A Life Cycle Model with Unemployment Traps

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Motivation

- Stock market disasters display life-cycle effects (Fagereng and Guiso, 2017).
- We study Personal Disaster Risk (PDR): rare but large reduction in the permanent component of individual earnings
- We calibrate the model to unemployment, rather than bankruptcy, since most workers face a small risk of falling in an unemployment trap (UT)
  - Unemployment by duration (2014)

>27 weeks	*>52 weeks*	*>99 weeks*
33.5%	**22%**	**11%**

  - Only 11% of the long-term unemployed finds a job a year later; exit from labor force is likelier (Krueger et al., 2014)
  - The risk is small, but uniform across education groups (Katz et al., 2016)
  - Earnings losses are persistent (Jacobson et al., 2005) and increase with unemployment duration (Keane and Wolpin, 1997; Arulampalam et al., 1993)

Model in a Nutshell

- No unemployment risk (Cocco et al., 2005): Permanent and transitory earnings shocks
- Unemployment risk (Bremus and Kuzin, 2014): Three-state Markov chain
  - A young, employed agent either remains employed or becomes unemployed;
  - Next, if she stays unemployed, *her earnings fall*;
  - .....
- *Unemployment Trap*: % loss in the permanent component of earnings.
  - Deterministic: set to 0.6, including losses due to exit from the labor force.
  - Stochastic: expected loss at 0.2 delivers same results.
    - Beta distribution, with shape parameter putting most probability mass on low realizations of this loss
  - Transition matrix conservatively matches unemployment by duration

Unemployment Trap versus Other Cases

A rare personal disaster risk

- increases optimal savings and cautiousness when young: grandma's advice!
- flattens the optimal investment profile, due to higher uncertainty when young
- reduces the average skewness of consumption growth

- shrinks heterogeneity in optimal portfolios despite unequal career histories
- amplifies welfare losses of sub-optimal default investment rules (3-10 times as large), due to excess (insufficient) consumption when young (old)
- dampens sensitivity to (both inter-temporal and across assets) correlation due to skewness-inducing disaster

Contribution

- Normative analysis of the economics behind negative skewness in earnings
  - relevant in the data (Guvenen, Karahan, Ozkan and Song, 2015; Catherine, 2018; Galvez, 2017; Shen, 2018).
- Average implied skewness of consumption growth becomes negative, without reinforcing change in the labor income process
  - this improves asset pricing in Constantinides and Ghosh, 2014, and Schmidt, 2016
- Portfolio choice with non-Gaussian returns to human capital
  - instead of non-Gaussian financial returns (Guidolin and Timmerman, 2008)
- We add the rare personal disaster dimension to the following insights:
  - Resolution of uncertainty over working years
    - Bagliano et al., (2014); Hubener, Maurer and Mitchell (2016); Chang, Hong and Karabarbounis (2017)
  - Precautionary savings and employment insurance (Low, Meghir and Pistaferri, 2010)

The Model

- Finite horizon with uncertain lifespan

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j-1}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j})\, b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right]$$

- C_{it} level of consumption at time t; X_{it} wealth the investor leaves as bequest
- $b \geq 0$ strength of the bequest motive; $\beta < 1$ discount factor; γ CRRA.

Investment opportunities, with short-sales and borrowing constraints:

Portfolio return:

$$R_{it}^P = \alpha_{it}^s R_t^s + (1-\alpha_{it}^s)\, R^f \tag{1}$$

- R_t^f one-period risk-free return; α_{it}^s share invested in stocks; stock return:

$$\tilde{R}_t^s = R^f + \mu^s + \nu_t^s; \nu_t^s \sim N\left(0, \sigma_s^2\right)$$

Cash on hand

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{2}$$

Labor and Retirement Income

- Labor income process

$$Y_{it} = H_{it} N_{it} \qquad t_0 \leq t \leq t_0 + K \tag{3}$$

  - $H_{it} = F(t, \mathbf{Z}_{it}) P_{it}$ permanent income component
  - $F(t, \mathbf{Z}_{it}) \equiv F_{it}$ deterministic trend component
  - $\log(N_{it})$ is $N(0, \sigma_\varepsilon^2)$
  - Stochastic permanent component:

$$\log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

  - ω_{it} is $N(0, \sigma_\omega^2)$

- Retirement income

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+I}}\right) P_{it_{0+I}} \qquad t_0 + K < t \leq T \tag{5}$$

  - $t_0 + I$ last working period ; $t_0 + K$ retirement age
  - λ of the permanent component of labor income in the last working year

Labor Market Dynamics and Income

- Transition matrix

$$\begin{bmatrix} \pi_{e,e} & 1-\pi_{e,e} & 0 \\ \pi_{u_1,e} & 0 & 1-\pi_{u_1,e} \\ \pi_{u_2,e} & 0 & 1-\pi_{u_2,e} \end{bmatrix}$$

- Labor income depends on past working history, $0 \leq \Psi_j \leq 1$:

$$H_{it} = \begin{cases} H_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \quad (6)$$

- Cocco et al: $\pi_{e,e}=1$; Bremus et al: $\Psi_j = 0$

- Unemployment benefit

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\ 0 & \text{if } s_t = u_2 \text{ and } s_{t-1} = u_1 \text{ and } s_{t-2} = e \end{cases} \qquad t = t_0, ..., t_0 + K \quad (7)$$

Maximization problem

individual problem value function

Value function in each possible labor market state

maximization problem

Calibration: U.S. Unemployment and Benefits

- Transition matrix between labor market states implies conservative short (4.7%) and long-term (0.8%) unconditional probability of being unemployed:

$$\begin{bmatrix} \pi_{e,e} & 1-\pi_{e,e} & 0 \\ \pi_{u_1,e} & 0 & 1-\pi_{u_1,e} \\ \pi_{u_2,e} & 0 & 1-\pi_{u_2,e} \end{bmatrix} = \begin{bmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.85 & 0 & 0.15 \end{bmatrix}$$

- Unemployment benefits: $\xi_1 = 0.3$ Average before 26 weeks. After: $\xi_2 = 0$

- $\psi_1 = 0\$$; $\psi_2 = 60\%$
  - Persistent earning losses: 43-66% (Jacboson et al., 2005)
  - After 24 months: 40 % probability of finding a job; and 88%of exiting the labor force (Katz et al., 2016)

- Stochastic earnings loss (expected value 10%-20%, st.dev 20%-30%)

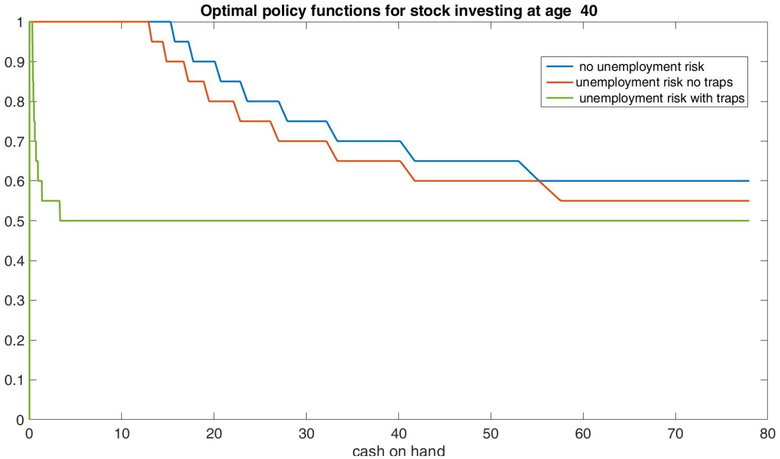
Calibration

Table 1. Calibration parameters

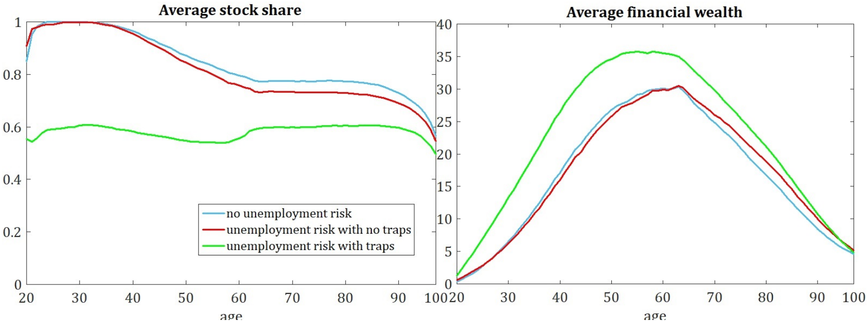
Working life (max)	20 -65
Retirement (max)	65 -100
Discount factor (β)	0.96
Risk aversion (γ)	5
Replacement ratio (λ)	0.68
Variance of permanent shocks to labour income (σ^2_ε)	0.0106
Variance of transitory shocks to labour income (σ^2_n)	0.0738
Riskless rate	2%
Excess returns on stocks (μ^s)	4%
Variance of stock returns innovations (σ^2_s)	0.025
Stock ret./permanent lab. Income shock correlation (ρ_{sY})	0

	No unemployment risk	**Unemployment risk with no traps**	**Unemployment traps**
Unemployment benefit			
Short term unemployed (ξ_1)	-	0.3	0.3
Long term unemployed (ξ_2)	-	0	0
Human capital erosion			
Short term unemployed (Ψ_1)	-	-	0
Long term unemployed (Ψ_2)	-	-	0.6

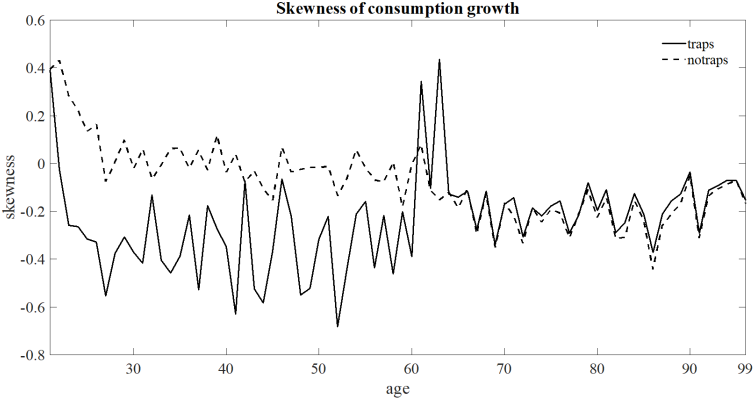
Optimal stock shares - all models



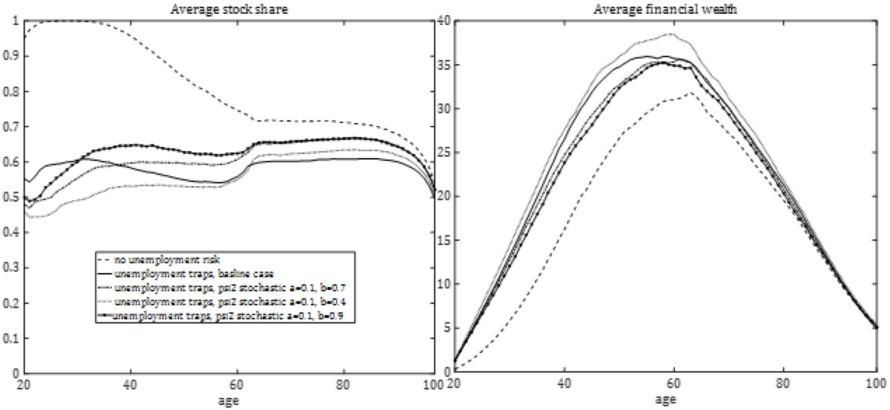
Optimal Life Cycle Profiles



Average Skewness of Consumption Growth



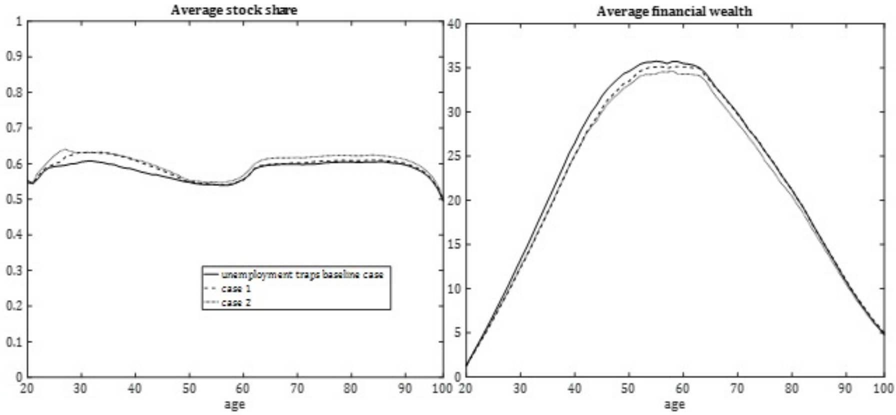
Stochastic loss due to long-term unemployment



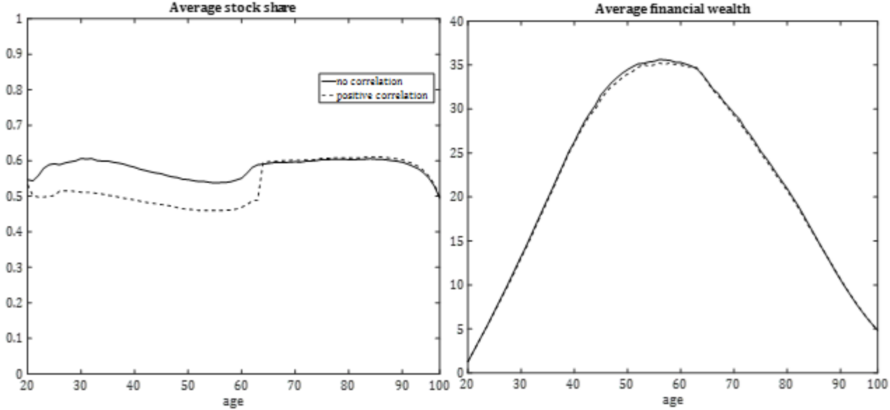
Skewness induces Robustness

- age-dependent unemployment risk
  - In 2015, U.S. overall unemployment rate 5.7%, LTU 1.7%
  - 20% 16-24 years old
  - 35% 25-55 years old
  - 41% over 55 years old
- Correlation between earnings and stock returns
- Epstein Zin preferences

Age-dependent long-term unemployment

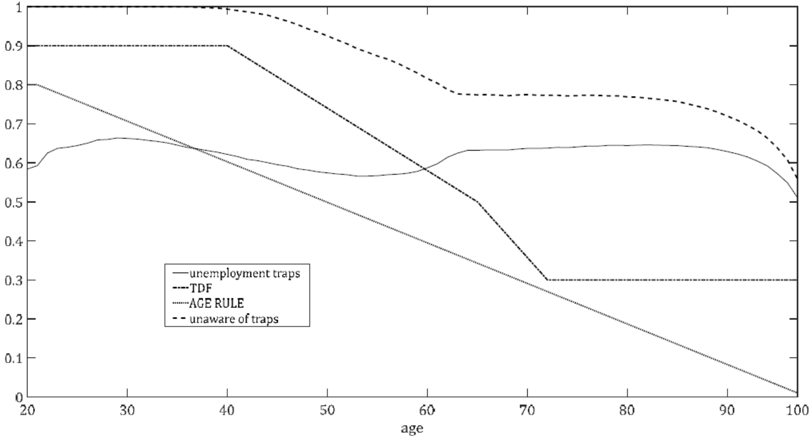


Correlation between earnings and stock returns



Default Investment Rules

Optimal and suboptimal life-cycle portfolio share profiles



Welfare Analysis of Default Investment Rules

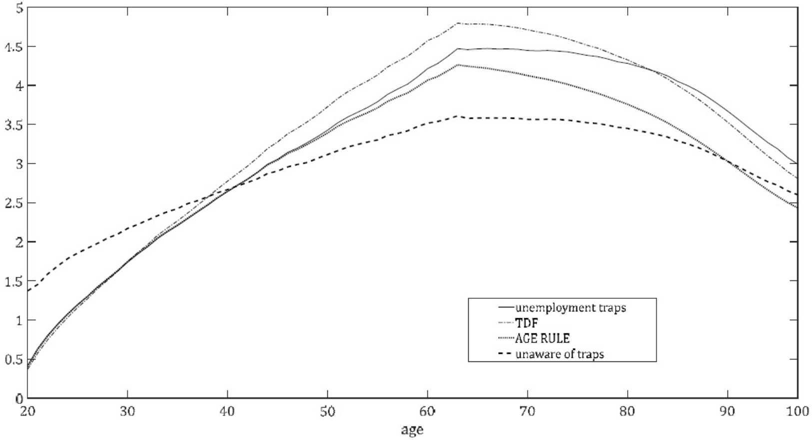
a. Distribution of welfare gains (% points)

	Age Rule	TDF rule	Unaware of Traps
mean	3.3	12.0	642.5
median	3.3	11.8	215.8
5th	1.5	8.0	-40.5
95th	5.4	17.0	573.6

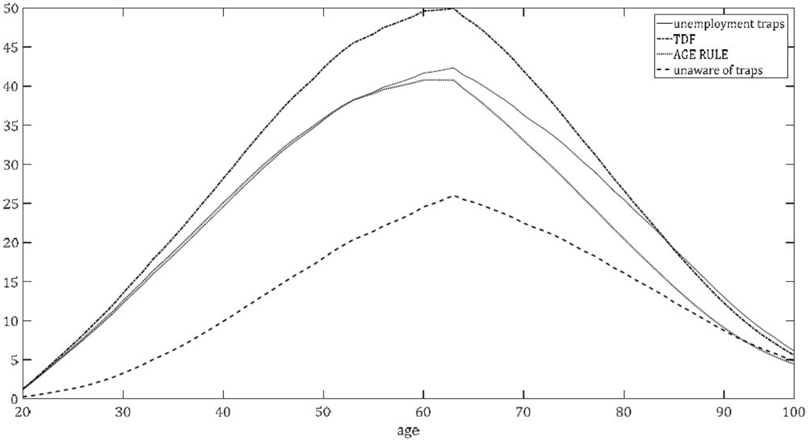
b. Welfare gains conditional on income at age 64 (% points)

	Age Rule	TDF rule	Unaware of Traps
Below 5^{th} percentile	1.6	9.5	1024.0
Above 95^{th} percentile	2.4	12.3	218.2

Optimal and Default Consumption Profiles



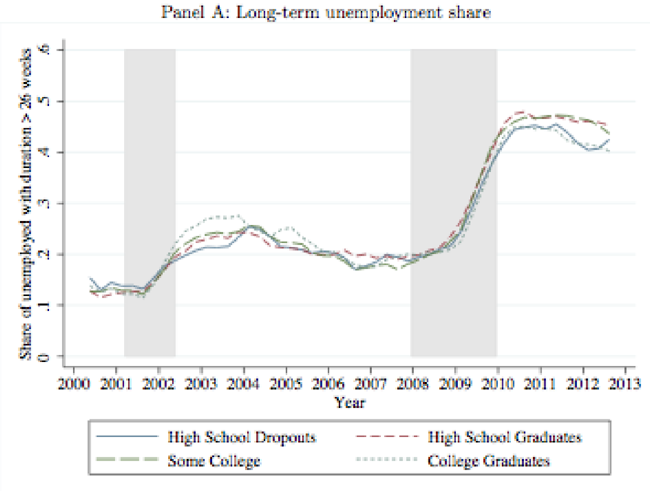
Optimal and Default Wealth Profiles



Conclusions

- We show the life-cycle implications of a rare personal disaster risk during working life (uninsured bankruptcy or unemployment)
    - inducing skewness in labor income and consumption
    - robustly changings the optimal savings/ risk-taking profile
- Calibrations to US long-term unemployment show that common Default Investment Rules may generate large welfare losses.

LTU Share and Education, 2000-13



Source: Katz et al., 2016

Maximization Problem

Individual's optimal program

$$\max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^{s}\right\}_{t_0}^{T-1}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^{j} \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right.\right.\right.$$
$$\left.\left.\left. + \left(1 - p_{t_0+j-1}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \quad (8)$$

$$s.t. \quad X_{it+1} = \left(X_{it} - C_{it}\right)\left(\alpha_{it}^{s} R_t^{s} + \left(1-\alpha_{it}^{s}\right) R^{f}\right) + Y_{it+1} \quad (9)$$

Dynamic Programming Form

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\left\{\alpha_{it}^{s}\right\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t\left[p_t V_{it+1}\left(X_{it_{+1}}, P_{it+1}, s_{it+1}\right) \right.\right.$$
$$\left.\left. + \left(1-p_t\right) b \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right)$$

individual problem | maximization problem

Value Function

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\left\{C_{it}\right\}_{t_0}^{T-1}, \left\{\alpha_{it}^{s}\right\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta \left[p_t \sum_{s_{it+1}=e_1,u_2} \pi\left(s_{it+1}|s_{it}\right)\right.\right.$$

$$\left.\left.\widetilde{E_t V}_{it+1}\left(X_{it_{+1}}, P_{it+1}, s_{it+1}\right) + \left(1-p_t\right) b \sum_{s_{it+1}=e_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right.$$

value function | maximization problem

Value function in each possible labor market state

$$V(X_{it}, P_{it}, e) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} \widetilde{E_t V}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{e,e} \\ \text{with } \; P_{it+1} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it+1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} \widetilde{E_t V}(X_{it+1}, P_{it+1}, u_1) \quad \text{with prob. } \pi_{e,u_1} \\ \text{with } \; P_{it+1} = (1 - \Psi_1) P_{it} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + \xi_1 F_{it} P_{it} \end{cases} \end{cases}$$

$$V(x_{it}, P_{it}, u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} \widetilde{E_t V}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_1) P_{it-1} \, e^{\omega_{it+1}} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} \widetilde{E_t V}(x_{it+1}, P_{it+1}, u_2) \quad \text{with prob. } \pi_{u_1,u_2} \\ \text{with } \; P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} = (1 - \Psi_2) P_{it} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} \end{cases} \end{cases}$$

$$V(X_{it}, P_{it}, u_2) = u(X_{it}) + \beta p_t \begin{cases} \widetilde{E_t V}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{u_2,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} e^{\omega_{it+1}} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-2} P_{it+1} e^{\varepsilon_{it+1}} \end{cases}$$