A Risk-centric Model of Demand Recessions and Macroprudential Policy

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Abstract

We theoretically analyze the interactions between asset prices, financial speculation, and macroeconomic outcomes when output is determined by aggregate demand. If the interest rate is constrained, a rise in the risk premium lowers asset prices and generates a demand recession. This reduces earnings and generates a feedback loop between asset prices and aggregate demand. The recession is exacerbated by speculation due to heterogeneous asset valuations during the ex-ante low-risk-premium period. Macroprudential policy that restricts speculation can Pareto improve welfare. We also provide empirical support for the mechanisms by comparing impulse responses to house price shocks within and outside the Eurozone.

JEL Codes: **E00, E12, E21, E22, E30, E40, G00, G01, G11**

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1. Introduction

Prices of risky assets, such as stocks and houses, fluctuate considerably without meaningful changes in underlying payoffs such as earnings and rents. These fluctuations, which are typically described as the result of a "time-varying risk premium," have been the subject of a large finance literature exploring their scope and causes (see Cochrane (2011); Campbell (2014) for recent reviews). Asset price fluctuations driven by the time-varying risk premium matter not only for financial markets but also for macroeconomic outcomes. Low asset prices are worrisome because they can create or exacerbate recessions. In influential work, Mian and Sufi (2014) demonstrate that the decline in U.S. house prices explains much of the aggregate job losses during the Great Recession. High asset prices also generate concerns because they are often associated with excessive leverage and speculation, which might exacerbate the damage when prices decline. Central banks are aware of these connections between financial markets and macroeconomic outcomes. For example, Cieslak and Vissing-Jorgensen (2017) conduct a textual analysis of 184 FOMC minutes during the 1994-2016 period and find extensive reference to stock market developments, which had significant explanatory power for monetary policy. The Fed's stated rationale for these policy reactions emphasizes the negative impact of severe stock markets declines on aggregate consumption and investment.

There is an extensive literature at the intersection of macroeconomics and corporate finance that highlights a variety of financial frictions and balance sheet mechanisms that capture the connection between asset prices and the macroeconomy. The bulk of the mechanisms in this literature operate through the supply side of the economy, as the tightening of financial constraints reallocates resources from more productive to less productive agents (see Gertler et al. (2010) for a review). With few exceptions, there is no special role for aggregate demand or monetary policy. In this paper, we provide a model that complements this literature by focusing on the role of the aggregate demand channel in causing recessions driven by a rise in the risk premium. We also study the interaction of these recessions with financial speculation and derive the implications for macroprudential policy. In order to isolate our insights, we remove all financial frictions.

Our model is set in continuous time with diffusion productivity shocks and Poisson shocks that move the economy between high and low risk premium states. The supply side is a stochastic endowment economy with sticky prices (which we extend to an endogenous growth model when we add investment). The demand side has risk-averse consumer-investors that demand goods and risky assets. We focus on "interest rate frictions" and "financial *speculation*." By interest rate frictions, we mean factors that might constrain or delay the adjustment of the risk-free interest rate to shocks. For concreteness, we work with a zero lower bound on the policy interest rate, but our mechanism is also applicable with other interest rate constraints such as a currency union or a fixed exchange rate. By financial speculation, we mean the trade of risky financial assets among investors that have heterogeneous valuations of these assets. We capture speculation by allowing investors to have *belief disagreements* with respect to the transition probabilities between high and low risk premium states.

To fix ideas, consider an increase in perceived volatility. This is a "risk premium shock" that

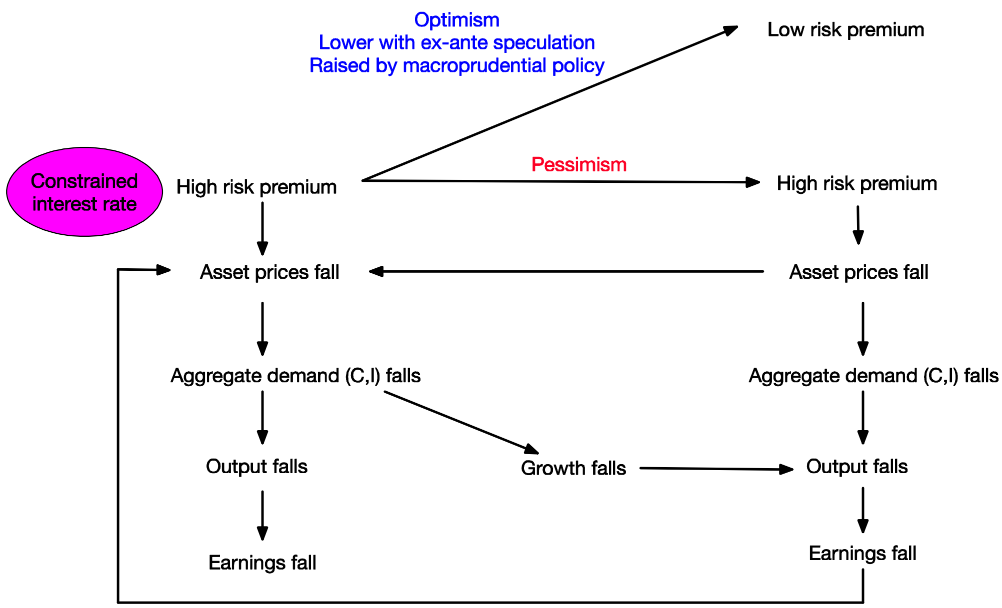

Figure 1: Output-asset price feedbacks during a risk-centric demand recession.

exerts downward pressure on risky asset prices without a change in current productivity (the supply-determined output level). If the monetary authority allows asset prices to decline, then low prices induce a recession by reducing aggregate demand through a wealth effect. Consequently, monetary policy responds by reducing the interest rate, which stabilizes asset prices and aggregate demand. However, if the interest rate is constrained, the economy loses its natural line of defense. In this case, the rise in the risk premium reduces asset prices and generates a demand recession.

Dynamics play a crucial role in this environment, as the recession is exacerbated by feedback mechanisms. In the main model, when the higher risk premium is expected to persist, the decline in future demand lowers expected earnings, which exerts further downward pressure on asset prices. With endogenous investment, there is a second mechanism, as the decline in current investment lowers the growth of potential output, which further reduces expected earnings and asset prices. In turn, the decline in asset prices feeds back into current consumption and investment, generating scope for severe spirals in asset prices and output. Figure 1 provides a graphical illustration of these dynamic mechanisms. The feedbacks are especially powerful when investors are *pessimistic* and interpret the higher risk premium as very persistent. Hence, beliefs matter in our economy not only because they have a direct impact on asset prices but also because they determine the strength of the amplification mechanism.

In this environment, speculation during the low risk premium phase (boom) further exacerbates the recession when there is a transition to the high risk premium phase. With heterogeneous asset valuations, which we capture with belief disagreements, the economy's degree of optimism depends on the share of wealth in the hands of optimists (or high-valuation investors). During recessions, the economy benefits from wealthy optimists because they raise asset valuations, increasing aggregate demand. However, disagreements naturally lead to speculation during booms, which depletes optimists' wealth during recessions. Specifically, optimists take on risk by selling insurance contracts to pessimists that enrich optimists if the boom persists but lead to a large reduction in their wealth when there is a transition to recession. This reallocation of wealth lowers asset prices and leads to a more severe recession.

These effects motivate *macroprudential policy* that restricts speculation during the boom. We show that macroprudential policy that makes optimistic investors behave as-if they were more pessimistic (implemented via portfolio risk limits) can generate a *Pareto improvement* in social welfare. This result is not driven by paternalistic concerns—the planner respects investors' own beliefs, and the result does not depend on whether optimists or pessimists are closer to the truth. Rather, the planner improves welfare by internalizing *aggregate demand externalities.* The depletion of optimists' wealth during a demand recession depresses asset prices and aggregate demand. Optimists (or more broadly, high-valuation investors) do not internalize the effect of their risk taking on asset prices and aggregate demand during the recession. This leads to excessive risk taking that is corrected by macroprudential policy. Moreover, our model supports *procyclical macroprudential policy.* While macroprudential policy can be useful during the recession, by restricting speculation, these benefits can be outweighed by its immediate negative impact on asset prices. This decline can be offset by the interest rate policy during the boom but not during the recession.

While there is an extensive empirical literature supporting the components of our model (see Section 7 for a brief summary), we extend this literature by presenting empirical evidence consistent with our results. We focus on three testable implications. First, our model predicts that negative risk premium shocks generate a more severe demand recession when the interest rate is constrained. Second, the recession reduces firms' earnings and leads to a further reduction in asset prices. Third, the recession is more severe when the shock takes place in an environment with more speculation.

To test these predictions, we assemble a quarterly panel data set of 21 advanced countries between 1990 and 2017, and subdivide the panel into countries that are part of the Eurozone or the European Exchange Rate Mechanism (*the Euro/ERM sample*) and those that have their own currencies (*the non-Euro/ERM sample*). The first group has constrained interest rate with respect to local asset price shocks, since they share a common monetary policy. The second group has less constrained interest rate. We use a local projection method as in Jordà (2005) to estimate impulse responses to surprise house price changes (our risk premium shock variable) separately for each sample. We absorb time and country fixed effects, and include controls for recent output growth and monetary policy. Thus the identification comes from comparing the outcomes in a country that experiences a house price change unrelated to its recent economic activity or monetary policy with the outcomes in countries in the same (Euro/ERM or non-Euro/ERM) sample but do not experience a change.

We find that a negative house price shock in a non-Euro/ERM country is associated with an initial decline in economic activity, followed by a decline in the policy interest rate and output stabilization. In contrast, a similar shock in a Euro/ERM country is associated with no interest rate response (compared to other Euro/ERM countries), which is followed by a more persistent and larger decline in economic activity. This supports our prediction that risk premium shocks generate a more severe demand recession in economies with constrained interest rate. We also find that the house price shock is followed by a larger decline in earnings and stock prices of publicly traded firms in the Euro/ERM sample than in the other sample (although the standard errors are larger for these results). This spillover effect supports our prediction that shocks are amplified due to the endogenous earnings response. Finally, we find that past bank credit expansion is associated with more severe outcomes following the house price shock in the Euro/ERM sample (but not in the other sample). Interpreting bank credit as a broad measure of speculation, both because banks are relatively high-valuation investors (due to their greater capacity and expertise to handle risk) and because they lend to optimists in the housing market, this provides some support for our final prediction that speculation amplifies the severity of the recession.

Literature review. Our paper is part of a large literature that emphasizes the links between asset prices and macroeconomic outcomes. Our model contributes to this literature by establishing a relationship between asset prices and aggregate demand even without financial frictions. This relates our paper to strands of the New-Keynesian literature that emphasize demand shocks that might drive business cycles while also affecting asset prices, such as "noise shocks" (Lorenzoni (2009); Blanchard et al. (2013)), "confidence shocks" (Ilut and Schneider (2014)), "uncertainty shocks" (Basu and Bundick (2017); Fernández-Villaverde et al. (2015)), and "disaster shocks" (Gourio (2012)). Aside from the modeling novelty (ours is a continuous time macrofinance model), we provide an integrated treatment of these and related forces. We refer to them as "risk premium shocks" to emphasize their close connection with asset prices and the finance literature on time-varying risk premia. Accordingly, we also make asset prices the central object in our theoretical and empirical analyses, breaking with convention in the New-Keynesian literature without financial frictions. More substantively, we show that heterogeneity in asset valuation matters in these environments. This heterogeneity matters because it leads to speculation that exacerbates demand recessions and provides a distinct rationale for macroprudential regulation.

The interactions between heterogeneous valuations, risk-premia, and interest rate lower bounds are also central themes of the literature on structural safe asset shortages and safety traps (see, for instance, Caballero and Farhi (2017); Caballero et al. (2017b)). Aside from emphasizing a broader set of factors that can drive the risk premium (in addition to safe asset scarcity), we contribute to this literature by focusing on dynamics. We analyze the connections between boom and recession phases of recurrent business cycles driven by risk premium shocks. Among other things, we show that speculation between "optimists" and "pessimists" during the boom exacerbates a future risk-centric demand recession, and derive the implications for macroprudential policy. In contrast, Caballero and Farhi (2017) show how "pessimists" can create a demand recession in otherwise normal times, and derive the implications for fiscal and unconventional monetary policies. More broadly, our paper is related to an extensive literature on liquidity traps that has exploded since the Great Recession (see, for instance, Tobin (1975); Krugman (1998); Eggertsson and Woodford

(2006); Eggertsson and Krugman (2012); Guerrieri and Lorenzoni (2017); Werning (2012); Hall (2011); Christiano et al. (2015); Eggertsson et al. (2017); Rognlie et al. (2017); Midrigan et al. (2016); Bacchetta et al. (2016)).

At a methodological level, our paper belongs in the new continuous time macrofinance literature started by the work of Brunnermeier and Sannikov (2014, 2016a) and summarized in Brunnermeier and Sannikov (2016b) (see also Basak and Cuoco (1998); Adrian and Boyarchenko (2012); He and Krishnamurthy (2012, 2013); Di Tella (2012); Moreira and Savov (2017); Silva (2016)). This literature seeks to highlight the full macroeconomic dynamics induced by financial frictions. While the structure of our economy shares many similarities with theirs, our model has no financial frictions, and the macroeconomic dynamics stem not from the supply side (relative productivity) but from the aggregate demand side.

Our results on macroprudential policy are related to recent work that analyzes the implications of aggregate demand externalities for the optimal regulation of financial markets. For instance, Korinek and Simsek (2016) show that, in the run-up to deleveraging episodes that coincide with a zero-lower-bound on the interest rate, policies targeted at reducing household leverage can improve welfare (see also Farhi and Werning (2017)). In these papers, macroprudential policy works by reallocating wealth across agents and states so that agents with higher marginal propensity to consume hold relatively more wealth when the economy is more depressed due to deficient demand. The mechanism in our paper is different and works through *heterogeneous asset valuations.* The policy operates by transferring wealth to optimists during recessions, not because optimists spend more than other investors, but because they raise asset valuations and induce *all investors to spend more* (while also increasing aggregate investment).[1]

The macroprudential literature beyond aggregate demand externalities is mostly motivated by the presence of pecuniary externalities that make the competitive equilibrium constrained inefficient (e.g., Caballero and Krishnamurthy (2003); Lorenzoni (2008); Bianchi and Mendoza (2013); Jeanne and Korinek (2010)). The friction in this literature is market incompleteness or collateral constraints that depend on asset prices (see Davila and Korinek (2016) for a detailed exposition). We show that a decline in asset prices is damaging not only because of the reasons emphasized in this literature, but also because it lowers aggregate demand through standard wealth and investment channels.

Our analysis of heterogeneous valuations and speculation via belief disagreements is related to a large finance literature (e.g., Lintner (1969); Miller (1977); Harrison and Kreps (1978); Varian (1989); Harris and Raviv (1993); Chen et al. (2002); Scheinkman and Xiong (2003); Fostel and Geanakoplos (2008); Geanakoplos (2010); Simsek (2013a,b); Iachan et al. (2015)). One strand of this literature emphasizes that disagreements can exacerbate asset price fluctuations by creating endogenous fluctuations in agents' wealth distribution (see, for instance, Basak (2000, 2005); Detemple and Murthy (1994); Zapatero (1998); Cao (2017); Xiong and Yan (2010); Kubler and Schmedders (2012); Korinek and Nowak (2016)). Our paper features similar forces but explores

[1] Also, see Farhi and Werning (2016) for a synthesis of some of the key mechanisms that justify macroprudential policies in models that exhibit aggregate demand externalities.

them in an environment where output is not necessarily at its supply-determined level due to interest rate rigidities.

The rest of the paper is organized as follows. In Section 2 we present an example that illustrates the main mechanism and motivates the rest of our analysis. Section 3 presents the general environment and defines the equilibrium. Section 4 characterizes the equilibrium in a benchmark setting with homogeneous beliefs. This section illustrates how risk premium shocks can lower asset prices and induce a demand recession, and how the recession is further exacerbated by feedback loops between asset prices and aggregate demand. Section 5 characterizes the equilibrium with belief disagreements, and illustrates how speculation exacerbates the recession. Section 6 illustrates the aggregate demand externalities associated with optimists' risk taking and establishes our results on macroprudential policy. Section 7 presents our empirical analysis on the relationship between house price shocks and demand recessions, and summarizes other supporting evidence from the related literature. Section 8 concludes. The (online) appendices contain the omitted derivations and proofs as well as the details of our empirical analysis.

2. A stepping-stone example

Here we present a simple (largely static) example that serves as a stepping stone into our main (dynamic) model. We start with a representative agent setup and illustrate the basic aggregate demand mechanism. We then consider belief disagreements and illustrate the role of speculation.

A two-period risk-centric aggregate demand model. Consider an economy with two dates, $t \in \{0,1\}$, a single consumption good, and a single factor of production—capital. For simplicity, capital is fixed (i.e., there is no depreciation or investment) and it is normalized to one. *Potential* output is equal to capital's productivity, z_t, but the actual output can be below this level due to a shortage of aggregate demand, $y_t \leq z_t$. For simplicity, we assume output is equal to its potential at the last date, $y_1 = z_1$, and focus on the endogenous determination of output at the previous date, $y_0 \leq z_0$. We assume the productivity at date 1 is uncertain and log-normally distributed so that,

$$\log y_1 = \log z_1 \sim N\left(g - \frac{\sigma^2}{2}, \sigma^2\right). \tag{1}$$

We also normalize the initial productivity to one, $z_0 = 1$, so that g captures the (log) expected growth rate of productivity, and σ captures its volatility.

There are two types of assets. There is a "market portfolio" that represents claims to the output at date 1 (which accrue to production firms as earnings), and a risk-free asset in zero net supply. We denote the price of the market portfolio with Q, and its log return with,

$$r^m(z_1) = \log \frac{z_1}{Q}. \tag{2}$$

We denote the log risk-free interest rate with r^f.

For now, the demand side is characterized by a representative investor, who is endowed with the initial output as well as the market portfolio (we introduce disagreements at the end of the section). At date 0, she chooses how much to consume, c_0, and what fraction of her wealth to allocate to the market portfolio, ω^m (with the residual fraction invested in the risk-free asset). When asset markets are in equilibrium, she will allocate all of her wealth to the market portfolio, $\omega^m = 1$, and her portfolio demand will determine the risk premium.

We assume the investor has Epstein-Zin preferences with the discount factor, $e^{-\rho}$, and the relative risk aversion coefficient (RRA), γ. For simplicity, we set the elasticity of intertemporal substitution (EIS) equal to 1. Later in this section, we will show that relaxing this assumption leaves our conclusions qualitatively unchanged. In the dynamic model, we will simplify the analysis further by setting RRA as well as EIS equal to 1 (which leads to time-separable log utility).

The supply side of the economy is described by New-Keynesian firms that have pre-set fixed prices. These firms meet the available demand at these prices as long as they are higher than their marginal cost (see Appendix B.1.2 for details). These features imply that output is determined by the aggregate demand for goods (consumption) up to the capacity constraint,

$$y_0 = c_0 \leq z_0. \tag{3}$$

Since prices are fully sticky, the real interest rate is equal to the nominal interest rate, which is controlled by the monetary authority. We assume that the interest rate policy attempts to replicate the supply-determined output level. However, there is a lower bound constraint on the interest rate, $r^f \geq 0$. Thus, the interest rate policy is described by, $r^f = \max\left(r^{f*}, 0\right)$, where r^{f*} is the natural interest rate that ensures output is at its potential, $y_0 = z_0$.

To characterize the equilibrium, first note that there is a tight relationship between output and asset prices. Specifically, the assumption on the EIS implies that the investor consumes a fraction of her lifetime income,

$$c_0 = \frac{1}{1 + e^{-\rho}} \left(y_0 + Q\right). \tag{4}$$

Combining this expression with Eq. (3), we obtain the following equation,

$$y_0 = e^{\rho} Q. \tag{5}$$

We refer to this equation as *the output-asset price relation*—generally, it is obtained by combining the consumption function (and when there is investment, also the investment function) with goods market clearing. The condition says that asset prices increase aggregate wealth and consumption, which in turn leads to greater output.

Next, note that asset prices must also be consistent with equilibrium in risk markets. In Appendix A.1, we show that, up to a local approximation, the investor's optimal weight on the market portfolio is determined by,

$$\omega^m \sigma \simeq \frac{1}{\gamma} \frac{E\left[r^m\left(z_1\right)\right] + \frac{\sigma^2}{2} - r^f}{\sigma}. \tag{6}$$

In words, the optimal portfolio risk (left side) is proportional to "the Sharpe ratio" on the market portfolio (right side). The Sharpe ratio captures the reward per risk, where the reward is determined by the risk premium: the (log) expected return in excess of the (log) risk free rate. This is the standard risk-taking condition for mean-variance portfolio optimization, which applies exactly in continuous time. It applies approximately in the two-period model, and the approximation becomes exact when there is a representative household and the asset markets are in equilibrium ($\omega^m = 1$).

In particular, substituting the asset market clearing condition, $\omega^m = 1$, and the expected return on the market portfolio from Eqs. (1) and (2), we obtain the following equation,

$$\sigma = \frac{1}{\gamma} \frac{g - \log Q - r^f}{\sigma}. \tag{7}$$

We refer to this equation as *the risk balance condition*—generally, it is obtained by combining investors' optimal portfolio allocations with asset market clearing and the equilibrium return on the market portfolio. It says that, the equilibrium level of the Sharpe ratio on the market portfolio (right side) needs to be sufficiently large to convince investors to hold the risk generated by the productive capacity (left side).

Next, consider the supply-determined equilibrium in which output is equal to its potential, $y_0 = z_0 = 1$. Eq. (5) reveals that this requires the asset price to be at a particular level, $Q^* = e^{-\rho}$. Combining this with Eq. (7), the interest rate also needs to be at a particular level,

$$r^{f*} = g + \rho - \gamma \sigma^2. \tag{8}$$

Intuitively, the monetary policy needs to lower the interest rate to a low enough level to induce sufficiently high asset prices and aggregate demand to clear the goods market.

Now suppose the initial parameters are such that $r^{f*} > 0$, so that the equilibrium features Q^*, r^{f*} and supply-determined output, $y_0 = z_0 = 1$. Consider a "risk premium shock" that raises the volatility, σ, or risk aversion, γ. The immediate impact of this shock is to create an imbalance in the risk balance condition (7). The economy produces too much risk (left side) relative to what investors are willing to absorb (right side). In response, the monetary policy lowers the risk-free interest rate (as captured by the decline in r^{f*}), which increases the risk premium and equilibrates the risk balance condition (7). Intuitively, the monetary authority lowers the opportunity cost of risky investment and induces investors to absorb risk.

Next suppose the shock is sufficiently large so that the natural interest rate becomes negative, $r^{f*} < 0$, and the actual interest rate becomes constrained, $r^f = 0$. In this case, the risk balance condition is re-established with a decline in the price of the market portfolio, Q. This increases the expected return on risky investment, which induces investors to absorb risk. However, the decline in Q reduces aggregate wealth and induces a demand-driven recession. Formally, we combine Eqs. (5) and (7) to obtain,

$$\log y_0 = \rho + \log Q \text{ where } \log Q = g - \gamma \sigma^2. \tag{9}$$

Note also that, in the constrained region, asset prices and output become sensitive to beliefs about future prospects. For instance, an increase in the expected growth rate, g (optimism)—rational or otherwise—increases asset prices and mitigates the recession.

More general EIS. Now consider the same model with the difference that we allow the EIS, denoted by ε, to be different than one. Appendix A.2 analyzes this case and shows that the analogue of the output-asset price relation is given by [cf. Eq. (5)],

$$y_0 = e^{\rho\varepsilon} \left(R^{CE}\right)^{1-\varepsilon} Q. \tag{10}$$

Here, R^{CE} denotes the investor's certainty-equivalent portfolio return that we formally define in the appendix. The expression follows from the fact that consumption is not only influenced by a wealth effect, as in the baseline analysis, but also by substitution and income effects. When $\varepsilon > 1$, the substitution effect dominates. All else equal, a decline in the attractiveness of investment opportunities captured by a reduction in R^{CE} tends to reduce savings and increase consumption, which in turn increases output. Conversely, when $\varepsilon < 1$, the income effect dominates and a decline in R^{CE} tends to increase savings and reduce consumption and output.

We also show that the risk balance condition (7) remains unchanged (because the EIS does not affect the investor's portfolio problem). Furthermore, we derive the equilibrium level of the certainty-equivalent return as,

$$\log R^{CE} = g - \log Q - \frac{1}{2}\gamma\sigma^2. \tag{11}$$

As expected, R^{CE} decreases with the volatility, σ, and the risk aversion, γ.

These expressions illustrate that a risk premium shock that increases σ or γ affects consumption and aggregate demand through two channels. As before, it exerts a downward influence on asset prices, which reduces consumption through a wealth effect. But in this case it also exerts a downward influence on the certainty-equivalent return, which affects consumption further depending on the balance of income and substitution effects. When $\varepsilon > 1$, the second channel works against the wealth effect because investors substitute towards consumption. When $\varepsilon < 1$, the second channel reinforces the wealth effect.

In Appendix A.2, we complete the characterization of equilibrium and show that the net effect on *aggregate demand* is qualitatively the same as in the baseline analysis *regardless of the level of EIS*. In particular, a risk premium shock that increases γ or σ reduces "rstar" (see Eq. $(A.9)$).[2]

[2] The effect of this risk premium shock on Q^* is more subtle (see Eq. $(A.8)$). When $\varepsilon > 1$, Q^* declines, which means that "rstar" does not need to fully accommodate the risk premium shock. The reason is that the substitution effect supports current consumption and reduces the burden on wealth to support aggregate demand. The opposite

When the interest rate is constrained, $r^f = 0$, the shock reduces the equilibrium level of output y_0, as well as the asset price, Q (see Eq. $(A.10)$). When $\varepsilon > 1$, the substitution effect mitigates the magnitude of these declines but it does not overturn them—that is, the wealth effect ultimately dominates. Since the purpose of our model is to obtain qualitative insights, in the dynamic model we assume $\varepsilon = 1$ and isolate the wealth effect.[3]

Belief disagreements and speculation. Let us go back to the baseline case with $\varepsilon = 1$ and illustrate the role of speculation. Suppose that there are two types of investors with heterogeneous beliefs about productivity growth. Specifically, there are optimists and pessimists that believe $\log z_1$ is distributed according to, respectively, $N\left(g^o - \frac{\sigma^2}{2}, \sigma^2\right)$ and $N\left(g^p - \frac{\sigma^2}{2}, \sigma^2\right)$. We assume $g^o > g^p$ so that optimists perceive greater growth. Beliefs are dogmatic, that is, investors know each others' beliefs and they agree to disagree (and it does not matter for our mechanism whether any of them is closer to truth than the other). Optimists are endowed with a fraction α of the market portfolio and of date 0 output (and pessimists are endowed with the remaining fraction). Hence, α denotes the wealth share of optimists. The rest of the model is unchanged.

Following similar steps to those as in the baseline case, we solve for "rstar" as follows (see Appendix A.3),

$$r^{f*} \simeq \alpha g^o + (1-\alpha) g^p + \rho - \gamma\sigma^2. \tag{12}$$

When $r^{f*} < 0$, the interest rate is constrained and $r^f = 0$, so we have a demand recession with,

$$\log y_0 = \rho + \log Q, \text{ where } \log Q \simeq \alpha g^o + (1-\alpha) g^p - \gamma\sigma^2. \tag{13}$$

Hence, equilibrium prices and output depend on optimists' wealth share, α. During the recession, increasing α improves outcomes because optimists increase asset prices, which increases aggregate wealth and *everyone*'s spending. In our dynamic model, α will be endogenous because investors will (ex-ante) speculate on their different beliefs. Moreover, speculation will reduce α during the recession because optimists think the risk premium shock is unlikely. This will exacerbate the recession and motivate macroprudential policy. Next, we turn to a formal analysis of dynamics.

3. Dynamic environment and equilibrium

In this section we first introduce our general dynamic environment and define the equilibrium. We then describe the optimality conditions and provide a partial characterization of equilibrium. In subsequent sections we will further characterize this equilibrium in various special cases of interest. Throughout, we simplify the analysis by abstracting away from investment. In Appendix D.1, we extend the environment to introduce investment and endogenous growth. We discuss additional

happens when $\varepsilon > 1$, where the substitution effect is dominated by the income effect. In this case Q^* needs to rise to support aggregate demand, which is achieved by a larger decline in "rstar" following the risk premium shock.

[3] We further simplify the dynamic model by setting $\gamma = 1$ (which leads to log utility), because $\gamma \neq 1$ leads additional dynamic hedging motives that are not central for our analysis.

results related to investment at the end of Section 4.

Potential output and risk premium shocks. The economy is set in infinite continuous time, $t \in [0, \infty)$, with a single consumption good and a single factor of production, capital. Let $k_{t,s}$ denote the capital stock at time t and in the aggregate state $s \in S$. Suppose that, when fully utilized, $k_{t,s}$ units of capital produce $Ak_{t,s}$ units of the consumption good. Hence, $Ak_{t,s}$ denotes the potential output in this economy. Capital follows the process,

$$\frac{dk_{t,s}}{k_{t,s}} = gdt + \sigma_s dZ_t. \tag{14}$$

Here, g denotes the expected productivity growth, which is an exogenous parameter in the main text (it is endogenized in Appendix D.1 that introduces investment). The term, dZ_t, denotes the standard Brownian motion, which captures "aggregate productivity shocks."[4]

The states, $s \in S$, differ only in terms of the volatility of aggregate productivity, σ_s. For simplicity, there are only two states, $s \in \{1, 2\}$, with $\sigma_1 < \sigma_2$. State $s = 1$ corresponds to a low-volatility state, whereas state $s = 2$ corresponds to a high-volatility state. At each instant, the economy in state s transitions into the other state $s' \neq s$ according to a Poisson process. We use these volatility shocks to capture the time variation in the risk premium due to various unmodeled factors (see Section 2 for an illustration of how risk, risk aversion, or beliefs play a similar role in our analysis).

Transition probabilities and belief disagreements. We let $\lambda_s^i > 0$ denote the perceived Poisson transition probability in state s (into the other state) according to investor $i \in I$. These probabilities capture the degree of investors' (relative) optimism or pessimism. For instance, greater λ_2^i corresponds to greater optimism because it implies the investor expects the current high-risk-premium conditions to end relatively soon. Likewise, smaller λ_1^i corresponds to greater optimism because it implies the investor expects the current low-risk-premium conditions to persist longer. We set up the model for investors with heterogeneous beliefs (and in fact, this is the only exogenous source of heterogeneity). We first analyze the special case with common beliefs (Section 4) and then investigate belief disagreements and speculation (Section 5). When investors disagree, they have dogmatic beliefs (formally, investors know each others' beliefs and they agree to disagree).

Menu of financial assets. There are three types of financial assets. First, there is a market portfolio that represents a claim on all output (which accrues to production firms as earnings as we describe later). We let $Q_{t,s}k_{t,s}$ denote the price of the market portfolio, so $Q_{t,s}$ denotes the price per unit of capital. We let $r_{t,s}^m$ denote the instantaneous expected return on the market portfolio

[4] Note that fluctuations in $k_{t,s}$ generate fluctuations in potential output, $Ak_{t,s}$. We introduce Brownian shocks to capital, $k_{t,s}$, as opposed to total factor productivity, A, since this leads to a slightly more tractable analysis when we extend the model to include investment (see Appendix D). In the main text, we could equivalently introduce the shocks to A and conduct the analysis by normalizing all relevant variables with $A_{t,s}$ as opposed to $k_{t,s}$.

conditional on no transition. Second, there is a risk-free asset in zero net supply. We denote its instantaneous return by $r^f_{t,s}$. Third, in each state s, there is a contingent Arrow-Debreu security that trades at the (endogenous) price $p^{s'}_{t,s}$ and pays 1 unit of the consumption good if the economy transitions into the other state $s' \neq s$. This security is also in zero net supply and it ensures that the financial markets are dynamically complete.

Price and return of the market portfolio. Absent transitions, the price per unit of capital follows an endogenous but deterministic process,[5]

$$\frac{dQ_{t,s}}{Q_{t,s}} = \mu^Q_{t,s} dt \text{ for } s \in \{1,2\}. \tag{15}$$

Combining Eqs. (14) and (15), the price of the market portfolio (conditional on *no* transition) evolves according to,

$$\frac{d\left(Q_{t,s}k_{t,s}\right)}{Q_{t,s}k_{t,s}} = \left(g + \mu^Q_{t,s}\right) dt + \sigma_s dZ_t.$$

This implies that, absent state transitions, the volatility of the market portfolio is given by σ_s, and its expected return is given by,

$$r^m_{t,s} = \frac{y_{t,s}}{Q_{t,s}k_{t,s}} + g + \mu^Q_{t,s}. \tag{16}$$

Here, $y_{t,s}$ denotes the endogenous level of output at time t. The first term captures the "dividend yield" component of return. The second term captures the (expected) capital gain conditional on no transition, which reflects the expected growth of capital as well as of the price per unit of capital.

Eqs. $(15-16)$ describe the prices and returns conditional on *no* state transition. If there is a transition at time t from state s into state $s' \neq s$, then the price per unit of capital jumps from $Q_{t,s}$ to a potentially different level, $Q_{t,s'}$. Therefore, investors that hold the market portfolio experience instantaneous capital gains or losses that are reflected in their portfolio problem.

Consumption and portfolio choice. There is a continuum of investors denoted by $i \in I$, who are identical in all respects except for their beliefs about state transitions, λ^i_s. They continuously make consumption and portfolio allocation decisions. Specifically, at any time t and state s, investor i has some financial wealth denoted by $a^i_{t,s}$. She chooses her consumption rate, $c^i_{t,s}$; the fraction of her wealth to allocate to the market portfolio, $\omega^{m,i}_{t,s}$; and the fraction of her wealth to allocate to the contingent security, $\omega^{s',i}_{t,s}$. The residual fraction, $1 - \omega^{m,i}_{t,s} - \omega^{s',i}_{t,s}$, is invested in the risk-free asset. For analytical tractability, we assume the investor has log utility. The investor then solves a relatively standard portfolio problem that we formally state in Appendix B.1.1.

[5] In general, the price follows a diffusion process and this equation also features an endogenous volatility term, $\sigma^Q_{t,s} dZ_t$. In this model, we have $\sigma^Q_{t,s} = 0$ because we work with complete financial markets, constant elasticity preferences, and no disagreements aside from the probability of state transitions. These features ensure that investors allocate identical portfolio weights to the market portfolio (see Eq. (25) later in the section), which ensures that their relative wealth shares are not influenced by dZ_t. The price per capital can be written as a function of investors' wealth shares so it is also not affected by dZ_t.

Equilibrium in asset markets. Asset markets clear when the total wealth held by investors is equal to the value of the market portfolio before and after the portfolio allocation decisions,

$$\int_I a_{t,s}^i di = Q_{t,s} k_{t,s} \text{ and } \int_I \omega_{t,s}^{m,i} a_{t,s}^i di = Q_{t,s} k_{t,s}. \tag{17}$$

Contingent securities are in zero net supply, which implies,

$$\int_I a_{t,s}^i \omega_{t,s}^{s',i} di = 0. \tag{18}$$

The market clearing condition for the risk-free asset (which is also in zero net supply) holds when conditions (17) and (18) are satisfied.

Nominal rigidities and the equilibrium in goods markets. The supply side of our model features nominal rigidities similar to the standard New Keynesian model. We relegate the details to Appendix B.1.2. There is a continuum of monopolistically competitive production firms that own the capital stock and produce intermediate goods (which are then converted into the final good). For simplicity, these production firms have pre-set nominal prices that never change (see Remark 1 below for the case with partial price flexibility). The firms choose their capital utilization rate, $\eta_{t,s} \in [0,1]$, which leads to output, $y_{t,s} = \eta_{t,s} A k_{t,s}$. We assume firms can increase factor utilization for free until $\eta_{t,s} = 1$ and they cannot increase it beyond this level.

As we show in Appendix B.1.2, these features imply that output is determined by aggregate demand for goods up to the capacity constraint. Combining this with market clearing in goods, output is determined by aggregate consumption (up to the capacity constraint),

$$y_{t,s} = \eta_{t,s} A k_{t,s} = \int_I c_{t,s}^i di, \text{ where } \eta_{t,s} \in [0,1]. \tag{19}$$

Moreover, all output accrues to production firms in the form of earnings.[6] Hence, the market portfolio can be thought of as a claim on all production firms.

Interest rate rigidity and monetary policy. Our assumption that production firms do not change their prices implies that the aggregate nominal price level is fixed. The real risk-free interest rate, then, is equal to the nominal risk-free interest rate, which is determined by the interest rate policy of the monetary authority. We assume there is a lower bound on the nominal interest rate, which we set at zero for convenience,

$$r_{t,s}^f \geq 0. \tag{20}$$

[6] In this model, firms own the capital so the division of earnings in terms of return to capital and monopoly profits is indeterminate. Since there is no investment, this division is inconsequential. When we introduce investment in Appendix D, we make additional assumptions to determine how earnings are divided between return to capital and monopoly profits.

The zero lower bound is motivated by the presence of cash in circulation (which we leave unmodeled for simplicity).

We assume that the interest rate policy aims to replicate the level of output that would obtain without nominal rigidities subject to the constraint in (20). Without nominal rigidities, capital is fully utilized, $\eta_{t,s} = 1$ (see Appendix B.1.2). Thus, we assume that the interest rate policy follows the rule,

$$r^f_{t,s} = \max\left(0, r^{f,*}_{t,s}\right) \text{ for each } t \geq 0 \text{ and } s \in S. \tag{21}$$

Here, $r^{f,*}_{t,s}$ is recursively defined as the (instantaneous) natural interest rate that obtains when $\eta_{t,s} = 1$ and the monetary policy follows the rule in (21) at all future times and states.

Definition 1. *The equilibrium is a collection of processes for allocations, prices, and returns such that capital evolves according to* (14)*, price per unit of capital evolves according to* (15)*, its instantaneous return is given by* (16)*, investors maximize expected utility (cf. Appendix B.1.1), asset markets clear (cf. Eqs.* (17) *and* (18)*), production firms maximize earnings (cf. Appendix B.1.2), goods markets clear (cf. Eq.* (19)*), and the interest rate policy follows the rule in* (21)*.*

Remark 1 (Partial Price Flexibility)**.** *Our assumption of a fixed aggregate nominal price (or inflation) is extreme. However, allowing nominal price flexibility does not necessarily circumvent the bound in* (20)*. In fact, if monetary policy follows an inflation targeting policy regime, then partial price flexibility leads to price deflation during a demand recession. This strengthens the bound in* (20) *and exacerbates the recession (see Werning (2012); Korinek and Simsek (2016); Caballero and Farhi (2017) for further discussion, and Footnote 10 for a discussion of how partial price flexibility would also strengthen our results with belief disagreements).*

In the rest of this section, we provide a partial characterization of the equilibrium.

Investors' optimality conditions. We derive these optimality conditions in Appendix B.1.1. In view of log utility, the investor's consumption is a constant fraction of her wealth,

$$c^i_{t,s} = \rho a^i_{t,s}. \tag{22}$$

Moreover, the investor's weight on the market portfolio is determined by,

$$\omega^{m,i}_{t,s}\sigma_s = \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \lambda^i_s \frac{1/a^i_{t,s'}}{1/a^i_{t,s}} \frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}\right). \tag{23}$$

That is, she invests in the market portfolio up to the point at which the risk of her portfolio (left side) is equal to the "Sharpe ratio" of the market portfolio (right side). This is similar to the optimality condition in the two period model (cf. Eq. (6)) with the difference that the dynamic model also features state transitions. Our notion of the Sharpe ratio accounts for potential revaluation gains or losses from state transitions (the term, $\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}}$) as well as the adjustment of marginal utility in case there is a transition (the term, $\frac{1/a^i_{t,s'}}{1/a^i_{t,s}}$).[7]

Finally, the investor's optimal portfolio allocation to the contingent securities implies,

$$\frac{p^{s'}_{t,s}}{\lambda^i_s} = \frac{1/a^i_{t,s'}}{1/a^i_{t,s}}. \tag{24}$$

The portfolio weight, $\omega^{s',i}_{t,s}$, is implicitly determined as the level that ensures this equality. The investor buys contingent securities until the price-to-(perceived)probability ratio of a state (or the state price) is equal to the investor's relative marginal utility in that state.

Substituting (24) into (23) shows that investors allocate identical portfolio weights to the market portfolio, $\omega^{m,i}_{t,s} = \omega^m_{t,s}$. Intuitively, investors express their differences in beliefs through their holdings of contingent securities. Combining this observation with Eq. (17), we further obtain that, in equilibrium, these identical portfolio weights are equal to one,

$$\omega^{m,i}_{t,s} = 1 \text{ for each } i. \tag{25}$$

Output-asset price relation. We next show that there is a tight relationship between output and asset prices as in the two period model. Combining Eqs. (22) and (17) implies that aggregate consumption is a constant fraction of aggregate wealth,

$$\int_I c^i_{t,s} di = \rho Q_{t,s} k_{t,s}. \tag{26}$$

Combining this with Eq. (19), we obtain the output-asset price relation,

$$A\eta_{t,s} = \rho Q_{t,s}. \tag{27}$$

As before, full factor utilization, $\eta_{t,s} = 1$, obtains only if the price per unit of capital is at a particular level $Q^* \equiv A/\rho$. This is the efficient price level that ensures the implied consumption clears the goods market. Likewise, the economy features a demand recession, $\eta_{t,s} < 1$, if and only if the price per unit of capital is strictly below Q^*.

Using the output-asset price relation (and $y_{t,s} = A\eta_{t,s} k_{t,s}$), we can rewrite Eq. (16) as,

$$r^m_{t,s} = \rho + g + \mu^Q_{t,s}. \tag{28}$$

In equilibrium, the dividend yield on the market portfolio is equal to the consumption rate ρ.

Combining the output-asset price relation with the interest rate policy in (21), we also summa-

[7] The presence of state transitions makes the Sharpe ratio in our model slightly different than its common definition, which corresponds to the expected return in excess of the risk-free rate normalized by volatility.

rize the goods market with,

$$Q_{t,s} \leq Q^*, r^f_{t,s} \geq 0, \text{ where at least one condition is an equality.} \tag{29}$$

In particular, the equilibrium at any time and state takes one of two forms. If the natural interest rate is nonnegative, then the interest rate policy ensures that the price per unit of capital is at the efficient level, $Q_{t,s} = Q^*$, capital is fully utilized, $\eta_{t,s} = 1$, and output is equal to its potential, $y_{t,s} = Ak_{t,s}$. Otherwise, the interest rate is constrained, $r^f_{t,s} = 0$, the price is at a lower level, $Q_{t,s} < Q^*$, and output is determined by aggregate demand according to Eq. (27).

For future reference, we also characterize the first-best equilibrium without interest rate rigidities. In this case, there is no lower bound constraint on the interest rate, so the price per unit of capital is at its efficient level at all times and states, $Q_{t,s} = Q^*$. Combining this with Eq. (28), we obtain $r^m_{t,s} = \rho + g$. Substituting this into Eq. (23) and using Eq. (25), we solve for "rstar" as,

$$r^{f*}_s = \rho + g - \sigma^2_s \text{ for each } s \in \{1, 2\}. \tag{30}$$

Hence, in the first-best equilibrium the risk premium shocks are fully absorbed by the interest rate. Next, we characterize the equilibrium with interest rate rigidities.

4. Common beliefs benchmark and amplification

In this section, we analyze the equilibrium in a benchmark case in which all investors share the same belief. That is, $\lambda^i_s \equiv \lambda_s$ for each i. We also normalize the total mass of investors to one so that individual and aggregate allocations are the same. We use this benchmark to illustrate how the spirals between asset prices and output exacerbate the recession.

Because the model is linear, we conjecture that the price and the interest rate will remain constant within states, $Q_{t,s} = Q_s$ and $r^f_{t,s} = r^f_s$ (in particular, there is no price drift, $\mu^Q_{t,s} = 0$). Since the investors are identical, we also have $\omega^m_{t,s} = 1$ and $\omega^{s'}_{t,s} = 0$. In particular, the representative investor's wealth is equal to aggregate wealth, $a_{t,s} = Q_{t,s}k_{t,s}$. Combining this with Eq. (23) and substituting for $r^m_{t,s}$ from Eq. (28), we obtain the following risk balance conditions,

$$\sigma_s = \frac{\rho + g + \lambda_s \left(1 - \frac{Q_s}{Q_{s'}}\right) - r^f_s}{\sigma_s} \text{ for each } s \in \{1, 2\}. \tag{31}$$

These equations are the dynamic counterpart to Eq. (7) in the two-period model. They say that, in each state, the total risk in the economy (the left side) is equal to the Sharpe ratio perceived by the representative investor (the right side). Note that the Sharpe ratio accounts for the fact that the aggregate wealth (as well as the marginal utility) will change if there is a state transition.[8]

[8] To see this, observe that the term, $\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s'}}$, in the equation is actually equal to, $\frac{Q_{t,s}}{Q_{t,s'}} \frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}$. Here, $\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}$ denotes the capital gains and $\frac{Q_{t,s}}{Q_{t,s'}}$ denotes the marginal utility adjustment when there is a representative investor

The equilibrium is then characterized by finding four unknowns, $\left(Q_1, r_1^f, Q_2, r_2^f\right)$, that solve the two equations (31) together with the two goods market equilibrium conditions (29). We solve these equations under the following parametric restriction.

Assumption 1. $\sigma_2^2 > \rho + g > \sigma_1^2$.

In view of this restriction, we conjecture an equilibrium in which the low-risk-premium state 1 features positive interest rates, efficient asset prices, and full factor utilization, $r_1^f > 0, Q_1 = Q^*$ and $\eta_1 = 1$, whereas the high-risk-premium state 2 features zero interest rates, lower asset prices, and imperfect factor utilization, $r_2^f = 0, Q_2 < Q^*$ and $\eta_2 < 1$. In particular, the analysis with common beliefs reduces to finding two unknowns, $\left(Q_2, r_1^f\right)$, that solve the two risk balance equations (31) (after substituting $Q_1 = Q^*$ and $r_2^f = 0$).

Equilibrium in the high-risk-premium state. After substituting $r_2^f = 0$, the risk balance equation (31) for the high-risk-premium state $s = 2$ can be written as,

$$\sigma_2 = \frac{\rho + g + \lambda_2 \left(1 - \frac{Q_2}{Q^*}\right)}{\sigma_2}. \tag{32}$$

In view of Assumption 1, if the price were at its efficient level, $Q_2 = Q^*$, the risk (the left side) would exceed the Sharpe ratio (the right side). As in the two period model, the economy generates too much risk relative to what the investors are willing to absorb at the constrained level of the interest rate. As before, the price per unit of capital, Q_2, needs to decline to equilibrate the risk markets. Rearranging the expression, we obtain a closed form solution,

$$Q_2 = Q^* \left(1 - \frac{\sigma_2^2 - (\rho + g)}{\lambda_2}\right). \tag{33}$$

As this expression illustrates, we require a minimum degree of optimism to ensure an equilibrium with positive price and output.

Assumption 2. $\lambda_2 > \sigma_2^2 - (\rho + g)$.

This requirement is a manifestation of an amplification mechanism that we describe next.

Amplification from endogenous output and earnings. In the two period model of Section 2, the return on the market portfolio is $r^m(z_1) = z_1/Q$ [cf. Eq. (2)], which is decreasing in Q, whereas in the current model the expected return on the market portfolio absent state transitions is, $r^m = \rho + g$ [cf. Eq. (28)], which is constant. Hence, a decline in asset prices does not help to increase the market return any more (aside from state transitions). To see why this happens, note that the dividend yield term in Eq. (32) can be rewritten as $\frac{y_{t,2}}{Q_2 k_{t,2}}$, where $y_{t,2} = \rho Q_2 k_{t,2}$ [cf. (16)]. This illustrates that, if output (and firms' earnings) remained at the first-best level, $y_{t,2}^* = \rho Q^* k_{t,2}$,

(see (23)).

then a decline in the price per unit of capital would increase the dividend yield as well as the return on the market portfolio—a stabilizing force as in the two-period model. However, output in this setting is not constant and is increasing in the current price per unit of capital. A lower price reduces output and economic activity, which reduces firms' earnings and leaves the dividend yield constant. Hence, the output-asset price relation overturns an important stabilizing force from price declines and opens the door for amplification of these declines.

In view of this amplification mechanism, one might wonder how the risk market ever reaches equilibrium once the price, Q_2, starts to fall below its efficient level, Q^*. The stabilizing force is captured by the last term in Eq. (32), $\lambda_2 \left(1 - \frac{Q_2}{Q^*}\right)$. A decline in the price increases the expected capital gain from transition into the recovery state $s = 1$, which increases the expected return to capital as well as the Sharpe ratio. Note that the stabilizing force is stronger when investors are more optimistic and perceive a higher transition probability into the recovery state, λ_2. Assumption 2 ensures that the stabilizing force is sufficiently strong to counter the impact of the risk premium shock. If this assumption were violated, a risk premium shock would trigger a downward price spiral that would lead to an equilibrium with zero asset prices and zero output.

Finally, consider the comparative statics of the equilibrium price with respect to the exogenous shifter of the risk premium, σ_2^2 [cf. (30)]. Using Eq. (33), we obtain $\frac{dQ_2}{d\sigma_2^2} = -\frac{1}{\lambda_2}$. Hence, risk premium shocks reduce asset prices (and output) by a greater magnitude when investors are more pessimistic about recovery (lower λ_2). These observations illustrate that beliefs matter in this environment not only because they have a direct impact on asset prices but also because they determine the strength of the amplification mechanism.

Equilibrium in the low-risk-premium state. Following similar steps for the low-risk-premium state $s = 1$, we also obtain a closed form solution for the interest rate in this state,

$$r_1^f = \rho + g - \sigma_1^2 - \lambda_1 \left(\frac{Q^*}{Q_2} - 1\right). \tag{34}$$

Intuitively, given the expected return on capital, the interest rate adjusts to ensure that the risk balance condition is satisfied with the efficient price level, $Q_1 = Q^*$. For our conjectured equilibrium, we also assume an upper bound on λ_1 which ensures that the implied interest rate is positive.

Assumption 3. $\lambda_1 < \left(\rho + g - \sigma_1^2\right) / \left(Q^*/Q_2 - 1\right)$, where Q^*/Q_2 is given by Eq. (33).

Note also that Eq. (34) implies r_1^f is decreasing in the transition probability, λ_1, as well as in the asset price drop conditional on transition, Q^*/Q_2. Intuitively, interest rates are kept relatively low by the fact that investors fear a recession triggered by an increase in the risk premium and constrained interest rate (an endogenous "disaster").

The following result summarizes the characterization of equilibrium in this section. The testable predictions regarding the effect of risk premium shocks on consumption and output follow from combining the characterization with Eqs. (26) and (27).

Proposition 1. *Consider the model with two states, $s \in \{1, 2\}$, with common beliefs and Assumptions 1-3. The low-risk-premium state 1 features a positive interest rate, efficient asset prices and full factor utilization, $r_1^f > 0, Q_1 = Q^*$ and $\eta_1 = 1$. The high-risk-premium state 2 features zero interest rate, lower asset prices, and a demand-driven recession, $r_2^f = 0, Q_2 < Q^*$, and $\eta_2 < 1$, as well as a lower level of consumption and output, $c_{t,2}/k_{t,2} = y_{t,2}/k_{t,2} = \rho Q_2$. The price in state 2 and the interest rate in state 1 are given by Eqs. (33) and (34).*

Equilibrium with investment and endogenous growth. In Appendix D.1, we extend the baseline environment to incorporate investment. This leads to two main changes. First, the growth rate in (14) becomes endogenous, $g_{t,s} = \varphi(\iota_{t,s}) - \delta$, where $\iota_{t,s} = \frac{i_{t,s}}{k_{t,s}}$ denotes investment rate per capital, $\varphi(\cdot)$ denotes a neoclassical production technology for capital, and δ denotes the depreciation rate. Second, under the simplifying assumption that output accrues to agents in the form of return to capital (i.e., no monopoly profits), optimal investment is an increasing function of the price per unit of capital, $Q_{t,s}$.[9] Moreover, using a convenient functional form for $\varphi(\cdot)$, we obtain a linear relation between the investment rate and the price, $\iota(Q_{t,s}) = \psi(Q_{t,s} - 1)$ for some $\psi > 0$.

In this setting, aggregate demand consists of the sum of consumption and investment. Using the expression for optimal investment, we also generalize the output-asset price relation (27) to,

$$A\eta_{t,s} = \rho Q_{t,s} + \psi(Q_{t,s} - 1). \tag{35}$$

Hence, output is increasing in asset prices not only because asset prices generate a wealth effect on consumption but also because they increase investment through a marginal-Q channel. Substituting optimal investment into the endogenous growth expression, we further obtain,

$$g_{t,s} = \psi q_{t,s} - \delta, \text{ where } q_{t,s} = \log Q_{t,s}. \tag{36}$$

Hence, this setting also features a *growth-asset price relation*: lower asset prices reduce investment, which translates into lower endogenous growth and lower potential output in future periods. The rest of the model is unchanged (see Appendix D.1 for details).

In Appendix D.2, we characterize the equilibrium in this extended environment and generalize Proposition 1. We find that risk premium shocks—captured by a transition to state 2—generate a decline in investment (and endogenous growth) as well as consumption and output as in the baseline version of the model. We test these predictions in Section 7. We also find that the decline in investment generates a second amplification mechanism that reinforces the mechanism we described earlier. Specifically, the recession lowers asset prices further not only by reducing output and earnings but also by reducing investment and growth (in potential output and earnings). Figure 1 in the introduction presents a graphical illustration of the two amplification mechanisms.

[9] Without this assumption, investment would be a function of $\tilde{Q}_{t,s} \leq Q_{t,s}$, which represents a claim on the rental rate of capital in future periods (excluding monopoly profits). The difference, $Q_{t,s} - \tilde{Q}_{t,s}$, captures the price of a claim on monopoly profits. Hence, allowing for profits would have a quantitative impact on investment, though we believe it would leave our qualitative results unchanged. We leave an investigation of this issue for future research.

5. Belief disagreements and speculation

Going back to the baseline model, we next investigate the effect of belief disagreements. We show that *speculation* induced by belief disagreements exacerbates recessions and motivates macroprudential policy.

We restrict attention to two types of investors, optimists and pessimists, with beliefs denoted by, $\left\{\left(\lambda_1^i, \lambda_2^i\right)\right\}_{i\in\{o,p\}}$. We normalize the mass of each belief type to one so that $i = o$ and $i = p$ denote, respectively, the representative optimist and pessimist. We assume the beliefs satisfy the following:

Assumption 4. $\lambda_2^o > \lambda_2^p$ and $\lambda_1^o \leq \lambda_1^p$.

When the economy is in the high-risk-premium state, optimists find the transition into the low-risk-premium state relatively likely ($\lambda_2^o > \lambda_2^p$); when the economy is in the low-risk-premium state, optimists find the transition into the high-risk-premium state relatively unlikely ($\lambda_1^o \leq \lambda_1^p$). Hence, optimism and pessimism are *relative*: an optimist is someone who is optimistic relative to a pessimist. In fact, we do not need to specify the "objective distribution" for our theoretical results (including the welfare results). We do, however, need the relative optimism and pessimism to be persistent across the two risk premium states (see Remark 2 at the end of this section).

To characterize the equilibrium, we define the wealth-weighted average transition probability,

$$\overline{\lambda}_{t,s} \equiv \overline{\lambda}_s\left(\alpha_{t,s}\right) \equiv \alpha_{t,s}\lambda_s^o + \left(1-\alpha_{t,s}\right)\lambda_s^p, \text{ where } \alpha_{t,s}^o = \frac{a_{t,s}^o}{k_{t,s}Q_{t,s}}. \tag{37}$$

Here, $\alpha_{t,s}$ denotes optimists' wealth share, and it is the payoff-relevant state variable in this economy. The notation, $\overline{\lambda}_s\left(\alpha_{t,s}\right)$, describes the wealth-weighted average belief in state s as a function of optimists' wealth share, and $\overline{\lambda}_{t,s}$ denotes the belief at time t and state s. This belief is central to the analysis because the following analogue of the risk balance condition (31) holds in this setting (see Appendix B.3),

$$\sigma_s = \frac{1}{\sigma_s}\left(\rho + g + \mu_{t,s}^Q + \overline{\lambda}_{t,s}\left(1 - \frac{Q_{t,s}}{Q_{t,s'}}\right) - r_{t,s}^f\right) \text{ for each } s \in \{1,2\}. \tag{38}$$

In particular, the equilibrium in risk markets is determined according to the wealth-weighted average belief. When $\alpha_{t,s}$ is greater, optimists exert a greater influence on asset prices. Note also that the expected return to the market portfolio features the price drift term, $\mu_{t,s}^Q$ [cf. (28)], which is not necessarily zero in this section because optimists' wealth share is nonstationary.

We must now characterize the dynamics of optimists' wealth share, $\alpha_{t,s}$ (and thus, the dynamics of $\overline{\lambda}_{t,s}$). Eq. (25) implies investors' weights on the market portfolio satisfy $\omega_{t,s}^{m,o} = \omega_{t,s}^{m,p} = 1$. In Appendix B.3, we also solve for investors' weights on the contingent securities,

$$\omega_{t,s}^{s',o} = \lambda_s^o - \overline{\lambda}_{t,s} = \left(\lambda_s^o - \lambda_s^p\right)\left(1-\alpha_{t,s}\right). \tag{39}$$

Thus, investors settle their disagreements on the jump risk by trading the contingent securities.

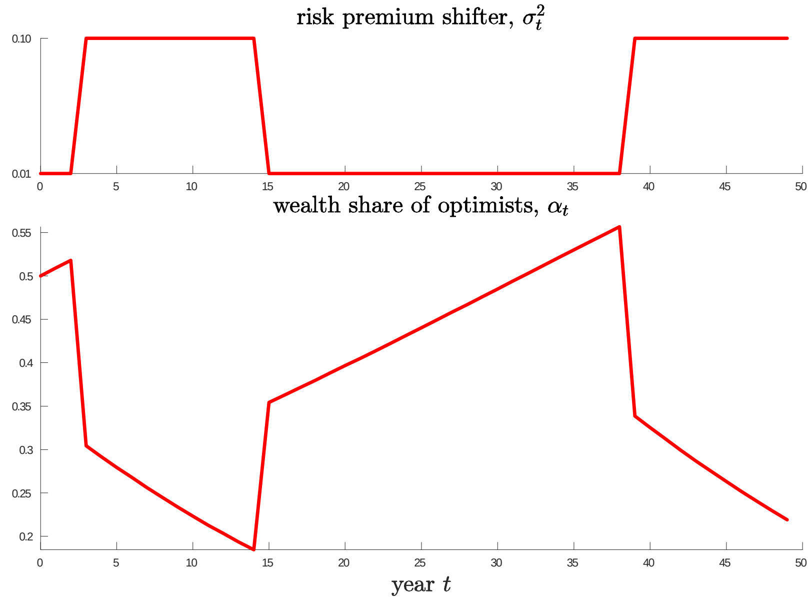

Figure 2: A simulation of the dynamics of optimists' wealth share over time.

Optimists take a positive position on a contingent security whenever their belief for the transition probability exceeds the weighted average belief. This implies that their wealth share evolves according to [cf. Eqs. $(B.13)$ and $(B.14)$],

$$\begin{cases} \dot{\alpha}_{t,s} = \left(\lambda_s^p - \lambda_s^o\right)\alpha_{t,s}\left(1-\alpha_{t,s}\right), & \text{if there is no state change,} \\ \alpha_{t,s'}/\alpha_{t,s} = \lambda_s^o/\overline{\lambda}_{t,s}, & \text{if there is a state change to } s'. \end{cases} \tag{40}$$

Here, $\dot{\alpha}_{t,s} = \frac{d\alpha_{t,s}}{dt}$ denotes the derivative with respect to time. As long as the economy remains in the boom state, optimists' wealth share drifts upwards (since $\lambda_1^o < \lambda_1^p$), because they make profits from selling insurance—contingent contracts that pay in the recession state. If there is a jump to the recession state, optimists' wealth share makes a downward jump. Conversely, optimists' wealth share drifts downwards in the recession state, and it makes an upward jump if there is a transition to the boom state. Figure 2 illustrates the dynamics of optimists' wealth share for a particular parameterization and realization of uncertainty.

These observations also imply that the weighted average belief in (37) (that determines asset prices) is effectively *extrapolative* in the sense that good realizations increase effective optimism whereas bad realizations reduce it. Specifically, as the boom state persists, optimists' wealth share increases and the aggregate belief becomes more optimistic. After a transition to the recession state, the aggregate belief becomes less optimistic. Similarly, the aggregate belief becomes less optimistic as the recession persists, and it becomes more optimistic after a transition into the boom.

Eq. (40) determines the evolution of optimists' wealth share (and thus, the weighted average belief) regardless of the level of asset prices and output. The equilibrium is determined by jointly solving this expression together with the risk balance condition (38) and the goods market equilibrium condition (29). To make progress, we suppose Assumptions 1-3 from the previous section hold according to both belief types. This ensures that, regardless of the wealth shares, the low-risk-premium state 1 features a positive interest rate, efficient price level, and full factor utilization, $r^f_{t,1} > 0, Q_{t,1} = Q^*, \eta_{t,1} = 1$, and the high-risk-premium state 2 features a zero interest rate, a lower price level, and insufficient factor utilization, $r^f_{t,2} = 0, Q_{t,2} < Q^*, \eta_{t,2} < 1$. We next characterize this equilibrium starting with the high-risk-premium state. In this as well as the next section, we also find it convenient to work with the log of the price level, $q_{t,s} \equiv \log Q_{t,s}$.

Equilibrium in the high-risk-premium state. Consider the risk balance equation (38) for state $s = 2$. Using $\mu^Q_{t,2} = \frac{dQ_{t,2}/dt}{Q_{t,2}} = \dot{q}_{t,2}$, we obtain the following analogue of Eq. (32),

$$\sigma_2 = \frac{1}{\sigma_2}\left(\rho + g + \dot{q}_{t,2} + \overline{\lambda}_{t,2}\left(1 - \frac{Q_2}{Q^*}\right)\right). \tag{41}$$

Combining this with Eq. (40), we obtain a differential equation system that describes the joint dynamics of the log price and optimists' wealth share, $(q_{t,2}, \alpha_{t,2})$, conditional on no transition. In Appendix B.3, we show that this system is saddle path stable: for any initial wealth share, $\alpha_{t,2} \in (0,1)$, there exists a unique equilibrium price level, $q_{t,2} \in [q^p_2, q^o_2)$, such that the solution satisfies $\lim_{t\to\infty} \alpha_{t,2} = 0$ and $\lim_{t\to\infty} q_{t,2} = q^p_2$. Here, q^i_2 denotes the log price level with common beliefs characterized in Section 4 corresponding to type i investors' belief. The system is also stationary, which implies that the price can be written as a function of optimists' wealth share. The price function, $q_2(\alpha)$, is characterized as the solution to the following differential equation in α-domain,

$$q'_2(\alpha)(\lambda^o_2 - \lambda^p_2)\alpha(1-\alpha) = \rho + g + \overline{\lambda}_2(\alpha)\left(1 - \frac{\exp(q_2(\alpha))}{Q^*}\right) - \sigma^2_2, \tag{42}$$

with boundary conditions, $q_2(0) = q^p_2$ and $q_2(1) = q^o_2$. We further show that $q_2(\alpha)$ is strictly increasing in α. As in the previous section, greater optimism increases the asset price in the high-risk-premium state. The left panel of Figure 3 illustrates the equilibrium price function for a particular parameterization.[10]

Equilibrium in the low-risk-premium state. Following similar steps for the risk balance condition for the low-risk-premium state $s = 1$, we obtain,

$$r^f_1(\alpha) = \rho + g - \overline{\lambda}_1(\alpha)\left(\frac{Q^*}{\exp(q_2(\alpha'))} - 1\right) - \sigma^2_1 \text{ where } \alpha' = \frac{\alpha\lambda^o_1}{\overline{\lambda}_1(\alpha)}. \tag{43}$$

Here, $r^f_1(\alpha)$ denotes the interest rate when optimists' wealth share is equal to α. The term, α', denotes optimists' wealth share after an immediate transition into the high-risk-premium state [cf.

[10] Introducing partial nominal price flexibility along the lines discussed in Remark 1 would create a second channel by which increasing optimists' wealth share would increase real asset prices. In that environment, pessimists would perceive lower expected inflation than optimists (because they believe the economy is more likely to stay in recession), which would lead to a greater perceived real interest rate and lower real asset valuations.

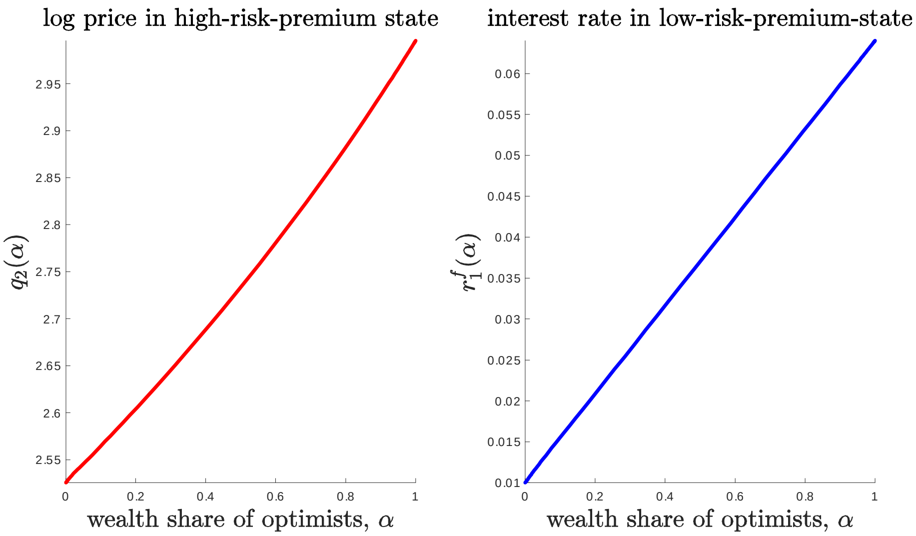

Figure 3: Equilibrium price and interest rate functions with heterogeneous beliefs.

Eq. (40)]. The interest rate depends on (among other things) the weighted average transition probability into the high-risk-premium state, $\overline{\lambda}_1(\alpha)$, as well as the price level that would obtain after transition, $q_2(\alpha')$. It is easy to check that $r_1^f(\alpha)$ is increasing in α, since, as in the previous section, greater optimism increases asset prices. The right panel of Figure 3 illustrates the interest rate function.

The following proposition summarizes the characterization of equilibrium. The last part, which follows by combining the characterization with Eqs. (26) and (27), shows that greater optimists' wealth share in the high-risk-premium state mitigates the severity of the recession.

Proposition 2. *Consider the model with two belief types. Suppose Assumptions 1-3 hold for each belief, and that beliefs are ranked according to Assumption 4. Then, optimists' wealth share evolves according to Eq.* (40)*. The equilibrium log-price and interest rate can be written as a function of optimists' wealth share,* $q_1(\alpha), r_1^f(\alpha), q_2(\alpha), r_2^f(\alpha)$*. In the low-risk-premium state,* $q_1(\alpha) = q^*$*, and* $r_1^f(\alpha)$ *is an increasing function of* α *given by Eq.* (43)*. In the high-risk-premium state,* $r_2^f(\alpha) = 0$*, and* $q_2(\alpha)$ *is an increasing function of* α *that solves the differential equation* (42) *with* $q_2(0) = q_2^p$ *and* $q_2(1) = q_2^o$*. Greater optimists' wealth share in the high-risk-premium state,* $\alpha_{t,2}$*, increases the price per capital,* $Q_{t,2}$*, as well as consumption and output,* $c_{t,2}/k_{t,2} = y_{t,2}/k_{t,2} = \rho Q_{t,2}$*.*

Amplification from speculation. We next illustrate how speculation further amplifies the business-cycle driven by risk premium shocks. To this end, we fix investors' beliefs and simulate the equilibrium for a particular realization of uncertainty over a 50-year horizon. We choose the (objective) simulation belief to be in the "middle" of optimists' and pessimists' beliefs in terms of the relative entropy distance.[11] Figure 4 illustrates the dynamics of equilibrium variables (except

[11] This ensures that there is a non-degenerate long-run wealth distribution in which neither optimists nor pessimists permanently dominate, which helps to visualize the destabilizing effects of speculation without taking a stand on

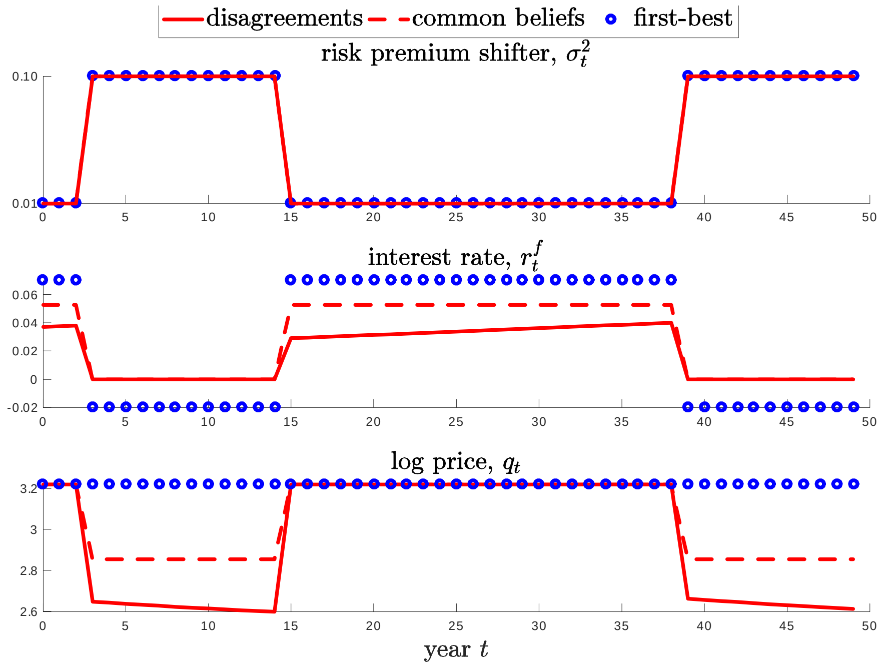

Figure 4: A simulation of the dynamics of equilibrium variables over time with belief disagreements (solid red line), with common beliefs (dashed red line), and the first-best benchmark (circled blue line).

for optimists' wealth share, which we plot in Figure 2). For comparison, the dashed red line plots the equilibrium that would obtain in the common-beliefs benchmark if all investors shared the "middle" simulation belief, and the circled blue line plots the first-best equilibrium that would obtain without interest rate rigidities.

The figure illustrates two points. First, consistent with our baseline analysis in the previous section, the price per unit of capital is more volatile and the interest rate is more compressed than in the first-best equilibrium. In the high-risk-premium state, the interest rate cannot decline sufficiently to equilibrate the risk balance condition, which leads to a drop in asset prices and a demand recession. In the low-risk-premium state, the fear of transition into the recessionary high-risk-premium state keeps the interest rate lower than in the first-best benchmark.

Second, risk-centric recessions are more severe when investors have belief disagreements (and this also leads to more compressed interest rates). The intuition follows from Figures 2 and 3. Speculation in the low-risk-premium state decreases optimists' wealth share once the economy

whether optimists and pessimists are "correct." Our welfare results in the next section do not require this assumption since we evaluate investors' expected utilities according to their own beliefs.

Formally, given two probability distributions $(p(\tilde{s}))_{\tilde{s}\in S}$ and $(q(\tilde{s}))_{\tilde{s}\in S}$, relative entropy of p with respect to q is defined as $\sum_{\tilde{s}} p(\tilde{s}) \log\left(\frac{p(\tilde{s})}{q(\tilde{s})}\right)$. Blume and Easley (2006) show that, in a setting with independent and identically distributed shocks (and identical discount factors), only investors whose beliefs have the maximal relative entropy distance to the true distribution survive. Since our setting features Markov shocks, we apply their result state-by-state to pick the simulation belief that ensures conditional transition probabilities satisfy the necessary survival condition for optimists as well as pessimists.

transitions into the high-risk-premium state, as illustrated by Figure 2, which translates into lower asset prices and a more severe demand recession, as illustrated by Figure 3 and Proposition 2. Speculation also increases optimists' wealth share if the boom continues, but this effect does not translate into higher asset prices or output since it is (optimally) neutralized by the interest rate response. The adverse effects of speculation on demand recessions motivates the analysis of macroprudential policy, which we analyze in the next section.

Remark 2 (Interpretatation of Belief Disagreements). *As this discussion suggests, what matters for our results on speculation is* persistent heterogeneous valuations *for risky assets that ensure: (i) during the boom, high-valuation investors absorb relatively more of the recession risks, and (ii) during the recession, greater wealth share of high-valuation investors increases the (relative) price of risky assets. Belief disagreements generate these features naturally, under the mild assumption that optimists and pessimists do not flip roles across booms and recessions,*[12] *but other sources of heterogeneous valuations would lead to similar results. For example, with heterogeneity in risk aversion, more risk tolerant agents take on more aggregate risk (i.e., they insure less risk tolerant agents), which reduces their wealth share and the (relative) price of risky assets following negative shocks to fundamentals (see, for instance, Garleanu and Pedersen (2011); Longstaff and Wang (2012)). From this perspective, belief disagreements can also capture institutional reasons for heterogeneous valuations such as capacity or mandates for handling risk. Investment banks, for example, have far larger capacity to handle and lever risky positions than pensioners and money market funds.*

6. Welfare analysis and macroprudential policy

Since our model features constrained monetary policy, most of the aggregate demand boosting policies that have been discussed in the New Keynesian literature are also effective in our environment. We skip a discussion of these policies for brevity (our results would still apply as long as these policies are imperfect). Instead, we focus on *macroprudential* policy interventions that impose restrictions on risk market participants, which play a central role in our analysis, with the objective of obtaining macroeconomic benefits. In practice, most macroprudential policies restrict risk taking by banks—especially large ones. Interpreting banks as relatively high-valuation investors (see Remark 2) or as lenders to such investors (see Section 7), we capture these policies in reduced form by imposing portfolio risk limits on relatively optimistic investors.

Formally, using the model with two belief types from the previous section, we characterize investors' value functions in equilibrium. This establishes the determinants of welfare and illustrates the aggregate demand externalities. We then show that macroprudential policy that induces optimists to act more pessimistically (via appropriate portfolio risk limits), but that otherwise does not distort allocations, can generate a *Pareto* improvement of social welfare. We focus on macroprudential policy in the boom (low-risk-premium) state and provide a brief discussion of the

[12] This assumption is supported by an extensive psychology literature that documents the prevalence of optimism, as well as its heterogeneity and persistence, since it is largely a personal trait (see Carver et al. (2010) for a review).

macroprudential policy in the recession (high-risk-premium) state.

Value function in equilibrium. Because the model is linear, investors' expected utility can be written as (see Appendix B.1.1),

$$V^i_{t,s}\left(a^i_{t,s}\right) = \frac{\log\left(a^i_{t,s}/Q_{t,s}\right)}{\rho} + v^i_{t,s}. \tag{44}$$

Here, $v^i_{t,s}$ denotes the normalized value function per unit of capital stock. In Appendix C.1, we further characterize it as the solution to the following differential equation system,

$$\rho v^i_{t,s} - \frac{\partial v^i_{t,s}}{\partial t} = \log\rho + q_{t,s} + \frac{1}{\rho}\left(\begin{array}{c} g - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right)\end{array}\right) + \lambda^i_s\left(v^i_{t,s'} - v^i_{t,s}\right). \tag{45}$$

The equilibrium price, $q_{t,s}$, affects investors' welfare since it determines output and consumption [cf. Eqs. (26) and (27)]. Consumption growth, g, and volatility, σ_s^2, also affect welfare. Finally, speculation affects investors' (perceived) welfare. This is captured by the term, $-\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right)$, which is zero with common beliefs, and strictly positive with disagreements.

Gap value function. To facilitate the policy analysis, we break down the value function into two components,

$$v^i_{t,s} = v^{i,*}_{t,s} + w^i_{t,s}. \tag{46}$$

Here, $v^{i,*}_{t,s}$ denotes *the first-best value function* that would obtain if there were no interest rate rigidities. It is characterized by solving Eq. (45) with the efficient price level, $q_{t,s} = q^*$, for each t, s. The residual, $w^i_{t,s} = v^i_{t,s} - v^{i,*}_{t,s}$, denotes *the gap value function*, which captures the loss of value due to interest rate rigidities and demand recessions. As we will see below, the first-order impact of macroprudential policy on social welfare depends only on the gap value function. Using Eq. (45), we characterize the gap value function as the solution to the following system,

$$\rho w^i_{t,s} = q_{t,s} - q^* + \frac{\partial w^i_{t,s}}{\partial t} + \lambda^i_s\left(w^i_{t,s'} - w^i_{t,s}\right). \tag{47}$$

This illustrates that, in view of the output-asset price relation (27), the gap value function depends on the asset prices relative to the efficient level. Recall also that the equilibrium features $q_{t,1} = q^*$ and $q_{t,2} < q^*$. Thus, the key objective of policy interventions in this environment is to increase the asset price in the high-risk-premium state (so as to mitigate the demand recession).

Aggregate demand externalities. In Appendix C.1, we show that the gap value function can be written as a function of optimists' wealth share, $w^i_s(\alpha)$. Combining Eqs. (47) and (40), we also characterize this function as the solution to the following system in α-domain,

$$\rho w_s^i(\alpha) = q_s(\alpha) - q^* - (\lambda_s^o - \lambda_s^p)\,\alpha(1-\alpha)\frac{\partial w_s^i(\alpha)}{\partial \alpha} + \lambda_s^i\left(w_{s'}^i(\alpha') - w_s^i(\alpha)\right), \tag{48}$$

where $\alpha' = \alpha\lambda_s^o/\overline{\lambda}_s(\alpha)$. Recall that the price function in the high-risk-premium state, $q_2(\alpha)$, is increasing in optimists' wealth share [cf. Figure 3]. This leads to the following result.

Lemma 1. *The gap value function satisfies,* $\frac{dw_s^i(\alpha)}{d\alpha} > 0$ *for each* s, i *and* $\alpha \in (0,1)$.

Intuitively, optimists' wealth share is a scarce resource that brings asset prices and output in the high-risk-premium state closer to its first-best level. Thus, the gap value function in the high-risk-premium state is increasing in optimists' wealth share. The gap value function in the other state is also increasing, because the economy can always transition into the high-risk-premium state, where optimists' wealth share is useful (see Lemma 2 below for a ranking of the marginal value of optimists' wealth share across the two states).

The result also illustrates the *aggregate demand externalities.* Optimists' wealth share is an endogenous variable that fluctuates due to investors' portfolio decisions [cf. Figure 2]. Individual optimists that take positions in contingent markets—and pessimists that take the other side of these positions—do not take into account the impact of their decisions on asset prices and social welfare. This leads to inefficiencies that can be corrected by macroprudential policy.

Equilibrium and gap value functions with macroprudential policy. To evaluate the direction of the inefficiency, we consider a constrained policy exercise where the planner can induce optimists to choose allocations as if they have less optimistic beliefs.[13] Specifically, optimists are constrained to choose allocations *as-if* they have the beliefs, $\lambda^{o,pl} \equiv \left(\lambda_1^{o,pl}, \lambda_2^{o,pl}\right)$, that satisfy, $\lambda_1^{o,pl} \geq \lambda_1^o$ and $\lambda_2^{o,pl} \leq \lambda_2^o$. Pessimists continue to choose allocations according to their own beliefs. Throughout, we use $\lambda_s^{i,pl}$ to denote investors' as-if beliefs and λ_s^i to denote their actual beliefs (for pessimists, the two beliefs coincide). We also use $\overline{\lambda}_s^{pl}(\alpha) = \alpha\lambda_s^{o,pl} + (1-\alpha)\lambda_s^p$ to denote the weighted average as-if belief.

In Appendix C.2, we show that the planner can implement this policy by imposing inequality restrictions on optimists' portfolio weights, while allowing them to make unconstrained consumption-savings decisions. Specifically, when the risk premium is low, the policy constrains optimists from taking too negative a position on the contingent security that pays if there is a transition to the high-risk-premium state, $\omega_{t,1}^{2,o} \geq \underline{\omega}_{t,1}^{2,o}$ (restrictions on selling "put options"). When the risk premium is high, the policy constrains optimists from taking too large a position on the contingent security that pays if there is a transition to the low-risk-premium state, $\omega_{t,2}^{1,o} \leq \overline{\omega}_{t,2}^{1,o}$ (restrictions on buying "call options"). Finally, in either state, the policy also constrains optimists' weight on the market portfolio not to exceed the market average, $\omega_{t,s}^{m,o} \leq 1$ (since otherwise optimists start to speculate by increasing their exposure to the market portfolio).

[13] For simplicity, we restrict attention to time-invariant policies. The planner commits to a policy at time zero, $\left(\lambda_1^{o,pl}, \lambda_2^{o,pl}\right)$, and implements it throughout.

The characterization of equilibrium with policy is then the same as in Section 5. In particular, Eqs. (40) and (41) still hold with the only difference that investors' beliefs are replaced with their as-if beliefs, $\lambda_s^{i,pl}$. We denote the resulting price functions with $q_s^{pl}(\alpha)$ to emphasize that they are determined by as-if beliefs (as opposed to actual beliefs). On the other hand, the equation system that characterizes the gap value function is given by,

$$\rho w_s^i(\alpha) = q_s^{pl}(\alpha) - q^* - \left(\lambda_s^{o,pl} - \lambda_s^p\right)\alpha(1-\alpha)\frac{\partial w_s^i(\alpha)}{\partial \alpha} + \lambda_s^i\left(w_{s'}^i\left(\alpha'^{,pl}\right) - w_s^i(\alpha)\right) \tag{49}$$

where $\alpha'^{,pl} = \alpha\lambda_s^{o,pl}/\overline{\lambda}_s^{o,pl}(\alpha)$. Comparing this with Eq. (48) illustrates that the macroprudential policy can affect the gap value through two potential channels. First, it might affect the equilibrium asset prices (captured by the term, $q_s^{pl}(\alpha)$). Second, the policy affects the dynamics of optimists' wealth share, which in turn influence the gap value. For example, in the low-risk-premium state $s=1$, the policy increases $\lambda_1^{o,pl}$, which induces optimists to increase their position on the contingent security that pays if there is a transition into the high-risk-premium state [cf. Eq. (39)]. This increases optimists' wealth share after a transition (captured by the term, $\alpha'^{,pl}$) at the expense of reducing optimists' wealth share in case there is no transition (captured by the term, $-\left(\lambda_s^{o,pl} - \lambda_s^p\right)$).

Planner's Pareto problem. To trace the Pareto frontier, we allow the planner to make a one-time wealth transfer among the investors at time zero. In Appendix C.2, we show that the planner's Pareto problem can then be reduced to,

$$\max_{\lambda^{o,pl}} v_{0,s}^{pl} = \alpha_{0,s} v_{0,s}^o + (1-\alpha_{0,s}) v_{0,s}^p. \tag{50}$$

Hence, the planner maximizes a wealth-weighted average of investors' normalized values (where the wealth shares correspond to Pareto weights). We also decompose the planner's value function into first-best and gap value components, $v_{0,s}^{pl} = v_{0,s}^{pl,*} + w_{0,s}^{pl}$. A key observation is that, in view of the First Welfare Theorem, the marginal impact of the policy on the planner's first-best value function is zero, $\left.\frac{\partial v_{0,s}^{pl,*}}{\partial \lambda^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} = 0$.[14] Thus, the first order impact of the policy is characterized by its impact on the planner's gap value function,

$$w_{0,s}^{pl} = \alpha_{0,s} w_{0,s}^o + (1-\alpha_{0,s}) w_{0,s}^p. \tag{51}$$

Macroprudential policy in the low-risk-premium state. Now suppose the economy is in the low-risk-premium state $s=1$. The planner can use macroprudential policy in the current state, $\lambda_1^{o,pl} \geq \lambda_1^o$ (she can induce optimists to act as if transition into the recession is more likely), but not in the other state $\lambda_2^{o,pl} = \lambda_2^o$ (she cannot influence optimists' actions in the recession state).

[14] This is because our model features complete markets and no frictions other than interest rate rigidities. Hence, the First Welfare Theorem applies to the first-best allocations that also correct for these rigidities. This implies that the marginal impact on the first-best value must be zero, since otherwise the first-best allocations could be Pareto improved by appropriately changing optimists' as-if beliefs.

Effectively, this policy induces optimists to sell less of the contingent security that pays in case there is a transition to the high-risk-premium state, while also preventing optimists from increasing their position in the market portfolio.

For small changes, this policy does not affect the price function in the current state, $q_1^{pl}(\alpha) = q^*$ (since Assumption 3 continues to hold with as-if beliefs). Hence, the policy affects the gap value only through its impact on optimists' wealth dynamics and the associated aggregate demand externalities. Differentiating Eq. (49) (for $s = 1$) with respect to optimists' as-if beliefs and evaluating at the no-policy benchmark ($\lambda_1^{o,pl} = \lambda_1^o$), we obtain,

$$\left(\rho + \lambda_1^i\right) \frac{\partial w_1^i(\alpha)}{\partial \lambda_1^{o,pl}} = \alpha(1-\alpha)\left[-\frac{\partial w_1^i(\alpha)}{\partial \alpha} + \frac{\lambda_1^i}{\overline{\lambda}_1(\alpha)} \frac{\lambda_1^p}{\overline{\lambda}_1(\alpha)} \frac{\partial w_2^i(\alpha')}{\partial \alpha}\right] + \lambda_1^i \frac{\partial w_2^i(\alpha)}{\partial \lambda_1^{o,pl}}, \tag{52}$$

where $\alpha' = \alpha\lambda_1^o/\overline{\lambda}_1(\alpha)$. Here, the two terms inside the brackets capture the direct impact of the policy on welfare through aggregate demand externalities. The second term illustrates that the policy generates positive aggregate demand externalities—because it increases optimists' wealth share if there is a transition into the high-risk-premium state. On the other hand, the first term illustrates that the policy also generates negative aggregate demand externalities—because it reduces optimists' wealth share in case there is no transition. Eq. (52) describes the balance of these externalities when optimists are required to purchase the contingent security at equilibrium prices.

This illustrates that, in a dynamic setting, macroprudential policy in the low-risk-premium state is associated with some costs as well as benefits. The costs emerge from the fact that the policy prevents optimists from accumulating wealth that could be useful in a *future recession*. However, intuition suggests the benefits should outweigh the costs as long as future recessions are not too different from an imminent recession. The following lemma verifies this for the special case, $\lambda_1^o = \lambda_1^p$.

Lemma 2. *When $\lambda_1^o = \lambda_1^p$, the gap value function satisfies $\frac{dw_2^i(\alpha')}{d\alpha} > \frac{dw_1^i(\alpha)}{d\alpha}$ for each i and $\alpha \in (0,1)$.*

That is, optimists' wealth share increases the gap value more when there is an immediate transition into the high-risk-premium state, in which case the benefits appear immediately. Any delay in such transition reduces the benefits by postponing them. Combining this lemma with Eq. (52) provides a heuristic derivation of our main result in this section (see Appendix C.2 for the proof).

Proposition 3. *Consider the model with two belief types that satisfy $\lambda_1^o = \lambda_1^p$. Consider the macroprudential policy in the boom state, $\lambda_1^{o,pl} \geq \lambda_1^o$ (and suppose $\lambda_2^{o,pl} = \lambda_2^o$). The policy increases the planner's gap value (and thus, also the total value),*

$$\left.\frac{\partial v_1^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda_1^{o,pl}=\lambda_1^o} = \left.\frac{\partial w_1^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda_1^{o,pl}=\lambda_1^o} > 0 \text{ for each } \alpha \in (0,1).$$

In particular, regardless of the planner's Pareto weight, there exists a Pareto improving macroprudential policy.

What happens when we relax the assumption, $\lambda_1^o = \lambda_1^p$? This is largely a technical assumption. We conjecture that Proposition 3 also holds when $\lambda_1^o < \lambda_1^p$ (under appropriate technical assumptions) but we are unable to provide a proof. There are two distinct challenges. First, we cannot generalize Lemma 2, although the ranking is intuitive and should hold unless there are strong nonlinearities in the gap value function.[15] Second, in the more general case pessimists and optimists disagree about the benefits of macroprudential policy (captured by λ_1^i in the bracketed terms of (52)). The planner takes a weighted average of these perceptions, which complicates the analysis.[16] These challenges notwithstanding, we have not yet encountered a counterexample in our numerical simulations.

Figure 5 illustrates the result for our earlier parameterization (that features $\lambda_1^o < \lambda_1^p$). We fix the optimists' wealth share at a particular level ($\alpha = \frac{1}{2}$) and calculate the effect of macroprudential policy on the planner's value function as well as on its components. The policy reduces the planner's first-best value function, since it distorts investors' allocations according to their own beliefs. However, the magnitude of this decline is small (due to the First Welfare Theorem). The policy also generates a relatively sizeable increase in the planner's gap value function. This increase is sufficiently large that the policy increases the actual value function and generates a Pareto improvement. As the policy becomes larger, the gap value continues to increase whereas the first-best value decreases. Moreover, the decline in the first-best value is negligible for small policy changes but it becomes sizeable for large policy changes. The (constrained) optimal macroprudential policy obtains at an intermediate level.

The result is reminiscent of the analysis in Korinek and Simsek (2016), in which macroprudential policy improves outcomes by increasing the wealth of high marginal propensity to consume (MPC) households when there is a demand-driven recession. While both results are driven by aggregate demand externalities, the mechanism here is different and operates via asset prices. In fact, in our setting, all investors have the same MPC equal to ρ. Optimists improve aggregate demand in the high-risk-premium state not because they spend more than pessimists, but because they increase asset prices and induce *all investors* to spend more.

Macroprudential policy in the high-risk-premium state. The analysis so far concerns macroprudential policy in the low-risk-premium state and maintains the assumption that $\lambda_2^{o,pl} = \lambda_2^o$.

[15] Specifically, the proof of Lemma 2 establishes,

$$\frac{\partial w_1^i\left(b_{0,1}\right)}{\partial b} = \frac{\lambda_1^i}{\lambda_1^i + \rho} \int_0^\infty e^{-\left(\rho + \lambda_1^i\right)t} \left(\rho + \lambda_1^i\right) \frac{\partial w_2^i\left(b_{t,2}\right)}{\partial b} dt,$$

where $b_{0,1}$ denotes a transformed version of α at the initial state, and $b_{t,2}$ denotes the same variable after a transition into the high-risk-premium state after a period of length t. When $\lambda_1^o = \lambda_1^p$, we also have $b_{t,2} = b_{0,1}$ (since there is no speculation in the low-risk state), which yields $\frac{\partial w_1^i(b_{0,1})}{\partial b} = \frac{\lambda_1^i}{\lambda_1^i+\rho} \frac{\partial w_2^i(b_{0,1})}{\partial b} < \frac{\partial w_2^i(b_{0,1})}{\partial b}$. When $\lambda_1^o < \lambda_1^p$, the same result holds and the ranking remains unchanged if the value function is linear in the transformed variable b. Hence, the ranking can fail only if there are sufficiently large nonlinearities in the gap value function.

[16] When $\lambda_1^o = \lambda_1^p$, we actually have the stronger result that $\frac{\partial w_1^i(\alpha)}{\partial \lambda_1^{o,pl}} > 0$ for each i, that is, the policy increases the gap value according to optimists and pessimists (see Eq. (C.18)). We state the weaker version of the result in Proposition 3 because the stronger version might conceivably fail according to optimists (e.g., if λ_1^o is close to zero).

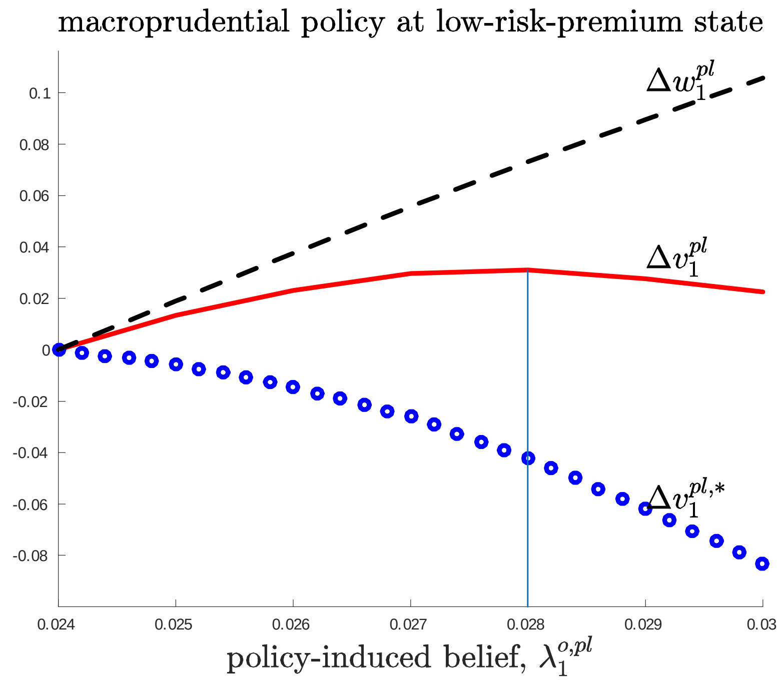

Figure 5: Effect of macroprudential policy in the low-risk-premium state on the planner's value function and its components.

In Appendix C.2, we also analyze the polar opposite case when the economy is in the high-risk-premium state $s = 2$, and the planner can apply macroprudential policy in this state, $\lambda_2^{o,pl} \leq \lambda_2^o$ (she can induce optimists to act as if the recovery is less likely), but not in the other state, $\lambda_1^{o,pl} = \lambda_1^o$. Proposition 4 in the appendix shows that, in contrast to Proposition 3, this policy can reduce social welfare. Consider the two counteracting forces. First, similar to before, macroprudential policy increases the gap value by increasing optimists' wealth share if the economy stays at the high-risk-premium state. However, unlike before, macroprudential policy also reduces current asset prices because the price is below the first-best level, $q_2^{pl}(\alpha) < q^*$, and it is increasing in optimists' as-if optimism, $\lambda_2^{o,pl}$ (see Eq. (33)). This channel reduces the gap value function (see Eq. (49)). When optimists' wealth share is large ($\alpha \to 1$), the latter channel is dominant and macroprudential policy reduces social welfare. Even when the latter channel does not dominate, it suggests that the macroprudential policy in the recession state is less useful than in the boom state (which we verify in numerical simulations).

It is useful to emphasize that macroprudential policy in the low-risk-premium state does not lower asset prices due to the monetary policy response. Specifically, while the asset price in this state is not influenced by policy, $q_1^{pl}(\alpha) = q^*$, the interest rate, $r_1^f(\alpha)$, is decreasing in optimists' as-if pessimism, $\lambda_1^{o,pl}$ (see Eq. (34)). Intuitively, as macroprudential policy reduces the demand for risky assets, monetary policy lowers the interest rate to dampen its effect on asset prices and aggregate demand.

Taken together, our analysis provides support for *procyclical* macroprudential policy. In states where output is not demand constrained (in our model, the boom state $s = 1$), macroprudential policy that restricts high-valuation investors' (in our model optimists') risk taking is desirable. This policy improves welfare by ensuring that high-valuation investors bring more wealth to the demand-constrained states, which increases asset prices and output. In states where output is demand constrained (in our model, the recession state $s = 2$), macroprudential policy is less useful because it has an immediate negative impact on asset prices and aggregate demand.

7. Empirical evidence

Our empirical analysis focuses on three predictions. First, our model predicts that risk premium shocks generate an interest rate reduction when the interest rate is not constrained, and a more severe demand recession when the interest rate is constrained. Second, the recession reduces firms' earnings and leads to a further reduction in asset prices. Third, the recession is more severe when the shock takes place in an environment with more speculation. To test these predictions, we compare the response to house price shocks in Eurozone countries (which have constrained interest rate with respect to national shocks) to the response in non-Eurozone developed countries (which have less constrained interest rate). At the end of the section, we discuss empirical evidence from the recent literature which suggests that similar results apply for price shocks to other asset classes, such as stocks, as well as for other constraints on the interest rate, such as the zero lower bound.

While our model relies on the zero lower bound constraint, the mechanisms are more general, and we find it more convenient to work with the currency-union constraint in our empirical analysis. The zero lower bound has only recently become a practical constraint, generating data limitations, and it calls for an asymmetric specification that requires separate responses to positive and negative price shocks (since the monetary policy can raise the interest rate in response to positive shocks, especially if the economy is close to full capacity utilization). In contrast, individual Eurozone countries have had constrained interest rates (with respect to national shocks) for much longer, and the constraint has been symmetric with respect to the direction of shocks.

A major challenge in this exercise is the identification of the risk premium shock that drives asset prices. As we clarify in Section 2, the exact source of the shock is not important for our mechanisms (e.g., risk, risk aversion, or beliefs have similar effects). Therefore, our strategy is to control for factors that *do not* act as a risk premium shock according to our model. In particular, we attempt to control for supply shocks and monetary policy shocks, and interpret the residual change in asset prices as a plausibly exogenous risk premium shock.

Our model has a single type of capital, which can be interpreted as a value-weighted average of housing, stocks, and other assets. We focus on house prices for two reasons. First, housing wealth is large and its size (relative to output) is comparable between Eurozone and non-Eurozone developed countries (see Table 3 in Appendix E). In contrast, stock markets in Eurozone countries are typically much smaller than in non-Eurozone developed countries, which makes stocks less suitable for our empirical strategy (see Table 4). Second, house prices are less volatile and seem to react to news with some delay (see Figure 15 in Appendix E). These features make it easier to control for other drivers of house prices. Specifically, we control for supply shocks using past realizations of output growth, and for monetary policy shocks using past realizations of policy interest rates.[17] We interpret the residual house price change as a risk premium shock. We also interpret the contemporaneous or future changes in interest rates as the monetary policy response to the shock, which enables us to test a key prediction of our model. This strategy works less well for stocks, because stock prices react to monetary policy news quickly, which might create a correlation between prices and interest rates with the opposite sign (since stock price declines driven by policy-news shocks are typically followed by interest rate *hikes*—the opposite of risk premium shocks).[18]

Data sources. We assemble a quarterly cross-country panel data set of financial and economic variables for advanced economies. We obtain data on house price indices from the quarterly dataset described in Mack et al. (2011). We obtain data on macroeconomic activity such as GDP, investment, and consumption from the OECD. We also obtain financial market data such as the policy interest rate, stock price indices, and earnings (of publicly traded firms) from Global Financial Data (GFD) and the Bank for International Settlements (BIS). Appendix E describes the details of data sources and variable construction.

Sample selection. Our sample covers 21 advanced economies from the first quarter of 1990 until the last quarter of 2017. Our selection of countries is driven by the availability of consistent house price data. We start the sample in 1990 because monetary policy in most advanced economies had shifted from focusing on stabilizing inflation to stabilizing output by this time, as in our model. Our results are robust to alternative sample selections.[19]

To capture interest rate constraints, we divide the data into two categories. The first category, which we refer to as *the Euro/ERM sample*, consists of country-quarters in which the country was a member of the Euro area or the European Exchange Rate Mechanism (ERM) for most of the calendar year. The ERM system, which was introduced as a precedent to the Euro, requires the member countries to keep their exchange rates within a narrow band of a central currency. This system constrains countries' relative policy interest rates (albeit imperfectly) and most member

[17] While our controls for supply shocks are imperfect, we also report the differential effects of these shocks in Eurozone countries compared to their effects outside the Eurozone, which provides additional robustness. For example, our model illustrates that permanent supply shocks (e.g., an increase in A) shift asset prices and output regardless of whether the interest rate is constrained (see Sections 3 and 4). This suggests that omitted supply shocks would lead to a similar bias inside and outside the Eurozone that is mitigated by focusing on the differential responses.

[18] Formally, we assume house prices react to monetary policy news with a delay of at least one quarter. Figure 15 in the appendix plots impulse responses to shocks to the policy interest rate and provides support for this assumption. Specifically, a surprise increase in the policy interest rate is followed by a decline in house prices, but the response starts after the first quarter and takes several quarters to complete. The same figure also shows that the assumption is clearly violated for stock prices. A surprise increase in the policy interest rate also reduces stock prices, but all of the response takes place in the same quarter as the shock.

[19] Figures 13 and 14 in the appendix show that starting the sample in 1980 leaves our results (except for the effect on inflation) qualitatively unchanged.

countries eventually adopted the Euro. The countries in the Euro area share the same policy interest rate (determined by the European Central Bank). The second category, which we refer to as *the non-Euro/ERM sample*, consists of the remaining country-quarters. Table 1 in Appendix E describes the Euro/ERM status by country and year.

Empirical specification. To describe how the economy behaves after house price shocks, we follow the local projection method developed by Jordà (2005). In particular, we regress several outcome variables at various horizons after time t on (residual) house price changes at time t. Specifically, we estimate equations of the type,

$$Y^h_{j,t+h} - Y^h_{j,t-1} = \alpha^h_j + \gamma^h_t + \beta^{p,h}\left(-\Delta \log P_{j,t}\right) + \beta^{c,h}\text{controls}_{j,t} + \varepsilon^h_{j,t}, \tag{53}$$

where j denotes the country, t denotes the quarter, h denotes the horizon, Y denotes an outcome variable, P denotes the (real) house price index, and $\Delta \log P_{j,t} = \log P_{j,t} - \log P_{j,t-1}$ denotes its quarterly log change. We include time as well as country fixed effects so our "house price shock" is a decline in house prices in a quarter, after accounting for the average price decline in the sample countries as well as various other controls within the country. Our control variables include 12 lags (3 years) of the first difference of log GDP—to control for supply shocks, and 12 lags of the policy interest rate—to control for recent monetary policy. We also include 12 lags of the first difference of log house prices—to capture the momentum in house prices, and 12 lags of the first difference of the outcome variable—to control for other dynamics that might influence the outcomes. We weight each regression with countries' relative GDP, and estimate (53) for horizons 0 to 12.

To evaluate the responses within and outside the Eurozone, we also include indicator variables for Euro/ERM and non-Euro/ERM status, and we interact all right-hand-side variables (including the fixed effects) with these indicators. We let $\beta^{p,h}_{euro}$ and $\beta^{p,h}_{non}$ denote the coefficient on the interaction of the price shock with the corresponding indicator. Our specification is equivalent to running the regressions separately within the Euro/ERM and non-Euro/ERM samples.[20] We report the sequence of coefficients, $\left\{\beta^{p,h}_{euro}\right\}^{12}_{h=0}$ and $\left\{\beta^{p,h}_{non}\right\}^{12}_{h=0}$, which provide an estimate of the impulse response functions for the respective samples. We also report 95% confidence intervals calculated according to Newey and West (1987) standard errors with a bandwidth of 20 quarters.

Our outcome variables include terms for which our model makes a clear prediction, such as the policy interest rate, the unemployment rate (a proxy for factor underutilization), the logs of GDP, investment, and consumption. We also include the log (core) CPI. Even though it is constant in our model (by assumption), variants of our model predict that it should decline in a demand recession. We also analyze public firms' earnings and log stock prices to investigate spillover and amplification effects, as well as log house prices to investigate the price dynamics following the

[20] The point estimates from our regression are identical to those obtained from running separate regressions within each sample. However, because our standard errors account for autocorrelation of the residuals, the joint regression will have slightly different standard errors (for example, the joint regression will account for the fact that residuals are correlated from before and after Greece joined the ERM). The joint regression is preferable to separate regressions, because it uses more data and thus gives more precise standard errors.

initial shock. All relevant variables except for the policy interest rate are adjusted for inflation to focus on real effects, as in our model. For earnings, we use the ratio of earnings to the initial stock price level as our dependent variable (which helps to obtain meaningful units).[21]

Table 2 in Appendix E describes the summary statistics by Euro/ERM status for the variables that enter our regression analysis. The Euro/ERM sample has 821 country-quarters and the non-Euro/ERM sample has 1120 country-quarters.[22] Both samples are unbalanced because a few countries have imperfect data coverage in earlier years (and because a few countries transition between samples). The two samples are comparable except that the non-Euro/ERM sample experienced slightly faster growth over the sample period.

House price shocks and demand recessions. Figure 6 plots the estimated sequences of coefficients by Euro/ERM status (see Figure 10 in Appendix E for the differenced coefficients). The panels at the top two rows illustrate our main empirical findings. The top left panel shows that, in the non-Euro/ERM sample (dashed blue line), a decline in house prices is followed by a sizeable and persistent decline in the policy interest rate. By contrast, in the Euro/ERM sample (solid red line), a decline in house prices does not lead to an additional decline in the country's interest rate relative to other Euro/ERM countries, illustrating the interest rate constraint.[23] The remaining panels in the top two rows illustrate that the shock is followed by a more severe demand recession in an Euro/ERM country than in a non-Euro/ERM country. In fact, the panels on GDP, investment, and consumption suggest that the shock initially leads to similar effects in both samples but is eventually followed by milder outcomes in the non-Euro/ERM sample.

These results are consistent with our prediction that risk premium shocks lead to a more severe demand recession when the interest rate is constrained. From the lens of our model, the interest rate policy mitigates a demand recession driven by a local risk premium shock outside the Eurozone but not within the Eurozone.[24]

[21] Earnings sometimes take a negative value (e.g., for Greece in recent years) which makes a log transformation problematic. Instead, we change the specification in (53) slightly so that the dependent variable is $(\text{earnings}_{t+h} - \text{earnings}_{t-1})/(\text{stock price}_{t-1})$. Likewise, we adjust the control variables that feature earnings by dividing them with the stock price at quarter $t-1$.

[22] These are the sample sizes for our baseline regression in which the outcome variable is the policy interest rate and the horizon is 0 (see (53)). For some regressions, the sample size is slightly smaller, because we estimate outcomes at future horizons (that removes some data from the end of the sample period) and because some variables do not have complete coverage.

[23] For the Euro era, the Euro/ERM-wide policy interest rate response is common to all countries and is captured by our time-fixed effects. And during the ERM era, there were severe cross-country monetary policy constraints. Figure 12 in Appendix E illustrates the results from the same regression without time-fixed effects. The figure shows that a negative house price shock in the Euro/ERM sample leads to a decline in the Euro/ERM-wide policy interest rate, but the magnitude of this decline is smaller than in the other sample. This is because house price shocks have a national (or idiosyncratic) component, and the Euro/ERM-wide policy interest rate arguably responds only to the Euro/ERM-wide (or systematic) component of these shocks.

[24] In our model, risk premium shocks generate a less severe recession in unconstrained countries because the interest rate policy response leads to a smaller decline in asset prices. This suggests that asset price changes might provide an inaccurate measure of the underlying shock. We believe our analysis is robust to this concern for three reasons. First, to the extent that this concern is relevant, it biases the empirical analysis against finding support for our mechanisms, because it implies that an equivalent magnitude of asset price decline corresponds to a larger underlying shock if the country has unconstrained interest rate. Second, the concern is less relevant in practice than in our model because the

Spillover effects and amplification. The panels at the bottom row of Figure 6 illustrate the effect of the house price shock on asset markets. The panels on earnings and stock prices establish that there are spillover effects to the stock market: specifically, earnings as well as stock prices decline more in the Euro/ERM sample than in the other sample (although the estimates are imprecise due to the high volatility of earnings and prices). The remaining panel illustrates that, after the initial shock, house prices decline more persistently and by a greater magnitude in the Euro/ERM sample.

These results are consistent with our prediction that the demand recession reduces firms' earnings and leads to a further decline in asset prices. From the lens of the model, stock prices (resp. house prices) decline less in the non-Euro/ERM sample due to the interest rate response, which not only increases the price to earnings ratio (resp. price to rent ratio) but also mitigates the recession and supports earnings (resp. rents).[25]

Speculation and further amplification. We need a proxy for speculation to test the final prediction of our model. We choose a measure of bank credit, which is a major catalyst of speculation in housing markets. First, banks can be thought of as the high-valuation investors ("optimists"), because they have a greater capacity and expertise to handle risk relative to non-institutional investors, and they have real estate exposures through mortgage loans. Under this interpretation, bank credit provides a measure of banks' exposure to the housing market. Second, banks also lend to other high-valuation investors in the housing markets such as optimistic homebuyers that use bank credit to purchase larger homes or second homes. When bank credit is easily available, perhaps because of banks' optimism about house prices, these high-valuation investors wield a greater influence in the housing market (see Simsek (2013a) for a formalization). Thus, bank credit provides a broad proxy for speculation in the housing market.

Our specific measure of bank credit comes from Baron and Xiong (2017), who construct a variable, "credit expansion", defined as the change in the bank credit to GDP ratio in the last three years. They standardize the variable by its mean and standard deviation within each country so that the measure is high when bank credit expansion in a country has been high in recent years relative to its historical trends. They show that their standardized measure predicts the likelihood of a large decline in bank equity prices, and despite the elevated risk, it also predicts lower average returns on bank equity. Their preferred interpretation is that bank equity investors are excessively optimistic or neglect crash risk, which in our framework would translate into greater speculation (by banks or their borrowers).

We use the BIS data on bank credit to households and nonfinancial firms to construct a close analogue of Baron and Xiong's standardized credit expansion variable (see Appendix E for details).

interest rate policy affects all assets, which implies that risk-driven price declines in one asset class (such as housing) are partially absorbed by price increases in other asset classes. Third, the concern is also less relevant for house prices because they seem to react to interest rate changes with some delay (see Figure 15 in Appendix E). In fact, the panel of Figure 6 on house prices suggests that the interest rate response only partially stabilizes risk-driven house price changes and with some delay.

[25] We cannot test the predictions on rents because we do not have reliable data.

Figure 6: Results from the regression specification in (53) with the addition of the indicator variables for Euro/ERM and non-Euro/ERM status as well as the interaction of all right-hand-side variables with these indicators. The solid red (resp. dashed blue) lines plot the coefficients corresponding to the the negative log house price variable when the Euro/ERM status is equal to 1 (resp. 0). For the units, "percent" corresponds to 0.01 log units (i.e., it is approximate) and "pp" corresponds to percentage points. All regressions include time and country fixed effects; 12 lags of the first difference of log GDP, 12 lags of the level of the policy interest rate, 12 lags of the first difference of log house prices, and 12 lags of the first difference of the outcome variable. The dotted lines show 95% confidence intervals calculated according to Newey-West standard errors with a bandwidth of 20 quarters. All regressions are weighted by countries' PPP-adjusted GDP in 1990. Data is unbalanced quarterly panel that spans 1990Q1-2017Q4. All variables except for those in the top panel are adjusted for inflation. Earnings are normalized by the stock price at the quarter before the shock (see Footnote 21). The sources and the definitions of variables are described in Appendix E.

We then run the same regressions as in (53), but we also include the interaction of the price shock with standardized credit expansion. That is, we estimate,

$$Y_{j,t+h}^{h} - Y_{j,t-1}^{h} = \alpha_j^h + \gamma_t^h + \left[\begin{array}{c} \beta^{p,h}\left(-\Delta \log P_{j,t}\right) \\ +\beta^{pc,h}\left(-\Delta \log P_{j,t}\right) \times \text{credit expansion-std} \end{array} \right] + \beta^{c,h}\text{controls}_{j,t} + \varepsilon_{j,t}. \tag{54}$$

In addition to the earlier controls, we include 12 lags of standardized credit expansion to capture its direct impact. As before, we also interact all right-hand-side variables with the Euro/ERM and the non-Euro/ERM status indicators. We let $\beta_{euro}^{pc,h}$ and $\beta_{non}^{pc,h}$ denote the coefficient on the interaction of the shock and credit with these indicators. The sequence of coefficients, $\left\{\beta_{euro}^{pc,h}\right\}_{h=0}^{12}$ and $\left\{\beta_{non}^{pc,h}\right\}_{h=0}^{12}$, provide an estimate of the *additional* effect of the shock when credit expansion has been one standard deviation above average (relative to its baseline effect with average credit).

Figure 7 plots these sequences and illustrates our findings (see Figure 11 in the appendix for the differenced coefficients). The panels at the first two rows show that, in the Euro/ERM sample, house price shocks lead to a greater decline in economic activity when credit expansion has been high in recent years. In contrast, credit expansion does not seem to change the effect of the house price shock in the non-Euro/ERM sample. These results support our prediction that risk premium shocks lead to a more severe demand recession (in constrained economies) when they take place in an environment with elevated speculation.

On the other hand, the panels at the bottom row of Figure 7 present largely inconclusive results that do not necessarily support (or refute) our predictions. We do not find meaningful differences for the additional effect of house price shocks on earnings or house prices when credit expansion has been high (in either sample). We do find a negative effect on stock prices for the Euro/ERM sample, but the effect is not statistically significantly different from the other sample. That said, since standard errors are large, we cannot rule out sizeable effects either. Hence, while we tentatively conclude that speculation proxied by credit expansion is associated with deeper risk-centric demand recessions, further empirical research should verify the robustness of this conclusion as well as the precise channels by which speculation affects the recession.

Other supporting evidence. Our mechanisms are supported by a growing empirical literature. Our empirical analysis is related to Mian and Sufi (2014), who use regional data within the U.S. to argue that house price declines explain much of the job losses during the Great Recession. Our results for the Euro/ERM sample suggest that similar results hold in cross-country data. Our results for the non-Euro/ERM sample suggest that monetary policy can mitigate the adverse effects of house price shocks. Moreover, while Mian and Sufi (2014) emphasize household deleveraging as the key channel by which house price declines cause damage, some of our empirical results (e.g., the investment response) suggest this is unlikely to be the only relevant mechanism. As our model demonstrates, house price declines could lower aggregate demand even without household deleveraging or other financial frictions—although these additional ingredients would naturally amplify the effects.

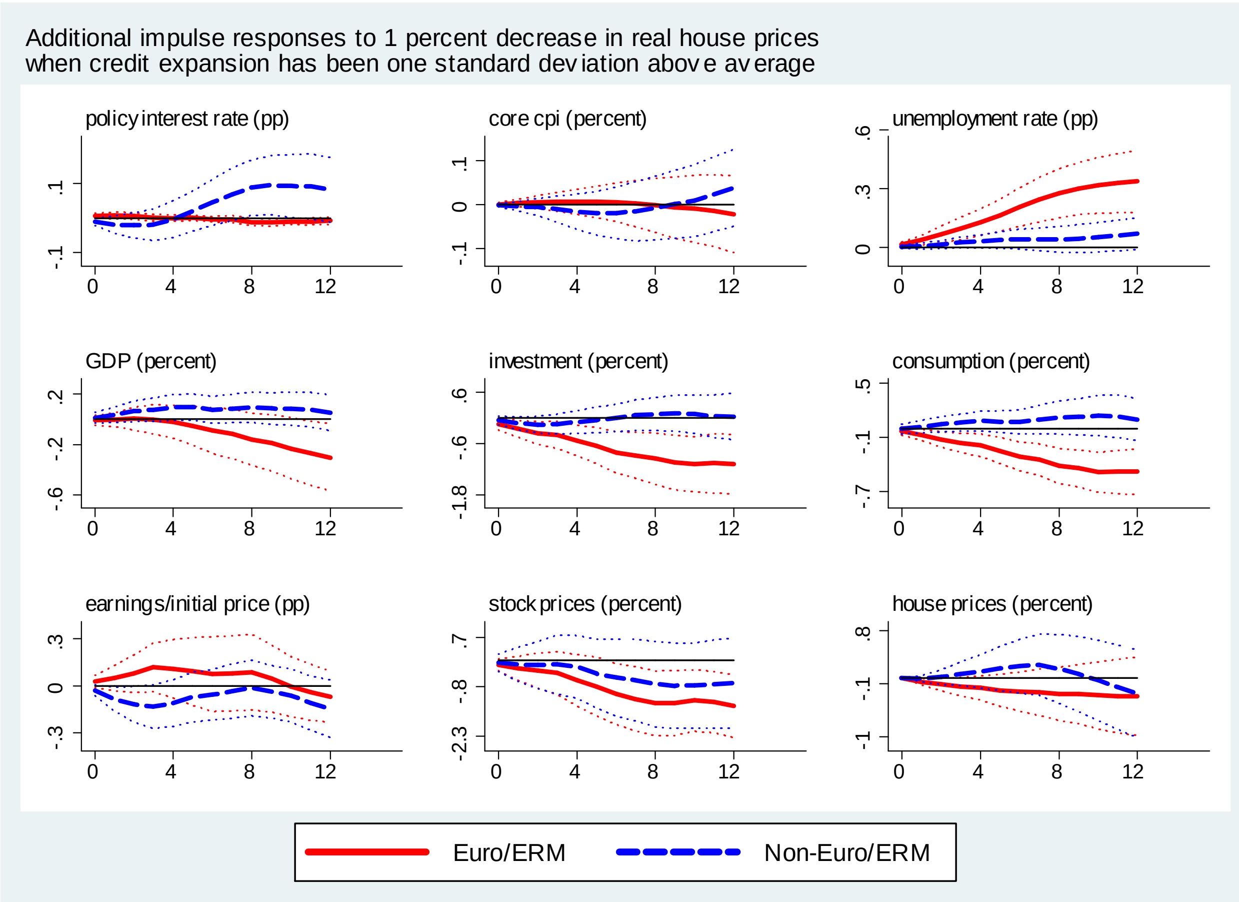

Figure 7: Results from the regression specification in (53) with the addition of the indicator variables for Euro/ERM and non-Euro/ERM status as well as the interaction of all right-hand-side variables with these indicators. The solid red (resp. dashed blue) lines plot the coefficients corresponding to the interaction of the negative log house price and the standardized credit expansion variables when the Euro/ERM status is equal to 1 (resp. 0). For the units, "percent" corresponds to 0.01 log units (i.e., it is approximate) and "pp" corresponds to percentage points. All regressions include time and country fixed effects; 12 lags of the first difference of log GDP, 12 lags of the level of the policy interest rate, 12 lags of the first difference of log house prices, 12 lags of the first difference of the outcome variable, and 12 lags of standardized credit expansion. The dotted lines show 95% confidence intervals calculated according to Newey-West standard errors with a bandwidth of 20 quarters. All regressions are weighted by countries' PPP-adjusted GDP in 1990. Data is unbalanced quarterly panel that spans 1990Q1-2017Q4. All variables except for those in the top panel are adjusted for inflation. Earnings are normalized by the stock price at the quarter before the shock (see Footnote 21). The sources and the definitions of variables are described in Appendix E.

In more recent work, Pflueger et al. (2018) argue that our mechanisms are likely to be relevant for stock prices, too. Specifically, they construct a measure of risk appetite for the U.S. as the price of high (idiosyncratic) volatility stocks relative to low volatility stocks. They show that a decrease in their measure of risk appetite is followed by a slowdown in economic activity as well as a decline in the risk-free rate—similar to our results for the non-Euro/ERM sample. Pflueger et al. (2018) go on to argue that their risk appetite measure explains almost half of the variation of the one year risk-free rate in the U.S. since 1970. This suggests that the time varying risk premium is a quantitatively important driver of the risk-free rate in practice. Likewise, focusing on a value-weighted average of house and stock prices, Jordà et al. (2017) argue that low frequency fluctuations in the risk premium in developed economies have typically been associated with a collapse of the safe rates (as opposed to a spike in risky rates). Focusing on more recent years, Del Negro et al. (2017) provide a comprehensive empirical evaluation of the different mechanisms that have put downward pressure on interest rates and show that risk and liquidity considerations played a central role (see also Caballero et al. (2017a)).

Finally, our mechanisms are supported by a literature that investigates the macroeconomic impact of "uncertainty shocks." Using vector autoregressions (VARs), Bloom (2009) shows that an increase in the volatility index in the U.S. is followed by a slowdown in economic activity. Moreover, although his model does not emphasize monetary policy, his empirical analysis shows that the shock is followed by a decline in the federal funds rate. This response suggests the effects could be more severe if the interest rate is constrained. Recent empirical work verifies this intuition and shows that uncertainty shocks in the U.S. are associated with a greater decline in economic activity when the federal funds rate is close to zero, arguably because of the zero lower bound constraint on the interest rate (see, for instance, Caggiano et al. (2017); Plante et al. (2018)).

8. Final remarks

The key tension in our model is that asset prices must both equilibrate risk markets and support aggregate demand. When these roles are inconsistent, the risk market equilibrium prevails. Interest rate policy takes over the role of equilibrating risk markets, which leaves asset prices free to balance the goods markets. However, if the interest rate is constrained, the dual role problem reemerges and asset prices are driven primarily by risk market equilibrium considerations. An increase in the risk premium lowers asset prices and triggers a demand recession, which further drives down asset prices. Speculation during the ex-ante boom phase exacerbates the recession because it depletes high-valuation investors' wealth once the risk premium rises, which leads to a greater decline in asset prices. Macroprudential policy (in the boom phase) improves outcomes by restricting speculation and preserving high-valuation investors' wealth during the recession. This leads to a Pareto improvement because it forces speculators to internalize aggregate demand externalities.

Interest rate cuts in our model improve the market's Sharpe ratio. From this perspective, any policy that reduces perceived market volatility and prevents sudden asset price drops should have similar effect, rendering support for various policies implemented during the aftermath of the subprime and European crises.

In our model, we use a lower bound constraint as the interest rate friction, but as we stated earlier our mechanisms are also applicable if the interest rate is constrained for other reasons. Also, when the interest rate has an upper bound as well as a lower bound (such as in a currency union or fixed exchange rate regime), our results often become stronger. In this setting, speculation creates damage not only by lowering asset prices during the recession but also by raising asset prices during the boom, when the aggregate demand is stretched above its natural level, which typically exacerbates the inefficiency. Moreover, in this case macroprudential policy during the boom is beneficial not only because it preserves high-valuation investors' wealth for a future recession but also because it immediately contains the excessive rise in asset prices.

In the main text, we did not take a stand on whether optimists or pessimists are right about the transition probabilities. The core of our analysis does not depend on this. For example, we could think of optimists as rational agents and pessimists as Knightian agents (see, e.g., Caballero and Krishnamurthy (2008); Caballero and Simsek (2013)). Absent any direct mechanism to alleviate Knightian behavior during severe recessions, the key point that reducing optimists's risk taking during the boom leads to Pareto improvements survives this alternative motivation.

As we noted earlier, our modeling approach belongs to the literature spurred by Brunnermeier and Sannikov (2014), although our analysis does not feature financial frictions. However, if we were to introduce these realistic frictions in our setting, many of the themes in that literature would reemerge and be exacerbated by aggregate demand feedbacks. For instance, in an incomplete markets setting, optimists take leveraged positions on the market portfolio and induce endogenous volatility in asset prices. In this case, a sequence of negative diffusion shocks that make the economy deeply pessimistic can lead to extreme tail events (we analyze the incomplete markets case in a companion paper).

Finally, while this is mostly an applied theory paper, we also surveyed some of the extensive empirical evidence supporting our analysis, and provided our own evidence by contrasting the local response to risk premium shocks (captured by surprise house price changes) of (constrained) Euro/ERM countries to that of (unconstrained) non-Euro/ERM countries. Our evidence suggests that risk premium shocks lead to more severe recessions when the interest rate is constrained, as in our model. The evidence also supports our model's prediction that recessions reduce firms' earnings and lead to a further reduction in asset prices. Finally, we found some evidence consistent with our prediction that recessions are more severe when the shock takes place in an environment with high speculation (as measured by the size of the bank credit expansion before the shock).

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Online Appendices: Not for Publication

A. Appendix: Omitted Derivations for the Two Period Model

This appendix presents the derivations and proofs omitted from the main text for the two period model that we analyze in Section 2. We start by the case analyzed in the main text. We then analyze the case in which EIS is different than one, as well as the case with belief disagreements. Throughout, recall that the market portfolio is the claim to all output at date 1. Combining Eqs. (1) and (2), the return on the market portfolio is also log normally distributed, that is,

$$r^m(z_1) = \log\left(\frac{Q_1}{z_1}\right) \sim N\left(g - \log Q - \frac{\sigma^2}{2}, \sigma^2\right). \tag{A.1}$$

A.1. Baseline two period model

For this case, most of the analysis is provided in the main text. Here, we formally state the investor's problem and derive the optimality conditions. The investor takes the returns as given and solves the following problem,

$$\begin{aligned} &\max_{c_0, a_0, \omega^m} \log c_0 + e^{-\rho} \log U_1 \\ &\text{where } U_1 = \left(E\left[c_1(z_1)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ \text{s.t.} \quad & c_0 + a_0 = y_0 + Q \\ \text{and} \quad & c_1(z_1) = a_0\left(\omega^m \exp(r^m(z_1)) + (1-\omega^m)\exp\left(r^f\right)\right). \end{aligned}$$

Here, $c_1(z_1)$ denotes total financial wealth, which equals consumption (since the economy ends at date 1). Note that the investor has Epstein-Zin preferences with EIS coefficient equal to one and the RRA coefficient equal to $\gamma > 0$. The case with $\gamma = 1$ is equivalent to time-separable log utility as in the dynamic model.

In view of the Epstein-Zin functional form, the investor's problem naturally splits into two steps. Conditional on savings, a_0, she solves a portfolio optimization problem, that is, $U_1 = R^{CE} a_0$, where

$$\begin{aligned} R^{CE} &= \max_{\omega^m}\left(E\left[(R^p(z_1))^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ \text{and } R^p(z_1) &= \left(\omega^m \exp(r^m(z_1)) + (1-\omega^m)\exp\left(r^f\right)\right). \end{aligned} \tag{A.2}$$

Here, we used the observation that the portfolio problem is linearly homogeneous. The variable, $R^p(z_1)$, denotes the realized portfolio return per dollar, and R^{CE} denotes the optimal certainty-equivalent portfolio return. In turn, the investor chooses asset holdings, a_0, that solve the intertemporal problem,

$$\max_{a_0} \log(y_0 + Q - a_0) + e^{-\rho}\log\left(R^{CE} a_0\right). \tag{A.3}$$

The first order condition for this problem implies Eq. (4) in the main text. That is, regardless of her certainty-equivalent portfolio return, the investor consumes and saves a constant fraction of her lifetime wealth.

It remains to characterize the optimal portfolio weight, ω^m, as well as the certainty-equivalent return, R^{CE}. Even though the return on the market portfolio is log-normally distributed (see Eq. $(A.1)$), the portfolio return, $R^p(z_1)$, is in general not log-normally distributed (since it is the sum of a log-normal variable and a constant). Following Campbell and Viceira (2002), we assume the investor solves an approximate version of the portfolio problem ($A.2$) in which the log portfolio return is also normally distributed. To state the problem, let $\pi^p \equiv \log E[R^p] - r^f$ and $(\sigma^p)^2 \equiv var(\log R^p)$ to denote respectively the risk premium and the variance of the market portfolio (measured in log returns). Then, the approximate portfolio return satisfies,

$$\begin{aligned} \pi^p &= \omega^m \pi^k \\ \text{where } \pi^k &\equiv \log\left(E\left[\exp\left(r^m(z_1)\right)\right]\right) - r^f = E\left[r^m(z_1)\right] - r^f + \frac{\sigma^2}{2}. \end{aligned} \tag{A.4}$$

Hence, the risk premium on the portfolio return depends linearly on the risk premium on the market portfolio (measured in log returns). We also have,

$$\sigma^p = \omega^m \sigma. \tag{A.5}$$

Thus, the volatility of the portfolio also depends linearly on the volatility of the market portfolio (measured in log returns). These identities hold exactly in continuous time. In the two period model, they hold approximately when the period time-length is small. Moreover, they become exact for the level the risk premium that ensures equilibrium, $\omega^m = 1$, since in this case the portfolio return is actually log-normally distributed.

Taking the log of the objective function in problem ($A.2$), and using the log-normality assumption, the problem can be equivalently rewritten as,

$$\log R^{CE} - r^f = \max_{\omega^m} \pi^p - \frac{1}{2}\gamma\left(\sigma^p\right)^2, \tag{A.6}$$

where π^p and σ^p are defined in Eqs. ($A.4$) and ($A.5$). It follows that, up to an approximation (that becomes exact in equilibrium), the investor's problem turns into standard mean-variance optimization. Taking the first order condition, we obtain Eq. (6) in the main text. Substituting $\omega^m = 1$ and $E[r^m(z_1)] = g - \log Q - \frac{\sigma^2}{2}$ [cf. Eq. ($A.1$)] into this expression, we further obtain Eq. (7) in the main text. Substituting these expressions into ($A.6$), we also obtain the closed form solution for the certainty-equivalent return in (11).

A.2. More general EIS

In this case, the representative investor solves the following problem,

$$\begin{aligned} \max_{c_0, a_0, \omega^m, \{c_1(z_1)\}} \quad & U_0 = \frac{c_0^{1-1/\varepsilon} - 1}{1 - 1/\varepsilon} + e^{-\rho}\frac{U_1^{1-1/\varepsilon} - 1}{1 - 1/\varepsilon} \\ \text{where} \quad & U_1 = \left(E\left[c_1(z_1)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ \text{s.t.} \quad & c_0 + a_0 = y_0 + Q \\ \text{and} \quad & c_1(z_1) = a_0\left(\omega^m \exp\left(r^m(z_1)\right) + (1-\omega^m)\exp\left(r^f\right)\right). \end{aligned}$$

Here, ε denotes the elasticity of substitution. The case with $\varepsilon = 1$ is equivalent to the earlier problem.

Most of the analysis remains unchanged. As before, the investor's problem splits into two parts. The portfolio problem ($A.2$) as well as its solution remains unchanged. In particular, Eqs. (6), (7), (11) from the main text continue to apply.

The main difference concerns the intertemporal problem $(A.3)$, which is now given by,

$$\max_{a_0} (y_0 + Q - a_0)^{1-1/\varepsilon} + e^{-\rho} \left(R^{CE} a_0\right)^{1-1/\varepsilon}.$$

Taking the first order condition and rearranging terms, we obtain the consumption function,

$$c_0 = \frac{1}{1 + e^{-\rho\varepsilon} \left(R^{CE}\right)^{(\varepsilon-1)}} (y_0 + Q).$$

Combining this expression with the aggregate resource constraint, $y_0 = c_0$, we obtain the output-asset price relation (10) in the main text. The main difference from the earlier analysis is that consumption (and savings) also depends on income and substitution effects, in addition to the wealth effect in the main text. When $\varepsilon > 1$, the substitution effect dominates and all else equal an increase in the certainty-equivalent return reduces consumption (increases savings). This in turn lowers aggregate demand and output. Conversely, when $\varepsilon < 1$, the income effect dominates and an increase in certainty-equivalent return increases consumption, aggregate demand, and output.

The equilibrium is found by jointly solving Eq . (10) together with Eqs. (7) and (11), as well as the constrained policy interest rate. Collecting the equations together, the equilibrium tuple, $\left(y_0, Q, R^{CE}, r^f\right)$, is the solution to the following system,

$$\begin{aligned} \log y_0 &= \rho\varepsilon + (1-\varepsilon) \log R^{CE} + \log Q \qquad \text{(A.7)} \\ \log R^{CE} &= g - \log Q - \frac{1}{2}\gamma\sigma^2 \\ \sigma &= \frac{1}{\gamma} \frac{g - \log Q - r^f}{\sigma} \\ r^f &= \max\left(r^{f*}, 0\right) \text{ where } r^{f*} \text{ ensures } y_0 = z_0. \end{aligned}$$

To characterize the solution further, consider the case in which the equilibrium is supply determined, $y_0 = z_0 = 1$. Substituting this into the first two equations, we solve for the first-best price level of the market portfolio as,

$$\log Q^* = -\rho + \frac{(\varepsilon - 1)}{\varepsilon} \left(g - \frac{1}{2}\gamma\sigma^2\right). \qquad \text{(A.8)}$$

Substituting this into the last equation, we further obtain an expression for "rstar",

$$\begin{aligned} r^{f*} &= \rho + g - \gamma\sigma^2 - \frac{(\varepsilon-1)}{\varepsilon} \left(g - \frac{1}{2}\gamma\sigma^2\right) \qquad \text{(A.9)} \\ &= \rho + \frac{g}{\varepsilon} - \frac{1}{2}\gamma \left(1 + \frac{1}{\varepsilon}\right) \sigma^2. \end{aligned}$$

Note that setting $\varepsilon = 1$ recovers Eq. (8) in the main text. The main difference is that "rstar" is now also influenced by the attractiveness of investment opportunities, captured by the term $g - \frac{1}{2}\gamma\sigma^2$ (that shifts $\log R^{CE}$). When $\varepsilon > 1$, reducing the attractiveness of investment opportunities induces the representative household to consume more and save less due to a substitution effect. This requires an increase in the risk-free rate to equilibrate the goods market. In this case, a risk premium shock that increases γ or σ reduces aggregate wealth, which tends to reduce the interest rate as before, but it also reduces the attractiveness of investment opportunities, which tends to raise the interest rate. When $\varepsilon < 1$, the two channels work in the same direction. The second line of Eq. $(A.9)$ collects similar terms together and shows that the risk shocks lower "rstar" as in the baseline setting regardless of the level of ε. When $\varepsilon > 1$, the effect is quantitatively weaker due to the substitution channel but it is qualitatively the same.

Now consider the case in which the interest rate is at its lower bound, $r^f = 0$. Substituting this into the equation system $(A.7)$, we obtain,

$$\begin{aligned} \log Q &= g - \gamma\sigma^2 \\ \text{and } \log y_0 &= \varepsilon\left(\rho + \log Q - \frac{(\varepsilon-1)}{\varepsilon}\left(g - \frac{1}{2}\gamma\sigma^2\right)\right) \\ &= \varepsilon\left(\rho + \frac{g}{\varepsilon} - \frac{1}{2}\gamma\left(1+\frac{1}{\varepsilon}\right)\sigma^2\right). \end{aligned} \tag{A.10}$$

In this case, the additional effect of the changes in the attractiveness of investment opportunities is absorbed by output, because the interest rate does not respond. An increase in $\gamma\sigma^2$ tends to reduce the output by reducing the aggregate wealth, as in the baseline setting, but it also affects output through substitution or income effects. The last line in $(A.10)$ illustrates that the wealth effect dominates regardless of the level of ε. When $\varepsilon > 1$, the substitution effect mitigates the quantitative impact of the wealth effect relative to the baseline setting but it does not overturn it. When $\varepsilon < 1$, the income effect amplifies the quantitative impact of the wealth effect.

A.3. Belief disagreements and speculation

We denote optimists and pessimists respectively with superscript $i \in \{o, p\}$. With a slight abuse of notation, we also let $\alpha^o \equiv \alpha$ and $\alpha^p \equiv 1-\alpha$ denote respectively optimists' and pessimists' wealth shares. Recall that investors are identical except possibly their beliefs about aggregate growth. Then, type i investors solve the following problem,

$$\begin{aligned} \max_{c_0, a_0, \omega^m, \{c_1(z_1)\}} \quad & \log c_0 + e^{-\rho}\log U_1 \\ \text{where} \quad & U_1 = \left(E^i\left[c_1(z_1)^{1-\gamma}\right]\right)^{1/(1-\gamma)} \\ \text{s.t.} \quad & c_0 + a_0 = \alpha^i(y_0 + Q) \\ \text{and} \quad & c_1(z_1) = a_0\left(\omega^m \exp(r^m(z_1)) + (1-\omega^m)\exp\left(r^f\right)\right). \end{aligned} \tag{A.11}$$

Note that we set the EIS equal to one as in the baseline setting. Note also that the asset market clearing condition requires,

$$\omega^{m,o} a_0^o + \omega^{m,p} a_0^p = Q, \tag{A.12}$$

that is, the total amount of wealth invested in the market portfolio equals the value of the market portfolio. The rest of the model is the same as in the baseline setting.

In this case, the investor's portfolio problem $(A.2)$ remains unchanged. Applying the log-normal approximation that we described previously, we obtain Eq. (6) as in the main text, that is,

$$\omega^{m,i}\sigma \simeq \frac{1}{\gamma}\frac{E^i\left[r^m(z_1)\right] + \frac{\sigma^2}{2} - r^f}{\sigma}.$$

Substituting $E^i[r^m(z_1)] = g^i - \log Q - \frac{\sigma^2}{2}$ [cf. Eq. $(A.1)$] into this expression, we further obtain,

$$\omega^{m,i}\sigma \simeq \frac{1}{\gamma}\frac{g^i - \log Q - r^f}{\sigma}. \tag{A.13}$$

As before, investors choose their share of the market portfolio so that their optimal portfolio risk is proportional to the Sharpe ratio. The difference is that the Sharpe ratio is calculated according to investors' own beliefs (and it is greater for optimists since $g^o > g^p$).

The intertemporal problem $(A.3)$ also remains unchanged. Taking the first order condition, we obtain,

$$c_0^i = \frac{1}{1+e^{-\rho}}\alpha^i(y_0 + Q) \tag{A.14}$$

Aggregating this equation across investors, and using the aggregate resource constraint (3), shows that the output-asset price relation (5) continues to apply in this setting. Belief heterogeneity does not affect this equation since investors share the same discount rate, ρ.

Next note that combining $(A.12), (A.14)$ and (5), the asset market clearing condition can be rewritten as,

$$\alpha\omega^{m,o} + (1-\alpha)\omega^{m,p} = 1. \tag{A.15}$$

Investors' wealth-weighted average portfolio weight on the market portfolio is equal to one. Combining this with Eq. $(A.13)$, we obtain the following analogue of Eq. (7),

$$\sigma \simeq \frac{1}{\gamma}\frac{\alpha^o g^o + \alpha^p g^p - \log Q - r^f}{\sigma}. \tag{A.16}$$

Hence, the risk balance condition continues to apply with the difference that the expected growth rate is determined according to a weighted average belief. Another difference is that the condition is typically not exact because investors' shares of the market portfolio typically deviate from one (and thus, their return is typically not log-normal). Specifically, the equilibrium portfolio shares satisfy, $\omega^o > 1 > \omega^p$: optimists' make a leveraged investment in the market portfolio by issuing some risk-free debt, whereas pessimists invest only a fraction of their wealth in the market portfolio (and invest the rest of their wealth in the risk-free asset issued by optimists).

Next consider the supply-determined equilibrium in which output is equal to its potential, $y_0 = z_0 = 1$. By Eq. (5), this requires the asset price to be at a particular level, $Q^* = e^{-\rho}$. Combining this with Eq. $(A.16)$ we obtain Eq. (12) in the main text that characterizes "rstar." The level of "rstar" is increasing in optimists' wealth share, α. This is because increasing optimists' wealth share tends to increase asset prices, aggregate demand, and output. In a supply-determined equilibrium, the monetary policy increases the interest rate to neutralize the impact of optimists on aggregate demand and output.

Finally, consider the case in which the interest rate is at its lower bound, $r^f = 0$. Substituting this into the risk balance condition $(A.16)$, and using the output-asset price relation (5), we obtain Eq. (9) in the main text that characterizes the equilibrium level of output in a demand recession. In this case, increasing optimists' wealth share translates into an actual increase in asset prices, aggregate demand, and output, because the monetary policy cannot neutralize these effects due to the constraint on the interest rate.

B. Appendix: Omitted Derivations for the Dynamic Model

This appendix presents the derivations and proofs omitted from the main text for the dynamic model that we present and analyze in Sections 3-5. The subsequent appendix C presents the details of the welfare analysis for the same model.

B.1. Omitted derivations in Section 3

B.1.1. Portfolio problem and its recursive formulation

The investor's portfolio problem (at some time t and state s) can be written as,

$$V^i_{t,s}\left(a^i_{t,s}\right) = \max_{\left[\tilde{c}_{\tilde{t},\tilde{s}},\tilde{\omega}^m_{\tilde{t},\tilde{s}},\tilde{\omega}^{\tilde{s}'}_{\tilde{t},\tilde{s}}\right]_{\tilde{t}\geq t,\tilde{s}}} E^i_{t,s}\left[\int_t^\infty e^{-\rho\tilde{t}}\log \tilde{c}^i_{\tilde{t},\tilde{s}}d\tilde{t}\right]$$

$$\text{s.t.} \begin{cases} da^i_{t,s} = \left(a^i_{t,s}\left(r^f_{t,s} + \tilde{\omega}^m_{t,s}\left(r^m_{t,s} - r^f_{t,s}\right) - \tilde{\omega}^{\tilde{s}'}_{t,\tilde{s}}\right) - \tilde{c}_{t,s}\right)dt + \tilde{\omega}^m_{t,s}a^i_{t,s}\sigma_s dZ_t & \text{absent transition,} \\ a^i_{t,s'} = a^i_{t,s}\left(1 + \tilde{\omega}^m_{t,s}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} + \tilde{\omega}^{s'}_{t,s}\frac{1}{p^{s'}_{t,s}}\right) & \text{if there is a transition to state } s' \neq s. \end{cases} \quad \text{(B.1)}$$

Here, $E^i_{t,s}[\cdot]$ denotes the expectations operator that corresponds to the investor i's beliefs for state transition probabilities. The HJB equation corresponding to this problem is given by,

$$\begin{aligned} \rho V^i_{t,s}\left(a^i_{t,s}\right) = & \max_{\tilde{\omega}^m,\tilde{\omega}^{s'},\tilde{c}} \log\tilde{c} + \frac{\partial V^i_{t,s}}{\partial a}\left(a^i_{t,s}\left(r^f_{t,s} + \tilde{\omega}^m\left(r^m_{t,s} - r^f_{t,s}\right) - \tilde{\omega}^{s'}\right) - \tilde{c}\right) \\ & + \frac{1}{2}\frac{\partial^2 V^i_{t,s}}{\partial a^2}\left(\tilde{\omega}^m a^i_{t,s}\sigma_s\right)^2 + \frac{\partial V^i_{t,s}\left(a^i_{t,s}\right)}{\partial t} \\ & + \lambda^i_s\left(V^i_{t,s'}\left(a^i_{t,s}\left(1 + \tilde{\omega}^m\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} + \frac{\tilde{\omega}^{s'}}{p^{s'}_{t,s}}\right)\right) - V^i_{t,s}\left(a^i_{t,s}\right)\right). \end{aligned} \quad \text{(B.2)}$$

In view of the log utility, the solution has the functional form in (44), which we reproduce here,

$$V^i_{t,s}\left(a^i_{t,s}\right) = \frac{\log\left(a^i_{t,s}/Q_{t,s}\right)}{\rho} + v^i_{t,s}.$$

The first term in the value function captures the effect of holding a greater capital stock (or greater wealth), which scales the investor's consumption proportionally at all times and states. The second term, $v^i_{t,s}$, is the normalized value function when the investor holds one unit of the capital stock (or wealth, $a^i_{t,s} = Q_{t,s}$). This functional form also implies,

$$\frac{\partial V^i_{t,s}}{\partial a} = \frac{1}{\rho a^i_{t,s}} \text{ and } \frac{\partial^2 V^i_{t,s}}{\partial a^2} = \frac{-1}{\rho\left(a^i_{t,s}\right)^2}.$$

The first order condition for $\tilde{c}$ then implies Eq. (22) in the main text. The first order condition for $\tilde{\omega}^m$ implies,

$$\frac{\partial V^i_{t,s}}{\partial a}a^i_{t,s}\left(r^m_{t,s} - r^f_{t,s}\right) + \lambda^i_s\frac{\partial V^i_{t,s'}\left(a^i_{t,s'}\right)}{\partial a}a^i_{t,s}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} = -\frac{\partial^2 V^i_{t,s}}{\partial a^2}\omega^m_{t,s}\left(a^i_{t,s}\sigma_s\right)^2.$$

After substituting for $\frac{\partial V^i_{t,s}}{\partial a}, \frac{\partial V^i_{t,s'}}{\partial a}, \frac{\partial^2 V^i_{t,s}}{\partial a^2}$ and rearranging terms, this also implies Eq. (23) in the main text.

Finally, the first order condition for $\tilde{\omega}^{s'}$ implies,

$$\frac{p_{t,s}^{s'}}{\lambda_s^i} = \frac{\frac{\partial V_{t,s'}^i\left(a_{t,s'}^i\right)}{\partial a}}{\frac{\partial V_{t,s}^i\left(a_{t,s}^i\right)}{\partial a}} = \frac{1/a_{t,s'}^i}{1/a_{t,s}^i},$$

which is Eq. (24) in the main text. This completes the characterization of the optimality conditions.

B.1.2. New Keynesian microfoundation for nominal rigidities

The supply side of our model features nominal rigidities similar to the standard New Keynesian setting. There is a continuum of measure one of monopolistically competitive production firms denoted by ν. These firms own the capital stock (in equal proportion) and produce differentiated goods, $y_{t,s}(\nu)$, subject to the technology,

$$y_{t,s}(\nu) = A\eta_{t,s}(\nu)\, k_{t,s}. \tag{B.3}$$

Here, $\eta_{t,s}(\nu) \in [0,1]$ denotes the firm's choice of capital utilization. We assume utilization is free up to $\eta_{t,s}(\nu) = 1$ and infinitely costly afterwards. The production firms sell their output to a competitive sector that produces the final output according to the CES technology,

$$y_{t,s} = \left(\int_0^1 y_{t,s}(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, \tag{B.4}$$

for some $\varepsilon > 1$. Thus, the demand for the firms' goods implies,

$$y_{t,s}(\nu) \leq p_{t,s}(\nu)^{-\varepsilon} y_{t,s}, \text{ where } p_{t,s}(\nu) = P_{t,s}(\nu)/P. \tag{B.5}$$

Here, $p_{t,s}(\nu)$ denotes the firm's relative price, which depends on its nominal price, $P_{t,s}(\nu)$, as well as the ideal nominal price index, $P_{t,s} = \left(\int P_{t,s}(\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$. We write the demand constraint as an inequality because an individual firm can in principle refuse to meet the demand for its goods.

Without price rigidities, the firm chooses $\eta_{t,s}(\nu) \in [0,1], y_{t,s}(\nu), p_{t,s}(\nu)$ to maximize its earnings, $p_{t,s}(\nu)\, y_{t,s}(\nu)$, subject to the supply constraint in $(B.3)$ and the demand constraint, $(B.5)$. In this case, the demand constraint holds as equality (because otherwise the firm can always raise its price to keep its production unchanged and raise its earnings). By combining the constraints, the firm's problem can be written as,

$$\max_{p_{t,s}(\nu),\eta_{t,s}(\nu)} p_{t,s}(\nu)^{1-\varepsilon} y_{t,s} \text{ s.t. } 0 \leq \eta_{t,s}(\nu) = \frac{p_{t,s}(\nu)^{-\varepsilon} y_{t,s}}{Ak_{t,s}} \leq 1.$$

Inspecting this problem reveals that the solution features full factor utilization, $\eta_{t,s}(\nu) = 1$. This is because, when $\eta_{t,s}(\nu) < 1$, the marginal cost of production is zero. Thus, the firm can always lower its price and increase its demand and production, which in turn increases its earnings. Hence, at the optimum, the firms set $\eta_{t,s}(\nu) = 1$ and $y_{t,s}(\nu) = Ak_{t,s}$. To produce at this level, they set the relative price level, $p_{t,s}(\nu) = \left(\frac{y_{t,s}}{Ak_{t,s}}\right)^{-1/\varepsilon}$. Since all firms are identical, we also have $p_{t,s}(\nu) = 1$ and $y_{t,s} = y_{t,s}(\nu) = Ak_{t,s}$. In particular, output is determined by aggregate supply at full factor utilization.

Now consider the alternative setting in which firms have a preset nominal price that is equal for all firms, $P_{t,s}(\nu) = P$. This also implies the relative price of a firm is fixed and equal to one, $p_{t,s}(\nu) = 1$. The firm chooses the remaining variables, $\eta_{t,s}(\nu) \in [0,1], y_{t,s}(\nu)$, to maximize its earnings, $y_{t,s}(\nu)$, subject to the supply constraint in $(B.3)$ and the demand constraint, $(B.5)$. Combining the constraints and using

$p_{t,s}(\nu) = 1$, the firm's problem can be written as,

$$\max_{\eta_{t,s}(\nu)} A\eta_{t,s}(\nu) k_{t,s} \text{ s.t. } 0 \leq \eta_{t,s}(\nu) \leq 1 \text{ and } A\eta_{t,s}(\nu) k_{t,s} \leq y_{t,s}.$$

The solution is given by, $\eta_{t,s}(\nu) = \min\left(1, \frac{y_{t,s}}{Ak_{t,s}}\right)$. Intuitively, when $\eta_{t,s}(\nu) < 1$ and $A\eta_{t,s}(\nu) k_{t,s} < y_{t,s}$, the marginal cost of production is zero and there is some unmet demand for firms' goods. The firm optimally increases its production until the supply or the demand constraints bind. Combining this observation with the production technology for the final output, we also obtain, $y_{t,s} \leq Ak_{t,s}$. This implies that the demand constraint holds as equality also in this case. In particular, we have $\eta_{t,s}(\nu) = \frac{y_{t,s}}{Ak_{t,s}} \leq 1$.

In sum, when the firms' nominal prices are fixed, aggregate output is determined by aggregate demand subject to the capacity constraint, which verifies Eq. (19) in the dynamic model (and Eq. (3) in the two period model).

Note also that, in equilibrium, firms' equilibrium earnings are equal to aggregate output, $y_{t,s}$. Since firms own the capital (and there is no rental market for capital), the division of these earnings between return to capital and monopoly profits is indeterminate. This division does not play an important role in our baseline model but it matters when we introduce investment. In Appendix D with endogenous investment (that we present subsequently), we use slightly different microfoundations that ensure earnings accrue to firms in the form of return to capital, i.e., there are no monopoly profits, which helps to simplify the exposition.

B.2. Omitted derivations in Section 4

Proof of Proposition 1. Provided in the main text.

B.3. Omitted derivations in Section 5

We derive the equilibrium conditions that we state and use in Section 5. First note that, using Eq. (24), the optimality condition (23) can be written as,

$$\omega_{t,s}^{m,i} \sigma_s = \frac{1}{\sigma_s} \left(r_{t,s}^m - r_{t,s}^f + p_{t,s}^{s'} \frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}} \right). \tag{B.6}$$

Note also that Eq. (25) implies,

$$\omega_{t,s}^{m,o} = \omega_{t,s}^{m,p} = 1. \tag{B.7}$$

Next note that by definition, we have

$$a_{t,s}^o = \alpha_{t,s} Q_{t,s} k_{t,s} \text{ and } a_{t,s}^p = (1 - \alpha_{t,s}) Q_{t,s} k_{t,s} \text{ for each } s \in \{1, 2\}.$$

After plugging these into Eq. (24), using $k_{t,s} = k_{t,s'}$ (since capital does not jump), and aggregating over optimists and pessimists, we obtain,

$$p_{t,s}^{s'} = \overline{\lambda}_{t,s} \frac{Q_{t,s}}{Q_{t,s'}}, \tag{B.8}$$

where $\overline{\lambda}_{t,s}$ denotes the wealth-weighted average belief defined in (37).

Next, we combine Eqs. $(B.6), (B.7)$, and $(B.8)$ to obtain

$$\sigma_s = \frac{1}{\sigma_s} \left(r_{t,s}^m - r_{t,s}^f + \overline{\lambda}_{t,s} \left(1 - \frac{Q_{t,s}}{Q_{t,s'}} \right) \right) \text{ for each } s \in \{1, 2\}. \tag{B.9}$$

Substituting for $r^m_{t,s}$ from Eq. (28), we obtain the risk balance condition (38) in the main text.

We next characterize investors' equilibrium positions. Combining Eq. $(B.1)$ with Eqs. $(B.7)$ and $(B.8)$, investors' wealth after transition satisfies,

$$\frac{a^i_{t,s'}}{a^i_{t,s}} = \frac{Q_{t,s'}}{Q_{t,s}} \left(1 + \frac{\omega^{s',i}_{t,s}}{\overline{\lambda}_{t,s}}\right). \tag{B.10}$$

From Eq. (24), we have $\frac{p^{s'}_{t,s}}{\lambda^i_s} = \frac{1/a^i_{t,s'}}{1/a^i_{t,s}}$. Substituting this into the previous expression and using Eq. $(B.8)$ once more, we obtain,

$$\omega^{s',i}_{t,s} = \lambda^i_s - \overline{\lambda}_{t,s} \text{ for each } i \in \{o,p\}. \tag{B.11}$$

Combining this with Eq. (37) implies Eq. (39) in the main text.

Finally, we characterize the dynamics of optimists' wealth share. Combining Eqs. $(B.10)$ and $(B.11)$ implies,

$$\frac{a^i_{t,s'}}{a^i_{t,s}} = \frac{\lambda^i_s}{\overline{\lambda}_{t,s}} \frac{Q_{t,s'}}{Q_{t,s}}. \tag{B.12}$$

Combining this with the definition of wealth shares as well as $k_{t,s} = k_{t,s'}$, we further obtain,

$$\frac{\alpha_{t,s'}}{\alpha_{t,s}} = \frac{\lambda^o_s}{\overline{\lambda}_{t,s}}. \tag{B.13}$$

Thus, it remains to characterize the dynamics of wealth conditional on no transition. To this end, we combine Eq. $(B.1)$ with Eqs. $(B.7), (28), (22)$ to obtain,

$$\frac{da^o_{t,s}}{a^o_{t,s}} = \left(g + \mu^Q_{t,s} - \omega^{s',i}_{t,s}\right) dt + \sigma_s dZ_t.$$

After substituting $a^o_{t,s} = \alpha_{t,s} Q_{t,s} k_{t,s}$, and using the observation that $\frac{dQ_{t,s}}{Q_{t,s}} = \mu^Q_{t,s} dt$ and $\frac{dk_{t,s}}{k_{t,s}} = g dt + \sigma_s dZ_t$, we further obtain,

$$\frac{d\alpha_{t,s}}{\alpha_{t,s}} = -\omega^{s',o}_{t,s} dt = -\left(\lambda^o_s - \overline{\lambda}_{t,s}\right) dt. \tag{B.14}$$

Combining Eqs. $(B.13)$ and $(B.14)$ implies Eq. (40) in the main text.

Proof of Proposition 2. First consider the high-risk-premium state, $s = 2$. Combining Eqs. (40) and (41), we obtain the differential equation system,

$$\begin{aligned} \dot{q}_{t,2} &= -\left(\rho + g + \overline{\lambda}_2 (\alpha_{t,2}) \left(1 - \frac{\exp(q_2)}{Q^*}\right) - \sigma_2^2\right), \\ \dot{\alpha}_{t,2} &= -\left(\lambda^o_2 - \lambda^p_2\right) \alpha_{t,2} (1 - \alpha_{t,2}). \end{aligned} \tag{B.15}$$

This system describes the joint dynamics of the price and optimists' wealth share, $(q_{t,2}, \alpha_{t,2})$, conditional on there not being a transition. We next analyze the solution to this system using the phase diagram over the range $\alpha \in [0,1]$ and $q_2 \in [q^p_2, q^o_2]$. Here, recall that q^i_2 corresponds to the equilibrium log price with common beliefs characterized in Section 4 corresponding to type i investors' belief.

First note that the system has two steady states given by, $(\alpha_{t,2} = 0, q_{t,2} = q^p_2)$, and $(\alpha_{t,2} = 1, q_{t,2} = q^o_2)$. Next note that the system satisfies the Lipschitz condition over the relevant range. Thus, the vector flows that describe the law of motion do not cross. Next consider the locus, $\dot{q}_2 = 0$. By comparing Eqs. (41)

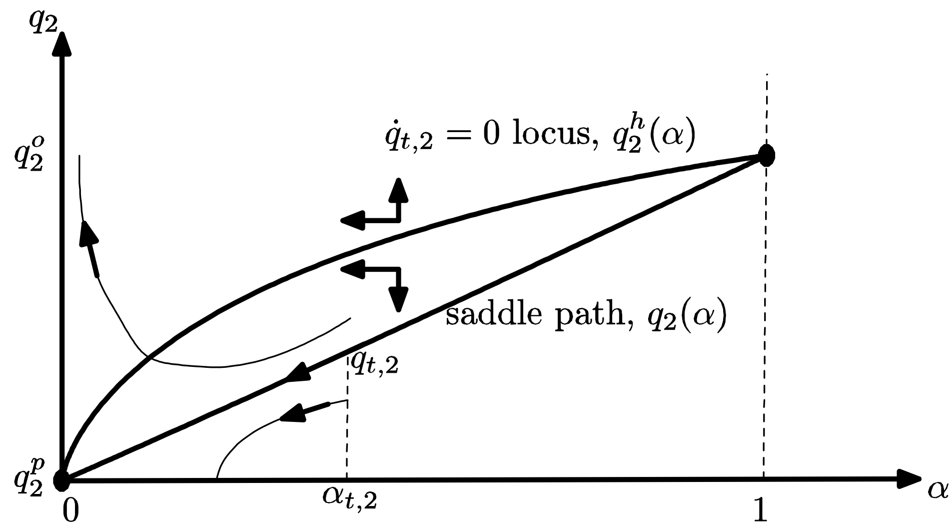

Figure 8: The phase diagram that describes the equilibrium with heterogeneous beliefs.

and (32), this locus is exactly the same as the price that would obtain if investors shared the same wealth-weighted average belief, denoted by $q_2 = q_2^h(\alpha)$. Using our analysis in Section 4, we also find that $q_2^h(\alpha)$ is strictly increasing in α. Moreover, $q_2 < q_2^h(\alpha)$ implies $\dot{q}_{t,2} < 0$ whereas $q_2 > q_2^h(\alpha)$ implies $\dot{q}_{t,2} > 0$. Finally, note that $\dot{\alpha}_{t,2} < 0$ for each $\alpha \in (0,1)$.

Combining these observations, the phase diagram has the shape in Figure 8. This in turn implies that the system is saddle path stable. Given any $\alpha_{t,2} \in [0,1)$, there exists a unique solution, $q_{t,2}$, which ensures that $\lim_{t\to\infty} q_{t,2} = q_2^p$. We define the price function (the saddle path) as $q_2(\alpha)$. Note that the price function satisfies $q_2(\alpha) < q_2^h(\alpha)$ for each $\alpha \in (0,1)$, since the saddle path cannot cross the locus, $\dot{q}_{t,2} = 0$. Note also that $q_2(1) = q_2^o$, since the saddle path crosses the other steady-state, $(\alpha_{t,2} = 1, q_{t,2} = q_2^o)$. Finally, recall that $q_2 < q_2^h(\alpha)$ implies $\dot{q}_{t,2} < 0$. Combining this with $\dot{\alpha}_{t,2} < 0$, we further obtain $\frac{dq_2(\alpha)}{d\alpha} > 0$ for each $\alpha \in (0,1)$.

Next note that, after substituting $\dot{q}_{t,2} = q_2'(\alpha)\dot{\alpha}_{t,2}$, Eq. $(B.15)$ implies the differential equation (42) in α-domain. Thus, the above analysis shows there exists a solution to the differential equation with $q_2(0) = q_2^p$ and $q_2(1) = q_2^o$. Moreover, the solution is strictly increasing in α, and it satisfies $q_2(\alpha) < q_2^h(\alpha)$ for each $\alpha \in (0,1)$. Note also that this solution is unique since the saddle path is unique. The last part of the proposition follows from Eqs. (26) and (27).

Next consider the low-risk-premium state, $s = 1$. In the conjectured equilibrium, we have $Q_{t,1} = Q^*$, which also implies $\mu_{t,1}^Q = 0$. Substituting these expressions into Eq. (38), we obtain the risk balance condition in this state,

$$\sigma_1 = \frac{1}{\sigma_1}\left(g + \rho - r_{t,1}^f + \overline{\lambda}_{t,1}\left(1 - \frac{Q^*}{Q_{t,2}}\right)\right).$$

Writing the equilibrium variables as a function of optimists' wealth share, we obtain $r_{t,1}^f = r_1^f(\alpha)$ and $\overline{\lambda}_{t,1} = \overline{\lambda}_1(\alpha)$ and $Q_{t,2} = \exp(q_2(\alpha'))$, where $\alpha' = \alpha\lambda_1^o/\overline{\lambda}_1(\alpha)$ denotes optimists' wealth share after a transition [cf. Eq. (40)]. Substituting these expressions into the risk balance condition and rearranging terms, we obtain Eq. (43) in the main text that, which we replicate here,

$$r_1^f(\alpha) = \rho + g - \overline{\lambda}_1(\alpha)\left(\frac{Q^*}{\exp(q_2(\alpha'))} - 1\right) - \sigma_1^2.$$

Note also that $\frac{dr_1^f(\alpha)}{d\alpha} > 0$ because $\overline{\lambda}_1(\alpha)$ is decreasing in α (in view of Assumption 4), and $q_2(\alpha')$ is strictly increasing in α. The latter observation follows since $\alpha' = \frac{\alpha\lambda_1^o}{\alpha\lambda_1^o+(1-\alpha)\lambda_1^p}$ is increasing in α (in view of Assumption 4) and $q_2(\cdot)$ is a strictly increasing function. Note also that $r_1^f(\alpha) > r_1^f(0) > 0$, where the latter inequality follows since Assumptions 1-3 holds for the pessimistic belief. Thus, the interest rate in state 1 is always positive, which verifies our conjecture and completes the proof. □

C. Appendix: Omitted Derivations for the Welfare Analysis

This appendix presents the omitted derivations and proofs for the welfare analysis of the dynamic model that we present in Section 6. Section C.1 establishes the properties of the equilibrium value functions that are used in the main text. Section C.2 describes the details of the equilibrium with macroprudential policy, presents the analyses omitted from the main text (e.g., macroprudential policy in the high-risk-premium state), and presents omitted proofs.

C.1. Value functions in equilibrium

We first derive the HJB equation that describes the normalized value function in equilibrium and derive Eqs. (45). We then derive the differential equations in α-domain that characterize the value function and its components, and derive Eq. (48). We then prove Lemmas 1 and 2 that are used in the analysis.

Characterizing the normalized value function in equilibrium. Consider the recursive version of the portfolio problem in $(B.2)$. Recall that the value function has the functional form in Eq. (44). Our goal is to characterize the value function per unit of capital, $v_{t,s}^i$ (corresponding to $a_{t,s}^i = Q_{t,s}$). To facilitate the analysis, we define,

$$\xi_{t,s}^i = v_{t,s}^i - \frac{\log Q_{t,s}}{\rho}. \tag{C.1}$$

Note that $\xi_{t,s}^i$ is the value function per unit wealth (corresponding to $a_{t,s}^i = 1$), and that the value function also satisfies $V_{t,s}^i\left(a_{t,s}^i\right) = \frac{\log\left(a_{t,s}^i\right)}{\rho} + \xi_{t,s}^i$. We first characterize $\xi_{t,s}^i$. We then combine this with Eq. $(C.1)$ to characterize our main object of interest, $v_{t,s}^i$.

Consider the HJB equation $(B.2)$. We substitute the optimal consumption rule from Eq, (22), the contingent allocation rule from Eq. (24), and $a_{t,s}^i = 1$ (to characterize the value per unit wealth) to obtain,

$$\begin{aligned}\rho\xi_{t,s}^i = \log\rho + \frac{1}{\rho}\left(r_{t,s}^f + \omega_{t,s}^{m,i}\left(r_{t,s}^m - r_{t,s}^f\right) - \frac{1}{2}\left(\omega_{t,s}^{m,i}\right)^2\sigma_s^2 - \rho - \omega_{t,s}^{s',i}\right) \\ + \frac{\partial\xi_{t,s}^i}{\partial t} + \lambda_s^i\left(\frac{1}{\rho}\log\left(\frac{\lambda_s^i}{p_{t,s}^{s'}}\right) + \xi_{t,s'}^i - \xi_{t,s}^i\right).\end{aligned} \tag{C.2}$$

As we describe in Section 5, the market clearing conditions imply the optimal investment in the market portfolio and contingent securities satisfies, $\omega^m = 1$ and $\tilde{\omega}_{t,s}^{s',i} = \lambda_s^i - \overline{\lambda}_{t,s}$, and the price of the contingent security is given by, $p_{t,s}^{s'} = \overline{\lambda}_{t,s}\frac{1/Q_{t,s'}}{1/Q_{t,s}}$. Here, $\overline{\lambda}_{t,s}$ denotes the weighted average belief defined in (37). Using these conditions, the HJB equation becomes,

$$\begin{aligned}\rho\xi_{t,s}^i = \log\rho + \frac{1}{\rho}\left(\begin{array}{c} r_{t,s}^m - \rho - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda_s^i - \overline{\lambda}_{t,s}\right) + \lambda_s^i\log\left(\frac{\lambda_s^i}{\overline{\lambda}_{t,s}}\right)\end{array}\right) \\ + \frac{\partial\xi_{t,s}^i}{\partial t} + \lambda_s^i\left(\frac{1}{\rho}\log\left(\frac{Q_{t,s'}}{Q_{t,s}}\right) + \xi_{t,s'}^i - \xi_{t,s}^i\right).\end{aligned} \tag{C.3}$$

After substituting the return to the market portfolio from (28), the HJB equation can be further simplified as,

$$\rho\xi^i_{t,s} = \begin{array}{c} \log\rho + \frac{1}{\rho}\left(\begin{array}{c} g+\mu^Q_{t,s}-\frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right) \end{array}\right) \\ +\frac{\partial \xi^i_{t,s}}{\partial t} + \lambda^i_s\left(\frac{1}{\rho}\log\left(\frac{Q_{t,s'}}{Q_{t,s}}\right) + \xi^i_{t,s'} - \xi^i_{t,s}\right) \end{array}.$$

Here, the term inside the summation on the second line, $-\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right)$, is zero when there are no disagreements, and it is strictly positive when there are disagreements. This illustrates that speculation increases the expected value for optimists as well as pessimists.

We finally substitute $v^i_{t,s} = \xi^i_{t,s} + \frac{\log Q_{t,s}}{\rho}$ (cf. (C.1)) into the HJB equation to obtain the differential equation,

$$\rho v^i_{t,s} = \begin{array}{c} \log\rho + \log\left(Q_{t,s}\right) + \frac{1}{\rho}\left(\begin{array}{c} g-\frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_{t,s}\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_{t,s}}\right) \end{array}\right) \\ +\frac{\partial v^i_{t,s}}{\partial t} + \lambda^i_s\left(v_{t,s'} - v_{t,s}\right) \end{array}.$$

Here, we have canceled terms by using the observation that $\frac{\partial \xi^i_{t,s}}{\partial t} = \frac{\partial v^i_{t,s}}{\partial t} - \frac{1}{\rho}\frac{\partial \log Q_{t,s}}{\partial t} = \frac{\partial v^i_{t,s}}{\partial t} - \frac{1}{\rho}\mu^Q_{t,s}$. We have thus obtained Eq. (45) in the main text.

Differential equations for the value functions in α-domain. The value function and its components, $\left\{v^i_{t,s}, v^{i,*}_{t,s}, w_{t,s}\right\}_{s,i}$, can be written as functions of optimists' wealth share, $\left\{v^i_s\left(\alpha\right), v^{i,*}_s\left(\alpha\right), w_s\left(\alpha\right)\right\}_{s,i}$, that solve appropriate ordinary differential equations. We next represent the value functions as solutions to the differential equations in α-domain. Recall that the price level in each state can be written as a function of optimists' wealth shares, $q_{t,s} = q_s\left(\alpha\right)$ (where we also have, $q_1\left(\alpha\right) = q^*$). Plugging in these price functions, and using the dynamics of $\alpha_{t,s}$ from Eq. (40), the HJB equation (45) can be written as,

$$\rho v^i_s\left(\alpha\right) = \begin{array}{c} \log\rho + q_s\left(\alpha\right) + \frac{1}{\rho}\left(\begin{array}{c} g-\frac{1}{2}\sigma_s^2 \\ -\left(\lambda^i_s - \overline{\lambda}_s\left(\alpha\right)\right) + \lambda^i_s \log\left(\frac{\lambda^i_s}{\overline{\lambda}_s\left(\alpha\right)}\right) \end{array}\right) \\ -\frac{\partial v^i_s}{\partial \alpha}\left(\lambda^o_s - \lambda^p_s\right)\alpha\left(1-\alpha\right) + \lambda^i_s\left(v^i_{s'}\left(\alpha\frac{\lambda^o_s}{\overline{\lambda}_s\left(\alpha\right)}\right) - v^i_s\left(\alpha\right)\right) \end{array}.$$

For each $i \in \{o, p\}$, the value functions, $\left(v^i_s\left(\alpha\right)\right)_{s\in\{1,2\}}$, are found by solving this system of ODEs. For $i = o$, the boundary conditions are that the values, $\left\{v^o_s\left(1\right)\right\}_s$, are the same as the values in the common belief benchmark characterized in Section 4 when all investors have the optimistic beliefs. For $i = p$, the boundary conditions are that the values, $\left\{v^p_s\left(0\right)\right\}_s$, are the same as the values in the common belief benchmark when all investors have the pessimistic beliefs.

Likewise, the first-best value functions, $\left(v^{i,*}_s\left(\alpha\right)\right)_{s\in\{1,2\}}$, are found by solving the analogous system after replacing $q_s\left(\alpha\right)$ with q^* (and changing the boundary conditions appropriately). Finally, substituting the price functions into Eq. (47), the gap-value functions, $\left(w^i_s\left(\alpha\right)\right)_{s,i}$, are found by solving the system in (48).

For the proofs in this section (as well as in some subsequent sections), we find it useful to work with the transformed state variable,

$$b_{t,s} \equiv \log\left(\frac{\alpha_{t,s}}{1-\alpha_{t,s}}\right), \text{ which implies } \alpha_{t,s} = \frac{1}{1+\exp\left(-b_{t,s}\right)}. \tag{C.4}$$

The variable, $b_{t,s}$, varies between $(-\infty, \infty)$ and provides a different measure of optimism, which we refer to as "bullishness." Note that there is a one-to-one relation between optimists' wealth share, $\alpha_{t,s} \in (0,1)$, and the bullishness, $b_{t,s} \in \mathbb{R} = (-\infty, +\infty)$. Optimists' wealth dynamics in (40) become particularly simple when expressed in terms of bullishness,

$$\begin{cases} \dot{b}_{t,s} = -\left(\lambda_s^o - \lambda_s^p\right), & \text{if there is no state change,} \\ b_{t,s'} = b_{t,s} + \log \lambda_s^o - \log \lambda_s^p, & \text{if there is a state change.} \end{cases} \tag{C.5}$$

With a slight abuse of notation, we also let $q_2(b), w_s^i(b)$, and so on, denote the equilibrium functions in terms of bullishness. Note also that, since $\frac{db}{d\alpha} = \frac{1}{\alpha(1-\alpha)}$, we have the identities,

$$\frac{\partial q_2(b)}{\partial b} = \alpha(1-\alpha)\frac{\partial q_2(\alpha)}{\partial b} \text{ and } \frac{\partial w_s^i(b)}{\partial b} = \alpha(1-\alpha)\frac{\partial w_s^i(\alpha)}{\partial \alpha}. \tag{C.6}$$

Using this observation, the differential equation for the price function, Eq. (42), can be written in terms of bullishness as,

$$\frac{\partial q_2(b)}{\partial b}\left(\lambda_2^o - \lambda_2^p\right) = \rho + g + \overline{\lambda}_2(\alpha)\left(1 - \frac{Q_2}{Q^*}\right) - \sigma_2^2. \tag{C.7}$$

Likewise, the differential equation for the gap value function, Eq. (48), can be written in terms of bullishness as,

$$\rho w_s^i(b) = q_s(b) - q^* - \left(\lambda_s^o - \lambda_s^p\right)\frac{\partial w_s^i(b)}{\partial b} + \lambda_s^i\left(w_{s'}^i(b') - w_s^i(b)\right). \tag{C.8}$$

Proof of Lemma 1. To show that the gap value function is increasing, consider its representation in terms of bullishness, $w_s^i(b)$ [cf. $(C.4)$], which solves the system in $(C.8)$. We will first describe this function as a fixed point of a contraction mapping. We will then use this contraction mapping to establish the properties of the function.

Recall that, in the time domain, the gap value function solves the HJB equation (47). Integrating this equation forward, we obtain,

$$w_s^i(b_{0,s}) = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(q_s(b_{t,s}) - q^* + \lambda_s^i w_{s'}^i(b_{t,s'})\right)dt, \tag{C.9}$$

for each $s \in \{1,2\}$ and $b_{0,s} \in \mathbb{R}$. Here, $b_{t,s}$ denotes bullishness conditional on there not being a transition before time t, whereas $b_{t,s'}$ denotes the bullishness if there is a transition at time t. Solving Eq. $(C.5)$ (given beliefs, λ^i) we further obtain,

$$\begin{aligned} b_{t,s} &= b_{0,s} - t\left(\lambda_s^o - \lambda_s^p\right), \\ b_{t,s'} &= b_{0,s} - t\left(\lambda_s^o - \lambda_s^p\right) + \log \lambda_s^o - \log \lambda_s^p. \end{aligned} \tag{C.10}$$

Hence, Eq. $(C.9)$ describes the value function as a solution to an integral equation given the closed form solution for bullishness in $(C.10)$.

Implicitly differentiating the integral equation $(C.9)$ with respect to $b_{0,s}$, and using Eq. $(C.10)$, we also obtain,

$$\frac{\partial w_s^i(b_{0,s})}{\partial b} = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(\frac{\partial q_s(b_{t,s})}{\partial b} + \lambda_s^i\frac{\partial w_{s'}^i(b_{t,s'})}{\partial b}\right)dt. \tag{C.11}$$

We next let $B\left(\mathbb{R}^2\right)$ denote the set of bounded value functions over $\mathbb{R}^2$. Given some continuation value function, $\left(\frac{\partial \tilde{w}_s^i(b)}{\partial b}\right)_s \in B\left(\mathbb{R}^2\right)$, we define the function, $\left(T\frac{\partial \tilde{w}_s^i(b)}{\partial b}\right)_s \in B\left(\mathbb{R}^2\right)$, so that

$$T\frac{\partial \tilde{w}_s^i\left(b_{0,s}\right)}{\partial b} = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(\frac{\partial q_s\left(b_{t,s}\right)}{\partial b} + \lambda_s^i \frac{\partial \tilde{w}_{s'}^i\left(b_{t,s'}\right)}{\partial b}\right)dt,$$

for each s and $b_{0,s} \in \mathbb{R}$. Note also that the resulting value functions are bounded since the derivative of the price functions, $\left(\frac{\partial q_s(b_{t,s})}{\partial b}\right)_s$, are bounded (see Eq. $(C.7)$). Thus, Eq. $(C.11)$ describes the derivative functions, $\left(\frac{\partial w_s^i(b_{0,s})}{\partial b}\right)_s$, as a fixed point of a corresponding operator T over bounded functions. It can be checked that this operator is a contraction mapping with respect to the sup norm. Thus, it has a unique fixed point that corresponds to the derivative functions. Moreover, since $\frac{\partial q_s(b_{t,s})}{\partial b} > 0$ for each b, and $\lambda_s^i > 0$ for each s, it can further be seen that the fixed point satisfies, $\frac{\partial w_s^i(b_{0,s})}{\partial b} > 0$ for each b and $s \in \{1,2\}$. Using Eq. $(C.6)$, we also obtain $\frac{\partial w_s^i(\alpha)}{\partial \alpha} > 0$ for each $\alpha \in (0,1)$ and $s \in \{1,2\}$, completing the proof. □

Proof of Lemma 2. Consider the analysis in Lemma 1 for the special case, $\lambda_1^o = \lambda_1^p$. Applying Eq. $(C.11)$ for $s = 1$, we obtain [since $q_1\left(b_{t,s}\right) = q^*$ is constant],

$$\frac{\partial w_1^i\left(b_{0,1}\right)}{\partial b} = \int_0^\infty e^{-\left(\rho+\lambda_1^i\right)t}\lambda_1^i \frac{\partial w_2^i\left(b_{t,2}\right)}{\partial b}dt.$$

Note also that $\lambda_1^o = \lambda_1^p$ and Eq. $(C.10)$ imply $b_{t,2} = b_{0,1}$ (since there is no speculation). Substituting this into the displayed equation, we obtain $\frac{\partial w_1^i(b_{0,1})}{\partial b} = \frac{\lambda_1^i}{\rho+\lambda_1^i}\frac{\partial w_2^i(b_{0,1})}{\partial b} < \frac{\partial w_2^i(b_{0,1})}{\partial b}$. Combining this with Eq. $(C.6)$ completes the proof. □

C.2. Equilibrium with macroprudential policy

Recall that macroprudential policy induces optimists to choose allocations as if they have more pessimistic beliefs, $\lambda^{o,pl} \equiv \left(\lambda_1^{o,pl}, \lambda_2^{o,pl}\right)$, that satisfy, $\lambda_1^{o,pl} \geq \lambda_1^o$ and $\lambda_2^{o,pl} \leq \lambda_2^o$. We next show that this allocation can be implemented with portfolio restrictions on optimists. We then show that the planner's Pareto problem reduces to solving problem (50) in the main text. We also derive the equilibrium value functions that result from macroprudential policy. We then analyze macroprudential policy in the recession state, which complements the analysis in the main text (that focuses on the boom state), and present Proposition 4. Finally, we present the proofs of Propositions 3 and 4.

Implementing the policy with risk limits. Consider the equilibrium that would obtain if optimists had the planner-induced beliefs, $\lambda_s^{o,pl}$. Using our analysis in Section 5, optimists' equilibrium portfolios are given by,

$$\omega_{t,s}^{m,o,pl} = 1 \text{ and } \omega_{t,s}^{s',o,pl} = \lambda_s^{o,pl} - \overline{\lambda}_{t,s}^{pl} \text{ for each } t, s. \tag{C.12}$$

We first show that the planner can implement the policy by requiring optimists to hold exactly these portfolio weights. We will then relax these portfolio constraints into inequality restrictions (see Eq. $(C.14)$).

Formally, an optimist solves the HJB problem $(B.2)$ with the additional constraint $(C.12)$. In view of log utility, we conjecture that the value function has the same functional form (44) with potentially different normalized values, $\xi_{t,s}^o, v_{t,s}^o$, that reflect the constraints. Using this functional form, the optimality condition for consumption remains unchanged, $c_{t,s} = \rho a_{t,s}^o$ [cf. Eq. (22)]. Plugging this equation and the portfolio holdings in $(C.12)$ into the objective function in $(B.2)$ verifies that the value function has the conjectured functional form. For later reference, we also obtain that the optimists' unit-wealth value function satisfies [cf. Eq. $(C.1)$],

$$
\begin{aligned}
\xi^o_{t,s} &= \log\rho + \frac{1}{\rho}\left(r^f_{t,s} + \omega^{m,o,pl}_{t,s}\left(r^m_{t,s} - r^f_{t,s}\right) - \rho - \omega^{s',o,pl}_{t,s}\right) \\
&\quad -\frac{1}{2\rho}\left(\omega^{m,o,pl}_{t,s}\sigma_s\right)^2 + \frac{\partial \xi^o_{t,s}}{\partial t} + \lambda^o_s\left(\frac{1}{\rho}\log\left(\frac{a^o_{t,s'}}{a^o_{t,s}}\right) + \xi^o_{t,s'} - \xi^o_{t,s}\right).
\end{aligned}
\tag{C.13}
$$

Here, $\frac{a^o_{t,s'}}{a^o_{t,s}} = 1 + \omega^{m,o,pl}_{t,s}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}} + \frac{\omega^{s',o,pl}_{t,s}}{p^{s'}_{t,s}}$ in view of the budget constraints $(B.1)$. Hence, the value function has a similar characterization as before [cf. Eq. $(C.2)$] with the difference that optimists' portfolio holdings reflect the portfolio constraints.

Since pessimists are unconstrained, their optimality conditions are unchanged. It follows that the equilibrium takes the form in Section 5 with the difference that investors' beliefs are replaced by their as-if beliefs, $\lambda^{i,pl}_s$. This verifies that the planner can implement the policy using the portfolio restrictions in $(C.12)$. We next show that these restrictions can be relaxed to the following inequality constraints,

$$
\begin{aligned}
\omega^{m,o,pl}_{t,s} &\leq 1 \text{ for each } s, \\
\omega^{2,o,pl}_{t,1} &\geq \underline{\omega}^{2,o}_{t,1} \equiv \lambda^{o,pl}_1 - \overline{\lambda}^{pl}_{t,1} \text{ and } \omega^{1,o,pl}_{t,2} \leq \overline{\omega}^{1,o}_{t,2} \equiv \lambda^{o,pl}_2 - \overline{\lambda}^{pl}_{t,2}.
\end{aligned}
\tag{C.14}
$$

In particular, we will establish that all inequality constraints bind, which implies that optimists optimally choose the portfolio weights in Eq. $(C.12)$. Thus, our earlier analysis continues to apply when optimists are subject to the more relaxed restrictions in $(C.14)$.

The result follows from the assumption that the planner-induced beliefs are more pessimistic than optimists' actual beliefs, $\lambda^{o,pl}_1 \geq \lambda^o_1$ and $\lambda^{o,pl}_2 \leq \lambda^o_2$. To see this formally, note that the optimality condition for the market portfolio is given by the following generalization of Eq. (23),

$$
\omega^{m,o,pl}_{t,s}\sigma_s \leq \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \lambda^o_s \frac{a^o_{t,s}}{a^o_{t,s'}}\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}}\right) \text{ and } \omega^{m,o,pl}_{t,s} \leq 1,
\tag{C.15}
$$

with complementary slackness. Note also that,

$$
\lambda^o_s \frac{a^o_{t,s}}{a^o_{t,s'}}\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s}} = \lambda^o_s \frac{\overline{\lambda}^{pl}_{t,s}}{\lambda^{o,pl}_s}\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s'}} \geq \overline{\lambda}^{pl}_{t,s}\frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s'}} \text{ for each } s.
$$

Here, the equality follows because Eq. $(B.12)$ in Appendix B.3 applies with as-if beliefs. The inequality follows by considering separately the two cases, $s \in \{1, 2\}$. For $s = 2$, the inequality holds since $Q_{t,s'} - Q_{t,s} > 0$ and the beliefs satisfy, $\lambda^o_s \geq \lambda^{o,pl}_s$. For $s = 1$, the inequality holds since $Q_{t,s'} - Q_{t,s} < 0$ and the beliefs satisfy, $\lambda^{o,pl}_s \geq \lambda^o_s$. Note also that in equilibrium the return to the market portfolio satisfies Eq. $(B.9)$, which we replicate here,

$$
\sigma_s = \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \overline{\lambda}^{pl}_{t,s}\left(1 - \frac{Q_{t,s}}{Q_{t,s'}}\right)\right).
$$

Combining these expressions implies, $\sigma_s \leq \frac{1}{\sigma_s}\left(r^m_{t,s} - r^f_{t,s} + \lambda^o_s \frac{a^o_{t,s}}{a^o_{t,s'}}\frac{Q_{t,s'}-Q_{t,s}}{Q_{t,s}}\right)$, which in turn implies the optimality condition $(C.15)$ is satisfied with $\omega^{m,o,pl}_{t,s} = 1$. A similar analysis shows that optimists also choose the corner allocations in contingent securities, $\omega^{2,o,pl}_{t,1} = \underline{\omega}^{2,o}_{t,1}$ and $\omega^{1,o,pl}_{t,2} = \overline{\omega}^{1,o}_{t,2}$, verifying that the portfolio constraints $(C.12)$ can be relaxed to the inequality constraints in $(C.14)$.

Simplifying the planner's problem. Recall that, to trace the Pareto frontier, we allow the planner to do a one-time wealth transfer among the investors at time 0. Let $V^i_{t,s}\left(a^i_{t,s}|\left\{\lambda^{o,pl}_t\right\}\right)$ denote type i investors' expected value in equilibrium when she starts with wealth $a^i_{t,s}$ and the planner commits to implement the policy, $\left\{\lambda^{o,pl}_t\right\}$. Then, the planner's Pareto problem can be written as,

$$\max_{\tilde{\lambda}^{o,pl},\tilde{\alpha}_{0,s}} \gamma^o V^o_{0,s}\left(\tilde{\alpha}_{0,s}Q_{0,s}k_{0,s}|\tilde{\lambda}^{o,pl}\right)+\gamma^p V^p_{0,s}\left(\left(1-\tilde{\alpha}_{0,s}\right)Q_{0,s}k_{0,s}|\tilde{\lambda}^{o,pl}\right). \quad \text{(C.16)}$$

Here, $\gamma^o,\gamma^p \geq 0$ (with at least one strict inequality) denote the Pareto weights, and $Q_{0,s}$ denotes the endogenous equilibrium price that obtains under the planner's policy.

Next recall that the investors' value function with macroprudential policy has the same functional form in (44) (with potentially different $\xi^o_{t,s}, v^o_{t,s}$ for optimists that reflect the constraints). After substituting $a^i_{t,s}=\alpha^i_{t,s}k_{t,s}Q_{t,s}$, the functional form implies,

$$V^i_{t,s}=v^i_{t,s}+\frac{\log\left(\alpha^i_{t,s}\right)+\log\left(k_{t,s}\right)}{\rho}.$$

Using this expression, the planner's problem $(C.16)$ can be rewritten as,

$$\max_{\tilde{\lambda}^{o,pl},\tilde{\alpha}_{0,s}} \left(\gamma^o v^o_{0,s}+\gamma^p v^p_{0,s}\right)+\frac{\gamma^o\log\left(\tilde{\alpha}^o_{0,s}\right)+\gamma^p\log\left(1-\tilde{\alpha}^o_{0,s}\right)}{\rho}+\frac{\left(\gamma^o+\gamma^p\right)\log\left(k_{0,s}\right)}{\rho}.$$

Here, the last term (that features capital) is a constant that doesn't affect optimization. The second term links the planner's choice of wealth redistribution, $\alpha^o_{0,s},\alpha^p_{0,s}$, to her Pareto weights, γ^o,γ^p. Specifically, the first order condition with respect to optimists' wealth share implies $\frac{\gamma^o}{\gamma^p}=\frac{\alpha_{0,s}}{1-\alpha_{0,s}}$. Thus, the planner effectively maximizes the first term after substituting γ^o and γ^p respectively with the optimal choice of $\alpha_{0,s}$ and $1-\alpha_{0,s}$. This leads to the simplified problem (50) in the main text.

Characterizing the value functions with macroprudential policy. We first show that the normalized value functions, $v^i_{t,s}$, are characterized as the solution to the following differential equation system,

$$\rho v^i_{t,s}-\frac{\partial v^i_{t,s}}{\partial t}=\log\rho+q_{t,s}+\frac{1}{\rho}\left(\begin{array}{c} g-\frac{1}{2}\sigma^2_s \\ -\left(\lambda^{i,pl}_s-\overline{\lambda}^{pl}_{t,s}\right)+\lambda^i_s\log\left(\frac{\lambda^{i,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\right)\end{array}\right)+\lambda^i_s\left(v^i_{t,s'}-v^i_{t,s}\right). \quad \text{(C.17)}$$

This is a generalization of Eq. (45) in which investors' positions are calculated according to their as-if beliefs, $\lambda^{i,pl}_s$, but the transition probabilities are calculated according to their actual beliefs, λ^i_s.

First consider the pessimists. Since they are unconstrained, their value function is characterized by solving the earlier equation system $(C.13)$. In this case, equation $(C.17)$ also holds since it is the same as the earlier equation.

Next consider the optimists. In this case, the analysis in Appendix B.3 applies with as-if beliefs. In particular, we have [cf. Eqs. $(B.12)$ and $(B.13)$],

$$\frac{a^o_{t,s'}}{a^o_{t,s}}=\frac{\lambda^{o,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\frac{Q_{t,s'}}{Q_{t,s}}.$$

Plugging this expression as well as Eq. $(C.12)$ into Eq. $(C.13)$, optimists' unit-wealth value function satisfies,

$$\begin{aligned}\xi^o_{t,s} &= \log\rho + \frac{1}{\rho}\left(\begin{array}{c} r^m_{t,s} - \rho - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda_s^{o,pl} - \overline{\lambda}^{pl}_{t,s}\right) + \lambda^o_s \log\left(\frac{\lambda^{o,pl}_{t,s}}{\overline{\lambda}^{pl}_{t,s}}\right)\end{array}\right) \\ &+\frac{\partial \xi^o_{t,s}}{\partial t} + \lambda^o_s\left(\frac{1}{\rho}\log\left(\frac{Q_{t,s'}}{Q_{t,s}}\right) + \xi^o_{t,s'} - \xi^o_{t,s}\right),\end{aligned}$$

This is the same as Eq. $(C.13)$ with the difference that the as-if beliefs, $\lambda_s^{o,pl}$, are used to calculate their positions on (and the payoffs from) the contingent securities, whereas the actual beliefs, λ^o_s, are used to calculate the transition probabilities. Using the same steps after Eq. $(C.13)$, we also obtain $(C.17)$ with $i = o$.

We next characterize the first-best and the gap value functions, $v^{i,*}_{t,s}$ and $w^i_{t,s}$, that we use in the main text. By definition, the first-best value function solves the same differential equation $(C.17)$ after substituting $q_{t,s} = q^*$. It follows that the gap value function $w^i_{t,s} = v^i_{t,s} - v^{i,*}_{t,s}$, solves,

$$\rho w^i_{t,s} - \frac{\partial w^i_{t,s}}{\partial t} = q_{t,s} - q^* + \lambda^i_s\left(w^i_{t,s'} - w^i_{t,s}\right),$$

which is the same as the differential equation (47) without macroprudential policy. The latter affects the path of prices, $q_{t,s}$, but it does not affect how these prices translate into gap values.

Note also that, as before, the value functions can be written as functions of optimists' wealth share, $\left\{v^i_s(\alpha), v^{i,*}_s(\alpha), w_s(\alpha)\right\}_{s,i}$. For completeness, we also characterize the differential equations that these functions satisfy in equilibrium with macroprudential policy. Combining Eq. $(C.17)$ with the dynamics of optimists' wealth share conditional on no transition, $\dot{\alpha}_{t,s} = -\left(\lambda^{o,pl}_s - \lambda^p_s\right)\alpha_{t,s}(1-\alpha_{t,s})$, the value functions, $\left(v^i_s(\alpha)\right)_{s,i}$, are found by solving,

$$\rho v^i_s(\alpha) = \left[\begin{array}{c} \log\rho + q^{pl}_s(\alpha) + \frac{1}{\rho}\left(\begin{array}{c} g - \frac{1}{2}\sigma_s^2 \\ -\left(\lambda^{i,pl}_s - \overline{\lambda}^{pl}_{t,s}\right) + \lambda^i_s\log\left(\frac{\lambda^{i,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\right)\end{array}\right) \\ -\frac{\partial v^i_s}{\partial\alpha}\left(\lambda^{o,pl}_s - \lambda^p_s\right)\alpha(1-\alpha) + \lambda^i_s\left(v^i_{s'}\left(\alpha\frac{\lambda^{o,pl}_s}{\overline{\lambda}^{pl}_{t,s}}\right) - v^i_s(\alpha)\right)\end{array}\right],$$

with appropriate boundary conditions. As in the main text, we denote the price functions with $q^{pl}_s(\alpha)$ to emphasize that they are determined by as-if beliefs. Likewise, the first-best value functions, $\left(v^{i,*}_s(\alpha)\right)_{s\in\{1,2\}}$, are found by solving the analogous system after replacing $q_s(\alpha)$ with q^*. Finally, combining Eq. (47) with the dynamics of optimists' wealth share, the gap-value functions, $\left(w^i_s(\alpha)\right)_{s,i}$, are found by solving Eq. (49) in the main text

Macroprudential policy in the recession state. The analysis in the main text concerns macroprudential policy in the boom state and maintains the assumption that $\lambda^{o,pl}_2 = \lambda^o_2$. We next consider the polar opposite case in which the economy is currently in the recession state $s = 2$, and the planner can apply macroprudential policy in this state, $\lambda^{o,pl}_2 \leq \lambda^o_2$ (she can induce optimists to act as if the recovery is less likely), but not in the other state, $\lambda^{o,pl}_1 = \lambda^o_1$. We obtain a sharp result for the special case in which optimists' wealth share is sufficiently large.

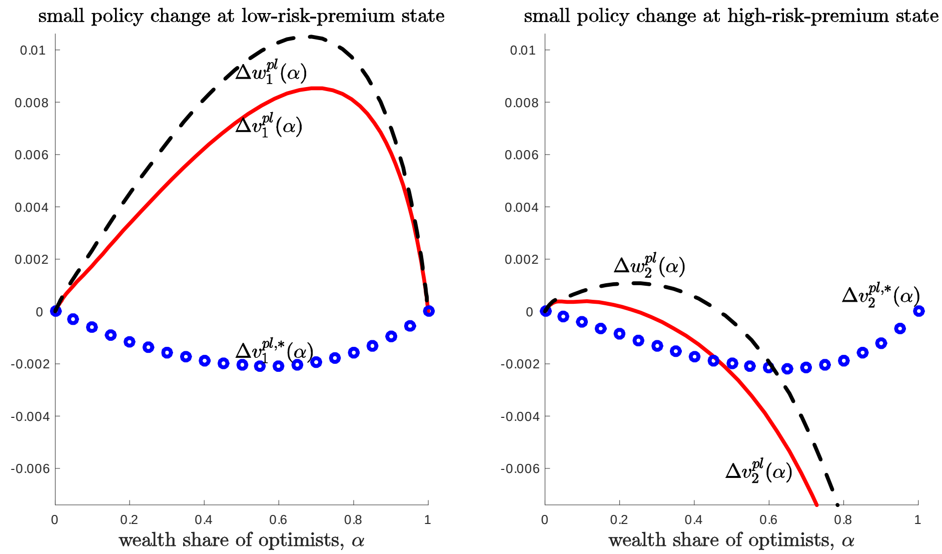

Figure 9: The left (resp. the right) panel illustrates the effect of a small change in macroprudential policy in the boom (resp. the recession) state.

Proposition 4. *Consider the model with two belief types. Consider the macroprudential policy in the recession state, $\lambda_2^{o,pl} \leq \lambda_2^o$ (and suppose $\lambda_1^{o,pl} = \lambda_1^o$). There exists a threshold, $\overline{\alpha} < 1$, such that if $\alpha \in (\overline{\alpha}, 1]$, then the policy reduces the gap value according to each belief, that is,*

$$\left.\frac{\partial w_2^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}\right|_{\lambda_2^{o,pl}=\lambda_2^o} < 0 \text{ for each } i \in \{o,p\}.$$

Thus, for $\alpha \in (\overline{\alpha}, 1]$, the policy also reduces the planner's value, $\left.\frac{\partial v_s^{pl}(\alpha)}{\partial(-\lambda_2^{o,pl})}\right|_{\lambda_2^o} = \left.\frac{\partial w_s^{pl}(\alpha)}{\partial(-\lambda_2^{o,pl})}\right|_{\lambda_2^o} < 0.$

Thus, in contrast to Proposition 3, macroprudential policy in the recession state can actually reduce the social welfare. The intuition can be understood by considering two counteracting forces. First, as before, macroprudential policy in the recession state is potentially valuable by reallocating optimists' wealth from the boom state $s = 1$ to the recession state $s = 2$. Intuitively, optimists purchase too many call options that pay if there is a transition to the boom state but that impoverish them in case the recession persists. They do not internalize that, if they keep their wealth, they will improve asset prices if the recession lasts longer.

However, there is a second force that does not have a counterpart in the boom state: Macroprudential policy in the recession state also affects the current asset price level, with potential implications for social welfare. It can be seen that making optimists less optimistic in the recession state shifts the price function downward, $\frac{\partial q_2^{pl}(\alpha)}{\partial(-\lambda_2^{o,pl})} < 0$ (as in the common-belief benchmark we analyzed in Section 4). Hence, the price impact of macroprudential policy is welfare reducing. Moreover, as optimists dominate the economy, $\alpha \to 1$, the price impact of the policy is still first order, whereas the beneficial effect from reshuffling optimists' wealth is second order. Thus, when optimists' wealth share is sufficiently large, the net effect of macroprudential policy is negative.

This analysis also suggests that, even when the policy in the recession state exerts a net positive effect, it would typically increase the welfare by a smaller amount than a comparable policy in the boom state. Figure

9 confirms this intuition. The left panel plots the change in the planner's value function in the boom state resulting from a small macroprudential policy change. Note that the policy slightly reduces the planner's first-best value function but increases the gap value function as well as the actual value function, illustrating Proposition 3 (see also Figure 5). The right panel illustrates the effect of the macroprudential policy in the recession state that would generate a similar distortion in the first-best equilibrium as the policy in the boom state.[26] Note that a small macroprudential policy in the recession state has a smaller positive impact when optimists' wealth share is small, and it has a negative impact when optimists' wealth share is sufficiently large, illustrating Proposition 4.

Proof of Proposition 3. We will prove the stronger result that

$$\left.\frac{\partial w_s^i(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} > 0 \text{ for each } i, s \text{ and } \alpha \in (0,1)\,. \tag{C.18}$$

That is, a marginal amount of macroprudential policy in the low-risk-premium state increases the gap value according to each investor (and in either state). Combining this with the definition of the planner's gap value function in (51) implies $\left.\frac{\partial w_s^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} > 0$. Combining this with $\left.\frac{\partial v_{0,s}^{pl,*}}{\partial \lambda^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} = 0$ (which follows from the First Welfare Theorem) and $v_{0,s}^{pl} = v_{0,s}^{pl,*} + w_{0,s}^{pl}$ implies $\left.\frac{\partial w_s^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o} = \left.\frac{\partial v_s^{pl}(\alpha)}{\partial \lambda_1^{o,pl}}\right|_{\lambda^{o,pl}=\lambda^o}$ for each s and $\alpha \in (0,1)$. Applying this result for state $s=1$ proves the proposition.

It remains to prove the claim in $(C.18)$. To this end, fix a belief type i and consider the representation of the gap value function in terms of bullishness, $w_s^i(b)$ [cf. $(C.4)$]. Following similar steps as in Lemma 1, we describe this as solution to the integral function,

$$w_s^i(b_{0,s}) = \int_0^\infty e^{-\left(\rho+\lambda_s^i\right)t}\left(q_s^{pl}(b_{t,s}) - q^* + \lambda_s^i w_{s'}^i(b_{t,s'})\right)dt, \tag{C.19}$$

for each $s \in \{1,2\}$ and $b_{0,s} \in \mathbb{R}$, where the bullishness has the closed form solution,

$$\begin{aligned} b_{t,s} &= b_{0,s} - t\left(\lambda_s^{o,pl} - \lambda_s^p\right), \\ b_{t,s'} &= b_{0,s} - t\left(\lambda_s^{o,pl} - \lambda_s^p\right) + \log\lambda_s^{o,pl} - \log\lambda_s^p. \end{aligned} \tag{C.20}$$

The main difference from the analysis in Lemma 1 is that the dynamics of bullishness is influenced by policy, as illustrated by the as-if beliefs in $(C.10)$. In addition, we denote the price functions with $q_s^{pl}(b)$ to emphasize they are in principle determined by as-if beliefs.

Next note that in this case the price functions $q_s^{pl}(b)$ are actually not affected by the as-if belief, $\lambda_1^{o,pl}$. The price function in the low-risk-premium state is not affected because $q_1^{pl}(b) = q^*$ (because the beliefs continue to satisfy Assumption 3 for small changes). The price function in the high-risk-premium state is also not affected because $\lambda_1^{o,pl}$ does not enter the differential equation that characterizes $q_2^{pl}(b)$ [see. Eq. (42) or Eq. $(C.7)$].

Using this observation, we implicitly differentiate the integral equation $(C.19)$ with respect to $\lambda_1^{o,pl}$, and

[26] Specifically, we calibrate the policy-induced belief change in the recession state so that the maximum decline in the planner's first-best value function is the same in both cases plotted in Figure 9, $\max_\alpha \left|\Delta v_2^{pl,*}(\alpha)\right| = \max_\alpha \left|\Delta v_1^{pl,*}(\alpha)\right|$.

use Eq. $(C.20)$, to obtain,

$$\begin{aligned}\frac{\partial w_1^i(b_{0,1})}{\partial \lambda_1^{o,pl}} &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_1^i\left(\frac{\partial w_2^i(b_{t,2})}{\partial \lambda_1^{o,pl}}+\frac{\partial w_2^i(b_{t,2})}{\partial b}\frac{db_{t,2}}{d\lambda_1^{o,pl}}\right)dt,\\ \frac{\partial w_2^i(b_{0,2})}{\partial \lambda_1^{o,pl}} &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_2^i\frac{\partial w_1^i(b_{t,1})}{\partial \lambda_1^{o,pl}}dt.\end{aligned}$$

Note also that, using Eq. $(C.20)$ implies, $\frac{db_{t,2}}{d\lambda_1^{o,pl}} = -t + \frac{1}{\lambda_1^{o,pl}}$. Plugging this into the previous system, and evaluating the partial derivatives at $\lambda_1^{o,pl} = \lambda_1^o$, we obtain,

$$\begin{aligned}\frac{\partial w_1^i(b_{0,1})}{\partial \lambda_1^{o,pl}} &= h(b_{0,1}) + \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_1^i\frac{\partial w_2^i(b_{t,2})}{\partial \lambda_1^{o,pl}}dt,\\ \frac{\partial w_2^i(b_{0,2})}{\partial \lambda_1^{o,pl}} &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_2^i\frac{\partial w_1^i(b_{t,1})}{\partial \lambda_1^{o,pl}}dt,\\ \text{where } h(b_{0,1}) &= \int_0^\infty e^{-(\rho+\lambda_1^i)t}\lambda_1^i\frac{\partial w_2^i(b_{t,2})}{\partial b}\left(-t+\frac{1}{\lambda_1^o}\right)dt.\end{aligned} \tag{C.21}$$

Note that the function, $h(b)$, is bounded since the derivative function, $\frac{\partial w_2^i(b)}{\partial b}$, is bounded (see $(C.11)$). Hence, Eq. $(C.21)$ describes the partial derivative functions, $\left(\frac{\partial w_s^i(b)}{\partial \lambda_1^{o,pl}}|_{\lambda_1^{o,pl}=\lambda_1^o}\right)_s$, as a fixed point of a corresponding operator T over bounded functions. Since $h(b)$ is bounded, it can be checked that the operator T is also a contraction mapping with respect to the sup norm. In particular, it has a fixed point, which corresponds to the partial derivative functions.

The analysis so far applies generally. We next consider the special case, $\lambda_1^o = \lambda_1^p$, and show that it implies the partial derivatives are strictly positive. In this case, $\lambda_1^i \equiv \lambda_1$ for each $i \in \{o,p\}$. In addition, Eq. $(C.10)$ implies $b_{t,2} = b_{0,2}$. Using these observations, for each $b_{0,1}$, we have,

$$\begin{aligned}h(b_{0,1}) &= \frac{\partial w_2^i(b_{0,2})}{\partial b}\int_0^\infty e^{-(\rho+\lambda_1)t}\lambda_1\left(-t+\frac{1}{\lambda_1}\right)dt\\ &= \frac{\partial w_2^i(b_{0,2})}{\partial b}\left(-\frac{\lambda_1}{\rho+\lambda_1}\frac{1}{\rho+\lambda_1}+\frac{1}{\rho+\lambda_1}\right) > 0.\end{aligned}$$

Here, the inequality follows since $\frac{\partial w_2^i(b_{0,2})}{\partial b} > 0$ [cf. Lemma 1]. Since $h(b) > 0$ for each b, and $\lambda_s^i > 0$, it can further be seen that the fixed point that solves $(C.21)$ satisfies $\frac{\partial w_s^i(b)}{\partial \lambda_s^{o,pl}} > 0$ for each b and $s \in \{1,2\}$. Using Eq. $(C.6)$, we also obtain $\frac{\partial w_s^i(\alpha)}{\partial \lambda_1^{o,pl}} > 0$ for each $s \in \{1,2\}$ and $\alpha \in (0,1)$. Since the analysis applies for any fixed belief type i, this establishes the claim in $(C.18)$ and completes the proof. □

Proof of Proposition 4. A similar analysis as in the proof of Proposition 3 implies that the partial derivative function, $\frac{\partial w_s^i(b)}{\partial(-\lambda_2^{o,pl})}$, is characterized as the fixed point of a contraction mapping over bounded functions (the analogue of Eq. $(C.21)$ for state 2). In particular, the partial derivative exists and it is bounded. Moreover, since the corresponding contraction mapping takes continuous functions into continuous functions, the partial derivative is also continuous over $b \in \mathbb{R}$. Using Eq. $(C.6)$, we further obtain that the partial derivative, $\frac{\partial w_s^i(\alpha)}{\partial(-\lambda_2^{o,pl})}$, is continuous over $\alpha \in (0,1)$.

Next note that $w_s^i(1) \equiv \lim_{\alpha\to 1} w_s^i(\alpha)$ exists and is equal to the value function according to type i beliefs when all investors are optimistic. In particular, the asset prices are given by $q_1^{pl} = q^*$ and $q_2^{pl} = q^o$, and the transition probabilities are evaluated according to type i beliefs. Then, following the same steps as in our analysis of value functions in Appendix C.1, we obtain,

$$\begin{aligned}\rho w_s^i(1) &= \beta_s^i q_s^o + \left(1-\beta_s^i\right) q_{s'}^o - q^*,\\ \text{where } \beta_s^i &= \frac{\rho + \lambda_{s'}^i}{\rho + \lambda_{s'}^i + \lambda_s^i}.\end{aligned}$$

Here, β_s^i can be thought of as the expected discount time the investor spends in state s according to type i beliefs. We consider this equation for $s = 2$ and take the derivative with respect to $\left(-\lambda_2^{o,pl}\right)$ to obtain,

$$\frac{\partial w_2^i(1)}{\partial\left(-\lambda_2^{o,pl}\right)} = \beta_2^i \frac{dq_2^o}{d\left(-\lambda_2^{o,pl}\right)} < 0.$$

Here, the inequality follows since reducing optimists' optimism reduces the price level in the common belief benchmark (see Section 4).

Note that the inequality, $\frac{\partial w_2^i(1)}{\partial\left(-\lambda_2^{o,pl}\right)} < 0$, holds for each belief type i. Using the continuity of the partial derivative function, $\frac{\partial w_2^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}$, we conclude that there exists $\overline{\alpha}$ such that $\left.\frac{\partial w_2^i(\alpha)}{\partial\left(-\lambda_2^{o,pl}\right)}\right|_{\lambda_2^{o,pl}=\lambda_2^o} < 0$ for each i, s and $\alpha \in (\overline{\alpha}, 1)$, completing the proof. □

D. Appendix: Extension with investment and endogenous growth

Our baseline setup in the main text assumes there is no investment and the expected growth rate of capital is exogenous. In this appendix, we analyze a more general environment that relaxes these assumptions. We first present the environment, define the equilibrium, and provide a partial characterization. We then characterize this equilibrium when investors have common beliefs and generalize Proposition 1 to this setting.

D.1. Environment and equilibrium with investment

We focus on the components that are different than the baseline setting described in Section 3.

Potential output and endogenous growth. We modify the equation that describes the dynamics of capital (14) as follows,

$$\frac{dk_{t,s}}{k_{t,s}} = g_{t,s}dt + \sigma_s dZ_t \qquad \text{where } g_{t,s} \equiv \varphi\left(\iota_{t,s}\right) - \delta. \tag{D.1}$$

Here, $\iota_{t,s} = \frac{i_{t,s}}{k_{t,s}}$ denotes the investment rate, $\varphi\left(\iota_{t,s}\right)$ denotes a neoclassical production function for capital (we will work with a special case that will be described below), and δ denotes the depreciation rate. Hence, the growth of capital is no longer exogenous: it depends on the endogenous level of investment as well as depreciation.

Investment firms. To endogenize investment, we introduce a new set of firms, which we refer to as investment firms, that own and manage the aggregate capital stock. These firms rent capital to production firms to earn the instantaneous rental rate, $R_{t,s}$. They also make investment decisions to maximize the value of capital. Letting $\tilde{Q}_{t,s}$ denote the price of capital, the firm's investment problem can generally be written as,

$$\max_{\iota_{t,s}} \tilde{Q}_{t,s}\varphi\left(\iota_{t,s}\right)k_{t,s} - \iota_{t,s}k_{t,s}. \tag{D.2}$$

As before, we denote the price of the market portfolio per unit of capital with $Q_{t,s}$. In this case, the market portfolio represents a claim on investment firms as well as production firms. Hence, we have the inequality $\tilde{Q}_{t,s} \leq Q_{t,s}$, where the residual price, $Q_{t,s} - \tilde{Q}_{t,s}$, corresponds to the value of production firms per unit of capital. We make assumptions (that we describe below) so that output accrues to the investment firms in the form of return to capital, $y_{t,s} = R_{t,s}k_{t,s}$, and there are no monopoly profits. This in turn implies that the value of the market portfolio is equal to the value of capital (and the value of production firms is zero), that is,

$$Q_{t,s} = \tilde{Q}_{t,s}. \tag{D.3}$$

This simplifies the analysis by ensuring that we have only one price to characterize. Considering a different division of output between return to capital and profits will have a quantitative effect on investment, as illustrated by problem (D.2), but we conjecture that it would leave our qualitative results on investment unchanged. We leave a systematic exploration of this issue for further research.

Return of the market portfolio. The price of the market portfolio per unit of capital follows the same equation (15) as in the main text. The volatility of the market portfolio (absent state transitions) is also unchanged and given by σ_s. However, the return on the market portfolio conditional on no transition is slightly modified and given by,

$$r^m_{t,s} = \frac{y_{t,s} - \iota_{t,s} k_{t,s}}{Q_{t,s} k_{t,s}} + \left(g_{t,s} + \mu^Q_{t,s}\right). \tag{D.4}$$

Hence, the dividend yield is now net of the investment expenditures the (investment) firms undertake. In addition, the expected growth of the price of the market portfolio is now endogenous and given by $g_{t,s}$.

Nominal rigidities and equilibrium in goods markets. As before, the supply side of our model features nominal rigidities similar to the standard New Keynesian model that ensure output is determined by aggregate demand. In this case, demand comes from investment as well as consumption so we modify Eq. (19) as,

$$y_{t,s} = \eta_{t,s} A k_{t,s} = \int_I c^i_{t,s} di + k_{t,s}\iota_{t,s}, \text{ where } \eta_{t,s} \in [0,1]. \tag{D.5}$$

We also modify the microfoundations that we provide in Section B.1.2 so that all output accrues to investment firms as return to capital and there are no monopoly profits, that is,

$$R_{t,s} = A\eta_{t,s} \text{ and thus } y_{t,s} = R_{t,s} k_{t,s}. \tag{D.6}$$

We relegate a detailed description of these microfoundations to the end of this appendix.

Combining Eqs. $(D.5)$, $(D.4)$, (22) and (17), we can also rewrite the instantaneous (expected) return to the market portfolio as,

$$r^m_{t,s} = \rho + g_{t,s} + \mu^Q_{t,s}.$$

Hence, as in the main text, the equilibrium dividend yield is equal to the consumption rate ρ.

The rest of the model is the same as in Section 3. We formally define the equilibrium as follows.

Definition 2. *The equilibrium with investment and endogenous growth is a collection of processes for allocations, prices, and returns such that capital evolves according to* (14)*, the price of market portfolio per capital evolves according to* (15)*, its instantaneous return (conditional on no transition) is given by* $(D.4)$*, investment firms maximize (cf. Eqs.* $(D.7)$*), investors maximize (cf. Appendix B.1.1), asset markets clear (cf. Eqs.* (17) *and* (18)*), production firms maximize (cf. Appendix D.3), goods markets clear (cf. Eq.* (19)*), all output accrues to agents in the form of return to capital* $(D.6)$*, the price of the market portfolio per unit of capital is the same as the price of capital (cf. Eq.* $(D.3)$*), and the interest rate policy follows the rule in* (21)*.*

We next provide a partial characterization of the equilibrium with investment.

Investors' optimality conditions. Eqs. $(22 - 25)$ in the main text remain unchanged.

Investment firms' optimality conditions. Under standard regularity conditions for the capital production function, $\varphi(\iota)$, the solution to problem $(D.2)$ is determined by the optimality condition,

$$\varphi'(\iota_{t,s}) = 1/Q_{t,s}.$$

We will work with the special and convenient case proposed by Brunnermeier and Sannikov (2016b): $\varphi(\iota) = \psi \log\left(\frac{\iota}{\psi}+1\right)$. In this case, we obtain the closed form solution,

$$\iota(Q_{t,s}) = \psi(Q_{t,s}-1). \tag{D.7}$$

The parameter, ψ, captures the sensitivity of investment to asset prices.

Growth-asset price relation. Note also that the amount of capital produced is given by,

$$\varphi(\iota(Q_{t,s})) = \psi q_{t,s}, \text{ where } q_{t,s} \equiv \log(Q_{t,s}). \tag{D.8}$$

The log price level, $q_{t,s}$, will simplify some of the expressions. Combining Eq. (D.8) with Eq. (14), we obtain Eq. (36) in the main text, which we replicate here for ease of exposition,

$$g_{t,s} = \psi q_{t,s} - \delta.$$

Hence, the expected growth rate of capital (and potential output) is now endogenous and depends on asset prices. Lower asset prices reduce investment, which translates into lower growth and lower potential output in future periods. As we will describe, this mechanism provides a new source of amplification.

Output-asset price relation. As in the main text, there is a tight relationship between output and asset prices as in the two period model. Specifically, Eq. (26) in the main text continues to apply and implies that aggregate consumption is a constant fraction of aggregate wealth. Plugging this into Eq. (19) and using the investment equation (D.7), we obtain Eq. (35) in the main text, which we replicate here for ease of exposition,

$$A\eta_{t,s} = \rho Q_{t,s} + \psi(Q_{t,s}-1) = (\rho+\psi) Q_{t,s} - \psi.$$

In this case, factor utilization (and output) depends on capital not only because consumption depends on asset prices through a wealth effect but also because investment depends on asset prices through a standard marginal-Q channel. Full factor utilization, $\eta_{t,s} = 1$, obtains only if the price of capital is at a particular level

$$Q^* \equiv \frac{A+\psi}{\rho+\psi}.$$

This is the efficient price level that ensures that the implied consumption and investment clear the goods market. Likewise, the economy features a demand recession, $\eta_{t,s} < 1$, if and only if the price of capital is strictly below Q^*.

Combining the output-asset price relation (together with $y_{t,s} = A\eta_{t,s}k_{t,s}$) with Eq. ($D$.7), we obtain $\frac{y_{t,s}-\iota_{t,s}k_{t,s}}{Q_{t,s}k_{t,s}} = \rho$. Using this expression along with Eq. (36), we can rewrite Eq. (16) as,

$$r^m_{t,s} = \rho + \psi q_{t,s} - \delta + \mu^Q_{t,s}. \tag{D.9}$$

Hence, a version of Eq. (28) in the main text continues to apply. In equilibrium, the dividend yield on the market portfolio is equal to the consumption rate ρ. Moreover, the growth rate of dividends is endogenous and is determined by the growth-asset price relation.

Combining the output-asset price relation with the interest rate policy in (21), we also summarize the goods market side of the economy with (29) as in the main text. In particular, the equilibrium at any time and state takes one of two forms. If the natural interest rate is nonnegative, then the interest rate policy ensures that the price per unit of capital is at the efficient level, $Q_{t,s} = Q^*$, capital is fully utilized, $\eta_{t,s} = 1$, and output is equal to its potential, $y_{t,s} = Ak_{t,s}$. Otherwise, the interest rate is constrained, $r^f_{t,s} = 0$, the price is at a lower level, $Q_{t,s} < Q^*$, and output is determined by aggregate demand according to Eq. (27).

As a benchmark, we characterize the first-best equilibrium without interest rate rigidities. In this case, there is no lower bound constraint on the interest rate, so the price of capital is at its efficient level at all times and states, $Q_{t,s} = Q^*$. Combining this with Eq. (D.9), we obtain $r^m_{t,s} = \rho + \psi q^* - \delta$, where $q^* = \log Q^*$. Substituting this into Eq. (23) and using Eq. (25), we solve for "rstar" as,

$$r^{f*}_s = \rho + \psi q^* - \delta - \sigma^2_s \text{ for each } s \in \{1, 2\}. \tag{D.10}$$

Hence, in the first-best equilibrium the risk premium shocks are fully absorbed by the interest rate. We next characterize the equilibrium with interest rate rigidities for the case in which investors have common beliefs.

D.2. Common beliefs Benchmark with Investment

Suppose investors have common beliefs (that is, $\lambda^i_s \equiv \lambda_s$ for each i). Substituting Eq. (D.9) into (23), we obtain the following analogue of the risk balance conditions (31),

$$\sigma_s = \frac{\rho + \psi q_s - \delta + \lambda_s \left(1 - \frac{Q_s}{Q_{s'}}\right) - r^f_s}{\sigma_s} \text{ for each } s \in \{1, 2\}. \tag{D.11}$$

The only difference is that the growth rate in each state is endogenous and described by the growth-asset price relation, $g_s \equiv \psi q_s - \delta$, where recall that $q_s = \log Q_s$ [cf. Eq. (36)]. We also make the following analogue of Assumption 1.

Assumption 1$^{\text{I}}$. $\sigma^2_2 > \rho + \psi q^* - \delta > \sigma^2_1$.

With this assumption, we conjecture that the low-risk-premium state 1 features positive interest rates, efficient asset prices, and full factor utilization, $r^f_1 > 0, q_1 = q^*$ and $\eta_1 = 1$, whereas the high-risk-premium state 2 features zero interest rates, lower asset prices, and imperfect factor utilization, $r^f_2 = 0, q_2 < q^*$ and $\eta_2 < 1$.

Equilibrium in the high-risk-premium state and amplification from the growth-asset price relation. Under our conjecture, the risk balance condition (D.11) for the high-risk state $s = 2$ can be written as,

$$\sigma_2 = \frac{\rho + \psi q_2 - \delta + \lambda_2 \left(1 - \frac{Q_2}{Q^*}\right)}{\sigma_2}. \tag{D.12}$$

As before, this equation illustrates an amplification mechanism: Since the recession reduces firms' earnings, a lower price level does not increase the dividend yield (captured by the constant dividend yield, $\rho = \frac{\rho Q_2}{Q_2}$). Unlike before, Eq. ($D$.12) illustrates a second amplification mechanism captured by the growth-asset price relation, $g_2 = \psi q_2 - \delta$. In particular, a lower price level lowers investment, which reduces the expected growth of potential output and profits, which in turn lowers the return to capital. The strength of this second mechanism depends on the sensitivity of investment to asset prices, captured by the term ψq_2. Figure 1 in the introduction presents a graphical illustration of the two amplification mechanisms.

The stabilizing force from price declines comes from the expected transition into the low-risk-premium state captured by the term, $\lambda_2 \left(1 - \frac{Q_2}{Q^*}\right)$. As before, to ensure that there exists an equilibrium with positive prices, we need a minimum degree of optimism, which we capture with the following analogue of Assumption 2.

Assumption 2^{I}. $\lambda_2 \geq \lambda_2^{\min}$, where $\lambda_2^{\min}$ is the unique solution to the following equation over the range $\lambda_2 \geq \psi$:

$$\rho + \psi q^* - \delta + \lambda_2^{\min} - \psi + \psi \log\left(\psi/\lambda_2^{\min}\right) = \sigma_2^2.$$

This assumption ensures that there exists a unique $Q_2 \in (0, Q^*)$ that solves Eq. $(D.12)$ (see the proof at the end of this section).

Equilibrium in the low-risk-premium state. Under our conjecture, the risk balance condition $(D.11)$ can be written as,

$$r_1^f = \rho + \psi q^* - \delta - \sigma_1^2 + \lambda_1 \left(1 - \frac{Q^*}{Q_2}\right) \tag{D.13}$$

As before, the interest rate adjusts to ensure that the risk balance condition is satisfied with the efficient price level, $Q_1 = Q^*$. For our conjectured equilibrium, we also assume an upper bound on λ_1 so that the implied interest rate is positive, $r_1^f > 0$, which we capture with the following analogue of Assumption 3.

Assumption 3^{I}. $\lambda_1 < \left(\rho + \psi q^* - \delta - \sigma_1^2\right) / \left(Q^*/Q_2 - 1\right)$, where $Q_2 \in (0, Q^*)$ solves Eq. (33).

As before, Eq. $(D.13)$ implies that r_1^f is decreasing in the transition probability, λ_1, as well as in the asset price drop conditional on transition, Q^*/Q_2.

The following result summarizes the characterization of equilibrium and generalizes Proposition 1. The testable predictions regarding the effect of risk premium shocks on consumption, investment, and output follow by combining the characterization with Eqs. $(26), (D.7), (35)$, and .

Proposition 5. *Consider the extended model with investment with two states, $s \in \{1, 2\}$, with common beliefs and Assumptions 1^I-3^I. The low-risk-premium state 1 features a positive interest rate, efficient asset prices and full factor utilization, $r_1^f > 0, Q_1 = Q^*$ and $\eta_1 = 1$. The high-risk state 2 features zero interest rate, lower asset prices, and a demand-driven recession, $r_2^f = 0, Q_2 < Q^*$, and $\eta_2 < 1$, as well as a lower level of consumption, $c_{t,2}/k_{t,2} = \rho Q_2$, investment, $i_{t,2}/k_{t,2} = \psi (Q_2 - 1)$, output, $y_{t,2}/k_{t,2} = (\rho + \psi) Q_2 - \psi$, and growth, $g_2 = \psi q_2 - \delta$. The price of capital in state 2 is characterized as the unique solution to Eq. $(D.12)$, and the risk-free rate in state 1 is given by Eq. $(D.13)$.*

Proof. Most of the proof is provided in the discussion leading to the proposition. The remaining step is to show that Assumptions 1^{I}-2^{I} ensure there exists a unique solution, $Q_2 \in (0, Q^*)$ (equivalently, $q_2 < q^*$) to Eq. $(D.12)$.

To this end, we define the function,

$$f(q_2, \lambda_2) = \rho + \psi q_2 - \delta + \lambda_2 \left(1 - \frac{\exp(q_2)}{Q^*}\right) - \sigma_2^2.$$

The equilibrium price is the solution to, $f(q_2, \lambda_2) = 0$ (given λ_2). Note that $f(q_2, \lambda_2)$ is a concave function of q_2 with $\lim_{q_2 \to -\infty} f(q_2, \lambda_2) = \lim_{q_2 \to \infty} f(q_2, \lambda_2) = -\infty$. Its derivative is,

$$\frac{\partial f(q_2, \lambda_2)}{\partial q_2} = \psi - \lambda_2 \exp(q_2 - q^*).$$

Thus, for fixed λ_2, it is maximized at,

$$q_2^{\max}(\lambda_2) = q^* + \log(\psi/\lambda_2).$$

Moreover, the maximum value is given by

$$\begin{aligned} f\left(q_2^{\max}\left(\lambda_2\right),\lambda_2\right) &= \rho-\delta+\psi\left(q^*+\log\left(\psi/\lambda_2\right)\right)+\lambda_2\left(1-\exp\left(\log\left(\psi/\lambda_2\right)\right)\right)-\sigma_2^2 \\ &= \rho-\delta+\psi q^*+\psi\log\left(\psi/\lambda_2\right)+\lambda_2-\psi-\sigma_2^2. \end{aligned}$$

Next note that, by Assumption 1^{I}, the maximum value is strictly negative when $\lambda_2=\psi$, that is, $f\left(q_2^{\max}\left(\psi\right),\psi\right)<0$. Note also that $\frac{df\left(q_2^{\max}\left(\lambda_2\right),\lambda_2\right)}{d\lambda_2}=1-\frac{\psi}{\lambda_2}$, which implies that the maximum value is strictly increasing in the range $\lambda_2\geq\psi$. Since $\lim_{\lambda_2\to\infty}f\left(q_2^{\max}\left(\lambda_2\right),\lambda_2\right)=\infty$, there exists $\lambda_2^{\min}>\psi$ that ensures $f\left(q_2^{\max}\left(\lambda_2^{\min}\right),\lambda_2^{\min}\right)=0$. By Assumption 2^{I}, the transition probability satisfies $\lambda_2\geq\lambda_2^{\min}$, which implies that $f\left(q_2^{\max}\left(\lambda_2\right),\lambda_2\right)\geq 0$. By Assumption 1^{I}, we also have that $f\left(q^*,\lambda_2\right)<0$. It follows that, under Assumptions 1^{I}-2^{I}, there exists a unique price level, $q_2\in[q_2^{\max},q^*)$, that solves the equation, $f\left(q_2,\lambda_2\right)=0$. □

D.3. New Keynesian microfoundations for nominal rigidities with investment

In the rest of this appendix, we present the microfoundations for nominal rigidities that lead to Eqs. $(D.5)$ and $(D.6)$. The production structure is the same as in Appendix B.1.2. Specifically, there is a continuum of monopolistically competitive production firms that produce intermediate goods according to $(B.3)$, and there is a competitive sector that produces the final good according to $(B.4)$. This also implies the demand for production firms is given by$(B.5)$. One difference is that production firms do not own the capital but they rent it from investment firms at rate $R_{t,s}$. Hence, they choose how their capital input $k_{t,s}\left(\nu\right)$, in addition to their factor utilization rate, $\eta_{t,s}\left(\nu\right)$, as well as production and pricing decisions, $y_{t,s}\left(\nu\right),p_{t,s}\left(\nu\right)$.

These features ensure that the production firm's output will be split between their capital expenditures (that they pay to investment firms) and monopoly profits. To simplify the analysis, we make assumptions so that there are no monopoly profits in equilibrium (and all output accrues to investment firms as return to capital). Specifically, we assume the government taxes the firm's profits lump sum, and redistributes these profits to the firms in the form of a linear subsidy to capital.

Formally, we let $\Pi_{t,s}\left(\nu\right)$ denote the equilibrium pre-tax profits of firm ν (that will be characterized below). We assume each firm is subject to the lump-sum tax determined by the average profits of all firms,

$$T_{t,s}=\int_\nu \Pi_{t,s}\left(\nu\right)d\nu. \tag{D.14}$$

We also let $R_{t,s}-\tau_{t,s}$ denote the after-subsidy cost of renting capital, where $R_{t,s}$ denotes the equilibrium rental rate paid to investment firms, and $\tau_{t,s}$ denotes a linear subsidy paid by the government. We assume the magnitude of the subsidy is determined by the government's break-even condition,

$$\tau_{t,s}\int_\nu k_{t,s}\left(\nu\right)d\nu=T_{t,s}. \tag{D.15}$$

Without price rigidities, the firm chooses $p_{t,s}\left(\nu\right),k_{t,s}\left(\nu\right),\eta_{t,s}\left(\nu\right)\in[0,1],y_{t,s}\left(\nu\right)$, to maximize its (pre-tax) profits,

$$\Pi_{t,s}\left(\nu\right)\equiv p_{t,s}\left(\nu\right)y_{t,s}\left(\nu\right)-\left(R_{t,s}-\tau_{t,s}\right)k_{t,s}\left(\nu\right), \tag{D.16}$$

subject to the supply constraint in $(B.3)$ and the demand constraint in $(B.5)$. As in Appendix B.1.2, the demand constraint holds as equality. Then, the optimality conditions imply,

$$\eta_{t,s}(\nu) = 1 \text{ and } p_{t,s}(\nu) = \frac{\varepsilon}{\varepsilon - 1} \frac{R_{t,s} - \tau_{t,s}}{A}.$$

That is, the firm utilizes its capital at full capacity (as before) and it increases its capital input and production up to the point at which its price is a constant markup over its after-subsidy marginal cost. In a symmetric-price equilibrium, we further have, $p_{t,s}(\nu) = 1$. Using Eqs. $(B.3)$ and $(D.15)$, this further implies,

$$y_{t,s}(\nu) = y_{t,s} = Ak_{t,s} \text{ and } R_{t,s} = \frac{\varepsilon - 1}{\varepsilon} A + \tau_{t,s} = A. \tag{D.17}$$

That is, output is equal to potential output, and capital earns its marginal contribution to potential output (in view of the linear subsidies).

Now consider the alternative setting in which the firms have a preset nominal price that is equal across firms, $P_{t,s}(\nu) = P$. In particular, the relative price of a firm is fixed and equal to one, $p_{t,s}(\nu) = 1$. The firm chooses the remaining variables, $k_{t,s}(\nu), \eta_{t,s}(\nu) \in [0,1], y_{t,s}(\nu)$, to maximize its (pre-tax) profits, $\Pi_{t,s}(\nu)$, subject to the supply constraint in $(B.3)$ and the demand constraint, $(B.5)$. Combining the constraints and using $p_{t,s}(\nu) = 1$, the firm's problem can be written as,

$$\max_{\eta_{t,s}(\nu), k_{t,s}(\nu)} A\eta_{t,s}(\nu) k_{t,s}(\nu) - (R_{t,s} - \tau_{t,s}) k_{t,s}(\nu) \text{ s.t. } 0 \leq \eta_{t,s}(\nu) \leq 1 \text{ and } A\eta_{t,s}(\nu) k_{t,s}(\nu) \leq y_{t,s}.$$

We conjecture an equilibrium in which $R_{t,s} = \tau_{t,s}$ and firms choose symmetric capital inputs, $k_{t,s}(\nu) = k_{t,s}$. Under this equilibrium, the marginal cost of renting capital is zero, $R_{t,s} - \tau_{t,s} = 0$. This verifies that it is optimal for firms to choose symmetric inputs, $k_{t,s}(\nu) = k_{t,s}$. After substituting these expressions, the firm's problem becomes equivalent to its counterpart in Appendix B.1.2. Following the same steps there, the optimal factor utilization is given by $\eta_{t,s}(\nu) = \frac{y_{t,s}}{Ak_{t,s}} \leq 1$. Hence, output is determined by aggregate demand, $y_{t,s}$, subject to the capacity constraint, $\eta_{t,s}(\nu) \leq 1$.

In the conjectured equilibrium, the production firms choose the same level of inputs and factor utilization rates and produce the same level of output as each other. Therefore, they also have the same level of pre-tax profits. Using Eqs. $(D.16)$ together with $R_{t,s} = \tau_{t,s} = 0$, we also calculate the pre-tax profit level as $\Pi_{t,s} = y_{t,s}$. Substituting this into Eqs. $(D.14)$ and $(D.15)$, we obtain $\tau_{t,s} = y_{t,s}/k_{t,s} = \eta_{t,s}A$. Substituting this into Eq. $(D.16)$, we further obtain $R_{t,s} = y_{t,s}/k_{t,s} = \eta_{t,s}A$. This verifies the conjecture, $R_{t,s} = \tau_{t,s}$.

In sum, when the firms' nominal prices are fixed, aggregate output is determined by aggregate demand subject to the capacity constraint, which verifies Eq. $(D.5)$. Moreover, thanks to lump-sum costs to profits and linear subsidies to capital, all output accrues to the investment firms as return to capital, which verifies Eq. $(D.6)$.

E. Appendix: Data Details and Omitted Empirical Results

This appendix presents the details of the data sources and variable construction used in Section 7, and presents the empirical results (tables and figures) omitted from the main text.

House price index. We rely on the cross-country quarterly panel dataset described in Mack et al. (2011). The dataset is regularly updated and publicly available at https://www.dallasfed.org/institute/houseprice. We use the inflation-adjusted (real) house price index measure to construct the shock variable in our regression analysis (see (53)). Our country coverage is to a large extent determined by the availability of this measure, e.g., we exclude a few developed countries such as Portugal and Austria for which we do not have consistent data on real house prices.

Euro or Exchange Rate Mechanism (Euro/ERM) status. We hand-collect this data from various online sources. A country-quarter is included in the Euro/ERM sample if the country is a member of the Euro or the European Exchange Rate mechanism in most of the corresponding calendar year. Table 1 describes the Euro/ERM status by year for all countries in our sample.

GDP, consumption, investment. We obtain this data from the OECD's quarterly national accounts dataset (available at https://stats.oecd.org). We use the variables calculated according to the expenditure approach. The corresponding OECD subject codes are as follows:

- GDP: "B1_GE" (Gross domestic product – expenditure approach).
- Consumption: "P31S14_S15" (Private final consumption expenditure)
- Investment: "P51" (Gross fixed capital formation)

For each of these variables, we use the measures that are adjusted for inflation as well as seasonality. The OECD measure code is: "LNBQRSA" (National currency, chained volume estimates, national reference year, quarterly levels, seasonally adjusted).

Relative GDP (with PPP-adjusted prices in a common base year). We obtain an alternative GDP measure from the OECD's annual national accounts dataset (available at https://stats.oecd.org). We use the variable calculated according to the expenditure approach (with subject code "B1_GE"), measured with PPP-adjusted prices in a common base year. The OECD measure code is: "VPVOB" (Current prices, constant PPPs, OECD base year). We use the value of this measure in 1990 to weight all of our regressions (see (53)).

CPI. We obtain this data from the OECD's prices and purchasing power parities dataset (available at https://stats.oecd.org). We use the core CPI measure that excludes food and energy. The OECD subject code is: "CPGRLE" (Consumer prices - all items non-food, non-energy). We use the annual measure, which is less subject to seasonality, and we linearly interpolate this to obtain a quarterly measure.

Unemployment rate. We obtain this data from the OECD's key short-term economic indicators database (available at https://stats.oecd.org). We use the harmonized unemployment rate measure with seasonal adjustment and at quarterly frequency. The OECD subject code is "LRHUTTTT" (Harmonised unemployment rate: all persons, s.a).

The policy interest rate. Obtaining the policy interest rate is not as trivial as it might sound since different central banks conduct monetary policy in terms of different target rates (and sometimes without specifying a target rate, or by monitoring multiple rates). On the other hand, the selection does not substantially affect the results since short-term risk-free rates within a developed country are often highly correlated. Following Romer and Romer (2018), we use announced policy target rates when available, and otherwise we use collateralized short-term market rates (such as Repo rates or Lombard rates). For Eurozone countries, we use the local collateralized rate until the country joins the Euro, and we switch to the European Central Bank's (ECB) main refinancing operations (MRO) rate after the country joins the Euro.

For most of the countries, we construct our own measure of the policy interest rate according to the above selection criteria by using data from the Global Financial Data's GFDATABASE (GFD). This is a proprietary database that contains a wealth of information on various asset prices (see https://www.globalfinancialdata.com for details).

For a few countries (specified below), we instead rely on the Bank for International Settlements's (BIS) database on central bank policy interest rates (publicly available at https://www.bis.org/statistics/cbpol.htm). We switch to the BIS measure when we cannot construct an appropriate measure using the GFD; or when the BIS measure has greater coverage than ours and the two measures are highly correlated. From either database, we obtain monthly data and convert to quarterly data by averaging over the months within the quarter.

- United States: GFD ticker "IDUSAFFD" (USA Fed Funds Official Target Rate).
- United Kingdom: GFD ticker "IDGBRD" (Bank of England Base Lending Rate).
- Australia: GFD ticker "IDAUSD" (Australia Reserve Bank Overnight Cash Rate).
- South Korea: GFD ticker "IDKORM" (Bank of Korea Discount Rate).
- Germany: GFD ticker "IDDEULD" (Germany Bundesbank Lombard Rate) until the country joins the Euro. Afterwards, we use the ECB MRO rate. The corresponding GFD ticker is: "IDEURMW" (Europe Marginal Rate on Refinancing Operations).
- New Zealand: GFD ticker "IDNZLD" (New Zealand Reserve Bank Official Cash Rate).
- France: GFD ticker "IDFRARD" (Bank of France Repo Rate) until the country joins the Euro.
- Denmark: We use the BIS measure (highly correlated with our measure and greater coverage).
- Finland: GFD ticker "IDFINRM" (Bank of Finland Repo Rate) until the country joins the Euro.
- Sweden: GFD ticker "IDSWERD" (Sweden Riksbank Repo Rate).
- Israel: GFD ticker "IDISRD" (Bank of Israel Discount Rate).
- Italy: GFD ticker "IDITARM" (Bank of Italy Repo Rate) until the country joins the Euro.
- Spain: GFD ticker "IDESPRM" (Bank of Spain Repo Rate) until the country joins the Euro.
- Ireland: GFD ticker "IDIRLRD" (Bank of Ireland Repo Rate) until the country joins the Euro.
- Belgium: GFD ticker "IDBELRM" (Belgium National Bank Repo Rate) until the country joins the Euro.
- Greece: GFD ticker "IDGRC"D (Bank of Greece Discount Rate) until the country joins the Euro.

- Netherlands: GFD ticker "IDNLDRD" (Netherlands Bank Repo Rate) until the country joins the Euro.
- Norway: GFD ticker "IDNORRD" (Bank of Norway Sight Deposit Rate).
- Japan: GFD ticker "IDJPNCM" (Japan Target Call Rate). GFD data is missing from March 2001 until July 2006. BIS data is also missing for most of this period. We use other sources to hand-fill the interest rate over this period as being equal to 0% (see for instance, the data from St. Louis Fed at https://fred.stlouisfed.org/series/IRSTCI01JPM156N).
- Switzerland: We use the BIS measure (cannot identify an appropriate rate from the GFD).
- Canada: We use the BIS measure (highly correlated with our measure and greater coverage).

Stock prices. We obtain this data from the GFD. For each country, we try to pick the most popular stock price index (based on Internet searches). We obtain daily data and convert to quarterly data by averaging over all (trading) days within the quarter. We then divide this with our core CPI measure (see above) to obtain a real stock price series.

- United States: GFD ticker "_SPXD" (S&P500 Index)
- United Kingdom: GFD ticker "_FTSED" (UK FTSE100 Index).
- Australia: GFD ticker "_AXJOD" (Australia S&P/ASX 200 Index).
- South Korea: GFD ticker "_KS11D" (Korea SE Stock Price Index (KOSPI)).
- Germany: GFD ticker "_GDAXIPD" (Germany DAX Price Index).
- New Zealand: GFD ticker "_NZ15D" (NZSX-15 Index).
- France: GFD ticker "_FCHID" (Paris CAC-40 Index).
- Denmark: GFD ticker "_OMXC20D" (OMX Copenhagen-20 Index).
- Finland: GFD ticker "_OMXH25D" (OMX Helsinki-25 Index).
- Sweden: GFD ticker "_OMXS30D" (OMX Stockholm-30 Index).
- Israel: GFD ticker "_TA125D" (Tel Aviv SE 125 Broad Index).
- Italy: GFD ticker "_BCIJD" (Milan SE MIB-30 Index).
- Spain: GFD ticker "_IBEXD" (Madrid SE IBEX-35 Index).
- Ireland: GFD ticker "_ISEQD" (Ireland ISEQ Overall Price Index).
- Belgium: GFD ticker "_BFXD" (Belgium CBB Bel-20 Index).
- Greece: GFD ticker "_ATGD" (Athens SE General Index).
- Netherlands: GFD ticker "_AEXD" (Amsterdam AEX Stock Index).
- Norway: GFD ticker "_OSEAXD" (Oslo SE All-Share Index).
- Japan: GFD ticker "_N225D" (Nikkei 225 Stock Index).
- Switzerland: GFD ticker "_SSMID" (Swiss Market Index).

- Canada: GFD ticker "_GSPTSED" (Canada S&P/TSX 300 Index).

Earnings. We obtain monthly data on the price-earnings ratio of publicly traded firms from the GFD (typically constructed for a broad sample of stocks chosen by the GFD). We then combine this information with our nominal price index (using the price at the last trading day of the month) to construct a monthly series for earnings. We convert this to a quarterly measure by averaging over the months within the quarter. We then divide this by our core CPI measure to obtain a quarterly real earnings series for publicly traded firms.

GFD ticker for the price earnings ratio typically has the form "SY-three digit country code-PM" (e.g., the ticker for the United States is "SYUSAPM"). One exception is the United Kingdom for which the corresponding GFD code is "_PFTASD" (UK FT-Actuaries PE Ratio).

Credit expansion. Our measure of bank credit is based on Baron and Xiong (2017), who construct a variable, *credit expansion*, defined as the annualized past three-year change in bank credit to GDP ratio. Mathematically, it is expressed as

$$\text{credit expansion} = \frac{\Delta\left(\frac{\text{bank credit}}{\text{GDP}}\right)_t - \Delta\left(\frac{\text{bank credit}}{\text{GDP}}\right)_{t-12}}{12} \times 4, \tag{E.1}$$

where t denotes a quarter. Baron and Xiong (2017) construct this measure by merging data from two sources. Their main source is the "bank credit" measure from the BIS, which covers a large set of countries but is generally available only for postwar years. For this reason, Baron and Xiong (2017) also supplement it with the "bank loans" measure from Schularick and Taylor (2012), which covers fewer countries but more years. Since our panel starts in 1990, we ignore the second source and rely entirely on the BIS measure.

Specifically, we use the quarterly BIS database on credit to the nonfinancial sector (publicly available at https://www.bis.org/statistics/totcredit.htm). We obtain the measure "bank credit to the private nonfinancial sector" expressed in units of percentage of GDP (the corresponding BIS code is "Q:5A:P:B:M:770:A"), which enables us to construct the variable in (E.1). We verify that our variable is highly correlated with the measure constructed by Baron and Xiong (2017) (who generously shared their data with us)—the correlation coefficient for the available country-quarters is 0.975.

Following Baron and Xiong (2017), we also construct a "credit expansion-std" variable by standardizing the measure in (E.1) by its mean and standard deviation within each country. Since Baron and Xiong (2017) focus on predicting stock prices, they calculate the mean and the standard deviation using only past data so as to avoid any look-ahead bias. Since our focus is different, we ignore this subtlety and calculate the sample statistics using the entire data for the corresponding country (in the BIS database).

Table 1: Euro/ERM status by country and year

Country	1990	1991	1992	1993	1994	1995	1996	1997-2017
Belgium	1	1	1	1	1	1	1	1
Denmark	1	1	1	1	1	1	1	1
Finland	0	0	0	0	0	0	0	1
France	1	1	1	1	1	1	1	1
Germany	1	1	1	1	1	1	1	1
Greece	0	0	0	0	0	0	1	1
Ireland	1	1	1	1	1	1	1	1
Italy	1	1	1	0	0	0	0	1
Netherlands	1	1	1	1	1	1	1	1
Spain	1	1	1	1	1	1	1	1
Australia	0	0	0	0	0	0	0	0
Canada	0	0	0	0	0	0	0	0
Israel	0	0	0	0	0	0	0	0
Japan	0	0	0	0	0	0	0	0
Korea	0	0	0	0	0	0	0	0
NZL	0	0	0	0	0	0	0	0
Norway	0	0	0	0	0	0	0	0
Sweden	0	1	1	0	0	0	0	0
Switzerland	0	0	0	0	0	0	0	0
UK	0	1	1	0	0	0	0	0
USA	0	0	0	0	0	0	0	0

Euro status. Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Spain adopted the Euro in 1999. Greece adopted in 2001. Denmark hasn't adopted the Euro but is a member of the ERM.

Table 2: Summary statistics by ERM for the baseline regression sample

	ERM sample		Non-ERM sample		Difference	
	Mean	Std.Deviation	Mean	Std.Deviation	Mean	Std.Error
Δ log house prices (real)	0.0040	0.0183	0.0053	0.0181	-0.0013	(0.0023)
Δ log GDP (real)	0.0043	0.0128	0.0065	0.0093	-0.0022	(0.0010)
policy interest rate (nominal)	0.0232	0.0194	0.0352	0.0288	-0.0119	(0.0038)
Δ log CPI (core)	0.0041	0.0029	0.0046	0.0039	-0.0004	(0.0005)
Δ unemployment rate	-0.0000	0.0042	-0.0002	0.0030	0.0002	(0.0004)
Δ log investment (real)	0.0030	0.0535	0.0070	0.0297	-0.0040	(0.0021)
Δ log consumption (real)	0.0035	0.0103	0.0069	0.0095	-0.0034	(0.0008)
earnings to price ratio	0.0616	0.0409	0.0585	0.0227	0.0031	(0.0039)
Δ log stock prices (real)	0.0011	0.0974	0.0108	0.0820	-0.0097	(0.0047)
credit expansion	0.0175	0.0557	0.0136	0.0298	0.0040	(0.0079)
credit expansion-std	0.1715	1.2657	-0.0346	1.1128	0.2062	(0.1957)
Observations	821		1120		1941	

Δ represents quarterly change. Standard errors are Newey-West standard errors with a bandwidth of 20 quarters.

Table 3: Private housing wealth in 2005 (% of GDP) by Euro/ERM status

Country (Euro/ERM)	Housing wealth	Country (Non-Euro/ERM)	Housing wealth
Spain	414.33	Australia	301.32
Italy	271.25	USA	199.77
France	253.74	Korea	179.55
Netherlands	222.03	Japan	169.74
Germany	186.77	Canada	146.51
Denmark	168.45	Norway	139.48
		Sweden	132.10
Average	252.76	Average	181.21
GDP-weighted average	255.29	GDP-weighted average	191.64

Table 4: Stock market capitalization in 2005 (% of GDP) by Euro/ERM status

Country (Euro/ERM)	Market cap	Country (Non-Euro/ERM)	Market cap
Finland	102.48	Switzerland	229.68
Netherlands	87.37	Canada	129.84
Spain	82.95	UK	126.75
France	80.07	Australia	121.32
Belgium	74.47	Sweden	116.08
Denmark	67.30	USA	103.83
Greece	58.57	Korea	96.16
Ireland	53.90	Israel	86.04
Italy	43.08	Norway	79.94
Germany	42.01	Japan	61.89
Average	69.22	Average	115.16
GDP-weighted average	61.84	GDP-weighted average	120.26

Data sources. We obtain housing wealth to GDP ratio from the World Inequality Database (WID) which is publicly available at https://wid.world/. We construct the ratio by combining yearly series on "private housing assets" (WID indicator, "mpwhou") and "gross domestic product (WID indicator, "mgdpro").

We obtain stock market capitalization to GDP ratio as yearly series from the GFD. The corresponding ticker has the form "CM.MKT.LCAP.GD.ZS three digit country code" (e.g., the ticker for the United States is "CM.MKT.LCAP.GD.ZS USA").

For both tables, we construct the GDP-weighted averages by using our relative GDP measure (in 2005) described earlier in this appendix.

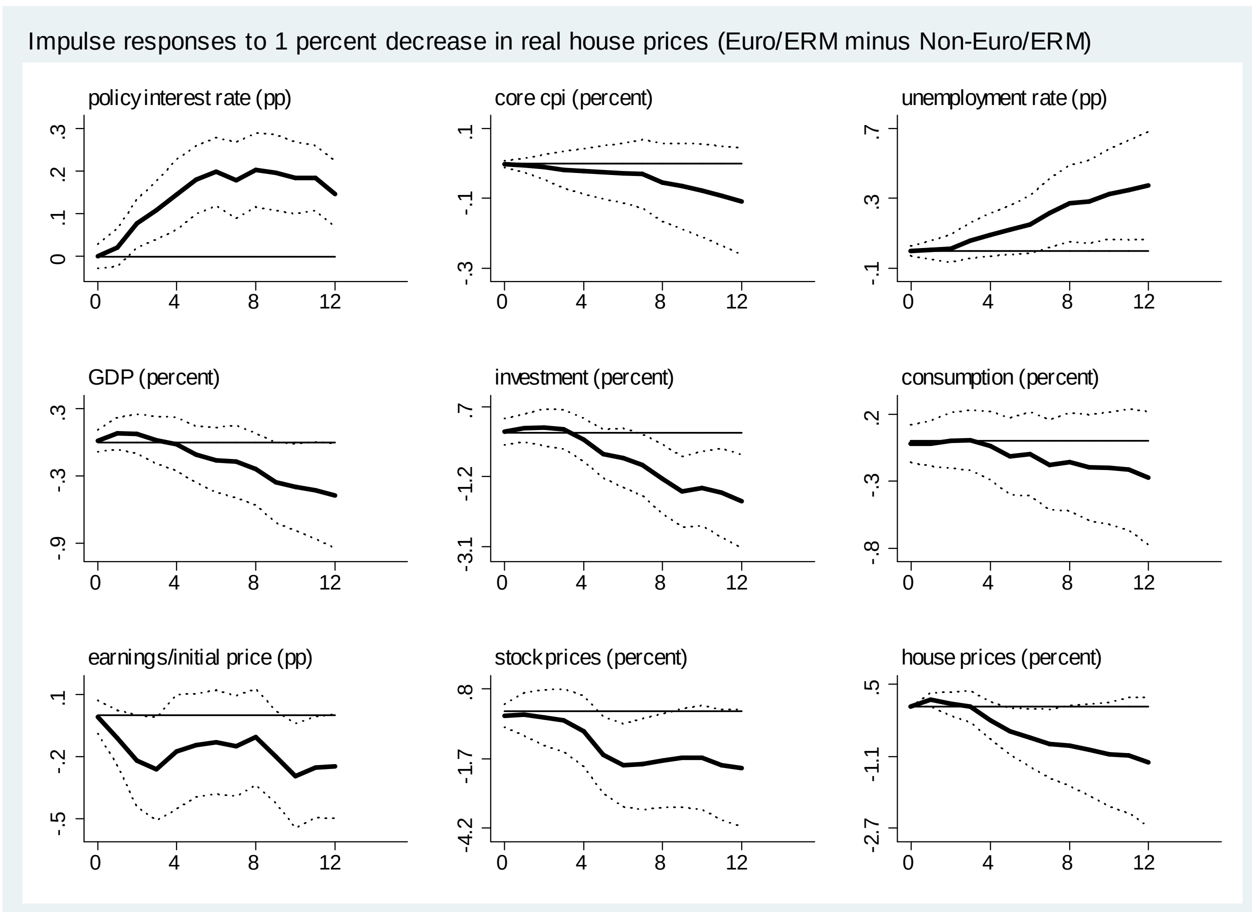

Figure 10: Differences in coefficients between the ERM and the non-ERM samples corresponding to the baseline regression results in Figure 6.

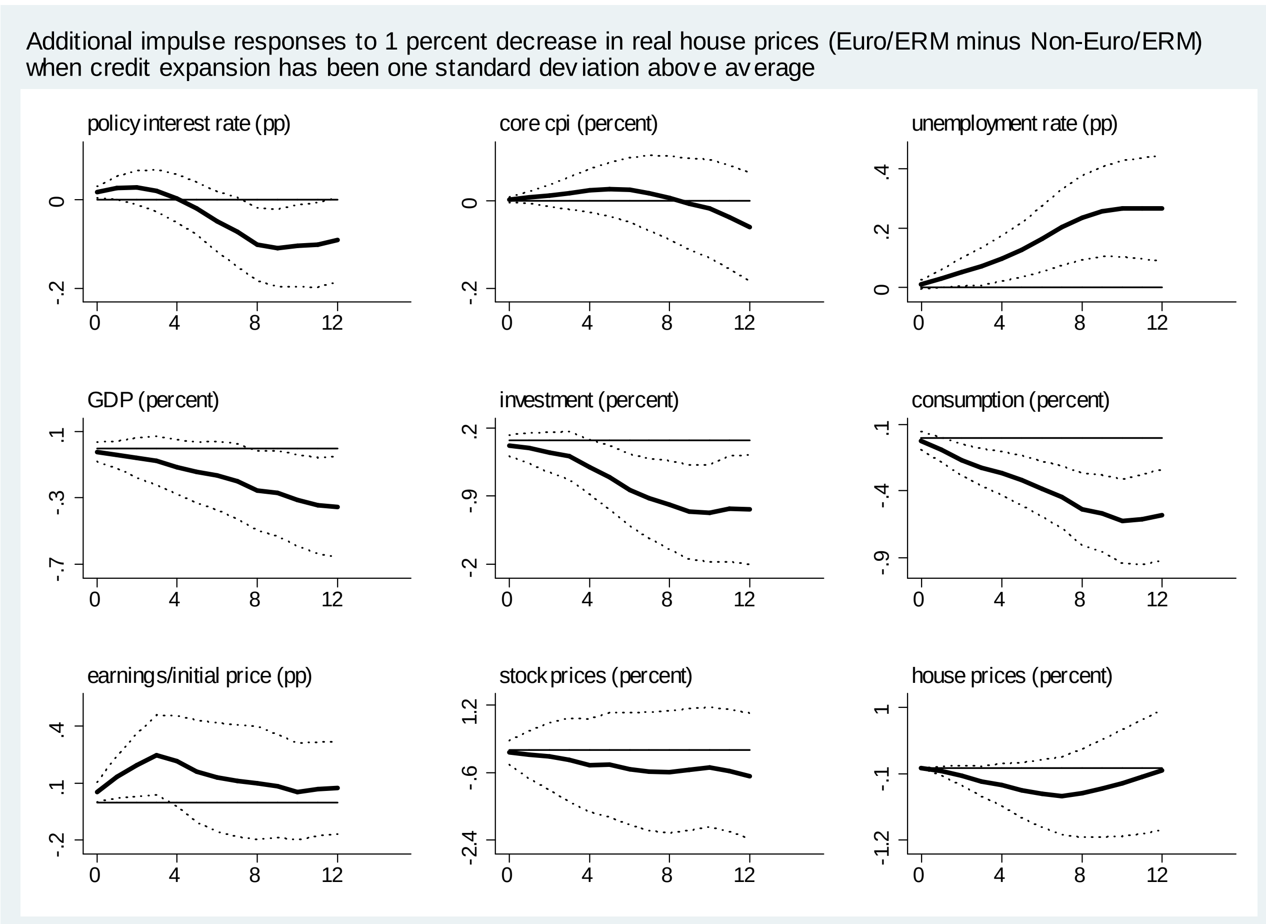

Figure 11: Differences in coefficients between the ERM and the non-ERM samples corresponding to the regression results with credit interaction in Figure 7.

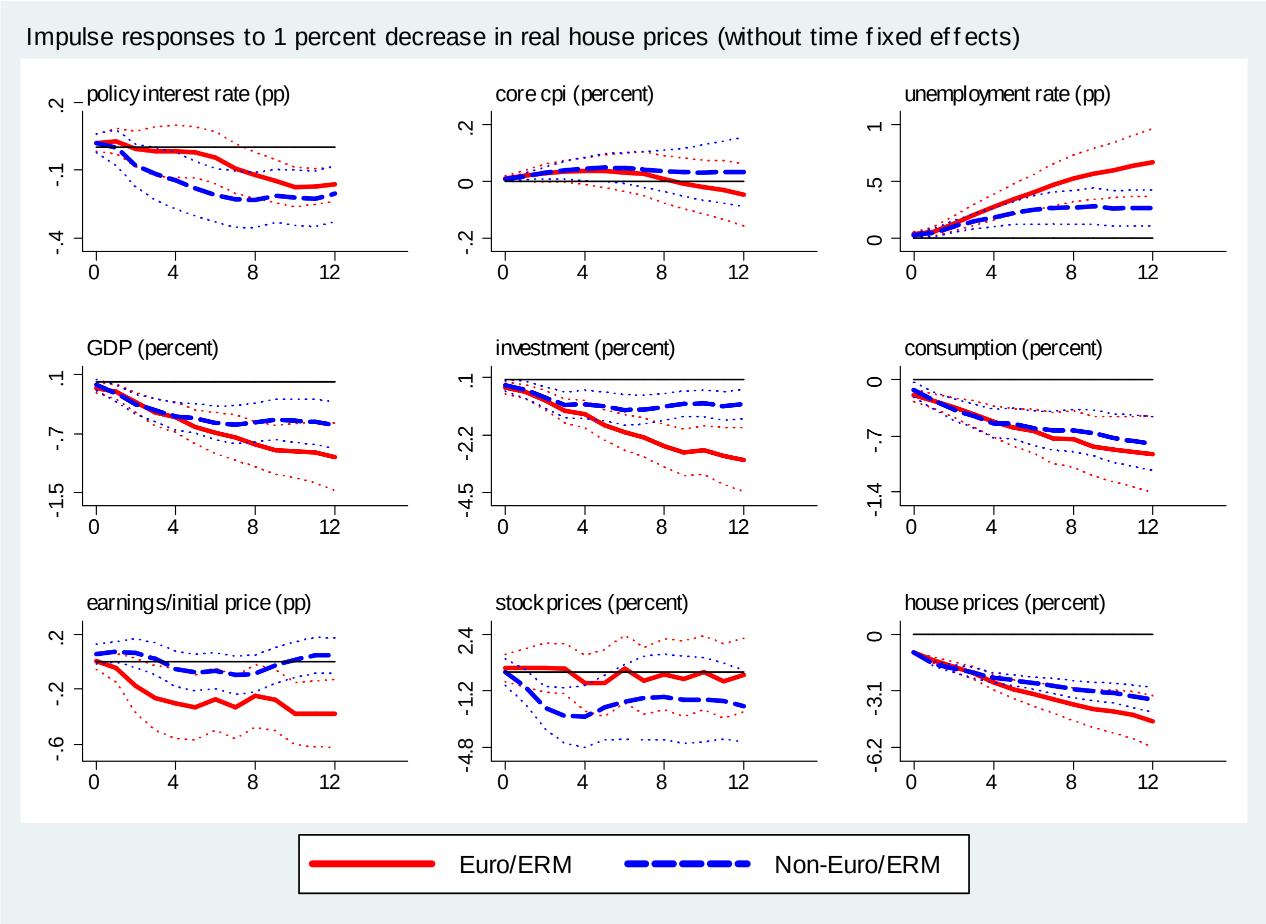

Figure 12: The analogues of the baseline regression results in Figure 6 with the difference that time fixed effects are excluded from the regressions.

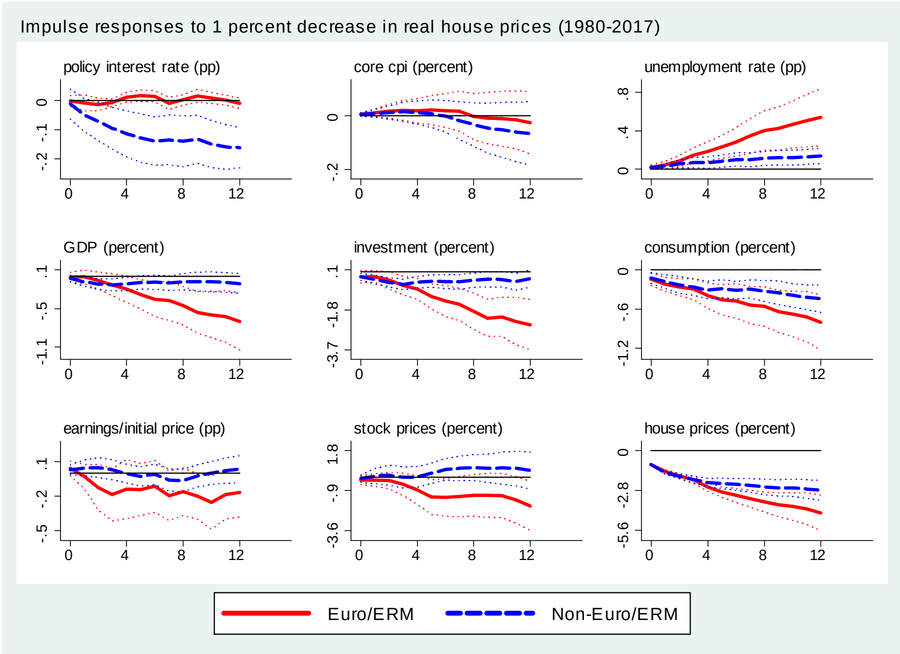

Figure 13: The analogues of the results in Figure 6 with a sample that starts in 1980Q1 (as opposed to 1990Q1).

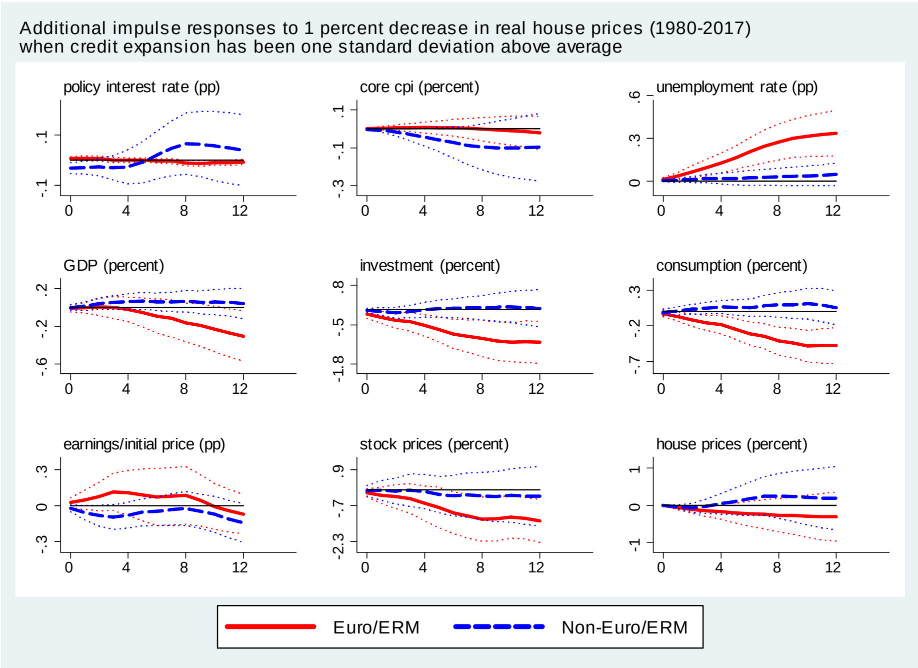

Figure 14: The analogues of the results in Figure 7 with a sample that starts in 1980Q1 (as opposed to 1990Q1).

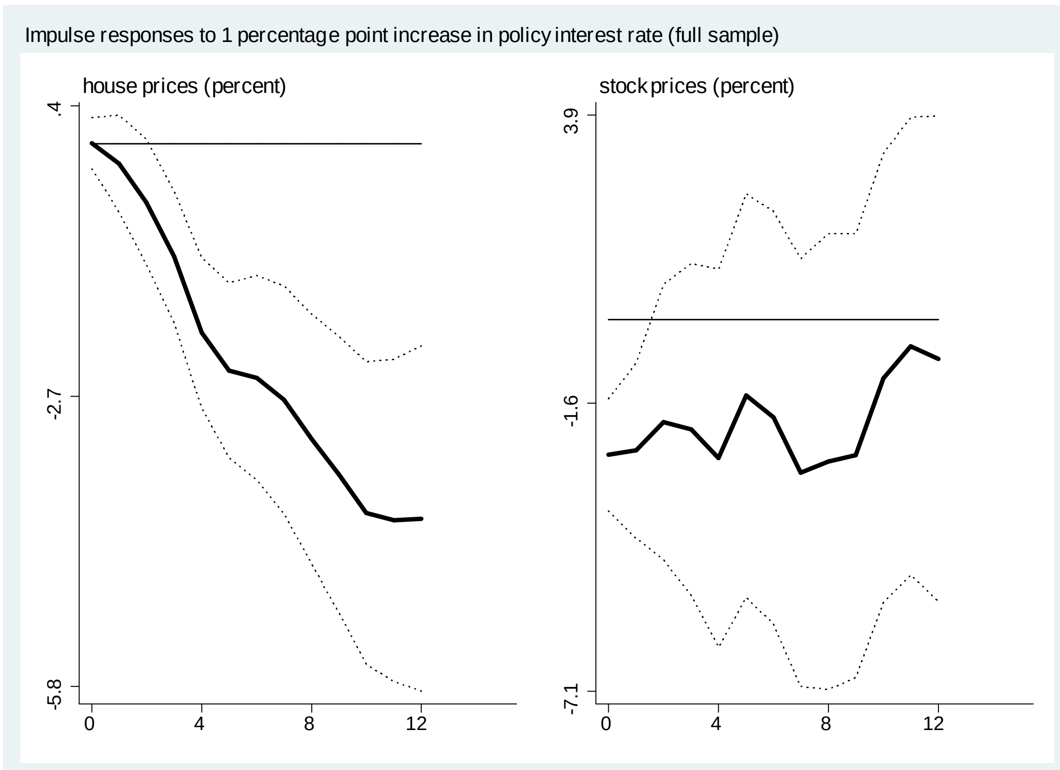

Figure 15: The analogues of the baseline regression results in Figure 6, where we consider shocks to the policy interest rate as opposed to house prices. Specifically, we run the analogue of the specification in (53) (on the full sample) where the shock variable is the level of the policy interest rate and the outcome variable is log house prices (left panel) or log stock prices (right panel). The solid lines plot the coefficients corresponding to the the policy interest rate variable. All regressions include time and country fixed effects; 12 lags of the level of the policy interest rate, 12 lags of the first difference of log GDP, 12 lags of the first difference of log house prices, and 12 lags of the first difference of log stock prices. The dotted lines show 95% confidence intervals calculated according to Newey-West standard errors with a bandwidth of 20 quarters. All regressions are weighted by countries' PPP-adjusted GDP in 1990. Data is unbalanced quarterly panel that spans 1990Q1-2017Q4. All variables except for the policy interest rate are adjusted for inflation. The sources and the definitions of variables are described earlier in this appendix.