Neutral Bargaining in Financial Over-The-Counter Markets

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December 9, 2018

Abstract

I study bargaining over prices between two investors in financial over-the-counter markets with asymmetric information. I focus on environments in which an asset owner has private information about both her liquidity state and asset quality, and so a buyer is uncertain about the owner's true motive for selling—whether it is because of a liquidity need or because of a low asset valuation. I apply the concept of neutral bargaining solution to characterize the prices at which the investors trade with each other. I illustrate the implications for asset prices in over-the-counter markets where private information may be prevalent.

JEL classification: C78, D82, G12, G14.

Keywords: Bargaining, asymmetric information, mechanism design, interim efficiency, neutral bargaining solution, asset pricing, over-the-counter markets.

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In financial over-the-counter (OTC) markets, investors bargain over the prices at which assets may be bought or sold. An asset owner may wish to sell because of a holding cost arising from low liquidity or because of a low valuation for holding a poor-quality asset. A buyer may want to identify the asset owner's motive for selling because the seller's liquidity state would not affect the buyer's value for acquiring the asset while the asset quality would. But if the asset owner has private information about both her liquidity state and the asset quality, she might have incentives to hide her motive for selling in an attempt to get a more favorable price; hence the bargained price will be influenced by such incentives.

In this paper, I consider a simple bargaining model that captures this information asymmetry, and provide a reasonable notion of bargained prices for OTC markets. Specifically, I use the cooperative solution concept of neutral bargaining solution (Myerson 1984) to delimit bargaining mechanisms under which the expected bargained prices are fair and efficient. To illustrate, I present a numerical example and compute explicitly the neutral price for which investors would reasonably bargain. I compare it to other prices on the Pareto frontier, and show that the neutral price differs from the efficient price predicted by the Nash bargaining solution. I discuss the implications for asset pricing in OTC markets with asymmetric information.

This paper relates to the literature on asset pricing models of OTC markets initiated by Duffie, Gârleanu and Pedersen (2005).[1] In their model, an investor's type is characterized by whether he owns the asset or not and whether he has high or low liquidity, and the investors have a commonly known constant value for holding the asset; so the presence of private information about the investor's type is irrelevant because trade occurs only between low-liquidity owners and high-liquidity nonowners. Hence it is natural to adopt the Nash bargaining solution for computing the prices that form a dynamic stationary equilibrium.[2]

[1] Duffie, Gârleanu and Pedersen (2005) build a dynamic model with search-and-bargaining frictions and risk-neutral agents. Duffie, Gârleanu and Pedersen (2007) consider an extension to risky asset pricing.

[2] Tsoy (2018*b*) adds endogenous bargaining delays to Duffie, Gârleanu and Pedersen's (2007) model and argues that the Nash double limit should be used instead of the Nash bargaining solution in environments with precise private information and coarse public information. See also Tsoy (2018*a*).

In my model, the investor's private information matters; nonowners are essentially uncertain about why owners want to sell, and so trade can also occur between high-liquidity owners and nonowners. The novelty of my paper is in the application of the neutral bargaining solution to pin down asset prices in such environments.

The contribution is twofold. First, this paper clarifies an appropriate definition of asset prices for OTC markets with asymmetric information. The model here is not intended to provide a complete analysis of asset pricing under incomplete information; but to suggest that there is a caution to be added to the approach that uses the Nash bargaining solution for characterizing the terms of trade in OTC markets where information asymmetry between investors may be pervasive. Second, this paper demonstrates that neutral bargaining mechanisms are computed by the tractable set of conditions, and are insightful and easy to use from a practical perspective in applications to bargaining in OTC markets.

I. Model

A. The Environment

There are two agents: a dealer (seller, she) and a trader (buyer, he). The dealer is initially endowed with 1 unit of a given asset. The trader does not own any asset and has high liquidity.

The dealer is characterized by two-dimensional types. First, the dealer can have either "high" or "low" liquidity. A low-liquidity dealer incurs a positive holding cost of δ when trade fails, whereas a high-liquidity dealer has no such holding cost. Second, the asset that the dealer initially owns can have either "good" or "bad" quality. The dealer values the asset of good quality at c_g and the asset of bad quality at $c_b < c_g$. The trader has values v_g and v_b for acquiring the asset of good quality and bad quality, respectively, where $v_b < v_g$. The asset is always worth more to the trader than to the dealer: $v_g > c_g$ and $v_b > c_b$.

The dealer has private information about her types, and the trader has prior beliefs about the dealer's type. Two agents meet and bargain over the price of the asset at which they may trade with each other. The trader cannot verify any claims that the dealer might make about her type, and the two agents do not expect to make any further transactions in the future.

The environment described above can be formulated as a Bayesian bargaining problem à la Myerson (1984). The set of possible bargaining outcomes is $\mathcal{D} = \{(q, y) | 0 \leq q \leq 1, y \in \mathbb{R}\}$, where, for each $(q, y) \in \mathcal{D}$, q represents the probability that the asset is sold to the trader and y represents the amount of money that the trader pays to the dealer. The set of dealer types is $\mathcal{T} = \{lb, lg, hb, hg\}$, where the first letters denote the dealer's liquidity-type, and the second letters denote the dealer's asset-quality-type. For simplicity, I assume that the dealer's types are independent. The trader believes that the probability of the dealer being of type $t \in \mathcal{T}$ is p_t such that $\sum_{t \in \mathcal{T}} p_t = 1$. I assume that all types have positive probability, so $p_t > 0$ for all $t \in \mathcal{T}$.

Let u_1 and u_2 denote respectively the dealer's and the trader's utility function from $\mathcal{D} \times \mathcal{T}$ into $\mathbb{R}$, such that $u_i((q, y), t)$ is the utility payoff which agent i would get if $(q, y) \in \mathcal{D}$ were chosen and if t were the dealer's type. Let $d^* \equiv (q, y) = (0, 0)$ represent the outcome of bargaining breakdown, where trade fails and the dealer pays nothing. The low-liquidity dealer's payoff from d^* is $-\delta$, whereas the high-liquidity dealer and the trader receive zero payoffs from d^*. For ease of exposition and without loss of generality, I normalize utility payoff scales so that every agent's payoff from d^* is always zero. Under this formulation, the utility functions are defined by the formula

$$u_1((q, y), t) = y - (c_{t(2)} - \delta \mathbf{1}_{t \in \{lb, lg\}}) q,$$
$$u_2((q, y), t) = v_{t(2)} q - y,$$

for all $t \in \{lb, lg, hb, hg\}$, where the subscript $t(2) \in \{b, g\}$ is an index function that returns the second letter in t, and $\mathbf{1}_{t \in \{lb, lg\}}$ is an indicator function that equals one if $t \in \{lb, lg\}$ and zero otherwise.

B. Efficient Bargaining Mechanism

By the revelation principle, I can set up the bargaining problem as a direct-revelation mechanism, without loss of generality. That is, the agents do not have to agree on a specific price; instead they may agree on some bargaining mechanism.

A pair $(Q(\cdot), Y(\cdot))$ represents a *bargaining mechanism* for determining the bargaining outcome as a function of the dealer's reported type, where $Q(t)$ is the probability that the asset is transferred from the dealer to the trader (i.e., the probability of trade) and $Y(t)$ is the expected transfer payment from the trader to the dealer if the dealer's reported type is t. This mechanism must satisfy $0 \leq Q(t) \leq 1$ for all $t \in \mathcal{T}$. If $Q(t) > 0$, then $P(t) \equiv Y(t)/Q(t)$ represents an expected price per unit of the asset when the dealer's type is t.

The expected utilities for the dealer of type t and for the trader if (Q, Y) is implemented are $U_1(Q, Y|t) = Y(t) - (c_{t(2)} - \delta \mathbf{1}_{t \in \{lb, lg\}})Q(t)$ and $U_2(Q, Y) = \sum_{t \in \mathcal{T}} p_t(v_{t(2)}Q(t) - Y(t))$, respectively.

A mechanism (Q, Y) is *feasible* iff it is incentive compatible and individually rational, in the sense of conditions $U_1(Q, Y|t) \geq Y(s) - (c_{t(2)} - \delta \mathbf{1}_{t \in \{lb, lg\}})Q(s)$ $\forall t \in \mathcal{T}$, $\forall s \in \mathcal{T}$; and $U_1(Q, Y|t) \geq 0$ $\forall t \in \mathcal{T}$, $U_2(Q, Y) \geq 0$. By the revelation principle, there is no loss of generality in focusing on feasible mechanisms.

A feasible mechanism (Q, Y) is *interim incentive efficient* (IIE) iff there is no other feasible mechanism (Q', Y') such that $U_1(Q', Y'|t) \geq U_1(Q, Y|t)$ $\forall t \in \mathcal{T}$ and $U_2(Q', Y') \geq U_2(Q, Y)$, with at least one strict inequality. Because bargaining takes place when the dealer has private information, the agents would reasonably agree on an IIE bargaining mechanism.

C. Neutral Bargaining Mechanism

Myerson (1984) proposes a generalization of the Nash bargaining solution for bargaining problems with incomplete information, called the *neutral bargaining solution*. This solution concept is axiomatically derived, and can be characterized as an incentive-feasible mechanism that is equitable and efficient in terms of players' virtual utilities.[3] Importantly, Myerson's neutral bargaining solution captures the idea of an "inscrutable intertype compromise," which I explain below in the context of my model.

If the feasible mechanism that is best for the dealer depends on what her type is, then demanding a particular IIE mechanism might convey information about the dealer's type that could be unfavorable to her bargaining position. Hence, no matter what her type is, the dealer should maintain an inscrutable facade in the bargaining process (see Myerson (1983) for the *inscrutability principle*). To do so, the dealer must make some sort of equitable compromise between what she really wants and what she might have wanted if her type had been different, due to the conflicting incentives of different possible types of the dealer.

I present a variant of Myerson's characterization theorem for neutral mechanisms in my model, and define neutral price for which agents should reasonably bargain.

REMARK 1: *A mechanism (Q, Y) is neutral if and only if, for each positive number ε, there exist vectors $\lambda = \big((\lambda_1(t))_{t\in\mathcal{T}}, \lambda_2\big)$, $\alpha = \big(\alpha(s|t)\big)_{t\in\mathcal{T}, s\in\mathcal{T}}$, and $\omega = \big((\omega_1(t))_{t\in\mathcal{T}}, \omega_2\big)$ such that $\lambda_1(t) > 0$, $\lambda_2 > 0$, $\alpha(s|t) \geq 0$, $\forall s \in \mathcal{T}$, $\forall t \in \mathcal{T}$;*

$$\Big(\big(\lambda_1(t) + \sum_{s\in\mathcal{T}} \alpha(s|t)\big)\omega_1(t) - \sum_{s\in\mathcal{T}} \alpha(t|s)\omega_1(s)\Big)/p_t \tag{1}$$

$$= \max_{(q,y)\in\mathcal{D}} \sum_{i\in\{1,2\}} \frac{v_i((q,y),t,\lambda,\alpha)}{2},\ \forall t \in \mathcal{T},$$

$$\lambda_2\omega_2 = \sum_{t\in\mathcal{T}} p_t \max_{(q,y)\in\mathcal{D}} \sum_{i\in\{1,2\}} \frac{v_i((q,y),t,\lambda,\alpha)}{2};$$

$$U_1(Q,Y|t) \geq \omega_1(t) - \varepsilon,\ \forall t \in \mathcal{T},\ U_2(Q,Y) \geq \omega_2 - \varepsilon; \tag{2}$$

where $v_1((q,y),t,\lambda,\alpha) = \big((\lambda_1(t) + \sum_{s\in\mathcal{T}} \alpha(s|t))u_1((q,y),t) - \sum_{s\in\mathcal{T}} \alpha(t|s)u_1((q,y),s)\big)/p_t$ and $v_2((q,y),t,\lambda,\alpha) = \lambda_2 u_2((q,y),t)$.

[3] I omit detailed expositions of the axioms, which can be found in Myerson (1984).

Given a neutral mechanism (Q, Y), I say that, for $t \in \mathcal{T}$ such that $Q(t) > 0$, $P(t) = Y(t)/Q(t)$ is a *neutral price* at which the dealer sells to the trader if they trade when the dealer's reported type is t. For $t \in \mathcal{T}$ such that $Q(t) = 0$, I do not need to specify any price because the agents would not trade when such t were reported.

II. Example

I select the parameters of the model for a numerical illustration of IIE and neutral bargaining mechanisms: $\delta = 10$, $c_b = 20$, $c_g = 40$, $v_b = 30$, $v_g = 50$, $p_{lb} = 0.4$, $p_{hb} = 0.3$, $p_{lg} = 0.2$, and $p_{hg} = 0.1$.

For this example, it can be shown that if (Q, Y) is a feasible mechanism, then $0.1U_1(Q, Y|lb) + 0.2U_1(Q, Y|hb) + 0.5U_1(Q, Y|lg) + 0.2U_1(Q, Y|hg) + U_2(Q, Y) \leq 8$, and that if

$$0.1U_1(Q, Y|lb) + 0.2U_1(Q, Y|hb) + 0.5U_1(Q, Y|lg) + 0.2U_1(Q, Y|hg) + U_2(Q, Y) = 8, \quad (3)$$

then (Q, Y) is an IIE mechanism.

Consider the following four IIE mechanisms that satisfy equation (3): •

(Q^s, Y^s) – $Q^s(lb) = 1$, $Y^s(lb) = 30$,

$Q^s(hb) = 1$, $Y^s(hb) = 30$,

$Q^s(lg) = 1/3$, $Y^s(lg) = 50/3$,

$Q^s(hg) = 1/3$, $Y^s(hg) = 50/3$.

(Q^m, Y^m) – $Q^m(lb) = 1$, $Y^m(lb) = 20$,

$Q^m(hb) = 1/2$, $Y^m(hb) = 15$,

$Q^m(lg) = 1/6$, $Y^m(lg) = 25/3$,

$Q^m(hg) = 1/6$, $Y^m(hg) = 25/3$.

(Q^n, Y^n) – $Q^n(lb) = 1$, $Y^n(lb) = 20$,

$Q^n(hb) = 2/3$, $Y^n(hb) = 50/3$,

$Q^n(lg) = 1/6$, $Y^n(lg) = 20/3$,

$Q^n(hg) = 1/9$, $Y^n(hg) = 5$.

(Q^b, Y^b) – $Q^b(lb) = 1$, $Y^b(lb) = 10$,

$Q^b(hb) = 0$, $Y^b(hb) = 0$,

$Q^b(lg) = 0$, $Y^b(lg) = 0$,

$Q^b(hg) = 0$, $Y^b(hg) = 0$.

Note that there are many other IIE mechanisms.

Among all of the IIE mechanisms, (Q^m, Y^m) is the unique neutral mechanism. It can be verified that the conditions in Remark 1 are satisfied for all ε by the parameters $\lambda_1(lb) = 0.1$, $\lambda_1(hb) = 0.2$, $\lambda_1(lg) = 0.5$, $\lambda_1(hg) = 0.2$, $\lambda_2 = 1$, $\alpha(hb|lb) = 0.3$, $\alpha(lg|hb) = 0.4$, $\alpha(hg|lg) = 0.1$, and all other $\alpha(\cdot|\cdot)$ equal to zero. Because all IIE mechanisms satisfy equation (3), they must all have the same λ and α. With these parameters, condition (1) has the unique solution $\omega_1(lb) = 10$, $\omega_1(hb) = 5$, $\omega_1(lg) = 10/3$, $\omega_1(hg) = 5/3$, and $\omega_2 = 4$, which satisfy condition (2) for every positive ε only for (Q^m, Y^m) among all IIE mechanisms.

III. Discussion

The example enables a discussion of the implications of the model for reasonable bargained prices in OTC markets with asymmetric information.

The four mechanisms yield the following expected price of the asset if the agents trade for each $t \in \mathcal{T}$:

t	lb	hb	lg	hg
$P^s(\cdot)$	30	30	50	50
$P^m(\cdot)$	20	30	50	50
$P^n(\cdot)$	20	25	40	45
$P^b(\cdot)$	10	*	*	*

An asterisk is indicated for when the probability of trade is zero.

Under (Q^s, Y^s), the price is exactly the reservation value to the trader for acquiring the asset. So this mechanism gives expected utility payoff zero to the trader, and is the best feasible mechanism for all types of the dealer. Under (Q^b, Y^b), only the dealer of type lb sells at a price exactly equal to her reservation value of $c_b - \delta$; and there is no trade for all other types. So this mechanism always gives expected utility payoff zero to the dealer, and is the best feasible mechanism for the trader.[4]

An interesting observation is that the neutral mechanism (Q^m, Y^m) is a 50-50 randomization between the dealer- and trader-best mechanisms; i.e., for each $t \in \mathcal{T}$, $Q^m(t) = .5Q^s(t) + .5Q^b(t)$ and $Y^m(t) = .5Q^s(t) + .5Q^b(t)$. The neutral mechanism (Q^m, Y^m) stipulates that, if the agents trade, the neutral price of the asset be

$$P^m(lb) = \tfrac{1}{2}v_b + \tfrac{1}{2}(c_b - \delta)$$
$$\text{and } P^m(t) = v_{t(2)} \text{ for } t \in \{hb, lg, hg\}.$$

The underlying intuition is as follows. The dealer of type hg has the highest reservation value among all types, and so would try to bargain as if she is maximizing her "virtual" utility that exaggerates the difference from type lg with the next highest reservation value who wants to mimic her. Because the trader never pays a price higher than his reservation value for acquiring the asset, the maximal exaggeration type hg could do is to act as if she has a virtual reservation value equal to the trader's reservation value of $v_g = 50$, instead of her actual reservation value of $c_g = 40$. In a similar fashion, the lg-type dealer has a virtual value of $v_g = 50$ instead of $c_g - \delta = 30$; and the hb-type dealer has a virtual value of $v_b = 30$ instead of $c_b = 20$. Hence, in those cases the resulting bargained price is exactly the trader's reservation value. Only the lb-type dealer's virtual reservation value is the same as her actual value of $c_b - \delta = 10$, so the bargained price is exactly halfway between the reservation values of trader and dealer. The neutral price reflects the idea of inscrutable

[4] In the terminology of Myerson (1983), (Q^s, Y^s) is the unique strong solution for the dealer, and is the most reasonable solution for the dealer if she could dictatorially choose the mechanism. If the trader were the dictator, he would choose his best mechanism (Q^b, Y^b).

intertype compromise: The dealer acts strategically in the bargaining process, implicitly contemplating a compromise among the conflicting preferences of alternative types, which leads to the price that is most favorable to the dealer when her type is $t \in \{hb, lg, hg\}$.

Let us now turn to mechanism (Q^n, Y^n), which stipulates that the price (if trade) be

$$P^n(t) = \tfrac{1}{2}v_{t(2)} + \tfrac{1}{2}\left(c_{t(2)} - \delta \mathbf{1}_{t \in \{lb, lg\}}\right) \ \forall t \in \mathcal{T}.$$

This is the price attained in the generalized Nash bargaining solution when the agents have equal bargaining power. Also note that the Nash bargaining solution selects P^n as a unique Pareto-efficient price for the complete-information version of this example. Because the price at which agents trade is always halfway between the reservation values for the trader and for the dealer, this "Nash" mechanism (Q^n, Y^n) can be considered both equitable and efficient in real utility terms. However it is not equitable in virtual utility terms, because dealers of types hb, lg, and hg are selling the asset for prices that are less than their virtual values under the terms of mechanism (Q^n, Y^n).

The asset price predicted by the Nash mechanism (or any other IIE mechanism) may be considered as a reasonable approximation of the price for which investors bargain in OTC markets, but only reasonable in the sense of Pareto efficiency. The asset quality may reflect various factors that affect investors' valuations for the asset but that are not captured by public information. So if an asset owner has private information about asset qualities, then private information about her intrinsic liquidity state also matters. With the information asymmetry on both asset quality and liquidity state, the owner has incentives to conceal information about her true reason for selling the asset from a potential trading partner in order to receive a better price; so investors should reasonably bargain for the price that respects not only Pareto efficiency but also fair intertype compromise. The neutral bargaining solution gives a unique prediction of such price.

The illustration in this paper suggests caution in using the (generalized) Nash bargaining solution to compute prices, and instead one should use the neutral bargaining solution for asset pricing in OTC markets with asymmetric information.[5] In future work it would be useful to generalize to a dynamic model with asymmetric information and explore neutral bargaining in the presence of search frictions and investors' impatience for liquidity.

References

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen. 2005. "Over-the-Counter Markets." *Econometrica*, 73(6): 1815–1847.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen. 2007. "Valuation in Over-the-Counter Markets." *The Review of Financial Studies*, 20(5): 1865–1900.

Kim, Jin Yeub. 2017. "Interim Third-Party Selection in Bargaining." *Games and Economic Behavior*, 102: 645–665.

Myerson, Roger B. 1983. "Mechanism Design by an Informed Principal." *Econometrica*, 51(6): 1767–1797.

Myerson, Roger B. 1984. "Two-Person Bargaining Problems with Incomplete Information." *Econometrica*, 52(2): 461–488.

Tsoy, Anton. 2018*a*. "Alternating-Offer Bargaining with the Global Games Information Structure." *Theoretical Economics*, 13: 869–931.

Tsoy, Anton. 2018*b*. "Bargaining without Common Knowledge and its Application to Over-the-Counter Markets." Working Paper.

[5] In the context of conflict settings in which two privately-informed parties choose a mediator, Kim (2017) elaborates the analytical power of the neutral optimum as a solution concept for bargaining problems with incomplete information.