# RANDOM ASSIGNMENT WITH NON-RANDOM PEERS: A STRUCTURAL APPROACH TO COUNTERFACTUAL TREATMENT ASSESSMENT

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Efforts to leverage peer effects by creative assignment have fallen short due, in part, to endogenous peer choice. To address this, I build a two-part model: agents form a network via continuous linking decisions; conditional on the network, outcomes are determined allowing for peer effects. I provide a method to recover parameter estimates controlling for unobserved heterogeneity, leveraging new theory and identification results. I estimate the model using data from a randomized study in Indian schools, assessing its predictions against realized outcomes. This paper contributes new methodology for estimating peer effects, while advancing theory and econometrics of network formation.

KEYWORDS: Social networks, Peer effects, Endogenous networks.

## 1. INTRODUCTION

A rich literature in economics and related fields shows that individuals' choices and outcomes are correlated with the choices, outcomes, and characteristics of those they interact with. Further, papers leveraging random assignment credibly make the case that these peer effects can be given a causal interpretation (Epple and Romano, 2011; Sacerdote, 2011). For example, prominent studies have exploited random assignment to university dorms (Sacerdote, 2001; Stinebrickner and Stinebrickner, 2006), university class sections (DeGiorgi, Pelllizzari and Radaelli, 2010), second-grade classrooms in rural Kenya (Duflo, Dupas and Kremer, 2011), and squadrons at the Air Force Academy (Carrell, Fullerton and West, 2009; Carrell, Hoekstra and West, 2011). These studies often find large and statistically significant peer effects on a variety of outcomes (Epple and Romano, 2011). This robust evidence for the existence of peer effects suggests that creative peer assignment may be a powerful policy tool to influence individual choices and outcomes. That is, if peer effects are sufficiently strong, simply changing the composition of peer groups may substantially change measurable outcomes.

However, efforts to leverage peer effects to improve outcomes have proven difficult due to, among other reasons, a failure to account for endogenous sorting (see, e.g., Angrist, 2014;

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Carrell, Sacerdote and West, 2013). That is, policy interventions designed to change peer group composition may also affect patterns of interaction: even with random assignment, agents still may choose with whom they interact. The importance of this channel is highlighted by a number of recent studies documenting experimental interventions that change network structure (see Banerjee et al., 2018; Comola and Prina, 2018; Delavallade, Griffith and Thornton, 2016; Vasilaky and Leonard, 2018).

Further, even random assignment to treatment may be insufficient to predict the effects of novel treatment assignments. That is, as shown most starkly by Carrell, Sacerdote and West (2013), while random assignment may facilitate identification of average treatment effects within the support of an experiment, predicting the effects of off-support assignments requires more. In such a setting, the researcher must carefully account for the effects of assignments not only *through* peers but also on the *choice* of peers.[1] Thus, several recent papers have suggested pairing models of peer effects with models of peer choice that can be taken to data (Blume et al., 2015; Graham, 2015). This paper develops and estimates just such a model.

Network formation models are notoriously difficult to estimate, however, due to related issues of theory, identification, and computation.[2] The bulk of the theory on network formation posits links as binary: links either exist or do not.[3] The discrete nature of these games motivates the widespread use of pairwise stability as an equilibrium concept, in preference to Nash equilibrium.[4] These games tend to be characterized by the existence of multiple equilibria, a feature that complicates analysis, leading to partial identification (see de Paula, Richards-Shubik and Tamer, 2018; Leung, 2017; Sheng, 2012) or the need to specify complex equilibrium selection rules (see Badev, 2017; Christakis et al., 2010; Mele, 2017). Further, the discrete nature of the problem implies the need to calculate high-dimensional inequalities (see, e.g. Sheng, 2012), leading to a curse of dimensionality in estimation.

To surmount these difficulties, I model the network formation process as a static, simultaneous game in which players make *continuous* linking decisions. Additionally, linking decisions are made subject to a budget constraint, which necessarily builds in tradeoffs between forming links. The continuous nature of the game allows for the use of Nash equilibrium rather

[1]Of course, if the goal is simply to learn the effect of alternative policies on some outcome, researchers may design studies to cover a wide range of possible alternative assignments while remaining agnostic about network endogeneity, but such a strategy is often impractical due to institutional or funding limitations. One notable exception is Booij, Leuven and Oosterbeek (2016).

[2]Chandrasekhar (2015) and Graham (2015) provide comprehensive overviews of the current literature on the identification and estimation of network formation models.

[3]A few exceptions do exist (Baumann, 2016; Bloch and Dutta, 2009; Boucher, 2015).

[4]This recognizes the basic idea that forming links requires mutual consent while severing may be done unilaterally (Jackson, 2008; Jackson and Wolinsky, 1996).

than pairwise stability; that is, individuals may unilaterally choose to put more or less effort into a given link.[5] I show that the model has a unique strictly positive Nash equilibrium, in which all agents link positively to all other agents. This crucial feature facilitates point identification of the network formation game without the need to specify a sequential meeting process.

The strictly positive equilibrium is characterized by linear best-response functions which can then be used for identification and estimation. Additionally, the tradeoffs implied by the budget constraint motivate the relevance of a budget-set instrument to identify parameters of the network formation game.[6] With sufficient variation in exogenous characteristics, parameters of the network formation model are point identified. Further, individual-specific unobserved variables are identified as the size of each observed network grows. The use of large-network asymptotics is thus related to the results in Graham (2017) and Leung (2017). However, in contrast to Graham (2017), identification here requires neither conditional link independence nor specification of a likelihood function.

Next, I model outcomes conditional on the realized network. I generalize a reduced-form version of the linear-in-means model (see Manski, 1993) by including additively-separable unobserved or "latent" effects (Goldsmith-Pinkham and Imbens, 2013; Hsieh and Lee, 2016; Jackson, 2014; Auerbach, 2016) as well as non-linear peer effects (Carrell, Sacerdote and West, 2013). This approach explicitly models network endogeneity as an omitted-variable problem, and failure to account for the latent characteristics may bias estimates of the peer effects model. Crucially, the unobserved variables that cause bias due to network endogeneity are the same unobserved individual-specific parameters that are identified in the network-formation process. Conditional on their identification, I show that the parameters of the peer effects model are identified even in the presence of network endogeneity. Further, in contrast to Auerbach (2016), the fact of identifying parameters of the network formation process as well as unobserved heterogeneity through a structural model facilitates counterfactual analysis.[7]

I then take these identification results to data gathered as part of a randomized trial of a girls' empowerment program in a state in northern India. The study design consists of a treatment that assigns 13 girls (out of approximately 44) in each of 10 schools to participate in an after-school program, while an equal number of control schools do not

[5]This is consistent with other authors who employ Nash equilibrium in characterizing equilibrium in continuous network formation models (Baumann, 2016; Bloch and Dutta, 2009; Boucher, 2015). Boucher (2015) uses a stronger concept that rules out pairwise deviations.

[6]In the language of Manski (2000), interdependence occurs because of *constraint interactions* (see Boucher, 2015, for discussion).

[7]Graham (2017) makes a similar point in discussing the benefits of identifying latent unobserved factors.

receive any programming. As part of this effort, we collected especially rich data on social networks, consisting of a pairwise network census with detailed data on network connections along a number of dimensions. From this rich data, I construct a continuous measure of connectedness.

I first estimate the network-formation model to recover the individual-specific unobservables.[8] These estimates indicate that the structural unobservables are quite important in determining network structure. I then plug these into the peer-effects model, which allows for consistent estimation of the parameters of that model even in the presence of network endogeneity. For two program outcomes, I show that the individual-specific unobservables affect realized outcomes, a finding that is significant both substantively and statistically.

As a further check, I employ the estimated parameters in simulating outcomes for comparison to a treatment arm that was not included in the estimation sample. This step shows that, for one outcome at least, the model performs well in out-of-sample prediction, a validation step proposed by Todd and Wolpin (2006) and also pursued in development contexts by, for example Kaboski and Townsend (2011) and Bryan, Chowdhury and Mobarak (2014). Finally, while analytically solving for optimal assignment is beyond the scope of this paper, I randomly simulate a large number of alternative assignments to shed light on what an optimal assignment might entail.

This paper's primary contribution lies in providing a method for identifying peer effects models in the presence of endogenous network formation. Such estimates can then be used to predict the effects of alternative policies while simultaneously accounting for the effect of those policies on network structure. As a necessary step in developing this method, I make two additional contributions. First, I advance the theoretical literature on network formation, particularly in the context of agents making continuous linking decisions. Second, building upon this novel theoretical model, I provide an important advance in the econometric literature on the identification of network-formation games. Finally, in the empirical application, I contribute to the literature building bridges between structural and experimental approaches to program evaluation, especially in development contexts (see, e.g., Attanasio, Meghir and Santiago, 2011; Duflo, Hanna and Ryan, 2012; Todd and Wolpin, 2006), as well as adding to work comparing randomized to non-randomized assignments, as demonstrated, for example, in Shadish, Clark and Steiner (2008).

This paper proceeds as follows. Section 2 provides the peer effects model that posits network endogeneity as an omitted-variable problem. Section 3 then develops the network-formation

[8]The results derived here speak to *identification*. Identification here is constructive and implies moments that can be used for estimation within a GMM framework. Accordingly, inference is based on standard GMM results, in contrast to Auerbach (2016), Graham (2017), and Johnsson and Moon (2017), who derive novel estimators and asymptotic results.

model as a means of controlling for these unobserved confounding variables. Section 4 describes the program under study as well as deriving a number of key reduced-form facts that are consistent with the structural model. Section 5 presents results of structural estimation of the two-part model, using estimators suggested by the identification results. Using these parameter estimates, Section 6 compares predicted outcomes to outcomes from a treatment arm not used for estimation, in order to assess the model's performance in out-of-sample prediction. Section 7 then simulates a large number of random assignments in order to shed light on features of an optimally-designed assignment policy. Section 8 concludes and discusses implementation of the method derived here in related contexts.

## 2. PEER EFFECTS MODEL

The model of network formation and outcome determination is part of a two-step process. Networks are formed, then outcomes are determined conditional on the realized network. I discuss the second part of the model first in order to motivate the necessity of the (logically prior) network formation model.

### 2.1. *The Problem of Endogenous Networks*

The peer effects model begins with Equation (1), a reduced-form version of the standard linear-in-means model (see, e.g., Manski, 1993).[9] The "peer effect" is identified by the parameter $\alpha_2$.[10]

$$y_{is} = \alpha_0 + \alpha_1 P_{is} + \alpha_2 \bar{P}_{is} + u_{is} \tag{1}$$

In Equation (1), $y_{is}$ is some outcome for individual $i$ in school $s$. $P_{is}$ is an indicator for individual $i$ in school $s$ being chosen to participate in some treatment, and $\bar{P}_{is}$ is individual $i$'s peer group mean participation. The variable $u_{is}$ is unobserved.[11]

Equation (1) requires a definition of the peer group mean variable $\bar{P}_{is}$. This in turn requires the choice of how to weight peers. Suppose that in each school $s$ we observe a matrix $\mathbf{G_s}$ of directed links between individuals $i$ and $j$, where $g_{ijs} \in \mathbb{R}^+$, element $(i, j)$ of $\mathbf{G}$, corresponds

[9]The canonical model from Manski (1993) includes an endogenous peer effect $\bar{y}_{is}$, which complicates identification (see Blume et al. (2015) for a generalization of various identification results derived since Manski (1993)). As discussed in Carrell, Sacerdote and West (2013), with some assumptions the canonical model can be rewritten as Equation (1).

[10]Note that this combines the three different effects identified in Manski (1993). However, this is immaterial to the goal here as in Carrell, Sacerdote and West (2013) which is not to identify peer effects per se but to control for peer effects as a confounder in predicting effects of alternative policies.

[11]This model effectively assumes that peer influence is characterized by the peer group mean. Some recent studies, in contrast, have presented evidence of the importance of the variance in peer ability (e.g. Booij, Leuven and Oosterbeek, 2016; Lyle, 2009).

to individual $i$'s link to $j$. So, $\overline{P}_{is}$ is a weighted average of $i$'s links' participation:

$$\bar{P}_{is} = \sum_{j \neq i} \frac{w_{ijs}(\mathbf{G_s})}{\sum_{k \neq i} w_{iks}(\mathbf{G_s})} P_{js} \tag{2}$$

That is, $w_{ijs}(\mathbf{G_s})$ is a function from the link matrix $\mathbf{G_s}$ that defines the weight for the link between individuals $i$ and $j$. For example, when links are binary and symmetric, one particular weighting function could be $w_{ijs}(\mathbf{G_s}) = g_{ijs} \in \{0, 1\}$ and thus $\bar{P}_{is}$ is merely the fraction of an individual's peers who are also chosen to participate. If link values are continuous, then $\bar{P}_{is}$ may weight "closer" or "stronger" links more. For purposes of estimation, I take the weighting function as given, but note that there are many possible ways to weight continuous, asymmetric links.[12] See Appendix C for a fuller discussion of the issue of weighting.

I augment this model in Equation (1) by decomposing the error term $u_{is}$ in a manner similar to Goldsmith-Pinkham and Imbens (2013) and Hsieh and Lee (2016), who effectively include the unobserved variable $a_{is}$. In contrast to those papers, however, I follow the suggestion by Bramoullé (2013) to include a peer effect in the unobserved variable, which amounts to including $\bar{a}_{is}$ as an additional regressor in the structural peer effects model.[13]

Let $u_{is} = \alpha_3 a_{is} + \alpha_4 \bar{a}_{is} + v_{is}$. Accordingly, Equation (1) becomes Equation (3).

$$y_{is1} = \alpha_0 + \alpha_1 P_{is} + \alpha_2 \bar{P}_{is} + \alpha_3 a_{is} + \alpha_4 \bar{a}_{is} + v_{is} \tag{3}$$

With this formulation, network endogeneity biases peer effects estimates whenever $\bar{P}_{is}$ is correlated with either $a_{is}$ or $\bar{a}_{is}$. That is, if one estimates Equation (1) without controlling for $a_{is}$ and $\bar{a}_{is}$, estimates of $\alpha_2$ will be biased due to correlation between $\bar{P}_{is}$ and $u_{is}$ (which includes $(a_{is}, \bar{a}_{is})$).

As an example for when $\text{cov}(\bar{P}_{is}, a_{is}) \neq 0$, suppose that $a_{is}$ is unobserved academic ability, and this unobserved ability is positively associated with outcome $y_{is}$. Endogeneity arises when $a_{is}$ *also* plays a part in the network formation process, such as if those with higher ability also are more likely to link with participants. Therefore, those with higher $a_{is}$ will tend to have more of their links be with participants, bringing about positive correlation between $\bar{P}_{is}$ and $a_{is}$.[14] Note that this endogeneity may arise even when $P_{is}$ is exogenous, such as the case when participation is assigned randomly. That is, even with random assignment, estimation that does not account for unobserved $a_{is}$ may be biased in the presence of endogenous network

[12]Particularly, I take as given the weighting function $w_{ijs}(\mathbf{G_s}) = g_{ijs} + g_{jis}$.

[13]$\bar{a}_{is}$ is defined analogously to $\bar{P}_{is}$: $\bar{a}_{is} = \sum_{j \neq i} \frac{w_{ijs}(\mathbf{G_s})}{\sum_{k \neq i} w_{iks}(\mathbf{G_s})} a_{js}$. Other peer-group mean variables are defined similarly.

[14]It is not sufficient that those with higher ability link more (or less) with all students. Endogeneity arises because ability leads to differential valuation of network links based on ability.

formation.

In addition to accounting for network endogeneity, I further allow for the possibility of non-linear peer effects. As in Carrell, Sacerdote and West (2013), these non-linear peer effects account for the fact that peer means $\bar{P}_{is}$ and $\bar{a}_{is}$ may affect different types of individuals differently. This is accounted for by the variables $I_{isk}(z_{is}), k = 1, ..., K$, which define a set of $K$ indicators for being in different categories of the population. Such a partition could be defined by grade level, gender, baseline outcome, or any other function of exogenous characteristics $z_{is}$. With this additional step, Equation (3) becomes Equation (4).

$$y_{is} = \sum_{k=1}^{K} I_{isk} \left(\alpha_{0k} + \alpha_{1k} P_{is} + \alpha_{2k} \bar{P}_{is} + \alpha_{3k} a_{is} + \alpha_{4k} \bar{a}_{is}\right) + v_{is} \tag{4}$$

Non-linear peer effects are captured by the coefficients $\alpha_{2k}$ and $\alpha_{4k}$ varying with different values of $k$.[15]

It should be noted, however, that these coefficients do not have a direct interpretation in this context. In contrast to a case in which, for example, $\bar{P}_{is}$ is defined as the average participation of those in the same classroom as student $i$, $\bar{P}_{is}$ and $\bar{a}_{is}$ are determined by an endogenous network-formation process. Accordingly, the peer effects coefficients are intermediate parameters that play a part in determining final outcomes under counterfactual assignments.

### 2.2. *Identification Results for the Peer Effects Model*

With the outcome equation formulated as in Equation (4), identification is straightforward. Define the parameter vector $\alpha = (\alpha_{01}, ..., \alpha_{41}, ..., \alpha_{0K}, ..., \alpha_{4K})$. Let $N_s$ be the number of students in school $s$. For each $s$, define $\mathbf{P_s} = (P_{1s}, ..., P_{N_s s})'$ and $\mathbf{A_s} = (a_{1s}, ..., a_{N_s s})'$. Conditional on independence of observations across schools as well as exogeneity of participation $(\mathbf{P_s})$, the unobserved confounders $(\mathbf{A_s})$, and the network $(\mathbf{G_s})$, $\alpha$ is identified. This result is formalized in Proposition 1.

PROPOSITION 1 *Suppose that*

1. $(P'_{is}, \bar{P}_{is}, a_{is}, \bar{a}_{is}) \perp\!\!\!\perp (P_{jt}, \bar{P}_{jt}, a_{jt}, \bar{a}_{jt}) \; \forall \; s \neq t$.
2. $\alpha \in \mathbf{\Theta}_\alpha \subset \mathbb{R}^{5K}$, *where* $\mathbf{\Theta}_\alpha$ *is compact.*
3. $(P_{is}, \bar{P}_{is}, a_{is}, \bar{a}_{is}) \in \mathbf{X} \subset \mathbb{R}^4$, *where* $\mathbf{X}$ *is compact.*
4. $\mathbb{E}[v_{is} | \mathbf{P_s}, \mathbf{G_s}, \mathbf{A_s}] = 0 \; \forall \; j$
5. $\mathbb{E}[D_{is} D'_{is}]$ *is of rank* $5K$,

[15] This model allows for non-linear *direct* effects (captured by $\alpha_{1k}$ and $\alpha_{3k}$) as well as non-linear *peer* effects (captured by $\alpha_{2k}$ and $\alpha_{4k}$).

*where* $D_{is} = [I_{is1}(1, P_{is}', \bar{P}_{is}, a_{is}, \bar{a}_{is})', ..., I_{isK}(1, P_{is}', \bar{P}_{is}, a_{is}, \bar{a}_{is})'] \in \mathbb{R}^{5K}$.
*Then the parameter vector* $\alpha$ *of Equation (4) is identified as* $S \to \infty$.

PROOF: See Appendix A.

This model generalizes both the standard linear-in-means model (with no endogenous peer effect) as well as the more general model used by Carrell, Sacerdote and West (2013). The authors of that paper essentially assumed that $\alpha_{3k} = \alpha_{4k} = 0$ for all $k$. The standard linear-in-means model typically further assumes that $K = 1$ and thus $I_{is1} = 1 \ \forall \ i, s$, implying no non-linear effects. Accordingly, the identification result in Equation (1) states weaker conditions than those previously used in the literature on peer effects.

Further, the peer effects model here combines two approaches. First, it relies upon arbitrary latent characteristics $a_{is}$ that must be accounted for, thus drawing parallels to the "latent space" models of Goldsmith-Pinkham and Imbens (2013) and Hsieh and Lee (2016). Second, identification of the parameters in the presence of endogeneity takes a control function approach. These strategies are discussed in more detail in Appendix E.

Finally, I note that identification and thus consistent estimation in the presence of network endogeneity depends crucially on an initial estimate of unobserved $a_{is}$. This estimate is obtained from estimation of the network-formation process, which is described in the next section. Conditional on $a_{is}$ and given the assumptions of Proposition 1, we can recover the true parameters of the peer effects model.

## 3. A STRUCTURAL MODEL OF NETWORK FORMATION

The prior section showed that, conditional on the observed network and unobserved variables $a_{is}$, the parameters of the peer-effects model are identified. This section demonstrates how these unobeserved variables $a_{is}$ are identified through observation of the network-formation process.

### 3.1. *Simple Model*

To fix ideas and intuition, I first develop a simple version of the network-formation model. This simple model sets aside the unobserved variables $a_{is}$ that will be added into the model later.

#### 3.1.1. *Players, Strategy Space, and Utility*

For a given school $s$, there are $N_s$ players in the network formation game. $N_s$ is assumed to be determined exogenously. Each player $i$ in school $s$ chooses whether to be linked to each of the other $N_s - 1$ players. More formally, each player $i$ in school $s$ chooses a vector of actions

$g_{is} \in \mathbb{R}_+^{N_s-1}$. Link intensity is continuous: $g_{ijs} \in [0, \infty)$. That is, each player chooses a weakly positive amount to invest in each link.

Modeling links as continuous choices is not standard in the literature on network formation, which tends to model networks links as binary choices. Two earlier examples of continuous models are Bloch and Dutta (2009) and Rogers (2006). Baumann (2016) presents a more recent model, and provides a good discussion of such models in both economics and related fields.

Individuals make their linking choices subject to a total effort constraint as spelled out in Assumption 1. Each individual's objective is to maximize utility subject to this constraint.

Assumption 1 *For each* $i = 1, ...N_s$, $\sum_{j \neq i} c_{ijs} g_{ijs} \leq M_{is}$, *where* $c_{ijs}$ *is the cost to individual* $i$ *of forming a link with* $j$ *and* $M_{is}$ *is individual* $i$*'s endowment. Further,* $M_{is} \in [\underline{M}, \overline{M}] \subset \mathbb{R}_{++}$ *and* $c_{ijs} \in [\underline{c}, \overline{c}] \subset \mathbb{R}_{++}$.

The budget constraint serves two purposes in the model.[16] First, it imposes a structured way in which individuals trade off the costs and benefits of different linking strategies. If individual $i$'s constraint is binding and she chooses to increase $g_{ijs}$ (her link to $j$), then she must decrease some $g_{iks}$ (her link to another student $k$). Second, $M_{is}$ may vary across students and may depend on observed or unobserved characteristics. Accordingly, $M_{is}$ allows for out-degree heterogeneity: individuals with higher $M_{is}$ have a higher effort endowment and thus will tend to have more out-links in equilibrium, conditional on other variables in the model. Finally, note that the lower bound on cost implicitly imposes the restriction that network size is bounded above for each individual: even as the size of a person's school grows infinitely, the sum of links can only grow so much: $\sum_{j \neq i} g_{ijs} \leq \frac{M_{is}}{\underline{c}}$. Compact support of $M_{is}$ implies further that network size is bounded above across individuals.

Utility for individual $i$ in school $s$ is a function of the realized network $\mathbf{G_s}$ as well as exogenous characteristics of all students in school $s$, $\mathbf{X_s} = (X'_{1s}, ...X'_{N_s s})'$. Following prior models (e.g., Badev, 2017; Mele, 2017), I assume that the utility of links is additive. Similar to these models, I assume that individuals derive different utilities depending upon how "mutual" their links are. The utility to individual $i$ of a network $\mathbf{G_s}$ is given in Equation (5).

$$\begin{aligned} U_{is}(\mathbf{G_s}, \mathbf{X_s}) &= \sum_{j \neq i} u_{ijs} \\ &= \sum_{j \neq i} g_{ijs}^{\alpha} g_{jis}^{\beta} e^{f(X_{is}, X_{js})} \end{aligned} \tag{5}$$

[16] Budget constraints are rare in models of network formation, but have found application in both continuous (Baumann, 2016; Bloch and Dutta, 2009) and discrete (Boucher, 2015) cases. Further, the restriction of bounded degree in de Paula, Richards-Shubik and Tamer (2018) restricts networks in a similar manner.

The utility to individual $i$ from his link to $j$ depends upon both his linking strategy $g_{ijs}$ and on $j$'s linking strategy via $g_{jis}$. The Cobb-Douglas function imposes complementarity in linking strategies. Further, the functional form implies that all links are marginally valuable, except when $g_{jis} = 0$. Hence, in the absence of a budget constraint, a weakly dominant strategy for any individual is to link infinitely to all other agents.

Assumption 2 *The following restrictions hold:*

1. $X_{is}$ *and* $f()$ *are bounded in* $\mathbb{R}^k$ *and* $\mathbb{R}$, *respectively.*
2. $0 < \beta < (1 - \alpha) < 1$

Assumption 2 imposes additional structure on the utility function, which has important implications for equilibrium. First, bounded utility is useful in generating interior equilibria. Second, I deviate from the bulk of the literature on network formation by requiring the utility function to be *concave* in own strategy, which is implied here by the restriction $\alpha < 1$ (see, e.g., Bloch and Dutta, 2009). As pointed out by Boucher (2015), an assumption of *convexity* in own strategy leads to equlibria in which actors form few strong links, and the equilibrium set is qualitatively similar to the case when strategy sets are discrete.[17] In contrast, concavity has vastly different implications for the set of equilibria, as demonstrated below.

### 3.1.2. *Model Limitations*

Identification and estimation of network formation models tends to be complicated by related issues of multiplicity, partial identification, and high computational burdens. Accordingly, tractability and computational difficulties demand simplifying assumptions that may be more or less innocuous depending on application.

Before discussing equilibrium, I note that the model is limited in two important ways. First, utility from given links depends *only* on the link between those two individuals as well as their characteristics. Importantly, the utility to $i$ of linking to $j$ does *not* depend upon $j$'s links, other than his link to $i$. Thus, this model does not allow for utility from linking to popular individuals or congestion externalities, whereby a link to a given individual is less valuable when that individual has more links. While such externalities are allowed in the models of Mele (2017) and Badev (2017), this substantially complicates equilibrium characterization and, in turn, identification and estimation.

Accordingly, it is common to rule out these externalities, often by modeling network formation as a dyadic process. This is a common assumption that is made by, for example, Breza et al. (2017), Graham (2017), and Johnsson and Moon (2017). It has further been made

[17]As a counterpoint, Baumann (2016) does assume concavity, but also assumes $\beta = (1 - \alpha)$, which leads to different sets of equilibrium strategy profiles.

in papers that estimate network formation as a way of controlling for network endogeneity in peer effects contexts (See Graham, 2017; Goldsmith-Pinkham and Imbens, 2013; Hsieh and Lee, 2016). In the language of Manski (2000), I rule out certain forms of preference interactions. However, I go beyond these network models in two ways. First, I do not require symmetry in network links or in the utility function. Second, I allow for interactions via the budget constraint. That is, since the budget constraint binds in equilibrium, linking more with one individual necessitates linking less with at least one other.

A second limitation here is that, in contrast to the structure discussed by Bramoullé (2013) and Blume et al. (2015) and explicitly modeled by Badev (2017), individuals do not consider final outcomes $y_{is}$ in making their linking decisions. This is a crucial assumption of the model that greatly aids in tractability, and its plausibility will clearly be context- and outcome-specific. I note two suggetive factors here, however. First, the findings in Carrell, Sacerdote and West (2013) show that students in their study tend to choose peers by homophily. However, their reduced-form results suggest that those predicted to be in the lowest tercile would benefit from the exact opposite strategy: they should be seeking out links with those in the highest tercile from whom they benefit via peer effects. This provides suggestive evidence that optimizing final academic outcomes is of relatively little importance to these individuals in making network links. In addition, in the empirical application here, final outcomes are attitudes and other "soft skills," which individuals may freely choose (in contrast to test scores) and which would seem to play a very small role in peer choice. Accordingly, in these contexts, the assumption that individuals do not consider final outcomes in choosing peers is much more plausible. In other situations, such as when the outcome of interest is teenage smoking decisions (Badev, 2017), a model that allows actors to consider the effects of peers on outcomes may be needed.[18]

#### 3.1.3. *Equilibrium*

As spelled out above, each individual chooses a vector of links $g_{is} \in \mathbb{R}^{N_s-1}$ to maximize utility subject to others' linking decisions. As discussed in the Introduction, as agents work within continuous action spaces, I use Nash equilibrium as the solution concept: the game is in Nash equilibrium when all players simultaneously choose $g_{is}$ that maximizes their utility subject to other players' strategies. This is consistent with Bloch and Dutta (2009) and Baumann (2016), both of whom employ Nash equilibrium in network formation models with continuous action spaces. In contrast, Boucher (2015) uses the stronger concept that he defines as *bilateral equilibrium*, which allows for pairwise deviations but must also assume

[18]This assumption, along with the other functional form assumptions made throughout, provides further impetus for the need to perform out-of-sample testing, which I do here in Section 6.

*convexity* in order to characterize the equilibrium set.

Proposition 2 provides the primary equilibrium existence result. Existence is guaranteed by the concavity of the game, a result that dates back at least to Rosen (1965). However, the Nash Equilibrium is not necessarily unique. For example, there exists an equilibrium in which each person is connected only to one other person, on whom he exhausts his entire endowment of effort. Further, a completely empty network, in which $g_{ijs} = g_{jis} = 0$ for all $i, j \neq i$ is an equilibrium.

Accordingly, to refine the set of equilibria, I define a *strictly positive equilibrium* as a Nash equilibrium in which each person's strategy profile exhibits strictly positive links. That is, for a strategy profile to be a strictly positive equilibrium, it must be a Nash equilibrium and $g_{ijs} > 0$ for every $i, j \neq i$. Proposition 2 shows that a strictly positive equilibrium exists. Intuitively, this existence result relies heavily on the Inada condition inherent in the game: as $g_{ijs} \to 0^+$, the marginal utility of $i$ investing in a link with $j$ approaches infinity. A full proof is in Appendix A.

PROPOSITION 2 *There exists a Nash equilibrium for the network-formation game. Further, there exists a strictly positive equilibrium.*

PROOF: See Appendix A.

A necessary condition for a strictly positive equilibrium is that the following first-order conditions hold:

$$\frac{\partial U_{is}}{\partial g_{ijs}} = \alpha g_{ijs}^{\alpha-1} g_{jis}^{\beta} e^{f(X_{is},X_{js})} - c_{ijs}\lambda_{is} = 0 \ \ \forall \, i, \ j \neq i \tag{6}$$

$$\frac{\partial U_{is}}{\partial \lambda_{is}} = M_{is} - \sum_{j \neq i} c_{ijs} g_{ijs} = 0 \ \ \forall \, i \tag{7}$$

Importantly, there is only one interior equilibrium, as stated in Proposition 3. Intuitively, uniqueness derives from the concavity of the network-formation game. The result states that there is a *unique* solution to the First-Order Conditions in Equations (6) and (7) that characterize the strictly positive equilibrium.

PROPOSITION 3 *The strictly positive equilibrium of the game is unique.*

PROOF: See Appendix A.

This uniqueness result is quite important for estimation and simulation. First, identification and estimation proceed by assuming we observe the network in this unique state. Second, conditional on the parameters of the model, we can simulate counterfactuals by finding any solution to these conditions, with the knowledge that no others exist.

### 3.2. *Identification Results for the Simple Model*

The prior subsection showed that there exists a unique strictly positive Nash equilibrium. Identification proceeds by assuming that we observe $S$ networks in this equilibrium state.[19] As a preliminary matter, Assumption 3 states what is observed. I note that this assumption allows for observations to be arbitrarily dependent *within* schools.

ASSUMPTION 3 *For each $s = 1, ..., S$, $i = 1, ..., N_s$, we observe a vector of characteristics and links $(X'_{is}, g'_{is}) \in \mathbf{X} \times \mathbf{G}$, where $\mathbf{X} \subset \mathbb{R}^m$ and $\mathbf{G} \subset \mathbb{R}^{N_s-1}$ are compact, $m = dim(X_{is})$, and $N_s$ is the number of agents in school $s$. Further, $(X'_{is}, g'_{is}) \perp\!\!\!\perp (X'_{jt}, g'_{jt})\ \forall\ s \neq t$.*

#### 3.2.1. *Identification Arguments for Networks*

Before proceeding to identification results, I here discuss identification in network contexts, where dependence among observed links complicates asymptotics. This dependence among links arises for two reasons, which Manski (2000) refers to as refers to these two sources as *preference* and *constraint* interactions, respectively.[20] First, since utility depends upon the mutual-ness of links, individual $i$'s link choice to $j$ depends on $j$'s choice to $i$. So, $g_{ijs}$ depends on $g_{jis}$, where $j \neq i$. Second, the budget constraint imposes dependence among all of an individual's links. That is, $g_{ijs}$ depends on $g_{iks}$, where $j, k \neq i$. Accordingly, we require identification arguments that account for these cross-sectional dependencies.

To account for these dependencies, identification results in network-formation models have taken two different strategies, both of which I employ here. These strategies are often referred to as "many network asymptotics" and "single network asymptotics" (see, e.g., Graham, 2017, especially Footnote 7). Many network asymptotics depend upon observation of a number of different, generally independent networks.[21] That is, in our context, identification is achieved as $S$, the number of schools, approaches infinity. Such arguments can be employed to identify parameters that are common across networks.

[19] In other words, I assume a deterministic equilibrium selection rule or, alternatively, that only the strictly positive equilibrium is played in practice. In the empirical setting discussed below, the average school has 44 girls, making the assumption that all individuals are positive linked to all others more plausible, especially since agents may have very weak (but nonzero) links. In different settings, in which school size is much larger, such as in AddHealth (Harris, 2009), a selection rule that allows for zero links may be preferable.

[20] Manski (2000) also identifies a third source of dependence, expectations interactions, which becomes relevant in settings with imperfect information.

[21] Similar issues arise in IO contexts, where asymptotic arguments may be based on a large number of small markets ("many market asymptotics") or a small number of large markets ("large market asymptotics") (Armstrong, 2016; Berry, Linton and Pakes, 2004; Freyberger, 2015).

In contrast, identification of parameters that are only observed within a single network requires observation of arbitrarily large networks. As discussed in Graham (2017), for a network with $N_s$ agents, the econometrician observes $N_s - 1$ linking decisions per agent. Importantly in our context, we need to identify individual-specific parameters $a_{is}$ that are only observed within a single school. Identification of these parameters leverages such single network asymptotics, where parameters are identified as the size of the network $s$—which contains individual $i$—grows.

### 3.2.2. *Instrumentation Strategy and Identification*

The network formation model as spelled out above has two sources of endogeneity, for which I employ two distinct strategies. First, I difference out endogenous variables that depend only on $i$. Second, to control for endogeneity of individual $j$'s network choice, I employ a budget set instrument, whereby exogenous variation is obtained via variation in the utility of potential links. I then show that, conditional on appropriate exogeneity assumptions and rank conditions, crucial parameters of the network-formation model are identified.

Before proceeding to results, I rearrange the first-order conditions and redefine some variables. First, Equation (6) becomes Equation (8) and then Equation (9).[22]

$$\log g_{ijs} = \frac{\log \alpha}{1-\alpha} + \frac{\beta}{1-\alpha} \log g_{jis} + \frac{f(X_{is}, X_{js})}{1-\alpha} - \frac{\log \lambda_{is}}{1-\alpha} - \frac{\log c_{ijs}}{1-\alpha} \tag{8}$$

$$\tilde{g}_{ijs} = \tilde{\alpha} + \tilde{\beta}\tilde{g}_{jis} + \tilde{f}(X_{is}, X_{js}) - \tilde{\lambda}_{is} - \tilde{c}_{ijs} \tag{9}$$

Importantly, the parameters $\alpha$ and $\beta$ are subsumed into a composite parameter $\frac{\beta}{1-\alpha}$, defined hereafter as $\tilde{\beta}$. Additionally, assume the following functional form:

$$\tilde{f}(X_{is}, X_{js}) = \gamma_1 X_{is} + \delta_1 X_{is} X_{js} + \gamma_3 X_{js} \tag{10}$$

In the data as described below, all $X_{is}$ are binary variables. Accordingly, homophily corresponds to the coefficient $\delta_1$ being positive (and possibly $\gamma_1$ and $\gamma_3$ being negative).[23] Substitution and rearrangement of terms yields the following:

$$\tilde{g}_{ijs} = \tilde{\beta}\tilde{g}_{jis} + (\tilde{\alpha} + \gamma_1 X_{is} - \tilde{\lambda}_{is}) + \delta_1 X_{is} X_{js} + \gamma_3 X_{js} - \tilde{c}_{ijs} \tag{11}$$

[22] To get from Equation (8) to Equation (9), define and substitute $\tilde{g}_{ijs} = \log g_{ijs}$, $\tilde{\alpha} = \frac{\log \alpha}{1-\alpha}$, $\tilde{f}(X_{is}, X_{js}) = \frac{f(X_{is}, X_{js})}{1-\alpha}$, $\tilde{\lambda}_{is} = \frac{\log \lambda_{is}}{1-\alpha}$, and $\tilde{c}_{ijs} = \frac{\log c_{ijs}}{1-\alpha}$.

[23] A common alternative specification is $\tilde{f}(X_{is}, X_{js}) = \eta_0 + \eta_1 |X_{is} - X_{js}|$, where homophily is identified by $\eta_1 < 0$. If $X_{is}$ is binary (as in the empirical application in Sections 4 to 7), then it is simple to show that this alternative is a special case of Equation (10), where $\eta_0 = \gamma_1 + \delta_1 + \gamma_3 = 0$ and $\eta_0 + \eta_1 = \gamma_1 = \gamma_3$.

The econometric issue is to identify and estimate the parameters of Equation (11).

Identification is complicated due to two sources of endogeneity in Equation (11). First, $\tilde{\lambda}_{is}$, which identifies the (log) shadow value of additional effort endowment, necessarily depends upon $\tilde{c}_{ijs}$, the cost of linking. Second, whenever $\tilde{\beta} > 0$, $\tilde{g}_{jis}$ depends upon $\tilde{g}_{ijs}$, which depends on $\tilde{c}_{ijs}$. I solve these issues by using two different strategies.

The first strategy leverages the "panel" nature of the data by applying a tweak to a standard transformation. Instead of the standard two dimensions of individuals $i$ and time $t$, here we have two dimensions "out" $i$ and "in" $j$. For all variables in Equation (11), perform a "within $i$" transformation. That is, define $\bar{g}^i_{ijs} = \frac{1}{N_s-1}\sum_{k\neq i}\tilde{g}_{iks}$ and $\dot{g}^i_{ijs} = \tilde{g}_{ijs} - \bar{g}^i_{ijs}$. Other variables are defined similarly, leading to Equation (12).

$$\dot{g}^i_{ijs} = \tilde{\beta}\dot{g}^i_{jis} + \delta_1 X_{is}\dot{X}^i_{js} + \gamma_3\dot{X}^i_{js} - \dot{c}^i_{ijs} \tag{12}$$

This transformation eliminates all terms that vary only with $i$, including the necessarily endogenous term $\tilde{\lambda}_{is}$.

Second, I employ novel instruments for the necessarily endogenous $\dot{g}^i_{jis}$ term in Equation (12). The instrument relies upon tradeoffs between different linking strategies, which in turn relies upon the non-dyadic structure of the network formation model. Intuitively, due to the budget constraint, individual $j$'s linking decision to $i$ depends upon his alternative options for links. That is, it depends upon the utility he derives from linking to *other* individuals $k$, where $k \neq i, j$, which in turn depends upon $k$'s characteristics. Crucially, the instrument works through the budget constraint and thus the shadow value of effort.

Simple algebra shows how these instruments are relevant. First, take the mirror image of Equation (11), replacing $i$ with $j$ and $j$ with $i$, leading to Equation (13).

$$\tilde{g}_{jis} = \tilde{\beta}\tilde{g}_{ijs} + (\tilde{\alpha} + \gamma_1 X_{js} - \tilde{\lambda}_{js}) + \delta_1 X_{js}X_{is} + \gamma_3 X_{is} - \tilde{c}_{jis} \tag{13}$$

Next, perform the "within $i$" transformation, leading to Equation (14), and note that $\dot{g}^i_{jis}$ on the left-hand side is the same as the endogenous regressor in Equation (12).

$$\dot{g}^i_{jis} = \tilde{\beta}\dot{g}^i_{ijs} + \gamma_1\dot{X}^i_{js} - \dot{\lambda}^i_{js} + \delta_1 X_{is}\dot{X}^i_{js} - \dot{c}^i_{jis} \tag{14}$$

The terms on the right-hand side of Equation (14) suggest instrument candidates. However, $\dot{g}^i_{ijs}$ is the dependent variable in Equation (12) and thus necessarily depends on $\tilde{c}_{ijs}$. Further, $\dot{X}^i_{js}$ and $X_{is}\dot{X}^i_{js}$ are on the right-hand side of that same equation and thus not excludable. Accordingly, instruments must come through the term $\dot{\lambda}^i_{js}$.

Relevant instruments are revealed by decomposing the term $\dot{\lambda}^i_{js}$. This shows that

$$\dot{\lambda}^i_{js} = \tilde{\lambda}_{js} - \frac{1}{N_s - 1}\sum_{k \neq i} \tilde{\lambda}_{ks} = \underbrace{\tilde{\lambda}_{js}}_{\text{Equation (16)}} - \underbrace{\frac{1}{N_s - 1}\sum_{k} \tilde{\lambda}_{ks}}_{\text{constant}} + \underbrace{\frac{1}{N_s - 1}\tilde{\lambda}_{is}}_{\text{Equation (17)}} \tag{15}$$

The middle term is constant for all $i$ and $j$ within the school $s$. However,

$$\tilde{\lambda}_{js} = \frac{1}{N_s - 2}\sum_{k \neq i, k \neq j} \left(-\tilde{g}_{jk} + \tilde{g}_{jk}\tilde{\beta} + X_{ks}\gamma_1 + X_{js}X_{ks}\delta_1 - c_{kj}\right) \tag{16}$$

$$\tilde{\lambda}_{is} = \frac{1}{N_s - 2}\sum_{k \neq i, k \neq j} \left(-\tilde{g}_{ik} + \tilde{g}_{ik}\tilde{\beta} + X_{ks}\gamma_1 + X_{is}X_{ks}\delta_1 - c_{kj}\right) \tag{17}$$

Equations (16) and (17) motivate the use of the following instruments:

1. $\frac{1}{N_s-2}\sum_{k\neq i, k\neq j} X_{ks}$
2. $\frac{1}{N_s-2}\sum_{k\neq i, k\neq j} X_{js}X_{ks}$
3. $\frac{1}{N_s-2}\sum_{k\neq i, k\neq j} X_{is}X_{ks}$

These instruments are the mean characteristics of individuals other than $i$ and $j$ within school $s$, as well as those characteristics interacted with $i$'s and $j$'s characteristics.

To provide intuition for these instruments, I employ a brief example. Suppose there are three individuals in a given school: $i$, $j$, and $k$. Students come in two types: Wolverines and Spartans, and variable $X$ is an indicator for being a Wolverine. Wolverine students exhibit strong homophily ($\delta_1 > 0$). Suppose $i$ and $j$ are both type Wolverines. Variation in $k$'s type clearly affects $i$ and $j$'s link decisions to each other: if $k$ is also a Wolverine, then both $i$ and $j$ will link more to $k$ than if $k$ is a Spartan. Due to the budget constraint, linking more to $k$ necessitates that they link less to each other. Accordingly, variation in characteristics of *other* students serves as a relevant instrument in determining $i$'s and $j$'s linking strategies toward each other.

Now that relevance has been established, Assumption 4 provides the primary excludability assumption. This assumes mean independence of unobserved costs from *all* covariates, both those of the two individuals involved with the specific link and others. Independence of unobserved costs from all covariates is necessary for the instruments discussed above to be valid.

Assumption 4 $\mathbb{E}[\log c_{ijs}|X_{ks}] = 0 \;\forall\; k$.

Assumption 5 $(\tilde{\beta}, \delta_1', \gamma_3') \in \Theta$, *a compact subset of* $\mathbb{R}^{2m+1}$, *where* $m = dim(X_{is})$.

The simple model's main identification result is stated in Proposition 4. I note that, due to the "within $i$" transformation, parameters for terms that vary only with $i$ are not identified. Importantly, $\lambda_{is}$, $\tilde{\alpha}$, and $\gamma_1$ are not identified, but this amounts to non-identification of the scale of each individual's utility. In contrast, parameters that identify the utility *tradeoffs* that $i$ makes in her linking decisions—particularly, $\tilde{\beta}$, $\delta_1$, and $\gamma_3$— are identified.

PROPOSITION 4 *Define* $z_{ijs} = [X_{is}\dot{X}^i_{js}, \dot{X}^i_{js}, \frac{1}{N_s-2}\sum_{k\neq i,j}[X_{ks}, X_{is}X_{ks}, X_{js}X_{ks}]]$ *and* $b_{ijs} = [\dot{g}^i_{jis}, X_{is}\dot{X}^i_{js}, \dot{X}^i_{js}]$. *Given Assumptions 3, 4, and 5,* $(\tilde{\beta}, \delta_1', \gamma_3')$ *is identified if* $\mathbb{E}[z'_{ijs}b_{ijs}]$ *is of rank* $2m+1$.

PROOF: See Appendix A.

There are at least two situations in which identification fails the hypotheses of Proposition 4. First, if $\delta_1$ and $\gamma_3$ are both zero, then the constructed instruments are irrelevant, since then the $X$ characteristics are irrelevant to the link-formation process. Second, the instruments may be collinear with the exogenous regressors. Importantly, if $X$ is an indicator for treatment that is assigned by school—that is, for each $s$, $X_{is} = X_{js} = X_{ks}\ \forall\ i, j, k$—the instruments will be collinear. In both situations, the rank condition stated in Proposition 4 fails. Identification, therefore, requires that exogenous characteristics vary *within* schools and that these exogenous characteristics are relevant for network formation.

## 3.3. *Adding in Scalar Unobservables*

Recall that the purpose of the network formation model is to recover the unobserved variables $a_{is}$ for each $i$ in school $s$, in order to control for network endogeneity in the peer effects model. Having derived results for the simple model, I now add these into the model.

### 3.3.1. *Equilibrium and Functional Form*

Scalar unobservables $a_{is}$ and $a_{js}$ are included in the model as part of the function $f$.[24] Functionally, they enter utility exactly the same way as $X_{is}$ and $X_{js}$. That is, these scalar unobservables change the relative utilities of the various linking strategies. I make the following assumption on the functional form of $f$:

$$\begin{aligned}\tilde{f}(X_{is}, X_{js}, a_{is}, a_{js}) = \gamma_1 X_{is} + \gamma_2 a_{is} + \delta_1 X_{is}X_{js} + \delta_2 X_{is}a_{js} \\ + \delta_3 a_{is}X_{js} + \delta_4 a_{is}a_{js} + \gamma_3 X_{js} + \gamma_4 a_{js}\end{aligned} \tag{18}$$

[24] As such, the utility function shares features of those employed by Graham (2017) and Breza et al. (2017). In contrast to Graham (2017), however, allowing $a_{is}$ and $a_{js}$ to interact with each other and also with observable characteristics $X_{is}$ and $X_{js}$ implies that unobservables can affect the *types* of links rather than just the *number* (degree). In contrast to Breza et al. (2017) and Hsieh and Lee (2016), I model unobserved characteristics as scalar rather than vector-valued.

In the empirical estimation in the following sections, the vector of observed variables $X_{is}$ contains a participation indicator $P_{is}$. In order for omitted $a_{is}$ to bias estimates of the peer effects model, it must change the *relative* utilities derived from links conditional on $X$ (and thus $P$). Note the centrality of the *interactions* between $(a_{is}, a_{js})$ and $(X_{js}, X_{is})$ here. For example, if $\delta_3$ is positive, then individuals with higher $a_{is}$ derive more utility from linking with participant individuals (for whom $P_{js} = 1$) than those without such characteristic (for whom $P_{js} = 0$). This leads them to have higher $\bar{P}_{is}$ in the outcome equation, which is clearly correlated positively with $a_{is}$. This continues to hold even if $P_{is}$ is randomly assigned.

With the additional assumption that $a_{is}$ is bounded, the equilibrium results for the simple case extend to the case with scalar unobservables. That is, the results in Propositions 2 through 3 hold. Equilibria exist, and the strictly positive equilibrium is unique.

3.3.2. *Identification Results with Scalar Unobservables*

The simple model effectively assumes $a_{is} = 0$ for every individual. This rules out the primary source of endogenous network formation that leads to bias in the peer effects estimates. Adding these back into the model, Equation (12) becomes Equation (19).

$$\dot{g}^i_{ijs} = \tilde{\beta}\dot{g}^i_{jis} + \delta_1 X_{is}\dot{X}^i_{js} + \delta_2 X_{is}\dot{a}^i_{js} + \delta_3 a_{is}\dot{X}^i_{js} + \delta_4 a_{is}\dot{a}^i_{js} + \gamma_3\dot{X}^j_{is} + \gamma_4 a_{is}\dot{a}^i_{js} - \dot{c}^i_{ijs} \tag{19}$$

Again, the mirror image of Equation (19) provides instruments for endogenous $\dot{g}^i_{jis}$.

$$\dot{g}^j_{jis} = \tilde{\beta}\dot{g}^j_{ijs} + \delta_1 X_{js}\dot{X}^j_{is} + \delta_2 X_{js}\dot{a}^j_{is} + \delta_3 a_{js}\dot{X}^j_{is} + \delta_4 a_{js}\dot{a}^j_{is} + \gamma_3\dot{X}^j_{js} + \gamma_4 a_{js}\dot{a}^j_{is} - \dot{c}^j_{jis} \tag{20}$$

As in the simpler case, the instruments work through the (log) shadow value of the network constraint $\tilde{\lambda}_{is}$.

Assumption 6 provides exogeneity assumptions for the full model.

Assumption 6 *The following conditions hold:*

1. $\mathbb{E}[\log c_{ijs}|\mathbf{X_{ks}}, a_{ks}] = 0 \;\forall\, k, s$
2. $\mathbb{E}[a_{is}|\mathbf{X_{ks}}] = 0 \;\forall\, k, s$
3. $\mathbb{E}[a_{is}|a_{js}] = 0 \;\forall\, j \neq i \;\forall\, s$

The first part of the assumption is similar to Assumption 4 and implies that unobserved costs are (mean) independent of individual-level observed and unobserved variables. The second part serves to separate the composite term $(\gamma_3 X_{js} + \gamma_4 a_{js})$,[25] while the third part of Assumption 6 rules out correlation among these unobserved variables.

[25] This assumption imposes independence between observed and unobserved variables, an assumption also made by Hsieh and Lee (2016).

ASSUMPTION 7 *$(\tilde{\beta}, \delta_1', \delta_2', \delta_3', \delta_4, \gamma_3', \gamma_4) \in \mathbf{\Theta}$, a compact set in $\mathbb{R}^{4m+3}$, where $m = dim(X_{is})$. Further, $a_{js} \in \mathbf{\Omega}\ \forall\ j, s$, where $\mathbf{\Omega}$ is a compact set in $\mathbb{R}$.*

Identification results are analogous to those of the simpler model. Proposition 5 states the first result.

PROPOSITION 5 *Define $z_{ijs} = [X_{is}\dot{X}^i_{js}, \dot{X}^i_{js}, \frac{1}{N_s-2}\sum_{k\neq i,j}[X_{ks}, X_{is}X_{ks}, X_{js}X_{ks}]]$ and $b_{ijs} = [\dot{g}^i_{jis}, X_{is}\dot{X}^i_{js}, \dot{X}^i_{js}]$. Given Assumptions 3, 6, and 7, the parameters $\tilde{\beta}$, $\gamma_1$, and $\delta_1$ are identified if $\mathbb{E}[z'_{ijs}b_{ijs}]$ is of rank $l \geq 2m+1$, where $m = dim(X_{is})$.*

PROOF: See Appendix A.

Conditional variance assumptions are necessary to identify the remaining network-formation parameters, as stated in Assumption 8.

ASSUMPTION 8 *The following conditional variance restrictions hold:*

1. $\mathbb{E}[a^2_{is}|\mathbf{X_{ks}}] = \sigma^2_a\ \forall\ k$
2. $\mathbb{E}[a^2_{is}|a_{js}] = \sigma^2_a\ \forall\ j \neq i$

Given Assumption 8, Proposition 6 provides conditions under which the parameters are identified, but only to scale. These parameters are only identified to scale due to the fact that we can re-scale them by correspondingly re-scaling the latent variable $a_{is}$. For a given normalization, such as $\sigma^2_a = 1$, these parameters are identified absolutely.

PROPOSITION 6 *Given Assumptions 6 and 8, the parameters $\gamma_2$, $\delta_2$, $\delta_3$, and $\delta_4$ are identified to scale if the following rank conditions hold:*

1. *rank*$(\mathbb{E}[z'_{ijs}b_{ijs}]) = 2m+1$, *where* $b_{ijs} = [\dot{g}^i_{jis}, X_{is}\dot{X}^i_{js}, \dot{X}^i_{js}]$
2. *rank*$(\mathbb{E}[z'_{ijs}b^2_{ijs}]) = m+1$, *where* $b^2_{ijs} = [1, X_{js}]$
3. *rank*$(\mathbb{E}[z'_{ijs}X_{is}]) = m$
4. *rank*$(\mathbb{E}[z'_{ijs}]) = 1$

PROOF: See Appendix A.

Proposition 6 only identifies the global parameters. identification of the the peer effects model relies upon controlling for latent variables $a_{is}$, and thus we need to recover these variables in order to identify its parameters. While results to this point have relied upon "many market" asymptotics, identification of $a_{is}$ relies upon "large market" asymptotics. For an individual in a school of size $N_s$, we observe $N_s - 1$ links.[26]

[26] Average school size in the data is 44 girls, and thus we have, on average, 86 data points with which to estimate $a_{is}$ for each $i$, corresponding to $i$'s 43 decisions to link to others and the 43 others' decisions to link to $i$. If, despite this, the prior estimation returns noisy but unbiased estimates of $a_{is}$ and $\bar{a}_{is}$, this should induce attenuation in estimates of $\alpha_{3k}$ and $\alpha_{4k}$.

PROPOSITION 7 *For a given s,* $\gamma_4 + \delta_2 \underset{i \neq j}{\mathbb{E}}[X_{is}] \neq 0 \Rightarrow a_{js}$ *is identified to scale for all* $j$ *as* $N_s \to \infty$.

PROOF: See Appendix A.

Proposition 7 provides the main identification result for scalar unobservables $a_{is}$. The condition that $\gamma_4 + \delta_2 \underset{i \neq j}{\mathbb{E}}[X_{is}] \neq 0$ is a relevance condition requiring that unobserved $a_{is}$ actually play a part in the network-formation process. If $X_{is}$ includes *all* variables involved in determining network links, then this condition will fail, but this seems quite unlikely. Note that, as in Proposition 6, each $a_{is}$ is only identified to scale, a scale that can be fixed with a convenient normalization.

Finally, note that identification of $a_{js}$ is achieved by observing *in* links, rather than out. That is, if, conditional on global parameters and observables $X_{js}$, others are linking more with $j$ than to other with similar observables, then we infer that $a_{js}$ is larger.

### 3.3.3. *Discussion of Identification Results*

Propositions 5 through 7 provide the primary identification results for the model with scalar unobservables. Observation of many networks provides identification of parameters common to all networks, as given in Propositions 5 and 6. Note again that, similar to the simple case in the prior subsection, parameters that involve variables that vary only with $i$ are not identified, but this essentially amounts to inability to identify scale factors in the utility function.

Observation of a large number of linkages within each network provides identification of the vector of individual-specific parameters $a_{is}$ for each $i$ in each $s$, as given in Proposition 7. Identification of the scalar unobservables here is similar to that derived in Graham (2017). As the size of a network $N_s$ grows, we observe $N_s(N_s - 1)$ network links and $N_s$ parameters. Therefore, the number of observations grows much faster than the number of parameters.[27]

There is currently a divide in the literature between positing models that generate dense or sparse networks. Mele (2017) and Badev (2017) propose models that generate dense networks, as does Breza et al. (2017). These rely on Bayesian methods for identification and estimation. Similarly, the joint maximum likelihood estimator derived in Graham (2017) as well as the sieve estimator in Johnsson and Moon (2017) similarly require dense network sequences, whereby the average degree increases proportionally to the number of agents. In contrast, Leung (2017) and Manresa (2016) require sparsity in links.

In contrast, identification here involves a sort of middle ground between dense and sparse

[27]In contrast, Breza et al. (2017) do inference on latent variables in a Bayesian framework that does not rely on network size growing.

networks. While total network size—defined by the sum of network intensities—is bounded, in the strictly positive equilibrium each individual is linked to a growing number of others as the number of others increases.[28]

Further, identification here relies upon looking at average *ratios* rather than absolute link values. That is, ratios may converge as the size of the network grows even as all link values converge to zero.[29] This problem is qualitatively similar to the problem of identification and inference encountered in the industrial organization literature on differentiated products, where inference is performed as the number of products grows to infinity while average market shares necessarily converge to zero (See, e.g. Armstrong, 2016; Berry, Linton and Pakes, 2004; Freyberger, 2015).

## 4. EMPIRICAL SETTING AND DATA DESCRIPTION

### 4.1. *Girls' Empowerment Program*

An NGO partner operates a Girls' Parliament program in government schools in rural districts of a state in northern India. As part of the program, 13 girls in grades 6 through 8 are chosen to form a Girls' Parliament. The program focuses on developing so-called "soft skills." These skills include leadership and self-confidence as well as attitudes and aspirations about education, age at marriage, and gender roles. The larger goal of the program is for girls to employ these skills as a means of overcoming barriers to their own education, such as early marriage.[30]

The intervention consists of a series of five "games" during which village volunteers work through increasingly difficult scenarios. Through activities such as role playing, girls are taught to develop their own voice in difficult situations such as, for example, their parents desiring to have them marry young or end their schooling. The parliaments meet biweekly over a span of approximately six months during the academic year, for a total intervention time of approximately 25-50 hours. Participating girls are encouraged to share their learning and experiences with their classmates who are not participants, and effects spilling over to non-participants is a key feature of the implementing organization's theory of change.

Under the NGO's preferred assignment rule, the 13 participants girls in each school are chosen through elections involving all students in grades 6-8, including boys. These elections lead to non-random selection, a fact that is documented in Delavallade, Griffith and Thornton (2016).

[28] That is, the average number of positive links $\frac{1}{N}\sum_i \sum_{j\neq i} \mathbf{1}\{g_{ijs} > 0\} \to \infty$, while the average link value $\frac{1}{N}\sum_i \sum_{j\neq i} g_{ijs} \to 0$.

[29] I have shown identification in this section, but leave inference on $a_{is}$ to further study, while noting that large-network asymptotics is a very active research area.

[30] This program is discussed in more detail in Delavallade, Griffith and Thornton (2016).

### 4.2. *Study Design and Data Collection*

As part of the rollout of the program to a new district during the 2013-14 academic year, a study team designed and implemented a randomized trial. A sample of thirty schools was chosen, none of which had ever had a Girls' Parliament prior to the start of the study.

Prior to treatment assignment, three data collection activities were conducted in each school. First, elections were held in all schools, including those that would later serve as controls. Second, girls in each school filled out an extensive questionnaire that gathered background demographic information as well as data on attitudes, aspirations, and expectations along a number of dimensions. Third, prior to treatment assignment, a pairwise network census was collected among all girls in each of the 30 schools, the form of which is described below.

After baseline data collection, schools were assigned to one of three treatment groups. In Elected Treatment schools, the program was conducted with the girls chosen by election, as is customary for the program as implemented by the NGO. In Random Treatment schools, girls were randomly chosen to participate, which selection rule superseded the elections that were previously held. Finally, Control schools did not receive the program in any form.

The program was implemented over a period of approximately six months. At the conclusion of program implementation, enumerators returned to each school to conduct an endline survey that measured outcomes similar to those measured at baseline. Further, in order to assess the effect of the program on networks, we conducted an additional pairwise network census. Accordingly, this data allows us to measure the program's effects on both endline outcomes—as measured by aspirations and attitudes—and endline networks..

### 4.3. *Demographics and Outcomes at Baseline*

Table I provides descriptive statistics for the full sample of 1319 girls at baseline.[31] Note that approximately 28 percent of the girls were elected to participate, out of an average of approximately 44 girls in each school. Enrollment is slightly skewed toward girls in Grade 6 (the omitted category), which suggests school dropout during the covered ages. Finally, lower castes represent the overwhelming majority of the sample, as 37% of the sample are members of Scheduled Castes/Scheduled Tribes, while 44.5% are in Other Backwards Castes. The omitted caste category, General or upper castes, comprises 18.5% of the sample.

This paper focuses on two outcomes: Educational Aspirations and Gender Roles attitudes. These outcome measures are constructed as the normalized first principal component of all

[31]The sample consists of all girls who have non-missing data on the covariates in Panel A. This consists of more than 99% of eligible girls.

relevant survey questions.[32] Girls have higher educational aspirations if, for example, they indicate they would like to complete university, as compared to stating they would like to complete only eighth grade. Girls have higher Gender Roles attitudes if, for example, they say it is okay for a wife to disagree with her husband in public.

TABLE I
Baseline Variable Descriptives

| | Mean | S.D. |
|---|---|---|
| *Panel A: Baseline Covariates* | | |
| Elected | 0.281 | 0.449 |
| Grade 7 | 0.318 | 0.466 |
| Grade 8 | 0.306 | 0.461 |
| Scheduled Caste | 0.252 | 0.435 |
| Scheduled Tribe | 0.118 | 0.323 |
| Other Backward Caste | 0.445 | 0.497 |
| | | |
| *Panel B: Baseline Outcomes* | | |
| Educational Aspirations | -0.197 | 1.036 |
| Gender Roles | 0.115 | 0.984 |

Notes: Sample is all girls in all schools. N = 1319 in 30 schools. Baseline Outcomes normalized among *all* students in the data (including boys).

Baseline outcomes are summarized in Table I, Panel B. Since the mean of the outcome variables is zero by construction in the data among *all* students (including boys), these means indicate that girls have below average Educational Aspirations and above average Gender Roles attitudes. This conforms to our priors that girls have lower Educational Aspirations than boys but higher Gender Roles attitudes.[33]

Further, the outcomes in Panel B vary substantially by baseline characteristics in Panel A, as shown in Appendix Table A.3. Elected girls have higher Educational Aspirations on average, while lower-caste girls (SC, ST, and OBC) have substantially lower levels of both baseline outcomes. Accordingly, if the program is to be targeted at those most "at need," it may make sense to target lower-caste girls for participation in the program rather than the more popular girls who are chosen by election.

[32]That is, among all students in the sample, the mean is set to zero with variance of one.

[33]Baseline balance is presented in Appendix B. Table A.1 shows balance across treatment arms, while Table A.2 shows within-school balance between girls (randomly) selected to participate in Random Treatment and those not selected.

### 4.4. *Baseline Network Descriptives, and a Continuous Measure of Connectedness*

The network data come from a pairwise network census, conducted at both baseline and endline. This procedure consisted of each girl in the sample answering a series of binary questions about every other girl in her school in grades 6 through 8. I categorize these variables by whether they are choices, such as being friends, or static variables, such as living in close proximity. These are described in Table II in Panels A and B, respectively. I have highlighted the "She is a friend" measure in gray, as that is the link definition commonly reported in the literature on networks and peer effects. Note that the friendship networks are quite dense: on average, girls indicate that 45.8 percent of other girls in their school are friends, as shown in the shaded row of Table II. Other measures of connectedness, on the other hand, suggest less dense networks. For example, only 23.8% of girls say that they have spent time outside school with the other in the past week.

TABLE II
ENDLINE NETWORK VARIABLE DESCRIPTIVES

| | Mean | In/Out Correlation | Factor Loading |
|---|---|---|---|
| *Panel A: Choice Network Variables* | | | |
| She is a friend | 0.458 | 0.334 | 0.324 |
| I speak with her regularly | 0.373 | 0.291 | 0.343 |
| In the past week, spent time outside school | 0.238 | 0.244 | 0.310 |
| I think she is clever | 0.397 | 0.176 | 0.334 |
| She has a lot of friends / is popular | 0.384 | 0.257 | 0.347 |
| She is very shy/quiet | 0.399 | 0.139 | 0.270 |
| I think she is very confident | 0.275 | 0.214 | 0.367 |
| I wish I could be like her | 0.210 | 0.171 | 0.337 |
| I can trust her to keep my secrets | 0.245 | 0.244 | 0.359 |
| *Panel B: Static Network Variables* | | | |
| She is a relative | 0.154 | 0.434 | |
| We are in the same caste | 0.208 | 0.704 | |
| I can walk to her home in less than 10 minutes | 0.248 | 0.282 | |

Notes: Sample is all pairs of students. N = 78,238 in 30 schools. Missing data imputed via iterative EM algorithm (see Appendix D). First principal component explains 47.3% of variation.

While the bulk of the economics literature on networks treats links as binary, the additional measures of connectedness allow for the capture of more granular link intensity. Further, the structural network formation model developed in Section 3 requires a continuous measure of connectedness to implement. In order to exploit this additional information—and as an input into the structural estimation process described in later sections of this paper—I construct a "continuous" measure of connectedness by collapsing the measures in Table II, Panel A into a single index. While this provides for more exploitable variation, these variables are highly correlated with each other. To account for the covariance structure, I take the first principal component, scaled such that the constructed continuous measure has minimum zero and unit variance.[34] The final column of Table II provides the factor loadings for each variable in Panel A; the first component accounts for 47.3% of the variation in the included variables.

In Table III, I compare the continuous measure of connectedness to the binary one. For the latter, I follow the bulk of the literature in using the student's response to "She is a friend" as a binary link measure. Panel A shows the probability that a student in the group identified on the y-axis indicates an individual on the x-axis is a friend. For example, the likelihood that a member of a Scheduled Caste names another Scheduled Caste member as a friend is 57.7%, while the likelihood of her naming a member of General Castes is 42.2%. Comparison of (shaded) elements along the diagonal with others in the same row suggests individuals are much more likely to claim as friends others of their own population grouping. The final column provides the p-value of a test of the equality between the probability of an individual in that row indicating a same-type other individual is a friend with the probability of her indicating an individual in a different category is a friend. Note that this test suggests strong homophily among members of Scheduled Castes and General Castes, but provides weaker evidence for Scheduled Tribes and Other Backwards Castes under the binary link definition.

Panel B performs the same exercise as Panel A, except with means of the continuous measure of connectedness. From this, we see similar patterns of homophily: Scheduled Castes and General castes continue to show substantial homophily, with weaker evidence for Other Backwards Castes. Interestingly, the continuous measure also suggests that we can reject the null of no homophily for Scheduled Tribes, in contrast to the case of the binary link measure. In all, these results suggest that the continuous measure of connectedness reflects similar network patterns to the binary "She is a friend" measure that has received the bulk of attention in the literature on the economics of networks.

Appendix Table A.4 presents additional features of the measured networks. These results are presented for descriptive purposes, making no claims as to causation, in order to demonstrate that the network data is consistent with the network formation model as developed in Section 3. Three main patterns emerge. First, Panel A shows that the size of an individual's

[34]If connectedness is indeed a latent continuous measure, an additional motivation for use of the first principal component is to reduce measurement error. See Appendix F for a more formal measurement error model that draws heavily on Black and Smith (2006).

TABLE III

Homophily by Population Group

*Panel A: Binary Network Definition*

| | SC | ST | OBC | General | P-value of Test for Homophily |
|---|---|---|---|---|---|
| SC | 0.577 | 0.386 | 0.407 | 0.422 | 0.000 |
| | (0.021) | (0.029) | (0.026) | (0.035) | |
| ST | 0.407 | 0.484 | 0.412 | 0.348 | 0.131 |
| | (0.039) | (0.039) | (0.066) | (0.041) | |
| OBC | 0.388 | 0.374 | 0.440 | 0.446 | 0.068 |
| | (0.045) | (0.067) | (0.055) | (0.038) | |
| General | 0.362 | 0.314 | 0.402 | 0.564 | 0.000 |
| | (0.034) | (0.040) | (0.037) | (0.025) | |

*Panel B: Continuous Link Intensity Definition*

| | SC | ST | OBC | General | P-value of Test for Homophily |
|---|---|---|---|---|---|
| SC | 1.294 | 0.864 | 0.895 | 0.936 | 0.003 |
| | (0.087) | (0.078) | (0.102) | (0.093) | |
| ST | 0.978 | 1.363 | 1.023 | 0.844 | 0.004 |
| | (0.098) | (0.048) | (0.185) | (0.100) | |
| OBC | 0.883 | 0.848 | 1.000 | 0.987 | 0.117 |
| | (0.110) | (0.151) | (0.138) | (0.086) | |
| General | 0.822 | 0.710 | 0.915 | 1.399 | 0.000 |
| | (0.083) | (0.078) | (0.095) | (0.111) | |

Notes: N = 78,238 in 30 schools. Values indicate mean value of link for individual in group on the y-axis with respect to individual in group on the x-axis. Robust standard errors in parentheses, clustered by school. Final column presents p-value of test that mean value of link is equal for same type and other types. For example, for SC, it tests that the mean link value to other SCs is the same as the average link value of ST, OBC, and General pooled together. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste.

network is increasing in the size of her school, as defined by the number of girls in grades 6-8. For the binary measure, one additional student in one's school is associated with 0.576 more claimed friends. Analogously, under the continuous measure, each additional student is associated with 0.392 higher sum of links. Second, *average link value* is decreasing in school size, as shown by Appendix Table A.4, Panel B. That is, average link value is not proportional to school size, as would be suggested by dyadic models of network formation (Comola and Prina, 2018; Goldsmith-Pinkham and Imbens, 2013; Graham, 2017). Third, linking decisions are complementary but not symmetric. For a given pair $i, j \neq i$, if $i$ links more with $j$, then $j$ is likely to link more with $i$.[35] In other words, one's "In" and "Out" links are correlated but may be different.

### 4.5. *Reduced-Form Treatment Effects*

Before proceeding to structural estimation, here I present reduced-form treatment effects, on both outcomes and networks. I restrict this exercise to Random Treatment and Control schools so that we can interpret differences between those chosen for participation under Random Treatment and those not chosen as causal. These results reveal that the program has negative but insignificant average effects on endline outcomes, especially for participants, and that selection to participation has a significant and substantively meaningful effect on individuals' networks.

#### 4.5.1. *Effects on Outcomes*

First, I estimate reduced-form treatment effects with specifications as in Equation (21). The omitted category in these regressions is all students in Control schools.

$$\begin{aligned} y_{is} = \beta_0 &+ \beta_1 RandomTreat_s \times Participant_{is} \\ &+ \beta_2 RandomTreat_s \times NonParticipant_{is} + \epsilon_{is} \end{aligned} \tag{21}$$

Table IV shows reduced-form treatment effects. While noting possible lack of statistical power to detect small differences, first observe that the point estimates of the program's effects are negative in all specifications. Further, the point-estimated effects of approximately -0.2 standard deviations are substantively meaningful, at least among participants. Additionally, the effect on both outcomes is more negative for participants than non-participants, although both are insignificant in all specifications.

[35] Even if links are indeed symmetric, measurement error in the network measure would tend to attenuate the estimated coefficient away from one. However, measurement error would need to be extraordinarily large in relation to the variance in link values to account for coefficients of 0.129 and 0.224

TABLE IV
Reduced-From Treatment Effects

| | Education | | Gender Roles | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Random Treat × Participant | -0.181 | -0.194 | -0.228 | -0.200 |
| | (0.185) | (0.150) | (0.194) | (0.183) |
| Random Treat × Non-Participant | -0.093 | -0.117 | -0.039 | -0.023 |
| | (0.134) | (0.101) | (0.157) | (0.146) |
| Baseline Outcome | | 0.335*** | | 0.126** |
| | | (0.042) | | (0.056) |
| Constant | -0.028 | 0.034 | 0.076 | 0.053 |
| | (0.097) | (0.073) | (0.067) | (0.071) |
| R-squared | 0.004 | 0.124 | 0.006 | 0.024 |

Notes: Estimation restricted to Random Treatment and Control. N = 920 students in 20 schools in all specifications. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Omitted category is all girls in Control.

The average treatment effects in Table IV may be masking important heterogeneity. Heterogeneity may occur along many dimensions, such as those defined by the variables in Table I. In order to reduce dimensionality, I use the Control schools to predict endline outcomes conditional on baseline outcomes and individual-level covariates. This takes the form of regression results presented in Appendix Table A.5. In a sense, this uses the Control group as a counterfactual to predict what would have occurred in treatment schools in the absence of treatment, conditional on variables observed at baseline.

Using the predicted outcomes from this regression, I then group students into predicted outcome terciles. Low, Middle, and High predicted terciles are denoted by $\hat{L}$, $\hat{M}$, and $\hat{H}$, respectively.[36] This functional form is analogous to that specified in Carrell, Sacerdote and West (2013), who create terciles of predicted Grade Point Average.

Using these predicted outcome terciles, I estimate heterogeneous treatment effects with regressions of the form in Equation (22).

$$\begin{aligned} y_{is} = \sum_{k=1}^{3} I_k(\beta_{0k} + \beta_{1k} RandomTreat_s \times Participant_{is} \\ + \beta_{2k} RandomTreat_s \times NonParticipant_{is}) + \epsilon_{is} \end{aligned} \tag{22}$$

[36]While the table presents the coefficient estimates used to predict $\hat{L}$, $\hat{M}$, and $\hat{H}$ in both Treatment arms, I use a leave-one-out procedure suggested in Abadie, Chingos and West (2014) to predict outcome terciles for students in Control. Abadie, Chingos and West (2014) show through simulation that such a procedure solves the overfitting bias that results from endogenous stratification.

In this specification, $I_k$ is an indicator for being in each predicted tercile.[37] Results for this specification appear in Table A.6. I also estimate versions of Equation (22) that include baseline outcomes interacted with $\hat{L}$, $\hat{M}$, and $\hat{H}$. Note that there are strongly negative effects for Gender Roles among Participants in Random Treatment, but only for those in the middle predicted tercile. This presents suggestive evidence of heterogeneous treatment effects for Gender Roles in Random Treatment schools.

4.5.2. *Effects on Networks*

While I find suggestive evidence for negative treatment effects on outcomes, there is much stronger evidence for treatment effects on networks. Since we have random within-school variation in Random Treatment schools, I present reduced-form treatment effect estimates broken down by whether each node involved in the link is chosen for participation. To do this, I estimate Equation (23).

$$\begin{aligned} L_{ijs} = \gamma_0 + \gamma_1 Participant_{is} + \gamma_2 Participant_{js} \\ + \gamma_3 Participant_{is} \times Participant_{js} + u_{ijs} \end{aligned} \tag{23}$$

These results are presented in Table V, where Columns (2) and (4) additionally control for baseline link values.

Results for the binary link definition show effects for the interaction term $\gamma_3$ that are both statistically and substantively meaningful. If both students are chosen to participate, then those students are approximately 10 percentage points more likely to be linked at endline than if either one or neither is chosen. Similar patterns hold for the continuous definition in Columns (3) and (4), where we see much larger average link value if both are chosen. Further, for the continuous link definition, we see evidence for *substitution* of links: if only one is chosen, the average link value decreases.

These results contain powerful implications for evaluation of counterfactual assignment policies. Participation in the program has a substantial effect on the identity of others with whom individuals interact. That is, if program effects diffuse through networks, then failing to account for the effect of the program *on the structure of the network itself* may lead to erroneous estimates. That is, any attempt to predict outcomes under counterfactual assignments needs to account for the effect of the program on networks.

[37] That is, $I_1 = \hat{L}$, $I_2 = \hat{M}$, $I_3 = \hat{H}$.

TABLE V

REDUCED-FORM TREATMENT EFFECTS ON NETWORKS

| Network Definition | Binary | | Continuous | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Participant (Self) | -0.031 | -0.032 | -0.059 | -0.102** |
| | (0.027) | (0.021) | (0.039) | (0.040) |
| Participant (Alter) | 0.012 | 0.004 | 0.078 | 0.021 |
| | (0.015) | (0.013) | (0.044) | (0.034) |
| Participant (Both) | 0.117** | 0.096** | 0.308** | 0.220** |
| | (0.037) | (0.031) | (0.110) | (0.085) |
| Baseline Measure (Self) | | 0.251*** | | 0.240*** |
| | | (0.014) | | (0.014) |
| Baseline Measure (Alter) | | 0.125*** | | 0.096*** |
| | | (0.010) | | (0.019) |
| Constant | 0.536*** | 0.309*** | 1.231*** | 0.920*** |
| | (0.015) | (0.013) | (0.032) | (0.040) |
| R-squared | 0.003 | 0.086 | 0.007 | 0.092 |
| P-value of Test | 0.041 | 0.082 | 0.048 | 0.163 |

Notes: Estimation restricted to Random Treatment. N = 19,430 in 20 schools in all specifications. Robust standard errors in parentheses, clustered by school. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Dependent variable is existence/intensity of link between $i$ (self) and $j$ (alter), as indicated by $i$ at endline. Test is a test of significance of sum of coefficients for Participant (self), Participant (Alter), and Participant (Both), against a null that the sum is zero. Missing network data imputed via algorithm described in Appendix D.

## 5. STRUCTURAL ESTIMATION RESULTS

Armed with the identification results from Sections 2 and 3, I now proceed to structural estimation. For the purposes of estimation, I restrict attention to Random Treatment and Control schools, setting aside Elected Treatment for use in the validation exercise in Section 6.

Structural estimation consists of two steps. First, I estimate the parameters of the network formation game. Next, conditional on these parameters—particularly the estimated structural unobservables $a_{is}$—I estimate the parameters of the outcome equation which accounts for peer effects.

### 5.1. *Network Formation Estimation*

This section estimates the network formation model within a GMM framework using moments motivated by the identification results above. Before doing so, I discuss how I handle missing network data and zeros in the network data.

5.1.1. *Missing Network Data*

As described in Section 4 above, network data was collected via school visits. Accordingly, there is missing network data for two reasons. First, some students were not present in school on the date of the survey. Second, students may not have properly answered the survey questions, for example by leaving some lines blank. Of the estimation sample in the two treatment arms used for structural estimation (Control and Random Treatment), link values for approximately 40% of possible link pairs are missing.

Missing network data has the potential to confound estimation for a number of reasons. If data is missing non-randomly, listwise deletion leads to biased estimates of even network-level descriptive statistics (see, e.g., Chandrasekhar and Lewis, 2011). In the specific model outlined here, missing network data means that we do not observe an individual's entire vector of network choices. If certain types of students, defined by observed or unobserved characteristics, are more likely to be absent on the day of the network survey, then we need a way of accounting for these students.

Accordingly, a method of reconstructing the missing network data is needed. Chandrasekhar and Jackson (2014), using a model arising from the random graphs literature, provide a method that reconstructs networks based upon the probability of observing given dyadic and triadic relationships in the data. Williams (2016) recently extended this method to allow for missingness to vary by observed characteristics. He shows that the method does a reasonable job in reconstructing missing data in AddHealth with 75% missing data, as in his application.[38] He then applies the method to simulate missing network data at the Air Force Academy. A key limitation of this method, however, is that it does not model tradeoffs between linking strategies.

Fortunately, in this context, the network formation model can be pressed into service to fill in missing data. In his application at the Air Force Academy, Williams (2016) does not model network formation; rather, he models only outcomes conditional on the observed network. He then uses the network-formation model implicit in Chandrasekhar and Jackson (2014) to impute missing network data. In contrast, the structural model developed here posits a specific model of network formation that I use to reconstruct missing data.

Accordingly, I employ an iterative multiple imputation EM algorithm that uses the network model itself to simulate missing data. The overall methodology is described in Cameron and Trivedi (2005). While less common in economics, multiple imputation methods are often

[38]While the network reconstruction technique I employ is different, I note that my network data has a much higher response rate (60% vs. 25%) than the data employed by Williams (2016).

employed in related fields such as statistics (Little and Rubin, 2002) and political science (King et al., 2001). The basic structure is as follows, with more detail in Appendix D. First, fill in the missing data arbitrarily. Second, estimate the network formation model with this dataset. Third, using these estimated parameters and implied distributions of unobserved data, simulate values for missing network data. Repeat the second and third steps for sufficient iterations to converge to the distribution of both the simulated networks and the estimated parameters. This generates a Markov Chain of simulated networks and estimated parameters.[39] After a sufficient burn-in period, I take draws from this chain as the simulated parameters and full networks. To shorten calculation time and reduce serial correlation of simulated estimates, I simulate multiple parallel Markov chains.

### 5.1.2. *Zeros in the Data*

Identification of the parameters of the network-formation game depends on observing the *strictly positive* equilibrium. This implies that no pairs of students choose a zero link to each other. In the actual data, however, there are a number of students who answer all of the link questions negatively, leading to the constructed continuous link measure being zero. In the raw link data, of 58,530 dyads used for estimation, approximately 28.5% are zeros.

I attribute these zeros to measurement error. That is, the actually-observed continuous network measure is a noisy version of the true measure.[40] It is constructed from nine binary questions. Presumably, especially given potential networks that average 44 students, if we asked substantially more link questions, the answers to some questions would be positive. Accordingly, in order to account for this, whenever zeros appear in the constructed continuous link measure, I replace this value with an imputed value that is drawn randomly and uniformly between 0 and the minimum link measure observed in the actual (non-zero) data.[41]

[39] While the raw data consists of discrete network measures, the network formation model operates at the level of continuous link values. Accordingly, the imputation algorithm—which employs the network formation model—directly imputes the continuous measure.

[40] I note that measurement error is theoretically a issue for all network data, not just those that are observed as zero in the data. A more formal model might account for error in constructing the continuous network measure, for example, analogously to Cunha, Heckman and Shennach (2010) in their study of educational skills formation. Future projects with the continuous network link models will explore this issue further. See Appendix F.2 for a fuller discussion of measurement error and its relationship to observed zeros in the data.

[41] Estimates are not sensitive to simulation error. That is, the estimated network formation parameters are quite similar across many different draws of the algorithm. Further, estimates do not substantially differ between this imputation method and simply adding a small number, such as 0.001, to each observed link value.

5.1.3. *Network Formation Parameters*

Having discussed the data issues, I now move on the estimation. Essentially, this consists of finding values of the structural parameter $(\tilde{\beta}, \gamma, \delta)$ and the scalar unobservables $a_{is}$ such that the sample analogues of the assumed moment conditions hold empirically.[42] I estimate this via GMM, with moments motivated by the identification results. As discussed in the prior subsection, in order to correct for missing data, the GMM routine is the minimization step of the iterative EM algorithm. Standard errors are adjusted to account for imputation error as well as allowing for arbitrary within-school correlation of unobservables.[43]

Estimated parameters of the network formation game are given in Table VI. In Panel A, we see that estimated $\tilde{\beta}$ is positive and highly significant, indicating that effort levels of the two actors forming a link are strongly complementary, consistent with the reduced-form facts. Importantly, this is true even when controlling for a large set of observed and unobserved characteristics. Further $\tilde{\beta}$, estimated at 0.281, is substantially less than one,[44] as required for the network-formation process to have a unique strictly positive equilibrium.

Additionally, the point estimate of $\gamma_2$ shows that scalar unobservables $a_{is}$ are important in link decisions. The parameter $\gamma_2$ identifies the additional utility derived to individual $i$ from linking with $j$ when $j$'s unobserved $a_{js}$ increases by one standard deviation. Note that the effect of a one standard deviation change, 0.725, is of the same order of magnitude of the effect of homophily for many characteristics: for example, two students in Grade 7 derive 0.807 units more utility than if either is not in Grade 7.

Panel B presents parameter estimates that show how observed and unobserved variables interact in determining the utility of network links. The $\gamma_1$ parameters identify the difference in utility to individual $i$ from linking with $j$ when $j$ has the indicated characteristic versus $j$ not having it. For example, if $j$ is elected, then $i$ derives 0.433 units more utility than if $j$ is not elected. I note that the negative point estimates on members of lower castes suggest less utility from linking with them but that these effects are mitigated somewhat when they are chosen to participate, as indicated by positive and significant coefficients for interactions of SC and ST with the participation indicator.

The second column indicates substantial homophily along a number of dimensions, as shown by the $\delta_1$ estimates. Those in Grades 7 and 8 derive more utility from linking with

[42]Recall that identification results showed that $a_{is}$ and any parameters that interact with $a_{is}$ are only identified to scale. I have set this scale by setting the variance of estimated $a_{is}$ to one and the sign of $\delta_2$ to positive. The first assumption is simply a convenient normalization, while the latter imposes the condition that higher $a_{is}$ implies more utility to others from linking with agent $i$.

[43]That is, at the end of each chain of the EM algorithm, I calculate variance using the standard sandwich estimator that allows for arbitrary within-cluster correlation of unobservables. I then construct the final variance (and hence standard errors) according to the formula in Little and Rubin (2002).

[44]A one-sided test strongly rejects the null that $\tilde{\beta} \geq 1$.

TABLE VI

Structural Network Formation Parameter Estimates

| *Panel A: Parameters Not involving Covariates* | | | | |
|---|---|---|---|---|
| $\tilde{\beta}$ | 0.281*** | | | |
| | (0.062) | | | |
| $\gamma_2$ | 0.725*** | | | |
| | (0.026) | | | |
| $\delta_4$ | 0.097*** | | | |
| | (0.012) | | | |
| *Panel B: Parameters involving Covariates* | | | | |
| *X Variable* | $\gamma_1$ | $\delta_1$ | $\delta_2$ | $\delta_3$ |
| Elected | 0.433*** | 0.024 | -0.035 | 0.173*** |
| | (0.029) | (0.045) | (0.023) | (0.023) |
| Grade 7 | -0.097*** | 0.807*** | 0.016 | 0.001 |
| | (0.025) | (0.074) | (0.019) | (0.020) |
| Grade 8 | -0.148*** | 0.873*** | -0.015 | 0.041** |
| | (0.027) | (0.080) | (0.019) | (0.020) |
| SC | -0.753*** | 0.883*** | -0.251*** | -0.028 |
| | (0.041) | (0.082) | (0.027) | (0.032) |
| ST | -0.732*** | 0.940*** | -0.272*** | -0.023 |
| | (0.045) | (0.126) | (0.034) | (0.045) |
| OBC | -0.297*** | 0.206*** | -0.078*** | 0.030 |
| | (0.034) | (0.038) | (0.021) | (0.022) |
| Participant | -0.044 | 0.033 | -0.412*** | 0.393*** |
| | (0.125) | (0.082) | (0.124) | (0.120) |
| Participant × Elected | -0.067 | -0.012 | 0.030 | -0.040 |
| | (0.062) | (0.128) | (0.059) | (0.058) |
| Participant × Grade 7 | -0.126* | -0.298 | 0.001 | -0.117* |
| | (0.070) | (0.184) | (0.070) | (0.069) |
| Participant × Grade 8 | 0.149* | -0.274** | 0.021 | -0.059 |
| | (0.079) | (0.118) | (0.068) | (0.062) |
| Participant × SC | 0.368*** | -0.313 | 0.244* | -0.193 |
| | (0.126) | (0.189) | (0.126) | (0.133) |
| Participant × ST | 0.292** | -0.159 | 0.236* | -0.337** |
| | (0.130) | (0.165) | (0.127) | (0.134) |
| Participant × OBC | -0.029 | 0.201* | 0.200* | -0.359*** |
| | (0.112) | (0.110) | (0.116) | (0.121) |

Notes: Estimation restricted to Random Treatment and Control. N = 58,530 in 20 schools. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1.Missing data imputed and estimates adjusted via algorithm described in Appendix D. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste. Omitted Categories are Not Elected, Grade 6, and General.

their classmates, and members of Scheduled Castes, Scheduled Tribes, and Other Backwards Castes similarly get more utility from linking to others in the same population grouping. Interestingly, among those in Grades 7 and 8 as well as Scheduled Tribes and Scheduled Castes, being chosen to participate seems to mitigate homophilic tendencies, as indicated by the negative and point estimates of interactions between Participant and these characteristics, some of which are statistically significant as well.

The final two columns show estimates of the effects of interactions between observed characteristics and unobserved $a_{is}$. Many of these estimated coefficients are highly significant and large in magnitude. This suggests that these interactions are quite important in individuals' decisions about network formation. Accordingly, failure to account for these interactions has the potential to crucially bias estimates of the parameters of the peer effects model.

## 5.2. *Peer Effects Estimates*

This section presents estimates of the peer effects model, as specified by Equation (4). Similar to the network formation case, I first describe treatment of missing data, then present the estimated parameters as well as the results of cross-equation specification tests.

### 5.2.1. *Missing Outcome Data*

Similar to the network link variables, I encounter missing data for two reasons: (2) some girls were not present on the day that the endline questionnaires were administered; (2) even when present, some students did not answer the relevant questions. To account for this possibly non-random missing data, I employ an iterative EM algorithm. Estimation is done by OLS, which then imputes outcomes according to the estimated distribution of unobserved variables. Importantly, the parameters of the peer effects model are estimated conditional on a realized network and unobserved parameters $a_{is}$. Accordingly, to account for variance in imputing the network data, I take draws from the imputed distribution of networks and unobserved $a_{is}$, as these were calculated as part of the network formation estimation process. Conditional on each draw of the network and $a_{is}$, I iterate the algorithm 500 times to minimize sensitivity to starting values. This algorithm is described in more detail in Appendix D.

### 5.2.2. *Peer Effects Parameter Estimates*

Table VII provides the structural peer effects estimates. These estimates are calculated via OLS conditional on the realized network and estimated $\hat{a}_{is}$. From these, I construct peer mean variables $\overline{\text{Participant}}$ and $\bar{a}_{is}$. All estimates include interactions of Baseline outcomes and peer mean baseline outcomes with $\hat{L}$, $\hat{M}$, and $\hat{H}$, while Columns (2) and (4) further include interactions with unobserved $a_{is}$ and $\bar{a}_{is}$. Standard errors are adjusted to account for missing data imputation, generated regressors ($a_{is}$ and $\bar{a}_{is}$), and arbitrary within-school correlation in unobservables.[45]

Looking first at Educational Aspirations in Columns (1) and (2), we see positive and significant coefficients for the interaction between $a_{is}$ and all three tercile indicators. Since $a_{is}$ is normalized to have standard deviation of one, this means that, among those in the lowest predicted outcome tercile, one standard deviation higher unobserved $a_{is}$ leads to 0.064 standard deviations higher predicted Educational Aspirations. We see larger positive effects for those in the other predicted terciles (0.142 and 0.141 for $\hat{M}$ and $\hat{H}$, respectively). Recall that, from the network formation estimates, individuals derive more utility from linking to those with higher $a_{is}$. Accordingly, this suggests that those who are more desirable as friends also have higher unobserved factors that affect their Educational Aspirations. While noting less power,[46] I note that the effect of $\bar{a}_{is}$ is only significant for those in the middle predicted outcome tercile ($\hat{M}$).

In addition to showing that the omitted $a_{is}$ variables are important in determining outcomes in Column (2), I also test cross-equation restrictions between Equations (1) and (2). These test the equality of each set of interactions with $\hat{L}$, $\hat{M}$, and $\hat{H}$: for example, the first tests whether the three coefficients on Participant $\times\,\hat{L}$, Participant $\times\,\hat{M}$, and Participant $\times\,\hat{H}$ are equal between Columns (1) and (2). The p-values presented at the bottom of the table suggest that we cannot reject the null that these are equal, suggesting that omission of terms including $a_{is}$ and $\bar{a}_{is}$ in Column (1) does not lead to substantially biased parameter estimates. I further test the joint significance of the interactions of $\hat{L}$, $\hat{M}$, and $\hat{H}$ with $a_{is}$ and $\bar{a}_{is}$, and note that the interactions with $a_{is}$ are jointly significant, while those with $\hat{a}_{is}$ are not.

Results for Gender Roles attitudes are presented in Columns (3)-(4) of Table VII. In Column (4), we see that the effect of unobserved $a_{is}$ is significant only for those in the highest predicted outcome tercile ($\hat{H}$). At the bottom of the table, the p-value 0.026 reveals that these interactions are jointly significant. However, the interactions with $\hat{a}_{is}$ are not significant, and I fail to reject the equality of coefficients in Columns (3) and (4).

Taken together, these peer effects estimates reveal mixed results for the importance of

[45]Mechanically, at each step of the imputation algorithm, estimation of the network formation and all specifications of the peer effects parameters is performed in a single GMM minimization problem, with variance calculated using the standard cluster-robust sandwich estimator. To account for imputation error, final variance is calculated by the standard formula in Cameron and Trivedi (2005) and Little and Rubin (2002).

[46]Standard errors are much higher for the $\bar{a}_{is}$ variables than the $a_{is}$ variables. For example, compare the standard errors on the coefficients in Column (2) on $\bar{a} \times \hat{L}$ (0.036) versus $a_{is} \times \hat{L}$ (0.189). This is likely due to the fact that $\bar{a}_{is}$ is constructed from many estimates of $a_{is}$, all of which are noisily estimated.

TABLE VII
STRUCTURAL PEER EFFECTS ESTIMATES

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| *Outcome* | *Educational Aspirations* | | *Gender Roles* | |
| Participant × $\hat{L}$ | -0.285*** | -0.292*** | 0.122*** | 0.099*** |
| | (0.047) | (0.038) | (0.029) | (0.028) |
| Participant × $\hat{M}$ | -0.001 | 0.031 | -0.475*** | -0.470*** |
| | (0.053) | (0.043) | (0.032) | (0.035) |
| Participant × $\hat{H}$ | 0.277*** | 0.234*** | 0.281*** | 0.277*** |
| | (0.039) | (0.048) | (0.046) | (0.054) |
| $\overline{\text{Participant}}$ × $\hat{L}$ | -0.688*** | -0.656*** | -0.137 | -0.080 |
| | (0.108) | (0.125) | (0.106) | (0.117) |
| $\overline{\text{Participant}}$ × $\hat{M}$ | -0.140 | -0.146 | -0.164 | -0.148 |
| | (0.089) | (0.107) | (0.120) | (0.140) |
| $\overline{\text{Participant}}$ × $\hat{H}$ | -0.644*** | -0.639*** | -0.747*** | -0.753*** |
| | (0.182) | (0.190) | (0.069) | (0.078) |
| $a_{is} \times \hat{L}$ | | 0.064* | | -0.024 |
| | | (0.036) | | (0.032) |
| $a_{is} \times \hat{M}$ | | 0.142*** | | 0.040 |
| | | (0.030) | | (0.029) |
| $a_{is} \times \hat{H}$ | | 0.141*** | | 0.091*** |
| | | (0.037) | | (0.035) |
| $\bar{a}_{is} \times \hat{L}$ | | 0.012 | | 0.122 |
| | | (0.189) | | (0.151) |
| $\bar{a}_{is} \times \hat{M}$ | | -0.275** | | -0.111 |
| | | (0.130) | | (0.164) |
| $\bar{a}_{is} \times \hat{H}$ | | -0.075 | | 0.077 |
| | | (0.136) | | (0.166) |
| *P-value of cross-equation test:* | | | | |
| Interactions with Participant | | 0.519 | | 0.925 |
| Interactions with $\overline{\text{Participant}}$ | | 0.992 | | 0.943 |
| *P-value of joint significance test:* | | | | |
| Interactions with $a_{is}$ | | 0.000 | | 0.026 |
| Interactions with $\bar{a}_{is}$ | | 0.334 | | 0.858 |

Notes: Estimation restricted to Random Treatment and Control. N = 920 in 20 schools in all specifications. Coefficients for $\hat{L}$, $\hat{M}$, and $\hat{H}$ suppressed. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Missing data imputed and estimates adjusted via algorithm described in Appendix D. Standard error calculations account for variance in estimating generated regressors $a_{is}$ and $\bar{a}_{is}$. All specifications include interactions of Baseline Outcome and $\overline{\text{Baseline Outcome}}$ with $\hat{L}$, $\hat{M}$, and $\hat{H}$.

network endogeneity. First, we see that unobserved heterogeneity is a significant predictor of endline outcomes, as shown by joint significance tests in Columns (2) and (4). However, in comparing estimates that omit unobserved heterogeneity to those that include it, the addition of unobserved $a_{is}$ does not substantially change estimated parameters. These two findings suggest that, at least in this context, omission of unobserved heterogeneity may not prove fatal to out-of-sample prediction. However, this may be a special case, and estimates of the full model are robust to this type of network endogeneity.

## 6. OUT-OF-SAMPLE VALIDATION

While a structural model allows for out-of-sample prediction, confidence in the model can be bolstered by comparison of the model's predictions to realized out-of-sample outcomes. Fortunately here, the study design allows for such a validation step, similar to Todd and Wolpin (2006).[47] In Elected Treatment schools, which were not used in structural estimation in the prior section, participation in the program was assigned by election rather than randomly, as was done in Random Treatment schools. Therefore, having used Random Treatment and Control to estimate the model, I now use the estimated parameters to predict outcomes conditional on all participants being chosen by election. Comparing these predictions to the actual realized outcomes in Elected Treatment schools provides a check on the model's predictive power.

### 6.1. *Simulation Method*

Counterfactual simulation relies upon simulation of unobserved variables. The network-formation model includes three such unobservables: $c_{ijs}$, $M_{is}$, and $a_{is}$. The cost variables $c_{ijs}$ are by construction independent of all observables and $a_{is}$. In simulation, I do allow for correlation between $c_{ijs}$ and $c_{jis}$, so for each pair $i, j < i$, the pair $(\log c_{ijs}, \log c_{jis})$ is drawn from a normal distribution with variance $\hat{\sigma}_c^2$ and covariance $\hat{\rho}_c$, where $\hat{\sigma}_c^2$ and $\hat{\rho}_c$ are estimated from residuals in estimation. Scalar variable $a_{is}$ is drawn from an independent normal distribution with mean zero and variance 1 (recall that this mean and variance are imposed as moment conditions in estimation). Finally, $M_{is}$ is drawn from a log-normal distribution allowing for some dependence on observed characteristics.[48] I take the distribution

[47] I note here that I perform a slightly different validation step than Todd and Wolpin (2006), who estimate the model using non-experimental observations to compare to experimental ones. I estimate the model using random (experimental) assignment to compare to non-random assignment.

[48] In practice, simulating $M_{is}$ is a three-step process as follows: (1) for each $i$ and $s$, recover $\hat{M}_{is}$ from the estimation routine, where $\hat{M}_{is} = \sum_{j \neq i} \hat{c}_{ijs} g_{ijs}$, (2) regress $\log \hat{M}_{is}$ on the same observed variables that appear in Table VI, (3) with these parameter estimates and implied variance of residuals $\hat{\sigma}_M^2$, simulate $\log M_{is}$, drawing the residuals from the a normal distribution with variance $\hat{\sigma}_M^2$.

of observed characteristics in Elected Treatment as given in all simulations in order to avoid any possible composition issues.

After the network-formation process is simulated, I simulate outcomes. This employs the parameters in Table VII and simulated $a_{is}$. Again, to avoid any composition bias, all simulations are done on 10 schools with the exact distribution of observed covariates as found in Elected Treatment schools.

In order to facilitate comparisons, I simulate outcomes under three different specifications, as outlined in Table VIII. Model 3 is the full model in which $a_{is}$ plays a part in both network formation and outcome determination. Model 2 is a restricted version in which this unobserved heterogeneity is only relevant for network formation. Model 1 is a further restricted model in which unobserved heterogeneity $a_{is}$ is relevant for neither network formation nor endline outcomes. Clearly, Models 1 and 2 are special cases of Model 3. Estimates used to generate simulations for Models 2 and 3 are in Tables VI and VII, while those for Model 1 are available upon request from the author.

TABLE VIII
Out-of-Sample Validation Models for Comparison

| | Network Model Restrictions | Peer Effects Model Restrictions |
|---|---|---|
| Model 1 | $\gamma_2 = \delta_2 = \delta_3 = \delta_4 = 0$ | $\alpha_{3k} = \alpha_{4k} = 0 \; \forall \; k$ |
| Model 2 | None | $\alpha_{3k} = \alpha_{4k} = 0 \; \forall \; k$ |
| Model 3 | None | None |

### 6.2. *Comparison to Elected Treatment*

Simulation results are presented in Table IX.[49] Simulations for Educational Aspirations are presented in Panel A, which shows that all three models are overly optimistic about mean Educational Aspirations. While all three models predict substantially similar means, I note that the full model (Model 3) is closer to the true realized mean in the entire population.

Further, the model does a good job of getting at treatment effect heterogeneity. Consistent with the realized outcomes, we see that elected girls do better than those who were not elected, as do members of General castes. Members of lower castes are predicted to have substantially lower Educational Outcomes at endline, which matches the patterns from Elected Treatment.

Moving on to Gender Roles attitudes in Panel B, all three models do a better job of predicting the overall mean. Further, all three models predict that Elected girls have worse

[49]This exercise is analogous to that undertaken in Tables 12-15 in Todd and Wolpin (2006).

endline Gender Roles attitudes than Not Elected girls. However, all three fail to pick up the patterns of heterogeneity by caste grouping.

In sum, the model does a credible job of matching many features of the out-of-sample realized outcomes. However, while I credibly match means within the entire sample for both outcomes, the model only correctly predicts the patterns of heterogeneity for Educational Aspirations. This is qualitatively similar to the out-of-sample fit results in Todd and Wolpin (2006), who find that their model predicts average school attendance reasonably well for some subgroups but not others.[50]

TABLE IX

Comparison of Realized to Predicted in Elected Treatment Schools

| ***Panel A: Educational Aspirations*** | | | | | | | |
|---|---|---|---|---|---|---|---|
| | All | Elected | Not Elected | SC | ST | OBC | General |
| *Observed in Elected Treatment* | | | | | | | |
| Mean | -0.279 | -0.053 | -0.398 | -0.364 | -0.489 | -0.392 | 0.085 |
| S.E. of Mean | 0.151 | 0.180 | 0.163 | 0.233 | 0.179 | 0.189 | 0.234 |
| N | 330 | 114 | 216 | 64 | 32 | 153 | 81 |
| *Mean of Simulated Means* | | | | | | | |
| Model 1 | -0.170 | -0.099 | -0.208 | -0.259 | -0.308 | -0.252 | 0.108 |
| Model 2 | -0.181 | -0.113 | -0.217 | -0.268 | -0.327 | -0.262 | 0.100 |
| Model 3 | -0.186 | -0.127 | -0.218 | -0.273 | -0.327 | -0.262 | 0.081 |
| ***Panel B: Gender Roles Attitudes*** | | | | | | | |
| | All | Elected | Not Elected | SC | ST | OBC | General |
| *Observed in Elected Treatment* | | | | | | | |
| Mean | -0.022 | -0.085 | 0.012 | 0.066 | 0.085 | -0.339 | 0.462 |
| S.E. of Mean | 0.147 | 0.210 | 0.149 | 0.240 | 0.239 | 0.121 | 0.100 |
| N | 332 | 116 | 216 | 65 | 33 | 153 | 81 |
| *Mean of Simulated Means* | | | | | | | |
| Model 1 | -0.018 | -0.057 | 0.002 | -0.077 | -0.033 | 0.016 | -0.030 |
| Model 2 | -0.008 | -0.030 | 0.004 | -0.073 | -0.026 | 0.025 | -0.011 |
| Model 3 | -0.007 | -0.033 | 0.007 | -0.074 | -0.015 | 0.030 | -0.019 |

Notes: Estimates for Model 1 on file with author. Estimates for Model 2 are presented in Table VI and Columns (2) and (4) of Table VII. Estimates for Model 3 are presented in Table VI and Columns(2) and (4) of Table VII. Simulations based upon 100,000 repetitions, with residuals drawn from random normal. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste.

[50] A formal test for model fit as well as development of the theory of statistical power for such a test is beyond the scope of this paper but will be investigated in future work.

## 7. FROM ESTIMATES TO OPTIMAL TREATMENT ASSIGNMENT

### 7.1. *Problem Description*

Formal numerical optimization is beyond the scope of this paper for reasons discussed in this section. Assessment of the effects of counterfactual assignment policies in this context is an exercise in statistical treatment assignment (see Manski, 2004; Smith and Staghøj, 2009). That is, we search for statistical rules that maximize some function of outcomes, conditional on observable characteristics of individuals. As pointed out by Bhattacharya (2009), the maximand under optimal assignment weakly dominates the maximand under any feasible assignment. Accordingly, we need some way of assessing the effects of alternative assignments across the entire class of feasible alternative assignments. Dehejia (2005) formulates the problem in a Bayesian framework, drawing inferences from comparing features of the posterior predictive distributions. Similarly, Bhattacharya (2009) investigates the assignment of freshmen to dorms as a linear programming problem, providing results for maximizing the mean or any quantile of the outcome of interest.

The hypothetical optimal assignment problem here is complicated, however, by at least three factors. First, the presence of a budget constraint—only 13 girls per school can be assigned to program participation—complicates analysis. That is, even in settings in which agents' outcomes are independent, the need to estimate the threshold assignment rule—possibly including which covariates to include in this estimation—adds an important dimension of uncertainty that must be accounted for (Bhattacharya and Dupas, 2012).

Second, treatment externalities in the form of peer effects increase the complexity of the assignment problem. That is, identification and inference in the models from the econometrics literature typically rely upon independence across observations (see, e.g., Bhattacharya and Dupas, 2012; Manski, 2004). When, on the other hand, agents' outcomes are not independent, it may be impossible to derive a closed-form solution to the optimization problem. Accordingly, solving for the optimal assignment may necessitate the use of high-dimensional numerical programming procedures, an approach taken by Carrell, Sacerdote and West (2013).

Finally, and most pertinent to the central theme of this paper, the addition of the network formation process vastly increases the computational complexity. The approach taken in Carrell, Sacerdote and West (2013) already faced a very high computational burden. The model developed here is much richer due to explicit modeling of network formation. This additional complexity substantially increases the computational burden of numerical optimization: due to the non-linear nature of the two-part model developed here, solving for optimal treatment assignment—while minimizing simulation error—requires a large number simulations of the network *at each iteration* of a numerical solver. Accordingly, formal numerical optimization is beyond the scope of this paper.

## 7.2. *Simulations*

### 7.2.1. *Simulation Procedure*

Given the extreme computational burden entailed in analytically solving for optimal treatment assignment, I pursue a more indirect approach. I take a large number $V$ of random simulations of the two-part process leading from treatment assignment to final outcomes,[51] and in this section present features of the distribution of these simulations. Simulations employ the estimated parameters presented in Tables VI and VII as well as the $\hat{a}_{is}$ estimated by the same process.[52] This allows for analysis of features of the distribution of outcomes in these simulated random assignments.

Recall that heterogeneity in the peer effects model is based upon predicted outcome tercile, which is based upon regression results from Section 4. These results are used to generate predictions of what would have happened in the absence of treatment, which are then binned into Highest, Middle, and Lowest terciles. For this section, I refer to students predicted to be in the lowest outcome tercile as "type-$\hat{L}$", those in the middle predicted tercile as "type-$\hat{M}$", and those in the highest predicted tercile as "type-$\hat{H}$."[53]

In order to isolate the effect of assignment to participation from composition effects, I hold the composition of students within schools fixed. That is, in all simulated assignments, the distribution of student characteristics across schools is the same, and I use the same $\hat{a}_{is}$—which estimates unobserved heterogeneity $a_{is}$—in each repetition. For each simulation draw, within each school I randomly draw 13 girls for participation. This necessarily constrains the set of feasible assignments. For example, due to distribution of individual characteristics across schools, it is not possible to assign all type-$\hat{L}$ or all type-$\hat{H}$ students to participate, as some schools do not have 13 students in one or more of these categories.

### 7.2.2. *Average Marginal Effects*

Here, I present regression results that analyze the average marginal effect of changing the composition of assigned students. These results show the average effect of substituting one type of student for another on average outcomes. From this exercise, we see that there are tradeoffs in who benefits from different assignments for Educational Aspirations, while all students have higher Gender Roles attitudes on average when more type-$\hat{H}$ students are

[51]In the simulations presented here, $V = 1,000,000$.

[52]In practice, this means that the simulations are performed on Control and Random Treatment, the estimation sample, consisting of 920 girls in 20 schools.

[53]Note that predicted outcome terciles are outcome-specific. Students may be predicted to be in different outcome terciles for Educational Aspirations and Gender Roles attitudes.

chosen to participate.

To set up the results, for each simulation draw $v = 1, ...V$, define $\%\hat{L}P_v$ as the percentage of participants who are in the lowest predicted tercile for a given outcome. Define $\%\hat{M}P_v$ and $\%\hat{H}P_v$ similarly. I estimate Equation (24), where $\bar{y}_v$ is the mean outcome among some sub-population in simulation $v$.[54]

$$\bar{y}_v = \eta_1 \%\hat{L}P_v + \eta_2 \%\hat{M}P_v + \eta_3 \%\hat{H}P_v + u_v \tag{24}$$

Note that each coefficient identifies the effect of adding *more* of that type to the set of participants, while holding all else constant. Given the constraint that there must be 13 participating girls per school, this is not the relevant parameter in analyzing counterfactuals. Rather, in thinking about alternative *feasible* assignments, we need to look at substitutions of one type for another. Therefore, we must analyze *differences* in parameters. The marginal effect of substituting type-$\hat{H}$ for type-$\hat{L}$ is therefore $\eta_3 - \eta_1$, the effect of substituting type-$\hat{H}$ for type-$\hat{M}$ is $\eta_3 - \eta_2$, etc.

TABLE X

Regression Results for Simulations of Random Assignments

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Dep. Var: $\bar{y}_v$ restricted to | All Girls | $\hat{L}$ | $\hat{M}$ | $\hat{H}$ |
| ***Panel A: Educational Aspirations*** | | | | |
| $\eta_1$ ($\%\hat{L}P_v$) | -0.196 | -0.691 | -0.104 | 0.210 |
| $\eta_2$ ($\%\hat{M}P_v$) | -0.104 | -0.441 | -0.077 | 0.209 |
| $\eta_3$ ($\%\hat{H}P_v$) | -0.050 | -0.446 | -0.102 | 0.405 |
| ***Panel B: Gender Roles Attitudes*** | | | | |
| $\eta_1$ ($\%\hat{L}P_v$) | 0.036 | 0.027 | 0.035 | 0.045 |
| $\eta_2$ ($\%\hat{M}P_v$) | -0.128 | -0.158 | -0.142 | -0.083 |
| $\eta_3$ ($\%\hat{H}P_v$) | 0.080 | 0.038 | 0.063 | 0.140 |

Notes: Simulations based on 1,000,000 random draws, with residuals drawn from random normal. Simulation results in Panel A correspond to Table VI and Column (2) of Table VII. Simulation results in Panel B correspond to Table VI and Column (4) of Table VII.

These regression results for Educational Aspirations are presented in Panel A of Table X.[55] In Column (1), where the dependent variable is mean Educational Aspirations for all girls,

[54] I omit the constant terms in these regressions since it is collinear with the regressors, which mechanically must sum to 1.

[55] I omit standard errors since they are necessarily dependent on number of simulations draws, and can be made arbitrarily small by increasing the number of draws.

the average effect of substituting type-$\hat{H}$ girl for type-$\hat{L}$ is the difference 0.146 = (-0.050) - (-0.196). Thus, making such a substitution on the margin will tend to *increase* average outcomes among all girls.

Columns (2), (3), and (4) present estimates of the effect of participation assignment on average Educational Aspirations among type-$\hat{L}$, type-$\hat{M}$, and type-$\hat{H}$, respectively. In Column (2), we see that type-$\hat{L}$ students do better when they participate *less* in the program, with about equal gains of substituting from type-$\hat{L}$ to type-$\hat{M}$ or type-$\hat{H}$.[56] In contrast, type-$\hat{M}$ and type-$\hat{H}$ girls seems to benefit from own-type participation, as the largest coefficients in Columns (3) and (4) are for higher own-type participation.

A different picture emerges for Gender Roles attitudes, as presented in Panel B. In Column (1), we see that the largest coefficient is 0.080, suggesting that substitution of type-$\hat{H}$ into participation for either of the two lower types is an improvement for average outcomes. Further, the largest coefficient in each of Columns (2)-(4) is found in the row for $\%\hat{H}P_v$, which suggests that the largest marginal improvement for *all three* subgroups comes from substituting toward more type-$\hat{H}$ girls being assigned to participate.

Taken together, these results imply that whether a particular posited alternative assignment improves outcomes on average may depend crucially on the target population. That is, there may be tradeoffs. For example, for Educational Aspirations, type-$\hat{M}$ girls benefit from substitution of more type-$\hat{M}$ participants and fewer type-$\hat{H}$, while this same substitution makes type $\hat{H}$ students worse off on average. However, such tradeoffs need not be present, and the existence of tradeoffs will depend on the pattern of coefficients in Equation (24). In this setting, there are no tradeoffs for Gender Roles, where substitution toward more type-$\hat{H}$ students participating makes all three sub-populations better off on average.

### 7.2.3. *Extrema of the Distributions of Simulations*

The regressions in Table X give the effect of marginal changes on averages in the entire set of simulated outcomes. However, if interest lies in optimizing the average outcome among some subset of the population, then these results may be less informative due to non-linear relationships between assignments and outcomes. Accordingly, in this section I look at features of the extrema of the distributions of outcomes of interest. To this end, I present mean characteristics of the upper and lower 0.1% of the distribution of mean outcomes for various target populations.[57] As a point of comparison, I also present analogous features of the 0.1%

[56] The effect of these substitutions are $0.250 = -0.441 - (-0.691)$ and $0.245 = -0.446 - (-0.691)$ respectively.

[57] That is, for a given average outcome of interest $\bar{y}_v$, I present average characteristics among simulations where $F(\bar{y}_v) > 0.999$ and $F(\bar{y}_v) < 0.001$, where $F()$ is the empirical c.d.f. of the distribution of the variable of interest $\bar{y}_v$ in the $V$ simulations.

of the distribution clustered around the median.

For each outcome, I consider four variables of interest: the mean simulated outcome for (A) All girls, (B) type-$\hat{L}$ girls, (C) type-$\hat{M}$ girls, and (D) type-$\hat{H}$ girls. Note that (B) corresponds to the target population in Carrell, Sacerdote and West (2013). The target populations (A)-(D) are labeled in Table XI.

Results for Educational Aspirations are presented in Table XI, Panel A. From Column (1), we see that, for the highest 0.1% of simulations, the mean Educational Aspirations among All Girls (A) is -0.013, while the mean of the lowest 0.1% of simulations is -0.226. Further, those simulations in the highest 0.1% tend to have a higher percentage of type-$\hat{H}$ participants as compared to simulations in the lowest 0.1% (0.306 vs. 0.296), while also having fewer type-$\hat{L}$ girls (0.341 vs. 0.352).

Next, if the target population is type-$\hat{L}$ (B), analogous to the optimand in Carrell, Sacerdote and West (2013), we see that draws with the highest simulated means for this group have fewer type-$\hat{L}$ participants and more of both type-$\hat{H}$ and type-$\hat{M}$ participants. In contrast, the highest simulated means for type-$\hat{M}$ (C) have slightly more type-$\hat{M}$ participants and fewer of types $\hat{L}$ and $\hat{H}$. Similarly, the highest simulated means for type-$\hat{H}$ have more type-$\hat{H}$ and fewer participatns of the other two types. This suggests that both type-$\hat{M}$ and type-$\hat{H}$ benefit from participating, while type-$\hat{L}$ does best when fewer type-$\hat{L}$ girls are participating.

The patterns for Gender Roles attitudes, presented in Panel B, are different. For All Girls (A), the highest simulated means are found in schools with higher percentages of *both* type-$\hat{L}$ and type-$\hat{H}$ participants, and thus necessarily a lower percentage of type-$\hat{M}$. Further, this pattern holds when the target population is any of type-$\hat{L}$, -$\hat{M}$ or -$\hat{H}$. For all three subsets of the population, means in the highest 0.1% of the simulated distribution correspond to higher percentages of *both* type-$\hat{L}$ and type-$\hat{H}$ students participating.

These results are roughly similar to the regression results presented above. For Educational Aspirations, the highest average outcomes among type-$\hat{L}$ girls are found in simulations with fewer type-$\hat{L}$ participants, while the highest for type-$\hat{M}$ and type-$\hat{H}$ have more of those types participating. Further, in Panel B, the highest simulations for all three subgroups tend to include higher numbers of type-$\hat{L}$ and type-$\hat{H}$ participants and fewer of type-$\hat{M}$.

## 8. CONCLUSION AND DISCUSSION

The very existence of peer effects implies that individuals' outcomes and choices may be affected by the presence or absence of others. This suggests that, in settings where policymakers have control over assignments, simply changing the assignment rule may change outcomes, and such interventions should be relatively costless to implement. However, prior efforts to design and implement such assignment rules have fallen short due to, among other

TABLE XI
Summary of Extrema in Simulations of Random Assignments

| | | (1) | (2) | (3) | (4) |
|---|---|---|---|---|---|
| ***Panel A: Educational Aspirations*** | | | | | |
| Target Pop. | Quantile of Simulations | Mean Outcome in Target Pop. | $\%\hat{L}P$ | $\%\hat{M}P$ | $\%\hat{H}P$ |
| (A) All Girls | Highest 0.1% | -0.013 | 0.341 | 0.353 | 0.306 |
| | Middle 0.1% | -0.120 | 0.347 | 0.351 | 0.301 |
| | Lowest 0.1% | -0.226 | 0.352 | 0.352 | 0.296 |
| (B) Type-$\hat{L}$ | Highest 0.1% | -0.345 | 0.339 | 0.357 | 0.304 |
| | Middle 0.1% | -0.529 | 0.348 | 0.351 | 0.301 |
| | Lowest 0.1% | -0.714 | 0.355 | 0.346 | 0.299 |
| (C) Type-$\hat{M}$ | Highest 0.1% | 0.082 | 0.346 | 0.354 | 0.300 |
| | Middle 0.1% | -0.094 | 0.347 | 0.352 | 0.301 |
| | Lowest 0.1% | -0.270 | 0.347 | 0.352 | 0.302 |
| (D) Type-$\hat{H}$ | Highest 0.1% | 0.451 | 0.343 | 0.349 | 0.309 |
| | Middle 0.1% | 0.269 | 0.348 | 0.351 | 0.302 |
| | Lowest 0.1% | 0.086 | 0.350 | 0.354 | 0.296 |
| ***Panel B: Gender Roles Attitudes*** | | | | | |
| Target Pop. | Quantile of Simulations | Mean Outcome in Target Pop. | $\%\hat{L}P$ | $\%\hat{M}P$ | $\%\hat{H}P$ |
| (A) All Girls | Highest 0.1% | 0.099 | 0.355 | 0.323 | 0.322 |
| | Middle 0.1% | -0.005 | 0.352 | 0.332 | 0.316 |
| | Lowest 0.1% | -0.109 | 0.348 | 0.342 | 0.310 |
| (B) Type-$\hat{L}$ | Highest 0.1% | 0.148 | 0.354 | 0.326 | 0.320 |
| | Middle 0.1% | -0.031 | 0.352 | 0.332 | 0.316 |
| | Lowest 0.1% | -0.210 | 0.349 | 0.338 | 0.313 |
| (C) Type-$\hat{M}$ | Highest 0.1% | 0.160 | 0.354 | 0.327 | 0.319 |
| | Middle 0.1% | -0.015 | 0.353 | 0.332 | 0.315 |
| | Lowest 0.1% | -0.190 | 0.349 | 0.339 | 0.312 |
| (D) Type-$\hat{H}$ | Highest 0.1% | 0.214 | 0.353 | 0.327 | 0.320 |
| | Middle 0.1% | 0.032 | 0.351 | 0.335 | 0.314 |
| | Lowest 0.1% | -0.149 | 0.351 | 0.338 | 0.311 |

Notes: Simulations based upon 1,000,000 repetitions, with residuals drawn from random normal. Simulation results in Panel A correspond to Table VI and Column (2) of Table VII. Simulation results in Panel B correspond to Table VI and Column (4) of Table VII. Highest 0.1% represent means of simulations above the 0.999 quantile, Middle 0.1% represent means of simulations between quantiles 0.4995 and 0.5005, Lowest 0.1% represent means of simulations below the 0.001 quantile.

factors, endogenous peer selection. To account for this, a model of peer selection is needed, but modeling and estimating such models presents many difficulties.

My approach explicitly models outcomes as the result of a two-step process. In the first step, agents choose peers within a continuous action space subject to a budget constraint. I show that this greatly simplifies equilibrium characterization and identification: under certain conditions, there is a unique strictly positive Nash equilibrium, and the necessary first-order conditions can be employed for identification and estimation. The structure of the game motivates the use of a budget-set instrument to identify the model's parameters, and I provide conditions under which identification holds. Crucially, the model provides for identification of individual-specific unobserved variables that affect both network structure and outcomes.

In the second step, outcomes are determined conditional on the realized network. Here, network endogeneity is modeled explicitly as an omitted variable issue. Conditional on these unobserved variables—which are identified in the network formation model—the parameters of the peer effects model is identified, even under certain types of network endogeneity.

With these methodological results in hand, I then estimate the model using innovative new data from a randomized trial in rural northern India. I find that the unobserved variables play a large role in determining both network structure and outcomes conditional on the network. With the estimated parameters in hand, I move to out-of-sample validation and counterfactual simulation. First, by comparing predicted outcomes to realized out-of-sample means, I show that the model performs well in out-of-sample prediction. Next, while I cannot solve analytically for optimal assignments, I present features of the extrema of distributions of means, shedding light on the relationship between assignment rules and optimal outcomes.

With this paper, I provide a method to account for network endogeneity when estimating peer effects, developing an explicit model for how network endogeneity biases results that neglect to account for endogenous network structure. This further leads to a method to predict the effects of alternative assignments while accounting for network endogeneity. As necessary steps in developing this methodology, I make further contributions to the theory of network formation as well as providing new econometric results for the identification of network formation games.

Importantly, the methodology developed here is not context-specific. Rather, it has broad applicability in settings where assignment rules may influence outcomes both directly and through changing network structure. In order to apply the methods used here, researchers need to collect data on the outcome of interest and demographics, as well as sufficiently rich network data from which to construct a continuous network measure. Indeed, the results here suggest an additional impetus to collect rich network data.

While such detailed network data may seem rare, high-dimensional network data has been gathered in a variety of contexts. For example, for all nominated friends, AddHealth asks a series of five follow-up questions such as whether a problem was talked about or the pair spent time together on weekends (Harris, 2009). Further, Banerjee et al. (2012), in their study of microfinance in 75 rural villages in rural India, ask a series of binary questions about 13 different types of links, including borrowing, lending, and friendships. More granular measures of connectedness may be created from this 13-dimensional binary data in a way similar to the empirical strategy used in this paper.

Indeed, high-dimensional network data is routinely collected in field projects—mostly in developing countries—studying peer effects, information diffusion, and related issues. Tjernström (2017) asks 18 questions related to relationships, whom individuals talk to about various processes, and knowledge about others. Beaman et al. (2015) ask respondents to name friends, those with whom they share food, and those with whom they discuss agriculture. Ngatia (2015) asks about friends, family members, and those who are admired. Accordingly, the data required to implement the strategy developed in this paper is increasingly available. With this data, researchers can use the methods developed here to generate predictions of the effects of out-of-sample assignments that are robust to certain types of network endogeneity.

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## APPENDIX A: PROOF OF PROPOSITIONS.

*Proposition 1*

PROOF: This is a standard OLS result other than the fact that it relies upon consistent estimates of $a_{is}$. *Q.E.D.*

*Proposition 2*

PROOF: Existence of equilibrium follows directly from Rosen (1965). Given each other player's strategies, each player's uility function is concave in his own strategy $g_{is}$. Therefore, existence of equilibrium follows from Theorem 1 of Rosen (1965).

I show existence of a strictly positive equilibrium in four steps. First, I show existence of equilibrium in a version of the game in which players' strategy sets are bounded below by $\underline{g} > 0$ (Step 1). Second, I show that, for sufficiently small $\underline{g}$, the lower bound is non-binding (Step 2). Next, I show that there exists a strategy profile $\mathbf{g}^*$ that is an equilibrium of the game for *all* sufficiently small $\underline{g}$ (Step 3). Finally, I show that $\mathbf{g}^*$ is an equilibrium of the game when $\underline{g} = 0$ (Step 4).

*Step 1: Existence with Strictly Positive Strategy Sets*

Define a network-formation game in which individuals maximize utility as defined by the text. Different than the game defined in the text, however, they must form strictly positive links with each individual. That is, for each $i, j \neq i$, $g_{ijs} \geq \underline{g}$, where $\underline{g} > 0$ (strictly). Set $\underline{g}$ sufficiently small that each player's strategy set is non-void: $\underline{g} \in (0, \frac{\overline{M}}{(N-1)\underline{c}})$.

As defined by Rosen (1965) and Ui (2008), for each $i$, $U_{is}(g_{is}, g_{-is})$ is concave in $g_{is}$ (his own strategy) for every $g_{-is}$. Accordingly, the game is a smooth concave game on a compact strategy set. Thus, by Lemma 1 in Ui (2008) and the notes afterward, a Nash Equilibrium of this game exists.

*Step 2: Lower Bound is Non-Binding for Sufficiently Small $\underline{g}$* Define $h = \left( \left( \frac{\underline{M}}{(N-1)\overline{c}} \right)^{\alpha-1} \left( \frac{\overline{M}}{\underline{c}} \right)^{\beta} \frac{f_{max}\overline{c}}{f_{min}\underline{c}} \right)^{\frac{1}{1-\alpha-\beta}}$, where $f_{min} \leq e^{f(X_{is},X_{js})} \leq f_{max}\ \forall\ i, j \neq i$.[58] Suppose $\underline{g} \in (0, h)$. The result in Step 1 applies for any $\underline{g} > 0$ such that strategy sets are non-void, so there exists an equilibrium of this restricted game.

Suppose further that the constraint binds for some pair $i, j \neq i$ and thus $g_{ijs} = \underline{g}$.

(1) Show necessary inequality when $g_{ijs} = \underline{g}$ for some $k \neq i, j$.

[58] $f_{min}$ and $f_{max}$ are well-defined and finite due to compactness of the range of the function $f$ and continuity of the exponential function.

Define $\lambda_{is}$ as the LaGrange Multiplier for the budget constraint, and $\mu_{ij}$ as the LaGrange Multiplier for the lower-bound constraint $g_{ijs} - \underline{g} \geq 0$ for $i, j \neq i$. Therefore, the following Kuhn-Tucker conditions hold for individual $i$ and all $j, k \neq i$:

$$\alpha g_{ijs}^{\alpha-1} g_{jis}^{\beta} e^{f(X_{is},X_{js})} - \lambda_{is} c_{ijs} + \mu_{ijs} = 0 \tag{A.1}$$

$$\alpha g_{iks}^{\alpha-1} g_{kis}^{\beta} e^{f(X_{is},X_{ks})} - \lambda_{is} c_{iks} + \mu_{iks} = 0 \tag{A.2}$$

Since utility is increasing in $g_{iks}$ whenever $\underline{g} > 0$, the budget constraint must bind in equilibrium. So, $\sum_{k \neq i} c_{iks} g_{iks} = M_{is}$. Therefore, since $\underline{g} < \frac{\underline{M}}{(N-1)\underline{c}}$, the lower-bound constraint must *not* bind for some $k \neq j, i$. Thus, $g_{iks} > \underline{g}$ and $\mu_{iks} = 0$. Combine Equations (A.1) and (A.2) through $\lambda_{is}$ as follows:

$$\alpha g_{ijs}^{\alpha-1} g_{jis}^{\beta} \frac{e^{f(X_{is},X_{js})}}{c_{ijs}} + \frac{\mu_{ijs}}{c_{ijs}} = \alpha g_{iks}^{\alpha-1} g_{kis}^{\beta} \frac{e^{f(X_{is},X_{ks})}}{c_{iks}} \tag{A.3}$$

Since $\mu_{ijs} > 0$,

$$g_{ijs}^{\alpha-1} g_{jis}^{\beta} \frac{e^{f(X_{is},X_{js})}}{c_{ijs}} < g_{iks}^{\alpha-1} g_{kis}^{\beta} \frac{e^{f(X_{is},X_{ks})}}{c_{iks}} \tag{A.4}$$

<u>(2) Derive upper bound on RHS of Equation (A.4)</u>

W.l.o.g., choose $k$ such that $g_{iks} \geq g_{ils}\ \forall\ l \neq i$. Due to the budget constraint holding with equality,

$$M_{is} = \sum_{l \neq i} c_{ils} g_{ils} \tag{A.5}$$

$$\leq \sum_{l \neq i} \bar{c} g_{iks} = (N-1)\bar{c} g_{iks} \tag{A.6}$$

So, $g_{iks} \geq \frac{M_{is}}{(N-1)\bar{c}} \geq \frac{\underline{M}}{(N-1)\bar{c}}$. Since $\alpha - 1 < 0$, $g_{iks}^{\alpha-1} \leq \left(\frac{\underline{M}}{(N-1)\bar{c}}\right)^{\alpha-1}$. Further, $g_{kis} \leq \frac{\bar{M}}{\underline{c}}$. Therefore,

$$g_{iks}^{\alpha-1} g_{kis}^{\beta} \frac{e^{f(X_{is},X_{ks})}}{c_{iks}} \leq \left(\frac{\underline{M}}{(N-1)\bar{c}}\right)^{\alpha-1} \left(\frac{\bar{M}}{\underline{c}}\right)^{\beta} \frac{f_{max}}{\underline{c}} \tag{A.7}$$

<u>(3) Derive lower bound for LHS of Equation (A.4)</u>

On the left-hand side of Equation (A.4), $g_{ijs} = \underline{g}$. Further, $g_{jis} \geq \underline{g}$ and $\frac{e^{f(X_{is},X_{js})}}{c_{ijs}} \geq \frac{f_{min}}{\bar{c}}$.

This implies the lower bound for the left-hand side as follows:

$$\underline{g}^{\alpha+\beta-1}\frac{f_{min}}{\bar{c}} \leq g_{ijs}^{\alpha-1}g_{jis}^{\beta}\frac{e^{f(X_{is},X_{js})}}{c_{ijs}} \tag{A.8}$$

<u>(4) Combine Parts (2) and (3) to find contradiction</u>

Combining Equations (A.7) and (A.8) through Equation (A.4), we see that

$$\underline{g}^{\alpha+\beta-1}\frac{f_{min}}{\bar{c}} < \left(\frac{\underline{M}}{(N-1)\bar{c}}\right)^{\alpha-1}\left(\frac{\bar{M}}{\underline{c}}\right)^{\beta}\frac{f_{max}}{\underline{c}} \tag{A.9}$$

Since $\alpha+\beta-1<0$, this implies

$$\underline{g} > \left(\left(\frac{\underline{M}}{(N-1)\bar{c}}\right)^{\alpha-1}\left(\frac{\bar{M}}{\underline{c}}\right)^{\beta}\frac{f_{max}}{f_{min}}\frac{\bar{c}}{\underline{c}}\right)^{\frac{1}{\alpha+\beta-1}} = h \tag{A.10}$$

This implies a contradiction since we assumed that $\underline{g} \in (0,h)$. Accordingly, when $\underline{g} \in (0,h)$, the constraint $g_{ijs} \geq \underline{g}$ does *not* bind for any pair $i, j \neq i$. Therefore, when $\underline{g} \in (0,h)$, there exists an equilibrium in which $g_{ijs} > \underline{g}\ \forall\, i, j \neq i$ (strictly).

*<u>Step 3</u>: There exists a strategy profile* $\mathbf{g}^*$ *that is an equilibrium for all* $\underline{g}$ *sufficiently small.*

Step 2 above demonstrates that, whenever $\underline{g} \in (0,h)$, there exists an equilibrium in which $\underline{g}$ does not bind. For an arbitrary $\underline{g} \in (0,h)$, define one such equilibrium as $\mathbf{g}^*$. At this equilibrium, $\mu_{ijs} = 0\ \forall\, i, j \neq i$. Therefore, at $\mathbf{g}^*$, the following equalities hold

$$\alpha(g^*_{ijs})^{\alpha-1}(g^*_{jis})^{\beta}e^{f(X_{is},X_{js})} - \lambda^*_{is}c_{ijs} = 0\ \forall\, i, j \neq i \tag{A.11}$$

$$\sum_{k\neq i} c_{ijs}(g^*_{ijs}) = M_{is} \tag{A.12}$$

Note that these conditions do *not* depend on $\underline{g}$, as long as $\underline{g} \in (0,h)$. Therefore, since they hold for some $\underline{g} \in (0,h)$, they hold for *all* $\underline{g} \in (0,h)$. Thus, the strategy profile $\mathbf{g}^*$ is an equilibrium of the game for *all* $\underline{g} \in (0,h)$.

*<u>Step 3</u>: The strategy profile* $\mathbf{g}^*$ *is an equilibrium of the game when* $\underline{g} = 0$.

This proof proceeds by contradiction. Suppose $\mathbf{g}^*$ is *not* an equilibrium of the game when $\underline{g} = 0$. Therefore, for some $i$, there exists $g'_{is} \neq g^*_{is}$ such that $U_{is}(g'_{is}, g^*_{-is}) > U_{is}(g^*_{is}, g^*_{-is})$

<u>Case 1</u>: $g'_{ijs} > 0\ \forall\, j \neq i$

Define $\underline{g}'_{is} = \min_{k \neq i}\{g'_{iks}\}$. By assumption, $U_{is}(g'_{is}, g^*_{-is}) > U_{is}(g^*_{is}, g^*_{-is})$. Since this is a feasible strategy for player $i$ whenever $0 < \underline{g}_1 < \underline{g}'_{is}$, this implies that $\mathbf{g}^*$ is not an equilibrium of the game when $\underline{g}_1 \in (0, \underline{g}'_{is}) \cap (0, h)$, contradicting Step 3. Therefore, it cannot be the case that $g'_{ijs} > 0 \; \forall \; j \neq i$.

<u>Case 2</u>: $g'_{ijs} = 0$ for some $j \neq i$, $g'_{iks} > 0$ for some $k \neq i, j$

<u>(1) Calculate $U_{is}(g^*_{is}, g^*_{-is})$.</u>

The First-Order Conditions that characterize $\mathbf{g}^*$ require the following to hold:

$$\alpha (g^*_{ijs})^{\alpha-1} (g^*_{jis})^{\beta} e^{f(X_{is}, X_{js})} = \lambda^*_{is} c_{ijs} \; \forall j \neq i \tag{A.13}$$

Next, multiply by $(g^*_{ijs})$. So,

$$\alpha (g^*_{ijs})^{\alpha} (g^*_{jis})^{\beta} e^{f(X_{is}, X_{js})} = \lambda^*_{is} c_{ijs} g^*_{ijs} \; \forall j \neq i \tag{A.14}$$

From this, we see that

$$U_{is}(g^*_{is}, g^*_{-is}) = \sum_{j \neq i} (g^*_{ijs})^{\alpha} (g^*_{jis})^{\beta} e^{f(X_{is}, X_{js})} = \frac{1}{\alpha} \sum_{j \neq i} \lambda_{is} c_{ijs} g^*_{ijs} \tag{A.15}$$

$$= \lambda_{is} \frac{M_{is}}{\alpha} \tag{A.16}$$

<u>(2) $U_{is}(g'_{is}, g^*_{-is}) \leq U_{is}(g''_{is}, g^*_{-is})$</u>

It is clear that the utility to $i$ from playing $g'_{is}$ is bounded above by the utility of the strategy $g''_{is}$ defined by

$$g''_{is} = \text{argmax}\{U_{is}(g_{is}, g^*_{-is}) | \sum_{k \neq i} c_{iks} g_{iks}, g'_{ijs} = 0 \Rightarrow g_{ijs} = 0\} \tag{A.17}$$

That is, $g''_{is}$ maximizes utility subject to the budget constraint and the restriction that $g''_{ijs} = 0$ whenever $g'_{ijs} = 0$. Since $g'_{is}$ is in the feasible set, utility from playing $g'_{is}$ must be weakly lower than utility from $g''_{is}$. Therefore,

$$U_{is}(g'_{is}, g^*_{-is}) \leq U_{is}(g''_{is}, g^*_{-is}) \tag{A.18}$$

<u>(3) Characterize $U_{is}(g''_{is}, g^*_{-is})$</u>

Since $g''_{iks}$ maximizes utility to $i$ of the restricted problem, for all $g'_{ijs} > 0$, the following condition holds:

$$\alpha(g''_{iks})^{\alpha-1}(g^*_{kis})^{\beta}e^{f(X_{is},X_{ks})} = \lambda''_{is}c_{iks} \tag{A.19}$$

where $\lambda''_{is}$ is the LaGrange multiplier on the budget constraint of the restricted problem. Multiply Equation (A.19) by $g''_{iks}$, yielding

$$\alpha(g''_{iks})^{\alpha}(g^*_{kis})^{\beta}e^{f(X_{is},X_{ks})} = \lambda''_{is}c_{iks}g''_{iks}\ \forall\ k \neq i \tag{A.20}$$

So,

$$U_{is}(g''_{is}, g^*_{-is}) = \sum_{j\neq i} 1\{g'_{ijs} > 0\}(g''_{iks})^{\alpha}(g^*_{kis})^{\beta}e^{f(X_{is},X_{ks})} \tag{A.21}$$

$$= \frac{1}{\alpha}\sum_{j\neq i} 1\{g'_{ijs} > 0\}\lambda''_{is}g''_{ijs}c_{ijs} \tag{A.22}$$

$$= \lambda''_{is}\frac{M_{is}}{\alpha} \tag{A.23}$$

<u>(4) Show that $\lambda''_{is} < \lambda^*_{is}$</u>

Whenever $g'_{iks} > 0$, combining Equations (A.13) and (A.19) yields

$$\frac{g''_{iks}}{g^*_{iks}} = \left(\frac{\lambda''_{is}}{\lambda^*_{is}}\right)^{\frac{1}{\alpha-1}} \tag{A.24}$$

This implies that $\frac{g''_{iks}}{g^*_{iks}}$ is constant across all $k$ for whom $g'_{ijs} > 0$. Further, since there exists some $j$ for whom $g'_{iks} = 0$ but $g^*_{ijs} > 0$,

$$\sum_{k\neq i} c_{iks}g^*_{iks} = M_{is} > \sum_{k\neq i} 1\{g'_{iks} > 0\}c_{iks}g^*_{iks} \tag{A.25}$$

Further, since the budget constraint holds at equality at $g''_{is}$,

$$M_{is} = \sum_{k\neq i} c_{iks}g''_{iks} \tag{A.26}$$

$$= \sum_{k\neq i} 1\{g'_{iks} > 0\}c_{iks}g''_{iks} \tag{A.27}$$

$$= \left(\frac{\lambda''_{is}}{\lambda^*_{is}}\right)^{\frac{1}{\alpha-1}} \sum_{k\neq i} 1\{g'_{iks} > 0\}c_{iks}g*_{iks} \tag{A.28}$$

where the last substition follows from Equation (A.24). Combining this with (A.25) shows

$$M_{is} < \left(\frac{\lambda''_{is}}{\lambda^*_{is}}\right)^{\frac{1}{\alpha-1}} M_{is} \tag{A.29}$$

$$1 < \left(\frac{\lambda''_{is}}{\lambda^*_{is}}\right)^{\frac{1}{\alpha-1}} \tag{A.30}$$

$$\Rightarrow \lambda''_{is} < \lambda^*_{is} \tag{A.31}$$

where the last line follows since $0 < \alpha < 1$.

(5) Finally, bring it all together

Bringing these parts together, we see that

$$U_{is}(g'_{is}, g^*_{-is}) \leq U_{is}(g''_{is}, g^*_{-is}) \qquad \text{Part (2)} \tag{A.32}$$

$$= \lambda''_{is}\frac{M_{is}}{\alpha} \qquad \text{Part (3)} \tag{A.33}$$

$$< \lambda^*_{is}\frac{M_{is}}{\alpha} \qquad \text{Part (4)} \tag{A.34}$$

$$= U_{is}(g*_{is}, g^*_{-is}) \qquad \text{Part (1)} \tag{A.35}$$

which is a contradiction to the supposition that $U_{is}(g'_{is}, g^*_{-is}) > U_{is}(g*_{is}, g^*_{-is})$.

Therefore, there does not exist a deviation $g'_{ijs} = 0$ for some $i, j \neq i$ and $g'_{iks} > 0$ for some $k \neq i, j$ where $U_{is}(g'_{is}, g^*_{-is}) > U_{is}(g*_{is}, g^*_{-is})$

Case 3: $g'_{ijs} = 0 \;\forall\; j \neq i$.

In this case, $U_{is}(g'_{is}, g^*_{-is}) = 0$, while $U_{is}(g^*_{is}, g^*_{-is}) > 0$. So, $U_{is}(g'_{is}, g^*_{-is}) < U_{is}(g^*_{is}, g^*_{-is})$. Thus, when $g'_{ijs} = 0 \;\forall\; i, j \neq i$, utility to $i$ is less than utility from $\mathbf{g}^*$.

Bringing it all together, Cases 1-3 together show that, when $\underline{g} = 0$, there does not exist any $g'_{is}$ such that $U_{is}(g'_{is}, g^*_{-is}) > U_{is}(g^*_{is}, g^*_{-is})$ for some $i$. Thus, $\mathbf{g}^*$ is a Nash equilibrium when $\underline{g} = 0$.

*Q.E.D.*

*Proposition 3*

PROOF: Suppose there are two equilibria $(g, \lambda)$ and $(g', \lambda')$, where $g = (g_{12s}, g_{13s}, ..., g_{NN-1s})$ and $\lambda = (\lambda_{1s}, ..., \lambda_{Ns})$. Equations (6) and (7), the First Order necessary conditions for strictly positive equilibrium, imply

$$(\alpha - 1)(\log g_{ijs} - \log g'_{ijs}) + \beta(\log g_{jis} - \log g'_{jis}) - (\log \lambda_{is} - \log \lambda'_{is}) = 0 \;\forall\; i, j \neq i \tag{A.36}$$

$$\sum_{j \neq i} c_{ijs}(g_{ijs} - g'_{ijs}) = 0 \;\forall\; i \tag{A.37}$$

Define $\tilde{\beta} = \frac{\beta}{1-\alpha}$ and $\tilde{\lambda}_{is} = \frac{\log \lambda_{is}}{1-\alpha}$. After substitution and rearrangement, Equation (A.36) becomes

$$(\log g_{ijs} - \log g'_{ijs}) = \tilde{\beta}(\log g_{jis} - \log g'_{jis}) - (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is}) \;\forall\; i, j \neq i \tag{A.38}$$

By symmetry,

$$(\log g_{jis} - \log g'_{jis}) = \tilde{\beta}(\log g_{ijs} - \log g'_{ijs}) - (\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) \;\forall\; i, j \neq i \tag{A.39}$$

Substitute Equation (A.39) into Equation (A.38) and rearrange, yielding

$$(\log g_{ijs} - \log g'_{ijs}) = -\frac{1}{1 - \tilde{\beta}^2}\left(\tilde{\beta}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) + (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is})\right) \;\forall\; i, j \neq i \tag{A.40}$$

Since the log function is continuously differntiable for all positive values, the Mean Value Theorem $\Rightarrow \exists\; g^*_{ijs} \in [g_{ijs}, g'_{ijs}]$, where $\log g_{ijs} - \log g'_{ijs} = \frac{1}{g^*_{ijs}}(g_{ijs} - g'_{ijs})$ and $g^*_{ijs} > 0$. Make this substitution and multiply by $-(1 - \tilde{\beta}^2) g^*_{ijs} c_{ijs}$:

$$-(1 - \tilde{\beta}^2) c_{ijs}(g_{ijs} - g'_{ijs}) = c_{ijs} g^*_{ijs}\left(\tilde{\beta}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) + (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is})\right) \;\forall\; i, j \neq i \tag{A.41}$$

Next, sum across $j \neq i$, substitute and rearrange:

$$-(1 - \tilde{\beta}^2)\sum_{j \neq i} c_{ijs}(g_{ijs} - g'_{ijs}) = \sum_{j \neq i} c_{ijs} g^*_{ijs}\left(\tilde{\beta}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) + (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is})\right) \;\forall\; i \tag{A.42}$$

$$0 = (\sum_{j \neq i} c_{ijs} g^*_{ijs})(\tilde{\lambda}_{is} - \tilde{\lambda}'_{is}) + \tilde{\beta}\sum_{j \neq i} c_{ijs} g^*_{ijs}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) \;\forall\; i \tag{A.43}$$

This defines a linear system of $N$ equations and $N$ unknowns, as defined by $\mathbf{Ab} = 0$ in Equation (A.44):

$$\begin{bmatrix} (\sum_{j\neq 1} c_{1js}g^*_{1js}) & \tilde{\beta}c_{12s}g^*_{12s} & ... & \tilde{\beta}c_{1Ns}g^*_{1Ns} \\ \tilde{\beta}c_{21s}g^*_{21s} & (\sum_{j\neq 2} c_{2js}g^*_{2js}) & ... & \tilde{\beta}c_{2Ns}g^*_{2Ns} \\ . & . & . & . \\ \tilde{\beta}c_{N1s}g^*_{N1s} & . & ... & (\sum_{j\neq 1} c_{Njs}g^*_{Njs}) \end{bmatrix} \begin{bmatrix} \tilde{\lambda}_{1s} - \tilde{\lambda}'_{1s} \\ \tilde{\lambda}_{2s} - \tilde{\lambda}'_{2s} \\ . \\ . \\ \tilde{\lambda}_{Ns} - \tilde{\lambda}'_{Ns} \end{bmatrix} = 0 \tag{A.44}$$

Clearly, $\mathbf{A}$ being invertible will guarantee $\tilde{\lambda}_{is} - \tilde{\lambda}'_{is} = 0 \; \forall \; i$.

Suppose $\mathbf{A}$ is not invertible. Therefore, 0 is an eigenvalue of $\mathbf{A}$ with an associated eigenvector $\mathbf{v}$. Let $v_m$ be the largest element of $\mathbf{v}$ and, w.l.o.g., $v_m > 0$. So, $v_m \geq v_j \geq -v_m \; \forall j \neq m$. Now,

$$v_m(\sum_{j\neq m} c_{mjs}g^*_{mjs}) + \tilde{\beta}\sum_{j\neq m} v_j c_{mjs}g^*_{mjs} \geq v_m(\sum_{j\neq m} c_{mjs}g^*_{mjs}) - \tilde{\beta}v_m \sum_{j\neq m} c_{mjs}g^*_{mjs} \tag{A.45}$$

$$> v_m(\sum_{j\neq m} c_{mjs}g^*_{mjs})(1-\tilde{\beta}) > 0 \tag{A.46}$$

This contradicts that 0 is an eigenvalue. Therefore, $\mathbf{A}$ is invertible, and $\tilde{\lambda}_{is} = tilde\lambda'_{is}\forall i$.

Finally, from Equation (A.40), we see that $\tilde{\lambda}_{is} - \tilde{\lambda}'_{is} = 0 \; \forall \; i, j \neq i \; \Rightarrow (\log g_{ijs} - \log g'_{ijs}) = 0 \; \forall \; i, \; j \neq i$. Therefore, $(g, \lambda) = (g', \lambda')$ and the equilibrium is unique.

*Q.E.D.*

*Proposition 4*

PROOF: This result is a typical panel IV result, allowing for arbitrary correlation of variables within clusters. Let $S$ be the number of schools (potential networks) observed and $N$ be the number of actors per school. Starting with Equation (12) and the instrument set $z_{ijs}$, we see that

$$\begin{aligned} z'_{ijs}\dot{g}^i_{ijs} &= z'_{ijs}\dot{g}^i_{jis}\tilde{\beta} + z'_{ijs}X_{is} \odot \dot{X}^i_{js}\delta_1 + z'_{ijs}\dot{X}^i_{js}\gamma_3 - z'_{ijs}\dot{c}^i_{ijs} \\ &= z'_{ijs}b_{ijs}\theta - z'_{ijs}\dot{c}^i_{ijs} \end{aligned} \tag{A.47}$$

where $\theta = (\tilde{\beta}, \delta_1', \gamma_3')'$. Next, sum across all schools and pairs of students:

$$\begin{aligned}\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} b_{ijs}' z_{ijs} z_{ijs}' \dot{g}_{ijs}^i &= \sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{SN(N-1)} b_{ijs}' z_{ijs} z_{ijs}' b_{ijs}\theta \\ &- \frac{1}{Sn(n-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} b_{ijs}' z_{ijs} z_{ijs}' \dot{c}_{ijs}^i\end{aligned} \tag{A.48}$$

Since $\dot{c}_{ijs}^i$ is a linear combination of terms that are assumed to be independent across $s$, $\dot{c}_{ijs}^i$ is also independent across $s$. Further, all terms are bounded and thus have finite variance. Let $w_{ijs}$ be an element of one of the matrices in Equation (A.48). For any such variable, $\mathbb{C}[w_{ijs}, w_{klt}] = 0$ whenever $s \neq t$. Further, since each $w_{ijs}$ is identically distributed within a school, $\mathbb{V}[w_{ijs}] = \mathbb{V}[w_{lks}] \forall i, j, k, l$. Further,

$$\begin{aligned}\mathbb{V}[\bar{w}_{ijs}] &= \mathbb{V}[\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} w_{ijs}] \\ &= \frac{1}{S^2N^2(N-1)^2}\left(SN(N-1)\mathbb{V}[w_{ijs}] + S(\sum_{i=1}^{N}\sum_{j\neq i})(\sum_{k=1}^{N}\sum_{l\neq j})\mathbb{C}[w_{ijs}, w_{kls}]\right) \\ &\leq \frac{1}{S}\left(\frac{1}{N(N-1)} + 1\right)\mathbb{V}[w_{ijs}]\end{aligned} \tag{A.49}$$

where the final line applies the Cauchy Schwarz Inequality ($\mathbb{C}[w_{ijs}, w_{kls}] \leq \mathbb{V}[w_{ijs}]$). Therefore,
$\lim_{S\to\infty} \mathbb{V}[\bar{w}_{ijs}] = 0$ and by Chebyshev's Inequality,

$$\operatorname*{plim}_{S\to\infty} \bar{w}_{ijs} = \mathbb{E}[w_{ijs}] \tag{A.50}$$

I here note that all terms in Equation (A.48) are sample averages. Therefore, we can apply Equation (A.50) to each element. Thus, $\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} b_{ijs}' z_{ijs} z_{ijs}' \dot{g}_{ijs}^i \to_p \mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' \dot{g}_{ijs}^i]$, etc. Now, replacing the terms in Equation (A.48) with probability limits,

$$\mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' \dot{g}_{ijs}^i] = \mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' b_{ijs}]\theta - \mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' \dot{c}_{ijs}^i] \tag{A.51}$$

Assumption 4 implies that the final term in Equation (A.51) is zero, while the rank condition of Proposition 4 implies invertibility of $\mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' b_{ijs}]$. Therefore,

$$\theta = (\mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' b_{ijs}])^{-1}\mathbb{E}[b_{ijs}' z_{ijs} z_{ijs}' \dot{g}_{ijs}^i] \tag{A.52}$$

and thus the parameters $\tilde{\beta}$, $\delta_1$, and $\gamma_3$ are identified.

*Q.E.D.*

*Proposition 5*

PROOF: This proof is very similar to Proposition 4 but relies upon the additional exogeneity conditions in Assumption 6. Starting with Equation (19) and the instrument set $z_{ijs}$, we see

$$\begin{aligned} z'_{ijs}\dot{g}^i_{ijs} &= z'_{ijs}\dot{g}^i_{jis}\tilde{\beta} + z'_{ijs}X_{is}\dot{X}^i_{js}\delta_1 + z'_{ijs}X_{is}\dot{a}^i_{js}\delta_2 + z'_{ijs}a_{is}\dot{X}^i_{js}\delta_3 + z'_{ijs}a_{is}\dot{a}^i_{js}\delta_4 \\ &\quad + z'_{ijs}\dot{X}^j_{is}\gamma_3 + z'_{ijs}a_{is}\dot{a}^i_{js}\gamma_4 - z'_{ijs}\dot{c}^i_{ijs} \end{aligned} \tag{A.53}$$

Rearrangement of terms shows that

$$z'_{ijs}\dot{g}^i_{ijs} = z'_{ijs}b_{ijs}\theta + z'_{ijs}X_{is}\dot{a}^i_{js}\delta_2 + z'_{ijs}a_{is}\dot{X}^i_{js}\delta_3 + z'_{ijs}a_{is}\dot{a}^i_{js}\delta_4 + z'_{ijs}a_{is}\dot{a}^i_{js}\gamma_4 - z'_{ijs}\dot{c}^i_{ijs} \tag{A.54}$$

where $\theta = (\tilde{\beta}, \delta'_1, \gamma_3)'$. Next, sum across all schools and all pairs of students. So,

$$\begin{aligned} \frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} b'_{ijs}z_{ijs}z'_{ijs}\dot{g}^i_{ijs} &= \frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i}(b'_{ijs}z_{ijs}z'_{ijs}b_{ijs}\theta \\ &\quad + b'_{ijs}z_{ijs}z'_{ijs}X_{is}\dot{a}^i_{js}\delta_2 + b'_{ijs}z_{ijs}z'_{ijs}a_{is}\dot{X}^i_{js}\delta_3 + b'_{ijs}z_{ijs}z'_{ijs}a_{is}\dot{a}^i_{js}\delta_4 \\ &\quad + b'_{ijs}z_{ijs}z'_{ijs}a_{is}\dot{a}^i_{js}\gamma_4 - b'_{ijs}z_{ijs}z'_{ijs}\dot{c}^i_{ijs}) \end{aligned} \tag{A.55}$$

Since $\dot{c}^i_{ijs}$ is a linear combination of terms that are assumed to be independent across $s$, $\dot{c}^i_{ijs}$ is also independent across $s$. Further, all terms are bounded and thus have finite variance. Let $w_{ijs}$ be an element of one of the matrices in Equation (A.55). For any such variable, $\mathbb{C}[w_{ijs}, w_{klt}] = 0$ whenever $s \neq t$. Further,

$$\begin{aligned} \mathbb{V}[\bar{w}_{ijs}] &= \mathbb{V}[\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} w_{ijs}] \\ &= \frac{1}{S^2N^2(N-1)^2}\left(SN(N-1)\mathbb{V}[w_{ijs}] + S(\sum_{i=1}^{N}\sum_{j\neq i})(\sum_{k=1}^{N}\sum_{l\neq j})\mathbb{C}[w_{ijs}, w_{kls}]\right) \\ &\leq \frac{1}{S}\left(\frac{1}{N(N-1)} + 1\right)\mathbb{V}[w_{ijs}] \end{aligned} \tag{A.56}$$

where the final line applies the Cauchy-Schwarz Inequality ($\mathbb{C}[w_{ijs}, w_{kls}] \leq \mathbb{V}[w_{ijs}]$). Therefore,

$\lim_{S\to\infty} \mathbb{V}[\bar{w}_{ijs}] = 0$ and by Chebyshev's Inequality,

$$\underset{S\to\infty}{\text{plim}}\, \bar{w}_{ijs} = \mathbb{E}[w_{ijs}] \tag{A.57}$$

I here note that all terms in Equation (A.55) are sample averages. Therefore, we can apply Equation (A.57) to each element. Thus, $\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j\neq i} b'_{ijs} z_{ijs} z'_{ijs} \dot{g}^i_{ijs} \to_p \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{g}^i_{ijs}]$, etc. Now, replacing the terms in Equation (A.55) with probability limits,

$$\begin{aligned} \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{g}^i_{ijs}] &= \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} b_{ijs}]\theta + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} X_{is} \dot{a}^i_{js}]\delta_2 + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \dot{X}^i_{js}]\delta_3 \\ &\quad + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \dot{a}^i_{js}]\delta_4 + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{a}^i_{js}]\gamma_4 - \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{c}^i_{ijs}] \end{aligned} \tag{A.58}$$

The first part of Assumption 6 implies that the final term in Equation (A.58) is zero. Note that $z_{ijs}$ and $b_{ijs}$ are simply functions of $x_{ks}$. Therefore, application of L.I.E. implies that $\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} X_{is} \dot{a}^i_{js}] = \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} X_{is} \mathbb{E}[\dot{a}^i_{js} | b_{ijs}, z_{ijs}]] = 0$. By similar argument, $\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \dot{X}^i_{js}] = 0$ and $\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{a}^i_{js}] = 0$. Further, the third part of Assumption 6 implies

$$\begin{aligned} \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \dot{a}^i_{js}] &= \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \dot{a}^i_{js}] \\ &= \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \mathbb{E}[\dot{a}^i_{js} a_{is} | x_{ks}]] \\ &= \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \mathbb{E}[\dot{a}^i_{js} \mathbb{E}[a_{is} | x_{ks}, a_{ls}] | x_{ks}]] \\ &= 0 \end{aligned} \tag{A.59}$$

where we condition on all $k$ and $l \neq i$. Substituting these results into Equation (A.58) shows that

$$\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{g}^i_{ijs}] = \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} b_{ijs}]\theta \tag{A.60}$$

The rank condition guarantees the existence of $(\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} b_{ijs}])^{-1}$ and thus

$$\theta = (\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} b_{ijs}])^{-1} \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \dot{g}^i_{ijs}] \tag{A.61}$$

So, $\theta = (\tilde{\beta}, \delta'_1, \gamma'_3)$ is identified. *Q.E.D.*

*Proposition 6*

The first rank condition, together with Assumption 6 and Proposition 5, imply that $\tilde{\beta}$ is identified. I prove the rest of the proposition in three steps: (1) Scale identification of $\delta_2$ and

$\gamma_2$, (2) Scale identification of $\delta_3$, and (3) Scale identification of $\delta_4$.

*Step 1: Scale identification of $\delta_2$ and $\gamma_2$*

PROOF: To begin, multiply Equation (19) by $z'_{ijs}a_{js}$ and sum across $SN(N-1)$ observations. So,

$$\begin{aligned}\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} z'_{ijs}a_{js}\dot{g}^i_{ijs} = \frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i}(z'_{ijs}a_{js}\dot{g}^i_{jis}\tilde{\beta} \\ + z'_{ijs}a_{js}X_{is}\dot{X}^i_{js}\delta_1 + z'_{ijs}a_{js}X_{is}\dot{a}^i_{js}\delta_2 + z'_{ijs}a_{js}a_{is}\dot{X}^i_{js}\delta_3 \\ + z'_{ijs}a_{js}a_{is}\dot{a}^i_{js}\delta_4 + z'_{ijs}a_{js}\dot{X}^j_{is}\gamma_3 + z'_{ijs}a_{js}a_{is}\dot{a}^i_{js}\gamma_4 - z'_{ijs}a_{js}\dot{c}^i_{ijs})\end{aligned} \tag{A.62}$$

Since $\dot{c}^i_{ijs}$ is a linear combination of terms that are assumed to be independent across $s$, $\dot{c}^i_{ijs}$ is also independent across $s$. Further, all terms are bounded and thus have finite variance. Let $w_{ijs}$ be an element of one of the matrices in Equation (A.62). For any such variable, $\mathbb{C}[w_{ijs}, w_{klt}] = 0$ whenever $s \neq t$. Further,

$$\begin{aligned}\mathbb{V}[\bar{w}_{ijs}] &= \mathbb{V}[\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} w_{ijs}] \\ &= \frac{1}{S^2N^2(N-1)^2}\left(SN(N-1)\mathbb{V}[w_{ijs}] + S(\sum_{i=1}^{N}\sum_{j\neq i})(\sum_{k=1}^{N}\sum_{l\neq j})\mathbb{C}[w_{ijs}, w_{kls}]\right) \\ &\leq \frac{1}{S}\left(\frac{1}{N(N-1)} + 1\right)\mathbb{V}[w_{ijs}]\end{aligned} \tag{A.63}$$

where the final line applies the Cauchy-Schwarz Inequality ($\mathbb{C}[w_{ijs}, w_{kls}] \leq \mathbb{V}[w_{ijs}]$). Therefore,
$\lim_{S\to\infty}\mathbb{V}[\bar{w}_{ijs}] = 0$ and by Chebyshev's Inequality,

$$\operatorname*{plim}_{S\to\infty}\bar{w}_{ijs} = \mathbb{E}[w_{ijs}] \tag{A.64}$$

I here note that all terms in Equation (A.55) are sample averages. Therefore, we can apply Equation (A.64) to each term. So, $\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} z'_{ijs}a_{js}\dot{g}^i_{ijs} \to_p \mathbb{E}[z'_{ijs}a_{js}\dot{g}^i_{ijs}]$, etc. Now, replacing the terms in Equation (A.55) with probability limits,

$$\begin{aligned}\mathbb{E}[z'_{ijs}a_{js}\dot{g}^i_{ijs}] &= \mathbb{E}[z'_{ijs}a_{js}\dot{g}^i_{jis}]\tilde{\beta} + \mathbb{E}[z'_{ijs}a_{js}X_{is}\dot{X}^i_{js}]\delta_1 + \mathbb{E}[z'_{ijs}a_{js}X_{is}\dot{a}^i_{js}]\delta_2 \\ &+ \mathbb{E}[z'_{ijs}a_{js}a_{is}\dot{X}^i_{js}]\delta_3 + \mathbb{E}[z'_{ijs}a_{js}a_{is}\dot{a}^i_{js}]\delta_4 + \mathbb{E}[z'_{ijs}a_{js}\dot{X}^j_{is}]\gamma_3 \\ &+ \mathbb{E}[z'_{ijs}a_{js}a_{is}\dot{a}^i_{js}\gamma_4] - \mathbb{E}[z'_{ijs}a_{js}\dot{c}^i_{ijs}] \end{aligned} \tag{A.65}$$

Assumption 6 implies that $\mathbb{E}[z'_{ijs}a_{js}\dot{c}^i_{ijs}] = 0$.

By Assumption 6 and L.I.E., $\mathbb{E}[z'_{ijs}a_{js}X_{is}\dot{X}^i_{js}] = \mathbb{E}[z'_{ijs}X_{is}\dot{X}^i_{js}\mathbb{E}[a_{js}|z_{ijs}, X_{is}, \dot{X}^i_{js}]] = 0$. Similarly, $\mathbb{E}[z'_{ijs}a_{js}\dot{X}^j_{is}] = \mathbb{E}[z'_{ijs}\dot{X}^j_{is}\mathbb{E}[a_{js}|z_{ijs}, \dot{X}^j_{is}] = 0$. Independence of $a_{js}$ and $a_{ks}$ when $k \neq j$ implies $\mathbb{E}[z'_{ijs}a_{js}a_{is}\dot{X}^i_{js}] = \mathbb{E}[z'_{ijs}\dot{X}^i_{js}\mathbb{E}[a_{js}a_{is}|z_{ijs}, \dot{X}^i_{js}] = 0$, and similarly $\mathbb{E}[z'_{ijs}a_{js}a_{is}\dot{a}^i_{js}] = 0$. Next,

$$\begin{aligned}\mathbb{E}[z_{ijs}a_{js}X_{is}\dot{a}^i_{js}] &= \mathbb{E}[z'_{ijs}a^2_{js}X_{is}] - \sum_{k\neq i}\mathbb{E}[z'_{ijs}a_{ks}X_{is}] \\ &= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}a^2_{js}X_{is}] - \sum_{k\neq i, k\neq j}\mathbb{E}[z'_{ijs}a_{ks}a_{js}X_{is}] \\ &= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}a^2_{js}X_{is}] \\ &= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}X_{is}]\sigma_a \end{aligned} \tag{A.66}$$

where $\sigma_a$ is the variance of the scalar unobservable $a_{is}$. Similarly,

$$\begin{aligned}[z_{ijs}a_{js}\dot{a}^i_{js}] &= \mathbb{E}[z'_{ijs}a^2_{js}] - \sum_{k\neq i}\mathbb{E}[z'_{ijs}a_{ks}] \\ &= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}a^2_{js}] - \sum_{k\neq i, k\neq j}\mathbb{E}[z'_{ijs}a_{ks}a_{js}] \\ &= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}a^2_{js}] \\ &= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}]\sigma_a \end{aligned} \tag{A.67}$$

Combining these results and substituting into Equation (A.65), now

$$\begin{aligned}\mathbb{E}[z'_{ijs}a_{is}\dot{g}^i_{ijs}] &= \mathbb{E}[z'_{ijs}a_{is}\dot{g}^i_{jis}]\tilde{\beta} + \frac{N-2}{N-1}\sigma_a(\mathbb{E}[z'_{ijs}X_{is}]\delta_2 + \mathbb{E}[z'_{ijs}]\gamma_2) \\ &= \mathbb{E}[z'_{ijs}a_{is}\dot{g}^i_{jis}]\tilde{\beta} + \frac{N-2}{N-1}\sigma_a\mathbb{E}[z'_{ijs}b^1_{ijs}]\begin{bmatrix}\delta_2 \\ \gamma_2\end{bmatrix} \end{aligned} \tag{A.68}$$

Next, assume there exists $\theta_1 = (\tilde{\beta}, \delta_2, \gamma_2)$ and $\theta'_1 = (\tilde{\beta}', \delta'_2, \gamma'_2)$. Further, let $\sigma^2_a$ and $(\sigma'_a)^2$ both be finite.

From Equation (A.68), it must be true that

$$0 = \mathbb{E}[z'_{ijs} a_{is} \dot{g}^i_{jis}](\tilde{\beta} - \tilde{\beta}') + \frac{N-2}{N-1}\left(\sigma_a^2 \mathbb{E}[z'_{ijs} b^1_{ijs}] \begin{bmatrix} \delta_2 \\ \gamma_2 \end{bmatrix} - (\sigma'_a)^2 \mathbb{E}[z'_{ijs} b^1_{ijs}] \begin{bmatrix} \delta'_2 \\ \gamma'_2 \end{bmatrix}\right) \tag{A.69}$$

From above, $\tilde{\beta}$ is identified, and thus $(\tilde{\beta} - \tilde{\beta}') = 0$. Therefore,

$$0 = \mathbb{E}[z'_{ijs} b^2_{ijs}] \left(\sigma_a^2 \begin{bmatrix} \delta_2 \\ \gamma_2 \end{bmatrix} - (\sigma_a)^2 \begin{bmatrix} \delta'_2 \\ \gamma'_2 \end{bmatrix}\right) \tag{A.70}$$

The second rank condition implies that there exists some $(m+1) \times l$ matrix $\mathbf{A_1}$ such that $\mathbf{A_1}\mathbb{E}[z'_{ijs} b^2_{ijs}]$ is of rank $2m$. Therefore, $(\mathbf{A_1}\mathbb{E}[z'_{ijs} b^1_{ijs}])^{-1}$ exists and

$$0 = \left(\sigma_a^2 \begin{bmatrix} \delta_2 \\ \gamma_2 \end{bmatrix} - (\sigma_a)^2 \begin{bmatrix} \delta'_2 \\ \gamma'_2 \end{bmatrix}\right) \tag{A.71}$$

Accordingly, $\delta_2$ and $\gamma_2$ are identified up to the scale factor $\sigma_a^2$.

*Step 2: Scale identification of* $\delta_3$ Multiply Equation (19) by $z'_{ijs} a_{is}$. So,

$$\begin{aligned} z'_{ijs} a_{is} \dot{g}^i_{ijs} &= z'_{ijs} a_{is} \dot{g}^i_{jis} \tilde{\beta} + z'_{ijs} a_{is} X_{is} \dot{X}^i_{js} \delta_1 + z'_{ijs} a_{is} X_{is} \dot{a}^i_{js} \delta_2 + z'_{ijs} a^2_{is} \dot{X}^i_{js} \delta_3 \\ &\quad + z'_{ijs} a^2_{is} \dot{a}^i_{js} \delta_4 + z'_{ijs} a_{is} \dot{X}^j_{is} \gamma_3 + z'_{ijs} a_{is} \dot{a}^i_{js} \gamma_4 - z'_{ijs} a_{is} \dot{c}^i_{ijs} \end{aligned} \tag{A.72}$$

Next, take the mean over all $SN(N-1)$ observations. So,

$$\begin{aligned} \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ijs} a_{is} \dot{g}^i_{ijs} &= \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} (z'_{ijs} a_{is} \dot{g}^i_{jis} \tilde{\beta} \\ &\quad + z'_{ijs} a_{is} X_{is} \dot{X}^i_{js} \delta_1 + z'_{ijs} a_{is} X_{is} \dot{a}^i_{js} \delta_2 + z'_{ijs} a^2_{is} \dot{X}^i_{js} \delta_3 + z'_{ijs} a^2_{is} \dot{a}^i_{js} \delta_4 \\ &\quad + z'_{ijs} a_{is} \dot{X}^j_{is} \gamma_3 + z'_{ijs} a_{is} \dot{a}^i_{js} \gamma_4 - z'_{ijs} a_{is} \dot{c}^i_{ijs}) \end{aligned} \tag{A.73}$$

By the same argument as in Step 1, $\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ijs} a_{is} \dot{g}^i_{ijs} \to_p \mathbb{E}[z'_{ijs} a_{is} \dot{g}^i_{ijs}]$, etc. So, replace the matrices in Equation (A.73) with their probability limits.

$$\mathbb{E}[z'_{ijs} a_{is} \dot{g}^i_{ijs}] = \mathbb{E}[z'_{ijs} a_{is} \dot{g}^i_{jis}]\tilde{\beta} + \mathbb{E}[z'_{ijs} a_{is} X_{is} \dot{X}^i_{js}]\delta_1 + \mathbb{E}[z'_{ijs} a_{is} X_{is} \dot{a}^i_{js}]\delta_2 \tag{A.74}$$

$$+ \mathbb{E}[z'_{ijs} a^2_{is} \dot{X}^i_{js}]\delta_3 + \mathbb{E}[z'_{ijs} a^2_{is} \dot{a}^i_{js}]\delta_4 + \mathbb{E}[z'_{ijs} a_{is} \dot{X}^j_{is}]\gamma_3 + \mathbb{E}[z'_{ijs} a_{is} \dot{a}^i_{js}]\gamma_4 - \mathbb{E}[z'_{ijs} a_{is} \dot{c}^i_{ijs}] \tag{A.75}$$

By Assumption 6, $\mathbb{E}[z'_{ijs}a_{is}\dot{c}^{i}_{ijs}] = 0$. Further, by L.I.E., $\mathbb{E}[z_{ijs}a_{is}X_{is}\dot{X}^{i}_{js}]$ $= \mathbb{E}[z'_{ijs}X_{is}\dot{X}^{i}_{js}\mathbb{E}[a_{is}|z_{ijs}, X_{is}, \dot{X}^{i}_{js}]] = 0$ and similarly $\mathbb{E}[z_{ijs}a_{is}\dot{X}^{i}_{js}] = 0$. Independence of $a_{is}$ and $a_{js}$ from each other and from $X$ implies $\mathbb{E}[z_{ijs}a_{is}X_{is}\dot{a}^{i}_{js}]$ $= \mathbb{E}[z'_{ijs}X_{is}\mathbb{E}[\dot{a}^{i}_{js}\mathbb{E}[a_{is}|\dot{a}^{i}_{js}, X_{is}, z_{ijs}]|X_{is}, z_{ijs}]] = 0$, and by similar logic $\mathbb{E}[z'_{ijs}a_{is}\dot{a}_{js}] = 0$. Further,

$$\begin{aligned}
\mathbb{E}[z'_{ijs}a^2_{is}\dot{a}^{i}_{js}] &= \mathbb{E}[z'_{ijs}a^2_{is}a_{js}] - \sum_{k\neq i}\mathbb{E}[z'_{ijs}a^2_{is}a_{ks}] \\
&= \mathbb{E}[z'_{ijs}a^2_{is}\mathbb{E}[a_{js}|a_{is}]] - \sum_{k\neq i}\mathbb{E}[z'_{ijs}a^2_{is}\mathbb{E}[a_{ks}|a_{is}]] \\
&= 0 \qquad \text{(A.76)}
\end{aligned}$$

From Assumption 8, it follows that $\mathbb{E}[z'_{ijs}a^2_{is}\dot{X}^{i}_{js}] = \mathbb{E}[z'_{ijs}\dot{X}^{i}_{js}]\sigma^2_a$, where $\sigma^2_a$ is the variance of $a_{is}$. Now, substittuion of these results into Equation (A.75) yields

$$\mathbb{E}[z'_{ijs}a_{is}\dot{g}^{i}_{ijs}] = \mathbb{E}[z'_{ijs}a_{is}\dot{g}^{i}_{jis}]\tilde{\beta} + \mathbb{E}[z'_{ijs}\dot{X}^{i}_{js}]\sigma^2_a\delta_3 \qquad \text{(A.77)}$$

Now, assume there exists some parameter vector $\theta_2 = (\tilde{\beta}, \gamma_3)$ and $\theta'_2 = (\tilde{\beta}', \gamma'_3)$. These vectors are associated with finite $\sigma^2_a$ and $(\sigma_a)^2$. So,

$$0 = \mathbb{E}[z'_{ijs}a_{is}\dot{g}^{i}_{jis}](\tilde{\beta} - \tilde{\beta}') + \mathbb{E}[z'_{ijs}\dot{X}^{i}_{js}](\sigma^2_a\delta_3 - (\sigma^2_a)'\delta'_3) \qquad \text{(A.78)}$$

Identification of $\tilde{\beta}$ implies $\tilde{\beta} = \tilde{\beta}'$. So,

$$0 = \mathbb{E}[z'_{ijs}\dot{X}^{i}_{js}](\sigma^2_a\delta_3 - (\sigma^2_a)'\delta'_3) \qquad \text{(A.79)}$$

The third rank condition further implies that there exists some $mxl$ matrix $\mathbf{A_2}$ such that $\mathbf{A_2}\mathbb{E}[z'_{ijs}\dot{X}^{i}_{js}]$ is of full rank $m$. Therefore, $(\mathbf{A_2}\mathbb{E}[z'_{ijs}\dot{X}^{i}_{js}])^{-1}$ exists. So,

$$0 = \sigma^2_a\delta_3 - (\sigma^2_a)'\delta'_3 \qquad \text{(A.80)}$$

Accordingly, the parameter vector $\delta_3$ is identified up to the scale factor $\sigma^2_a$.

*Step 3: Scale identification of $\delta_4$*

Finally, multiply Equation (19) by $z'_{ijs}a_{is}a_{js}$. So,

$$z'_{ijs}a_{is}a_{js}\dot{g}^i_{ijs} = z'_{ijs}a_{is}a_{js}\dot{g}^i_{jis}\tilde{\beta} + z'_{ijs}a_{is}a_{js}X_{is}\dot{X}^i_{js}\delta_1 + z'_{ijs}a_{is}a_{js}X_{is}\dot{a}^i_{js}\delta_2 \tag{A.81}$$

$$+ z'_{ijs}a_{is}a^2_{js}\dot{X}^i_{js}\delta_3 + z'_{ijs}a^2_{is}a_{js}\dot{a}^i_{js}\delta_4 + z'_{ijs}a_{is}a_{js}\dot{X}^j_{is}\gamma_3 + z'_{ijs}a_{is}a_{js}\dot{a}^i_{js}\gamma_4$$
$$- z'_{ijs}a_{is}a_{js}\dot{c}^i_{ijs} \tag{A.82}$$

Next, take the mean over all $SN(N-1)$ observations. So,

$$\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} z'_{ijs}a_{is}a_{js}\dot{g}^i_{ijs} = \frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i}(z'_{ijs}a_{is}a_{js}\dot{g}^i_{jis}\tilde{\beta}$$
$$+ z'_{ijs}a_{is}a_{js}X_{is}\dot{X}^i_{js}\delta_1 + z'_{ijs}a_{is}a_{js}X_{is}\dot{a}^i_{js}\delta_2 + z'_{ijs}a_{is}a^2_{js}\dot{X}^i_{js}\delta_3$$
$$+ z'_{ijs}a^2_{is}a_{js}\dot{a}^i_{js}\delta_4 + z'_{ijs}a_{is}a_{js}\dot{X}^j_{is}\gamma_3 + z'_{ijs}a_{is}a_{js}\dot{a}^i_{js}\gamma_4$$
$$- z'_{ijs}a_{is}a_{js}\dot{c}^i_{ijs}) \tag{A.83}$$

By the same argument as in Step 1, $\frac{1}{SN(N-1)}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\neq i} z'_{ijs}a_{is}\dot{g}^i_{ijs} \to_p \mathbb{E}[z'_{ijs}a_{is}\dot{g}^i_{ijs}]$, etc. So, replace the matrices in Equation (A.83) with their probability limits, yielding

$$\mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{g}^i_{ijs}] = \mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{g}^i_{jis}]\tilde{\beta} + \mathbb{E}[z'_{ijs}a_{is}a_{js}X_{is}\dot{X}^i_{js}]\delta_1 + \mathbb{E}[z'_{ijs}a_{is}a_{js}X_{is}\dot{a}^i_{js}]\delta_2$$
$$+ \mathbb{E}[z'_{ijs}a_{is}a^2_{js}\dot{X}^i_{js}]\delta_3 + \mathbb{E}[z'_{ijs}a^2_{is}a_{js}\dot{a}^i_{js}]\delta_4 + \mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{X}^j_{is}]\gamma_3$$
$$+ \mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{a}^i_{js}]\gamma_4 - \mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{c}^i_{ijs}] \tag{A.84}$$

Assumption 6 implies $\mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{c}^i_{ijs}] = \mathbb{E}[z'_{ijs}a_{is}a_{js}\mathbb{E}[\dot{c}^i_{ijs}|z_{ijs}, a_{is}, a_{js}]] = 0$. Application of Assumption 6 and L.I.E. together imply $\mathbb{E}[z'_{ijs}a_{is}a_{js}X_{is}\dot{X}^i_{js}]$, $\mathbb{E}[z'_{ijs}a_{is}a_{js}X_{is}\dot{a}^i_{js}]$, $\mathbb{E}[z'_{ijs}a_{is}a^2_{js}\dot{X}^i_{js}]$, $\mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{X}^j_{is}]$, and $\mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{a}^i_{js}]$ are also zero. Further,

$$\begin{aligned}
\mathbb{E}[z'_{ijs}a^2_{is}a_{js}\dot{a}^i_{js}] &= \mathbb{E}[z'_{ijs}a^2_{is}a^2_{js}] - \sum_{k\neq i}\mathbb{E}[z'_{ijs}a^2_{is}a_{js}a_{ks}] \\
&= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}a^2_{is}a^2_{js}] - \sum_{k\neq i,k\neq j}\mathbb{E}[z'_{ijs}a^2_{is}a_{js}a_{ks}] \\
&= \frac{N-2}{N-1}\mathbb{E}[z'_{ijs}a^2_{is}a^2_{js}] \\
&= \frac{N-2}{N-1}(\sigma^2_a)^2\mathbb{E}[z'_{ijs}]
\end{aligned} \tag{A.85}$$

Substitution into Equation (A.83) yields

$$\mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{g}^i_{ijs}] = \mathbb{E}[z'_{ijs}a_{is}a_{js}\dot{g}^i_{jis}]\tilde{\beta} + \frac{n-2}{n-1}(\sigma^2_a)^2\mathbb{E}[z'_{ijs}]\delta_4 \tag{A.86}$$

Assume there exist parameter vectors $\theta_3 = (\tilde{\beta}, \delta_4)$ and $\theta_3' = (\tilde{\beta}', \delta_4')$, with associated $\sigma_a^2$ and $(\sigma_a')^2$. Equation (A.86) thus implies that

$$0 = \mathbb{E}[z_{ijs}' a_{is} a_{js} \dot{g}_{jis}^i](\tilde{\beta} - \tilde{\beta}') + \frac{n-2}{n-1}(\sigma_a^2)^2 \mathbb{E}[z_{ijs}'](\delta_4 - \delta_4') \tag{A.87}$$

Identification of $\beta$ implies $(\tilde{\beta} - \tilde{\beta}') = 0$. Further, the fourth rank condition implies that there exists some $1 \times l$ matrix $\mathbf{A_3}$ such that $\mathbf{A_3}\mathbb{E}[z_{ijs}']$ is of rank 1. Therefore, $0 = (\sigma_a^2)^2\delta_4 - ((\sigma_a')^2)^2\delta_4'$, and $\delta_4$ is identified to up to the scale factor $\sigma_a^2$.

*Q.E.D.*

*Proposition 7*

The prior propositions have provided conditions under which $\tilde{\beta}$, $\delta$, and $\gamma$ are identified. So, I proceed under the assumption that these parameters are identified. I now show that, conditional on these parameters being identified, $a_{js}$ is identified for all $j$ as $s \to \infty$.

First, for any $i, j, k$,

$$\begin{aligned}(\tilde{g}_{ijs} - \tilde{g}_{iks}) - \tilde{\beta}(\tilde{g}_{jis} - \tilde{g}_{kis}) &= \delta_1 X_{is}(X_{js} - X_{ks}) + \delta_2 X_{is}(A_{js} - A_{ks}) \\ &+ \delta_3 A_{is}(X_{js} - X_{ks}) + \delta_4 A_{is}(A_{js} - A_{ks}) \\ &+ \gamma_3(X_{js} - X_{ks}) + \gamma_4(A_{js} - A_{ks}) - (\tilde{c}_{ijs} - \tilde{c}_{jis})\end{aligned} \tag{A.88}$$

Since every element on the right-hand side of Equation (A.88) is bounded, $(\tilde{g}_{ijs} - \tilde{g}_{iks}) - \tilde{\beta}(\tilde{g}_{jis} - \tilde{g}_{kis})$ is also bounded. Therefore, it has finite variance. Note further that it does not depend on $N$. Note that $\frac{1}{N-1}\sum_{k \neq i}\Big((\tilde{g}_{ijs} - \tilde{g}_{iks}) - \tilde{\beta}(\tilde{g}_{jis} - \tilde{g}_{kis})\Big) = \dot{g}_{ijs}^i - \tilde{\beta}\dot{g}_{jis}^i$.

Summing over $i \neq j$ and with slight rearrangement of Equation (19), for any $j$, we thus see

$$\begin{aligned}\frac{1}{(N-1)}\sum_{i \neq j}(\dot{g}_{ijs}^i - \tilde{\beta}\dot{g}_{jis}^i) &= \frac{1}{(N-1)}\sum_{i \neq j}(\delta_1 X_{is}\dot{X}_{js}^i + \delta_2 X_{is}\dot{a}_{js}^i + \delta_3 a_{is}\dot{X}_{js}^i + \delta_4 a_{is}\dot{a}_{js}^i \\ &+ \gamma_3 \dot{X}_{is}^j + \gamma_4 \dot{a}_{js}^i - \dot{c}_{ijs}^i)\end{aligned} \tag{A.89}$$

Finite variance and independence implies that $\frac{1}{(N-1)}\sum_{i \neq j}\Big(\dot{g}_{ijs}^i - \tilde{\beta}\dot{g}_{jis}^i\Big)$ $= \underset{i \neq j}{\mathbb{E}}[\Big(\dot{g}_{ijs}^i - \tilde{\beta}\dot{g}_{jis}^i\Big)] + o_p(1)$ for any $j$. Similarly,

- $\frac{1}{(N-1)}\sum_{i \neq j} X_{is}\dot{X}_{js}^i = X_{js}\underset{i \neq j}{\mathbb{E}}[X_{is}] - \underset{i \neq j}{\mathbb{E}}[X_{is}^2] + o_p(1)$
- $\frac{1}{(N-1)}\sum_{i \neq j} X_{is}\dot{a}_{js}^i = a_{js}\underset{i \neq j}{\mathbb{E}}[X_{is}] + o_p(1)$
- $\frac{1}{(N-1)}\sum_{i \neq j} a_{is}\dot{X}_{js}^i = o_p(1)$

- $\frac{1}{(N-1)}\sum_{i\neq j} a_{is}\dot{a}^i_{js} = o_p(1)$
- $\frac{1}{(N-1)}\sum_{i\neq j} \dot{X}^i_{js} = X_{js} - \underset{i\neq j}{\mathbb{E}}[X_{is}] + o_p(1)$
- $\frac{1}{(N-1)}\sum_{i\neq j} \dot{a}^i_{js} = a_{js} + o_p(1)$
- $\frac{1}{(N-1)}\sum_{i\neq j} \dot{c}^i_{ijs} = o_p(1)$

Therefore, in the limit, Equation (A.89) becomes

$$\begin{aligned}\underset{i\neq j}{\mathbb{E}}[\dot{g}^i_{ijs} - \tilde{\beta}\dot{g}^i_{jis}] = \delta_1(X_{js}\underset{i\neq j}{\mathbb{E}}[X_{is}] - \underset{i\neq j}{\mathbb{E}}[X^2_{is}]) + \delta_2 a_{js}\underset{i\neq j}{\mathbb{E}}[X_{is}] + \gamma_3(X_{js} - \underset{i\neq j}{\mathbb{E}}[X_{is}])\\ + \gamma_4 a_{js} + o_p(1)\end{aligned} \tag{A.90}$$

Rearrangement yields

$$\begin{aligned}a_{js}(\gamma_4 + \delta_2\underset{i\neq j}{\mathbb{E}}[X_{is}]) = \underset{i\neq j}{\mathbb{E}}[\dot{g}^i_{ijs} - \tilde{\beta}\dot{g}^i_{jis}] - \delta_1(X_{js}\underset{i\neq j}{\mathbb{E}}[X_{is}] - \underset{i\neq j}{\mathbb{E}}[X^2_{is}])\\ - \gamma_3(X_{js} - \underset{i\neq j}{\mathbb{E}}[X_{is}]) + o_p(1)\end{aligned} \tag{A.91}$$

Now, suppose there exist $a'_{js} \neq a_{js}$. From Equation (A.91), we see that $(a'_{js} - a_{js})(\gamma_4 + \delta_2\underset{i\neq j}{\mathbb{E}}[X_{is}]) = o_p(1)$. Therefore, $(\gamma_4 + \delta_2\underset{i\neq j}{\mathbb{E}}[X_{is}]) \neq 0 \Rightarrow (a'_{js} - a_{js}) = o_p(1)$ and thus $a_{js}$ is point identified.

## APPENDIX B: SUPPLEMENTARY TABLES AND FIGURES

TABLE A.1
Baseline Balance Across Schools

| | Elected Treatment | Random Treatment | Control | P-value of Balance Test |
|---|---|---|---|---|
| *Panel A: Baseline Covariates* | | | | |
| Elected | 0.321 | 0.291 | 0.240 | 0.603 |
| | (0.056) | (0.037) | (0.061) | |
| Grade 7 | 0.331 | 0.311 | 0.315 | 0.817 |
| | (0.016) | (0.041) | (0.029) | |
| Grade 8 | 0.346 | 0.272 | 0.303 | 0.237 |
| | (0.030) | (0.034) | (0.016) | |
| SC | 0.195 | 0.267 | 0.285 | 0.572 |
| | (0.057) | (0.068) | (0.084) | |
| ST | 0.118 | 0.175 | 0.073 | 0.353 |
| | (0.033) | (0.082) | (0.028) | |
| OBC | 0.459 | 0.459 | 0.423 | 0.939 |
| | (0.063) | (0.069) | (0.092) | |
| *Panel B: Baseline Outcomes* | | | | |
| Education Aspirations | -0.217 | -0.146 | -0.201 | 0.892 |
| | (0.144) | (0.105) | (0.096) | |
| Gender Roles | 0.135 | 0.042 | 0.186 | 0.759 |
| | (0.171) | (0.183) | (0.086) | |

Notes: Robust standard errors in parentheses, clustered by school. Sample is 1319 students in 30 schools.

TABLE A.2
Baseline Balance Within Random Treatment Schools

| | Participant | Non-Participant | P-value of Balance Test |
|---|---|---|---|
| *Panel A: Baseline Covariates* | | | |
| Elected | 0.362 | 0.260 | 0.135 |
| | (0.055) | (0.037) | |
| Grade 7 | 0.244 | 0.340 | 0.060 |
| | (0.055) | (0.040) | |
| Grade 8 | 0.339 | 0.242 | 0.204 |
| | (0.071) | (0.032) | |
| SC | 0.283 | 0.260 | 0.779 |
| | (0.084) | (0.074) | |
| ST | 0.205 | 0.161 | 0.335 |
| | (0.102) | (0.074) | |
| OBC | 0.433 | 0.470 | 0.620 |
| | (0.091) | (0.068) | |
| *Panel B: Baseline Outcomes* | | | |
| Educational Aspirations | -0.083 | -0.174 | 0.489 |
| | (0.142) | (0.111) | |
| Gender Roles | 0.006 | 0.058 | 0.706 |
| | (0.169) | (0.202) | |

Notes: Robust standard errors in parentheses, clustered by school. Sample is 412 students in 10 Random Treatment schools.

TABLE A.3
BASELINE OUTCOME HETEROGENEITY

| | Educational Aspirations (1) | Gender Roles (2) |
|---|---|---|
| Elected | 0.168** | -0.042 |
| | (0.078) | (0.073) |
| Grade 7 | 0.024 | 0.125* |
| | (0.080) | (0.070) |
| Grade 8 | 0.077 | 0.172** |
| | (0.099) | (0.084) |
| Scheduled Caste | -0.234* | -0.337*** |
| | (0.118) | (0.120) |
| Scheduled Tribe | -0.185 | -0.544*** |
| | (0.118) | (0.111) |
| Other Backwards Caste | -0.284*** | -0.078 |
| | (0.087) | (0.102) |
| Constant | -0.068 | 0.219** |
| | (0.092) | (0.080) |
| R-squared | 0.018 | 0.040 |

Notes: N = 1,319 in 30 schools in all specifications. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1.

TABLE A.4
Network Size and Compementarity

*Panel A: Relationship between School Size and Link Count*

| Network Definition | Binary | Continuous |
|---|---|---|
| | (1) | (2) |
| School Size | 0.576*** | 0.392*** |
| | (0.030) | (0.029) |
| Constant | 20.988*** | 8.448*** |
| | (2.913) | (1.510) |
| R-squared | 0.481 | 0.642 |

*Panel B: Relationship between School Size and Link Value*

| Network Definition | Binary | Continuous |
|---|---|---|
| | (1) | (2) |
| School Size | -0.002*** | -0.004*** |
| | (0.000) | (0.001) |
| Constant | 0.684*** | 1.289*** |
| | (0.021) | (0.082) |
| R-squared | 0.024 | 0.035 |

*Panel C: Relationship between In- and Out-Link Values*

| Network Definition | Binary | Continuous |
|---|---|---|
| | (1) | (2) |
| In-Link Value | 0.129*** | 0.224*** |
| | (0.016) | (0.039) |
| Constant | 0.471*** | 0.729*** |
| | (0.033) | (0.042) |
| R-squared | 0.017 | 0.050 |

Notes: N = 1,319 in 30 schools in Panel A, N = 78,238 in 30 schools in Panels B and C. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Dependent variable for Panel A is sum (or count) of links under appropriate definition. Dependent variable for Panels B and C is value of out-link under appropriate definition. Unit of observation is individual student in Panel A, dyad (pair of students) in Panels B and C.

TABLE A.5
DEFINING PREDICTED OUTCOME TERCILES

| | Educational Aspirations (1) | Gender Roles (2) |
|---|---|---|
| Elected | 0.039 | -0.009 |
| | (0.113) | (0.058) |
| Grade 7 | 0.181 | 0.087 |
| | (0.203) | (0.196) |
| Grade 8 | 0.106 | 0.166 |
| | (0.189) | (0.164) |
| Scheduled Caste | -0.509** | -0.165 |
| | (0.168) | (0.161) |
| Scheduled Tribe | -0.517* | -0.804 |
| | (0.229) | (0.484) |
| Other Backwards Caste | -0.359 | -0.272*** |
| | (0.208) | (0.059) |
| Baseline Outcome | 0.302*** | 0.027 |
| | (0.059) | (0.057) |
| Constant | 0.288* | 0.222 |
| | (0.145) | (0.175) |
| Observations | 393 | 395 |
| R-squared | 0.164 | 0.052 |

Notes: Estimation restricted to Control schools. Robust standard errors in parentheses, clustered by school. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

TABLE A.6
Treatment Effect Heterogeneity by Predicted Outcome Tercile

| | Education | | Gender Roles | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $\hat{L}$ | -0.417** | 0.061 | -0.021 | -0.018 |
| | (0.191) | (0.178) | (0.152) | (0.149) |
| $\hat{M}$ | -0.064 | -0.057 | 0.124* | 0.107 |
| | (0.098) | (0.093) | (0.069) | (0.074) |
| $\hat{H}$ | 0.346*** | 0.251*** | 0.108 | 0.038 |
| | (0.054) | (0.056) | (0.099) | (0.116) |
| Participant in Random Treat × $\hat{L}$ | -0.323 | -0.422 | -0.048 | -0.029 |
| | (0.277) | (0.250) | (0.215) | (0.217) |
| Participant in Random Treat × $\hat{M}$ | -0.090 | -0.112 | -0.610*** | -0.582*** |
| | (0.127) | (0.124) | (0.168) | (0.158) |
| Participant in Random Treat × $\hat{H}$ | 0.139 | 0.120 | 0.135 | 0.114 |
| | (0.201) | (0.205) | (0.327) | (0.291) |
| Non-Participant in Random Treat × $\hat{L}$ | -0.061 | -0.110 | 0.038 | 0.049 |
| | (0.198) | (0.177) | (0.183) | (0.183) |
| Non-Participant in Random Treat × $\hat{M}$ | -0.035 | -0.074 | 0.004 | 0.006 |
| | (0.126) | (0.119) | (0.214) | (0.202) |
| Non-Participant in Random Treat × $\hat{H}$ | -0.090 | -0.098 | -0.176 | -0.188 |
| | (0.146) | (0.144) | (0.238) | (0.209) |
| Baseline Outcome Interactions | NO | YES | NO | YES |
| R-squared | 0.120 | 0.147 | 0.024 | 0.042 |
| Test 1 P-value | 0.354 | 0.209 | 0.009 | 0.011 |
| Test 2 P-value | 0.922 | 0.978 | 0.587 | 0.492 |

Notes: Estimation restricted to Random Treatment and Control. N = 920 in 20 schools in all specifications. Omitted category is all girls in Control. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. $\hat{L}$, $\hat{M}$, and $\hat{H}$ predicted from baseline variables (see Table A.5). Baseline Outcome Interactions include interactions of Baseline Outcome with $\hat{L}$, $\hat{M}$, and $\hat{H}$. Test 1 is a test of equality of the interactions of $\hat{L}$, $\hat{M}$, and $\hat{H}$ with Participant in Random Treat. Test 2 is a test of equality of the interactions of $\hat{L}$, $\hat{M}$, and $\hat{H}$ with Non-Participant in Random Treat.

## APPENDIX C: WEIGHTING IN THE CONSTRUCTION OF PEER MEANS

In the standard setting with binary directed link data, peer weighting is a near-trivial matter and thus construction of peer means is fairly straightforward. In a binary setting, there are four obvious link definitions between individuals $i$ and $j$:

1. An "OUT" link exists if individual $i$ indicates that $j$ is a friend.
2. An "IN" link exists if individual $j$ indicates that $i$ is a friend.
3. An "OR" link exists if either an "OUT" link or an "IN" link exists.
4. An "AND " link exists if both an "OUT" link and an "IN" link exist.

Note that the first two are necessarily directed, while the third and fourth are symmetric. For purposes of the reduced-form analysis in this paper, and to be consistent with the continuous results, I employ the "OUT" definition for binary network links.

Peer weighting is much more complicated when link intensities are continuous, as posited in the structural model developed in this paper. The following general assumptions on all weights will be maintained throughout. While in principle these weights could be estimated, in order to preserve computational power, I assume that the function is known. Letting $g_{ijs}$ be the intensity of $i$'s link toward $j$, and $g_{jis}$ be the intensity of $j$'s link toward $i$, the following three definitions seem natural

1. "OUT" link weight is $g_{ijs}$
2. An "IN" link weight is $g_{jis}$
3. An "SUM" link weight is $g_{ijs} + g_{jis}$

Once these weight are constructed, they are normalized so that the sum of the weights for a given individual $i$ is one. For purposes of this paper, I employ the "SUM" weight definition for continuous link intensities. Future work will investigate the sensitivity of results to a choice of different weighting functions.

## APPENDIX D: MISSING DATA ALGORITHM

### *D.1: Network Data*

Data imputation requires a model. As briefly discussed in the main body of text, given that I have a model of network formation, I use this model to impute missing network data. Imputation proceeds via an iterative EM algorithm. The algorithm proceeds as follows:

1. For the continuous network measure $g_{ij}$, impute missing data arbitrarily.
2. Using the imputed data, estimate the parameters of the network formation model. Recover moments of distributions of unobserved $a_{is}$, $M_{is}$ and $c_{ijs}$
3. Using the implied distributions of the unobserved variables $a_{is}$, $M_{is}$ and $c_{ijs}$, impute missing data. This step requires iteration of the network-formation process until an equilibrium consistent with the First-Order Conditions is reached.
4. Iterate Steps 2 and 3 sufficiently to reach convergence to a stable distribution of parameters and networks.
5. Take draws from this stable distribution. Construct point and variance estimates that properly adjust for imputation error. This adjustment is discussed in Cameron and Trivedi (2005) Section 27.7.

### *D.2: Outcome Data*

Equation (4) provides the model's equation whereby outcomes are determined conditional on networks, observed variables, and unobserved $a_{is}$. Data imputation here proceeds from the imputed full networks as follows:

1. Take $m$ draws from the converged distribution of networks and estimated parameters $a_{is}$.
2. For each draw

   2.1 Arbitrarily impute missing outcome data.

   2.2 Using imputed outcomes as well as the draw of networks and $a_{is}$, estimate the parameters of Equation (4).

   2.3 Using implied distribution of residuals from Step 2.2, impute outcome data where missing

   2.4 Iterate Steps 2.2 and 2.3 sufficient to reach convergence to stationary distribution. Take one draw from this distribution.

3. Given the final parameter values in Step 2.4, construct point and variance estimates that properly adjust for imputation error as well as error in estimating $a_{is}$.

## APPENDIX E: PEER EFFECTS MODEL RELATION TO LATENT SPACE AND CONTROL FUNCTION APPROACHES

The peer effects model here combines two approaches that have received substantial attention in the statistics and econometrics literature. First, I specify arbitrary latent characteristics $a_{is}$ that must be accounted for. Second, conditional on these latent characteristics, the model posits a parametric control function approach. These twin approaches allow for identification of the parameters of the peer effects model in the presence of certain types of network endogeneity.

First, I posit $a_{is}$ as an unobserved, "latent" characteristic, making this model similar to the "latent space" models summarized in Jackson (2014). Such models posit that unobserved "latent" characteristics play a part in the process being modeled. As Jackson (2014) discusses, a key feature of such models is that the latent characteristic may be *any* unobserved—and possibly difficult-to-measure—characteristic, such as "ability" or "ambition."[59] Goldsmith-Pinkham and Imbens (2013) model the latent characteristic as a single binary variable, while Hsieh and Lee (2016) allow for continuous multi-dimensional unobservables.

Second, conditional on these latent characteristics $a_{is}$, identification of the model's parameters is achieved via a parametric control function approach. Rearrangement of Equation (4) shows this.

$$y_{is} = \sum_{k=1}^{K} I_{isk}\left(\alpha_{0k} + \alpha_{1k}P_{is} + \alpha_{2k}\bar{P}_{is}\right) + \sum_{k=1}^{K} I_{isk}\left(\alpha_{3k}a_{is} + \alpha_{4k}\bar{a}_{is}\right) + v_{is} \tag{A.92}$$

$$y_{is} = \sum_{k=1}^{K} I_{isk}\left(\alpha_{0k} + \alpha_{1k}P_{is} + \alpha_{2k}\bar{P}_{is}\right) + f(a_{is}, \bar{a}_{is}, z_{is}) + v_{is} \tag{A.93}$$

$$y_{is} = \sum_{k=1}^{K} I_{isk}\left(\alpha_{0k} + \alpha_{1k}P_{is} + \alpha_{2k}\bar{P}_{is}\right) + u_{is} \tag{A.94}$$

The control function is $f(a_{is}, \bar{a}_{is}, Z_{is})$ in Equation (A.93). Endogeneity arises because $P_{is}$ and $\bar{P}_{is}$ may depend on $a_{is}$ and $\bar{a}_{is}$. This implies correlation between these regressors and $u_{is}$, leading to biased estimates of $\alpha_{1k}$ and $\alpha_{2k}$ if estimating Equation (A.94). On the other hand, estimating the control function $f$ in Equation (A.93) allows for identification in the presence of this endogeneity. That is, the parameters of the model are identified under strictly weaker exogeneity assumptions than are typically assumed in the literature. For example, Carrell, Sacerdote and West (2013) effectively assume exogeneity of $u_{is}$ (and thus of $a_{is}$ and $\bar{a}_{is}$) in Equation (A.94), while the method here only requires the exogeneity of $v_{is}$ in Equation

[59] "Latent space" models have also been heavily used in industrial organization, for example in Berry, Levinsohn and Pakes (1995), in their pathbreaking methodology for demand estimation.

(A.93).

Identification in the presence of endogeneity via control functions has found wide application in applied econometrics, and the model here follows in this tradition. As pointed out by Bramoullé (2013), the model in Goldsmith-Pinkham and Imbens (2013) is similar in spirit to the canonical Heckman selection model (Heckman, 1979), which itself uses a parametric control function approach to identification. Employment of control functions to account for unobserved heterogeneity has also found widespread application in industrial organization, particularly in the estimation of production functions (Ackerberg, Caves and Frazer, 2015; Levinsohn and Petrin, 2003; Olley and Pakes, 1996).

## APPENDIX F: MEASUREMENT ERROR AND PRINCIPAL COMPONENTS

### *F.1: Measurement Error Model Derivation*

As a further justification for the use of the first principal component as a measure of connectedness, I employ results analogous to those in Black and Smith (2006). Suppose there is some (unobserved) true measure of individual $i$'s connectedness to $j$. Call this measure $g^*_{ijs}$. Instead of observing $g^*_{ijs}$ directly, we observe $K$ noisy proxies $g^k_{ijs}$, where $k = 1, ..., K$. Make the following normalization: $\mathbb{E}[g^k_{ijs}] = \mathbb{E}[g^*_{ijs}] = 0$. Since each is unbiased, $g^k_{ijs} = g^*_{ijs} + u^k_{ijs}$, where we further assume that $\mathbb{E}[u^k_{ijs}] = 0 \; \forall \; k$ and $\mathbb{E}[u^k_{ijs} g^*_{ijs}] = 0$. Unlike Black and Smith (2006), I allow for correlation between the errors across measures. So, $\mathbb{E}[u^k_{ijs} u^l_{ijs}]$ is not restricted to be zero whenever $k \neq l$.

The goal is to construct a measure $\hat{g}_{ijs}$ as a linear combination of $g^k_{ijs}$ that is the closest to the "true" latent variable $g^*_{ijs}$ as possible, in a mean squared error sense. That is, we want to estimate $\alpha \in \mathbb{R}^K$ to minimize

$$\mathbb{E}[(g^*_{ijs} - \hat{g}_{ijs})^2] = \mathbb{E}[(g^*_{ijs} - \sum_{k=1}^{K} \hat{\alpha}_k g^k_{ijs})^2] \tag{A.95}$$

It is simple to show that the First-Order necessary conditions to minimize (A.95) are

$$\mathbb{V}[g^*_{ijs}] - \sum_{l=1}^{K} \hat{\alpha}_l \mathbb{V}[g^*_{ijs}] - \sum_{l=1}^{K} \hat{\alpha}_l \mathbb{C}[u^k_{ijs} u^l_{ijs}] = 0 \; \forall \; k = 1, .., K \tag{A.96}$$

This defines a system of equations $\mathbf{A}\hat{\alpha} = \mathbb{V}[g^*_{ijs}]\iota$, where $\iota$ is a $K \times 1$ vector of ones. Further,

$$\begin{aligned} \mathbf{A}_{(k,l)} &= \mathbb{V}[g^*_{ijs}] + \mathbb{C}[u^l_{ijs}, u^k_{ijs}] \\ &= \mathbb{C}[g^k_{ijs}, g^l_{ijs}] \; \forall \; k, l \end{aligned} \tag{A.97}$$

So, $\hat{\alpha}$ is defined (to scale) by the inverse covariance of all the noisy proxies, which corresponds to the solution derived from factor analysis. The scale is typically set in factor analysis by the further restriction that $\sum_{k=1}^{K} \hat{\alpha}_k^2 = 1$.

Accordingly, a raw measure of $\hat{g}_{ijs}$ is justified by a simple measurement error model, where $\hat{g}_{ijs}$ is formed as a linear combination of the noisy proxies using the first principal component. Such a measure will, by construction, have mean zero. In order to convert this such that $\hat{g}_{ijs} \in [0, \infty)$ as the model dictates, after deriving the $\hat{\alpha}$, I construct $\hat{g}$ slightly differently by the following two steps:

1. $\hat{g}^{raw}_{ijs} = \sum_{k=1}^{K} \hat{\alpha}_k g^k_{ijs}$
2. $\hat{g}_{ijs} = \frac{\hat{g}^{raw}_{ijs}}{\sqrt{\mathbb{V}[\hat{g}^{raw}_{ijs}]}}$

This leads to $\hat{g}_{ijs}$ that has minimum 0 and variance one.

*F.2: Zeros in the Data*

Clearly, as $K$ increases, assuming each new $K$ is linearly independent, unobserved $g^*_{ijs}$ will be better approximated, thus leading to less measurement error. This point is made in Footnote 19 of Black and Smith (2006).

Note, however, an additional complication here. The procedure to construct $\hat{g}_{ijs}$ leads to some observed values of zero in the data. This is inconsistent with the strictly positive equilibrium. I attribute this to measurement error due to the fact that we were only able to collect finitely many noisy proxies $g^k_{ijs}$. To make the point more concretely, suppose we observe a number of (binary) proxy variables $g^k_{ijs}$ of an underlying continuous measure $g^*_{ijs}$, with reporting rule as follows:

$$g^k_{ijs} = (1 - a_k)\mathbf{1}\{g_{ijs} > x_k > 0\} + a_k(1 - \mathbf{1}\{g_{ijs} > x_k > 0\}), a_k \in (0, \frac{1}{2}) \tag{A.98}$$

That is, $g^k_{ijs}$ maps from $g^*_{ijs}$ via a threshold rule, but with some error rate $a_k$. Mechanically, the more such questions that are asked, the less likely that any individual $i$ claims that $g^k_{ijs} = 0$ for all $k$ with respect to some other individual $j$. Substantively, we are gathering more detailed data on links that allows for differentiation among more specific types of links.

For example, a question in the data, "I speak with her regularly" has a mean of 0.373 at baseline. A broader question, such as "I have spoken with her before," would have a much higher probability of being answered in the affirmative, corresponding with a lower $x_k$ for the latter question. Similarly, we could ask "I have met her" in addition to "She is a friend." If we gathered sufficiently many such questions, then the number of zeros in the data would decrease, eventually approaching zero, while still accounting for differences in the *intensity* of connections.

Note that this assumption is much more reasonable in schools of the size under study. The schools, which are located in remote rural villages, on average have 44 girls in the relevant grades. In this setting, it seems reasonable that individuals would have some connection to all others, even if such connection is extremely limited. If we were to ask much more detailed network questions, it would be possible to reveal positive but weak links between all pairs of girls.