Model-Free International SDFs

AEA Annual Meeting 2018

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Outline

Motivation

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The foreign or domestic Euler equation is given by

$$E[M_f \mathbf{R}_f] = \mathbf{1} = E[M_d \mathbf{R}_d]$$

In complete markets and with consumption SDFs

$$X = \ln(M_f / M_d)$$

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In complete markets and with consumption SDFs

$$X = \ln(M_f / M_d) = \gamma (\Delta c_d - \Delta c_f)$$

Many puzzles:

1. **Volatility puzzle**: $\sigma(x) << \gamma\sigma(\Delta c_d - \Delta c_f)$ [Brandt et al. '06].

2. **Cyclicality puzzle**: $corr(x, \Delta c_d - \Delta c_f) \approx 0$ [Backus & Smith '93].

3. **Forward premium anomaly** [Hansen & Hodrick '80, Fama '84]:

   $$E[x] - (r_{f0} - r_{d0}) >> 0 \iff r_{f0} - r_{d0} << 0 \ .$$

   Systematic deviations from UIP, not explained by cross-sectional differences in consumption volatility.

What can we do?

We can change SDF M_i in complete markets:

- Long-run risk (Colacito & Croce (2011, 2013, etc.)), habit (Verdelhan (2010) & Stathopoulos (2017)), rare disasters (Farhi and Gabaix (2016)), etc.

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Or we can bring in some form of market segmentation/limited participation:

- Chien, Lustig, & Naknoi (2015), Dou & Verdelhan (2015), Gabaix & Maggiori (2016).

What we do

- Let the data choose the "optimal" SDF using **asset prices** in an incomplete markets setting.
- Only condition we impose is **no-arbitrage**.
- Look at different degrees of market segmentation by varying the menu of assets foreign and domestic investors can trade.

We then ask

- What are the properties of these SDFs? → Highly correlated permanent SDF components
- What does market segmentation buy us? → More realistic SDFs (less volatile)
- Can we link our SDFs to observables? → Financial intermediary wealth/VaR constraints

Theory

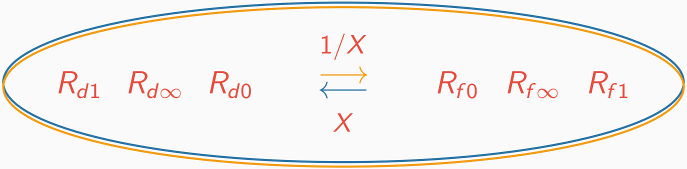
Complete Markets and Symmetry

- When markets are complete, domestic and foreign SDFs are uniquely defined.
- In integrated markets, the Euler pricing restrictions uniquely pin down the exchange rate return as the ratio between foreign and domestic SDFs:

$$X = M_f / M_d,$$

  i.e. **the asset market view holds**.
- International financial markets are called **symmetric** whenever $span(\mathbf{R}_d) = span(\mathbf{R}_f X)$, where $span(\mathbf{R}_d)$ ($span(\mathbf{R}_f X)$) is the linear span of portfolio returns generated by domestic returns (foreign returns converted in domestic currency).

Degrees of Financial Market Integration

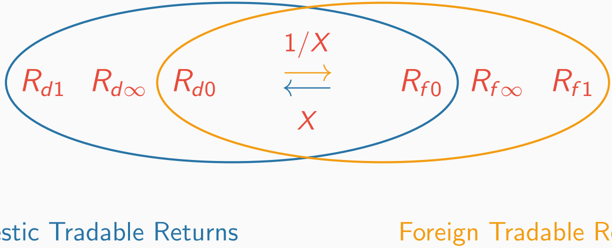


Domestic Tradable Returns

Foreign Tradable Returns

Full Symmetry = No Market Segmentation

Degrees of Financial Market Integration



Asymmetry = Segmented Long-Term Bond and Stock Markets

Minimum Dispersion SDFs in Incomplete Markets

- Suppose markets are incomplete.
- Return vector $\mathbf{R}_i = (R_{i0}, \ldots, R_{iK_i})'$ with risk-free rate R_{i0} for market $i = d, f$.
- For fixed $\alpha \in \mathbb{R}$, the minimum dispersion SDF M_i solves:

$$M_i(\alpha) := \arg\min_{M_i} \frac{\log E[(M_i / E[M_i])^{\alpha}]}{\alpha(\alpha - 1)} \ , \qquad (1)$$

$$\text{s.t. } E[M_i \mathbf{R}_i] = \mathbf{1} \ ; \ M_i > 0 \ .$$

$\rightarrow$ Different choices of α correspond to different dispersion measures.

Closed-Form Minimum Dispersion SDFs

Proposition 1

The minimum dispersion SDF is given in closed-form by

$$M_i^\star(\alpha) = R_{\lambda_i^*}^{-1/(1-\alpha)} / E[R_{\lambda_i^*}^{-\alpha/(1-\alpha)}] \,, \tag{2}$$

where optimal return $R_{\lambda_i^*} = \sum_{k=1}^{K_i} \lambda_{ik}^* R_{ik} + (1 - \sum_{k=1}^{K_i} \lambda_{ik}^*) R_{i0}$ solves the (dual) maximization problem

$$R_{\lambda_i^*} = \arg\max_{\lambda_i} - \frac{\log E\left[R_{\lambda_i}^{\alpha/(\alpha-1)}\right]}{\alpha} \,, \tag{3}$$

$$\text{s.t. } R_{\lambda_i} > 0 \,.$$

- Simple empirical estimation with method of moments.
- Various minimum dispersion SDF bounds in incomplete markets.

Examples

1. **Minimum variance** SDF ($\alpha = 2$): tightest upper bound on the maximal Sharpe ratio and single **tradable** minimum dispersion SDF:

$$M_i(2) = R_{\lambda_i^*} / E(R^2_{\lambda_i^*}) \; .$$

2. **Minimum entropy** SDF ($\alpha = 0$): optimal growth portfolio and single **numéraire invariant** minimum dispersion SDF:

$$M_i(0) = R^{-1}_{\lambda_i^*} \; .$$

Numéraire Invariance in Incomplete Markets

Remember that in symmetric and complete markets

$$X = M_f / M_d$$

But what about incomplete markets?

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$$X = (M_f / M_d) \exp(\eta)$$

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We show

Proposition 2

Let international financial markets be symmetric but incomplete and $\alpha_d = \alpha_f =: \alpha$. It then follows:

(i) The asset market view of exchange rates holds with respect to minimum entropy SDFs ($\alpha = 0$): $X = M_f^*(0)/M_d^*(0)$.

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Proposition 2

Let international financial markets be symmetric but incomplete and $\alpha_d = \alpha_f =: \alpha$. It then follows:

(i) The asset market view of exchange rates holds with respect to minimum entropy SDFs ($\alpha = 0$): $X = M_f^*(0)/M_d^*(0)$.

(ii) The asset market view of exchange rates does not hold with respect to minimum dispersion SDFs different from minimum entropy SDFs: $X \neq M_f^*(\alpha)/M_d^*(\alpha)$ for $\alpha \neq 0$.

SDF Components

- We factorize SDFs into permanent and transitory components:

$$M_i = M_i^P M_i^T .$$

- In line with Alvarez and Jermann (2005), we identify the permanent component with the normalization: $E[M_i^P] = 1$.
- The transitory component is the inverse of the return of the infinite maturity bond: $M_i^T := 1/R_{i,\infty}$.
- Exchange rate changes are now determined by:

$$X = \frac{M_f}{M_d} \exp(\eta) = \frac{M_f^P}{M_d^P} \frac{R_{d,\infty}}{R_{f,\infty}} \exp(\eta).$$

Why Market Segmentation?

$$X = M_f / M_d$$

- Asset market view forces SDFs to be very highly correlated.
- We show that this holds under symmetry both in complete and incomplete markets!

$\Rightarrow$ Market incompleteness does not help us to lower the co-movement of SDFs internationally!

- **Market segmentation** buys us less volatile SDFs which can co-move less.

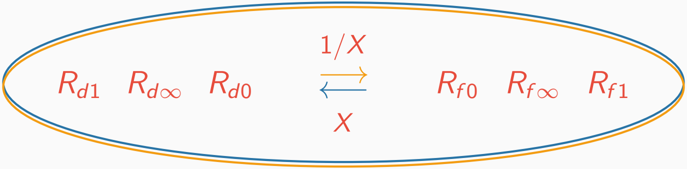
Empirical Analysis

Data

1. Real monthly returns, from Jan 1975 to Dec 2015:
   - R_{i0} (1M LIBOR), $R_{i\infty}$ (10Y gov. bond return) and R_{i1} (MSCI country index stock return).
   - 1 domestic currency (USD) and 7 foreign currencies (GBP, CHF, JPY, EUR, AUD, CAD, NZD).
   - Exchange rate returns X in terms of USD prices of foreign currencies.
2. Allow investors to trade all assets (full symmetry) and only short-term bond (asymmetry = segmented long-term bond and equity markets).

   - FS: $\mathbf{R}_i = (R_{i0}, R_{i1}, R_{i\infty}, R^e_{i0}, R^e_{i\infty}, R^e_{i1})'$, where $R^e_{dk,t+1} := R_{fk,t+1}X_{t+1}$ ($R^e_{fk,t+1} := R_{dk,t+1}(1/X_{t+1})$), $k = \{0, \infty, 1\}$.
   - AS: $\mathbf{R}_d = (R_{d0}, R_{d1}, R_{d\infty}, R^e_{d0})'$, where $R^e_{d0,t+1} := R_{f0,t+1}X_{t+1}$ and $\mathbf{R}_f = (R_{f0}, R_{f1}, R_{f\infty}, R^e_{f0})'$, where $R^e_{f0,t+1} := R_{d0,t+1}(1/X_{t+1})$.

Full Symmetry



Properties of SDFs

	US	UK	US	CH	US	JP	US	EU	US	AU	US	CA	US	NZ
$E[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
$Std(M_i)$	0.841	0.872	0.979	0.926	0.740	0.694	0.690	0.681	0.919	0.951	0.726	0.720	0.639	0.557
$corr(M_i, M_j)$		**0.992**		**0.989**		**0.989**		**0.985**		**0.992**		**0.994**		**0.981**

- Standard deviation of SDFs is large and clearly exceeds maximum Sharpe ratio in each country.
- Correlation among SDFs is almost perfect.

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$\mathrm{Std}(M_i)$	0.841	0.872	0.979	0.926	0.740	0.694	0.690	0.681	0.919	0.951	0.726	0.720	0.639	0.557
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.917	0.948	1.048	0.951	0.814	0.707	0.774	0.725	1.029	1.065	0.823	0.827	0.681	0.625
$\mathrm{corr}(M_i^T, M_i^P)$	-0.454	-0.498	-0.407	-0.233	-0.519	-0.155	-0.549	-0.502	-0.411	-0.636	-0.506	-0.607	-0.317	-0.634
$\mathrm{corr}(M_i, M_j)$		**0.992**		**0.989**		**0.989**		**0.985**		**0.992**		**0.994**		**0.981**

- Standard deviation of SDFs is large and clearly exceeds maximum Sharpe ratio in each country.
- Correlation among SDFs is almost perfect.
- In line with Alvarez and Jermann (2005), variability of SDFs is dominated by the permanent component.

The Three Puzzles

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- By construction, all risk premia are matched and in particular, currency risk premia are perfectly matched. UIP violation ✓
- The large SDF comovement is related to the low volatility puzzle of Brandt, Cochrane, and Santa-Clara (2006).
- Recall that

$$X_{t+1} \frac{R_{f\infty,t+1}}{R_{d\infty,t+1}} = \frac{M^P_{f,t+1}}{M^P_{d,t+1}} \exp(\eta_{t+1})$$

- The low volatility of the LHS is obtained if
    1. wedges and permanent component ratios are not too volatile
    2. wedges and permanent component ratios are strongly negatively correlated
    3. a combination of 1. and 2.

Wedge Summary Statistics (Unrestricted Trading)

	Minimum Variance			
	E[η]	Std(η)	Sk(η)	K(η)
UK	-0.007	0.059	-0.259	11.62
CH	-0.019	0.120	-6.146	68.40
JP	-0.009	0.083	-4.483	36.18
EU	0.000	0.064	-1.130	11.47
AU	0.005	0.075	2.110	24.82
CA	-0.001	0.034	-0.538	5.772
NZ	-0.031	0.216	-15.61	268.3

$\Rightarrow$ Wedge dispersion is clearly smaller than SDF volatility.

Correlation of Permanent Components

	UK	CH	JP	EU	AU	CA	NZ
$\alpha = 0$	0.972***	0.984***	0.981***	0.978***	0.987***	0.976***	0.968***
	[0.019]	[0.007]	[0.004]	[0.007]	[0.006]	[0.005]	[0.006]
$\alpha = 2$	0.968***	0.982***	0.977***	0.974***	0.98***	0.973***	0.965***
	[0.009]	[0.003]	[0.004]	[0.006]	[0.005]	[0.004]	[0.005]

- Almost perfect correlation among permanent components.
- Minimum dispersion SDFs are highly correlated and disperse due to their highly correlated and disperse permanent components.
- Low volatility driven by high correlation of permanent components:

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- Low volatility driven by high correlation of permanent components: Low vol puzzle ✓

Backus and Smith (1993) Puzzle

$$m_{f,t+1} - m_{d,t+1} = \delta + \beta x_{t+1} + u_{t+1},$$
$$m^U_{f,t+1} - m^U_{d,t+1} = \delta^U + \beta^U x_{t+1} + u^U_{t+1}, \quad U = T, P,$$

	US/UK	
	$\alpha = 0$	$\alpha = 2$
β	1.000***	1.022***
	[0.000]	[0.0261]
β^P	1.085***	1.065***
	[0.068]	[0.0742]
β^T	-0.084	-0.084
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$\Rightarrow$ Estimates for permanent component basically $= 1$ but estimates for transitory component zero and insignificant.

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Backus & Smith puzzle ✓

Three Puzzles Summary

Three puzzles can be jointly addressed in an economy with unrestricted trading

- Martingale components are highly volatile and almost perfectly correlated while
- Differences in transitory components are uncorrelated with changes in real exchange rate.

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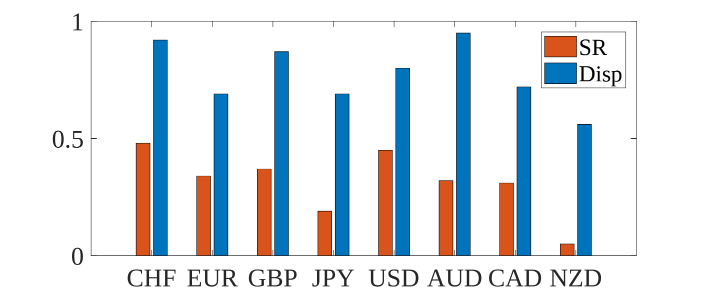
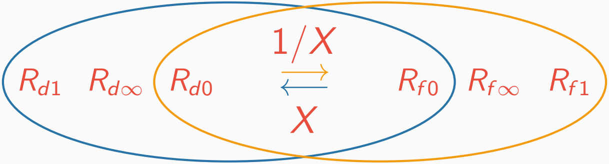
- Differences in transitory components are uncorrelated with changes in real exchange rate.
- **BUT...**



Segmented Long-Term Bond and Stock Markets

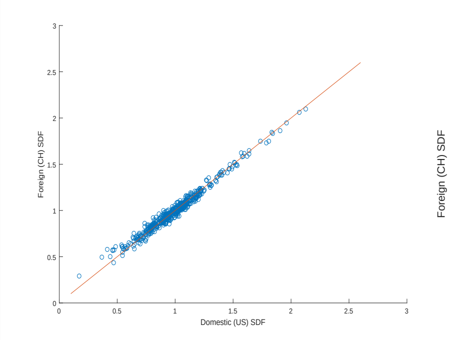


Trading in Short-Term Bonds Only

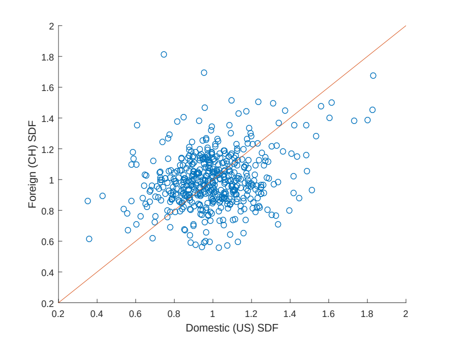
- To lower SDF dispersion, we allow investors to trade only short-term bonds internationally.
- Currency risk premia are still matched.
- SDF dispersion drops considerably between 40% (Switzerland) and 50% (New Zealand).
- Deviation from AMV implies more volatile wedge

	Minimum Entropy				Minimum Variance			
	$E[\eta]$	$Std(\eta)$	$Sk(\eta)$	$K(\eta)$	$E[\eta]$	$Std(\eta)$	$Sk(\eta)$	$K(\eta)$
UK	0.003	0.636	-0.646	13.55	0.042	0.814	1.074	9.239
CH	-0.006	0.682	-0.367	6.270	-0.021	0.826	-0.019	3.724
JP	-0.123	0.545	1.446	8.938	-0.149	0.612	-0.259	5.417
EU	-0.048	0.439	0.265	4.026	-0.059	0.517	-0.554	5.011
AU	0.104	0.581	-0.181	5.573	0.129	0.716	1.051	6.714
CA	-0.036	0.490	0.148	9.963	-0.040	0.561	0.305	5.082
NZ	-0.020	0.413	0.362	4.556	-0.029	0.442	0.178	3.834

What does market segmentation buy us?



(a) Full Symmetry



(b) Asymmetry

Financial Intermediary Wealth

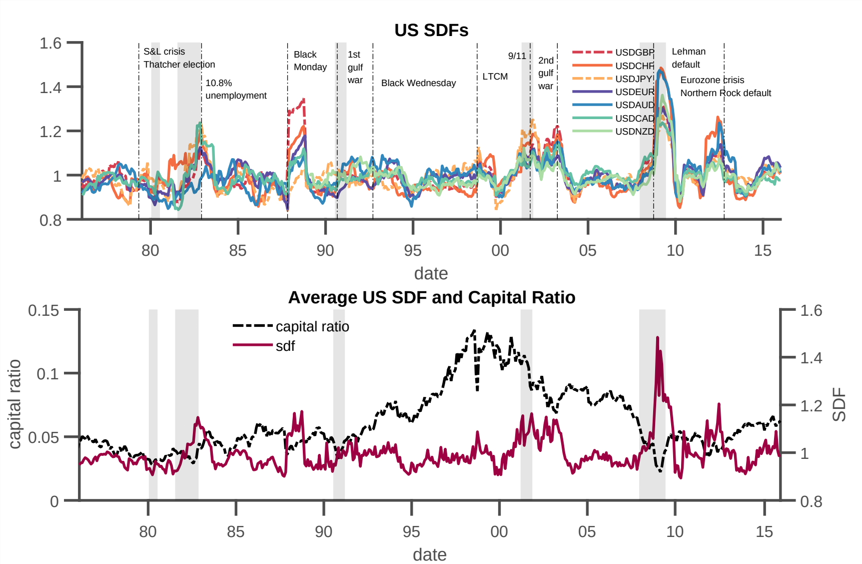
Intermediation in the FX Market

- Market for FX is highly intermediated and concentrated.

rank	bank	mrkt share	cumulative
1	Citibank	10.74%	
2	JP Morgan	10.34%	21.08%
3	UBS	7.56%	28.64%
4	Bank of America	6.73%	35.37%
5	Deutsche Bank	5.68%	41.05%
6	HSBC	4.99%	46.04%
7	Barclays	4.69%	50.73%
8	Goldman Sachs	4.43%	55.16%
9	Standard Chartered	4.26%	59.42%
10	BNP Paribas	3.73%	63.15%

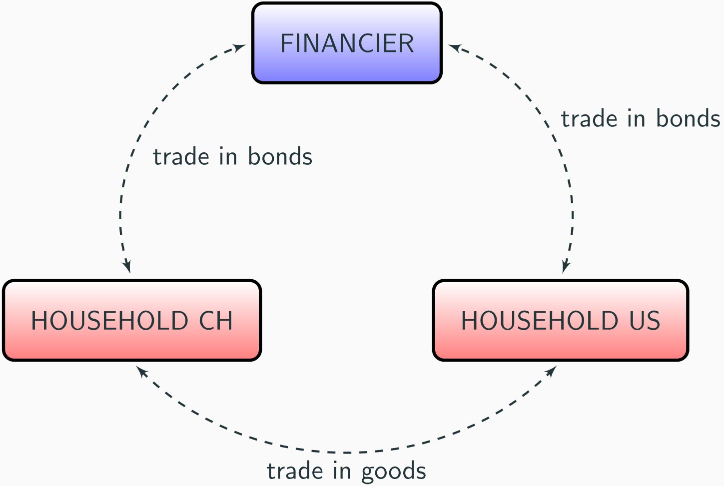
- Is also concentrated across currencies: the first two most traded pairs (USDEUR & USDJPY) account for 40% of the total market share.

SDFs and Financial Intermediary Wealth



Financial Intermediaries in Segmented Markets

Simplified version of Gabaix and Maggiori (2016).



Intermediary SDF

- Intermediary maximizes her wealth subject to a **Value-at-Risk** constraint:

$$\begin{aligned} \max_{Q_t} \quad & E_t[V_{t+1}] \\ \text{s.t.} \quad & \mathbb{P}_t(V_{t+1} \leq -\epsilon_t) \leq c_t, \end{aligned} \tag{4}$$

  where ϵ_t is the Value-at-Risk of next period financier's wealth for confidence level c_t.
- In this case, the intermediary SDF is linear in wealth.
- We can run linear regressions to test this relationship in the data.

SDFs and Financial Intermediary Wealth

$$M_{t+1} = \alpha + \beta_k \, \Delta\text{intermediary wealth}_{t+1} + \beta_v \, \Delta\text{VIX}_{t+1} + \epsilon_{t+1}$$

	USDGBP	**USDCHF**	**USDJPY**	**USDEUR**	**USDAUD**	**USDCAD**	**USDNZD**
α	1.550***	2.086***	0.744***	1.169***	1.116***	0.697***	0.799
	(11.08)	(8.57)	(3.27)	(7.59)	(4.15)	(5.48)	(1.44)
β_k	−0.949***	−1.587***	-0.054	−0.437***	-0.559	-0.153	-0.101
	(-6.31)	(-5.63)	(-0.12)	(-2.43)	(-1.60)	(-0.72)	(-0.12)
β_v	0.395***	0.497***	0.306***	0.264**	0.438***	0.450***	0.298***
	(5.43)	(3.03)	(5.17)	(2.36)	(2.96)	(5.25)	(3.56)
R-Squared	0.46	0.35	0.09	0.24	0.20	0.31	0.04

- SDFs load negatively on intermediary wealth
- SDFs load positively on VIX

Conclusion

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- The three exchange rate puzzles are addressed by SDFs with high permanent components when short-term bonds are internationally tradable.
- However, under perfect symmetry, this comes at the cost of highly disperse SDFs. Market segmentation lowers the dispersion.
- Successful models should therefore consist of two ingredients:
    1. Large and positively correlated martingale components
    2. Mild market segmentation.
- Models that incorporate financial intermediaries seem promising.

Thank you!

Appendix

Stochastic Wedges

Corollary 1

In symmetric international financial markets, the AMV holds with respect to minimum entropy SDFs:

$$X = \frac{M_f(0)}{M_d(0)} = \frac{M_f(2)}{M_d(2)} \cdot \frac{M_f(0)/M_f(2)}{M_d(0)/M_d(2)} =: \frac{M_f(2)}{M_d(2)} \cdot \exp(\eta) \ ,$$

with a minimum variance Backus et al. '01 **stochastic wedge** given by:

$$\eta = \ln(M_f(0)/M_f(2)) - \ln(M_d(0)/M_d(2)).$$

- The stochastic wedge captures **unspanned** exchange rate risks induced by the component of minimum entropy SDFs that cannot be replicated using asset returns.
- Exchange rates are larger:
  - ⇒ due to **mean-variance trade-off** between domestic and foreign markets.
  - ⇒ due to **higher moment trade-off**.

Summary Statistics

	CHF	EUR	GBP	JPY	USD	AUD	CAD	NZD
	Panel A: Bonds							
1M	2.81	4.33	7.39	2.61	5.36	8.25	6.31	6.68
10Y	1.79	2.26	3.23	2.31	1.91	2.19	2.04	3.94
	Panel B: Excess stock returns							
Mean	7.39	6.89	6.23	3.49	7.08	5.71	5.15	0.84
Std	15.42	20.08	16.99	18.31	15.71	17.76	16.77	18.23
SR	48	34	37	19	45	32	31	5
	Panel C: Exchange rates							
Mean	2.96	0.03	-0.65	2.85		-0.86	-0.48	0.76
Std	12.12	10.56	10.20	11.32		10.93	6.78	11.92
	Panel D: Inflation							
Mean	1.76	2.22	4.74	1.57	3.69	4.83	3.71	5.57
Std	1.24	1.60	2.12	1.78	1.28	1.22	1.45	1.71

Wedge Cyclicality

	corr(η, m_i)	SE	corr(η, m_i^P)	SE	corr(η, m_i^T)	SE
US	0.658***	[0.039]	0.651***	[0.039]	-0.356***	[0.049]
UK	-0.617***	[0.052]	-0.602***	[0.057]	0.338***	[0.053]
US	0.541***	[0.026]	0.569***	[0.029]	-0.431***	[0.038]
CH	-0.594***	[0.051]	-0.585***	[0.053]	0.077	[0.054]
US	0.728***	[0.039]	0.759***	[0.042]	-0.532***	[0.058]
JP	-0.201***	[0.054]	-0.200***	[0.056]	-0.029	[0.061]
US	0.552***	[0.047]	0.546***	[0.050]	-0.273***	[0.058]
EU	-0.296***	[0.084]	-0.324***	[0.909]	0.402***	[0.052]
US	0.523***	[0.031]	0.441***	[0.040]	0.278***	[0.039]
NZ	-0.465***	[0.075]	-0.508***	[0.071]	0.606***	[0.057]

SDF Components

Two issues when decomposing SDFs into transitory and permanent components:

1. Hansen and Scheinkman (2009): Alvarez and Jermann (2005) decomposition is not necessarily unique. It is, however, unique when state variables are stationary. Extensions to semi-martingales in Qin and Linetsky (2017).
2. Ten-year bond may be a bad approximation for the infinite maturity bond.

We estimate transitory and permanent components of the Perron-Frobenius problem. Given the eigenvector ρ and eigenfunction ϕ, the permanent and transitory components can be recovered as follows:

$$\frac{M^P_{t+\tau}}{M^P_t} = \rho^{-\tau} \frac{M_{t+\tau}}{M_t} \frac{\phi(X_{t+\tau})}{\phi(X_t)}, \qquad \frac{M^T_{t+\tau}}{M^T_t} = \rho^{\tau} \frac{\phi(X_t)}{\phi(X_{t+\tau})}. \tag{5}$$

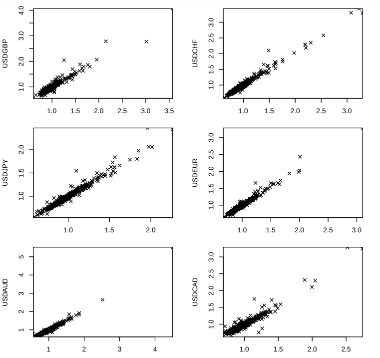

Table 1: Properties of SDFs components (Nonparametric estimates)

	USDGBP	USDCHF	USDJPY	USDEUR	USDAUD	USDCAD	USDNZD
Std(M_d^P)	0.786	0.970	0.725	0.683	0.907	0.660	0.601
Std(M_d^T)	0.126	0.065	0.057	0.039	0.143	0.140	0.112

All results remain the same when we use non-parametrically estimated transitory and permanent components.