic that of the main specification. The key differences are that inflation after the change in the inflation target is at its target, while the output gap is zero, and the optimal policy analysis no longer consists of choosing a point along a stationary Phillips curve.

A.5.1 Regime Shifts

The model is described by:

$$\pi_{j,t} = \beta E_t \pi_{j,t+1} + \kappa x_{j,t} + (1 - \beta)\pi_j^\star \tag{195}$$

$$x_{j,t} = E_t x_{j,t+1} - \sigma^{-1}(i_{j,t} - E_t \pi_{j,t+1}) \tag{196}$$

$$i_{j,t} = \phi_\pi \pi_{j,t} + \phi_{\pi'} E_t \pi_{j,t+1} + \phi_x x_{j,t} + \pi_j^\star(1 - \phi_\pi - \phi_{\pi'}) \tag{197}$$

First consider the outcome in regime two, which by assumption remains an absorbing state of the Markov process. As there are no shocks and regime two is in a steady state, (195) and (196) become

$$\pi_2 = \beta \pi_2 + \kappa x_2 + (1 - \beta)\pi_2^\star \Rightarrow \pi_2(1 - \beta) = \kappa x_2 + (1 - \beta)\pi_2^\star \tag{198}$$

$$x_2 = x_2 - \sigma^{-1}(i_2 - \pi_2) \Rightarrow i_2 = \pi_2 \tag{199}$$

Plugging everything into (197),

$$\begin{aligned}\pi_2(1-\phi_\pi-\phi_{\pi'}) &= \phi_x x_2 + \pi_2^\star(1-\phi_\pi-\phi_{\pi'}) \\ \Rightarrow \kappa\pi_2(1-\phi_\pi-\phi_{\pi'}) &= \phi_x(\pi_2(1-\beta)-(1-\beta)\pi_2^\star) + \kappa\pi_2^\star(1-\phi_\pi-\phi_{\pi'}) \\ \Rightarrow \pi_2(\kappa(1-\phi_\pi-\phi_{\pi'})-\phi_x(1-\beta)) &= \pi_2^\star(\kappa(1-\phi_\pi-\phi_{\pi'})-\phi_x(1-\beta)) \Rightarrow \pi_2 = \pi_2^\star.\end{aligned} \tag{200}$$

Then by (198),

$$x_2 = 0 \tag{201}$$

Hence in the absorbing regime, inflation is always at it's target and the output gap is zero. Long run neutrality of money holds in the absorbing regime, but as will soon be shown expectations of a regime shift will lead to none neutrality of money in regime one, albeit with the caveat that regime one is by its very nature not a long run outcome.

As in the main specification, regime one is in a steady state. Adopting steady state notation and solving (195) for π_1,

$$\pi_1 = \frac{\beta\lambda\pi_2^\star + \kappa x_1 + (1-\beta)\pi_1^\star}{1-\beta(1-\lambda)} \tag{202}$$

The regime one Philips curve depends not just on the inflation target in the other regime but also the current inflation target. This will play a major role in the welfare analysis. Plugging in (197) into (196) and solving for π_1,

$$\pi_1 = -\frac{\sigma\lambda+\phi_x}{\phi_\pi+(1-\lambda)(\phi_{\pi'}-1)}x_1 - \frac{\lambda(\phi_{\pi'}-1)}{\phi_\pi+(1-\lambda)(\phi_{\pi'}-1)}\pi_2^\star - \frac{1-\phi_\pi-\phi_{\pi'}}{\phi_\pi+(1-\lambda)(\phi_{\pi'}-1)}\pi_1^\star, \tag{203}$$

which is nearly identical to (52) except for the coefficient on $\pi_2^\star$. Solving (202) and (203) for x_1 and π_1,

$$\pi_1 = \frac{\lambda(\beta(\sigma\lambda+\phi_x)+\kappa(1-\phi_{\pi'}))\pi_2^\star + ((1-\beta)(\sigma\lambda+\phi_x)+\kappa(\phi_\pi+\phi_{\pi'}-1))\pi_1^\star}{\kappa(\phi_\pi+\kappa(1-\lambda)(\phi_{\pi'}-1))+(1-\beta(1-\lambda))(\sigma\lambda+\phi_x)} \tag{204}$$

$$x_1 = \frac{\lambda(1-\phi_{\pi'}-\beta\phi_\pi)\pi_2^\star + \lambda(\beta\phi_\pi+\phi_{\pi'}-1)\pi_1^\star}{\kappa(\phi_\pi+\kappa(1-\lambda)(\phi_{\pi'}-1))+(1-\beta(1-\lambda))(\sigma\lambda+\phi_x)} \tag{205}$$

With active policy ($\phi_\pi+\phi_{\pi'}-1>0$),

- $\frac{\partial\pi_1}{\partial\pi_2^\star} > 0 \Leftrightarrow \beta\sigma\lambda+\beta\phi_x+\kappa(1-\phi_{\pi'}) > 0$, which is the same as without indexing except a smaller constant term. Qualitatively, a higher $\pi_2^\star$ still increases π_1 unless the response to expected inflation is so strong that a large large recession reduces marginal costs sufficiently for firms to reduce prices despite the higher anticipated future prices.

- $\frac{\partial \pi_1}{\partial \pi_1^\star} > 0$, which is the same as without indexing.
- $\frac{\partial x_1}{\partial \pi_2^\star} < 0 \Leftrightarrow \phi_{\pi'} + \beta\phi_\pi > 1$, which is a weaker condition that for the main specification. Under the main specification, a higher inflation target in regime two results in both a change in the real interest rate and the output level in regime two. The higher x_2 implies a consumption smoothing motive that pushes up x_1. For x_1 to fall, real interest rates must rise enough so that the savings motive is greater than the consumption smoothing motive. With price indexing, the consumption smoothing force no longer exists, and therefore any increase in the real interest rates is sufficient to push output down.
- $\frac{\partial x_1}{\partial \pi_1^\star} > 0 \Leftrightarrow \phi_{\pi'} + \beta\phi_\pi > 1$, whereas in the main specification this partial derivative is always positive with active policy.

The zero inflation outcome in regime one can be achieved by lowering the inflation target in regime one such that

$$\pi_1^\star = -\frac{\lambda(\beta(\sigma\lambda + \phi_x) + \kappa(1 - \phi_{\pi'}))}{(1-\beta)(\sigma\lambda + \phi_x) + \kappa(\phi_\pi + \phi_{\pi'} - 1)}\pi_2^\star \tag{206}$$

Optimal policy in this setup is complicated by the movement of the Philips Curve in response to changes in the current regime's inflation target. While it is possible to derive the optimal policy by substituting in the expressions for inflation and output in regime one into the loss function, it is no longer simply a function of the inflation target, but changing the other policy parameters will effect the inflation-volatility tradeoff which in the numerical analysis section is shown to be the dominant determinant of welfare. Instead here I provide a broad intuition for the range the optimal response may take. First of by changing the other policy parameters, the slope and intercept of (205) will change and the optimal response will generally involve at least minor modifications of these parameters. Increasing $\pi_1^\star$ shifts both (204) and (205) up, increasing inflation but with an ambiguous effect on the output gap. If $\phi_{\pi'} + \beta\phi_\pi > 1$, then both inflation and output in regime one increase. This preserves the same qualitative trade off as simply moving along the Philips curve: more positive inflation vs a less negative output gap. Therefore, the central bank still chooses some point between zero inflation and a negative output gap and positive inflation and a zero output gap, but achieving zero inflation requires a less negative output gap and achieving a zero output gap requires a larger inflation rate than simply moving along the Philips curve without price indexing. If $\phi_{\pi'} + \beta\phi_\pi < 1$, then decreasing $\pi_1^\star$ results in both lower inflation and a less negative output gap, improving welfare among both dimensions until inflation is at zero. Further decreasing the inflation target creates a trade-off of a more negative inflation rate vs a less negative output gap. Therefore the optimal policy will choose some point between

a zero inflation rate and a negative output gap and a zero output gap and a negative inflation rate.

A.5.2 Perfect Foresight

The only difference from the main specification described in section 8.1 is that equations (105) becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + (1-\beta)\pi_t^\star \tag{207}$$

The recursive formulation of the variables between $0 \leq t < T$, remains unchanged because for t in this range $\pi_t^\star = 0$. To solve for the one period in advance outcome first note that prior to the announcement and after the inflation target changes the model is in a single regime setting identical to regime two. Therefore by (200) $\pi = \pi^\star$ and from (201) $x = 0$. The $t = 0$ allocation can be obtained by using the recursive formulation (108) with $t = 1$,

$$\begin{bmatrix} \pi_0 \\ x_0 \end{bmatrix} = \frac{1}{1+\kappa\sigma^{-1}\phi_\pi} \begin{bmatrix} \beta + \kappa\sigma^{-1} & \kappa \\ -\beta\sigma^{-1}\phi_\pi + \sigma^{-1} & 1 \end{bmatrix} \begin{bmatrix} \pi^\star \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\beta+\kappa\sigma^{-1}}{1+\kappa\sigma^{-1}\phi_\pi}\pi^\star \\ \frac{\sigma^{-1}(1-\beta\phi_\pi)}{(1+\kappa\sigma^{-1}\phi_\pi)}\pi^\star \end{bmatrix} \tag{208}$$

Following the same procedure as in the main specification results in a modified version of proposition 1. If equation (208) holds and $\pi^\star > 0$, then

1. $\pi_0 \in (0, \pi^\star)$ if $\phi_\pi > 1 - \frac{1-\beta}{\kappa\sigma^{-1}}$ and $\pi_0 \geq \pi^\star$ otherwise
2. $x_0 < 0$ if $\phi_\pi > \frac{1}{\beta}$ and $x_0 \geq 0$ otherwise
3. $\frac{\partial x_0}{\partial \phi_\pi} < 0$ and $\frac{\partial \pi_0}{\partial \phi_\pi} < 0$

The T period in advance announcement solution remains very similar to the solution under the main specification and the approach to solve for it is the same. First rewriting the equations

$$\kappa x_t = E_t \kappa x_{t+1} - \kappa\sigma^{-1}(\phi_\pi \pi_t - (\phi_\pi - 1)\pi_t^\star - E_t\pi_{t+1}) \tag{209}$$

$$\kappa x_t = \pi_t - \beta E_t \pi_{t+1} - (1-\beta)\pi_t^\star \tag{210}$$

Now combine and rearrange terms,

$$\pi_t(1+\kappa\sigma^{-1}\phi_\pi) - (1+\beta+\kappa\sigma^{-1})E_t\pi_{t+1} + \beta E_t \pi_{t+2} = \kappa\sigma \underbrace{(\phi_\pi - 1)\pi_t^\star}_{\hat{i}_t} + \underbrace{(1-\beta)(\pi_t^\star - E_t \pi^s tar_{t+1})}_{\eta_t} \tag{211}$$

Adopting lag notation and rearranging,

$$E_t(1 - \frac{1+\beta+\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}L^{-1} + \frac{\beta}{1+\sigma^{-1}\kappa\phi_\pi}L^{-2})\pi_t = \frac{-\sigma^{-1}\kappa\hat{i}_t + \eta_t}{1+\sigma^{-1}\kappa\phi_\pi}, \tag{212}$$

which except for the η_t term is identical to (122). (123) becomes

$$E_t(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})\pi_t = \frac{-\sigma^{-1}\kappa\hat{i}_t + \eta_t}{1+\sigma^{-1}\kappa\phi_\pi}, \tag{213}$$

where

$$\lambda = \frac{1+\beta+\sigma^{-1}\kappa \pm \sqrt{(1+\beta+\sigma^{-1}\kappa)^2 - 4\beta(1+\sigma^{-1}\kappa\phi_\pi)}}{2(1+\sigma^{-1}\kappa\phi_\pi)}, \tag{214}$$

which are the same as in the main specification. If $\phi_\pi > 1$, then $|\lambda_1|$ and $|\lambda_1|$ are less then one and the model can be solved forward. Doing so,

$$\begin{aligned}\pi_t &= -E_t \frac{1}{(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})} \frac{\sigma^{-1}\kappa\hat{i}_t + \eta_t}{1+\sigma^{-1}\kappa\phi_\pi} \\ &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2} E_t(-\lambda_1 \sum_{j=0}^{\infty} \lambda_1^j \hat{i}_{t+j} + \lambda_2 \sum_{j=0}^{\infty} \lambda_2^j \hat{i}_{t+j}) \\ &\quad + \frac{1}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2} E_t(-\lambda_1 \sum_{j=0}^{\infty} \lambda_1^j \eta_{t+j} + \lambda_2 \sum_{j=0}^{\infty} \lambda_2^j \eta_{t+j})\end{aligned} \tag{215}$$

If $E_t\hat{i}_{t+j} = E_t\eta_{t+j} = 0 \; \forall j$, then $\pi_t = 0$ and $x_t = 0$, which is the pre-announcement steady state. Simplifications of equation (216) depend on whether λ_1 and λ_2 are real or complex numbers. They are real if

$$\begin{aligned}&(1+\beta+\sigma^{-1}\kappa)^2 - 4\beta(1+\sigma^{-1}\kappa\phi_\pi) \geq 0 \\ &\qquad \Leftrightarrow \frac{(1-\beta)^2}{\kappa\sigma^{-1}} + \kappa\sigma^{-1} \geq 2(\beta\phi_\pi - 1) + 2\beta(\phi_\pi - 1),\end{aligned} \tag{216}$$

which remains unchanged from (126). Assuming equation (216) holds, I can rewrite

equation (215) for period $T-t$ and then simplify to get

$$\begin{aligned}
\pi_{T-t} &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_{T-t}(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\hat{i}_{T-t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\hat{i}_{T-t+j}) \\
&\quad+\frac{1}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_{T-t}(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\eta_{T-t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\eta_{T-t+j}) \\
&= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}(-\lambda_1\sum_{j=t}^{\infty}\lambda_1^j\hat{i}+\lambda_2\sum_{j=t}^{\infty}\lambda_2^j\hat{i})+\frac{\lambda_2^t-\lambda_1^t}{\lambda_2-\lambda_1}\frac{(1-\beta)\pi^\star}{1+\sigma^{-1}\kappa\phi_\pi} \\
&= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}(-\lambda_1^{t+1}\sum_{j=0}^{\infty}\lambda_1^j+\lambda_2^{t+1}\sum_{j=0}^{\infty}\lambda_2^j)\hat{i}+\frac{\lambda_2^t-\lambda_1^t}{\lambda_2-\lambda_1}\frac{(1-\beta)\pi^\star}{1+\sigma^{-1}\kappa\phi_\pi} \\
&= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}\frac{-\lambda_1^{t+1}(1-\lambda_2)+\lambda_2^{t+1}(1-\lambda_1)}{(1-\lambda_1)(1-\lambda_2)}\pi^\star(1-\phi_\pi)+\frac{\lambda_2^t-\lambda_1^t}{\lambda_2-\lambda_1}\frac{(1-\beta)\pi^\star}{1+\sigma^{-1}\kappa\phi_\pi} \\
&= \pi^\star\frac{(1-\phi_\pi)\sigma^{-1}\kappa}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1+\sigma^{-1}\kappa\phi_\pi}{(\phi_\pi-1)\sigma^{-1}\kappa}\frac{-\lambda_1^{t+1}(1-\lambda_2)+\lambda_2^{t+1}(1-\lambda_1)}{-1(\lambda_2-\lambda_1)}+\frac{\lambda_2^t-\lambda_1^t}{\lambda_2-\lambda_1}\frac{(1-\beta)\pi^\star}{1+\sigma^{-1}\kappa\phi_\pi} \\
&= \pi^\star\frac{-\lambda_1^{t+1}(1-\lambda_2)+\lambda_2^{t+1}(1-\lambda_1)}{\lambda_2-\lambda_1}+\frac{\lambda_2^t-\lambda_1^t}{\lambda_2-\lambda_1}\frac{(1-\beta)\pi^\star}{1+\sigma^{-1}\kappa\phi_\pi}, \qquad (217)
\end{aligned}$$

x_{T-t} is implicitly defined by $x_{T-t}=\frac{\pi_{T-t}-\beta E_{T-t}\pi_{T-t+1}}{\kappa}$. The simplification of the infinite sum of η_{T-t+j} relies on noting that

$$E_{T-t}\eta_{T-t+j}=(1-\beta)(\pi^\star_{T-t+j}-\pi^\star_{T-t+j+1})=\begin{cases}0, & \text{if } j\neq t-1\\ -(1-\beta)\pi^\star, & \text{if } j=t-1\end{cases} \qquad (218)$$

Therefore,

$$\begin{aligned}
&\frac{1}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_{T-t}(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\eta_{T-t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\eta_{T-t+j}) \\
&\quad=\frac{1}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}(-\lambda_1\lambda_1^{t-1}(-(1-\beta)\pi^\star)+\lambda_2\lambda_2^{t-1}(-(1-\beta)\pi^\star) \\
&\quad=\frac{\lambda_2^t-\lambda_1^t}{\lambda_2-\lambda_1}\frac{(1-\beta)\pi^\star}{1+\sigma^{-1}\kappa\phi_\pi} \qquad (219)
\end{aligned}$$

The solution for π_{T-t} when equation (216) is violated proceeds in a similar manner. Since $\lambda_{1,2}$ are unchanged r and ω are also unchanged. Then (215) can be simplified by looking at the two terms separately. The first term involving $\hat{i}_{t+j}$ is unchanged and simplifies to (133). Following the same steps as in (132),

$$\frac{1}{1+\sigma^{-1}\kappa\phi_\pi}\frac{1}{\lambda_1-\lambda_2}E_t(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\eta_{t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\eta_{t+j}) \qquad (220)$$

can be expressed as

$$\frac{1}{1+\sigma^{-1}\kappa\phi_\pi}E_t(\sum_{j=0}^{\infty}r^j\frac{\sin(\omega(j+1))}{\sin\omega}\eta_{t+j}), \tag{221}$$

which simplifies to

$$\frac{\phi_\pi-1}{1+\sigma^{-1}\kappa\phi_\pi}r^{t-1}\frac{\sin(\omega t)}{\sin\omega}\pi^\star, \tag{222}$$

because $\eta_{T-t+j}=0$ for all $j\neq t-1$. Combining the two parts,

$$\pi_{T-t}=\pi^\star\frac{\sigma^{-1}\kappa(\phi_\pi-1)}{1+\sigma^{-1}\kappa\phi_\pi}r^t\sum_{j=0}^{\infty}r^j\frac{\sin(\omega(j+1+t))}{\sin\omega} + \frac{\phi_\pi-1}{1+\sigma^{-1}\kappa\phi_\pi}r^{t-1}\frac{\sin(\omega t)}{\sin\omega}\pi^\star \tag{223}$$

Together (223) and (217) form the solutions for inflation depending on whether the roots of the polynomial equation (212) are real or complex. The broad properties of the solution are unchanged. When the roots are complex, the solution exhibits cyclical properties. Qualitatively the main difference is whether after the inflation target increases the output gap is zero or greater than zero as in the main specification. If the roots are real, the solution exhibits more monotonic properties. Unfortunately, I am unable to prove a version of proposition three for the price indexing model. However, computational analysis shows that if $\phi_\pi<1-\frac{1-\beta}{\kappa\sigma^{-1}}$, then the further in the future the announcement is made the more explosive the solutions for inflation and output are. Unlike the no indexing case, there does not exist a case for which inflation and output jump to the new inflation target steady sate values and remain at them forever. For $\frac{1}{\beta}>\phi_\pi>1-\frac{1-\beta}{\kappa\sigma^{-1}}$, a variety of dynamics are possible, but as ϕ_π increases in this range the outcome changes from both inflation and the output gap increasing past their future steady state values if the announcement is made atleast a few period in advance to the output gap falling while inflation increasing but by less than $\pi^\star$. For $\frac{1}{\beta}<\phi_\pi$ but with real roots to the polynomial equation, inflation increases at the announcement and monotonically increases as the inflation target change period approaches. For announcements made sufficiently in advance, the output gap at the announcement declines and continues to decline for some amount of periods, but as the period when the inflation target increases approaches the output gap will eventually begin to monotonically grow and approach zero.

A.6 Neither Regime is Absorbing

In this section, I briefly go though the analytical solution when the assumption that regime two is absorbing is relaxed. The key difference is that regime two outcomes are no longer independent of regime one policy and have to be solved jointly. For simplicity I assume interest rates respond only to inflation, but allow different coefficients in the two

regimes. That is let the policy response function (9) be

$$i_{j,t} = \phi_{\pi,j}\pi_{j,t} + (\phi_{\pi,j} - 1)\pi_j^\star \equiv \phi_{\pi,j}\pi_{j,t} + \bar{i}_1 \tag{224}$$

and the Markov process

$$\Pi = \begin{bmatrix} 1-\lambda & \lambda \\ \gamma & 1-\gamma \end{bmatrix}.$$

As with an absorbing second regime, both regimes are still in a steady state. The two regime specific Phillips Curves after rearranging terms and using steady state notation from section 8.2 are:

$$\pi_1(1 - \beta(1-\lambda)) = \beta\lambda\pi_2 + \kappa x_1 \tag{225}$$

$$\pi_2(1 - \beta(1-\gamma)) = \beta\gamma\pi_1 + \kappa x_2 \tag{226}$$

And the aggregate supply relationships are:

$$\lambda x_1 = \lambda x_2 - \sigma^{-1}((\phi_{\pi,1} + \lambda - 1)\pi_1 - \lambda\pi_2 + \bar{i}_1) \tag{227}$$

$$\gamma x_2 = \gamma x_1 - \sigma^{-1}((\phi_{\pi,2} + \gamma - 1)\pi_2 - \gamma\pi_1 + \bar{i}_2) \tag{228}$$

Next, combine these four equations into three equations in terms of $x_1 - x_2$, π_1, and π_2 by rearranging (227) and (228) and combining (225) with (226);

$$x_1 - x_2 = \frac{-\sigma^{-1}}{\lambda}((\phi_{\pi,1} + \lambda - 1)\pi_1 - \lambda\pi_2 + \bar{i}_1) \tag{229}$$

$$x_1 - x_2 = \frac{\sigma^{-1}}{\gamma}((\phi_{\pi,2} + \gamma - 1)\pi_2 - \gamma\pi_1 + \bar{i}_2) \tag{230}$$

$$x_1 - x_2 = \frac{1}{\kappa}(\pi_1(1 - \beta(1-\lambda-\gamma)) - \pi_2(1 - \beta(1-\lambda-\gamma))) \tag{231}$$

Combining (230) and (231),

$$\frac{\sigma^{-1}}{\gamma}((\phi_{\pi,2} + \gamma - 1)\pi_2 - \gamma\pi_1 + \bar{i}_2) = \frac{1}{\kappa}(\pi_1(1 - \beta(1-\lambda-\gamma)) - \pi_2(1 - \beta(1-\lambda-\gamma)))$$

$$\Rightarrow \pi_2 = \frac{\pi_1\gamma(1 - \beta(1-\gamma-\lambda) + \sigma^{-1}\kappa) - \sigma^{-1}\kappa\bar{i}_2}{\gamma(1 - \beta(1-\gamma-\lambda)) + \sigma^{-1}\kappa(\phi_{\pi,2} + \gamma - 1)} \tag{232}$$

or

$$\pi_1 = \frac{\pi_2\gamma(1 - \beta(1-\gamma-\lambda) + \sigma^{-1}\kappa(\phi_{\pi,2} + \gamma - 1)) + \sigma^{-1}\kappa\bar{i}_2}{\gamma(1 - \beta(1-\gamma-\lambda) + \sigma^{-1}\kappa)} \tag{233}$$

Combining (229) and (231),

$$\frac{-\sigma^{-1}}{\lambda}((\phi_{\pi,1}+\lambda-1)\pi_1-\lambda\pi_2+\bar{i}_1)=\frac{1}{\kappa}(\pi_1(1-\beta(1-\lambda-\gamma))-\pi_2(1-\beta(1-\lambda-\gamma)))$$
$$\Rightarrow \pi_1=\frac{\pi_2\lambda(1-\beta(1-\gamma-\lambda)+\sigma^{-1}\kappa)-\sigma^{-1}\kappa\bar{i}_1}{\lambda(1-\beta(1-\gamma-\lambda))+\sigma^{-1}\kappa(\phi_{\pi,1}+\lambda-1)} \tag{234}$$

or

$$\pi_2=\frac{\pi_1\lambda(1-\beta(1-\gamma-\lambda)+\sigma^{-1}\kappa(\phi_{\pi,1}+\lambda-1))+\sigma^{-1}\kappa\bar{i}_1}{\lambda(1-\beta(1-\gamma-\lambda)+\sigma^{-1}\kappa)} \tag{235}$$

Combining (229) and (230),

$$\frac{-\sigma^{-1}}{\lambda}((\phi_{\pi,1}+\lambda-1)\pi_1-\lambda\pi_2+\bar{i}_1)=\frac{\sigma^{-1}}{\gamma}((\phi_{\pi,2}+\gamma-1)\pi_2-\gamma\pi_1+\bar{i}_2)$$
$$\Rightarrow \pi_1=-\frac{\lambda(\phi_{\pi,2}-1)\pi_2+\lambda\bar{i}_2+\gamma\bar{i}_1}{\gamma(\phi_{\pi,1}-1)} \tag{236}$$

or

$$\pi_2=-\frac{\gamma(\phi_{\pi,1}-1)\pi_1+\lambda\bar{i}_2+\gamma\bar{i}_1}{\lambda(\phi_{\pi,2}-1)} \tag{237}$$

Setting (237) equal to (232)

$$-\frac{\gamma(\phi_{\pi,1}-1)\pi_1+\lambda\bar{i}_2+\gamma\bar{i}_1}{\lambda(\phi_{\pi,2}-1)}=\frac{\pi_1\gamma(1-\beta(1-\gamma-\lambda)+\sigma^{-1}\kappa)-\sigma^{-1}\kappa\bar{i}_2}{\gamma(1-\beta(1-\gamma-\lambda))+\sigma^{-1}\kappa(\phi_{\pi,2}+\gamma-1)}, \tag{238}$$

which simplifies to

$$\pi_1=-\frac{\lambda(z+\sigma^{-1}\kappa)\bar{i}_2+(\gamma z+\sigma^{-1}\kappa(\phi_{\pi,2}+\gamma-1))\bar{i}_1}{\lambda(z+\sigma^{-1}\kappa)(\phi_{\pi,2}-1)+\gamma z+\sigma^{-1}\kappa(\phi_{\pi,2}+\gamma-1)(\phi_{\pi,1}-1)}, \tag{239}$$

where $z=1-\beta(1-\lambda-\gamma)$. Similarly, setting (236) equal to (234) and simplifying results in

$$\pi_2=-\frac{\gamma(z+\sigma^{-1}\kappa)\bar{i}_1+(\lambda z+\sigma^{-1}\kappa(\phi_{\pi,1}+\lambda-1))\bar{i}_2}{\gamma(z+\sigma^{-1}\kappa)(\phi_{\pi,1}-1)+\gamma z+\sigma^{-1}\kappa(\phi_{\pi,1}+\lambda-1)(\phi_{\pi,2}-1)} \tag{240}$$

The regime specific output gaps are implicitly defined by (225) and (226) and can be shown to be of the form $x_j=f_{j,1}(\phi_{\pi,1}\phi_{\pi,2})\bar{i}_1+f_{j,2}(\phi_{\pi,1}\phi_{\pi,2})\bar{i}_2$, where $f_{j,k}$ are functions of the policy parameters.

From (239) and (240), we can observe that if policy is active in both regimes then inflation in each regime is the weighted average of the inflation targets in the two regimes and is increasing in each inflation target. Therefore, to reduce inflation generated from an expectation of a future shift to a higher inflation regime the central bank needs to reduce the current inflation target just as when regime two is absorbing. However, reducing $\pi_1^\star$ will now also reduce inflation in regime two thereby changing how much the inflation target in the current regime needs to be adjusted. By setting $\pi_1=0$ in (239), I can solve

for the regime one inflation target that results in zero inflation in regime one. Doing so,

$$\pi_1^\star = \frac{-\lambda(z+\sigma^{-1}\kappa)}{\lambda(z+\sigma^{-1}\kappa)+\sigma^{-1}\kappa(\phi_{\pi,2}-1+\gamma)}\frac{1-\phi_{\pi,2}}{1-\phi_{\pi,1}}\pi_2^\star \tag{241}$$

As in the main specification with active policy (and in the absence of responding to expected inflation), if policy is more responsive to inflation in the current regime then inflation doesn't change by as much and the inflation target does not need to be lowered by as much. However, the responsiveness of policy in the other regime determines by how much changing $\pi_1^\star$ effects inflation in the other regime, which in turn effects how expected inflation changes and how much $\pi_1^\star$ needs to be changed by. I do not solve for the optimal policy in this setting as conceptually it is not clear what real world scenario such an optimization captures and is not likely to provide useful expressions.