The Zero Lower Bound and Financial Stability: A Role for Central Banks *

(Preliminary and incomplete)

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Abstract

Financial stability objectives have taken a new place in central bank preferences next to the prime objectives of monetary policy – stabilizing inflation and output. When deciding to raise interest rates from the zero lower bound (ZLB), central banks must take into account the effects of an increase in nominal interest rates on financial stability not only via bank profitability but also via aggregate default in the economy. We develop a general equilibrium model with a financial sector, an autonomous central bank, and collateral default to analyze (a) how monetary policy and financial stability objectives affect optimal policies, and (b) how a lift-off from the ZLB affects liquidity and default (i) when the central bank cares only about monetary policy objectives, and (ii) when the central bank cares also about financial stability objectives. A lift-off from the ZLB exacerbates default but mitigates default-induced deflation when the central bank has control over one instrument for each policy objective. A dual mandate without considering financial stability concerns increases the variance in targeted policy outcomes across states of nature. Pursuing financial stability objectives on top of the dual mandate makes optimal monetary policy less pro-cyclical in our model.

Keywords: Financial stability, monetary & macroprudential policy trade-off, central bank optimization, zero lower bound, default.
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1 Introduction

The Global Financial Crisis (GFC) in 2008 triggered a paradigm shift for both monetary authorities and regulators. The former came to acknowledge the unprecedented role of a stable financial system in guaranteeing structural stability.[1] The latter recognized in particular the systemic nature of financial fragility. It highlighted the need to improve macroprudential policy instruments in order to mitigate risks like default. Trichet (2009) picked up on the role of financial stability in the ECB's dual mandate in one of his speeches shortly after the financial crisis: "*Given that our mandate requires the maintenance of price stability [...], we consider it important to monitor the slow accumulation of unsustainable financial imbalances which pose a threat to macroeconomic and price stability over the longer term. Maintaining a mediumterm orientation, keeping a close eye on monetary and credit dynamics, and adopting a broader, stability-oriented view of policy making – which are key elements of the ECB's monetary policy strategy – supports this approach.*" We are interested in how this potential shift from a *de jure* dual mandate to a *de facto* "ternary mandate" has affected optimal monetary policy and policy outcomes in the financial intermediation sector and the private sector.[2] In light of the recent prolonged zero interest rate environment, it is of particular interest to examine how and if policy objectives drive interest rates away from the ZLB. We employ a monetary general equilibrium (GE) model with incomplete markets and default to study the decision of a consolidated monetary and regulatory authority (henceforth called the central bank) to lift the future monetary policy rate from the ZLB. The central bank takes into account its monetary policy objectives – preserving price and output stability – as well as financial stability.

The trade-off between monetary policy and financial stability objectives (Cesa-Bianchi & Rebucci, 2017; Peek et al., 2016; Woodford, 2012) depends on the opposite effects that accommodative monetary policy has, in theory, on the stability of the economy versus the stability of the financial system. The consensus view in times of below-target inflation and economic growth is to keep interest rates close to zero. It is expected that spending and investment will consequently increase. At the same time, it may be desirable from the standpoint of a macroprudential regulator to increase interest rates from the ZLB in order to bolster banks' profit margins and disincentivize banks from investing into riskier asset classes in search for yield (Dell'Ariccia, Laeven, & Marquez, 2014; Döttling, 2018; Martinez-Miera & Repullo, 2017). In yet another twist of turns, higher interest rates may exacerbate the debt-servicing costs of private borrowers and decrease the liquidity in the economy.[3] A higher probability of default beyond a threshold may trigger fire sales of assets, which could further depress prices and increase the real burden of debt (Eggertsson & Krugman, 2012; Goodhart, Tsomoco, & Vardoulakis, 2010; Lin, Tsomocos, & Vardoulakis, 2015). Debt-deflation might ensue if interest rates were pushed too high too soon. Based on these scenarios, it is therefore not clear whether a much-demanded departure from the ZLB would achieve either of the central bank's objectives.

[1] For an excellent discussion of the question whether financial stability is relevant to monetary policy making, see for example Peek, Rosengren, and Tootell (1999) and Woodford (2012).

[2] Peek, Rosengren, and Tootell (2016) provide empirical evidence that the Federal Reserve Bank's policies have revealed a ternary mandate.

[3] This is not too unlikely a case in light of the still very high debt burden of private households and high levels of non-performing loans (NPLs) that the banking sector is burdened with.

Our model thus addresses two of the big questions in macro-finance that characterize the post-GFC era: (a) How do the monetary policy and financial stability objectives affect optimal monetary policy and policy outcomes, and how much should central banks consequently incorporate these objectives into their policy implementation, and (b) how does a lift-off from the ZLB affect liquidity and default (i) when the central bank cares only about monetary policy objectives, and (ii) when the central bank cares also about financial stability objectives?[4] We consider an exchange economy where the ZLB is the status quo in the first period. We then assess how the policy trade-off facing the central bank affects optimal monetary policy and the policy outcome of a lift-off from the ZLB in the second period. Our research framework allows us to explore how the relative importance of monetary policy versus financial stability objectives in the decision problem of the central bank matter. In particular, we are interested in how they affect the impact of a departure from the ZLB on the debt sustainability of the private and financial sector. We examine whether specific weights on either objective can exacerbate the debt-deflationary effect in the economy.

This paper relates to several strands of literature. One of them investigates the role of financial frictions and financial stability concerns in setting the monetary policy rate in dynamic macroeconomic models (Adrian & Duarte, 2017; Aikman, Giese, Kapadia, & McLeay, 2018; P. Benigno & Paciello, 2014; Christiano, Ilut, Motto, & Rostagno, 2011; Cúrdia & Woodford, 2010, 2011, 2016; Svensson, 2017; Woodford, 2012). Another growing body of literature then discusses the adoption of specific macroprudential policy instruments and the interaction of such instruments with monetary policy (Angelini, Neri, & Panetta, 2014; Angeloni & Faia, 2013; Beau, Cahn, Clerc, & Mojon, 2013; G. Benigno, Chen, Otrok, Rebucci, & Young, 2013; Cecchetti & Kohler, 2014; Kannan, Rabanal, & Scott, 2014; Nelson & Pinter, 2018; Quint & Rabanal, 2014). There is also a body of work that examines the different policy objectives that central bankers communicate to the public and the *de facto* objectives that they follow with their policies. Both monetary policy and financial stability goals as well as internal balance sheet constraints (Hall & Reis, 2015; Reis, 2013) and concerns for profitability (Goncharov, Ioannidou, & Schmalz, 2017) have been identified by the literature as factors affecting the policy stance and outcome. Empirical evidence on the interaction of monetary and macroprudential policy at or away from the ZLB is still in its infancy. Several papers investigate the effects of negative deposit rates on banks (Heider, Saidi, & Schepens, 2018; Lopez, Rose, & Spiegel, 2018), the resulting optimal macroprudential policy (Döttling, 2018; Richter, Schularick, & Shim, 2018), and the effects of a lift-off from the ELB (Drechsler, Savov, & Schnabl, 2017).

Our modelling framework mainly builds on previous work by Goodhart et al. (2010), Goodhart, Sunirand, and Tsomocos (2005, 2006), Tsomocos (2003), Lin et al. (2015), and Kotak, Ozsoylev, and Tsomocos (2017), who develop computable GE models that allow for deposits, endogenous default on loans, and interbank markets. Goodhart et al. (2010) model an exchange economy with a mortgage market. Heterogeneous households trade in a durable and a perishable consumption good and maximize their expected utility by smoothing consumption. Cash-in-advance constraints generate a demand for money and thus a demand for loans from a financial intermediary. Incorporating the mortgage market provides rich implications about default on bank lending and house

[4]Note that we do *not* answer the question which specific policy *tools* should be employed. Instead we analyze the interaction of policy *objectives* in the decision problem of the central bank.

price dynamics. We use a similar GE model of an exchange economy with two periods, default, incomplete markets, and heterogeneous agents in which households deposit funds and borrow a collateralized loan from a bank to smooth their consumption. Default emerges as an equilibrium phenomenon. However, we modify their model in the following ways.

First, we simplify the model to include two heterogeneous agents, one commercial bank, and no interbank market. Second, rather than assuming an ad hoc monetary authority that intervenes in the interbank market, we augment the model by providing micro foundations to an autonomous central bank. The central bank optimizes its own objective function that reflects its monetary policy and financial stability objectives subject to its balance sheet constraints. It does so by setting the future policy rate on central bank loans and by setting an ex-ante capital requirement on the risky assets held by the bank. The central bank is subject to the same type of uncertainty as the financial and the private sector. As it turns out, macroprudential policy is a crucial factor for financial stability. Finally, we allow the commercial bank to not only obtain short-term loans from the central bank but to also deposit excess reserves with the central bank. We are interested in the spread between lending and borrowing rates because we want to assess the monetary policy channel via bank profitability.

Augmenting an otherwise "standard" banking model by the central bank's optimization problem renders the search for a general equilibrium solution substantially more complex. This complexity stems from two interrelated issues. First, the central bank maximizes its objectives subject to setting the policy rate. However, the policy rate only *indirectly* affects the central bank's objectives via other variables determined in equilibrium. Second, the model setup does not allow for closed-form analytical equilibrium solutions to all endogenous variables. It is extremely difficult, if not impossible, to derive equations for prices and allocations in terms of the policy rate and exogenous parameters only. To the best of our knowledge, no previous paper has analytically derived optimality conditions for a central bank that endogenously depend on the policy rate and a macroprudential instrument without resorting to reduced form equations in infinite horizon models. We circumvent this analytical problem by designing a two-step numerical solution method. We first retrieve numerical starting values for partial derivatives in the central bank's first order conditions by deriving a set of envelope conditions with respect to each control variable. Using these starting values and the system of equilibrium conditions as well as envelope conditions, we numerically obtain a general equilibrium under constrained central bank optimization.[5] Envelope Condition Methods (ECM) have for example been designed by Arellano, Maliar, Maliar, and Tsyrennikov (2016) for dynamic programming problems in the context of large-scale macroeconomic models.

Our results suggest that a lift-off from the ZLB has three distinct general equilibrium effects in our benchmark calibration with equal weights on monetary policy and financial stability objectives. First, at the ZLB, mortgage default generates deflationary dynamics. A rise in the interest rate corridor at the ZLB raises the debt servicing costs of private borrowers and exacerbates their debt burden. The decision to default on long-

[5]We recognize the limitations of our approach relative to other solution methods of dynamic infinite-horizon GE models such as approximations of policy functions. However, the benefits of retrieving a general equilibrium including all four agents in the economy outweigh the costs – making very strong assumptions – in our view.

term debt is endogenously determined through the value of the underlying collateral. A higher interest rate depresses the nominal value of goods by the Quantity Theory of Money since the velocity of money stays constant in our model. Losses due to default and deflation increase and output falls in bad times as the interest rate corridor at the ZLB widens. Second, away from the ZLB, deflation is lower but default increases. This is because the central bank's optimal monetary policy away from the ZLB is to conduct monetary easing in good times so as to stimulate activity and decrease deflation. At the same time, optimal monetary policy is contractionary in bad times because the central bank intends to limit credit expansion through a high policy rate. Contractionary monetary policy is the result of a trade-off between achieving lower deflation on the one hand and higher losses due to default on the other hand. Optimal credit spreads between borrowing and lending rates are narrower in good times and wider in bad times, accounting for default and liquidity premia. Third, monetary policy becomes more pro-cyclical as a result, the further the economy departs from the ZLB. At the same time, the optimal capital requirement is laxer away from the ZLB. The macroprudential policy instrument therefore complements the monetary policy instrument. Third, the argument that rates should be raised from the ZLB to restore bank profitability does not obtain in our simulations. Bank profits decrease as the economy moves away from the ZLB both in good and in bad times since the increase in interest rates dampens aggregate demand for credit in the economy and worsens losses due to default.

We hasten to add to the debate on monetary policy versus financial stability objectives by noting two additional findings from our simulations. First, the higher the weight that the central bank places on targeting inflation relative to other objectives, the larger is the variance between the achieved objectives – bank profitability, default, inflation, and output – across states of nature. For example, if the central bank reverts to a plain vanilla dual mandate and neglects financial stability concerns altogether, inflation will be even closer to target in booms but deflation will surge even more in busts. This is because the central bank does not take into account the adverse effects that default will have on the price level in the bad states of nature. A positive weight on financial stability objectives in the central bank's objective function "smoothes" the outcomes of interest across states of nature. Second, optimal monetary policy is less pro-cyclical under a ternary mandate than under a dual mandate. That is, when the central bank cares about financial stability, the prescriptions about optimal monetary easing in booms and tightening in busts are not as extreme as compared to when it only considers inflation targeting and the output gap. Finally, whether the central bank cares about financial stability or not does *not* affect the *direction* that changes in endogenous variables in equilibrium take when the economy departs from the ZLB. The importance placed on monetary policy versus financial stability objectives only affects the *magnitude* of changes.

However, there are two modelling choices that qualify these results. First, the comparative statics exercise is, by design, limited to *exogenous* shocks to the interest rate on excess reserves that anchors the ZLB in our model. We do not endogenize these shocks since the reasons for the economy to be at the ZLB are outside the scope of our model. It is therefore not possible to make quantitative statements about the *dynamic* behavior of the central bank and its *endogenous* decision to lift the interest rate on excess reserves from the ZLB. Note however that while the ZLB is not endogenously determined, the width of the interest rate corridor is endogenously chosen by the central bank. Second, the simulations implicitly assume that the central bank has perfect knowledge over all agents' optimality conditions. However it is "boundedly rational" in the sense that it cannot fully comprehend the indirect effects of its actions on all endogenous variables in the economy. An approach grounded in economic theory would take into account the central bank's imperfect knowledge in gauging the effects of its policy on the economy.

Our findings parallel those of Lin et al. (2015) who examine the role that monetary policy plays in the decision to default in a GE model with collateralized loans and production. They show that contractionary monetary policy via default, foreclosure, and higher borrowing costs can lead to debt-deflation dynamics. The main difference to our model without production is that we model explicitly the propagation of debt-deflationary dynamics of default via a financial sector which intermediates between the policy objectives of the central bank and households. Moreover, contractionary monetary policy in our model exacerbates losses due to default but it mitigates deflationary tendencies as already explained.

Along the lines of Dubey and Geanakoplos (1992), we show that monetary policy is non-neutral in equilibrium. Outside money (Dubey & Geanakoplos, 2003a, 2003b, 2006) and default (Lin, Tsomocos, & Vardoulakis, 2016) ensure positive nominal interest rates and price level determinacy. Because money and default are frictions in the economy, the *de facto* output level will always deviate from potential output obtained under a barter economy that achieves Pareto optimality. The output gap is therefore always negative. The goal of the central bank is to minimize deviations of actual from potential output while maintaining price stability and financial stability. This is to say that it aims to minimize the friction created by our transaction technology, or the distance between the first best and constrained best outcome.

The paper proceeds as follows. Section 2 explains the conduct of monetary policy in practice which motivates our modelling choices. Section 3 introduces the model setup including all agents' opimization problems and the market clearing conditions. Section 3 defines the equilibrium and Section 4 characterizes the equilibrium in the economy. Section 5 reduces the model to two states of nature and presents the calibration and results of the comparative statics exercise whereby the economy lifts off from the ZLB. Finally, Section 6 concludes.

2 Central banking in practice and in theory

One of the questions that we want to understand is whether a lift-off from the ZLB, the way it has been conducted by the US Federal Reserve Bank, imperils the stability of the financial system if the central bank does not consider the effects of its policy on financial stability in its decision to raise the interest rate. Monetary policy in our model closely mimics the way the Federal Reserve implemented its (conventional) monetary policy during the period from October 2008 to December 2015. Prior to 2008, the Federal Reserve set a target for the federal funds rate and steered towards this target through temporary Open Market Operation (OMOs) and lending from the discount window against eligible collateral. The federal funds rate was bound below by the interest rate on required reserves (effectively the ZLB because the Federal Reserve was legally not allowed to pay interest) and above by the discount rate – a so called *channel/corridor system*. The Federal Reserve started paying interest on excess reserves (IOER) for the first time on October 6, 2008. The IOER provided a new lower bound for the federal funds rate. It subsequently became the main policy tool since large-scale injections of reserves into the financial system put downward pressure on the federal funds rate with the start of the QE program.[6] This is illustrated in Figure 1 where the federal funds rate closely traces the IOER.[7] The targeting of the federal funds rate gradually moved from a channel/corridor system to a *floor system*.

Figure 1: The Federal Funds Rate tracking the IOER (daily (7-day) time series)

Despite the IOER having become the main steering wheel of monetary policy since 2008, the central bank in our model has discretion over setting the federal funds rate (henceforth called the "policy rate").[8] This allows us to exogenously impose the lift-off of the IOER from the ZLB in a comparative statics exercise. We assume the IOER to be predetermined because the causes for the economy to be at the ZLB lie outside the scope of our model. We want to investigate the relevance of different policy objectives when a lift-off is inevitable. We choose to distinguish between the IOER and the policy rate in order to demonstrate the importance of the wedge between borrowing and lending rates for monetary policy transmission. Inevitably, this positive wedge generates seigniorage income for the central bank. To close the system, the seigniorage income is assumed to be redistributed to households as a lump-sum transfer in our baseline model. It is thus considered as an expected money-financed fiscal transfer to households. This assumption is not far from reality since central banks re-distribute profits to their shareholders.[9] In

[6]The IOER provided a floor because banks had no incentive to lend in the federal funds market at a rate lower than the IOER. They could earn more by depositing their excess reserves with the Federal Reserve. In theory, the IOER also anchored the federal funds rate and thus provided a ceiling. If the federal funds rate was higher than the IOER, lending in the overnight market would become relatively more attractive than depositing excess reserves with the Federal Reserve, thus putting downward pressure on the federal funds rate.

[7]We note a technical point that explains why the effective federal funds rate is not at or above the IOER in Figure 1 as arbitrage theory would suggest. This phenomenon is called "leaky floor". The leaky floor results from the discrepancy in rates at which institutions trade that are eligible to earn interest on excess reserves and non-bank institutions that are not eligible. For more details, see Martin (2017).

[8]The central bank in our model deploys an interest rate setting instrument as opposed to a monetary base instrument for monetary policy. That is, it sets the price rather than the quantity. Goodhart, Sunirand, and Tsomocos (2011) show that an interest rate instrument is preferable as a tool to maintain financial stability in times of financial crisis.

[9]For example, the ECB redistributes profits from its monetary operations to member countries of the Euro area.

most cases the main shareholder is the government which may distribute any income from the central bank to households (Archer & Moser-Boehm, 2013).

One reason that induces the bank in our model to hold excess reserves rather than lend them out is that excess reserves are the only long-term investment available to the bank that is riskless. Moreover, they "absorb" part of the losses from mortgage default in the second period. Other reasons may include increasing risk aversion in response to a financial crisis as well as counterparty risk, adverse selection, and search frictions in the interbank market (Cui & Radde, 2014; Heider, Hoerova, & Holthausen, 2009; Malherbe, 2014). The resulting problem of identifying solvent borrowers in the interbank market can culminate in an interbank market liquidity freeze (Brunnermeier & Pedersen, 2009) and precautionary hoarding of liquidity either as cash on the bank's balance sheet or with the central bank. For the sake of simplicity, we do not model the reasons for liquidity hoarding.

In practice, the policy rate is targeted through temporary OMOs. These repurchase agreements are essentially overnight loans by the central bank to depository institutions, secured by safe, liquid collateral that is repurchased the following day. We simplify collateralized repos to short-term unsecured loans from the central bank in order to keep the model parsimonious and tractable.[10][11]

To choose the rule that guides the optimization problem of the central bank, Rudebusch and Svensson (1998) emphasize the distinction between an instrument rule and a targeting rule for monetary policy. Under an *instrument rule*, "the monetary policy instrument is expressed as an explicit function of available information". Such is for example the Taylor rule for the federal funds rate. Under a *targeting rule* "the central bank is assigned to minimize a loss function that is increasing in the deviation between a target variable and the target level for this variable". Such a welfare loss function is often given by the expected discounted sum of period loss functions (Woodford, 2005):

$$W_t \equiv E_t \sum_{s=t}^{\infty} \beta^s L_s \tag{1}$$

$$L_s = \frac{1}{2}[\,(\,\pi_s - \pi^*)^{\,2} + \lambda(\,y_s - y_s^*)^{\,2}] \tag{2}$$

where the period loss function is the (weighted) sum of the squared inflation gap and real output gap with some weight $\lambda > 0$. Rotemberg and Woodford (1998) use a quadratic approximation to translate the expected utility of the representative household in the Calvo model into the loss function given by (2). They thus provide a possible link between the social planner's canonical objective, i.e. maximizing the weighted sum of utilities, and the central bank's objective of stabilizing inflation and output.

[10]The intra-period loan could also be obtained from other banks in the interbank market. However, we model the single bank ϕ as representing the entire financial intermediation sector and therefore abstract from the existence of an interbank market. Collateral in the form of government bonds may be added as an extension at a later stage. This would allow us to investigate the effects of QE on price dynamics, default, and the term structure of interest rates.

[11]We consider central bank loans to be *intra*-period loans in our model as opposed to *inter*-period loans in the models in Goodhart et al. (2005) and Goodhart et al. (2010). Intra-period central bank loans correspond to the short-term nature of emergency liquidity injections after 2008.

To the best of our knowledge, attempts to model the (constrained) optimization problem of the central bank have largely been limited to dynamic infinite-horizon models that employ a targeting rule as in (2).[12] We use a targeting rule in a two-period GE model in order to micro-found the optimization problem of the central bank. However, we augment the targeting rule by financial stability objectives. The central bank is assumed to maximize a criterion function (minimize a loss function) whose key components - the deviation of expected inflation from target inflation and the real output gap - directly derive from the Taylor rule. Additionally, the central bank aims to maximize the profitability of banks which serves as a proxy for the overall health of the financial sector. It also wants to minimize the aggregate loss due to default in the economy (Kotak et al., 2017) because it cares about the repayment of claims.

Central banks, unlike commercial banks, do not seek to maximize their own profits, nor do they face the same balance sheet constraints as financial intermediaries. In practice, central banks are never insolvent. In theory however, they may become insolvent if their income is far from sufficient to pay dividends to their shareholders (usually the government), causing an explosion of debt to commercial banks (Hall & Reis, 2015). We do not model the possibility of insolvency by the central bank, nor do we model a government. Instead, we assume that the central bank in our model cares about at least breaking even. Goncharov et al. (2017) provide global evidence that central banks are more likely to report small positive profits than small negative profits. Few other authors explicitly model central bank constraints arising from their balance sheet in finite horizon. In the three-period model by McMahon, Peiris, and Polemarchakis (2018), a consolidated monetary-fiscal authority sets the interest rate and manages its asset portfolio subject to a present value budget constraint. Its objective merely is to break even.

The famous "Tinbergen Principle" (Tinbergen, 1956) prescribes that any *target* in the economy addressed by a monetary authority should be tackled with a separate policy *instrument*.[13] Cesa-Bianchi and Rebucci (2017) argue that the absence of a second policy instrument at the advent of the GFC was a reason why monetary policy could not achieve efficient allocations and faced a trade-off between monetary policy and financial stability objetives. Our model contains both liquidity and default frictions - or equivalently both economic and financial stability as targets. Yet, only the monetary policy rate is available as an instrument so far. To obtain an additional tool to address financial stability, we therefore let the central bank also control the capital requirement set on the bank's risk-weighted assets.

3 The Model

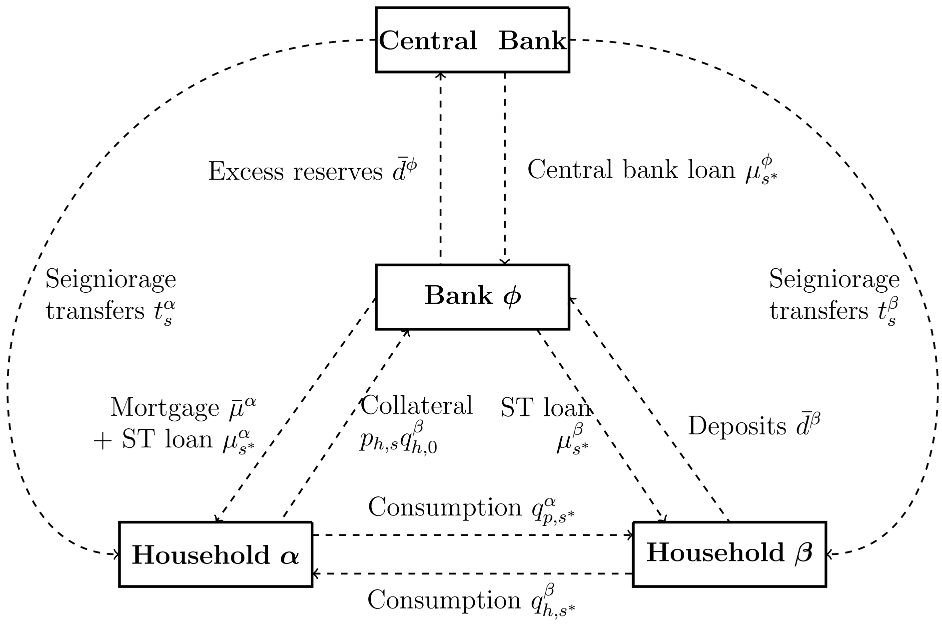
We embed the monetary policy decision problem of the central bank in an otherwise standard GE model with heterogeneous agents, collateralized mortgages, default, and trade in fiat money following Goodhart et al. (2010). Let there be an exchange economy that

[12] The advantage of (constrained) optimization in this type of model is that the objective function is linear and constraints (the IS-LM relation and the Phillips curve) are linearized. A suite of computational tools is available for the researcher to solve for the equilibrium path which limits the analytical burden. For an example of a DSGE model that does not rely on linearization around the steady state, see for example Bénassy (2006).

[13] For an excellent review of Tinbergen's policy recommendations, see Arrow (1958).

operates at the ZLB of the nominal interest rate on excess reserves (henceforth referred to as IOER rate). There exist two commodities, one perishable consumption good and one durable capital good. The economy is populated by two types of households, long-term borrowers and lenders who buy and sell commodities. All contracts are denominated in money which is the postulated means of exchange. Hence, households do not trade commodities directly with each other but trade with fiat money through a financial intermediary, henceforth called the commercial bank.[14] The commercial bank maximizes profits by granting loans to and taking on deposits from the households but does not discriminate between the types of households. A central bank controls liquidity in the financial system by setting the monetary policy rate. It takes on excess reserve deposits from the commercial bank and extends loans to the commercial bank. The income that the central bank earns from charging a premium on its lending activities over the deposit rate is distributed to households as a lump-sum payment at the end of the second period. Figure 2 summarizes the main actors in the model.

Figure 2: Model Setup



There is one contract traded which is defaultable. Default is discontinuous (Geanakoplos, 2002; Geanakoplos & Zame, 2014) such that agents have no discretion over the exact amount they default on but only over whether they default or not. If they default, their collateral is seized and resold in the market at the current market price. This is the case for the collateralized non-recourse mortgage that the household enters with the commercial bank.

The model features two time periods $t \in T = (0, 1)$. Uncertainty is resolved in the second period, which we for now assume to consist of a finite number of states of nature, $s \in S$. The set of all states is therefore given by $s \in S^* = \{0\} \cup S$. For computational tractability, the number of states will be reduced to $s \in (1, 2)$ in Section

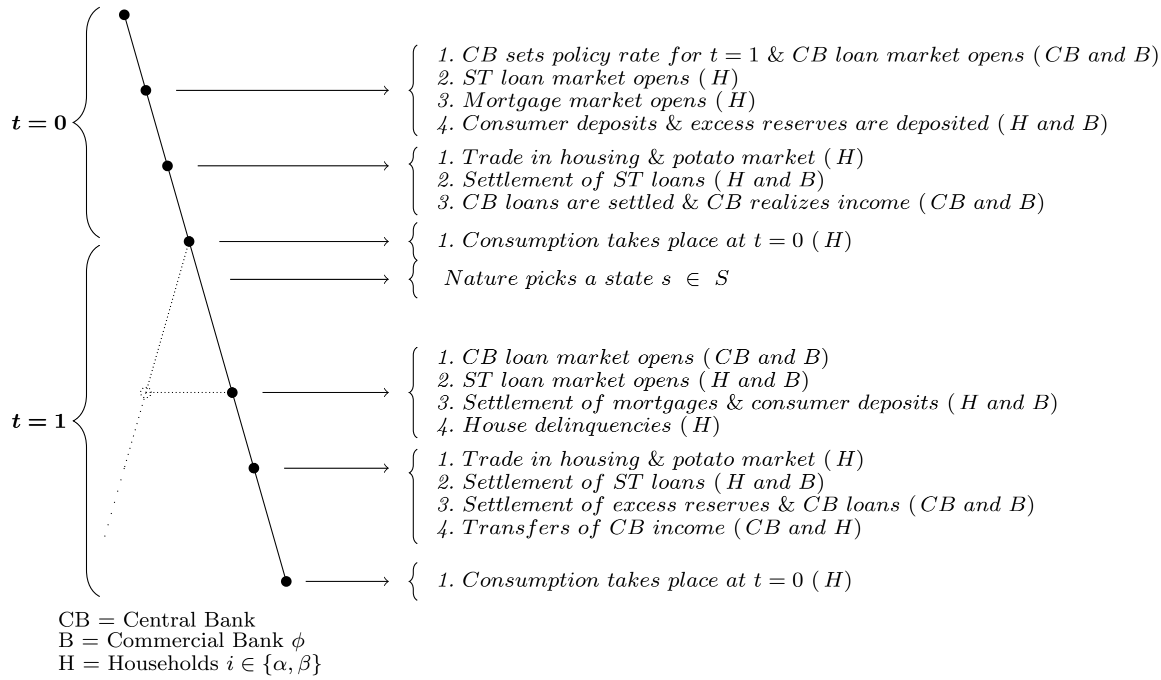
[14] Gains from trading with money exist because without money there is no trade in goods and no consumption smoothing. These gains rationalize positive interest rates (Dubey & Geanakoplos, 2003a)

5. This comprises of one "good" state in which the household does not default and one "bad" state in which the household defaults on its mortgage.

3.1 Markets and their time structure

Six different markets span each period and across periods. The intra-period markets include the market for commodities, short-term loans, and for central bank loans. The inter-period markets comprise the mortgage and consumer deposit market as well as the market for excess reserves.[15] Figure 3 summarizes the timing of events in each period and across states of nature $s \in S$.

Figure 3: Timeline



3.2 Households α and β

The household problem is akin to the model of mortgage crises of Goodhart et al. (2010). Let there be a set of heterogeneous private sector agents $i \in \{\alpha, \beta\}$. Households hold an initial stock of money $m^i_{s*} \geq 0$ that is free and clear of any debt obligations in each period and each state of nature. The economy is endowed with a durable capital good (housing) and a perishable consumption good (potatoes), both of which are infinitely divisible. Households maximize the sum of present and discounted future utility from consuming both goods. They use their monetary endowment and credit from the bank to trade in

[15] The assumption that the maturity of excess reserves is long-term rather than short-term is not too far from reality. Interest on excess reserves (IOER) with the Fed is paid over a reserve maintenance period of 14 days (Board of Governors of the Federal Reserve System, 2013). It therefore clearly differs in maturity from short-term OMOs which are overnight.

the respective markets for housing and potatoes. There is a need for fiat money as a transaction technology to intermediate goods because the proceeds from selling potatoes cannot be immediately used to buy housing and vice versa. The liquidity friction thus enters in the form of a cash-in-advance constraint. The cash-in-advance constraint in the spirit of Clower (1967) and the existence of a banking sector generate a transaction demand for money (Dubey & Geanakoplos, 1992).

Households differ in their endowment structure but not in their beliefs about the probability γ_s of state s occurring. Household α is endowed only with potatoes e^{α}_{p,s^*} in every $s \in S^*$ while household β is endowed only with housing $e^{\beta}_{h,0}$ at $t = 0$. Both households also have access to intra-period loans $\mu^i_{s^*}$ granted by the bank at a nominal interest rate $r^{ST}_{s^*}$. They both receive an expected share of lump-sum seigniorage transfers t^i_s at the end of the second period.[16]

Household $\boldsymbol{\alpha}$ sells q^{α}_{p,s^*} units of potatoes to agent β and consumes the remainder of her endowment $e^{\alpha}_{p,s^*} - q^{\alpha}_{p,s^*}$ herself. She finances the purchase of $q^{\alpha}_{h,0}$ units of housing with $b^{\alpha}_{h,0}$ units of fiat money by taking out a fixed-rate mortgage with face value $\bar{\mu}^{\alpha}$ from the bank which is fully collateralized by the house. The mortgage has to be repaid at the end of the second period.[17] The fixed interest rate charged by the bank on this long-term debt obligation is $\bar{r}^m$. Household α can also borrow short-term from the bank through an intra-period loan with face value $\mu^{\alpha}_{s^*}$ at interest rate $r^{ST}_{s^*}$ that she repays in full at the end of each period. At the start of the second period, household α can again decide how many units of housing q^{α}_{h,s^*} to buy with b^{α}_{h,s^*} units of fiat money.

At the end of the second period, household α chooses whether she defaults on the mortgage but she has no discretion over the amount that she defaults on. In that sense, default is discontinuous. She repays the mortgage in full if the value of the housing pledged as collateral exceeds the amount of the mortgage plus interest to be repaid, i.e. $p_{h,s} q^{\alpha}_{h,0} \geq \bar{\mu}^{\alpha}$. She defaults on her obligation if the value of housing is lower than the value of the mortgage plus interest repayment. The bank thus receives $\min[\, p_{h,s} q^{\alpha}_{h,0},\, \bar{\mu}^{\alpha}]$. The on-the-verge condition for default depends on the interest rate on the mortgage and prices. Let $S^{\alpha}_g \subset S$ be the set of good states in which agent α does not default on her mortgage. For analytical tractability, households are assumed to have a constant relative risk aversion utility function of the form $u(c^i_s) = (1-\rho)^{-1}(c^i_s)^{1-\rho} : \mathbb{R}_+ \mapsto \mathbb{R},\ \forall s \in S^*,\ i \in \{\alpha, \beta\}$ with the first and second derivatives given by $u'(c^i_s) = (c^i_s)^{-\rho}$, $u''(c^i_s) = (-\rho)(c^i_s)^{-\rho-1}$, and a time discount factor $0 < \boldsymbol{\beta}^{\alpha} \leq 1$. The optimization problem of agent α is thus given by:

$$\begin{aligned}\max_{q^{\alpha}_{p,s^*},\, b^{\alpha}_{h,s^*},\, \mu^{\alpha}_{s^*},\, \bar{\mu}^{\alpha},} U^{\alpha} = \quad & u(\, e^{\alpha}_{p,0} - q^{\alpha}_{p,0}) + u\left(\frac{b^{\alpha}_{h,0}}{p_{h,0}}\right) + \boldsymbol{\beta}^{\alpha} \sum_{s \in S} \gamma_s u(\, e^{\alpha}_{p,s} - q^{\alpha}_{p,s}) \\ & + \boldsymbol{\beta}^{\alpha} \sum_{s \in S_g} \gamma_s u\left(\frac{b^{\alpha}_{h,0}}{p_{h,0}} + \frac{b^{\alpha}_{h,s}}{p_{h,s}}\right) + \boldsymbol{\beta}^{\alpha} \sum_{s \notin S_g} \gamma_s u\left(\frac{b^{\alpha}_{h,s}}{p_{h,s}}\right) \quad ,\end{aligned}$$

whereby the amount of money $b^{\alpha}_{h,0} = p_{h,0} q^{\alpha}_{h,0}$ spent on housing at the beginning of period $t = 0$ cannot exceed the sources of funds, i.e. the intra-period loan, the long-term

[16] The timing of the seigniorage distribution matters crucially for price level determinacy (see Bloise and Polemarchakis (2006) and McMahon et al. (2018)).

[17] If we are willing to assume that mortgages have an adjustable rate rather than a fixed rate, we could discount the mortgage over both periods and allow the bank to adjust the interest rate on the mortgage in the second period.

mortgage, and the initial private monetary endowment,

$$b^{\alpha}_{h,0} \leq \frac{\mu^{\alpha}_0}{(1+r^{ST}_0)} + \frac{\bar{\mu}^{\alpha}}{(1+\bar{r}^m)} + m^{\alpha}_0 \ , \tag{Λ^{α}_1}$$

whereby Λ^{h}'s are the corresponding Lagrange multipliers. Agent α cannot take out more short-term debt than she is able to repay with the proceeds of the sale of potatoes at the end of the period, independently of whether she defaults on the mortgage or not. Hence,

$$\mu^{\alpha}_0 \leq p_{p,0} q^{\alpha}_{p,0} \ . \tag{Λ^{α}_2}$$

(a) If there is **no mortgage default**, agent α spends additional money on housing in $t = 1$ and repays the mortgage at the end of the period. These expenses cannot exceed the funds available from the intra-period loan, the private monetary endowment, and the lump-sum government transfer in each state of nature,

$$b^{\alpha}_{h,s} + \bar{\mu}^{\alpha} \leq \frac{\mu^{\alpha}_s}{(1+r^{ST}_s)} + m^{\alpha}_s + t^{\alpha}_s \quad , \forall s \in S^{\alpha}_g \ , \tag{Λ^{α}_3}$$

facing again the cash-in-advance constraint

$$\mu^{\alpha}_s \leq p_{p,s} q^{\alpha}_{p,s} \quad , \forall s \in S^{\alpha}_g \ . \tag{Λ^{α}_4}$$

(b) If there is **mortgage default**, money spent on additional housing at $t = 2$ cannot exceed the funds available from the intra-period loan, the private monetary endowment, and the lump-sum government transfer in each state of nature

$$b^{\alpha}_{h,s} \leq \frac{\mu^{\alpha}_s}{(1+r^{ST}_s)} + m^{\alpha}_s + t^{\alpha}_s \quad , \forall s \notin S^{\alpha}_g \ , \tag{Λ^{α}_5}$$

facing again the cash-in-advance constraint

$$\mu^{\alpha}_s \leq p_{p,s} q^{\alpha}_{p,s} \quad , \forall s \notin S^{\alpha}_g \ . \tag{Λ^{α}_6}$$

Also, there are no short sales of goods allowed in any period and any state, thus

$$q^{\alpha}_{p,s^*} \leq e^{\alpha}_{p,s^*} \quad , \forall s \in S^*. \tag{Λ^{α}_7}$$

Budget constraints (Λ^{α}_1) - (Λ^{α}_6) for agent α as well as for the other agents are all binding because we consider an economy with fiat money. Money does not enter the utility function but only the set of affordable allocations. Thus, agents do not derive utility from holding idle cash. They either spend it or lend it out, provided that interest rates are positive.

Agent $\boldsymbol{\beta}$'s optimization problem differs by virtue of the endowment structure. Since he is only endowed with $e^{\beta}_{h,0}$ units of the durable good in the first period, he sells q^{β}_{h,s^*} units of housing in every period and in every state of nature to buy potatoes for consumption with $b^{\beta}_{p,s*}$ units of fiat money from agent α in the goods market. We assume that the economy is relatively more endowed with potatoes than with housing. The relative price for potatoes versus housing is low, making it relatively inexpensive for agent β to buy potatoes. He thus has an incentive to deposit the sales receipts, in units of fiat money, in a consumer deposit account with the bank in period $t = 0$ to be used for consumption at $t = 1$. Deposits $\bar{d}^{\beta}$ and accrued interest at the nominal deposit rate $\bar{r}^{d}$ are only settled at the end of the second period before consumption takes place. Agent β also has access to the short-term money market. He can take out an intra-period loan μ^{β}_{s*} at the same rate r^{ST}_{s*} as agent α. This gives rise to the following optimization problem for agent β:

$$\begin{aligned}\max_{q^{\beta}_{h,s*},\, b^{\beta}_{p,s*},\, \bar{d}^{\beta},\, \mu^{\beta}_{s*}} U^{\beta} = \quad & u\left(\frac{b^{\beta}_{p,0}}{p_{p,0}}\right) + u(\, e^{\beta}_{h,0} - q^{\beta}_{h,0}) \\ & + \boldsymbol{\beta}^{\beta} \sum_{s\in S} \gamma_s \left\{ u\left(\frac{b^{\beta}_{p,s}}{p_{p,s}}\right) + u(\, e^{\beta}_{h,0} - q^{\beta}_{h,0} - q^{\beta}_{h,s}) \right\} \quad ,\end{aligned}$$

whereby the amount of money spent on potatoes at the beginning of period $t = 0$ and the funds deposited long-term cannot exceed the sources of funds, i.e. the intra-period loan, and the initial private monetary endowment,

$$b^{\beta}_{p,0} + \bar{d}^{\beta} \leq \frac{\mu^{\beta}_{0}}{(\,1 + r^{ST}_{0})} + m^{\beta}_{0} \ . \tag{Λ^{β}_{1}}$$

Agent β cannot take out more short-term debt than she is able to repay with the proceeds of the sale of housing at the end of the period

$$\mu^{\beta}_{0} \leq p_{h,0} q^{\beta}_{h,0} \ . \tag{Λ^{β}_{2}}$$

The amount of money spent on potatoes at the beginning of period $t = 1$ cannot exceed the sources of funds, i.e. the intra-period loan, the deposits plus accrued interest that the bank repays, the private monetary endowment, and the seigniorage transfer in all states of nature

$$b^{\beta}_{p,s} \leq \frac{\mu^{\beta}_{s}}{(\,1 + r^{ST}_{s})} + \bar{d}^{\beta}(\,1 + \bar{r}_{d}) + m^{\beta}_{s} + t^{\beta}_{s} \quad , \forall s \in S \ , \tag{Λ^{β}_{3}}$$

facing again the cash-in-advance constraint

$$\mu^{\beta}_{s} \leq p_{h,s} q^{\beta}_{h,s} \quad , \forall s \in S^{*} \ . \tag{Λ^{β}_{4}}$$

Note that there are no short sales of goods allowed in any period or any state

$$q_{h,0} \leq e_{h,0} \tag{Λ^{β}_{5}}$$

$$q_{h,s} \leq e_{h,0} - q_{h,0} \quad , \forall s \in S \ . \tag{Λ^{β}_{6}}$$

2.3 Bank ϕ

There is one commercial bank ϕ that operates in the market for long-term consumer deposits, $\bar{\mu}_d^{\phi}$, intra-period loans to households, $m_{s^*}^{ST}$, and long-term collateralized mortgages, $\bar{m}^{LT}$. To balance the difference in assets in liabilities at the aggregate level, bank ϕ also obtains funding in the form of an intra-period loan with face value $\mu_{s^*}^{\phi}$ granted by the central bank. It pays an interest rate ρ_{s^*} on the intra-period loan. Bank ϕ also deposits excess reserves $\bar{d}^{\phi}$ at the central bank from the first to the second period.[18] Since mortgage default makes investing in mortgages inherently risky, the excess reserves are the only risk-free long-term investment opportunity that the bank has available. Other reasons that induce the bank to hold excess reserves rather than lend it out are described in section 2. Bank ϕ earns an interest rate $\bar{r}^{CB}$ on excess reserves which we denote the IOER.

Bank ϕ is also endowed with an amount $e_{s^*}^{\phi}$ of equity capital in all states of nature that is denominated in fiat money and offers zero return. The equity may absorb part of the losses from mortgage default by agent α. We implicitly assume that the market for equity is exogenous such that endowments are in positive net supply and the bank cannot raise more equity.[19] The central bank imposes a capital requirement CAR^* on the ratio of equity to risk-weighted assets that bank ϕ has to hold in the first period. Since the amount of equity is fixed, the bank has to adjust its risk-weighted assets to fulfil the capital requirement. Risk weights $\{w_{ST}, w_{LT}\}$ are placed on short-term credit and on long-term mortgages.[20] If the bank fails to comply with the capital requirement, it incurs a default penalty λ_0^{ϕ} that is proportional to the discrepancy between the required and the realized capital ratio, $(CAR^* - CAR_0)$. However, in our numerical analysis, we only consider cases where the bank does not violate the capital requirement and $CAR^* = CAR_0$ since our focus is on the departure from the ZLB. Finally, for the sake of simplicity, banks are not allowed to default. Table 1 summarizes the aggregate assets and liabilities held by bank ϕ and the respective nominal interest rates received and paid on these balance sheet items. Figure 4 presents an overview of the different maturities of balance sheet items.

Table 1: Bank ϕ's stylized balance sheet

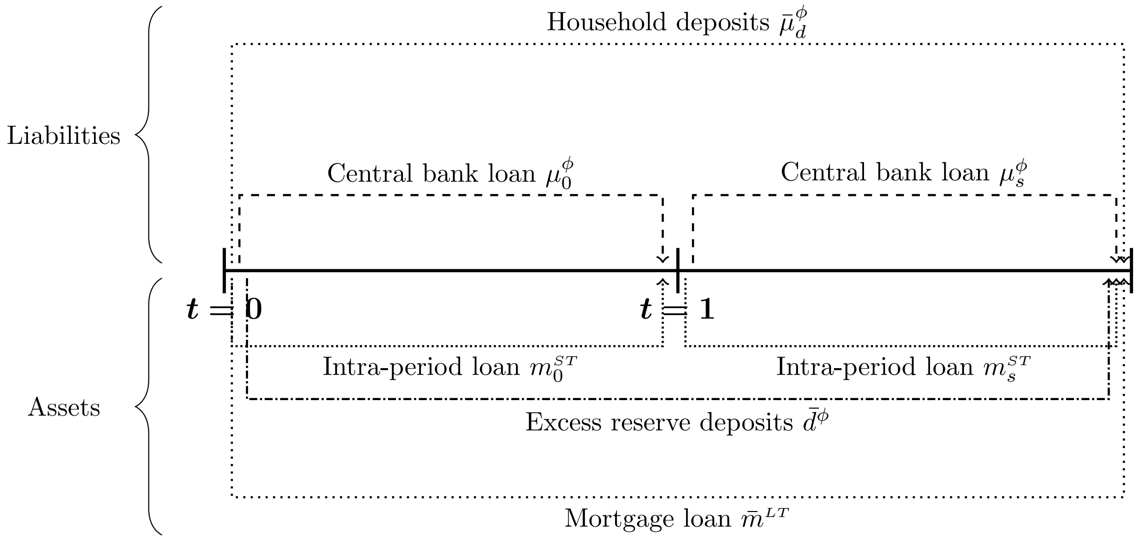
Assets	Interest rate	Liabilities	Interest rate
Excess reserves with CB $\bar{d}^{\phi}$	$\bar{r}^{CB}$	Consumer deposits $\bar{\mu}_d^{\phi}$	$\bar{r}^d$
ST loans $m_{s^*}^{ST}$	$r_{s^*}^{ST}$	Central bank loan $\mu_{s^*}^{\phi}$	ρ_{s^*}
LT mortgage $\bar{m}^{LT}$	$\bar{r}^m$	Equity $e_{s^*}^{\phi}$	

[18] We distinguish between inter-period deposits and intra-period borrowing from the central bank in our model in order to emphasize the various short-term and long-term sources of funding that a commercial bank has.

[19] One may think of the equity endowment as a form of outside money that is free and clear of any debt obligations, for example, because it is income earned from overseas subsidiaries outside our model.

[20] Although short-term loans in our model are non-defaultable and hence bear no risk as opposed to mortgages, we place a positive risk weight on short-term credit extended to households to prevent an inflation of short-term credit by the bank.

Figure 4: Maturities of the Balance Sheet of bank ϕ



Bank ϕ maximizes the sum of expected discounted profits $\Pi_{s^*}^{\phi}$ by implicitly earning a credit spread on its lending and borrowing activities in both periods. We can think of $\bar{r}_s^m - \bar{r}^d$ as the spread of private sector long-term loans (i.e. mortgages) over deposits that the bank earns. Similarly, $(1+\rho_0)(1+\rho_{s^*}) - (1+\bar{r}^{CB})$ can be seen as the premium paid by the bank on borrowing from the central bank over depositing excess reserves with it over both periods. The term structure is endogenously determined and driven by liquidity premia. First, the bank pays weakly more on borrowing from the central bank than it earns on lending to it.[21] Second, the bank earns at least as much interest on its assets as it pays on its liabilities in order to stay solvent. Third, long-term interest rates are weakly higher than short-term interest rates due to a liquidity risk premium. These conditions are summarized as:

$$\bar{r}^{CB} \leq \rho_{s^*} \leq r_{s^*}^{ST} \leq \bar{r}^m$$

Finally, long-term interest rates on lending must be at least as high as long-term interest rates on borrowing from households:

$$\bar{r}_d \leq \bar{r}^m$$

In summary, the nominal spread of interest earned on assets over liabilities is non-negative to induce the bank to provide credit to the private sector.

Bank ϕ has a quadratic objective function (i.e. mean-variance preferences), parameterized by a^{ϕ}, to reflect the nature of the bank's risk aversion and the benefits of diversification stemming from the bank's portfolio choice problem.[22] The regulatory

[21] Otherwise the bank could earn a profit by borrowing from the central bank and depositing those funds at the same time. The central bank has an incentive to keep the bank from hoarding cash, so it will set the deposit rate below the lending rate

[22] Although the theoretical literature often assumes banks to be risk neutral, we believe that real-world behavior by systemically important banks displays substantially more risk aversion than risk-neutral preferences would generate. From a practical point of view, using quadratic utility yields analytically

penalty λ_0^ϕ enters linearly. Moreover, beliefs about the probability of the different states of nature are the same across all agents. We distinguish between two different cases: the set of good states $S_g = S_g^\alpha \subset S$ in which agent α does not default on her collateralized debt and the set of bad states $S_b = \{s \in S \,|\, s \notin S_g\}$ in which agent α defaults. The bank has the following optimization problem subject to its balance sheet constraints:

$$\max_{m_{s*}^{ST},\, \bar{m}^{LT},\, \bar{d}^\phi,\, \bar{\mu}_d^\phi,\, \mu_{s*}^\phi} \mathbb{P}^\phi = \quad [\,\Pi_0^\phi - a^\phi(\,\Pi_0^\phi)^{\,2}] - \lambda_0^\phi \max[\,0,\, CAR^* - CAR_0\,]$$
$$+ \; \boldsymbol{\beta}^\phi \sum_{s \in S} \gamma_s [\,\Pi_s^\phi - a^\phi(\,\Pi_s^\phi)^{\,2}] \quad ,$$

whereby in period $\boldsymbol{t = 0}$ total short term credit and long-term credit extended to households as well as excess reserves that the bank decides to hold with the CB must be financed by consumer deposits, central bank loans, and equity capital,

$$m_0^{ST} + \bar{m}^{LT} + \bar{d}^\phi \leq \frac{\bar{\mu}_d^\phi}{(\,1 + \bar{r}^d)} + \frac{\mu_0^\phi}{(\,1 + \rho_0)} + e_0^\phi \,. \tag{Λ_1^ϕ}$$

The repayment to the central bank at the end of the period must be fully covered by the receipt of short-term loans plus interest from households

$$\mu_0^\phi \leq m_0^{ST}(\,1 + r_0^{ST}) \;\;, \tag{Λ_2^ϕ}$$

and the difference accrues as bank profits,

$$\Pi_0^\phi \leq \Delta(\Lambda_2^\phi) = m_0^{ST}(\,1 + r_0^{ST}) \,-\, \mu_0^\phi \,. \tag{Λ_3^ϕ}$$

The bank's capital ratio can be no less than the regulatory capital ratio CAR^*,

$$CAR^* \leq CAR_0 = \frac{e_0^\phi}{w^{ST}(\sigma) m_0^{ST} + w^{LT}(\sigma)\bar{m}^{LT}} \;, \tag{Λ_4^ϕ}$$

where σ summarizes future macroeconomic factors affecting the economy at $t = 1$. Note that we will only consider cases where $CAR^* = CAR_0$ so that constraint (Λ_4^ϕ) is binding and no penalty is incurred.

(a) If there is **no collateral default** by agent α at the end of period $\boldsymbol{t = 1}$ in the good states $s \in S_g$,

$$\bar{m}^{LT}(\,1 + \bar{r}^m) \,\leq p_{h,s} q_{h,0}^\alpha \;,$$

then, total short-term credit extended to households and consumer deposits paid back to agent β must be financed by a central bank loan, the repayment of excess reserves plus

tractable solutions. The convexity of the optimization problem allows us to consider interior solutions. Moreover, concave production functions may be thought of as approximations of the limited liability legal clause that holds for banks.

interest from the central bank, the effective repayment of the mortgage plus interest, and equity,

$$m_s^{ST} + \bar{\mu}_d^{\phi} \leq \frac{\mu_s^{\phi}}{(1+\rho_s)} + \bar{d}^{\phi}(1+\bar{r}^{CB}) + \bar{m}^{LT}(1+\bar{r}_s^{m}) + e_s^{\phi} \quad ,\forall s \in S_g \; . \tag{Λ_5^{ϕ}}$$

The repayment of the loan to the central bank must be fully covered by the receipt of short-term loans plus interest from households,

$$\mu_s^{\phi} \leq m_s^{ST}(1+r_s^{ST}) \; , \tag{Λ_6^{ϕ}}$$

and the difference accrues as bank profits at the end of the final period

$$\Pi_s^{\phi} \leq \Delta(\Lambda_6^{\phi}) = m_s^{ST}(1+r_s^{ST}) - \mu_s^{\phi} \quad ,\forall s \in S_g \; . \tag{Λ_7^{ϕ}}$$

(b) If there is **collateral default** by agent α in period $\boldsymbol{t=1}$ in the bad states $s \notin S_g$,

$$p_{h,s}q_{h,0}^{\alpha} \leq \bar{m}^{LT}(1+\bar{r}^{m}) \; ,$$

then again, total short term credit extended to households and consumer deposits paid back to agent β must be financed by central bank loans, the repayment of excess reserves plus interest from the central bank, the effective repayment of the mortgage plus interest, and equity,

$$m_s^{ST} + \bar{\mu}_d^{\phi} \leq \frac{\mu_s^{\phi}}{(1+\rho_s)} + \bar{d}^{\phi}(1+\bar{r}^{CB}) + \bar{m}^{LT}(1+\bar{r}_s^{m}) + e_s^{\phi} \quad ,\forall s \notin S_g \; . \tag{Λ_8^{ϕ}}$$

Note, however, that the interest rate on the mortgage in this case reflects the *realized* interest rate, $\bar{r}_s^m$ for $s \notin S_g$, after taking mortgage default into account. Bank ϕ does not get repaid the mortgage plus interest but instead seizes the housing collateral and resells it immediately at the prevailing market price.

Again, the repayment of the loan to the central bank must be fully covered by the receipt of short-term loans plus interest from households,

$$\mu_s^{\phi} \leq m_s^{ST}(1+r_s^{ST}) \quad ,\forall s \notin S_g \; , \tag{Λ_9^{ϕ}}$$

and the difference accrues as bank profits at the end of the final period,

$$\Pi_s^{\phi} \leq \Delta(\Lambda_9^{\phi}) = m_s^{ST}(1+r_s^{ST}) - \mu_s^{\phi} \quad ,\forall s \notin S_g \; . \tag{Λ_{10}^{ϕ}}$$

2.4 Central Bank

There is a central bank in the economy who takes excess reserve deposits D^{CB} from bank ϕ at the reserve rate $\bar{r}^{CB}$. It also grants intra-period loans to bank ϕ at the policy rate, ρ_{s^*}. It has full discretion over setting the future policy rate, ρ_s, in period $t = 1$ in order to maximize its objective function. However, we set both the nominal interest rate on excess reserves (the IOER equivalent), $\bar{r}^{CB}$, and the initial policy rate, ρ_0, exogenously to "impose" the ZLB onto the economy ex ante. We do so because the reasons for the economy to be at the ZLB are outside the scope of our model but we are interested in equilibria at the ZLB. The model will be calibrated for two scenarios, one where the economy starts off at $\bar{r}^{CB} = 0.0\%$ and $\rho_0 = 1\%$ at the ZLB and one where the IOER rate departs from the ZLB. The premium paid by bank ϕ for borrowing from the central bank above and beyond what it earns on its excess reserve deposits can be interpreted as the income $w_{s^*}^{CB}$ that the central bank earns on its notes and coins in circulation. The central bank income $w_{s^*}^{CB}$ is distributed to households at the end of the second period. Lastly, the central bank is also endowed with equity capital $m_{s^*}^{CB}$ in each state of nature. Equity capital is free and clear of any debt obligations, i.e. a form of outside money. An overview of the central bank's balance sheet is given in Table 2.

Table 2: The central bank's stylized balance sheet

Assets	Interest rate	Liabilities	Interest rate
Loans to the bank $M_{s^*}^{CB}$	ρ_{s^*}	Excess reserve deposits D^{CB}	$\bar{r}^{CB}$
		Endowment m_s^{CB}	

The central bank's prime objectives combine the classical goals of monetary policy – stable inflation and output – with the goal of maintaining financial stability. In search for the simplest and most micro-founded objective function of the central bank, we augment the canonical welfare loss function in (1) - (2) by two financial stability objectives in the spirit of Kotak et al. (2017): maximizing profits of the financial sector while minimizing the aggregate loss due to default.

The real output gap is measured as the deviation of the current level of GDP from the "natural level of GDP". The natural level of GDP, or potential output, is reached when all factors of production are utilized efficiently to their full capacity. We proxy the current level of GDP by the total real volume of trades in the durable and perishable goods markets in the economy. To obtain a measure of the GDP level under full capacity utilization in a model without production, we consider a version of the model in which trade in the economy is frictionless. In fact, money is the main friction in our model. By removing money and the cash-in-advance constraint from the model, we reduce the economy to a frictionless barter economy. Dubey and Geanakoplos (2003a) show that such a barter economy achieves the Pareto optimal outcome when there are gains to trade. By contrast, the introduction of money and a banking sector creates an inefficiency.[23] The output gap in our model can therefore be interpreted as the distance to the Pareto optimal outcome caused by monetary frictions. Details are summarized in Appendix A.

[23]The existence of trade in fiat money and positive interest rates creates a wedge between buying and

The central bank also cares about the health of the banking sector and the repayment of claims. It therefore has a macroprudential policy tool in addition to its monetary policy instrument, the policy rate ρ_s, at its disposal. The central bank controls the capital requirement CAR^* that bank ϕ has to achieve in the first period in order to avoid a violation penalty. It thus complies with the "Tinbergen Principle" of employing as many policy instruments as targets exist. While the capital requirement entrusts the central bank with power over regulating the size of risky mortgage extension and short-term credit, it does not give it the power to directly control collateral default on mortgages. The latter could be controlled through strict loan-to-value ratios which we exclude for now to keep the number of equations manageable.

Each objective $o \in \{\phi, \lambda, \pi, y\}$ of financial stability and monetary policy, respectively, bears a relative weight θ_o in the objective function that will be varied in the simulation. The goal of this exercise is to understand how the relative weighing of monetary policy versus financial stability goals affects liquidity, default, quantities traded, and prices in the economy. At the start of the first period, the central bank maximizes total expected future "welfare", W^{CB}, in the second period, where all terms enter linearly for convenience. It does so by choosing the future nominal interest rate ρ_s on central bank loans as well as the capital requirement CAR^*:

$$\begin{aligned}\max_{\rho_s, CAR^*} W^{CB} = \;& \overbrace{\frac{1}{2}\theta_\phi \sum_{s\in S} \gamma_s \Pi_s^\phi}^{(1)} - \overbrace{\frac{1}{2}\theta_\lambda \sum_{s\in S} \gamma_s (\max[\,\bar{\mu}^\alpha - p_{h,s} q_{h,0}^\alpha\,, 0\,]\,)^2}^{(2)} \\ & - \underbrace{\frac{1}{2}\theta_\pi \sum_{s\in S} \gamma_s (\,\pi_s - \pi^*)^{\,2}}_{(3)} - \underbrace{\frac{1}{2}\theta_y \sum_{s\in S} \gamma_s (\,y_s - y^*)^{\,2}}_{(4)} \end{aligned} \qquad (\Lambda_1^{CB})$$

Terms (1) and (2) in the equation represent the financial stability objectives, i.e. bank profitability and the aggregate loss due to default in the second period, respectively. Terms (3) and (4) represent the monetary policy objectives, inflation targeting and the output gap, which correspond to the loss function in (2). The variables in these four terms comprise:

(1) Bank ϕ's profit function Π_s^ϕ given by (Λ_3^ϕ), and (Λ_7^ϕ).

(2) The loss from default on the mortgage by household α, $\max[\,\bar{\mu}^\alpha - p_{h,s} q_{h,0}^\alpha\,, 0\,]$ for $s \notin S_g$.

(3) The target rate of inflation $\pi^* = 2\,\%$ as adopted by many central banks and the expected rate of inflation defined by the change in the aggregate commodity price index, p_s, from the first to the second period,[24]

$$\pi_s = p_s/p_0 - 1 \quad \forall s \in S\ .$$

selling prices which hinders trade in commodities. Trade only takes place if gains to trade are sufficiently large.

[24] See Section 5.3 for a definition of the price index.

(4) Real GDP y_s, defined as the total real volume of trade in the durable and perishable goods markets in the economy in a given state,

$$y_s = \sum_{i\in\{\alpha,\beta\}} \left[q^i_{p,s} + q^i_{h,s} \right] = \sum_{i\in\{\alpha,\beta\}} \left[\frac{b^i_{p,s}}{p_{p,s}} + \frac{b^i_{h,s}}{p_{h,s}} \right] \quad \forall s \in S^*.$$

We assume that the central bank is constrained by its balance sheet because it cares about its profitability (Goncharov et al., 2017) and its solvency (Reis, 2013). It can only give out as many loans to bank ϕ on the asset side as it can finance through excess reserve deposits and its monetary endowment on the liability side at the beginning of $t = 0$:

$$M_0^{CB} \leq \frac{D^{CB}}{1 + r^{CB}} + m_0^{CB} \; . \tag{Λ_2^{CB}}$$

Central bank income at the end of period $t = 0$ equals the repayment of the short-term central bank loan including interest and the funds that remained idle on the balance sheet at the start of the period,

$$w_0^{CB} = M_0^{CB}(1 + \rho_0) + \Delta(\Lambda_2^{CB}) \; . \tag{Λ_3^{CB}}$$

The loan to bank ϕ in the next period is financed by central bank income carried over from the previous period and the monetary endowment. Hence,

$$\begin{aligned} & M_s^{CB} \leq w_0^{CB} + m_s^{CB} \quad \forall s \in S \\ \Rightarrow \quad & M_s^{CB} \leq \rho_0 M_0^{CB} + \frac{D^{CB}}{1 + \bar{r}^{CB}} + m_0^{CB} + m_s^{CB} \; . \end{aligned} \tag{Λ_4^{CB}}$$

At the end of the second period, excess reserves D^{CB} must be paid back with the repayment of the central bank loan plus interest and the funds that remained idle on the balance sheet at the start of period $t = 1$,

$$\begin{aligned} & D^{CB} \leq \quad M_s^{CB}(1 + \rho_s) + \Delta(\Lambda_4^{CB}) \\ \Rightarrow \quad & D^{CB} \leq \quad \rho_s M_s^{CB} + \rho_0 M_0^{CB} + \frac{D^{CB}}{1 + \bar{r}^{CB}} + m_0^{CB} + m_s^{CB} \quad \forall s \in S \; . \end{aligned} \tag{Λ_5^{CB}}$$

All remaining funds after repayment of excess reserves to the bank accrue as central bank income at the end of period $t = 1$:

$$\begin{aligned} & w_s^{CB} = \Delta(\Lambda_5^{CB}) \geq 0 \quad \forall s \in S \quad \text{and} \\ & w_s^{CB} = \rho_s M_s^{CB} + \rho_0 M_0^{CB} + m_0^{CB} + m_s^{CB} - \frac{\bar{r}^{CB}}{1 + \bar{r}^{CB}} D^{CB} \quad \forall s \in S \; . \end{aligned} \tag{Λ_6^{CB}}$$

Essentially, the interest that bank ϕ has to pay on central bank loans in period $t = 0$ over what it receives on its excess reserves is retained by the central bank as seigniorage income. A fraction x of the remaining seigniorage income w_s^{CB} is distributed as a lump-sum transfer t_s^i to households $i \in \{\alpha, \beta\}$ according to some predetermined share κ:[25]

$$x\, w_s^{CB} = t_s^\alpha + t_s^\beta \quad \forall s \in S \;, \tag{Λ_7^{CB}}$$

where $t_s^\alpha = \kappa\, x\, w_s^{CB}$ and $t_s^\beta = (\,1-\kappa)\, x\, w_s^{CB}$. Households take the lump-sum transfer as given.

2.5 Market Clearing

Six markets in our model must clear in order for prices to be uniquely determined: the market for the consumption good, housing, intra-period loans, mortgages, excess reserves, consumer deposits, and central bank loans.

2.5.1 Goods market

The market for the perishable good, potatoes, clears when the amount of fiat money offered for the good is exchanged for the quantity of goods sold,

$$p_{p,0} = \frac{b_{p,0}^\beta}{q_{p,0}^\alpha} \quad \text{and} \tag{MC_1}$$

$$p_{p,s} = \frac{b_{p,s}^\beta}{q_{p,s}^\alpha} \quad \forall s \in S. \tag{MC_2}$$

2.5.2 Housing market

The market for the durable good, housing, clears when the amount of fiat money offered for housing is exchanged for the quantity of housing sold,

$$p_{h,0} = \frac{b_{h,0}^\alpha}{q_{h,0}^\beta} \;, \tag{MC_3}$$

$$p_{h,s} = \frac{b_{h,s}^\alpha}{q_{h,s}^\beta} \quad \forall s \in S_g \;, \tag{MC_4}$$

$$p_{h,s} = \frac{b_{h,s}^\alpha}{q_{h,s}^\beta + C} \quad \forall s \notin S_g \;. \tag{MC_5}$$

where $C = b_{h,0}^\alpha / p_{h,0}$ is the amount of housing collateral pledged per unit of the mortgage that will be seized and resold by bank ϕ in the market if the borrower defaults at $t = 1$.

[25] The central bank does not redistribute the full amount of its income to households since this would render the price level indeterminate (see Bloise & Polemarchakis, 2006).

2.5.3 Intra-period loan market

The market for intra-period loans clears when the amount of short-term loans demanded by households is exchanged for the amount of short-term credit offered by the bank,

$$1 + r_0^{ST} = \frac{\mu_0^\alpha + \mu_0^\beta}{m_0^{ST}} \quad \text{and} \tag{MC_6}$$

$$1 + r_s^{ST} = \frac{\mu_s^\alpha + \mu_s^\beta}{m_s^{ST}} \quad \forall s \in S. \tag{MC_7}$$

2.5.4 Mortgage market

The market for mortgages clears when the amount of long-term mortgages demanded by households is exchanged for the amount of long-term credit offered by the bank,

$$1 + \bar{r}^m = \frac{\bar{\mu}^\alpha}{\bar{m}^{LT}} \ .$$

However, because we allow for collateral default, the *ex ante* return on the mortgage does not equal its effective return in the bad states of nature $s \notin S_g$. The effective return depends on the minimal payoff of the mortgage $\min[\, p_{h,s} q_{h,0}^\alpha,\, \bar{\mu}^\alpha] = \min[\, p_{h,s} \frac{b_{h,0}^\alpha}{p_{h,0}},\, \bar{\mu}^\alpha]$. Recall that S_g is the set of good states in which agent α does not default on the mortgage since $p_{h,s} q_{h,0}^\alpha \geq \bar{\mu}^\alpha$. The effective return is thus given by:

$$1 + \bar{r}_{s,m} = 1 + \bar{r}^m \quad , \forall s \in S_g \quad \text{and} \tag{MC_8}$$

$$1 + \bar{r}_{s,m} = (\, 1 + \bar{r}^m) \, \frac{p_{h,s} \frac{b_{h,0}^\alpha}{p_{h,0}}}{\bar{\mu}^\alpha} \quad , \forall s \notin S_g \ , \tag{MC_9}$$

where in the bad states the effective return is determined by the ratio of the salvaged value of the collateral plus interest over the total face value of the mortgage to be repaid.

2.5.5 Excess reserve market

The market for excess reserves clears when the amount of excess reserves supplied by the bank is exchanged for the "demand" of excess reserve deposits by the central bank,

$$1 + \bar{r}^{CB} = \frac{D^{CB}}{\bar{d}^\phi} \quad , \forall s \in S \ . \tag{MC_{10}}$$

2.5.6 Consumer deposit market

The market for consumer deposits clears when the amount of long-term credit demanded by the bank is exchanged for the amount of long-term credit offered by households,

$$1 + \bar{r}_d = \frac{\bar{\mu}_d^\phi}{\bar{d}^\beta} \ . \tag{MC_{11}}$$

2.5.7 Central bank loan market

The market for loans from the central bank clears when the amount of short-term credit demanded by the bank is exchanged for the amount of short-term credit offered by the central bank

$$1+\rho_0 = \frac{\mu_0^{\phi}}{M_0} \quad \text{and} \tag{MC_{12}}$$

$$1+\rho_s = \frac{\mu_s^{\phi}}{M_s} \quad , \forall s \in S \tag{MC_{13}}$$

2.5.8 Seignorage transfers

Finally, the market for seigniorage transfers consists of the amount of seigniorage income earned by the central bank in the second period and is distributed as lump-sum transfers to households according to their shares,

$$x\, w_s^{CB} = t_s^{\alpha} + t_s^{\beta} \quad , \forall s \in S\ . \tag{MC_{14}}$$

where $t_s^{\alpha} = \kappa\, x\, w_s^{CB}$ and $t_s^{\beta} = (\,1-\kappa)\ x\, w_s^{CB}$.

3 Definition of Equilibrium

Let the consumption and investment plans $\boldsymbol{\sigma}^j$, i.e. vectors of decision variables, for each agent $j \in \{\alpha, \beta, \phi, CB\}$ be defined over an n-dimensional space $\mathbb{R}^n$, where

$$\begin{aligned}
\boldsymbol{\sigma}^{\alpha} \ &= \{q_{p,s}^{\alpha}, b_{h,s}^{\alpha}, \mu_s^{\alpha}, \bar{\mu}^{\alpha}\} \quad \in \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+\ , \\
\boldsymbol{\sigma}^{\beta} \ &= \{q_{h,s}^{\beta}, b_{p,s}^{\beta}, \mu_s^{\beta}, \bar{d}^{\beta}\} \quad \in \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+\ , \\
\boldsymbol{\sigma}^{\phi} \ &= \{\Pi_s^{\phi}, m_s^{ST}, \bar{m}^{LT}, \bar{d}^{\phi}, \mu_s^{\phi}, \bar{\mu}_d^{\phi}\} \quad \in \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+\ , \quad \text{and} \\
\boldsymbol{\sigma}^{CB} &= \{\rho_s, CAR^*\} \quad \in \mathbb{R}_+^{s} \times \mathbb{R}_+^{s}\ .
\end{aligned}$$

Let $\boldsymbol{\eta} = \{p_{h,s}, p_{p,s}, r_s^{ST}, \bar{r}^m, \bar{r}^d, \bar{r}^{CB}, \rho_s\} \in \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+^{s^*} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^{s^*}$ denote the set of prices in the macroeconomy which are determined in equilibrium and which households and the bank take as given in their decision problem. Furthermore, let agents' budget sets be given by

$$\begin{aligned}
B^{\alpha}(\boldsymbol{\eta}) \ &= \{\sigma^{\alpha} : (\Lambda_1^{\alpha}) - (\Lambda_7^{\alpha}) \text{ hold}\}\ , \\
B^{\beta}(\boldsymbol{\eta}) \ &= \{\sigma^{\beta} : (\Lambda_1^{\beta}) - (\Lambda_6^{\beta}) \text{ hold}\}\ , \\
B^{\phi}(\boldsymbol{\eta}) \ &= \{\sigma^{\phi} : (\Lambda_1^{\phi}) - (\Lambda_{10}^{\phi}) \text{ hold}\}\ , \quad \text{and} \\
B^{CB}(\boldsymbol{\eta}) &= \{\sigma^{CB} : (\Lambda_2^{CB}) - (\Lambda_6^{CB}) \text{ hold}\}\ .
\end{aligned}$$

We say that the set of allocations and prices $(\,\boldsymbol{\sigma}^j, \boldsymbol{\eta}\,)$ is a *monetary General Equilibrium with incomplete markets (GEI), collateral, banks, and default under optimal monetary policy and macroprudential policy* iff:

(i) $\boldsymbol{\sigma}^i \in \operatorname{Arg\,max}_{\boldsymbol{\sigma}^i \in B^i(\boldsymbol{\eta})} \Pi^i \quad , i \in \{\alpha, \beta\} \quad ,$

(ii) $\boldsymbol{\sigma}^\phi \in \operatorname{Arg\,max}_{\boldsymbol{\sigma}^\phi \in B^\phi(\boldsymbol{\eta})} \Pi^\phi \quad ,$ and

(iii) $\boldsymbol{\sigma}^{CB} \in \operatorname{Arg\,max}_{\boldsymbol{\sigma}^{CB} \in B^{CB}(\boldsymbol{\eta})} \Pi^{CB} \quad ,$

that is, all households, the bank, and the central bank optimize by choosing an optimal allocation within the set of feasible allocations given their constraints, and

(iv) all markets in (MC_1) - (MC_{14}) clear.

Under conditions (i) - (iv), the equilibrium is characterized by rational expectations, competitive markets and market clearing. Given their information set and their budget set, agents rationally anticipate current and future prices, interest rates, and repayment rates in choosing their optimal allocation. A formal proof of the existence of an equilibrium follows, *mutatis mutandis*, from Dubey and Geanakoplos (2003a), Geanakoplos and Zame (2014), and Tsomocos (2003).

4 Characterization of the Equilibrium

Before presenting a numerical solution to a partial equilibrium and comparative statics, we first discuss the fundamental properties of our model in equilibrium. Hence, we characterize the monetary policy transmission channel, the coordination of monetary policy and macroprudential policy, as well as the way prices are determined.

To gauge how a lift-off of the nominal interest rate on excess reserves will affect equilibrium prices, allocations, and default, it is pivotal to examine how interest rate changes by the central bank affect interest rates that the financial sector charges. Propositions 1 and 2 establish the transmission of monetary policy to the real economy.

Proposition 1 (Monetary Policy Transmission Mechanism: Channel 1)**.** *In equilibrium, in every future state of nature $s \in S$, in the absence of default by households on short-term loans, the interest rate paid by households on short-term loans must equal the interest rate paid by the bank on the short-term loan with the central bank,*

$$r_s^{ST} = \rho_s \quad \forall s \in S\ .$$

Therefore, any change in the monetary policy rate will have a direct effect on households' budget constraints and consumption at $t = 1$.

The proof of Proposition 1 follows directly from the optimality conditions of bank ϕ given in Appendix C. It is straightforward why Proposition 1 holds only for the transmission at $t = 1$ but not in the first period. At $t = 0$, bank ϕ faces a "cost" to extending additional short-term credit in the form of a capital requirement. In order to extend more short-term credit while fulfilling the requirement, the bank would need to lower the size of mortgage loans extended. This additional cost draws a wedge between the interest rate that it pays and the interest rate that it demands on short-term credit at $t = 0$.

Proposition 2 (Monetary Policy Transmission Mechanism: Channel 2)**.** *In equilibrium, in every state of nature $s \in S^*$, in the absence of default on deposits, the interest rate paid by the bank on consumer deposits must equal the interest rate on excess reserves (IOER) paid by the central bank,*

$$\bar{r}^d = \bar{r}^{CB} \quad \forall s \in S^*$$

Therefore, a departure of the deposit rate from the ZLB will directly pass through to the depositing household's budget constraint and consumption.

Again the proof of Proposition 2 follows directly from the optimality conditions of bank ϕ given in Appendix C. If default was permitted, it would generate a wedge between the rate at which the bank borrows long-term from the household and the rate at which it lends to the central bank given the same maturity. In the absence of default, the wedge is zero.

Lemma 1 establishes how relative prices, real allocations, and short-term interest rates are related to each other. Proposition 3 establishes the existence of price level determinacy and the Quantity Theory of Money. Proposition 4 stipulates under which condition price level determinacy fails to hold. While the former two are classical results of monetary GE models with incomplete markets obtained by Dubey and Geanakoplos (1992, 2003a, 2003b, 2006), the latter result stems from the novel properties of our model.

Lemma 1 (Relative prices, allocations, and short-term interest rates)**.** *For agent α who borrows in the short-term money market and in the long-term market for mortgages, sells the perishable good and purchases the durable good, the ratio of marginal utilities of consuming the perishable and durable goods in both periods, respectively, is given by*

$$\frac{u'(\,e^{\alpha}_{p,0} - q^{\alpha}_{p,0})}{u'(\,q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\alpha}\gamma_1 u'(\,q^{\beta}_{h,0} + q^{\beta}_{h,s})} = \frac{p_{p,0}}{p_{h,0}(\,1 + r^{ST}_0)} \quad \forall s \in S_g\,,$$

$$\frac{u'(\,e^{\alpha}_{p,s} - q^{\alpha}_{p,s})}{u'(\,q^{\beta}_{h,0} + q^{\beta}_{h,s})} = \frac{p_{p,s}}{p_{h,s}(\,1 + \rho_s)} \quad \forall s \in S\,, \textit{ and}$$

$$\frac{\boldsymbol{\beta}^{\alpha}\gamma_s u'(\,q^{\beta}_{h,0} + q^{\beta}_{h,s})}{u'(\,q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\alpha}\gamma_s u'(\,q^{\beta}_{h,0} + q^{\beta}_{h,s})} = \frac{p_{h,s}}{p_{h,0}(\,1 + \bar{r}^m)} \quad \forall s \in S_g\,.$$

All derivations are relocated to Appendix C. A similar set of relations holds for agent β. Lemma 1 implies that the monetary policy rate has a direct effect on households' marginal rate of substitutions and on real allocations at $t = 1$ and an indirect effect at $t = 0$. The Quantity Theory of Money obtains as it is clear from Proposition 3.

Proposition 3 (Price Level Determinacy)**.** *The size of endowments in the economy restricts the size of the central bank's balance sheet in the absence of income transfers. The presence of outside money and default guarantee that the price level is determinate. The Quantity Theory of Money holds in equilibrium.*

We provide a proof of Proposition 3 in Appendix C. For a general proof, see Dubey and Geanakoplos (2006). To understand Proposition 3, note that a positive stock of outside money free of any debt obligations, i.e. monetary endowments, makes it possible to repay interest on borrowing in excess of lending in the presence of a cash-in-advance constraint. At the end of the last period, all money must exit the economy again. In the absence of outside money, repayment of claims would not be possible and the nominal interest rate would hence be zero. Thus, outside money (Dubey & Geanakoplos, 2003a, 2003b, 2006) and default (Lin et al., 2016) guarantee the existence of positive nominal interest rates and price level determinacy.[26] Money is therefore non-neutral. Fiat money has positive value because of its role in facilitating transactions by virtue of the transaction technology (Dubey & Geanakoplos, 1992). The cash-in-advance structure ensures that interest rates formalize the price for liquidity. The cash-in-advance constraints and fiat money imply that aggregate income, i.e. the value of all goods sales, is linked to short-term credit extension. Both prices and quantities are thus affected by changes in money supply. This is the crux of the Quantity Theory of Money (QTM) that obtains in our model.

Moreover, a finite, positive amount of real endowments in the economy prevents the central bank's balance sheet from growing infinitely large in the absence of income transfers. The size of endowments restricts the purchasing and selling power that households have in both periods due to the scarcity of collateral. Cash-in-advance constraints limit the amount of credit that households can obtain from the proceeds of their endowment sales. Because household credit is restricted, the amount of borrowing from the central bank that the commercial bank requires to fund these household loans is bounded from above. Proposition 4 presents the conditions under which this does not hold.

Proposition 4 (Price Level Indeterminacy)**.** *If the central bank redistributes its profits to households on a per period basis and if its balance sheet is unconstrained, the price level is indeterminate.*

Bloise and Polemarchakis (2006) provide a theoretical treatment under more general assumptions. The intuition behind 4 follows the aforementioned argument. The transfer of funds from the central bank to households is equivalent to an additional injection of outside money. This outside money is free of any offsetting obligations because households do not need to repay it. The additional outside money relaxes households' budget constraints, enabling them to spend more fiat money until the budget constraint binds again. Higher expenditures of fiat money require more short-term credit due to cash-in-advance constraints. To finance this additional credit demanded by households, the commercial bank needs to borrow more heavily from the central bank. This leads to higher profits of the central bank as long as the central bank is unrestricted in its lending. The resulting larger expected transfers to household further prop up credit and central bank income. Since real endowments are limited, the only way for expenditures of fiat money and credit to increase is through price inflation. In the limit, the price level is therefore indeterminate.

Lastly, the Fisher effect holds in our model. Agents in the model receive (or pay) the gross real interest rate $(1+\tilde{r}_s) = (1+r_s)/(1+\pi_s^e)$ where r_s stands for any of the per period nominal interest rates in our model. The gross real interest rate approximately equals the nominal interest rate less expected inflation.

[26]Note that by positive nominal interest rates we mean positive lending rates in the private sector since nominal deposit rate is set equal to zero by assumption.

Having established these general properties of our monetary GEI, we proceed by reducing the model to a manageable size in order to investigate its properties numerically. It will become clear in the following sections how the monetary policy transmission channel and the debt deflation channel operate. The propositions of this section will allow us to analyze and assess our computational results.

5 A Two-Period Model with Two States of Nature

In this section, we discuss a reduced version and the computational method of the two-period model in Section 3. We first present the set of simplifying assumptions that make the model tractable. We subsequently calibrate the initial model parameters to retrieve the values of endogenous variables in a partial equilibrium. The overall goal of this section is to simplify the model to a manageable set of equations that yields a numerical solution to a general equilibrium of the model.

5.1 Assumptions and central bank rationality

Several steps are taken to reduce complexity of the model while retaining its core features. First, we limit the number of possible states in the second period to two, i.e. one good state $s = 1$ without mortgage default and one bad state $s = 2$ with mortgage default. Second, we assume that at the beginning of period $t = 0$, the central bank sets the interest rates on loans to bank ϕ, henceforth denoted the "policy rate", only for period $t = 1$ for each state of nature. It does not set the policy rate for $t = 0$. This type of "forward guidance" allows us to calibrate the initial policy rate ρ_0 at a level similar to post-GFC levels close to the ZLB. We are interested in understanding how an (endogenous) increase in the policy rate in the second period affects the economy.[27] Third, we exogenously set the interest rate on excess reserves (IOER) at a level close to zero to impose the ZLB onto our model. We do so because the reasons for the economy to reach the ZLB lie outside the scope of our model. Fourth, the central bank sets the capital requirement CAR^* only for the first period. We assume that the CAR^* is not binding in the second period as banks have accumulated a capital buffer in the first period.

As for the central bank's optimal behavior, we attempt to model it as closely as possible to a social planner's welfare function that consists of the aggregate sum of agents' utility functions. The central bank in our model possesses information about all agents' optimality conditions, budget constraints, and market clearing conditions but it does not internalize all of the second order effects of its actions. We assume that the central bank internalize the *direct* effect of setting the policy rate ρ_s in state $s \in S$ and the capital requirement CAR^* at $t = 0$ on endogenous variables in all states of nature. However, it does not consider the *indirect* effects via other endogenous variables in the system.

[27]Therefore, the exogeneity of the policy rate at $t = 0$ implies $\partial\rho_0 \backslash \partial\rho_s = 0$ for $s \in \{1, 2\}$.

5.2 A computational solution method

As the attentive reader may have noticed, the policy rate, ρ_s, does not show up in all four terms (1)-(4) in equation (Λ_1^{CB}). For example, it only implicitly affects inflation via prices and real output via quantities in terms (3) and (4). Due to the non-linearity of the optimality conditions, we cannot derive a closed-form analytical solution for prices and quantities. Put differently, determining the change in inflation and output in response to a change in the policy rate, $\partial \pi_s / \partial \rho_s$ and $\partial y_s / \partial \rho_s$, and in response to the capital requirement, $\partial \pi_s / \partial CAR^*$ and $\partial y_s / \partial CAR^*$, therefore poses a major analytical challenge.

We proceed by retrieving *numerical* solutions to each partial derivative in the central bank's first order conditions. Our point of departure is the fact that the optimality conditions for each agent must hold in equilibrium after any change in the policy rate has occurred. That is, all changes in each optimality condition and in the market clearing conditions must net out in equilibrium. This reasoning allows us to use the envelope theorem. Loosely speaking, it allows us to hold the optimal values for choice variables constant while only evaluating the effect of a change in the policy rate on the objective function. Thus, it allows us to only consider *direct* effects of the policy change and ignore *indirect* effects via other endogenous variables. We subsequently solve for numerical solutions to a system of partial derivatives. Appendix D provides an example of our solution method.

We use three steps in the following two sections: (1) we first solve for an initial partial equilibrium for the policy rates ρ_1 and ρ_2 and the capital requirement CAR^* by neglecting the central bank's decision problem, (2) we then use the initial equilibrium calibration of the endogenous variables to retrieve numerical values of the partial changes with respect to the policy rates from a linear system of partial derivatives derived from the optimality conditions, and (3) we finally solve for a general equilibrium using all optimality conditions, envelope conditions, and numerical values of the partial derivates plugged into the central bank's FOCs as the initial calibration. Our computational approach is compatible with DSGE solution methods (see for example Arellano et al. (2016) for dynamic programming problems in the context of large-scale macroeconomic models).

5.3 Calibration

The model is parameterized in Table 3 to reflect the post-GFC environment of low interest rates. The model parameters are moreover chosen to mirror the implications of the subprime mortgage crisis and its aftermath, following Goodhart et al. (2010), because we are interested in the effect of monetary policy on default and prices via the collateral channel. The general equilibrium values obtained under central bank optimization at the ZLB are presented in Table 4. In addition, we present a list of all variables of the model in Appendix B

At $t = 0$, the economy is calibrated with borrowing and lending rates at, or close to, the ZLB, respectively. Household α is relatively "poorer" than household β because the value of her endowment of potatoes is lower than the value of the housing endowment of household β in nominal terms. The poorer household α is therefore a net borrower while the richer household β is a net lender to bank ϕ. The initial loan-to-value ratio

Table 3: Exogenous Variables

Real Endowments	Money Endowments	CB Parameters	Other Parameters
$e^{\alpha}_{p,0} = 10.0$	$m^{\alpha}_0 = 0.1$	$\bar{r}^{CB} = 0.0\%$	$\beta^{\alpha} = 0.99$
$e^{\alpha}_{p,1} = 10.0$	$m^{\alpha}_1 = 5.0$	$\rho_0 = 1.0\%$	$\beta^{\beta} = 0.99$
$e^{\alpha}_{p,2} = 3.0$	$m^{\alpha}_2 = 0.1$	$\kappa = 0.6$	$\beta^{\phi} = 0.99$
$e^{\beta}_{h,0} = 5.3$	$m^{\beta}_0 = 3.5$	$\varkappa = 0.6$	$\gamma_1 = 0.9$
$e^{\phi}_0 = 5.0$	$m^{\beta}_1 = 2.0$	$y^*_1 = 6.19$	$\gamma_2 = 0.1$
$e^{\phi}_1 = 5.0$	$m^{\beta}_2 = 0.2$	$y^*_2 = 3.32$	$\sigma = 1.3$
$e^{\phi}_2 = 1.5$	$m^{CB}_0 = 80$	$\pi^* = 2.0\%$	$a^{\phi} = 0.007$
	$m^{CB}_1 = 0.1$	$\theta_{\pi} = 1$	$x = 0.01$
	$m^{CB}_2 = 0.1$	$\theta_{\lambda} = 0.001$	
		$\theta_{\phi} = 0.001$	
		$\theta_{\theta} = 0.001$	
		$w_{ST} = 0.3$	
		$w_{LT} = 1.0$	

of the collateralized mortgage that household α takes out is only about 31%. Bank ϕ has a high equity buffer approximately equal to 30% of the value of its risky assets, the mortgage. The parameterization is chosen to motivate intra-temporal borrowing from the central bank and inter-temporal depositing of excess reserves. The central bank has a sufficiently high initial endowment to lend out more short-term credit to bank ϕ than it is able to finance through excess reserve deposits. The interest rates on excess reserves $\bar{r}^{CB}$ and on central bank loans ρ_0 are chosen to reflect the interest rate "corridor" at the ZLB.

Two scenarios are assessed at $t = 1$. In the good state $s = 1$, since household α's monetary endowment increases relative to the first period and she anticipates an income transfer from the central bank, she sells less of the same endowment of potatoes and consumes more of it herself. Consequently, the price of potatoes increases in state $s = 1$. Household α can sell enough potatoes to pay back her mortgage to bank ϕ and no mortgage default occurs. At the same time, agent α's additional demand for housing at $t = 1$ decreases because borrowing short-term credit from the bank becomes more expensive.[28] The drop in demand for housing makes houses relatively cheaper. Household β sells less housing in state $s = 1$ relative to $t = 0$ and buys the relatively more expensive potatoes. Facing the same high interest rate for short-term credit, it uses the relatively higher monetary endowment and anticipated income transfer to afford potatoes.

In the bad state $s = 2$, an adverse supply shock occurs. Household α is endowed with much fewer potatoes relative to the previous period and state $s = 1$. She therefore sells only a fifth of what she sold in the first period, which results in inflation in potato

[28] Remember that households cannot immediately use the proceeds from their endowment sales to buy goods due to cash-in-advance constraints. They hence have to borrow short-term credit from the bank.

Table 4: Initial Equilibrium

Prices	Quantities	Money spent	ST credit	LT credit	CB variables
$p_{p,0} = 6.50$	$q^{\alpha}_{p,0} = 5.69$	$b^{\alpha}_{h,0} = 51.61$	$\mu^{\alpha}_0 = 36.99$	$\mu^{\alpha} = 17.65$	$M^{CB}_0 = 82.70$
$p_{p,1} = 10.60$	$q^{\alpha}_{p,1} = 4.28$	$b^{\alpha}_{h,1} = 29.39$	$\mu^{\alpha}_1 = 45.41$	$\bar{m}^{LT} = 15.88$	$M^{CB}_1 = 59.25$
$p_{p,2} = 19.66$	$q^{\alpha}_{p,2} = 1.36$	$b^{\alpha}_{h,2} = 25.88$	$\mu^{\alpha}_2 = 26.83$	$\bar{d}^{\beta} = 16.20$	$M^{CB}_2 = 32.22$
$p_{h,0} = 31.81$	$q^{\beta}_{h,0} = 1.62$	$b^{\beta}_{p,0} = 36.99$	$\mu^{\beta}_0 = 51.61$	$\bar{\mu}^{\phi}_d = 16.20$	$D^{CB} = 2.70$
$p_{h,1} = 23.96$	$q^{\beta}_{h,1} = 1.23$	$b^{\beta}_{p,1} = 45.41$	$\mu^{\beta}_1 = 29.39$	$\bar{d}^{\phi} = 2.70$	$w^{CB}_1 = 86.48$
$p_{h,2} = 9.35$	$q^{\beta}_{h,2} = 1.15$	$b^{\beta}_{p,2} = 26.83$	$\mu^{\beta}_2 = 10.72$		$w^{CB}_2 = 82.90$
$r^{ST}_0 = 3.86\%$			$\mu^{\phi}_0 = 83.52$		$\pi_1 = -4.08\%$
$r^{ST}_1 = 9.38\%$			$\mu^{\phi}_1 = 64.80$		$\pi_2 = -6.54\%$
$r^{ST}_2 = 6.11\%$			$\mu^{\phi}_2 = 34.19$		$y_1 = 5.51$
$\rho_1 = 9.38\%$					$y_2 = 2.51$
$\rho_2 = 6.11\%$					$\Pi^{\phi}_1 = 10.00$
$\bar{r}^d = 0.0\%$					$\Pi^{\phi}_2 = 3.35$
$\bar{r}^m_1 = 11.10\%$					$CAR^* = 12.05\%$
$\bar{r}^m_2 = -4.55\%$					

prices. To afford the relatively more expensive potatoes without being endowed with much money at $s = 2$, household β now sells more of his leftover housing endowment than he otherwise would have. House prices fall even further relative to period $t = 0$ and state $s = 1$. The decline in house prices unfolds up to a critical point, where the value of housing that household α initially pledged as collateral at $t = 0$ falls below the value of the mortgage to be repaid. This triggers default on the mortgage and foreclosure of the collateral. In the spirit of Fisher (1933), falling prices raise the real burden of debt which leads to a further decline in prices – a debt-deflation "spiral" ensues. Bank ϕ only receives the value of the collateral that is immediately resold in the market and suffers a loss equal to the discrepancy between the mortgage and the value of the collateral at the prevailing price in the bad state. Since additional housing collateral is sold in the market for housing, house prices experience an additional downward boost. Bank ϕ's profits experience a large drop, being endowed with equity capital close to nil and suffering losses from mortgage default.

What do these dynamics in the second period imply for the central bank's objectives, *ceteris paribus*? First, bank profitability rises in the good state compared to the first period. High levels of trade and credit extension as well as high returns on the mortgage boost bank profits. By contrast, bank profitability drops by almost 34% in the bad state. Although bank ϕ does not earn negative profits and become insolvent, it still suffers considerably from mortgage default and subdued credit demand at $s = 2$. Second, losses due to default are high in the bad state. The recovery value on the mortgage is 86% of the face value. Third, the rate of inflation deviates by at least 6% from the target level of 2% in both periods. Deflation dynamics in both states of nature depend on the relative weights of perishable and durable goods in the consumer price index. We assume that the consumer price index (CPI), p_s, takes the following simple form:

$$p_s = \varkappa p_{p,s} + (1-\varkappa)p_{h,s} \quad \text{for} \quad s \in S = \{1,2\}$$

where we place a weight of $\varkappa = 0.6$ on the consumption good. The economy experiences deflation in both states of nature. Deflation is more severe in the bad state $s = 2$.[29] By virtue of the Fisher effect, the combination of deflation and high real short-term interest rates explains why the nominal short-term rate on loans must rise in the second period.

Finally, the output gap is negative in both states of nature. This is due to the fact that we have calibrated the natural level of output to reflect the total volume of trade that would be achieved in a barter economy without financial frictions (see Appendix A). Since our economy achieves at most a constrained Pareto suboptimal result due to the frictions embedded in it, the output gap will always be negative. The goal of the central bank then is to minimize deviations of actual from potential output by bringing the economy closer to first best.

5.4 Comparative Statics

We have reduced our GEI model to a manageable two-period model with two states of nature. We have established the properties that characterize the equilibrium and numerically solved for an equilibrium of the model. We now investigate (1) how a lift-off of the IOER from the ZLB affects the equilibrium prices, allocations, and default levels, and (2) how the relative importance of monetary policy and financial stability objectives affects the equilibrium. In order to examine the former, we start by addressing how optimal policy is affected by the interest rate corridor between borrowing and lending rates set by the central bank at the ZLB.

5.4.1 Optimal policy at the ZLB

Given our initial general equilibrium presented in Section 5.3 with the interest rate on excess reserves $\bar{r}^{CB}$ at the ZLB, we first run simulations by exogenously varying the interest rate on central bank loans in the first period, ρ_0, by increments of 20 basis points. This means that we vary the interest rate corridor between the rate at which the central bank borrows and the rate at which it lends to bank ϕ in the first period.[30] At every iteration

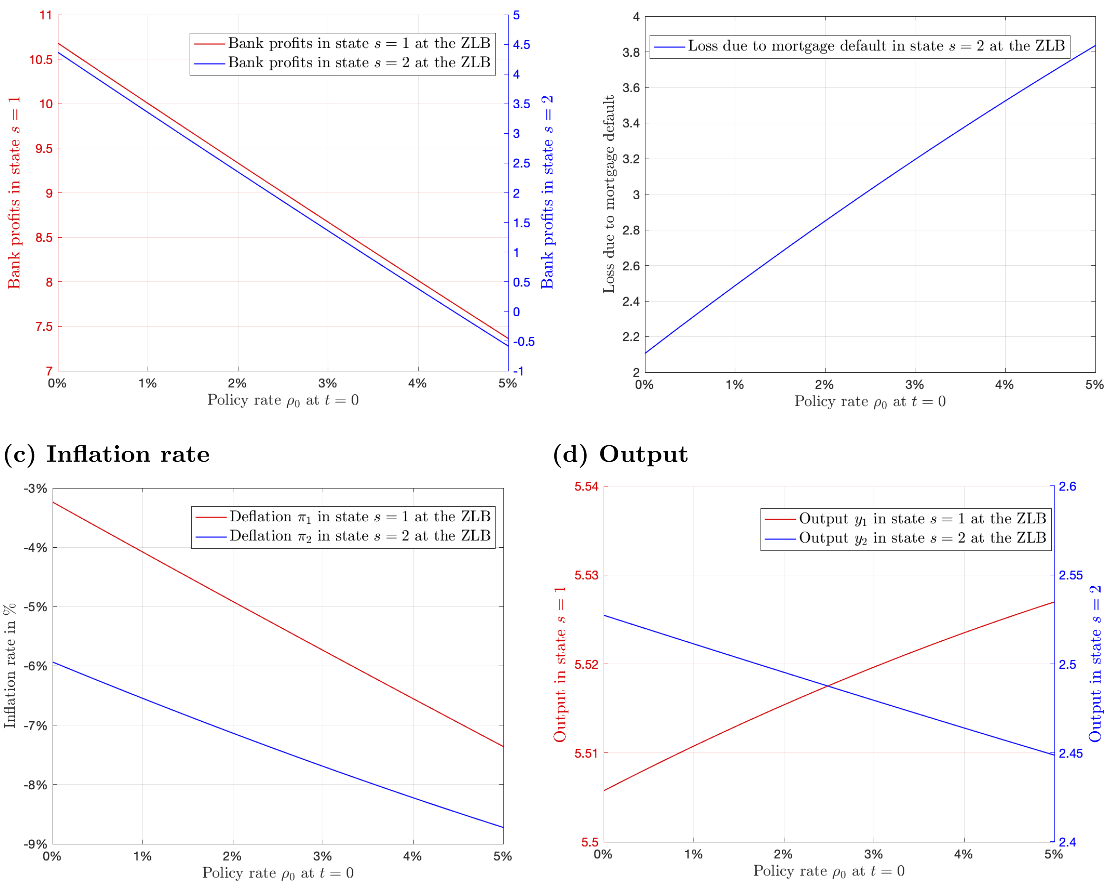
[29] We acknowledge that the results are sensitive to the weight on the price of the perishable good in the CPI. The UK Office for National Statistics (2015) published in a 2015 report that it placed weights of 11%, 4.3%, 7%, and 12.8% on food and non-alcoholic beverages, alcoholic beverages and tobacco, clothing, and housing, respectively. The Office for National Statistics (2018) has progressively increased the weight on housing expenditures to 30% in 2018. We therefore think that a value of $\varkappa = 0.6$ of the perishable good in the CPI is still modest. The "appropriate" parameter value for $\varkappa$ depends on whether one is willing to interpret potatoes as comprising all consumption goods and housing as comprising all durable goods in our model economy.

[30] We admit that our definition of the interest rate corridor, $(\rho_0 - \bar{r}^{CB})$, is not fully accurate. In fact, while the ceiling rate is a short-term rate, the floor rate is a long-term rate in our model. Technically, the correct definition of an interest rate corridor would be the policy rate compounded over both periods

of the simulation, we re-calculate the equilibrium values of all endogenous variables given the equilibrium values of the previous iteration as starting values.[31] Equipped with the simulated equilibrium values under different sizes of the interest rate corridor at the ZLB, we proceed to analyze (1) how each of the four central bank objectives – bank profits, default, inflation, and output – is affected by the change in the interest rate corridor (Figure 5), and (2) how optimal capital regulation at $t = 0$ and optimal monetary policy rates at $t = 1$ change with the size of the corridor (Figure 6).

Figure 5: Central bank objectives in future states of nature $s \in \{1, 2\}$ under different interest rate corridors when the economy is at the ZLB.

(a) Bank profits **(b) Loss due to Default** in state $s = 2$



(c) Inflation rate **(d) Output**

An increase in the interest rate corridor at the ZLB increases losses due to mortgage default in the second period as shown in panel (b) in Figure 5. In accordance with our initial hypothesis, a larger credit spread exacerbates the debt servicing costs of private borrowers. As a result, demand of short-term credit by households falls. Suffering from that sluggish demand in short-term credit and losses incurred from mortgage default in bad times, bank ϕ earns lower profits in both states of nature as can be seen in panel (a).

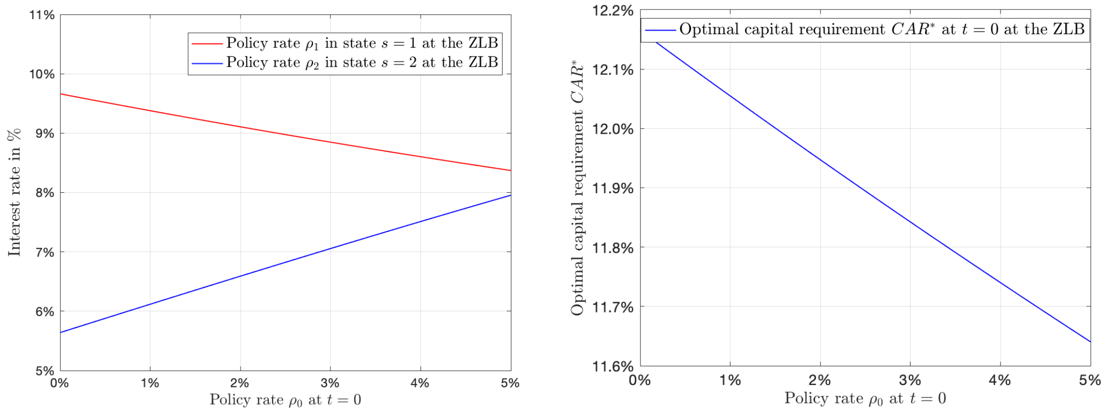
minus the IOER, $(1 + \rho_0)(1 + \rho_s) - (1 + \bar{r}^{CB})$, or the IOER compounded back to a one-period rate. For the ease of exposition, however, we continue referring to the interest rate corridor as the former definition. Moreover, as long as $\bar{r}^{CB}$ is zero, the distinction does not matter for the purpose of the analysis.

[31] Even without updating the starting values in the solution algorithm at each iteration, we get a set of local, stable equilibria.

Figure 6: Optimal policies at the ZLB under different interest rate corridors.

(a) Optimal policy rates in both states

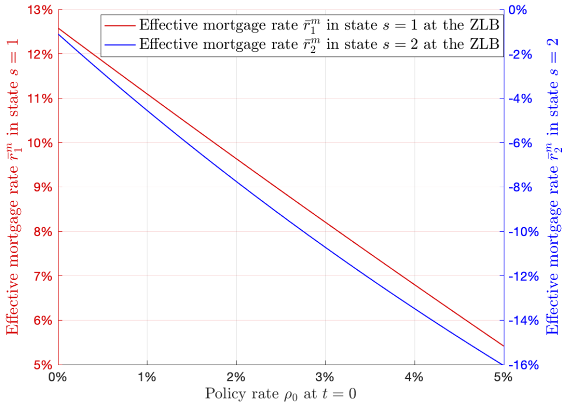
(b) Optimal capital requirement at $t = 0$



In bad times, bank profits even turn negative under a large interest rate corridor. The drop in demand for short-term credit by households in both states of nature depresses the price level. Panel (c) displays increasing deflation in both states of nature. These debt-deflation effects are in line with the findings obtained in the general equilibrium model with collateral default of Lin et al. (2015). Although output in panel (d) slightly rises in good times, this increase is not substantial. The central bank's optimal response is to ease future monetary policy in good times and to tighten monetary policy in bad times as the initial interest rate corridor widens at $t = 0$ as illustrated in panel (a) of Figure 6. Monetary policy in this scenario is therefore *pro-cyclical*. This seems paradoxical, however it is due to the fact that our initial equilibrium is deflationary. The central bank is therefore more strongly interested in easing monetary policy in good times so as to promote inflation. Having rational expectations about higher default under a wider initial interest rate corridor, the central bank tightens monetary policy even further in bad times in order to limit credit extension in the economy.

Moreover, tighter monetary policy in bad times is complemented by looser macro-prudential policy in the first period. Panel (b) shows that the optimal capital requirement falls as the interest rate corridor increases. The increase in the credit spread is partly passed on to the interest rate paid by households on short-term loans in the first period. The rest is absorbed by the bank in the form of a higher capital ratio. Less demand for short-term credit allows the bank to invest more funds in mortgages, while satisfying the capital requirement. It therefore internalizes the rise in the rate that it pays on loans from the central bank, and increases the rate on short-term household loans by less than one-to-one. Lastly, the losses due to mortgage default in bad times result in a further decline in the effective mortgage rate, presented in Figure 7. Household α defaults on a larger fraction of the mortgage the larger the interest rate corridor, i.e. the tighter initial monetary policy at $t = 0$.

Figure 7: Effective mortgage rates in both states of nature at the ZLB.



5.4.2 Optimal policy away from the ZLB

The results of the previous section provide an interesting case for keeping the interest rate corridor narrow in the first period when the economy is at the ZLB. We now examine the case when the interest rate on excess reserves lifts off from the ZLB. We simulate several exogenous increases of 20 basis points in the policy rate ρ_0 in the first period. Yet, this time we also raise the interest rate on excess reserves $\bar{r}^{CB}$ by steps of 20 basis points at each iteration, allowing for different sizes of the interest rate corridor away from the ZLB. This essentially gives us a grid of different interest rate corridors over which we simulate the model.

Table 5: Marginal positive ($\uparrow$) and negative ($\downarrow$) changes of endogenous variables in equilibrium with respect to an exogenous increase in the interest rate on excess reserves $\bar{r}^{CB}$ from the ZLB.

Variable	$t = 0$	$s = 1$	$s = 2$
Excess reserves D^{CB}	$\downarrow$		
Mortgage loan $\bar{\mu}^{\alpha}$	$\uparrow$		
Price for housing p_{h,s^*}	$\downarrow$	$\downarrow$	$\downarrow$
Price for potatoes p_{p,s^*}	$\downarrow$	$\downarrow$	$\downarrow$
Quantity of housing q_{h,s^*}^{β}	$\uparrow$	$\downarrow$	$\downarrow$
Quantity of potatoes q_{p,s^*}^{α}	$\downarrow$	$\downarrow$	$\uparrow$
Demand for short-term credit $m_{s^*}^{ST}$	$\downarrow$	$\downarrow$	$\downarrow$
Money supply/ liquidity $M_{s^*}^{CB}$	$\downarrow$	$\uparrow$	-
Short-term household loan rate r_0^{ST}	$\uparrow$		
Optimal policy rate ρ_s		$\downarrow$	$\uparrow$
Capital requirement CAR^*	$\downarrow$		
Real short-term rate $\tilde{\rho}_s$		$\downarrow$	$\uparrow$
Bank profits Π_s^{ϕ}		$\downarrow$	$\downarrow$
Loss due to default		-	$\uparrow$
Inflation π_s		$\uparrow$	$\uparrow$
Output y_s		$\downarrow$	$\downarrow$

In the following figures, we plot our variables of interest as a function of $\bar{r}^{CB}$ and ρ_0, i.e. as a function of the interest rate corridor $(\rho_0 - \bar{r}^{CB})$ at $t = 0$. The resulting surfaces over the interest rate grid are characterized by a blue diagonal "threshold" line. The threshold marks the separation between those equilibria that we are interested in on the right hand side (RHS) where $\bar{r}^{CB} \leq \rho_0$, i.e. where bank ϕ has an incentive to lend out loans to households rather than just to deposit its funds with the central bank. We therefore focus on the RHS of the threshold and trace the slope of the blue line as the interest rate on excess reserves $\bar{r}^{CB}$ increases. Table 5 summarizes the direction of changes of endogenous variables as the economy moves along the blue line, i.e. as $\bar{r}^{CB}$ and ρ_0 increase.

Figure 8: Bank profits in both states of nature away from the ZLB under different interest rate corridors.

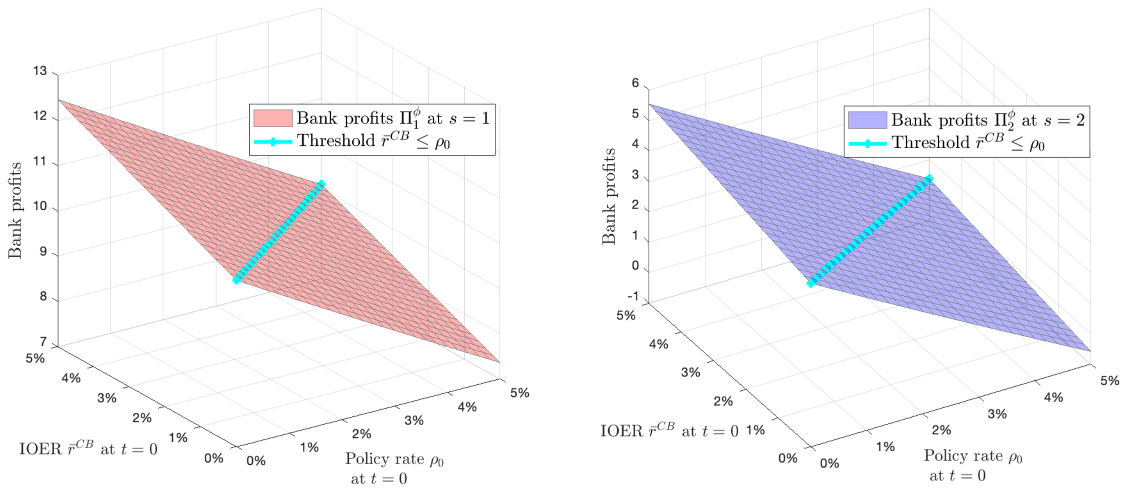
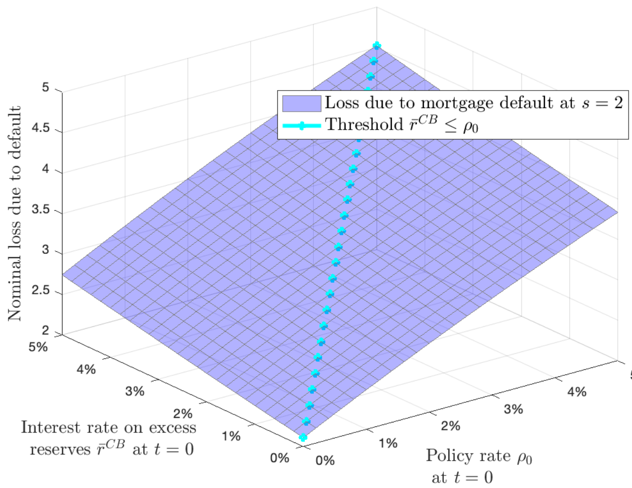


Figure 9: Loss due to mortgage default in state $s = 2$ away from the ZLB under different interest rate corridors.



Figures 8 to 11 display the main results of the simulation for the four central bank objectives. Losses due to mortgage default in bad times are exacerbated by a departure from the ZLB. They also steadily increase with the size of the interest rate corridor. By contrast, future bank profits in both states of nature suffer when the economy departs from the ZLB in the first period. This stands at odds with the current popular expectation that a departure from the ZLB would improve banks' profit margins and enhance financial stability. What factors drive these results? On the one hand, the deflation rate in both good and bad times decreases, coming much closer to positive inflation in the good state of nature. Output in good times remains almost stable but decreases in bad times due to the drag on the economy caused by higher default. These trends highlight that monetary and macroprudential policy may actually be effective in combatting deflationary tendencies; however, this occurs at the expense of not being able to achieve financial stability objectives, i.e. lower default and higher bank profitability.

Figure 10: Inflation in both states of nature away from the ZLB under different interest rate corridors.

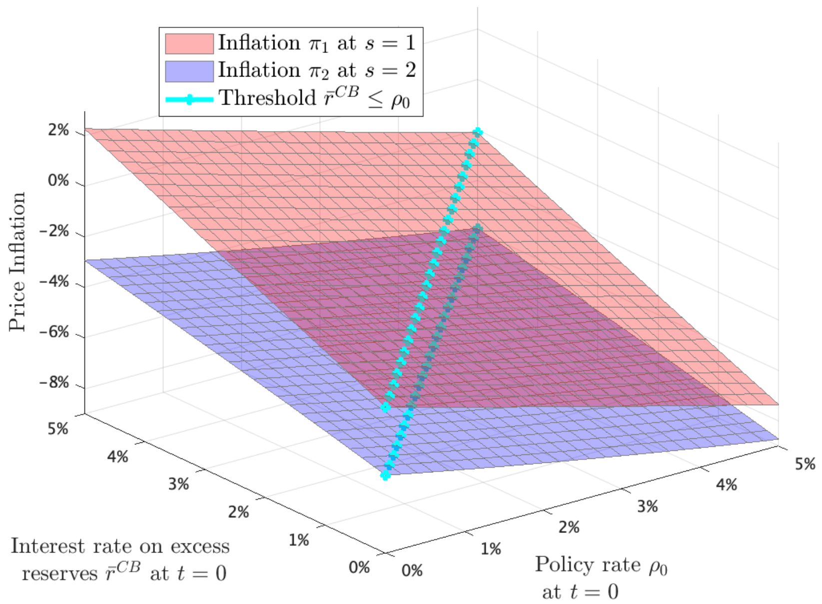


Figure 11: Output in both states of nature away from the ZLB under different interest rate corridors.

(a) Output in state $s = 1$ **(b) Output in state $s = 2$**

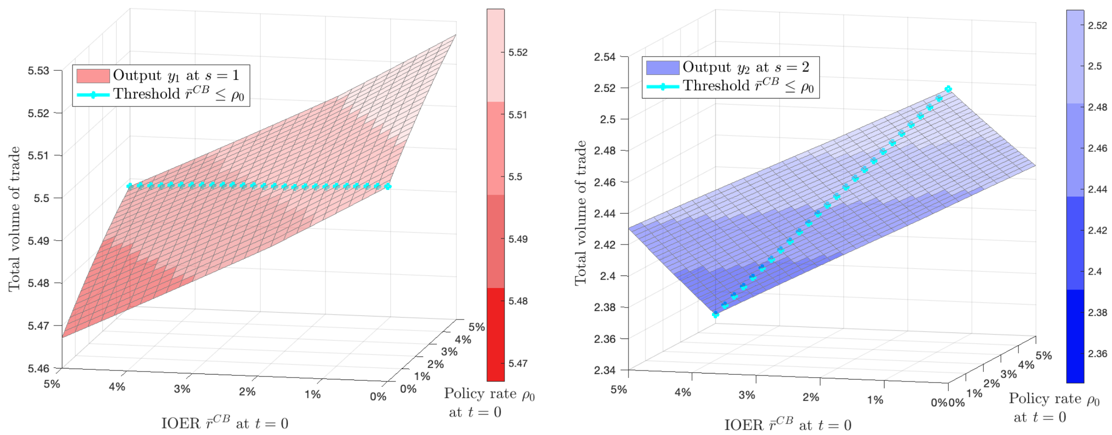
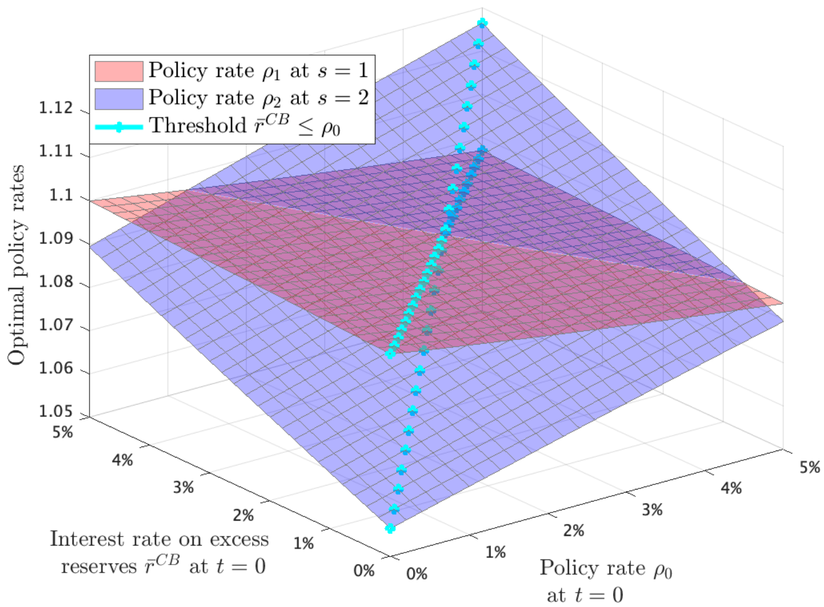


Figure 12 shows that the optimal response of monetary policy is slightly more expansionary in good times and more contractionary in bad times away from the ZLB. Put differently, the optimal future spread between deposit and loan rates should be narrower in booms and wider in busts the further away the economy is from the ZLB. This result is intuitive since the central bank has rational expectations about the likelihood of future default and deflation. If the economy is expected to fare well, it will want to set the policy rate at a lower level to fuel inflation. If the economy is expected to slump, the central bank will want to raise the policy rate so as to dampen credit extension that would otherwise increase default.

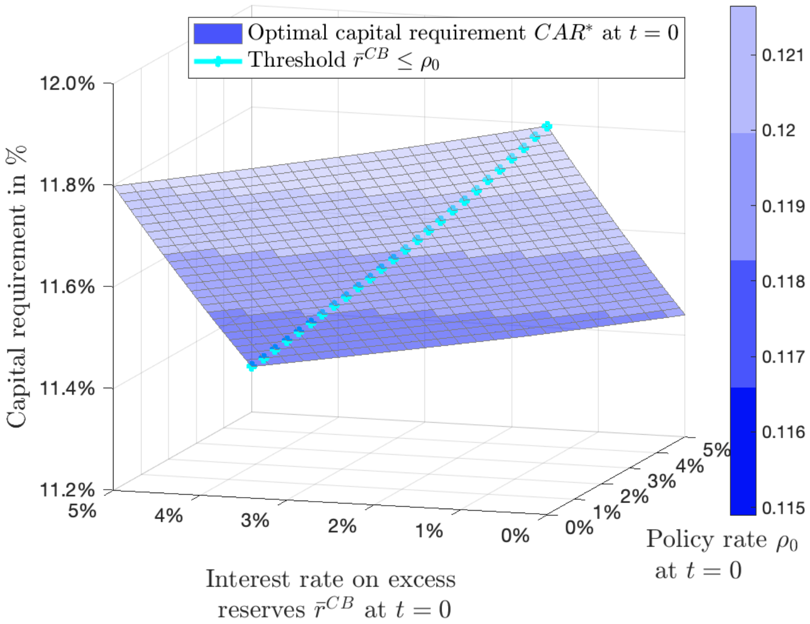
As seen in Figure 13, the further away the economy is from the ZLB, i.e. the more contractionary monetary policy is in general, the looser should the optimal capital requirement be. In other words, the optimal *ex post* capital requirement is counter-cyclical. Optimal monetary and macroprudential policies may therefore be complementary as obtained, for example, in Angelini et al. (2014), Kashyap, Tsomocos, and Vardoulakis (2017), and Martinez-Miera and Repullo (2019). The high level of nominal policy rates combined with the low, albeit apparent deflation away from the ZLB, results in a high real rate of interest on short-term credit, in particular in the bad state of nature, according to the Fisher effect. A high real rate of interest incentivizes households to postpone consumption and exacerbates the real burden of debt. This explains both the increase in default in Figure 9 and the fall in output in bad times in Figure 11 when the economy lifts off from the ZLB. As household spending and the demand for short-term credit fall and the rate charged on central bank funding increases in bad times, bank profits are squeezed. This is because the commercial bank is perfectly competitive and by Proposition 1 does not charge a spread on short-term loans to households.

Figure 12: Optimal policy rates in both states of nature away from the ZLB under different interest rate corridors.



In sum, the results suggest an over-emphasis on targeting inflation by the central bank when the economy departs from the ZLB. On the one hand, from the perspective of a monetary policy maker who cares about price developments, a departure from the ZLB may be desirable to limit deflation in the economy. However, on the other hand, from the perspective of a macroprudential regulator, the health of the banking sector suffers and losses due to default increase in bad times. The looser capital requirement combined with the falling demand for short-term credit allows the bank to invest more in the risky mortgage at a lower expected mortgage rate. High expected policy rates exacerbate the debt servicing costs of both the bank and households which fuels default in bad times. Thus, the trade-off between monetary policy and financial stability objectives obtains.

Figure 13: Optimal capital requirement CAR^* at $t = 0$ away from the ZLB under different interest rate corridors.



5.4.3 Optimal policy under different weighing of central bank objectives

In our benchmark calibration in the previous section, we imposed "equal" weights on each of the four objectives of the central bank.[32] In the following analysis, we examine first what happens when zero weights are placed on financial stability objectives. That is, we consider optimal monetary policy changes when the central bank adheres to its dual mandate of targeting inflation and the output gap. We then marginally increase the weight on financial stability objectives. We are not only interested in the relative changes in endogenous variables when the preferences of the central bank change but also whether financial stability concerns affect the set of optimal policies when the economy departs from the ZLB.

Figures 14 to 17 compare the equilibrium levels of bank profits, loss due to default, inflation, and output under the benchmark weights with the levels obtained under zero

[32]By "equal" we mean that we put equal weights of 1 on each objective after scaling down bank profits, default, and output to a decimal number to account for the fact that inflation is a percentage and therefore smaller in size. We acknowledge that this approach is rather ad hoc but note that we are not interested in absolute welfare levels but rather in relative changes due to changes in weights.

weights on financial stability objectives, i.e. $\theta_\phi = \theta_\lambda = 0$.[33] Under a dual mandate, bank profits are higher in good times but lower in bad times compared to a ternary mandate. Deadweight losses due to default surge when the central bank does not care about financial stability. Figure 16 shows that the central bank is clearly trading off proximity to target inflation in good times against soaring deflation in bad times. Likewise, output approaches full potential in booms but falls considerably in busts. Thus, a dual mandate increases the variance in the macro variables of interest between good and bad times.

Figure 14: Bank profits in both states of nature away from the ZLB under different weights on monetary policy (MP) and macroprudential policy (MaP).

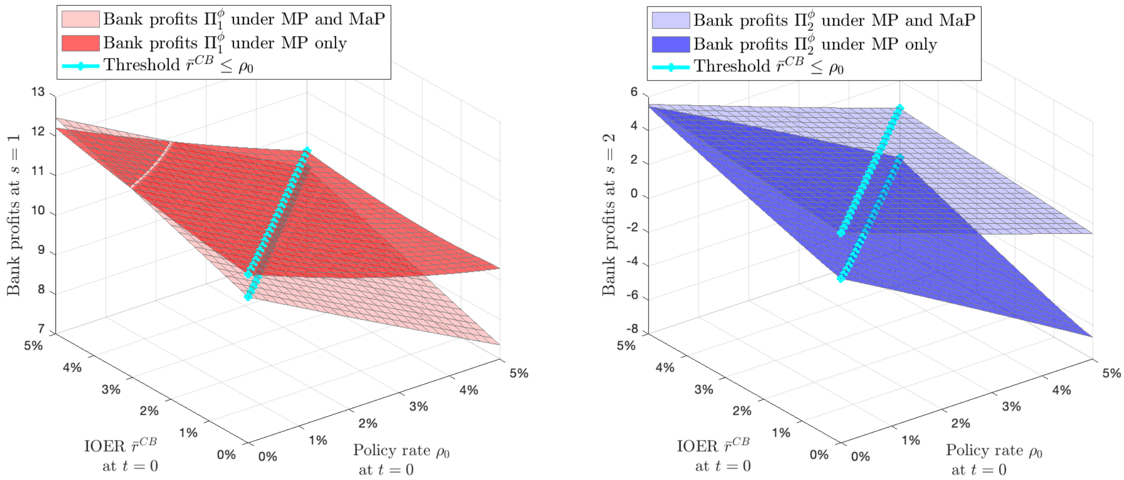
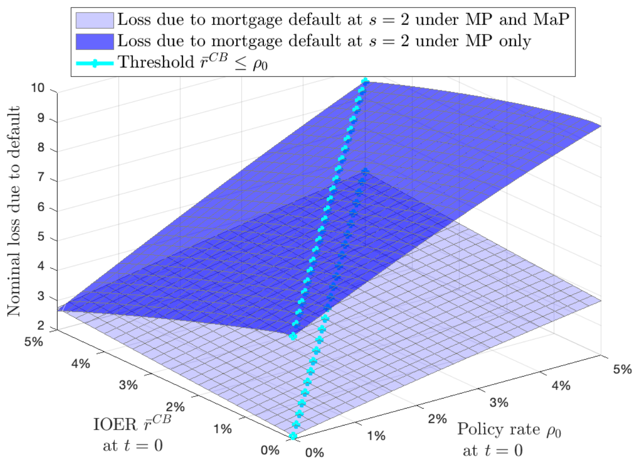


Figure 15: Loss due to default at $s = 2$ away from the ZLB under different weights on monetary policy (MP) and macroprudential policy (MaP).



How do the optimal levels of policy instruments play into this? Figures 18 and 19 present the comparison between a dual and a ternary mandate for the optimal level of policy rates in both states of nature and optimal capital requirements. It is optimal for monetary policy to be even more expansionary in booms and more contractionary in busts compared to the benchmark case. Since the central bank cares even more about

[33] Note however that, although we remove one of the central bank's targets, we do not remove the endogenous macroprudential tool from the model in the simulation.

Figure 16: Inflation in both states of nature away from the ZLB under different weights on monetary policy (MP) and macroprudential policy (MaP).

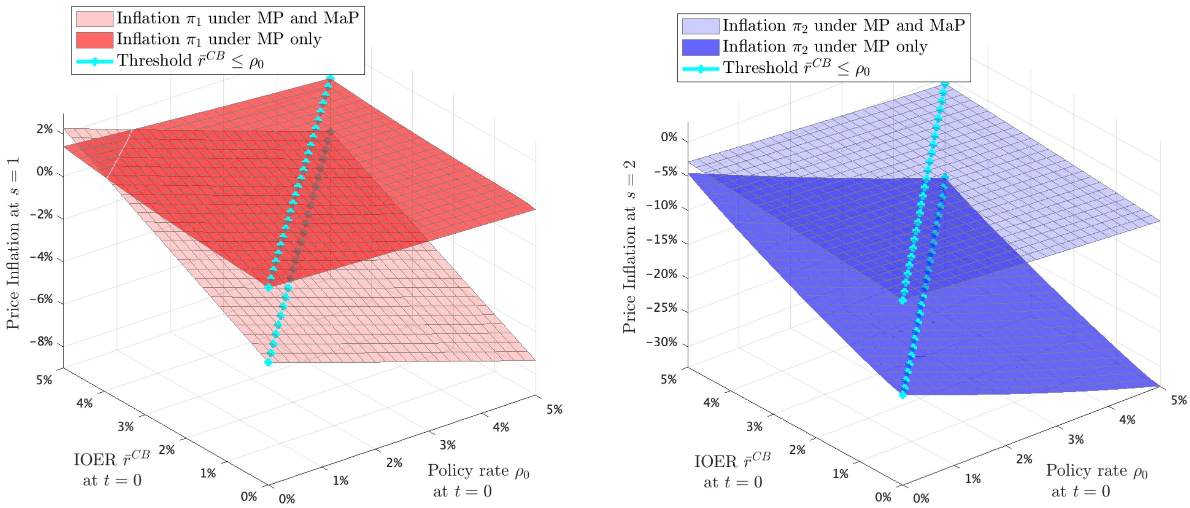
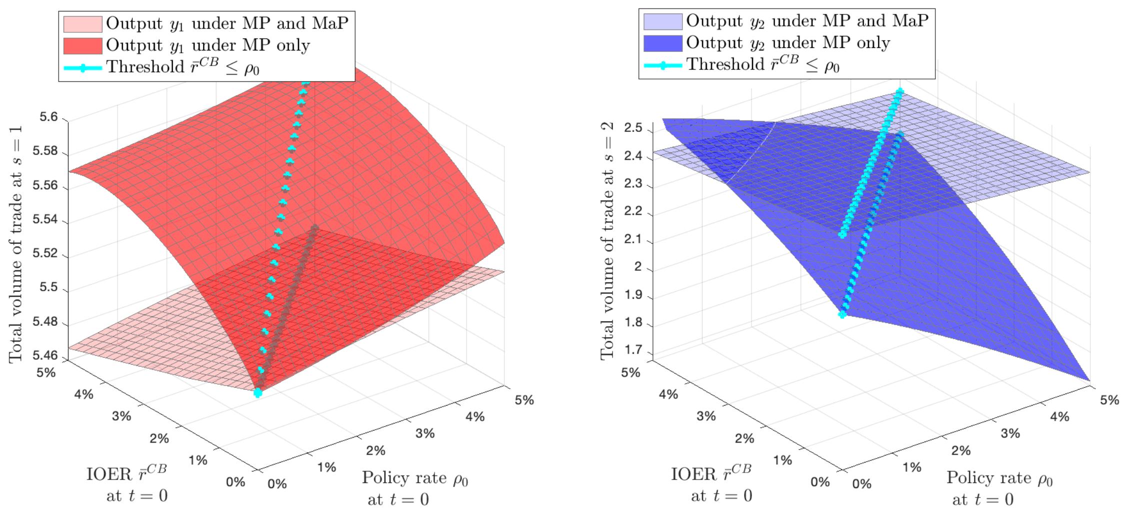


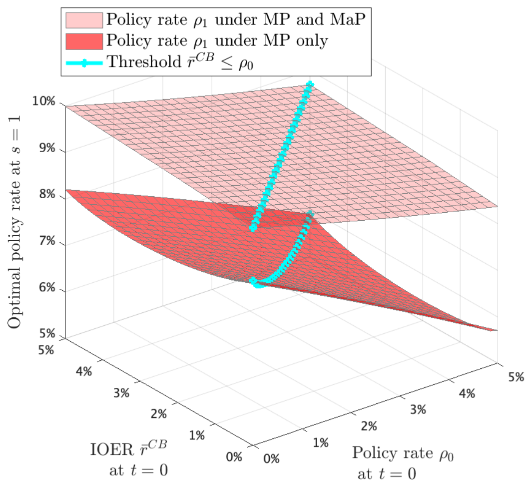
Figure 17: Output in both states of nature away from the ZLB under different weights on monetary policy (MP) and macroprudential policy (MaP).

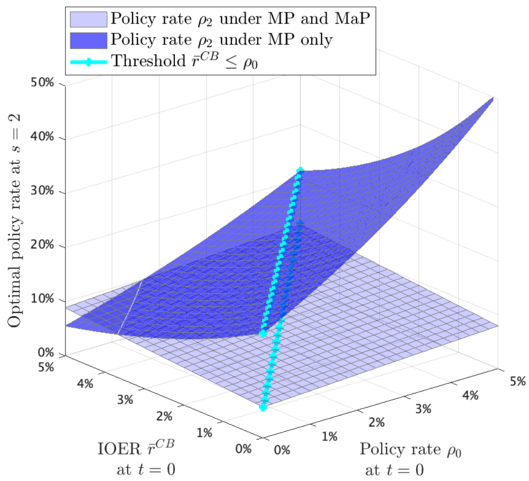


supporting inflation after an initial first period of deflation, a dual mandate exacerbates the pro-cyclicality of monetary policy tremendously. The optimal policy rate in bad times is nevertheless higher than under a ternary mandate because it reflects both the default and liquidity premium that the central bank does not explicitly take into account. The larger the initial interest rate corridor ($\rho_0 - \bar{r}^{CB}$), the higher future expected losses due to default, and therefore the higher the policy rate in bad times must be in order to limit credit expansion. The optimal *ex post* capital requirement is even looser under a dual mandate because the central bank does not internalize that a relaxation increases mortgage extension and consequently default.

Let θ_o^{BC} be the weight placed on objective $o \in \{\phi, \lambda, \pi, y\}$ in the benchmark equilibrium case under monetary and financial stability objectives. Let θ_o^{MP} and θ_o^{MaP} be the weights placed on the objectives under a a stronger preference for monetary policy and financial stability objectives, respectively. Table 6 summarizes the marginal changes in endogenous variables with respect to a departure of $\bar{r}^{CB}$ from the ZLB for two cases: (i) when larger weights are placed on monetary policy objectives relative to the benchmark case, i.e. $\theta_\pi^{MP}, \theta_y^{MP} \gg \theta_\pi^{BC}, \theta_y^{BC}$, and (ii) when larger weights are placed on financial stability objectives relative to the benchmark case, i.e. $\theta_\phi^{MaP}, \theta_\lambda^{MaP} \gg \theta_\phi^{BC}, \theta_\lambda^{BC}$, *ceteris paribus.* Comparing column (i) and (ii), we establish that the relative weighing on monetary policy versus financial stability objectives does not affect the *direction* of changes of most endogenous variables with respect to a positive change in $\bar{r}^{CB}$. Only relative prices of housing and potatoes move in opposite directions depending on which objectives receive a larger weight. Yet, this does not affect the direction of the change in overall inflation as can be seen in the penultimate row. Therefore, whether the central bank cares more about monetary policy objectives than about financial stability objectives does not affect the recommendation whether to depart from the ZLB or not. Nevertheless, Figures 14 to 17 and additional simulations indicate that the *magnitude* of changes in the targeted objectives with respect to a change in the width of the interest rate corridor ($\rho_0 - \bar{r}^{CB}$) is larger when the central bank cares less about financial stability. A ternary mandate therefore smoothes the large effects that a lift-off from the ZLB has on bank profits, losses due to default, inflation, and output across states of nature. It seems that it seeks the middle ground between trading off higher bank profits, inflation, and output in booms against lower profits and output as well as higher deflation and default in busts. Moreover, a ternary mandate mitigates the pro-cyclicality of monetary policy to some extent. Relative to a dual mandate, it calls for a wider spread between borrowing and lending rates in good times and a narrower spread in bad times.

Figure 18: Optimal policy rates in both states of nature away from the ZLB under different weights on monetary policy (MP) and macroprudential policy (MaP).





In conclusion, whether the central bank cares about financial stability or not does not radically change the direction that a departure from the ZLB means for optimal monetary and macroprudential policy *instruments*. However, the magnitude of effects of a departure from the ZLB on policy *targets* – inflation, output, bank profits, and default – is substantially affected by the choice and relevance of objectives of the central bank.

Figure 19: Optimal capital requirement at $t = 0$ away from the ZLB under different weights on monetary policy (MP) and macroprudential policy (MaP).

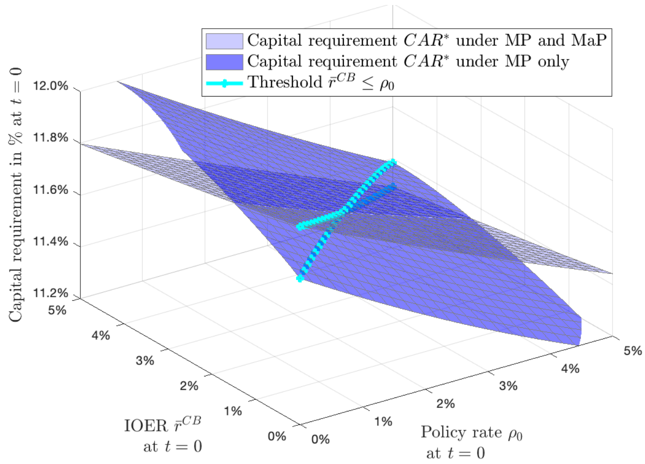


Table 6: Marginal changes of endogenous variables in equilibrium with respect to an increase in $\bar{r}^{CB}$ from the ZLB under different weights on monetary policy (MP) versus macroprudential policy (MaP) objectives relative to the benchmark case in Table 5.

Variable	**(i) Larger weight on MP**			**(ii) Larger weight on MaP**		
	$t = 0$	$s = 1$	$s = 2$	$t = 0$	$s = 1$	$s = 2$
Excess reserves D^{CB}	$\downarrow$			$\downarrow$		
Mortgage loan $\bar{\mu}^{\alpha}$	$\uparrow$			$\uparrow$		
Price for housing p_{h,s^*}	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Price for potatoes p_{p,s^*}	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Quantity of housing q^{β}_{h,s^*}	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
Quantity of potatoes q^{α}_{p,s^*}	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
Demand for short-term credit $m^{ST}_{s^*}$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Money supply/ liquidity $M^{CB}_{s^*}$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
Short-term household loan rate r^{ST}_0	$\uparrow$			$\uparrow$		
Optimal policy rate ρ_s		$\downarrow$	$\uparrow$		$\downarrow$	$\uparrow$
Capital requirement CAR_0	$\downarrow$			$\downarrow$		
Real short-term rate $\tilde{\rho}_s$		$\downarrow$	$\uparrow$		$\downarrow$	$\uparrow$
Bank profits Π^{ϕ}_s		$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$
Loss due to default		-	$\uparrow$		-	$\uparrow$
Inflation π_s		$\uparrow$	$\uparrow$		$\uparrow$	$\uparrow$
Output y_s		$\uparrow$	$\downarrow$		$\downarrow$	$\downarrow$

6 Conclusion

This paper addresses two major questions of the post-GFC decade. First, we investigate how optimal monetary policy and policy outcomes are affected when the central bank does not only care about classical monetary policy objectives but also about financial stability objectives. Second, we examine how a (static) lift-off from the ZLB affects default and liquidity when the central bank also cares about financial stability. We develop a monetary GE model with incomplete markets, heterogeneous agents, financial intermediation, and default to study the impact of policy rates on bank profitability and debt-deflationary dynamics. The central bank in our model explicitly maximizes its objective function subject to its balance sheet constraints. We augment an otherwise standard Taylor-rule type of criterion function by financial stability objectives. The central bank thus cares not only about targeting inflation and the output gap but also about bank profitability and aggregate losses due to default.

We propose two different channels of direct monetary policy transmission that operate in theory: (1) the transmission from the interest rate on excess reserves (the "ZLB") to the interest rate on household deposits, and (2) the transmission from the expected future policy rate paid on central bank loans to the expected future interest rate on short-term household credit. These transmission channels are characterized by a direct one-to-one passthrough of the rate set by the central bank to the rate received or paid by households. We hasten to add that this does not correspond to real-world transmission channels that are commonly characterized by default risk, market power of banks, and other frictions. However, we still believe that the channels in our simplified representation of the economy are important because they inform about the impact that monetary policy has. Moreover, classical propositions of monetary GEI models obtain in our benchmark model. Default and outside money safeguard the existence of positive nominal interest rates and price level determinacy. Monetary policy is non-neutral in equilibrium. A non-trivial Quantity Theory of Money obtains – prices and allocations directly respond to changes in money supply.

We simulate the general equilibrium values of endogenous variables and central bank objectives over a grid of different levels of the interest rate on excess reserves (which anchors the ZLB) and the predetermined initial policy rate that define the interest rate corridor in the first period. The results suggest that a default-induced debt-deflation cannot be effectively mitigated by monetary policy only. A departure from the ZLB lowers the rate of deflation and brings the economy closer to target inflation. However, this happens at the expense of higher losses due to default and lower output in bad times as well as lower bank profitability overall. The findings do not corroborate the popular claim made by some central bankers that departing from the ZLB would bolster bank's profit margins. Optimal future monetary policy is more pro-cyclical away from the ZLB whereas the optimal *ex post* capital requirement is counter-cyclical with respect to a departure from the ZLB. This confirms findings in the literature that monetary and macroprudential policy instruments are in fact complementary.

We give a first tentative answer to the relative importance of financial stability versus monetary policy objectives. The greatest source of welfare losses in our model are potential debt-deflationary dynamics. This stems from the higher debt-servicing costs and default risk that private borrowers face under higher interest rates. When the central bank places relatively more weight on monetary policy objectives compared to financial stability objectives, inflation and output during booms come close to target but they suffer an even bigger economic recession. The more important inflation is to the central bank, the narrower the optimal spread between borrowing and lending rates away from the ZLB in good times, and the wider the optimal spread in bad times. A dual mandate does not change the direction that optimal monetary policy should take but it amplifies the magnitude of the targeted objectives in both states of nature. In contrast, under a ternary mandate, the central bank is able to smooth welfare losses and gains incurred through inflation and default across states of nature. When the central bank cares about financial stability, monetary policy is less pro-cyclical compared to when financial stability concerns are irrelevant. Therefore, a case can be made for macroprudential policy to dampen the volatility in monetary policy outcomes across states of nature.

An initial motivation for this paper was the public debate about a down-sizing of Quantitative Easing (QE) by central banks, the so called "tapering". Tapering bears important implications for the term structure of interest rates as well as pre-existing debt obligations of agents. Our model could be extended by enabling the central bank to purchase mortgages or mortgage-backed securities from the commercial bank, so as to assess the conduct of QE.

In sum, our findings question the arguments that are brought forward by popular advocates of a departure from the ZLB. We highlight the need to take the risk of a debt-deflation seriously and consider debt levels in optimal monetary policy setting. Finally, we learn that is it pivotal to let the central bank control a macroprudential policy instrument if it pursues goals intended to stabilize the financial system. Our model does not provide insights into the *dynamic* decision to lift-off from the ZLB. Further research is warranted as to how the central bank endogenously chooses the optimal interest rate corridor at or away from the ZLB and the optimal level of the macroprudential policy instrument in a dynamic infinite horizon model.

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Appendices

Appendix A The Natural Level of Output

To obtain an estimate for the natural level of output in our endowment economy, we change our model to a barter economy without frictions like money, default, and cash-in-advance constraints. In such a model, households trade directly with each other, obliterating the need for a financial sector and a central bank who injects money in the economy. The two optimization problems of households $i \in \{\alpha, \beta\}$ therefore simplify substantially. Agents are endowed with the same amounts of the durable and perishable consumption goods at in the GEI economy. Agent α chooses have many units q^{α}_{p,s^*} of the perishable good to sell. Agent β decides how many units q^{β}_{h,s^*} of the durable good to sell in each state of nature. Prices are determined in equilibrium when the markets for both goods clear. There is no need for money and hence no interest rate.

In the absence of frictions, the optimization problem of **household $\boldsymbol{\alpha}$** is given by

$$\max_{q^{\alpha}_{p,s^*}, q^{\beta}_{h,s^*}} U^{\alpha} = \quad u(e^{\alpha}_{p,0} - q^{\alpha}_{p,0}) + u(\, q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\alpha} \sum_{s \in S} \gamma_s \Big[u(\, e^{\alpha}_{p,s} - q^{\alpha}_{p,s}) + u(\, q^{\beta}_{h,0} + q^{\beta}_{h,s}) \Big]$$

subject to

$$p_{h,0} q^{\beta}_{h,0} \leq p_{p,0} q^{\alpha}_{p,0} \tag{$\Gamma 1^{\alpha}$}$$

$$p_{h,1} q^{\beta}_{h,1} \leq p_{p,1} q^{\alpha}_{p,1} \tag{$\Gamma 2^{\alpha}$}$$

$$p_{h,2} q^{\beta}_{h,2} \leq p_{p,2} q^{\alpha}_{p,2} \tag{$\Gamma 3^{\alpha}$}$$

The optimization problem of **household $\boldsymbol{\beta}$** is given by:

$$\max_{q^{\beta}_{p,s^*}, q^{\beta}_{h,s^*}} U^{\beta} = \quad u(\, q^{\alpha}_{p,0}) + u(e^{\beta}_{h,0} - q^{\beta}_{h,0}) \, + \boldsymbol{\beta}^{\beta} \sum_{s \in S} \gamma_s \Big[u(\, q^{\alpha}_{p,s}) + u(\, e^{\beta}_{h,0} - q^{\beta}_{h,0} - q^{\beta}_{h,s}) \Big]$$

Subject to

$$p_{p,0} q^{\alpha}_{p,0} \leq p_{h,0} q^{\beta}_{h,0} \tag{$\Gamma 1^{\beta}$}$$

$$p_{p,1} q^{\alpha}_{p,1} \leq p_{h,1} q^{\alpha}_{h,1} \tag{$\Gamma 2^{\beta}$}$$

$$p_{p,2} q^{\alpha}_{p,2} \leq p_{h,2} q^{\alpha}_{h,2} \tag{$\Gamma 3^{\beta}$}$$

Solving for the equilibrium allocations yields an optimal set of quantities traded, $q^{\alpha^*}_{h,s}$ and $q^{\beta^*}_{p,s}$. Given the same set of endowments of potatoes and housing presented in Section 5.3, the natural level of output is defined as the total volume of trade in each state of nature:

$$y^*_s = q^{\alpha}_{h,s} + q^{\beta}_{p,s} \quad \text{for} s \in S = \{1, 2\}$$

Solving the above program yields $y^*_1 = 6.1862$ and $y^*_2 = 3.3232$.

Appendix B List of variables

Table 8: List of variables

e^{α}_{p,s^*}	Endowment of the perishable good, potatoes p, of agent α in state $s \in S^*$
$e^{\beta}_{h,0}$	Endowment of the durable good, housing h, of agent β in period $t = 0$
q^{α}_{p,s^*}	Amount of the perishable good, potatoes p, offered for sale by agent α in $s \in S^*$
q^{β}_{h,s^*}	Amount of the durable good, housing h, offered for sale by agent β in $s \in S^*$
b^{α}_{h,s^*}	Amount of money spent by agent α to buy housing h in state $s \in S^*$
b^{β}_{p,s^*}	Amount of money spent by agent β to buy potatoes p in state $s \in S^*$
$m^{i}_{s^*}$	Monetary endowment of household $i \in \{\alpha, \beta\}$ in state $s \in S^*$
p_{p,s^*}	Price of potatoes p in state $s \in S^*$
p_{h,s^*}	Price of housing h in state $s \in S^*$
$r^{ST}_{s^*}$	Short-term loan rate demanded by bank ϕ in state $s \in S^*$
$\bar{r}^{m}$	Long-term mortgage rate demanded by bank ϕ
$\bar{r}_{d}$	Long-term deposit rate offered by bank ϕ
$\mu^{i}_{s^*}$	Face value of intra-period borrowing by agent $i \in \{\alpha, \beta\}$ in state $s \in S^*$
$\boldsymbol{\beta}^{i}$	Subjective discount factor of agent $i \in \{\alpha, \beta\}$
$\gamma^{i}_{s^*}$	Subjective probability of agent $i \in \{\alpha, \beta\}$ of state $s \in S$ occurring
$\bar{\mu}^{\alpha}$	Face value of mortgage borrowing paid by agent α at $t = 1$
$\bar{d}^{\beta}$	Deposits by agent β held with bank ϕ at $t = 0$
t^{i}_{s}	Seignorage transfer to household $i \in \{\alpha, \beta\}$ at $t = 1$ in state $s \in S$
$\Pi^{\phi}_{s^*}$	Profit per period of bank ϕ in state $s \in S^*$
a^{ϕ}	Risk aversion coefficient of bank ϕ
$m^{ST}_{s^*}$	Total amount of short-term loans granted by bank ϕ in state $s \in S^*$
$\bar{m}^{LT}$	Total amount of long-term loans granted by bank ϕ at $t = 0$
$\bar{d}^{\phi}$	Amount of excess reserves deposited by bank ϕ with the central bank at $t = 0$
$\bar{\mu}^{\phi}_{d}$	Face value of long-term deposits borrowed by bank ϕ from household β to be repaid at $t = 1$
$\mu^{\phi}_{s^*}$	Face value of short-term borrowing by bank ϕ from the central bank in $s \in S^*$
	Long-term deposit repayment rate of bank ϕ in state $s \in S$
$e^{\phi}_{s^*}$	Equity endowment of bank ϕ in state $s \in S^*$
$\bar{r}^{CB}$	Interest rate paid by the central bank on excess reserves deposited by bank ϕ
ρ_{s^*}	Interbank rate or interest rate on central bank loans paid by bank ϕ in $s \in S^*$
θ_{o}	Weight put on policy objective $o \in \{\phi, \lambda, i, y\}$ in the CB's objective function
π_{s}	Rate of inflation in state state $s \in S$
π^{*}	Target rate of inflation
y_{s}	Level of GDP proxied by the aggregate volume of trade in state $s \in S$
y^{*}_{s}	Natural level of output achieved in a frictionless barter economy in state $s \in S$
$M^{CB}_{s^*}$	Total credit granted by the central bank in the form of repo loans in state $s \in S^*$
D^{CB}	Borrowing by the central bank in the form of excess reserve deposits in $s \in S^*$
w^{CB}_{s}	Seignorage income earned by the central bank in state $s \in S^*$
κ	Share of the seigniorage transfer to the relatively "poorer" household α at $t = 1$

Appendix C Proofs of Propositions

Due to the length of the Appendix, all derivations of equilibrium conditions are available on request from the corresponding author `Tatjana.Schulze@sbs.ox.ac.uk`. We present here the proofs that rely on agents' optimality conditions. We provide proofs for Propositions 1, Proposition 2, and Lemma 1 in Section 4 by first proving for two states of nature and then generalizing this to multiple states of nature.

Proof of Proposition 1. From the two simplified first order conditions (FOC) of bank ϕ,

$$\begin{aligned} \beta^{\phi}\gamma_1(r_1^{ST}-\rho_1)\,[\,1-2a^{\phi}\Pi_1^{\phi}] &= 0 \\ \beta^{\phi}\gamma_2(r_2^{ST}-\rho_2)\,[\,1-2a^{\phi}\Pi_2^{\phi}] &= 0 \end{aligned}$$

we immediately see that either $\rho_s = r_s^{ST}$ or $[\,1-2a^{\phi}\Pi_s^{\phi}] = 0$ for $s \in [1,2]$. However, zero future marginal profits would imply in the following FOC,

$$(1+\rho_0)(1-2a^{\phi}\Pi_0^{\phi}) - (1+\bar{r}^d)\boldsymbol{\beta}^{\phi}\Big[\gamma_1(1+\rho_1)(\,1-2a^{\phi}\Pi_1^{\phi}) + \gamma_2(1+\rho_2)(\,1-2a^{\phi}\Pi_2^{\phi})\Big] = 0\ ,$$

that also period-0 marginal profits would need to be zero. This in turn would mean by the following FOC

$$\begin{aligned} &(\,1+\rho_0)\,(\,1-2a^{\phi}\Pi_0^{\phi})\ -(1+\bar{r}_1^m)\boldsymbol{\beta}^{\phi}\gamma_1(1+\rho_1)(\,1-2a^{\phi}\Pi_1^{\phi}) \\ &-(1+\bar{r}_2^m)\boldsymbol{\beta}^{\phi}\gamma_2(1+\rho_2)(\,1-2a^{\phi}\Pi_2^{\phi}) + w^{LT}CAR_0\Lambda_4^{\phi} \quad = \quad 0 \end{aligned}$$

that either the capital requirement or the Lagrange multiplier would need to be zero. A zero Lagrange multiplier would imply by complementary slackness that the capital requirement would not bind. Since we rule out non-binding and zero capital requirements, the only way for all optimality conditions to hold is that $\rho_s = r_s^{ST}$ for$s \in [1,2]$. This can be easily generalized to $s \in S^*$. □

Proof of Proposition 2. Eliminating the Lagrange multipliers in the FOCs of bank ϕ by substitution, and subtracting the resulting equations

$$\begin{aligned} &(1+\rho_0)(1-2a^{\phi}\Pi_0^{\phi}) - (1+\bar{r}^d)\boldsymbol{\beta}^{\phi}\Big[\gamma_1(1+\rho_1)(\,1-2a^{\phi}\Pi_1^{\phi}) + \gamma_2(1+\rho_2)(\,1-2a^{\phi}\Pi_2^{\phi})\Big] = 0 \\ &(1+\rho_0)(1-2a^{\phi}\Pi_0^{\phi}) - (1+\bar{r}^{CB})\boldsymbol{\beta}^{\phi}\Big[\gamma_1(1+\rho_1)(\,1-2a^{\phi}\Pi_1^{\phi}) + \gamma_2(1+\rho_2)(\,1-2a^{\phi}\Pi_2^{\phi})\Big] = 0 \end{aligned}$$

immediately results in $\bar{r}^{CB} = \bar{r}^d \quad \forall s \in S^*$ which must hold in equilibrium if household β rationally optimizes. This can be generalized to a finite number of states $s \in S^*$. □

Proof of Lemma 1. We first plug the market clearing conditions (MC_1) - (MC_5) into household α's FOCs, rearrange, and substitute. We can then trivially extend to multiple states of nature as long as we distinguish between non-default states S_g and default states

$$\frac{u'(e^{\alpha}_{p,0} - q^{\alpha}_{p,0})}{u'(q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\alpha}\gamma_1 u'(q^{\beta}_{h,0} + q^{\beta}_{h,s})} = \frac{p_{p,0}}{p_{h,0}(1+\rho_0)} \quad \forall s \in S_g$$

$$\frac{u'(e^{\alpha}_{p,s} - q^{\alpha}_{p,s})}{u'(q^{\beta}_{h,0} + q^{\beta}_{h,s})} = \frac{p_{p,s}}{p_{h,s}(1+\rho_s)} \quad \forall s \in S$$

$$\frac{\boldsymbol{\beta}^{\alpha}\gamma_s u'(q^{\beta}_{h,0} + q^{\beta}_{h,s})}{u'(q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\alpha}\gamma_s u'(q^{\beta}_{h,0} + q^{\beta}_{h,s})} = \frac{p_{h,s}}{p_{h,0}(1+\bar{r}^m)} \quad \forall s \in S_g$$

We repeat the same steps for household β and then trivially extend to multiple states of nature

$$\frac{u'(e^{\beta}_{h,0} - q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\beta}\sum_s \gamma_s u'(e^{\beta}_{h,0} - q^{\beta}_{h,0} - q^{\beta}_{h,s})}{u'(q^{\alpha}_{p,0})} = \frac{p_{h,0}}{p_{p,0}(1+r^{ST}_0)} \quad \forall s \in S$$

$$\frac{u'(e^{\beta}_{h,0} - q^{\beta}_{h,0} - q^{\beta}_{h,s})}{u'(q^{\alpha}_{p,s})} = \frac{p_{h,s}}{p_{p,s}(1+\rho_s)} \quad \forall s \in S$$

$$u'(e^{\beta}_{h,0} - q^{\beta}_{h,0}) + \boldsymbol{\beta}^{\beta}\sum_s \gamma_s u'(e^{\beta}_{h,0} - q^{\beta}_{h,0} - q^{\beta}_{h,s}) = \frac{(1+\bar{r}^d)}{(1+r^{ST}_0)}\beta^{\beta}\sum_s \gamma_s \frac{p_{h,0}}{p_{p,s}} u'(q^{\alpha}_{p,s}) \quad \forall s \in S$$

□

Proof of Proposition 3. In order to prove the first statement in Proposition 3, we start by noting that the central bank's balance sheet constraint in the first period is given by equation (Λ^{CB}_2)

$$M^{CB}_0 \le \frac{D^{CB}}{1+\bar{r}^{CB}} + m^{CB}_0$$

which must be binding in equilibrium. Substituting the market clearing conditions (MC_{10}) and (MC_{12}) and rearranging, we get

$$\frac{\mu^{\phi}_0}{(1+\rho_0)} - \bar{d}^{\phi} = m^{CB}_0$$

We then use the budget constraint of bank ϕ given by equation (Λ^{ϕ}_5) and plug market clearing conditions (MC_6), (MC_8), and (MC_{11}) into it

$$m^{ST}_0 + \bar{m}^{LT} + \bar{d}^{\phi} - \frac{\bar{\mu}^{\phi}_d}{(1+\bar{r}^d)} - \frac{\mu^{\phi}_0}{(1+\rho_0)} - e^{\phi}_0 = 0$$

$$\frac{\mu^{\alpha}_0 + \mu^{\beta}_0}{(1+\rho_0)} + \frac{\bar{\mu}^{\alpha}}{(1+\bar{r}^m)} + \bar{d}^{\phi} - \bar{d}^{\beta} - \frac{\mu^{\phi}_0}{(1+\rho_0)} - e^{\phi}_0 = 0$$

Rearranging this and plugging it into the central bank's budget constraints above, we get

$$\frac{\mu_0^\alpha}{(1+\rho_0)} + \frac{\bar{\mu}^\alpha}{(1+\bar{r}^m)} + \frac{\mu_0^\beta}{(1+\rho_0)} - \bar{d}^\beta = m_0^{CB} + e_0^\phi$$

Noting that we can substitute household α's budget constraint in (Λ_1^α) into the first two terms above and household β's budget constraint in (Λ_1^β) into the middle two terms, we get

$$p_{h,0}q_{h,0}^\beta - m_0^\alpha + p_{p,0}q_{p,0}^\alpha - m_0^\beta = m_0^{CB} + e_0^\phi$$
$$\Rightarrow \quad p_{h,0}q_{h,0}^\beta + p_{p,0}q_{p,0}^\alpha = m_0^\alpha + m_0^\beta + m_0^{CB} + e_0^\phi$$

The central bank's budget constraint is therefore determined by the set of available endowments in the economy in the first period. Similar results can be derived for the second period. Moreover, the Quantity Theory of Money holds since the circulation of money in the economy directly affects prices and allocations. □

Appendix D The "Envelope Theorem Approach"

In this appendix, we sketch the basic approach that we attempt to take in order to retrieve numerical solutions to the partial derivatives - the changes of endogenous variables with respect to a change in the policy rates ρ_1, ρ_2, and CAR_0 in the optimization problem of the central bank - presented in Section 2.4. Our point of departure is the envelope theorem. Losely speaking, the envelope theorem for unconstrained optimization can be summarized as follows. For a function $f(x,p)$ with an endogenous variable x and a parameter p that we want to maximize, we retrieve a unique solution $x^*(p)$ for which the function achieves a maximum $f^*(p) = f(x^*(p),p)$. If we want to find the derivative of the value function f^* with respect to the parameter p, then we must only consider the direct effect of the parameter on the function, holding the value of x fixed at its optimal level $x^*(p)$. We can neglect the indirect effect that results from the change in the optimal value of x caused by a change in the parameter p. Formally,

$$\frac{d}{dp} f(x^*(p),p) = \frac{\partial}{\partial p} f(x^*(p),p)$$

where the indirect effect $f'(x^*(p),p)\dfrac{\partial x^*(p)}{\partial p}$ is zero. A similar reasoning applies to constrained optimization problems. We leave a more formal treatment to economics textbooks. The basic idea in the context of our model is to treat the policy rates ρ_1 and ρ_2 as parameters and the endogenous variables in (Λ_1^{CB}) as value functions, i.e. as functions that depend on other endogenous variables but are already optimized in equilibrium. Let's take loans from the central bank, μ_s^{ϕ}, as an example. We want to know how central bank lending to the bank in equilibrium changes when the policy rate changes, i.e. $\partial\mu_s^{*\phi}/\partial\rho_s$. The envelope theorem approach means that we do not care about the indirect effect of the policy rate via other endogenous variables

$$\frac{\partial\mu_s^{*\phi}}{\partial m_s^{*ST}} \frac{\partial m_s^{*ST}}{\partial\mu_s^{*\alpha}} \frac{\partial\mu_s^{*\alpha}}{\partial x} \cdots \frac{\partial x}{\partial\rho_s}$$

but only about the direct effect $\partial\mu_s^{*\phi}/\partial\rho_s$. The second point we want to make is that based on the envelope theorem, we can apply the chain rule to all optimality conditions, i.e. all first order conditions of all agents and market clearing conditions. All optimality conditions must hold in equilibrium. They must still hold in equilibrium after a change in the policy rate has taken place. Therefore, all changes in endogenous variables with respect to a change in the policy rate must add up to zero for a given set of optimal endogenous variables, $x^*(p)$.

We illustrate our approach in the following example. We plug the market clearing conditions into one of the first order conditions of household α, which gives [34]

$$u'(\, e_{p,0}^{\alpha} - q_{p,0}^{\alpha}) \;-\; \frac{p_{p,0}}{p_{h,0}(\,1+r_0^{ST})}\Big[u'(\, q_{h,0}^{\beta}) + \boldsymbol{\beta}^{\alpha}\gamma_1 u'(\, q_{h,0}^{\beta} + q_{h,1}^{\beta})\Big] \quad = 0$$

[34] For both households α and β we assume CRRA utility of the form $u(c_s^i) = (1-\rho)^{-1}(c_s^i)^{1-\rho}$ with the first and second derivatives given by $u'(c_s^i) = (c_s^i)^{-\rho}$, $u''(c_s^i) = (-\rho)(c_s^i)^{-\rho-1}$. Moreover, we neglect income transfers from the central bank to households for now in order to make the problem more tractable.

Applying the chain rule, we write the above optimality condition as a sum of partial changes with respect to the policy rates ρ_1 and ρ_2. Invoking the envelope theorem, we neglect all indirect effects via other endogenous variables in the system:

$$\begin{aligned}
&-u''(\,e^{\alpha}_{p,0}-q^{\alpha}_{p,0})\,\frac{\partial q^{\alpha}_{p,0}}{\partial \rho_1}-\frac{p_{p,0}}{p_{h,0}(1+r^{ST}_0)}\Bigg[u''(\,q^{\beta}_{h,0})\,\frac{\partial q^{\beta}_{h,0}}{\partial \rho_1}+\boldsymbol{\beta}^{\alpha}\gamma_1 u''(\,q^{\beta}_{h,0}+q^{\beta}_{h,1})\,\Big(\frac{\partial q^{\beta}_{h,0}}{\partial \rho_1}\\
&+\frac{\partial q^{\beta}_{h,1}}{\partial \rho_1}\Big)\Bigg]-\Big[u'(\,q^{\beta}_{h,0})\,+\boldsymbol{\beta}^{\alpha}\gamma_1 u'(\,q^{\beta}_{h,0}+q^{\beta}_{h,1})\Big]\Bigg[\frac{1}{p_{h,0}(1+r^{ST}_0)}\frac{\partial p_{p,0}}{\partial \rho_1}\\
&-\frac{p_{p,0}}{(p_{h,0})^2(1+r^{ST}_0)}\frac{\partial p_{h,0}}{\partial \rho_1}-\frac{p_{p,0}}{p_{h,0}(1+r^{ST}_0)^2}\frac{\partial r^{ST}_0}{\partial \rho_1}\Bigg]\quad=\quad 0
\end{aligned}$$

We repeat these steps for all first order conditions of agent α, agent β, and bank ϕ, and also with respect to the capital requirement CAR^*. We thus obtain a system of equations that are *linear* in partial derivatives. In theory, the endogenous variables that serve as coefficients on the partial derivatives to be solved for can be considered as "exogenous" because they are already optimized in our initial partial equilibrium. Note that all period-0 and period-1 variables change both in response to ρ_1, ρ_2, and CAR^*. Plugging all of these equations and the original optimality condition into a computer program, one can jointly solve for numerical values of all endogenous variables and all partial derivatives. The latter are inserted into the two FOCs of the central bank.

The downside of the "envelope theorem approach" is that we must make a number of strong assumptions. First, we assume that the initial partial equilibrium is already a general equilibrium such that all optimality conditions are fulfilled. However, we think that using computational methods to iteratively improve on the initial partial equilibrium will bring us closer to the true general equilibrium. Of course any equilibrium is local in nature and depends on the initial calibration used. Second, unless we make additional assumptions about the rationality of the central bank and its information set, the dimension of the system of simultaneous linear and non-linear equations will explode. We therefore assume that the central bank cannot understand the *indirect* effects of change in the policy rate on all endogenous variables in the system. It can only gauge the *direct* effects on the optimality conditions on which it has perfect information. Lastly, we abstract from strategic behavior. Households and the bank take the central bank's actions as given.

In conclusion, we believe that this approach is most flexible to being tailored to economic behavior under different assumptions about the beliefs of the central bank. It makes it possible to retrieve numerical values to solve for first order conditions of the central bank, and, possibly endogenously solve for a general equilibrium. However, we realize that an adequate initial equilibrium that serves as starting values is crucial to finding a solution. This makes the computational process more tedious and time-consuming.