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Parental Education Investment Decision with Imperfect Talent Signal

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Introduction

- classic parental education investment model treats parental human capital investment as intergenerational transfer
  - and parents are willing to invest in children's education until MC=MB (Glomm, 1997)
  - education investment on children with high ability will be higher (Raut and Tran, 2005)
- Education behaviors do not always follow this prediction
  - remedial programs (Dizon-Ross, 2013)
  - disadvantaged children (Heckman, 2006)

Summary

- Question:
  - how would parental education investment change with respect to change in talent?
- Model:
  - Discontinuous utility function
    - Additional bonus at certain threshold
  - Uncertainty
    - The signal of talent parents observed is not the true talent
- Findings:
  - The correlation between parental education investment and observed talent is not monotone
    - General: Positive
    - When close to the threshold: Negative
  - Students close to thresholds are less likely to drop out of school
  - The correctness of signals doesn't change the main conclusion
    - Perfect signal: Jumps & kinks
    - Imperfect signal: Smooth curve

Set up

- Parents' optimization equation:

$$u = U(C) + V(t, EI), \text{ st. } C + EI = I$$

  where:
  - I : the endowment
  - $U(C)$: the utility from consumption
  - $V(t, EI)$: the utility from the child's school performance

- Assumptions:
  - Assumption 1: $U' > 0, U'' < 0$
  - Assumption 2: $V(t, EI)) = R(t, EI) + k \cdot 1\{R(t, EI) > Th\}$ where Th is the threshold for additional bonus

$$1\{R(t, EI) \geq Th\} = \begin{cases} \text{1, if R(t,EI)} \geq \text{Th} \\ \text{0, Otherwise} \end{cases}$$

  - Assumption 3: $\frac{\partial R}{\partial t} > 0, \frac{\partial R}{\partial EI} > 0, \frac{\partial R^2}{\partial t^2} < 0, \frac{\partial R^2}{\partial EI^2} < 0, \frac{\partial R^2}{\partial EI \partial t} > 0$

Perfect Signal: Binary

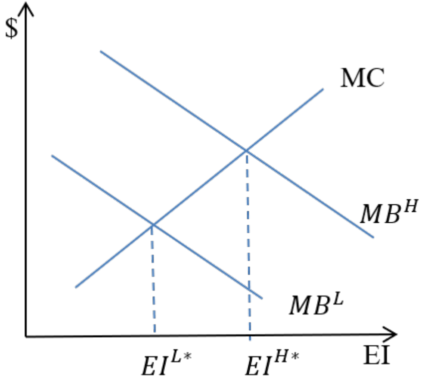
$$talent = t^L, t^H$$

- Th^L, Th^H: Thresholds
- k^L, k^H: Bonuses for reaching the thresholds
- $EI^L_{MC=MB}, EI^H_{MC=MB}$: The education investment level at which marginal cost is equal to marginal benefit
- EI^L_{Th}, EI^H_{Th}: The education investment level which ensure the child to reach the threshold
- EI^{L*}, EI^{H*}: The optimal education investment levels

Perfect Signal: Binary

Case 1: Both types choose their MC=MB points

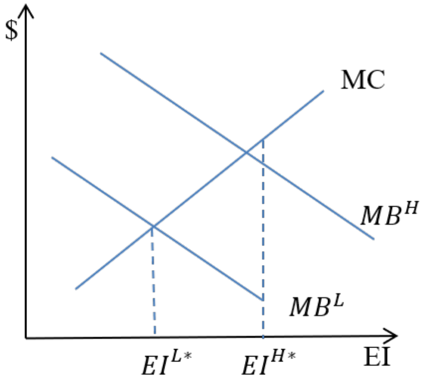
$$EI^{L*} = EI^{L}_{MC=MB} < EI^{H}_{MC=MB} = EI^{H*}$$



Perfect Signal: Binary

Case 2: High type chooses threshold point, low type chooses its MC=MB point

$$EI^{L*} = EI^{L}_{MC=MB} < EI^{H}_{MC=MB} < EI^{H}_{Th} = EI^{H*}$$

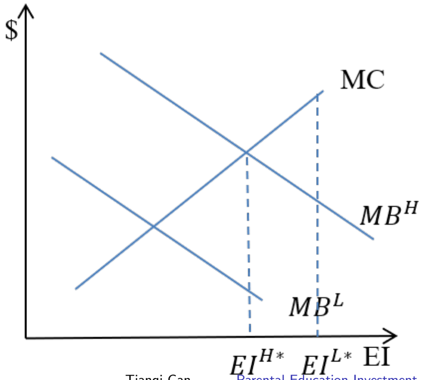


Perfect Signal: Binary

Case 3: High type chooses its MC=MB point, low type chooses threshold point

$EI^{L*} > EI^{H*}$ if

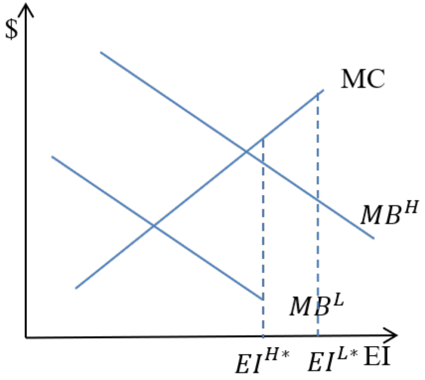
- The threshold of the low-talent type is way higher than the MC=MB point
- the gap between the marginal effect of education investment on years of schooling is large;



Perfect Signal: Binary

Case 4: Both types choose their threshold points
$EI^{L*} > EI^{H*}$ when

- the difference between thresholds is relatively small (the extreme case will be the threshold is the same for both types);
- the education investment is much more efficient for the high type



Perfect Signal: Continuous

- When the optimal point is at the MC=MB point, a marginal increase of talent will increase the optimal education investment

$$\frac{\partial U}{\partial EI_{MC=MB}} = \frac{\partial R}{\partial EI_{MC=MB}}$$

$$\frac{\partial EI_{MC=MB}}{\partial t} = [\frac{\partial R}{\partial EI_{MC=MB}} \cdot \frac{\partial R}{\partial t} + \frac{\partial^2 R}{\partial EI_{MC=MB} \partial t}] \cdot (\frac{\partial U}{\partial EI_{MC=MB}})^{-1} > 0$$

- When the optimal point is at the threshold point, the correlation depends on the values of $\frac{\partial Th}{\partial t}$ and $\frac{\partial R}{\partial t}$

$$Th = R(t, EI_{th})$$

$$\frac{\partial EI_{Th}}{\partial t} = (\frac{\partial Th}{\partial t} - \frac{\partial R}{\partial t}) \cdot (\frac{\partial R}{\partial EI_{Th}})^{-1}$$

Assumption 4 $\frac{\partial Th}{\partial t} = 0$

$$\frac{\partial EI_{Th}}{\partial t} = -\frac{\partial R}{\partial t} \cdot (\frac{\partial R}{\partial EI_{Th}})^{-1} < 0$$

Perfect Signal: Continuous

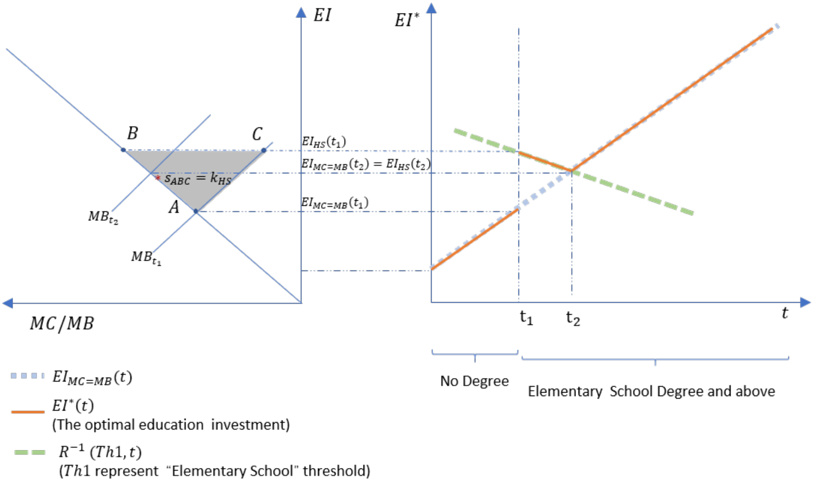
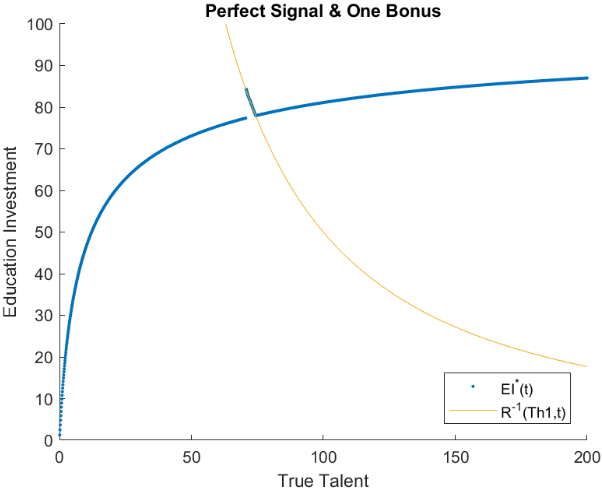


Figure: The correlation between t and EI^* with one threshold

Perfect Signal: Continuous

Simulation Result



Perfect Signal: Continuous

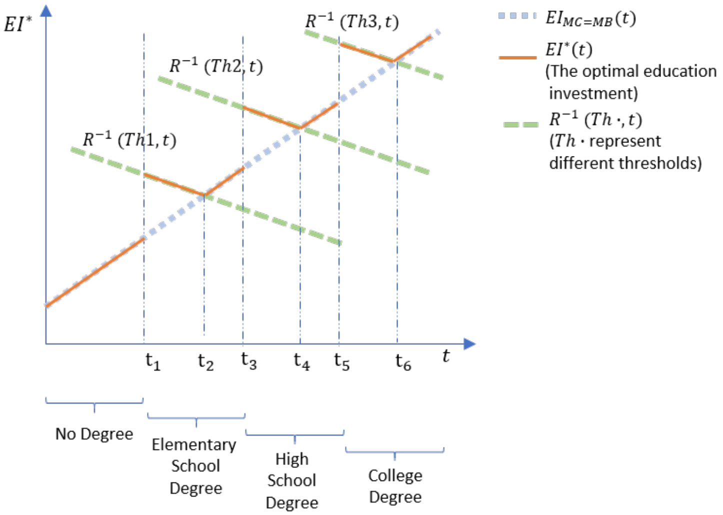
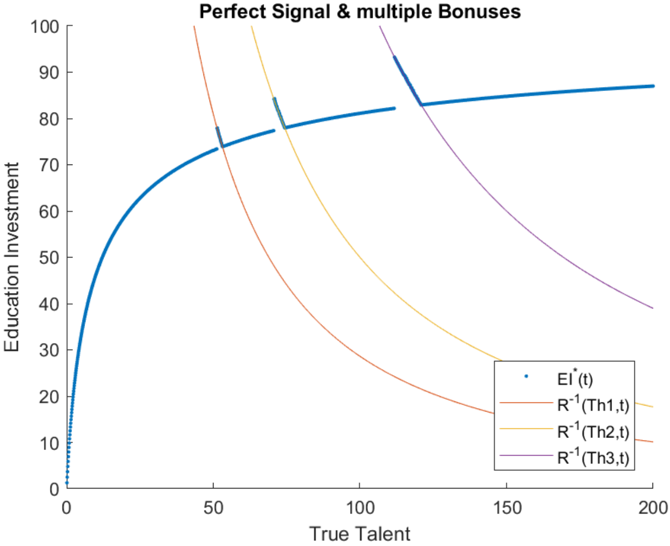


Figure: The correlation between t and EI^* with multiple thresholds

Perfect Signal: Continuous

Simulation results:



Imperfect Signal: Continuous

Now assume parents observe the signal of talent $\hat{t}$, and they know the conditional probability distribution of the true talent t.

$$\max_{EI} E[u|\hat{t}] = U(I - EI) + \int R(t, EI) \cdot f(t|\hat{t}) dt + k \cdot [1 - F(R^{-1}(Th, EI)|\hat{t})]$$

FOC: $$\frac{\partial E[u|\hat{t}]}{\partial EI} = -U^{'} + \int \frac{\partial R}{\partial EI} \cdot f(t|\hat{t}) dt - k \cdot f(R^{-1}(Th, EI)|\hat{t}) \cdot \frac{\partial R^{-1}}{\partial EI}$$

SOC: $$\frac{\partial^2 E[u|\hat{t}]}{\partial^2 EI} = U^{''} + \int \frac{\partial^2 R}{\partial^2 EI} \cdot f(t|\hat{t}) dt - k \cdot [f^{'}(R^{-1}(Th, EI)|\hat{t}) \cdot (\frac{\partial R^{-1}}{\partial EI})^2 + f(R^{-1}(Th, EI)|\hat{t}) \cdot \frac{\partial^2 R^{-1}}{\partial^2 EI}]$$

Imperfect Signal: Continuous

Comparative Statics

- Change in bonus

$$\frac{\partial EI^*}{\partial k} = -\frac{\frac{\partial FOC}{\partial k}}{SOC} = -\frac{-f(R^{-1}(Th, EI)|\hat{t}) \cdot \frac{\partial R^{-1}}{\partial EI}}{SOC}$$

SOC will be negative so $\frac{\partial EI^*}{\partial k} \geq 0$.

- Change in talent signal

$$\frac{\partial EI^*}{\partial \hat{t}} = -\frac{\frac{\partial FOC}{\partial \hat{t}}}{SOC} = -\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}} dt - k \cdot \frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} \cdot \frac{\partial R^{-1}}{\partial EI}}{SOC}$$

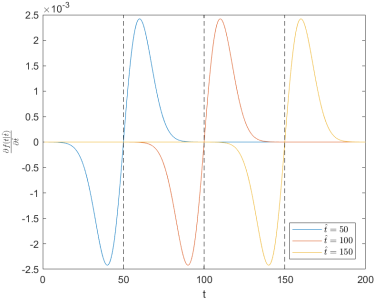
Imperfect Signal: Continuous

$$\frac{\partial EI^*}{\partial \hat{t}} = -\frac{\frac{\partial FOC}{\partial \hat{t}}}{SOC} = -\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}} dt - k \cdot \frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} \cdot \frac{\partial R^{-1}}{\partial EI}}{SOC}$$

Assumption 6

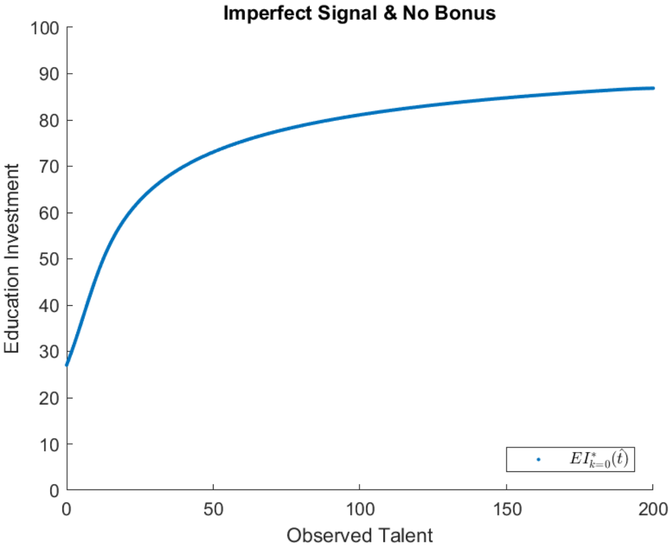
$$f(t|\hat{t}) = \frac{\phi(\frac{t-\hat{t}}{\sigma})}{\sigma[\Phi(\frac{\bar{t}-\hat{t}}{\sigma}) - \Phi(\frac{\underline{t}-\hat{t}}{\sigma})]} = \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\hat{t})^2}{2\sigma^2}}}{\int_{\underline{t}}^{\bar{t}} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\hat{t})^2}{2\sigma^2}} dx}$$

- If $k = 0$, positive
- If $k > 0$,



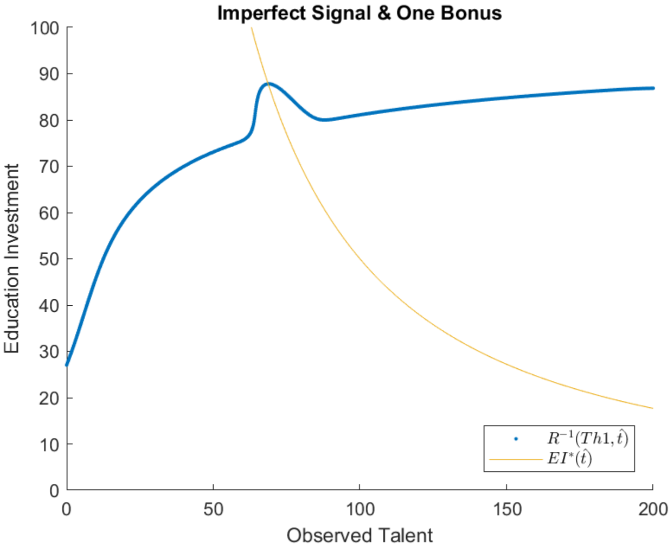
Imperfect Signal: Continuous

If $k = 0$,



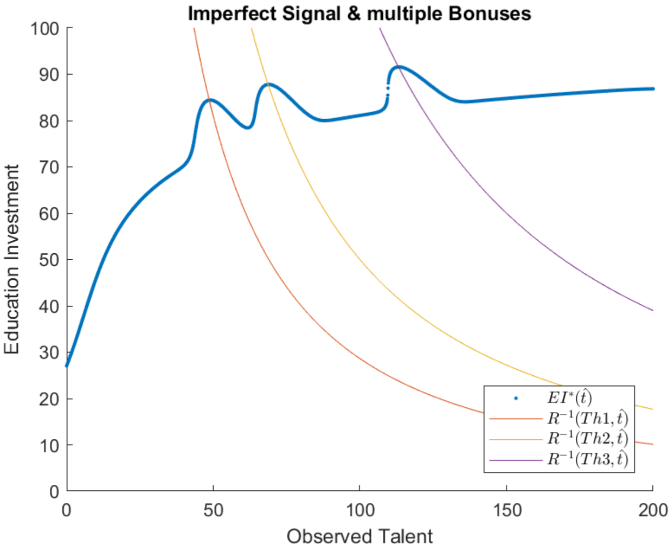
Imperfect Signal: Continuous

If $k > 0$,



Imperfect Signal: Continuous

If $k > 0$ and there are multiple thresholds



Future Steps

- Dynamic
- Empirical

Conclusion

Findings:

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  - General: Positive
  - When close to the threshold: Negative
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- The correctness of signals doesn't change the main conclusion
  - Perfect signal: Jumps & kinks
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Thank you!