

# Public Debt, Interest Rates, and Negative Shocks \*

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## Abstract

Debt-to-GDP ratios across developed economies are at historically high levels and government borrowing rates have remained persistently low. Blanchard (2019) provides evidence that the fiscal costs are low of increased government debt in low interest rate environments and that long-run average welfare effects can be positive. This paper attempts to replicate Blanchard's main results and tests their robustness to some key assumptions about risk in the model. This study finds that the attempted replication of Blanchard's stated approach results in no long-run average welfare gains from increased government debt and that those welfare losses are exacerbated if some strong risk-reducing assumptions are relaxed to more realistic values. Furthermore, I argue that the Blanchard calibration strategy also biases the results toward more beneficial government debt.

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# 1 Introduction

Then outgoing President of the American Economic Association, Olivier Blanchard, gave the AEA Presidential Address at the January 2019 annual meeting on a timely topic on which a consensus has not yet been established in the field and among policy makers. [Blanchard \(2019\)](#) provides evidence that the fiscal and welfare costs of public debt may be very small in economic environments of low interest rates. A significant contribution of his paper shows that the United States is in a prolonged period of low interest rates, calculates a careful measure of average borrowing rate for U.S. government debt, and provides evidence that this low-interest-rate environment is likely to persist. This topic of fiscal and welfare costs of public debt is also timely because debt-to-GDP ratios among developed countries are historically high, and the policy response to increased debt has been varied since the 2008-2009 global recession.

It is mechanically true that the fiscal cost of expanded public debt is low in a low-interest-rate environment. That is, if the borrowing rate for government debt is less than the rate of economic growth  $r_t < g_t$  and if new debt from the primary deficit  $x_{t-1}$  does not outsize the natural reduction in debt-to-GDP from its previous stock  $d_{t-1}$ , then the future debt-to-GDP ratio  $d_t$  falls.

$$d_t = \left( \frac{1 + r_t}{1 + g_t} \right) d_{t-1} + x_{t-1}$$

Despite the many interesting questions having to do with the dynamics of fiscal costs on the government budget constraint, this paper only addresses them indirectly. Instead, I focus on the welfare effect of increased debt in a low interest rate environment.

This paper attempts to replicate the stated approach of the [Blanchard \(2019\)](#) paper and explores the robustness of its “strong argument for using fiscal policy to sustain demand” in a persistent low-interest-rate environment with respect to two of the paper’s main assumptions. First, in my attempted replication of the Blanchard approach, I am not able to find any positive long-run average welfare gains from increasing public debt in any of the suggested calibrations.<sup>1</sup> Next, using Blanchard’s calibration strategy, I test whether his long-run average welfare effects of increased debt survive realistic increases in risk. In his model, Blanchard makes a strong assumption that forces the risk from public debt to be low. He assumes that each agent receives a “manna from heaven” consumption endowment when young that is large enough to preclude any form of government default on its commitment to transfer resources from the young to the old. The size of this assumed transfer is equal to the average wage an individual would expect to earn in a regime in which the government makes no fiscal tax on the young. Furthermore, this endowment does not enter into any government budget constraint or resource constraint and, therefore, provides a costless safety net to both individuals and government. This is a very strong assumption about risk exposure in this model economy.

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<sup>1</sup>Although the code for [Blanchard \(2019\)](#) is publicly available at <https://piie.com/system/files/documents/wp19-4.0.zip>, I was not able to isolate why his results and my results differed. However, there do seem to be inconsistencies in the listed axes and calibration values in his Figures 7 through 10.

A more subtle assumption of [Blanchard \(2019\)](#) is his calibration approach. The model is calibrated to match low average risky returns, low average riskless interest rates, and small average spreads between the two. The interaction between this calibration approach and the endowment assumption previously discussed bias Blanchard’s results toward positive welfare effects of increased debt.

A large literature connects fiscal stress to increasing equity premia or spreads between the risky return and riskless return. The [Blanchard \(2019\)](#) modeling approach is nearly identical to the approach of [Evans et al. \(2013\)](#), who show that increased government debt leads to more frequent default which in turn increases the interest rate spread. In particular, [Evans et al. \(2013\)](#) find that the equity premium increases as the economy gets closer to a default event.

[Rebelo et al. \(2019\)](#) study a model in which rare disasters generate increased hedging and savings behavior and increased credit spreads. [Tsai and Wachter \(2015\)](#) provide a broad survey of the rare disaster literature and its effect on asset prices, especially building off of the work by [Gourio \(2012\)](#) and [Barro \(2009\)](#). All of these papers find that rare negative events generate higher equity premia, more insurance and hedging behavior, and lower overall utility, even when the economy is most often in a moderate macroeconomic range.

The modeling assumptions of [Blanchard \(2019\)](#) doubly bias the results toward welfare improvements from increased debt in low interest rate environment. First, the assumption of an endowment that precludes government default gets rid of any catastrophic rare events. Furthermore, the calibration of the model to an assumed low interest rate spread implements a parameterization that is associated with low fiscal stress. The quantitative results of this paper provide evidence that public debt has significant welfare costs in many different calibrations—evidence counter to the findings of [Blanchard \(2019\)](#).

## 2 Economic Model

A detailed specification and derivation of the model is available in the online technical appendix.<sup>2</sup> The economic environment is an overlapping generations model with two-period-lived agents for which age  $s$  is indexed by  $s = 1$  for young and  $s = 2$  for old. Agents supply a unit of labor inelastically for the market wage  $w_t$  when young and are retired and supply no labor when old. Agents choose how much to consume when they are young  $c_{s=1,t}$  and old  $c_{s=2,t+1}$ , and they choose how much to save when young  $k_{s=2,t+1}$  which comes back to them at the risky interest rate when old. The household

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<sup>2</sup>See online technical appendix at [https://github.com/OpenSourceEcon/PubDebtNegShocks/blob/master/Evans2020\\_TechApp.pdf](https://github.com/OpenSourceEcon/PubDebtNegShocks/blob/master/Evans2020_TechApp.pdf).

optimization problem is the following,

$$\max_{k_{2,t+1}} (1 - \beta) \ln(c_{1,t}) + \beta \frac{1}{1 - \gamma} \ln\left(E_t[(c_{2,t+1})^{1-\gamma}]\right) \quad \forall t \quad (1)$$

$$\text{such that } c_{1,t} + k_{2,t+1} = w_t + x_1 - H_t \quad (2)$$

$$\text{and } c_{2,t+1} = R_{t+1}k_{2,t+1} + H_{t+1} \quad (3)$$

$$\text{and } c_{1,t}, c_{2,t+1}, k_{2,t+1} > 0 \quad (4)$$

where  $R_t$  is the gross return on risky savings and  $w_t$  is the wage on the unit of inelastically supplied labor by the young.

The functional form for lifetime utility in (1) is the Epstein-Zin-Weil utility used in [Blanchard \(2019\)](#).<sup>3</sup> The value  $x_1$  in the young age  $s = 1$  budget constraint (2) is the endowment that the young receive, and  $H_t$  is the lump sum government transfer taken from the young and given to the old each period. In general,  $H_t$  equals the promised amount  $\bar{H}$ . In [Blanchard \(2019\)](#), the endowment  $x_1$  guarantees that this is always the case. But I will allow  $x_1$  to be small enough that the government might not always be able to collect  $\bar{H}$  in every period, as is the case in [Evans et al. \(2013\)](#). I will specify  $H_t$  in more detail in Equation (12). The resulting Euler equation for optimal risky savings  $k_{2,t+1}$  is the following.

$$\frac{1 - \beta}{c_{1,t}} = \beta \frac{E_t \left[ R_{t+1} (c_{2,t+1})^{-\gamma} \right]}{E_t \left[ (c_{2,t+1})^{1-\gamma} \right]} \quad \forall t \quad (5)$$

We can independently derive the equilibrium price of a riskless bond, the exogenous supply of which is arbitrarily set to zero, as is shown in the technical appendix. Let  $\bar{R}_t$  be the return on the riskless bond (the inverse of the price). The derived Euler equation characterizing the equilibrium riskless bond return in each period is the following.

$$\bar{R}_t = \left( \frac{1 - \beta}{\beta} \right) \frac{E_t \left[ (c_{2,t+1})^{1-\gamma} \right]}{(c_{1,t}) E_t \left[ (c_{2,t+1})^{-\gamma} \right]} \quad \forall t \quad (6)$$

I assume a unit measure of identical perfectly competitive firms that rent capital  $K_t$  at rental rate  $r_t$  and hire labor  $L_t$  at wage  $w_t$  to produce consumption good output  $Y_t$  and maximize profits according to a constant elasticity of substitution production function with stochastic total factor productivity,

$$Y_t = F(K_t, L_t, z_t) = A_t \left[ \alpha (K_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha) (L_t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \quad \text{where } A_t \equiv e^{z_t} \quad (7)$$

where the capital share of income is given by  $\alpha \in (0, 1)$  and  $\varepsilon \geq 1$  is the constant elasticity of substitution between capital and labor in the production process. Total factor productivity  $A_t \equiv e^{z_t}$  is distributed log normally, and  $z_t$  follows a normally distributed  $AR(1)$  process.

$$z_t = \rho z_{t-1} + (1 - \rho)\mu + \epsilon_t \quad \text{where } \rho \in [0, 1) \quad \text{and } \epsilon_t \sim N(0, \sigma) \quad (8)$$

<sup>3</sup>See also [Epstein and Zin \(2013\)](#) and [Weil \(1990\)](#).

Two important special parameterizations of the production function (7) are the unit elasticity case  $\varepsilon = 1$  in which the limit is the Cobb-Douglas production function and the perfectly elastic case  $\varepsilon = \infty$  in which the production function is linear in  $K_t$  and  $L_t$  (perfect substitutes).

The firm's problem each period is to choose how much capital  $K_t$  to rent and how much labor  $L_t$  to hire in order to maximize profits,

$$\max_{K_t, L_t} Pr_t = F(K_t, L_t, z_t) - w_t L_t - R_t K_t \quad \forall t \quad (9)$$

where the marginal cost of capital is the gross interest rate  $R_t$  because the depreciation rate is assumed to be 100 percent. Profit maximization implies that the wage and interest rate are determined by the standard first order conditions for the firm.

$$R_t = \alpha(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[ \frac{Y_t}{K_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (10)$$

$$w_t = (1 - \alpha)(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[ \frac{Y_t}{L_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (11)$$

As can be seen from first order conditions (10) and (11), in the case of perfect substitutes (linear production,  $\varepsilon = \infty$ ), the first order conditions are independent of capital and labor.

Because the interest rate  $R_t$  in (10) is not defined when the capital stock is zero  $K_t = 0$ , the wage  $w_t$  in (11) is not defined when aggregate labor is zero  $L_t = 0$ , and output  $Y_t$  is not defined when capital or labor are less-than-or-equal-to zero, we know that both values must be strictly positive  $K_t, L_t > 0$  in equilibrium.

The government has committed to a balanced-budget lump-sum transfer each period  $\bar{H} \geq 0$  from the young to the old subject to feasibility of the transfer. Let  $c_{min} > 0$  and  $K_{min} > 0$  be minimum positive levels of consumption and aggregate capital. Then the government transfer rule characterizing  $H_t$  is that it equals  $\bar{H}$  except in periods when the promised transfer is greater than the total income minus minimum values of consumption and aggregate capital.<sup>4</sup>

$$H_t \equiv \begin{cases} \bar{H} & \text{if } w_t \geq \bar{H} - x_1 + c_{min} + K_{min} \\ w_t + x_1 - c_{min} - K_{min} & \text{if } w_t < \bar{H} - x_1 + c_{min} + K_{min} \end{cases} \quad \forall t \quad (12)$$

$$= \min(\bar{H}, w_t + x_1 - c_{min} - K_{min}) \quad \forall t$$

In equilibrium, the aggregate capital, labor, riskless assets, and goods markets

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<sup>4</sup>I remain agnostic about what happens after the government defaults on its promised transfer  $\bar{H}$  in any period in which  $w_t < \bar{H} - x_1 + c_{min} + K_{min}$  as shown in the second case in (12). This case forces the consumption of young agents to be the minimum value  $c_{1,t} = c_{min}$ . Technically, that household can survive beyond the default period because consumption is positive. [Evans et al. \(2013\)](#) study cases in which the government default causes either a complete economic shut down and reversion to autarky or cases in which it causes a regime shift to a new tax regime.

must clear. The goods market clearing condition (16) is redundant by Walras' Law.

$$K_t = k_{2,t} \quad \forall t \quad (13)$$

$$L_t = 1 \quad \forall t \quad (14)$$

$$0 = b_{2,t} \quad \forall t \quad (15)$$

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t \quad \forall t \quad (16)$$

$$\text{where } C_t \equiv c_{1,t} + c_{2,t}$$

Equilibrium is defined as stationary allocation functions  $a$  and price functions of the state for which household optimality conditions hold (5) and (6), firm optimality conditions hold (10) and (11), markets clear (13) and (14), and government transfers follow the feasible transfer rule (12).

### 3 Blanchard Calibration

The online technical appendix provides a detailed description and derivation of the calibration.<sup>5</sup> Table 1 shows the values of variables in the Blanchard (2019) calibration. Blanchard calibrates the capital share of income parameter  $\alpha = 1/3$ . He calibrates the annual standard deviation of the normally distributed component of  $z_t$  the total factor productivity process to be  $\sigma_{an} = 0.2$ , consistent with U.S. stock market returns historical average, which implies a model 25-year standard deviation of  $\sigma \approx 0.615$ .

**Table 1: Blanchard (2019) calibration values**

Variable	Value(s)	Variable	Value(s)	Variable	Value(s)
$\alpha$	0.33	$E[R_{t+1,an}]$	[0.00, 0.04]	$\beta$	func. of $E[R_{t+1}]$
$\varepsilon$	1.0 or $\infty$	avg. $\bar{R}_{t,an}$	[-0.02, 0.01]	$x_1$	func. of $E[R_{t+1}]$
$\rho_{an}$	0.95	$\mu$	func. of $E[R_{t+1}]$	avg. $k_{2,t}$	func. of $E[R_{t+1}]$
$\rho$	0.21	$\gamma$	func. of $E[R_{t+1}]$	$\bar{H}$	[0, 0.05(avg. $k_{2,t}$ )]
$z_0$	$\mu$		and avg. $\bar{R}_t$		
$\sigma_{an}$	0.200				
$\sigma$	0.615				

Given a calibrated value for  $\sigma$ , Blanchard (2019, p. 1213) identifies the value of  $\mu$  independently of  $\beta$  using the linear production ( $\varepsilon = \infty$ ) expression for the average value of the risky return, derived from marginal product of capital (10),

$$E_t[R_{t+1}] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} \quad \forall t \quad (17)$$

and calibrates the value for  $\gamma$  given  $\sigma$  from equilibrium expression for the spread between the log average risky return and the log riskless return derived from (10) and

<sup>5</sup>See online technical appendix at [https://github.com/OpenSourceEcon/PubDebtNegShocks/blob/master/Evans2020\\_TechApp.pdf](https://github.com/OpenSourceEcon/PubDebtNegShocks/blob/master/Evans2020_TechApp.pdf).

$$(6) \quad \ln\left(E_t[R_{t+1}]\right) - \ln(\bar{R}_t) = \gamma\sigma^2 \quad \forall t \quad (18)$$

For higher values of average risky returns  $E[R_{t+1}]$  the calibrated value of  $\mu$  is higher, which reduces risk and counterbalances the higher risky returns. And for larger average interest rate spreads, agents have higher risk aversion  $\gamma$ . Despite using these two specifications of the production function to calibrate  $\mu$  and  $\gamma$ , Blanchard analyses the cases of both the Cobb-Douglas production function ( $\varepsilon = 1$ ) and the perfect substitutes production function ( $\varepsilon = \infty$ ), separately.

Blanchard (2019) uses the Cobb-Douglas specification of the model ( $\varepsilon = 1$ ) to identify  $\beta$  independent of  $\mu$  and as a function of the average risky return.

$$\beta = \left(\frac{\alpha}{1-\alpha}\right) \frac{1}{2E[R_{t+1}]} \quad (19)$$

One of the main focuses of this paper is Blanchard's inclusion and calibration of the endowment to all young individuals  $x_1$ . He calibrates this value to be 100 percent of the average wage in the model in which the transfer is set to zero  $\bar{H} = 0$ .

$$x_1 = \left[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}(2\beta)^\alpha\right]^{\frac{1}{1-\alpha}} \quad (20)$$

This value constitutes a large safety net, and guarantees that the promised transfer never induces a default  $w_t \geq \bar{H} - x_1 + c_{min} + K_{min}$ . It is the effect of reducing this value  $x_1$  that will be the main experiment of this paper.

## 4 Simulations

The primary experiment of Blanchard (2019) is to measure the average change in realized lifetime utility of agents across simulations of the model from a baseline version of the model in which there is no government transfer program  $\bar{H} = 0$  to an economy in which the government transfer equals 5 percent of average savings  $\bar{H} = 0.05(\text{avg. } k_{2,t})$ .<sup>6</sup> I simulate 15 independent time series of 25 periods each and take averages.

Table 2 shows the percent change in average lifetime utility across simulations for nine different calibrations of the model based on all permutations of three values of average risky interest rates and average riskless interest rates and their implied spreads. The left-side panel of 3-by-3 results in Table 2 is a replication of Figure 7 in Blanchard (2019), and the right-side panel of 3-by-3 results is the replication of Figure 9 in Blanchard (2019).

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<sup>6</sup>I show the results in Tables 2 through 5 in percent change in average lifetime utility in which the levels used to calculate the units are in utils in order to remain consistent with the results in Blanchard (2019). However, it is probably more appropriate to show the results in percent change in consumption equivalent compensating variation, the solution of which is a trivial transformation of lifetime utility.

**Table 2: Percent change in average lifetime utility from increased transfer  $\bar{H}$ : constant  $\mu = 1.0786$**

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{r}$ (annual)			average $\bar{r}$ (annual)		
		-2.0%	-0.5%	1.0%	-2.0%	-0.5%	1.0%
average	0.0%	-0.59%	-0.59%	n/a	-0.78%	-0.77%	n/a
$r_t$	2.0%	-0.73%	-0.73%	-0.73%	-1.62%	-1.58%	-1.54%
(annual)	4.0%	-0.86%	-0.86%	-0.86%	-3.35%	-3.23%	-3.10%

\* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled “n/a” because that combination of average risky rate and average riskless rate implies a negative spread  $\text{avg. } R_t < \text{avg. } \bar{R}_t$ , which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

Notable is that the percent change in long-run average utility from an increase in the promised transfer  $\bar{H}$  is nowhere positive. Another notable difference in these results from Blanchard’s is that, although the qualitative relationship between welfare changes and respective risky and riskless interest rate changes are the same, the percent change in long-run average utility is most sensitive to different average risky returns and is relatively non responsive to different average riskless returns. This is opposite of Blanchard’s findings and is almost certainly a result of the calibrated parameter values shown in Table 1 being mostly functions of average risky returns and only  $\gamma$  being a function of average riskless returns.

It is unclear why Blanchard (2019, Figure 7) keeps  $\mu$  constant at 1.0786 in the simulations, which I attempted to replicate in Table 2, given that the calibration strategy in Equation (17) suggests that  $\mu$  should be a function of the average risky rate  $E[R_{t+1}]$ . The difference in  $\mu$  values is striking with calibrated values in the range  $\mu \in [0.91, 1.90]$  for average risky asset values in the range  $E[r_{t+1,an}] \in [0.00, 0.04]$ . Table 3 shows the percent change in average lifetime welfare when the calibrated value of  $\mu$  adjusts with the assumed average risky rate indicated in the different rows of the table.

**Table 3: Percent change in average lifetime utility from increased transfer  $\bar{H}$ : variable  $\mu$  as a function of  $E[R_{t+1}]$**

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{r}$ (annual)			average $\bar{r}$ (annual)		
		-2.0%	-0.5%	1.0%	-2.0%	-0.5%	1.0%
average	0.0%	-0.66%	-0.66%	n/a	-1.00%	-0.98%	n/a
$r_t$	2.0%	-0.31%	-0.31%	-0.31%	-0.52%	-0.51%	-0.49%
(annual)	4.0%	-0.16%	-0.16%	-0.16%	-0.32%	-0.31%	-0.30%

\* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled “n/a” because that combination of average risky rate and average riskless rate implies a negative spread  $\text{avg. } R_t < \text{avg. } \bar{R}_t$ , which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.



As with Table 2, all of the percent changes in average welfare from the increased transfer are negative. However, the direction of the relationship changes between percent changes in welfare and the calibrated average risky return. At higher average risky returns, the loss in welfare becomes smaller. A higher risky return is more than offset by the corresponding increase in  $\mu$  demanded by the calibration. It seems likely that, under this calibration strategy, there exists a higher risky return that would result in an increase in welfare from the increased transfer. But it is likely that this calibration strategy is not ideal.

I now proceed to test how the results of Table 3 change when more riskiness is added to the model. I first study the effect of reducing the endowment  $x_1$ . Table 4 shows the percent change in average lifetime utility across simulations from an increase in the transfer given the same calibrations of the model from Table 3 but with an endowment to the young that is equal to 50 percent of the average wage from the model in which there is no transfer—half the size of the endowment  $x_1$  in the Blanchard calibration. In this setting, the government can default on its promised transfer, which default implies minimal consumption for the young in the default period. And some simulations default before the maximal 25 periods. Table 5 shows the results for the highest risk environment in which the young agent endowment is completely removed  $x_1 = 0$ . In both Tables 4 and 5, the government can default on its promised transfers, which default happens more often than in the simulation from Table 4.

**Table 4: Percent change in average lifetime utility from increased transfer  $\bar{H}$ : variable  $\mu$  as a function of  $E[R_{t+1}]$ ,  $x_1 = 0.5x_{1,orig}$**

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{r}$ (annual)			average $\bar{r}$ (annual)		
		-2.0%	-0.5%	1.0%	-2.0%	-0.5%	1.0%
average	0.0%	-1.44%	-1.44%	n/a	-3.14%	-3.08%	n/a
$r_t$	2.0%	-0.55%	-0.55%	-0.55%	-1.28%	-1.23%	-1.19%
(annual)	4.0%	-0.27%	-0.27%	-0.27%	-0.71%	-0.68%	-0.65%

\* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled “n/a” because that combination of average risky rate and average riskless rate implies a negative spread  $\text{avg. } R_t < \text{avg. } \bar{R}_t$ , which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

The direction of welfare effects in Tables 4 and 5 with respect to different average risky and riskless asset calibrations remains the same as in Table 3. And the losses in welfare from the increased transfer become larger as the young agent endowment is reduced. The welfare losses from the transfer become particularly large in the Table 5 case in which the endowment is completely removed and in which average risky rates are zero (first row).

I tested the effect of holding the original endowment  $x_1$  constant and instead adding risk by implementing a mean preserving spread of the TFP shock. The effects

**Table 5: Percent change in average lifetime utility from increased transfer  $\bar{H}$ : variable  $\mu$  as a function of  $E[R_{t+1}]$ ,  $x_1 = 0$**

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{r}$ (annual)			average $\bar{r}$ (annual)		
		-2.0%	-0.5%	1.0%	-2.0%	-0.5%	1.0%
average	0.0%	-20.59%	-20.59%	n/a	-39.87%	-38.30%	n/a
$r_t$	2.0%	-1.83%	-1.83%	-1.83%	-19.05%	-18.01%	-17.00%
(annual)	4.0%	-0.73%	-0.73%	-0.73%	-5.84%	-5.43%	-5.04%

\* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled “n/a” because that combination of average risky rate and average riskless rate implies a negative spread avg.  $R_t < \text{avg. } \bar{R}_t$ , which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

of this type of increase in risk were predictably small and are reported separately in the online technical appendix.<sup>7</sup>

## 5 Conclusion

This paper attempts to replicate the modeling and calibration approaches of [Blanchard \(2019\)](#) and finds contrasting results that no calibrated parameterizations of the model produce positive long-run average utility changes from an increase in public debt. Furthermore, I find that those negative long-run welfare effects are exacerbated when Blanchard’s strong assumption of a large endowment to young agents is relaxed. Reducing the endowment results in an economic environment in which rare negative economic events can occur. A large literature described in the introduction has shown that rare negative events can produce large equity premia and welfare losses, even in moderate times leading up to the negative shocks. Finally, I argue that Blanchard’s calibration approach based on small equity premia or interest rate spreads further biases the results toward long-run average welfare enhancing government debt expansion.

Blanchard’s results provide support for governments to expand government debt in times of economic growth as long as interest rates are low enough. I provide evidence in this paper that the long-run welfare costs of expansionary debt policies might be significant for a wider range of model parameterizations than previously known.

## References

**Barro, Robert J.**, “Rare Disasters, Asset Prices, and Welfare Costs,” *American Economic Review*, March 2009, 99 (1), 243–64.

<sup>7</sup>See online technical appendix at [https://github.com/OpenSourceEcon/PubDebtNegShocks/blob/master/Evans2020\\_TechApp.pdf](https://github.com/OpenSourceEcon/PubDebtNegShocks/blob/master/Evans2020_TechApp.pdf).

- Blanchard, Olivier**, “Public Debt and Low Interest Rates,” *American Economic Review*, April 2019, *109* (4), 1197–1229.
- Epstein, Larry G. and Stanley E. Zin**, “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” in Leonard C. MacLean and William T. Ziemba, eds., *Handbook of the Fundamentals of Financial Decision Making*, New Jersey: World Scientific, 2013, pp. 207–239.
- Evans, Richard W., Laurence J. Kotlikoff, and Kerk L. Phillips**, “Game Over: Simulating Unsustainable Fiscal Policy,” in Alberto Alesina and Francesco Giavazzi, eds., *Fiscal Policy after the Financial Crisis*, University of Chicago Press, 2013, chapter 5.
- Gourio, François**, “Disaster Risk and Business Cycles,” *American Economic Review*, October 2012, *102* (6), 2734–2766.
- Rebelo, Sergio, Neng Wang, and Jinqiang Yang**, “Rare Disasters, Financial Development, and Sovereign Debt,” NBER Working Paper No. 25031, National Bureau of Economic Research May 2019.
- Tsai, Jerry and Jessica A. Wachter**, “Disaster Risk and Its Implications for Asset Pricing,” *Annual Review of Financial Economics*, 2015, *7*, 219–252.
- Weil, Philippe**, “Nonexpected Utility in Macroeconomics,” *Quarterly Journal of Economics*, February 1990, *105* (1), 29–42.