

The Maturity Premium ^{*}

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Abstract

We analyze asset-pricing implications of debt maturity. Firms with long debt maturities have weaker incentives to delever after negative shocks and therefore exhibit high leverage and high betas during downturns when the market price of risk is high. They also increase leverage less aggressively during booms. Thus, the betas of firms with longer debt maturities covary more with the market price of risk. As a result, they generate higher expected returns, controlling for average exposure to systematic risk. We demonstrate this in a model and document empirically a 0.21% monthly premium for buying long-maturity financed firms and selling those with shorter debt maturities.

JEL Classifications: G12, G32, G33.

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1 Introduction

We explore the asset-pricing implications of corporate debt maturity. What happens to the risk of a firm's equity if it decides to shorten the maturity of its debt claims? On the one hand, this increases the refinancing needs in the future and the firm becomes more exposed to rollover risk. This is so since refinancing conditions change with the firm's uncertain profitability. Deteriorating profitability leads to deteriorating debt refinancing conditions and thus forces equityholders to cut dividends or inject more equity. This rollover effect is more pronounced for short maturity firms, which implies that, *ceteris paribus*, their equity-holders walk away at higher cash flow thresholds than those of long maturity firms, as demonstrated by [Leland and Toft \(1996\)](#). On the other hand, short debt maturity causes less debt overhang, which encourages equityholders to delever if profits fall, as shown by [Dangl and Zechner \(2016\)](#). Unlike long-term financed firms burdened with debt overhang, short-maturity financed firms delever by not fully rolling over expiring debt when profits deteriorate, thereby reducing default risk.

This paper provides a first analysis of the effect of debt overhang associated with long maturities on equity returns. The classical view of debt overhang is that outstanding debt creates a conflict of interest between shareholders and existing bond holders, leading to under-investment ([Myers, 1977](#)). However, debt overhang can also distort decisions on the liability side of the balance sheet. In particular, it discourages reductions in leverage. This effect is referred to as the 'leverage ratchet effect' ([Admati et al., 2018](#)). It means that absent a-priori commitment, firms do not actively reduce their outstanding debt. When profits deteriorate, a reduction in leverage would decrease the default probability, increasing the value of both equity and still outstanding debt. This externality implies that equityholders' incentives to actively reduce debt are weaker than first-best. Therefore, shareholders of long-maturity financed firms expect market leverage to increase and stay elevated if profits fall. Shareholders of short-maturity financed firms, on the other hand, expect leverage to spike in response to a sudden drop in profitability, but then to revert to normal levels, as they do not fully rollover maturing short-maturity debt if fundamentals deteriorate ([Dangl and Zechner, 2016](#); [DeMarzo and He, 2018](#)).

The leverage dynamics of long-maturity financed firms drives co-movement between firms' betas and the market price of risk. Since longer debt maturities expose firms more to debt overhang, their leverage increases more and for a longer period during economic downturns, when the market price of risk is high.

The resulting co-movement between beta and the market price of risk generates a premium for holding equity of firms financed with long debt maturities. We call it a maturity premium. In our paper, we explore this premium both theoretically and empirically.

First, we show theoretically how differences in maturities give rise to a maturity premium using the framework introduced in [Dangl and Zechner \(2016\)](#) and [DeMarzo and He \(2018\)](#). Unlike classical models of rollover debt ([Leland and Toft, 1996](#)), we do not assume that firms can precommit to rollover decisions before issuing debt. This allows us to analyze the optimal rollover decisions of firms for bonds with different maturities. We demonstrate that an instantaneous deterioration of profitability leads to a sharper instantaneous market leverage increase for short-maturity financed firms. This confirms the standard intuition that having to rollover a higher fraction of debt imposes more short-term exposure to risk. However, we also demonstrate that firms financed with short-maturity debt optimally reduce the face value of debt following a deterioration of profitability, whereas long-maturity financed firms fail to do so. Thus, for longer holding periods, firms with long-maturity debt become riskier. Note that this result is strictly due to the endogenous rollover decisions, and not to the average level of leverage ([Choi, 2013](#)). Our numerical analysis demonstrates that this effect generates a maturity risk premium for plausibly calibrated model parameters.

We find that in a simulated panel of firms, debt maturity is positively related to equity returns, controlling for the average level of beta. Hence, the maturity premium is not due to the differences in average levels of leverage between long- and short-maturity financed firms. These would be accounted for by differences in average levels of beta. Rather, it is due to the co-movement between betas of long-maturity financed firms and the market price of risk. While the conditional CAPM holds in our setting, the co-movement between beta and the market price of risk shows up as alpha in the unconditional CAPM model. Thus, the alpha found in the unconditional model is not an anomaly, but a compensation for the risk of adverse increases in leverage and default probability in downturns, caused by long-maturity debt financing.

Furthermore, we examine the required returns of shareholders over various holding periods. We find that over a short period, short-maturity financed firms are more risky because of rollover risk and thus shareholders require higher expected returns. However, over longer holding horizons shareholders of long-maturity financed firms anticipate leverage increases in downturns and require compensation for that. Hence, required equity returns over longer holding horizons are higher for long-maturity financed firms. For all

firms, the expected equity returns increase with the holding horizon, because firms are more likely to either increase their leverage or default than delever.

Using the insights of our model, we examine empirically whether firms that are financed with longer maturity indeed earn higher returns than firms financed with shorter maturities. We analyze equity returns of firms in CRSP, which we match with firms' fundamentals from COMPUSTAT, from January 1976 to December 2017. We follow the standard procedure when constructing a factor by pre-sorting on size. We document a monthly 0.21% risk-adjusted premium for a portfolio that buys long-maturity financed firms and sells short-maturity financed firms. When we only consider firms with substantial leverage (top 20% of the most levered firms), the monthly maturity premium increases to 0.38%. The premium remains statistically significant after controlling for the size and value factors.

We examine the systematic risk-exposure of our maturity premium portfolio on a monthly basis. We document that long-maturity financed firms have larger increases in beta than short-maturity financed firms in months when the market risk-premium is high. However, the sharper is the drop in the market returns, the smaller is the difference between short- and long-maturity financed firms. That is, during severe market drops, short-term financed firms become as risky as long-term financed firms. This is a realization of rollover risk, as short-maturity firms have to re-finance in unfavorable market conditions and equity loses its value. Conversely, in months in which the market is contracting more slowly, long-maturity financed firms perform worse. This is consistent with the risk of not deleveraging in downturns because of debt overhang. We document statistically-significant increases in beta during market downturns for the long-short maturity portfolio. Our analysis demonstrates that accounting for time-variation in beta on a monthly basis explains at least a part of the maturity premium as measured by CAPM.

In our model the risk of increases in financial leverage generates a maturity premium. This finding relates to papers that have analyzed operating leverage as a possible source of risk and as an explanation for the value premium. Since value firms have exercised their growth options they tend to exhibit higher operating leverage, whereas growth firms tend to have low overhead costs and operating leverage (Zhang, 2005; Cooper, 2006). If operating leverage is sticky, then decreasing revenues drive the equity of value firms closer to zero than that of growth firms due to the difference in their operating leverage. Consequently, the beta of growth firms is mostly constant in time, while the beta of value firms increases substantially in

crises (Lettau and Ludvigson, 2001). Due to the fact that value firms are riskier in crises, they command an unconditionally higher required rate of return on their assets. While being plausible, the operating leverage alone cannot account for the entire size of value premium observed empirically (Clementi and Palazzo, 2015). To match the magnitude of the value premium, an extreme assumption of investment irreversibility is required, which contradicts empirical evidence on the sales of assets in the secondary market by at least 15% of firms in every given year.

In our paper, we demonstrate how long-maturity financial leverage contributes to the value premium. Financial leverage makes firms more sensitive to cash flow fluctuations in bad times, but only if the firm does not optimally delever. Short-maturity financed firms delever quickly, while long-maturity financed firms delever slowly or not at all. Therefore, the extent to which financial leverage can give rise to a value premium, depends on the difference in maturity choices of value and growth firms. Empirically, growth firms borrow with shorter maturities than value firms (Barclay and Smith, 1995; Barclay et al., 2003; Custódio et al., 2013). This can be attributed to lower cash-flow risk of value firms who have implemented their growth options.¹ The maturity choice is arguably driven by a trade-off between smaller investment debt-overhang (Myers, 1977) or financial debt overhang (Dangl and Zechner, 2016; DeMarzo and He, 2018) of short-term debt and higher transaction costs and higher default probability of long-term debt (Leland and Toft, 1996). Higher risk tilts the choice towards shorter maturities, that is why risky growth firms tend to borrow with short-maturity debt. Therefore, book-to-market acts as a noisy proxy of firms' maturity choices. Thus, long-maturity debt of value firms creates a convex shape of equity's beta as a function of the aggregate state. It is precisely this time variation in beta, which is not captured by the standard unconditional CAPM equation, that creates a value premium through a maturity and leverage dynamics channel.

More broadly, our paper contributes to the literature exploring asset pricing implications of corporate decisions. For example, Choi (2013) shows that a higher level of financial leverage of value firms contributes to the value premium. We argue that beyond the current level of debt the debt maturity plays a crucial role in generating an equity premium. Friewald et al. (2018) document an equity premium for rollover risk of firms with a larger fraction of their debt maturing within one year. While this result might appear to contradict our

¹ The presence of growth options increases the risk of firm's assets (Meckling and Jensen, 1976; Berk et al., 1999). The effect of options on equity risk is partially offset by the endogenously higher financial leverage of mature firms (Barclay et al., 2006), but not entirely — equity of growth firms is still riskier than equity of value firms, both in systematic (Shin and Stulz, 2000) and idiosyncratic (Cao et al., 2008) dimensions.

findings, in fact it is fully consistent with our hypothesis that short-maturity financed firms are risky over short holding horizons. [Friewald et al. \(2018\)](#) isolate the effect of rollover risk on firms over short horizons, considering leverage as fixed. Our analysis focuses on the combination of debt maturity and the dynamic adjustments of leverage. [Berk et al. \(1999\)](#), [Gomes and Schmid \(2010\)](#), [Kuehn and Schmid \(2014\)](#), and [Babenko et al. \(2016\)](#), among others, explore the implications of investment decisions and exercised growth options on equity returns.

[Chen et al. \(2018\)](#) analyze the distress risk puzzle based on a dynamic capital structure model. In their model firms are exposed to time-varying indirect distress costs, which drive the apparent under-performance of distressed firms. In contrast to their paper, we focus on the role of finite debt maturity, while their firms issue perpetual debt. Our setup provides a complementary rational explanation for the distress risk puzzle. In our model, short-maturity financed firms have higher leverage and default probabilities, but their betas co-vary less with the market price of risk. Relative to the unconditional CAPM, short-maturity financed firms seem to under-perform long-maturity financed firms, consistent with the return pattern that gave rise to the distress risk puzzle.

Finally, other aspects of corporate policy decisions, such as the fraction of secured and convertible debt ([Valta, 2016](#)), cash holdings ([Simutin, 2010](#)), debt capacity ([Hahn and Lee, 2009](#)), and competition in the production chain ([Gofman et al., 2018](#)) have been shown to be related to equity risk premia. We contribute to this literature by demonstrating that the maturity choices by firms influence future leverage dynamics and therefore command an equity premium. Capital structure adjustments in our model vary over the business cycle. [Hackbarth et al. \(2006\)](#) also derive a model where firms capital structures vary with the business cycle. In contrast to our paper, they do not model the effects of debt maturity or the resulting equity return dynamics.

We also contribute to the dynamic corporate finance literature by extending the framework of [Dangl and Zechner \(2016\)](#) and [DeMarzo and He \(2018\)](#) by explicitly modeling time-varying market risk premia and analyzing the asset-pricing implications of leverage dynamics in such a setting.

2 Model

In this section, we analyze the implications of different debt maturities for the dynamics of firm leverage. The key feature of our model is the ability of firms to choose the debt roll-over intensity. It means that firms optimally decide what fraction of their maturing debt to re-finance. We build on the models of [Dangl and Zechner \(2016\)](#) and [DeMarzo and He \(2018\)](#). Following [DeMarzo and He \(2018\)](#), equityholders cannot credibly commit to future leverage adjustments via contractual obligations. Thus, at every instant we allow equityholders to optimally choose the amount of new debt to be issued or repurchased. This setup allows for a tractable model of the link between debt maturity and leverage dynamics, accounting for debt overhang effects. While the model lacks features such as transactions costs or different debt seniority, it allows us to analyze the effect of debt maturity on equity risk premia.

2.1 Cash Flow

We consider a market comprised of heterogeneous firms. An individual firm's cash flow before paying interest and taxes, $Y_{i,t}$, is the product of two components, namely $Y_{i,t} = X_t \cdot I_{i,t}$. First, cash flows of all firms are driven by an aggregate productivity factor, X_t , which follows a geometric Brownian motion with time-varying drift $\mu(X_t, t)$ and volatility σ_X

$$dX_t = \mu(X_t, t)X_t dt + \sigma_X X_t dW_{X,t}^P \quad (1)$$

$$\mu(X_t, t) = \mu_0 - k [\log(X_t) - (\mu_0 - \sigma_X^2/2)t]. \quad (2)$$

Thus, the growth rate of the aggregate process is mean-reverting with a speed of k to its time-average of μ_0 .² The drift's deviations from μ_0 are due to X_t diverging from its expected growth path. During periods where X_t is above (below) the expected trajectory, expected growth rates are reduced (increased). Thus, the drift component introduces cyclical growth, i.e., a business cycle.

The firm-specific cash flows are orthogonal to the aggregate state variable, and are determined by a firm-specific idiosyncratic factor $I_{i,t}$, which is independent across firms. It follows a geometric Brownian motion

² Note that while $\mathbb{E}_0[\mu(t, X_t)] = \mu_0$, it is not true that $\mathbb{E}_0[X_t] = X_0 e^{\mu_0 t}$. In fact, $\mathbb{E}_0[X_t] < X_0 e^{\mu_0 t}$, and the reason the aggregate process grows at a smaller rate than it would if the drift-process was not mean-reverting is in the negative covariance between μ_t and X_t . $\mathbb{E}_0[X_t e^{\mu_t t}] = \text{Cov}(X_t, e^{\mu_t t}) + \mathbb{E}_0[X_t] \mathbb{E}_0[e^{\mu_t t}] < \mathbb{E}_0[X_t] e^{\mu_0 t}$. It is also true that $\mathbb{E}_0[e^{\mu_t t}] < e^{\mu_0 t}$ due to Jensen's inequality.

without a drift:

$$dI_{i,t} = \sigma_i I_{i,t} dW_{i,t}^P. \quad (3)$$

Given the multiplicative combination of the variables, the resulting cash flow $Y_{i,t}$ of a firm i also follows a geometric Brownian motion (under the physical measure)

$$dY_{i,t} = \mu(X_t, t) Y_{i,t} dt + \sigma_Y Y_{i,t} dW_{Y_{i,t}}^P, \quad (4)$$

where $\sigma_Y = \sqrt{\sigma_X^2 + \sigma_i^2}$ and $dW_{Y_{i,t}}^P = (\sigma_X dW_{X,t}^P + \sigma_i dW_{i,t}^P) / \sigma_Y$ govern the stochastic part. Moreover, under the risk neutral measure, a firm's cash flows are given by

$$dY_{i,t} = \mu_Y Y_{i,t} dt + \sigma_Y Y_{i,t} dW_{Y_{i,t}}^Q, \quad (5)$$

where $\mu_Y < r$. In Section 3.1, we further specify the Girsanov kernel associated with this measure change from no-arbitrage conditions for the market portfolio, and characterize μ_Y . We take the consumption process of the representative consumer in the economy as given, so under our assumptions the financing decisions of a firm do neither impact the change of measure nor the market price of risk.

2.2 Debt and Equity Valuation

Consider a firm that issues debt with face (book) value $F_{i,t}$. The bond pays a fixed coupon rate c that is tax-deductible. The marginal tax rate is denoted by τ . In the spirit of finite maturity debt models (e.g., [Leland \(1994\)](#) and [Leland \(1998\)](#), among others), we consider a debt structure, where a constant fraction m_i of outstanding bonds matures every period. The average maturity of outstanding debt is $1/m_i$, which is constant even if the firm stops rolling over maturing debt. Hence, cash flows to debt holders in the absence of default are given by the coupon payments and the retirement of debt $(c + m_i)F_t dt$. In default we assume a zero recovery. When the firm is founded, the firm chooses a debt maturity, which is then held constant throughout the firm's life. Since it can be shown that the firm founders are indifferent between alternative debt maturities we take maturity as an exogenous parameter.

The firm can issue new debt with a face value $G_{i,t}$. Negative values of $G_{i,t}$ represent voluntary retirements. As long as $G_{i,t}$ is less than or equal to the maturing debt, $m_i F_{i,t}$, then the firm's total face value of debt is either reduced or stays constant. In contrast to [Dangl and Zechner \(2016\)](#) and in accordance with [DeMarzo and He \(2018\)](#), firms in our model are also allowed to increase debt smoothly by issuing more than the maturing fraction of debt, i.e., choosing $G_{i,t} > m_i F_{i,t}$. Consequently, the dynamics of the outstanding face value of debt are given by:

$$dF_{i,t} = (G_{i,t} - m_i F_{i,t}) dt. \quad (6)$$

Next, we take a look at the distributions to equity owners. We abstract from transaction costs of issuing either debt or equity. Hence, the residual cash flow net of debt-related payments and taxes, given by

$$\Pi_{t,t+dt}^i = \{Y_{i,t}(1 - \tau) + \tau c F_{i,t} - (c + m_i) F_{i,t} + G_{i,t} v_{i,t}^D\} dt \quad (7)$$

is distributed to equityholders. The first term represents the operating cash flows before interest. As the coupons are tax deductible the tax benefit of debt, expressed by the second term, is added. The third and fourth term are related to the leverage adjustments. First, the currently outstanding debt $F_{i,t}$ has to be serviced by paying coupons and retiring the maturing portion. Second, new debt is issued (or bought back if $G_{i,t}$ is negative) at market prices $v_{i,t}^D$.

The market values of equity and debt claims, $V_{i,t}^E$ and $V_{i,t}^D$, are given by the conditional expectations of their respective future cash flows under the risk-neutral measure Q :

$$V_i^E(Y_{i,t}, F_{i,t}) = \mathbb{E}_t^Q \left[\int_t^{t_b} e^{-r(s-t)} \Pi_{t,s}^i ds \right], \text{ and} \quad (8)$$

$$V_i^D(Y_{i,t}, F_{i,t}) = \mathbb{E}_t^Q \left[\int_t^{t_b} e^{-(r+m_i)(s-t)} (c + m_i) ds \right] F_{i,t}, \quad (9)$$

where t_b denotes the time when the equity owners endogenously decide to declaring default of the firm.

We restrict the solution space to policy functions $G_{i,t}$ which are continuous in the state variables, i.e., the debt issuance policy is smooth. The equity maximization problem involves solving the Hamilton-Jacobi-Bellman equation, which is homogeneous in the face value of debt $F_{i,t}$. Therefore, we scale every variable

by $1/F_{i,t}$, and use lower case letters to indicate the scaled version, e.g., $y_{i,t} = Y_{i,t}/F_{i,t}$ throughout.

Using the valuation principles from DeMarzo and He (2018), we find the scaled value of equity³:

$$v_i^E(y_{i,t}) = \frac{1-\tau}{r-\mu_Y} y_{i,t} - \frac{c(1-\tau)+m_i}{r+m_i} \left(1 - \frac{1}{1+\gamma_i} \left(\frac{y_{i,t}}{y_{b,i}} \right)^{-\gamma_i} \right), \quad (10)$$

$$\gamma_i = \frac{(\mu_Y + m_i - \sigma_Y^2/2) + \sqrt{(\mu_Y + m_i - \sigma_Y^2/2)^2 + 2\sigma_Y^2(r+m_i)}}{\sigma_Y^2} > 0,$$

$$y_{b,i} = \frac{\gamma_i}{1+\gamma_i} \frac{r-\mu_Y}{r+m_i} \left(c + \frac{m_i}{1-\tau} \right),$$

where $y_{b,i}$ denotes the endogenously chosen scaled cash flow where the equityholders default.

Moreover, from the solution to the equity-maximization problem we can derive the value of debt. Given the fact that equityholders can adjust the outstanding amount of debt freely, the equilibrium price of debt $v_i^D(y_{i,t})$, i.e., the marginal benefit from debt issuance, will equal the marginal cost of future obligations, given by $-\partial V^E(Y, F)/\partial F$. Hence, the price of debt per unit of face value equals

$$v_i^D(y_{i,t}) = \frac{c(1-\tau)+m_i}{r+m_i} \left(1 - \left(\frac{y_{i,t}}{y_{b,i}} \right)^{-\gamma_i} \right). \quad (11)$$

2.3 Debt Issuance Policy and Leverage Dynamics

The optimal debt issuance policy function $g_{i,t}$ is a key driver of leverage dynamics. As shown in Appendix A the debt issuance policy function is given by

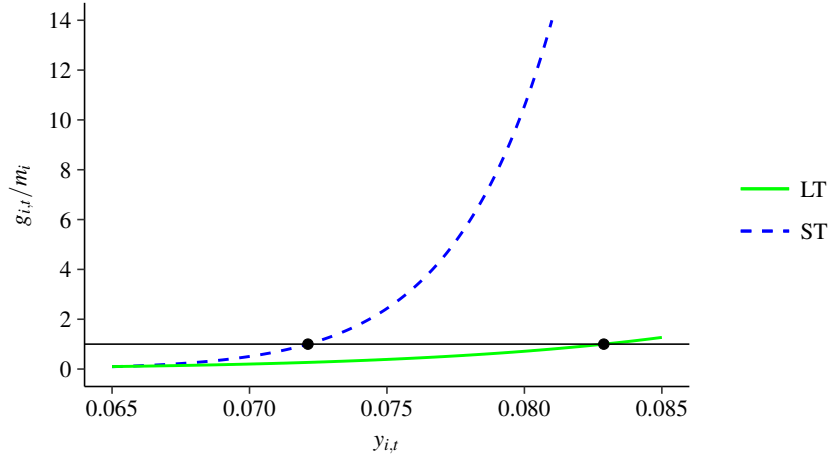
$$g_i(y_{i,t}) = m_i \left(\frac{y_{i,t}}{y_{m,i}} \right)^{\gamma_i}, \quad (12)$$

where $y_{m,i}$ denotes the scaled cash flow level at which the firm's issuance rate is exactly equal to the maturity rate m_i . It equals to:

$$y_{m,i} = y_{b,i} \left(\gamma_i \frac{c(1-\tau)+m_i}{(r+m_i)\tau c} m_i \right)^{1/\gamma_i}. \quad (13)$$

³ While we present only the closed-form solutions in this section, Appendix A presents the details on how to solve the model.

Figure 1: Optimal Roll-Over Rate. This graphs show the optimal roll-over rate of debt, which is given by the issuance policy $g_{i,t}$ scaled by the maturity rate m_i . This ratio equals one when the firm's net issuance is zero. The short- and long-maturity financed firms are characterized by $m_i = 0.5$ and $m_i = 0.2$, i.e., a debt maturity of 2 (ST) and 5 (LT) years, respectively. The volatility of the cash flows is $\sigma_X = 0.15$ and $\sigma_i = 0.15$. The solid (dashed) line represents the roll-over rate of the LT (ST) firm.



At the scaled cash flow level $y_{m,i}$ the firm keeps the outstanding amount of debt constant. Hence, for any level of cash flows $Y_{i,t}$, the face value of debt that results in the scaled level of cash flows of $y_{m,i}$, i.e., $F_{m,i,t} = Y_{i,t}/y_{m,i}$, is the target face value of debt.

Equation (12) implies that the net debt issuance is non-negative. This means that shareholders never actively repurchase debt, even though there are no associated transaction costs. This illustrates the leverage ratchet effect of [Admati et al. \(2018\)](#), and the debt-overhang problem that existing debt creates. Second, the roll-over rate positively depends on cash flow shocks, meaning that firms with higher cash flows per unit of face value issue more debt. Figure 1 illustrates the optimal debt issuance policy functions graphically for different levels of cash flow shocks and different maturities of debt. The long and short-term financed firms have different levels of optimal leverage. As short-term financed firms have higher target leverage levels, there are cash flow values $y_{i,t}$ for which short-term financed firms issue debt, while long-term financed firms reduce leverage through partial roll-over, everything else equal. However, the short-term financed firms respond more aggressively to changes in cash flows than long-term financed firms. They are relatively more aggressive at both increasing the leverage after positive cash flow shocks, and decreasing leverage after negative cash flow shocks.

The market leverage in our model, given by:

$$L_{i,t} = \frac{v_{i,t}^D}{v_{i,t}^E + v_{i,t}^D}. \quad (14)$$

changes over time for two reasons — the firm actively manages the face value of debt outstanding $F_{i,t}$, and the value of the firm's assets changes. The face value of debt can increase or decrease over time, as the firm sometimes decides to issue additional debt, while at other times optimally lets the debt mature and does not roll it over completely. The dynamics of $F_{i,t}$ depend on the realized path of the cash flow process in the following way

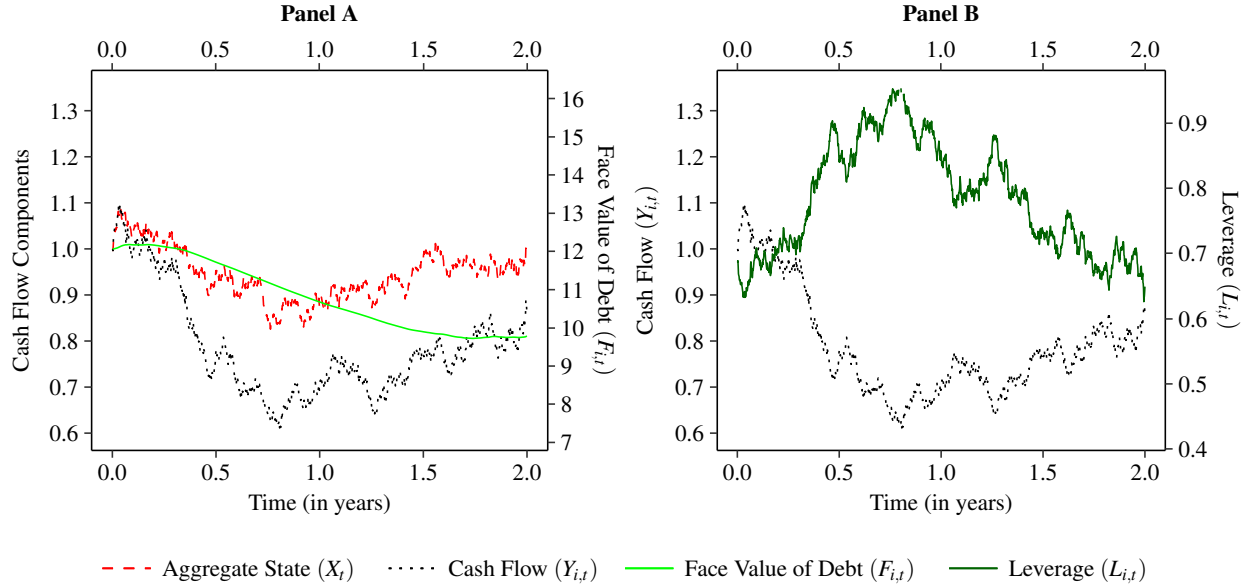
$$F_{i,t} = \left(\int_0^t \gamma_i m_i \left(\frac{Y_{i,s}}{y_{m,i}} \right)^{\gamma_i} e^{\gamma_i m_i (s-t)} ds \right)^{1/\gamma_i}. \quad (15)$$

Let us consider the dynamics of the face value of debt of a firm that first experiences first a decrease and then an increase as illustrated in Figure 2. The graph in Panel A depicts the realizations of the aggregate process and the cash flow process. The difference between the two lines is due to the idiosyncratic risk component. The firm is long-term financed, with an average bond maturity of 5 years. The right-hand y-axis depicts the evolution of the face value of debt. Following a decrease in cash flows, the firm starts reducing its outstanding debt. The reduction process is gradual and slow, in each period the firm is rolling over only a fraction of its maturing debt. When cash flows increase, the firm starts issuing debt. The corresponding evolution of market leverage is shown in Panel B of Figure 2. Its path follows that of the face value of debt, with fluctuations around that path reflecting changes in the market value of equity and debt due to the stochastic cash flow shocks.

2.4 The Leverage Ratchet Effect and Maturity

The goal of our theoretical model is to establish the effect of different debt maturities on the dynamics of leverage over a profitability cycle. In this subsection, we look at the evolution of market leverage of two firms — one financed with long-term debt (low m_i) and one financed with short-term debt (high m_i) — that were hit with the same sequence of cash flow realizations. Our focus is on the difference in leverage responses between the two firms.

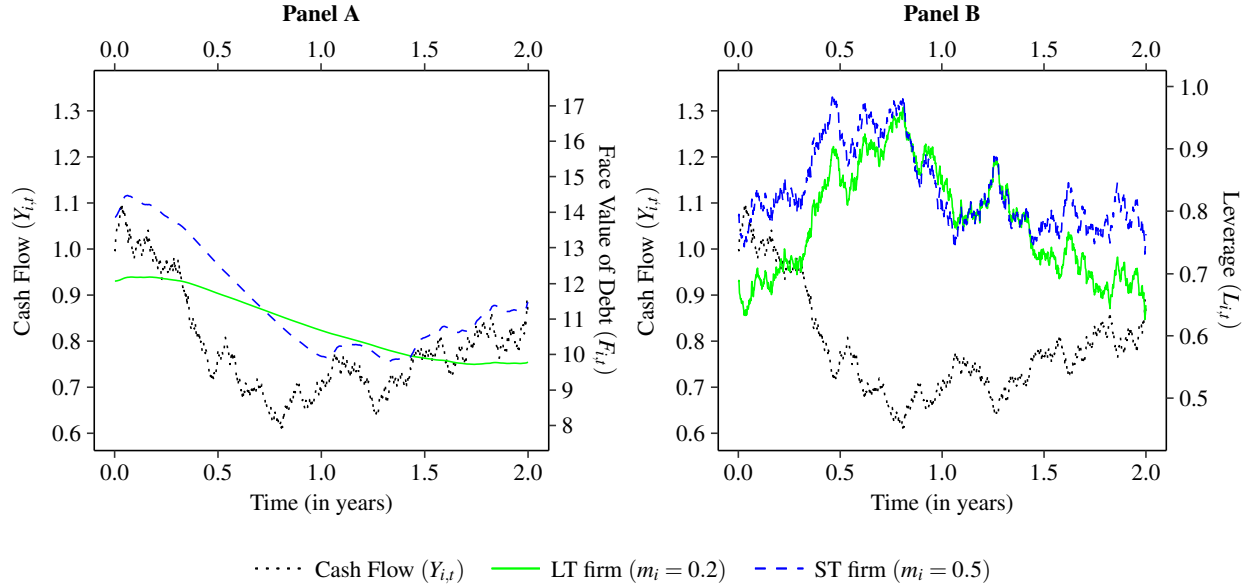
Figure 2: Evolution of Leverage. This figure illustrates the dynamics of cash flows and leverage for one firm. The graph in Panel A depicts dynamics of the aggregate state process X_t , the cash flows $Y_{i,t}$, and the face value of debt $F_{i,t}$. Panel B shows the dynamics of leverage. The parameters for this simulation are: $\mu_0 = 5\%$, $k = 0.25$, $\sigma_X = 15\%$, $\sigma_i = 15\%$, $r = 5\%$, $\delta = 4\%$, $c = r/(1 - \tau)$, $\tau = 30\%$. The LT firm is has an average maturity rate of $m_i = 0.2$ (5 years), while the ST firm has $m = 0.5$ (2 years).



Following [Admati et al. \(2018\)](#), we define the ratchet effect of leverage as shareholders not willing to actively repurchase debt following a deterioration of market conditions. In the notation of our model, we see that $g_{i,t} > 0$, which means that firms never actively repurchase debt, even though it is frictionless to doing so (no transaction costs on repurchasing of debt). The reason for this lies in the debt overhang that existing debt imposes on shareholders. However, as pointed out by [Dangl and Zechner \(2016\)](#) and by [DeMarzo and He \(2018\)](#), this intuition does not apply one-to-one to the refinancing of maturing debt. Shareholders sometimes find it optimal to roll over only a fraction of maturing debt, effectively reducing their leverage. Therefore, the amount of maturing bonds is the maximum by which the firm reduces its outstanding debt. Long-term financed firm are slow to decrease debt, while short-term financed firm respond relatively fast to negative profitability shocks. We illustrate this intuition in [Figure 3](#).

The graph in [Figure 3](#) Panel A illustrates the different adjustments of the face value of debt between a short-term and a long-term financed firm, where both firms experience the same cash flows. The face value of debt for the short-term financed firm follows ups and downs of the cash flows process very closely. This

Figure 3: Debt Maturity and the Leverage Ratchet Effect. This figure illustrates the differences in leverage dynamics for a short- (dashed lines) and long-maturity (solid lines) financed firms (referred to ST and LT, respectively). Panel A (Panel B) shows the face value of debt $F_{i,t}$ (leverage $L_{i,t}$) for two firms facing the same cash flow process $Y_{i,t}$. The parameters for this simulation are: $\mu_0 = 5\%$, $k = 0.25$, $\sigma_X = 15\%$, $\sigma_i = 15\%$, $r = 5\%$, $\delta = 4\%$, $c = r/(1 - \tau)$, $\tau = 30\%$. The LT firm has an average maturity rate of $m_i = 0.2$ (5 years), while the ST firm has $m = 0.5$ (2 years).



is not the case for the long-term financed firm. Its face value responds less to cash-flow fluctuations, which is most noticeable when cash flows decrease — the face value of debt also decreases, but much slower. As a result, we see in Panel B that the leverage of the long-term financed firm increases much more than the leverage of the short-term financed firm due to deterioration of cash flows. These dynamics are due to the leverage ratchet effect, which manifests itself in the slow deleveraging process for the long-term financed firm.

3 Asset Pricing Implications of Debt Maturity

In this section, we explore the asset-pricing implications of different maturities of debt. The focus of our analysis are the differences in leverage dynamics, their effect on the dynamics of equity betas, and the resulting perceived alphas.

3.1 Market Return and the Market Price of Risk

We consider the market to be populated by many firms, not only those that we analyze in the previous section. Individual firm's decisions and composition of surviving firms does not affect the dynamics of the market portfolio in our analysis, reminiscent of our assumptions that firms' financing decisions do not affect the market price of risk. The market portfolio $M(X_t)$ is driven by the aggregate productivity level X_t , which is defined in Equation (1). This market portfolio is traded and its return over a time increment is:

$$r_{t,t+dt}^M = (\mu(X_t, t) + \delta) dt + \sigma_X dW_{X,t}^P, \quad (16)$$

where $\delta > 0$ represents aggregate dividends. Assuming no-arbitrage and complete markets we change to the risk neutral measure. Given that the market portfolio is traded, its risk neutral drift equals the risk-free rate r . The market price of risk is therefore given by a Girsanov transformation as:

$$\lambda_t = \frac{(\mu(X_t, t) + \delta - r)}{\sigma_X}. \quad (17)$$

It is time-varying due to the variation in $\mu(X_t, t)$, as shown in Equation (2). Furthermore, we denote by η_t the market risk premium for bearing systematic risk, which equals $\eta_t = \sigma_X \lambda_t$.

The risk-neutral drift of a firm's cash flows, μ_Y , consistent with the no-arbitrage condition is given by $\mu_Y = r - \delta$. It follows from writing the cash flow process under the risk-neutral measure that:

$$\frac{dY_t}{Y_t} = \mu(X_t, t) dt + \sigma_X dW_{x,t}^P + \sigma_i dW_{Y_i,t}^P = (\mu(X_t, t) - \sigma_X \lambda_t) dt + \sigma_X dW_{x,t}^Q + \sigma_i dW_{Y_i,t}^Q = \mu_Y dt + \sigma_Y dW_{Y_i,t}^Q. \quad (18)$$

3.2 Equity Returns and Equity Beta

Next, we turn our attention to the analysis of the link between leverage and systematic exposure of the firm, i.e., its beta. Instantaneous equity returns to equityholders can be computed as:

$$r_{t,t+dt}^E = \frac{dV_t^E + \Pi_{t,t+dt}}{V_t^E}. \quad (19)$$

Utilizing the equity-pricing equation (see details in Appendix B):

$$r_{t,t+dt}^E = r dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \eta_t dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \sigma_Y dW_{Y,t}^P, \quad (20)$$

we arrive at a decomposition of equity returns that consists of three components: the risk-free rate, the market price of risk times the exposure to the systematic risk, and a random component.

Under the risk-neutral measure the expected value of equity returns is just the risk-free rate r .⁴ Under the physical measure it is:

$$\begin{aligned} \mathbb{E}_t^P [r_{t,t+dt}^E] &= \mathbb{E}_t^P \left[r dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \eta_t dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \sigma_Y dW_{Y,t}^P \right] \\ &= r dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \eta_t dt. \end{aligned} \quad (21)$$

This expression illustrates that the conditional CAPM holds in our setting. The asset beta is normalized to one in our setting, and the equity beta is then one plus debt over equity, i.e., $\beta_{i,t} = 1 + \frac{v_{i,t}^D}{v_{i,t}^E}$, while η_t represents the time-varying market risk premium.⁵

We think about $\beta_{i,t}$ as representing a scaling of each firm's asset betas. In our model asset beta is normalized to one, but in reality firms differ substantially in the systemic exposure of their physical assets. The variations in beta that we analyze are on top of any differences in asset betas. While betas in our setting are by construction larger than one, we think of them as representing an amplifying factor relative to the asset beta of each firm. For example, a beta of 1.3 in our setting corresponds to an equity beta of a real firm that is 30% larger than its asset beta, which is due to financial leverage. Therefore, while all betas in our model are above one, our model nevertheless is consistent with real data, once heterogeneity in asset betas is taken into account.

⁴ $\mathbb{E}_t^Q [r_{t,t+dt}^E] = \mathbb{E}_t^Q \left[r dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \sigma_Y dW_{Y,t}^Q \right] = r dt.$

⁵ Naturally, we obtain the same result if we derive beta using a classical formula $\beta_{i,t} = \frac{\text{Cov}_t(r_{t,t+dt}^E, r_{t,t+dt}^M)}{\text{Var}_t(r_{t,t+dt}^M)}$. Details can be found in the Appendix.

3.3 Shocks and Beta

The dynamics of beta in our setting is determined by the dynamics of financial leverage. We have already established, that due to the ratchet leverage effect, long-term financed firms have larger increases in leverage following negative cash flows shocks. We therefore expect the beta of long-term financed firms to increase more in bad times.

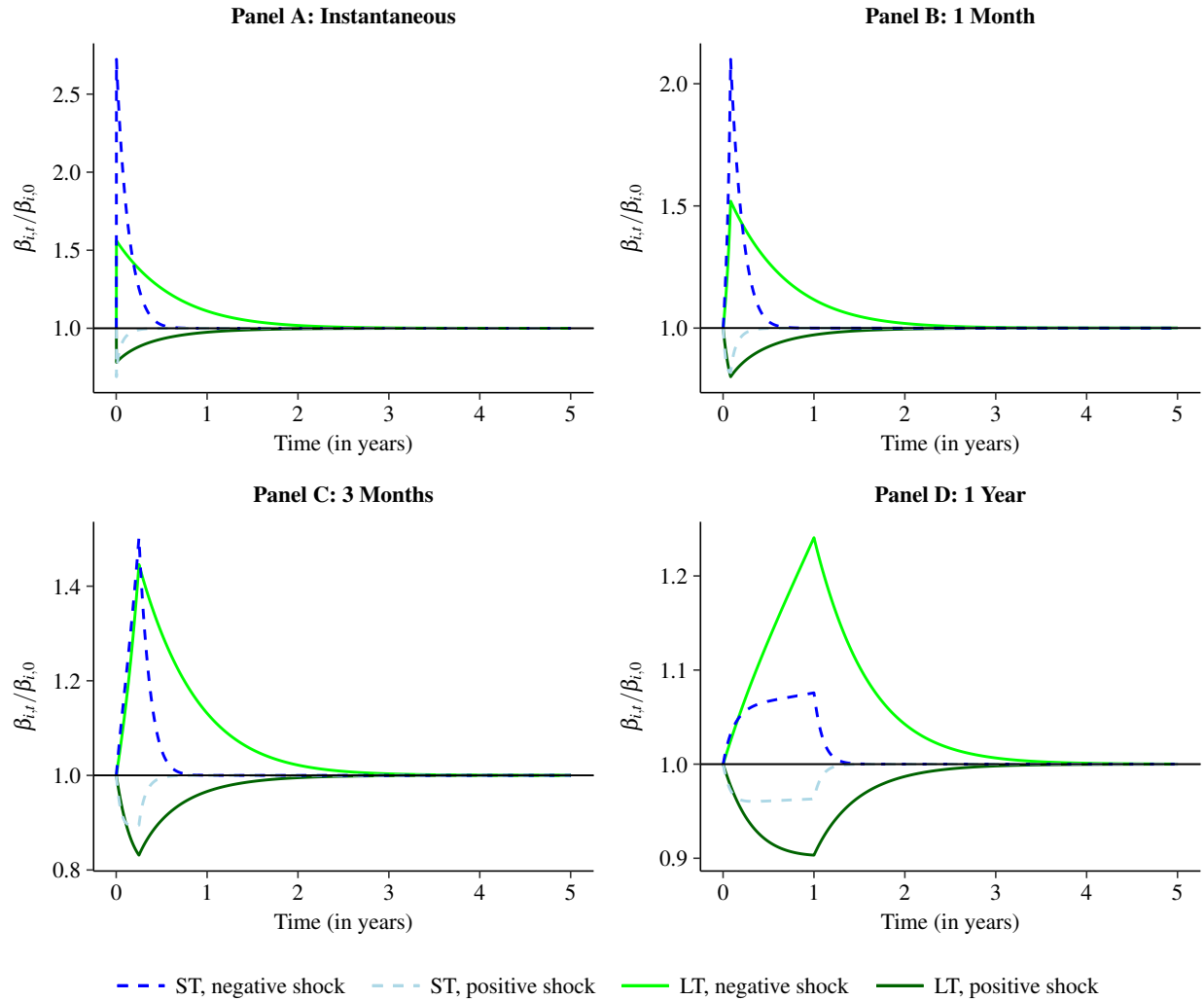
To visualize the difference between how the beta of short and long-term financed firms responds to cash flow shocks, we analyze alternative scenarios where we consider specific cash flow paths. We start with an instantaneous increase or decrease in cash flows by 15% and hold the subsequent cash flows constant at these shocked levels.⁶ The results are plotted in Figure 4 in the top left-hand subplot. Firms' initial leverage ratios are chosen so that they are at their targets, i.e., at the initial cash flow level, each firm rolls over exactly 100% of its expiring debt. For an instantaneous negative shock, the short-term financed firm experiences a larger spike in leverage and therefore beta, but it quickly reduces the face value of debt by not rolling over the entire amount. Its beta falls quickly within a year after the negative cash flow shock. The opposite is true for a long-term financed firm. It experiences a smaller initial spike in leverage, but it takes substantially longer, more than three years, to reduce its leverage back to the target level.

Next we investigate how beta responds if cash flow shocks are more gradual. We consider cash-flows where the change takes place linearly over a month, three months and a year. After that period, cash-flows are again held constant, while firms adjust their leverage by issuing or retiring maturing debt. The plots in Figure 4 demonstrate that the more gradual the shock is, the more pronounced is the difference between the impact on leverage and betas of long-term financed firms compared to short-term financed firms. With an decrease in cash flows over a year, short-term financed firms delever by not rolling over their debt, so their leverage increases much less than that of long-term financed firms. Moreover, the leverage of long-term financed firms stays elevated for more than 4 years after the shock, while the leverage of short-term financed firms goes back to normal after 2 years.

To summarize, an instantaneous deterioration of cash flows initially affects short-term financed firms more severely, raising their cash flows and thus equity betas more sharply. While long-term debt financed

⁶Note that this is the cash flow path that we consider in our simulation but, of course, the firms in our simulations do not anticipate that the cash flows will remain constant as they move through time.

Figure 4: Evolution of Beta Following Cash Flow Shocks. This figure shows beta evolutions for linear cash flow increases and decreases of 15% over different time intervals. After the cash flow has completed the change it is held constant, but the firms continue rolling over debt. In the Panel A the shock is instantaneous, while Panels B and C are based on cash flow shocks over 1 and 3 months, respectively. Finally, in Panel D the shock happens over an entire year. The solid (dashed) lines represent $\beta_{i,t}/\beta_{i,0}$ for a firm with $\sigma_i = 0.15$ (while $\sigma_X = 0.15$) and $m_i = 0.2$ ($m_i = 0.5$) — i.e., a debt maturity of 5 (LT) and 2 (ST) years, respectively. The initial beta $\beta_{i,0}$ is chosen such that the firm rolls over the amount of debt that matures. The lines featuring initial spikes (drops) represent reactions to cash flow decreases (increases).



firms' initial leverage and equity beta spike is more modest, their leverage and betas remain elevated for a long time following the initial cash flow shock. If the cash flow deterioration is more gradual, then short-term debt financed firms' leverage and equity betas never rise that much, since these firms reduce debt levels

quickly in response to decreasing cash flows. By contrast, long-term financed firms' leverage and betas rise more, as their debt reductions are very slow. They exhibit elevated levels of leverage and equity betas for a long period of time.

3.4 The Expected Equity Returns Over Holding Horizons

Instantaneous expected equity returns in Equation (21) are time-varying. The dynamics of a firm's leverage together with the time-varying price of risk determine the evolution of conditional expected returns. We compare the behaviour of expected equity returns over different time horizons $\mathbb{E}_0 \left[r_{0,\tau}^E \right]$ for firms with short- and long-maturity debt. Visually this is illustrated in Figure 5. The two firms, financed with long- and short-term debt, start at the same exposure to systematic risk, and therefore, have the same instantaneous expected equity returns.⁷

Leverage responds to cash flow shocks in an asymmetric way. Following good cash flow shocks, firms are more eager to increase the face value of debt than they are to decrease it following negative shocks because of debt overhang. Therefore, going forward, we on average expect the leverage of firms to go up. The expected upward trend in leverage means that shareholders require a higher return on equity, which explains the positive slopes in Figure 5.

Short-term financed firms are quicker at adjusting leverage both up and down. Following a bad cash flow shock, they delever more quickly than long-term financed firms because they have a smaller fraction of debt outstanding, and hence, are subject to a smaller debt-overhang. Following good cash flow shocks, short-term financed firms do not hesitate to increase the face value of debt, as, through short maturity, they have the commitment to delever when needed. Hence, over a short horizon (up to 2 years), we expect a larger increase in leverage for short-term firms. Short-term financed firms require a higher premium on their equity than long-term financed firms in the near future.

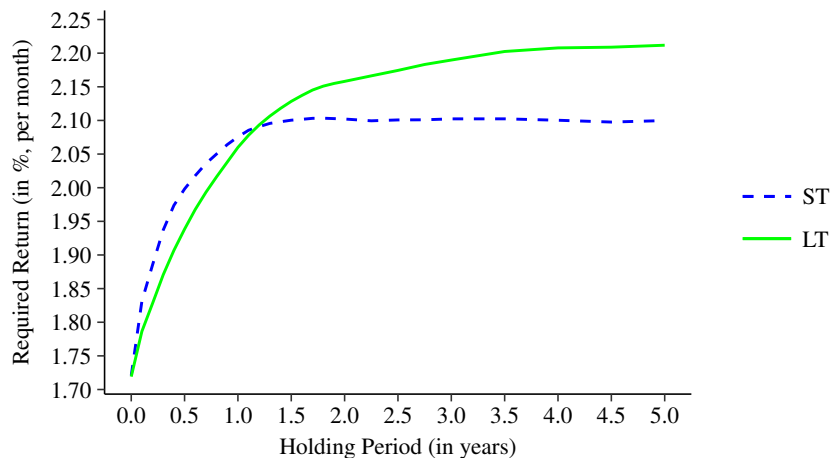
However, over a longer horizon, long-term financed firms are more risky. They are expected to increase their leverage more than short-term firms, and shareholders require a higher premium.

The positive co-movement between beta (leverage) and market price of risk makes the slopes of equity

⁷ In fact, we let the firms start at the leverage level at which the firms issue exactly as much debt as matures. Moreover, the idiosyncratic volatility of the short-term financed firm is chosen such that the firms' leverage levels result in the same initial $\beta_{i,0}$ for both firms.

yield curves steeper. The more leverage increases exactly when the market price of risk is high, the higher is the required compensation for bearing this risk.

Figure 5: Expected Equity Returns Over Holding Horizons. This graphs show the expected equity returns $\mathbb{E}_0 \left[r_{0,\tau}^E \right]$ over different investment horizons starting at $t = 0$ and ending in period τ . The expectations are calculated as averages based on the simulated changes in $\beta_{i,t}$ and η_t . The simulated panel consists of 2,000 firms per economy and 10,000 economies. Defaulted firms are not replaced. The solid (dashed) line represents the function for a long-maturity (short-maturity) financed firm with $\sigma_i = 0.1$ ($\sigma_i = 0.1945$) and $m_i = 0.2$ ($m_i = 0.5$) — i.e., a debt maturity of 5 (LT) and 2 (ST) years, respectively. The (m_i, σ_i) -pairs produce the same $\beta_{i,0}$ initially. Other parameters are as in the benchmark case, i.e., $\mu_0 = 0.05$, $k = 0.25$, $\sigma_X = 0.15$, $\delta = 0.04$, $r = 0.05$, $\tau = 0.3$.



3.5 Idiosyncratic Volatility

In our analysis so far we have only considered the difference between firms financed with long and short-term debt. In the setting that we consider, the choice of maturity is irrelevant for the firm a-priori. In other words, this setting is inadequate to analyze the optimal choice of maturity, as it ignores many important features that would be relevant for it, for example, transaction costs of issuing debt. However, in data we observe that firms with higher idiosyncratic volatility tend to be financed with shorter maturity debt (e.g., [Custódio et al., 2013](#)). This is consistent with predictions of [Dangl and Zechner \(2016\)](#) that firms with higher volatility will choose shorter maturity.

3.6 Unconditional CAPM and Alpha

We can re-write the expression for the conditional expected equity return stated in Equation (21) using $\beta_{i,t}$ to arrive at a notation similar to the CAPM as

$$\mathbb{E}_t [r_{i,t+dt}^E] = r dt + \left(1 + \frac{v_{i,t}^D}{v_{i,t}^E}\right) \sigma_X \lambda_t dt = r dt + \beta_{i,t} \eta_t dt, \quad (22)$$

where $\eta_t = \sigma_X \lambda_t = \mu(X_t, t) + \delta - r$ is the time-varying market risk premium.

In our model, the conditional version of the CAPM holds, period by period. However, an unconditional CAPM does not hold because $\beta_{i,t}$ and η_t are related through the evolution of the aggregate state X_t . And unconditional alpha, according to Lewellen and Nagel (2006), can be calculated as:

$$\alpha_i = \left[1 - \frac{\eta^2}{\sigma_M^2}\right] Cov(\beta_{i,t}, \eta_t) - \frac{\eta}{\sigma_M^2} Cov(\beta_{i,t}, (\eta_t - \eta)^2), \quad (23)$$

where $\eta = \mathbb{E}[\eta_t]$ is the unconditional mean of the market risk premium, and $\sigma_M^2 = \sigma_X^2 + \sigma_\eta^2$ is the unconditional variance of the market return. Note that in our model $\sigma_{i,M} = \sigma_X$, that is, the conditional market volatility is constant in time.⁸

In our setting, $Cov(\beta_{i,t}, \eta_t)$ is non-zero because of the time-varying market price of risk λ_t . In the downturns, when market risk premium η_t is high because of low aggregate productivity X_t , the firm's leverage is high, and correspondingly its systematic risk exposure $\beta_{i,t}$ is high. Therefore, there is a positive relationship between the market risk premium η_t and the firm's exposure to risk $\beta_{i,t}$. This co-movement is not captured by the unconditional CAPM and appears as α in CAPM regressions.

Short- and long-maturity firms have different dynamics of leverage and therefore different dynamics in their exposure to systematic risk. In particular, long-maturity firms experience larger increases in leverage and it remains elevated longer during recessions. This implies that there is more co-movement between

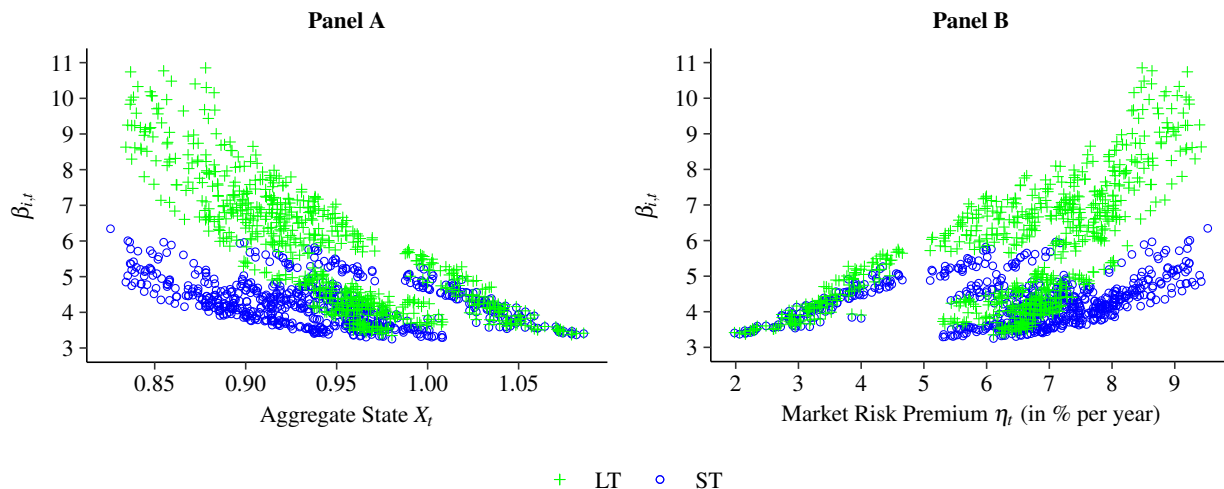
⁸ The formula in 23 is for an annual alpha with $dt = 1$. Generally speaking, alpha over increments of time dt is

$$\alpha_{i,dt} dt = \left[1 - \frac{(\eta dt)^2}{\sigma_M^2 dt}\right] Cov(\beta_{i,t}, \eta_t dt) - \frac{\eta dt}{\sigma_M^2 dt} Cov(\beta_{i,t}, (\eta_t dt - \eta dt)^2),$$

and its annualized version is:

$$\alpha_{i,dt} = \left[1 - \frac{\eta^2}{\sigma_M^2} dt\right] Cov(\beta_{i,t}, \eta_t) - \frac{\eta}{\sigma_M^2} Cov(\beta_{i,t}, (\eta_t - \eta)^2) dt.$$

Figure 6: Beta, Aggregate State, and Market Risk Premium. This figure shows the endogenous development of $\beta_{i,t}$ over the aggregate state variable X_t in Panel A and over the market risk premium η_t in Panel B. The crosses (circles) depict $\beta_{i,t}$ for a firm with $\sigma_i = 0.10$ ($\sigma_i = 0.20$) and $m_i = 0.2$ ($m_i = 0.5$) — i.e., a debt maturity of 5 (LT) and 2 (ST) years, respectively. The underlying simulated cash flow shocks are the same. Other parameters are as in the benchmark case, i.e., $\mu_0 = 0.05$, $k = 0.25$, $\sigma_X = 0.15$, $\delta = 0.04$, $r = 0.05$, $\tau = 0.3$.



betas and the market price of risk for long-maturity firms than for short-maturity firms. This is illustrated in Figure 6. In Panel A we see a simulated scatter-plot of beta over the aggregate state X_t for firms financed with long- and short-term debt, which also differ in their idiosyncratic volatility. In particular, we consider long-term firms with low idiosyncratic volatility and short-financed firms with high idiosyncratic volatility, so that their target leverage level is the same. When the aggregate productivity process is low, long-financed firms exhibit larger betas than short-maturity firms, despite the fact that during high-productivity states betas of these firms are very similar. Panel B in Figure 6 depicts the same relation in a scatter plot of beta on market risk premium. Long-term financed firms have more co-movement between beta $\beta_{i,t}$ and the market risk premium η_t .

As can be seen from the expression in squared brackets in Equation (23), whether the covariance between beta and the market risk premium translates into an increase or a decrease of alpha depends on the market's squared Sharpe ratio. If the Sharpe ratio is below one, then the covariance between beta and the market price of risk leads to an increase in alpha. Since empirical estimates for Sharpe ratios are normally well below

one⁹, this condition will hold under plausible market conditions.

The second term in Equation (23) denotes the covariance between beta and the squared deviation of the market price of risk from its mean. If η_t is distributed symmetrically around its mean, as is the case in our model, this term will be zero. This is also found in our numerical simulations below, which reveal that this second term is insignificant. Summarizing, the observed α_t should be a scaled version of the beta's covariance with the market risk premium.

3.7 Simulation: Maturity Premium

Finally, we conduct a simulation study of the maturity effect for CAPM alphas. We simulate the capital structure model introduced in Section 2 to assess the asset pricing implications. In total we simulate 5,000 economies of 1,000 firms for 10 years. At origination of the analysis all firms start at their target leverage levels. Then, we average the quantities of interest over firms in every economy and then over economies. In total, we apply this procedure for different specifications of (m_i, σ_i) -pairs. In Table 1 we show the parameters used in the simulation.

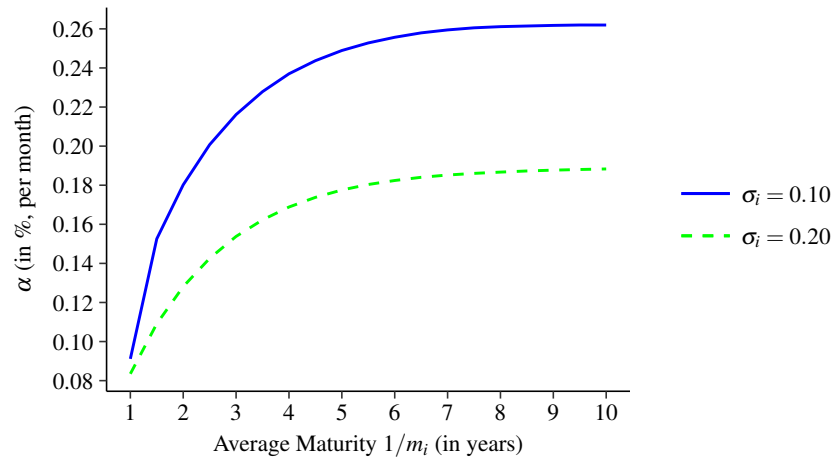
Table 1: Simulation Parameters. This table details the parameters of the simulation study. We group them into three categories. First, we present cash flow parameters associated with $Y_{i,t}$ under both measures. Second, we show parameters used for three rates and debt related parameters. Finally, we provide details on the simulation setting.

Cash Flow	μ_0	k	σ_X	σ_i	μ_Y
	0.05	0.25	0.15	[0.1, 0.2]	0.01
Rates & Debt	r	δ	τ	$1/m$	c
	0.05	0.04	0.30	[1, 10]	0.07
Simulation	economies	firms	years	Δt	
	5,000	1,000	10	1/1200	

The relation between the perceived CAPM alpha and the maturity of debt ($1/m_i$) is illustrated in Figure 7. The longer the maturity of debt, the larger is the CAPM alpha. This means that the ratchet effect of leverage indeed makes firms more risky in downturns and that this dominates the roll-over risk of short-term firms, which may lead to short-term spikes in leverage and betas. Of course this result depends on the underlying

⁹ Using the market excess return from Prof. French's homepage, the market's annual Sharpe ratio for our time interval equals 0.52.

Figure 7: Maturity Premium. In this figure we present the resulting alpha from unconditional CAPM regression of simulations for different maturity-volatility pairs (over 5,000 economies of 1,000 firms each). Alphas are represented in % per month. All parameters underlying this simulation are detailed in Table 1.

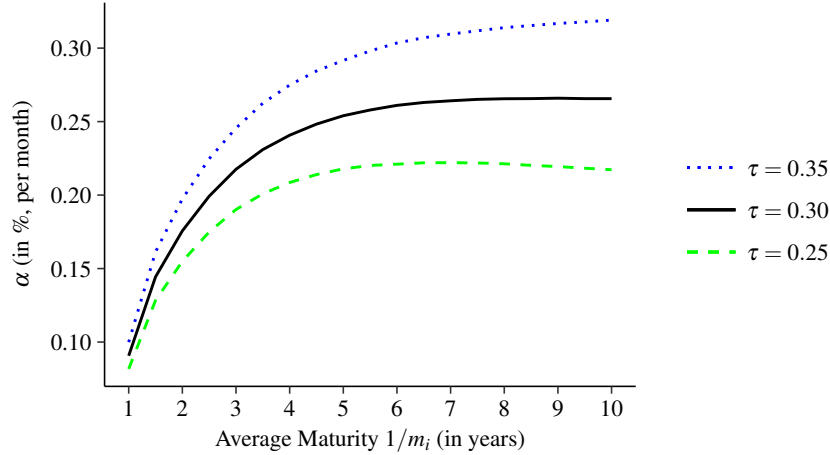


process of economic uncertainty. For example, if one would add the possibility of crashes in the X process, then presumably this would hurt short-term financed firms more than long-term financed firms, as argued above, thereby mitigating the maturity premium.

Figure 7 shows that the largest effects of debt maturity on alpha occur for expected maturity increases from one to six years. Additional maturity increases beyond six years have a relatively moderate additional effect. The reason for this result is the inverse relation between debt maturity and target leverage. Firms with very long-term debt, optimally lever less. This is so, since they rationally anticipate that they will not delever when profitability decreases, thereby creating bankruptcy risk. Thus, as we move to very long debt maturities, the additional covariance between beta and the market price of risk for given leverage tends to be offset by the lower target leverage ratios.

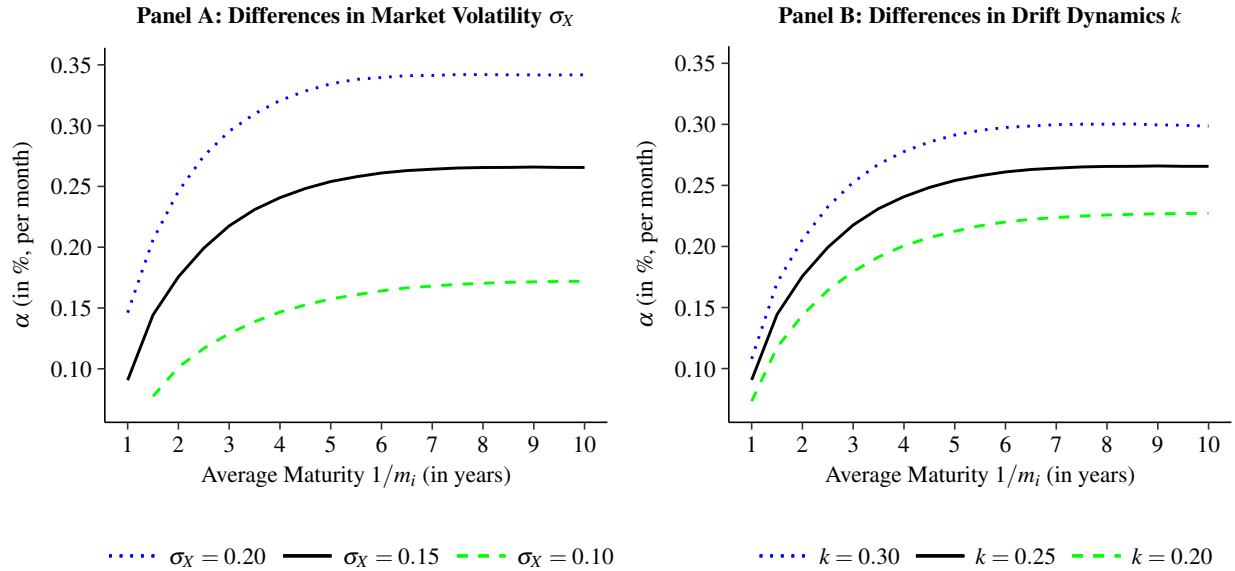
The numerical results also reveal that firms with more idiosyncratic risk exhibit a smaller maturity premium. This happens due to an inverse relation between idiosyncratic volatility and target leverage. Thus, for high-risk firms the maturity premium is mitigated since they choose lower target leverage ratios.

Figure 8: Maturity Premium - Tax Benefits of Debt. In this figure we present the alpha from unconditional CAPM regression on simulated panels of firms (over 1,000 economies of 500 firms each) for different maturities and tax rates. Alphas are represented in % per month. The black solid line represents the base case scenario. We use an idiosyncratic volatility of $\sigma_i = 0.1$. The rest of the model parameters are in Table 1.



To provide additional insights on how the maturity premium is related to firms' characteristics, we study the maturity premium for different levels of tax benefits of debt. The higher the tax benefits of debt, the higher the target debt level. We therefore vary the tax rate τ in the simulations displayed in Figure 8. First we note that the higher the tax rate, the higher the alpha. Moreover, for all tax levels we find that unconditional alphas increase substantially as we increase a firm's average debt maturity. The increase is much more pronounced for firms with larger tax rates. For example, for a tax rate of 0.35, the monthly alpha increases from less than 10 basis points per month to over 30 basis points as we move from a one year debt maturity to a six year debt maturity. If the tax benefit of debt is only 0.25, then the alpha increases to only slightly above 20 basis points as we move to a six year debt maturity. Thus, we would expect firms with a significant tax benefit of debt, and therefore higher leverage ratios, to exhibit larger maturity premia.

Figure 9: Maturity Premium - Market Risk and Mean Reversion. In this figure we present the alpha from unconditional CAPM regression of simulations (over 1,000 economies of 500 firms each) for different maturities and three levels of market volatility (Panel A) and three values of mean reversion k in the productivity drift process (Panel B). Alphas are represented in % per month. The black solid lines in both panels represent the base case scenario. We use an idiosyncratic volatility $\sigma_i = 0.1$. The rest of the model parameters are in Table 1.



We move on to investigating the effects of the productivity process' parameters on the maturity premium levels. We run several additional simulations and show the results in Figure 9. In Panel A illustrates the effect of the volatility of the productivity process σ_X . As can be seen, the maturity premium increases with σ_X . This is the opposite reaction to an increase in the idiosyncratic volatility σ_i , as shown in Figure 7. As the volatility of the productivity process increases, the market price of risk becomes more volatile. Hence, it co-moves more with betas of long-maturity financed firms, which we capture as higher unconditional alphas. In contrast, an increase in idiosyncratic volatility has no effect on the volatility of the market price of risk.

Furthermore, Panel B in Figure 9 shows the effect of the mean reversion k in the drift process. The larger k , the larger the maturity premium. This result is also due to the increased volatility of the market price of risk. To understand the intuition, consider the polar case $k = 0$, when the market price of risk does not change. Then there is no co-movement between beta and the market price of risk, and no maturity premium.

4 Empirical Results

In the empirical part of this study we investigate the role of long-term debt in the cross-section of equity returns. Our theory predicts that firms with longer debt maturities have a risk of increased systematic risk exposures in downturns and therefore earn higher returns than firms with short-term financing. We show evidence that this prediction holds. Furthermore, we find that it is especially valid for small firms and firms with high leverage.

4.1 Data and Summary Statistics

In our empirical analysis, we use monthly stock market data from the Center for Research in Security Prices (CRSP) and firm accounting data from COMPUSTAT's North America Fundamentals Annual file. We use CRSP's monthly returns on common equity of US-based enterprises from NYSE, AMEX, and NASDAQ. Firms are included when all items for computing a firm's debt maturity are available.¹⁰ This restriction limits our sample, as COMPUSTAT does not provide all items required for the debt maturity proxy for fiscal years ending before 1974. To ensure consistency, we truncate the matched sample by excluding observations before January 1976.¹¹

Given the data input, we compute several metrics for each firm to conduct the empirical study.¹² For every firm we compute leverage (L) as the ratio of book debt to the sum of book debt and market equity (see [Danis et al., 2014](#)). A key variable in our analysis is debt maturity (DM). We compute it following [Barclay and Smith \(1995\)](#) as the relative amount of long-term debt maturing in more than 3 years. Moreover, we compute market capitalization (ME) as the price per share times the number of shares outstanding. Following [Fama and French \(1992, 1993\)](#) we compute book equity and the book-to-market ratio (BM). For the BM -ratio book equity for the fiscal year ending in year t is related to market equity as of December of year t . Apart from the requirement of non-missing debt maturity, we require positive values for book equity, debt maturity and leverage. The final sample consists of 1,840,640 firm-month observations for a total of

¹⁰In the current analysis we include financial and utility companies. However, the main results are robust to excluding them from the sample.

¹¹ This truncation has to be interpreted under consideration of the procedure for matching accounting data and returns. Consequently, to ensure that necessary items are available for all firms, we have to drop two additional years from the matched return series.

¹² For detailed definitions, including the exact items used, we refer to [Appendix D](#).

18,392 unique firms over a time horizon from January 1976 until December 2017. A set of moments is computed for the final data set and presented in Table 2.

Table 2: Summary Statistics. We compute mean, standard deviation, as well as the 25%-, 50%-, and 75%-quantiles of several firm characteristics monthly for the cross-section. The table presents time series averages of the monthly statistics. Excess returns, leverage and debt maturity are displayed in % and market equity in million USD. The underlying data set comprises matched observations from CRSP and COMPUSTAT over the time horizon January 1976 until December 2017. In total, the panel consists of 1,840,640 firm-month observations of 18,392 unique firms.

	Mean	SD	Q_{25}	Median	Q_{75}
Excess Returns	0.92	15.64	-6.08	0.01	6.45
Market Equity (ME)	2365.89	9903.90	59.10	260.15	1111.80
Book-to-Market Ratio (BM)	0.93	0.94	0.43	0.74	1.16
Leverage (L)	31.33	23.92	10.71	27.08	48.55
Debt Maturity (DM)	53.15	33.87	21.55	58.87	83.32

We ensure that accounting information on debt maturity, leverage, and book equity is publicly available upon portfolio assignment by following the procedures by [Fama and French \(1992, 1993\)](#). Thus, we consider information from year t for portfolio assignments at the end of June of year $t + 1$ onwards until the following June.

4.2 Debt Maturity and the Time-Series of Stock Returns

We start investigating our predictions regarding the behavior of stock returns by constructing debt maturity-sorted portfolios. The tests to follow use the same portfolios in different test settings. The main test portfolios we employ are shown in Table 3.

We construct portfolios by sorting stocks into 5 size buckets and 5 conditional debt maturity buckets. We have to consider conditional sorts within each size group due to the variation in debt maturity across size portfolios. In our sample smaller firms tend to borrow with shorter-maturity debt as opposed to their larger counterparts. Not considering this heterogeneity would result in picking up a size effect unrelated to our predictions.

The first prediction we test is the presence of alpha in unconditional CAPM regressions of long-maturity minus short-maturity portfolio returns. Our theoretical model predicts that time-variation in systematic risk

exposure of the long-short maturity portfolio should produce alpha only relative to CAPM. However, it could be the case that other asset pricing factors established in the literature capture this time-variation, driving the alpha to zero in 3 or 5-factor models.

In particular, we therefore pay close attention to the role of the value factor. In fact, our model may imply a relation between the time-series evolution of leverage and the value premium. If value firms exhibit longer debt maturities, this will discourage them from reducing debt when profitability decreases, which leads to an increase in leverage. Hence, we will test if the value factor eliminates any premium for long debt maturities.

As stated above, we construct 25 portfolios by splitting the sample into 5 size buckets at the median market equity, and 5 debt maturity buckets. We then compare the performance of the strategy going long in firms with long maturity and short firms with short maturity along the size categories as well as equal weighting between the 5 size groups.

As predicted by our model, we find that firms with longer debt maturities earn higher risk-adjusted returns compared to firms with short horizon debt, as shown in Panel B of Table 3. The CAPM features an excess return of 0.21% per month, while both models of Fama and French (1993, 2015) report a somewhat smaller risk-adjusted return of 0.15% and 0.10% respectively. Thus, we empirically confirm the higher risk associated with long-term debt, reflected in higher required returns. The increased adverse reaction of leverage to negative shocks coupled with the slower adjustment seems to constitute an additional source of risk. This risk is not accounted for by the standard factors, such as the FF 3 factors or the FF 5 factors. The factor loadings reported in Panel B of Table 3 indicate a positive relationship with the value premium.

Table 3 also shows that the effect of longer debt maturities on the riskiness of equity seems to be larger among small firms. The CAPM reports a 0.43% monthly adjusted return for the smallest firms.

Table 3: Debt Maturity-Sorted Portfolios. Panel A shows the average excess return of the individual value-weighted portfolios. The last row contains the long-maturity minus short-maturity portfolios (LMS). The portfolios are formed by double sorts on size (5 buckets at 20%, 40%, 60%, and 80%-percentile) and debt maturity (5 buckets at 20%, 40%, 60%, and 80%-percentile) conditional within each size group. Panel B examines the LMS portfolios within each size bucket (LMS1 for small to LMS5 for large firms) and average return of these five portfolios the last column (LMS itself). The long-short portfolios are represented by excess returns (r^e) as well as alpha estimates from CAPM-regressions (α^{CAPM}), the 3-factor model by Fama and French (1993) (α^{FF3}) and the 5-factor model by Fama and French (2015) (α^{FF5}). Moreover, risk factor loadings for FF5 are shown. We report t-statistics based on standard errors following Newey and West (1987, 1994) in parentheses. The underlying data set comprises matched observations from CRSP and COMPUSTAT from January 1976 until December 2017.

Panel A: Portfolio Sorts

		Size				
		Small	Medium	Large		
Debt Maturity	Short	0.59	0.71	0.72	0.81	0.62
		0.66	0.82	0.86	0.89	0.68
	Medium	0.75	0.82	0.92	0.89	0.66
		0.86	0.89	0.99	0.93	0.60
	Long	0.92	0.91	0.86	0.86	0.64
LMS		0.33	0.20	0.14	0.05	0.01

Panel B: Debt Maturity (LMS)

	LMS1	LMS2	LMS3	LMS4	LMS5	LMS
r^e	0.33** (2.01)	0.20 (1.25)	0.14 (1.21)	0.05 (0.59)	0.01 (0.09)	0.15* (1.93)
α^{CAPM}	0.44*** (2.98)	0.32** (2.03)	0.22* (1.82)	0.04 (0.41)	0.03 (0.26)	0.21*** (2.93)
α^{FF3}	0.35*** (2.69)	0.20 (1.44)	0.09 (0.82)	0.00 (0.04)	0.11 (0.94)	0.15** (2.31)
α^{FF5}	0.30** (2.27)	0.10 (0.59)	-0.06 (-0.60)	-0.11 (-1.22)	0.25** (2.57)	0.10* (1.67)
β^M	-0.08** (-1.99)	-0.04 (-1.17)	0.02 (0.75)	0.08*** (3.62)	-0.13*** (-4.45)	-0.03* (-1.78)
β^{SMB}	-0.17*** (-3.09)	-0.23*** (-3.53)	-0.12*** (-2.76)	-0.01 (-0.28)	0.07 (1.41)	-0.09*** (-2.97)
β^{HML}	0.36*** (5.44)	0.33*** (3.80)	0.33*** (5.25)	0.00 (0.08)	-0.13 (-1.32)	0.18*** (3.92)
β^{RMW}	0.20*** (2.73)	0.22 (1.51)	0.36*** (5.18)	0.18** (2.50)	-0.26*** (-3.55)	0.14*** (2.65)
β^{CMA}	-0.21* (-1.89)	0.05 (0.46)	0.01 (0.07)	0.20** (2.21)	-0.16 (-1.41)	-0.02 (-0.45)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

4.3 Conditional CAPM and Beta Dynamics

The existence of a maturity premium in our model relies on the fact that long-maturity financed firms experience larger and more prolonged increases in their exposure to systematic risk in downturns than short-maturity financed firms. This premium represents a compensation for the positive covariance between betas of long-term financed firms and the market price of risk. In this subsection we provide direct empirical evidence that the beta of long-term financed firms increases in crises and that this increase is larger than that experienced by short-term financed firms.

To estimate the dynamics of the LMS-portfolio's beta we consider the following conditional version of the CAPM:

$$r_t^{LMS} = \alpha + \beta_0 r_t^M + \beta_1 Z_{t-1} r_t^M + \varepsilon_t, \quad (24)$$

where Z_{t-1} is the lagged conditioning variable. The average exposure to systematic risk of the portfolio is captured by the value β_0 , as in the classical CAPM. Moreover, the time-variation in beta is captured by the third coefficient, i.e., β_1 .

Our instrument Z consists of variables that are likely to drive the countercyclical market risk premium. As predictors we use the dividend yield (DY), the default spread (DS), the term spread (TS), and the T-Bill rate (TB). These variables are commonly employed in the literature (see [Choi, 2013](#)) and obtained from Amit Goyal's homepage.

We follow a two-step regression design to estimate the predictive variable Z_{t-1} in Equation (24). The first step is to fit a one-month ahead predictive regression to span the observed market return by the predictors mentioned above, i.e.,

$$r_t^M = \delta_0 + \delta_1 DY_{t-1} + \delta_2 DS_{t-1} + \delta_3 TS_{t-1} + \delta_4 TB_{t-1} + \varepsilon_t^M = \eta_t + \varepsilon_t^M. \quad (25)$$

Results of the fitting estimation are presented in Table 4. In the first column, we use all four macro-variables to forecast the market return. As we see, only two out of four are significant. Hence, we re-estimate the model using only the significant dividend yield and t-bill rate as explanatory variables. Model (2) of the same table contains the estimates of this specification.

Table 4: Predictive Variable. In this table shows the result of the predictive regression of the market's excess return on lagged predictors. The lagged explanatory variables are the dividend yield (DY), the default spread (DS), the term spread (TS), and the T-Bill rate (TB). In Model (1) we include all predictors, while in Model (2) only the significant variables from the first test are used. We report t-statistics based on standard errors following Newey and West (1987, 1994) in parentheses. The time horizon lasts from January 1976 until December 2017.

	Model (1)	Model (2)
DY	2.03*** (2.69)	1.60** (2.43)
DS	-22.51 (-0.27)	
TS	-17.25 (-0.97)	
TB	-25.78** (-2.30)	-18.99** (-2.37)
Intercept	9.89*** (2.81)	7.40*** (2.74)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Although the two regression coefficients reported for model (2) are statistically highly significant, the overall predictive power of the regression is small. It explains only approximately 2% of the overall variation in the market risk premium. Thus, despite the statistical significance of dividend yield and t-bill rate, the predicted market returns are quite noisy.

In the next step, we use the coefficient estimates reported in Table 4 to calculate the predictor Z_{t-1} for the conditional model presented in Equation (24). We note that Z_{t-1} is high in times when the dividend yield is high and the t-bill rate is low, consistent with a countercyclical market risk premium. The results of this conditional CAPM are reported in Table 5. The first column contains estimates of a standard unconditional CAPM. We find that the LMS portfolio exhibits a negative unconditional market beta and a positive alpha. The latter is by construction identical to the alpha reported in Panel B of Table 3.

The second column of Table 5 reports estimates of our conditional version of the CAPM. We interact the market return with our estimated market risk premium to assess the time-variation in beta of the LMS portfolio. The interaction term β_1 is positive and statistically significant at the 1% level. This suggests that the beta of long-maturity financed firms increases in times when the market price of risk is high, i.e., in

Table 5: Maturity Premium in a Conditional CAPM. This table presents the result of an unconditional and a conditional version of the CAPM, respectively. The dependent variable is the LMS portfolio’s return as constructed in Table 3. In the conditional version we use the predicted market risk premium as a conditioning variable and interact it with the market return. The predicted risk premium is constructed using lagged predictors and the coefficient estimates shown in Model (2) of Table 4. Whereas β_0 corresponds to the unconditional estimate, β_1 represents the coefficient on the interaction between the market’s excess return and the conditioning variable. We report t-statistics based on standard errors following [Newey and West \(1987, 1994\)](#) in parentheses. The time horizon lasts from January 1976 until December 2017.

	Unconditional	Conditional
α	0.21*** (2.93)	0.19*** (2.60)
β_0	-0.09*** (-6.29)	-0.13*** (-6.03)
β_1		0.06*** (2.69)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

recessions or crises times, more than the beta of short-maturity financed firms. Hence, the overall exposure to market risk of the LMS portfolio increases when the market price of risk increases. This is exactly in line with our theoretical predictions. Long-term financed firms have an increased exposure to systematic risk in times when the market risk premium is particularly high.

Moreover, we also find that the estimated alpha decreases from 0.21 to 0.19 once we introduce our conditioning variable. Thus, the time-variation in beta that we capture via our estimated market price of risk explains part of the maturity premium that we observe in the unconditional CAPM.

The magnitude of the reduction in alpha between the unconditional and the conditional CAPM is rather small though. However, one must consider that our conditioning variable Z_{t-1} , is a rather poor predictor of future market risk premia. Thus, the moderate drop in alpha when moving from the unconditional to the conditional model is likely due to the limited ability of our predictors (dividend yield and t-bill rate) to forecast market returns. Since dominating alternative conditional models are not readily available, we rely on the standard model to forecast market risk premia. Using rolling-window estimates of beta is not a good solution either, as there is not enough time-variation in the beta estimates, as discussed in [Lewellen and Nagel \(2006\)](#).

Our model also predicts a relationship between the risk reaction of firms with different debt maturity

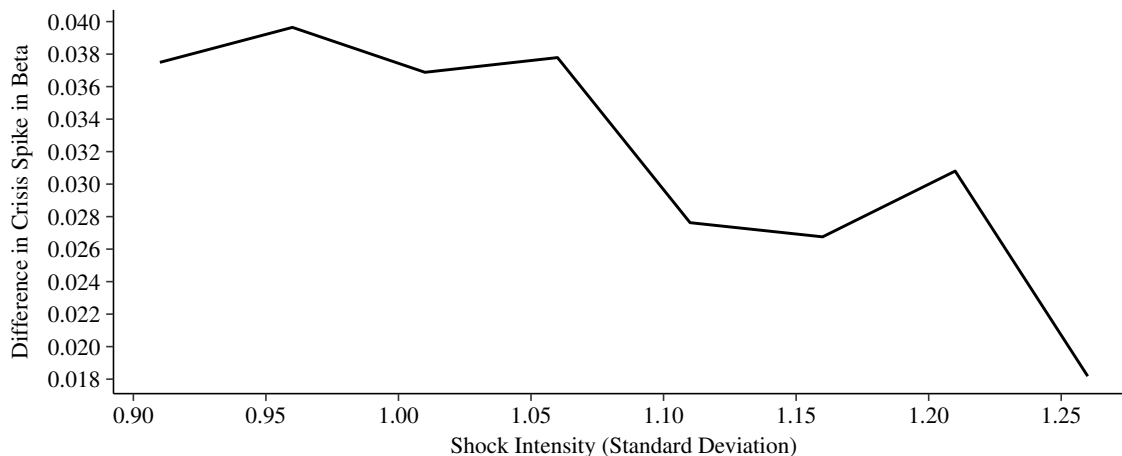
profiles and how gradual a shock is. As the severity of the systematic shock increases, the short-maturity financed firms experience a larger increase in the market risk exposure than their long-maturity financed counterparts. We investigate this prediction empirically by a different version of a conditional CAPM

$$r_{p,t} = \alpha + \beta_0 r_t^M + \beta_1 1_{R,t} r_t^M + \beta_2 1_{R,t} 1_{LT,t} r_t^M + \varepsilon_{p,t}, \quad (26)$$

where $p \in \{ST, LT\}$ for the the short- and long-leg of the LMS portfolio. This regression features two dummy variables capturing recessions $1_{R,t}$ and long-maturity financed firms $1_{LT,t}$, which is one if $p = LT$. We form several recession dummies by varying the recession classification threshold. A month t is classified as a recession (i.e., $1_{R,t} = 1$) if the monthly market return is below the recession threshold. This cut-off is defined in standard-deviations below the average market return. The larger the distance to the average market return (i.e., the lower the threshold), the more severe a shock. We vary the recession threshold from 0.9 to 1.25.

From each regression with varying cut-off we take the estimates of β_2 . This coefficient measures the increase in beta experienced during a downturn by long-term financed firms above the increases in beta for short-term financed firms. We depict the resulting relation between the estimated spikes in beta and recession cut-off levels in Figure 10. From the figure we see that the more severe the market drop, the smaller is the difference between responses in beta of long-maturity and short-maturity financed firms.

Figure 10: Beta Reactions in Market Downturns and Debt Maturity. This graph shows the effect of downturns on betas of long-maturity financed firms above the reaction of short-maturity financed firms' betas as represented by β_2 of Equation (26). The recession indicator variable is defined based on the market's excess return. The indicator is one if the market return in a given month t is x standard deviations below its mean. The shock intensity on the horizontal axis shows the cut-off multiplier x . The larger x the lower the recession cut-off. The time horizon lasts from January 1976 until December 2017.



4.4 Interaction of Debt Maturity and Leverage

As we demonstrate in Section 3.7, our model predicts that a firm's leverage should be related to the maturity premium. Specifically, we find that firms with high idiosyncratic risk, and thus lower target leverage, exhibit lower debt maturity premia. Similarly, firms with low net benefits of debt also exhibit lower target leverage and lower maturity premia. Intuitively, this is easy to understand. When a firm has very little debt, then even a very long maturity will only marginally affect the covariance of its beta with the market price of risk. In this subsection we investigate this prediction empirically.

In general, any firm with long-term debt is impacted by negative shocks and the resulting slow adjustment of debt. Yet firms with high leverage will show a stronger reaction compared to firms with low levels of debt. Thus, we look at the interaction of leverage and debt maturity by conducting another set of conditional double sorts based on leverage and debt maturity.

Indeed, we find evidence that the effects of long debt maturities on equity risk are stronger among firms with high leverage. In Table 6 Panel B, the risk-adjusted returns for the portfolio that is long firms with long debt maturities and short firms with short-term debt in the high leverage bucket (i.e., LMS5) show positive alphas. Firms with long-term debt produce risk-adjusted excess returns between 0.38% and 0.41%

per month in all factor models, which are statistically-significant at the 10% level. Moreover, the alpha estimate for highly levered firms exceeds the magnitude of the LMS premium reported in Table 3.

For firms with low leverage ratios, our model predicts that the risk associated with longer debt maturities is reduced. We find support for this in the data. For firms with the lowest leverage ratios (i.e., LMS1) we do not find a premium significantly different from 0.

5 Conclusion

In this paper we show theoretically and empirically that long-maturity financed firms have higher expected returns than short-maturity financed firms. We provide evidence that this is due to the risk of leverage increases in downturns, which are more severe for long-maturity financed firms. Short-maturity financed firms are more exposed to rollover risk and may therefore be more risky during a sharp and instantaneous decline, while for the more progressive declines commonly associated with recessions long-maturity firms are more risky.

While a conditional CAPM holds in our model, increases in leverage during downturns generate a co-movement between firms' betas and the market price of risk, which appears as alpha in unconditional CAPM regressions. Empirically, we document a monthly premium of 0.21% for a strategy that goes long long-term financed firms and short short-term financed firms, which we call maturity premium.

Our paper sheds light on the contribution of leverage dynamics to asset pricing patterns, that appear as anomalies relative to the unconditional CAPM. While the role of operating leverage and investment irreversibility has been shown to explain the value premium, they alone can't match the magnitude. We show that long-term maturity is what makes financial leverage hard to reverse because of debt overhang. Hence, long-maturity financial leverage gives rise to a maturity premium. As value firms tend to be financed with long-term debt, the book-to-market ratio proxies for maturity choice. We therefore demonstrate that long-term financial leverage contributes to the value premium. However, controlling for the value factor, the portfolio of long minus short maturity financed firms still generates an unconditional alpha. This means that the maturity factor is distinct from the value factor and captures the important risk of leverage increases in downturns. We believe that a fuller exploration of the effects of dynamic corporate decisions on equity pricing is a highly attractive area for future research.

Table 6: Value-Weighted Returns for Dependent Sorts on Debt Maturity and Leverage. Panel A shows the average excess return of the individual value-weighted portfolios. The last row contains the long-maturity minus short-maturity portfolios (LMS). The portfolios are formed by double sorts on leverage (5 buckets at 20%, 40%, 60%, and 80%-percentile) and debt maturity (5 buckets at 20%, 40%, 60%, and 80%-percentile) conditional within each leverage group. Panel B examines the LMS portfolios within each leverage bucket (LMS1 for firm with low to LMS5 for firms with high leverage). The long-short portfolios are represented by excess returns (r^e) as well as alpha estimates from CAPM-regressions (α^{CAPM}), the 3-factor model by Fama and French (1993) (α^{FF3}) and the 5-factor model by Fama and French (2015) (α^{FF5}). Moreover, risk factor loadings for FF5 are shown. We report t-statistics based on standard errors following Newey and West (1987, 1994) in parentheses. The underlying data set comprises matched observations from CRSP and COMPUSTAT from January 1976 until December 2017.

Panel A: Portfolio Sorts

		Leverage				
		Low	·	Medium	·	High
Debt Maturity	Short	0.78	0.89	0.84	0.79	0.77
	·	0.40	0.74	0.78	0.79	0.87
	Medium	0.60	0.71	0.88	0.77	0.84
	·	0.55	0.63	0.81	0.69	0.84
	Long	0.61	0.71	0.83	0.81	0.94
LMS		-0.17	-0.18	-0.01	0.02	0.17

Panel B: Debt Maturity (LMS)

	LMS1	LMS2	LMS3	LMS4	LMS5
r^e	-0.17	-0.18	-0.01	0.02	0.17
	(-1.10)	(-1.18)	(-0.09)	(0.13)	(0.91)
α^{CAPM}	-0.08	-0.23	-0.00	0.18	0.38*
	(-0.57)	(-1.36)	(-0.00)	(0.99)	(1.81)
α^{FF3}	-0.11	-0.10	0.11	0.19	0.41**
	(-0.70)	(-0.73)	(0.77)	(1.06)	(2.20)
α^{FF5}	-0.35**	0.00	0.10	0.24	0.34*
	(-2.31)	(0.01)	(0.62)	(1.44)	(1.89)
β^M	0.01	0.06	0.02	-0.24***	-0.38***
	(0.16)	(1.39)	(0.47)	(-4.12)	(-6.22)
β^{SMB}	-0.19**	-0.37***	-0.37***	-0.07	0.42***
	(-2.34)	(-5.09)	(-5.88)	(-0.63)	(4.27)
β^{HML}	0.07	-0.03	-0.18**	-0.12	-0.45***
	(0.89)	(-0.32)	(-2.22)	(-0.91)	(-2.65)
β^{RMW}	0.48***	-0.10	-0.01	-0.23**	-0.06
	(5.43)	(-0.80)	(-0.10)	(-2.20)	(-0.39)
β^{CMA}	0.14	-0.31**	0.10	0.26	0.51**
	(1.42)	(-2.27)	(0.87)	(1.56)	(2.56)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

APPENDIX

A Model Solution

For the valuation of the equity claim consider the Hamilton-Jacobi-Bellman equation (*HJB* below)¹³ associated with the expected future dividends shown in Equation (8). The required return is equal to the risk-free rate r when the firm issues the optimal amount of debt at any point in time, which in turn determines the dynamics of the total face value of debt, $dF_{i,t}$, as defined in Equation (6). Hence, we need to solve the following HJB equation for the optimal $G_{i,t}$

$$rV^E(Y_{i,t}, F_{i,t}) = \max_{G_{i,t}} \{Y_{i,t}(1 - \tau) + \tau c F_{i,t} - (c + m)F_{i,t} + G_{i,t}v_{i,t}^D + (G_{i,t} - mF_{i,t})V_F^E(Y_{i,t}, F_{i,t})\} \quad (\text{A-1})$$

$$+ \mu_Y V_Y^E(Y_{i,t}, F_{i,t}) + 1/2 \sigma_Y^2 V_{YY}^E(Y_{i,t}, F_{i,t}).$$

Issuing a marginal unit of debt generates benefits of $v_{i,t}^D$ to equityholders and costs of $V_F^E(Y_{i,t}, F_{i,t})$ for future payments to debt holders. Assuming that debt is issued smoothly at the discretion of equityholders, equating marginal benefits and marginal costs results in the following first-order-condition (FOC)

$$v_{i,t}^D + V_F^E(Y_{i,t}, F_{i,t}) = 0. \quad (\text{A-2})$$

DeMarzo and He (2018) lay out optimality conditions for the debt issuance policy, which are met in our setup. Using the FOC from Equation (A-2) in the HJB shown in Equation (A-1) yields the following HJB that does not depend on $G_{i,t}$

$$rV^E(Y_{i,t}, F_{i,t}) = Y_{i,t}(1 - \tau) + \tau c F_{i,t} - (c + m)F_{i,t} - mF_{i,t}V_F^E(Y_{i,t}, F_{i,t}) \quad (\text{A-3})$$

$$+ \mu_Y V_Y^E(Y_{i,t}, F_{i,t}) + 1/2 \sigma_Y^2 V_{YY}^E(Y_{i,t}, F_{i,t}).$$

Now we divide both state variables by the face value of debt $F_{i,t}$, which leaves one state variable constant. From here onwards, lower case letters refer to scaled versions of the upper case variables (e.g., the scaled

¹³ Here we use subscripts Y and F for the functions of the market value, where superscripts denote that it is either the equity or debt market value, respectively, to denote partial derivatives with respect to those variables to save on notation.

cash flow level $y_{i,t} = Y_{i,t}/F_{i,t}$ and the scaled issuance policy $g_{i,t} = G_{i,t}/F_{i,t}$). Subsequently, the dynamics of the firm's scaled cash flow process under the risk-neutral measure from Equation (5) accounting for the scaling by $1/F_{i,t}$ are given by

$$dy_{i,t} = (\mu_Y + m_i - g_{i,t})y_{i,t} dt + \sigma_Y y_{i,t} dW_{Y_{i,t}}^Q, \quad (\text{A-4})$$

which also changes the HJB from Equation (A-3) to

$$(r + m_i)v_i^E(y_{i,t}) = y_{i,t}(1 - \tau) + c\tau - (c + m_i) + (\mu_Y + m_i)y_{i,t}v_Y^E(y_{i,t}) + 1/2\sigma_Y^2 y_{i,t}^2 v_{YY}^E(y_{i,t}). \quad (\text{A-5})$$

To solve Equation (A-5) we impose the boundary condition for $y_{i,t} \rightarrow \infty$, where the equity value should converge to the perpetuity of the after-tax cash flows plus the coupons tax shield less the bond's perpetuity value. Furthermore, at the cash flow level where equityholders default y_b , equity is worth nothing. Finally, the optimal default boundary is determined by the smooth-pasting condition, i.e., $v_Y^E(y_b) = 0$. Then, the equity value function is given by

$$v_i^E(y_{i,t}) = \frac{1 - \tau}{r - \mu_Y} y_{i,t} - \frac{c(1 - \tau) + m_i}{r + m_i} \left(1 - \frac{1}{1 + \gamma_i} \left(\frac{y_{i,t}}{y_{b,i}} \right)^{-\gamma_i} \right), \quad (\text{A-6})$$

with the exponent equal to

$$\gamma_i = \frac{(\mu_Y + m_i - \sigma_Y^2/2) + \sqrt{(\mu_Y + m_i - \sigma_Y^2/2)^2 + 2\sigma_Y^2(r + m_i)}}{\sigma_Y^2} > 0, \quad (\text{A-7})$$

and the default boundary

$$y_{b,i} = \frac{\gamma_i}{1 + \gamma_i} \frac{r - \mu_Y}{r + m_i} \left(c + \frac{m_i}{1 - \tau} \right). \quad (\text{A-8})$$

The scaled value of debt, i.e., the price per unit of face value, follows from the FOC in Equation (A-2)

as

$$v_i^D(y_{i,t}) = \frac{c(1-\tau) + m_i}{r + m_i} \left(1 - \left(\frac{y_{i,t}}{y_{b,i}} \right)^{-\gamma_i} \right). \quad (\text{A-9})$$

While we initially did not have to specify the debt issuance policy $G_{i,t}$, we still need to derive it. We do so by considering the HJB for the value of debt. The value of debt can also be based on the expectation of future principal payments and coupons paid to debt holders as shown in Equation (9), like

$$\begin{aligned} rv^D(Y_{i,t}, F_{i,t}) &= \\ &= c + m(1 - v^D(Y_{i,t}, F_{i,t})) + (G_{i,t} - mF_{i,t})v_F^D(Y_{i,t}, F_{i,t}) + \mu_Y v_Y^D(Y_{i,t}, F_{i,t}) + 1/2 \sigma_Y^2 v_{YY}^D(Y_{i,t}, F_{i,t}). \end{aligned} \quad (\text{A-10})$$

Next, we impose the FOC from Equation (A-2) on the derivative with respect to the debt level $F_{i,t}$ of the HJB for equity in Equation (A-3) to find another HJB for the price of debt, which is equal to

$$\begin{aligned} -rv^D(Y_{i,t}, F_{i,t}) &= \\ &= \tau c - (c + m) + mv^D(Y_{i,t}, F_{i,t}) + mF_{i,t}v_F^D(Y_{i,t}, F_{i,t}) - \mu_Y v_Y^D(Y_{i,t}, F_{i,t}) - 1/2 \sigma_Y^2 v_{YY}^D(Y_{i,t}, F_{i,t}). \end{aligned} \quad (\text{A-11})$$

Finally, adding Equations (A-10) and (A-11) results in the following expression for the optimal debt issuance policy (in its scaled version)

$$g_i(y_{i,t}) = \frac{(r + m_i)\tau c}{c(1-\tau) + m_i} \frac{1}{\gamma_i} \left(\frac{y_{i,t}}{y_{b,i}} \right)^{\gamma_i}. \quad (\text{A-12})$$

We can see that there is a cash flow level, which we denote by $y_{m,i}$, at which the firm rolls over exactly the maturing amount of debt m_i , which keeps the face value of debt constant. This cash flow level equals

$$y_{m,i} = y_{b,i} \left(\frac{c(1-\tau) + m_i}{(r + m_i)\tau c} m_i \right)^{1/\gamma_i}. \quad (\text{A-13})$$

and can be used to restate Equation (A-12) from above to the version of the main text in Equation (12).

In the end, the evolution of the face value of debt $F_{i,t}$ is the result of debt issuance decisions and the constantly maturing portion of debt. By combining the law of motion of $F_{i,t}$ presented in Equation (6) with

the optimal debt issuance function from Equation (12), we find that

$$dF_{i,t} = \left(m_i \frac{Y_{i,t}^\gamma}{y_{m,i}^\gamma} F_{i,t}^{1-\gamma} - mF_{i,t} \right) dt. \quad (\text{A-14})$$

While Equation (A-14) is not linear in $F_{i,t}$ we can substitute $H = F^\gamma$. Then we can find a solution to the differential equation for $dH = \gamma F^{\gamma-1} dF$ given that $H_0 = 0$. In the end, we can insert $F_{i,t}$ back into the general solution and find

$$F_{i,t} = \left(\int_0^t \gamma_i m_i \left(\frac{Y_{i,s}}{y_{m,i}} \right)^\gamma e^{\gamma m_i (s-t)} ds \right)^{1/\gamma}. \quad (\text{A-15})$$

B Return on Equity

In this subsection we analyze equity returns in detail and demonstrate that under the risk-neutral measure the expected value of equity returns is r , consistent with FOC of equity pricing, while innovations to cash flows are amplified by a firm's financial leverage v^D/v^E .

$$r_{t,t+dt}^E = \frac{dV_t^E + \Pi_{t,t+dt}}{V_t^E}. \quad (\text{B-1})$$

$$\begin{aligned}
r_{t,t+dt}^E &= \frac{V_F^E dF_t + V_Y^E \mu_Y Y_t dt + V_Y^E \sigma_Y Y_t dW_{Y,t}^Q + \frac{1}{2} V_{YY}^E \sigma_Y^2 Y_t^2 dt + \Pi_{t,t+dt}}{V^E(Y_t, F_t)}, \\
\frac{\partial V^E(Y, F)}{\partial F} &= \frac{\partial}{\partial F} \left(V^E \left(\frac{Y}{F}, 1 \right) F \right) = -\frac{Y}{F^2} \frac{\partial V^E \left(\frac{Y}{F}, 1 \right)}{\partial \frac{Y}{F}} F + V^E \left(\frac{Y}{F}, 1 \right) \\
V_F^E &= -y v_y^E + v^E
\end{aligned} \tag{B-2}$$

$$\begin{aligned}
rV^E(Y, F) &= \max_G \Pi_{t,t+dt} + V_F^E dF_t + V_Y^E \mu_Y Y_t dt + \frac{1}{2} V_{YY}^E \sigma_Y^2 Y_t^2 dt \\
r_{t,t+dt}^E &= \frac{1}{V^E(Y_t, F_t)} \left(rV^E dt + V_Y^E \sigma_Y Y_t dW_{Y,t}^Q \right) \\
&= r dt + \frac{V_Y^E Y_t}{V^E(Y_t, F_t)} \sigma_Y dW_{Y,t}^Q; \text{ divide by } F \\
&= r dt + \frac{v_y^E Y_t}{v^E(Y_t, F_t)} \sigma_Y dW_{Y,t}^Q; \text{ and using Equation (B-2) we arrive at} \\
&= r dt + \frac{v^E - V_F^E}{v^E(Y_t, F_t)} \sigma_Y dW_{Y,t}^Q; \text{ using FOC from Equation (A-2)} \\
&= r dt + \frac{v^E + v^D}{v^E} \sigma_Y dW_{Y,t}^Q; \\
&= r dt + \left(1 + \frac{v^D}{v^E} \right) \sigma_Y dW_{Y,t}^Q
\end{aligned} \tag{B-3}$$

Under the physical measure we find:

$$\begin{aligned}
r_{t,t+dt}^E &= r dt + \left(1 + \frac{v^D}{v^E} \right) \sigma_Y \lambda_t dt + \left(1 + \frac{v^D}{v^E} \right) \sigma_Y dW_{Y,t}^P \\
r_{t,t+dt}^E &= r dt + \left(1 + \frac{v^D}{v^E} \right) \eta_t dt + \left(1 + \frac{v^D}{v^E} \right) \sigma_Y dW_{Y,t}^P.
\end{aligned} \tag{B-4}$$

C Detailed Beta Derivation

The equity beta can be calculated as:

$$\beta_{i,t} = \frac{Cov_t(r_{t,dt}^E, r_{t,dt}^M)}{Var_t(r_{t,dt}^M)} \quad (C-1)$$

$$= \frac{1}{Var_t(r_{t,dt}^M)} Cov_t \left(\left(1 + \frac{v_i^D(y_t)}{v_i^E(y_t)} \right) \sigma_Y dW_{Y,t}^P; \sigma_x dW_{x,t}^P \right) \quad (C-2)$$

$$= \frac{1}{\sigma_x^2} \sigma_Y \sigma_x \left(1 + \frac{v_i^D(y_t)}{v_i^E(y_t)} \right) Cov_t(dW_{Y,t}^P, dW_{x,t}^P) \quad (C-3)$$

$$= \frac{1}{\sigma_x} \sigma_Y \left(1 + \frac{v_i^D(y_t)}{v_i^E(y_t)} \right) Cov_t \left(\frac{1}{\sigma_Y} (\sigma_x dW_{x,t}^P + \sigma_i dW_{i,t}^P), dW_{x,t}^P \right) \quad (C-4)$$

$$= 1 + \frac{v_i^D(y_t)}{v_i^E(y_t)}. \quad (C-5)$$

D Definition of Variables

In this section we provide definitions for the metrics and proxies used in the empirical part. For each item used from either COMPUSTAT or CRSP we identify the source. The item abbreviations are matched to variable descriptions in Table C-1.

Table C-1: COMPUSTAT & CRSP Item Description. Items from COMPUSTAT are listed below in capital letters, all variables from CRSP are listed using lower case letters.

Item Name	Variable Description
<i>CSHO</i>	Common Shares Outstanding
<i>DD1</i>	DD1 – Long-Term Debt Due in One Year
<i>DD2</i>	DD2 – Debt Due in 2nd Year
<i>DD3</i>	DD3 – Debt Due in 3rd Year
<i>DLC</i>	Debt in Current Liabilities - Total
<i>DLTT</i>	Long-Term Debt - Total
<i>PRCC_F</i>	Price Close - Annual - Fiscal
<i>PSTKRV</i>	Preferred Stock Redemption Value
<i>PSTKL</i>	Preferred Stock Liquidating Value
<i>PSTK</i>	Preferred/Preference Stock (Capital) - Total
<i>TXDITC</i>	Deferred Taxes and Investment Tax Credit
<i>altprc</i>	Price Alternate
<i>shROUT</i>	Number of Shares Outstanding

We define *leverage* as the ratio of book debt to book debt plus market equity, as in [Danis et al. \(2014\)](#).

$$L := \frac{DLC + DLTT}{DLC + DLTT + PRCC.F * CSHO} \quad (C-1)$$

Next, we define a proxy for *debt maturity* by looking at the share of debt maturing in more than 3 years to the total amount of book debt, as proposed by [Barclay and Smith \(1995\)](#). To measure debt maturing in more than 3 years we subtract debt maturing in the 2nd and 3rd year (items *DD2* and *DD3*, respectively) from the total of long-term debt.

$$DM := \frac{DLTT - DD2 - DD3}{DLC + DLTT} \quad (C-2)$$

Market equity is defined as the price per share times shares outstanding scaled by a factor 10^{-3} .

$$ME := \frac{|altprc| * shrout}{1000} \quad (C-3)$$

The *book value of equity* is defined in line with [Fama and French \(1992, 1993\)](#) as the book value of stockholder's equity adjusted for the value of tax effects of deferred taxes and investment credit and subtracting the book value of preferred stock. The value of preferred stock (abbreviated [*BVPS*]) is determined by taking redemption, liquidation, or par value (from COMPUSTAT *PSTKRV*, *PSTKL*, or *PSK*, respectively) depending on availability in the given order.

$$BE := SEQ + TXDITC - [BVPS] \quad (C-4)$$

Finally, the *book-to-market* ratio is calculated as proposed by [Fama and French \(1992, 1993\)](#). This means to relate book equity as computed by the fiscal year ending in year *t* to market equity as of December of year *t*.

$$BM := \frac{BE}{ME} \quad (C-5)$$

For the conditional version of the CAPM, we take predictors from Amit Goyal's homepage. The divi-

dividend yield (denoted by DY) is the log difference between dividends and lagged prices. Where the dividends are the 12-month moving sum of dividends on the S&P 500. The default spread (denoted by DS) is the difference between the yields on BAA and AAA-rated corporate bonds. The term spread (denoted by TS) is defined as the yield difference between long-term and short-term government bonds. T-bill rate is denoted as TB .

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