Strategic Complexity

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Abstract

This paper explores the incentives of product designers to produce complex products, and the resulting implications for overall product quality. In our model, there is a consumer who can accept or reject a product proposed by a designer, who jointly chooses the quality and the complexity of the product. While the product's quality determines the direct benefits of the product to the consumer, the product's complexity primarily affects the information she can extract about the product's quality. Examples include banks that design financial products that they later offer to retail investors, or policymakers who propose policies for approval by voters. We find that complexity is not necessarily a feature of bad quality products. For example, while an increase in alignment between the consumer and the designer leads to more complex but better quality products, higher demand or lower competition among designers leads to more complex and worse quality products. We discuss how our findings can rationalize the observed trends in complexity of financial products and of regulation.

Keywords: strategic complexity, information frictions, financial products, product design, regulation.

JEL Codes: D82, D83, G18, P16, D78.

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1 Introduction

In recent decades, the issue of rapidly increasing complexity has been raised by both financial market participants and policymakers. In the financial industry, for instance, it is argued that the products sold to retail investors have become increasingly complex – with product descriptions that contain jargon and complicated or vague explanations (Carlin, 2009; Carlin and Manso, 2011; Célérier and Vallée, 2015). Similarly, in the regulatory or the legislative process, it has become more common to encounter policy proposals lacking specificity, with broadly worded or ambiguous provisions (Davis, 2017). Such increase in complexity could be a concern if it prevents consumers from evaluating the quality of the financial products that they buy, or of the policies that they support. This, in turn, could foster the proliferation of bad quality products and policies. Consistent with this argument, Célérier and Vallée (2015) find that issuers of lower quality financial products deliberately increase their complexity in order to hide risks from investors.[1]

In many situations, consumers evaluate a product before deciding whether to accept it or not. Some examples include retail investors who evaluate financial products, the median-voter who evaluates a policy proposal, the Editor of a journal who evaluates a paper. To make a decision, a consumer gathers information from several sources, such as the description of the product attributes, reviews, media reports, etc. The product designer, in turn, can influence the quality of the information the consumer receives by designing a more or less complex product.[2] For example, a product can be made more complex by adding unnecessary attributes and contingencies, or by opting for complicated jargon and lengthy and ambiguous descriptions. In such an environment, however, if all (and only) bad products were complex, it would be very easy for a consumer to reject all products whose attributes she doesn't understand well. This logic seems to contradict the increasing and prevalent provision of complex financial products and complex regulation witnessed in the last two decades. Motivated by this puzzle, we develop a framework to understand the drivers of product complexity, and its implications for the production and proliferation of bad quality products.

In the model, there is one consumer who wants a product that can only be produced by a product designer. When designing a product, the designer privately and separately chooses the product's output and complexity. While a product's output determines the direct payoff to the consumer, which can be good or bad, a product's complexity influences the consumer's

[1] Also related, Christoffersen and Musto (2002) show that financial institutions use distortions in transparency in order to price discriminate among investors.

[2] We think of financial intermediaries as the designers of financial products, and of policymakers as the designers of legislation and policy proposals.

ability to learn the product's output. We model this by assuming that when a product is more complex, the consumer is more likely to extract noisy information about the product's output. We focus on cases in which the objective of the product designer is to have his product *accepted*, while the objective of the consumer is to accept a good product. For example, the objective of a bank is to convince a retail investor to accept a given savings account; while a policymaker wants voters' approval for his tax reform proposal.[3] Finally, we suppose that product designers are misaligned with the consumer, as they receive a higher payoff from having a bad product accepted. This misalignment captures in reduced form differences between the interest of the consumer and that of the designer, stemming from good products being more costly to design, career concerns, ideological preferences, or privately negotiated sales commission incentives. The timing of the game is as follows. First, a designer privately chooses a product's output and complexity, and proposes the designed product to the consumer. Then, the consumer obtains information about product output and decides whether to accept or reject the product. If the consumer accepts, product payoffs are realized; otherwise, everyone gets their outside options.

An important contribution of our paper is to model the joint decision of choosing a product's quality and complexity, which we view as two attributes that a designer can control separately.[4] For example, the quality (i.e., output) of a financial product should be determined by the net present value (NPV) that it generates to an investor. There are, however, many financial contracts that generate the same NPV. Thus, for a given NPV, a financial product can be made more complex by adding contingencies that generate zero NPV to the investor, by using ambiguous words in the product's description, by linking payments to financial indeces that the consumer is unlikely to know, etc. A similar argument can be made for policymakers in charge of writing policy proposals. By studying both attributes separately, we are able to gain a better understanding of the incentives to produce good/bad quality products vs. complex/simple products. For simplicity, we also assume that a product's complexity does not directly affect the consumer's payoff from the product. We show that our qualitative results do not depend on this assumption in Section 6, where we allow for deviations from some "natural level of complexity" to be costly for the consumer.

Our framework delivers several powerful insights. We show that complexity is not always a feature of bad quality products. In fact, designers of good quality products will sometimes choose to complexify them, and designers of bad quality products will sometimes choose to

[3]Our objective as paper designers is to have the Editor accept our proposed paper for publication.

[4]In a different context, Bar-Isaac et al. (2010) study incentives to produce different product attributes by exploring firms integrated strategy for marketing, pricing, and investment in quality, where marketing affects the information consumers' receive about the product's other attributes.

simplify them. The model generates novel implications for the relationship between product quality and complexity. In particular, as product designers become more aligned with the consumer, both product quality and complexity increase. On the other hand, as the demand for a product increases and/or competition among product designers decreases, product quality falls and complexity increases. With these results, we are able to discuss possible (and novel) drivers of the recent trend towards complexification in finance and regulation, and to better understand the resulting implications for product quality.

A key insight of our model is that incentives to design complex or simple products depend crucially on the consumer's acceptance strategy in the absence of information. In particular, all designers have incentives to design a complex product when the consumer would accept the product in the absence of information –in this case, we say the consumer is *optimistic*. This result, though surprising at first, is intuitive: information can only increase the chances of a product being rejected, since it is already accepted with probability one in the absence of information. In contrast, when the consumer would reject a product in the absence of information –i.e., the consumer is *pessimistic*,– the designers have incentives to design simple products. Now, information weakly increases the chances of a product being accepted, since it is rejected with probability one in the absence of information.[5]

It is essential for our results that communication is imperfect: in particular, the product designer cannot ensure that the consumer receives perfect information. By choosing a simple product, a designer can only decrease the likelihood that the consumer receives noisy information.[6] This is natural, since in most settings product designers cannot fully control the information set of the consumer. The latter may feel comfortable with the words used in the product description due to her education, but she could also misinterpret what she reads in it; she may have access to reviews from other consumers or media articles that further simplify or confuse her understanding of the product's quality. Thus, our model suggests that in some situations, when designers of good quality products cannot perfectly reveal their quality to consumers, they may be better off by making their products complex.

Our model predicts that all designers have incentives to complexity their products when (i) alignment between the consumer and the designer is high, or (ii) the consumer's outside option is sufficiently low, which we show can reflect high product demand or low competition

[5] A related finding is present in Perez-Richet and Prady (2011), who consider a setting with a privately informed sender that can "complicate to persuade" a receiver to obtain a certification, where complication increases the cost of the receiver to acquire information.

[6] We show that when the designer can choose an infinitely precise signal, there is always a perfect communication equilibrium where only good products products are designed, and this is revealed to the consumer through a perfectly informative signal.

among designers. At the other extreme, all designers have incentives to simplify, while in intermediate cases bad products are made complex and good products are made simple.

We then examine the implications for the design of bad quality products. A designer trades-off a higher probability of acceptance (with a good product) with a higher payoff conditional on acceptance (with a bad product). As a result, when the consumer's acceptance strategy is very strict, designers have higher incentives to produce good products. In equilibrium, of course, the strictness of the consumer's acceptance strategy depends also on how complex products are. We show that in equilibrium, the designer produces bad products with probability $m \in (0, 1)$; i.e. the equilibrium is in mixed strategies. Thus, changes in this probability determine overall product quality. We show that products are more likely to be of worse quality when (i) the alignment between the consumer and the designer is low, (ii) demand for the product is high, or (iii) competition among designers is low.

Finally, we combine our previous results to obtain new predictions about the relationship between product quality and complexity in equilibrium. First, as the alignment between the consumer and the designer increases, products will tend to be better quality but more complex, suggesting a positive relation between quality and complexity. Second, as the consumer's demand for the product increases, or competition among designers decreases, products will tend to be worse quality and more complex, suggesting a negative relation between quality and complexity. Therefore, in order to understand the relationship between quality and complexity of products, it is essential to understand the underlying drivers of product heterogeneity.

We show that our results are robust to several extensions. First, we introduce aligned designers to the model; that is, designers who obtain a higher payoff from producing good products. We show that a (relatively) large fraction of aligned designers is needed to change the equilibrium, in which case both quality and complexity increase. Second, we suppose that the consumer can perfectly observe the designer's choice of complexity (as in Perez-Richet and Prady (2011)). We show that in this scenario, two pooling equilibria co-exist: all designers simplify or they all complexity. Furthermore, when the consumer is sufficiently optimistic (pessimistic), all designers are better off in the complex (simple) equilibrium. An advantage of our approach is that it eliminates the multiplicity of equilibria that is common to signaling games, due to the freedom in setting off-equilibrium beliefs; this in turn allows us to obtain rich comparative statics. Third, we consider direct costs of complexity by supposing that deviations from some "natural level" of complexity are costly to the consumer. We show that our main results remain qualitatively unchanged, with the intuitive new prediction that complexity decreases (increases) as it becomes more (less) costly to the consumer. Finally, we show that our model can be re-stated, though at a loss in tractability, as an optimal information extraction problem, where it is more costly for the consumer to reduce uncertainty (entropy) about the quality of a more complex product.

Even though our model is stylized and abstracts from many institutional details of real-world settings, we nevertheless discuss our model's predictions within the context of our two concrete applications: financial products and regulatory policy. In financial markets, intermediaries design financial products to offer to retail investors, such as savings account and asset-backed securities. The design of a product consists of determining a set of cash flows for different states of the world (price, future payments, fees, contingencies, etc.) and of writing these contract terms down. The investor evaluates a product and decides whether to invest/borrow or not. If she does not, her outside option is to either search for another financial advisor, or to do nothing. In this context, our model suggests that the increase in the complexity of financial products documented by Célérier and Vallée (2015) could be driven by (i) an increase in investor's trust in financial advisors, or (ii) an increased demand for financial products. Both of these features were characteristics of financial markets prior to the 2008/09 crisis, and while the trust in the financial system may have fallen in response to the crisis, the high demand for relatively safe financial products still persists. This suggests that the observed proliferation of worse and more complex products could be an endogenous response of product designers to an increasing demand for relatively safe financial products. These results are complementary to those in Pagano and Volpin (2012), who show that securitizers may have incentives to increase the opacity of their products in order to draw unsophisticated investors into the market. We obtain a similar prediction of increased complexity, but through a different mechanism. In their model, opacity is used because information release would create a winner's curse problem to unsophisticated investors. Meanwhile, in our model, designers complexify products in response to high demand, because consumers in markets with high demand are more likely to be optimistic, and thus more inclined to accept complex products.

In the political sphere, politicians are the designers in charge of proposing policies, such as plans for taxation or regulation. We can interpret the consumer in such a setup as the median voter, from whom the politician must obtain approval for policy proposals. A more complex policy proposal by a politician may, for instance, take the form of less specific promises and hazy details on the exact implementation of the policy goal. This idea is present in the literature of strategic ambiguity, deliberate vagueness or noise by politicians (Alesina and Cukierman, 1990; Aragones and Postlewaite, 2002; Espinosa and Ray, 2018). Our model suggests that policy proposals are more likely to be complex when (i) public opinion about the politician's alignment with the median voter is high; and (ii) there is urgency to pass a given policy; that is, the status-quo is costly. Both of these features were present when complex policies, such as the Affordable Care Act and the Dodd-Frank Act were passed. Moreover, in policy areas where public opinion of politicians is low, more policy proposals tend to be simple and to delegate subsequent details and lawmaking authority to federal agencies, for example to the FDA in the case of the pharmaceutical industry.

Our paper is closely related to the literature on obfuscation (Carlin, 2009; Ellison and Ellison, 2009; Carlin and Manso, 2011; Ellison and Wolitzky, 2012), and shrouding attributes (Gabaix and Laibson, 2006) or limited awareness (Auster and Pavoni, 2018). These papers study the incentives of sellers of homogeneous goods to obfuscate by limiting the buyers' ability to observe certain product attributes or to compare attributes with those of competing products. In common with these papers, in our model the producer can take an action to affect the information that the consumer receives. Our approach, however, differs in several respects. First, in our setting, the designer jointly chooses the quality and the complexity of the product, resulting in heterogeneous goods being offered in equilibrium. This allows us to study the incentives to produce complex *and* bad products. Second, the consumer in our setting is Bayesian, and she rationally processes all of the information she receives; i.e., there are no hidden-attributes or information search costs. The interaction of complexity added by the sender and the receiver's learning from a noisy signal relates our model to Dewatripont and Tirole (2005); however, their focus is on costly communication in a setting with moral hazard in teams.

Finally, our paper relates to the broader literature on strategic information transmission (Crawford and Sobel, 1982; Grossman, 1981; Milgrom, 1981; Kartik, 2009), and on Bayesian persuasion (Green and Stokey, 2007; Kamenica and Gentzkow, 2011). These papers focus on the optimal information structure with private information (ex-post design) or with commitment (ex-ante design), which is not the focus of our paper. Although in our setting, information transmission (i.e. complexity) is strategic and chosen ex-post, the designer does not optimally choose an information structure. In particular, he cannot perfectly transmit, nor perfectly obfuscate, information about the quality of the product he has chosen, which as we have argued is essential for our results. Instead, we view our approach as closer to the literature on rational inattention, as in Sims (2003), if we interpret product complexity as an attribute that limits the consumer's ability to process information.[7]

The rest of the paper is organized as follows. In Section 2, we present the setup of the

[7] For a recent survey of the literature see Wiederholt et al. (2010).

model. In Section 3, we illustrate our main results and in Section 5, we present the model comparative statics, which we discuss in the context of our leading applications in Section 6. In Section 7 we show that our model is robust to several extensions. Section 8 concludes. All of the proofs are relegated to the Appendix.

2 The Model

We consider the following interaction between a consumer and a product designer. The consumer needs a product, that only the designer can produce. The designer privately takes two actions $\{y, \kappa\}$, where $y \in \{\mathbf{G}ood, \mathbf{B}ad\}$ determines the product's output, and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ determines the product's complexity, which is defined in detail below. After this, the designer proposes the product to the consumer, who evaluates it and decides whether to accept it $(a = 1)$ or take an outside option $(a = 0)$.

A Product's Output. By taking action y, the designer affects the output of the proposed product. We assume that the payoff to the consumer from accepting a product with output y (which we refer to as a y-product) is $w(y)$, and her outside option if no product is accepted is w_0. The designer receives payoff $v(y)$ from having a y-product being accepted, and zero otherwise. We make the following assumptions on the payoffs:

Assumption 1 *The payoffs satisfy the following properties:*

1. $w(G) > w_0 > w(B) \geq 0$.

2. $v(B) > v(G) > 0$.

The first assumption states that the consumer wants to accept a G-product but reject a B-product. The second assumption states that the designer is *misaligned* with the consumer, as he prefers to have a B-product being accepted. A natural interpretation of the latter assumption is that it is more costly to design good quality products than bad quality ones.[8]

A Product's Complexity. The complexity of a product determines how difficult it is for the consumer to understand the product's output, y. Formally, we suppose that after the designer proposes product (y, κ), the consumer is able to extract a binary signal $S \in \{b, g\}$ about the

[8]In the financial products industry, misalignment can also arise due to financial advisors receiving higher fees for selling products that are not necessarily the best fit for their clients (i.e., fixed vs. adjustable-rate mortgages). In the policy sphere, misalignment of policymakers vis-à-vis the public may also arise due to ideological differences, lobbying, or career concerns.

product's output with some noise $z \equiv \mathbb{P}(y = G|S = b) = P(y = B|S = g)$, where $z \sim f(\cdot|\kappa)$ with full support on $\left[0, \frac{1}{2}\right]$ for all κ with the property that $\frac{f(z|\bar{\kappa})}{f(z|\underline{\kappa})}$ increasing in z (MLRP). That is, the signal that the consumer extracts is more likely to be noisy when the product is complex.[9] That complexity, κ, is not perfectly observed by consumers is convenient but not essential for our main results: it rules out multiplicity of equilibria, typical of signaling games, that arise from the freedom in specifying off-equilibrium beliefs. This facilitates comparative statics, and it also has the natural interpretation that consumers are not able to perfectly observe the underlying actions of the designers towards complexification. Nevertheless, we show that our main results do not depend on this assumption in Section 7.1.

Remark 1 *We have assumed that complexity does not directly affect the consumer's payoffs. This is convenient, as it allows us to isolate the strategic role of complexity in deterring information acquisition or learning by consumers. For example, a product could be made "more" complex by the use of complicated words and jargon in its description, without necessarily affecting the product's attributes and consumers' payoffs. Nevertheless, we incorporate direct costs of complexity (or simplicity) to the consumer in Section 6.2 and Appendix D.*

The Consumer's Problem. The consumer has to decide whether to accept the designer's product or not. Before making her decision, the consumer observes the signal realization s with noise z, and forms her posterior beliefs about the output y, denoted by $\mu(s, z) \equiv \mathbb{P}(y = G|s, z)$. The consumer's acceptance strategy maximizes her expected payoff, given by

$$W(a|s, z) \equiv a \cdot [\mu(s, z) \cdot w(G) + (1 - \mu(s, z)) \cdot w(B)] + (1 - a) \cdot w_0. \tag{1}$$

The Designer's Problem. The designer's expected payoff is given by

$$V(y, \kappa) \equiv \mathbb{P}(a = 1|y, \kappa) \cdot v(y) \tag{2}$$

where $\mathbb{P}(a = 1|y, \kappa)$ denotes the probability that product $\{y, \kappa\}$ is accepted by the consumer. The designer chooses $y \in \{G, B\}$ and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ to maximize (2). We denote the designer's strategy by $\{m, \sigma_G, \sigma_B\}$, where $m = \mathbb{P}(y = G)$ is the probability with which the designer chooses the G-product, and $\sigma_y = \mathbb{P}(\kappa = \bar{\kappa}|y)$ is the probability with which he chooses high complexity, conditional on him also choosing the y-product.

[9] The restriction to binary-symmetric signals facilitates tractability but is not crucial for our main results (see Appendix 7.2).

Equilibrium Concept. We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, given her beliefs, the consumer's acceptance strategy must maximize her expected payoff (*Consumer Optimality*). Second, the designer's strategy must maximize his expected payoff, given the consumer's strategy (*Designer Optimality*). Finally, the consumer's beliefs must be consistent with the designer's strategy and updated using Bayes' rule when possible (*Belief Consistency*).

2.1 Benchmarks

Before we proceed to the equilibrium analysis, we find it useful to establish two benchmarks against which our results can be contrasted. First, we consider the allocations that would arise in the absence of asymmetric information (*Perfect information*). Second, we consider the allocations that would arise if the designers were able to perfectly communicate their product's attributes to the consumer (*Perfect communication*).

The findings of this section are summarized in the following proposition.

Proposition 1 *In both the perfect information and perfect communication benchmarks, there exists an equilibrium in which only G-products are produced by the designer, and where the consumer perfectly observes the product's output.*

Perfect information. Suppose that the consumer perfectly observes the output, y, chosen by the designer. Since the consumer's outside option, w_0, is greater than the payoff she obtains from a B-product, $w(B)$, and smaller than the payoff she obtains from a G-product (Assumption 1), it follows that she will accept a proposed product if and only if its output is $y = G$. Given this acceptance strategy, it is clear that it is optimal to only choose G-products.

Perfect communication. Suppose that the designer can perfectly communicate the output of her product to the consumer. Formally, assume the designer can freely choose the noise $z \in [0, \frac{1}{2}]$ of the signal. Then, there is always an equilibrium with perfect communication: the designer produces G-product and chooses $z = 0$. To see this, first, note that it is optimal for the designer of a G-product to perfectly reveal the product's output, i.e. choose $z = 0$, since in that case his product is accepted with probability one.[10] Assume that the designer of a G-product indeed chooses $z = 0$. Then, the consumer's interim belief after observing noise $z \neq 0$ is $\mu(z) = 0$; that is, the consumer infers that she has been proposed a B-product when the

[10] We assume that the consumer's posterior belief after observing a perfectly informative signal is $\mu(g, 0) = 1$ and $\mu(b, 0) = 0$ for all prior beliefs $\mu \in [0, 1]$.

designer does not perfectly communicate his product's output. But then, since the consumer rejects any product with $z \neq 0$, it is optimal for the designer to only produce G-products, which is in turn consistent with the consumer's beliefs. Note that this argument does not depend on the information structure as long as perfect communication is possible.

3 Equilibrium

In this section, we characterize PBE of our game. First, we consider the consumer's acceptance strategy, given her beliefs about the product proposed by the designer (Section 3.1). Second, we analyze the designer's strategy: his choice of output (Section 3.3) and of complexity (Section 3.2), given the consumer's optimal acceptance strategy. Finally, we impose belief consistency to obtain the equilibria of our model (Section 3.4).

3.1 Consumer's Acceptance Strategy

From the consumer's problem, as given in (1), we see that she follows a threshold strategy: she accepts the product, $a(s,z) = 1$, if and only if her posterior belief about the product having good output is sufficiently high, $\mu(s,z) \geq \omega$, where $\omega \equiv \frac{w_0 - w(B)}{w(G) - w(B)}$ captures the relative value of the consumer's outside option.[11]

Thus, to understand the consumer's acceptance strategy, we need to analyze the determinants of her posterior belief. Let $\mu \equiv \mathbb{P}(y = G)$ denote the consumer's prior belief. After the designer proposes his product, the consumer observes signal s with noise z about the product's output. Since z is informative about the choice of κ, it may contain information about output y. Let $\mu(z)$ denote the consumer's interim belief upon observing z,

$$\mu(z) \equiv \mathbb{P}(y = G|z) = \frac{\mu}{\mu + (1-\mu)\,\ell(z)} \tag{3}$$

where prior belief μ and likelihood ratio $\ell(z) \equiv \frac{\mathbb{P}(z|y=B)}{\mathbb{P}(z|y=G)}$ are computed using the designer's equilibrium strategies, which the consumer takes as given. As a result, the consumer's posterior belief, $\mu(s,z)$, after observing signal s with noise z, is as follows

$$\mu(s,z) = \frac{\mathbb{P}(S = s|y = G) \cdot \mu(z)}{\mathbb{P}(S = s|y = G) \cdot \mu(z) + \mathbb{P}(S = s|y = B) \cdot (1 - \mu(z))}. \tag{4}$$

The consumer's acceptance strategy is contingent on the observed signal only if she accepts

[11] If indifferent, we assume without loss that the consumer accepts the product.

the product when she observes a good signal, $S = g$, and rejects it when she observes a bad signal, $S = b$. For this to be optimal, the signal has to be informative enough so that:

$$\mu(b, z) \leq \omega \leq \mu(g, z) \tag{5}$$

Now, consider the threshold noise level $\bar{z}$ at which $\mu(s, \bar{z}) = \omega$ for some $s \in \{b, g\}$. This threshold determines the maximum noise level at which the consumer makes her acceptance strategy contingent on the signal.[12] The following definition will be useful in what follows.

Definition 1 *We say that the consumer is* **optimistic** *when the threshold $\bar{z}$ is given by condition $\omega = \mu(b, \bar{z})$, and that the consumer is* **pessimistic** *when it is given by $\omega = \mu(g, \bar{z})$.*

That is, the consumer is optimistic when in the absence of information she accepts the proposed product. Intuitively, the consumer is more likely to be optimistic when "trust" in the designer, captured by her prior belief μ, is high, or when her relative outside option, ω, is low. Consistent with this, when the noise of the signal is relatively high ($z > \bar{z}$) the consumer disregards her signal: she always accepts the product if she is *optimistic*, and rejects it if she is *pessimistic*. Conversely, when the signal is sufficiently informative ($z \leq \bar{z}$), the consumer makes her acceptance decision contingent on the signal: she approves the product after observing a **g**ood signal, and rejects it after a **b**ad signal. These results are formalized in the following lemma.

Lemma 1 *When the consumer is optimistic, her acceptance strategy is:*

$$a = \begin{cases} \mathcal{I}_{\{S=g\}} & \textit{if} \quad z \leq \bar{z} \\ 1 & \textit{if} \quad z > \bar{z} \end{cases}, \tag{6}$$

whereas when the consumer is pessimistic, her acceptance strategy is:

$$a = \begin{cases} \mathcal{I}_{\{S=g\}} & \textit{if} \quad z \leq \bar{z} \\ 0 & \textit{if} \quad z > \bar{z} \end{cases} \tag{7}$$

where $\mathcal{I}_{\{S=s\}}$ is the indicator equal to one when the signal is equal to s.

[12] In principle, $\bar{z}$ could take multiple values. In the Appendix we impose a condition on the distribution of z to ensure that in equilibrium $\mu(b, z)$ is increasing in z. This guarantees the existence of a unique $\bar{z}$, since $\mu(g, z)$ can be shown to always be decreasing in z, $\mu(g, 0) = 1$, $\mu(b, 0) = 0$, and $\mu(g, 0.5) = \mu(b, 0.5)$. Intuitively, the condition boils down to assuming that the information content of the signal, S, is greater than the information content of observing the realization of the noise itself, z. Though not essential for our main results, it considerably facilitates the analysis.

It is important to highlight the role of information in each scenario. When the consumer is optimistic, more precise information (weakly) increases the chances that any product is rejected, because a product is always accepted when information is sufficiently noisy ($z > \bar{z}$). The opposite holds when the consumer is pessimistic, where more precise information (weakly) increases the chances that any product is accepted.

Before we move on, it is important to note that whether the consumer is optimistic or pessimistic and the actual value of $\bar{z}$ in each case are endogenous, as they depend on the beliefs μ and $\mu(s, z)$ that need to be consistent with the designer's strategy and Bayes' rule.

3.2 The Designer's Choice of Complexity

We next consider the designer's choice of complexity, given his choice of output, y, and the consumer's acceptance strategy as described in the previous section. From the designer's objective in (2), it follows that a designer who chooses a y-product also chooses low complexity, $\underline{\kappa}$, whenever

$$\mathbb{P}(a = 1|y, \underline{\kappa}) \geq \mathbb{P}(a = 1|y, \bar{\kappa}). \tag{8}$$

Otherwise, the designer chooses high complexity, $\bar{\kappa}$. Using the results from Lemma 1, we can compute the probability of acceptance of a y-product conditional on a signal noise level, z, which we denote by $\pi(y, z)$. In the optimistic consumer case (o),

$$\pi^o(G, z) = \begin{cases} 1 - z & \text{if } z < \bar{z} \\ 1 & \text{if } z \geq \bar{z} \end{cases} \quad \text{and} \quad \pi^o(B, z) = \begin{cases} z & \text{if } z < \bar{z} \\ 1 & \text{if } z \geq \bar{z}, \end{cases} \tag{9}$$

whereas in the pessimistic consumer case (p),

$$\pi^p(G, z) = \begin{cases} 1 - z & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z} \end{cases} \quad \text{and} \quad \pi^p(B, z) = \begin{cases} z & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z}. \end{cases} \tag{10}$$

Thus, the designer's expected probability of a (y, κ) product being accepted depends on whether the consumer is optimistic ($j = o$) or pessimistic ($j = p$) and is given by:

$$\mathbb{P}(a = 1|y, \kappa) = \int_0^{\frac{1}{2}} \pi^j(y, z) \cdot f(z|\kappa) \cdot dz. \tag{11}$$

The following proposition provides the optimal choice of complexity of a designer that has produced a y-product. We say that the designer *simplifies* when he chooses $\underline{\kappa}$ ($\sigma_y = 0$), and

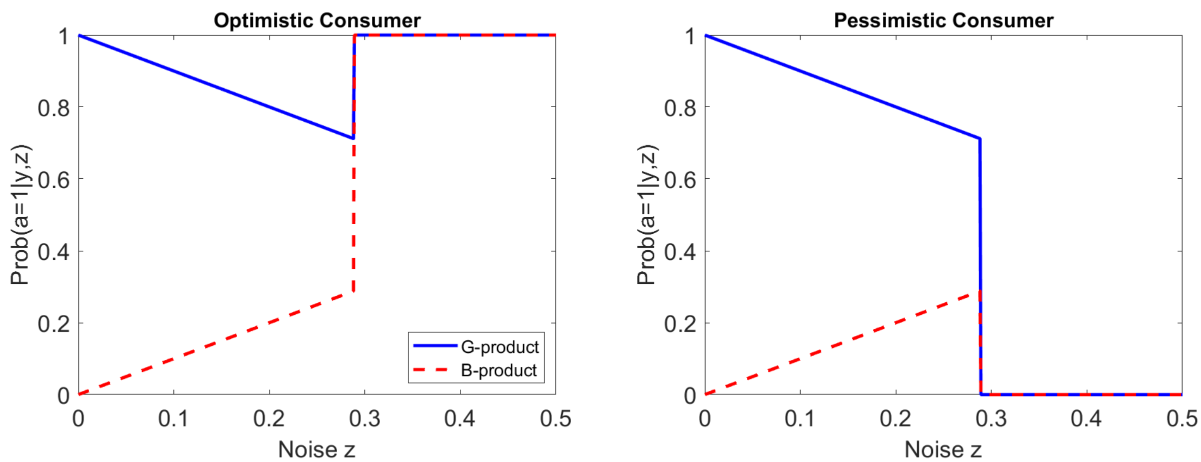

Figure 1: Illustrates the probability of acceptance of a y-product as a function of the signal's noise, z. The left panel depicts the acceptance strategy of an optimistic consumer. The right panel depicts the acceptance strategy of a pessimistic consumer.

that he *complexifies* when he chooses $\bar{\kappa}$ ($\sigma_y = 1$).

Proposition 2 *Let $\hat{z}$ denote the unique solution to $\int_0^{\hat{z}} z \cdot f(z|\underline{\kappa})dz = \int_0^{\hat{z}} z \cdot f(z|\bar{\kappa})dz$. Then, when the consumer is optimistic,*

$$\sigma_B = 1 \quad \textit{and} \quad \sigma_G \begin{cases} = 1 & \textit{if } \bar{z} < \hat{z} \\ \in [0,1] & \textit{if } \bar{z} = \hat{z} \\ = 0 & \textit{if } \bar{z} > \hat{z}, \end{cases} \tag{12}$$

whereas, when the consumer is pessimistic,

$$\sigma_B \begin{cases} = 0 & \textit{if } \bar{z} < \hat{z} \\ \in [0,1] & \textit{if } \bar{z} = \hat{z} \\ = 1 & \textit{if } \bar{z} > \hat{z} \end{cases} \quad \textit{and} \quad \sigma_G = 0. \tag{13}$$

The proposition shows that when the consumer is optimistic (pessimistic), there is a tendency to complexify (simplify). The intuition for this result can be obtained from Figure 1, which illustrates the acceptance probabilities $\pi^j(y,z)$ as a function of z, in each region $j \in \{o,p\}$. First, a B-product's probability of acceptance increases in the noise of the signal when the consumer is optimistic. As a result, the designer of a B-product chooses to complexify in such scenario. Second, a G-product's probability of acceptance decreases in the noise of the signal when the consumer is pessimistic. As a result, the designer of a G-product chooses to simplify in such scenario. The effect of z on the probability of acceptance, however, is non-monotonic for a B-product when the consumer is optimistic and for a G-product when the consumer is pessimistic. Here, the choice of complexity depends critically on the consumer's acceptance strategy, summarized by the threshold $\bar{z}$, which will be determined in equilibrium.

3.3 The Designer's Choice of Output

We now study the designer's problem of choosing product output. When choosing the product's output, the designer faces a trade-off between increasing the product's acceptance probability (by choosing $y = G$) or increasing his payoff conditional on acceptance (by choosing $y = B$). For a given acceptance strategy of the consumer, the net expected payoff to the designer from choosing the G-product over the B-product is:

$$\gamma \equiv \max_{\kappa} \mathbb{P}(a = 1|G, \kappa) \cdot v(G) - \max_{\kappa} \mathbb{P}(a = 1|B, \kappa) \cdot v(B). \tag{14}$$

The first term is the expected payoff from choosing the G-product given the corresponding best choice of complexity, as characterized in Proposition 2. The second term is the expected payoff from choosing the B-product given the corresponding best choice of complexity. The probabilities in each scenario are computed as in equation (11), given the consumer's acceptance strategy as characterized by the optimism/pessimism region and the threshold $\bar{z}$ described in Lemma 1. The next result then follows immediately.

Proposition 3 *Given the consumer's acceptance strategy, the designer chooses the G-product with probability*

$$m = \begin{cases} = 1 & \text{if } \gamma > 0 \\ \in [0, 1] & \text{if } \gamma = 0 \\ = 0 & \text{if } \gamma < 0, \end{cases}$$

where γ is given by (14).

3.4 Characterization of Equilibria

In Section 3.1, we characterized the consumer's acceptance strategy given her prior belief μ and interim belief $\mu(z)$. In Sections 3.2 and 3.3, we characterized the designer's choice of output and complexity, given the consumer's acceptance strategy. We now impose belief consistency to characterize the equilibria of our model.

We find it instructive to proceed in two steps. In the first step, we take the equilibrium distribution of product output (i.e., μ) as given, and we require that the consumer's interim belief, $\mu(z)$, be consistent with the designer's choice of complexity. This allows us to characterize the choices of complexity that are consistent with an equilibrium belief of μ. In the second step, we require that belief μ be also consistent with the designer's choice of output. This two-step procedure will help clearly isolate the determinants of product output and complexity, and how these two product attributes are related in equilibrium.

3.4.1 Consistency of Interim Beliefs

For a given equilibrium belief, μ, the consumer's interim beliefs are computed using the designer's strategies and Bayes rule, as given in equation (3). Since we have shown that complexity may be informative about the product's output (Proposition 2), the consumer's interim belief, $\mu(z)$ depends on $\{\sigma_y\}$ through the likelihood ratio as follows

$$\ell(z) = \frac{\mathbb{P}(z|y=B)}{\mathbb{P}(z|y=G)} = \frac{\sigma_B f(z|\bar{\kappa}) + (1-\sigma_B) f(z|\underline{\kappa})}{\sigma_G f(z|\bar{\kappa}) + (1-\sigma_G) f(z|\underline{\kappa})}. \tag{15}$$

Interim belief consistency requires that a y-product designer's choice of complexity, $\{\sigma_y\}$, be consistent with the consumer's acceptance strategy, computed using interim beliefs, which in turn depend on $\{\sigma_y\}$. Note that the consumer's acceptance strategy only depends on $\{\sigma_y\}$ when the equilibrium features separation in complexity levels. Otherwise, when $\sigma_G = \sigma_B$, noise z does not provide information about product output and thus $\mu(z) = \mu, \forall z$. This is consistent with an equilibrium if the acceptance strategy induced by such belief results in all product designers optimally choosing the same level of complexity. When $\sigma_B > \sigma_G$ (the other inequality never arises in equilibrium), the implied acceptance strategy of the consumer, which now does depend on $\{\sigma_y\}$, needs to be consistent with the designers choosing different complexity levels. The following proposition characterizes the choices of complexity that are consistent with an equilibrium belief $\mu \in (0,1)$.

Proposition 4 *For a given equilibrium belief, $\mu \in (0,1)$, there exist belief thresholds $\mu_1 - \mu_4$, which are given by (39)-(48), such that:*

1. *If $\mu \leq \mu_1$, all products are simple, $\sigma_B = \sigma_G = 0$.*

2. *If $\mu \in (\mu_1, \mu_2]$, G-products are simple, $\sigma_G = 0$, whereas B-products are complex with probability*

$$\sigma_B = \left(\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})} - 1\right)^{-1} \left(\frac{1-\hat{z}}{\hat{z}} \frac{\mu}{1-\mu} \frac{1-\omega}{\omega} - 1\right).$$

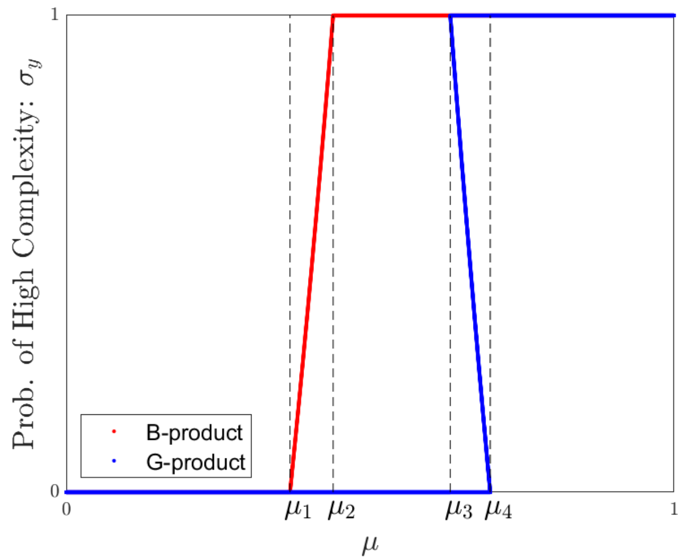

Figure 2: Illustrates the choice of complexity of a designer who chooses y-product, as it depends on the consumer's belief μ.

3. *If $\mu \in (\mu_2, \mu_3]$, G-products are simple, $\sigma_G = 0$, whereas B-products are complex, $\sigma_G = 1$.*

4. *If $\mu \in (\mu_3, \mu_4)$, G-products are simple with probability*

$$\sigma_G \in \left\{0,\ 1 - \left(1 - \frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})}\right)^{-1} \left(1 - \frac{1-\hat{z}}{\hat{z}} \frac{1-\mu}{\mu} \frac{\omega}{1-\omega}\right),\ 1\right\},$$

   whereas B-products are complex, $\sigma_B = 1$.

5. *If $\mu \geq \mu_4$, all products are complex, $\sigma_B = \sigma_G = 1$.*

These results are illustrated in Figure 2. When μ is relatively low, all products are made simple. This is because, when the consumer's belief is sufficiently low, she rejects the product after observing very noisy signals; thus, the designer benefits from producing simple products, independently of their quality. Instead, when μ is relatively high, all products are made complex. This is because, when the consumer's belief is sufficiently high, she accepts the product after observing very noisy signals; thus, the designer benefits from producing complex products. In addition, since the benefit of complexity is higher for B-products, for intermediate belief μ, G-products are made simple and B-products are made complex, i.e. $\sigma_B > \sigma_G$.

Finally, it is worth noting that there is a region of beliefs, $\mu \in (\mu_3, \mu_4)$, where multiple choices of complexity are consistent with a given belief μ. As we will see next, once we impose consistency of belief μ, complexity will be uniquely pinned down.

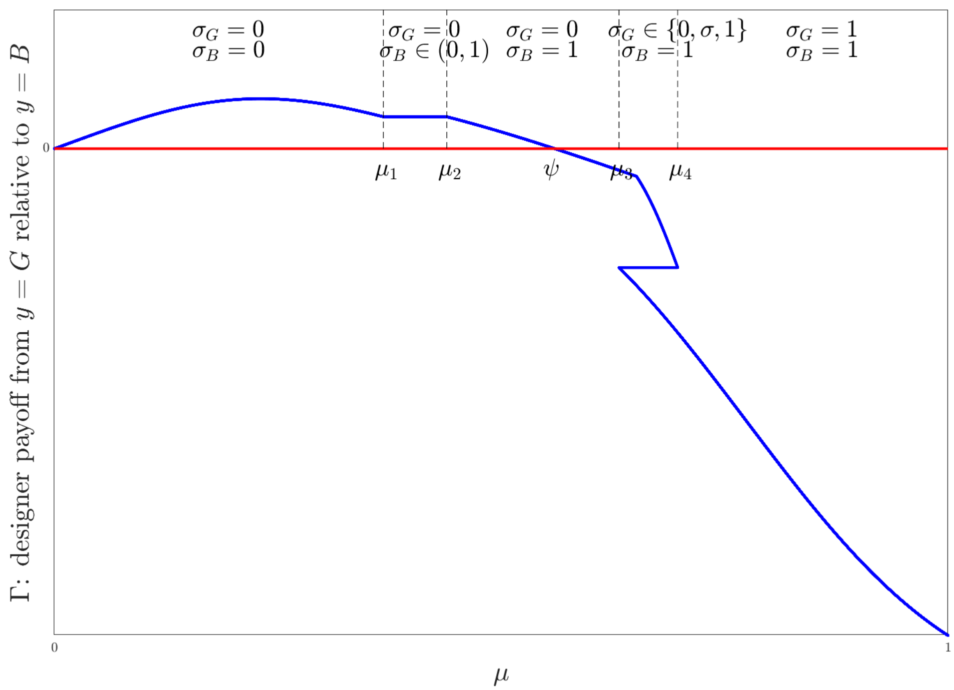

Figure 3: Illustrates the designer's net payoff from choosing the G-product, as it depends on belief μ.

3.4.2 Equilibrium

We now find the PBE of our model by requiring that the consumer's belief μ be consistent with the designer's choice of output, i.e. $\mu = m$. We begin the analysis by making explicit the dependence of the designer's net payoff $\gamma(\mu)$ from choosing the G-product (as defined in (14)) on the consumer's belief μ. The latter determines complexity strategies, $\{\sigma_y\}$ (Proposition 4), and the consumer's acceptance strategy. We define a correspondence $\Gamma : [0, 1] \to 2^{\mathbb{R}}$, where $\Gamma(\mu)$ is the set of $\gamma(\mu)$ implied by all the complexity choices consistent with an equilibrium with belief μ. The next lemma establishes some properties of this correspondence, which will be used to determine the set of all PBE.

Lemma 2 *The set $\{\mu > 0 : 0 \in \Gamma(\mu)\}$ is non-empty and generically has only one element, $\psi \in (0, 1)$. Furthermore, $\gamma(\mu) \gtreqless 0$ if and only if $\mu \lesseqgtr \psi$.*

Figure 3 illustrates the behavior of this correspondence, which is single-valued for $\mu \notin (\mu_3, \mu_4)$. When μ is low (i.e., $\mu < \psi$), the designer is better off producing a G-product, $\gamma(\mu) > 0$, as doing so increases his probability of acceptance since the consumer is more likely to reject when her beliefs are low. For the same reason, in this region products are made simple. Conversely, when μ is high (i.e., $\mu > \psi$), the designer prefers to produce a B-product, $\gamma(\mu) < 0$, as the consumer is more likely to accept when her beliefs are high. Furthermore, this is the region where products are made complex.

The following proposition provides a characterization of the equilibrium.

Proposition 5 *There are generically only two equilibria, which have the following properties:*

1. *Zero trade equilibrium: the designer produces bad products with probability one,* $m = 0$*, and complexity is indeterminate as all products are rejected with probability one.*

2. *Positive trade equilibrium: the designer produces good products with probability* $m = \psi$ *defined in Lemma 2, and complexity is as in Proposition 4 s.t.* $\mu = \psi$ *and* $\gamma(\psi) = 0$*.*

Consider first the *zero trade equilibrium*, in which the designer produces bad products with probability one, i.e. $m = 0$. In such an equilibrium, it is optimal for the consumer to reject products with probability one. But then, it is indeed optimal for the designer to produce such products; finally, product complexity is irrelevant in this equilibrium.[13]

Next, consider the *positive trade equilibrium*, in which the designer produces good products with positive probability, i.e. $m > 0$. Note that, if the designer were to produce good products with probability one, i.e. $m = 1$, then the consumer would accept products with probability one, but then the designer would want to deviate to produce bad products as those yield a higher payoff when accepted, $\gamma(1) < 0$, a contradiction. Thus, we have $m \in (0, 1)$ and the designer must be indifferent between the two products, i.e. $m = \psi$ defined in Lemma 2. Finally, the equilibrium complexity can be computed from Proposition 4 by setting $\mu = \psi$. Note that when $\psi \in (\mu_3, \mu_4)$, complexity is pinned down uniquely by the indifference condition $\gamma(\psi) = 0$. Figure 3 illustrates the workings of Propositions 4 and 6.

4 Introducing aligned designers

We now complete the model by introducing product designers whose payoffs are *aligned* with those of the consumer. In particular, we suppose that with probability $p \in [0, 1]$ the consumer can encounter an aligned product designer, who obtains a higher payoff from having a G-product being accepted, $\bar{v}(G) > \bar{v}(B) > 0$. With probability $1 - p$, however, the consumer meets a misaligned designer, and thus the model analyzed in Section 3 is the particular case of the general model with $p = 0$. Importantly, whether the designer is aligned or misaligned is not

[13] If there were a small cost of production, then no production would take place in this equilibrium.

observable to the consumer. An aligned designer chooses product output, y, and complexity κ, to maximize his expected payoff: $\mathbb{P}(a = 1|y, \kappa) \cdot \bar{v}(y)$.

First, for an aligned designer, both the probability of acceptance and the payoff conditional on acceptance are higher with a G-product, for any choice of complexity. Thus, for any belief μ, aligned designers produce G-products with probability one. Second, the choice of product complexity only depends on the product's output, and not on whether the designer is aligned or misaligned. As a result, the choice of complexity for the G-product is as in Proposition 4, as κ_G is chosen to maximize $\mathbb{P}(a = 1|G, \kappa)$.

The presence of aligned designers changes the equilibrium analyzed in Section 3 through the likelihood of a G-product being offered in equilibrium. Consistent with this, the new belief consistency condition is $\mu = p + (1 - p) \cdot m$. As before, however, to pin down the level of μ consistent with an equilibrium we need to analyze the incentives of misaligned product designers to produce G-products, m, which continues to be characterized by Proposition 3. The following proposition fully characterizes how the equilibrium changes with the presence of aligned designers.

Proposition 6 *In the presence of aligned designers, $p \in [0, 1]$, an equilibrium always exists, there are generically at most two equilibria with positive trade, with the following properties:*

1. *If $p \leq \psi$, the Positive Trade equilibrium of Proposition 5 with $m > 0$ and $\mu = \psi$ exists.*
2. *If $p > \psi$ or if $\mu_3 \leq p \leq \psi$ and $\inf \Gamma(\mu_3) \leq 0$, an equilibrium in which the misaligned designer produces the G-product w.p. $m = 0$, and thus $\mu = p$, exists. Complexity is given by Proposition 4 for $\mu = p$.*

When the fraction of aligned designers is small, $p > \psi$, they have no effect on aggregate equilibrium outcomes, as their presence only increases the probability with which misaligned designers produce B-products. This fully offsets the positive effect of aligned designers, as $\mu = \psi$ remains unchanged, and so do the corresponding choices of complexity. When the fraction of aligned designers is sufficiently high, $p > \psi$, the equilibrium changes as now all misaligned designers produce B-products, with overall quality determined by the fraction of aligned designers in the economy, $\mu = p$. Finally, for $p \in [\mu_3, \psi]$, both equilibria can co-exists: one in which $m > 0$ and $\mu = \psi$, and one in which $m = 0$ and $\mu = p$. This type of multiplicity arises due to the presence of multiple choices of complexity being consistent with beliefs $\mu \in (\mu_3, \mu_4)$ (see Proposition 4).

The results in Propositions 4 and 6 provide a full characterization of the equilibrium of the model with aligned and misaligned designers. In what follows, we analyze in more detail how overall quality and complexity vary with changes in the relative outside option, ω, and different measures of alignment, p and $v(B) - v(G)$.

5 Comparative Statics

In this section, we study the comparative statics properties of our model, which we later interpret in the context of two applications (Section 6). We will say that product quality increases when the probability of a product having a good output, $\mathbb{P}[y = G] = \mu$, increases; and that overall complexity increases when the probability of a product being complex, $\mathbb{E}[\sigma_y] = \mu\sigma_G + (1 - \mu)\sigma_B$, increases. We also discuss the quality and complexity of accepted products by conditioning these expectations on a product being accepted.[14]

We begin by considering the effect of a decrease in the consumer's relative outside option. A decrease in ω could result from a decrease in the consumer's payoff when the product is rejected, w_0, or by an increase in the consumer's payoffs when the product is accepted, i.e., higher $w(G)$ and/or $w(B)$. That is, a decrease in ω reflects that the net value of the product to the consumer increases, making the product more attractive.

Proposition 7 *As the consumer's relative outside option, ω, decreases, product quality falls, while product complexity increases. Moreover, all designers complexify if ω is sufficiently low.*

Intuitively, when ω is sufficiently large, the consumer is very selective in accepting products. This gives all designers an incentive to produce simple G-products, reflected in $\mu = \psi$ increasing and average complexity decreasing. As ω decreases, however, incentives to design simpler and better products fall, since the consumer's acceptance strategy becomes less strict. When ω is sufficiently low, the consumer accepts almost all products. Thus, all designers complexify and misaligned designers exclusively produce B-products, reflected in $\mu = p$ constant and high complexity. Figure 4 presents these results graphically, and shows that the quality and complexity of accepted products follow the same pattern.

Next, we consider the effect of an increase in the measure of aligned designers, p.

Proposition 8 *As the measure of aligned designers, p, increases, product quality increases, while complexity decreases for μ sufficiently low, and increases otherwise. Moreover, all designers complexity if p is sufficiently large.*

[14]Throughout this section, if multiple equilibria arise (i.e. only if $\mu \in (\mu_3, \mu_4)$), we focus on the equilibrium with the lowest level of complexity. Our qualitative results, however, remain unchanged if we focus on any of the other possible equilibria.

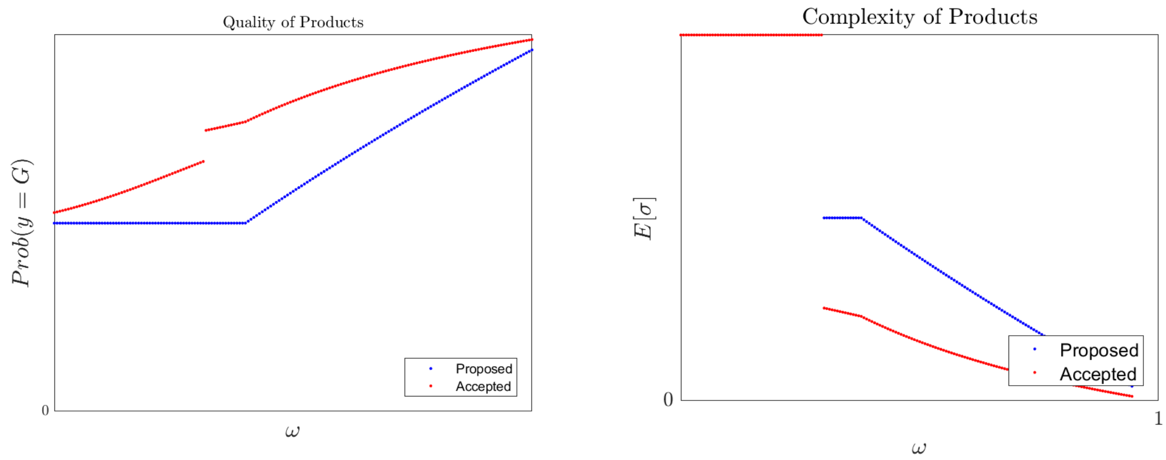

Figure 4: Product quality and complexity of proposed and of approved products as a function of the consumer's relative outside option $\omega \in [0, 1]$.

Not surprisingly, as the measure of aligned designers increases, so does product quality, since they always produce G-products.[15] The downside, however, is that the increase in average quality relaxes the acceptance strategy of the consumer, which increases designers' incentives to produce complex products. Consistent with this, when p is sufficiently large, the consumer will accept almost all products, and thus all designers complexify. Even though the quality of proposed products increases in p, the resulting increase in complexity worsens the consumers' ability to identify a G-product, which can result in a decrease in the quality of accepted products, as shown in Figure 5.

We conclude by analyzing the effect of an increase in the payoff to the misaligned designer from producing a G-product, $v(G)$.

Proposition 9 *As $v(G)$ increases, both product quality and complexity increase for large changes in $v(G)$, though they may be non-monotonic locally. Moreover, all products are made complex if $v(G)$ is sufficiently high.*

As $v(G)$ increases, the net payoff to the misaligned designer from producing a G-product increases, captured by an upward shift of the correspondence $\Gamma(\cdot)$. Generally, such an upward shift leads to an increase in ψ, and thus improvement in overall product quality for $p < \psi$. We say generally, however, because as ψ approaches μ_4, a further increase in $v(G)$ generates an upward jump in equilibrium complexity, from separation to pooling at complexity, which

[15]For $p < \psi$ such initial increase in quality improves the incentives of the misaligned designer to produce B-products, resulting in an decrease in m and a constant overall quality, $\mu = \psi$.

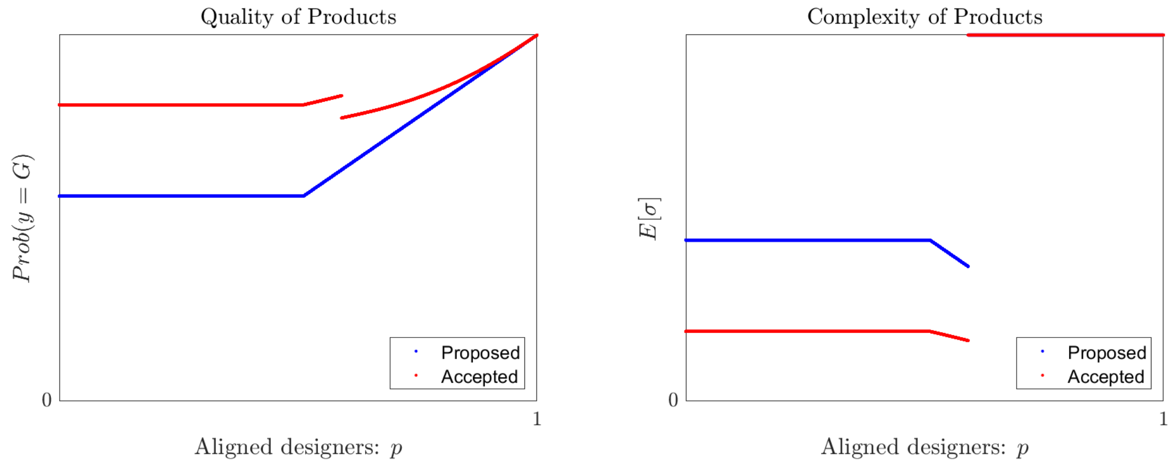

Figure 5: Quality of approved products: output and complexity as a function of p

requires a downward jump in ψ from μ_4 to μ_3. This can be seen by inspecting Figure 6. Note, however, that product quality is increasing in $v(G)$ before and after the jump.

An increase in $v(G)$ weakly increases the incentives of the designer to make products complex, i.e., σ_G and σ_B weakly increase in $v(G)$. The effect on average complexity, however, is non-monotonic as it depends on equilibrium μ. In particular, when the initial equilibrium features $\sigma_G < \sigma_B$, average complexity decreases as the designer produces simple G-products with higher probability than complex B-products. Otherwise, complexity increases in $v(G)$. This non-monotonicity can be seen graphically in Figure 6. Finally, as $v(G)$ becomes sufficiently high, all products are made complex.

The results in Propositions 7 to 9 have implications for the relationship between product quality and complexity, and highlight the importance of understanding the underlying drivers of product heterogeneity. We have shown that while more alignment between the consumer and the designer (captured by an increase in the measure of aligned designers, p, or by an increase in the payoff to misaligned designers from producing G-products), results in better and more complex products, higher demand for the products results in worse and more complex products. In what follows, we interpret the results of this section in the context of two applications.

6 Applications

We focus on two main applications that are at the center of the policy and academic discussion on issues related to complexity, and that have motivated our work: the design of financial

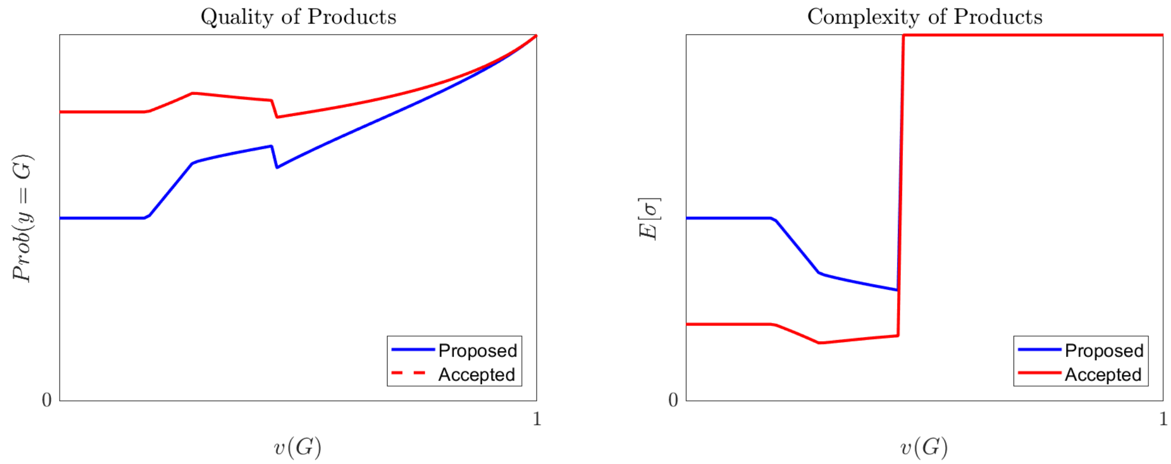

Figure 6: Quality of approved products: output and complexity as a function of $v(G)$

products and of regulatory policies. In what follows, we describe in detail how to map our model to these applications, and the resulting implications. Even though our model is stylized and not able to capture the richness of the institutional details of these environments, we present simple extensions to address particular issues of interest that arise within a particular application.

6.1 Financial products

Banks and other financial intermediaries design financial products that they offer retail investors, such as savings and retirement accounts, mortgages, credit lines, etc. When a retail investor (i.e., the consumer in our model) approaches a financial advisor (i.e., the product designer), the latter chooses which product to offer to the consumer. In practice, financial advisors may receive different payments from selling a given financial product to investors, and those products that give the financial advisor a higher commission need not be the best ones for the retail investor. When incentives are not aligned, the financial advisor faces a trade-off as the one faced by the designer in our model: to increase the probability of acceptance by offering the product that best suits the needs of the investor, or to increase his payoff conditional on acceptance by choosing the product with a higher commission. In turn, the choice of product design within financial institutions will crucially depend on the financial advisor's ability to sell different types of financial products, such as more or less complex. In what follows, we examine the drivers of product quality (i.e. the net present value of a financial product to the investor) and of the complexity in financial products (i.e., the multidimensionality of the contract) through the lenses of our model.[16]

Trust in financial advisors. Proposition 8 states that as the measure of aligned financial advisors, p, increases (which we view as an increase in the fraction of advisors whom retail investors trust, or that are "honest advisors") the overall quality of financial product increases, but so does their complexity. In our model, retail investors are rational and they do not misperceive the distribution of financial advisors. However, if we allowed for such deviations, an unjustified increase in the trust in financial advisors would generate a decrease in product quality accompanied by an increase in complexity. This observation is consistent with the behavior of several financial intermediaries that during the 2000s were allegedly designing and selling increasingly worse and more complex financial products to overly optimistic retail investors, a behavior that contributed to the financial crisis and resulted in multiple lawsuits.[17]

Motivated by this conflict of interest, the Securities and Exchange Comission (SEC) has increased its efforts to forbid the use of the term "financial adviser" for those managing brokerage accounts (particularly retirement funds) unless the broker has formally accepted a fiduciary duty to act in the investor's best interest.[18] If we interpret such policy as increasing the fraction of aligned financial advisors, our model predicts that it would not only increase the quality of financial products (as intended by the SEC), but also their complexity.

Competition in financial markets. To capture competition in financial markets, we extend our model to a dynamic search setting to capture the effects of competition. In particular, we now suppose that, if the consumer rejects a product, she searches for a new designer, whom she finds with probability $\beta \in (0, 1)$. The new designer proposes a product to the consumer, and the game repeats until the consumer accepts an offered product. In this setting, a higher β implies lower search frictions and, hence, more competition in the market for designers.

In a stationary equilibrium, in which U denotes the consumer's equilibrium value, we have:

$$U = \mathbb{E}\left[\max_{a\in\{0,1\}} \{a \cdot [\mu(s, z) \cdot w(G) + (1 - \mu(s, z)) \cdot w(B)] + (1 - a) \cdot \beta U\}\right]. \tag{16}$$

[16] While the net present value of a financial product is related to its dimensionality, it is also possible that two products provide the same net present value to an investor but vary in their dimensionality, as we have modeled it. We consider the case of complexity affecting the consumer's payoff directly in Section 6.2.

[17] In 2011, the Federal Housing Finance Agency filed lawsuits against some of the largest US financial institutions, involving allegations of securities law violations and fraud in the packaging and sale of mortage-backed-securities. For a detailed description, see https://www.fhfa.gov/SupervisionRegulation/LegalDocuments/Pages/Litigation.aspx.

[18] *"Fiduciary Rule" Poised for Second Life Under Trump Administration*, article by Dave Michaels on the Wall Street Journal, January 10th, 2018: https://www.wsj.com/articles/fiduciary-rule-poised-for-second-life-under-trump-administration-1515580200.

If we set $w_0 = \beta U$, then the static equilibrium is fully characterized in Section 3.4.

Proposition 10 *An equilibrium exists, and in it $\beta U \in (w(B), w(G))$ provided that β is not too low. Furthermore, βU is increasing in β.*

Comparative statics with respect to β are qualitatively similar to the comparative statics with respect to ω in Proposition 7. This is because, in a search environment, βU is the consumer's effective outside option, which we have shown increases in β. Thus, just as in our baseline model, an increase in competition, β, leads to an increase in overall quality and a decrease in complexity of products.

Demand for financial products. An increase in the relative payoff of given financial product to the consumer is captured in our model by a decrease in the consumer's outside option (w_0), or by an increase in the payoffs associated with a given product ($w(G)$ and $w(B)$). As shown in Proposition 7, our model suggests that as retail investors' demand for a given financial product increases, the quality of such products falls while their complexity increases. These predictions are consistent with the observed trend in financial products that were perceived as "safe" before the financial crisis. The increase in investor demand for these products, sometimes blamed on the so-called *savings glut*, could have been an important driver of their worsening quality and increased complexity, as exemplified by mortgage-backed-securities.[19]

Compensation structures. Our model can also be used to examine the role played by the financial advisors' compensation structure. If the compensation of financial advisors is linked to the volume or the characteristics of financial products that they sell, then the advisors are more likely to be misaligned with retail investors. In our model, we interpret this friction as the difference between the payoff associated with selling good vs. bad product: $v(B) - v(G)$. As shown in Proposition 9, as the difference in payments across products is reduced and the incentives of the financial advisor become more aligned with those of the investor, there is an increase provision of good financial products. Interestingly, such a change in compensation structures could result in more complexity when the effect on overall quality is strong enough. Note, however, that complexity is not too detrimental when the overall quality of financial products is very high.

[19]For evidence on the increasing demand for safe products; see Bernanke (2005), on the worsening quality of securitized products see Jaffee et al. (2009); and on the increasing complexity of financial products see Célérier and Vallée (2015).

6.2 Regulatory Complexity

Politicians design and propose policies to achieve a policy agenda. In doing so, they balance their own preferences (determined by ideology, informational lobbying etc.) and the need to obtain voter approval for that policy. In this environment, politicians (i.e., product designers) propose policies to voters (i.e., the consumer) who may accept or reject them. For instance, if the policy in question is a tax reform, the policy's quality is given by whether it implies "higher or lower taxes" or "more or less redistribution." In contrast, the policy is more complex if it contains unnecessarily complicated wording, several unlikely contingencies etc. An illustrative example of such complexity comes from the regulatory framework proposed by the Basel Committee on Banking Supervision. An analysis of its text has shown that an average sentence in the Basel documents consists of 25.7 words, significantly longer than the average 21 words in a sentence of the British National Corpus.[20] Moreover, the second sentence of the very first document published by the Basel Committee on Banking supervision spans over 72 words.[21]

Public opinion. In most cases, politicians do not have to wait until election time to learn about voters' support. Public opinion data provides politician's with real time information about voters' perceptions. Public opinion in our model would be captured by the voters' belief about the politicians being aligned with their interests in a certain policy area, captured by μ. Our model suggests that when public opinion is high, politicians have incentives to complexify policies, while simplicity should arise when public opinion is low. A variant of this implication has been used by legal scholars to explain why policy proposals coming out of the US Congress in domains in which public opinion of politicians is low tend to be simpler, leaving it to federal agencies to propose additional policies and to draft rules for these industries.[22] For instance, voters have higher distrust of politicians when it comes to policies pertaining to industries that are major lobbyists and contributors to campaigns, such as the financial services or pharmaceutical industries, and politicians tend to delegate more of that lawmaking to regulators rather than proposing complex policies themselves.

In terms of the quality of proposed policies, our model predicts that the quality of policies should be high when the improvement in public opinion is rational, but it may low if voters are being overly optimistic about the alignment between their interests and those of the politicians.

[20] The British National Corpus is a collection of texts covering a broad range of modern British English.

[21] Analysis performed by Neue Zurcher Zeitung, as cited by Marie-Jose Kolly and Jurg Muller, https://www.endofbanking.org/2018/05/22/how-banking-regulation-has-grown-out-of-all-proportions

[22] See Stiglitz (2017).

Urgency. The pressure or urgency to pass a given policy may vary depending on the reform under discussion. For example, there was a strong sense of urgency to pass financial regulation reform after the 2008-09 crisis. One possible reason being that the public did not feel they could trust the financial system otherwise. Other type of policies, such as environmental policies, do not seem to be considered with the urgency that they maybe deserve. Through the lenses of our model, urgency could be captured as the voters' outside option, a measure of the status-quo: i.e., the payoff to voters if the reform does not pass. In light of this, our model suggests that when there is urgency to pass a given reform, policies will tend to be of worse quality and more complex; while lack of urgency would result in better quality and simpler policies. This prediction is consistent with the mere observation that in the U.S., reforms passed in times of urgency, such as the Dodd-Frank Act or the Affordable Care Act, have been described as overly complex, while those passed in normal times, such as the Clean Air Act, seem to be seen as much simpler.

Direct costs of complex or simple rules. In the context of regulation, it is natural to think that the level of complexity may have a direct impact on the voter. On the one hand, complex policies may be worse if they imply a higher costs of compliance, e.g. hiring accountants and lawyers. On the other hand, more complex policies may be better when regulating complex systems, such as the financial system. In view of this, in Appendix D we extend our model to introduce what we refer to as a "natural level of complexity," which we denote by κ^n, where deviations from this natural level are costly to the consumer. Specifically, we suppose that the consumer pays a cost $c(\kappa) > 0$ when a product with complexity $\kappa \neq \kappa^n$ is accepted, and zero otherwise, where $\kappa^n \in \{\underline{\kappa}, \bar{\kappa}\}$. Then, given information (s, z) and acceptance decision $a \in \{0, 1\}$, the consumer's payoff is:

$$W(a|s,z) \equiv a \cdot E\left[w(y) - c(\kappa)|s,z\right] + (1-a) \cdot w_0 \tag{17}$$

In contrast to our baseline model, the consumer's acceptance strategy is now modified to incorporate the direct cost of complexity (or simplicity). The consumer's acceptance rule is again contingent on the signal only when the signal is sufficiently informative, i.e., $z < \bar{z}$, but with an adjusted threshold $\bar{z}$. The equilibrium analysis is analogous to the baseline model, with the not surprising prediction that the equilibrium level of complexity will be lower (higher) if complexity (simplicity) is costlier. Hence, our model helps us understand the policymakers' or regulators' strategic motives for designing more or less complex products relative to what would otherwise be natural, i.e. optimal for the voters.[23]

7 Extensions

7.1 Observable Complexity

We now consider the case in which the designer's choice of $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ is observable to the consumer. The rest of the model is as in the baseline setup. When κ is observed by the consumer, it can act as a signal of the product's output. Note that the noise z, in turn, no longer provides any information since κ is directly observed. Thus, given prior belief μ, the consumer does a first belief update after observing κ to interim belief

$$\mu(\kappa) = \Pr(y = G|\kappa) = \frac{\mu}{\mu + (1-\mu)\frac{\Pr(\kappa|y=B)}{\Pr(\kappa|y=G)}}, \tag{18}$$

and a second belief update after observing the signal to posterior belief $\mu(s, z, \kappa)$, given by (4) where interim belief $\mu(z)$ is replaced by $\mu(\kappa)$. The first important result is that, since a B-product is never accepted by the consumer, i.e., there are no gains from trade for the B-product, then separation cannot be obtained in equilibrium.

Lemma 3 *In any equilibrium, there is pooling on complexity; that is, $\sigma_G = \sigma_B \in \{0, 1\}$.*

Thus, even though complexity can in principle be used as a signaling device by designers, those designing B-products choose to always mimic the complexity choice of G-product designers. Intuitively, any equilibrium in which different product types come with different complexities can be ruled out by requiring belief consistency from the consumer. However, the freedom of assigning off-equilibrium beliefs gives rise to multiple equilibria in this setting. As a result, both equilibrium with complex and equilibrium with simple products can be supported, as we formalize in the following proposition.

Proposition 11 *There is always two equilibria with positive trade with the following features:*

- *Simple equilibrium (S): $\sigma_G^S = \sigma_B^S = 1$, with $\mu^S = \max\{\psi_S, p\}$.*
- *Complex equilibrium (C): $\sigma_G^C = \sigma_B^C = 0$, with $\mu^C = \max\{\psi_C, p\}$.*

where $\psi^C < \psi^S$ are given by equations (71)-(72) respectively.

[23] It is straightforward to show that under full information, i.e. if (y, κ) were observable to the consumer, then all designers would produce (G, κ^n) products.

Therefore, even when complexity is observable, there exists an equilibrium in which G-products are made complex, and an equilibrium in which B-products are made simple. In what follows, we show that, as in the baseline model, when the consumer is sufficiently optimistic, all designers are better off when producing complex products.

Proposition 12 *If p is large enough or ω is low enough, all designers are better off in the complex equilibrium.*

Thus, the main insights of our model are robust to a setting where the choice of κ is observable. When the fraction of aligned designers is sufficiently large, or the consumer's relative outside option relatively low, all designers benefit from the consumer having access to less precise information, as she will accept all products when information is sufficiently noisy. Furthermore, we can show that for a given equilibrium (complex or simple) the comparative statics on product quality, μ, are as described in the baseline model (see Section 5).

7.2 Optimal Information Extraction with Limited Attention

In this section, we show that our information structure captures the idea that learning about a more complex product requires more of the consumer's attention. To do so, we follow the rational attention literature by assuming that the consumer can choose how much uncertainty about the product output to reduce subject to an entropy-reduction cost, where entropy measures the consumer's uncertainty (Sims, 2003). In particular, a more complex product has a higher cost of entropy reduction. As in our baseline model, we refine the set of equilibria by supposing that the mapping between the designer's action and the actual entropy cost faced by the consumer is imperfect. The advantage of this approach is that it micro-founds the relation between a product's complexity and the type of information received by the consumer. The drawback, however, is that we can no longer obtain as sharp of an equilibrium characterization as in our baseline setting. Nevertheless, we show next that the model's main mechanisms remain robust under the optimal information choice by the consumer.

The main difference with the baseline setup is that now the consumer chooses the distribution of the signal about product output that she receives. The uncertainty faced by a consumer with belief $\tilde{\mu} = P(y = g)$ is measured by the entropy function, as given by:

$$H(\tilde{\mu}) = -\left(\tilde{\mu} \cdot log(\tilde{\mu}) + (1 - \tilde{\mu}) \cdot log(1 - \tilde{\mu})\right), \tag{19}$$

which reaches a minimum of zero at $\tilde{\mu} \in \{0, 1\}$ and a maximum of $-log\left(\frac{1}{2}\right)$ at $\tilde{\mu} = \frac{1}{2}$. As before, let S denote the signal observed by the consumer and s denote its realization. The signal has a distribution conditional on the product's output, $\pi(s|y) \equiv \mathbb{P}(S = s|y)$, which determines the consumer's posterior belief:

$$\tilde{\mu}(s) \equiv P(y = G|s) = \frac{\pi(s|G) \cdot \tilde{\mu}}{\pi(s|G) \cdot \tilde{\mu} + \pi(s|B) \cdot (1 - \tilde{\mu})}. \tag{20}$$

The entropy associated with the posterior belief is $H(\tilde{\mu}(s))$.

We measure the amount of information that the consumer obtains from a particular information structure π as the expected reduction in entropy:

$$I(\pi) = H(\tilde{\mu}) - \int_s H(\tilde{\mu}(s)) \cdot \pi(s) \cdot ds, \tag{21}$$

and we assume that the consumer faces a cost $c \cdot I(\pi)$ of entropy reduction for some $c > 0$.

We suppose that more complex products are more attention intensive. And, as before, we refine the set of equilibria by assuming that the mapping from complexity κ to the cost of entropy reduction is imperfect. In particular, we assume that the parameter c is random and satisfies $c \sim F(c|\kappa)$ with pdf $f(c|\kappa)$ that has full support on some interval $[\underline{c}, \bar{c}]$ and where $\frac{f(c|\bar{\kappa})}{f(c|\underline{\kappa})}$ is increasing in c (MLRP).

Since the consumer's action is binary, i.e. she chooses to accept or reject the product, it is without loss of generality to focus on binary signals $S \in \{b, g\}$ (Woodford, 2009; Yang, 2015). Let π_y denote the probability that the consumer receives signal g, conditional on the designer producing a y-product. Let $\mu(c)$ be the consumer's interim belief after observing the cost c. For a given c, the consumer's information extraction problem is to choose π_G and π_B to maximize her expected payoff:

$$\mu(c) \cdot \pi_G \cdot (w(G) - w_0) + (1 - \mu(c)) \cdot \pi_B \cdot (w(B) - w_0) - c \cdot I(\pi) \tag{22}$$

where

$$I(\pi) = H(\mu(c) \cdot \pi_G + (1 - \mu(c)) \cdot \pi_B) - \mu(c) \cdot H(\pi_G) - (1 - \mu(c)) \cdot H(\pi_B). \tag{23}$$

Figure 7 depicts the solution to the consumer's problem for a given prior belief μ and an interim belief $\mu(c) = \mu$, i.e. the figure assumes that the equilibrium features pooling at complexity level. As we can see, the resulting probabilities of acceptance are closely related to those in our baseline model, as depcited in Figure 1. When the cost c is low enough, the consumer extracts an informative signal and makes her decision contingent on its realization.

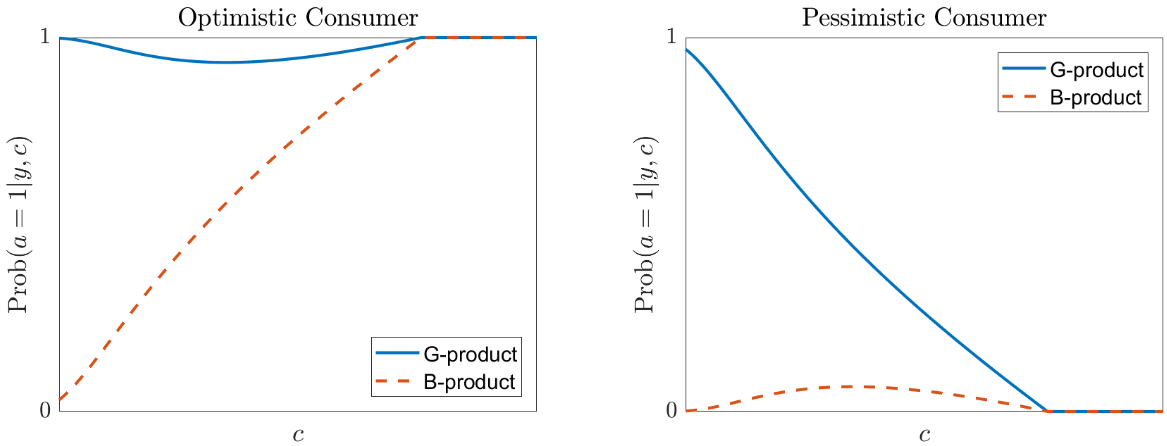

Figure 7: Illustrates the probability of acceptance of a y-product as a function of the entropy-reduction cost, c. The left panel depicts the acceptance strategy of an optimistic consumer. The right panel depicts the acceptance strategy of a pessimistic consumer

Otherwise, the consumer makes her decision solely based on her interim belief. Finally, observe that when c is high enough, then the consumer either accepts the product w.p.1 (left panel) or she rejects it w.p.1 (right panel). As in our baseline model, which of the two scenarios arises depends on whether the consumer is optimistic or pessimistic; that is, what she would do in the absence of information.[24]

Naturally, an equilibrium requires that the consumer's prior belief μ and her interim belief $\mu(c)$ be consistent with the designer's strategies $\{m, \sigma_G, \sigma_B\}$ and Bayes' rule. Although a full analytical characterization of the equilibrium set is now difficult to obtain, we show next (numerically) that the equilibrium set of the model with optimal information extraction resembles closely that of our baseline model.

Figure 8 is the analogue of the Figures 2 and 3, in the model with optimal information extraction. The left panel illustrates the choices of complexity σ_G and σ_B that are consistent with an equilibrium belief μ. As in our baseline model (see Figure 2), if the consumer's belief μ is relatively low (high), then both products are made simple (complex), while G-products are made simple and B-products are made complex if μ is intermediate. The right panel depicts the correspondence Γ as defined in Section 3.4.2. Given strategy σ_y and the correspondence Γ, equilibrium product quality and complexity can be found as in the baseline model using Propositions 5 and 6.

[24]In this setting, the consumer is optimistic if $\mu(\bar{c}) \geq \omega$, and she is pessimistic otherwise.

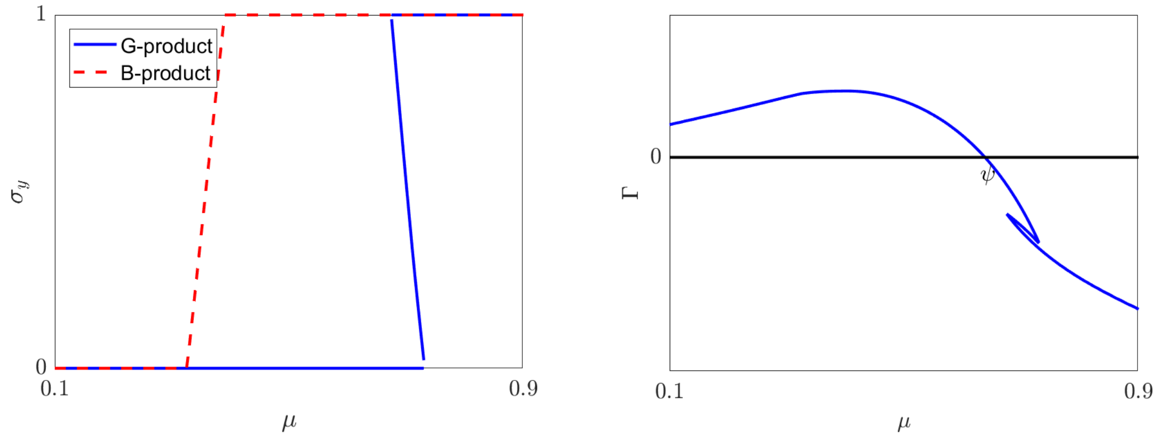

Figure 8: The left panel illustrates the choice of complexity of a designer who produces a y-product and is consistent with a consumer's belief of μ. The right panel illustrates the misaligned designer's net payoff from choosing the G-product given consumer's belief μ.

8 Conclusion

We presented a model of complexity, in which both product quality and complexity are a strategic choice made by product designers. The model sheds light on the incentives of designers to add complexity to products in order to increase their chance of acceptance by a consumer. The model delivers two powerful insights. First, complexity is not necessarily a feature of worse quality products. The designer may complexify good products, or simplify bad products. We show how this choice depends crucially on the consumer's beliefs about the average quality of products and the consumer's outside option. Second, the relationship between average product quality and complexity depends on the underlying drivers of product heterogeneity. More alignment between the consumer and the designer increases average quality as well as average complexity. More competition between designers, however, increases average product quality and decreases average complexity.

Our model provides a tractable framework for analyzing the joint choice of a product's quality and complexity, and it can be extended along several dimensions. In particular, future work may examine the evolution of quality and complexity over time for financial products and policies which are subject to amendments or renegotiations.

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A Appendix: Proofs and Complementary Lemmas

A.1 Proofs for Section 3

Proof of Lemma 1. See text. ■

Proof of Proposition 2. We begin by studying the designer's optimal choice of κ in the optimistic region, as in Definition 1.

Case 1 (consumer is optimistic). In this region, the designer is accepted w.p.1 when information is noisy enough, $z > \bar{z}$. So, his optimal choice of κ solves:

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{z}} P(s = g|y) \cdot f(z|\kappa) dz + \int_{\bar{z}}^{1/2} f(z|\kappa) dz. \tag{24}$$

Thus, it is optimal for the designer of B-product to choose $\bar{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\bar{\kappa}) dz \geq \int_0^{\bar{z}} z \cdot f(z|\underline{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\underline{\kappa}) dz, \tag{25}$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \tag{26}$$

But, note that we have:

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz > (1 - \bar{z})(F(\bar{z}|\underline{\kappa}) - F(\bar{z}|\bar{\kappa})) > 0. \tag{27}$$

for $\bar{z} > 0$, as will be the case in equilibrium. Thus, condition (25) is satisfied with strict inequality, and it is uniquely optimal for the designer of the B-product to choose $\bar{\kappa}$.

On the other hand, it is optimal for the designer of G-product to choose $\bar{\kappa}$ if

$$\int_0^{\bar{z}} (1 - z) \cdot f(z|\bar{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\bar{\kappa}) dz \geq \int_0^{\bar{z}} (1 - z) \cdot f(z|\underline{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\underline{\kappa}) dz, \tag{28}$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \tag{29}$$

Condition (28) is satisfied if $\bar{z} \leq \hat{z}$, and holds with strictly inequality if $\bar{z} < \hat{z}$. Thus, if $\bar{z} < \hat{z}$, it is uniquely optimal for the designer of G-product to choose $\bar{\kappa}$. Otherwise, if $\bar{z} = \hat{z}$, the designer is indifferent to the choice of κ, and if $\bar{z} > \hat{z}$, it is uniquely optimal to choose $\underline{\kappa}$.

Next, we study the designer's optimal choice of κ in the pessimistic region.

Case 2 (consumer is pessimistic). In the region, the designer is rejected if information is too noisy, $z > \bar{z}$. So, his optimal choice of κ solves:

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{z}} P(s = g|y) \cdot f(z|\kappa) dz. \tag{30}$$

Thus, it is optimal for the designer of B-product to choose $\underline{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa}) dz \leq \int_0^{\bar{z}} z \cdot f(z|\underline{\kappa}) dz, \tag{31}$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \tag{32}$$

Condition (31) is satisfied if $\bar{z} \leq \hat{z}$, and holds with strict inequality if $\bar{z} < \hat{z}$. Thus, if $\bar{z} < \hat{z}$, it is uniquely optimal for the designer of B-product to choose $\underline{\kappa}$. Otherwise, if $\bar{z} = \hat{z}$, the designer is indifferent to the choice of κ, and if $\bar{z} > \hat{z}$, it is uniquely optimal to choose $\bar{\kappa}$.

On the other hand, it is optimal for the designer of the G-product to choose $\underline{\kappa}$ if

$$\int_0^{\bar{z}} (1 - z) \cdot f(z|\bar{\kappa}) dz \leq \int_0^{\bar{z}} (1 - z) \cdot f(z|\underline{\kappa}) dz, \tag{33}$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \tag{34}$$

Re-writing the above condition, we have

$$\int_0^{\bar{z}} (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz > \int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz, \tag{35}$$

which immediately implies that condition (33) is satisfied for all $\bar{z} > 0$, as will be the case in equilibrium, and it is uniquely optimal for the designer of the G-product to choose $\underline{\kappa}$. ■

Proof of Proposition 3. The designer's net benefit from choosing the G-product is:

$$\gamma(\mu) = \max_{\kappa} \mathbb{P}_{\mu}(a = 1|G, \kappa) \cdot v(G) - \max_{\kappa} \mathbb{P}_{\mu}(a = 1|B, \kappa) \cdot v(B). \tag{36}$$

Since again $\mathbb{P}_{\mu}(a = 1|G, \kappa) \geq \mathbb{P}_{\mu}(a = 1|B, \kappa) > 0$ and $v(G) > v(B)$, we can have $\gamma(\mu) \lesseqgtr 0$. Thus, the designer chooses the G-product whenever $\gamma(\mu) > 0$, the B-product whenever $\gamma(\mu) < 0$, and he is indifferent whenever $\gamma(\mu) = 0$. ■

Proof of Proposition 4. Suppose that, in equilibrium, the consumer's belief that the designer has produced a G-product is $\mu \in (0, 1)$.

Pooling equilibria. Consider first the candidate pooling equilibrium in which $\sigma_B = \sigma_G = 0$. By Proposition 2, this requires that $\mu \leq \omega$ and $\bar{z} \leq \hat{z}$. On equilibrium path, the consumer does not update from observation of z and, thus, threshold $\bar{z}$ is given by

$$\mu(g, \bar{z}) = \omega, \tag{37}$$

which is equivalent to:

$$\bar{z} = \frac{(1-\omega) \cdot \mu}{(1-\omega) \cdot \mu + \omega \cdot (1-\mu)}. \tag{38}$$

This is an equilibrium if and only if $\bar{z} \leq \hat{z}$, which is equivalent to:

$$\mu \leq \frac{\omega \cdot \frac{\hat{z}}{1-\hat{z}}}{\omega \cdot \frac{\hat{z}}{1-\hat{z}} + 1 - \omega} \equiv \mu_1. \tag{39}$$

Consider next the candidate pooling equilibrium in which $\sigma_B = \sigma_G = 1$. By Proposition 2, this requires that $\mu \geq \omega$ and $\bar{z} \leq \hat{z}$. On equilibrium path, the consumer does not update from observation of z and, thus, threshold $\bar{z}$ is given by

$$\mu(b, \bar{z}) = \omega, \tag{40}$$

which is equivalent to:

$$\bar{z} = \frac{\omega \cdot (1-\mu)}{(1-\omega) \cdot \mu + \omega \cdot (1-\mu)}. \tag{41}$$

This is an equilibrium if and only if $\bar{z} \leq \hat{z}$, which is equivalent to:

$$\mu \geq \frac{\omega \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} \equiv \mu_3. \tag{42}$$

Therefore, $\sigma_B = \sigma_G = 0$ is an equilibrium if and only if $\mu \in (0, \mu_1]$, whereas $\sigma_B = \sigma_G = 1$ is an equilibrium if and only if $\mu \in [\mu_3, 1)$.

Separating equilibria. Consider the candidate *separating equilibrium* in which $\sigma_B = 1$ and $\sigma_G = 0$. There are two cases to consider, depending on whether the consumer is optimistic or pessimistic.

First, suppose that

$$\mu\left(g, \frac{1}{2}\right) = \mu\left(b, \frac{1}{2}\right) = \frac{\mu}{\mu + (1-\mu)\cdot \ell\left(\frac{1}{2}\right)} \leq \omega, \tag{43}$$

where $\ell(\cdot) = \frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$. Then, the consumer must be pessimistic. On equilibrium path, there is updating from observation of z, and thus threshold $\bar{z}$ is given by

$$\mu\left(g, \bar{z}\right) = \frac{\mu}{\mu + (1-\mu)\cdot \ell(\bar{z}) \cdot \frac{\bar{z}}{1-\bar{z}}} = \omega. \tag{44}$$

This is an equilibrium if and only if also $\bar{z} \geq \hat{z}$, i.e.

$$\mu_2 \equiv \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{\hat{z}}{1-\hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{\hat{z}}{1-\hat{z}} + 1 - \omega} \leq \mu \leq \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} \equiv \tilde{\mu}. \tag{45}$$

Second, suppose that

$$\mu\left(g, \frac{1}{2}\right) = \mu\left(b, \frac{1}{2}\right) = \frac{\mu}{\mu + (1-\mu)\cdot \ell\left(\frac{1}{2}\right)} > \omega, \tag{46}$$

Then, the consumer must be optimistic. The threshold $\bar{z}$ is now given by

$$\mu(b, \bar{z}) = \frac{\mu}{\mu + (1-\mu)\cdot \ell(\bar{z}) \cdot \frac{1-\bar{z}}{\bar{z}}} = \omega. \tag{47}$$

This is an equilibrium if and only if also $\bar{z} \geq \hat{z}$, i.e.

$$\tilde{\mu} = \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} < \mu \leq \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} \equiv \mu_4. \tag{48}$$

Therefore, $\sigma_B = 0$ and $\sigma_G = 1$ is an equilibrium if and only if $\mu \in [\mu_2, \mu_4]$.

Semi-separating equilibria. Consider the candidate semi-separating equilibrium, in which $\sigma_B \in (0, 1)$. By Proposition 2, such an equilibrium requires that the consumer be pessimistic and so $\sigma_G = 0$. On equilibrium path, there is updating from observation of z, and threshold

$\bar{z}$ must exactly equal $\hat{z}$ so that the designer of B-product is indifferent to the choice of κ (Proposition 2) and is willing to mix:

$$\mu\left(g,\hat{z}\right)=\frac{\mu}{\mu+(1-\mu)\cdot(\sigma_B\cdot\ell(\hat{z})+1-\sigma_B)\cdot\frac{\hat{z}}{1-\hat{z}}}=\omega, \tag{49}$$

which in turn implies that:

$$\sigma_B=\frac{\frac{1-\hat{z}}{\hat{z}}\cdot\frac{\mu}{1-\mu}\cdot\frac{1-\omega}{\omega}-1}{\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})}-1}. \tag{50}$$

Since the posterior belief $\mu\left(g,\hat{z}\right)$ is continuous and decreasing in σ_B (MLRP implies that $\ell(\hat{z})>1$), this equilibrium exists if and only if:

$$\mu(g,\hat{z})|_{\sigma_B=1}<\omega<\mu(g,\hat{z})|_{\sigma_B=0}, \tag{51}$$

which is equivalent to:

$$\mu_1=\frac{\omega\cdot\frac{\hat{z}}{1-\hat{z}}}{\omega\cdot\frac{\hat{z}}{1-\hat{z}}+1-\omega}<\mu<\frac{\omega\cdot\ell(\hat{z})\cdot\frac{\hat{z}}{1-\hat{z}}}{\omega\cdot\ell(\hat{z})\cdot\frac{\hat{z}}{1-\hat{z}}+1-\omega}=\mu_2. \tag{52}$$

Therefore, $\sigma_G=0$ and $\sigma_B\in(0,1)$ is an equilibrium if and only if $\mu\in(\mu_1,\mu_2)$.

Consider the candidate semi-separating equilibrium in which $\sigma_G\in(0,1)$. By Proposition 2, such an equilibrium requires that the consumer be optimistic and so $\sigma_B=1$. On equilibrium path, there is updating from observation of z, and threshold $\bar{z}$ must exactly equal $\hat{z}$ so that the designer of G-product is indifferent to the choice of κ and is willing to mix::

$$\mu\left(b,\hat{z}\right)=\frac{\mu}{\mu+(1-\mu)\cdot\frac{1-\hat{z}}{\hat{z}}\cdot\frac{1}{\sigma_G+(1-\sigma_G)\cdot\ell(\hat{z})^{-1}}}=\omega. \tag{53}$$

which in turn implies that

$$\sigma_G=1-\frac{1-\frac{1-\hat{z}}{\hat{z}}\cdot\frac{1-\mu}{\mu}\cdot\frac{\omega}{1-\omega}}{1-\frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})}}. \tag{54}$$

Since the posterior belief $\mu\left(b,\hat{z}\right)$ is continuous and increasing in σ_G, this equilibrium exists if and only if:

$$\mu(b,\hat{z})|_{\sigma_G=0}<\omega<\mu(b,\hat{z})|_{\sigma_G=1}, \tag{55}$$

which is equivalent to:

$$\mu_3 = \frac{\omega \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} = \mu_4. \tag{56}$$

Therefore, $\sigma_G \in (0,1)$ and $\sigma_B = 1$ is an equilibrium if and only if $\mu \in (\mu_3, \mu_4)$.

We have thus characterized all the possible equilibrium $\{\sigma_y\}$, as a function of equilibrium belief μ:

1. If $\mu \leq \mu_1$, the equilibrium is $\sigma_B = \sigma_G = 0$.
2. If $\mu \in (\mu_1, \mu_2)$, then $\sigma_G = 0$ and $\sigma_B = \frac{\frac{1-\hat{z}}{\hat{z}} \cdot \frac{\mu}{1-\mu} \cdot \frac{1-\omega}{\omega} - 1}{\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})} - 1}$.
3. If $\mu \in [\mu_2, \mu_4]$, then $\sigma_G = 0$ and $\sigma_B = 1$.
4. If $\mu \in (\mu_3, \mu_4)$, $\sigma_B = 1$ and $\sigma_G \in \left\{0,\ 1 - \frac{1 - \frac{1-\hat{z}}{\hat{z}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})}}\ , 1\right\}$.
5. If $\mu \geq \mu_4$, then $\sigma_B = \sigma_G = 1$.

This establishes the stated result. ■

Proof of Lemma 2. The designer's expected net payoff from choosing G-product is:

$$\gamma(\mu) = \max_{\kappa} \mathbb{P}_\mu(a = 1|G, \kappa) \cdot v(G) - \max_{\kappa} \mathbb{P}_\mu(a = 1|B, \kappa) \cdot v(B). \tag{57}$$

Consider the correspondence $\Gamma(\mu)$ as defined in text. First, note that $0 \in \Gamma(0)$, and it is its unique element, since $\gamma(0) = 0$ always, as all products are rejected with probability one when the consumer's belief is $\mu = 0$. Next, let us look at $\mu \in (0, 1]$.

The values $\max_\kappa \mathbb{P}_\mu(a = 1|G, \kappa)$ and $\max_\kappa \mathbb{P}_\mu(a = 1|B, \kappa)$ depend on the corresponding equilibrium $\{\sigma_y\}$, given in Proposition 4. Note that $\Gamma(\mu)$ is a singleton for $\mu \notin (\mu_3, \mu_4)$, since equilibrium $\{\sigma_y\}$ corresponding to such μ are unique. On the other hand, $\Gamma(\mu)$ consists of three elements when $\mu \in (\mu_3, \mu_4)$, since then the equilibrium can feature either ($\sigma_G = 0$ and $\sigma_B = 1$), ($\sigma_G \in (0, 1)$ and $\sigma_B = 1$), or ($\sigma_G = 1$ and $\sigma_B = 1$).

In this proof, we will heavily rely on the results in Proposition 4 and the thresholds $\mu_1 - \mu_4$ defined in its proof, without referencing them. To follow this proof, please read the proposition and its proof in advance.

Case $\mu \leq \mu_1$. In this region, the equilibrium has $\sigma_G = \sigma_B = 0$, and the consumer is pessimistic, since $\mu_1 < \tilde{\mu}$. Furthermore, $\Gamma(\mu)$ is single-valued and given by:

$$\Gamma(\mu) = v(G) \cdot \int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa})\, dz - v(B) \cdot \int_0^{\bar{z}(\mu)} z f(z|\underline{\kappa})\, dz. \tag{58}$$

Therefore:

$$\Gamma'(\mu) = [v(G) - (v(G)+v(B)) \cdot \bar{z}(\mu)] \cdot f(z|\underline{\kappa}) \cdot \frac{d\bar{z}}{d\mu}. \tag{59}$$

where $\bar{z}(\mu)$ is given by (38). It is easy to check that $\frac{d\bar{z}}{d\mu} > 0$, $\bar{z}(0) = 0$, and $\bar{z}(\mu_1) = \hat{z}$. As a result, for μ sufficiently small, $\Gamma'(\mu) > 0$ and thus $\Gamma(\mu) > 0$. Next, consider μ_v such that:

$$\bar{z}(\mu_v) = \frac{v(G)}{v(G)+v(B)} \implies \mu_v \equiv \frac{v(G)\omega}{v(G)\omega + v(B)(1-\omega)}. \tag{60}$$

Observe that, if $\mu_v > \mu_1$, then $\Gamma'(\mu) > 0\ \forall \mu \in (0, \mu_1)$. Otherwise, $\Gamma'(\mu) > 0$ for $\mu \in (0, \mu_v)$ and $\Gamma'(\mu) < 0$ for $\mu \in (\mu_v, \mu_1)$.

Case $\mu \in (\mu_1, \mu_2]$. In this region, the equilibrium has $\sigma_G = 0$ and $\sigma_B \in (0,1)$, and the consumer is pessimistic, i.e. $\mu_2 < \tilde{\mu}$. Most importantly, in this case $\bar{z} = \hat{z}$ and, thus, $\Gamma(\mu)$ is single-valued and given by:

$$\Gamma(\mu) = v(G) \cdot \int_0^{\hat{z}} (1-z) f(z|\underline{\kappa})\, dz - v(B) \cdot \int_0^{\hat{z}} z f(z|\underline{\kappa})\, dz \tag{61}$$

Since $\bar{z}(\mu_1) = \hat{z}$, it is immediate that Γ is continuous at μ_1 and constant on interval (μ_1, μ_2).

Case $\mu \in (\mu_2, \mu_3]$. In this region, the equilibrium has $\sigma_G = 0$ and $\sigma_B = 1$. The consumer is pessimistic if $\mu < \tilde{\mu}$, and she is optimistic otherwise.

Suppose that $\mu < \tilde{\mu}$. Then, $\Gamma(\mu)$ is single-valued and given by:

$$\Gamma(\mu) = v(G) \cdot \int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa})\, dz - v(B) \cdot \int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz. \tag{62}$$

Therefore:

$$\Gamma'(\mu) = [v(G) \cdot (1-\bar{z}(\mu)) \cdot f(\bar{z}(\mu)|\underline{\kappa}) - v(B) \cdot \bar{z}(\mu) \cdot f(\bar{z}(\mu)|\bar{\kappa})] \frac{d\bar{z}}{d\mu}. \tag{63}$$

where $\bar{z}(\mu)$ is given by (44). Since $\bar{z}(\mu_2) = \hat{z}$, Γ is contiuous at μ_2. Furthermore, $\Gamma'(\mu) \geq 0$ iff

$$\bar{z}(\mu) \leq \frac{1}{1 + \frac{v(B)}{v(G)} \cdot \ell(\bar{z}(\mu))} \iff \frac{(1-\mu)\cdot\omega}{\mu \cdot (1-\omega)} \geq \frac{v(B)}{v(G)} \iff \mu \leq \mu_v, \tag{64}$$

and $\Gamma'(\mu) > 0$ if $\mu < \mu_v$. Furthermore, since $\frac{(1-\mu)\cdot\omega}{\mu\cdot(1-\omega)} \geq \frac{v(B)}{v(G)}$ is decreasing in μ and equal to $\ell(\frac{1}{2}) < 1$ when $\mu = \tilde{\mu}$, it follows that $\mu_v < \tilde{\mu}$ and thus $\Gamma'(\tilde{\mu}) < 0$.

Next, suppose that $\mu > \tilde{\mu}$. Now, $\bar{z}(\mu) \geq \hat{z}$ and is given by (47), and note that $\frac{d\bar{z}}{d\mu} < 0$. Therefore:

$$\begin{aligned}\Gamma(\mu) =& v(G) \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa})\, dz + (1 - F(\bar{z}|\underline{\kappa}))\right] - v(B) \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa}))\right] \\ =& v(G) - v(G) \cdot \int_0^{\bar{z}(\mu)} z f(z|\underline{\kappa})\, dz - v(B) \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa}))\right], \qquad (65)\end{aligned}$$

and thus:

$$\Gamma'(\mu) = \left[v(B) \cdot (1 - \bar{z}(\mu)) \cdot f(\bar{z}(\mu)|\bar{\kappa}) - v(G) \cdot \bar{z}(\mu) \cdot f(\bar{z}(\mu)|\underline{\kappa})\right] \frac{d\bar{z}}{d\mu} < 0, \qquad (66)$$

where we use the fact that $\ell(\bar{z}(\mu)) \geq \ell(\hat{z}) > 1$. Recall that $\tilde{\mu}$ is the threshold between the region where the consumer is pessimistic and the region where she is optimistic. Since $\bar{z}(\tilde{\mu}) = \frac{1}{2}$, it is easy to check that Γ is continuous at $\tilde{\mu}$.

To summarize, we have shown that $\Gamma(\mu)$ is positive and (weakly) increasing for $\mu \in (0, \mu_v]$, and (weakly) decreasing for $\mu \in (\mu_v, \mu_3]$, with $\mu_v \in (0, \tilde{\mu})$. We are left to consider regions $\mu \in (\mu_3, \mu_4)$ and $\mu \in [\mu_4, 1]$

Case $\mu \geq \mu_4$. In this region, the equilibrium has $\sigma_G = \sigma_B = 1$, and the consumer is optimistic. Moreover, $\bar{z}(\mu)$ given by (41). We have that Γ is single-valued and given by

$$\Gamma(\mu) = v(G) \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}(\mu)|\bar{\kappa}))\right] - v(B) \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}(\mu)|\bar{\kappa}))\right].$$

Then, Γ decrease in μ for $\mu > \mu_4$.

$$\Gamma'(\mu) = [v(B) \cdot (1 - \bar{z}) - v(G) \cdot \bar{z}] \cdot f(z|\bar{\kappa}) \cdot \frac{d\bar{z}}{d\mu} < 0.$$

Finally, note that $\Gamma(1) = v(G) - v(B) < 0$.

Case $\mu \in (\mu_3, \mu_4)$. In this region, $\Gamma(\mu)$ is not single-valued. For $j \in \{S, M, P\}$, let $\gamma^j(\mu) \in \Gamma(\mu)$ denotes the net expected payoff to the L-type from choosing G-product, when the equilibrium complexity is given respectively by (i) *S*eparating on complexity $\sigma_G = 0, \sigma_B = 1$, (ii) *M*ixing of the H-type designer with $\sigma_G \in (0, 1)$, $\sigma_B = 1$, and (iii) *P*ooling on complexity $\sigma_G = \sigma_B = 1$.

We have already shown that the functions $\gamma^S(\mu)$ (i.e., given by $\Gamma(\mu)$ analyzed for $\mu \in (\tilde{\mu}, \mu_3)$

above) and $\gamma^P(\mu)$ (i.e., given by $\Gamma(\mu)$ analyzed for $\mu > \mu_4$ above) are decreasing in μ. We are therefore left to consider $\gamma^M(\mu)$. For the producer of a G-product to mix, it must be that $\bar{z}(\mu) = \hat{z}$ for all $\mu \in (\mu_3, \mu_4)$. Since the consumer is optimistic in this region, we have that:

$$\gamma^M(\mu) = v(G) - v(G) \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa})\, dz - v(B) \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa})\, dz - v(B) \cdot (1 - F(\hat{z}|\bar{\kappa})), \quad (67)$$

which is independent of μ. Furthermore, since $\bar{z}(\mu_3) = \hat{z}$ when pooling at complexity and $\bar{z}(\mu_4) = \hat{z}$ when separating at complexity, it follows that $\gamma^M(\mu) = \gamma^P(\mu_3) = \gamma^S(\mu_4)$. Furthermore, it is straightforward that $\lim_{\mu \uparrow \mu_3} \Gamma(\mu) = \gamma^S(\mu_3)$ and $\lim_{\mu \downarrow \mu_4} \Gamma(\mu) = \gamma^P(\mu_4)$.

To summarize, we have that $\Gamma(\mu)$ is single-valued for $\mu < \mu_3$, with $\lim_{\mu \to 0} \Gamma(\mu) > 0$ and $\Gamma(\mu)$ increasing for $\mu \leq \mu_v$ and decreasing for $\mu \in (\mu_v, \mu_3)$. We have also shown that (i) $\lim_{\mu \uparrow \mu_3} \Gamma(\mu) = \gamma^S(\mu_3)$, with $\gamma^S(\mu)$ decreasing in μ, (ii) $\gamma^S(\mu_4) = \gamma^M(\mu) = \gamma^P(\mu_3)$, and (iii) $\gamma^P(\mu)$ decreasing for $\mu > \mu_3$, and (iv) $\Gamma(\mu) = \gamma^P(\mu)$ for $\mu > \mu_4$, and thus single-valued and decreasing until $\Gamma(1) < 0$. These results are depicted in Figure 3.

Since Γ is single-valued and positive for μ small, then increases until it reaches a maximum, and decreases (i.e. does not jump up in the region where it is not single-valued) and becomes single-valued and negative for μ relatively large, it follows that Γ intersects 0 generically once.

■

Proof of Proposition 6. First, consider the *zero trade equilibrium*. If $\mu = 0$, then the consumer rejects all proposed products. This is consistent with the designer only producing bad products, $m = 0$, which is consistent with the consumer's belief. Thus, the zero trade equilibrium always exists.

Now consider an equilibrium in which G-products are produced with positive probability, $m > 0$. Since in equilibrium $\mu = m$, if $m < \psi$, then $\gamma(\mu) \in \Gamma(\mu) > 0$, i.e., the designer produces G-product with probability one ($m = 1$), which is consistent with a belief of $\mu = 1$, reaching a contradiction since $\psi < 1$. Thus, there cannot be an equilibrium with $\mu \in (0, \psi)$. If instead $m > \psi$, $\gamma(\mu) \in \Gamma(\mu) < 0$, i.e., the designer produces B-products with probability zero ($m = 0$), which is consistent with a belief of $\mu = 0$, reaching a contradiction as well. Thus, there cannot be an equilibrium with $\mu \in (\psi, 1]$. Thus, the equilibrium requires the designer to follow a mixed strategy $m = \psi$, so that the designer is indifferent between producing a B- or a G-product. Lemma 2 shows that generically there is a unique $\psi \in (0, 1)$ such that $\gamma(\psi) \in \Gamma(\psi) = 0$.

Note that if $\psi \in (\mu_3, \mu_4)$, there could be multiple complexity levels consistent with an equilibrium belief of $\mu = \psi$. However, there is only one choice of complexity consistent with

$\gamma(\psi) \in \Gamma(\psi) = 0$, and thus the equilibrium is pinned down uniquely. ∎

A.2 Proofs for Section 5

For the purpose of generality, the proofs for the model's comparative statics are done for the more general case in which the consumer can encounter an aligned designer (one with $\bar{v}(G) > \bar{v}(B)$), with probability $p \in [0, 1]$. The results stated in Proposition 7 and 9 are obtained immediately by supposing that $p = 0$, as we did in our baseline model. A full characterization of the equilibrium with $p > 0$ can be found in the Proof of Proposition 8 below.

Proof of Proposition 7. For this comparative static, we focus on the simplest equilibrium as multiple equilibria with positive trade may arise when $p > 0$ (see Proof of Proposition 8 for details). It is easy to check that an increase in ω affects thresholds $\mu_1 - \mu_4$ as follows

$$\frac{d\mu_1}{d\omega} > 0; \frac{d\mu_2}{d\omega} > 0; \frac{d\tilde{\mu}}{d\omega} > 0; \frac{d\mu_3}{d\omega} > 0; \frac{d\mu_4}{d\omega} > 0.$$

but has no effect effect on Γ otherwise; that is, $\Gamma(\mu)$ is unchanged if we condition on the complexity choices. Thus, it follows that ψ also weakly increase in ω, but note that it will always stay in the same region; that is, at $\mu = \psi$ the complexity choices of the designers remain unchanged (even as ψ increases with ω).

If $p \leq \psi$ (e.g. $p = 0$), an increase in ω generates an improvement in average quality as $\mu = \psi$ increases. In this scenario, however, σ_y do not change since ψ has not changed complexity regions (e.g. if it is between (μ_2, μ_3) it continues to be in this region after the change in ω, even though thresholds have changed). Thus, average complexity increases as μ increases only when $\sigma_G < \sigma_B$; that is, for $\mu = \psi \in (\mu_1, \mu_4)$.

If $p > \psi$, an increase in ω does not affect average quality which stays constant at $\mu = p$. It may, however, result in a decrease in average complexity, as μ stays constant but thresholds μ_1, μ_4, and thus $\sigma_y(\mu)$, increase. ∎

Proof of Proposition 9. For this comparative static, we focus on the simplest equilibrium as multiple equilibria with positive trade may arise when $p > 0$ (see Proof of Proposition 8 for details). Note that an increase in $v(G)$ does not affect the choices of complexity for a given μ. That is, thresholds $\mu_1 - \mu_4$ and $\sigma_y(\mu)$ remain unchanged (where $\sigma_y(\mu)$ denotes the mapping from beliefs, μ, to complexity choices given in Proposition 4, where simplicity is the chosen equilibrium for $\mu \in (\mu_3, \mu_4)$). We can re-state Γ as follows

$$\Gamma(\mu) = P(a = 1|\sigma_G(\mu))v(G) - P(a = 1|\sigma_B(\mu))v(B) \tag{68}$$

It is immediate that since σ_y is independent of $v(G)$, $\Gamma(\mu)$ increases in $v(G)$ for all $\mu \in (0, 1)$. Thus, ψ given by $\Gamma(\psi) = 0$ must increase as well*.

When $p \leq \psi$ (e.g. $p = 0$), an increase in $v(G)$ generates an increase in average quality, since $\mu = \psi$ has increased. In contrast, when $p > \psi$, an increase in $v(G)$ does not affect average quality which stays constant at $\mu = p$.

Since the thresholds $\mu_1 - \mu_4$ remain unchanged, when average quality increases, σ_y (weakly) increase as a result (otherwise complexity remains unchanged). As a result, the effect on average complexity may be non-monotonic. In particular, the increase in μ has no effect on complexity if $\mu < \mu_1$, as both designers simplify in this region, average complexity decreases if $\sigma_G(\mu) < \sigma_B(\mu)$, which occurs for $\mu \in (\mu_2, \mu_4)$, but eventually increases as all designers produce complex products when μ is sufficiently high, i.e., $\mu > \mu_4$.

*Caveat: when $p < \psi \in (\mu_3, \mu_4)$, an increase in $v(G)$ can generate a jump from the separating equilibrium to pooling at complexity. In this case, an increase in $V(G)$ can generate a downward shift in average quality ψ from μ_4 to μ_3. Note, however, that average quality is increasing in $v(G)$ before and after the jump. ■

Proof of Proposition 8. We now extend the analysis of our model to incorporate aligned designers. We suppose that with probability p the consumer can encounter a designer with payoffs $\bar{v}(G) > \bar{v}(B)$. The aligned designer's net benefit from choosing the G-product is:

$$\gamma(\mu) = \max_{\kappa} \mathbb{P}_\mu(a = 1|G, \kappa) \cdot \bar{v}(G) - \max_{\kappa} \mathbb{P}_\mu(a = 1|B, \kappa) \cdot \bar{v}(B). \tag{69}$$

Since $\mathbb{P}_\mu(a = 1|G, \kappa) \geq \mathbb{P}_\mu(a = 1|B, \kappa) > 0$ for all κ and $\bar{v}(G) > \bar{v}(G)$, it follows that the net benefit for the aligned designer is positive, and it is optimal for him to produce only a G-product. Thus, belief consistency now requires that $\mu = p + (1 - p)m$.

First consider the case of $p > \psi$. From Lemma 2, there exists a $\gamma(p) \in \Gamma(p) < 0$. Then, there exists an equilibrium in which the misaligned designer produces a B-product with probability $m = 1$, consistent with an equilibrium belief of $\mu = p$.

Suppose instead that $p < \mu_3$, then $\gamma(p) \in \Gamma(p) > 0$. Thus, if $\mu = p$, the misaligned designer wants to produce a G-product with probability one, i.e., $m = 1$, which is consistent with a belief of $\mu = 1$, reaching a contradiction. Thus, there cannot be an equilibrium with $\mu < \mu_3$. Thus, for $p < \mu_3$ the equilibrium requires the misaligned designer to follow a mixed strategy,

$m \in (0,1)$ such that the designer is indifferent between producing a B- or a G-product: $\mu = \psi$, which indicates there exists a $\gamma(\psi) \in \Gamma(\psi) = 0$. Note that this is the unique equilibrium of our baseline model with $p = 0$.

Finally, consider $p \in [\mu_3, \psi]$. If $\min\{\Gamma(\mu_3)\} > 0$, then the equilibrium is as the one described above for $p < \mu_3$, as all $\gamma(\mu) \in \Gamma(\mu) > 0$ for $\mu < \psi$. If instead $\min\{\Gamma(\mu_3)\} < 0$, multiple equilibria arise. In particular, (i) $\mu = \psi$ is consistent with an equilibrium in which the misaligned designer follows the mixed strategy, $m = \frac{\psi - p}{1-p}$, since there exists a j such that $\gamma^j(\psi) \in \Gamma(\psi) = 0$, (ii) $\mu = p[\mu_3, \psi]$ is consistent with an equilibrium since there exists $\gamma^j(p) \in \Gamma(p) < 0$, where $j \in \{S, M, P\}$, denotes the different choices of complexity, Separating, Mixing, Pooling, consistent with an equilibrium belief of μ.

Furthermore, if $\mu \in (\mu_3, \mu_4)$, there could be multiple complexity levels consistent with an equilibrium belief of μ if $\gamma^j(\mu) \in \Gamma(\mu) \leq 0$ for more than one $j \in \{S, M, P\}$.

For the comparative statics on p we focus on the simplest equilibrium. We consider an increase from p to p'.

If $p' \leq \psi$, equilibrium average quality stays constant at $\mu = \psi$, with complexity given by Proposition 4. As the fraction of H-types increases, so does the fraction of L-types producing B-products, m_L, and thus $\mu = \psi$, and complexity, are constant.

If $p' > \psi$, then the equilibrium features $m_L = 1$, and average quality $\mu = p' > \max\{p, \psi\}$. Thus, average quality has increased. As for the effect on average complexity,

$$E[\sigma] = \mu \sigma_G(\mu) + (1-\mu)\sigma_B(\mu),$$

note that σ_y (weakly) increases in μ. Thus, the increase in μ has no effect on complexity if $\mu < \mu_1$, as both designers simplify in this region, average complexity decreases if $\sigma_G(\mu) < \sigma_B(\mu)$, i.e., for $\mu \in (\mu_2, \mu_4)$, and eventually increases as all designers produce complex products when μ is sufficiently high, i.e., $\mu > \mu_4$. ■

B Proofs of Section 6

Proof of Proposition 10. For the existence result, it is without loss of generality to focus on the most simple equilibrium (proof analogous for the most complex). For each $U \in [w(B), w(G)]$, consider the map $T_\beta : U \mapsto \mathbb{R}$ defined by:

$$T_\beta(U) = \mathbb{E}\left\{ \max_{a \in \{0,1\}} \{a \cdot (\mu(s,z) \cdot w(G) + (1-\mu(s,z)) \cdot w(B)) + (1-a) \cdot \beta U\} \right\}, \tag{70}$$

where $\mu(s, z)$ is the consumer's equilibrium belief that the proposed product has output G, given signal s with noise z. For an exogenously given consumer value U, which pins down the consumer's outside option $w_0 = \beta U$, this map gives us a new ex-ante welfare $T_\beta(U)$. An equilibrium is a fixed point of this map, and we denote it by U^*. Since we have selected the simplest equilibrium, $T_\beta(\cdot)$ is single-valued. Since we have selected the simplest equilibrium, $T_\beta(\cdot)$ is single-valued.

Since, in equilibrium, the consumer has some information, it must be that $T_\beta(w(B)) > pw(G) + (1-p)w(B)$. Also, in equilibrium, B-products are produced with positive probability and the consumer's information is imperfect, it must be that $T_\beta(w(G)) < w(G)$. Thus, if $\beta(pw(G) + (1-p)w(B)) > w(B)$, then the outside option βU^* will satisfy Assumption 1, i.e. $\beta U^* \in (w(B), w(G))$. We assume that β is not too low, so that this condition holds.[25]

Therefore, to show that an equilibrium exists, it suffices to show that $T_\beta(\cdot)$ is increasing. But, the consumer's ex-ante welfare increases in the outside option βU, and for three reasons. First, there is the direct effect of the outside option. Second, in equilibrium, σ_B and σ_G are decreasing in the outside option. To see this, recall that the thresholds $\mu_1 - \mu_4$ are increasing in the outside option (see proof of Proposition 7) and, when $\mu \in (\mu_1, \mu_2]$, which is the only case where complexity is interior in the most simple equilibrium, σ_B is decreasing in the outside option and $\sigma_G = 0$. Finally, also by Proposition 7, the probability that the designers propose G-product, μ, is increasing in the outside option.

For comparative statics, note that, for a given U, an increase from β to some β' is equivalent to an increase in the consumer's outside option. But then again, it must be that $T_\beta(U) < T_{\beta'}(U)$. If there is a unique solution to $T_{\beta'}(U) = U$, then the solution must be higher at β' than any solution at β, since $T_\beta(\cdot)$ is increasing. If there are multiple solutions to $T_{\beta'}(U) = U$, then the statement holds for the maximal solution. ∎

C Proofs of Section 7

Proof of Lemma 3. First, note that $\mu(\kappa) = 1$ is not consistent with an equilibrium for any $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$. If that was the case, the L-type designer would produce a B-product with complexity κ, since it would be accepted with probability one, which implies a contradiction. Second, note that $\mu(\kappa) = 0$ is not consistent for any equilibrium complexity level $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$. Suppose WLOG that $\mu(\underline{\kappa}) = 0$, that is, only the B-type designer chooses $\underline{\kappa}$ in equilibrium. If this was the case, the B-product designer would be better off by choosing complexity $\bar{\kappa}$, since it

[25]Otherwise, when β is small enough, then the consumer will accept all products and the equilibrium strategies and payoffs will be independent of β in that region.

would give him a positive expected payoff (note that $\mu(\bar{\kappa}) > 0$), which is higher than the payoff of zero associated with the choice of $\underline{\kappa}$, which reaches a contradiction. Thus, it must be that in any equilibrium either (1) $\sigma_G \in (0,1)$ and $\sigma_B \in (0,1)$, or (2) $\sigma_G = \sigma_B \in \{0,1\}$. Suppose (1) holds. Since the designer of either product is mixing, he must be indifferent between choosing a complex or a simple product. However, it is easy to check that for any interim belief function, if the B-product designer is indifferent, then the G-product designer strictly prefers simple products; while if the G-product designer is indifferent, the B-product designer strictly prefers complex products, a contradiction. Thus, (2) must holds in equilibrium. ■

Proof of Proposition 11. Lemma 3 has established that there are only two candidates for complexity levels in equilibrium. To support either complexity level as part of an equilibrium, suppose that off-equilibrium beliefs are such that they assign a deviation to off-equilibrium complexity levels to the B-product designer. Then, any deviations from equilibrium complexity yield a payoff of zero and are never optimal. Finally, it remains to show that with such beliefs, both equilibria exist. The proof is analogous to the proof of Proposition 6. Consider the equilibrium with pooling at $\bar{\kappa}$. Following in the footsteps for that proof, it is straightforward to show that there exists a unique $\psi^C > 0$ such that

$$\gamma_C(\psi^C) = \mathbb{P}(a=1|G,\bar{\kappa})v(G) - \mathbb{P}(a=1|B,\bar{\kappa})v(B) = 0 \tag{71}$$

with $\gamma_C(\mu) > 0$ for $\mu < \psi^C$ and $\gamma_C(\mu) < 0$ for $\mu > \psi^C$, and where the probabilities are computed with a consumer's prior belief μ. We denote by γ_j the function in the equilibrium with complexity $j \in \{C, S\}$. Similarly, in the equilibrium with pooling at simplicity, $\underline{\kappa}$, there exists a unique $\psi_S = 0$ such that

$$\gamma_S(\psi^S) = \mathbb{P}(a=1|G,\underline{\kappa})v(G) - \mathbb{P}(a=1|B,\underline{\kappa})v(B) = 0, \tag{72}$$

with $\gamma_S(\mu) > 0$ for $\mu < \psi^S$ and $\gamma_S(\mu) < 0$ for $\mu > \psi^S$, and where the probabilities are computed with a consumer's prior belief μ. Therefore, it is optimal for the designer to produce G-product w.p. $m = \psi^j$ for $j \in \{C, S\}$ respectively, which is in turn consistent with the consumer's prior belief $\mu = \psi^j$.

We are left to show that $\psi^C < \psi^S$, which will follow from the MLRP property of the $f(z|\kappa)$. There are two cases to consider.

Case 1. If $\psi_S \geq \omega$, the simple equilibrium is in the optimistic region. If $\psi_C < \omega$, then the result follows. Otherwise, suppose that $\psi_C \geq \omega$ as well. In the simple equilibrium we have

$$\gamma_S\left(\psi^S\right) = v(G) - v(B) - \int_0^{\bar{z}\left(\psi^S\right)} \left(v(G)\cdot z - v(B)\cdot(1-z)\right)\cdot f\left(z|\underline{\kappa}\right)dz = 0. \tag{73}$$

Since $v(G)\cdot z - v(B)\cdot(1-z) < 0$ and increasing in z, the MLRP property implies that

$$\gamma_C\left(\psi^S\right) = v(G) - v(B) - \int_0^{\bar{z}\left(\psi^S\right)} \left(v(G)\cdot z - v(B)\cdot(1-z)\right)\cdot f\left(z|\bar{\kappa}\right)dz < 0. \tag{74}$$

and thus $\bar{z}\left(\psi^C\right) > \bar{z}\left(\psi^S\right)$. Since in the optimistic region $\frac{d\bar{z}}{d\mu} < 0$, it follows that $\psi^C < \psi^S$.

Case 2. If $\psi^S < \omega$, the simple equilibrium is in the pessimistic region. We have

$$\gamma_S\left(\psi^S\right) = \int_0^{\bar{z}(\psi^S)} \left(v(G)\cdot(1-z) - v(B)\cdot z\right)\cdot f\left(z|\underline{\kappa}\right)dz = 0, \tag{75}$$

Since $v(G)\cdot(1-z) - v(B)\cdot z$ is decreasing in z, the MLRP property implies:

$$\int_0^{\bar{z}(\psi^S)} \left(v(G)\cdot(1-z) - v(B)\cdot z\right)\cdot f\left(z|\bar{\kappa}\right)dz < 0, \tag{76}$$

This implies that $\gamma_C(\psi_S) < 0$, and given the shown properties of the γ function (Lemma 2), that $\psi^C < \psi^S$. ■

Proof of Proposition 12. First, we show that if $p > \max\{\mu_3, \psi_S\}$, defined in (42) and (72) respectively, then all designers are better off in the complex equilibrium. Second, we show that this condition holds for p large enough or ω small enough. Remember that $\psi^C < \psi^S$, as shown in Proposition 11.

Suppose that $p > \max\{\mu_3, \psi_S\}$. Then, we have that $\mu^S = \mu^C = p$ and that both equilibria (C and S) are in the optimistic region, as $p > \mu_3 \geq \omega$. Given this, the differential payoff to the aligned (A) designer from being in the complex relative to the simple equilibrium equilibrium is as follows

$$\Delta U^A = \bar{v}(G)\cdot\left[\int_0^{\bar{z}(p)} (1-z)\cdot\left(f\left(z|\bar{\kappa}\right) - f\left(z|\underline{\kappa}\right)\right)dz + \int_{\bar{z}(p)}^{\frac{1}{2}} \left(f\left(z|\bar{\kappa}\right) - f\left(z|\underline{\kappa}\right)\right)\cdot dz\right] \tag{77}$$

$$= \bar{v}(G)\cdot\int_0^{\bar{z}(p)} z\cdot\left(f\left(z|\underline{\kappa}\right) - f\left(z|\bar{\kappa}\right)\right)dz. \tag{78}$$

Then, from MLPR we have that $\Delta U^A > 0$ whenever $\bar{z}(p) < \hat{z}$, defined in Proposition 2, i.e.,

$$\frac{\omega\,(1-p)}{\omega\,(1-p)+p\cdot(1-\omega)} < \hat{z} \tag{79}$$

$$\iff p > \frac{\omega\,(1-\hat{z})}{\omega\,(1-\hat{z})+\hat{z}\,(1-\omega)} = \mu_3. \tag{80}$$

Then, the aligned designer prefers the equilibrium with complex products.

Deriving the relative benefit of being in the complex equilibrium for the misaligned designer (M), who produces B-products with probability one, we obtain

$$\Delta U^M = v(B)\cdot\left[\int_0^{\bar{z}(p)} z\cdot\left(f\left(z|\bar{\kappa}\right)-f\left(z|\underline{\kappa}\right)\right)dz + \int_{\bar{z}(p)}^{\frac{1}{2}}\left(f\left(z|\bar{\kappa}\right)-f\left(z|\underline{\kappa}\right)\right)\cdot dz\right] \tag{81}$$

$$= v(G)\cdot\int_0^{\bar{z}(p)}(1-z)\cdot\left(f\left(z|\underline{\kappa}\right)-f\left(z|\bar{\kappa}\right)\right)dz > 0. \tag{82}$$

Thus, the misaligned designer is always better off in the equilibrium with complex products.

Finally, note that $\psi_S \in (0,1)$, $\mu_3 \in (0,1)$, and both are independent of p. Thus, condition $p > \max\{\mu_3, \psi_S\}$ holds for p large enough. Analogously, as $\psi_S \to 0$ and $\mu_3 \to 0$ as $\omega \to 0$ (this is easy to check), continuity of both ψ_S and μ_3 on ω imply that the condition also holds for small enough ω. ■

D Costly Complexity

In this extension, we incorporate a direct cost for the consumer if the product deviates from a 'natural level of complexity.' We suppose that the consumer pays a cost $c(\kappa)$ when a product with complexity κ is approved. The cost is $c(\kappa) = 0$ if $\kappa = \kappa_n$ and $c(\kappa) = \bar{c} > 0$ otherwise. Thus, the consumer's utility becomes:

$$W(a|s,z) \equiv a\cdot E\left[w(y)-c(\kappa)|s,z\right]+(1-a)\cdot w_0 \tag{83}$$

We continue to study the case of binary complexity, and we make the following assumptions for payoffs.

Assumption 2 *The payoffs satisfy the following property:* $w(G) - \bar{c} > w_0$.

The assumption states that the cost of complexity is not too large, so that a product is approved if the consumer is sufficiently confident that it is a G-product. We begin by considering the case in which $\kappa_n = \underline{\kappa}$. Later, we discuss the converse case when $\kappa_n = \bar{\kappa}$. The only difference with our model described in Section 3 is that the consumer's acceptance strategy is now modified to incorporate this direct cost of complexity. In particular, the consumer's expected payoff can be re-written as follows,

$$a \cdot \{\mu(s,z) \cdot [w(G) - \sigma_G(z) \cdot \bar{c}] + (1-\mu(s,z)) \cdot [w(B) - \sigma_B(z) \cdot \bar{c}]\} + (1-a) \cdot w_0, \tag{84}$$

where

$$\sigma_y(z) \equiv \mathbb{P}(\kappa = \bar{\kappa}|y,z) = \frac{\sigma_y f(z|\bar{\kappa})}{\sigma_y f(z|\bar{\kappa}) + (1-\sigma_y) f(z|\underline{\kappa})}. \tag{85}$$

In contrast to our baseline model, the consumer now also updates her beliefs about the complexity of the product after observing noise z, since $z \sim f(z|\kappa)$. Note that $\sigma_y(z)$ weakly increases in z for all y.

By inspection of (84), we see that the consumer approves the product if:

$$\mu(s,z) \geq \frac{w_0 - w(B) + \sigma_B(z) \cdot \bar{c}}{w(G) - w(B) + (\sigma_B(z) - \sigma_G(z)) \cdot \bar{c}} \equiv \omega(z). \tag{86}$$

The left-hand side of condition (86) is the consumer's posterior belief about being offered a G-product, and it is the same as in (4). The right-hand side incorporates the fact that complexity is costly to the consumer. The new relative outside option, $\omega(z)$, differs from ω in Section 3 for two reasons. First, it is higher for all z, to reflect the fact that complex products are more costly to the consumer. Second, its value weakly increases in the noise of the signal.

As in Section 3, we let $\bar{z}$ be defined as the solution to (86) with equality for some s. Then, the solution to $\omega(\bar{z}) = \mu(b, \bar{z})$ gives the threshold $\bar{z}$ when the consumer is optimistic, and the solution to $\omega(\bar{z}) = \mu(g, \bar{z})$ gives the threshold $\bar{z}$ when the consumer is pessimistic.

The following Lemma then characterizes the consumer's acceptance strategy.

Lemma 4 *The threshold $\bar{z}$ is unique provided that $\bar{c}$ is not too large, and the consumer's acceptance strategy is as in Lemma 1. Furthermore, threshold $\bar{z}$ is increasing in the cost of complexity, $\bar{c}$, when the consumer is optimistic, and decreasing when the consumer is pessimistic.*

The acceptance strategy depends, through beliefs $\mu(z)$ and $\sigma_y(z)$, on the t-designer's equilibrium strategies $\{m_t, \sigma_t\}$, which the consumer takes as given. As in the no cost model, the consumer's acceptance strategy rule is contingent on the signal only when the signal is sufficiently informative, i.e. $z < \bar{z}$. However, when complexity is costly, the consumer's decision rule becomes more tight as she tends to reject products more often. The optimistic consumer relies more on the signal than in the no cost case making acceptance less likely to occur, while the pessimistic consumer relies less on the signal making rejection more likely to occur.

The equilibrium characterization follows analogously to the no cost case, and it is summarized in the following proposition.

Proposition 13 *The equilibrium characterization is as in Propositions 4 and 6, but with modified thresholds* μ_1-μ_4 *and* ψ *are given in* (90) − (92). *In particular, the equilibrium level of product complexity is decreasing in the cost* $\bar{c}$.

Thus, the main qualitative results of our model carry over to the case with a direct cost to complexity for the consumer, with the not very surprising prediction that the equilibrium level of complexity will be lower if it is costlier.

To complete the analysis, we now turn to the case when $\kappa_n = \bar{\kappa}$. The cost of complexity then becomes $c(\underline{\kappa}) = \bar{c}$ and $c(\bar{\kappa}) = 0$. The analysis is analogous to the above. Lemma 4 holds as before, and, perhaps surprisingly, the qualitative result of Proposition 13 continues to hold. An increase in $\bar{c}$ still results in lower equilibrium complexity. A higher cost of owning low complexity products increases the consumer's relative outside option, since buying the product yields a lower expected payoff. This in turn raises the threshold on the informativeness of the product that is needed for acceptance. Hence, in equilibrium, designers who want their products accepted simplify them in order make them more informative for the consumer, even if doing so is costlier for the consumer. We conclude, therefore, that having a cost to complexity implies a lower equilibrium level of complexity, regardless of the value of κ_n.

Proofs of Lemmas and Propositions in this Appendix

Proof of Lemma 4. To drive the value of $\bar{z}$, we must consider the possible equilibria in each region. Consider first the pessimistic consumer.

Case 1: Equilibrium with $\sigma_G = \sigma_B = 0$. In this case, $\bar{z}$ is given by (38).

Case 2: Equilibrium with $\sigma_G = 0$ and $\sigma_B \in (0, 1)$. In this case, for the designer of the B-product to mix, it must be that $\bar{z} = \hat{z}$.

Case 3: Equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. For this to be an equilibrium, it requires that

$$\frac{f\left(\frac{1}{2}|\underline{\kappa}\right)\cdot\mu}{f\left(\frac{1}{2}|\underline{\kappa}\right)\cdot\mu + f\left(\frac{1}{2}|\overline{\kappa}\right)\cdot(1-\mu)} < \frac{w_0 - w\left(B\right) + \bar{c}}{w\left(G\right) - w\left(B\right) + \bar{c}}.$$

Then, $\bar{z}$ is given by

$$\frac{(1-\bar{z})\cdot\mu\cdot\ell(\bar{z})}{(1-\bar{z})\cdot\mu\cdot\ell(\bar{z})+\bar{z}\cdot(1-\mu)}=\frac{w_0-w(B)+\bar{c}}{w(G)-w(B)+\bar{c}}, \tag{87}$$

where $\ell(\bar{z})$ is defined in (15). In (87), the solution $\bar{z}$ is unique under Assumption 2 about the magnitude of $\bar{c}$. Notice that the threshold $\omega(z)$ is increasing in $\bar{c}$. The left-hand side of equation (87) is decreasing in $\bar{z}$. Thus, as $\bar{c}$ increases, $\bar{z}$ decreases.

Consider next the optimistic consumer.

Case 1: Equilibrium with $\sigma_G=\sigma_B=1$. In this case, $\bar{z}$ is given by

$$\frac{\bar{z}\cdot\mu}{\bar{z}\cdot\mu+(1-\bar{z})\cdot(1-\mu)}=\frac{w_0-w(B)+\bar{c}}{w(G)-w(B)}. \tag{88}$$

Case 2: Equilibrium with $\sigma_B=1$ and $\sigma_G\in(0,1)$. In this case, for the holder of the G-product to mix, it must be that $\bar{z}=\hat{z}$.

Case 3: Equilibrium with $\sigma_G=0$ and $\sigma_B=1$. For this to be an equilibrium, it requires that

$$\frac{f\left(\frac{1}{2}|\underline{\kappa}\right)\cdot\mu}{f\left(\frac{1}{2}|\underline{\kappa}\right)\cdot\mu+f\left(\frac{1}{2}|\overline{\kappa}\right)\cdot(1-\mu)}\geq\frac{w_0-w(B)+\bar{c}}{w(G)-w(B)+\bar{c}}.$$

Then, $\bar{z}$ is given by

$$\frac{\bar{z}\cdot\mu\cdot\ell(\bar{z})}{\bar{z}\cdot\mu\cdot\ell(\bar{z})+(1-\bar{z})\cdot(1-\mu)}=\frac{w_0-w(B)+\bar{c}}{w(G)-w(B)+\bar{c}}. \tag{89}$$

In both (88) and (89), the solution $\bar{z}$ is unique under Assumption 2. The left-hand side of equations (88) and (89) is increasing in $\bar{z}$. Thus, as $\bar{c}$ increases, $\omega(z)$ increases, and thus $\bar{z}$ increases. ■

Proof of Proposition 13. The proof is analogous to the proof from the case with no cost. We highlight here the differences in the values of the thresholds $\mu_1-\mu_4$ given the cost parameter. For the equilibrium with $\sigma_G=\sigma_B=0$, the threshold for acceptance is the same as before, so μ_1 is given by (39). For the equilibrium with $\sigma_G=0$ and $\sigma_B\in(0,1)$, the threshold for acceptance is given by:

$$\mu(G;\hat{z})=\frac{(1-\hat{z})\cdot\mu}{(1-\hat{z})\cdot\mu+\hat{z}\cdot(1-\mu)\cdot(\sigma_B+(1-\sigma_B)\cdot\ell(\hat{z}))}.$$

This leads to threshold μ_2 :

$$\mu_2 = \frac{w_c^c \cdot \hat{z}}{w_c^c \cdot \hat{z} + (1 - w_c^c) \cdot \ell(\hat{z}) \cdot (1 - \hat{z})}, \tag{90}$$

where

$$w_c^c \equiv \frac{\omega_0 - \omega(B) + \bar{c}}{\omega(G) - \omega(B) + \bar{c}}.$$

For the equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$, the threshold at which the pessimistic consumer region ends is given by

$$\tilde{\mu} = \frac{w_c^c}{w_c^c + (1 - w_c^c) \frac{f(0.5|\underline{\kappa})}{f(0.5|\bar{\kappa})}}. \tag{91}$$

An equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$, exists in the optimistic consumer region for $\mu > \tilde{\mu}$ and $\mu < \mu_4$, with

$$\mu_4 = \frac{w^c}{w^c + (1 - w^c) \frac{\hat{z}}{1 - \hat{z}}}, \tag{92}$$

where

$$w^c \equiv \frac{\omega_0 - \omega(B) + \bar{c}}{\omega(G) - \omega(B)}.$$

The equilibrium with $\sigma_B = 1$ and $\sigma_G = 0$ exists in the optimistic consumer region between thresholds μ_3 and μ_4, with

$$\mu_3 = \frac{w_c^c}{w_c^c + (1 - w_c^c) \frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})} \frac{\hat{z}}{1 - \hat{z}}}. \tag{93}$$

As shown in the model with no cost of complexity, the equilibrium with $\sigma_B = 1$ and $\sigma_G \in (0, 1)$, exists between to thresholds μ_3 and μ_4.

From (90)-(92),

$$\frac{d\mu_2}{dw_c^c} > 0 \ , \ \frac{d\mu_3}{dw_c^c} > 0 \ , \ \frac{d\mu_4}{dw^c} > 0.$$

Also

$$\frac{dw_c^c}{d\bar{c}} > 0 \ , \ \frac{dw^c}{d\bar{c}} > 0.$$

Then, a marginal increase in c weakly increases thresholds $\mu_2 - \mu_4$, which implies a decrease in the size of the interval (μ_2, μ_4) over which $\sigma_B > 0$. Moreover, since μ_3 increases as well, the interval $[\mu_3, 1]$ over which $\sigma_G > 0$ decreases. Thus, an increase in $\bar{c}$ reduces expected equilibrium complexity. ■