

Common Ownership and the Secular Stagnation Hypothesis

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VERY PRELIMINARY

Recent work by [Gutiérrez and Philippon \(2016\)](#) has shown that investment by U.S. firms is low relative to measures of profitability and valuation, such as Tobin's Q. This fact is even more puzzling given that real interest rates have been at historic lows for over a decade ([Summers, 2016](#)). [Barkai \(2016\)](#) and [Karabarbounis and Neiman \(2018\)](#) have shown that both the labor and capital shares have declined in recent decades. Several observers have suggested that, at least in part, this pattern of “secular stagnation” can be explained by an increase in market power ([Summers, 2016](#); [Brun and González, 2017](#); [Gutiérrez and Philippon, 2017](#); [Eggertsson, Robbins and Getz Wold, 2018](#)).

In this paper, we explore this hypothesis by developing a macroeconomic model in which higher effective market concentration (including through common ownership) leads to lower equilibrium real interest rates. Our model is different from the ones that have been generally used in the literature on market power and macroeconomic outcomes in that it builds on models of oligopolistic competition from the industrial organization literature, as opposed to the monopolistic competition model of [Dixit and Stiglitz \(1977\)](#).

Thus, changes in markups in our model are driven by changes in market structure, such as the number of firms in the economy, or the level of common ownership among firms. In contrast, the macroeconomic literature has generally relied on changes in preference parameters (in particular,

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the elasticity of substitution parameter of the Dixit-Stiglitz utility function) to generate changes in market power over time.

Another new feature of our model is that firms are large and have market power in both product and factor markets, including labor and capital markets. This implies that the wedge between the marginal product of labor and the wage is not necessarily the same as the wedge between the marginal product of capital and the real interest rate, since the level of market power can be different in both markets.

We calibrate our model using market concentration measures from [Gutiérrez and Philippon \(2017\)](#) and [Rinz \(2018\)](#), and our own calibration for common ownership parameters. Our calibration results suggest that, without accounting for common ownership, an increase in concentration cannot explain (under plausible values for elasticity parameters) the decline in labor and capital shares in recent decades. However, when taking common ownership into account, the model implies a decline in the labor share that is similar to the actual decline, and a decline in the capital share that is somewhat larger than the actual decline.

1 Model

We develop a general equilibrium oligopoly model with two factors of production: labor and capital. The economy has a finite number J of firms and three types of people: workers, owners, and savers. We denote the set of savers I_S , the set of workers I_W , and the set of owners I_O , each of measure one.

There are two periods: an initial period, which we call period zero (“the past”), in which the savers have an endowment of output which they can consume or lend to the firms so they transform it into capital, and another period (“the present”) in which the firms produce by combining the capital with labor that they buy from the workers. All three types of agents consume in period one. There are four goods: consumption in the past, consumption in the present, leisure, and capital.

We assume that the owners are divided uniformly into J groups, one per firm, with owners in

group j owning $1 - \phi + \phi/J$ of firm j and ϕ/J of the other firms; here $\phi \in [0, 1]$. The utility of the owners is simply their consumption of the present period good, which they purchase with the profits that they receive from their ownership of the firms.

The workers have preferences over consumption in the present and leisure given by

$$U(C_{1,i}, L_i) = \frac{C_{1,i}^{1-\sigma}}{1-\sigma} - \chi \frac{L_i^{1+\xi}}{1+\xi}.$$

They sell their labor to the firms at a wage w , and use it to buy the present consumption good that the firms produce and sell at price p . Therefore, they face the budget constraint is $pC_1 \leq wL$.

The savers do not work or own the firms. They have an endowment E of output in the period 0 (i.e., in the past), and can decide whether to consume the output, or lend it to the firms so they can use it as capital in period 1 (i.e., in the present). The savers lend to the firms at a gross real interest rate r , so that a firm has to pay back r units of the period 1 good in period one for unit of period 0 good that they borrowed. Thus, the inter-temporal budget constraint of the savers is $C_{0,i} + \frac{C_{1,i}}{r} = E$. Their preferences exhibit constant elasticity of substitution between present and future consumption $1/\sigma$:

$$U(C_{0,i}, C_{1,i}) = \frac{C_{0,i}^{1-\sigma}}{1-\sigma} + \beta \frac{C_{1,i}^{1-\sigma}}{1-\sigma},$$

with σ and β in $(0, 1)$.

The firms transform the output that they purchase from the savers into productive capital at a 1:1 rate. They combine the capital with labor that they buy from the workers to produce in period 1 using the production function $Y_j = F(K_j, L_j)$, which we assume is a constant-returns to scale Cobb-Douglas: $Y_j = AK_j^{1-\alpha}L_j^\alpha$, $\alpha \in (0, 1)$.¹ The capital stock depreciates at a rate $1 - \delta$, and the firms can transform the capital that's left it into the consumption good in the present at a 1:1 rate. Thus, the profits of firm j (in terms of the consumption good in the present period) are:

$$\frac{\pi_j}{p} = F(K_j, L_j) - \frac{w}{p}L_j - (r - 1 + \delta)K_j.$$

¹This production function is twice continuously differentiable and concave, with $F_{KK} \leq 0$, $F_{LL} \leq 0$, and $F_{KK}F_{LL} - F_{KL}^2 \geq 0$.

We assume that the objective function of the firm is to maximize a share-weighted average of the utilities of its shareholders. In our context, that implies that its objective is to maximize $(\pi_j + \lambda \sum_{k \neq j} \pi_k) / p$, where $\lambda = \frac{(2-\phi)\phi}{(1-\phi)^2 J + (2-\phi)\phi}$ is the Edgeworth sympathy coefficient. The formula for λ is the same as that in [Azar and Vives \(2018\)](#), which provides a derivation.

We use the concept of Cournot-Walras equilibrium with shareholder representation from [Azar and Vives \(2018\)](#), which adapts the equilibrium concept from [Gabszewicz and Vial \(1972\)](#) to a context in which firms maximize a weighted average of shareholder utilities instead of maximizing profits. This solves the issue of dependence of the equilibrium on the choice of price normalization, since utilities depend only on relative prices. The idea of the Cournot-Walras equilibrium is the following: each possible production plans of the firms imply a competitive equilibrium allocation and relative price vector. Given this mapping from production plans to price vectors, the Cournot-Walras equilibrium (with shareholder representation) is a set of production plans for the firms that are mutual best responses (that is, a Nash equilibrium).

1.1 Competitive equilibrium conditional on firms' production plans

The first-order condition for worker i is:

$$(w/p)^{1-\sigma} L_i^{-\sigma} = \chi L^\xi.$$

Since all the workers are identical and of measure one, aggregate labor supply function is the same as the labor supply of an individual worker. The competitive equilibrium real wage (relative to the price of present consumption) is a function of the total employment plans by the firms, and is given by the aggregate inverse labor supply function, which we call $\omega(L)$:

$$\omega(L) = \chi^{\frac{1}{1-\sigma}} L^{\frac{\xi+\sigma}{1-\sigma}}.$$

with elasticity $\eta = (1 - \sigma) / (\xi + \sigma)$. The first-order conditions for the savers yield the Euler

equation:

$$C_{0,i}^{-\sigma} = \beta r C_{1,i}^{-\sigma}.$$

Combining the Euler equation and the budget constraint yields an expression for the level of savings as a function of the real interest rate:

$$S_i = E - C_{0,i} = E \frac{\beta^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}}}{1 + \beta^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}}}.$$

Since all savers are identical, the expression for the total supply of savings $S = \int_{i \in I_S} S_i$ is the same as that for S_i . Since $\sigma < 1$, savings are increasing in r . Market clearing implies that $S = K$ where K is the total investment of the firms. As was the case for the real wage and labor supply, the inverse of the savings function determines the competitive equilibrium real interest as a function of K , which we call $\rho(K)$ and is given by:

$$\rho(K) = \left(\frac{K}{E - K} \right)^{\frac{\sigma}{1-\sigma}} \left(\frac{1}{\beta} \right)^{\frac{1}{1-\sigma}}.$$

with elasticity $\varepsilon = \frac{\rho(K)}{\rho'(K)K} = \frac{1-\sigma}{\sigma}(1-s)$, where $s = K/E$ is the saving rate. The competitive equilibrium real interest rate is increasing in K , tending to 0 as $K \rightarrow 0$, and to ∞ as $K \rightarrow E^-$.

1.2 Cournot-Walras equilibrium

We start by establishing existence and characterizing the equilibrium:

Proposition 1. *A unique symmetric equilibrium exists and it is characterized by the solution to the system of equations:*

$$\frac{F_L\left(\frac{K}{J}, \frac{L}{J}\right) - \omega(L)}{\omega(L)} = \frac{H}{\eta},$$

$$\frac{F_K\left(\frac{K}{J}, \frac{L}{J}\right) - \rho(K) + (1-\delta)}{\rho(K) - (1-\delta)} = \frac{H}{\varepsilon} \left(1 - \frac{1-\delta}{\rho(K)}\right)^{-1},$$

where $H = 1/J + \lambda(1 - 1/J)$ is the modified Herfindahl-Hirschman index (which in this model is the same in the labor and capital markets).

The equilibrium is characterized by the markdown of real wages relative to the marginal product of labor being equal to the elasticity of the competitive equilibrium real wage with respect to firms' employment plans, multiplied by the MHHI. The new condition adds that the markdown of the real interest rate relative to the marginal product of capital (including the capital that is left over after depreciation) is equal to the elasticity of the competitive equilibrium real interest rate with respect to firms' investment plans, multiplied by the MHHI.

We show the following comparative statics result:

Proposition 2. *Suppose $\phi < 1$. Then either a decline in the number of firms J or an increase in the common ownership parameter ϕ leads to an equilibrium with lower:*

(a) capital stock K^ ; (b) employment L^* ; (c) real interest rate r^* ; (d) real wage $(w/p)^*$; (e) output; and (f) labor share of income.*

2 Multiple Sectors

In this section we extend the model to the case of multiple sectors. This case is similar to the one-sector case, except that the present consumption good is an aggregate of N goods $c_{1,ni}$:

$$C_{1,i} = \left[\left(\frac{1}{N} \right)^{1/\theta} \sum_{n=1}^N c_{1,ni}^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)},$$

where $\theta > 1$ is the elasticity of substitution indicating a preference for variety. Each good is produced by one sector, each with J firms. We assume that the workers and of savers have mass N . The savers can now provide a firm with a unit of the period zero good in exchange for r units of the composite good in period one. As in the one-sector model, the firm can transform the period zero good into capital at a 1:1 rate. In period one, each unit of capital after production can be transformed into $1 - \delta$ units of the composite good.

We assume that the ownership structure of the firms is as in [Azar and Vives \(2018\)](#), with the initial owners divided into NJ groups, and the initial owner group nj owning a fraction $1 - \phi - \tilde{\phi} \geq$

0 in firm nj , an index holding a fraction $\frac{\tilde{\phi}}{J}$ in each firm in sector n , and an index holding $\frac{\phi}{NJ}$ in every firm in the economy.

In this case, we can show that the equilibrium markdowns of wages relative to the marginal product of labor, and of the real interest rate relative to the marginal product of capital include two wedges: one that reflects the level of product market power, and one that reflects the level of market power in each factor market.

Proposition 3. *At a symmetric equilibrium, markdowns of wages and the return to capital are:*

$$1 + \mu_L^* = \frac{1 + H_{labor}/\eta}{1 - (H_{product} - \lambda_{inter})(1 - 1/N)/\theta}$$

$$1 + \mu_K^* = \frac{1 + H_{capital}/\varepsilon \cdot (1 - (1 - \delta)/\rho(K))^{-1}}{1 - (H_{product} - \lambda_{inter})(1 - 1/N)/\theta},$$

where H_{labor} , $H_{capital}$, and $H_{product}$ are the modified Herfindahl-Hirschman indices in the labor, capital, and product market, respectively, and λ_{inter} is the inter-industry Edgeworth sympathy coefficient.

The labor share is

$$\frac{\alpha}{1 + \mu_L^*}.$$

The capital share is

$$\frac{1 - \alpha}{1 + \mu_K^*}.$$

The profit share is the residual: $\frac{\mu_L^* + \mu_K^* - \mu_L^* \mu_K^*}{(1 + \mu_L^*)(1 + \mu_K^*)}$.

3 Calibration

We calculate average product, labor, and capital market HHIs using Compustat data. We calibrate the average MHHI delta in product and labor markets as $\lambda_{intra}(1 - HHI)$ (using the respective HHI), based on our estimate of the average intra-industry Edgeworth sympathy coefficient λ_{intra} .

For capital market MHHI delta, we do the same but using $\lambda_{inter}(1 - HHI)$

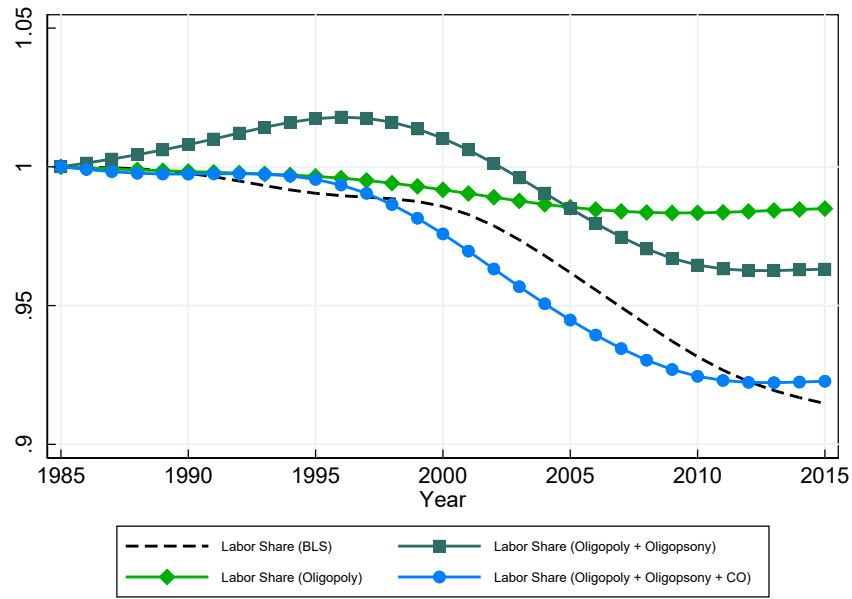
We calibrate θ to 3 following [Hobijn and Nechio \(2015\)](#), and η to 0.59 based on estimates from [Chetty et al. \(2011\)](#). We calibrate σ to 1/2, based on the estimate of the intertemporal elasticity of substitution by [Gruber \(2013\)](#) and [Nakamura et al. \(2013\)](#). We calibrate the savers' endowment E and productivity A to match the real interest rate of 1.071 in 1985 and the level of capital per worker in that year. We calibrate χ to match the employment-population ratio in 1985. We use $\alpha = 2/3$, $\delta = 0.1$, and $\beta = 0.99$, which are values commonly used in the literature.

The results are shown in [Figure 1](#). The increase in product market concentration without taking into account common ownership implies almost no decline in the labor or capital share. Adding labor market concentration actually implies an increase in the labor share, since the series from [Rinz \(2018\)](#) that we use implies HHIs that decline over time. However, the full model including common ownership implies a decline in the labor share that is roughly the observed decline. The model implies a decline in the capital share that is somewhat higher than the actual decline in the non-residential capital share according to [Karabarbounis and Neiman \(2018\)](#) and [Barkai \(2016\)](#).

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(a) Labor share



(b) Capital share

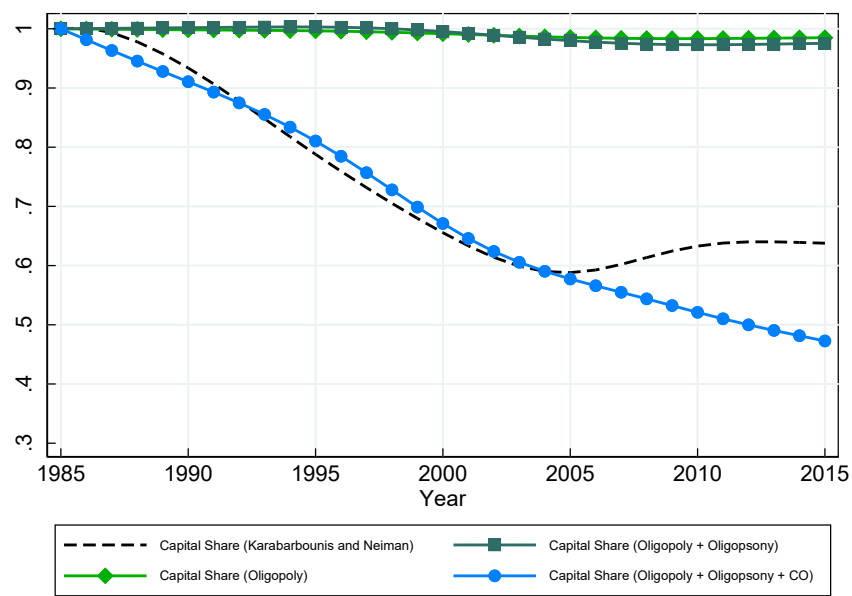


Figure 1. Model calibration results

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Appendix

PROOF OF PROPOSITION 1: The objective function of the firm is strictly concave. The second derivative of the objective function with respect to labor is:

$$F_{LL} - 2\omega' - \omega'' \cdot (L_j + \lambda L_{-j}) < 0$$

since $F_{LL} < 0$ and $-2\omega' - \omega'' \cdot (L_j + \lambda L_{-j}) < 0$ because we are assuming that labor supply is constant elasticity. The second derivative of the objective function with respect to capital is

$$F_{KK} - 2\rho' - \rho'' (K_j + \lambda K_{-j}) < 0$$

since $F_{KK} < 0$ and $-2\rho' - \rho'' (K_j + \lambda K_{-j}) < 0$. The latter inequality follows because $-2\rho' - \rho'' (K_j + \lambda K_{-j}) = -\rho'(K) \left[2 + \rho''(K)K/\rho'(K)(s_j^K + \lambda(1 - s_j^K)) \right]$, where s_j^K is firm j 's share of capital and the expression in brackets is positive because $\rho''(K)K/\rho'(K) \geq -1$. To see this, note that $\rho'(K) = \frac{\sigma}{1-\sigma} \frac{E}{E-K} \frac{\rho(K)}{K}$ and $\rho''(K) = \frac{\sigma}{1-\sigma} \frac{\rho(K)}{K^2} \frac{E}{E-K} \left[\frac{K}{E-K} + \frac{\rho'(K)K}{\rho(K)} - 1 \right]$. Since $\frac{\rho'(K)K}{\rho(K)} = \frac{\sigma}{1-\sigma} \frac{E}{E-K}$, then $\rho''(K)K/\rho'(K) = (K/E + \sigma/(1-\sigma))E/(E-K) - 1 \geq -1$.

The fact that $F_{LL} \cdot F_{KK} - F_{LK}^2$ is positive implies that the determinant of the matrix of second derivatives is positive, which is the last condition we needed to establish strict concavity of the objective function. From the first-order conditions, it is then clear that the reaction functions are continuous, and therefore a Nash equilibrium exists.

To prove that there is a unique symmetric equilibrium, we consider the system of FOCs when employment and capital are symmetric across firms, and show that there is a unique solution. From the FOC for labor, we can solve for labor as a function of capital, obtaining:

$$L = \left[\frac{A\alpha}{\chi^{\frac{1}{1-\sigma}} \left(1 + \frac{H}{\eta} \right)} \right]^{\frac{1}{1-\alpha+\frac{1}{\eta}}} K^{\frac{1-\alpha}{1-\alpha+\frac{1}{\eta}}}.$$

Replacing this in the FOC for capital, we obtain an implicit equation for capital:

$$A(1-\alpha) \left[\frac{A\alpha}{\chi^{\frac{1}{1-\sigma}} \left(1 + \frac{H}{\eta}\right)} \right]^{\frac{\alpha}{1-\alpha+\frac{1}{\eta}}} K^{-\frac{\alpha}{1-\alpha+\frac{1}{\eta}}} - [\rho(K)(1+H/\varepsilon(K)) - (1-\delta)] = 0.$$

The limit when $K \rightarrow 0^+$ of this expression is $+\infty$, while the limit when $K \rightarrow E^-$ is $-\infty$. The derivative of this expression with respect to K is negative, which implies that there is a unique solution to the equation. The two-equation characterization of the equilibrium obtains directly from imposing symmetry in the FOCs of the firm.

PROOF OF PROPOSITION 2:

(a) We start by noting that the number of firms J and the common ownership parameter ϕ enter the equilibrium equation for capital only through market concentration H . We then use the equilibrium equation for capital to define capital as an implicit function of $H \in (0, 1]$:

$$A(1-\alpha) \left[\frac{A\alpha}{\chi^{\frac{1}{1-\sigma}} \left(1 + \frac{H}{\eta}\right)} \right]^{\frac{\alpha}{1-\alpha+\frac{1}{\eta}}} K(H)^{-\frac{\alpha}{1-\alpha+\frac{1}{\eta}}} \equiv \rho(K(H))(1+H/\varepsilon(K(H))) - (1-\delta).$$

Taking log and derivative with respect to $\log H$ yields

$$-\frac{\alpha}{1-\alpha+\frac{1}{\eta}} \left(\frac{\frac{H}{\eta}}{1+\frac{H}{\eta}} + \frac{1}{\eta} \frac{\partial \log K}{\partial \log H} \right) = \frac{\rho \cdot (1+H/\varepsilon)}{\rho \cdot (1+H/\varepsilon) - (1-\delta)} \left[\frac{1}{\varepsilon} \frac{\partial \log K}{\partial \log H} + \frac{\frac{H}{\varepsilon}}{1+\frac{H}{\varepsilon}} \left(1 + \frac{\partial \log K}{\partial \log H} \frac{s}{1-s} \right) \right].$$

Solving for $\frac{\partial \log K}{\partial \log H}$:

$$\frac{\partial \log K}{\partial \log H} = -\frac{\frac{\alpha}{1-\alpha+\frac{1}{\eta}} \frac{\frac{H}{\eta}}{1+\frac{H}{\eta}} + \frac{\rho \cdot (1+H/\varepsilon)}{\rho \cdot (1+H/\varepsilon) - (1-\delta)} \frac{\frac{H}{\varepsilon}}{1+\frac{H}{\varepsilon}}}{\frac{\alpha}{1-\alpha+\frac{1}{\eta}} \frac{1}{\eta} + \frac{\rho \cdot (1+H/\varepsilon)}{\rho \cdot (1+H/\varepsilon) - (1-\delta)} \left(\frac{1}{\varepsilon} + \frac{\frac{H}{\varepsilon}}{1+\frac{H}{\varepsilon}} \frac{s}{1-s} \right)} < 0.$$

(b) We know that

$$L = \left[\frac{A\alpha}{\chi^{\frac{1}{1-\sigma}} \left(1 + \frac{H}{\eta}\right)} \right]^{\frac{1}{1-\alpha+\frac{1}{\eta}}} K^{\frac{1-\alpha}{1-\alpha+\frac{1}{\eta}}}.$$

which is decreasing in H and increasing in K . Since H increases when the number of firms decreases or common ownership increases, and K decreases with them, L must decline with both lower J and higher ϕ .

(c), (d), and (e) Since the equilibrium real wage and real interest rates are increasing in L and K , they also must decline when the number of firms decreases or common ownership increases. A lower level of employment and capital also implies lower output.

(f) The labor share of income is $\frac{\omega(L)L}{F(K,L)} = \frac{\alpha}{1+H/\eta}$. A decrease in the number of firms or an increase in the common ownership parameter ϕ increases H and therefore decreases the labor share.

PROOF OF PROPOSITION 3: