Optimal Deposit Insurance*

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Abstract

This paper studies the optimal determination of deposit insurance (DI) when bank runs are possible. In a variety of environments, the welfare impact of changes in the level of deposit insurance coverage exclusively depends on three sufficient statistics: the sensitivity of the likelihood of bank failure with respect to the level of DI, the utility gain induced by preventing the marginal bank failure, which can be expressed in terms of the drop in depositors' consumption, and the direct social cost of intervention in bank failure scenarios, which can be expressed in terms of the probability of bank failure, the marginal cost of public funds, and the mass of partially insured depositors. The same expression applies a) when banks face perfect ex-ante regulation and b) when banks are not allowed to react to policy changes. Under imperfect regulation, because banks do not internalize the fiscal impact of their actions, changes in the behavior of banks induced by varying the level of DI (often referred to as moral hazard) only affect the level of optimal DI directly through a fiscal externality, but not independently. We characterize the wedges that determine the optimal ex-ante regulation, which map to liability-side regulation (e.g., deposit insurance premia) and asset-side regulation. Finally, we explore the quantitative implications of our approach through a direct measurement exercise and a model simulation.

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1 Introduction

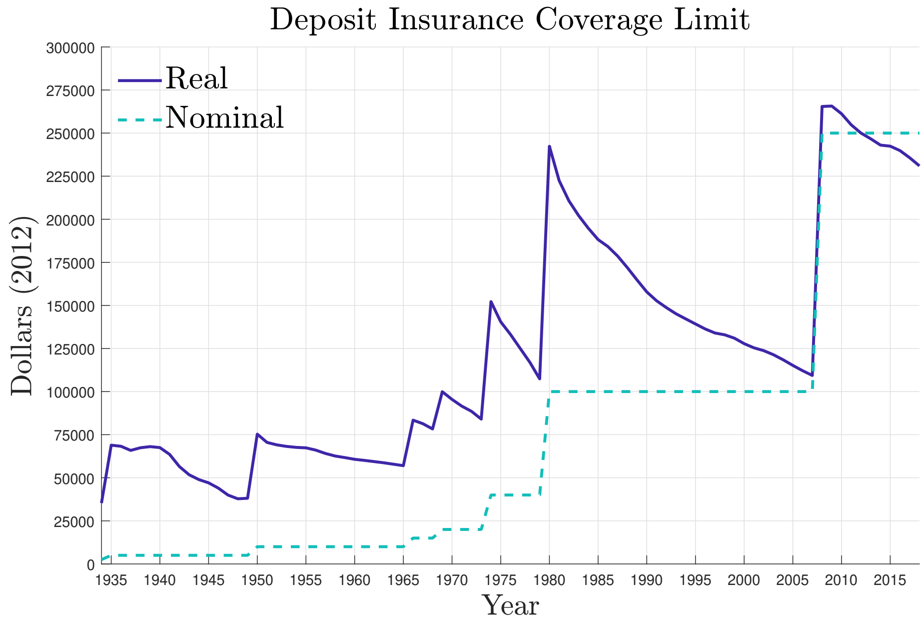
Bank failures have been a recurrent phenomenon in the United States and many other countries throughout modern history. A sharp change in the United States banking system occurs with the introduction of federal deposit insurance in 1934, which dramatically reduced the number of bank failures. For reference, more than 13,000 banks failed between 1921 and 1933, of which 4,000 banks failed in 1933 alone. In contrast, a total of 4,057 banks have failed in the United States between 1934 and 2014.[1] In many other countries, the design of deposit insurance schemes is still in progress and is the subject of ongoing debates; see, e.g., Demirgüç-Kunt, Kane and Laeven (2014) for a recent account of deposit insurance systems around the world. As of today, deposit insurance is a crucial pillar of financial regulation in most economies and represents the most salient explicit government guarantee to the financial sector.

Despite its success in reducing bank failures, deposit insurance entails costs when it has to be paid and affects the ex-ante behavior of market participants – these responses to the policy are often referred to as moral hazard. Hence, in practice, deposit insurance only guarantees a fixed level of deposits. As shown in Figure 1, this level of coverage has changed over time in the US. Starting from the original $2,500 in 1934, the nominal insured limit per account in the US has been $250,000 since October 2008. A natural question to ask is how the level of this guarantee should be determined to maximize social welfare. In particular, what is the optimal level of deposit insurance coverage? Are $250,000, the current value in the US, or €100,000, the current value in most European countries, the optimal levels of deposit insurance coverage for these economies? How should emerging economies set their insured limits? Which variables ought to be measured to optimally determine the level of deposit insurance coverage in a given economy?

This paper provides an analytical characterization, written as a function of observable or potentially recoverable variables, which directly addresses those questions. Although existing research has been effective at understanding several of the theoretical tradeoffs associated with deposit insurance, a general framework that incorporates the most relevant tradeoffs and that can be used to provide explicit guidance to policymakers when facing these questions has been missing. With this paper, we provide a first step in that direction.

We initially derive the main results of the paper in a version of the canonical model of bank runs of Diamond and Dybvig (1983), augmented to consider depositors who hold different levels of deposits. In our baseline framework, banks offer a predetermined interest rate on a deposit contract to share risks between early and late depositors in an environment with aggregate uncertainty about the profitability of banks' investments. Due to the demandable nature of the deposit contract, depending on the aggregate state, both fundamental-based and panic-based bank failures are possible. Mimicking actual deposit insurance arrangements, we assume that deposits are guaranteed by the government up to a deposit

[1] These values come from the FDIC Historical Statistics on Banking. Weighting bank failures by the level of banks' assets or correcting by the total number of banks still generates a significant discontinuity on the level of bank failures after the introduction of deposit insurance.



Note: Figure 1 plots the evolution of the level of deposit insurance coverage from 1934 until 2018 in nominal and real terms. Real values are reported in 2012 dollars using a consumption expenditure deflator.

Figure 1: Evolution of Deposit Insurance Coverage Limit

insurance limit of δ dollars and then focus on the implications for social welfare of varying the level of coverage δ. We also initially assume that any funding shortfall associated with deposit insurance payments entails a distortionary fiscal cost.

After characterizing how changes in the level of coverage δ affect equilibrium outcomes, in particular depositors' withdrawal choices and bank failure probabilities, we focus on the welfare implications of varying δ. It is best to introduce our results by describing the determinants of the marginal impact of changes in the level of coverage on social welfare, $\frac{dW}{d\delta}$, which can be written as a function of a few sufficient statistics as follows:

$$\begin{aligned}\frac{dW}{d\delta} = & -\text{Sensitivity of bank failure probability to a change in } \delta \times \text{Utility gain of preventing marginal failure} \\ & -\text{Probability of bank failure} \times \text{Expected marginal social cost of intervention}\end{aligned} \tag{1}$$

Equation (1), which embeds the fundamental tradeoffs regarding the optimal determination of deposit insurance, should be interpreted as a directional test to determine whether to increase or decrease the level of coverage, starting from a given level. On the one hand, when a marginal change in δ substantially reduces the likelihood of bank failure, at the same time that there are significant gains from avoiding a marginal bank failure, it is optimal to increase the level of coverage. On the other hand, when bank failures are frequent and when the social cost of ex-post intervention associated with them – for instance, because it is very costly to raise resources through distortionary taxation – is substantial, it is optimal to decrease the level of coverage.

Our formulation in terms of sufficient statistics is appealing for two reasons. First, conceptually, as we describe below, we show that the same characterization of the marginal welfare impact of change in the level of coverage is valid for a large set of primitives. In that sense, the high-level variables that we identify are not specific to a particular set of modeling assumptions. Second, in practice, it is possible to directly infer or recover the different elements that determinate $\frac{dW}{d\delta}$. By directly measuring the variables in Equation (1), our framework provides direct guidance to policymakers regarding which variables ought to be measured to determine the optimal level of deposit insurance. Once the relevant variables are known, the policymaker does not need any other information to consider changes in the level of coverage.

Our characterization can also be used to derive several analytical insights. In particular, we show that in an environment in which banks never fail and government intervention is never required in equilibrium, it is optimal to guarantee deposits fully. This result, which revisits the classic finding by Diamond and Dybvig (1983), follows from Equation (1) when the probability of bank failure tends towards zero. We also show that the conditions under which a non-zero or a maximal level of coverage are optimal are subtle, and crucially depend on the endogenous relation between the probability of bank failure and the level of coverage and the properties of the fiscal cost in case of failure.

Although we initially derive our results when banks' deposit rates are predetermined, we also study the scenarios in which banks face no ex-ante regulation or perfect ex-ante regulation. First, we show that the changes in the behavior of unregulated competitive banks to the policy (often referred to as moral hazard) only modify the optimal policy formula directly through a fiscal externality caused by banks.[2] Next, we use our framework to explore the optimal ex-ante regulation, which in practice corresponds to optimally setting deposit insurance premia or deposit rate regulations. In particular, we show that the optimal ex-ante regulation, which requires to jointly restrict banks' asset and liability choices, is designed so that banks internalize the fiscal externalities of their actions. We characterize the wedges that banks must face when the optimal ex-ante regulation is implemented, sharply distinguishing between the corrective and revenue-raising roles of ex-ante regulations. In practice, our results imply that deposit insurance premia, even if optimally determined, is not sufficient when banks can adjust their asset allocation, so regulating banks' asset allocations is necessary.[3] Our results also imply that fairly priced deposit insurance is neither necessary nor sufficient for the optimal regulation.

To show the applicability of our results in practice, we study the quantitative implications of our results for the optimal coverage level in the US. We approach the quantification process in two different ways. First, we provide direct measures of the sufficient statistics that we identify and implement the test that determines whether it is optimal to increase or decrease coverage. This approach has the advantage of sidestepping the need to specify model parameters and functional forms, but it faces

[2] We use the term fiscal externality to refer to the social resource cost associated with the need to raise funds through distortionary taxation, as in the public finance literature. This result does not contradict common wisdom, which emphasizes the role of moral hazard as the primary welfare loss created by having a deposit insurance system. Our results simply show that the changes in banks' behavior associated with changes in the level of coverage are subsumed into the sufficient statistics that we identify. In other words, even though high levels of coverage can induce unregulated banks to make decisions that will increase the likelihood and severity of bank failures, only its effects through the fiscal externality that we identify have a first-order impact on welfare.

[3] We briefly discuss the implications of our results for the recently implemented net stable funding ratios and liquidity coverage ratios.

significant challenges given the current state of measurement. Using the best empirical counterparts of the sufficient statistics that we can construct, our framework suggests that existing coverage levels may have been close to optimal, although there are potentially large welfare gains when failure probabilities are highly sensitive to the level of coverage.

Subsequently, we use the set of sufficient statistics that we identify to understand how specific changes in primitives affect the optimal deposit insurance policy within a fully parametrized version of our model. By explicitly computing the sufficient statistics in a parametrized model, we provide an intermediate step between primitives and welfare assessments. This approach should be of interest to the growing quantitative structural literature on banking, since our characterization allows us to provide further insights into how to interpret the normative implications of calibrated structural models. In particular, we i) explain why increasing the level of coverage may be Pareto improving when the level of coverage is low, ii) illustrate how a higher cost of public funds can increase in magnitude both the marginal benefit and marginal cost of higher coverage, and iii) describe how an increase in banks' riskiness differential affects the depositors' and taxpayers' welfare, with ambiguous effects on the optimal exemption level.

Finally, we show how the sufficient statistics of the baseline model continue to be valid exactly or suitably modified once we relax many of the model assumptions. First, we allow depositors to have a consumption-savings decision and portfolio decisions. We show that our optimal policy formulas remain unchanged: the identified sufficient statistics account for any change in banks' behavior induced by the policy along these dimensions. Second, we allow banks to have an arbitrary set of investment opportunities, with different liquidity and return properties. This possibility modifies the social cost of intervention in the case of bank failure, introducing a new fiscal externality term that accounts for how banks' investment decisions vary with the level of coverage. Third, we show that the sufficient statistics remain invariant to the introduction of alternative equilibrium selection mechanisms, for instance, global games. Finally, we allow for spillovers among banks and show that the optimal deposit insurance level features a macro-prudential correction when ex-ante regulation is not perfect. It is worth highlighting that the characterization provided in the baseline model (Equation (1)) remains valid under perfect regulation generally, so additional features only introduce new terms into our main characterization when banks are unregulated.

Related Literature This paper is directly related to the well-developed literature on banking and bank runs that follows Diamond and Dybvig (1983), which includes contributions by Allen and Gale (1998), Rochet and Vives (2004), Goldstein and Pauzner (2005), Uhlig (2010), or Keister (2016), among others. As originally pointed out by Diamond and Dybvig (1983), bank runs can be prevented by either modifying the trading structure, in particular by suspending convertibility, or by introducing deposit insurance. Both ideas have been further developed ever since. A sizable literature on mechanism design, including Peck and Shell (2003), Green and Lin (2003), or Ennis and Keister (2009), among others, has focused on the optimal design of contracts to prevent runs.[4] Instead, taking the contracts used as a primitive, we focus on the optimal determination of the deposit insurance limit, a policy measure

[4]See also the recent work by Schilling (2018) studying the optimal delay of bank resolution.

implemented in most modern economies. Purely from a theoretical perspective, our paper expands on previous work by developing a tractable framework with a rich cross-section of depositors. Allowing for depositors who hold different levels of deposits turns out to be a key element to study the optimal level of deposit insurance coverage, since changes in the level of coverage modify the identity of the set of fully insured depositors at the margin.

The papers by Merton (1977), Kareken and Wallace (1978), Pennacchi (1987, 2006), Chan, Greenbaum and Thakor (1992), Matutes and Vives (1996), Hazlett (1997), Freixas and Rochet (1998), Freixas and Gabillon (1999), Cooper and Ross (2002), Duffie et al. (2003), Acharya, Santos and Yorulmazer (2010), have explored different dimensions of the deposit insurance institution, in particular the possibility of moral hazard and the determination of appropriately priced deposit insurance for an imperfectly informed policymaker. More recently, Allen et al. (2018) show that government guarantees, including deposit insurance, are welfare improving within a global games framework, while Kashyap, Tsomocos and Vardoulakis (2019) study optimal asset and liability regulations with credit and run risk, but abstract from modeling deposit insurance. In this paper, we depart from the existing literature, which has exclusively provided theoretical results, by developing a general but tractable framework that provides direct guidance to policymakers regarding the set of variables that must be measured to set the level of deposit insurance optimally. Our approach crucially relies on characterizing optimal policy prescriptions as a function of potentially observable variables.

Our emphasis on measurement is related to a growing quantitative literature on the implications of bank runs and deposit insurance. Demirgüç-Kunt and Detragiache (2002), Ioannidou and Penas (2010), Iyer and Puri (2012), and Martin, Puri and Ulfier (2017) are examples of recent empirical studies that shed light on how deposit insurance affects the behavior of banks and depositors in practice. Lucas (2019) provides economic estimates of the magnitude of transfers associated with deposit insurance. Also closely related is the work of Egan, Hortaçsu and Matvos (2017), who quantitatively explore different regulations in the context of a rich empirical structural model of deposit choice. Using a macroeconomic perspective, Gertler and Kiyotaki (2015) have quantitatively assessed the convenience of guaranteeing bank deposits, but they have not characterized optimal policies, which is the focus of our paper.

Methodologically, we draw from the sufficient statistic approach developed in public finance, summarized in Chetty (2009), to tackle a core normative question for banking regulation. In the context of financial intermediation, Matvos (2013) follows a similar approach to measure the benefits of contractual completeness. Dávila (2019) uses a related approach to optimally determine the level of bankruptcy exemptions. Sraer and Thesmar (2018) build on similar methods to produce aggregate estimates from individual firm's experiments. More broadly, this paper contributes to the growing literature that seeks to inform financial regulation by designing adequate measurement systems for financial markets, recently synthesized in Haubrich and Lo (2013) and Brunnermeier and Krishnamurthy (2014).

Outline The remainder of this paper is organized as follows. Section 2 lays out the baseline model and introduces the main results when the deposit rate is predetermined. Section 3 allows for the endogenous determination of deposit rates and characterizes the optimal ex-ante regulation in different scenarios. Section 4 describes the implications of the model for direct measurement and when simulated. Section 5 extends the results in several dimensions and Section 6 concludes. All proofs and derivations are in the Appendix.

2 A Model of Bank Runs with Heterogeneous Depositors

This paper develops a framework suitable to determine the optimal level of deposit insurance coverage. We initially introduce our main results in a stylized model of bank runs with aggregate risk and heterogeneous depositors. Sections 3 and 5 show that our insights extend to richer environments.

2.1 Environment

Our model builds on Diamond and Dybvig (1983). Time is discrete, there are three dates $t = 0, 1, 2$, and a single type of consumption good (dollar), which serves as numeraire. There is a continuum of aggregate states realized at date 1, denoted by $s \in [\underline{s}, \overline{s}]$ and distributed according to a cdf $F(\cdot)$. The realization of the aggregate state becomes common knowledge at the beginning of date 1. The economy is populated by a double continuum of depositors, indexed by i, and a continuum of taxpayers, indexed by τ. We use the index j to denote both depositors and taxpayers. There are also a continuum of identical banks and a benevolent policymaker.

Depositors Every type i depositor is initially endowed with D_{0i} dollars, which are deposited in a bank. The cross-sectional holdings of deposits are distributed according to $G(\cdot)$, with support in $\left[0, \overline{D}\right]$, where $\overline{D} \leq \infty$. We denote the total mass of depositors by $\overline{G} = \int_0^{\overline{D}} dG(i)$.

Depositors, whose preferences are identical ex-ante, are uncertain about their preferences over future consumption. Some will be early depositors, who only want to consume at date 1, and some will be late depositors, who only want to consume at date 2. At date 0, depositors know the probability of being an early or a late depositor. At date 1, depositors privately learn whether they are of the early or the late type. Out of those depositors with initial deposits D_{0i}, the probability of being an early type corresponds to λ while the probability of being a late type corresponds to $1 - \lambda$. Under a law of large numbers, λ and $1 - \lambda$ are respectively the exact proportions of early and late depositors with initial deposits D_{0i}.[5]

Hence, the ex-ante utility of a type i depositor, V_i, is given by

$$V_i = \mathbb{E}_s \left[\mathbb{E}_\lambda \left[U\left(C_{ti}(s)\right)\right]\right] = \mathbb{E}_s \left[\lambda U\left(C_{1i}(s)\right) + (1-\lambda) U\left(C_{2i}(s)\right)\right], \tag{2}$$

where $C_{1i}(s)$ and $C_{2i}(s)$ respectively denote the consumption of early and late depositors with initial deposits D_{0i} for a given realization of the aggregate state s. Depositors' flow utility $U(\cdot)$ satisfies

[5] In previous versions of this paper, as in Wallace (1988, 1990) and Chari (1989), among others, we allowed for the fraction of early depositors $\lambda(s)$ to vary with the aggregate state. This introduces an additional source of aggregate risk but does not affect our conclusions. Similarly, we could allow for the fraction or early/late depositors to vary with the level of deposits.

standard regularity conditions: $U'(\cdot) > 0$, $U''(\cdot) < 0$, and $\lim_{C \to 0} U'(C) = \infty$. Because depositors have external sources of income, our model remains well-behaved even when their utility satisfies an Inada condition. Figure 2 illustrates the timeline of the model.

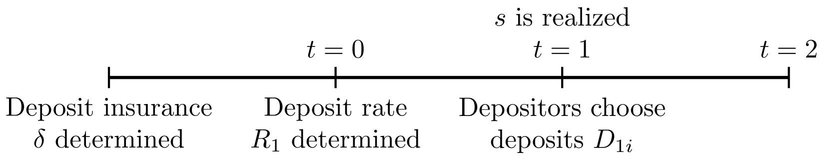


Figure 2: Timeline

Early depositors receive a stochastic endowment $Y_{1i}(s) > 0$ at date 1 while late depositors receive a stochastic endowment $Y_{2i}(s) > 0$ at date 2. The endowments at dates 1 and 2 capture the payoffs on the rest of the depositors' portfolios. Late depositors also have access to a storage technology between dates 1 and 2.

At date 1, after learning their type and observing the aggregate state s, depositors can change their deposit balance by choosing their new deposit level $D_{1i}(s)$: this is the only choice variable for depositors. We also assume that there is an iid sunspot at date 1 for every realization s of the aggregate state – this becomes relevant later on when dealing with multiple equilibria.

Banks' Technology and Deposit Contract At date 0, banks have access to a production technology with the following properties. Every unit of consumption good invested at date 0 is transformed into $\rho_1(s) \geq 0$ units of consumption good at date 1. Every unit of consumption good held by banks at the end of date 1 is transformed into $\rho_2(s) \geq 0$ units of consumption good at date 2.[6] For simplicity, we assume that banks do not have access to an additional storage technology at date 1 with returns that differ from $\rho_2(s)$. We assume that both $\rho_1(s)$ and $\rho_2(s)$ are continuous and increasing in the realization of the aggregate state s, so high (low) realizations of s correspond to states in which banks are more (less) profitable. We further assume i) that $\rho_2(\underline{s}) < 1$, which guarantees the existence of fundamental bank failures, and ii) that $\rho_1(s) \leq 1$ whenever $\rho_2(s) \leq 1$, which simplifies the exposition by limiting the cases to consider.

The only contract available to depositors is a deposit contract, which takes the following form. A depositor who deposits his endowment at date 0 is promised an uncontingent gross return $R_1 \geq 1$, which accrues at date 1. Hence, a depositor that deposits D_{0i} at date 0 is entitled to withdraw on demand up to $D_{0i}R_1$ dollars at date 1. At date 1, depositors can withdraw funds or leave them in the bank, but no new funds are added. This restricts depositors' choices to $D_{1i}(s) \in [0, D_{0i}R_1]$. When $D_{0i}R_1 > D_{1i}(s)$, a depositor withdraws a strictly positive amount of deposits at date 1. When $D_{0i}R_1 = D_{1i}(s)$, a depositor leaves his deposit balance unchanged. We denote aggregate net withdrawals in state s by $\Omega(s)$, defined

[6] Allowing $\rho_1(s)$ to take values different from 1 is necessary to guarantee that there are regions in which banks cannot fail even when all depositors withdraw their funds. Goldstein and Pauzner (2005) make an equivalent assumption to generate an upper-dominance region. By flexibly modeling $\rho_1(s)$ and $\rho_2(s)$ our setup accommodates illiquidity or insolvency scenarios.

as follows

$$\Omega(s) = \int_0^{\overline{D}} (D_{0i}R_1 - D_{1i}(s))\, dG(i) = D_0R_1 - D_1(s), \qquad \text{(Aggregate net withdrawals)}$$

where $D_0 = \int_0^{\overline{D}} D_{0i}dG(i)$ and $D_1(s) = \int_0^{\overline{D}} D_{1i}(s)\, dG(i)$ respectively denote the aggregate amount of bank deposits at dates 0 and 1.

Depositors make withdrawal decisions at date 1 simultaneously. Similarly to Allen and Gale (1998), funds are allocated proportionally in case of failure among all depositors. That is, if, given withdrawal decisions, banks anticipate being unable to satisfy all promised claims at date 1 or 2, they enter into a liquidation process in which funds are distributed on a proportional basis among claimants after the deposit insurance guarantee has been satisfied.[7] The actual payoff received by a given depositor at either date 1 or date 2 depends on the returns to bank investments, the promised deposit rate, the behavior of all depositors, and the level of deposit insurance – as described below. Consistently with models that build on Diamond and Dybvig (1983), depositors receive all remaining proceeds of banks' investments at date 2.

Deposit Rate Determination Throughout the paper, we consider three alternative assumptions regarding the determination of the deposit rate. First, we assume that the deposit rate R_1 is predetermined and invariant to the level of deposit insurance coverage δ. That is, we take R_1 as a primitive of the model. This assumption simplifies the characterization of the equilibrium and allows for a transparent derivation of the optimal policy formulas.

Subsequently, in Section 3, we re-derive our results in two scenarios in which the deposit rate is endogenously determined, allowing for changes in banks' choices induced by changing the level of coverage – this behavior is often referred to as moral hazard. We first study the scenario in which R_1 is chosen by competitive banks and then the case in which R_1 is chosen by a benevolent planner. Comparing both solutions allows us to study the optimal deposit rate regulation.

Deposit Insurance and Taxpayers The level of deposit insurance δ, measured in dollars (units of the consumption good), is the single instrument available to the planner. It is modeled to mimic actual deposit insurance policies: in any event, depositors are guaranteed the promised return on their deposits up to an amount δ, for any realization of the aggregate state s. The level of deposit insurance, which can take any value $\delta \geq 0$ and gets paid after a bank failure at date 1, is chosen under commitment at date 0 through a planning problem.

Any funds disbursed to pay for deposit insurance must be raised through taxation. We denote the funding shortfall generated by the deposit insurance system in state s by $T(s)$. Any dollar raised through taxation induces a resource loss of $\kappa(T(s)) \geq 0$ dollars, which represents the cost of public funds. We assume that $\kappa(\cdot)$ is a weakly increasing and convex function that satisfies $\kappa(0) = 0$ and

[7] In previous versions of this paper, we adopted a sequential service constraint, without affecting our conclusions. The current formulation, which is substantially more tractable, eliminates the need to keep track of which specific depositors are first in line when banks cannot pay back all depositors in full.

$\lim_{T\to\infty}\kappa(T)=\infty$.[8] We also assume that, whenever banks fail and deposit insurance has to actually be paid, the deposit insurance authority is only able to recover a fraction $\chi(s)\in[0,1]$ of any resources held by the banks. The remaining fraction $1-\chi(s)$ captures deadweight losses associated with bank failure. We allow for the recovery rate to vary with the realization of the state s and, to preserve the differentiability of the planner's problem, we assume that $\chi(s)$ is continuous and that $\chi(\underline{s})=0$.

Finally, we assume that all taxes and associated deadweight losses are borne by a continuum of identical taxpayers, who have the same flow utility $U(\cdot)$ as depositors. For simplicity, taxpayers only consume at date 1. Taxpayers have an endowment $Y_\tau(s)$ that is sufficiently large to pay for the full funding shortfall $T(s)$ generated by the deposit insurance policy in any state. Modeling depositors and taxpayers as distinct groups of agents highlights the fiscal implications of the policy.

Equilibrium Definition An equilibrium, for a given level of deposit insurance δ and a given deposit rate R_1, is defined as consumption allocations $C_{1i}(s)$ and $C_{2i}(s)$ and deposit choices $D_{1i}(s)$, such that depositors maximize their utility, given that other depositors behave optimally, and taxpayers cover the funding shortfall.

Remarks on the Environment Before characterizing the equilibrium, we would like to emphasize several features of our environment. First, following most of the literature on bank runs, we take the noncontingent nature of deposits and its demandability as primitives. With this, we depart from the well-established approach that regards deposit contracts as the outcome of a mechanism. The upside of our approach is that we can map banks' choices to observable variables, like deposit rates, as opposed to focusing on more abstract assignment procedures.

Second, we restrict our attention to a single policy instrument: the amount of deposit insurance coverage. Consequently, we are solving a second-best problem in the Ramsey tradition. More general policy responses, either explicit or implicit and potentially state-contingent, for instance, lender-of-last-resort policies, can bring social welfare closer to the first-best. Even when those policies are available, independently of whether they are chosen optimally, our main characterization and the insights associated with it remain valid as long as these additional policies do not restore the first-best, as we formally discuss in Section 5. We work under the assumption of full commitment throughout, which in practice may require credible fiscal backing.[9]

Finally, our paper departs from Diamond and Dybvig (1983) in three significant ways. First, we allow for a non-degenerate distribution of deposit holdings, which is crucial to capture the extensive margin effects of deposit insurance. Second, the profitability of banks' investments at dates 1 and 2 is subject to aggregate risk, which is necessary to observe bank failures in equilibrium under the optimal deposit insurance policy, as in Goldstein and Pauzner (2005). Finally, instead of a sequential service constraint, we adopt a proportional sharing rule for the distribution of funds in the case of bank failure.

[8] In the context of our model, it is trivial to make the fiscal distortion endogenous by endowing taxpayers with a labor supply choice and assuming that raising public funds distorts their consumption-leisure decision. The model also easily accommodates a cost of public funds that varies with the state s, by including s as an additional argument to the function $\kappa(\cdot)$.

[9] See Bonfim and Santos (2017) for evidence consistent with this view.

This formulation, similar to Allen and Gale (1998), allows us to eliminate the ex-post consumption heterogeneity among depositors of the same type that emerges under sequential service and to simplify the model solution, but it is otherwise inessential.

2.2 Equilibrium Characterization

We first characterize depositors' equilibrium choices at date 1. Subsequently, we study the planning problem that determines $\delta^{\star}$.

Depositors' Optimal Choices The level of aggregate deposit withdrawals determines the funds available to banks at date 2. Two possible scenarios arise, depending on the level of aggregate deposit claims after date 1, $D_1(s)$. In the no bank failure scenario, banks have sufficient funds to satisfy their commitments. In the bank failure scenario, banks do not have sufficient funds to satisfy their commitments to depositors either at date 1 or at date 2. In that case, banks fail and depositors make use of deposit insurance. Formally,

$$\begin{array}{ll} \text{Bank Failure,} & \text{if } \rho_2(s)(\rho_1(s) D_0 - \Omega(s)) < D_1(s) \\ \text{No Bank Failure,} & \text{if } \rho_2(s)(\rho_1(s) D_0 - \Omega(s)) \geq D_1(s), \end{array} \tag{3}$$

where the left-hand side of Equation (3) represents the total resources available to banks to satisfy deposits at date 2.

We must separately consider the behavior of i) early depositors, ii) fully insured late depositors, and iii) partially insured late depositors in both the failure and no failure scenarios. Under our assumptions, regardless of the actions of other depositors, it is optimal for early depositors to withdraw all their deposits at date 1 and set $D_{1i}^{\star}(s) = 0$, $\forall s$. Hence, the equilibrium consumption of early depositors is given by

$$C_{1i}(s) = \begin{cases} \min\{D_{0i}R_1, \delta\} + \alpha_F(s)\max\{D_{0i}R_1 - \delta, 0\} + Y_{1i}(s), & \text{Bank Failure} \\ D_{0i}R_1 + Y_{1i}(s), & \text{No Bank Failure,} \end{cases} \tag{4}$$

where $\alpha_F(s) \geq 0$ corresponds to the equilibrium recovery rate on uninsured deposits, which is defined and characterized in Equation (15) below.

Fully insured late depositors are those whose deposit holdings are (weakly) less than the level of deposit insurance coverage, that is, $D_{0i}R_1 \leq \delta$. Regardless of the actions of other depositors, fully insured late depositors are indifferent between withdrawing or leaving all their funds inside the banks in case of failure, as long as they have access to a perfect storage technology. We restrict our attention to equilibria in which these depositors leave all their funds in banks at date 1, so $D_{1i}^{\star}(s) = D_{0i}R_1$. This equilibrium behavior is consistent with a small fixed cost of withdrawing funds or an imperfect storage technology.[10]

[10]Note that we effectively assume that receiving insured funds does not involve other costs, like delayed access to funds. This is an appropriate assumption for the US. See Goldsmith-Pinkham and Yorulmazer (2010) for an account of the Northern Rock failure episode in the UK, which shows why partial and delayed coverage of deposits may be ineffective to stop runs.

Partially insured late depositors are those whose deposit holdings are larger than the level of deposit insurance coverage, that is, $D_{0i}R_1 > \delta$. If banks do not fail, it is optimal for these depositors to set $D^{\star}_{1i}(s) = D_{0i}R_1$, since they will receive a positive net return on their deposits between dates 1 and 2, as shown below. In the case of bank failure, we restrict our attention to equilibria in which these depositors leave up to the level of coverage in banks, setting $D^{\star}_{1i}(s) = \delta$. In net terms, this behavior is consistent with the recent evidence uncovered by Martin, Puri and Ulfier (2017), which shows that depositors rarely exceed the level of deposit insurance coverage when a bank failure is likely.[11]

Formally, the equilibrium consumption of both fully insured and partially insured late depositors can be expressed as

$$C_{2i}(s) = \begin{cases} \min\{D_{0i}R_1, \delta\} + \alpha_F(s)\max\{D_{0i}R_1 - \delta, 0\} + Y_{2i}(s), & \text{Bank Failure} \\ \alpha_N(s) D_{0i}R_1 + Y_{2i}(s), & \text{No Bank Failure,} \end{cases} \tag{5}$$

where $\alpha_N(s) \geq 1$ corresponds to the equilibrium gross return on deposits accrued between dates 1 and 2, which is fully characterized below. Note that the consumption of early and late depositors with the same deposit balance is identical in the case of bank failure. When banks don't fail, late depositors receive a higher return relative to early depositors, modulated by $\alpha_N(s)$, which is fully characterized in Equation (15) below.

Equilibria at Date 1 After characterizing the optimal individual behavior of depositors for a given level of aggregate withdrawals, we now show that two different types of equilibria may emerge at date 1, depending on the realization of s. We refer to the first type of equilibrium as a *no failure equilibrium.* In that equilibrium, partially insured depositors keep their deposits in banks, allowing banks to honor their promises at dates 1 and 2. We refer to the second type of equilibrium as a *failure equilibrium.* In that equilibrium, partially insured depositors withdraw all deposits in excess of the level of coverage, making banks unable to honor their promises either at date 1 or date 2. As shown above, in both types of equilibria early depositors find it optimal to withdraw all their funds, and fully insured late depositors find it optimal not to withdraw any of their funds.

Note that we can reformulate Equation (3), which determines the form of equilibrium, as follows:

$$\begin{array}{ll} \text{Bank Failure,} & \text{if } \tilde{D}_1(s) > D_1 \\ \text{No Bank Failure,} & \text{if } \tilde{D}_1(s) \leq D_1, \end{array} \tag{6}$$

where the failure threshold $\tilde{D}_1(s)$ is given by

$$\tilde{D}_1(s) = \begin{cases} \frac{(R_1 - \rho_1(s))D_0}{1 - \frac{1}{\rho_2(s)}}, & \text{if } \rho_2(s) > 1 \\ \infty, & \text{if } \rho_2(s) \leq 1, \end{cases} \tag{7}$$

[11] Martin, Puri and Ulfier (2017) provide the most detailed available evidence on the behavior of depositors in the case of bank failure in the US. They show that a fraction of existing depositors abandon the bank they study when it is close to failure. They also show that these depositors are replaced by new depositors who hold exactly up to the level of coverage. In net terms, which is the relevant dimension for the problem we study, our model is consistent with their evidence. Our model can also accommodate a type of failure equilibrium in which partially insured late depositors optimally set $D^{\star}_{1i}(s) = 0$, yielding similar conclusions.

and the variable D_1 corresponds to the level of deposits, which can potentially take two values, depending on the behavior of partially insured depositors.[12] If partially insured late depositors decide to withdraw their deposits up to the maximum level of coverage, the aggregate level of deposits is given by the total amount of insured deposits, that is,

$$D_1 = D_1^- (\delta, R_1) \equiv (1 - \lambda) \int_0^{\overline{D}} \min \{D_{0i} R_1, \delta\} \, dG (i) . \tag{8}$$

Alternatively, if partially insured late depositors decide to keep all their deposits, the aggregate level of deposits corresponds to

$$D_1 = D_1^+ (R_1) \equiv (1 - \lambda) D_0 R_1. \tag{9}$$

Figure 3 illustrates how Equation (6) determines whether there is a unique equilibrium or multiple equilibria. There are three possibilities. First, for sufficiently low realizations of s, both D_1^+ and D_1^- are less than the failure threshold $\tilde{D}_1 (s)$. Within this region, even if there are no withdrawals by late depositors, banks' profitability is so low that early depositors withdrawals make bank failure unavoidable. In this case, a unique failure equilibrium exists. We refer to bank failures in this region as fundamental failures.[13] Second, for intermediate realizations of s, if the level of aggregate deposits corresponds to D_1^+, banks are able to honor their promises, and a no failure equilibrium exists. However, if the level of aggregate deposits corresponds to D_1^-, banks are unable to honor their promises, and a failure equilibrium exists. Within this region, there are multiple equilibria. We refer to bank failures in this region as panic failures. Finally, for sufficiently high realizations of s, both D_1^+ and D_1^- are higher than the failure threshold $\tilde{D}_1 (s)$. Within this region, even if partially insured late depositors decide to withdraw all their funds, banks' profitability is high enough to be able to honor all promises, so a unique no failure equilibrium exists.

Figure 3 also illustrates the mechanism through which deposit insurance affects the set of equilibria. Since the value of D_1^- is increasing in δ, a higher level of deposit insurance coverage reduces the multiplicity region. Note that $\lim_{\delta \to \overline{D} R_1} D_1^- (\delta, R_1) = D_1^+ (R_1)$, so bank failure is possible even when all deposits are covered. In this case, when the realization of s is sufficiently low, the withdrawals of early depositors are sufficient to make banks fail. Note also that if $\delta \to 0$, the equilibrium still features three regions. For very low realizations of the aggregate state s, there is a unique fundamental failure equilibrium, while for very high realizations of s, there is a unique no failure equilibrium. In an intermediate region of s there are multiple equilibria. Therefore, high enough levels of deposit insurance eliminate the failure equilibrium as long as banks are not completely insolvent. Interestingly, the expression for $\tilde{D}_1 (s)$ features a "multiplier" $\frac{1}{1 - \frac{1}{\rho_2(s)}} > 1$. Intuitively, every dollar left inside the banks not only reduces the net loss on investments that must be liquidated, but also earns the extra marginal net return on banks' investments. This mechanism amplifies the effects of deposit insurance.

[12] Note that $\tilde{D}_1 (s)$ can be negative if $R_1 < \rho_1 (s)$. In that case, as we show below, only the no failure equilibrium exists.

[13] There exists a long tradition that distinguishes between fundamental failures (business cycle view) and panic failures (sunspot view). Our model purposefully accommodates both. See the earlier work by Chari and Jagannathan (1988), Gorton (1988), and Jacklin and Bhattacharya (1988), among others, as well as the more recent discussions by Allen and Gale (1998, 2007) and Goldstein (2012).

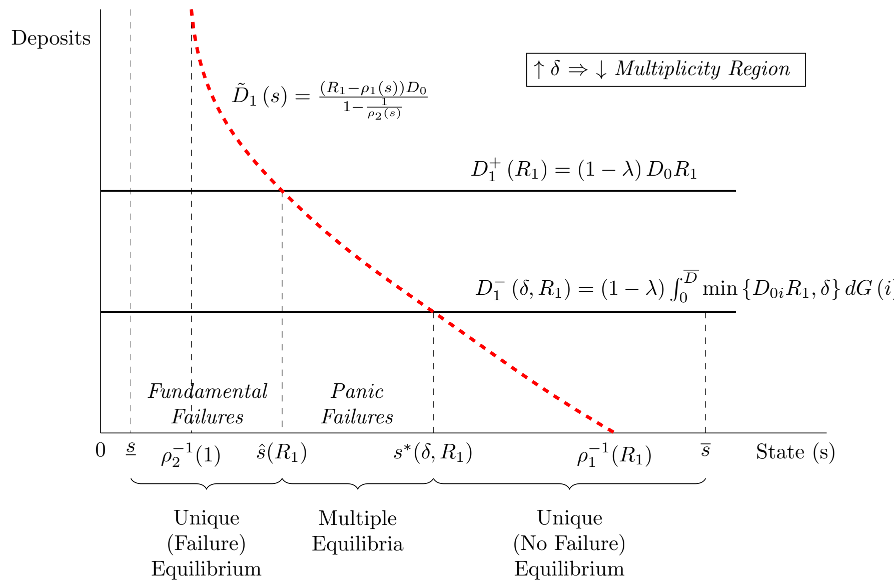


Figure 3: Equilibrium Regions

Note: Figure 3 illustrates, for a given level of deposit insurance coverage δ and for a given deposit rate R_1, whether there exists a unique equilibrium or multiple equilibria for different realizations of the aggregate state s. The red dashed line is defined in Equation (7). The black solid lines are defined in Equations (8) and (9). The intersections between the red dashed line and the black solid lines define the thresholds $s^*(\delta, R_1)$ and $\hat{s}(R_1)$, as described below and illustrated in Figure 4.

To characterize ex-ante behavior, it is useful to formally define the regions of the realization of s that determine the different type of equilibria that may arise at date 1. Formally,

$$\begin{array}{ll} \text{Unique (Failure) equilibrium,} & \text{if } \underline{s} \leq s < \hat{s}(R_1) \\ \text{Multiple equilibria,} & \text{if } \hat{s}(R_1) \leq s < s^*(\delta, R_1) \\ \text{Unique (No Failure) equilibrium,} & \text{if } s^*(\delta, R_1) \leq s \leq \overline{s}, \end{array}$$

where the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ are defined as follows

$$\hat{s}(R_1) = \left\{ s \,\middle|\, D_1^+(R_1) = \tilde{D}_1(s) \right\} \tag{10}$$

$$s^*(\delta, R_1) = \left\{ s \,\middle|\, D_1^-(\delta, R_1) = \tilde{D}_1(s) \right\}, \tag{11}$$

where $s^*(\delta, R_1) = \overline{s}$ whenever the Equation $D_1^-(\delta, R_1) = \tilde{D}_1(s)$ cannot be satisfied for any value of s. Figure 4 illustrates the three regions graphically. In the Appendix, we explicitly establish the relevant properties of the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$. In particular, we show that

$$\frac{\partial s^*}{\partial \delta} \leq 0, \quad \frac{\partial s^*}{\partial R_1} \geq 0, \quad \text{and} \quad \frac{\partial \hat{s}}{\partial R_1} \geq 0.$$

That is, that the region of multiplicity decreases with the level of deposit insurance while the region with a unique failure equilibrium is increasing in the deposit rate offered by banks. The region of multiplicity can increase or decrease with the deposit rate offered by banks.

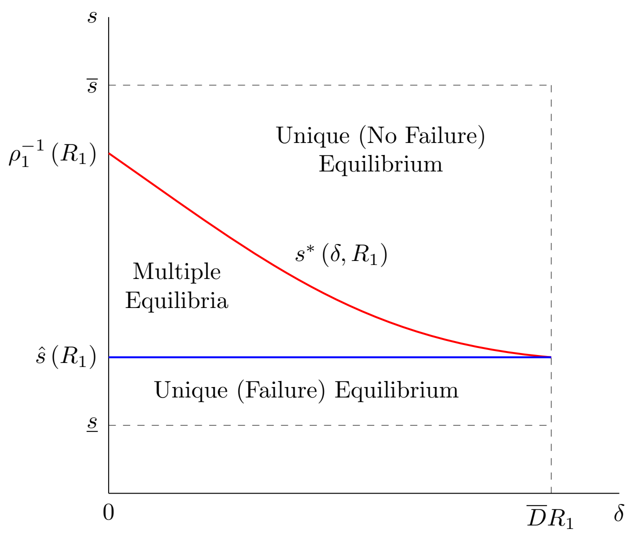


Figure 4: Regions Defined by $s^*(\delta, R_1)$ and $\hat{s}(R_1)$

Note: For a given deposit rate R_1, Figure 4 illustrates which realizations of the aggregate state s are associated with a unique equilibrium and its type, or with multiple equilibria, for different values of the deposit insurance limit δ.

Probability of Bank Failure In order to compute ex-ante welfare whenever there are multiple equilibria at date 1, we must take a stand on which equilibrium materializes for every realization of s. For now, a sunspot coordinates depositors' behavior: for a given realization of s, the failure equilibrium occurs with probability $\pi \in [0, 1]$ and the no failure equilibrium occurs with probability $1 - \pi$.[14] Alternatively, we could have introduced imperfect common knowledge of fundamentals, as in Goldstein and Pauzner (2005), which would allow us to endogenize the probability of bank failure. We show in Section 5 that the main insights of this paper extend to that case.

Therefore we can write the unconditional probability of bank failure in this economy, which we denote by $q^F(\delta, R_1)$, as

$$q^F(\delta, R_1) = F(\hat{s}(R_1)) + \pi [F(s^*(\delta, R_1)) - F(\hat{s}(R_1))]. \qquad \text{(Failure Probability)} \tag{12}$$

The unconditional probability of bank failure $q^F(\cdot)$ inherits the properties of $s^*(\cdot)$ and $\hat{s}(\cdot)$. Formally, we express the sensitivity of the probability of failure to a change in the level of coverage, $\frac{\partial q^F}{\partial \delta}$, which is a key input for the optimal determination of δ, and the sensitivity of the probability of failure to a change in R_1, $\frac{\partial q^F}{\partial R_1}$, as follows,

$$\frac{\partial q^F}{\partial \delta} = \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial \delta} \leq 0 \tag{13}$$

$$\frac{\partial q^F}{\partial R_1} = (1 - \pi) f(\hat{s}(R_1)) \frac{\partial \hat{s}}{\partial R_1} + \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial R_1} \geq 0. \tag{14}$$

Intuitively, holding the deposit rate constant, a higher level of deposit insurance coverage decreases the likelihood of bank failures in equilibrium, by reducing the region in which there are multiple equilibria. Similarly, holding the level of deposit insurance constant, higher deposit rates increase the likelihood of

[14]The sunspot probability π could be trivially made contingent on the aggregate state s as $\pi(s)$.

bank failure both by reducing the region with a unique no failure equilibrium, $\frac{\partial s^*}{\partial R_1} \geq 0$, and by enlarging the region with a unique failure equilibrium, $\frac{\partial \hat{s}}{\partial R_1} \geq 0$. Note that deposit insurance is more effective in reducing bank failures whenever depositors are more likely to coordinate in the failure equilibrium ($\pi \to 1$).

It follows immediately from Figure 4 that $\frac{\partial q^F}{\partial \delta}$ is weakly negative. We establish below that the marginal impact of a policy change, holding constant changes in bank behavior, is important to characterize the optimal deposit insurance policy.

Depositors' Equilibrium Consumption To determine depositors' consumption in equilibrium, it is necessary to characterize the equilibrium objects $\alpha_N(s)$ and $\alpha_F(s)$. As described in detail in the Appendix, the values of the recovery rate on uninsured claims in case of failure $\alpha_F(s)$ and the gross return in case of no failure $\alpha_N(s)$ are respectively given by

$$\alpha_F(s) = \frac{\max\left\{\chi(s)\rho_1(s)D_0 - \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i), 0\right\}}{\int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\}\, dG(i)} \quad \text{and} \quad \alpha_N(s) = \rho_2(s)\frac{\rho_1(s) - \lambda R_1}{(1-\lambda)R_1}. \tag{15}$$

Intuitively, the recovery rate on uninsured claims in case of failure is given by the ratio of total funds available after insurance payments to uninsured claims. The funds available after liquidation correspond to the difference between the total amount of bank resources $\chi(s)\rho_1(s)D_0$ and the level of insured payments, $\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i)$. The level of uninsured claims corresponds to $\int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\}\, dG(i)$. Note that for sufficiently low values of banks' profitability at date 1 or their recovery rate on assets $\chi(s)$, $\alpha_F(s)$ can be zero, implying that the recovery rate on uninsured deposits is zero. The funding shortfall will be positive in those scenarios. The value of $\alpha_F(s) \in [0,1)$ is decreasing in the deposit rate R_1 and in the level of coverage δ, and it is increasing in the realization of the aggregate state s.

The gross return in case of no failure corresponds to the ratio of available funds at date 2, given by $\rho_2(s)(\rho_1(s) - \lambda R_1)D_0$, to the level of date 1 deposits, $D_1 = (1-\lambda)D_0R_1$. The value of $\alpha_N(s)$ is increasing in the level of bank returns $\rho_2(s)$ and it is decreasing in λ and R_1.

As we show below, a key determinant of the optimal test for whether to increase or decrease the optimal level of deposit insurance is the consumption gap between failure and no-failure equilibria. Formally, for a given aggregate realization s, the gap between early and late depositors' consumption corresponds to

$$C_{1i}^N(s) - C_{1i}^F(s) = \underbrace{(1-\alpha_F(s))\max\{D_{0i}R_1 - \delta, 0\}}_{\text{Partially Recovered Uninsured Deposits}} \qquad \text{(Early Depositors)}$$

$$C_{2i}^N(s) - C_{2i}^F(s) = \underbrace{(\alpha_N(s) - 1)D_{0i}R_1}_{\text{Net Return}} + \underbrace{(1-\alpha_F(s))\max\{D_{0i}R_1 - \delta, 0\}}_{\text{Partially Recovered Uninsured Deposits}}. \quad \text{(Late Depositors)}$$

Note that the consumption gap between failure and no failure states is zero for early depositors who are fully insured. The gap for early depositors with uninsured claims corresponds to the fraction of funds that is not recovered in the case of bank failure. The gap for late depositors contains an additional term relative to early depositors that captures the net return on deposits between dates 1 and 2.

Funding Shortfall and Taxpayers Equilibrium Consumption Before turning to the question of the optimal determination of the level of deposit insurance coverage, we characterize the funding shortfall in state s, $T(s)$, which is given by

$$T(s) = \max\left\{\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i) - \chi(s)\rho_1(s) D_0, 0\right\}. \qquad \text{(Funding Shortfall)} \tag{16}$$

The funding shortfall is positive only when the total amount of deposit insurance claims exceeds the funds available after liquidation. Whenever the available funds $\chi(s)\rho_1(s)D_0$ are sufficient to pay for all insured claims $\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i)$, there is no need for taxation. Note that the deadweight loss of taxation $\kappa(T(s))$ is borne by taxpayers. Note also that whenever the funding shortfall is positive, $T(s) > 0$, the recovery rate on uninsured deposits is zero, $\alpha_F(s) = 0$.

Therefore we can express taxpayers' equilibrium consumption in failure and no failure scenarios as

$$C_\tau^F(s) = Y_\tau(s) - T(s) - \kappa(T(s)) \quad \text{and} \quad C_\tau^N(s) = Y_\tau(s),$$

where $T(s)$ is defined in Equation (16). The taxpayers' consumption gap between failure and no-failure equilibria is trivially given by the funding shortfall augmented by the deadweight loss of taxation, that is, $C_\tau^N(s) - C_\tau^F(s) = T(s) + \kappa(T(s))$.

2.3 Normative Analysis

After characterizing the equilibrium of this economy for a given level of deposit insurance coverage δ, we now study how changes in the level of coverage affect social welfare. In this section, we initially consider a scenario in which the deposit rate offered by banks is predetermined and invariant to the level of coverage δ. This case provides a tractable benchmark from which we subsequently study multiple departures.

We identify social welfare in this economy with the ex-ante expected utility of depositors and taxpayers.[15] We denote social welfare, expressed as a function of the level of deposit insurance, by $W(\delta)$. Formally, $W(\delta)$ corresponds to

$$W(\delta) = \int V_j(\delta, R_1)\, dj = \underbrace{\int_0^{\overline{D}} V_i(\delta, R_1)\, dG(i)}_{\text{Depositors}} + \underbrace{V_\tau(\delta, R_1)}_{\text{Taxpayers}}, \tag{17}$$

where $V_i(\delta, R_1)$ denotes depositors' ex-ante indirect utility for given levels of deposit insurance and the deposit rate, and $V_\tau(\delta, R_1)$ denotes taxpayers' indirect utility. Note that integrals over the index j account for all depositors and taxpayers, so the notation C_j could represent C_{1i}, C_{2i}, or C_τ. Formally,

[15]Our formulation attributes the same welfare weights to depositors and taxpayers. A more general formulation with general welfare weights yields similar insights. Because of diminishing marginal utility of consumption, the current formulation endogenously gives more weight to depositors with lower deposit balances, along the lines of the goals set for bank regulation by Dewatripont and Tirole (1994), which include protecting small and unsophisticated depositors. See Mitkov (2016) for recent work exploring the link between inequality and financial fragility.

we express $V_i(\delta, R_1)$ and $V_\tau(\delta, R_1)$ as follows

$$V_i(\delta, R_1) = \lambda \mathbb{E}_s[U(C_{1i}(s))] + (1-\lambda)\mathbb{E}_s[U(C_{2i}(s))] \qquad \text{(Depositors)} \tag{18}$$

$$V_\tau(\delta, R_1) = \mathbb{E}_s[U(Y_\tau(s) - T(s) - \kappa(T(s)))], \qquad \text{(Taxpayers)} \tag{19}$$

where early and late depositors' expected utility can be expressed as

$$\mathbb{E}_s[U(C_{ti}(s))] = \int_{\underline{s}}^{\hat{s}(R_1)} U\left(C_{ti}^F(s)\right) dF(s) + \int_{\hat{s}(R_1)}^{s^*(\delta,R_1)} \left(\pi U\left(C_{ti}^F(s)\right) + (1-\pi) U\left(C_{ti}^N(s)\right)\right) dF(s) + \int_{s^*(\delta,R_1)}^{\overline{s}} U\left(C_{ti}^N(s)\right) dF(s),$$

where $C_{ti}^F(s)$ and $C_{ti}^N(s)$ respectively represent consumption of depositors in failure and no failure equilibria, as described in Equations (4) and (5).[16] The thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ are defined in Equations (10) and (11), while π corresponds to the predetermined sunspot probability.

In Proposition 1, which presents the central result of this paper, we provide a test for whether to optimally increase or decrease the level of deposit insurance coverage.

Proposition 1. (Directional test for a change in the level of coverage δ) *The change in social welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ is given by:*

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial \delta} \int \left[U\left(C_j^N(s^*)\right) - U\left(C_j^F(s^*)\right)\right] dj + q^F \mathbb{E}_s^F\left[\int U'\left(C_j^F\right) \frac{\partial C_j^F}{\partial \delta} dj\right], \tag{20}$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

Proposition 1 characterizes the effect on welfare of a marginal change in the level of deposit insurance, and formalizes the tradeoffs that determine the optimal deposit insurance limit. The first of the two terms in Equation (20) can be interpreted as the marginal benefit of increasing the level of deposit insurance by a dollar. A marginal increase in the level of deposit insurance decreases the likelihood of bank failure by $\frac{\partial q^F}{\partial \delta}$.[17] The marginal welfare gain associated with such reduction in the probability of bank failure is captured by the differences in utilities between the failure and no failure equilibria evaluated at the marginal failure state s^*, $U\left(C_j^N(s^*)\right) - U\left(C_j^F(s^*)\right)$. While exact welfare assessments rely on computing utility differences, it is easier to compute the difference in consumption levels between both scenarios. To provide further intuition for the marginal benefit of increasing coverage, it is worth characterizing the behavior of the drop in aggregate consumption caused by bank failure for the marginal state, which is given by

$$\int \left[C_j^N(s^*) - C_j^F(s^*)\right] dj = \underbrace{(\rho_2(s^*) - 1)(\rho_1(s^*) - \lambda R_1) D_0}_{\text{Net Return Loss}} + \underbrace{(1-\chi(s^*))\rho_1(s^*) D_0}_{\substack{\text{Bank Failure} \\ \text{Deadweight Loss}}} + \underbrace{\kappa(T(s^*))}_{\substack{\text{Total Net Cost} \\ \text{of Public Funds}}}. \tag{21}$$

[16] Note that $V_\tau(\delta, R_1)$ can be written as

$$V_\tau(\delta, R_1) = \int_{\underline{s}}^{\hat{s}(R_1)} U\left(C_\tau^F(s)\right) dF(s) + \int_{\hat{s}(R_1)}^{s^*(\delta,R_1)} \left(\pi U\left(C_\tau^F(s)\right) + (1-\pi) U\left(C_\tau^N(s)\right)\right) dF(s) + \int_{s^*(\delta,R_1)}^{\overline{s}} U\left(C_\tau^N(s)\right) dF(s)$$

[17] As we show below, when deposit rates react to the level of δ, $\frac{dq^F}{d\delta} = \frac{\partial q^F}{\partial \delta} + \frac{\partial q^F}{\partial R_1}\frac{dR_1}{d\delta}$. We adopt the partial derivative notation, even though $\frac{dq^F}{d\delta} = \frac{\partial q^F}{\partial \delta}$ in the case we consider here, since it will become clear in Sections 3 and 5 that the partial derivative is the relevant object of interest more generally.

As shown below, Equation (21) corresponds to a first-order approximation to the social gain from avoiding the marginal bank failure. Its first term corresponds to the marginal net return loss caused by bank failure. Intuitively, at date 1, a bank failure forfeits the net return $\rho_2(s^*)-1$ per unit of available funds $(\rho_1(s^*)-\lambda R_1)D_0$. The second term corresponds to the deadweight loss on banks' assets associated with bank failure. The final term is the total cost of public funds, which is non-zero at the margin whenever banks do not have enough resources after liquidation to pay for all insurance claims at the marginal failure state s^*. Part of the marginal benefit of preventing a bank failure comes from avoiding fiscal distortions in some states of nature.

The second term in Equation (20) can be interpreted as the marginal cost of increasing the level of deposit insurance by a dollar. A marginal increase in the level of deposit insurance changes the consumption of depositors and taxpayers in the case of bank failure by $\frac{\partial C_j^F}{\partial \delta}$ over the set of failure states, which materialize with probability q^F. To provide further intuition, it is worth characterizing the aggregate marginal cost in case of failure in a given state s, which corresponds to

$$\int \frac{\partial C_j^F}{\partial \delta} dj = \begin{cases} -\overbrace{\kappa'(T(s))}^{\substack{\text{Mg. Cost} \\ \text{of DI Funds}}} \overbrace{\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i)}^{\substack{\text{Mass of} \\ \text{Partially Insured}}}, & \text{if } T(s)>0 \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

This value is strictly negative as long as distortionary taxation must be used to pay for deposit insurance claims. Intuitively, a marginal increase in δ reduces welfare by the expected fiscal cost of providing a dollar at the margin to the mass of partially insured depositors over the region of failure states. Explicitly modeling a rich cross-section of depositors allows us to highlight that a marginal increase in the level of coverage only affects partially insured depositors directly. Therefore, the marginal loss is increasing in the marginal cost of public funds $\kappa'(\cdot)$, as well as in the mass of partially insured depositors $\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i)$. In some deposit insurance systems, including the US, a deposit insurance fund provisioned by contributions of insured banks is responsible for paying insured depositors in case of failure. In this case, fiscal revenues may not be necessary in case of failure. We should then interpret the marginal cost of funds as the deadweight losses associated with keeping resources in the deposit fund (often invested in treasuries and other low yield securities) as opposed to inside the banks.

Equations (21) and (22) enter as inputs to the following tractable approximation, which expresses $\frac{dW}{d\delta}$ purely in terms of agent's consumption and failure probabilities.

Proposition 2. (Preference-free approximation of $\frac{dW}{d\delta}$) *The change in welfare induced by a marginal change in the level of deposit insurance can be expressed in dollars up to a first-order, that is, approximating $U(\cdot)$ linearly, as follows:*

$$\frac{dW}{d\delta} \approx -\frac{\partial q^F}{\partial \delta} \int \left[C_j^N(s^*)-C_j^F(s^*)\right] dj + q^F \mathbb{E}_s^F\left[\int \frac{\partial C_j^F}{\partial \delta} dj\right], \tag{23}$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure, and where $\int\left[C_j^N(s^*)-C_j^F(s^*)\right]dj$ and $\int \frac{\partial C_j^F}{\partial \delta} dj$, are given by Equations (21) and (22).

This approximation allows us to focus on the aggregate effects of the policy, sidestepping the distributional issues that emerge in normative problems with heterogeneous agents. This approximation is appealing in that it does not depend on the agents' utilities. Some of our remarks below are formalized under this first-order approximation. In Section 4, we further discuss some of the distributional issues that emerge when solving the model. By exploiting Equations (21) and (22), we can provide the following analytical insights.

Remark 1. *Sufficient statistics.* Propositions 1 and 2 provide a simple test for whether to increase or decrease the level of coverage that exclusively relies on changes in the level of consumption and failure probabilities. In particular, the probability of bank failure and its sensitivity to changes in the level of coverage, as well as the aggregate resource loss in marginal failure states along with the marginal resource loss in failure states caused by changing the level of coverage are the sufficient statistics that determine in which direction to adjust the level of coverage. These sufficient statistics can a) be potentially recovered from measured data, or b) used to shed light on the results of a calibrated structural model. We discuss both approaches in detail in Section 4. Even though we characterize $\frac{dW}{d\delta}$ locally, it is conceptually clear how to evaluate the welfare change caused by a non-local change in the level of coverage by integrating over the values of $\frac{dW}{d\delta}$. Formally, for a non-local policy change from δ to δ', we can write the welfare change as follows $W(\delta') - W(\delta) = \int_{\delta}^{\delta'} \frac{dW}{d\delta}\left(\tilde{\delta}\right) d\tilde{\delta}$, where $\frac{dW}{d\delta}(\cdot)$ is determined in Proposition 1. Therefore, direct measurement of these variables for different levels of δ is sufficient to assess the welfare impact of any change in the level of coverage.

Remark 2. *Diamond and Dybvig (1983) revisited.* Full insurance is optimal when no failure occurs under the optimal policy. Intuitively, when deposit insurance involves no payments in equilibrium, the optimal policy fully insures deposits. In an environment without aggregate risk, Diamond and Dybvig (1983) show that it is optimal to provide unlimited deposit insurance to avoid the bank failure equilibrium. In their model, unlimited deposit insurance eliminates all bank failures but, more importantly, deposit insurance never has to be paid in equilibrium. We can heuristically understand their results by setting $q^F = 0$ and assuming that $\frac{\partial q^F}{\partial \delta} > 0$. Because public funds are never raised to pay for deposit insurance in equilibrium, there is no cost of intervention, but increasing the level of coverage reduces the probability of failure, implying that the optimal level of insurance is the highest possible one.[18]

Remark 3. *Convexity and limiting results.* Our assumptions guarantee that the planner's problem is continuous and differentiable in δ. Although in our simulations we find that the planner's problem is well-behaved for standard functional forms, the convexity of the planner's problem is not guaranteed in general, as in most normative problems. Relatedly, it is worth highlighting that Equation (20) can be used to conclude whether a non-zero or a maximal level of coverage are desirable. First, if the marginal cost of funds of a small intervention is zero, $\kappa'(0) = 0$, but a small increase in the level of coverage is effective at reducing the probability of failure, $\left.\frac{\partial q^F}{\partial \delta}\right|_{\delta=0} > 0$, then Equation (20) implies that $\left.\frac{dW}{d\delta}\right|_{\delta=0} > 0$,

[18]While adopting the Diamond and Dybvig (1983) framework allows us to study a fully specified model, our insights extend beyond the specific assumptions of that framework. In the Appendix, we re-derive Proposition 1 under minimal assumptions.

so a strictly positive level of coverage is optimal.[19] Second, as long as banks fail in equilibrium when $\delta = \overline{D} R_1$ and fiscal costs are positive, $\kappa'(\cdot) > 0$, given that $\left.\frac{\partial q^F}{\partial \delta}\right|_{\delta = \overline{D} R_1} = 0$, a maximal level of coverage is not optimal, since in this case $\left.\frac{dW}{d\delta}\right|_{\delta = \overline{D} R_1} < 0$.

Remark 4. *Optimal level of coverage* $\delta^\star$*.* At at interior optimum, the optimal level of deposit insurance $\delta^\star$ satisfies the following relations exactly and as an approximation:

$$\delta^\star = \frac{\varepsilon^q_\delta \int \left[U\left(C^F_j(s^*)\right) - U\left(C^N_j(s^*)\right)\right] dj}{q^F \mathbb{E}^F_s \left[\int U'\left(C^F_j\right) \frac{\partial C^F_j}{\partial \delta} dj\right]} \approx \frac{\varepsilon^q_\delta \int \left[C^F_j(s^*) - C^N_j(s^*)\right] dj}{q^F \mathbb{E}^F_s \left[\int \frac{\partial C^F_j}{\partial \delta} dj\right]}, \tag{24}$$

where $\mathbb{E}^F_s[\cdot]$ stands for a conditional expectation over bank failure states, q^F denotes the unconditional probability of bank failure, and $\varepsilon^q_\delta = \frac{\partial q^F}{\partial \log(\delta)}$ denotes the change in the likelihood of bank failure induced by a one-percent change in the level of coverage. Intuitively, a high value for $\delta^\star$ is optimal when ε^q_δ and $U\left(C^F_j(s^*)\right) - U\left(C^N_j(s^*)\right)$ are large in magnitude, all else equal. If the reduction in the probability of bank failure is large at the same time that the welfare loss caused at the margin by a bank failure is also large, it is optimal to have a large level of deposit insurance. A low value for $\delta^\star$ is optimal when the probability of actually paying for deposit insurance is high, at the same time that the net marginal cost of public funds κ is high, and the recovery rate out of banks' investments is low. As it is common in optimal policy exercises, $\delta^\star$ cannot be written as a function of primitives, since all right-hand side variables in Equation (24) are endogenous to the level of δ.[20] Since solving for the fixed point that determines $\delta^\star$ in Equation (24) would require to find measures of all right-hand side variables for every level of δ, we focus on characterizing $\frac{dW}{d\delta}$, which can be computed for a given level of δ.

3 Endogenous Deposit Rate and Optimal Regulation

So far, we have considered the case in which the deposit rate R_1 offered by banks is predetermined. We now analyze two environments in which R_1 is endogenously determined. First, we consider an environment in which a regulator can directly determine the deposit rate offered by banks. Next, we consider a different environment in which competitive banks freely choose the deposit rate offered to depositors. Finally, by comparing the solution to both problems, we characterize the optimal regulation of bank deposit rates.

We draw three important conclusions from this analysis. First, we show that the equation that characterizes $\frac{dW}{d\delta}$ when deposit rates are fixed is *identical* to the equation that characterizes $\frac{dW}{d\delta}$ under the optimal deposit rate regulation. That is, in both scenarios, the same set of sufficient statistics is needed to determine the optimal policy. Second, we show that this equation only has to be augmented by the fiscal externality induced by banks' behavior when deposit rates can vary freely. Finally, we

[19] In a previous version of this paper, we explored a scenario in which $\left.\frac{\partial q^F}{\partial \delta}\right|_{\delta=0} = 0$ and $\kappa'(0) > 0$. In that case, low levels of coverage were welfare decreasing, since positive but small coverage levels generated fiscal costs in equilibrium without reducing the probability of failure.

[20] This logic is similar to classic characterizations of optimal taxes. For instance, optimal Ramsey commodity taxes are a function of demand elasticities, which are endogenous to the level of taxes – see Atkinson and Stiglitz (1980).

show that the optimal deposit rate regulation should be exclusively designed to counteract the fiscal externality caused by banks, regardless of whether deposit insurance is "fairly priced".

3.1 Regulated Deposit Rate

We now allow the policymaker to jointly determine the welfare maximizing deposit rate along with the optimal level of deposit insurance. Letting the planner choose the deposit rate directly is analogous to allowing for a rich set of ex-ante policies that modify banks' behavior at date 0. Deposit rate regulation has been commonly used in practice, in particular before the financial deregulation wave at the end of the last century. We first characterize the set of constrained efficient policies and then discuss possible decentralizations, for instance, imposing deposit rate ceilings or setting a deposit insurance premium.

The planner now chooses the level of δ and the deposit rate offered to households jointly. The optimal choice of $R_1 \in \left[1, \overline{R_1}\right]$ is characterized by the solution to $\frac{\partial W}{\partial R_1} = 0$, where $W(\cdot)$, introduced in Equation (17), is now defined as a function of both δ and R_1. Note that the planner fully internalizes the effect of changing R_1 on the funding shortfall $T(s)$. We formally describe in the Appendix the expression that characterizes the optimal rate and directly characterize the directional test for how social welfare varies with the level of coverage.

Proposition 3. (Directional test for δ under perfect ex-ante regulation) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ when R_1 is optimally determined by the planner is given by*

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial \delta} \int \left[U\left(C_j^N(s^*)\right) - U\left(C_j^F(s^*)\right)\right] dj + q^F \mathbb{E}_s^F \left[\int U'\left(C_j^F\right) \frac{\partial C_j^F}{\partial \delta} dj\right], \tag{25}$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<)\, 0$, it is optimal to locally increase (decrease) the level of coverage.

By comparing Equations (20) and (25), we observe that the marginal change in welfare caused by a change in the level of coverage can be expressed in identical form when R_1 is predetermined and when R_1 is optimally chosen by the planner. Once again, information about depositors and taxpayers' consumption and failure probabilities is sufficient to determine the welfare effect of changes in the level of coverage. Intuitively, any impact on welfare induced by the change in deposit rates generated by a change in δ must be 0 when R_1 is optimally chosen by perfectly regulated banks.

If one were to solve for the optimal value of δ by setting $\frac{dW}{d\delta} = 0$, the solutions when R_1 is predetermined and optimally chosen would differ, because the endogenous elements (consumption levels and failure probabilities) vary with the level of R_1. However, from the perspective of understanding the welfare impact of changes in the level of coverage, the set of relevant sufficient statistics is the same. This reasoning motivates the use of Equation (25) or, equivalently, Equation (20) for the purpose of direct measurement exercises, as we do in Section 4.1.

3.2 Unregulated Deposit Rate

We now allow banks to freely choose the deposit rate that they offer to their depositors. Banks set R_1 competitively at date 0 to maximize an average of depositors' expected utilities – this is the rate set by competitive banks under the veil of ignorance regarding the level of deposits holdings.[21] Our definition of equilibrium needs to be augmented to incorporate that R_1 is optimally chosen by zero-profit maximizing banks at date 0, for a given level of deposit insurance δ.

Banks anticipate the possibility of bank failure and internalize how the choice of the deposit rate affects the likelihood and severity of bank failure. Banks do not internalize how their actions affect the level of taxes that must be raised from taxpayers in the case of bank failure. Formally, the deposit rate $R_1 \in \left[1, \overline{R_1}\right]$ chosen by banks is given by

$$\arg\max_{R_1} \int_0^{\overline{D}} V_i dG\left(i\right),$$

where V_i is defined in Equation (2). The choice of R_1 determines the optimal degree of risk sharing between early and late types and across depositors, accounting for the level of aggregate uncertainty and incorporating the costs associated with bank failure. Overall, banks internalize that varying R_1 not only changes the consumption of depositors in both failure and no failure states (intensive margin terms) but also the likelihood of experiencing a bank failure (extensive margin terms). Importantly, banks do not take into account how their choice of R_1 affects the need to raise resources through taxation to pay for deposit insurance. For a given level of deposit insurance δ, under appropriate regularity conditions to preserve continuity and differentiability, discussed in the Appendix, the optimal $R_1^*\left(\delta\right)$ is given by the solution to $\frac{\partial V}{\partial R_1} = 0$, where $V = \int_0^{\overline{D}} V_i\left(\delta, R_1\right) dG\left(i\right)$, given by

$$\frac{\partial V}{\partial R_1} = \lambda \int_0^{\overline{D}} \frac{\partial \mathbb{E}_s\left[U\left(C_{1i}\left(s\right)\right)\right]}{\partial R_1} dG\left(i\right) + \left(1-\lambda\right) \int_0^{\overline{D}} \frac{\partial \mathbb{E}_s\left[U\left(C_{2i}\left(s\right)\right)\right]}{\partial R_1} dG\left(i\right) = 0, \tag{26}$$

and the marginal change in early and late depositors' utility can be expressed as

$$\begin{aligned}\frac{\partial \mathbb{E}_s\left[U\left(C_{ti}\left(s\right)\right)\right]}{\partial R_1} &= q^F \mathbb{E}_s^F\left[U'\left(C_{ti}^F\right)\frac{\partial C_{ti}^F}{\partial R_1}\right] + \left(1-q^F\right)\mathbb{E}_s^N\left[U'\left(C_{ti}^N\right)\frac{\partial C_{ti}^N}{\partial R_1}\right] \\ &+ \left(1-\pi\right)\left[U\left(C_{ti}^F\left(\hat{s}\right)\right) - U\left(C_{ti}^N\left(\hat{s}\right)\right)\right]\frac{\partial \hat{s}}{\partial R_1} f\left(\hat{s}\right) + \pi\left[U\left(C_{ti}^F\left(s^*\right)\right) - U\left(C_{ti}^N\left(s^*\right)\right)\right]\frac{\partial s^*}{\partial R_1} f\left(s^*\right),\end{aligned}$$

where $\mathbb{E}_s^F\left[\cdot\right]$ and $\mathbb{E}_s^N\left[\cdot\right]$ respectively denote conditional expectations over failure and no failure states. An increase in R_1 affects depositors' utility through intensive and extensive margins. The intensive margin effects are captured by $\frac{\partial C_{ti}^F}{\partial R_1}$ and $\frac{\partial C_{ti}^N}{\partial R_1}$. We show in the Appendix that $\frac{\partial C_{ti}^F}{\partial R_1}$ is weakly positive on aggregate, which can be interpreted as a form of moral hazard. Banks internalize that an increase in the deposit rate increases the consumption of insured depositors in failure states, at the expense of taxpayers. The term $\frac{\partial C_{ti}^N}{\partial R_1}$ takes on positive values for early depositors and negative values for late depositors. These effects capture the ex-ante risk sharing gains between early and late types generated by a higher deposit

[21] Our model can be augmented to allow banks to set different deposit rates R_{1i} for different types of depositors. See Jacewitz and Pogach (2018) for evidence consistent with this possibility.

rate.[22] On the extensive margin, banks take into account that offering a high deposit rate makes bank failures more likely. This is captured by the positive sign of $\frac{\partial \hat{s}}{\partial R_1}$ and $\frac{\partial s^*}{\partial R_1}$, which combined with the sign of $U\left(C_{ti}^F\right) - U\left(C_{ti}^N\right)$, which we show to be negative, makes increasing R_1 less desirable.

In principle, the equilibrium deposit rate R_1 can increase or decrease with the level of coverage δ, due to conflicting income effects and direct effects on the size of the failure/non-failure regions. However, in most cases, it is reasonable to expect R_1 to increases with δ, that is, $\frac{dR_1^*}{d\delta} > 0$. In a global games framework, Allen et al. (2018) explicitly find this result in a special case of our framework. Intuitively, we expect competitive banks to offer higher deposit rates when the level of coverage is higher, since they know that the existence of deposit insurance partially shields depositors' consumption. This result is a form of increased moral hazard by banks. We can now characterize the directional test for how social welfare varies with the level of coverage.

Proposition 4. (Directional test for δ without ex-ante regulation) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ when R_1 is determined by competitive banks is given by*

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial \delta} \int \left[U\left(C_j^N\left(s^*\right)\right) - U\left(C_j^F\left(s^*\right)\right)\right] dj + q^F \mathbb{E}_s^F \left[\int U'\left(C_j^F\right) \frac{\partial C_j^F}{\partial \delta} dj\right] + \underbrace{\frac{\partial V_\tau}{\partial R_1} \frac{dR_1}{d\delta}}_{\textit{Fiscal Externality}} \tag{27}$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states, q^F denotes the unconditional probability of bank failure, and $\frac{\partial V_\tau}{\partial R_1}$ can be expressed, in terms of a risk-neutral approximation as $\frac{\partial V_\tau}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s)+\kappa(T(s))]}{\partial R_1}$. If $\frac{dW}{d\delta} > (<) \, 0$, it is optimal to locally increase (decrease) the level of coverage.

It is clear that when banks choose their deposit rate freely, a new set of effects must be accounted for to understand the welfare impact of changes in the level of coverage. The derivation of Equation (27) repeatedly exploits the fact that banks choose the value of R_1 to provide insurance across types optimally, while taking into account how that may change the likelihood of bank failure. Its third term corresponds to the impact of the distortions on banks' behavior induced by the change in the level of deposit insurance. Under a risk-neutral approximation similar to one used in Proposition 2, $\frac{\partial V_\tau}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s)+\kappa(T(s))]}{\partial R_1}$, that is the direct impact on taxes of the change in rates induced by a change in the level of rates. The fiscal externality dimension features both an intensive and extensive margin. At the intensive margin, an increase in R_1 increases the level of claims that must be satisfied in failure states. At the extensive margin, an increase in R_1 increase the set of states in which bank failures occur and fiscal costs must be incurred.

We show in the Appendix that the fiscal externality term is negative, so the third term in Equation (27) increases the marginal cost of increasing the deposit insurance limit. Because it affects directly the funds that need to be raised by the government, we refer to it as a fiscal externality. It is worth emphasizing how "moral hazard" considerations affect our results in the following remark.

[22] When $s^* \to \underline{s}$ and $\hat{s} \to \underline{s}$, there are no bank failures in equilibrium, and Equation (26) defines the optimal arrangement that equalizes marginal rates of substitution across types with the expected marginal rate of transformation, determined by $\rho_2(s)$. In that case, banks set R_1 exclusively to provide insurance between early and late types across deposit levels.

Remark. Banks' changes in behavior (often referred to as moral hazard) only affect social welfare directly through the fiscal externality term. We indeed expect banks to quote higher deposit rates when the level of deposit insurance is higher, since they know the presence of deposit insurance partially shields depositors' consumption. However, because banks are competitive and maximize depositors' welfare, only the fiscal consequences of their change in behavior, which materializes when the fiscal authority actually has to pay for deposit insurance, matters. This result remains valid even when banks make endogenous liquidity and investment choices – see Section 5. Therefore, accounting for banks' moral hazard simply augments the directional test for δ by including a fiscal externality component. Indirectly, changes in bank behavior affect the level of gains from reducing bank failures (numerator of Equation (24)), the region in which deposit insurance is paid (denominator of Equation (24)) and the value attached to a dollar in the different states (captured by depositors and taxpayers marginal utilities), but these effects are subsumed into the identified sufficient statistics.

3.3 Optimal Deposit Rate Regulation

By comparing the optimal deposit rate chosen by the regulator and by competitive banks – Equations (38) and (40) in the Appendix – we can provide insights into the form of the optimal ex-ante regulation of deposit rates.

Proposition 5. (Optimal ex-ante deposit rate regulation) *The optimal corrective policy modifies the optimal choice of deposit rates by banks introducing a wedge in their deposit rate decision given by*

$$\tau_{R_1} = -\frac{\partial V_\tau}{\partial R_1} \approx \frac{\partial \mathbb{E}_s \left[T(s) + \kappa(T(s))\right]}{\partial R_1},$$

which is set to counteract the fiscal externality term defined in Proposition 4.

Proposition 5 shows how to correct banks' deposit rates so that they internalize the fiscal externality that their choices generate. Importantly, the existing literature has not previously identified this fiscal externality as the relevant object of interest that defines the optimal ex-ante regulation of banks. Consistent with Equation (27), an increase in the deposit rate offered by banks varies overall welfare according to $\frac{\partial V_\tau}{\partial R_1}$. Proposition 5 shows that this object can be expressed as the marginal change in the expected funding shortfall, augmented to include the cost of public funds. We show in the Appendix that this derivative accounts for the increased resource loss faced by taxpayers in the case of bank failure and the induced change in the unconditional probability of bank failure. Note that, even if there are no fiscal costs, so $\kappa(T) = 0$, there is a role for corrective regulation emerging from the fact that banks do not internalize the timing of taxation borne by taxpayers.[23]

In general, the implementation of the optimal ex-ante corrective policy is not unique, although in this particular case a single instrument affecting the choice of deposit rate is sufficient. Because the funds

[23]The exact expression for $\frac{\partial V_\tau}{\partial R_1}$, given in Equation (41) in the Appendix, shows that the optimal corrective policy must in general account for aggregate and systematic risk. In the context of optimally setting deposit insurance premia, a similar argument has been emphasized by Pennacchi (2006), Acharya, Santos and Yorulmazer (2010), and Lucas (2019), among others.

used to pay for deposit insurance are raised through distortionary taxation, any Pigovian corrective policy in which the deposit insurance authority raises revenue may generate a "double-dividend".[24] That is, a policy that corrects the ex-ante behavior of banks at the same time that reduces the need for raising revenue when required can improve welfare in two different margins. On the one hand, this argument supports an implementation of the optimal corrective policy through a deposit insurance fund financed with a deposit insurance premia paid by participating banks. On the other hand, if the return of the deposit insurance fund is less than the return earned by the banks themselves, it may be preferred to set a different type of ex-ante corrective policy, like a deposit ceiling. It is worth highlighting the distinction between the corrective role of ex-ante policies (optimal corrective deposit insurance premium) versus its revenue-raising role (fairly priced deposit insurance premium) in the following remark.

Remark. Optimal corrective regulation vs. Fairly priced deposit insurance. The existing literature has emphasized the study of deposit insurance schemes that are fairly priced or actuarially fair. A deposit insurance fund is said to be actuarially fair if deposit insurance premia are such that the deposit insurance fund breaks even. Our formulation shifts the emphasis from setting deposit insurance premia that covers the average fiscal cost to implementing regulations that distort banks' choices at the margin. This distinction is often blurred in existing discussions of deposit insurance premia. In the next section, we show how to account for risk choices in a more general framework, allowing for a form of risk-based deposit insurance.

Finally, note that we have considered two extreme scenarios. In one, there is no ex-ante regulation, so banks freely choose their deposit rate. In another one, regulation is perfectly targeted. Restriction on the set of feasible instruments available to the policymaker, which may arise from informational frictions about banks' characteristics or institutional or legal constraints, will deliver an intermediate outcome between the two considered.

4 Quantitative Implications

The approach developed in this paper allows us to link the theoretical tradeoffs that determine the optimal deposit insurance policy to a small number of observables. To show the applicability of our results in practice, we now study the quantitative implications of our framework for the optimal deposit insurance level. We approach this task in two different ways.

First, we illustrate how to use our directional test by directly measuring the sufficient statistics that we identify in this paper. This approach has the advantage of sidestepping the need to specify model parameters and functional forms, but it faces significant challenges given the current state of measurement, as we acknowledge below. Next, we use the set of sufficient statistics that we identify in this paper to understand how specific changes in primitives affect the optimal deposit insurance policy when numerically solving the model. Our theoretical characterization provides an intermediate step between primitives and welfare assessments. This approach has the potential to improve our understanding of

[24] See Goulder (1995) for a discussion of classic double-dividend arguments in the context of environmental regulation.

the implications of complex structural models of banking for deposit insurance.

4.1 Direct Measurement

Our first strategy seeks to find plausible empirical counterparts of the sufficient statistics identified in Proposition 2, which expresses welfare changes purely as a function of variables aggregated to the bank level. In order to further facilitate the measurement process, we focus on measuring the marginal welfare change per deposit account measured in dollars for a hypothetical representative bank, given by $\frac{dW_k}{d\delta}/\overline{G}_k$. Formally, starting from Proposition 2 applied to a given representative bank, indexed by k, we can express $\frac{dW_k}{d\delta}/\overline{G}_k$ as follows

$$\frac{\frac{dW_k}{d\delta}}{\overline{G}_k} \approx q_k^F \left(-\frac{\partial \log q_k^F}{\partial \delta} \frac{\int \left[C_{j,k}^N (s^*) - C_{j,k}^F (s^*) \right] dj}{\overline{G}_k} - \mathbb{E}_s^F \left[\kappa' (\cdot) \right] \frac{\int_{\frac{\delta}{R_1}}^{\overline{D}} dG_k (i)}{\overline{G}_k} \right), \tag{28}$$

where $\overline{G}_k = \int_0^{\overline{D}} dG_k (i)$ denotes the measure of accounts in bank k, q_k^F denotes the probability of bank failure, $\frac{\partial \log q_k^F}{\partial \delta} \equiv \frac{\partial q_k^F / q_k^F}{\partial \delta}$ denotes the semi-elasticity of bank failure with respect to a change in the level of coverage and $\mathbb{E}_s^F [\kappa' (\cdot)]$ denotes the expected marginal net cost of public funds. To find the marginal welfare change at the bank level, it is sufficient to multiply Equation (28) by the measure of account holders in a given bank. By appropriately extrapolating from a given representative bank to the whole banking sector, it is possible to produce aggregate welfare assessments.

We summarize our preferred measures of the sufficient statistics required to compute Equation (28) in Table 1. We interpret the horizon of the model as a one-year period in the data and rely on three sources of information: data on traded CDS (Credit Default Swaps) contracts on banks from Markit, FDIC's historical banking statistics, and the unique set of statistics on the composition of bank deposits for the representative bank reported by Martin, Puri and Ulfier (2017). A challenge for the empirical implementation is that all measures of sufficient statistics are in principle state- and time-dependent. Our baseline results, which correspond to the central element in Table 2, could be interpreted as applying to 2006/2007, consistently with the data of the placebo period (before the bank is in distress) analyzed in Martin, Puri and Ulfier (2017). Other estimates, as discussed below, may be more appropriate for different scenarios.[25]

We initially describe the marginal cost estimates, since they have more easily measurable counterparts. First, we look at forward-looking and historical estimates of bank failure probabilities. We use CDS data to compute average implied yearly default probabilities for a sample of banks – see the Appendix for a detailed explanation of these and other calculations with CDS data. We find average implied default probabilities of 0.23% per year around the reference year 2006. However, we also find

[25] By building a dynamic extension of the model and exploiting recursive methods, it is in principle possible to accommodate variation across time and states in the measured objects – see Dávila (2019) for a dynamic application of a similar methodology in a different context. We conjecture that a weighted sum across states of different estimates of Equation (28) will turn out to be the correct method to assess the welfare gains/losses of changing the level of coverage in a dynamic environment.

Table 1: Direct Measurement: Sufficient Statistics (Baseline)

Variable	Description	Value
	Marginal benefit	
$\frac{\partial \log q_k^F}{\partial \delta}$	Sensitivity of log failure probability to change in DI limit	$-\frac{0.129}{150{,}000}$
$\int \left[C_{j,k}^N(s^*) - C_{j,k}^F(s^*)\right] dj / \overline{G}_k$	Resource losses per account after failure	$7,840
	Marginal cost	
$\kappa'(\cdot)$	Net marginal cost of funds	13%
$\int_{\frac{\delta}{R_1}}^{\overline{D}} dG_k(i) / \overline{G}_k$	Fraction of partially insured depositors	6.4%
q_k^F	Probability of bank failure	0.75%

Note: Table 1 includes the baseline measures of the relevant sufficient statistics. The sensitivity of the probability of bank failure to a change in the coverage limit is computed using CDS data from Markit through WRDS. The measure of resource losses combines information from Martin, Puri and Ulfier (2017) with estimates from Granja, Matvos and Seru (2017) and Bennett and Unal (2015). The cost of public funds is consistent with Dahlby (2008). The fraction of partially insured depositors comes from Martin, Puri and Ulfier (2017). The probability of bank failure, as discussed in the text, combines FDIC's historical banking statistics with the CDS data.

average implied default probabilities of 1.58% during the post-crisis period 2012-2014. Alternatively, a direct estimate of historical bank failure probabilities, using the FDIC Historical Statistics on Banking starting in 1934, yields estimates of yearly failure probabilities of roughly 0.42%. In the interest of considering stable estimates, we decide to choose an average rate of 0.75% as our baseline estimate, which is closer to the long-run average of implied probability measures recovered from CDS data of 1.1%. We choose 0.23% (the long-run realized average failure probability) and 6.67% (the average implied failure in 10/3/2018) as low and high estimates for q_k^F in the sensitivity analysis in Table 2. Second, we use an average estimate of the net marginal cost of funds of around 13%, within the range of plausible values summarized in Dahlby (2008). As we describe above, this loss need not be literally interpreted as arising from distortionary taxation, but it could instead represent the marginal deadweight loss associated with the need to devote resources to build a Deposit Insurance Fund.[26] Third, we use 6.4% as the fraction of uninsured depositors, based on the representative bank studied in Martin, Puri and Ulfier (2017).

Next, we describe the estimates that determine the marginal benefit of changing the level of coverage, which are harder to identify – see the Appendix for a more detailed explanation. By using the change in the implied probability in bank failure around the last change in the level of coverage from $\$100,000$ to $\$250,000$ on October 3, 2008, we can provide a sense of how failure probabilities react to change in the level of coverage. In the Appendix, we document that the average implied probability of failure for the eight largest banks moved from 6.67% to 6.11% after the policy change, although there is substantial variation across banks. We use the change in the implied probability of failure among those banks for which their failure probabilities went down for our baseline estimate of the semi-elasticity:

[26] If optimally managed, the marginal cost of funds from taxation or from funding the deposit insurance fund should be identical.

Table 2: Direct Measurement: Sensitivity Analysis

	q_k^F: **0.23%**	**0.75%**	**6.68%**
$-\frac{\partial \log q_k^F}{\partial \delta}$: **0**	-0.86	-2.81	-25.08
0.13/150000	-0.16	-0.51	-4.60
0.40/150000	1.30	4.25	37.95

Note: Table 2 shows the value of $\frac{dW_k}{d\delta}$, computed as described in Equation (28), for different values of failure probabilities q_k^F and semi-elasticities $\frac{\partial \log q_k^F}{\partial \delta}$. Each element of this table measures the marginal dollar gain/loss induced by a one-dollar change in the level of coverage for the bank considered. It follows directly from Equation (28) that increasing (decreasing) the probability of failure q_k^F holding $\frac{\partial \log q_k^F}{\partial \delta}$ constant only increases (decreases) the magnitude of $\frac{\frac{dW_k}{d\delta}}{\overline{G}_k}$, but it does not change its sign.

$\frac{\partial \log q_k^F}{\partial \delta} \approx -\frac{0.13}{150{,}000} \approx -8.67 \times 10^{-7}$. We choose 0 (a theoretical lower bound) and 40% (the highest estimated individual sensitivity) as low and high estimates for $\frac{\partial \log q_k^F}{\partial \delta}$ in the sensitivity analysis in Table 2. We are aware that this approach to measurement is fraught with difficulties, since this specific policy change is far from a random event, and the change in the level of coverage is only one of the many measures that formed part of the Emergency Economic Stabilization Act passed on that date. That said, these are the best estimates of the desired elasticity that can be obtained with the existing data, so we hope that this paper spurs future measurement efforts on this topic. Finally, we compute the resource loss in case of a marginal failure, which we set at $\$7,840$, by combining the average deposit balance of $\$28,000$, as reported by Martin, Puri and Ulfier (2017), with a recovery rate on bank assets after failure of 72%, consistent with the estimates of Granja, Matvos and Seru (2017).[27] By using this estimate, Equation (21) implies that we are disregarding the fiscal savings associated with avoiding bank failure.

Combining all measures, we can use Equation (28) to compute the marginal welfare gain per deposit account associated with a one-dollar increase in the level of coverage. Given our baseline estimates, we find that $\frac{dW_k}{d\delta}/\overline{G} \approx -\$1.14 \times 10^{-5}$, which implies that a local decrease in coverage may be welfare improving. Similarly, we can compute the marginal welfare gain of a dollar increase in the level of coverage for the whole bank, given by

$$\frac{dW_k}{d\delta} \approx -\$1.14 \times 10^{-5} \times 45,000 \approx -\$0.51,$$

where $\overline{G}_k = 45,000$ denotes the measure (number) of accounts reported by Martin, Puri and Ulfier (2017). That is, our results imply that an increase in the level of coverage of $\$100,000$ is associated with a total loss related to the representative bank considered here of roughly $\$51,000$ dollars per year. When capitalized using a 3% discount rate, and expressed in relation to the asset size of the bank (2 billion), this seems like a small gain, of the order of $\frac{\frac{51000}{3\%}}{2\times 10^9} \approx 8.5 \times 10^{-4}$ (eight and a half bps) of total assets.

[27] See also Bennett and Unal (2015). Given that Martin, Puri and Ulfier (2017) report that the bank they study has 45,000 accounts, and the average deposit balance is roughly \$28,000, the bank's total deposits correspond to $28,000 \times 45,000 = 1.26$ billion. They also report that the bank has roughly 2 billion in assets.

We interpret these values as a manifestation that levels of coverage were set around the optimum in the baseline period.[28] However, in scenarios in which failure probability are large, and deposit insurance is more effective (bottom left element of Table 2), there are potentially large welfare gains from increasing the level of coverage. In this case, an increase of \$100,000 in the level of coverage is associated with a gain of $\frac{\frac{3795000}{3\%}}{2\times 10^9}0.06\% \approx 6.325\%$ of total assets. While we do not measure banks' changes in behavior directly, we appeal to our results in Section 3 and 5 to interpret this value as capturing the marginal welfare change induced when optimal regulation is also implemented.

Finally, note that the main drawback of the approach used here is the inherently local nature of the measurement exercise. Ideally, one could generate measures of the relevant sufficient statistics for any level of coverage. However, as just described, this is challenging to do in practice. Therefore, it may be useful to rely on a fully specified model to understand how social welfare varies with the level of coverage for deposit limits far from the existing ones. We provide a first step towards that goal in Section 4.2.

Since we aim to guide future measurement efforts, we conclude with the following remark.

Remark. Implications for future measurement. The main challenge of this direct measurement exercise is to find appropriate values for $\frac{\partial \log q_k^F}{\partial \delta}$, since changes in the level of coverage are often a response to banks' distress, which biases the coefficient of a simple regression of $\log q_k^F$ (or, similarly, an indicator of bank failure) on δ. Our approach suggests that finding quasi-experimental variation in δ, perhaps exploiting a change in the level of coverage unrelated to banks' profitability and failure probabilities, can be highly informative for policymakers. Alternative, policymakers may want to elicit how failure probabilities change in response to δ either directly from banks or other markets participants, or perhaps by doing some experimentation through policy changes.

4.2 Numerical Simulation

Our characterization in terms of sufficient statistics has also direct implications for quantitative modeling. Next, to illustrate how changes in primitives affect welfare assessments through the sufficient statistics identified in the paper in the context of a fully specified model, we explicitly solve our model for different parameter specifications. By explicitly computing the sufficient statistics in a parametrized model, we provide an intermediate step between primitives and welfare assessments. This approach results should be of interest to the growing quantitative structural literature on banking, since our characterization allows us to provide further insights into how to interpret the normative implications of calibrated structural models.

Functional Forms and Parameter Choices

To explicitly solve the model, we must make specific functional form assumptions that were not needed to derive our theoretical results or to conduct the direct measurement exercise. Table 3 in the Appendix

[28] In our numerical simulation, we show that the welfare losses associated with setting levels of coverage that are too large are lower than the losses associated with setting levels of coverage that are too low.

summarizes the choice of baseline parameters and functional forms, which we describe next. We select parameters plausibly consistent with US data and interpret a period in our model as a year.

First, we assume that depositors have isoelastic utility with an elasticity of intertemporal substitution $\frac{1}{\gamma}$, that is, $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$. In our baseline parametrization, we set $\gamma = -c\frac{u''(c)}{u'(c)} = 2$, a conventional choice. We also assume that the aggregate state is log-normally distributed according to $F(s)$, with a truncated support $[\underline{s}, \overline{s}]$. We set $\mu_s = 0.04$ and $\sigma_s = 0.01$ as parameters of the underlying normal distribution, and $[\underline{s}, \overline{s}] = [0.99, 1.1]$, which generates the distribution shown in Figure 12 that can be interpreted as the banks' annual return on assets. The parameters of the distribution $F(s)$, along with the choice of $\lambda = 0.05$, directly pin down the probability of fundamental failures, set at 1.43% per year in our parametrization. The choice of $\pi = 0.2$ implies that the probability of bank failure without deposit insurance is roughly 6.5% per year. We further assume that depositors outside sources of wealth scale proportionally with the level of their deposits, that is, $Y_{1i}(s) = y_1(s) D_{0i}$ and $Y_{2i}(s) = y_2(s) D_{0i}$, where $y_1(s), y_2(s) \geq 0$. The choice $y_1(s) = 3$ (implying that 25% of wealth is held as deposits) is within the set of estimates for households. Setting $y_2(s) = 3.05 > y_1(s)$ guarantees that banks are willing to offer deposit rates consistent with bank failures in equilibrium.

Next, we also need to make assumptions on i) the structure of bank returns $\rho_1(s)$ and $\rho_2(s)$, ii) the cost of public funds $\kappa(T)$, and iii) the recovery rate on bank assets after failure $\chi(s)$. First, we normalize the date 2 return to be $\rho_2(s) = s$ and express the date 1 return as

$$\rho_1(s) = 1 + \varphi(s-1), \quad \text{where} \quad \varphi \in [0,1],$$

and where we set $\varphi = 0.95$ in our baseline parametrization to modulate the size of the region with a unique (no failure) equilibrium. Second, we consider a marginal cost of public funds $\kappa(T)$ of the following exponential-affine form

$$\kappa(T) = \frac{\kappa_1}{\kappa_2}\left(e^{\kappa_2 T} - 1\right), \quad \text{where} \quad \kappa_1, \kappa_2 \geq 0.$$

The parameter $\kappa_1 = \kappa'(0)$ represents the marginal cost of public funds for a small intervention, which we set to $\kappa_1 = 0.13$, consistent with conventional estimates of marginal costs of public funds. The parameter $\kappa_2 = \frac{\kappa''(T)}{\kappa'(T)}$, is a measure of curvature, which we set to a small value of $\kappa_2 = 0.5$. Third, we model deadweight losses of failure as follows

$$\chi(s) = \chi_1 (s - \chi_3)^{\chi_2}, \quad \text{where} \quad \chi_1, \chi_2, \chi_3 \geq 0.$$

In this case, we select parameters so that $\chi(s_{min}) = 0$, $\chi(s_{max}) = 1$, and so that the average loss is equal to 28%, as measured by Granja, Matvos and Seru (2017).[29] Finally, we set the underlying parameters of the distribution of deposit holdings to be $\mu_D = 1$ and $\sigma_D = 2$ and set the maximum deposit to 8. When the units are interpreted in hundreds of thousands of dollars, these parameters respectively imply median and average deposits of \$128,000 and \$206,000, capturing the observed right skewness in deposit holdings in practice, e.g., (Martin, Puri and Ulfier, 2017) – see Figure 12 for the exact shape of the implied distribution of deposits.

[29] Formally, we set χ_1, χ_2, and χ_3 to satisfy $s_{min} = \chi_3$, $\chi_1 (s_{max} - \chi_3)^{\chi_2} = 1$, and $\chi_1 \int_{s_{min}}^{s_{max}} (s - \chi_3)^{\chi_2} \, dF(s) = 0.72$.

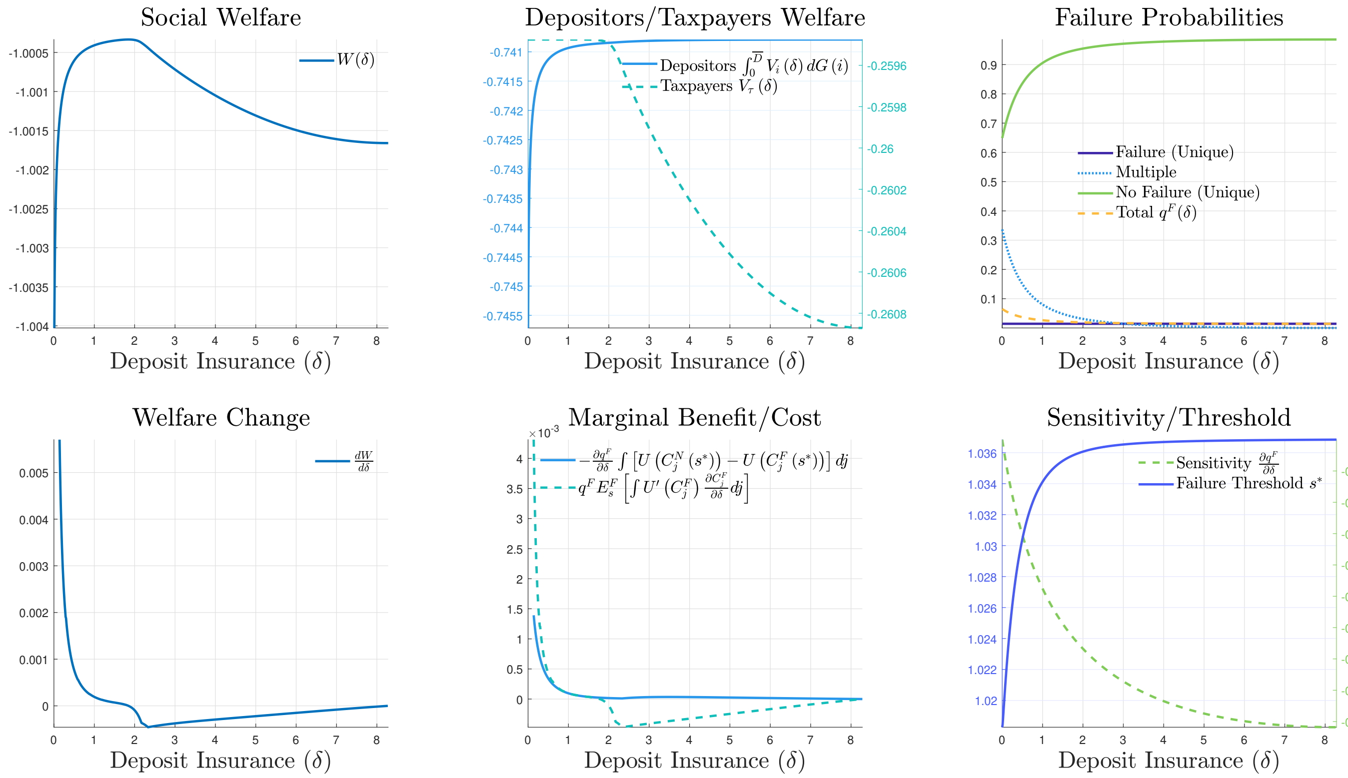


Figure 5: Social Welfare Decomposition

Note: The top left plot in Figure 5 shows social welfare $W(\delta)$ for different levels of deposit insurance coverage δ (measured in hundreds of thousands of dollars). The top middle plot separately shows the welfare of depositors and taxpayers, as defined in Equation (17). The top right plot shows the equilibrium probabilities of bank failure and its decomposition in unique and multiple equilibrium probabilities, as defined in Equation (12). The bottom left plot shows $\frac{dW}{d\delta}$, as defined in Equation (20). The bottom middle plot separately shows the marginal benefit and cost terms defined in Equation (20). The bottom right plot shows the sensitivity of the probability of failure and the failure threshold for different value of δ. The parameters used in all figures are: $\gamma = 2$, $\lambda = 0.05$, $\pi = 0.2$, $\mu_D = 1$, $\sigma_D = 2$, and $\overline{D} = 8$, $Y_{1i}(s) = 3D_{0i}$ and $Y_{2i}(s) = 3.05D_{0i}$, $\mu_s = 0.04$, $\sigma_s = 0.01$, and $[\underline{s}, \overline{s}] = [0.99, 1.1]$, $\varphi = 0.95$, $\chi_1 = 1.68$, $\chi_2 = 0.28$, and $\chi_3 = 0.99$, $\kappa_1 = 0.13$ and $\kappa_2 = 0.5$, $Y_\tau(s) = 3D_{0i}$ where $F(D_{0i}) = 0.5$, and $R_1 = 1.03$.

To more clearly illustrate the link between primitives and sufficient statistics, and how the latter influence the optimal deposit insurance policy, we focus on the case in which the deposit rate is fixed and set at $R_1 = 1.03$, although it is possible to solve the model under perfect and imperfect regulation.

Welfare Decomposition and Comparative Statics through the Sufficient Statistics

First, we leverage our decomposition of the marginal welfare change $\frac{dW}{d\delta}$ to illustrate how social welfare varies with the level of deposit insurance. Subsequently, we conduct two comparative statics exercises. We initially describe through the lens of Proposition 1 how the marginal value of changing the level of coverage and the optimal level of coverage vary with changes in the dispersion of banks' returns (changes in riskiness). Next, we use the same approach to describe how variation in the marginal cost of public funds affects the optimal level of coverage. Varying κ_1 exclusively affects the cost of intervention, while varying σ_s affects both benefits and costs of intervention.

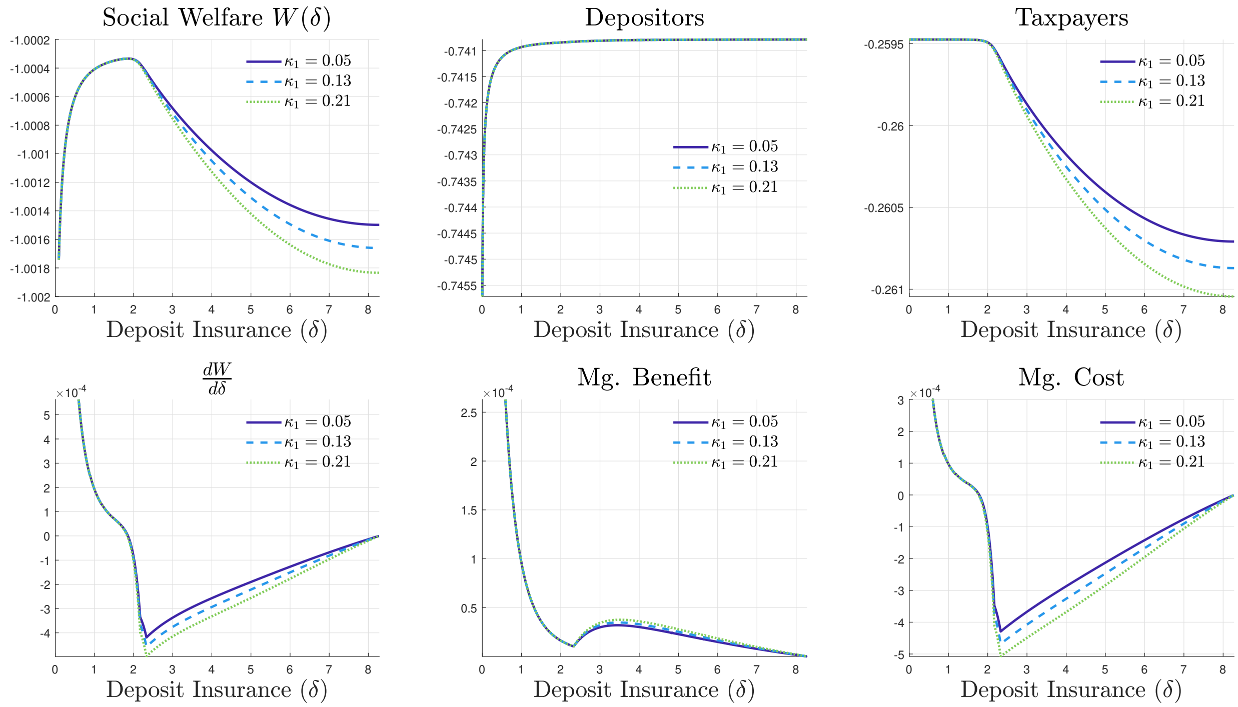


Figure 6: Comparative Statics: Cost of Public Funds (κ_1)

Note: The top left plot in Figure 6 shows social welfare $W(\delta)$ as a function of δ (measured in hundreds of thousands of dollars) for $\kappa_1 \in \{0.05, 0.13, 0.21\}$, as defined in Equation (17). The top middle and left plots respectively show the welfare of depositors and taxpayers, as defined in Equation (17), for the different values of κ_1. The bottom left plot shows $\frac{dW}{d\delta}$, as defined in Equation (20). The bottom middle and left plots respectively show the marginal benefit and cost terms defined in Equation (20), for the different values of δ. The optimal levels of coverage $\delta^\star$ respectively are $\delta^\star = 1.86$, $\delta^\star = 1.85$, and $\delta^\star = 1.84$ for $\kappa_1 = 0.05$, $\kappa_1 = 0.13$, and $\kappa_1 = 0.21$. The parameters used in all figures are: $\gamma = 2$, $\lambda = 0.05$, $\pi = 0.2$, $\mu_D = 1$, $\sigma_D = 2$, and $\overline{D} = 8$, $Y_{1i}(s) = 3D_{0i}$ and $Y_{2i}(s) = 3.05D_{0i}$, $\mu_s = 0.04$, $\sigma_s = 0.01$, and $[\underline{s}, \overline{s}] = [0.99, 1.1]$, $\varphi = 0.95$, $\chi_1 = 1.68$, $\chi_2 = 0.28$, and $\chi_3 = 0.99$, $\kappa_1 \in \{0.05, 0.13, 0.21\}$ and $\kappa_2 = 0.5$, $Y_\tau(s) = 3D_{0i}$ where $F(D_{0i}) = 0.5$, and $R_1 = 1.03$.

Figure 5 shows how $W(\delta)$ and $\frac{dW}{d\delta}$, as well as several of its determinants vary with δ – Sections A and E in the Appendix include additional illustrations of other relevant equilibrium objects. Two findings are worth highlighting. First, the slope of $W(\delta)$ (equivalently, the value of $\frac{dW}{d\delta}$) is particularly high for low levels of coverage, which implies that there are large gains from having positive levels of coverage. Two different channels account for this fact. First, when δ is low, the marginal impact of δ on reducing the probability of failure $\frac{\partial q^F}{\partial \delta}$ is large, which directly increases the marginal benefit of increasing the level of coverage (see the first term of Equation (20)). Second, when δ is low, the marginal cost of increasing the level of coverage (the second term of Equation (20)) is instead positive (a benefit), since the funding shortfall is zero because banks' resources are enough to cover insured depositors, but there is a redistributional benefit from transferring funds in case of failure to depositors with low deposit amounts and high marginal utility. For moderate and high levels of δ, the deadweight losses associated with covering the funding shortfall start to dominate, which yields an interior optimum for δ. The second finding corresponds to the observation that increasing δ is Pareto improving at low levels of coverage. It should not be surprising that depositors as a whole are better off when δ is higher, since they receive a transfer from taxpayers in case of failure.[30] However, taxpayers' welfare does not vary with δ when the level of coverage is sufficiently low because the resources of failing banks are sufficient to cover all deposit insurance claims, and the funding shortfall and its associated deadweight losses are zero.

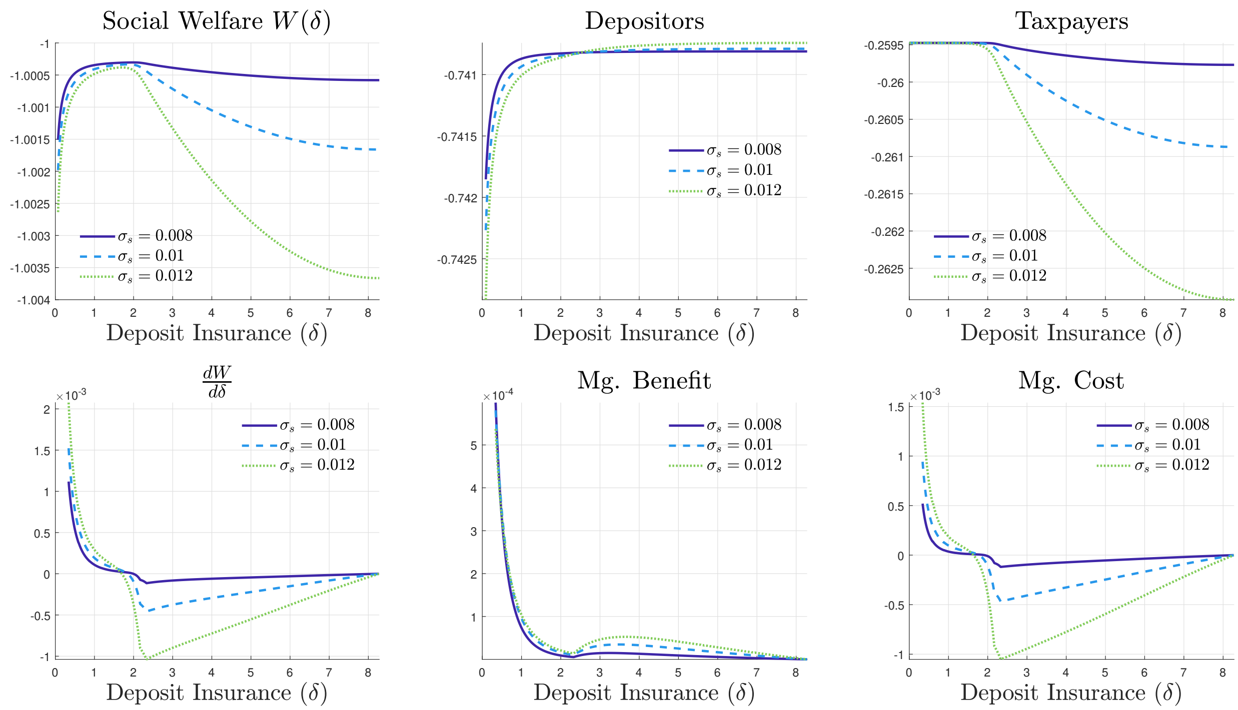


Figure 7: Comparative Statics: Banks' Riskiness (σ_s)

Note: The top left plot in Figure 6 shows social welfare $W(\delta)$ as a function of δ (measured in hundreds of thousands of dollars) for $\sigma_s \in \{0.08, 0.1, 0.12\}$, as defined in Equation (17). The top middle and left plots respectively show the welfare of depositors and taxpayers, as defined in Equation (17), for the different values of σ_s. The bottom left plot shows $\frac{dW}{d\delta}$, as defined in Equation (20). The bottom middle and left plots respectively show the marginal benefit and cost terms defined in Equation (20), for the different values of δ. The optimal levels of coverage $\delta^\star$ respectively are $\delta^\star = 1.99$, $\delta^\star = 1.85$, and $\delta^\star = 1.7$ for $\sigma_s = 0.05$, $\sigma_s = 0.13$, and $\sigma_s = 0.21$, illustrated in Figure 15. The parameters used in all figures are: $\gamma = 2$, $\lambda = 0.05$, $\pi = 0.2$, $\mu_D = 1$, $\sigma_D = 2$, and $\overline{D} = 8$, $Y_{1i}(s) = 3D_{0i}$ and $Y_{2i}(s) = 3.05D_{0i}$, $\mu_s = 0.04$, and $[\underline{s}, \overline{s}] = [0.99, 1.1]$, $\varphi = 0.95$, $\chi_1 = 1.68$, $\chi_2 = 0.28$, and $\chi_3 = 0.99$, $\kappa_1 = 0.13$ and $\kappa_2 = 0.5$, $Y_\tau(s) = 3D_{0i}$ where $F(D_{0i}) = 0.5$, and $R_1 = 1.03$.

Figure 6 illustrates the comparative static exercise of varying the linear parameter of the cost of public funds κ_1. Consistent with our analytical results, changes in the level of κ_1 exclusively affect taxpayers' welfare, leaving unchanged depositors' welfare. While increasing κ_1 significantly reduces overall welfare, the impact on $\frac{dW}{d\delta}$ and the optimal level of coverage is less pronounced. This can be seen by looking at the marginal benefit/cost decomposition in the bottom middle and left plots in Figure 6. On the one hand, increasing the cost of public funds increases the marginal benefit of increasing the level of coverage, since avoiding the gain of reducing deadweight losses for the case of the marginal failure is higher. On the other hand, since the cost of public funds is higher in bank failure states, increasing the total level of coverage becomes costlier. All considered, we find that higher κ_1 is associated with a lower

[30]Note that not all depositors are better off when δ is high. For moderate levels of δ, depositors with a large amount of uninsured funds may be worse off when δ increases, since the recovery rate on uninsured deposits is decreasing in δ.

optimal level of coverage $\delta^{\star}$.

Finally, Figure 7 illustrates the comparative exercise of increasing banks' riskiness. Increasing σ_s unambiguously reduces the welfare of taxpayers, since negative realizations of s, in which bank failure is more prevalent and costly, are likely to occur. Interestingly, an increase in σ_s has an ambiguous impact on depositors' utility, depending on the level of δ. When the level of coverage is low, the increased volatility generates worse and more frequent failures, lowering depositors' welfare. When the level of coverage is high, depositors benefit instead of the increase in volatility, since they receive all the upside when bank returns are high, but are shielded from bank failure by the generous level of coverage. In our simulation, the effects on taxpayers' and depositors' welfare combine so that higher riskiness is associated with lower levels of the optimal level of coverage, although one can conceive scenarios in which higher riskiness calls for optimally increasing the level of deposit insurance coverage.

5 Extensions

In this section, we extend the baseline model to show that our main findings remain valid more generally. Our goal in this Section is to show that Proposition 1 continues to be valid exactly or suitably modified once we relax many of the model assumptions. We explicitly incorporate flexible investment decisions by banks and multiple portfolio choices for depositors, an alternative equilibrium selection mechanism, and allow for aggregate spillovers. To ease the exposition, we study every extension separately, and focus on the characterization of marginal changes in the level of deposit insurance under perfect regulation of the deposit rate, although the analysis can be extended to other scenarios along the lines of Section 3. When appropriate, we discuss the implications for the optimal design of ex-ante regulation. Finally, we also explain how our results could be extended to incorporate additional features that may impact the optimal determination of deposit insurance.

5.1 Banks' Moral Hazard: General Portfolio and Investment Decisions

In our baseline formulation, neither depositors nor banks had portfolio decisions. Allowing for both sets of decisions is important to allow banks of depositors to adjust their risk-taking behavior in response to changes in the level of coverage – these effects are also often referred to as moral hazard. Depositors now have a consumption-savings decision at date 0 and a portfolio decision among K securities. In particular, depositors have access to $k = 1, 2, \ldots, K$ assets, with returns $\theta_{1k}(s)$ at date 1 in state s for early depositors and returns $\theta_{2k}(s)$ at date 2 in state s for late depositors. Hence, the resources of early and late depositors are respectively given by $Y_{1i}(s) = \sum_k \theta_{1k}(s)\, y_{ki}$ and $Y_2(s) = \sum_k \theta_{2k}(s)\, y_{ki}$. We preserve the structure of the distribution of deposits. Therefore, the budget constraint of depositors at date 0 is given by

$$\sum_k y_{ki} + D_{0i} + C_{0i} = Y_{0i}, \tag{29}$$

where Y_{0i}, which denotes the initial wealth of depositors, and D_{0i} are primitives of the model. Subject to Equation (29), the ex-ante utility of depositors now corresponds to

$$U\left(C_{0i}\right)+\mathbb{E}_s\left[\lambda U\left(C_{1i}\left(s\right)\right)+\left(1-\lambda\right)U\left(C_{2i}\left(s\right)\right)\right], \tag{30}$$

where $C_{1i}(s)$ and $C_{2i}(s)$ respectively denote the consumption of early and late depositors with initial deposits D_{0i} for a given realization of the aggregate state s. Depositors optimally choose their holdings of the different asset y_{ki} to maximize their expected utility.

Additionally, banks have access to $h=1,2,\ldots,H$ investment opportunities, which offer a gross return $\rho_{1h}(s)$ at date 1 and a return $\rho_{2h}(s)$ between dates 1 and 2 in state s. Hence, at date 0, banks must choose shares ψ_h for every investment opportunity such that $\sum_h \psi_h = 1$. We assume that banks liquidate an equal fraction of every type of investment at date 1. This is a particularly tractable formulation to introduce multiple investment opportunities. Our results could be extended to the case in which different investments have different liquidation rates at date 1 and banks have the choice of liquidating different investments in different proportions.

Given our assumptions, we show in the Appendix that the counterpart to the failure threshold $\tilde{D}_1(s)$ in Equation (7) is given by

$$\tilde{D}_1\left(s\right)=\frac{\left(R_1-\sum_h \rho_{1h}\left(s\right)\psi_h\right)D_0}{1-\frac{1}{\frac{\sum_h \rho_{2h}(s)\rho_1(s)\psi_h}{\sum_h \rho_{1h}(s)\psi_h}}}, \tag{31}$$

allowing us to characterize the equilibrium thresholds $\hat{s}$ and s^* as in the baseline model. It is equally straightforward to generalize the values taken by $\alpha_F(s)$, $\alpha_N(s)$, and $T(s)$. We characterize in the Appendix the optimal choices of y_{ki} and ψ_h by depositors and banks and focus again on the directional test for how welfare varies with the level of coverage.

Proposition 6. (Directional test for δ under general investment opportunities) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ under perfect regulation is given by*

$$\frac{dW}{d\delta}=-\frac{\partial q^F}{\partial \delta}\int\left[U\left(C_j^N\left(s^*\right)\right)-U\left(C_j^F\left(s^*\right)\right)\right]dj+q^F\mathbb{E}_s^F\left[\int U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial \delta}dj\right]$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta}>(<)\,0$, it is optimal to locally increase (decrease) the level of coverage.

Proposition 6 extends the results of the baseline model by showing that introducing a consumption-savings and portfolio choices for depositors does not modify the set of sufficient statistics already identified under perfect regulation. However, allowing unregulated banks to make investment choices requires accounting for a new set of fiscal externality terms. The new set of fiscal externalities, which capture the direct effects of banks' changes in behavior on taxpayers welfare, is now given by

$$\underbrace{\frac{\partial V_\tau}{\partial R_1}\frac{dR_1}{d\delta}}_{\text{Liability-side regulation}}+\underbrace{\sum_h\frac{\partial V_\tau}{\partial \psi_h}\frac{d\psi_h}{d\delta}}_{\text{Asset-side regulation}}. \tag{32}$$

As in Section 3.2, we expect more generous levels of coverage to increase the deposit rate, so $\frac{\partial V_\tau}{\partial R_1}\frac{dR_1}{d\delta} < 0$, making socially costlier to increase δ, since banks do not internalize the fiscal consequences of offering higher deposit rates. In principle, it is impossible to individually sign each of the H terms $\frac{\partial V_\tau}{\partial \psi_h}\frac{d\psi_h}{d\delta}$ that determine the regulation of banks' asset allocations. However, in most cases, it is reasonable to expect that the sum of all these terms takes negative values, since competitive banks have an incentives to increase their risk-taking when the level of coverage is higher. Previous research has nonetheless shown that the risk-taking behavior of banks is sensitive to the details of the market environment; see, for instance, Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010). In imperfectly competitive environments, it should not be surprising for the asset-side regulation term in Equation 32 to take positive values.

However, regardless of their sign, our results robustly point out that both liability-side regulations, controlling the deposit rate offered by banks, and asset-side regulations, controlling the investment portfolio of banks are in general needed to maximize social welfare when ex-ante policies are feasible.[31] The optimal corrective policy introduces wedges on banks' choices that can be approximated as follows

$$\tau_{R_1} = -\frac{\partial V_\tau}{\partial R_1} \approx \frac{\partial \mathbb{E}_s\left[T(s) + \kappa\left(T(s)\right)\right]}{\partial R_1} \quad \text{and} \quad \tau_{\psi_h} = -\frac{\partial V_\tau}{\partial \psi_h} \approx \frac{\partial \mathbb{E}_s\left[T(s) + \kappa\left(T(s)\right)\right]}{\partial \psi_h}. \tag{33}$$

As discussed above, restrictions on the set of ex-ante instruments available to the planner deliver intermediate outcomes between the two extremes analyzed here. Equation (33) provides direct guidance on how to set ex-ante policies to correct the ex-ante distortions on banks' behavior caused by deposit insurance.

5.2 Alternative Equilibrium Selection Mechanisms

In the baseline model, depositors coordinate following an exogenous sunspot. We now show that varying the information structure and the equilibrium selection procedure does not change the sufficient statistics we identify. We consider a global game structure in which late depositors observe at date 1 an arbitrarily precise private signal about the date 2 return on banks' investments, before deciding $D_{1i}(s)$. With that information structure, Goldstein and Pauzner (2005) show, in a model which can be mapped to our baseline model with no deposit insurance, that there exists a unique equilibrium in threshold strategies in which depositors withdraw their deposits when they receive a sufficiently low signal but leave their deposits in the bank otherwise.

Since our goal in this paper is to show the robustness of our optimal policy characterization and to directly use the set of sufficient statistics that we identify, we take the outcome of a global game as a primitive. In particular, we take as a prediction of the global game that there exists a threshold $s^G(\delta, R_1)$ such that when $s \leq s^G(\delta, R_1)$ there is a bank failure with certainty but when $s > s^G(\delta, R_1)$

[31] In practice, capital requirements and net stable funding ratios are forms of liability-side regulations, while liquidity coverage ratios are an example of asset-side regulations. See Diamond and Kashyap (2016) for a recent assessment of these policy measures in a model of runs and **?** for a study of its ex-ante consequence in an environment with strategic banks.

no failure occurs, with the following properties

$$\frac{\partial s^G}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^G}{\partial \delta} \leq 0.$$

Goldstein and Pauzner (2005) formally show that $\frac{\partial s^G}{\partial R_1} \geq 0$, while Allen et al. (2018) formally show that $\frac{\partial s^G}{\partial \delta} \leq 0$ in a special case of our framework. In fact, any model of behavior which generates a threshold with these properties, not necessarily a global game, is consistent with our results.

Therefore, given the behavior of depositors at date 1, the ex-ante welfare of depositors is now given by $\int_0^{\overline{D}} V_i (\delta, R_1) \, dG (i)$, where

$$V_i (\delta, R_1) = \lambda \mathbb{E}_s [U (C_{1i} (s))] + (1 - \lambda) \mathbb{E}_s [U (C_{2i} (s))], \tag{34}$$

and we define

$$\mathbb{E}_s [U (C_{ti} (s))] = \int_{\underline{s}}^{s^G(\delta, R_1)} U \left(C_{ti}^F (s) \right) dF (s) + \int_{s^G(\delta, R_1)}^{\overline{s}} U \left(C_{ti}^N (s) \right) dF (s),$$

where early and late depositors' consumption is exactly defined by Equations (4) and (5). We can then show that the characterization of $\frac{dW}{d\delta}$ remains valid in this context.

Proposition 7. (Directional test for δ under an alternative equilibrium selection) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ under perfect regulation is given by*

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial \delta} \int \left[U \left(C_j^N (s^*) \right) - U \left(C_j^F (s^*) \right) \right] dj + q^F \mathbb{E}_s^F \left[\int U' \left(C_j^F \right) \frac{\partial C_j^F}{\partial \delta} dj \right], \tag{35}$$

where $\mathbb{E}_s^F [\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) \, 0$, it is optimal to locally increase (decrease) the level of coverage.

The particular information structure considered and the equilibrium selection procedure only enter in the expression of $\frac{dW}{d\delta}$ through the sufficient statistics identified in this paper. In particular, even though the sensitivity of the probability of bank failure to changes in the level of coverage $\frac{\partial q^F}{\partial \delta}$ will depend on the assumptions on the informational structure of the economy. Studying a global game model, as in Allen et al. (2018), is appealing because it makes it possible understand how the probability of failure is endogenously determined However, Proposition 7 shows that it is enough to measure the sufficient statistics identified in this paper.the normative formulas that we characterize apply to alternative information structures.

5.3 Aggregate Spillovers/Macro-Prudential Considerations

In our baseline formulation, as in Diamond and Dybvig (1983), bank decisions do not affect aggregate variables, so our analysis so far can be defined as micro-prudential. When the decisions made by banks affect aggregate variables, for instance, asset prices, further exacerbating the possibility of a bank failure, the optimal deposit insurance formula may incorporate a macroprudential correction. These general equilibrium effects arise in models in which aggregate outcomes determined by decentralized choices directly interact with coordination failures. Our extension captures in a simple form the macro implications of banks' choices, which may operate through pecuniary externalities or aggregate demand externalities (Dávila and Korinek, 2018; Farhi and Werning, 2016).

Formally, we now assume that, given a level of aggregate withdrawals $\overline{\Omega}(s) = D_0 R_1 - D_1(s)$, banks must liquidate $\theta\left(\overline{\Omega}(s)\right)$ of their investments, where $\theta(\cdot) \geq 1$ is a well-behaved increasing function. By assuming that banks have to liquidate more than one-for-one the number of investments at a rate that increases with the aggregate level of liquidations, we capture the possibility of illiquidity in financial markets when many banks unwind existing investments. This is a parsimonious way of incorporating aggregate linkages, but there is scope for richer modeling of interbank markets as in, for instance, Freixas, Martin and Skeie (2011). Under this assumption, the level of resources available to banks with withdrawals $\Omega(s)$, when the level of total withdrawals is $\overline{\Omega}(s)$, is given by

$$\rho_2(s)\left(\rho_1(s) D_0 - \theta\left(\overline{\Omega}(s)\right)\Omega(s)\right). \tag{36}$$

Equation (36) generalizes the left-hand side of Equation (3). When $\theta(\cdot) > 1$, it captures that the unit price of liquidating investments is increasing in the aggregate level of withdrawals. Following the same logic used to solve the baseline model, we can define thresholds $\hat{s}$ and s^*, which now have $\overline{\Omega}(s)$ as a new argument. When the regulator sets δ optimally, he takes into account the effects of individual banks' choices on the aggregate level of withdrawals $\overline{\Omega}(s)$. Under these assumptions, we show that $\frac{dW}{d\delta}$ satisfies the same equation as in our baseline model when ex-ante regulation is available, although it must incorporate a macroprudential correction when ex-ante regulation is not available.

Proposition 8. (Directional test for δ incorporating aggregate spillovers) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ under perfect regulation is given by*

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial \delta}\int \left[U\left(C_j^N\left(s^G\right)\right) - U\left(C_j^F\left(s^G\right)\right)\right] dj + q^F \mathbb{E}_s^F\left[\int U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial \delta} dj\right], \tag{37}$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<)\, 0$, it is optimal to locally increase (decrease) the level of coverage.

In this case, ex-ante regulation can target directly the wedges caused by aggregate spillovers. In this case, the ex-ante regulation faced by banks partly addresses both the fiscal externality that emerges from the presence of deposit insurance and the externality induced by the aggregate spillovers caused by competitive deposit rate setting. Similar formulas would apply when banks have general portfolio decisions, as in our analysis earlier in this section.

As in the case of moral hazard, it is possible to correct the welfare impact of aggregate spillovers with ex-ante regulation. The optimal corrective policy can be expressed in this case as

$$\tau_{R_1} = -\underbrace{\frac{\partial V_\tau^P}{\partial R_1}}_{\text{Fiscal. Ext.}} - \underbrace{\int_0^{\overline{D}}\left(\frac{\partial V_i^P}{\partial R_1} - \frac{\partial V_i}{\partial R_1}\right) dG(i)}_{\text{Spillovers}},$$

where the superscript P corresponds to the welfare assessment from the planner's perspective, as described in the Appendix. The first term account for banks' fiscal externalities, as studied above. The second term, which accounts for the general equilibrium spillovers of banks decisions, is a function of the terms $\frac{\partial s_P^*}{\partial R_1} - \frac{\partial s^*}{\partial R_1}$ and $\frac{\partial \hat{s}_P}{\partial R_1} - \frac{\partial \hat{s}}{\partial R_1}$, which account for the fact that the planner acknowledges that when banks offer higher rates withdrawals are higher and bank failures more likely.

5.4 Additional Channels

Finally, we now discuss how to think about incorporating into our framework several significant features that are relevant for the determination of the optimal level of coverage. While explicitly modeling each one of them is beyond the scope of the paper, we would like to discuss how they may affect our main characterization. It is important to notice that, while extending our model in several dimensions may require additional information to account for social welfare changes, the channels identified in this paper do not vanish.

Lender of Last Resort. In our baseline formulation, we exclusively consider the level of deposit insurance coverage as the single policy instrument. In practice, in addition to the level of coverage, banks often receive alternative forms of government support, through lender of last resort policies or bailouts. Within our framework, we can interpret this form of intervention as a state-contingent policy that increases the resources available to banks in certain states. Formally, we can consider the following counterpart to Equation (3),

$$\begin{array}{ll} \text{Bank Failure,} & \text{if } \rho_2(s)(\rho_1(s) D_0 - \Omega(s)) + \Lambda(s) < D_1(s) \\ \text{No Bank Failure,} & \text{if } \rho_2(s)(\rho_1(s) D_0 - \Omega(s)) + \Lambda(s) \geq D_1(s), \end{array}$$

where $\Lambda(s)$ captures the size of the ex-post intervention in state s. Propositions 1 and 3, and the associated sufficient statistics, remain valid in this case when $\Lambda(s)$ is predetermined or when it can be optimally designed. However, for our results to be meaningful, it must be the case that the lender of last resort policy is imperfect and unable to fully eliminate the existence of coordination failures.

Multiple Deposit Accounts. Our baseline model does not explicitly allow a given depositor to have multiple accounts in different banks, although, in practice, deposit limits are defined at the account level in most countries. However, as long as there is a cost of switching/opening deposit accounts, making deposits partially inelastic, which is consistent with the evidence in Egan, Hortaçsu and Matvos (2017), Proposition 1 remains valid once suitably reinterpreted. In this case, as we discuss in the next section, the relevant marginal cost of varying δ account needs to account for the insured/uninsured status of a given account, not necessarily an individual depositor.[32]

Equityholders/Debtholders/Liquidity Benefits. Since we build on the Diamond and Dybvig (1983) framework, our baseline formulation does not explicitly incorporate a role for equityholders and debtholders, or does not allow for demand deposits to have a non-pecuniary liquidity benefit. Allowing for richer funding structures would call for extending Propositions 1 through 4 to include all stakeholders.

[32]See Shy, Stenbacka and Yankov (2015) for a model in which depositors can explicitly open multiple deposit accounts.

Beyond that, on aggregate, the sufficient statistics that we identify already capture differences in capital structure choices across banks. For instance, one would expect banks with more fragile capital structures – perhaps more likely to face debt rollover concerns – to be more likely to fail and potentially more sensitive to interventions.

Departures from Bank Value Maximization: Imperfect Competition and Agency Frictions. Both imperfect competition and agency frictions that depart from value maximization will introduce additional terms when extending Propositions 1 through 4, although its impact on the optimal level of coverage is a priori indeterminate. For instance, increasing the level of coverage when banks have market power can at the same time encourage bank managers to make safer investment decisions to preserve their franchise value but also to make less careful investment and borrowing choices, so it is not obvious whether the level of coverage should increase relative to the competitive benchmark in that case.[33] Similarly, if non-competitive banks happened to fund projects with negative net present values, ex-ante regulation would be needed. In general, if there are specific regulatory tools designed to ex-ante correct for the impact of imperfect competition or managerial distortions, it would be optimal to make use of them, allowing us to rely again on our baseline characterization.

Unregulated Sector. Throughout the paper, every bank is subject to deposit insurance and ex-ante regulation. Our framework implies that all sectors subject to coordination failures could benefit from deposit insurance guarantees. In general, the optimal level of coverage must account for whether depositors are able to shift funds from the regulated deposit sector into other unregulated sectors, and vice versa. As usual in models with imperfect instruments (see Diamond (1973), or Plantin (2014) and Ordoñez (2018) in the context of banking regulation), the optimal policy under perfect regulation studied in this paper becomes a key input when considering the optimal policy under imperfect enforcement.

6 Conclusion

We have developed a framework to study the tradeoffs associated with the optimal determination of deposit insurance coverage. Our analysis identifies the set of variables that have a first-order effect on welfare and become sufficient statistics for assessing changes in the level of deposit insurance coverage. Consequently, our results provide a step forward towards building a microfounded theory of measurement for financial regulation that can be applied to a wide variety of environments.

There are several avenues for further research that build on our results. From a theoretical perspective, exploring alternative forms of asset- or liability-side competition among banks or introducing dynamic considerations are non-trivial extensions that are worth exploring. However, the most promising implications of this paper for future research come from the measurement perspective. Recovering robust, well-identified, and credible estimates in different contexts of the sufficient statistics that we have uncovered in this paper, in particular, the sensitivity of bank failures to changes in the level of coverage and the relevant fiscal externalities associated with such a policy change, has the potential to

[33] See Corbae and D'Erasmo (2019) and Corbae and Levine (2018) for recent work exploring the impact of imperfectly competitive intermediaries.

directly discipline future regulatory actions.

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Appendix

A Proofs and Derivations

A.1 Proofs: Section 2

Proposition 1. (Directional test for a change in the level of coverage δ)

Social welfare in this economy $W(\delta)$, is given by Equation (17). Therefore, we can express $\frac{dW}{d\delta}$ as follows

$$\frac{dW}{d\delta} = \int_0^{\overline{D}} \frac{dV_i}{d\delta} dG(i) + \frac{dV_\tau}{d\delta},$$

where the marginal impact of varying δ on depositors' welfare is given by

$$\int_0^{\overline{D}} \frac{dV_i}{d\delta} dG(i) = \mathbb{E}_\lambda \left[\int_0^{\overline{D}} \frac{d\mathbb{E}_s \left[U\left(C_{ti}(s) \right) \right]}{d\delta} dG(i) \right],$$

and where, exploiting the envelope theorem, and the fact that $\frac{\partial C_{ti}^N}{\partial \delta} = 0$, we can express $\frac{d\mathbb{E}_s[U(C_{ti}(s))]}{d\delta}$ as

$$\begin{aligned}\frac{d\mathbb{E}_s \left[U\left(C_{ti}(s) \right) \right]}{d\delta} &= \overbrace{\int_{\underline{s}}^{\hat{s}} U'\left(C_{ti}^F \right) \frac{\partial C_{ti}^F}{\partial \delta} dF(s) + \pi \int_{\hat{s}}^{s^*} U'\left(C_{ti}^F \right) \frac{\partial C_{ti}^F}{\partial \delta} dF(s)}^{= q^F \mathbb{E}_s^F \left[U'\left(C_{ti}^F \right) \frac{\partial C_{ti}^F}{\partial \delta} \right]} \\ &\quad + \left[U\left(C_{ti}^F(s^*) \right) - U\left(C_{ti}^N(s^*) \right) \right] \underbrace{\pi f(s^*) \frac{\partial s^*}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}}.\end{aligned}$$

Note that while s^* is a function of δ and R_1, $\hat{s}$ is exclusively a function of R_1, but not δ. The objects q^F and $\frac{\partial q^F}{\partial \delta}$ are respectively defined in Equations (12) and (13) in the text.

The marginal impact of varying δ on taxpayers' welfare is given by

$$\begin{aligned}\frac{dV_\tau}{d\delta} &= \overbrace{\int_{\underline{s}}^{\hat{s}} U'\left(C_\tau^F(s) \right) \frac{\partial C_\tau^F}{\partial \delta} dF(s) + \pi \int_{\hat{s}}^{s^*} U'\left(C_\tau^F(s) \right) \frac{\partial C_\tau^F}{\partial \delta} dF(s)}^{= q^F \mathbb{E}_s^F \left[U'\left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial \delta} \right]} \\ &\quad + \left[U\left(C_\tau^F(s^*) \right) - U\left(C_\tau^N(s^*) \right) \right] \underbrace{\pi f(s^*) \frac{\partial s^*}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}}.\end{aligned}$$

Therefore, we can aggregate across agents and express $\frac{dW}{d\delta}$ as follows,

$$\frac{dW}{d\delta} = \int_0^{\overline{D}} \frac{dV_i}{d\delta} dG(i) + \frac{dV_\tau}{d\delta} = q^F \int \mathbb{E}_s^F \left[U'\left(C_j^F \right) \frac{\partial C_j^F}{\partial \delta} \right] dj + \frac{\partial q^F}{\partial \delta} \int \left[U\left(C_j^F(s^*) \right) - U\left(C_j^N(s^*) \right) \right] dj,$$

which corresponds to Equation (20) in Proposition 1.

Proposition 2. (Preference-free approximation of $\frac{dW}{d\delta}$)

Note that we can approximate $U\left(C_j^F\left(s^*\right)\right)-U\left(C_j^N\left(s^*\right)\right)$ around $C_j^F\left(s^*\right)$ as follows

$$U\left(C_j^F\left(s^*\right)\right)-U\left(C_j^N\left(s^*\right)\right)\approx U'\left(C_j^F\left(s^*\right)\right)\left(C_j^F\left(s^*\right)-C_j^N\left(s^*\right)\right).$$

Alternatively, we could have approximated the difference in utilities around $C_j^N\left(s^*\right)$. In this case, since $U'\left(C_j^F\left(s^*\right)\right)$ is approximated as a constant, the following relations apply

$$\frac{U\left(C_j^F\left(s^*\right)\right)-U\left(C_j^N\left(s^*\right)\right)}{U'\left(C_j^F\left(s^*\right)\right)}\approx C_j^F\left(s^*\right)-C_j^N\left(s^*\right)$$

$$\mathbb{E}_s^F\left[U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial\delta}\right]\approx\frac{\mathbb{E}_s^F\left[\frac{\partial C_j^F}{\partial\delta}\right]}{U'\left(C_j^F\left(s^*\right)\right)}.$$

Both approximations, when substituted in a version of Equation (20) expressed in money-metric terms, yields the result in Proposition 2.

A.2 Proofs: Section 3

Proposition 3. (Directional test for δ under perfect ex-ante regulation)

We can express social welfare explicitly as a function of R_1 and δ as follows

$$W\left(\delta,R_1\right)=\int V_j\left(\delta,R_1\right)dj=\int_0^{\overline{D}}V_i\left(\delta,R_1\right)dG\left(i\right)+V_\tau\left(\delta,R_1\right),$$

where $V_i\left(\delta,R_1\right)$ and $V_\tau\left(\delta,R_1\right)$ are defined in Equations (18) and (19) in the text, and where $C_j\left(\delta,R_1\right)$ should be interpreted as a function of both δ and R_1. In that case, we can write for depositors

$$\int_0^{\overline{D}}\frac{dV_i}{d\delta}dG\left(i\right)=\mathbb{E}_\lambda\left[\int_0^{\overline{D}}\frac{d\mathbb{E}_s\left[U\left(C_{ti}\left(s\right)\right)\right]}{d\delta}dG\left(i\right)\right],$$

where

$$\begin{aligned}\frac{d\mathbb{E}_s\left[U\left(C_{ti}\left(s\right)\right)\right]}{d\delta}&=\int_{\underline{s}}^{\hat{s}}U'\left(C_{ti}^F\right)\frac{dC_{ti}^F}{d\delta}dF\left(s\right)+\pi\int_{\hat{s}}^{s^*}U'\left(C_{ti}^F\right)\frac{dC_{ti}^F}{d\delta}dF\left(s\right)\\&+\left(1-\pi\right)\int_{\hat{s}}^{s^*}U'\left(C_{ti}^N\right)\frac{dC_{ti}^N}{d\delta}dF\left(s\right)+\int_{s^*}^{\overline{s}}U'\left(C_{ti}^N\right)\frac{dC_{ti}^N}{d\delta}dF\left(s\right)\\&+\left[U\left(C_{ti}^F\left(\hat{s}\right)\right)-U\left(C_{ti}^N\left(\hat{s}\right)\right)\right]\left(1-\pi\right)f\left(\hat{s}\right)\frac{d\hat{s}}{d\delta}+\left[U\left(C_{ti}^F\left(s^*\right)\right)-U\left(C_{ti}^N\left(s^*\right)\right)\right]\pi f\left(s^*\right)\frac{ds^*}{d\delta},\end{aligned}$$

and the impact on consumption can be decomposed as $\frac{dC_{ti}^F}{d\delta}=\frac{\partial C_{ti}^F}{\partial\delta}+\frac{\partial C_{ti}^F}{\partial R_1}\frac{dR_1}{d\delta}$ and $\frac{dC_{ti}^N}{d\delta}=\frac{\partial C_{ti}^N}{\partial R_1}\frac{dR_1}{d\delta}$, while the impact on the thresholds $\hat{s}$ and s^* satisfies $\frac{d\hat{s}}{d\delta}=\frac{\partial\hat{s}}{\partial R_1}\frac{dR_1}{d\delta}$, and $\frac{ds^*}{d\delta}=\frac{\partial s^*}{\partial\delta}+\frac{\partial s^*}{\partial R_1}\frac{dR_1}{d\delta}$.

In the case of taxpayers, we can express $\frac{dV_\tau}{d\delta}$ as follows

$$\begin{aligned}\frac{dV_\tau}{d\delta}&=\int_{\underline{s}}^{\hat{s}}U'\left(C_\tau^F\right)\frac{dC_\tau^F}{d\delta}dF\left(s\right)+\pi\int_{\hat{s}}^{s^*}U'\left(C_\tau^F\right)\frac{dC_\tau^F}{d\delta}dF\left(s\right)\\&+\left(1-\pi\right)\int_{\hat{s}}^{s^*}U'\left(C_\tau^N\right)\frac{dC_\tau^N}{d\delta}dF\left(s\right)+\int_{s^*}^{\overline{s}}U'\left(C_\tau^N\right)\frac{dC_\tau^N}{d\delta}dF\left(s\right)\\&+\left[U\left(C_\tau^F\left(\hat{s}\right)\right)-U\left(C_\tau^N\left(\hat{s}\right)\right)\right]\left(1-\pi\right)f\left(\hat{s}\right)\frac{d\hat{s}}{d\delta}+\left[U\left(C_\tau^F\left(s^*\right)\right)-U\left(C_\tau^N\left(s^*\right)\right)\right]\pi f\left(s^*\right)\frac{ds^*}{d\delta},\end{aligned}$$

where it is the case that $\frac{dC_\tau^F}{d\delta} = \frac{\partial C_\tau^F}{\partial \delta} + \frac{\partial C_\tau^F}{\partial R_1}\frac{dR_1}{d\delta}$, as well as $\frac{dC_\tau^N}{d\delta} = 0$. Note that, under the optimal regulation, R_1 must satisfy

$$\frac{\partial W}{\partial R_1} = \mathbb{E}_\lambda \left[\int_0^{\overline{D}} \frac{\partial \mathbb{E}_s \left[U \left(C_{ti} \left(s \right) \right) \right]}{\partial R_1} dG\left(i \right) \right] + \frac{\partial V_\tau}{\partial R_1} = 0, \tag{38}$$

where for depositors

$$\begin{aligned} \frac{\partial \mathbb{E}_s \left[U \left(C_{ti} \left(s \right) \right) \right]}{\partial R_1} &= \int_{\underline{s}}^{\hat{s}} U' \left(C_{ti}^F \right) \frac{\partial C_{ti}^F}{\partial R_1} dF\left(s \right) + \pi \int_{\hat{s}}^{s^*} U' \left(C_{ti}^F \right) \frac{\partial C_{ti}^F}{\partial R_1} dF\left(s \right) \\ &+ \left(1 - \pi \right) \int_{\hat{s}}^{s^*} U' \left(C_{ti}^N \right) \frac{\partial C_{ti}^N}{\partial R_1} dF\left(s \right) + \int_{s^*}^{\overline{s}} U' \left(C_{ti}^N \right) \frac{\partial C_{ti}^N}{\partial R_1} dF\left(s \right) \\ &+ \left[U \left(C_{ti}^F \left(s^* \right) \right) - U \left(C_{ti}^N \left(s^* \right) \right) \right] \pi f\left(s^* \right) \frac{\partial s^*}{\partial R_1} + \left[U \left(C_{ti}^F \left(\hat{s} \right) \right) - U \left(C_{ti}^N \left(\hat{s} \right) \right) \right] \left(1 - \pi \right) \frac{\partial \hat{s}}{\partial R_1} f\left(\hat{s} \right), \end{aligned} \tag{39}$$

and for taxpayers

$$\begin{aligned} \frac{\partial V_\tau}{\partial R_1} &= \int_{\underline{s}}^{\hat{s}} U' \left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial R_1} dF\left(s \right) + \pi \int_{\hat{s}}^{s^*} U' \left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial R_1} dF\left(s \right) \\ &+ \left(1 - \pi \right) \int_{\hat{s}}^{s^*} U' \left(C_\tau^N \right) \frac{\partial C_\tau^N}{\partial R_1} dF\left(s \right) + \int_{s^*}^{\overline{s}} U' \left(C_\tau^N \right) \frac{\partial C_\tau^N}{\partial R_1} dF\left(s \right) \\ &+ \left[U \left(C_\tau^F \left(s^* \right) \right) - U \left(C_\tau^N \left(s^* \right) \right) \right] \pi f\left(s^* \right) \frac{\partial s^*}{\partial R_1} + \left[U \left(C_\tau^F \left(\hat{s} \right) \right) - U \left(C_\tau^N \left(\hat{s} \right) \right) \right] \left(1 - \pi \right) \frac{\partial \hat{s}}{\partial R_1} f\left(\hat{s} \right), \end{aligned}$$

where $\frac{\partial C_\tau^N}{\partial R_1} = 0$. Therefore, given the optimal ex-ante regulation, we can express $\frac{dW}{d\delta}$ as follows

$$\begin{aligned} \frac{dW}{d\delta} &= \int_0^{\overline{D}} \frac{dV_i}{d\delta} dG\left(i \right) + \frac{dV_\tau}{d\delta} = \int_0^{\overline{D}} \frac{\partial V_i}{\partial \delta} dG\left(i \right) + \frac{\partial V_\tau}{\partial \delta}, \\ &= \int \underbrace{\left(\int_{\underline{s}}^{\hat{s}} U' \left(C_j^F \right) \frac{\partial C_j^F}{\partial \delta} dF\left(s \right) + \pi \int_{\hat{s}}^{s^*} U' \left(C_j^F \right) \frac{\partial C_j^F}{\partial \delta} dF\left(s \right) \right)}_{= q^F \mathbb{E}_s^F \left[U' \left(C_j^F \right) \frac{\partial C_j^F}{\partial \delta} \right]} dj \\ &+ \underbrace{\pi f\left(s^* \right) \frac{\partial s^*}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}} \int \left[U \left(C_j^F \left(s^* \right) \right) - U \left(C_j^N \left(s^* \right) \right) \right] dj, \end{aligned}$$

which corresponds exactly to Equation (25) in the text.

Proposition 4. (Directional test for δ without ex-ante regulation)

As before, we can express social welfare explicitly as a function of R_1 and δ as follows

$$W\left(\delta, R_1 \right) = \int V_j\left(\delta, R_1 \right) dj = \int_0^{\overline{D}} V_i\left(\delta, R_1 \right) dG\left(i \right) + V_\tau\left(\delta, R_1 \right),$$

where $V_i\left(\delta, R_1 \right)$ and $V_\tau\left(\delta, R_1 \right)$ are defined in Equations (18) and (19) in the text. In this case, we can express the optimal R_1 chosen by competitive banks, for a given δ, as

$$R_1^\star\left(\delta \right) = \arg\max_{R_1} \int_0^{\overline{D}} V_i\left(\delta, R_1 \right) dG\left(i \right).$$

Formally, R_1 must satisfy

$$\frac{\partial \int_0^{\overline{D}} V_i(\delta, R_1)\, dG(i)}{\partial R_1} = \mathbb{E}_\lambda \left[\int_0^{\overline{D}} \frac{\partial \mathbb{E}_s \left[U\left(C_{ti}(s) \right) \right]}{\partial R_1} dG(i) \right] = 0, \tag{40}$$

where the marginal change induced by a change in R_1 in early and late depositors' utility, $\frac{\partial \mathbb{E}_s[U(C_{ti}(s))]}{\partial R_1}$, is given by Equation (39) in the Appendix.

Therefore, we can express $\frac{dW}{d\delta}$ as follows

$$\begin{aligned}\frac{dW}{d\delta} &= \int_0^{\overline{D}} \frac{dV_i}{d\delta} dG(i) + \frac{dV_\tau}{d\delta} = \int_0^{\overline{D}} \frac{\partial V_i}{\partial \delta} dG(i) + \frac{\partial V_\tau}{\partial \delta} + \left(\int_0^{\overline{D}} \frac{\partial V_i}{\partial R_1} dG(i) + \frac{\partial V_\tau}{\partial R_1} \right) \frac{dR_1}{d\delta} \\ &= \int_0^{\overline{D}} \frac{\partial V_i}{\partial \delta} dG(i) + \frac{\partial V_\tau}{\partial \delta} + \frac{\partial V_\tau}{\partial R_1} \frac{dR_1}{d\delta},\end{aligned}$$

where $\frac{\partial V_\tau}{\partial R_1}$ is given by

$$\begin{aligned}\frac{\partial V_\tau}{\partial R_1} &= q^F \mathbb{E}_s^F \left[U'\left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial R_1} \right] + \left[U\left(C_\tau^F(s^*) \right) - U\left(C_\tau^N(s^*) \right) \right] \pi f(s^*) \frac{\partial s^*}{\partial R_1} \\ &\quad + \left[U\left(C_\tau^F(\hat{s}) \right) - U\left(C_\tau^N(\hat{s}) \right) \right] (1-\pi) \frac{\partial \hat{s}}{\partial R_1} f(\hat{s}),\end{aligned} \tag{41}$$

and where

$$q^F \mathbb{E}_s^F \left[U'\left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial R_1} \right] = \int_{\underline{s}}^{\hat{s}} U'\left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial R_1} dF(s) + \pi \int_{\hat{s}}^{s^*} U'\left(C_\tau^F \right) \frac{\partial C_\tau^F}{\partial R_1} dF(s).$$

Under a risk-neutral approximation similar to one used in Proposition 2, we can approximate V_τ by

$$V_\tau \approx -\mathbb{E}_s \left[T(s) + \kappa\left(T(s) \right) \right] + c,$$

where c denotes a constant, and consequently

$$\frac{\partial V_\tau}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s \left[T(s) + \kappa\left(T(s) \right) \right]}{\partial R_1}.$$

Note that Equation (47) guarantees that this fiscal externality term is negative, as described in the body of the paper.

Proposition 5. (Optimal ex-ante deposit rate regulation)

The choice of R_1 under perfect ex-ante regulation is given by

$$\frac{\partial \int_0^{\overline{D}} V_i(\delta, R_1)\, dG(i)}{\partial R_1} + \frac{\partial V_\tau}{\partial R_1} = 0.$$

The choice of R_1 under a linear utility penalty per unit of R_1 offered $(-\tau_{R_1} R_1)$ corresponds to

$$\frac{\partial \int_0^{\overline{D}} V_i(\delta, R_1)\, dG(i)}{\partial R_1} - \tau_{R_1} = 0.$$

Therefore, the optimal regulation is associated with a wedge

$$\tau_{R_1} = -\frac{\partial V_\tau}{\partial R_1},$$

which, as shown above, can be approximated as $\frac{\partial V_\tau}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s)+\kappa(T(s))]}{\partial R_1}$.

A.3 Proofs: Section 5

Proposition 6. (Directional test for δ under general investment opportunities)

First, we establish the new failure threshold, which corresponds to Equation (31) in the text. For a given common liquidation rate φ, the resources at date 2 for a bank are now given by

$$\begin{aligned}\sum_h \rho_{2h}(s)\left(\rho_{1h}(s)\psi_h D_0 - \varphi\rho_{1h}(s)\psi_h D_0\right) &= \sum_h \rho_{2h}(s)\left(\rho_{1h}(s)\psi_h D_0 - \frac{\rho_{1h}(s)\psi_h D_0}{\sum_h \rho_{1h}(s)\psi_h D_0}\Omega(s)\right)\\ &= \sum_h \rho_{2h}(s)\left(\rho_{1h}(s)\psi_h D_0 - \frac{\rho_{1h}(s)\psi_h}{\sum_h \rho_{1h}(s)\psi_h}\left(D_0 R_1 - D_1(s)\right)\right),\end{aligned}$$

where we use the fact that the level of withdrawals $\Omega(s)$ pins down the following liquidation rate $\varphi = \frac{\Omega(s)}{\sum_h \rho_{1h}(s)\psi_h D_0}$. It is therefore easy to show that the threshold for the level of deposits that delimits the probability of failure is

$$\tilde{D}_1(s) = \frac{\left(R_1 - \sum_h \rho_{1h}(s)\psi_h\right)D_0}{1 - \frac{1}{\frac{\sum_h \rho_{2h}(s)\rho_{1h}(s)\psi_h}{\sum_h \rho_{1h}(s)\psi_h}}},$$

which depends on R_1 and ψ_h. It is straightforward to compute consumption for early and late depositors, as in Equations (4) and (5). In this case

$$T(s) = \max\left\{\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i) - \sum_h \chi_h(s)\rho_{1h}(s)\psi_h D_0, 0\right\}.$$

As above, we can express $\frac{dW}{d\delta}$ as follows

$$\begin{aligned}\frac{dW}{d\delta} &= \int_0^{\overline{D}} \frac{dV_i}{d\delta} dG(i) + \frac{dV_\tau}{d\delta}\\ &= \int_0^{\overline{D}} \frac{\partial V_i}{\partial\delta} dG(i) + \frac{\partial V_\tau}{\partial\delta} + \left(\int_0^{\overline{D}} \frac{\partial V_i}{\partial R_1} dG(i) + \frac{\partial V_\tau}{\partial R_1}\right)\frac{dR_1}{d\delta} + \sum_h \left(\int_0^{\overline{D}} \frac{\partial V_i}{\partial \psi_h} dG(i) + \frac{\partial V_\tau}{\partial \psi_h}\right)\frac{d\psi_h}{d\delta}\\ &\quad + \left(\int_0^{\overline{D}} \sum_k \frac{\partial V_i}{\partial y_{ki}} dG(i) + \frac{\partial V_\tau}{\partial y_k}\right)\frac{dy_k}{d\delta},\end{aligned}$$

where

$$\int_0^{\overline{D}} \frac{\partial V_i}{\partial\delta} dG(i) + \frac{\partial V_\tau}{\partial\delta} = q^F \int \mathbb{E}_s^F\left[U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial\delta}\right] dj + \frac{\partial q^F}{\partial\delta}\int \left[U\left(C_j^F(s^*)\right) - U\left(C_j^N(s^*)\right)\right] dj,$$

which corresponds to $\frac{dW}{d\delta}$ under perfect regulation. The definition of V_i now corresponds to the updated utility specification (30), and it is subject to Equations (29) and $\sum_h \psi_h = 1$.

Unregulated banks optimally set $\int_0^{\overline{D}} \frac{\partial V_i}{\partial R_1} dG(i) = 0$, $\int_0^{\overline{D}} \frac{\partial V_i}{\partial \psi_h} dG(i) = 0$, $\forall h$, and $\frac{\partial V_i}{\partial y_{ki}} = 0$, $\forall i, k$. Since $T(s)$ is independent of y_{ki}, it is always the case that $\frac{\partial V_\tau}{\partial y_{ki}} = 0$. Therefore, in that case, $\frac{dW}{d\delta}$ corresponds to

$$\frac{dW}{d\delta} = \int_0^{\overline{D}} \frac{\partial V_i}{\partial\delta} dG(i) + \frac{\partial V_\tau}{\partial\delta} + \frac{\partial V_\tau}{\partial R_1}\frac{dR_1}{d\delta} + \sum_h \frac{\partial V_\tau}{\partial\psi_h}\frac{d\psi_h}{d\delta}.$$

We can express $\frac{\partial V_\tau}{\partial R_1}$ exactly as in Equation (41), and $\frac{\partial V_\tau}{\partial \psi_h}$ as follows

$$\frac{\partial V_\tau}{\partial \psi_h} = q^F \mathbb{E}_s^F \left[U'\left(C_\tau^F\right) \frac{\partial C_\tau^F}{\partial \psi_h} \right] + \left[U\left(C_\tau^F\left(s^*\right)\right) - U\left(C_\tau^N\left(s^*\right)\right) \right] \pi f\left(s^*\right) \frac{\partial s^*}{\partial \psi_h}$$
$$+ \left[U\left(C_\tau^F\left(\hat{s}\right)\right) - U\left(C_\tau^N\left(\hat{s}\right)\right) \right] (1-\pi) \frac{\partial \hat{s}}{\partial \psi_h} f\left(\hat{s}\right),$$

where

$$q^F \mathbb{E}_s^F \left[U'\left(C_\tau^F\right) \frac{\partial C_\tau^F}{\partial \psi_h} \right] = \int_{\underline{s}}^{\hat{s}} U'\left(C_\tau^F\right) \frac{\partial C_\tau^F}{\partial \psi_h} dF\left(s\right) + \pi \int_{\hat{s}}^{s^*} U'\left(C_\tau^F\right) \frac{\partial C_\tau^F}{\partial \psi_h} dF\left(s\right).$$

Under a risk-neutral approximation similar to one used in Proposition 2, we can write $V_\tau \approx -\mathbb{E}_s\left[T\left(s\right) + \kappa\left(T\left(s\right)\right)\right]$, and consequently

$$\frac{\partial V_\tau}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s\left[T\left(s\right) + \kappa\left(T\left(s\right)\right)\right]}{\partial R_1} \qquad \text{and} \qquad \frac{\partial V_\tau}{\partial \psi_h} \approx -\frac{\partial \mathbb{E}_s\left[T\left(s\right) + \kappa\left(T\left(s\right)\right)\right]}{\partial \psi_h}.$$

Proposition 7. (Directional test for δ under an alternative equilibrium selection)

Under the new equilibrium selection assumption, we can express social welfare as

$$W\left(\delta, R_1\right) = \int V_j\left(\delta, R_1\right) dj = \int_0^{\overline{D}} V_i\left(\delta, R_1\right) dG\left(i\right) + V_\tau\left(\delta, R_1\right),$$

where $V_i\left(\delta, R_1\right)$ is defined in Equation (34) in the text, and $V_\tau\left(\delta, R_1\right)$ is defined as

$$V_\tau\left(\delta, R_1\right) = \int_{\underline{s}}^{s^G\left(\delta, R_1\right)} U\left(C_\tau^F\left(s\right)\right) dF\left(s\right) + \int_{s^G\left(\delta, R_1\right)}^{\overline{s}} U\left(C_\tau^N\left(s\right)\right) dF\left(s\right).$$

Therefore, under perfect regulation, we can express $\frac{dW}{d\delta}$ as follows

$$\frac{dW}{d\delta} = \int_0^{\overline{D}} \frac{\partial V_i}{\partial \delta} dG\left(i\right) + \frac{\partial V_\tau}{\partial \delta},$$

where the marginal impact of a change in δ on depositors' welfare corresponds to

$$\int_0^{\overline{D}} \frac{\partial V_i}{\partial \delta} dG\left(i\right) = \mathbb{E}_\lambda \left[\int \frac{\partial \mathbb{E}_s\left[U\left(C_{ti}\left(s\right)\right)\right]}{\partial \delta} dG\left(i\right) \right],$$

where

$$\frac{\partial \mathbb{E}_s\left[U\left(C_{ti}\left(s\right)\right)\right]}{\partial \delta} = \left[U\left(C_{ti}^F\left(s^G\right)\right) - U\left(C_{ti}^N\left(s^G\right)\right) \right] \underbrace{f\left(s^G\right) \frac{\partial s^G}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}} + \underbrace{\int_{\underline{s}}^{s^G} U'\left(C_{ti}^F\right) \frac{\partial C_{ti}^F}{\partial \delta} dF\left(s\right)}_{= q^F \mathbb{E}_s^F \left[U'\left(C_{ti}^F\right) \frac{\partial C_{ti}^F}{\partial \delta} \right]},$$

since $\frac{\partial C_{ti}^N}{\partial \delta} = 0$. And where we can write

$$\frac{\partial V_\tau}{\partial \delta} = \left[U\left(C_\tau^F\left(s^G\right)\right) - U\left(C_\tau^N\left(s^G\right)\right) \right] \underbrace{f\left(s^G\right) \frac{\partial s^G}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}} + \underbrace{\int_{\underline{s}}^{s^G} U'\left(C_\tau^F\left(s\right)\right) \frac{\partial C_\tau^F}{\partial \delta} dF\left(s\right)}_{= q^F \mathbb{E}_s^F \left[U'\left(C_\tau^F\right) \frac{\partial C_\tau^F}{\partial \delta} \right]},$$

which corresponds to Equation (35) in the text.

Proposition 8. (Directional test for δ incorporating aggregate spillovers)

First, we establish the new failure threshold, which corresponds to Equation (36) in the text. That is, the total resources available to a given bank at date 2, given aggregate withdrawals $\overline{\Omega}(s)$, corresponds to $\rho_2(s)\left(\rho_1(s)D_0 - \theta\left(\overline{\Omega}(s)\right)\Omega(s)\right)$, which can be expressed as

$$\rho_2(s)\left(\theta\left(\overline{\Omega}(s)\right)D_1(s) + \left(\rho_1(s) - \theta\left(\overline{\Omega}(s)\right)R_1\right)D_0\right).$$

As in the baseline model, we can implicitly define a threshold level of deposits, denoted by $\tilde{D}_1(s)$ and given by

$$\tilde{D}_1(s) = \frac{\theta\left(\overline{\Omega}(s)\right)R_1 - \rho_1(s)}{\theta\left(\overline{\Omega}(s)\right) - \frac{1}{\rho_2(s)}}D_0,$$

which delimits the failure regions. When banks choose R_1 unregulated, they do not internalize that deposit rates affect $\theta(\cdot)$. In that case, we can define two types of thresholds. We denote the thresholds used by banks ex-ante to choose R_1 by $\hat{s}(R_1)$ and $s^*(\delta, R_1)$. Those perceived by the deposit insurance authority, incorporating the effects on aggregate withdrawals $\overline{\Omega}(s) = D_0R_1 - D_1(s)$, are denoted by $\hat{s}_P(R_1)$ and $s_P^*(\delta, R_1)$. In equilibrium, $\hat{s}(R_1) = \hat{s}_P(R_1)$ and $s^*(\delta, R_1) = s_P^*(\delta, R_1)$, even though, crucially, the partial derivatives of each set of thresholds with respect to R_1 are different.

As above, we can express $\frac{dW}{d\delta}$ as follows

$$\begin{aligned}
\frac{dW}{d\delta} &= \int_0^{\overline{D}} \frac{dV_i^P}{d\delta}dG(i) + \frac{dV_\tau^P}{d\delta} \\
&= \int_0^{\overline{D}} \frac{\partial V_i^P}{\partial\delta}dG(i) + \frac{\partial V_\tau^P}{\partial\delta} + \left(\int_0^{\overline{D}} \frac{\partial V_i^P}{\partial R_1}dG(i) + \frac{\partial V_\tau^P}{\partial R_1}\right)\frac{dR_1}{d\delta},
\end{aligned}$$

where we use the P notation to emphasize that V_i^P and V_τ^P are calculated from the perspective of a planner who uses thresholds $\hat{s}_P(R_1)$ and $s_P^*(\delta, R_1)$, which account for equilibrium spillovers. Note that the first two terms are given by

$$\begin{aligned}
\int_0^{\overline{D}} \frac{\partial V_i^P}{\partial\delta}dG(i) + \frac{\partial V_\tau^P}{\partial\delta} &= \int \underbrace{\left(\int_{\underline{s}}^{\hat{s}} U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial\delta}dF(s) + \pi\int_{\hat{s}}^{s^*} U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial\delta}dF(s)\right)}_{=q^F\mathbb{E}_s^F\left[U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial\delta}\right]}dj \\
&\quad + \underbrace{\pi f(s^*)\frac{\partial s^*}{\partial\delta}}_{=\frac{\partial q^F}{\partial\delta}}\int\left[U\left(C_j^F(s^*)\right) - U\left(C_j^N(s^*)\right)\right]dj.
\end{aligned}$$

We can express $\int_0^{\overline{D}} \frac{\partial V_i^P}{\partial R_1}dG(i) = \int_0^{\overline{D}}\left(\frac{\partial V_i^P}{\partial R_1} - \frac{\partial V_i}{\partial R_1}\right)dG(i)$ as follows

$$\begin{aligned}
\int_0^{\overline{D}} \frac{\partial V_i^P}{\partial R_1}dG(i) &= \mathbb{E}_\lambda\left[\int_0^{\overline{D}} \frac{\partial\mathbb{E}_s^P\left[U\left(C_{ti}(s)\right)\right]}{\partial R_1}dG(i)\right], \\
&= \mathbb{E}_\lambda\left[\int_0^{\overline{D}}\left[U\left(C_{ti}^F(s^*)\right) - U\left(C_{ti}^N(s^*)\right)\right]\pi f(s^*)\left(\frac{\partial s_P^*}{\partial R_1} - \frac{\partial s^*}{\partial R_1}\right)dG(i)\right] \\
&\quad + \mathbb{E}_\lambda\left[\int_0^{\overline{D}}\left[U\left(C_{ti}^F(\hat{s})\right) - U\left(C_{ti}^N(\hat{s})\right)\right](1-\pi)f(\hat{s}_P)\left(\frac{\partial\hat{s}_P}{\partial R_1} - \frac{\partial\hat{s}}{\partial R_1}\right)dG(i)\right],
\end{aligned}$$

where we use the fact that for depositors $\mathbb{E}_\lambda\left[\int_0^{\overline{D}} \frac{\partial \mathbb{E}_s[U(C_{ti}(s))]}{\partial R_1} dG(i)\right] = 0$. Similarly, we can express $\frac{\partial V_\tau^P}{\partial R_1}$ as follows

$$\frac{\partial V_\tau^P}{\partial R_1} = q^F \mathbb{E}_s^F\left[U'\left(C_\tau^F\right)\frac{\partial C_\tau^F}{\partial R_1}\right] + \left[U\left(C_\tau^F(s^*)\right) - U\left(C_\tau^N(s^*)\right)\right]\pi f(s^*)\frac{\partial s_P^*}{\partial R_1}$$
$$+\left[U\left(C_\tau^F(\hat{s})\right) - U\left(C_\tau^N(\hat{s})\right)\right](1-\pi)\frac{\partial \hat{s}_P}{\partial R_1} f(\hat{s}),$$

where $q^F \mathbb{E}_s^F\left[U'\left(C_\tau^F\right)\frac{\partial C_\tau^F}{\partial R_1}\right] = \int_{\underline{s}}^{\hat{s}} U'\left(C_\tau^F\right)\frac{\partial C_\tau^F}{\partial R_1} dF(s) + \pi \int_{\hat{s}}^{s^*} U'\left(C_\tau^F\right)\frac{\partial C_\tau^F}{\partial R_1} dF(s)$. Under a risk-neutral approximation similar to one used in Proposition 2, $V_\tau \approx -\mathbb{E}_s[T(s) + \kappa(T(s))] + c$, where c denotes a constant, and consequently $\frac{\partial V_\tau}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s)+\kappa(T(s))]}{\partial R_1}$.

Therefore, under perfect regulation, $\frac{dW}{d\delta} = \int_0^{\overline{D}} \frac{\partial V_i^P}{\partial \delta} dG(i) + \frac{\partial V_\tau^P}{\partial \delta}$, which corresponds to Equation (37) in the text. The optimal regulation of banks, set so that $\int_0^{\overline{D}} \frac{\partial V_i^P}{\partial R_1} dG(i) + \frac{\partial V_\tau^P}{\partial R_1} = 0$, now incorporates a correction that accounts for aggregate spillovers. The optimal regulation is set so that banks internalize their fiscal externality and their aggregate spillovers.

B Additional Analytical Results

In this section, to facilitate the understanding of the results, we provide detailed analytical characterizations of several outcomes of the model. Figures 8 through 12 provide a numerical illustration of our analytical results using the same parameters employed in Section 4.2.

B.1 Thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$

Figure 8 illustrates the results derived here. The threshold $\hat{s}(R_1)$ is given by the minimum between the value of s that satisfies

$$\frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}} = (1-\lambda) R_1, \tag{42}$$

and $\overline{s}$. Note that this threshold is not a function of δ. Similarly, the value of $s^*(\delta, R_1)$ is given by the minimum between the value of s that satisfies

$$\frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}} = (1-\lambda) R_1 \zeta(\delta, R_1), \tag{43}$$

and $\overline{s}$, where $\zeta(\delta, R_1) \equiv \frac{\int_0^{\overline{D}} \min\{D_{0i} R_1, \delta\} dG(i)}{D_0 R_1}$ denotes the fraction of insured deposits.[34] Note that $\zeta(\delta, R_1) \in [0,1]$, as well as $\frac{\partial \zeta}{\partial \delta} \geq 0$, and $\frac{\partial \zeta}{\partial R_1} \leq 0$.

The left-hand side of both equations, $z(s, R_1) \equiv \frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}}$, is a decreasing function of s, since both $\rho_1(s)$ and $\rho_2(s)$ are monotonically increasing in s. Since we have assumed that $\rho_2(\underline{s}) < 1$, it is always

[34] When $\rho_1(s) = 1$ and $\rho_2(s) = s$, the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ can be explicitly computed as

$$\hat{s}(R_1) = \frac{(1-\lambda) R_1}{1 - \lambda R_1} \quad \text{and} \quad s^*(\delta, R_1) = \max\left\{\rho_2^{-1}\left(\frac{(1-\lambda) R_1}{(1-\lambda) R_1 - \frac{r_1}{\zeta}}\right), \overline{s}\right\}.$$

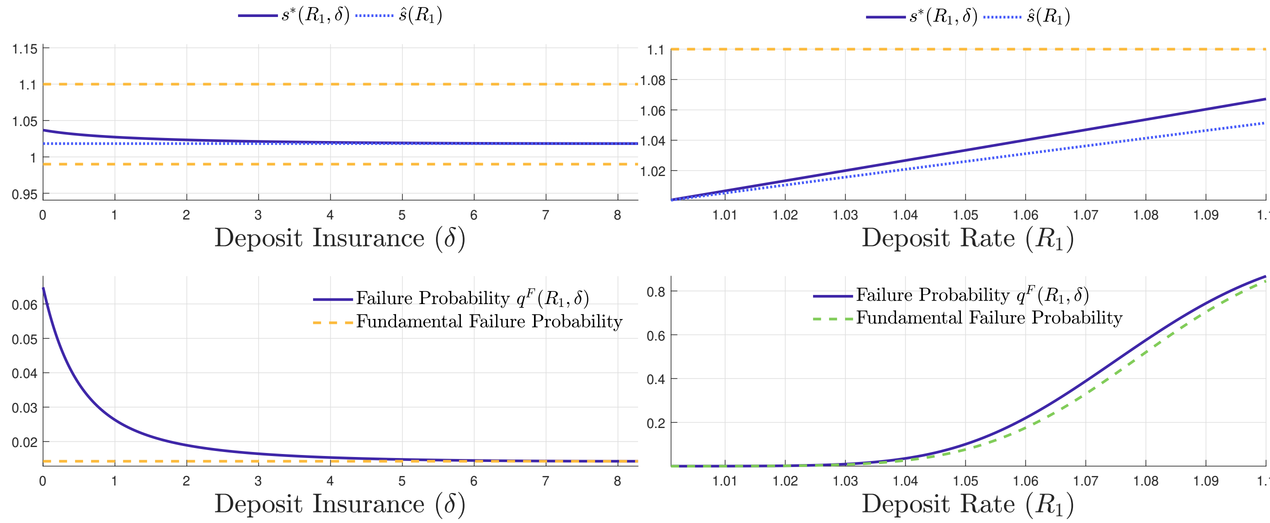


Figure 8: Equilibrium Thresholds and Failure Probabilities

Note: The top plots in Figure 8 show how s^* and $\hat{s}$ respectively vary with δ (measured in hundreds of thousands of dollars) and R_1. The bottom plots show the probability of bank failure and the probability of fundamental bank failure as a function of δ and R_1. All plots use the parameters described in Table 1.

guaranteed that $\hat{s}\,(R_1) > \underline{s}$. Note that

$$\lim_{\rho_2(s)\to 1+} z\,(s, R_1) = \infty \quad \text{and} \quad \lim_{\rho_1(s),\rho_2(s)\to\infty} z\,(s, R_1) < 0,$$

which is sufficient to establish that both Equation (42) and Equation (43) have a unique solution strictly higher than $\hat{s}\,(R_1)$. Since $\zeta\,(\delta, R_1) \in [0, 1]$, we can also conclude that $\hat{s}\,(R_1) \leq s^*\,(\delta, R_1)$, with equality only when all deposits are insured, $\delta \to \overline{D}R_1$, since $\lim_{\delta\to\overline{D}R_1} \zeta\,(\delta, R_1) = 1$.

In order for $s^* < \overline{s}$, as in Figure 3 in the text, it must be that $\rho_1\,(\overline{s}) > R_1$. In that case there are three regions (unique failure equilibrium, multiple equilibria, and unique no failure equilibrium) for any value of δ, including $\delta = 0$. If $\rho_1\,(\overline{s}) < R_1$, then there are only two regions (unique failure equilibrium and multiple equilibria) when $\delta = 0$ and for small value of δ.

The relevant comparative statics for $\hat{s}$ and s^* are the following. First, it follows directly from Equation (43) that

$$\frac{\partial s^*}{\partial \delta} \leq 0,$$

since its right-hand side is increasing in δ. The effect of δ on s^* is modulated by the behavior of $\zeta\,(\delta, R_1)$. Second, elementary arguments show that

$$\frac{\partial \hat{s}}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^*}{\partial R_1} \geq 0. \tag{44}$$

Finally, it also follows immediately from Equations (42) and (43) that $\frac{\partial \hat{s}}{\partial \lambda} \geq 0$ and $\frac{\partial s^*}{\partial \lambda} \geq 0$, since the right-hand side of both equations is decreasing in λ. Intuitively, all else constant, an increase in the mass of early depositors, who withdraw their deposits inelastically, increases bank fragility.

B.1.1 Parametric assumptions

Under the following parametric assumptions: $\rho_2(s) = s$ and $\rho_1(s) = 1 + \varphi(s-1)$, we can express the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ implied by Equations (42) and (43) as follows:

$$\hat{s}(R_1) = \left\{ s \middle| 0 = \varphi s^2 - (\lambda R_1 + \varphi - 1)s - (1-\lambda) R_1 \right\}$$
$$s^*(\delta, R_1) = \left\{ s \middle| 0 = \varphi s^2 - ((1 - \zeta + \zeta\lambda) R_1 + \varphi - 1)s - (1-\lambda) R_1 \zeta \right\},$$

where

$$\hat{s}(R_1) = \frac{\lambda R_1 + \varphi - 1 \pm \sqrt{(\lambda R_1 + \varphi - 1)^2 + 4\varphi(1-\lambda) R_1}}{2\varphi} \tag{45}$$
$$s^*(\delta, R_1) = \frac{(1 - \zeta + \zeta\lambda) R_1 + \varphi - 1 \pm \sqrt{((1 - \zeta + \zeta\lambda) R_1 + \varphi - 1)^2 + 4\varphi(1-\lambda) R_1 \zeta}}{2\varphi}. \tag{46}$$

Note that both quadratic equations have a unique positive root and that by setting $\zeta = 1$, Equation (46) collapses to Equation (45). Explicitly characterizing $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ simplifies the numerical computations.

B.2 Probability of failure $q^F(\delta, R_1)$

Making use of Equation (12) in the paper, we can express $\frac{\partial q^F}{\partial \delta}$ and $\frac{\partial q^F}{\partial R_1}$ as follows

$$\frac{\partial q^F}{\partial \delta} = \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial \delta} \leq 0$$
$$\frac{\partial q^F}{\partial R_1} = (1-\pi) f(\hat{s}(R_1)) \frac{\partial \hat{s}}{\partial R_1} + \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial R_1} \geq 0,$$

where the sign results follow from Equation (44). As explained in the text, higher levels of coverage reduce the probability of failure, holding R_1 constant, while higher deposit rates increase the probability of failure, holding δ constant. Figure 8 illustrates these results.

B.3 Fraction of insured/uninsured depositors

At the depositor level, it is the case that the total deposit claims can either be insured or uninsured. Formally, $D_{0i}R_1 = \min\{D_{0i}R_1, \delta\} + \max\{D_{0i}R_1 - \delta, 0\}$. In the aggregate, we can express the level of total insured and total uninsured deposits at date 1 as follows

$$\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\} \, dG(i) = \int_0^{\frac{\delta}{R_1}} D_{0i}R_1 dG(i) + \delta \int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i) \quad \text{(Insured Deposits)}$$
$$\int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i) = \int_{\frac{\delta}{R_1}}^{\overline{D}} (D_{0i}R_1 - \delta) \, dG(i), \quad \text{(Uninsured Deposits)},$$

where

$$\underbrace{\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\} \, dG(i)}_{\text{Insured Deposits}} + \underbrace{\int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i)}_{\text{Uninsured Deposits}} = \underbrace{D_0 R_1}_{\text{Total}}.$$

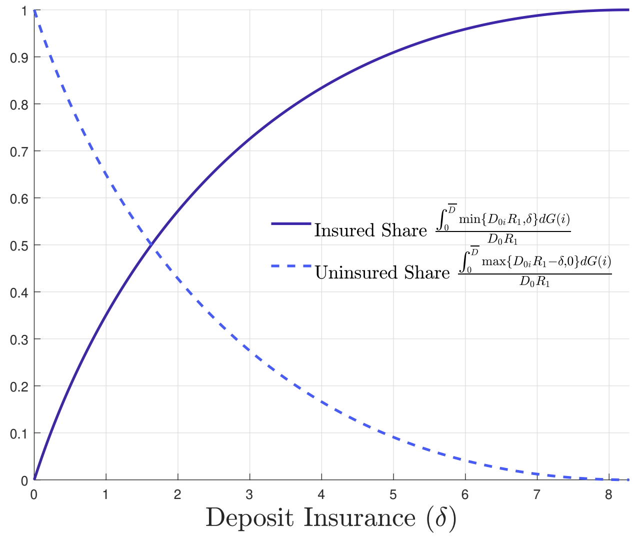


Figure 9: Fraction of Insured/Uninsured Deposits

Note: Figure 9 shows the share of insured deposits $\zeta(\delta, R_1) = \frac{\int_0^{\overline{D}} \min\{D_{0i}R_1,\delta\}dG(i)}{D_0R_1}$ and its complement, the share of uninsured deposits $1-\zeta(\delta,R_1) = \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\}\, dG(i)$, as a function of the level of deposit insurance coverage δ (measured in hundreds of thousands of dollars). This plot uses the distribution of deposits described in Table 1.

We define the fractions/shares of insured and uninsured deposits dividing by D_0R_1. Above, we defined the fraction of insured deposits as $\zeta(\delta, R_1) = \frac{\int_0^{\overline{D}} \min\{D_{0i}R_1,\delta\}dG(i)}{D_0R_1}$.

We repeatedly use the fact that

$$\frac{d\left(\int_0^{\overline{D}} \min\{D_{0i}R_1,\delta\}\, dG(i)\right)}{dR_1} = \int_0^{\frac{\delta}{R_1}} D_{0i}dG(i), \qquad \frac{d\left(\int_0^{\overline{D}} \max\{D_{0i}R_1-\delta,0\}\, dG(i)\right)}{dR_1} = \int_{\frac{\delta}{R_1}}^{\overline{D}} D_{0i}dG(i)$$

$$\frac{d\left(\int_0^{\overline{D}} \min\{D_{0i}R_1,\delta\}\, dG(i)\right)}{d\delta} = \int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i), \qquad \frac{d\left(\int_0^{\overline{D}} \max\{D_{0i}R_1-\delta,0\}\, dG(i)\right)}{d\delta} = -\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i).$$

Figure 9 illustrates the share of insured and uninsured deposits for different values of the level of coverage δ. Note that while $\hat{s}$ only depends on R_1 and λ, s^* also depends on those two objects in addition to the whole distribution of deposits, through its impact on the share of insured deposits.

B.4 Properties of depositors' and taxpayers' consumption

We can define the funding shortfall in state s as

$$T(s) = \max\left\{\int_0^{\overline{D}} \min\{D_{0i}R_1,\delta\}\, dG(i) - \chi(s)\,\rho_1(s)\, D_0, 0\right\}.$$

Note that we can respectively express depositors' equilibrium consumption in case of failure and no failure as follows

$$
\begin{aligned}
C_{ti}^{F}\left(\delta, R_1\right) &= \min\left\{D_{0i}R_1, \delta\right\} + \alpha_F\left(s\right)\max\left\{D_{0i}R_1 - \delta, 0\right\} + Y_{ti}\left(s\right) \\
C_{1i}^{N}\left(R_1\right) &= D_{0i}R_1 + Y_{1i}\left(s\right) \\
C_{2i}^{N}\left(R_1\right) &= \alpha_N\left(s\right)D_{0i}R_1 + Y_{2i}\left(s\right).
\end{aligned}
$$

The equilibrium objects $\alpha_F\left(s\right)$ and $\alpha_N\left(s\right)$ can be expressed as follows

$$
\begin{aligned}
&\alpha_F\left(s\right) = \frac{\max\left\{\chi\left(s\right)\rho_1\left(s\right)D_0 - \int_0^{\overline{D}}\min\left\{D_{0i}R_1, \delta\right\}dG\left(i\right), 0\right\}}{\int_0^{\overline{D}}\max\left\{D_{0i}R_1 - \delta, 0\right\}dG\left(i\right)} = \max\left\{1 - \frac{\left(R_1 - \chi\left(s\right)\rho_1\left(s\right)\right)D_0}{\int_0^{\overline{D}}\max\left\{D_{0i}R_1 - \delta, 0\right\}dG\left(i\right)}, 0\right\} \\
&\alpha_N\left(s\right) = \rho_2\left(s\right)\frac{\rho_1\left(s\right) - \lambda R_1}{\left(1-\lambda\right)R_1}.
\end{aligned}
$$

The rate $\alpha_N\left(s\right)$ captures the additional return obtained at date 2 by late depositors when there is no bank failure. The rate $\alpha_F\left(s\right)$ corresponds to the individual recovery rate on uninsured deposits in the case of bank failure. Note that $\alpha_F\left(s\right)$ is a function of s only through $\chi\left(s\right)\rho_1\left(s\right)$ and that $\alpha_N\left(s\right)$ is a function of s through $\rho_2\left(s\right)$ and $\rho_1\left(s\right)$. Note that $\alpha_N\left(s\right)D_{0i}R_1 = \rho_2\left(s\right)\frac{\rho_1(s)-\lambda R_1}{1-\lambda}D_{0i}$. The fact that $\rho_1\left(s\right) > R_1$ is incompatible with the existence of a failure equilibrium, which implies that $\alpha_F\left(s\right) < 1$. Note that we can also express $\alpha_F\left(s\right)$ as

$$
\alpha_F\left(s\right) = \frac{\max\left\{\chi\left(s\right)\rho_1\left(s\right)D_0 - \int_0^{\overline{D}}\min\left\{D_{0i}R_1, \delta\right\}dG\left(i\right), 0\right\}}{\int_0^{\overline{D}}\max\left\{D_{0i}R_1 - \delta, 0\right\}dG\left(i\right)},
$$

which implies that when $\alpha_F\left(s\right) > 0$, $T\left(s\right) = 0$, and when $\alpha_F\left(s\right) = 0$, $T\left(s\right) > 0$. Note also that $1 - \alpha_F\left(s\right) = \frac{(R_1-\chi(s)\rho_1(s))D_0}{\int_0^{\overline{D}}\max\{D_{0i}R_1-\delta,0\}dG(i)}$, whenever $\alpha_F\left(s\right) > 0$.

We can show that $\alpha_F\left(s\right)$ is decreasing in both R_1 and δ, as follows

$$
\frac{\partial\alpha_F\left(s\right)}{\partial R_1} = \begin{cases} -\dfrac{\int_0^{\frac{\delta}{R_1}} D_{0i}dG(i)\left(\int_0^{\overline{D}}\max\{D_{0i}R_1-\delta,0\}dG(i)\right)+\left(\chi(s)\rho_1(s)D_0-\int_0^{\overline{D}}\min\{D_{0i}R_1,\delta\}dG(i)\right)\int_{\frac{\delta}{R_1}}^{\overline{D}} D_{0i}dG(i)}{\left(\int_0^{\overline{D}}\max\{D_{0i}R_1-\delta,0\}dG(i)\right)^2} \leq 0, & \text{if } T\left(s\right) = 0 \\ 0, & \text{if } T\left(s\right) > 0 \end{cases}
$$

$$
\frac{\partial\alpha_F\left(s\right)}{\partial \delta} = \begin{cases} -\dfrac{(R_1-\chi(s)\rho_1(s))D_0\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i)}{\left(\int_0^{\overline{D}}\max\{D_{0i}R_1-\delta,0\}dG(i)\right)^2} = -\left(1-\alpha_F\left(s\right)\right)\dfrac{\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i)}{\int_0^{\overline{D}}\max\{D_{0i}R_1-\delta,0\}dG(i)} \leq 0, & \text{if } T\left(s\right) = 0 \\ 0, & \text{if } T\left(s\right) > 0. \end{cases}
$$

since $R_1 - \chi\left(s\right)\rho_1\left(s\right) \geq 0$ in any failure equilibrium.

In no failure states, depositors consumption varies with R_1 as follows

$$
\begin{aligned}
\frac{\partial C_{1i}^{N}}{\partial R_1} &= D_{0i} \geq 0 \\
\frac{\partial C_{2i}^{N}}{\partial R_1} &= -\rho_2\left(s\right)\frac{\lambda}{1-\lambda}D_{0i} \leq 0.
\end{aligned}
$$

In no failure states, depositors consumption is not directly affected by δ, so $\frac{\partial C_{ti}^{N}}{\partial \delta} = 0$.

In failure states, we can derive the following comparative statics, which are relevant inputs for the characterization of the optimal deposit insurance policy

$$\frac{\partial C_{ti}^{F}}{\partial R_1}(s) = \begin{cases} D_{0i} \geq 0, & \text{if } D_{0i}R_1 < \delta \\ \alpha_F(s) D_{0i} + \frac{\partial \alpha_F(s)}{\partial R_1}(D_{0i}R_1 - \delta) \gtreqless 0, & \text{if } D_{0i}R_1 \geq \delta \end{cases}$$

$$\frac{\partial C_{ti}^{F}}{\partial \delta}(s) = \begin{cases} 0, & \text{if } D_{0i}R_1 < \delta \\ 1 - \alpha_F(s) + \frac{\partial \alpha_F(s)}{\partial \delta}(D_{0i}R_1 - \delta) = (1 - \alpha_F(s))\left(1 - \frac{\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i)}{\int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} dG(i)}(D_{0i}R_1 - \delta)\right) \gtreqless 0, & \text{if } D_{0i}R_1 \geq \delta. \end{cases}$$

Hence, when $T(s) > 0$, $\alpha_F(s) = 0$ and $\frac{\partial \alpha_F(s)}{\partial \delta} = 0$, so $\frac{\partial C_{ti}^{F}}{\partial \delta}(s) = 1$ for all uninsured depositors – those for which $D_{0i}R_1 \geq \delta$. Note than when aggregated

$$\int_0^{\overline{D}} \frac{\partial C_{ti}^{F}}{\partial \delta} dG(i) = \begin{cases} 0, & \text{if } T(s) = 0 \\ \int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i), & \text{if } T(s) > 0. \end{cases}$$

Therefore, we can express $\int \int_0^{\overline{D}} \frac{\partial C_{ti}^{F}}{\partial \delta} dG(i) dF(s) = \int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i) \int \mathbb{I}[T(s) > 0] dF(s)$, where $\mathbb{I}[\cdot]$ denotes the indicator function. Although for some individual depositors $\frac{\partial C_{ti}^{F}}{\partial R_1}$ and $\frac{\partial C_{ti}^{F}}{\partial \delta}$ can take negative values (this is more likely to occur to depositors with large uninsured balances) since, as shown above, $\frac{\partial \alpha_F(s)}{\partial \delta} \leq 0$, we show below that the aggregate consumption response among depositors to δ and R_1 is positive.

We can derive similar comparative statistics for taxpayers' consumption as follows

$$\frac{\partial C_{\tau}^{F}(s)}{\partial R_1} = -(1 + \kappa'(\cdot)) \frac{\partial T(s)}{\partial R_1} = \begin{cases} 0, & \text{if } T(s) = 0 \\ -(1 + \kappa'(\cdot)) \int_0^{\frac{\delta}{R_1}} D_{0i} dG(i) \leq 0, & \text{if } T(s) > 0 \end{cases} \tag{47}$$

$$\frac{\partial C_{\tau}^{F}(s)}{\partial \delta} = -(1 + \kappa'(\cdot)) \frac{\partial T(s)}{\partial \delta} = \begin{cases} 0, & \text{if } T(s) = 0 \\ -(1 + \kappa'(\cdot)) \int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i) \leq 0, & \text{if } T(s) > 0. \end{cases}$$

We can also write $C_{ti}^{N}(s) - C_{ti}^{F}(s)$ as follows

$$\begin{aligned} C_{1i}^{N}(s) - C_{1i}^{F}(s) &= D_{0i}R_1 - \min\{D_{0i}R_1, \delta\} - \alpha_F(s) \max\{D_{0i}R_1 - \delta, 0\} \\ &= \underbrace{(1 - \alpha_F(s)) \max\{D_{0i}R_1 - \delta, 0\}}_{\text{Partial Recovery of Uninsured Deposits}} \end{aligned}$$

$$\begin{aligned} C_{2i}^{N}(s) - C_{2i}^{F}(s) &= \alpha_N(s) D_{0i}R_1 - \min\{D_{0i}R_1, \delta\} - \alpha_F(s) \max\{D_{0i}R_1 - \delta, 0\} \\ &= \underbrace{(\alpha_N(s) - 1) D_{0i}R_1}_{\text{Net Return}} + \underbrace{(1 - \alpha_F(s)) \max\{D_{0i}R_1 - \delta, 0\}}_{\text{Partial Recovery of Uninsured Deposits}}. \end{aligned}$$

Figure 10 illustrates the individual consumption in failure states for depositors in the 20%, 40%, 60%, and 80% of the deposit distribution, as well as the funding shortfall and the recovery rate on uninsured

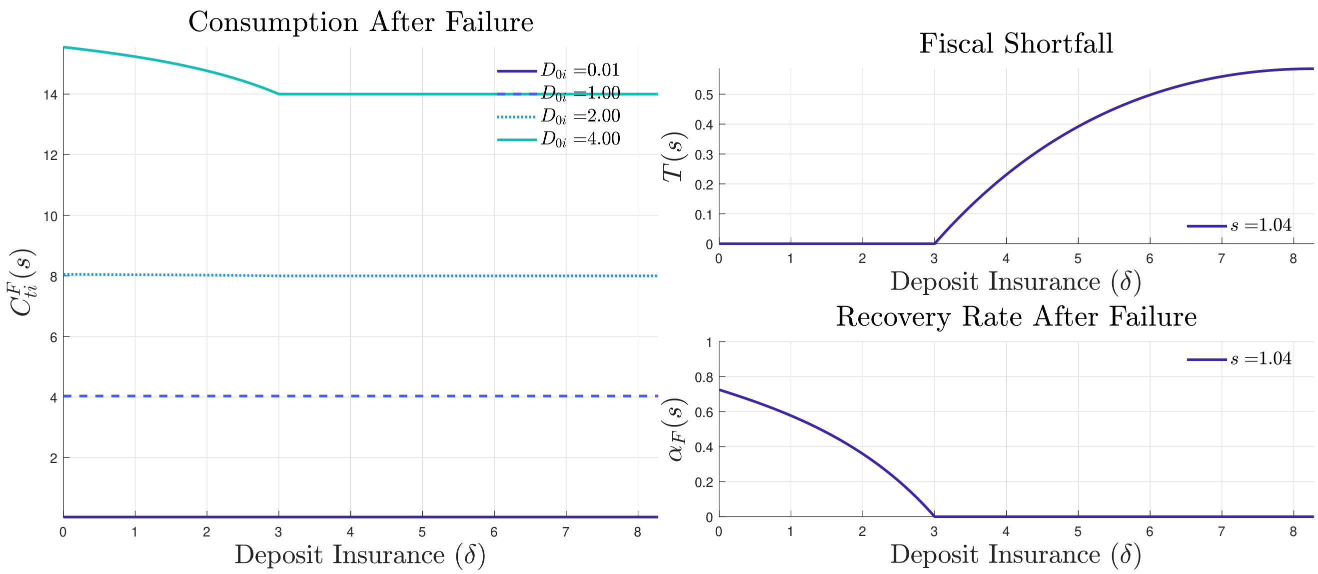


Figure 10: Consumption in Failure States

Note: The left plot in Figure 10 shows the consumption of depositors with deposit levels $\{0.01, 1, 2, 4\}$ in the case of bank failure. The right plots show the funding shortfall and the recovery rate on uninsured deposits when $s = 1.04$. All plots use the parameters described in Table 1.

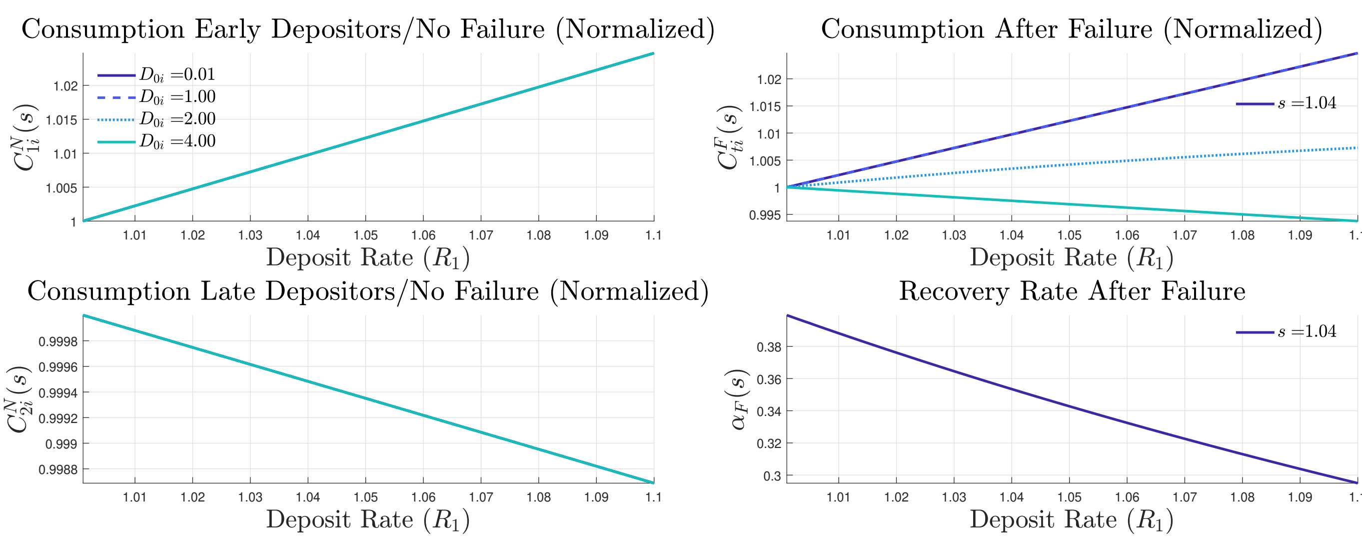


Figure 11: Consumption in Failure and No Failure States

Note: The left plot in Figure 11 shows the consumption of depositors with deposit levels $\{0.01, 1, 2, 4\}$ in the case of bank failure. The right plots show the funding shortfall and the recovery rate on uninsured deposits when $s = 1.04$. All plots use the parameters described in Table 1.

deposits for $s = 1.04$ for different levels of δ. Figure 11 illustrates the individual consumption in failure and no failure states for depositors in the 20%, 40%, 60%, and 80% of the deposit distribution, as well as the recovery rate after failure, for different levels of R_1.

Figure 10 shows that consumption for large depositors is initially decreasing in the level of coverage. When α_F is positive, Figure 11 shows that higher rates are associated with lower consumption.

B.5 Properties of aggregate consumption

The aggregate change in consumption among early depositors is given by

$$\begin{aligned}
\int_0^{\overline{D}} \left(C_{1i}^N(s) - C_{1i}^F(s)\right) dG(i) &= (1-\alpha_F(s)) \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i) \\
&= \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i) - \max\left\{\chi(s)\rho_1(s) D_0 - \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\} \, dG(i), 0\right\} \\
&= \begin{cases} (R_1 - \chi(s)\rho_1(s)) D_0, & \text{if } T(s) = 0 \\ \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i), & \text{if } T(s) > 0. \end{cases}
\end{aligned}$$

The aggregate change in consumption among late depositors is given by

$$\begin{aligned}
\int_0^{\overline{D}} \left(C_{2i}^N(s) - C_{2i}^F(s)\right) dG(i) &= (\alpha_N(s) - 1) D_0 R_1 + (1-\alpha_F(s)) \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i) \\
&= (\alpha_N(s) - 1) D_0 R_1 + \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i) \\
&\quad - \max\left\{\chi(s)\rho_1(s) D_0 - \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\} \, dG(i), 0\right\} \\
&= \begin{cases} (\alpha_N(s) R_1 - \chi(s)\rho_1(s)) D_0, & \text{if } T(s) = 0 \\ (\alpha_N(s) - 1) D_0 R_1 + \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i), & \text{if } T(s) > 0. \end{cases}
\end{aligned}$$

The aggregate change in consumption among depositors and taxpayers is given by

$$\begin{aligned}
\int \left(C_j^N(s) - C_j^F(s)\right) dj &= \lambda \int_0^{\overline{D}} \left(C_{1i}^N(s) - C_{1i}^F(s)\right) dG(i) + (1-\lambda) \int_0^{\overline{D}} \left(C_{2i}^N(s) - C_{2i}^F(s)\right) dG(i) + C_\tau^N(s) - C_\tau^F(s) \\
&= \begin{cases} \lambda (R_1 - \chi(s)\rho_1(s)) D_0 + (1-\lambda)(\alpha_N(s) R_1 - \chi(s)\rho_1(s)) D_0, & \text{if } T(s) = 0 \\ (1-\lambda)(\alpha_N(s) - 1) D_0 R_1 + \int_0^{\overline{D}} \max\{D_{0i}R_1 - \delta, 0\} \, dG(i) + T(s) + \kappa(T(s)), & \text{if } T(s) > 0 \end{cases} \\
&= \left[(\rho_2(s) - 1)(\rho_1(s) - \lambda R_1) + (1-\chi(s))\rho_1(s)\right] D_0 + \kappa(T(s)).
\end{aligned}$$

In the context of Equation (21) in the text, we express $\int \left(C_j^N(s) - C_j^F(s)\right) dj$ as follows

$$\int \left(C_j^N(s) - C_j^F(s)\right) dj = \left[(\rho_2(s) - 1)(\rho_1(s) - \lambda R_1) + (1-\chi(s))\rho_1(s)\right] D_0 + \kappa(T(s)),$$

where $T(s) = \max\left\{\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i) - \chi(s)\rho_1(s) D_0, 0\right\}$. Aggregate consumption among depositors in the case of bank failure is given by

$$\begin{aligned}\int_0^{\overline{D}} C_{ti}^F(\delta, R_1)\, dG(i) &= \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i) + \max\left\{\chi(s)\rho_1(s) D_0 - \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i), 0\right\} + \overline{Y}(s) \\ &= \begin{cases}\chi(s)\rho_1(s) D_0 + \overline{Y}(s), & \text{if } T(s) = 0 \\ \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i) + \overline{Y}(s), & \text{if } T(s) > 0\end{cases} \\ &= \max\left\{\chi(s)\rho_1(s) D_0, \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i)\right\} + \overline{Y}(s),\end{aligned}$$

where we define $\overline{Y}(s) = \int_0^{\overline{D}} Y_{ti}(s)\, dG(i)$.

Aggregate consumption among depositors and taxpayers in the case of bank failure is given by

$$\begin{aligned}\int_0^{\overline{D}} C_{ti}^F(\delta, R_1)\, dG(i) + C_\tau^F &= \max\left\{\chi(s)\rho_1(s) D_0, \int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i)\right\} \\ &\quad - \max\left\{\int_0^{\overline{D}} \min\{D_{0i}R_1, \delta\}\, dG(i) - \chi(s)\rho_1(s) D_0, 0\right\} - \kappa(T(s)) + \overline{Y}_j(s) \\ &= \chi(s)\rho_1(s) D_0 - \kappa(T(s)) + \overline{Y}_j(s),\end{aligned}$$

where we define $\overline{Y}_j(s) = \int_0^{\overline{D}} Y_{ti}(s)\, dG(i) + Y_\tau(s)$.

Therefore we can easily calculate $\int \frac{\partial C_j^F}{\partial \delta} dj$, which is a relevant input to set the optimal level of coverage, as well as $\int \frac{\partial C_j^F}{\partial R_1} dj$, as follows

$$\int \frac{\partial C_j^F}{\partial \delta} dj = \begin{cases} 0, & \text{if } T(s) = 0 \\ -\kappa'(\cdot) \int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i), & \text{if } T(s) > 0\end{cases}$$

$$\int \frac{\partial C_j^F}{\partial R_1} dj = \begin{cases} 0, & \text{if } T(s) = 0 \\ -\kappa'(\cdot) \int_0^{\frac{\delta}{R_1}} D_{0i} dG(i), & \text{if } T(s) > 0.\end{cases}$$

Aggregate consumption among depositors if there is no bank failure is given by

$$\begin{aligned}\int_0^{\overline{D}} C_{ti}^N(\delta, R_1)\, dG(i) &= \lambda D_0 R_1 + (1-\lambda)\alpha_N(s) D_0 R_1 + \overline{Y}(s) \\ &= \lambda D_0 R_1 + \rho_2(s)(\rho_1(s) - \lambda R_1) D_0 + \overline{Y}(s).\end{aligned}$$

B.6 Regularity conditions and limits

Continuity and differentiability of the problems faced by banks and regulators are guaranteed whenever distributions and parameters that vary with the realization of the aggregate state are sufficiently smooth. The one potential source of non-differentiability that emerges in the model is related to the form of the fiscal costs. To guarantee that social welfare is differentiable, it must be that $\frac{dW}{d\delta}$ is continuous. For this to be case, it must be that either $\min_s\{\chi(s)\rho_1(s)\} = 0$ or $\lim_{T\to 0} \kappa'(T) = 0$. Otherwise, for sufficiently low values of δ it is the case that there is no need to raise fiscal resources for any realization of s, so the second term in Equation (20) changes from 0 to a positive value at a point, preserving continuity, but inducing a non-differentiability.

As usual in normative exercises – see Atkinson and Stiglitz (1980) or Ljungqvist and Sargent (2004) for detailed discussions in different contexts, it is hard to guarantee the convexity of the planning problem in general: there are no simple conditions on primitives that guarantee that the regulator faces a convex problem. In practice, for natural parametrizations, as the one presented in Section 4.2, $W(\delta)$ is well-behaved and features an interior optimum. Similarly, it is not easy to establish the convexity of the problem solved by competitive banks to choose R_1, although the problem solved by banks is well-behaved for standard parametrizations, utility, and distributional choices.

In Remark 3, we make statements about the behavior of $\frac{dW}{d\delta}$ in the limit when $\delta \to 0$. Formally, we can write

$$\lim_{\delta\to 0^+}\frac{dW}{d\delta}=\lim_{\delta\to 0^+}-\frac{\partial q^F}{\partial \delta}\int\left[U\left(C_j^N\left(s^*\right)\right)-U\left(C_j^F\left(s^*\right)\right)\right]dj+\lim_{\delta\to 0^+}q^F\mathbb{E}_s^F\left[\int U'\left(C_j^F\right)\frac{\partial C_j^F}{\partial \delta}dj\right].$$

Since $U\left(C_j^N\left(s^*\right)\right)-U\left(C_j^F\left(s^*\right)\right)$ is non-negative for both depositors and taxpayers, the sign of the first element is given by $\lim_{\delta\to 0^+}\left(-\frac{\partial q^F}{\partial \delta}\right)$. Similarly, since q^F is strictly positive for any δ, including $\delta = 0$, the sign of the second term depends on whether when $\delta \to 0$, the average marginal cost of funds across states is zero or positive. In previous versions of this paper, we highlighted the fact that, at times, $\lim_{\delta\to 0^+}\frac{\partial s^*}{\partial \delta}=0$, implying that $\lim_{\delta\to 0^+}\left(-\frac{\partial q^F}{\partial \delta}\right)=0$. In that case, as long as there is a fiscal cost of paying for deposit insurance, that is, $\kappa(T)>0$, social welfare is decreasing when $\delta \to 0$. In the current formulation, $\lim_{\delta\to 0^+}\frac{\partial s^*}{\partial \delta}>0$, so $\lim_{\delta\to 0^+}\frac{dW}{d\delta}>0$.

C Directional Test: General Case

In this section, we provide a directional test for the level of coverage under minimal assumptions. While using the Diamond and Dybvig (1983) framework allows us to completely characterize a fully specified model, here we show that our insights extend more generally. As in the text, we focus on characterizing the welfare impact of a marginal change in the level of coverage under perfect regulation or when banks do not respond to the level of coverage. When banks are unregulated, our characterization needs to be augmented by the fiscal externality component(s).

Consider an economy in which the welfare of depositors and taxpayers is given by

$$W=\int V_j dj,$$

where V_j denotes the utility of depositors or taxpayers. We can then write

$$\begin{aligned}V_j &= \mathbb{E}\left[U_j\left(C_j\left(\delta,s\right);s\right)\right]\\ &=\int_{\mathcal{F}}U_j\left(C_j^F\left(\delta,s\right);s\right)dF\left(s\right)+\int_{\mathcal{N}}U_j\left(C_j^N\left(\delta,s\right);s\right)dF\left(s\right),\end{aligned}$$

where the second line decomposes the regions over the realization of the aggregate state between failure and no-failure regions. The value of C_j incorporates the final consumption by agent j at date 2. By making utility state-dependent, we can implicitly account for early and late types.

We can define the probability of bank failure as

$$q(\delta) = \int_{\mathcal{F}(\delta)} dF(s)$$

We can calculate the change in social welfare as

$$\begin{aligned}
\frac{dV_j}{d\delta} &= \int_{\mathcal{F}} U_j \left(C_j^F(\delta, s); s \right) dF(s) + \int_{\mathcal{N}} U_i \left(C_j^N(\delta, s); s \right) dF(s) \\
&= \left(U_j \left(C_j^F(s^*); s^* \right) - U_j \left(C_j^N(s^*); s^* \right) \right) \frac{\partial q^F}{\partial \delta} + q^F \mathbb{E}_{\mathcal{F}} \left[U_j' \left(C_j^F(\delta, s); s \right) \frac{\partial C_j^F}{\partial \delta} \right],
\end{aligned}$$

which is the same expression that we find in the paper whenever there are no fiscal externalities. When adding up across all agents, we can express $\frac{dW}{d\delta}$ as follows

$$\frac{dW}{d\delta} = \frac{\partial q^F}{\partial \delta} \int \left(\left(U_j \left(C_j^F(s^*); s^* \right) - U_j \left(C_j^N(s^*); s^* \right) \right) dj \right) + q^F \int \mathbb{E}_{\mathcal{F}} \left[U_i' \left(C_j^F(\delta, s); s \right) \frac{\partial C_j^F}{\partial \delta} \right] dj,$$

where we presume that C_j^N does not depend on δ directly. This equation is a direct generalization of Equations (20) or (25) in the text.

D Direct Measurement: Additional Material

Data Description For our calculations, we use Markit CDS data, as distributed by Wharton Research Data Services (WRDS). Our full sample includes daily data from January 2006 until December 2014. We focus on five-year CDS spreads (these are the most liquid) on the following banks (ticker in parentheses): Bank of America Corp (BACORP), Bank of NY Mellon (BK), Citigroup Inc (C), Goldman Sachs (GS), JP Morgan Chase (JPM), Merrill Lynch & Co Inc (MER), Morgan Stanley (MWD), State Street Corp (STT), Wachovia Corp (WB), and Wells Fargo & Co (WFC). We exclusively consider CDS contracts with CR (Complete Restructuring) as restructuring clause, so any restructuring event counts as a bank failure for our purposes. Similar results arise when using restructuring clauses MR (Modified Restructuring), MM (Modified Modified Restructuring), or XR (No Restructuring). We only consider CDS contracts on Senior Unsecured Debt and use the recovery rate provided by Markit.

Measurement The implied probability of failure can be read of spreads and recovery rates provided one is willing to make some assumptions. We use a simple constant hazard rate model (Hull, 2013), to calculate implied yearly default probabilities as follows:

$$\text{Implied Default Probability} = \frac{\text{5 Year Spread}}{1 - \text{Recovery Rate}}.$$

On October 3, 2008, President George W. Bush signed the Emergency Economic Stabilization Act of 2008, raising the limit on federal deposit insurance coverage from \$100,000 to \$250,000 per depositor. Initially, this change was temporary through the end of 2010, but it was made permanent by the Dodd-Frank Act in July 2010. We measure the jumps in the default probability caused by changes in the level of deposit insurance exactly on October 3, 2008. This R notebook contains replication code.

E Numerical Simulation: Additional Material

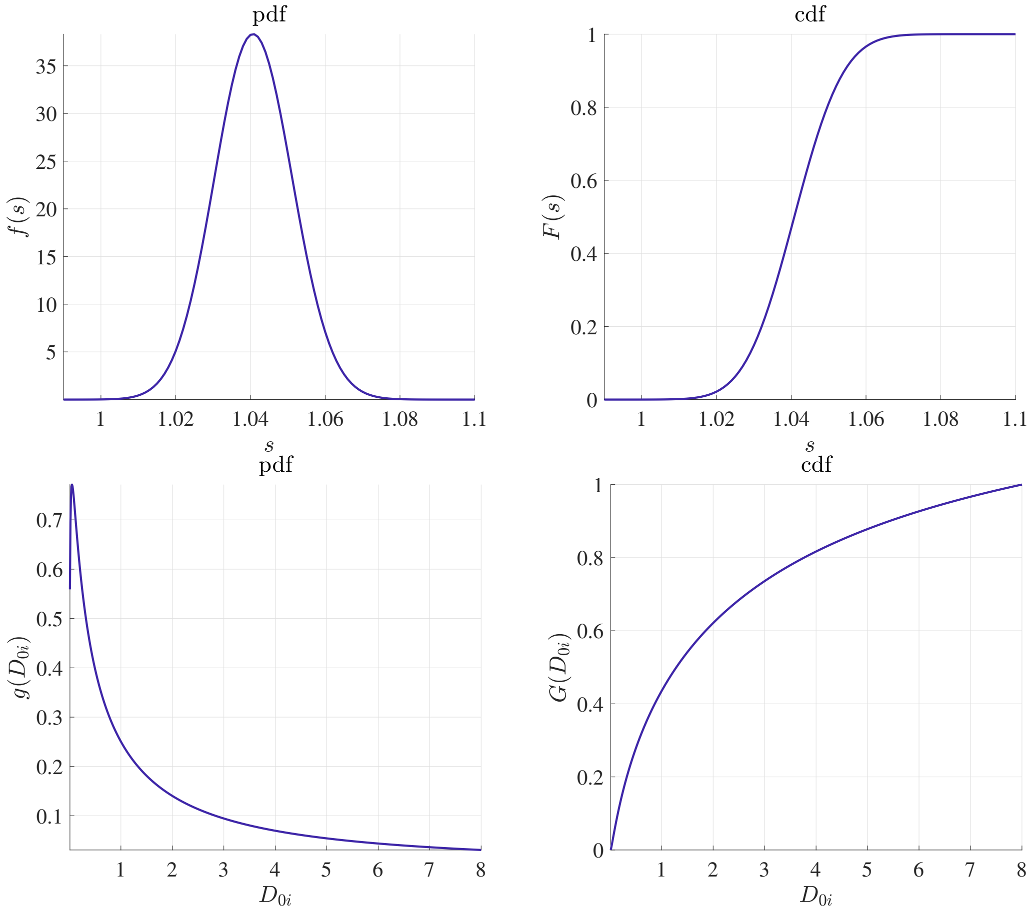


Figure 12: Distributions

Note: The left plots show the pdf (top) and cdf (bottom) of the distribution of banks' return on assets. The right plots show the pdf (top) and cdf (bottom) of the distribution of deposit accounts at date 0. Deposits are measured in hundreds of thousands of dollars.

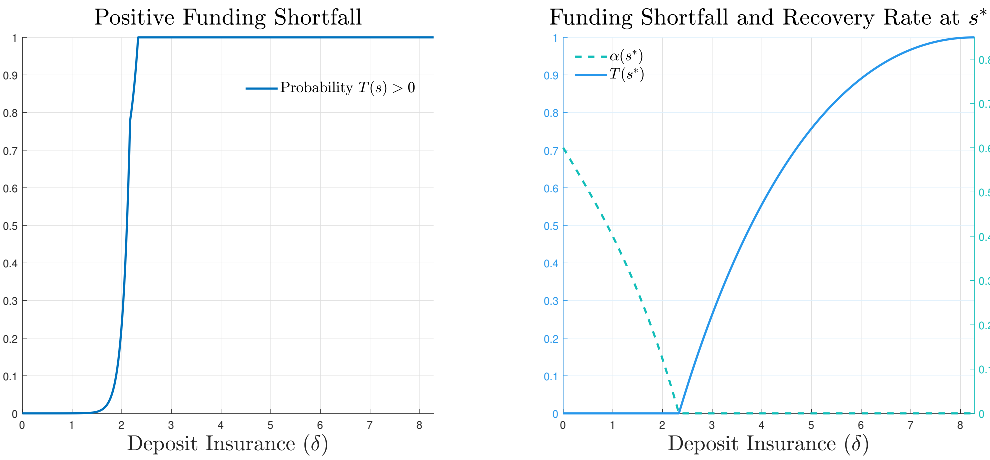


Figure 13: Funding Shortfall and Recovery Rate

Note: The left plot in Figure 13 shows the likelihood of facing a positive funding shortfall conditional on a bank failure for different levels of deposit insurance coverage δ (measured in hundreds of thousands of dollars). The right plot in Figure 13 shows the funding shortfall and the recovery rate on uninsured deposits at the threshold state, $T(s^*)$ and $\alpha(s^*)$, for different levels of deposit insurance coverage δ. Both plots use the parameters described in Table 1.

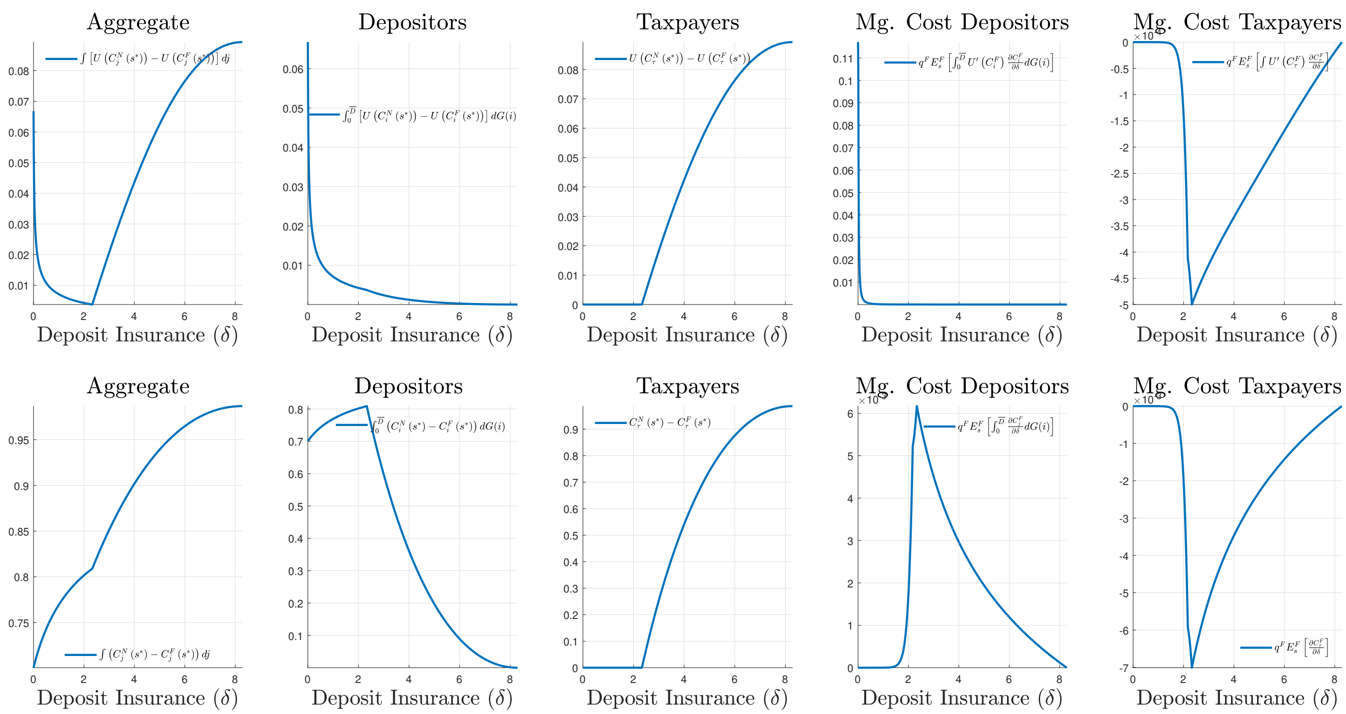
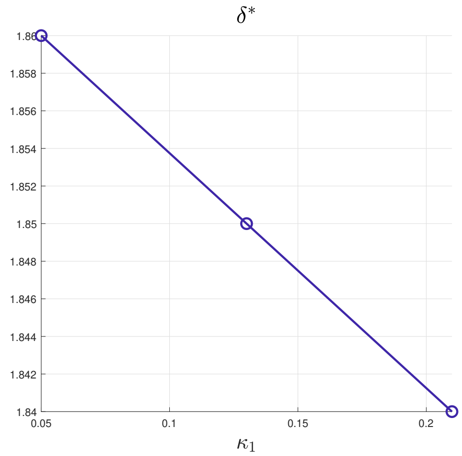


Figure 14: Determinants of Exact and Approximated Marginal Benefits and Costs

Note: The top plots in Figure 14 show individual determinants of marginal benefit and marginal costs when computed exactly. Each top plot maps to a different element of Equation (20). The bottom plots in Figure 14 show the same individual determinants of marginal benefits and costs when computed as an approximation. Each bottom plot maps to a different element of Equation (23). All plots use the parameters described in Table 1.



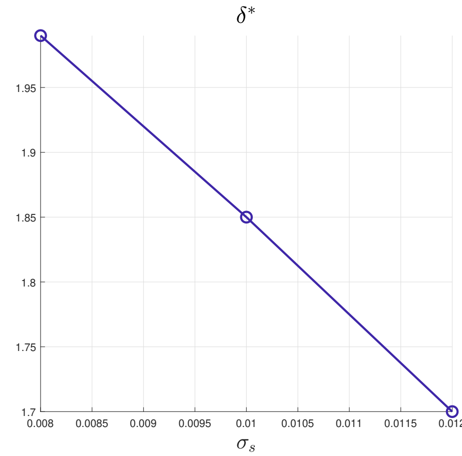


Figure 15: Optimal Coverage: Comparative Statics

Note: The left plot in Figure 15 shows the optimal level of deposit insurance coverage $\delta^\star$ for different values of $\kappa_1 \in \{0.05, 0.13, 0.21\}$. The right plot in Figure 15 shows the optimal level of deposit insurance coverage for different values of $\sigma_s \in \{0.08, 0.1, 0.12\}$. Both plots use as baseline parameters those described in Table 1.

Table 3: Simulation Parameters

Parameter	Definition	Value
	Depositors	
γ	Intertemporal Substitution	2
λ	Fraction of Early Depositors	0.05
π	Sunspot Failure Probability	0.15
$G(\cdot)$	Distribution of Deposits	$\mu_D = 1$, $\sigma_D = 2$, $\left[0, \overline{D}\right] = [0, 8]$
$Y_{1i}(s)$	Endowment Early Depositors	$3D_{0i}$
$Y_{2i}(s)$	Endowment Late Depositors	$3.05D_{0i}$
	Banks	
$F(\cdot)$, φ	Return on Assets	$\mu_s = 0.04$, $\sigma_s = 0.01$, $[\underline{s}, \overline{s}] = [0.99, 1.1]$, $\varphi = 0.95$
χ_1, χ_2, χ_3	Recovery Rate	1.68, 0.28, 0.99
	Taxpayers	
κ_1, κ_2	Deadweight Loss	0.13, 0.5
$Y_\tau(s)$	Endowment Taxpayers	$3D_{0i}$, where $F(D_{0i}) = 0.5$

Note: The distributions of deposits and return on assets are a truncated log-normal with μ_D, σ_D, μ_s, and σ_s denoting the parameters of the underlying normal distribution. In all simulations, $R_1 = 1.03$.