Demand Disagreement*

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Abstract

We develop an overlapping generations model with disagreement about the cross-sectional distribution of investors' preferences and beliefs. This disagreement implies different beliefs about future asset demand even if economic fundamentals are known and the resulting speculative trade leads to priced demand shocks. Demand disagreement also leads to low correlation between asset returns and economic fundamentals, excess stock market volatility, low means and volatilities of interest rates, valuation ratios predicting returns, Black's leverage effect, and high trading volume unrelated to economic fundamentals, even in a setting with i.i.d. consumption growth and without hedging demands, recursive preferences, habit formation, or disaster risk.

Keywords: Demand shocks, demand disagreement, heterogeneous beliefs and time preferences, asset pricing puzzles, trading volume.

JEL Classification: D51, G10, G11, G12

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I Introduction

When deciding on an investment in an asset you face the trade-off between risk and return, which requires a forecast of the asset's future cash flows and prices. Clearly, an investor might fear that the general macroeconomic conditions are weak when selling the asset in the future. However, the investor might also worry about "who is in the market" when selling the asset. Is the demand for the asset going to be strong or weak? This demand uncertainty might be caused by the lack of knowledge of investors' future time or risk preferences, factors that are unrelated to future macroeconomic conditions and that are difficult to predict.

Neoclassical asset pricing theories take investors' preferences as given and assume that they are not subject to change and so future asset demand is known given economic fundamentals. These theories imply a tight link between economic fundamentals and asset prices in equilibrium because all the variation in discount rates is due to shocks to economic fundamentals. While the workhorse asset pricing models solve many asset pricing puzzles which arise from the apparent disconnect between returns on financial assets and economic fundamentals by magnifying discount rate shocks, they still imply a high correlation between economic fundamentals and asset prices. This high correlation is inconsistent with the data and Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2005) refer to this feature of the data as the "correlation puzzle."

In this paper, we stick to the neoclassical assumption that investors' preferences are not subject to change but instead allow for changes in the cross-sectional distribution of investors with different preferences and beliefs. Moreover, investors have incomplete information about these changes and they also disagree on them which implies disagreement about the future aggregate asset demand even if economic fundamentals are known. This "demand disagreement" leads to speculative trade that is not based on macroeconomic fundamentals, yet is very important for asset prices. Specifically, we show that demand disagreement can account for several stylized features of asset market such as the low correlation between asset returns and economic fundamentals, a time varying equity premium, excess stock market volatility, a low mean and volatility of the risk-free rate, valuation ratios predicting returns, Black's leverage effect, and high trading volume unrelated to economic fundamentals, even in a setting with i.i.d. consumption growth and without hedging demands, preference for early/late resolution of uncertainty, habit formation, or risk of rare disasters.

We formalize the intuition for the effects of demand disagreement on asset prices in a simple heterogenous agent model with transparent economic forces. Specifically, we consider an overlapping generations economy (Blanchard (1985) and Gârleanu and Panageas (2015)) with two types of investors — patient and impatient. The fraction of patient and impatient investors entering and leaving the economy changes stochastically over time. We refer to shocks to this fraction as demand shocks because they lead to exogenous variation in aggregate asset demand. Hence, there is uncertainty about future asset demand that is unrelated to economic fundamentals without assuming a preference for early resolution of uncertainty about an investor's own future preferences as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016). Instead, demand uncertainty is due to stochastic variation in the population density of patient and impatient investors. Moreover, all investors have log-utility, receive the same endowment stream, and face complete markets. Although these assumptions can be relaxed—higher risk aversion would further increase the equity premium—they allow for closed form solutions and, more importantly, they isolate the effects of demand disagreement on asset prices from the effects of future variations in (i) hedging demands, (ii) preferences for early/late resolution of uncertainty, (iii) endowments of newborn investors, and (iv) market incompleteness.

Investors observe the risk-free rate and the wealth-consumption ratio and, thus, know the current fraction of newborn patient (α_t) and impatient ($1 - \alpha_t$) investors. However, they neither know the future realization of α_t nor its distribution, that is, there is incomplete information about the dynamics of α_t and therefore aggregate asset demand. To fix ideas, suppose investors believe that their own type is more prevalent than the other type in the long run, a bias that is well documented in the psychology literature (see Ross, Green, and House (1977)) where it is referred to as the false consensus bias. This assumption leads to constant disagreement about the long-run mean of α_t and to portfolios of patient (impatient) investors that increase (decrease) in value when there is a positive shock to α_t, which we refer to as a demand shock. Changing the sign of this bias and/or allowing for learning does not change the qualitative implications of our demand disagreement model. Moreover, disagreement about α_t also implies disagreement about the future belief of investors and so disagreement about asset demand is not only caused by disagreement about investors' future time preferences.

Investors differ with respect to their time preferences, their beliefs, and their age in our demand disagreement model which leads to trade and wealth shifts across investors. We are nevertheless able to fully describe the equilibrium with two stationary state variables: (i) the exogenous fraction of newborn patient investors α_t and (ii) the endogenous share of aggregate consumption that is consumed by all patient investors, in short, the consumption share f_t. While α_t is by assumption stationary, the consumption share f_t is stationary because investors do not live forever and, thus, neither patient investors nor investors with the correct belief dominate in the long run. Hence, our OLG model with finite life expectancy of investors allows us to compute unconditional moments of all equilibrium quantities which is often not possible in classical infinite horizon heterogeneous agent models.[1] Our demand disagreement model also completely overturns the implications of an infinite horizon model that impatient investors do not impact prices because they save less than patient investors which are therefore "naturally selected by the market" (Alchian (1950) and Friedman (1953)). Surprisingly, demand disagreement decreases the average consumption share of patient investors to levels even below 50% and, thus, impatient investors have a larger impact on prices than patient investors when there is a lot of disagreement. Moreover, demand disagreement and the resulting speculative trade lead to large wealth swings among speculators and so increase the consumption share volatility as well as the probability of very low or high consumption share realizations, which, as shown below, has important implications for asset pricing.

How does demand disagreement affect the risk-free interest rate and stock market valuations? Demand disagreement leads to shocks to the consumption/wealth share and thus time-variation in both the risk-free rate and stock market valuation because the resulting price impact of patient and impatient investors varies with these shocks. For instance, a negative demand shock leads to more price impact of impatient investors and hence to a higher effective discount rate which increases the risk-free rate to prevent excess-demand for current consumption and decreases the wealth-consumption ratio to prevent excess supply of the stock market portfolio. Moreover, the mean and the volatility of the risk-free rate increase with disagreement but they are nevertheless

[1] Yan (2008) shows that heterogenous beliefs and time preferences lead to degenerate consumption share distribution. Notable exceptions with stationary consumption shares in heterogenous agents models are Chan and Kogan (2002), Osambela (2015), Baker, Hollifield, and Osambela (2016), and Borovička (2019).

reasonably low and so we avoid the risk-free rate puzzle. In particular demand disagreement which is similar in magnitude to disagreement about GDP growth of professional forecasters leads to interest rate volatility that is 1.9% and which increases to 2.4% if we double disagreement. The impact of demand disagreement on the risk-free rate and the wealth-consumption ratio are distinctly different from the effects of uncertainty. While demand disagreement raises the average interest rate because it increases the price impact of impatient investors, uncertainty lowers the risk-free rate due to an increases in precautionary savings. Similarly, demand disagreement lowers the wealth-consumption ratio which is in stark contrast to discount rate uncertainty that increases it (Pastor and Veronesi (2009)). Finally, disagreement and the resulting speculative trade leads to large wealth shifts between patient and impatient investors which implies high and low valuation ratios without any changes in economic fundamentals.

How does demand disagreement affect the risk premium and volatility of the market portfolio? Demand disagreement leads to speculative trade among patient and impatient investors and thus to stock market volatility in excess of cash flow volatility. There is more speculative trade when there is more disagreement about future asset demand and so the stock market's exposure to demand shocks, and therefore stock market volatility, increases with disagreement. While the quantity of demand shock risk is always positive, the price of this risk depends on the belief about the cross-sectional population of investors. Specifically, patient investors are optimistic about the effects of demand shocks on the stock market since they overestimate the long run mean of patient investors whereas impatient investors are pessimistic about these effects. Hence, patient investors perceive the market price of demand shock risk to be negative whereas impatient investor perceive it to be positive; thus they trade with each other. Assuming the truth lies somewhere in between an outside observer or econometrician endowed with the consensus belief will estimate a positive risk premium for demand shocks if impatient investors have more price impact (i.e. impatient investors have a higher consumption share) and a negative risk premium if patient investors have more price impact (i.e. patient investors have a higher consumption share). While the conditional risk premium can be negative or positive depending on this price impact, we nevertheless find that the unconditional equity premium is typically positive and increases in disagreement. There are two reasons for this. First, higher disagreement lowers the consumption share of patient investors, and thus increases the unconditional price of risk for demand shocks. Second, the stock market exposure to demand shocks is more likely to decrease with the consumption share, and therefore stock market volatility typically decreases with the price of demand shocks. Hence, times when the price of demand shocks is high are also times when the stock market is more exposed to those shocks, thus leading on average to a positive equity premium.

Uncertainty about the cross-sectional distribution of future time-preferences leads to time-variation in aggregate asset demand and, thus, time-variation in both the risk-free rate and the wealth consumption ratio. However, in the absence of disagreement, demand shocks are neither priced nor do they directly impact consumption shares which implies a constant equity premium and stock market volatility. Speculative trade due to demand disagreement leads to time-variation in consumption/wealth shares and hence not surprisingly to a stochastic stock market risk premium and volatility. What is striking in this case, is that both variations in equity premiums and volatilities are consistently moving in the right direction with disagreement. Specifically, a low price-dividend ratio predicts high expected excess returns and the return on the stock market and its volatility are negatively correlated ("Black's leverage effect"). The economic mechanism is speculation due to demand disagreement and is, thus, different from the ones suggested in the literature such as countercyclical risk aversion, economic uncertainty, time-varying crash risk probabilities, time-varying investment opportunity sets, and/or market inefficiencies. Moreover, the variation in risk premiums and volatilities is accompanied with excess trading volume as patient and impatient agents optimally change their investment portfolios in response to demand shocks. Importantly, all time-variation in the risk premium, volatility, and trading volume is due to demand shocks and thus unrelated to shocks to cash flows. Hence, our demand disagreement model can reconcile the correlation puzzle.

Our demand disagreement model is consistent with several stylized features of asset markets. Yet, it is fairly simple and transparent allowing us to focus on the economic mechanism. We also consider two extensions to endogenize disagreement and aggregate consumption. First, we allow investors to learn from their own experience which is motivated by a large empirical literature and leads to disagreement even without any bias. While patient and impatient investors of the same birth cohort never disagree when there is no bias, there is disagreement across cohorts and thus the consumption shares of all patient and impatient cohorts matter for asset pricing. We are nevertheless able to characterize the equilibrium in a parsimonious way and show that pricing in equilibrium is as if there is time-varying disagreement between patient and impatient investors because patient investors save more and therefore they are typically wealthier when they are old and more accurate, creating a difference between the price impact of patient and impatient agents in equilibrium. Second, we introduce capital investment with adjustment cost and show that aggregate consumption loads negatively onto demand shocks in equilibrium which is in stark contrast to the stock market which has the opposite exposure. Importantly, the contribution of the demand risk compensation to the equity premium is still positive. This finding highlights the disconnect between the stock market and economic fundamentals.

Our paper relates to several strands of literature. First, it relates to the literature on heterogeneous beliefs such as the early work of Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), and Basak (2000); however, there are two important differences.[2] First, with the exception of Basak (2000) who considers disagreement about extraneous risk, the above papers consider disagreement about macroeconomic quantities such as consumption, inflation, and dividends. In contrast, our paper considers disagreement about the future prevalence of patient and impatient investors. Hence, it focuses on disagreement about the demand rather than the supply side of the economy and it addresses the correlation puzzle while capturing many important asset pricing stylized facts. Second, the disagreement literature has mostly focused on economies with infinitely lived agents.[3] These models are simpler to solve, which often comes at the cost of obtaining degenerate or non stationary consumption/wealth share distributions. There is a stationary consumption/wealth share distribution in our model which allows us to study unconditional asset pricing moments and it implies that impatient agents can have more price impact than patient agents which is in stark contrast to an infinite horizon economy where only the most patient agent

[2]Models with disagreement include, among many others, Scheinkman and Xiong (2003), Basak (2005), Berrada (2006), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic, Jouini, Malamud, and Napp (2012), Bhamra and Uppal (2014), Buraschi, Trojani, and Vedolin (2014), Cujean and Hasler (2017), and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018).

[3]Collin-Dufresne, Johannes, and Lochstoer (2017) and Ehling, Graniero, and Heyerdahl-Larsen (2018) consider overlapping generations models with heterogeneous beliefs.

impacts prices in the long run.

Second, our paper relates to the literature on heterogeneous preferences such as Dumas (1989), Wang (1996), Chan and Kogan (2002), Gollier and Zeckhauser (2005), Gomes and Michaelides (2008), Weinbaum (2009), Zapatero and Xiouros (2010), Cvitanic, Jouini, Malamud, and Napp (2012), Longstaff and Wang (2013), Bhamra and Uppal (2014), Gârleanu and Panageas (2015), and Gollier and Zeckhauser (2005). However, in contrast to these papers, we study the implications of demand disagreement for asset prices.

Third, our paper relates to the literature that focuses on the implications of preference shocks for asset pricing. Garber and King (1983) and Campbell (1993) are early examples.[4] Maurer (2012) and Albuquerque, Eichenbaum, Luo, and Rebelo (2016) use shocks to the time discount factor to rationalize the correlation puzzle. In their work, early resolution of uncertainty about an investor's own future preferences leads to priced demand shocks. In contrast, we show that demand shocks are priced even without recursive preferences or hedging demands when investors have different time discount rates and beliefs.

Our paper also relates to the literature on demand discovery, which is based on the early work of Grossman (1988) and studies how private information about future demand is transmitted through asset prices and trade. Recent work by Gallmeyer, Hollifield, and Seppi (2017) shows that prices reflect information about investors preferences and thus future asset demands. In contrast to this literature, in our model investors agree to disagree about future demand and there is nothing to learn from prices or trade which leads to more tractability and allows us to also study the quantitative implications of demand disagreement for many asset pricing stylized facts.

Our paper contributes to the literature that studies the asset pricing implication of OLG models. For instance, Constantinides, Donaldson, and Mehra (2002) show that mean and volatility of the equity premium increase when consumers are borrowing constraint. Gomes and Michaelides (2005) show that a life-cycle model with uninsurable labor income risk can simultaneously match stock market participation rates and asset allocation decisions. Gârleanu, Kogan, and Stavros (2012) show that due to the lack of inter-generational risk sharing, innovation creates a systematic risk

[4]Preference shocks have been frequently used in macroeconomics, international economics, and international finance (e.g. Pavlova and Rigobon (2007))

factor that explains the value premium and the high equity premium. Kogan, Papanikolaou, and Stoffman (2019) show that a high equity premium, return comovement, and cross-sectional differences in expected returns arise in a model with innovators and investors. Ehling, Graniero, and Heyerdahl-Larsen (2018) study asset prices and portfolio choice with agents that learn from experience to rationalize the negative correlation between the consensus belief about the risk premium and future excess returns. Most closely related to our paper is Gârleanu and Panageas (2019) who show that imperfect risk sharing across cohorts can explain many features of asset prices in a model without aggregate output risk. Our model also does not rely on aggregate output risk and contributes to the literature on OLG models by showing how disagreement about the cross-sectional distribution of future preferences and beliefs affects asset prices.[5]

II Model

In this section, we present and solve an equilibrium overlapping generations model with a continuum of investors who have different time preferences and beliefs.

A Overlapping Generations Model

We consider a continuous-time overlapping generations economy in the tradition of Blanchard (1985) and Gârleanu and Panageas (2015). Every agent lives until the stochastic time of death τ which is exponentially distributed with hazard rate, $\nu > 0$.[6] A new cohort of mass ν is born every period and therefore the total population size remains constant, that is,

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)}\, ds = 1, \tag{1}$$

where $\nu e^{-\nu(t-s)}$ denotes the population density. We set the birth/mortality rate to $\nu = 2\%$ in our baseline setting which implies a life expectancy of 50 years.[7]

[5] For a recent comprehensive review of the implications of heterogeneity and inequality for asset pricing see Panageas (2019).

[6] If $\nu = 0$ agents live forever and we are back to the classical infinite horizon economy.

[7] Agents start to trade immediately after they are born as in Gârleanu and Panageas (2015).

B Supply Side — the Endowment

Each agent born is entitled to an endowment stream until she dies.[8] The endowment of each agent at time t is equal to Y_t and, thus, aggregate output is

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_t \, ds = Y_t. \tag{2}$$

Suppose aggregate output growth is iid and, thus, Y_t follows the geometric Brownian motion

$$dY_t = \mu_Y Y_t \, dt + \sigma_Y Y_t \, dZ_{Y,t}, \tag{3}$$

where $Z_{Y,t}$ is a Brownian motion that represents the output or supply shock and μ_Y and σ_Y are the commonly observed expected output growth rate and output growth volatility, respectively. There is no disagreement about the supply shock and the parameters in our baseline setting are set to $\mu_Y = 2\%$ and $\sigma_Y = 3.3\%$, respectively.[9]

C Demand Side — Heterogenous Preferences and Beliefs

There are two types of agents with common risk-preferences but different time discount rates and beliefs. Patient type a agents have the low time discount rate ρ_a and belief $\mathbb{P}^a$ and impatient type b agents have the high time discount rate ρ_b and belief $\mathbb{P}^b$. The fraction of patient and impatient newborns are α_t and $1 - \alpha_t$, respectively. We set the time preference parameters of the patient investor to $\rho_a = 0$ and of the impatient investor to $\rho_b = 5\%$ in our baseline setting and define demand disagreement below.[10]

Patient and impatient investors observe the fraction α_t but they do not know its distribution. Moreover, they disagree on this distribution and, thus, form different beliefs about the prevalence of their type in the future, that is, they disagree on how many people share their preferences and beliefs

[8] We focus on the effects of demand disagreement on asset prices and thus we do not introduce heterogeneity between the endowment of existing agents and newborn agent as in Gârleanu and Panageas (2015).

[9] This is similar to the long sample in Campbell and Cochrane (1999).

[10] The effective time discount rates are $\nu + \rho_a = 2\%$ and $\nu + \rho_b = 7\%$ which are in line with the time discount rates in Bansal and Yaron (2004), Chan and Kogan (2002), and Campbell and Cochrane (1999) of 2.4%, 5.2%, and 11.6%, respectively.

in the future. We chose a parsimonious way to describe this disagreement and avoid learning and time-variation in disagreement in the baseline setting.[11] Specifically, let $\alpha_t = 1/(1+e^{-l_t}) \in (0,1)$, where the factor l_t follows a mean reverting Ornstein-Uhlenbeck process with investor specific long run mean $\bar{l}^a$ and $\bar{l}^b$ but common persistence κ and local volatility σ_l.[12] Specifically,

$$dl_t = \kappa \left(\bar{l}^i - l_t\right) dt + \sigma_l \, dZ^i_{\alpha,t}, \qquad i \in \{a, b\}, \tag{4}$$

where $Z^i_{\alpha,t}$ is a Brownian motion under the belief of agent i denoted by $\mathbb{P}^i$. An increase in l_t increases the fraction of patient investors and, as shown below, decreases the demand for current consumption. Hence, we refer to the Brownian motions $Z^a_{\alpha,t}$ and $Z^b_{\alpha,t}$ as the perceived demand shocks of patient and impatient investors, respectively. The demand shocks $Z^a_{\alpha,t}$ and $Z^b_{\alpha,t}$ are independent of the supply shock, $Z_{Y,t}$ and w.l.o.g. $\bar{l}^b \leq \bar{l}^a$. Hence, investors suffer from a false consensus bias because they overestimate the prevalence of their own type.[13] We set $\kappa = 0.01$ and $\sigma_l = 0.1$ in our baseline setting, which leads to a persistent α_t with an unconditional variance of 16%.[14]

How are the demand shocks perceived by patient and impatient investor linked to each other? Suppose $l_t = \bar{l}^b$ and $dl_t = 0$, which both types observe. In this case, impatient investors perceive no shock to demand whereas patient investors perceive it to be negative due to their positive drift assessment. More generally, investors observe the change dl_t and hence it follows from equation (4) that the demand shocks perceived by patient and impatient investors are linked by the constant disagreement parameter Δ. Specifically,

$$Z^b_{\alpha,t} - Z^a_{\alpha,t} = \Delta t, \qquad \text{where} \qquad \Delta \equiv \frac{\kappa}{\sigma_l}\left(\bar{l}^a - \bar{l}^b\right) \geq 0. \tag{5}$$

Moreover, it follows from Girsanov's theorem that the likelihood ratio, which allows to change

[11] We discuss a model with endogenous time-variation in disagreement due to learning from experience in Section IV.

[12] Investors observe the quadratic variation of the process l_t and thus they can directly infer the volatility parameter σ_l.

[13] We show in the Internet Appendix that our main results do not depend on this assumption.

[14] We set the long run mean of l_t under the dating generating belief to zero which we discuss in Section III.

belief between the two agents, is

$$\eta_t \equiv \frac{d\mathbb{P}^b}{d\mathbb{P}^a} |_{\mathcal{F}_t} = e^{-\frac{1}{2}\Delta^2 t - \Delta\, Z^a_{\alpha,t}}. \tag{6}$$

We consider three different cases for disagreement to study the asset pricing implications in the next section; no disagreement ($\Delta = 0$), medium disagreement ($\Delta = 0.4$), and high disagreement ($\Delta = 0.8$). For comparision, we link our model to a model where investors disagree about output growth. Specifically, a disagreement, Δ, of 0.4 and 0.8 in our demand disagreement model would correspond to disagreement about expected output growth of 1.32% and 2.64%. The first numbers is comparable to the time-series average over the interquartile range of one-year ahead GDP growth forecasts of 1.35% which is based on the Survey of Professional Forecasters (SPF).[15]

D Security Markets

As soon as investors are born they trade in a dynamically complete security market to share supply shock, demand shock, and mortality risk. Hence, we consider four different securities: (i) an instantaneously risk-free asset, (ii) an infinitely lived risky asset that is locally perfectly correlated with the supply shock and, thus, referred to as the supply asset, (iii) an infinitely lived risky asset that is locally perfectly correlated with the demand shock and, thus, referred to as the demand asset, and (iv) a life insurance/annuity contract.[16] The risk-free asset and the two risky assets are in zero-net-supply and the life insurance/annuity contract is offered by a competitive insurance industry at an actuarily fair rate. The two risky assets are tractable and simplify the intuition for the asset pricing implications of demand disagreement but every other two non-redundant risky assets would also dynamically complete this market.

Price process are posited below and verified to be an equilibrium in Theorem 1. The dynamics for the real risk free asset with price B_t are $dB_t = r_t B_t\, dt$, where r_t denotes the real short rate determined in equilibrium. The dynamics of the supply asset are $dP^Y_t = \mu^Y_{P,t} P^Y_t\, dt + \sigma_Y P^Y_t\, dZ_{Y,t}$

[15] When we convert disagreement Δ into disagreement about GDP growth rates, we multiply by output growth volatility, $\sigma_Y = 3.3\%$.

[16] The risky assets can be interpreted as continuously resettled contracts (e.g., futures contracts). The same asset structure is used, for example, in Karatzas, Lehoczky, and Shreve (1994) and Basak (2000).

where the expected rate of return $\mu^Y_{P,t}$ is determined in equilibrium and the volatility is w.l.o.g. set to output volatility since this asset is in zero-net supply. Of course, the volatility of the stock market which is a claim on future dividends is determined endogenously in equilibrium. The dynamics of the demand asset are

$$dP^\alpha_t = \mu^{i,\alpha}_{P,t} P^\alpha_t \, dt + P^\alpha_t dZ^i_{\alpha,t}, \quad i \in \{a, b\}, \tag{7}$$

where the expected rates of return $\mu^{a,\alpha}_{P,t}$, and $\mu^{b,\alpha}_{P,t}$ are determined in equilibrium.

Investors agree on the supply shock and, thus, agree on the expected rate of return of the supply asset. Investors also agree on the price P^α_t and, thus, its return. However, they perceive demand shocks differently and so they disagree on the expected rate of return of the demand asset. Hence, expected returns are linked through the disagreement parameter Δ. Specifically,

$$\left(\mu^{a,\alpha}_{P,t} - \mu^{b,\alpha}_{P,t}\right) dt = dZ^b_{\alpha,t} - dZ^a_{\alpha,t} = \Delta \, dt. \tag{8}$$

There is a life insurance/annuity contract as in Blanchard (1985) and Gârleanu and Panageas (2015) that is offered by a competitive insurance industry which pays the actuarily fair rate ν per unit of wealth in case of an annuity and charges it in case of life insurance. Specifically, investors with positive financial wealth can pledge it to the insurance company in return for an additional income stream proportional to their financial wealth. Similarly, investors with negative financial wealth can get life insurance in return for a stream of life insurance payments proportional to their financial wealth. Hence, the cash flows from this life insurance/annuity contract from the perspective of the investors are

$$d\mathcal{L}_t = \nu W^{\mathcal{L}}_t \, dt, \qquad \mathcal{L}_\tau = -W^{\mathcal{L}}_\tau, \qquad \forall t \leq \tau, \tag{9}$$

where $\mathcal{L}_t$ denotes the value of the life insurance contract at time t and $W^{\mathcal{L}}_t$ the amount of wealth at time t that is invested in the life insurance/annuity contract. It is optimal for an investor with positive financial wealth to annuitize all her wealth because she does not have any bequest motive and, thus, do not get any utility from dying with positive financial wealth (Yaari (1965)).

Moreover, there is no default in the model and, thus, investors have to buy life insurance for their negative financial wealth because they are no longer entitled to an income stream after death.

E Consumption-Portfolio Choice Problem

The lifetime expected utility of an agent of type i born at time s is

$$E_s^i \left[\int_s^\tau e^{-\rho_i(t-s)} \log\left(c_{s,t}^i\right) dt \right], \tag{10}$$

where $c_{s,t}^i$ denotes the time t consumption rate and $E_s^i[\cdot]$ the expectation w.r.t. to the belief of agent i born at time s, respectively. Agents maximize their lifetime utility given in equation (10) subject to their dynamic budget constraint given in equation (11) below. Specifically, let $\Phi_{s,t}^i$ and $\Psi_{s,t}^i$ denote the dollar amounts held in the risky supply and demand asset, respectively. Then the dynamics of financial wealth, $W_{s,t}$, of an agent born at time s are

$$\begin{aligned} dW_{s,t}^i &= \left(r_t W_{s,t}^i + \Phi_{s,t}^i \left(\mu_{P,t}^Y - r_t\right) + \Psi_{s,t}^i \left(\hat{\mu}_{P,t}^{i,\alpha} - r_t\right) + d\mathcal{L}_{s,t}^i + Y_t - c_{s,t}^i\right) dt \\ &\quad + \Phi_{s,t}^i \sigma_Y \, dZ_{Y,t} + \Psi_{s,t}^i \, dZ_{\alpha,t}^i, \qquad W_{s,s}^i = 0, \qquad W_{s,\tau}^i = W_{s,\tau^-}^i + \mathcal{L}_{s,\tau}^i = 0. \end{aligned} \tag{11}$$

Agents are born without any financial wealth and, thus, $W_{s,s}^i = 0$. Moreover, as long as agents are alive they consume at the rate $c_{s,t}^i$, receive the endowment Y_t, and make/receive life insurance/annuity payments $d\mathcal{L}_{s,t}^i = \nu W_{s,t}^i dt$. Agents' investments in the three financial securities may change their financial wealth and, thus, the financial wealth at death right before the life insurance contract is settled, denoted by W_{s,τ^-}^i, may be negative or positive. If W_{s,τ^-}^i is positive, then it is paid out to the insurance company and if it is negative, then this obligation is paid off by the insurance company. Hence, financial wealth—and, thus, total wealth—is always zero after the life insurance/annuity contract is settled, that is, $W_{s,\tau}^i = W_{s,\tau^-}^i + \mathcal{L}_{s,\tau}^i = 0$.

F Arrow-Debreu Price System

Before computing the equilibrium, it is convenient to summarize the price system in terms of investor-specific state price density, ξ_t^i, that capture the investor-specific beliefs, but common

Arrow-Debreu prices across investors. For instance, the price of an asset that pays $\tilde{x}$ units of the time t consumption good at time T is $\mathrm{E}^a\left[\frac{\xi_T^a}{\xi_t^a}\tilde{x}\right] = \mathrm{E}^b\left[\frac{\xi_T^b}{\xi_t^b}\tilde{x}\right]$. The dynamics of investor i's state price densities are

$$d\xi_t^i = -r_t\xi_t^i\,dt - \theta_{Y,t}\xi_t^i\,dZ_{Y,t} - \theta_{\alpha,t}^i\xi_t^i\,dZ_{\alpha,t}^i, \qquad i \in \{a,b\}, \tag{12}$$

where $\theta_{Y,t}$ denotes the market price of risk of the supply shock $Z_{Y,t}$ and $\theta_{\alpha,t}^i$ denotes the market price of risk of the demand shock $Z_{\alpha,t}^i$ perceived by investor i.[17] The individual state price densities are linked by the likelihood ratio and, thus, the perceived market prices of demand risk are linked through the disagreement parameter Δ, that is, $\xi_t^a = \eta_t\xi_t^b$ and $\theta_{\alpha,t}^a - \theta_{\alpha,t}^b = \Delta$.

G Defining the Equilibrium

We formally define and then compute in Subsection J the equilibrium of this economy. In a nutshell, agents maximize their lifetime expected utility subject to their dynamic budget constraint and consumption, financial securities, and insurance markets clear. The investor specific state price densities are pinned down by the real short rate, the market price of supply shocks, and the investor specific market prices of demand shocks and they price all assets in this economy.

Definition 1. *Given preferences, endowments, and beliefs, an equilibrium is a collection of allocations* $\left(c_{s,t}^i, \Phi_{s,t}^i, \Psi_{s,t}^i\right)$ *and a price system* $\left(r_t, \mu_{P,t}^Y, \hat{\mu}_{P,t}^{i,\alpha}\right)$ *such that the processes* $\left(c_{s,t}^i, \Phi_{s,t}^i, \Psi_{s,t}^i\right)$ *maximize utility given in Equation (10) subject to the dynamic budget constraint given in Equation (11) for* $i = a, b$*, and good markets* ($\int_{-\infty}^t \nu e^{-\nu(t-s)}(\alpha_s c_{s,t}^a + (1-\alpha_s)c_{s,t}^b)ds = Y_t$) *as well as financial markets* ($\int_{-\infty}^t \nu e^{-\nu(t-s)}\alpha_s\Phi_{s,t}^a + (1-\alpha_s)\Phi_{s,t}^b)ds = 0$, $\int_{-\infty}^t \nu e^{-\nu(t-s}\left(\alpha_s\Psi_{s,t}^a + (1-\alpha_s)\Psi_{s,t}^b\right)ds = 0$, $\int_{-\infty}^t \nu e^{-\nu(t-s)}(\alpha_s(W_{s,t}^a - \Phi_{s,t}^a - \Psi_{s,t}^a) + (1-\alpha_s)(W_{s,t}^b - \Phi_{s,t}^b - \Psi_{s,t}^b))ds$) *clear.*

All assets are in zero-net supply and, thus, Walras law implies that total financial wealth is always zero:

$$\int_{-\infty}^t \nu e^{-\nu(t-s)}\left(\alpha_s W_{s,t}^a + (1-\alpha_s)W_{s,t}^b\right)ds = 0. \tag{13}$$

[17] Investors within a patient or impatient cohort agree on the dynamics of α_t and so their beliefs and, thus, state price densities do not depend on their date of birth s. This is going to change in Section IV where we allow investors within a cohort to learn from their experience.

We already discussed that it is optimal for an agent to annuitize all her positive financial wealth because they do not have any bequest motive and that investors have to buy life insurance for all their negative financial wealth to avoid default. The insurance industry who annuitizes all the positive financial wealth of agents and provides life insurance for all agents with negative financial wealth breaks even since financial wealth adds up to zero; hence, the insurance market clears.

H Static Optimization Problem

After birth, patient and impatient cohorts face a dynamically complete market and so we solve the consumption-portfolio choice problem given in equations (10) and (11) by using the martingale methods developed in Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989). Specifically, the value of an agent's lifetime consumption stream has to be affordable and hence can not exceed the value of her endowment stream that agents receive over their lifetime, that is,

$$E_s^i\left[\int_s^\tau \frac{\xi_t^i}{\xi_s^i} c_{s,t}^i \, dt\right] \leq E_s^i\left[\int_s^\tau \frac{\xi_t^i}{\xi_s^i} Y_t \, dt\right]. \tag{14}$$

Investors have strictly increasing utility and the random time of death, τ, is exponentially distributed and independent of the supply and demand shock. Hence, instead of solving the dynamic consumption-portfolio choice problem for an agent of type i born at time s given in equations (10) and (11) we solve the static optimization problem,

$$\max_{\{c_{s,t}^i:\, s\leq t\}} \quad E_s^i\left[\int_s^\infty e^{-(\rho_i+\nu)(t-s)} \log\left(c_{s,t}^i\right) dt\right] \tag{15}$$

$$\text{s.t.}\quad E_s^i\left[\int_s^\infty e^{-\nu(t-s)} \frac{\xi_t^i}{\xi_s^i} c_{s,t}^i \, dt\right] = E_s^i\left[\int_s^\infty e^{-\nu(t-s)} \frac{\xi_t^i}{\xi_s^i} Y_t \, dt\right]. \tag{16}$$

From the first order conditions (FOCs) of this optimization problem, we get

$$\frac{e^{-(\rho_i+\nu)(t-s)}}{c_{s,t}^i} = \kappa_s^i e^{-\nu(t-s)} \frac{\xi_t^i}{\xi_s^i}, \qquad s \leq t, \tag{17}$$

where κ_s^i denotes the Lagrange multiplier of the static budget constraint given in Equation (16). The Lagrange multiplier, κ_s^i, is time-dependent which captures the different valuations of the endowment stream of new born agents who cannot hedge against endowment fluctuations prior to birth. Put it differently, while all agents receive the same endowment stream, the value of it depends on the state of the economy that unborn agents cannot insure against and, hence, differs across newborn agents. Using the FOCs in equation (17) leads to optimal consumption at time t of patient (type a) and impatient (type b) agent born at time s. Specifically,

$$c^i_{s,t} = c^i_{s,s} e^{-\rho_i (t-s)} \frac{\xi^i_s}{\xi^i_t}, \quad \forall s \leq t \leq \tau^i \qquad \text{with} \qquad c^i_{s,s} = \frac{1}{\kappa^i_s}, \qquad i \in \{a, b\}. \tag{18}$$

The individual consumption growth rate is inversely related to the investor's time discount rate and to the state price density. Moreover, the initial consumption share is inversely related to the Lagrange multiplier and the state price and, thus, needs to be solved for in equilibrium.

I Consumption Shares

Let $\hat{W}_t$ denote the time t value of the future endowment stream of all agents alive, that is, aggregate wealth and $\hat{W}^i_{s,t}$ denote time t total wealth—financial wealth plus the value of the future endowment stream—of agent i born at time s. Specifically,

$$\hat{W}^i_{s,t} = W^i_{s,t} + E^i_t \left[\int_t^\infty e^{-\nu(u-t)} \frac{\xi^i_u}{\xi^i_t} Y_u \, du \right] = W^i_{s,t} + \hat{W}_t. \tag{19}$$

Financial wealth differs across agents because it depends on their consumption/savings rates and their proceeds from speculative trading whereas the value of investors endowment is the same across agents. Every agent is born without financial wealth, that is, $W^i_{s,s} \equiv 0$, and hence her total wealth at birth is equal to the value of her endowment stream, that is, $\hat{W}_{s,s} = \hat{W}_s$.

Using the static budget constraint given in equation (16) and the consumption demand equation (18), we verify that the well know result of a constant consumption-wealth ratio with log utility also holds in our overlapping generations model with heterogeneous time preferences and beliefs. Specifically,

$$\hat{W}^i_{s,t} = E^i_t \left[\int_t^\infty e^{-\nu(u-t)} \frac{\xi^i_u}{\xi^i_t} c^i_{s,u} \, du \right] = \frac{1}{\nu + \rho^i} c^i_{s,t} \quad \Rightarrow \quad c^i_{s,t} = (\nu + \rho^i) \hat{W}^i_{s,t}, \quad \forall \, s \leq t,$$

where $\nu + \rho^i$ is the effective subjective time discount rate of agent i. Hence, patient investors consume less out of their wealth than impatient investors; 2% versus 7% in our baseline parameter specification. Moreover, both types consume more out of their wealth when ν increases because their life expectancy shortens. For instance, the fraction that patient and impatient investors consume out of their wealth decreases to 1% and 6%, respectively, if the life expectancy increases from 50 years to 100 years (the mortality rate ν decreases from 2% to 1%). We define and derive individual consumption per unit of output in the next proposition.

Proposition 1 (Individual consumption shares)**.** *The time t consumption share of a type i agent born at time s is*

$$\beta^i_{s,t} \equiv \frac{c^i_{s,t}}{Y_t} = \left(\nu + \rho^i\right) \frac{\hat{W}^i_{s,t}}{Y_t} = \beta^i_t + \left(\nu + \rho^i\right) \frac{W^i_{s,t}}{Y_t}, \quad \forall s \leq t \leq \tau^i, \tag{20}$$

where β^i_s is the initial individual consumption share. Specifically,

$$\beta^i_s \equiv \beta^i_{s,s} = (\nu + \rho^i) \frac{\hat{W}_s}{Y_s} = (\nu + \rho^i)\phi_s, \tag{21}$$

where ϕ_s denotes the aggregate wealth-consumption ratio.

The initial individual consumption share, β^i_s, depends on prevailing market conditions as measured by the wealth-consumption ratio. Hence, patient agents who are born in good times–when the wealth-consumption ratio is high–consume initially more than patient agents who are born in bad times. The same is true for impatient investors. Moreover, impatient investors initial consumption share is always higher than the initial consumption share of patient investors assuming they share the same birthday; otherwise the initial share of patient investors born in good times may exceed that of impatient investors born in bad times. The consumption share of patient investors after a few rounds of trading, $\beta^a_{s,t}$, may exceed the consumption share of impatient investors, $\beta^b_{s,t}$, even though the share the same birthday, because speculative trading due to demand disagreement allows them to capture financial wealth from each other.

So far we have only looked at the consumption share of individual patient and impatient investors. To study the implications of demand disagreement for asset pricing, we need to know how the consumption share (or wealth share) of all patient and impatient investors evolves over time. Consumption shares are defined next and wealth shares are discussed in the Internet Appendix.

Definition 2 (Consumption shares)**.** *Consumption shares of type a and type b cohorts are defined as*

$$f_t = \frac{1}{Y_t}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \alpha_s c^a_{s,t}\, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \alpha_s \beta^a_{s,t}\, ds \tag{22}$$

$$1 - f_t = \frac{1}{Y_t}\int_{-\infty}^{t} \nu e^{-\nu(t-s)} (1-\alpha_s) c^b_{s,t}\, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} (1-\alpha_s) \beta^b_{s,t}\, ds \tag{23}$$

respectively.

We show in the next section that all equilibrium quantities of our demand disagreement model are fully characterized by the endogenous state variable, f_t, and the exogenous state variable, α_t. The dynamics of the consumption share and its unconditional distribution are discussed in Section III.

J Equilibrium

We solve for the state price densities ξ^a_t and ξ^b_t by combining the market clearing condition for the consumption good given in Definition 1 with the optimal consumption demand of individual agents given in equation (18). These state price densities are given in closed form as solution of stochastic Voltera integral equations in Appendix A and we present them in the more familiar form as solutions of stochastic differential equations in the next theorem, together with equilibrium asset prices, optimal consumption allocations, and optimal portfolios.

Theorem 1 (Equilibrium)**.** *The dynamics of the investor specific state price densities in equilibrium are*

$$d\xi^i_t = -r_t \xi^i_t\, dt - \theta_{Y,t} \xi^i_t\, dZ_{Y,t} - \theta^i_{\alpha,t} \xi^i_t\, dZ^i_{\alpha,t}, \qquad i = a, b. \tag{24}$$

The equilibrium risk-free rate is

$$r_t = f_t \rho^a + (1 - f_t)\, \rho^b + \mu_Y - \sigma^2_Y + \nu \left(1 - \alpha_t \beta^a_t - (1 - \alpha_t)\, \beta^b_t\right), \tag{25}$$

where $\beta_t^i = (\nu + \rho^i)\phi_t$*. The wealth-consumption ratio is*

$$\phi_t = \frac{f_t}{\rho^a + \nu} + \frac{1 - f_t}{\rho^b + \nu}. \tag{26}$$

The equilibrium market prices of supply and demand shock risks are

$$\theta_{Y,t} = \sigma_Y, \qquad \theta^a_{\alpha,t} = \Delta(1 - f_t), \qquad \theta^b_{\alpha,t} = -\Delta f_t. \tag{27}$$

The expected excess return of the supply asset in equilibrium is σ_Y^2*. The equilibrium expected excess return of the demand asset perceived by the patient and impatient investor are* $\Delta(1 - f_t)$ *and* $-\Delta f_t$*, respectively. The equilibrium investment in the supply asset of a type i agent is* $\Phi^i_{s,t} = W^i_{s,t}$ *and the equilibrium investments in the demand asset of a patient and impatient agent are*

$$\Psi^a_{s,t} = \left(\hat{W}^a_{s,t} - (\phi^a - \phi^b) f_t Y_t\right) \Delta(1 - f_t) \quad \text{and} \quad \Psi^b_{s,t} = -\left(\hat{W}^b_{s,t} + (\phi^a - \phi^b)(1 - f_t) Y_t\right) \Delta f_t,$$

respectively. The equilibrium investment in the risk-free asset and the equilibrium consumption allocation of a type i agent are $-\Psi^i_{s,t}$ *and* $c^i_{s,t} = (\nu + \rho^i)\left(W^i_{s,t} + \hat{W}_t\right)$*, respectively. The equilibrium consumption and portfolio allocations are shown for an agent that is born at time s and dies at time* τ *and, thus, hold for all* $s \le t \le \tau$*.*

Theorem 1 confirms that allocations and prices depend on two state variables: the exogenous fraction of newborn patient investors α_t and the endogenous consumption share of patient investors f_t. The former has a direct effect on the risk-free rate and an indirect effect on all other quantities through its effect on the consumption share dynamics. Moreover, Theorem 1 shows that there are three economic effects driving equilibrium allocations and prices: (i) an effect coming from the overlapping generations structure, (ii) an effect coming from different time discount rates, (iii) and an effect coming from disagreement about the future prevalence of patient and impatient investors. We study the implications of all three economic effects on the consumption share, the wealth-consumption ratio, the risk-free interest rate, stock market returns, and trading volume in detail in the next section and conclude this section with a brief discussion of the equilibrium portfolio allocations.

Each of the risky securities loads only on one shock and so the optimal amount of wealth allocated to the supply and demand shock can be interpreted as the desired exposure of an investor to the supply and demand shock in excess of her exposure to these shocks through the value of the endowment stream. If there is no heterogeneity, then prices of all three securities adjust such that nobody holds them in equilibrium. In this case, there is no risk-sharing need because everybody has the same endowment, agents consume at the same rate out of their endowment, and, thus, aggregate wealth growth is inversely related to the state price density and time discount rate, that is, $\frac{\hat{W}_t}{\hat{W}_s} = e^{-\rho(t-s)}\frac{\xi_s}{\xi_t}$.

If there is no disagreement but investors have different time discount rates, then nobody is borrowing or lending at the risk-free rate and the demand asset is not traded; yet there is trade in the supply assets. Specifically, consider a patient and impatient investor from the same birth cohort. Initially, they have zero financial wealth and, thus, no exposure to the supply asset. But the impatient investor consumes at a higher rate out of his wealth than the patient investor consumes out of hers (7% versus 2% in our baseline parameter specification) and, thus, he needs to sell some of his endowment to her in order to finance his higher rate of consumption. Put it differently, the impatient investor shorts the supply asset to finance more consumption today while the patient investor saves by buying the supply asset to consume more in the future. There is no borrowing or lending in the risk-free asset to smooth consumption over time because investors have the same risk preferences and beliefs and therefore hold the same risky portfolio. If there is demand disagreement, then patient investors typically borrow at the risk-free rate and use the funds to long the demand asset and the impatient investors short the demand asset and use the funds to lend at the risk-free rate; they speculate on demand shocks.

It is difficult to compare equilibrium investments of patient and impatient investors across cohorts because (i) agents' of different cohorts are born with different wealth, (ii) heterogenous time preferences lead to different saving rates, and (iii) disagreement leads to different speculation profits/losses. However, disagreement always implies that patient investors put more money in the demand asset than impatient investors because they estimate a higher long run average of newborn patient investors and therefore profit from high returns on the demand asset.[18]

[18]We provide a proof of this statement in the Internet Appendix.

III Asset Pricing Implications

In the equilibrium model discussed in the previous section there are two state variables which form a Markov system: the exogenous fraction of newborn patient investors α_t (or equivalently l_t) and the endogenous consumption share of patient investors f_t. In contrast to most models with infinitely lived agents with heterogeneous preferences and beliefs, the consumption share is stationary in our overlapping generations model. Hence, we can compute the unconditional distribution of the consumption share and discuss the unconditional distribution of the wealth-consumption ratio, the risk-free rate, stock market returns, and trading volume.

A Consensus View

To study the asset pricing implications in our demand disagreement model we need to define the belief of an outside observer or econometrician. Specifically, let $\mathbb{P}$ denote the data generating measure or belief of the econometrician and $\eta_t^i = \frac{d\mathbb{P}^i}{d\mathbb{P}} \mid_{\mathcal{F}_t}$ the likelihood ratio that allows us to change belief from the econometrician to the patient (η_t^a) and impatient (η_t^b) investors, respectively. We assume that the belief of the econometrician coincides with the consensus belief among investors. Specifically,

$$dl_t = \kappa \left(\bar{l} - l_t\right)\, dt + \sigma_l \, dZ_{\alpha,t}, \qquad \bar{l} = \frac{\bar{l}^a + \bar{l}^b}{2}, \tag{28}$$

where $\bar{l}$ denotes the long run mean and $Z_{\alpha,t}$ the perceived demand shock under the consensus view. The belief of the econometrician and the belief of the type i investor inside the model are linked through the parameter Δ^i because everybody observes l_t. Specifically,

$$\frac{d\eta_t^i}{\eta_t^i} = \Delta^i \, dZ_{\alpha,t}, \qquad dZ_{\alpha,t}^i = dZ_{\alpha,t} - \Delta^i \, dt, \qquad \Delta^i = \frac{\kappa}{\sigma_l}\left(\bar{l}^i - \bar{l}\right). \tag{29}$$

The long run mean in our baseline setting is $\bar{l} = 0$ which implies that the unconditional mean and volatility of newborn patient investors is 0.5 and 0.16, respectively.[19] The values for medium and high disagreement are $\Delta = 0.4$ and $\Delta = 0.8$, respectively. These numbers are similar to disagreement about GDP growth as discussed in the previous section.

[19]The long run mean of α_t is not exactly 0.5 due to a Jensen's term but the effect of this term is negligible.

Our OLG model leads to a stationary distribution for the consumption share, f_t, that we study in the next subsection. We present the stochastic discount factor of this economy under the belief the econometrician denoted by ξ_t in the next proposition.

Proposition 2 (Stochastic Discount Factor)**.** *The stochastic discount factor under the belief of the econometrician is $\xi_t = \xi_t^a \eta_t^a = \xi_t^b \eta_t^b$ with dynamics*

$$\frac{d\xi_t}{\xi_t} = -r_t\, dt - \theta_{Y,t}\, dZ_{Y,t} - \theta_{\alpha,t}\, dZ_{\alpha,t}, \tag{30}$$

where the short rate, r_t, and market price of output risk, $\theta_{Y,t}$ are given in equation (25) and (27) of Theorem 1, respectively. The market price of demand shock risk perceived by the econometrician is

$$\theta_{\alpha,t} = -\frac{1}{dt}\mathrm{Cov}\left(\frac{d\xi_t}{\xi_t}, \frac{dP_t^\alpha}{P_t^\alpha}\right) = \Delta\left(\frac{1}{2} - f_t\right) = \theta_{\alpha,t}^a - \frac{1}{2}\Delta = \theta_{\alpha,t}^b + \frac{1}{2}\Delta, \tag{31}$$

where $\theta_{\alpha,t}^i = -\frac{1}{dt}\mathrm{Cov}^i\left(\frac{d\xi_t^i}{\xi_t^i}, \frac{dP_t^\alpha}{P_t^\alpha}\right)$ is the market price of demand shock risk perceived by patient $(i = a)$ and impatient $(i = b)$ investors given in equation (27) of Theorem 1.

The market price of demand shock risk perceived by the econometrician, $\theta_{\alpha,t}$, is an affine function of the consumption share and, thus, stationary. We will discuss its unconditional distribution in detail in Section E.E.1.

B Market Selection and the Consumption Share

The dynamics of the consumption share under the belief of the econometrician are summarized in the next proposition.

Proposition 3 (Consumption Share Dynamics)**.** *The dynamics of the consumption share of patient agents, f_t, in equilibrium are $df_t = \mu_{f,t}\, dt + \sigma_{f,t}\, dZ_{\alpha,t}$ with*

$$\begin{aligned} \mu_{f,t} = {} & \nu\left(\alpha_t \beta_t^a (1 - f_t) - (1 - \alpha_t)\beta_t^b f_t\right) + \left(\rho^b - \rho^a\right) f_t (1 - f_t) \\ & + \Delta^2 \left(\frac{1}{2} - f_t\right) f_t (1 - f_t) \qquad \text{and} \qquad \sigma_{f,t} = f_t(1 - f_t)\Delta. \end{aligned} \tag{32}$$

Output shocks do not effect the consumption share because investors have the same risk preferences and agree on the output shock, and, thus, invest the same fraction of wealth in the supply asset. The left graph of Figure 1 shows the consumption share volatility, $\sigma_{f,t}$, as a function of the consumption share, f_t, for different disagreement Δ. There are no shocks to the consumption share if there is no disagreement, that is, $\sigma_{f,t} = 0$ if $\Delta = 0$ because in this case there is no trade in the demand asset. The impact of shocks to the consumption share reaches its maximum when the consumption share is 0.5 and vanishes when it approaches either 0 or 1 because investors take large speculative positions of equal size in the former and have nobody to speculate with in the later. Moreover, speculative positions increase with disagreement Δ and, thus, magnify shocks to the consumption shares as wealth shifts from one speculator to the other.

There are three economic effects that influence the consumption share drift, $\mu_{f,t}$, that is shown for different disagreement Δ as a function of f_t in the right graph of Figure 1. Specifically, there is (i) an OLG effect, (ii) a preference heterogeneity effect, (iii) and a disagreement effect. The first term of the drift in equation (32), which is due to the OLG effect, leads to time variation of the drift even without disagreement and it does not vanish when the consumption share approaches either 0 or 1 because every instant new agents are born and the stochastic fraction, α_t, measures how many of them are patient. Hence, the OLG effect leads to reflecting boundaries at 0 or 1 and to a stationary distribution of the consumption share, f_t. Moreover, the push upward at the lower bound is on average smaller than the push downwards at the upper bound, that is, $-0.033dt$ vs $0.003dt$ in top right graph of Figure 1. Specifically,

$$- \mathrm{E}[\lim_{f_t \to 1} \mu_{f,t}] = \nu \left(1 - \mathrm{E}[\alpha_t]\right) (\rho^b + \nu)\phi^a \geq \nu \mathrm{E}[\alpha_t](\rho^a + \nu)\phi^b = \mathrm{E}[\lim_{f_t \to 0} \mu_{f,t}] > 0. \tag{33}$$

The average fraction of newborn patient investors is the same as for impatient investors and so the economic reason for the different boundary behavior is twofold. First, impatient investors consume at a higher rate out of their initial wealth and second initial wealth is more valuable at the extinction boundary for impatient investors where patient investors have all the price impact and so valuation ratios are higher.

The second term of the drift in equation (32) is due to investors having different time discount

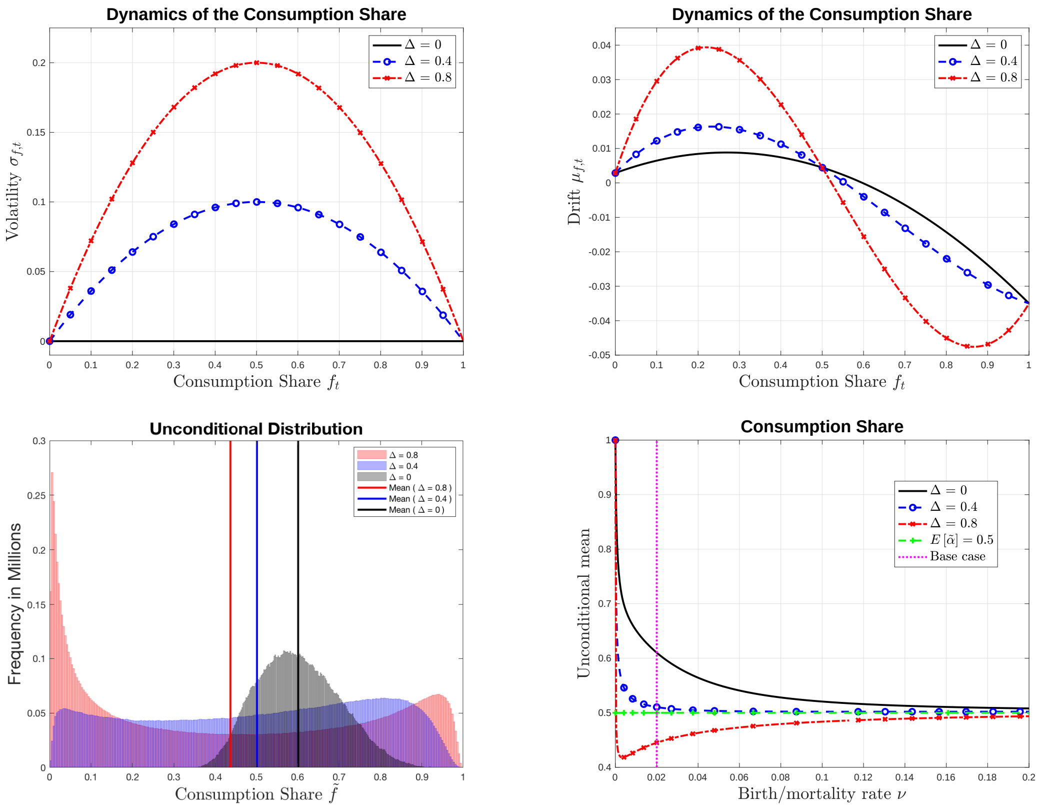


Figure 1: The consumption share. The top left graph shows the volatility $\sigma_{f,t}$ and the top right graph shows the drift $\mu_{f,t}$ of the consumption share dynamics as a function of the consumption share for three levels of disagreement Δ. The drift of the consumption share $\mu_{f,t}$ in the top right graph depends on the fraction of patient newborns α_t which is fixed at 0.5. The left graph shows the unconditional distribution of the consumption share of patient investors $\tilde{f}$ and the right graph shows the unconditional mean of the consumption share of patient investors $\mathrm{E}[\tilde{f}]$ as a function of the birth/mortality rate ν. The histograms and unconditional means in the bottom row are based one million years of monthly observations for each value of disagreement Δ. To guarantee a finite limit of the wealth-consumption ratio when ν converges to zero we set $\rho_a = 0.001$ in the right bottom graph.

rates and it is always positive because $\rho^b > \rho^a$. This upward drift reflects the market selection force that favors the patient agent who saves more than the impatient agent, which in an infinite horizon economy would lead to extinction of the impatient agent.

The third term of the drift in Equation (32) is due to disagreement and vanishes if the consumption share is either 0 or 1 because neither patient nor impatient investors have somebody to speculate with at the extinction boundaries. In stark contrast to the consumption share volatility, disagreement has no effect on the drift when the consumption share is 0.5 because from the econometricians' perspective speculating on the demand shock is a fair gamble in this case and, hence, neither patient nor impatient investors' financial wealth and, thus, consumption share is expected to change.

We determine the unconditional distribution of the consumption share by using the dynamics of the consumption share f_t to simulate a long time series, that is, one million months. In an infinite horizon economy this distribution does not exist and the consumption share would always be either zero or one. The bottom left graph of Figure 1 shows the histogram for the consumption share distribution for three different values of disagreement Δ.

If there is no disagreement, then the average consumption share is 60% which exceeds the unconditional average of newborn patient investors, $\mathrm{E}[\alpha_t] = 50\%$, because patient investors save more than impatient investors and, thus, have a higher consumption share in the long run; the market selection force. Moreover, the distribution is unimodal with no mass at the extinction boundaries.

Interestingly, disagreement lowers the average consumption share of patient investors to levels even below 50% and, thus, more than offsets the market selection force. The reason for this surprising result is that speculation leads to large and persistent wealth swings and, thus, more mass at the extinction boundaries. The different behavior of patient and impatient investors at the extinction boundaries (discussed above) leads to higher unconditional probabilities of very low consumption shares and, thus, to long run means below 50% when disagreement is large.[20] This

[20] We show in the Internet Appendix that an increase in the difference between the price-dividend ratios that would prevail in economies with only patient and impatient leads to a low unconditional mean of the patient investors' consumption share.

is shown in the bottom left graph of Figure 1 where the unconditional mean is decreasing from 60% with no disagreement ($\Delta = 0$, black line), to 50% with medium disagreement ($\Delta = 0.4$, blue line), and to 44% with high disagreement ($\Delta = 0.8$, red line). Moreover, disagreement increases the consumption share volatility from 10% to 28%, and 34%, respectively. Disagreement also has a strong effect on the shape of the unconditional distribution because it significantly increases the probability of having very low or very high consumption share realizations and, thus, it increases the likelihood of extreme low and high asset prices as the next sections demonstrate.

We show the unconditional mean as a function of the birth/mortality rate ν in the bottom right graph of Figure 1 to discuss the OLG effects on the consumption share distribution.[21] When ν approaches zero, the economy approaches that of an infinitely lived agent economy and therefore the most patient agent consumes everything, regardless of disagreement, as in Yan (2008). However, once the birth/mortality rate increases, the average consumption share declines and eventually converges to the unconditional mean of newborn patient investors because there is less time for the market selection force to work when the life expectancy of agents goes to zero. While the consumption share without disagreement monotonically decreases, this is not necessarily the case when there is disagreement as illustrated by the red line in the high disagreement case. Here the average consumption share drops quickly because consumption shares are very persistent for low mortality rates and disagreement increases the mass at the boundaries which has a larger effect on the consumption share of impatient than patient investors.

In the remainder of this section, we will use the random variables $\tilde{\alpha}$ and $\tilde{f}$ to denote the fraction of newborn patient agents and their consumption share in the long run drawn from their unconditional distribution.

C The Wealth-Consumption Ratio

The wealth-consumption ratio in a representative agent economy with log utility is constant and inversely related to the effective time discount rate; ρ^i in an infinite horizon economy and $\nu + \rho^i$

[21] To guarantee a finite limit of the wealth-consumption ratio when ν converges to zero, we set $\rho_a = 0.001$ in this example.

in an OLG economy. Hence, the wealth-consumption ratio in an OLG economy with only patient ($\tilde{\alpha} \equiv 1$) and impatient ($\tilde{\alpha} \equiv 0$) investors are $\phi^a \equiv \frac{1}{\nu+\rho^a}$ and $\phi^b \equiv \frac{1}{\nu+\rho^b}$, respectively. In our demand disagreement model, the wealth-consumption ratio is the consumption share weighted average of the wealth-consumption ratios that would prevail with only patient (ϕ^a) and impatient agents (ϕ^b). Specifically,

$$\tilde{\phi} = \tilde{f}\phi^a + (1-\tilde{f})\phi^b = \phi^b + (\phi^a - \phi^b)\tilde{f}. \tag{34}$$

The unconditional distribution of $\tilde{\phi}$ inherits all the properties of the consumption share distribution discussed in the previous subsection. For instance, it is bounded between $\phi^b = 14.29$ and $\phi^a = 47.62$ in our baseline setting. Moreover, a positive shock to the consumption share raises the wealth-consumption ratio and, hence, is good news for the stock market because the effective discount rate of patient investors is lower than that for impatient investors and, thus, valuation ratios are higher in the former than the latter. In an infinite horizon economy patient investors outsave impatient investors which leads to a high and constant valuation ratio in the long-run, that is, $\lim_{\nu\to 0}\tilde{\phi} = \phi_a$, a.s. In contrast, valuation ratios are stochastic and significantly lower in our demand disagreement model when the mortality rate is positive. The left graph of Figure 2 shows the unconditional distribution of the wealth-consumption ratio for no ($\Delta = 0$), medium ($\Delta = 0.4$), and high ($\Delta = 0.8$) disagreement and the right graph of Figure 2 shows its mean and volatility as a function of disagreement Δ. Disagreement lowers the average valuation ratio and makes it more volatile. Moreover, the probability of having a very low valuation ratio significantly increases with disagreement.

D The Risk-Free Interest Rate

We show in this section that average interest rates increase with disagreement but they are nevertheless reasonable low on average with low volatility. The consumption share is stationary and thus Theorem 1 implies that the risk-free rate is stationary and given by

$$\tilde{r} = \underbrace{\rho^a\tilde{f} + \rho^b(1-\tilde{f})}_{\mathcal{E}_{\tilde{f}}[\rho]} + \mu_Y - \sigma_Y^2 + \nu(1 - (\underbrace{\tilde{\alpha}\beta^a(\tilde{f}) + (1-\tilde{\alpha})\beta^b(\tilde{f})}_{\mathcal{E}_{\tilde{\alpha}}[\beta(\tilde{f})]})). \tag{35}$$

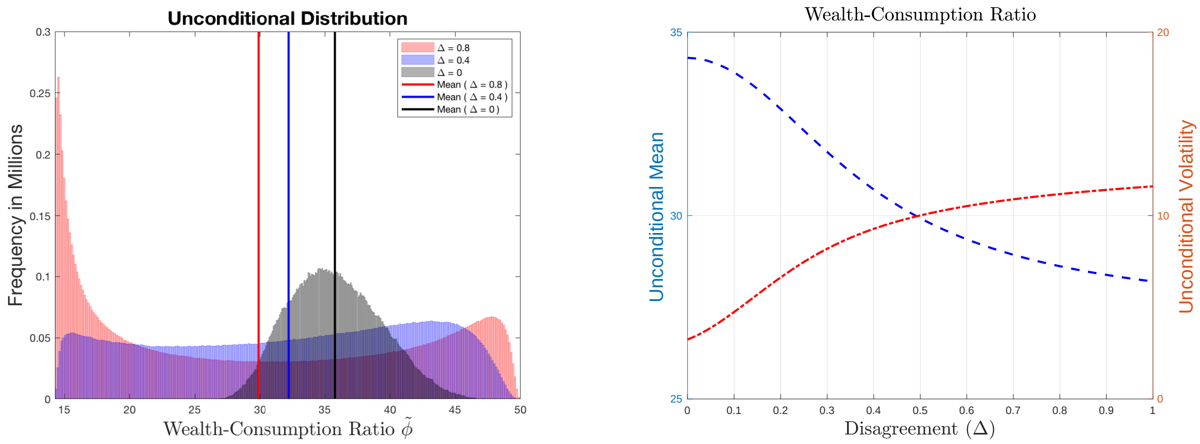


Figure 2: The wealth-consumption ratio. The left graph shows the unconditional distribution of the wealth-consumption ratio for different disagreement Δ and the right graph shows the unconditional mean and volatility of the wealth consumption ratio as a function of disagreement Δ. Disagreement lowers the average valuation ratio and increases its volatility. It also significantly increases the probability of a very low valuation ratio. The histograms and summary statistics are based on one million years of monthly observations for each value of disagreement Δ.

There are four economic effects that determine the risk-free interest rate: (i) a time-discounting effect, (ii) an intertemporal substitution effect, (iii) a precautionary savings effect, and (iv) and effect from the OLG structure. The market time discount rate, $\mathcal{E}_{\tilde{f}}[\rho]$, is the consumption share weighted average of the patient and impatient investors' time discount rate ρ_a and ρ_b and, thus, stochastic. The risk-free rate is low when the consumption share, $\tilde{f}$, is high because in this case patient investors consume a larger fraction of aggregate output and, therefore, their low demand for current consumption is more important in determining the interest rate than the high demand for current consumption of impatient investors. Hence, the risk-free rate is low to prevent patient investors from saving too much because otherwise the bond market would not clear. The intertemporal smoothing component captured by the expected output growth rate μ_Y raises interest rates and the precautionary saving motive captured by the output growth variance σ_Y^2 lowers the risk-free rate. Both effects are exactly the same as in a standard infinitely lived representative agent economy with log utility. The effect from the OLG structure is captured by the term

$$\nu(1 - \mathcal{E}_{\tilde{\alpha}}[\beta(\tilde{f})]) = \nu(1 - (\nu + \underbrace{\tilde{\alpha}\rho^a + (1-\tilde{\alpha})\,\rho^b}_{\mathcal{E}_{\tilde{\alpha}}[\rho]})\tilde{\phi}). \tag{36}$$

If there is no time preference heterogeneity, then this term is zero. This is no longer the case with preference heterogeneity as the expected consumption growth of agents currently alive no longer coincides with excepted output growth. Hence, a displacement effect must be taken into account because the short term interest rate reflects only expected consumption growth from agents currently alive. Specifically, if agents who are currently alive are expected to consume less on average than agents that are born next period, then the effective consumption growth is lower and, therefore, the interest rate is also lower.

Equation (35) shows that disagreement Δ has no direct effect on the interest rate. However, the interest rate depends on the consumption share and, thus, its unconditional distribution is affected by disagreement through its effect on the consumption share distribution. Specifically, Figure 3 shows that the average interest rate is increasing in demand disagreement. This is consistent with the results in Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018). However, the mechanism is quite different. In Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018), interest rates are higher due to the relative strength of the income and substitution effects and there would be no effect on the real rate with log utility. In our model with log utility, the reason for the increase is twofold. First, the unconditional average market time discount rate increases with disagreement. Second, disagreement lowers the unconditional average wealth-consumption ratio and consequently the consumption of newborns relative to the rest of the population. Both effects raise the average risk-free interest rate because disagreement counteracts the market selection force and, thus, impatient investors have more price impact in the long run. The left graph of Figure 3 shows the unconditional distribution of the risk-free rate for no ($\Delta = 0$), medium ($\Delta = 0.4$), and high ($\Delta = 0.8$) disagreement and the right graph of Figure 2 shows its unconditional mean and volatility as a function of disagreement Δ. Disagreement raises average interest rates and it makes them more volatile. However, in contrast to stock market volatility which significantly increases with disagreement, the effect of disagreement on interest rate volatility is moderate, that is it increases from 1.9% with medium disagreement ($\Delta = 0.4$) to 2.4% with high disagreement ($\Delta = 0.8$). Moreover, the probability of having a very low or high risk-free interest rate significantly increases with disagreement.

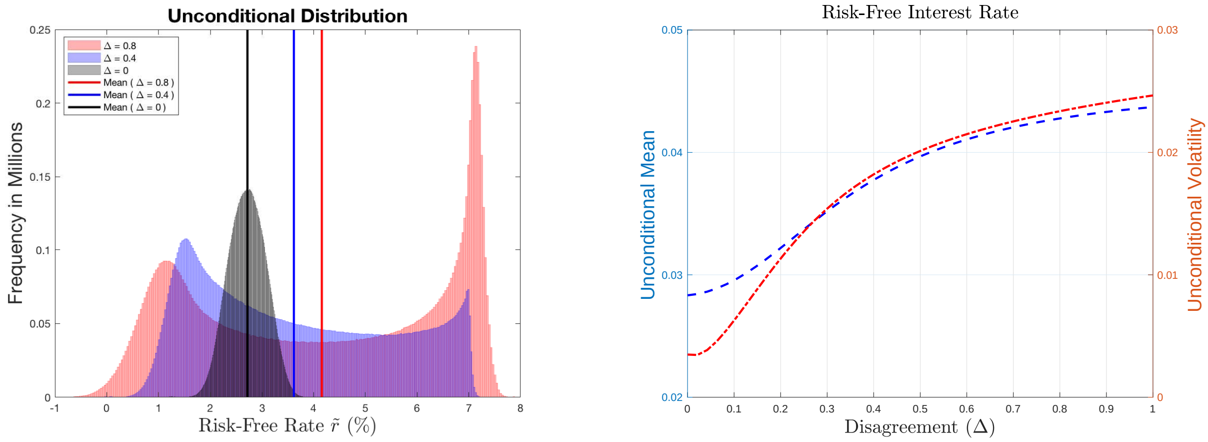


Figure 3: The risk-free rate. The left graph shows the unconditional distribution of the risk-free interest rate for different disagreement Δ and the right graph shows the unconditional mean and volatility of the risk-free rate as a function of disagreement Δ. Disagreement raises average interest rates and significantly increases the probability of a very low or very high interest rate scenario with a modest increase in volatility. The histograms and summary statistics are based on one million years of monthly observations for each value of disagreement Δ.

E Stock Market Return

The price of the aggregate wealth or stock market portfolio is $S_t \equiv \hat{W}_t = \phi_t Y_t$ and, hence, the instantaneous return of the dividend reinvested stock market price process is

$$dR_t \equiv \frac{dS_t + Y_t\,dt}{S_t} = (r_t + \lambda_{S,t})\;dt + \sigma^Y_{S,t}\,dZ_{Y,t} + \sigma^\alpha_{S,t}\,dZ_{\alpha,t}, \tag{37}$$

where $\lambda_{S,t}$ denotes the stock market risk premium, $\sigma^Y_{S,t}$ the stock market loading on the supply shock, and $\sigma^\alpha_{S,t}$ the stock market loading on the demand shock. The instantaneous correlation between changes in the valuation ratio and output growth is zero because supply and demand shocks are independent. The next proposition summarizes the results for the stock market.

Proposition 4 (Stock Market)**.** *The stock market loading on the supply shock is $\sigma^Y_{S,t} = \sigma_Y$. The stock market loading on the demand shock is*

$$\sigma^\alpha_{S,t} = \frac{\phi^a - \phi^b}{\phi_t} f_t\,(1 - f_t)\,\Delta \geq 0 \tag{38}$$

and it attains its maximum at $f^{max}_{\sigma_S} = \left(1 + \sqrt{\frac{\nu+\rho^b}{\nu+\rho^a}}\right)^{-1} \leq 0.5$. The stock market risk premium *perceived by the econometrician is*

$$\lambda_{S,t} = -\frac{1}{dt}\text{Cov}\left(\frac{d\xi_t}{\xi_t}, \frac{dS_t}{S_t}\right) = \sigma_Y^2 + \Delta^2 \frac{\phi^a - \phi_b}{\phi_t} f_t\,(1 - f_t)\left(\frac{1}{2} - f_t\right). \tag{39}$$

The stock market loading and market price of risk of the supply shock are the same as in a standard representative agent log-utility economy. If there is no disagreement, then the instantaneous stock market return is not exposed to the demand shock and, thus, the stock market volatility and risk premium are constant. Specifically, they are equal to $\sigma_Y = 3.3\%$ and $\sigma_Y^2 = 11bp$ as the black solid line in the left and right graph of Figure 4 show. Disagreement increases the exposure to the demand shock and so leads to stock market volatility levels in excess of output growth volatility as shown by the blue dashed circle line and the red chain-dotted crossed line in the left graph of Figure 4. Moreover, a positive demand shock is goods news for the stock market because it is goods news for patient investors who have long exposure to the demand shock and, thus, increases their consumption share which increases the price-dividend ratio. The stock market exposure to demand shocks vanishes if the consumption share approaches either 0 or 1 because in this case investors have nobody to speculate with. Hence, the stock market volatility approaches in both cases its minimum which is equal to output growth volatility of 3.3% in our baseline setting and so there is no excess volatility. The stock market volatility is maximized at the value $f_{\sigma_S}^{\max} = 0.35$ (vertical blue and red dotted line in the left graph of Figure 4) rather than at the peak of investors' speculative potential. Intuitively, the relative impact of speculation on stock market returns is higher for lower valuation ratios which coincides with low consumption shares of patient investors.

The market price of demand shock risks vanishes when the consumption share of patient and impatient investors are the same in which case the equity premium equals $11bp$ as in a standard infinite horizon log-utility model. The conditional equity premium exceeds $11bp$ if impatient investors consume more out of output and it is negative if patient investors consume more out of output. The reason for the low and even negative conditional equity premium is the negative market price of demand shock risk for high consumption shares that we discuss in detail in the next subsection. Hence, the stock market risk premium is not monotone in the consumption share.

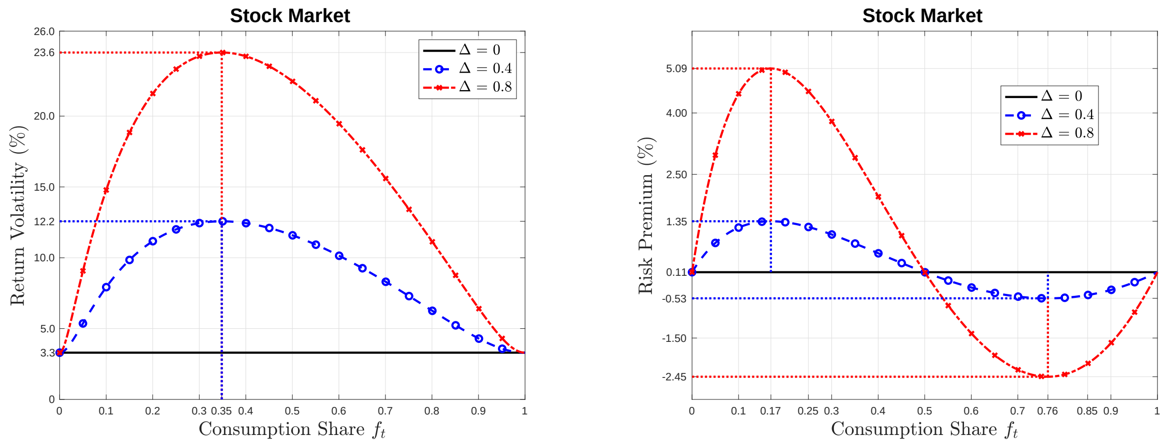


Figure 4: Conditional stock market volatility and risk premium. The left graph shows stock market volatility and the right graphs shows the equity premium conditional on the consumption share f_t for different disagreement Δ. Disagreement leads to stochastic stock market volatility in excess of output growth volatility and to a stochastic equity premium that is positive if impatient cohorts consume more out of output and negative if patient cohorts consume more out of output.

Specifically, it is zero at the extinction boundaries, reaches its maximum when impatient investors consume more out of output—$f^{\max}_{\lambda_S} = 0.17$ in our baseline setting—and reaches its minimum when patient investors consume more out of output—$f^{\min}_{\lambda_S} = 0.76$ in our baseline setting. Both critical consumption share values are independent of disagreement Δ as shown in the right graph of Figure 4. The intuition for this result is provided in the next subsections.

E.1 The Market Price of Risk for Demand Shocks

Proposition 4 implies that the equilibrium market price of demand shock risk is stationary and given by

$$\tilde{\theta}_\alpha = \frac{\kappa}{\sigma_l}(\underbrace{\bar{l}}_{\text{Consensus View}} - \underbrace{(\bar{l}^a \tilde{f} + \bar{l}^b(1-\tilde{f}))}_{\text{Market View, } \mathcal{E}_{\tilde{f}}[\bar{l}]}) = \Delta(0.5 - \tilde{f}). \tag{40}$$

The market view, $\mathcal{E}_{\tilde{f}}\left[\bar{l}\right]$, defined as the consumption share weighted average belief is the relevant belief for asset pricing and not the equally weighted average or consensus view, $\bar{l}$.[22] The econometrician is endowed with the consensus view and, thus, demand shocks are not priced if

[22] See Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018) for a detailed discussion of the market view.

and only if the market view coincides with the consensus view, that is, if the consumption share of patient and impatient investors coincides ($\tilde{f} = 0.5$). If impatient investors consume a larger share of output ($\tilde{f} < 0.5$), then their beliefs have a larger impact on prices and, thus, the market view does no longer coincide with the consensus view. In this case the econometrician perceives a positive market price of demand shock risk, that is, $\tilde{\theta}_\alpha > 0$. The opposite is true if patient investors consume a larger share of output.

What is the intuition for this result? We know that impatient investors overestimate the future demand for consumption of newborn agents (lower $\tilde{\alpha}$) and, thus, short the demand asset whereas patient investors underestimate this demand and, thus, long the demand asset. If impatient investors consume a larger share of output, then the price of the demand asset decreases to avoid excess supply of it. The econometrician who is endowed with the consensus view estimates a lower demand for future consumption of newborn agents (higher $\tilde{\alpha}$) than impatient investors and, thus, from her perspective the demand asset looks underpriced, that is, she estimates a higher covariance between returns on the demand asset and the pricing kernel. Hence, the market price of demand risk is positive from the econometricians point of view. On the other hand, if patient investors consume a larger fraction of output, then they push up the price of the demand asset. Hence, this asset looks overpriced from the perspective of the econometrician who therefore perceives the risk premium to be negative.[23] The left graph of Figure 5 shows the unconditional distribution of the market price of demand shock risk for no ($\Delta = 0$), medium ($\Delta = 0.4$), and high ($\Delta = 0.8$) disagreement and the right graph of Figure 5 shows its unconditional mean and volatility as a function of disagreement. The black vertical line verifies that without disagreement demand shocks are not priced. Demand shocks carry a negative risk premium for low disagreement ($\mathrm{E}\left[\tilde{\theta}_\alpha\right] = -15bp$ if $\Delta = 0.4$, blue solid line) and a positive risk premium for high disagreement ($\mathrm{E}\left[\tilde{\theta}_\alpha\right] = 5\%$ if $\Delta = 0.8$). The right graph shows that the average risk compensation for demand shocks initially decreases because for low disagreement the average consume share is high and, thus, the demand asset looks overpriced from the econometrician's perspective. However, after reaching its minimum it strictly increases with disagreement and becomes negative. The right graph of Figure 5 also

[23]The intuition is similar to Miller (1977) where an outside observer endowed with the consensus view among optimistic and pessimistic investors perceives the stock to be overpriced because short sale restrictions rule out the price impact of most pessimistic investors.

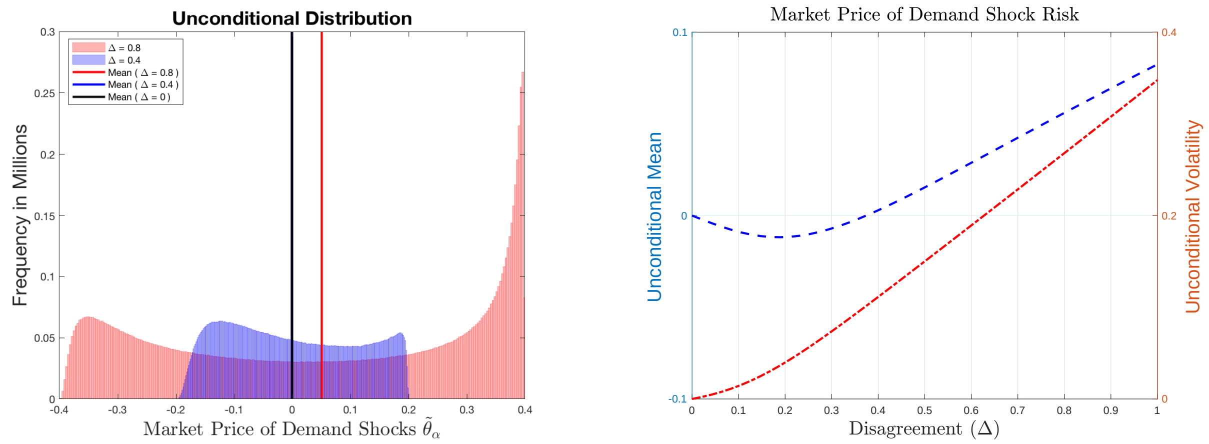


Figure 5: The market price of demand risk. The left graph shows the unconditional distribution of the market price of demand risk for different disagreement Δ and the right graph shows the unconditional mean and volatility of market price of demand risk as a function of disagreement Δ. Disagreement increases the volatility and the probability of a very low or very high realizations of the market price of risk. It does not always lead to a positive market price of risk even on average. The histograms and summary statistics are based on one million years of monthly observations.

shows that the volatility of the market price of demand shocks is strictly increasing with disagreement and the probability of having a high market price of risk is higher than the probability of having a very low price of risk.

E.2 Excess Stock Market Volatility

The stationary equilibrium stock market volatility is

$$\tilde{\sigma}_S = \sqrt{\sigma_Y^2 + \left(\tilde{\sigma}_S^\alpha\right)^2}, \qquad \tilde{\sigma}_S^\alpha = \frac{\phi^a - \phi^b}{\tilde{\phi}} \tilde{f}\left(1 - \tilde{f}\right)\Delta \geq 0. \tag{41}$$

Speculative trade leads to stochastic stock market volatility in excess of output growth volatility and so there is excess volatility. The left graph of Figure 6 shows the unconditional distribution of stock market volatility for no ($\Delta = 0$), medium ($\Delta = 0.4$), and high ($\Delta = 0.8$) disagreement and the right graph of Figure 6 shows its unconditional mean and volatility as a function of disagreement. Average stock market volatility is increasing with disagreement, that is, it more than doubles output growth volatility of 3.3% (black vertical line) to 9% when $\Delta = 0.4$ (blue vertical line) and it quadruples it to 14% when $\Delta = 0.8$ (red vertical line). The volatility of stock market volatility also increases with disagreement from 0 when there is no disagreement to 7% when $\Delta = 0.8$. Moreover, disagreement increases the probability of having an either very low or high stock market volatility.

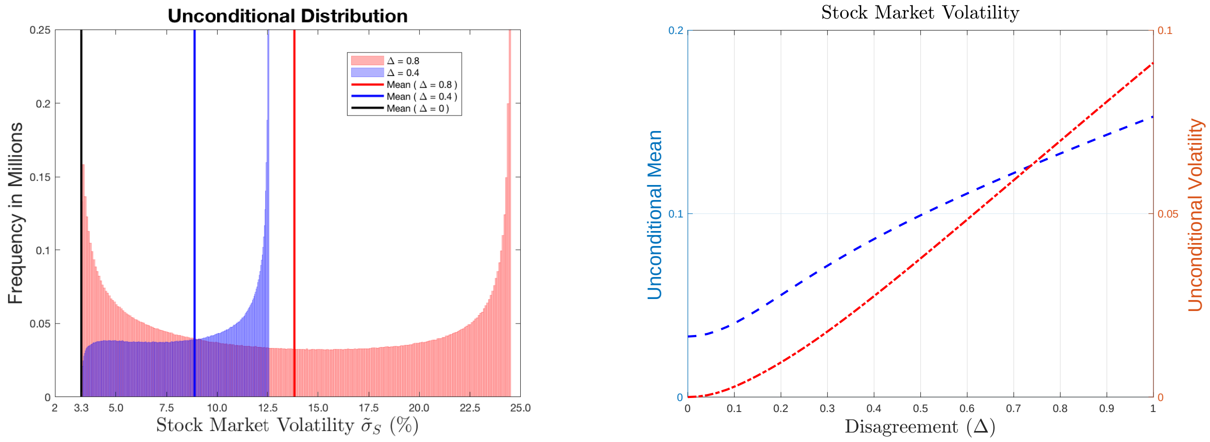


Figure 6: Excess stock market volatility. The left graph shows the unconditional distribution of the stock market volatility for different disagreement Δ and the right graph shows the unconditional mean and volatility of the stock market volatility as a function of disagreement Δ. Disagreement leads to excess stock market volatility because it significantly increase stock market volatility on average. It also raises the volatility of stock market volatility and significantly increases the probability of a very low or very high stock market volatility realization. The histograms and summary statistics are based on one million years of monthly observations for each value of disagreement Δ.

E.3 The Equity Premium

We show in this section that demand disagreement leads to a significantly higher equity premium than in infinite horizon economy with log-utility investors and a low and constant output growth volatility. Moreover, disagreement leads to a stochastic equity premium. The stationary equilibrium stock market risk premium is

$$\tilde{\lambda}_S = \sigma_Y^2 + \frac{\phi^a - \phi_b}{\tilde{\phi}} \tilde{f} \left(1 - \tilde{f}\right) \left(\frac{1}{2} - \tilde{f}\right) \Delta^2. \tag{42}$$

The left graph of Figure 7 shows the unconditional distribution of the stock market risk premium or equity premium for no ($\Delta = 0$), medium ($\Delta = 0.4$), and high ($\Delta = 0.8$) disagreement and the right graph of Figure 7 shows its unconditional mean and volatility as a function of disagreement

Δ.[24] Both the price and quantity of demand risk is zero without disagreement and, hence, the equity premium is the same as in a standard infinite horizon economy with log-utility investors, that is, $\sigma_Y^2 = 11bp$ in our baseline setting. While the stock market exposure to demand shocks is always positive and strictly increasing in disagreement the market price of risk is neither always positive nor strictly increasing in disagreement. Hence, it is at the first glance surprising that the average equity premium is positive and increasing in disagreement with the exception of an initial small decrease for low Δ. The reason for that is the positive unconditional covariance between the

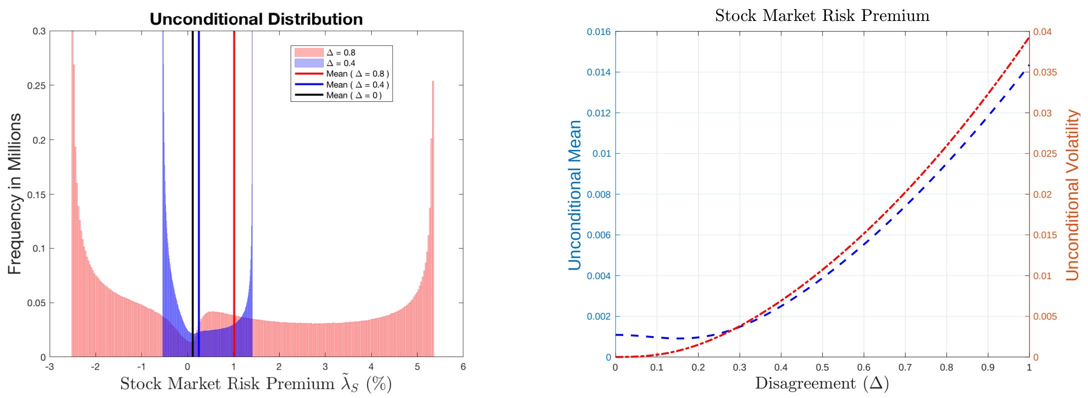


Figure 7: The equity premium. The left graph shows the unconditional distribution of the equity premium for different disagreement and the right graph shows the unconditional mean and volatility of the equity premium as a function of disagreement. Disagreement leads to a positive equity premium and it increases with disagreement with after a small initial decrease. Moreover, it significantly raises the volatility and the probability of very low or very high realizations of the equity premium. The histograms and summary statistics are based on one million years of monthly observations for each value of disagreement Δ.

price and the quantity of demand risk for low disagreement. Specifically,

$$\mathrm{E}\left[\tilde{\lambda}_S\right] = \sigma_Y^2 + \mathrm{E}\left[\tilde{\sigma}_S^\alpha\right]\mathrm{E}\left[\tilde{\theta}_\alpha\right] + \mathrm{Cov}\left[\tilde{\sigma}_S^\alpha, \tilde{\theta}_\alpha\right]. \tag{43}$$

The stock market's exposure to demand shocks is never negative but the average market price of demand shocks is negative when disagreement is low and, hence, $\mathrm{E}[\tilde{\sigma}_S^\alpha]\mathrm{E}[\tilde{\theta}_\alpha] < 0$ for low Δ. However, in this case it is more likely that patient investors consume more out of output than impatient investors, and, thus, the quantity of risk is more likely to decrease with the consumption

[24]We show in the Internet Appendix that an increase in the difference between the price-dividend ratios that would prevail in economies with only patient and impatient investors—without increasing the difference between the different time discount rates—raises the equity premium to levels we see in the data.

share (see Proposition 4 and the left graph of Figure 4). The market price of risk is always decreasing in the consumption share and so the covariance term, $\mathrm{Cov}[\tilde{\sigma}_S^\alpha, \tilde{\theta}_\alpha]$, is positive and it often outweighs the negative component leading to a positive equity premium even for low Δ. Moreover, disagreement leads to a stochastic equity premium that is predictable which we discuss in the next subsection. The volatility and the probability of an extreme high or low (even negative) equity premium is strictly increasing with disagreement and, thus, the model can capture high and low stock market valuations for reason that are unrelated to output levels.

E.4 Stock Market Predictability

There is a large literature which shows that excess stock market returns are predictable by valuation ratios such as price-dividend ratios, wealth-consumption ratios, price-earnings ratios, etc. In this section, we show that a high wealth-consumption or equivalently, price-dividend ratio predicts low future excess stock market returns consistent with this large literature. Specifically, we simulate one million years of monthly observations for disagreement Δ ranging from zero to one and regress stock market returns in excess of the risk-free rate on a constant and the price-dividend ratio for different horizons (see Cochrane (2008) for details). The left graph of Figure 8 shows that the slope coefficient of the predictability regression is negative and the right graph shows that the R^2 is increasing with the predictability horizon. Specifically, the slope is strictly decreasing and the R^2 is strictly increasing with disagreement. The intuition is straightforward when looking at the right graph of Figure 4. It shows that the equity premium is decreasing with the consumption share and, thus, decreasing with the price-dividend ratio except for very low or very high consumption share realizations which do not occur often. Moreover, the sensitivity of the equity premium to changes in the price-dividend ratio increases with disagreement and vanishes if there is no disagreement because in this case the equity premium is constant. Interestingly, we expect the predictive relation to change signs in times of very high or low stock market valuations.

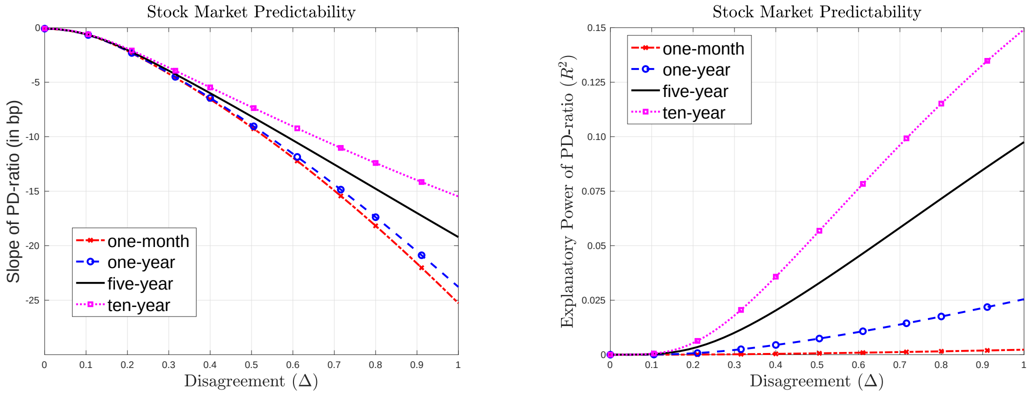


Figure 8: Stock Market Predictability. The left graph shows the slope and the right graph shows the R^2 for the price-divided regressions $Rx_{t,t+\tau} = a + b\phi_t + \epsilon_{t+\tau}$, where $Rx_{t,t+\tau}$ is the excess stock market return from t to $t+\tau$. An increase in the price-dividend ratio lowers expected stock market returns in excess of the risk-free rate. Moreover, the economic significance and explanatory power of this predictive regression is increasing in disagreement. The slope and R^2 are averages based on one million years of monthly observations for each value of disagreement Δ.

E.5 Black's "Leverage" Effect

One of the most well-established empirical regularities of equity markets is the negative correlation between the return of stocks and their volatility. In a seminal paper, Black (1976) provides financial leverage as compelling explanation for this phenomenon. While this effect, and the leverage-based explanation, have been empirically confirmed by a number of studies more recent papers show that financial leverage is not the only reason for the negative relation between the return of stocks and their volatility (e.g. Hasanhodzic and Lo (2011)). In our model, the negative relation between the stock market return and its volatility is due to the time-variation in consumption/wealth shares.

We derive the local correlation between stock market returns and changes in stock market volatility in Proposition 5. This correlation is usually set to be negative in the derivatives literature (e.g. Heston (1993)) and it is endogenous in our demand disagreement model. Specifically, we know from Proposition 4 and Figure 4 that the stock market volatility is strictly increasing in the consumption share until it attains its maximum at $f_{\sigma_S}^{\max} = 0.35$ and it is strictly decreasing after that. Positive shocks to the consumption share are good news for the stock market and, thus, the correlation between stock returns and their volatilities is positive for low consumption shares and negative otherwise. The left plot of Figure 9 provides a graphic illustration of this correlation conditional on the consumption share for different disagreement Δ.

Proposition 5 (Black's "Leverage" Effect)**.** *Let $f^{max}_{\sigma_S}$ denote the consumption share value that maximizes the conditional stock market volatility in Proposition 4. The local correlation between the return on the stock market and its volatility in equilibrium is*

$$\rho_{LE}(f_t) = corr\left(\frac{dS_t}{S_t}, \frac{d\sigma_{S,t}}{\sigma_{S,t}}\right) = \begin{cases} \frac{\sigma^{\alpha}_{S,t}}{\sigma_{S,t}} & if \quad f_t < f^{max}_{\sigma_S} \\ 0 & if \quad f_t = f^{max}_{\sigma_S} \\ -\frac{\sigma^{\alpha}_{S,t}}{\sigma_{S,t}} & if \quad f_t > f^{max}_{\sigma_S}. \end{cases} \tag{44}$$

The right graph of Figure 9 shows the unconditional mean of the correlation $\rho_{LE}\left(\tilde{f}\right)$ as a function of disagreement Δ. This correlation is zero without disagreement and becomes negative with low disagreement because in this case there is a low probability of high consumption share realizations of impatient investors. This probability increases when we increase disagreement and, thus, the correlation also decreases for high Δ after hitting its minimum around -0.54.

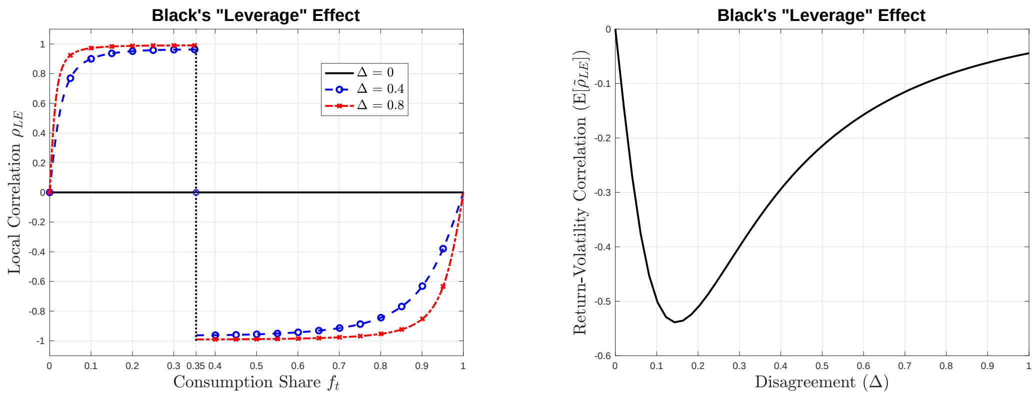


Figure 9: Black's "Leverage" Effect. The left graph shows the local correlation between the stock market return and its volatility as a function of the consumption share f_t for different disagreement Δ and the right graph shows the unconditional distribution of this correlation as a function of disagreement Δ. The unconditional statistic is based on one million years of monthly observations for each value of disagreement Δ.

E.6 Trading Volume and Demand Risk Exposure

Trading volume is difficult to define because it depends on the asset structure. Therefore, we introduce a measure that is independent of the asset structure but nevertheless captures the effects of disagreement on trading volume with the caveat that is not directly comparable to trade in a specific asset. Specifically, Definition 3 below introduces the concept of excess exposure to supply and demand shocks. It basically captures how investors' optimal portfolios deviate from two-fund separation. Based on this measure, we define trading volume as variations of excess exposure and determine it in Proposition 7 below. This measure captures the notion that large changes in excess exposure have to come from changes in underlying asset positions.

Definition 3 (Excess Exposure)**.** *Let $\mu^{\hat{W},i}_{s,t}$ denote the drift, $\sigma^{\hat{W},i}_{Y,s,t}$ the supply shock (output) exposure, and $\sigma^{\hat{W},i}_{\alpha,s,t}$ the demand shock exposure of a patient (type i=a) and impatient (type i=b) investor born at time s with total wealth dynamics*

$$\frac{d\hat{W}^i_{s,t}}{\hat{W}^i_{s,t}} = \mu^{\hat{W},i}_{s,t}\, dt + \sigma^{\hat{W},i}_{Y,s,t}\, dZ_{Y,t} + \sigma^{\hat{W},i}_{\alpha,s,t}\, dZ_{\alpha,t}. \tag{45}$$

Investors' supply and demand shock exposure in excess of their exposure to both shocks trough the value of their endowment stream is defined as

$$XE_{Y,t} = \int_{-\infty}^{t} \nu e^{\nu(t-s)} \left(\alpha_s \beta^{\hat{W},a}_{s,t} |\sigma^{\hat{W},a}_{Y,s,t}| + (1-\alpha_s)\, \beta^{\hat{W},b}_{s,t} |\sigma^{\hat{W},b}_{Y,s,t}|\right) ds - |\sigma^{Y}_{S,t}| \tag{46}$$

$$XE_{\alpha,t} = \int_{-\infty}^{t} \nu e^{\nu(t-s)} \left(\alpha_s \beta^{\hat{W},a}_{s,t} |\sigma^{\hat{W},a}_{\alpha,s,t}| + (1-\alpha_s)\, \beta^{\hat{W},b}_{s,t} |\sigma^{\hat{W},b}_{\alpha,s,t}|\right) ds - |\sigma^{\alpha}_{S,t}|, \tag{47}$$

respectively, where $\beta^{\hat{W},i}_{s,t} = \hat{W}^i_{s,t}/\hat{W}_t$ denotes the individual wealth share of a type i investor.

Total wealth $\hat{W}^i_{s,t}$ consist of financial wealth $W^i_{s,t}$ and the value of the endowment stream $\hat{W}_t$. Investors have the same exposure to the supply and demand shock through their endowments but they have different exposures through their financial wealth. We derive the excess exposure to supply and demand shocks in the next proposition.

Proposition 6 (Excess Exposure)**.** *The individual supply shock exposure, demand shock exposure, and drift of investors' total wealth $\hat{W}^i_{s,t} = W^i_{s,t} + \hat{W}_t$ is $\sigma^{\hat{W},i}_{Y,s,t} = \sigma_Y$, $\sigma^{\hat{W},i}_{\alpha,s,t} = \theta^i_{\alpha,t}$, and $\mu^{\hat{W},i}_{s,t} = r_t + \sigma^2_Y + \left(\theta^i_{\alpha,t}\right)^2 - \rho^i$, respectively. There is no excess exposure to supply shocks, that is, is $XE_{Y,t} = 0$*

but there is excess exposure to demand shocks. Specifically,

$$XE_{\alpha,t} = 2\frac{\phi_b}{\phi_t} f_t \left(1 - f_t\right) \Delta = 2\frac{\phi_b}{\phi_a - \phi_b} \sigma^{\alpha}_{S,t} \geq 0. \tag{48}$$

There is no disagreement on supply shocks and consequently investors do not speculate on it. Hence, investors' fractions of wealth invested in the market portfolio are equally exposed to supply shocks and, thus, there is no excess exposure to these shocks. If investors disagree on demand shocks, then they trade on this disagreement and by doing so invest different fractions of their wealth in the market portfolio even though they have the same risk preferences. The more they deviate from two-fund separation, the larger the excess exposure to demand shocks. Equation (48) shows that this exposure is proportional to the stock market loading onto demand shocks and, thus, inherits all properties of the stock market volatility discussed in Section E.2.

We define trading volume or trading intensity as the square root of the instantaneous quadratic variation of excess exposure to demand shocks. This is similar to the continuous time literature that typically use the quadratic variation of portfolio policies as a measure of the trading intensity (e.g. Grossman and Zhou (1996), Longstaff and Wang (2013), and Ehling and Heyerdahl-Larsen (2017)).

Proposition 7. *Let $QV_{t,T}$ denote the quadratic variation of excess demand risk exposure. Specifically,*

$$QV_{t,T} = \int_t^T dXE_{\alpha,u} dXE_{\alpha,u} = \int_t^T \left(TVol_u\right)^2 \, du. \tag{49}$$

Then $TVol_t$ denotes the trading volume or trading intensity. Specifically,

$$TVol_t = 2\phi^b \Delta^2 \frac{f_t(1-f_t)}{\phi_t^2} |\phi^B \left(1 - f_t\right)^2 - \phi^A f_t^2|. \tag{50}$$

The left graph of Figure 10 shows that trading volume is a non monotone function of the consumption share that is increasing in disagreement. Moreover, the right graph of Figure 10 shows that the unconditional mean and volatility of trading volume is strictly increasing in disagreement.

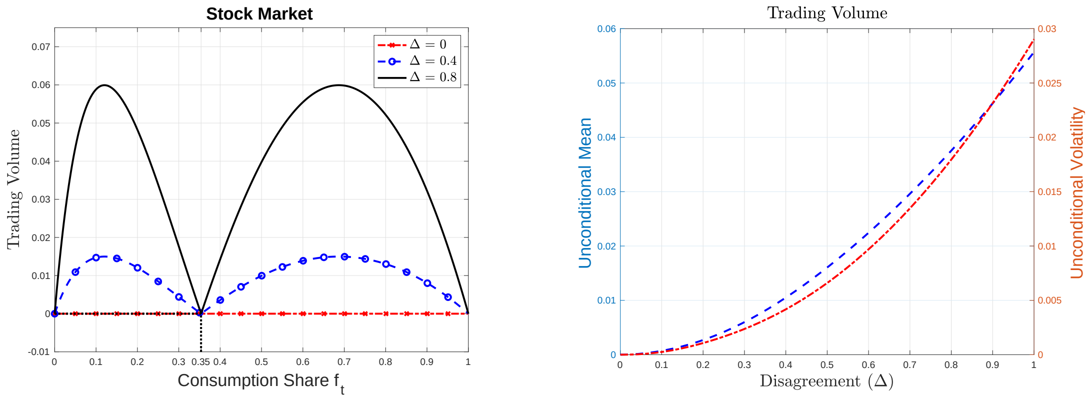
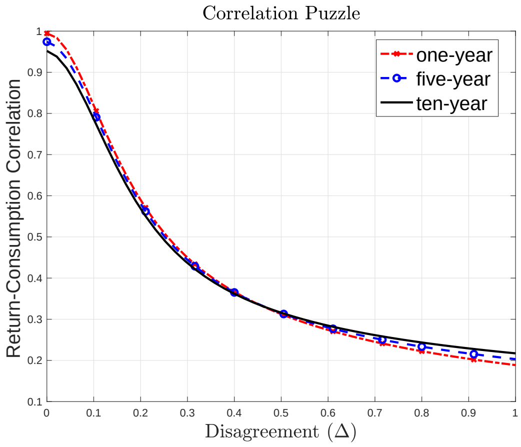


Figure 10: *Trading volume or trading intensity.* The left graph shows trading volume/intensity as a function of the consumption share f_t for different disagreement Δ and the right graph shows the unconditional mean and volatility of it as a function of disagreement Δ. Trading volume is a non monotone function of the consumption share but both its mean and volatility are strictly increasing in disagreement. The unconditional statistics are based on one million years of monthly observations.

F Correlation Puzzle

In classical asset pricing models, all shocks to fundamentals are shocks to the supply side of the economy, e.g. output shocks in Lucas (1978) type endowment economies or productivity shocks in Jermann (1998) type production economies. Hence, in these models stock market returns are highly correlated with measures of output growth, which is in stark contrast to the weak short and long term correlations between stock returns and both consumption and GDP growth observed in the data. Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2005) refer to this finding as the "correlation puzzle". For instance, Table 1 shows that the correlation between stock market returns and consumption at the one, five, and ten year horizon does not exceed 30% in the data. While the correlation between stock market returns and dividends is much higher and increasing with the horizon, it is with 65% still significantly lower than predicted by classical asset pricing models. Table 1 also shows that there is a similar disconnect between supply side fundamentals and both interest rates and trading volume. Similar to stock market returns, classic asset pricing models lead to a counterfactual high correlation between consumption growth and interest rates. The implications of classical asset pricing model for trade are twofold. There is either no trade, such as in the representative agent models of Campbell and Cochrane (1999), Bansal and Yaron (2004), and Tsai and Wachter (2015) or there is trade such as in asset pricing models in which agents have different preferences or beliefs (e.g. Chan and Kogan (2002), David (2008), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018), and Atmaz and Basak (2018)). The models in the latter case imply a high correlation between trading volume and measures of output growth, which is inconsistent with the weak short and long term correlations between stock market trading volume and dividends, as well as consumption growth reported in Table 1. Our



Horizon	Dividends	Consumption
	Stock Returns	
1 year	0.08	0.00
5 year	0.46	0.28
10 year	0.65	0.11
	Trading Volume	
1 year	0.21	0.29
5 year	0.35	0.38
10 year	0.31	0.40
	Interest Rate	
1 year	0.26	-0.16
5 year	0.20	0.04
10 year	0.24	0.06

Figure 11 & Table 1: Correlation puzzle. The figure shows the unconditional correlation between stock market returns and aggregate consumption for a one, five and ten year horizon as a function of disagreement Δ. The correlations are based on one million years of monthly observations. The table shows the correlation between dividend and consumption growth and (i) stock market returns, (ii) stock market trading volume, and (iii) the realized real interest rate. Stock market, dividend, consumption, and interest rate data are from Shiller's webpage and stock market trading volume is from the NYSE webpage. We consider annual frequency over the period beginning 1891 until end of 2009.

demand disagreement model can reconcile the low correlation between output growth and stock market returns and it leads to no correlation between output growth and both the risk-free rate and trading volume. Specifically, Figure 11 shows the correlation between stock market returns and aggregate consumption for the one, five, and ten year horizon as a function of disagreement Δ. The figure shows that without disagreement, the correlations are close to one. Hence, demand shocks alone are not sufficient to solve the correlation puzzle. When disagreement Δ increases, then the correlation decreases at an increasing rate, and thus for reasonable Δ, we get correlations comparable to the ones we see in the data.

What is the economic intuition for this low correlation in our model? Suppose there are no demand shocks, then the price dividend ratio is constant and stock market returns are perfectly correlated with output shocks. When there are commonly perceived demands shocks, then the price dividend ratio is stochastic but its dynamics are locally deterministic; hence short term correlations between stock market returns and consumption are close to one. The correlation is 0.95 when measured over ten years and, thus, the indirect effect of the demand shock through the drift of the consumption share is quantitatively small. The indirect effect is small because heterogeneous time preferences only lead to different consumption-savings rates. In contrast, when agents have different beliefs about demand shocks, they engage in speculative trade, thus changing their consumption-saving rates and portfolio compositions. Demand shocks, which are by assumption independent of output shocks, lead to shocks to the price-dividend ratio, the risk free rate, the volatility and risk premium of the stock market, and trading volume.[25] Hence, demand disagreement breaks the tight link between shocks to output growth and stock market returns and solves the correlation puzzle. Similar to the risk-free rate, disagreement and the resulting trade in the stock market is purely driven by demand shocks and, hence, the correlation between macroeconomic fundamentals and both the risk-free rate and trading volume, which is also low in the data, is by construction zero.[26]

An important implication of models with disagreement about fundamentals is that this disagreement is highly correlated with disagreement about future prices. This link seems difficult to break unless investors disagree on the functional form of the demand equation that links optimal consumption to output variables. Table 2 shows that it is necessary to break this link because disagreement about nominal interest rates is not spanned by disagreement about inflation and GDP growth even after accounting for a possible nonlinear relation as in Feldhütter, Heyerdahl-Larsen, and Illeditsch (2018) which we control for by including higher order terms. The R^2 of the disagreement regressions in Table 2 are significantly below one, which is consistent with the implications of our model that implies disagreement about the future time discount rate of the market that links aggregate consumption to state prices. Hence, our model also solves the "correlation puzzle" for disagreement.

[25]We endogenize this correlation in Section IV.B.

[26]It is straightforward to increases this correlation by adding disagreement about output growth.

Table 2: Disagreement correlation puzzle. This table shows the results from regressions of disagreement about the three-month Tbill rate on inflation disagreement (Δ_I) and disagreement about GDP growth (Δ_G). Disagreement is measured by the interquartile range of individual forecasts compiled by the Survey of Professional Forecasters (SPF). The t-statistics are reported in parenthesis. Quarterly sample: Q3-1981 – Q2-2017.

	Const.	Δ_G	Δ_I	Δ_G^2	Δ_I^2	$\Delta_G\Delta_I$	R^2_{adj}
Levels	-0.003	0.538	0.355				0.483
	(-1.676)	(3.600)	(2.327)				
	0.007	-0.595	-0.760	19.931	3.370	87.823	0.600
	(2.952)	(-1.167)	(-1.503)	(0.731)	(0.093)	(2.836)	
Changes	-0.000	0.065	0.312				0.073
	(-0.804)	(0.493)	(1.790)				
	-0.000	0.085	0.363	35.124	-35.407	32.046	0.090
	(-0.717)	(0.620)	(1.968)	(2.121)	(-1.131)	(0.839)	

IV Robustness and Extensions

In this section, we show that the predictions of our demand disagreement model are robust to endogenizing disagreement and aggregate consumption.

A Time-Varying Disagreement

So far we have assumed that the agents have dogmatic beliefs about the long-run mean of l_t, and thus the fraction of patient newborns α_t. In this section, we relax this assumption and nevertheless derive the equilibrium in closed form. Specifically, let $\Delta^i_{s,t}$ be the estimation error at time t of a type i agent born at time s and so the agents' perceived shocks are related to the true shock by the equation $dz^i_{s,t} = dz_{\alpha,t} - \Delta^i_{s,t}dt$. The likelihood ratio, $\eta^i_{s,t}$, which allows us to change from the belief of agent i born at time s to the data generating probability measure has the dynamics $d\eta^i_{s,t} = \Delta^i_{s,t}\eta^i_{s,t}dz_{\alpha,t}$. The next Proposition characterizes the dynamics of the stochastic discount factor and the stock price dynamics with this general disagreement process.

Proposition 8. *The dynamics of the state price density under the data generating measure, ξ_t, is given in Equation (30). Moreover, the real short rate, r_t, and the market price of supply shocks, $\theta_{Y,t}$, take the same form as before and are given in Equations (25) and (27), respectively. The* *market price of demand shocks under the data generating measure is*

$$\theta_{\alpha,t} = -\bar{\Delta}_t = -\left(f_t \bar{\Delta}^a_t + (1-f_t)\,\bar{\Delta}^b_t\right), \tag{51}$$

where f_t is the fraction of output consumed by all agents of type a, defined in Equation (22). The within type i consumption share weighted average estimation error is

$$\bar{\Delta}^i_t = \int_{-\infty}^{t} \bar{f}^i_{s,t} \Delta^i_{s,t} ds, \qquad i = a, b, \tag{52}$$

where the within group consumption share densities are

$$\bar{f}^a_{s,t} = \left(\frac{1}{f_t}\right) \nu e^{-(\rho^a+\nu)(t-s)} \alpha_s \beta^a_s \left(\frac{\eta^a_{s,t}}{\eta^a_{s,s}}\right) \left(\frac{Y_s}{Y_t}\right) \left(\frac{\xi_s}{\xi_t}\right) \tag{53}$$

$$\bar{f}^b_{s,t} = \left(\frac{1}{1-f_t}\right) \nu e^{-\left(\rho^b+\nu\right)(t-s)} (1-\alpha_s)\,\beta^b_s \left(\frac{\eta^b_{s,t}}{\eta^b_{s,s}}\right) \left(\frac{Y_s}{Y_t}\right) \left(\frac{\xi_s}{\xi_t}\right) \tag{54}$$

with $\beta^i_t = (\rho^i + \nu)\,\phi_t$ and the wealth-consumption ratio, ϕ_t, as before. The stock price dynamics are given in Proposition 4 where the constant disagreement, Δ, is replaced with the stochastic disagreement $\Delta_t = \bar{\Delta}^a_t - \bar{\Delta}^b_t$.

The consumption share, f_t, is still an endogenous state variable but the within group consumption share weighted estimation errors replace the constant estimation errors from the previous section. Importantly, the risk-free rate and market prices of risk take similar forms as in the case without learning. Although it is sufficient to know α_t, f_t and $\bar{\Delta}_t$ to characterize the equilibrium at any point in time, the dynamics of f_t and $\bar{\Delta}_t$ do not form a Markov system, but require the knowledge of every agent's belief. We now consider a special case where agents learn from their own experience.

A.1 Demand Disagreement with Learning from Experience

In the baseline, model we made two assumptions about preferences and beliefs. First, there is a strong link between preferences and beliefs as agents disagree across but not within preference types. Second, agents do not learn. In this section, we relax both of these assumptions by allowing agents to learn from their own experience as in Ehling, Graniero, and Heyerdahl-Larsen (2018).

We assume that an agent of type i born at time s has a normally distributed prior about the long-run mean of l_t, with mean $\bar{l}^i$ and variance V. Hence, everyone has the same prior variance, V, but patient and impatient agents might have different prior means, $\bar{l}^i$. The next proposition characterizes the time-t estimation error of an agent born at time s.

Proposition 9. *The time-t estimation error of an agent of type $i = a, b$ born at time s is*

$$\Delta^i_{s,t} = \frac{\sigma_A^2}{\sigma_A^2 + V(t-s)} \Delta^i_{s,s} + \frac{V}{\sigma_A^2 + V(t-s)} \left(Z_{\alpha,t} - Z_{\alpha,s} \right), \tag{55}$$

where $\sigma_A = \frac{\sigma_l}{\kappa}$ and $\Delta^i_{s,s} = \frac{\bar{l}^i - \bar{l}}{\sigma_A}$. Moreover, we have that $\lim_{t\to\infty} \Delta^i_{s,t} = 0$. Initial disagreement is $\Delta = \Delta_{s,s} = \Delta^a_{s,s} - \Delta^b_{s,s}$.

From Proposition 9, we see that agents eventually learn the true long-run mean. However, this will typically take a long time, and therefore agents keep making mistakes for many years. Moreover, young agents update their beliefs more aggressively than older agents. To examine the effects of learning, we consider three different values of initial disagreement $\Delta = (0, 0.4, 0.8)$. If the common prior variance, V, is zero, then agents do not learn and we are back to the dogmatic beliefs case. However, once V is positive, agents learn from their own experience and therefore different cohorts disagree, even without any initial disagreement Δ.

How does learning from experience affect the consumption share of patient and impatient investors? In contrast to the baseline model, investors learn over their lifetime and, thus, their beliefs are more accurate when they are older. Hence, more experienced investors are expected to gain wealth from less experienced investors. While this is true for both patient and impatient investors, the speculative profits are expected to be higher for patient than for impatient investors because they save more over time and therefore they are wealthier when their beliefs are more accurate. The left graph of Figure 12 confirms this intuition. Specifically, it shows that even without any initial disagreement (black solid line) the unconditional mean of the consumption share, $\tilde{f}$, is increasing with the prior variance except for an initial drop that is due to the fact that everybody is born with the true prior. Similarly, the average consumption share is increasing in the prior variance V if we allow for initial disagreement as the blue dashed circle line and the red chain-dotted crossed line in the left graph in Figure 12 show. In contrast to the slight initial drop of the average consumption share when everybody starts with the correct prior, there is a stark initial increase of the average consumption share because investors initially disagree and, thus, have incorrect initial beliefs.

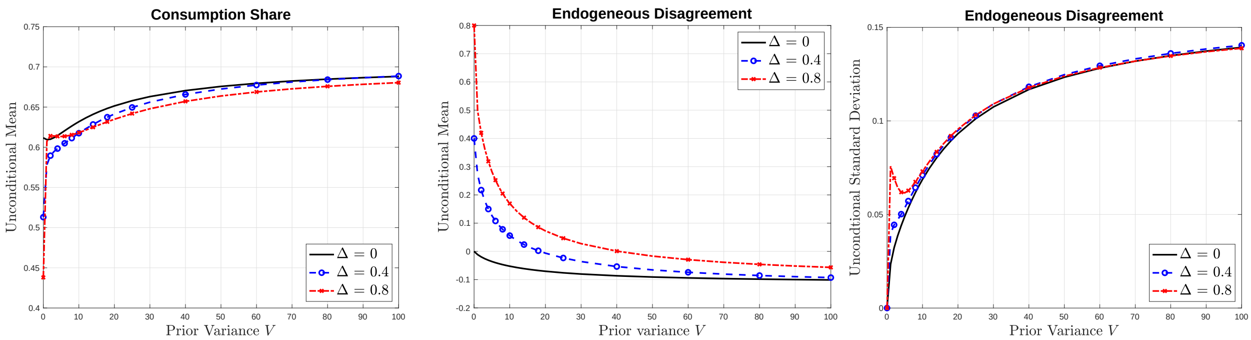


Figure 12: *Unconditional moments of the consumption share and disagreement.* This figure shows the unconditional mean of the consumption share (left), the unconditional mean of disagreement (middle) and the unconditional standard deviation of disagreement (right) as a function of the prior variance V for different initial disagreement Δ. All other parameters are as in the baseline calibration. The summary statistics are based on one million years of monthly observations for each prior variance value V.

The middle and right graph of Figure 12 show the unconditional mean and volatility of disagreement between patient and impatient investors Δ_t, respectively. As expected, this disagreement converges towards zero when $\Delta \neq 0$. However, this average is not zero even when all agents are born with the true prior (black solid line) because there is a correlation between the fraction of patient newborns α_t and disagreement Δ_t. Importantly, the right plot shows that the standard deviation is increasing even for the case with no initial disagreement. This implies that "typically" there is more disagreement when the prior variance, V, is high even when the two groups start we the same beliefs. Hence, the model with learning from experience endogenously creates disagreement between the two groups of investors and as we show below implies similar asset pricing implications as the case without learning.

Figure 13 shows the average risk free rate (left plot), the standard deviation of the market return (middle plot) and the risk premium on the market (right plot) as we vary the prior variance V for initial disagreement $\Delta = 0, 0.4, 0.8$. As the prior variance increases, initial disagreement has less impact. If there is no initial disagreement ($\Delta = 0$), then the standard deviation and risk premium are increasing in V which implies more disagreement and so is similar to the case without learning. In contrast, the risk-free rate is decreasing with disagreement measured by V because the average consumption share of patient investors increases.

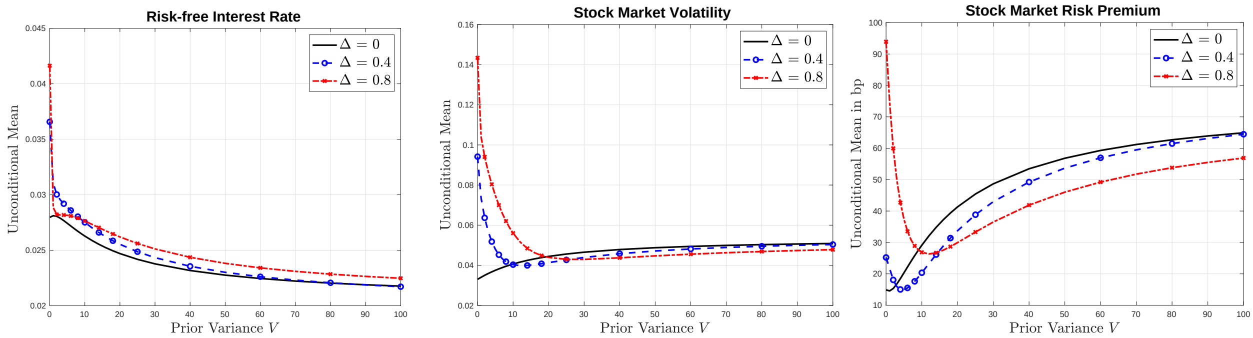


Figure 13: *Asset pricing with learning from experience.* The figure shows the unconditional mean of the risk free rate (left plot), stock market volatility (middle plot), and stock market risk premium (right plot) as a function of the prior variance (disagreement) for different initial disagreement Δ set to 0, 0.4 and 0.8. All other parameters are as in the baseline calibration. The summary statistics are based on one million years of monthly observations for each prior variance value V.

B Production economies

So far we have examined an exchange economy where the correlation between aggregate consumption and demand shocks is by assumption zero. Hence, it may not be surprising that there is no correlation puzzle in an exchange economy with preference shocks, even though we emphasize in Section III.F that adding these shocks without disagreement is not sufficient to solve the correlation puzzle. Hence, we endogenize aggregate consumption growth in this section and show that all the qualitative predictions of our demand disagreement model still hold. Interestingly, we find that there may be even a low negative correlation between aggregate consumption and valuation ratios for some adjustment cost parameters for investments, yet there is a positive risk premium for demand shocks.[27]

As in the exchange economy, patient and impatient newborns receive the same initial allocation.

[27] The production model of this section is based on the macroeconomic model with a financial sector in Brunnermeier and Sannikov (2014).

However, rather than getting a Lucas tree, they receive capital. The dynamics of capital are $dK_t = (\psi(\iota_t) + \mu_K - \delta) K_t dt + \sigma_K K_t dZ_{K,t}$. Each unit of capital produces a units of the consumption good, and therefore total output is $Y_t = aK_t$. The expected growth rate of capital consists of an exogenous part, $\mu_K - \delta$, and an endogenous part $\psi(\iota_t)$. The difference between the productivity growth of capital, μ_K, and the depreciation rate, δ, is exogenous and the function, $\psi(\iota) = \frac{log(\kappa\iota+1)}{\kappa}$, measures by how much an increase in the investment rate per unit of capital, ι_t, raise the expected growth rate of capital and is thus endogenous. The parameter, κ, measures adjustment cost and so there are no adjustment cost when κ approaches zero and the model of production converges to an exchange economy with output growth rate $\mu_K - \delta = 0.02$ and output volatility $\sigma_K = 0.033$ if it approaches infinity. Aggregate consumption C_t is equal to output Y_t minus aggregate investment I_t, that is, $C_t = Y_t - I_t = (a - \iota_t) K_t$. The equilibrium price of capital is denoted Q_t and hence the total value of capital at any point in time is $S_t = Q_t K_t$, which we refer to as the stock market value. We set the productivity parameter to $a = 0.05$ and vary the adjustment cost parameter between 0 and 1000, where the latter value gets us close to the exchange economy. We solve for the equilibrium in closed form and provide details in the Internet Appendix.

The dynamics of the consumption share, f_t, does not change if we move from an exchange economy to a production economy because endogenizing output does not change how it is split up among patient and impatient consumers. In contrast to the exchange economy, aggregate consumption is now exposed to demand shocks. Specifically, its dynamics are $dC_t = \mu_{C,t} C_t dt + \sigma^K_{C,t} C_t dZ_{K,t} + \sigma^\alpha_{C,t} C_t dZ_{\alpha,t}$. The lower right graph of Figure 14 shows the unconditional mean of the demand shock exposure of aggregate consumption, $\sigma^\alpha_{C,t}$ as a function of the adjustment cost parameter κ. Importantly, the demand shock exposure is negative and approaches zero as we increase κ because a demand shock increases the consumption share of patient investors and thus lowers the discount rate. A lower discount rate increases investment, but as aggregate output has no exposure to demand shocks this increase comes at the expense of lower consumption growth.

In contrast to consumption, demand shocks are good news for the stock markets as they lower discount rates. Hence, the unconditional mean of the stock market's exposure to demand shocks is positive as in the exchange economy which we show with the bottom left graph of Figure 14. While both consumption growth and stock returns have the same exposure to the productivity

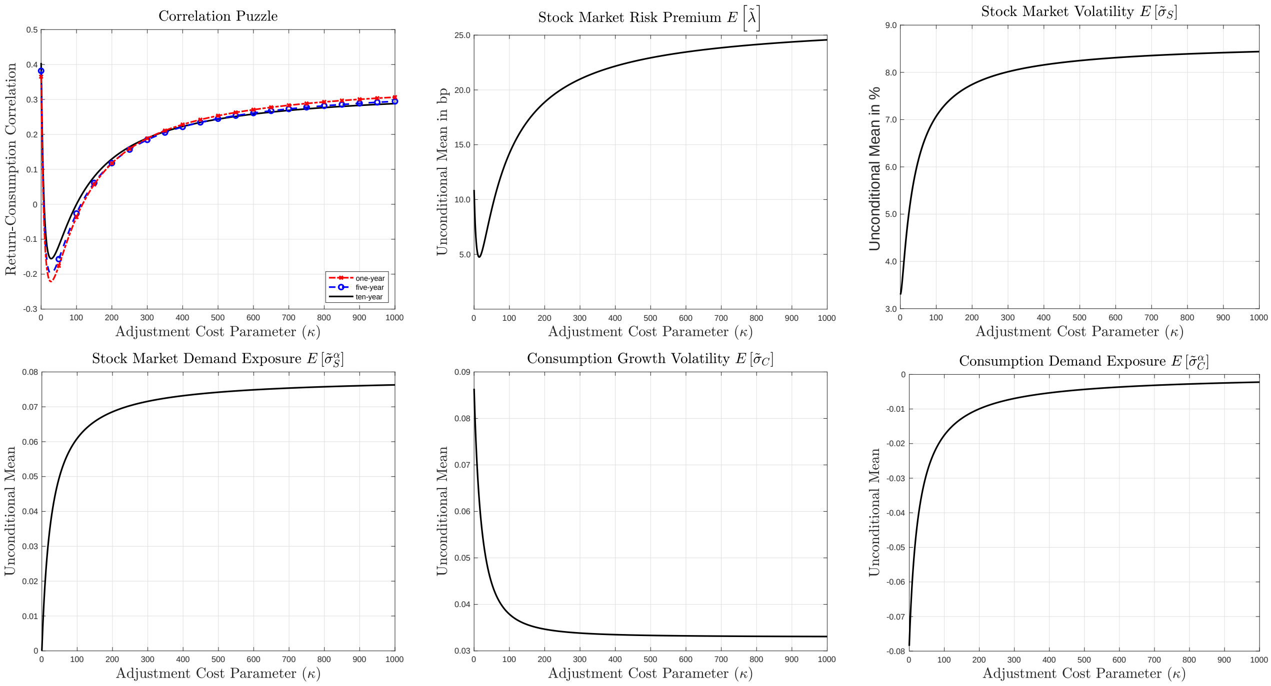


Figure 14: *Demand disagreement in a production model.* The graphs show the unconditional mean of the return-consumption correlation, the stock market risk premium, the stock market volatility, the stock market demand exposure, the consumption growth volatility, and the consumption demand exposure as a function of the adjustment cost parameter κ. The summary statistics are based on one million years of monthly observations for each value of κ.

of capital they have different exposure to demand shocks. The top left graph shows that this leads to a low correlation between stock returns and consumption growth that in contrast to an exchange may even be negative. Interestingly, the risk premium for demand shocks is still positive even though aggregate consumption loads negatively on them. The intuition is the same as in the baseline model with the exception that the negative loading partly offsets it. However, it is not strong enough to completely offset the effect, and therefore the risk premium is positive.

V Conclusion

We develop an overlapping generations model where the cross-sectional distribution of investors' preferences and beliefs changes over time. Investors have incomplete information and disagree on these changes which leads to disagreement about future asset demand even if economic fundamentals are known. This demand disagreement leads to speculative trade that is not based on macroeconomic fundamentals and to demand shocks that are priced. We show that demand disagreement can account for several stylized facts of financial market such as the low correlation between asset returns and economic fundamentals, a time varying equity premium, excess stock market volatility, a low mean and volatility of the risk-free rate, valuation ratios predicting returns, Black's leverage effect, and high trading volume unrelated to economic fundamentals, even in a setting with i.i.d. consumption growth and without hedging demands, preferences for early/late resolution of uncertainty, habit formation, or risk of rare disasters. We also consider two extensions of our demand disagreement model in order to endogenize demand disagreement and aggregate consumption and show that our results are robust to these extensions.

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A Proofs

Proof of Proposition 1. Optimal consumption satisfies $c^i_{s,t} = (\nu + \rho^i)\hat{W}_{s,t}$ and every investor is entitled to the same endowment stream worth $\hat{W}_t$ at time t and thus plugging in for $\beta^i_{s,t}$ proves equation (20). All investors are born with no financial wealth and thus plugging in for β_s proves equation (21). □

Proof of Theorem 1. To derive the wealth-consumption ratio, we multiply the consumption share f_t and $(1-f_t)$ with ϕ_a and ϕ_b, respectively. Then we add up both terms and impose market clearing, that is, total financial wealth of all investors alive has to be zero. This proves equation (26) of the wealth-consumption ratio. To derive the stochastic discount factor of the patient investor we plug optimal consumption of patient (type a) and impatient (type b) agents (see equation (18)) into the resource constraint

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s c^A_{s,t} + (1-\alpha_s) c^B_{s,t}\right) \, ds = Y_t \tag{56}$$

and use the fact that $\xi^b_t = \xi^a_t/\eta_t$ which leads to

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \beta^a_s e^{-\rho^a(t-s)} + (1-\alpha_s)\, \beta^b_s e^{-\rho^b(t-s)} \frac{\eta_t}{\eta_s}\right) Y_s \frac{\xi^a_s}{\xi^a_t} \, ds. \tag{57}$$

Let $\xi^a_t = \hat{X}^a_t / Y_t$ and multiply both sides of equation (57) with ξ^a_t. It follows that $\hat{X}^a_t$ has to solve the integral equation

$$\hat{X}^a_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \beta^a_s e^{-\rho^a(t-s)} + (1-\alpha_s)\, \beta^b_s e^{-\rho^b(t-s)} \frac{\eta_t}{\eta_s}\right) \hat{X}^a_s \, ds. \tag{58}$$

Moreover, $\hat{X}^a_t$ can be decomposed into two components, that is, $\hat{X}^a_t = \hat{X}^{a,a}_t + \hat{X}^{a,b}_t$ where $\hat{X}^{a,a}_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \alpha_s \beta^a_s e^{-\rho^a(t-s)} \hat{X}^a_s \, ds$ and $\hat{X}^{a,b}_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} (1-\alpha_s) \beta^b_s e^{-\rho^b(t-s)} \frac{\eta_t}{\eta_s} \hat{X}^a_s \, ds$, respectively. It follows that the consumption share of patient and impatient investors is $f_t = \hat{X}^a_t / \hat{X}^{a,a}_t$ and $1 - f_t = \hat{X}^a_t / \hat{X}^{a,b}_t$, respectively. Applying Itô's lemma to $\xi^a_t = \hat{X}^a_t / Y_t$ and using equation (58) leads to the dynamics of the stochastic discount factor ξ^a_t and, thus, the real short rate, the market price of supply shock risk, and market price of demand shocks perceived by patient investor. To proof of the dynamics of the state price density ξ^b_t is analogue to the one fore ξ^a_t and thus omitted. Plugging into to fundamental pricing equation leads to expected excess return of the supply asset and to the expected return of the demand asset perceived by patient and impatient investors. The return dynamics of total wealth are equal to the dynamics of consumption of every investor and so plugging in the FOC for consumption, applying Itô's lemma and comparing the exposures of individual consumption dynamics to supply and demand shocks with the exposures of the dynamic budget constraint to these shocks leads to the optimal investment in the supply asset, demand asset, and riskfree asset. This concludes the proof of this theorem. □

Proof of Proposition 2. Applying Itô's lemma to $\xi_t = \xi^a_t \eta^a_t$ leads to the dynamics of the SDF under the belief of the econometrician given in equation (30) with market price of demand shock risk given in equation (31). □

Proof of Proposition 3. Applying Itô's lemma to $f_t = \hat{X}_t^a/\hat{X}_t^{a,a}$ given in the proof of Theorem 1 and changing measure from $Z_{\alpha,t}^a$ to $Z_{\alpha,t}$ leads to the dynamics of the consumption share with drift and volatility given in equation (32). □

Proof of Proposition 4. The wealth-consumption ratio is $\phi_t = \phi_a f_t + \phi_b(1 - f_t)$ and thus applying Itô's lemma to the price of the market portfolio $S_t = Y_t\phi_t$ and using the dynamics of the consumption share f_t given in Proposition 3 leads to the exposures of stock market returns to the supply ($\sigma_{S,t}^Y$ and demand ($\sigma_{S,t}^\alpha$) shock as well as the risk premium $\lambda_{S,t}$. $\sigma_{S,t}^\alpha$ is a concave function of the consumption share f_t and, thus, taking the first derivative w.r.t. f leads to the global maximum of the demand shock exposure and thus the the conditional stock market volatility. □

Proof of Proposition 5. The conditional stock market volatility is a function of the consumption share f_t given in Proposition 4] and thus applying Itô's lemma to it and computing the quadratic covariation with the stock market returns leads to the conditional correlation given in equation (44). □

Proof of Proposition 6. By market clearing we have that total aggregate wealth is equal to the stock market and thus applying Itô's lemma to market clearing condition leads to zero exposure of investors' portfolios to supply shocks and to demand shock exposure given in equation (48) . □

Proof of Proposition 7. Applying Itô's lemma to $\mathrm{XE}_{\alpha,t}$ with dynamics given in equation (48) leads to the local volatility which is equal to trading volume in equation (50). □

We provide a detailed analysis of our two model extensions (i) time-varying disagreement and (ii) production economies in the internet appendix and thus the reader is referred to the online appendix for the proofs of Propositions 8 and 9.