A positive analysis of bank behaviour under capital requirements

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Abstract

We propose a theory of bank behaviour under capital requirements that accounts for both risk-shifting incentives and debt overhang considerations. A key result is that the bank's lending response to an increase in the requirement need not be negative. The sign and the magnitude of the response depend on the bank's balance sheet and economic prospects, and lending is typically U-shaped in the requirement. Using UK regulatory data, we find empirical support for the hypothesis that a bank mainly adjusts to a higher requirement by cutting lending when expected returns are low, but by raising capital when they are high. *(JEL Codes: G21, G28)*

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1 Introduction

In recent years there has been much discussion on the costs and benefits of capital regulation for banks. Normative considerations and general equilibrium effects have been centre stage in the theoretical literature.[1] A simpler question has received less attention: How do capital requirements affect an *individual* bank's joint capital issuance and lending decisions? This is perhaps surprising as answering this question is a key first step for a normative assessment in general equilibrium.[2]

The main contribution of this paper is to propose a new theory of bank behaviour under capital requirements. Our approach allows us to decompose analytically the effect of capital requirements on equilibrium lending and capital decisions, and derive empirical predictions. Specifically, we show how the relevant notions of marginal return and marginal cost are affected by the requirement. As one would expect, there are forces that push a bank to respond to an increase in capital requirements by cutting lending. However, we highlight an effect that pushes in the opposite direction and that can even be strong enough for the bank to increase lending in equilibrium. The net effect depends on the bank's balance sheet and economic conditions, as well as the initial level of the requirement itself.

Our second contribution is empirical. We exploit changes to UK banks' individual capital requirements over the period 1989 to 2007 to test the predictions of the model. Consistent with our predictions, we find that economic conditions matter: a way to summarise our findings is that the bank mainly cuts lending when expected returns are low, and raises capital when they are high.

[1] Two of the main current focuses are (i) the optimal overall level of capital requirement (see e.g. Admati et al. 2013, Martinez-Miera and Suarez 2014, Begenau (2015); (ii) time-varying adjustments (the so-called counter-cyclical buffers, see e.g. Kashyap and Stein 2004, Repullo and Suarez 2013, Malherbe 2015). Earlier, Thakor (1996) had studied the aggregate effect of capital regulation on lending, and Repullo and Suarez 2004 have studied the loan pricing implications of the Basel 2 IRB approach.

[2] A series of papers have looked at the effect of capital requirement on bank individual behaviour, but they have mostly focused on the distortions in the bank's portfolio decision (e.g. Koehn and Santomero 1980, Rochet 1992, Calem and Rob (1999)).

In our model, the bank chooses its level of capital and how much to lend in order to maximize the expected payoff of its initial shareholders. The bank starts with a given level of capital but can pay a dividend or issue new capital to risk-neutral investors. Besides the natural assumption that shareholders are protected by limited liability, several ingredients are important for the analysis. First, the bank has existing loans on its balance sheet. Second, the government guarantees deposits, which makes them a cheaper source of finance than capital (in this paper capital refers to loss-absorbing liabilities in general). Third, the bank faces a capital requirement. Fourth, the bank faces a downward sloping demand curve for loans. Therefore, bank lending presents diminishing expected marginal returns, which translates to there being an efficient level of lending.

To illustrate the main mechanisms, let us consider the response of lending to an increase in the capital requirement. We decompose it into three effects: (i) keeping the cost of capital constant, the bank's funding cost increases, from the point of view of shareholders, because (cheaper) deposits must be substituted for capital (a liability *composition* effect); (ii) keeping lending constant, the increase in the requirement decreases the marginal cost of bank capital because it reduces the bank's probability of default (a *price* effect); (iii) the increase in the requirement makes the bank internalise more of the downside risk, which decreases the shareholders' expected return of the marginal loan (a return *internalisation* effect). While the composition and internalisation effects induce a decrease in lending, the price effect pulls in the other direction. Since the bank's response is the sum of these three components, its sign depends on whether the price effect dominates the other two. Importantly, the relevant notions of marginal cost and marginal returns are their values *conditional on not defaulting* as that is what matters for shareholders.

In our model, capital requirement changes have symmetric effects. This means that the effects of a decrease are the opposite of those for an equivalent increase. For simplicity, we present our analysis in terms of increases in the capital requirement, and we refer to the bank response to such an increase as *the lending (or capital) response*.

We show that the sign of the lending response is ambiguous due to the interplay between excessive risk-taking incentives (because of government guarantees the bank does not fully internalise the downside from its lending) and debt overhang considerations (the bank does not fully internalise the upside either).[3] In fact, the level of lending is typically U-shaped in the capital requirement. At low capital requirements, an increase in the requirement generates a lending cut, but at higher levels, it generates an increase. We also show that changes in the expected return on loans shift this relationship mostly through changes in the composition and price effects (which we are able to disentangle via a numerical solution). Except in cases that are arguably extreme, we find that a higher expected return on loans increases both the lending and the capital response (that is, makes them less negative or more positive). A necessary condition for a positive lending response (that is, for an increase in capital requirements to increase lending) is that there is an overhang problem while the bank's probability of default is low. Perhaps counter-intuitively, this means that the overhang problem cannot be too severe.

Both the implicit subsidy from government guarantees and debt overhang problems (Myers 1977) have been recognized in the literature as important drivers of bank behaviour. It is well understood that government guarantees can distort investment decisions (Merton 1977, Kareken and Wallace 1978). Typically, the problem is cast in terms of asset substitutions, or risk-shifting, as in Jensen and Meckling (1976). When demand for loans is downward sloping, government guarantees are likely to induce negative net-present-value lending (see, for instance, Martinez-Miera and Suarez 2014 and Malherbe 2015), which can happen in our model. Studying related general equilibrium effects in a dynamic setup requires drastic assumptions that restrict bank ability to accumulate capital (Suarez, 2010). We conduct our analysis in partial equilibrium, but we allow capital issuance and dividend payments to be chosen freely.

A number of papers relate capital requirements to banks' debt overhang prob-

[3]A debt overhang problem occurs when the quality of current assets is such that a firm (here a bank) may not be able to repay its debt in full (Myers 1977). In this case, a fraction of the value created by the marginal loan is, in effect, transferred to debtholders. Since the firm does not fully internalize the returns, it ends up not taking up all positive-net-present-value lending opportunities.

lems. Considering the key ingredients of our model, the closest paper is Admati et al. (2016) as they also combine debt overhang and incentives to shift risk.[4] However, they mostly focus on (the lack of) incentives for firms to *voluntarily* deleverage, which in a dynamic setup gives rise to a leverage ratchet effect. To keep their model tractable they focus on a scale-invariant environment. In a sense, we have an opposite approach: we abstract from dynamic issues to be able to understand how requirements affect the optimal *level* of a bank's lending and capital.

Our approach allows us to disentangle and, therefore, highlight our three key mechanisms. Of these mechanisms, the idea behind the composition effect is not new and has, for instance, been used by those arguing against higher capital requirements. Admati et al. (2013) contends that this argument is often fallacious, for it ignores any price effect. Still, the composition effect is relevant as soon as, from the private point of view of the bank, capital is relatively costly. Policymakers are often concerned that bank capital is more costly in bad times. Hanson et al. (2011) point out that a reason for this is that overhang problems are then more salient, which makes banks more likely to make any needed upward adjustment to their capital ratios by cutting lending. This reasoning is equivalent to saying that the composition effect is particularly strong in bad times. At the same time, Aiyar et al. (2014a) state that an increase in a capital requirement can also help overcome a severe overhang problem.[5] One way to interpret this is that severe overhang problems are associated with particularly strong price effects. Our analysis clarifies that each argument captures part of the story. Both price and composition effects are stronger when the overhang problem is severe. However, we find that, with a severe overhang, the composition effect dominates and the lending response is negative. Conversely, when the overhang problem is mild, both effects are weaker but, in this case, the price ef-

[4]Following the recent crisis, a wave of papers have also studied overhang problems in the context of financial institutions with massive holdings of toxic assets. Examples include Tirole (2011), Philippon and Skreta (2012), and Philippon and Schnabl (2013). These works, however, abstract from capital requirements, which are central to our analysis. On the other hand, we abstract from adverse selection issues.

[5]Hanson et al. (2011) also point out that *ex-ante* higher capital requirement can decrease the likelihood and severity of *ex-post* overhang problem.

fect is the stronger of the two. The lending response is then positive, unless there is a sufficiently strong offsetting internalisation effect. Finally, to the best of our knowledge, the internalisation effect, as we define it, has been overlooked so far. However, it is not unrelated to the idea that increasing the capital requirement mitigates incentives to take on excessive risk.

To sum up, our main theoretical contribution is to clarify these three effects and combine them to provide a theory of bank behaviour under capital requirements. A key novelty is to highlight that the lending response need not be negative.

Taking the model to the data is the second contribution of the paper. We use UK bank regulatory data from 1989 to 2007 that includes the path for bank specific capital requirements and their balance sheet variables. Unfortunately, we cannot directly observe an individual bank's expected return on loans. Instead, we use an aggregate forward looking indicator as a proxy: the OECD's business confidence indicator for the UK.

On the basis of our theory, we expect upward sloping relationships between expected returns and the lending and capital responses. Our first empirical test is to estimate these relationships in a difference-in-differences set up at quarterly time horizons (up to ten quarters).[6] Our main objects of interest are the interaction terms between the change in capital requirement and the confidence indicator (these terms are our estimates for the slope of the relationships). We find that they both are positive and significant after a year.[7]

Our point estimates for lending suggest that, in periods of low confidence (when the indicator is 1 standard deviation (sd) below its sample mean), a 25 basis points (bp) increase in capital requirement leads to a cumulative cut in lending of 2% (of total stock of loans) after a year. In contrast, when confidence is high (i.e. the indicator is 1 sd above its mean), point estimates are close to (and not statistically significantly different from) zero. In fact one needs the confidence indicator to be around 1.5sd above its mean for an increase in capital

[6]Specifically, our approach follows the local projection methodology of Jordà (2005).

[7]These results hold for a variety of alternative proxies for expected returns, beyond business confidence, and are not sensitive to sample filtering criteria or sample time period.

requirement to generate a significant increase in lending. Assuming a normal distribution for the confidence indicator, this would correspond to a probability of 7%. The estimates for the slope of the capital response suggest that in periods of high confidence, increases in capital requirement are mainly met by increases in capital. Conversely, when confidence is low, we do not see a positive response, which suggests that the main adjustment takes place on the asset side of the balance sheet.

With the caveat that our estimates are based on a specific country under a specific regulatory regime; our empirical results speak to two additional issues beyond testing our model. First, our findings can help calibrate general equilibrium models, especially those models that allow for endogenous issuance of new capital. Second, our estimates are informative about how banks could adapt to higher capital requirements and respond in a time-varying regime, both of which are particularly relevant to the implementation of the latest wave of the Basel Capital Accord (Basel III).

There is a long history of papers studying bank responses to shocks to their capital. This literature runs, for instance, from Bernanke and Lown (1991), who use regional variation across the US to identify a role for capital losses in reducing subsequent credit provision, to a recent strand of literature that focusses on loan-level data. For example, Jimenez et al. (2012) demonstrate that "weak banks" with lower capital or liquidity ratios are less likely to grant a loan to a given applicant than stronger banks.[8]

Regulatory changes are another source of variation that can be exploited.[9] In this regard, the UK regime of discretionary changes in individual capital requirements provides a good testing ground, which recent papers have started to exploit. Building upon the data set in Bridges et al. (2014), we also consider this regime. While previous studies found a negative lending response on average

[8]Other papers studying bank responses to a shock to their capital include Peek and Rosengren (1997), Hancock and Wilcox (1994), and Hancock et al. (1995).

[9]Another alternative approach is to exploit how a macroeconomic shock interacts with capital regulation. Behn et al. (2016) use differences in the response of risk weighting models in German banks to the collapse of Lehman Brothers to identify how a a change in a capital requirements affects credit supply.

(see for instance Francis and Osborne (2012) and Aiyar et al. (2014b)), we clarify that a bank's response depends on economic conditions. In particular, we show that, in the right circumstances, it can be positive.

The relevance of economic conditions relates to Jiménez et al. (2016) who exploit heterogeneity in the bank-level impact of the introduction and subsequent changes of a common regulatory regime to identify changes in credit supply at the bank-borrower level. The main differences with our work is that: (i) While they separately assess different episodes, we consider a stable regulatory regime, in which we observe, at the bank level, many capital requirement changes that are directly comparable; (ii) we look at the response of both lending *and* capital; (iii) and we estimate these changes at the bank level. Consistent with our results, they find that bank behaviour differs depending on prevailing economic conditions. However, their main result is that it is the ability of firms to substitute between banks that differs over time, whereas we find that banks' own response are also very different.

Finally, in our model, there is no reason for banks to hold a voluntary capital buffer that would bring their capital ratio above the requirement. In fact, we could allow for voluntary buffers. What really matters for our analysis is that banks meet a change in requirement with an equivalent change in the capital ratio. We refer to this as the capital requirement being *essentially binding* and we show that, if this is the case, the slopes of the capital and lending responses are identical, and that the difference in intercepts equals the inverse of the initial capital ratio.[10] We use these novel empirical predictions to test the validity of our model. We find strong support for the essentially-binding requirement hypothesis: allowing for 2-3 quarters of transition, we find that, on average, a bank just replenishes its voluntary capital buffer following an increase in the

[10]That capital requirements are essentially binding is supported by the findings of the papers on UK data cited above. Some previous studies (see, for example, Flannery and Rangan (2008) and the discussion in Allen et al. (2011)) have found that requirements are a relatively unimportant determinant of a bank's choice of capital structure. However, as argued forcefully by Admati et al. (2013), bankers have reacted negatively to tighter capital regulation since the crisis, which in itself is evidence that capital regulation is relevant. Furthermore, better identified, recent studies relying on natural experiments and microdata show that capital regulation has an impact on bank behaviour (see Behn et al. (2016) and Jiménez et al. (2016)).

requirement (and goes no further).

2 Theory

2.1 The model

There are three dates, 0, 1, and 2. There is a bank and a continuum of households who own the bank's liabilities. Households are risk neutral and do not discount the future, they have an opportunity cost of funding of 1. We focus on the date-1 decision of the bank. The random variable A captures the realised state of the economy at date 2. It is distributed according to a function $f(A)$ with support $[A_L, A_H]$. Its date-1 expected value is μ.

Predetermined variables. Predetermined variables can be thought of as resulting from date 0 decisions. As of date 1, there are existing loans that will mature at date 2 and cannot be sold before then. We often refer to these as *legacy* loans. Their face value is z, and their date-2 payoff, denoted $Z(A)$, is increasing in the state of the economy ($Z_A(A) \geq 0$). The concept of bank capital that is relevant for our analysis is the book value of capital. Its value at the beginning of date 1 is denoted by e.

Decision variables. The bank chooses how much capital to issue. Seasoned capital is denoted s, and the corresponding date-2 total repayment is denoted S. This repayment is determined in equilibrium and can be contingent on any realised variable (see details and interpretation below). The bank also chooses how much to pay as an initial dividend d. Finally, the bank decides how much to lend. We denote the total amount of new lending by $x \geq 0$. For simplicity, new loans also mature at date 2. Their return is given by a function $X(A, x)$, which is increasing and strictly concave in x and strictly increasing in A; hence depends on the state of the economy. We assume that $E[X_x(A, 0)] > 1$, so that there are always positive net-present-value lending opportunities. Finally, $X_x(A, x)$ is non decreasing in A.

Deposit taking and the capital requirement. The bank can raise insured deposits. Assuming, without loss of generality, that the bank does not hold cash, the amount of deposits needed to fund an amount of lending x is $x + z - e - s + d$. Given deposits are insured, they pay a zero interest rate. There is no insurance premium, but the bank faces an exogenous capital requirement constraint that takes the form:

$$e + s - d \geq \gamma(x + z), \tag{1}$$

where $\gamma \in (0, 1)$ is a parameter (which we refer to as the *requirement*) set by the regulator, and where $x + z$ is the total value of the loans maturing at date 2, and $e + s - d$ is the bank's total capital at date 1. The requirement can be met with any type of capital, what matters is that capital is junior to deposits and will, therefore, absorb losses. To be allowed to operate at date 1, the bank must satisfy the capital requirement. If the bank does not satisfy the requirement, the regulator shuts down the bank. In this case, initial shareholders walk away with 0 and we impose $x = s = d = 0$.

Date-2 default. Denoting V the net worth of the bank at date 2, we have:

$$V \equiv X(A, x) + Z(A) - \underbrace{(x + z - e - s + d)}_{deposits}.$$

Because of limited liability, the corresponding total value of bank capital (from the point of view of shareholders and investors) is $\max\{0, V\}$. When $V < 0$, the bank defaults on deposits and no payment to any other liability is allowed. In this case, depositors are made whole by the government, which funds the transfer though a lump-sum tax on households.

Seasoned capital. The bank's contingent repayment to investors in seasoned capital is bounded below by 0 (the investors have limited liability) and above by V (deposits are senior and initial shareholders have limited liability). That is:

$$0 \leq S \leq \max\{0, V\}. \tag{2}$$

We assume that investors act competitively and that they have deep enough pockets, so that, in equilibrium, they just break even in expectation. That is:

$$E[S] = s. \tag{3}$$

We do not restrict seasoned capital to be a particular form of security. In practice, one can for instance think of it as seasoned equity or subordinated debt.[11] Note, finally, that, even though they all refer to households, we use different terms for holders of different bank-issued liabilities. Initial shareholders own the initial equity, investors hold seasoned capital, and depositors hold deposits.

2.2 Setting up the analysis

2.2.1 Initial shareholders' payoff

Assuming the bank operates (i.e. is not shut down by the regulator), the expected payoff to (or expected final *wealth* of) initial shareholders is:

$$w \equiv E^+[V - S] + d,$$

where $E^+[Y]$ denotes $E[\max\{0, Y\}]$.

Since $0 \leq S \leq \max\{0, V\}$, we have that $S = 0$ in states where $V \leq 0$. Hence, we have:

$$E^+[V - S] = E^+[V] - E[S].$$

Since seasoned capital investors' must break even, we have:

$$w = E^+[V] - (s - d).$$

This makes clear that, in our model, initial shareholders of a bank that op-

[11] For subordinated debt, the interest payment should compensate the loss of capital when the bank goes bust. Assume the bank stays solvent with probability π, then the expected return for the subordinated debt holders in these states should be $\frac{1}{\pi}$. In the case of seasoned equity, the logic is the same, but the mapping goes as follows: at date 1 the bank starts with e shares and issue e' additional shares at a unit price p in exchange of $s = e'p$. This gives investors the right to a payoff of $S = \frac{e'}{e+e'} \max\{0, V\}$. Hence, their break-even condition is $p = E\left[\frac{\max\{0,V\}}{e+e'}\right]$.

erates are indifferent between a continuum of combinations of s and d. Without loss of generality, we can therefore focus on net capital issuance: $s_{net} \equiv s - d$.[12]

Let us define w^* as the maximum expected payoff to the initial shareholders if the bank operates. That is,

$$w^* \equiv \max_{x \geq 0, s_{net}} w \tag{4}$$

$$\text{subject to: } e + s_{net} \geq \gamma(x + z).$$

If $w^* < 0$, initial shareholders are better off letting the regulator shut down the bank, in which case their payoff is 0.

2.2.2 Assumptions

For the purpose of our analysis, it is convenient to restrict the problem as follows.

The bank operates We are not interested in the cases where the regulator shuts down the bank. Therefore we assume that: $w^* \geq 0$.[13]

The capital requirement is binding If the bank is fully safe in equilibrium, the capital ratio is irrelevant (as in the Modigliani and Miller Theorem). In this case, the bank is locally indifferent between a continuum of mix of capital and deposits. For most of the analysis, we focus on the cases where the bank defaults at date 2 with strictly positive probability in equilibrium. In these cases, the capital requirement always binds because, from the bank's point of view, deposits are implicitly subsidised (depositors always break even, but sometimes at the expense of the taxpayer).

[12]If the regulator shuts down the bank, we have: $s_{net} = 0$.

[13]A sufficient condition for $w^* \geq 0$ is $e \geq \gamma z$.

2.2.3 The simplified problem of the bank

Assuming that the bank operates and that the capital requirement binds, we can define initial shareholders' payoff as a function of new lending:

$$w(x) \equiv E^{+}\left[X\left(A, x\right) + Z(A) - \left(1 - \gamma\right)\left(x + z\right)\right] - \gamma(x + z) + e.$$

Then, the bank's problem boils down to finding:

$$x^{*} \equiv \arg\max_{x \geq 0} w(x)$$

Equilibrium definition For a given vector of predetermined variables $\{e, z\}$, a given distribution for A, and a given capital requirement γ, an equilibrium is a pair $\{x^{*}, s^{*}_{net}\}$, where x^{*} is defined above and $s^{*}_{net} = \gamma(x^{*} + z) - e$.

2.3 Analysis: the effect of the capital requirement

The main research question of this paper is how capital requirements affect banks lending and capital decisions. In this section, we assume that x^{*} is pinned down by the first order condition and propose a comparative statics analysis based on it.[14] We first focus on equilibrium lending, as, given lending, equilibrium capital can always be backed out using the capital requirement given the assumptions above.

[14]Because the expectation term in $w(x)$ is truncated, due to limited liability, there may be knife-edge cases were x^{*} is not pinned down by the first order condition. In the numerical analysis that follows, we have verified that, in all cases, the first order condition was indeed pinning down the global maximum.

2.3.1 The wedge in the first order condition

It is useful to rewrite the initial shareholders' payoff $w(x)$ as:

$$w(x) = \underbrace{E\left[X\left(A,x\right) + Z(A) - (x+z)\right]}_{\text{economic surplus}} + \underbrace{\int_{A_L}^{A_0(x)} \left(\left(1-\gamma\right)\left(x+z\right) - X\left(A,x\right) - Z(A)\right) f(A)dA}_{\text{option value}} + e,$$

where $A_0(x)$ is the threshold realization for A below which the bank is insolvent, that is $\{A \mid V(x) = 0)\}$, if such an A exists. Otherwise, $A_0(x) = A_L$.

The first term is the economic surplus. The second captures the value of the bank's shareholders' put option, which arises from limited liability.[15] The term captures, in expectation, the shortfall in the bank's value compared to what is needed to repay depositors. Whenever there is such a shortfall, the net worth of the bank $V(A)$ is negative, but the bank's shareholders can walk away with zero. Therefore, it is as if they could sell for a price of zero something that has a negative value; hence the interpretation in terms of a put option (see Merton, 1977). The distortion occurs in the model because the value of this option is not priced in by the depositors as they are insured.

The first order condition with respect to x is:

$$E\left[X_x\left(A,x\right) - 1\right] + \underbrace{\int_{A_L}^{A_0(x)} \left(\left(1-\gamma\right) - X_x\left(A,x\right)\right) f(A)dA}_{\text{wedge}} = 0. \tag{5}$$

The first term is the derivative of the economic surplus with respect to x, and the second is the derivative of the option value. The latter term, therefore, constitutes a wedge against a natural efficiency benchmark: economic surplus maximisation. This term is a key object in our analysis. We refer to it as *the wedge in equation* (5) or, simply, *the wedge*. As we shall explain, it captures the interplay between risk-shifting incentives and debt overhang considerations. Even though it can have a very intuitive interpretation in special cases, this is

[15] Note that in the absence of deposit insurance the second term would be exactly offset by a term capturing the interest payments reflecting the riskiness of deposits.

not true in general. For now, it is however useful to note that equilibrium lending is increasing in the wedge. In particular, a positive wedge reflects over-lending (the bank funds negative net-present-value loans), and a negative wedge reflects under-lending (not all positive net-present-value loans are funded).

2.3.2 Decomposing the effect of γ on the wedge

The probability that the bank does not default at date 2 is

$$\pi \equiv \int_{A_0(x)}^{A_H} f(A)dA.$$

We can then rewrite the first order condition as

$$\underbrace{\frac{\int_{A_0(x)}^{A_H} X_x(A,x)f(A)dA}{\pi}}_{\equiv \overline{mr}_+} = \underbrace{(1-\gamma) + \frac{\gamma}{\pi}}_{\equiv \overline{mc}_+}. \tag{6}$$

This is a useful way to look at the optimality condition because it equates the *expected* marginal return on new loans and the bank's *expected* marginal cost of funds *conditional on not defaulting*. Given limited liability, the bank ignores the marginal effect of its decisions in states where it defaults (because its payoff is always 0 in these states). For simplicity, we refer to $\overline{mr}_+$ as the the marginal return and to $\overline{mc}_+$ as the marginal cost, but *it is important to keep in mind that these are conditional expectations.*

Assuming an interior solution, a change in γ affects equilibrium lending, x^*, as follows:

$$\frac{dx^*}{d\gamma} = \frac{\frac{\partial \overline{mc}_+(\gamma,x^*)}{\partial\gamma} - \frac{\partial \overline{mr}_+(\gamma,x^*)}{\partial\gamma}}{w_{xx}(\gamma,x^*)}. \tag{7}$$

Since the change in economic surplus does not depend on the capital requirement, the effect of γ on x^* goes entirely through the change in the wedge. Note also that the denominator of equation (7) is always negative since we assume that x^* is an interior maximum. Hence, the sign of the lending response is determined by the numerator. If the numerator is positive, an increase in γ is associated with a cut in lending.

Effect through the marginal cost The marginal cost $1-\gamma+\frac{\gamma}{\pi}$ reflects that the marginal loan is financed with $1-\gamma$ of deposits and γ of capital. When the bank does not default, it repays depositors one for one. However, it must compensate investors for the losses they make in case of default. For investors to break even, they must make a conditional expected return of $\frac{1}{\pi}$.

The effect on the marginal cost of a change in γ can be decomposed in two effects:

$$\frac{\partial \overline{mc}_+(\gamma, x^*)}{\partial \gamma} = \underbrace{\left(\frac{1}{\pi}-1\right)}_{\text{composition effect}} - \underbrace{\gamma \frac{\partial \pi(\gamma)}{\partial \gamma}\frac{1}{\pi^2}}_{\text{price effect}} .$$

First, increasing γ forces the bank to substitute deposit for capital. Since capital is the more expensive form of finance, this increases the marginal cost. We refer to this effect as the funding *composition effect*.

But a change in γ also affects the probability that the bank defaults, which in turn affects the equilibrium unit cost of capital. Keeping x constant, a higher capital requirement makes the bank safer because it is forced to operate with a larger buffer against losses. Hence, this makes capital cheaper and reduces the marginal cost. We refer to this effect as the capital unit *price effect*.

Effect through the marginal return The relevant distribution of A is a truncation, from below, of its unconditional distribution as limited liability makes the states in which the bank defaults irrelevant. This means that the expectation of A conditional on not defaulting is increasing in the probability of default. Put differently, the more states the bank defaults in, the better, in expectation, are the states it does *not* default in.[16] Now, this implies that, for a given x, an increase in γ decreases the marginal return.[17] This is because a higher require-

[16] To see this, consider a simple example where A takes the value of 10, 20, or 30, with equal probability. Its unconditional expectation is 20. Imagine that the bank fails if and only if $A=10$. Then, the expectation of A conditional on not defaulting is 25. If the bank also fails when $A=20$, it becomes 30.

[17] Our assumption that $X_x(A,x)$ is non decreasing in A plays a role here. Without it, it is possible to construct a function $X(A,x)$ such that, locally, the effect through the marginal return

ment makes the bank safer (as it operates with a larger buffer against losses). Hence, a higher γ makes the bank internalise more of the downside risk, which is why we call this effect the return *internalisation effect*.

The denominator The denominator is the second derivative of w with respect to x, evaluated at (γ, x^*) . While the sign of $\frac{dx^*}{d\gamma}$ only depends on the numerator, its magnitude also depends on the denominator, which captures the local curvature of the objective function.

2.3.3 Example 1: isolating the effects through the marginal cost

To understand how the capital requirement affects the equilibrium marginal cost, it is convenient to assume that new lending is safe. In this case, the marginal return does not depend on the state (formally: we assume $X(A,x) = X(\mu, x),\ \forall A$, where μ is a constant). This is sufficient to shut down the internalisation effect.

We first establish that the bank does not finance all positive net present value loans because of what is essentially an overhang problem. Then, we show that the composition effect worsens the problem and that the price effect mitigates it.

Proposition 1. *(Overhang problem) Assuming that new lending is safe, if the bank defaults with strictly positive probability in equilibrium, the wedge in (5) is strictly negative. Hence, the bank does not finance all the positive net present value loans (i.e. $x^* < x_1$, where x_1 denotes the level of lending that maximises economic surplus).*

Proof. All the proofs are in Appendix A. □

If the bank had no legacy loans, it would simply choose $x^* = x_1$, and the bank itself would be perfectly safe ($\pi = 1$). But with existing risks on the balance sheet, the situation is different. Imagine the bank chooses $x^* = x_1$ in equilibrium and defaults with strictly positive probability $1 - \pi$ (default will be caused by a low return on legacy loans). Then, the surplus generated by all new loans is only

has the opposite sign (see Supplementary Appendix SA1 for details).

captured by the shareholders with probability π: when the bank defaults, the surplus is used to repay depositors and capital is wiped out. Even though the mechanisms are different, the main logic is similar to the classic debt overhang model (Myers (1977)).[18]

As we have explained above, the banks' marginal cost, conditional on not defaulting, is $(1-\gamma)+\frac{\gamma}{\pi}$, which is strictly greater than 1 if $\pi<1$. This implies that the marginal return must also be greater than 1 in equilibrium. That is $X_x(A,x^*)>1$. Hence, we get under-lending: $x^*<x_1$, which is equivalent to saying that the wedge is negative:

$$\int_{A_L}^{A_0(x)} \left((1-\gamma)-X_x\left(A,x^*\right)\right) f(A)dA<0$$

In this example, the wedge has an intuitive interpretation. It captures, at the margin, the decrease in the expected amount of tax that will be needed to make depositors whole. In other words, this is a transfer to the taxpayer that reduces the private marginal surplus from lending.

Varying the capital requirement affects the wedge and, therefore, equilibrium lending. An increase in the wedge implies higher equilibrium lending, and vice versa.

The composition effect As we have explained above, for a given $\pi<1$, an increase in γ increases the marginal cost of lending. This makes the overhang problem worse: the bank contracts lending even further. And indeed, keeping π, and therefore $A_0(x)$, constant, the wedge is decreasing in γ.

The price effect Holding new x constant, raising γ forces the bank to operate at a larger buffer against legacy asset losses. Hence, it decreases $A_0(x)$, which

[18]In a textbook debt overhang model, the interest rate is set in a first period (which would correspond to date 0 in our model). Then, a new investment opportunity arises. If it is seized, it affects the probability of default of the firm, which typically implies a wealth transfer from shareholders to creditors. A key assumption is that the interest rate cannot be adjusted. In our model, interest rates are determined at date 1, but they do not adjust to default risk because deposits are insured.

means that the bank is safer (π increases). To see this, assume that $Z(A)$ is invertible to get the expression for $A_0(x)$ when it is greater than A_L:

$$A_0(x) = Z^{-1}\left((1-\gamma)(x+z) - X(\mu, x)\right),$$

and note that $Z^{-1}(.)$ is increasing. To simplify notation, from here on we omit the dependence of A_0 on x.

Hence, raising γ reduces the marginal unit cost of capital and increases equilibrium lending. By generating more surplus in all states, this feeds back into the probability of not defaulting and therefore adds a second round of decrease to the marginal cost.

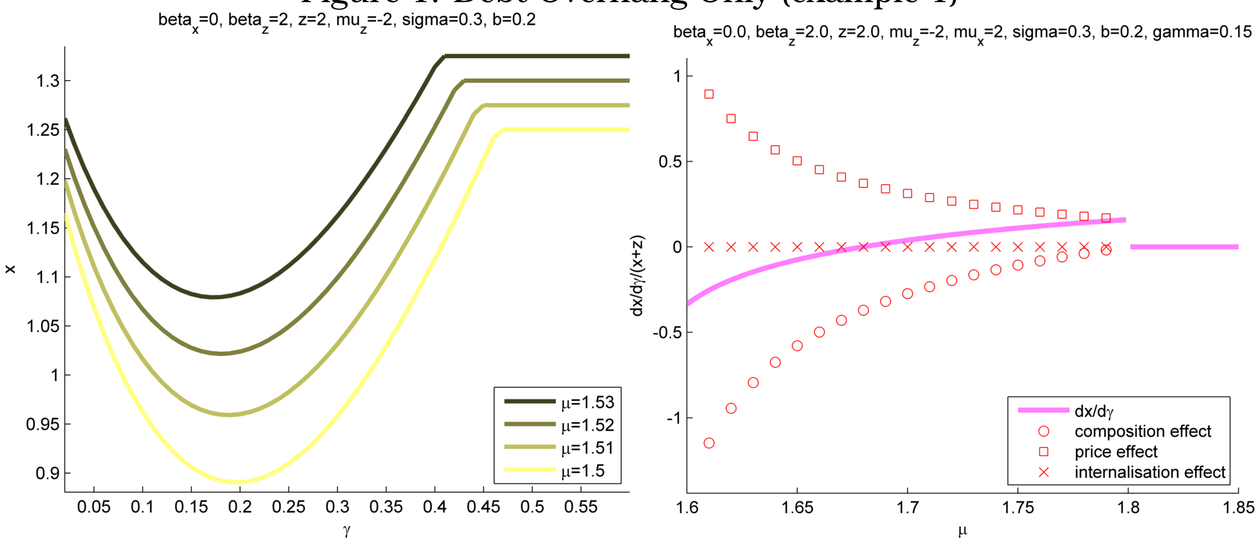
Numerical solution Unfortunately, first order condition (5) can generally not be solved for x in closed form.[19] Therefore we complement our analytical results with a numerical analysis.

Our numerical approach is described in detail in Supplementary Appendix SA3. In short, the starting point is to pick functional forms for $X(A, x)$ and $Z(A)$, with an associated set of parameters. The main idea is to pick these functional forms so that A can affect positively both the payoffs of new and legacy loans, but not necessarily in the same way. We calibrate the parameters to provide examples that are *qualitatively* representative of bank behaviour in the region where it chooses to operate. However, we leave *quantitative* results to our empirical analysis.

Which effect dominates and under which conditions? The left panel of Figure 1 displays the optimal level of lending x^* as a function of γ, for different values of μ. For each of the four levels of μ that we consider, there is a level of γ from which the bank is safe and the line is flat at the level that maximises economic surplus. At lower levels of γ, the bank defaults with strictly positive probability in equilibrium, and lends less (due to the overhang problem). As we can see, $x^*(\gamma)$ is U-shaped: it is initially sharply decreasing, but as γ increases,

[19]Expectations are truncated at A_0 which is itself not generally defined in closed form.

Figure 1: Debt Overhang Only (example 1)



Notes: This figure presents the results from our numerical solutions when new lending bears no risk. The left panel plots the relationship between x^* and γ for different levels of μ. The right panel plots the lending response, that is the relationship between $\frac{dx^*}{d\gamma}$ (scaled by total loans) and μ, and its decomposition into the three effects. For calibration, we set the distribution of A to a uniform with support: $(\mu - \sigma, \mu + \sigma)$. We select the following functional forms for the returns: $X(A, x) = \mu x - bx^2$ and $Z(A) = (\mu_z + \beta_z A)z - bz^2$. The calibration of the parameters is recorded at the top of the graphs.

it flattens and then becomes increasing until there is a kink and it becomes flat. Hence, depending on the initial level of γ, the lending response (i.e. $\frac{dx^*}{d\gamma}$) can be either positive or negative. Finally, changes in μ shift the U-shape. This means that, at a given initial level of γ, changes in μ affect the lending response $\left(\frac{dx^*}{d\gamma}\right)$.

The right panel of figure 1 explores this relationship further. In particular, the solid line plots $\frac{dx^*}{d\gamma}$ (scaled by the total amount of loans) as a function of μ for a given value of γ. We can see that this function is increasing (up to the point where the capital requirement becomes irrelevant). That is, starting from a case where an increase in γ is met by a lending cut, increasing μ increases the lending response (i.e. reduces the size of the cut). If μ is high enough, the response can even be positive (i.e. an increase in γ can even increase equilibrium lending).

The right panel also shows the decomposition of the lending response . We can see that a low μ is associated with both a strong composition effect (this is because π is low) and a strong price effect (π is more reactive to γ when the debt overhang is severe). Still, the composition effect dominates and the lending response is negative. As μ increases, both effects weaken and the lending response increases. At some point, the lending response becomes positive and keeps on increasing until it jumps to zero as we reach the point of (Modigliani and Miller) irrelevance. This means that high expected returns are a necessary condition for the price effect to dominate the composition effect. Assuming the relevant limit exists, we can establish the related analytical result:

Proposition 2. *(Positive lending response) Assuming that new lending is safe, if the bank defaults with strictly positive probability in equilibrium, then, as* $\pi \to 1$, $\frac{dx^*}{d\gamma} > 0$*. That is, the price effect dominates the composition effect and the wedge in (5) is increasing in* γ*.*

Remark 1. Proposition (2) considers the limit of an endogenous variable rather than conditions on the model's primitive. This is simply a compact way to summarize the similar effect changes in various primitives would have without have to choose functional forms for $X(x, A)$, $Z(A)$, and $f(A)$. In general, changes in parameters that raise expected returns will increase π in equilibrium. Similarly, as our numerical example illustrates, a sufficient increase in γ will typically raise π to values arbitrarily close to 1.

2.3.4 Example 2: incorporating the effect through the marginal return

When new lending is risky, the marginal return (conditional on not defaulting) is affected by the capital requirement because of limited liability. This part of the reason why mispriced government guarantees and limited liability generate an implicit subsidy that, typically, leads to risk-shifting (which materialises here through over-lending). Assuming away any overhang problem helps clarify how this mechanism works and how it is affected by the capital requirement.

Proposition 3. *(Risk-shifting) Assume* $Z(A_L) > (1 - \gamma)z$*. Then, there is no overhang and the wedge in (5) cannot be strictly negative in equilibrium. If the bank defaults with strictly positive probability in equilibrium, the wedge is strictly positive. Hence, the bank finances negative net present value loans (i.e.* $x^* > x_1$*).*

If $Z(A_L) > (1 - \gamma)z$ then legacy loans are good enough such that, in the worst case, the losses on these loans do not exceed the capital required to hold them.

This rules out any overhang problem: the bank will fund all the positive-net-present-value loans in equilibrium. Hence, the wedge must be weakly positive in equilibrium.

But the bank can still default because new loans are risky. When this happens with strictly positive probability in equilibrium, we get over-lending ($x^* > x_1$), which corresponds to a strictly positive wedge:

$$\int_{A_L}^{A_0} ((1-\gamma) - X_x(A, x^*)) f(A) dA > 0.$$

The bank finances negative-net-present-value loans because it does not fully internalise the down-side risk on new loans, due to the taxpayer footing the bill in the event of default.

In this case too, the wedge has an intuitive interpretation: it measures here, for the marginal loan, the expected losses that are shifted to the taxpayer. This transfer goes in the opposite direction than in Example 1. The wedge is positive because in all the states where the bank defaults $X_x(A, x^*) < 1 - \gamma$. The reason for this is that if the realised return on the marginal loan were enough to cover the $1 - \gamma$ unit of deposits issued to finance it, the bank could not default in the corresponding state. In contrast, this is not the case in Example 1 where default is caused by a low return on legacy loans rather than new lending.

In this second example, increasing the capital requirement forces the bank to internalise more of the downside risk, which decreases the wedge and, therefore, equilibrium lending.

Proposition 4. *Assume $Z(A_L) > (1-\gamma)z$. If the bank defaults with strictly positive probability in equilibrium, we have $\frac{dx^*}{d\gamma} < 0$. That is, the wedge in (5) is strictly decreasing in γ.*

This result also highlights that an overhang problem is necessary to have a positive lending response $\left(\frac{dx^*}{d\gamma} > 0\right)$. When we combine both sources of risk, the wedge becomes a rather subtle object as it blends the losses on the marginal loan that are shifted onto the taxpayer with an opposite transfer due to the overhang problem.

2.3.5 General case: combining risk-shifting and overhang

Even though the wedge itself is difficult to interpret in the general case, our numerical analysis still allows us to decompose the contribution of the three effects. We provide an example in Figure 2. First, consider the right panel. Compared to the case where new lending is safe (see Figure 1), the key difference is the presence of the internalisation effect. As we can see, it is negative, but it is relatively insensitive to μ. As a result, the overall shape of the lending response $\left(\frac{dx^*}{d\gamma}\right)$ is similar to that in Figure 1, but the internalisation effect shifts the lending response down. This means that in the general model it is less likely that the lending response is positive. As the left panel shows, both under- and over-lending are now possible. In other words, the wedge can be negative or positive (respectively) in equilibrium. An interpretation of the overall shape is that overhang problems tend to make the relationship U-shaped, and risk-shifting tends to steepen the slope, especially at low values of γ.

Last but not least, a general conclusion from our numerical exercise is that changes in μ shift the U-shape and affect the lending response. We delve into this point in Section 3 below, where we lay the ground for our empirical analysis.

Finally, we have investigated how changes in σ affect $\frac{dx^*}{d\gamma}$. Essentially, they produce the opposite effect to those produced by changes in μ: when loans become more risky, $\frac{dx^*}{d\gamma}$ decreases. That is, if we are in a region of the parameter space where increases in γ generate a cut in lending, an increase in the riskiness of fundamentals magnifies such a response.

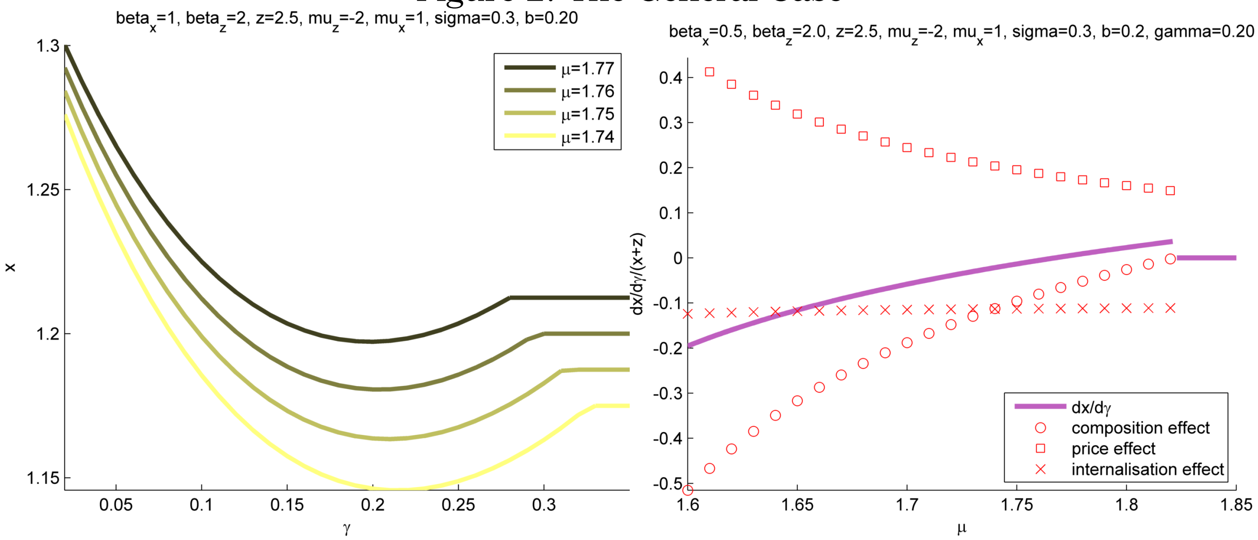
2.4 The capital response

Given optimal lending x^*, we can back out the optimal level of capital from the binding capital requirement.

$$\underbrace{s^*_{net} + e}_{\equiv e^*_{net}} = \gamma(x^* + z). \tag{8}$$

The main takeaways from the numerical analysis are straightforward: (i) typ-

Figure 2: The General Case



Notes: This figure presents results from our numerical solutions when both new lending and legacy loans are risky. The left panel plots the relationship between x^* and γ for different levels of μ. The right panel plots the lending response, that is the relationship between $\frac{dx^*}{d\gamma}$ (scaled by total loans) and μ, and its decomposition into the three effects. For each calibration, we set the distribution of A to a uniform with support: $(\mu-\sigma, \mu+\sigma)$. We select the following functional forms for the returns: $X(A,x) = (\mu_x + \beta_x A)x - bx^2$ and $Z(A) = (\mu_z + \beta_z A)z - bz^2$. The calibration of the parameters is recorded at the top of each graph.

ically e^*_{net} is increasing in γ. The bank must hold more capital per unit of new lending. Since the lending response can be negative, total capital held against new lending can go down. However, the bank must also raise capital against legacy loans. For any sensible calibration we get a positive capital response. (ii) the capital response is typically increasing in μ. In fact the slope of the capital response is proportional to that of the lending response. Taking derivatives with respect to γ on both side of equation (8) above, we get:

$$\frac{de^*_{net}/e^*_{net}}{d\gamma} = \frac{1}{\gamma} + \frac{dx^*/(x^*+z)}{d\gamma}. \tag{9}$$

That is, there is a linear relationship between the capital response and the lending response. Furthermore, to the extent that the capital requirement is not affected by μ, we have:

$$d\left(\frac{de^*_{net}/e^*_{net}}{d\gamma}\right)/d\mu = d\left(\frac{dx^*/(x^*+z)}{d\gamma}\right)/d\mu. \tag{10}$$

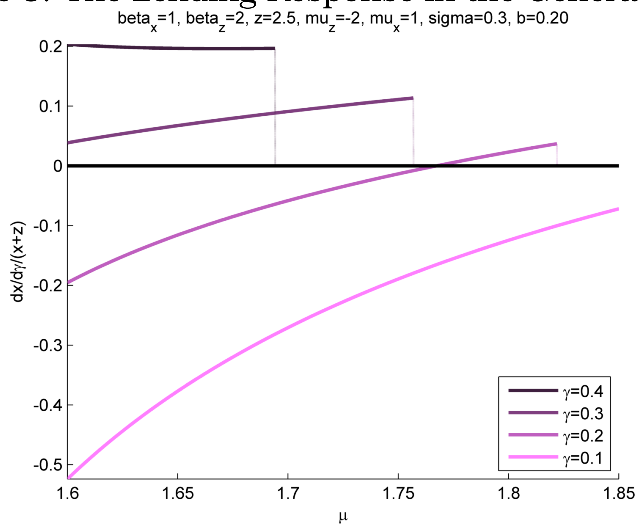
Hence, the effect of μ on the capital and lending responses is identical if we scale the lending response by total lending (this is what we do in the figures above).

3 Taking the model to the data

3.1 The importance of μ

We first elaborate on how μ affects the lending response $\left(\frac{dx^*}{d\gamma}\right)$. The right panels of Figures 1 and 2 display examples where the lending response is initially negative, it increases with μ, and becomes positive before finally jumping to zero at the point of (Modigliani-Miller) irrelevance. We find these examples particularly interesting, but all such features are not consistent across calibrations. To illustrate this, Figure 3 shows the same calibration as in Figure 2 (which corresponds to $\gamma = 0.2$) together with three others. As we can see, the lending response is typically increasing in μ. However, at relatively large values of γ, the lending response becomes flatter and can even become downward sloping. This latter case can happen when legacy loans are very bad but the bank is massively capitalised (note that $\gamma = 0.4$ in the corresponding example in Figure 3), so the probability of default is still low (and the overhang problem limited). Such a case is extreme in the sense that the range of μ for which the bank chooses to operate and defaults with positive probability is quite restricted. Whether it is relevant or not and, more generally, how the complex interaction of the mechanisms we have highlighted play out in reality, is ultimately an empirical question, which is the starting point of what follows. To give context to our empirical results it is useful to interpret μ : in the model it is the expected realisation of the state of the economy in the following period. In this regard we view it as capturing *economic prospects* (which we use as a synonym for μ).

Figure 3: The Lending Response in the General Case



Notes: This figure results from our numerical solutions when both new lending and legacy loans are risky. The figure plots the lending response, that is the relationship between $\frac{dx^*}{d\gamma}$ (scaled by total loans) and μ for alternative levels of γ. For calibration, we set the distribution of A to a uniform with support: $(\mu - \sigma, \mu + \sigma)$. We select the following functional forms for the returns: $X(A, x) = (\mu_x + \beta_x A)x - bx^2$ and $Z(A) = (\mu_z + \beta_z A)z - bz^2$. The calibration of the parameters is recorded at the top of each graph.

3.2 Empirical tests

3.2.1 Test 1

Our first test consists of estimating the slope (and intercept) of the lending and capital responses. That is, we estimate the relationship considered in Figure 3, and the equivalent for capital. Given the discussion above, we expect both responses to be increasing with economic prospects. Focusing on lending, the economic interpretation for this would be that, when economic prospects are weak, a bank responds to an increase in its individual capital requirement with a cut in lending. The worse the economic prospects the more severe the cut.

Would this prediction arise in models where capital requirements are binding due to different violations of the conditions for Modigliani and Miller's irrelevance theorem? In our model, the theorem is violated because deposits are insured (which is how we capture explicit and implicit government guarantees). Two other common departures from Modigliani-Miller that typically make capital requirements bind are: (i) tax deductible interest repayments on debt; and (ii) bank debt attracting a convenience premium (because it has money-like properties).

As we show in Supplementary Appendix SA2, in a model where interest payments are tax deductible, the lending response simply depends on the level of the nominal interest rate. If the Central Bank follows a forward looking Taylor rule with a positive coefficient on economic prospects, which seems reasonable assumption, the interest rate will also be increasing in economic prospects. This implies that the lending response is downward sloping in μ, which is the opposite of our prediction.

In a model with a convenience premium, if the premium is greater when prospects are weak, the prediction in terms of slope of the lending response would be similar to ours. The mechanism can be thought of in terms of equilibrium *price* and *composition effects*, as in our model, except that the price that changes is that of deposits rather than capital. To the best of our knowledge, there has not been a thorough investigation of the time series properties of the premium in the literature. Some works (Krishnamurthy and Vissing-Jorgensen (2012) and Begenau (2015)) assume that money-like instruments have more value when transactions are high. *Ceteris paribus* this demand effect implies a procyclical premium (which generates a prediction opposite to ours). Still, supply effects could overturn this, but we leave this as a question for future research.[20]

3.2.2 Test 2.

Our second test, related to the first, is to assess how likely it is that the lending response is positive. The theory tells us that this requires a combination of the bank having a mild overhang problem and strong enough prospects, so that the bank probability of default is low. Neither of the two alternative models described above can generate a positive lending response. Hence test 2 is a useful for discriminating between models.

[20]See van den Heuvel (2008) and Begenau (2015) for analysis of the effects of changes in aggregate capital requirements in this contexts.

3.2.3 Test 3.

Our third test relates to our analytical results linking the capital response to the lending response. Specifically, we test whether relationships (9) and (10) are borne out by the data. That is, we test, first, whether the slopes of the capital and lending responses are identical, and, second, whether the difference in their intercepts equals the inverse of the initial capital ratio. The economic interpretation of these tests is that the relative response between capital and lending is consistent with the bank's capital requirement being essentially binding, which is important for the validity of our theoretical analysis. What we mean by essentially binding is that banks meet a change in requirement with an equivalent change in the capital ratio. This will be the case if banks hold voluntary *buffers* for reasons outside of the model; for instance to avoid an accidental breach of the requirement (Rochet 2005; Milne and Whalley 2001). In this case, we have:

$$e^*_{net} = (\gamma + \text{buffer})(x^* + z).$$

If the buffer does not depend on the level of capital requirement, we have that:

$$\frac{de^*_{net}/e^*_{net}}{d\gamma} = \frac{1}{\gamma + \text{buffer}} + \frac{dx^*/(x^* + z)}{d\gamma}, \tag{11}$$

which means that Relationship (9) holds and, if the buffer does not depend on μ, Relationship (10) holds too.

These relationships have important implications for the interpretation of our empirical results. First, a necessary condition for relationship (10) to hold is if individual capital requirements are independent of μ.[21] If the relationship is not rejected by the data, this is reassuring in terms of identification. Second, our model also abstracts from a series of factors that would affect the relationship between e^*_{net} and x^*. For instance, different categories of loans have different risk weights. If the composition of the bank loan portfolio depends on economic prospects, Relationships (10) and (9) break down. We also ignore loans to finan-

[21] An exception would be if how the buffer responds to μ perfectly offsets how the capital requirement itself responds to it.

cial institutions and assets held for trading purposes. Again, if the proportion of these assets on the bank's balance sheet changes over time, these relationships break down. We ignore these factors for the sake of analytical tractability and because of a lack of reliable data. Test 3 is therefore also useful for assessing whether this is a valid approach.

3.2.4 An untested prediction

Testing the shape of the relationship between lending response and the level of the requirement would be interesting per se. In our model, it is typically U-shaped, but in the absence of an overhang problem, it is simply downward sloping. Unfortunately, this is not possible in practice with our data. This is due to the lack of a sample period with sufficient variation in the level of capital requirements, especially given the relatively large variation in economic prospects that one would need to control for. As we will discuss in more detail in section 4, our sample is best thought of as one where γ fluctuates with small increments but there are wide fluctuations in μ; this sample is better suited to the tests above.

4 Empirical Strategy

4.1 Institutional background: the regulatory regime

We exploit data from the period during which the first version of the Basel Accord was in effect in the United Kingdom (i.e. 1989Q1-2007Q4). This regulatory regime, dubbed Basel I, was relatively simple: bank capital was required to be at least 8% of risk-weighted assets (where the risk-weights corresponded to coarse time-invariant categories). The key feature, specific to the UK, is that the supervisor (first the Bank of England and then the Financial Services Authority (FSA)) could impose a requirement in excess of the 8% minimum. A breach of this requirement would then trigger supervisory intervention.[22] Crucially, the

[22]Alongside increased oversight, supervisory intervention can include restrictions on dividend payouts and asset growth. In extreme cases, the supervisor could shut down the institution.

supervisor had discretion and could set these requirements at different levels for different banks and could also change them over time. For the purpose of this analysis, we refer to these as the individual capital requirements, or simply the capital requirement. Banks were required to report their capital position and other relevant balance sheet variables in the supervisory returns that form the source of our data. Further details of this regulatory environment are described in Francis and Osborne (2009).[23]

4.2 Data

4.2.1 Bank data

We build upon the Bridges et al. (2014) data set. This source provides observations on a panel of UK supervised banks on a quarterly basis for our period of interest. The data set matches an individual bank's supervisory returns (i.e. filings used for microprudential purposes) which contain information on the bank's capital position including the requirement, with the individual bank monetary returns (the filings used for the purposes of constructing monetary aggregates) which include detailed information on bank lending.

Table 1 presents summary statistics for changes in individual capital requirements in our sample period. Changes in the requirements were fairly evenly balanced between increases and decreases with an average absolute size of around 50 basis points.

Figure 4 presents the distribution of changes over time. Changes happened throughout the sample period and many years saw increases as well as decreases. The concentration of changes in the late 1990s was due to the advent

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One implication of this is that while a change in the requirement came into force immediately, the supervisor could grant the bank some time to adjust before intervening.

[23]For those familiar with the details of the regime, we use the "trigger ratios" (which became the "individual capital requirements" in 2001). After 1998 the supervisor also had the option to set a separate capital requirements for the banking book and trading book separately. A drawback of the matched data set we rely upon is that the two have not been separated. Our capital requirement change is the weighted, by RWAs, average change in the requirement across the banking and trading book. Given our theory, it is appropriate to inspect the total capital requirement that the bank faces. However, we cannot do the natural robustness check of just focusing upon banking book changes.

Table 1: Summary Statistics

Breakdown of Capital Requirement Changes

Changes	Number	Average Absolute Size	Standard Deviation	Min	Max
Increases	22	59.3 bp	49.8 bp	7 bp	150 bp
Decreases	28	52.6 bp	34.0 bp	9 bp	129 bp
All Moves	50	55.5 bp	41.3 bp	7 bp	150 bp

Notes: Summary statistics on recorded changes in capital requirement changes over the period 1989-2007 for our sample of 18 UK banks. Moves less than 5 basis points in size are ignored.

Bank data summary

	Mean	Std dev.	25th %tile	75th %tile	Min	Max
Credit growth (%)	2.0	6.4	0.5	3.8	-46.5	45.7
Capital growth (%)	2.3	5.4	0.0	4.1	24.0	62.7
Capital Ratio (%)	12.2	2.2	10.9	13.0	8.0	25.3
Capital Requirement (%)	9.2	0.9	8.6	10.0	8.0	11.0
Number of Included Banks:	18					
Bank Quarter Observations:	589					

Notes: This table presents summary statistics for our filtered sample. The data is calculated at horizon 1 and is unweighted. As the horizon grows, we lose bank quarter observations (either because banks are discontinued, or changes occur to close to the end of our sample). At the longest horizon we consider (10 quarters) we have 380 observations.

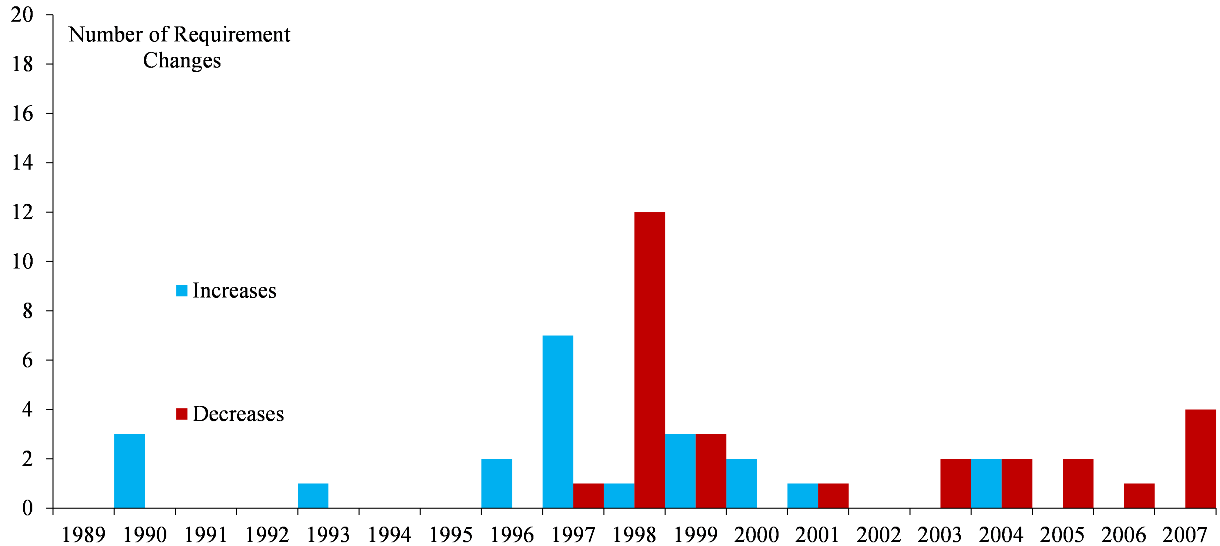
of the FSA. When we discussed this with supervisors who were in role at the time, they referred to this time as a “tidying up” period. There was a desire to eliminate any legacy inconsistencies in the way similar banks had been treated prior to the new authority taking over supervision.

Finally, the lower panel in table 1 shows the relevant summary statistics for lending and capital.

Our sample We use consolidated returns for the UK supervised portion of banking groups. This is the relevant unit of observation as data at the subsidiary bank level could be contaminated by intra-group reallocation of capital and lending. Mergers are dealt with by generating a new successor bank and discontinuing the two legacy institutions with a one quarter break in the data around the time of the merger.[24] The advantage of using lending data collected

[24]An alternative is to create synthetic merged institutions for the period before the merger, but there is then a difficulty of how to interpret a change in requirements that is only applied to one

Figure 4: Distribution of Capital Requirement Changes Through Time



Notes: The bar chart indicates the number of occasions policymakers increased (light blue bar) and decreased (dark red bar) a capital requirement on a bank in our sample in each year. Moves less than 5 basis point in size are ignored.

for monetary purposes is that it is possible to isolate a clean measure of the net flow of lending (new business less repayments on principal).[25] Changes in stocks, taken from a bank's balance sheet, can be contaminated by other effects such as write-offs and revaluations, which can lead to excessive volatility in time series data that do not reflect true lending decisions. Bridges et al. (2014) offer a detailed discussion. We consider the sum of loans to households and private non-financial firms.

We apply some filters to the sample. First, we ignore requirement changes smaller than 5bp (in absolute value) because they are likely to come from errors of recording or rounding errors in the raw data (requirements were sometimes reported in Sterling rather than in ratios). Second, we have decided to focus on larger entities with a substantial UK loan book. In particular, we drop any bank

portion of the synthetic entity. This approach would also not allow for differing behaviour of the two entities (i.e. fixed effects) prior to the merger.

[25] Bank monetary returns are included at a unconsolidated level in the raw data but when constructing the matched data set Bridges et al. (2014) aggregate the lending flows across members of the banking group.

with less than £50m credit outstanding in any quarter or any which average a market share less than 0.5% of total lending and we conduct our estimation weighting observations by the bank's share of UK lending.[26] As we show in Section 5.3, these filters do not substantially affect the results. We also drop banks with less than 8 quarters of supervisory or lending data as the number of observations is too low. This leaves us with an unbalanced panel of 18 institutions with 573 bank-quarter observations. 16 out of these 18 banks experience a requirement change during our sample period.

4.2.2 Indicator of economic prospects in our model

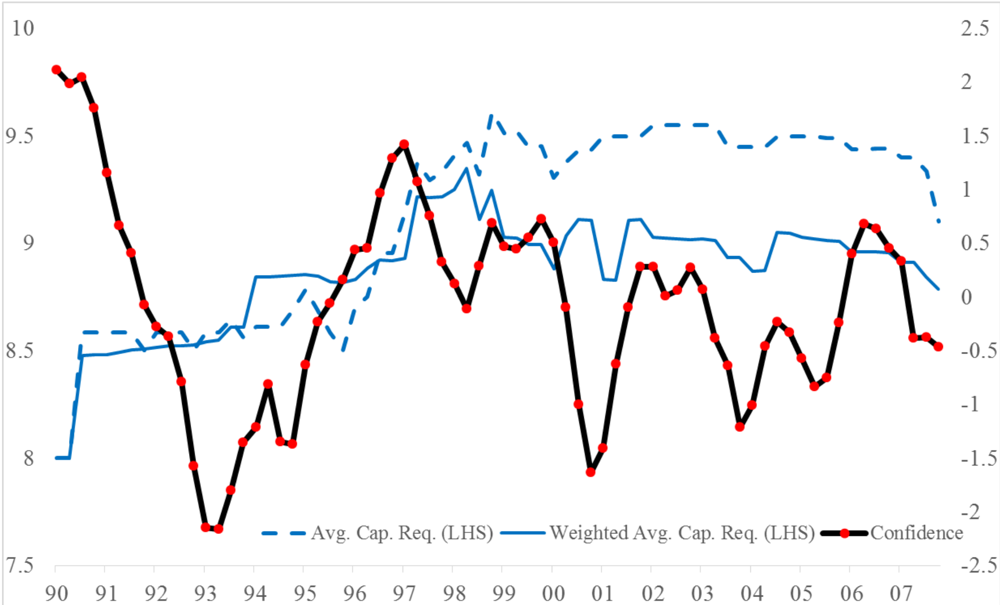
To conduct our model's empirical tests, we need a proxy for μ. That is, we need a variable that captures the expected return on the banks' existing and new loans. Unfortunately, we do not have reliable variables at the bank level that we could use. However, we believe that it is reasonable to assume that a bank's prospects are correlated with economic prospects in general. Furthermore, a change in μ in our model can be interpreted as a shift in demand for loans. This is why we have chosen to rely upon a leading indicator of the economic cycle in the UK. Specifically, we use the OECD's business confidence index for the UK (henceforth our *indicator of business confidence*).[27] The key reason for choosing this indicator is that it is forward looking, which is essential since we want to capture expected returns. Typical indicators of macroeconomic conditions, such as the output gap, capture *realised* returns (for the bank in our model, realised earnings are incorporated in date-1 initial capital e). Furthermore, an indicator based upon realised outcomes may peak after expected returns have started to fall. We do, however, show in Section 5.3 that our results also hold if we use a variety of alternative indicators.

Figure 5 plots the time series of our indicator alongside the unweighted and weighted (by lending stocks) average of the capital requirement.

[26]A larger proportion of these smaller institutions are subsidiaries of foreign banks which typically coexist with branches of the same foreign entity. The regulatory regime we study does not apply to such branches. As discussed in Aiyar et al. (2014b), lending can, and often does, switch between them and subsidiaries after a requirement change.

[27]Available at https://data.oecd.org/leadind/business-confidence-index-bci.htm

Figure 5: Business Confidence and the Average Capital Requirement



Notes: This chart shows time series plots for the simple cross-sectional average capital requirement across the banks in our filtered sample (left hand axis, as a % of risk weighted assets) both as a simple average (blue dashed line) and as a weighted average based on bank lending stock (blue solid line). On the right hand axis, we plot the OECD's UK business confidence leading indicator (black solid line, right hand axis), which is our economic prospect indicator and has been standardised by deducting the sample mean and dividing by the sample standard deviation.

First note that the requirement series appear to be unrelated to confidence (the correlations are -0.05 (unweighted) and 0.01).[28] Second, the confidence indicator has a clear cyclical pattern with an average of 9 quarters required to complete a full trough to trough cycle. While our sample period, from 1989 to 2007, encompasses a little over two UK business cycles, our confidence indicator has a higher periodicity. This is reflected by occasions where confidence falls, such as in 1997, that were not followed by the realisation of an economic recession. We have more time variation to exploit as a result, which is another advantage of using this indicator.

[28]Averaging the capital requirements over all banks in the data set (instead of our filtered sample) yield very similar correlations. A number of studies that focus on the 1998-2006 period have, however, found some positive correlation between the average requirement and other measures of economic conditions (see Aiyar et al. (2014b) for instance). Restricting our sample to the same period and allowing for phase shifts, we find a maximum correlation of 0.3 with our measure of business confidence.

4.3 Econometric Methodology

Our goal is to estimate the difference between actual bank behaviour following a change in their individual capital requirement and what it would have been without the change. Furthermore, we are interested in how this difference in behaviour interacts with economic prospects. To do so, we run regressions at different time horizons, interacting the change in capital requirement with our measure of economic prospects, and control for a series of variables that could have influenced supervisors' decisions and are likely to affect banks' future lending decisions (such as the pre-treatment bank credit growth), and for bank and time fixed effects. Because there may be lags in the bank reaction to requirement changes, we estimate our coefficients at different horizons, namely 1 to 10 quarters.[29] We run separate regressions at each time horizon because we want to focus on the interaction term, which makes an autoregressive specification less appropriate. Ramey (2016) discusses this issue in more detail in the context of dynamic models. This approach is known as a local projection (Jordà, 2005) and can be interpreted as a difference-in-differences estimation strategy (see Angrist et al. (2013) for a discussion).

4.3.1 Specification

We detail here our preferred econometric specification for the case where lending is the dependent variable. We then turn to the definition of other dependent variables.

Let $\Delta requirement_{j,t}$ be a change in capital requirement for bank j at date t. We estimate its cumulative effect on lending at an horizon of h quarters as follows:

$$lending_{j,t+h} = bank_j^h + time_{t+h} + (\alpha^h + \beta^h indicator_t)\Delta requirement_{j,t} + (\theta^h + \phi^h indicator_t) controls_{j,t} + \epsilon_{j,t}^h \tag{12}$$

where $lending_{j,t+h}$ is the cumulative net flow of lending, from period $t-1$ to period $t+h$, as a percentage of the initial stock of lending (at $t-1$). That is, $lending$

[29] Estimates at longer horizons are typically imprecise with such a methodology (Ramey and Zubairy (2014)). Indeed, given our relatively short panel, looking beyond 10 quarters leads to erratic results that are difficult to interpret.

corresponds to $dx/(x+z)$ in the theoretical model. The term $bank_j$ is a bank fixed effect to capture time invariant heterogeneity across banks. The term $time_{t+h}$ is a time fixed effect, controlling for the common response across banks over time. $Indicator_t$ is the value at time t of the business confidence indicator. We standardise $indicator_t$ by dividing by the sample standard deviation and deducting the sample mean. We do not need to include a linear term on $indicator_t$ because it is subsumed in $time_{t+h}$. The term $controls_{j,t}$ is a vector of bank specific controls, which we also interact with the $indicator$ of business confidence. They include (i) the current value and lag of the bank's capital ratio; (ii) the current and lagged rate of lending growth; (iii) the lag of the capital requirement; (iv) the bank's liquid asset to liability ratio to proxy its liquidity position;[30] (v) the ratio of provisions to the stock of loans.

Other dependent variables Our second variable of interest is *capital*. We define it as the cumulative growth rate in the stock of *regulatory capital* in log points from period $t-1$ to period $t+h$, so that it corresponds to de^*_{net}/e^*_{net} in the theoretical model. In this case, we expand the control set to include the current and past quarterly log change in regulatory capital. We denote our estimates for the parameters of interest as $\alpha^h_{capital}$ and $\beta^h_{capital}$ when we use *Capital* as a left hand side variable.

To put our analysis in context, we also report our estimates for the path of the *Capital requirement* itself, the bank *Capital ratio*, and the *Capital buffer*. These three variables are expressed in the cumulative difference from the period before the initial change in capital requirement. In these cases, we do not include bank fixed effect since we are already looking at differences and we control for the initial level.[31]

[30]Liquid assets are defined as holdings of cash, central bank reserves and government securities.

[31]In other words, we do not see why there would be unobserved time invariant heterogeneity at the bank level that determines the *rate of change* of the capital requirement or ratio. However, including a bank fixed effect does not meaningfully alter the messages below.

Inference Estimation is conducted using weighted least squares, with the stock of bank lending at time t as the weighting variable. We follow Auerbach and Gorodnichenko (2013) and conduct inference using Driscoll and Kraay (1998) standard errors, which addresses correlation in the error terms both within bank-levels observations across time and between banks. While, as discussed in the original paper, Driscoll and Kraay (1998) errors still perform well in panels with small cross-sections, we have also conducted inference using blocked (at the bank level) bootstrapped standard errors which give very similar findings to those presented below; these results are available upon request.[32]

4.3.2 Identification

We are not studying a controlled experiment and changes in a bank's individual capital requirements were not literally random. However, we argue that there is ground for considering them as exogenous with respect to banks' future lending and capital decisions. The key condition for a causal interpretation to be valid is that changes in individual capital requirements are not driven by expected changes in bank lending that we cannot observe and control for (this condition would not be satisfied if, for instance, supervisors typically adjusted capital requirements in response to soft information about future lending growth).

At this point, it is important to stress that individual capital requirements were not supposed to be set in response to changes in the quality of the bank's assets or expected changes in credit risk. Indeed, as described in Francis and Osborne (2009), credit risk was dealt with by the common minimum capital requirement of 8% and the assignment of risk weights to assets. Individual capital requirements were in turn designed to account for interest rate, operational, legal and reputational risks (which were overlooked by the Basel 1 regime). There

[32] Some of our empirical tests require inference over parameters across different local projections with alternative left-hand-side variables (specifically, capital and lending). To do this we set up the local projections as a system of two equations. We then estimate the system on an equation-by-equation basis but construct confidence intervals by conducting a block bootstrap at the bank level for the whole system. That is to say, we construct pseudo-samples by redrawing banks with replacement from the true sample and use the same pseudo sample to estimate the system. Collecting estimates across pseudo-samples gives the joint distribution of parameters.

are many reasons to believe that lending growth and credit risk were indeed not really relevant to the changes in capital requirements in the period we study. First, as described in Aiyar et al. (2014b,a), the review (Turner et al. (2009)) into the failures of the UK regulatory regime in the wake of the global financial crisis stated that supervisors where focussed more upon organisational structures, systems, and reporting procedures than bank business models when setting individual capital requirements. Second, the inquiry into the failure of the British Bank Northern Rock noted that the regulatory framework at the time did not require the supervisors to engage in financial analysis. Third, this is supported empirically by De Marco and Wieladek (2016) and Aiyar et al. (2014b), which for instance shows that there is little relationship between past or future loan write downs and changes in individual capital requirements.[33] Table 2 in the Supplementary Appendix SA5 presents similar evidence for our sample; in general, it is hard to relate changes in capital requirements with bank observable characteristics (if anything, supervisors may have been reacting to lending growth, but then with a substantial lag). Fourth, we were able to track some of the confidential official letters sent by the supervisor to the banks to notify them of their capital requirement decisions and we have interviewed some of the supervisors active at the time. Our findings largely confirm the above and interviewees described the approach as reactive, not forward looking (which is consistent our empirical finding in Table 1), and being focused on banks internal processes rather than the strength of their balance sheet. We did, nevertheless, find in the letters some instances where the supervisor expressed concerns related to a bank's fast rate of business expansion. However, these concerns were on whether the bank's relevant internal control structures such as compliance and internal audit have the necessary resources and skills to maintain a robust control environment. Furthermore, the main cases where such concerns would arise were those of external growth through mergers and acquisitions. Given that we consider the resulting banks as new entities, we de facto exclude these potentially problematic observations from the sample.

[33]We actually find that, if anything, provisions tend on average to decrease after an increase in the capital requirement (this is with weak statistical significance).

Still, if the supervisor was reacting to other unobserved information about further acceleration in lending growth rates, then this would bias estimates towards more positive lending responses. In particular, the interaction term would be directly affected if the supervisor reacted to such unobserved information in different ways depending on economic prospects. The persons we interviewed were convinced that it was not the case. According to them, the approach did not take the state of the economy into account, and decisions for a given bank would not be affected by the situation at other banks (increases in writedowns for instance). This is consistent with the lack of correlation between the average requirement and business confidence we have documented above.[34] Even if supervisors were taking economic conditions into account, this would not be a problem per se since we control for time fixed effects and we interact the bank specific controls with business confidence. This would only be a concern if the state of the economy would make the supervisor react differently to bank-specific future-lending relevant information that we cannot observe. If this was the case, we could arguably expect that the decisions to increase or decrease the requirement to be based on different grounds and therefore not to have a symmetric impact. As we will see their impact is remarkably symmetric, which helps alleviates such concerns. It is also worthwhile noting that our sample largely contains universal banks with a national footprint and similar business models, hence we would not expect substantial heterogeneity in the bank's response to aggregate conditions.

Another potential problem could be that the supervisor was reacting to factors that would affect a bank's future idiosyncratic demand for loans (time fixed effect deal with the aggregate demand conditions). However, we do not find it particularly convincing that such information would lead the supervisor to change the bank's requirement, and we did not find any hint of this happening in the letters we were able to consult. Note also that changes are confidential, so it is unlikely that the requirement changes would affect the demand for the bank's loans.

[34]Also, as we show below, the average future path for an individual capital requirement (conditional on an initial change) is essentially independent of business confidence. Meeks (2016) goes a step further and shows that, in an aggregate VAR analysis, macroeconomic shocks seemed to have little impact on the average level of the capital requirement across banks.

A final concern can be illustrated as follows. Imagine that the supervisor raises the requirement of a bank because of insufficient provisioning regarding some pending litigation (as described, legal risk, was part of the mandate of the requirement). Then, imagine that the verdict is unfavourable to the bank and results in the bank having to change its lending practice. Lending will most likely be affected. Still, one should note that the realisation of litigation risk was not a major concern during the period covered by our sample. UK banks were subject to fines and litigation in the post-crisis period, particularly regarding mortgage mis-selling, but not prior to the crisis. Also, we did not find any hint of this happening in the official letters we were able to consult.

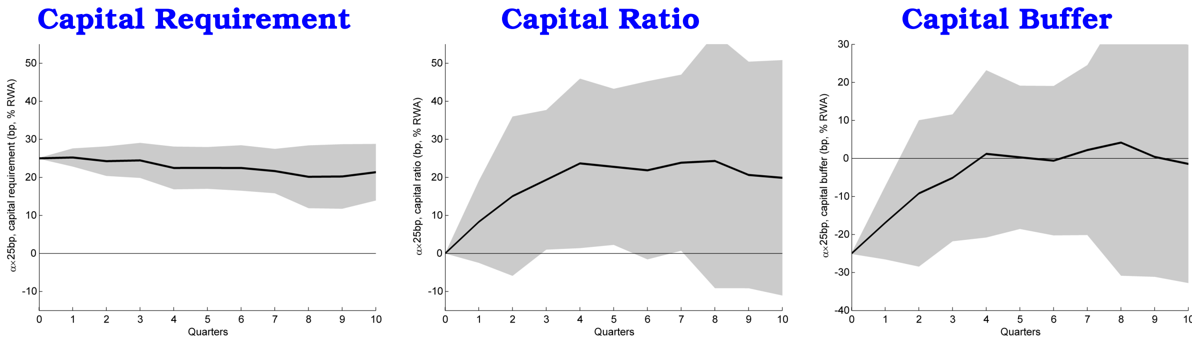
5 Empirical Results

To present our empirical results, we first have a look at how the capital requirement itself and the bank capital ratio evolve, on average, after an initial change in capital requirement. We then turn to our model's empirical tests. To make things concrete and easy to interpret, we present our estimates in the form of impulse responses. That is, in all the graphs below, the solid lines represent our point estimates over our considered range of time horizons. These point estimates correspond to a 25bp increase in capital requirements and our statements about statistical significance refer to the 5% level. Full regression tables at all horizons for our baseline specifications are in Supplementary Appendix SA5. There we also offer some brief commentary on the estimates for control variables.

5.1 Capital requirements and ratios

To set the stage, the left panel of Figure 6 illustrates the persistence of the changes in capital requirement (in line with our theoretical analysis, we do not include here an interaction term with $indicator_t$). The middle panel of the same figure displays the corresponding path for the bank capital ratio. We can see a gradual increase. In particular, it does not reach 25bp until the third quarter. It is feasible for the bank to smooth the transition if it operates with a sufficient

Figure 6: Bank capital requirements and ratios following a requirement increase



Notes: These graphs display the average path of the capital requirement itself, the capital ratio, and the capital buffer following an initial 25bp increase in capital requirement. In particular, we display the vector of estimates 0.25α, $\alpha \equiv \{\alpha^1,..., \alpha^{10}\}$, from regression $y_{j,t+h} = time_{t+h} + \alpha^h \Delta requirement_{j,t} + \theta^h controls_{j,t} + \epsilon^h_{j,t}$ at horizons h up to 10 quarters using panel data from 18 UK banks covering 1989-2007. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors. We consider the non-linear version of these graphs in section 5.3.3.

voluntary buffer, which is typically the case. And it turns out that, still on average, banks just replenish the voluntary buffer they had at the time of the change (see the right panel). Arguably, that the buffer response goes back to zero suggests that banks had not already built up an additional capital in *anticipation* of the requirement change. Recall that for our theoretical results to apply, it is important that the capital requirement is essentially binding. We explore this more formally when we evaluate Test 3 below; however, we can already see that the point estimates strongly suggest that banks do adjust their capital ratios one-to-one with a change in the requirement.

5.2 Our empirical tests

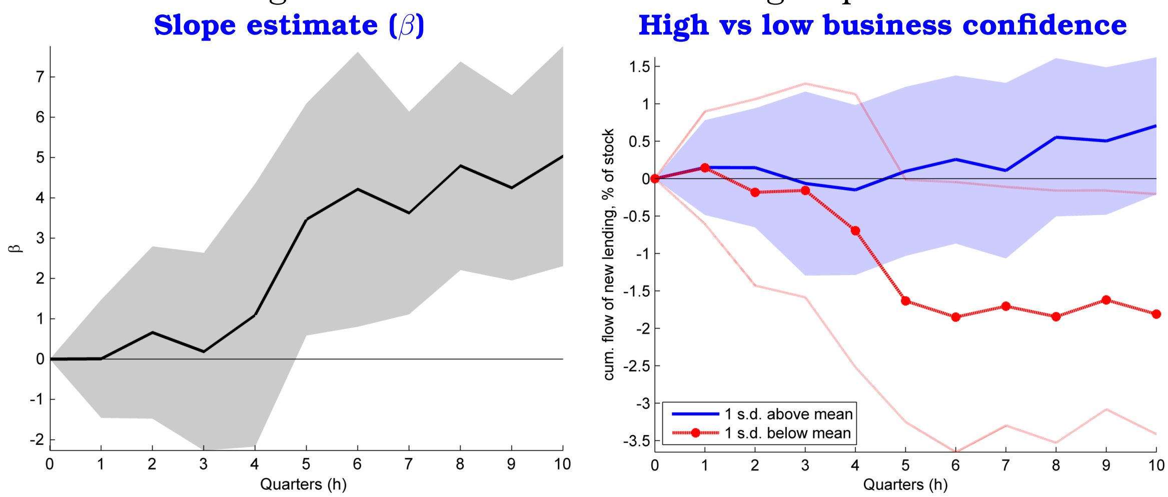
Test 1 consists of estimating the slope (and intercept) of the lending and capital responses. As we have explained, we expect the slope of the lending response to be positive. That is to say, we expect the interaction term between the capital requirement change and business confidence (i.e. β^h in Equation 12) to be positive. The left panel in Figure 7 shows our vector of estimates, $\beta \equiv \{\beta^1,...,\beta^{10}\}$, for these interaction terms. We find that they are essentially nil at short horizons. After a year, however, they become positive and statistically significant, which provides support for our prediction.[35] To get an economic interpretation of these estimates, it is useful to consider specific values for business confidence. The right panel of Figure 7 compares the lending response when business confidence is one standard deviation (*sd*) above and below the sample mean (i.e. we display the values for $\alpha+\beta$ and $\alpha-\beta$, where $\alpha \equiv \{\alpha^1,...,\alpha^{10}\}$, scaled by 25bp, the size of the requirement change). These values are arbitrary but we think they constitute a reasonable benchmark for periods of strong and weak economic prospects. As we explained above, our approach is essentially one of differences in difference. That is, the estimates aim at capturing the difference between actual bank behaviour following a change in its individual capital requirement and what it would have been without the change. What we can see is that, when prospects are weak (i.e. the indicator of business confidence is 1 sd below its mean), a 25bp increase in capital requirement is met by a significant cut in lending of 1.5 to 2% (of the stock of loans) after a year. In contrast, when the indicator of business confidence is 1 sd above its mean and prospects are strong, we do not see a significant lending response.

In terms of **Test 2** (i.e. how likely is it that the lending response is positive), this means that confidence being 1 standard deviation above the mean is not high enough. What level is high enough for a statistically significant positive response depends on the horizon. At $h = 5, 6, 7$ a positive response is a rare event: it requires confidence to be greater than 2.5sd above the mean. Assuming normality, this is something that happens less than 1% of the time. At horizons $h = 8, 9, 10$ then a positive response becomes a 1.2-1.5 standard deviation event, or something that happens 5-10% of the time.

Figure 8 shows the equivalent charts for the stock of bank capital. We also find positive estimates for the interaction term (they are statistically significant after a year, except at the 8 quarter horizon). This supports the hypothesis that the capital response is higher when economic prospects are stronger. To interpret the estimates, we consider in the right panel the same two scenarios

[35] It seems reasonable to think that it takes some time for a bank to adjust lending and capital decisions. Moreover, as mentioned above, banks that held a sufficient voluntary buffer before the shock do not necessarily need to adjust capital or lending immediately.

Figure 7: Estimates for the lending response



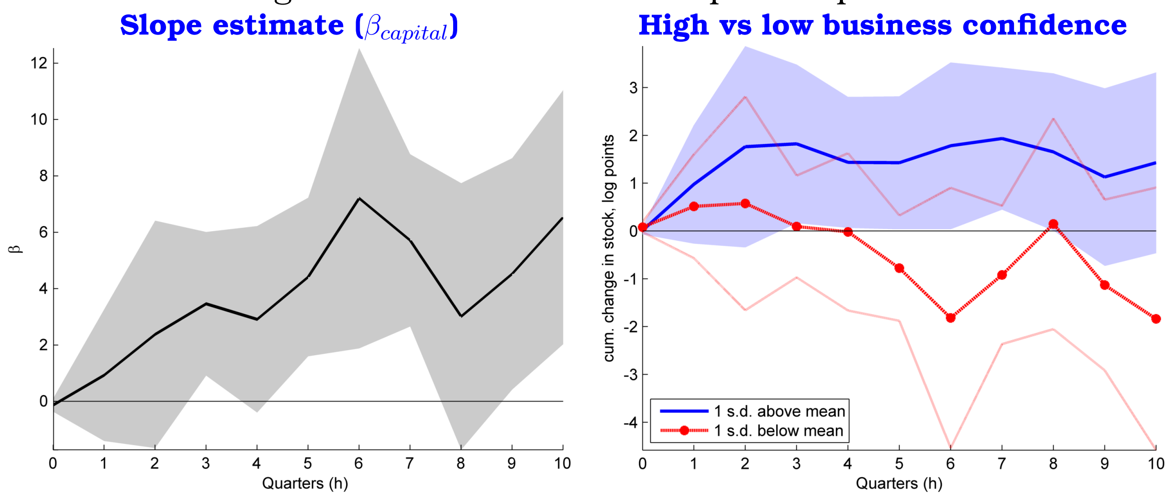
Notes: This figure displays the estimate of interest from our main specification (equation 12 at horizons h up to 10 quarters). We consider 25bp changes in capital requirements and the confidence indicator is standardised. The left panel shows the vector interaction terms between the change in capital requirement and the indicator of business confidence: $\beta \equiv \{\beta^1,...,\beta^{10}\}$. The right panel shows the corresponding lending response when business confidence is 1 standard deviation above or below its mean. That is, the solid **blue line** shows the estimates for $\alpha + \beta$, where $\alpha = \{\alpha^1,...,\alpha^{10}\}$, and the the solid **red line with circle markers** shows $\alpha - \beta$ both sets of estimates are multiplied by 25bp. The estimates are constructed from a panel local projection using quarterly data from 18 UK banks covering 1989-2007. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors.

as above. In times of high business confidence, the capital response is positive. Notably, it reaches about 2% (of the equity stock) after 2 quarters and seems then to stabilise. In contrast, in low confidence periods, while banks' capital levels seem to first go up, they then enter negative territory. However, there is not a time horizon at which the estimate is significantly different from zero.

Taking our estimates together, the data suggest that a bank facing a capital requirement increase is more likely to raise capital in times of strong prospects in response to an increase in requirements and cut lending in times of weak prospects. And conversely, if they face a decrease in capital requirement, they are more likely to increase lending when prospects are strong and decrease capital when prospects are weak. Linking to our model: the price effect is more likely to dominate when economic prospects are strong.

We now turn to **Test 3.** This has two parts; first, that changes in economic prospects affect the lending and capital responses in the same way. The formal prediction is that the slopes coefficients β and $\beta_{capital}$ are identical. Figure 9 (left

Figure 8: Estimates for the capital response

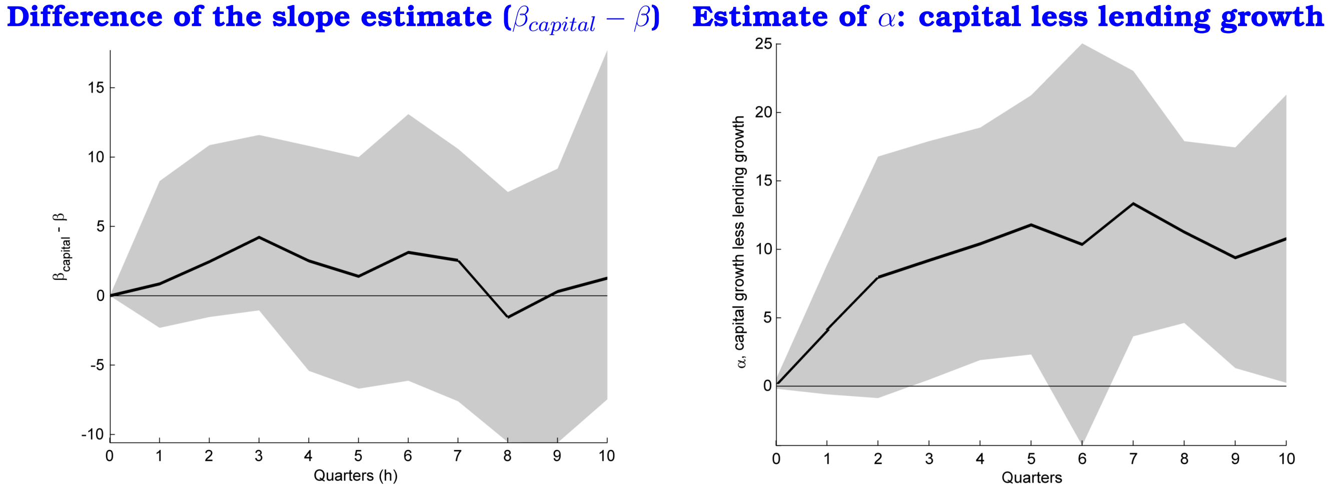


Notes: This figure displays the estimate of interest for capital (the specification corresponds to equation 12, still at horizons h up to 10 quarters, but with $capital_{j,t+h}$ as the dependent variable). We consider 25bp changes in capital requirements and the confidence indicator is standardised. The left panel shows the vector of interaction terms between the change in capital requirement and the indicator of business confidence: $\beta_{capital} \equiv \{\beta^1_{capital},...,\beta^{10}_{capital}\}$. The right panel shows the corresponding lending response when business confidence is 1 standard deviation above or below its mean. That is, the solid **blue line** shows the estimates for $(\alpha_{capital} + \beta_{capital})$, where $\alpha_{capital} = \{\alpha^1_{capital},...,\alpha^{10}_{capital}\}$, and the the solid **red line with circle markers** shows $(\alpha_{capital} - \beta_{capital})$ both sets of estimates are multiplied by 25bp. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors.

panel) provides the estimate for $\beta_{capital} - \beta$. There isn't a time horizon at which it is statistically different from zero, so we cannot reject this hypothesis. At short horizons, the value of the point estimates are not insignificant economically. However, comparing the path for the estimates of β and $\beta_{capital}$, we can see that the difference mostly comes from capital adjusting faster than lending initially; at longer horizons the differences have closed. Overall, our reading of the results is positive: despite the existence of factors that could invalidate it (see Subsection 3.2.3), we find support for the first part of the test in the data. This is reassuring for the validity of both our theoretical and empirical approach.

The second part of the test is that the intercepts for the capital and lending responses differ by a factor inversely proportional to the capital requirement. Formally, $\alpha_{capital} - \alpha = 1/\gamma$. Given an average capital requirement of 9.2%, our theoretical model predicts a value of 10.9. Figure 14 in Supplementary Appendix SA4 shows the difference in intercepts (i.e. $\alpha_{capital} - \alpha$). The point estimates are positive at all horizons; however, the estimates are imprecise and the test is

Figure 9: Estimates for Test 3



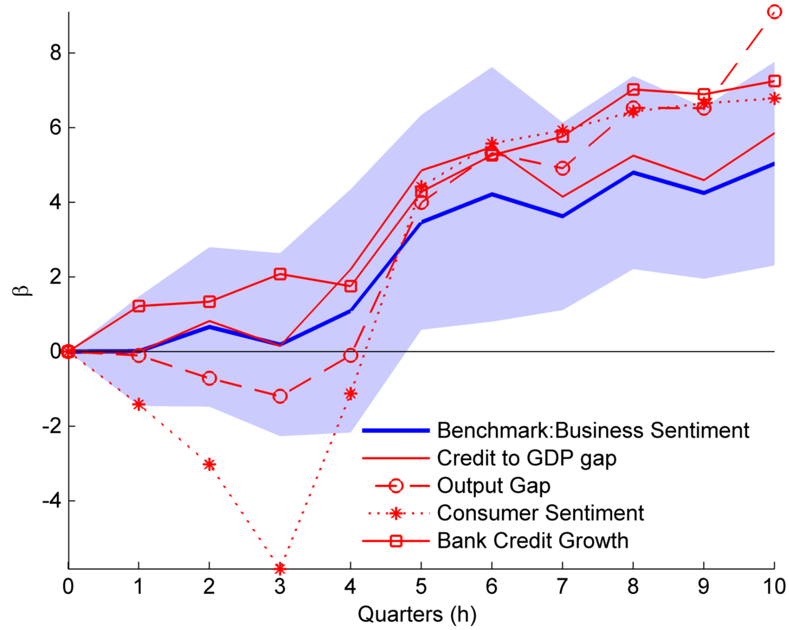
Notes: The left panel shows the difference in the vector interaction of terms between the change in capital requirement and the indicator of business confidence for the local projection using the change in the log of capital as a left hand side variable$\beta_{capital} \equiv \{\beta^1_{capital},...,\beta^{10}_{capital}\}$ and using lending growth as a left hand side variable $\beta \equiv \{\beta^1,...,\beta^{10}\}$. The error bands presented are 95% confidence intervals constructed using 1000 iterations of a block bootstrap carried out at the bank level. The right panel plots the estimates of $\alpha \equiv \{\alpha^1,..., \alpha^{10}\}$ for the model: $capital_{j,t+h} - lending_{j,t+h} = bank^h_j + time_{t+h} + (\alpha^h)\Delta requirement_{j,t} + (\theta^h) controls_{j,t} + \epsilon^h_{j,t}$ at horizons h up to 10 quarters. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors. Both sets of estimates are constructed from a panel local projection using quarterly data from 18 UK banks covering 1989-2007 at horizons h up to 10 quarters.

hardly conclusive. To tighten our estimates we can however exploit that the first part of Test 3 implies an additional restriction of the data. Taking the prediction that slopes are identical at face value, we can directly estimate a linear model for the difference between the capital and lending responses. The relevant coefficient is displayed in the right panel of Figure 9. As we can see, after the usual lag of a few quarters, point estimates are in the vicinity of the model's prediction and we do get statistical significance.[36]

As outlined in section 3, Test 3 relies on the assumption that the capital

[36]As explained in Subsection 3.2.3, there are several factors that could distort the predicted value of $\alpha_{capital} - \alpha$. First of all, banks hold voluntary buffers over and above the requirement. Accounting for this yields $\alpha_{capital} - \alpha = 1/(\gamma + \text{buffer})$. Calibrating the buffer to its sample mean, the relevant initial capital ratio is close to 12%, and the prediction becomes 8.3. Second, the capital requirement applies to risk-weighted assets. Risk-weights are time invariant in the Basel 1 regime, but the composition of the banks loan portfolio can vary. Assuming it does not, and using the average risk-weight on non-financial loans over the period for UK banks (0.65), brings the relevant initial ratio back to around 9%. In both cases, we are also in the vicinity of our point estimates, which suggests that the simplifications implied by our approach are not too consequential.

Figure 10: Estimates of β with alternative indicator series



Notes: These estimates are constructed from a panel local projection using quarterly data from 18 UK banks covering 1989-2007. The coefficient of interest is β^h in equation 12. The **blue line** is the estimate of $\beta \equiv \{\beta^1,...,\beta^{10}\}$ from the benchmark specification using business confidence. The **blue** shaded confidence interval denotes two Driscoll and Kraay (1998) standard errors in the benchmark specification. The **red lines** are the estimates of β using alternative series for $interaction_t$.

requirement is independent of economic prospects. The fact that the test's hypothesis is not rejected is more evidence (alongside figure 5) that this assumption is a reasonable description for how regulation was set in this time period.

5.3 Robustness

5.3.1 Alternative indicator series

We have tested whether alternative indicators for economic prospects (i.e. alternative proxies for the expected return on banks' loans) lead to different results. Namely, we have tried: (i) the OECD's consumer confidence indicator (the household analogue to business confidence indicator); (ii) the credit to GDP gap, that the Basel Committee for Banking Supervision has recommended as a key indicator of the financial cycle (we take the standard definition of this indicator and it is sourced from the Bank of England); (iii) the OECD's estimate of the output gap in the United Kingdom; (iv) the past rate of bank specific credit growth (defined as the average of the net flow over the stock over the previous four quarters). The intuition here being that a rapidly growing bank may perceive prospects for the returns on its loans to be good.[37]

Figure 10 summarises the estimates of β for these alternative indicators in comparison to the benchmark estimates. As $indicator_t$ enters our specification having been standardised by dividing by the sample standard deviation, these alternative estimates of β^h can be compared across a common scale. As can be seen, all four estimates present a similar pattern to the benchmark case and for the most part lie within the benchmark confidence interval. An exception is the estimates using consumer confidence at early horizons which have negative point estimates; however these estimates are not statistically different from zero. We have investigated how $\beta_{capital}$ is affected by our alternative indicator series and can report that our results for this variable are similarly robust.

5.3.2 Are we omitting financial conditions?

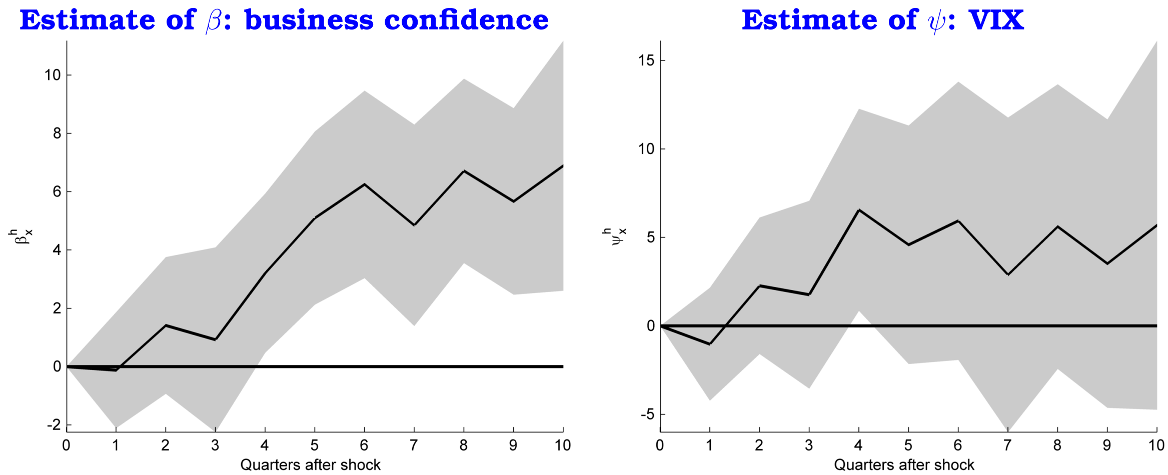
Throughout our empirical analysis we have assumed our indicator captures the expected return on both legacy and new loans. However, periods of low confidence may also coincide with turbulent financial conditions and difficulties for banks in funding their liabilities. To account for this, we estimate our model including a second indicator variable alongside our confidence indicator: the VIX index, which is often used to capture prevailing financial conditions. Specifically we estimate:

$$\begin{aligned} lending_{j,t+h} = \quad & bank_j^h + time_{t+h} + (\alpha^h + \beta^h indicator_t + \psi^h vix_t)\Delta requirement_{j,t} \\ & +(\theta^h + \phi^h indicator_t + \omega^h vix_t) controls_{j,t} + \epsilon_{j,t}^h, \end{aligned} \tag{13}$$

Where vix_t is the natural logarithm of the VIX, normalised as above. Figure 11 displays the estimates of β^h and ψ^h from this specification. As we can see, the coefficient on business confidence is essentially unaffected by the inclusion of

[37] We obtain very similar results using aggregate credit growth rather than bank specific credit growth. Results are available upon request.

Figure 11: Estimates including the VIX as an additional indicator



Notes: This figure displays the estimate of interest from equation 13 at horizons h up to 10 quarters. The confidence indicator and log VIX are both standardised. The left panel shows the vector of interaction terms between the change in capital requirement and the indicator of business confidence: $\beta \equiv \{\beta^1,...,\beta^{10}\}$. The right panel shows the vector interaction terms between the change in capital requirement and the logarithm of the VIX: $\psi^h \equiv \{\psi^1,...,\psi^{10}\}$. The estimates are constructed from a panel local projection using quarterly data from 18 UK banks covering 1989-2007. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors.

this second interacting term. The coefficient on the VIX itself is not statistically different from zero at every horizon but the fourth.

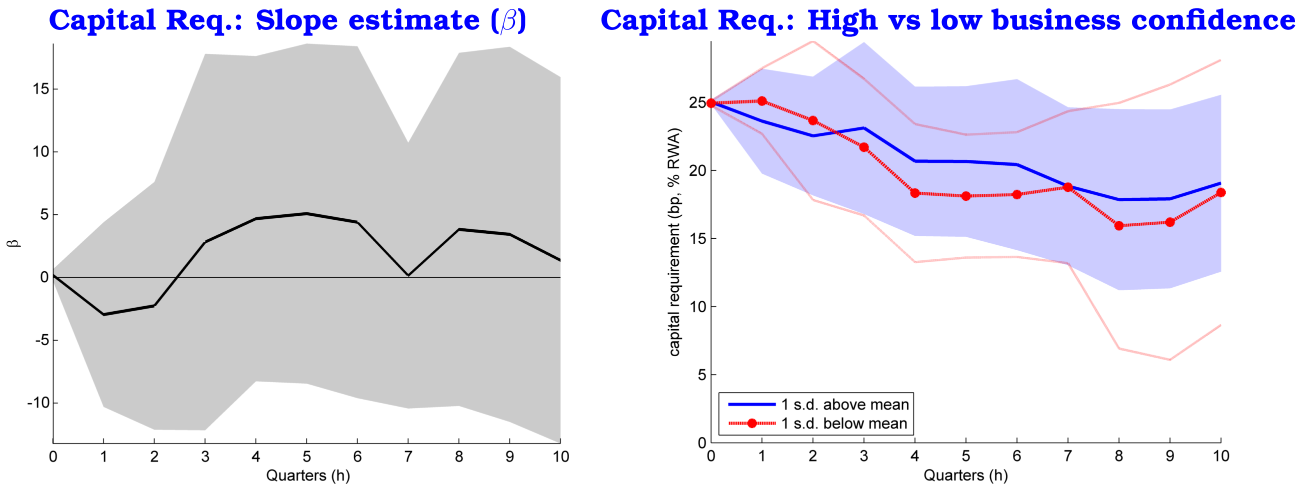
5.3.3 Do initial economic prospects affect the path of the capital requirements?

In section 5.1 we presented estimates for the path of banks' capital requirement and corresponding responses for the capital ratios in specifications that did not include an interaction term (with $indicator_t$). This is in line with our theoretical framework. To ensure this is valid, we explore in this subsection what happens when this interaction term is included.

The left panel in Figure 12 suggests prospects had little impact on the path of the capital requirement. The right panel in Figure 12 confirms this by presenting the path when business confidence is high and when it is low. This result provides further support for the notion that the setting of requirements was independent of economic prospects.

Figure 15 in Supplementary Appendix SA4 presents equivalent graphs for capital ratios and capital buffers. The statistical tests are not really conclusive. If anything, they suggest that banks take a bit longer to replenish their buffers when confidence is low.

Figure 12: Effect of confidence on the path for the requirement



Notes: This figure displays the estimate of interest from our main specification (equation 12 at horizons h up to 10 quarters) but with bank's capital requirement as the left hand side variable. We also exclude bank fixed effects from the specification. We consider 25bp changes in capital requirements and the confidence indicator is standardised. The left panel shows the vector of interaction terms between the change in capital requirement and the indicator of business confidence: $\beta \equiv \{\beta^1,...,\beta^{10}\}$. The right panel shows the corresponding lending response when business confidence is 1 standard deviation above or below its mean. That is, the solid **blue line** shows the estimates for $\alpha + \beta$, where $\alpha = \{\alpha^1,...,\alpha^{10}\}$, and the the solid **red line with circle markers** shows $\alpha - \beta$ both sets of estimates are multiplied by 25bp. The estimates are constructed from a panel local projection using quarterly data from 18 UK banks covering 1989-2007. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors.

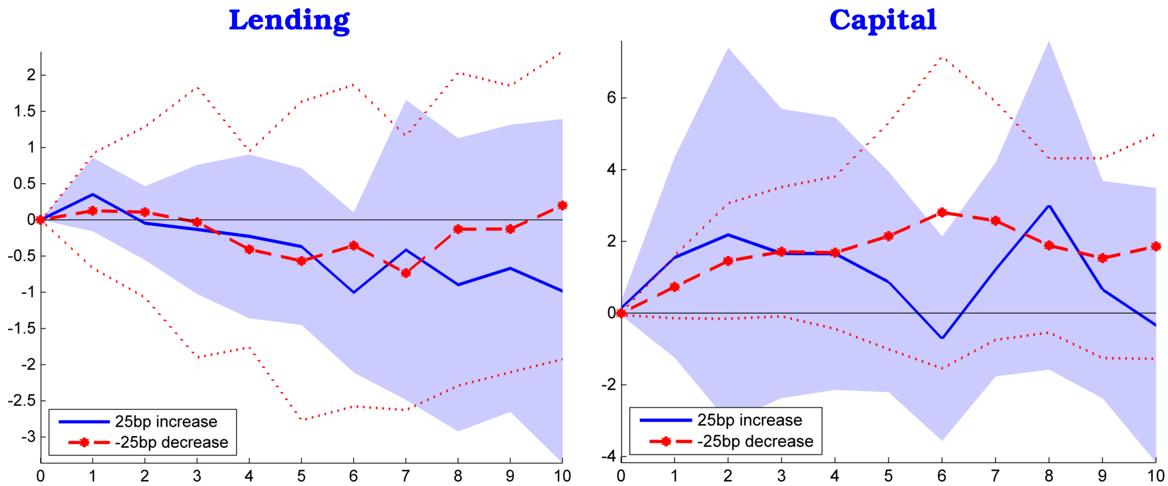
5.3.4 Are the effects of increases and decreases symmetric?

In line with our theoretical analysis, our empirical strategy presumes that changes in capital requirements have symmetric effects (i.e. a decreases has the opposite effect of an equivalent increase). We can test the empirical validity of this presumed symmetry by estimating the following equation for lending:

$$lending_{j,t+h} = bank_j^h + time_{t+h} + (\alpha^h + \beta^h I(\Delta requirement_{j,t} < 0))\Delta requirement_{j,t} + \theta^h controls_{j,t} + \epsilon_{j,t}^h, \quad (14)$$

where I is an indicator variable which takes a value of 1 if the capital requirement change is a decrease. The coefficient β^h now governs any asymmetry in the response of lending to a requirement change. We also estimate the model for

Figure 13: Increases versus decreases in capital requirements



Notes: We test for asymmetries in the response to a requirement change by estimating $lending_{j,t+h} = bank_j + time_{t+h} + (\alpha^h + \beta^h I(\Delta requirement_{j,t} < 0))\Delta requirement_{j,t} + \theta^h controls_{j,t} + \epsilon_{j,t}$ at horizons h up to 10 quarters. We consider 25bp changes in capital requirements. In the left panel the **blue line** is the estimate of $\alpha = \{\alpha^1,...,\alpha^{10}\}$, the **red dashed line** is the estimate of $\alpha + \beta$ where $\beta \equiv \{\beta^1,...,\beta^{10}\}$, both multiplied by 25bp. The right panel shows equivalent estimates when capital is a left hand side variable. The estimates are constructed from a panel local projection using quarterly data from 18 UK banks covering 1989-2007. Confidence intervals presented are two Driscoll and Kraay (1998) standard errors.

capital. Figure 13 shows the path of lending in response to a 25bp increase and a -25bp decrease (i.e. including the β^h term) in the requirement estimated using Equation (14). There is no obvious asymmetry. The point estimates display a similar path and our (unreported) estimates of β^h are not statistically different from zero either and fluctuate in sign across horizons.

We have explored whether asymmetries between increases and decreases are more important depending on economic prospects. There were no obvious asymmetric relationships that emerged from this analysis. However, slicing the data across these two dimensions significantly limits the amount of variation, which gives estimates that are both volatile and not statistically significant.

5.3.5 Alternative specifications and samples

To further explore the robustness of our results, we have examined the results under a series of alternative specifications. (All figures mentioned in this subsection are in Supplementary Appendix SA4.)

As discussed in section 4.2 we exclude a number of small banks from our sample. Figure 16 shows the results including smaller banks in the sample and using OLS (as opposed to weighting observations). The results are more volatile; however, our key findings regarding the size, magnitude and significance of β are sustained in this specification. The years 2006 and 2007 are potentially distorted by banks adjusting to the Basel II regime that was due to come into effect in 2008. Figure 17 shows the results when we end the sample in 2005. There is no noticeable impact on our empirical results from doing this. Figure 18 shows the results including increasing the lag order of the controls to two. Last, Figure 19 show the results excluding bank fixed effects.

For parsimony all these figures focus on the lending response (i.e. they are the counterpart of Figure 7 in our main specification). Estimates corresponding to other figures are available upon request.

6 Conclusion

We propose a model to study bank behaviour in the presence of capital requirements. Our analysis focuses on the bank's first order condition. It allows us to draw economic intuition and to formulate empirical predictions on how a capital requirement affects bank lending and capital decisions. The bank's behaviour depends on the interplay between a type of debt overhang problem and risk shifting incentives, both of which arise due to limited liability and government guarantees. A number of key insights emerge. First, there is typically a U-shaped relationship between the level of lending and the capital requirement. Second, and as a consequence, the sign of the response of lending to a change in capital requirements is ambiguous and, third, it depends on the prospects for new and existing loans.

We test this second insight from our model using UK bank regulatory data from 1989 to 2007. An interpretation of our main empirical results is that changes in bank specific capital requirements are mainly met by a cut in lending when economic prospects are poor, but by an increase in capital when prospects are strong. Notwithstanding general equilibrium effects, and with the caveat that our estimates are based on a specific country under a specific regulatory regime, the results speak to the current issues of how banks will adapt to higher capital requirements and how they would respond in a time-varying regime.

Our findings also have implications for future theoretical research. Studying the general equilibrium consequences of bank capital requirements (or market imposed constraints) in a tractable model necessitates making restrictive assumptions (see for Suarez, 2010 a discussion). A typical strategy is to impose specific decision rules on banks' net capital issuance (as, for instance, in Gertler and Kiyotaki (2015); Martinez-Miera and Suarez (2014); Malherbe (2015)). Our results, both theoretical and empirical, highlight that a bank's capital response to a change in capital requirement interacts positively with economic prospects. Future studies of capital requirements in general equilibrium may want to account for such a feature.

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A Proofs

Proposition. 1 *(Overhang problem) Assuming that new lending is safe, if the bank defaults with strictly positive probability in equilibrium, the wedge in (5) is strictly negative. Hence, the bank does not finance all the positive net present value loans (i.e. $x^* < x_1$, where x_1 denotes the level of lending that maximises economic surplus).*

Proof. Given that $X_x(\mu, x)$ is decreasing in x, the results directly follows from the first order condition.

□

Proposition. 2. *(Positive lending response) Assuming that new lending is safe, if the bank defaults with strictly positive probability in equilibrium, then, as $\pi \to 1$, $\frac{dx^*}{d\gamma} > 0$. That is, the price effect dominates the composition effect and the wedge in (5) is increasing in γ.*

Proof. We assume that the limit as $\pi \to 1$ of the numerator of equation 7 exist. We need to show that it is strictly negative:

$$\lim_{\pi \to 1} \left(\frac{1}{\pi} - 1 - \gamma \frac{\partial \pi(\gamma, x^*)}{\partial \gamma} \frac{1}{\pi^2} \right) < 0. \tag{15}$$

A sufficient condition is that $\lim_{\pi \to 1} \frac{\partial \pi(\gamma, x^*)}{\partial \gamma} > 0$. Applying the implicit function theorem to $X + Z(A_0) - (1-\gamma)(x+z) = 0$ gives:

$$\frac{dA_0}{d\gamma} = -\frac{x+z}{Z'(A_0)}.$$

Since $\frac{\partial \pi}{\partial \gamma} = \frac{-\partial A_0}{\partial \gamma} f(A_0)$, the sufficient condition becomes:

$$\lim_{A_0 \to^- A_L} \left(\frac{(x+z)}{Z'(A_0)} f(A_0) \right) > 0.$$

Since $Z^{'}(A_0)$, x, and z are positive by assumption, this condition is satisfied if $\lim_{A_0 \to^{-} A_L} (f(A_0)) > 0$. Since we assume full support for $f(A)$ over $[A_L, A_H]$, this is the case, which concludes the proof. Note, however, that the full support assumption is not needed here. For instance, if $f(A_0) = 0$, then one can use l'Hopital's Rule and show that Condition (15) still holds. Finally, if the limit in Condition (15) is not defined, one can still prove results similar in spirit. □

Proposition. 3. *(Risk-shifting) Assume $Z(A_L) > (1-\gamma)z$. Then, there is no overhang and the wedge in (5) cannot be strictly negative in equilibrium. If the bank defaults with strictly positive probability in equilibrium, the wedge is strictly positive. Hence, the bank finances negative net present value loans (i.e. $x^* > x_1$).*

Proof. The bank defaults for all realisations of A below A_0. Hence, for all $A \in [A_L, A_0(x)]$, we have

$$[X\left(A,x\right) + Z(A) - \left(1-\gamma\right)\left(x+z\right)] \leq 0;$$

which implies (since $Z(A) > z$)

$$\frac{X\left(A,x\right)}{x} \leq 1-\gamma. \tag{16}$$

As $X\left(A,x\right)$ is concave in x, we have that $X_x\left(A,x\right) \leq \frac{X(A,x)}{x}$. Hence $X_x\left(A,x\right) \leq 1-\gamma$, we have

$$\int_{A_L}^{A_0(x)} \left(\left(1-\gamma\right) - X_x\left(A,x\right)\right) f(A)dA \geq 0. \tag{17}$$

That is, the wedge is always positive; strictly if the bank defaults with strictly positive probability. In this case, $x^* > x_1$, which means that the bank funds negative NPV loans.

□

Proposition. 4. *Assume $Z(A_L) > (1-\gamma)z$. If the bank defaults with strictly positive probability in equilibrium, we have $\frac{dx^*}{d\gamma} < 0$. That is, the wedge in (5) is strictly decreasing in γ.*

Proof. We need the derivative of the wedge with respect to γ to be negative. That is

$$\frac{dA_0(x^*)}{d\gamma}\left(1-\gamma-X_x\left(A_0,x^*\right)\right)f(A_0)+\int_{A_L}^{A_0}(-1)\,f(A)dA<0.$$

From the proof of Proposition 3, we know that $X_x\left(A_0,x^*\right)<1-\gamma$. From the proof of Proposition 2, we know that $\frac{dA_0}{d\gamma}<0$. So the condition is satisfied as long as the bank defaults with strictly positive probability in equilibrium.

□