Bargaining and News*

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Abstract

We study a bargaining model in which a buyer makes frequent offers to a privately informed seller, while gradually learning about the seller's type from "news." We show that the buyer's ability to leverage this information to extract more surplus from the seller is remarkably limited. In fact, the buyer gains *nothing* from the ability to negotiate a better price despite the fact that a negotiation *must* take place in equilibrium. During the negotiation, the buyer engages in a form of costly "experimentation" by making offers that are sure to earn her negative payoffs if accepted, but speed up learning and improve her continuation payoff if rejected. We investigate the effects of market power by comparing our results to a setting with competitive buyers. Both efficiency and the seller's payoff can *decrease* by introducing competition among buyers.

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1 Introduction

A central issue in the bargaining literature is whether trade will be (inefficiently) delayed. What is often ignored, however, is that if trade is in fact delayed, new information may come to light.[1] Of course, the players' anticipation of this information may itself affect the amount of delay in the negotiation bargaining.

For example, consider a startup that has "catered" its innovation to a large firm with the aim of being acquired (an increasingly common strategy in entrepreneurship—see Wang (2015)). The longer the startup operates as an independent business, the more the large firm expects to learn about the quality of the innovation, which can influence the offers that it tenders. At the same time, delay is inefficient as the large firm can generate greater value from the innovation due to economies of scale and its portfolio of complementary products. We are interested in how the large firm's ability to learn about the startup over time affects its relative bargaining power, trading dynamics, and the amount of surplus realized from the potential acquisition.

Alternatively, consider the due diligence process associated with a corporate acquisition or commercial real estate transaction. This information gathering stage is inherently dynamic; the acquirer/purchaser must decide how long to continue gathering information, thereby delaying the transfer of ownership, as well as how to use the information acquired to maximize the profitability of the transaction. How does the acquirer's ability to conduct due diligence and renegotiate the price influence the eventual terms of sale and the profitability of the acquisition?

In this paper, we propose a framework to answer these questions. We study a model of bargaining in which the uninformed party (the "buyer") makes frequent offers to the informed party (the "seller") while simultaneously learning gradually about the seller's type from an observable *news* process. There is common knowledge of gains from trade, values are interdependent, and the seller is privately informed about the quality of the tradable asset (i.e., his type), which may be either high or low. Because of discounting, the efficient outcome is immediate trade. We pose the model directly in continuous time, which captures the idea that there are no institutional frictions in the bargaining protocol and facilitates a tractable analysis. News is modeled as a Brownian diffusion process with type-dependent drift.

We construct an equilibrium of the game and prove that it is the unique stationary equilibrium. In it, the buyer's ability to leverage her access to information in order to extract more surplus from the seller is remarkably limited. In particular, the buyer's equilibrium

[1] Fuchs and Skrzypacz (2010) is a notable exception, as we will discuss.

payoff is identical to what she would achieve if she were unable to negotiate the price based on new information. In addition, delay occurs if and only if there is an adverse-selection problem. Otherwise, the Coasian incentive to speed up trade overwhelms the buyer's desire to learn the seller's type and trade occurs immediately. The latter result extends existing no-delay results found in bargaining models without news (Fudenberg et al., 1985; Gul et al., 1986; Deneckere and Liang, 2006).

When trade is delayed the buyer engages in a form of costly "experimentation" by making offers that are sure to earn her negative payoffs if accepted. That is, the buyer makes some offers *hoping* that they will be rejected. Such rejections improve her information and expected continuation payoff. Yet, the buyer exhausts all of the benefits from this experimentation leaving her with precisely the same payoff she would obtain if she were unable to offer such prices. Thus, despite the fact that a negotiation takes place and the buyer responds to good (bad) news by adjusting her offer up (down), she is no better off by being able to do so. In fact, the sole beneficiary of this experimentation is the low-type seller, who earns strictly more than his value to the buyer.

We investigate the effects of market power by comparing our results to those of the competitive-buyer model of Daley and Green (2012) (hereafter, DG12). We find novel differences in both the pattern of trade and the resulting efficiency. With a single buyer, trading intensity with the low type is "smooth" at a rate proportional to dt, whereas trading intensity in DG12involves atoms and local time. The resulting equilibrium belief dynamics are also remarkably different. With a single buyer, the belief process follows an Ito Diffusion, whereas it has a lower reflecting boundary and discontinuous sample-paths in DG12. Perhaps most surprisingly, both efficiency and the seller's payoff can *decrease* by introducing competition among buyers.

Intuitively, the amount of delay is driven by the party that stands to gain from information revelation. With buyer competition, it is the high-value seller who gains from information being revealed as buyers bid up the price after good news. Without this competition, the (single) buyer is the player who determines the amount of delay. Because of his private information, the high-type seller is more optimistic about the realization of future news than is the buyer, which causes him to delay trade when facing competitive buyers in states where a single buyer would chose to trade immediately. This finding is most starkly illustrated in the no-adverse-selection case: with a single buyer trade is immediate, whereas it will be delayed with competitive buyers when the news process is sufficiently informative.

Our comparison of the single-buyer and competitive-buyer settings sheds new light on the interpretation of the Coasian force. One common interpretation of the Coasian force is that competition with one's future self is sufficient to simulate the competitive outcome. Yet as we have just seen, the single and competitive buyer outcomes are distinct in the presence of news. We therefore propose a different interpretation of the Coasian force: competition with one's future self renders attempts to screen through prices futile.

We formalize this finding by considering an auxiliary game, which we refer to as the "due diligence problem," in which the price is fixed at the high type seller's reservation value and the buyer's strategy is a stopping rule corresponding to a date at which to execute the transaction. We demonstrate the buyer's payoff in the due diligence problem is equal to her equilibrium payoff in the true game, while the low-type seller is strictly better off in the true game.

We employ our reinterpretation of the Coasian force to solve several extensions of the model. First, we consider an extension in which investigation is costly for the buyer. Second, we consider an extension in which the news process includes a "lumpy" component. In both cases, we construct the equilibrium by first solving for the buyer's value function in the analogous due diligence problemand then identifying the strategies and seller value function consistent with this payoff. The advantage of our approach is that the solution to the due diligence problem is independent of the seller's payoff and therefore the equilibrium can be constructed in relatively straightforward steps rather than through the usual, and sometimes arduous, fixed-point analysis. More generally, we believe our reinterpretation of the Coasian force may be instructive for solving other bargaining models with frequent offers.

Our work is related to Deneckere and Liang (2006) and Fuchs and Skrzypacz (2010) (hereafter, DL06 and FS10), both of which investigate frequent-offer, bilateral bargaining games. DL06 analyze an interdependent-value setting in the absence of news and show that the equilibrium is characterized by "bursts" of trade followed by periods of delay.[2] During a period of delay, the buyer's belief must be exactly such that the Coasian desire to speed up trade is absent, which is non-generic. The addition of learning via a diffusion process, even if arbitrarily noisy, means that the buyer's belief cannot remain constant at such a belief over any period of time. As a result, our findings are considerably different from theirs even in the limit as the news becomes completely uninformative (see Section 6.3). FS10 study the independent-value setting from the Coase conjecture literature, with the addition of a Poisson arrival of a game-ending "event." The primary interpretation given to the event is the arrival of a new trader, but it can also be interpreted as the arrival of a signal which reveals the informed party's private information. A critical difference is that in FS10 this information must alter the support of the uninformed party's belief, unlike our Brownian

[2] Fuchs and Skrzypacz (2013) show that trade becomes "smooth" and the buyer fails to capture any rents in the no-gap limit. In our model, there is a gap, the equilibrium features smooth trade prior to the end when there is a burst, and the buyer does capture rents, though not from screening.

news process. The possibility of the signal arrival in FS10 delays trade, but their results are consistent with our interpretation of the Coasian force.

Our work is also related to Ortner (2017), who analyzes a continuous-time model of a durable-good monopolist whose cost varies stochastically over time. A common feature is that the stochastic component (costs in Ortner (2017), information in our paper) can create an option value for the uninformed party to delay trade. Tsoy (2016, 2017) studies the effect of public information in a alternating-offer bargaining model with a global games information structure. Two recent papers, Ishii et al. (2017) and Ning (2017), explore the effect of learning via public information within symmetric information bargaining environments. Finally, DeMarzo and He (2017) study leverage dynamics of a firm, when the manager cannot commit not to issue more (or less) debt in the future. Our finding—that the buyer does not benefit from screening through price offers—is analogous to their finding that the firm's shareholders cannot benefit from its leverage policy.[3]

2 The Model

There are two players, a seller and a buyer, and a single durable asset of type $\theta \in \{L, H\}$, which is the seller's private information. Let $P_0 \in (0, 1)$ denote the prior probability that the buyer assigns to $\theta = H$. The seller's (opportunity) cost of parting with the asset is K_θ, where we normalize $K_L = 0 < K_H$, and the buyer's value for acquiring it is V_θ, with $V_H \geq V_L$. There is common knowledge of gains from trade: $V_\theta > K_\theta$ for each θ.

The equilibrium bargaining dynamics will depend on whether or not a static adverse selection problem can arise. As in DG12, we define the condition as follows:

Definition 1. *The **Static Lemons Condition (SLC)** holds if and only if $K_H > V_L$.*[4]

Until Section 7, we assume the SLC holds.

The game is played in continuous time, starting at $t = 0$ with an infinite horizon. At every time t, the buyer makes a price offer to the seller. If the seller accepts an offer of w at time t, the trade is executed and the game ends. The payoffs to the seller and the buyer respectively are $e^{-rt}(w - K_\theta)$ and $e^{-rt}(V_\theta - w)$, where $r > 0$ is the common discount rate. If trade never takes place, then both players receive a payoff of zero. Both players are risk-neutral, expected-utility maximizers.

[3] A similar property arises with respect to the large shareholder's trading strategy in the continuous time limit of DeMarzo and Urošević (2006).

[4] The SLC is related to the *Static Incentive Constraint* of DL06, which is satisfied if and only if $K_H \leq E[V_\theta | P_0]$. Hence, the SLC implies that there exists at least *some* non-degenerate P_0 such that this Static Incentive Constraint fails.

2.1 News Arrival

Prior to reaching an agreement, news about the seller's asset is revealed via a Brownian diffusion process. Regardless of type, the seller starts with an initial score X_0, normalized to 0. The news process then evolves according to

$$dX_t = \mu_\theta dt + \sigma dB_t \tag{1}$$

where $B = \{B_t, \mathcal{F}_t, 0 \leq t \leq \infty\}$ is standard Brownian motion on the canonical probability space $\{\Omega, \mathcal{F}, \mathcal{Q}\}$. At each time t, the entire history of news, $\{X_s, 0 \leq s \leq t\}$, is publicly observable. Without loss of generality, $\mu_H \geq \mu_L$. The parameters μ_H, μ_L and σ are common knowledge. Define the signal-to-noise ratio $\phi \equiv (\mu_H - \mu_L)/\sigma$. When $\phi = 0$, the news is completely uninformative. Larger values of ϕ imply more informative news. In what follows, we assume that $\phi > 0$, unless otherwise stated.

A heuristic description of the timing is depicted in Figure 1.

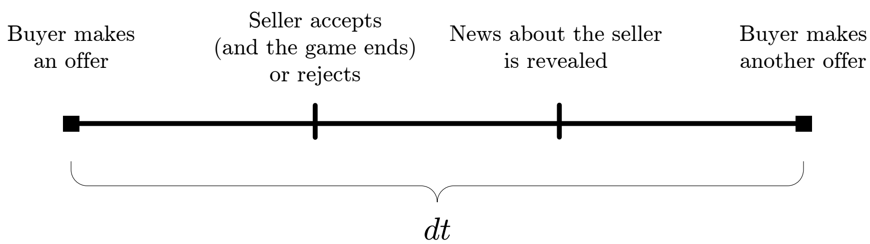


Figure 1: Heuristic Timeline of a Single "Period"

2.2 Equilibrium

Below we lay out the components of and requirements for equilibrium in turn, and collect them in Definition 2.

Stationarity In keeping with the literature, we focus on behavior that is stationary, using the uninformed party's belief as the state variable.[5] At every time t, if trade has not yet occurred, the buyer assigns a probability, $P_t \in [0, 1]$, to $\theta = H$. Analytically, it is convenient to track the belief in terms of its log-likelihood ratio, denoted $Z_t \equiv \ln\left(\frac{P_t}{1-P_t}\right) \in \overline{\mathbb{R}}$ (i.e., the extended real numbers).[6] This transformation from belief as a probability to a log-likelihood

[5]DL06 show that in discrete time, and without news, stationarity is a feature of all sequential equilibria given our assumption of common knowledge of strict gains from trade (i.e., the generalization of the "gap" case from independent-values models of Fudenberg et al. (1985) and Gul et al. (1986)).

[6]Degenerate beliefs $z = \pm\infty$ (i.e., $p = 0, 1$), are never reached in equilibrium and play no role in our analysis. Any reference to a generic state z should be intepreted as $z \in \mathbb{R}$ unless otherwise indicated.

ratio is injective, and therefore without loss. We will use z when referring to the state variable as opposed to the stochastic process Z (i.e., if $Z_t = z$, then the game is "in state z, at time t").

Formally, the belief process Z is adapted to the filtration $(\mathcal{H}_t)_{t\geq 0}$, where $\mathcal{H}_t$ is the σ-algebra generated by $\{X_s, 0 \leq s \leq t\}$. X and Z are stochastic processes defined over the probability space $\{\Omega', \mathcal{H}, \mathcal{P}\}$, where $\Omega' = \Omega \times \Theta$, $\mathcal{H} = \mathcal{F} \times 2^{\Theta}$ and $\mathcal{P} = \mathcal{Q} \times \nu$, where ν is the measure over $\Theta \equiv \{L, H\}$ defined implicitly by P_0.

Stationarity requires that both the current offer and the evolution of the belief depend only on the current belief.

Condition 1 (Stationarity)**.** *The buyer's offer in state z is given by $W(z)$, where $W : \overline{\mathbb{R}} \to \mathbb{R}$ is a Borel-measurable function, and Z is a time-homogenous $\mathcal{H}_t$-Markov process.*[7]

The Seller's Problem The seller takes the offer function, W, as given. A pure strategy for the type-θ seller is then an $\mathcal{H}_t$-adapted stopping time $\tau_\theta(\omega) : \Omega' \to \mathbb{R}_+ \cup \{\infty\}$.[8] A mixed strategy for the seller is a distribution over such times, which can be represented as a stochastic process $S^\theta = \{S^\theta_t, 0 \leq t \leq \infty\}$ also adapted to $(\mathcal{H}_t)_{t\geq 0}$. The process must be right-continuous and satisfy $0 \leq S^\theta_t \leq S^\theta_{t'} \leq 1$ for all $t \leq t'$. $S^\theta(\omega)$ is a CDF over the type-θ seller's acceptance time on $\mathbb{R}_+ \cup \{\infty\}$ along the sample path $X(\omega, \theta)$. A discontinuous increase in S^θ corresponds to acceptance with an atom.

Let $\mathcal{T}$ be the set of all $\mathcal{H}$-adapted stopping times. Given any offer function W and belief process Z, the type-θ seller faces a stopping problem.

$$\sup_{\tau \in \mathcal{T}} E^\theta \left[e^{-r\tau} (W(Z_\tau) - K_\theta) \right] \tag{SP_θ}$$

Recall that S^θ is a distribution over stopping times. Let $\mathcal{S}^\theta = supp(S^\theta)$. We say that S^θ solves (SP_θ) if all $\tau \in \mathcal{S}^\theta$ solve (SP_θ).[9]

Condition 2 (Seller Optimality)**.** *Given W and Z, S^θ solves (SP_θ).*

[7] This implies that Z is a time-homogenous Markov process with respect to the seller's information as well. For any t, s, because the distribution of Z_{t+s} given $\mathcal{H}_t$ depends only on Z_t, the distribution of Z_{t+s} given $\mathcal{H}_t$ and θ depends only on Z_t and θ, since $X(\cdot, \theta)$ has stationary, independent increments. In addition, while it is conventional to define stationarity as a restriction on strategies, which then has implications for beliefs through the *Belief Consistency* condition, Condition 1 is clearer in our model. That is, an alternative condition for *Stationarity* would replace the restriction on Z with a more notationally cumbersome, equivalent restriction on seller strategies.

[8] That is, τ_θ does not specify how to handle off-path offers, which is addressed by Condition 5.

[9] That is, for any $\tau_\theta \in \mathcal{S}^\theta$, $E^\theta \left[e^{-r\tau_\theta}(W(Z_{\tau_\theta}) - K_\theta)\right] = \sup_{\tau \in \mathcal{T}} E^\theta \left[e^{-r\tau}(W(Z_\tau) - K_\theta)\right]$.

Consistent Beliefs If trade has not occurred by time t, the buyer's belief, Z_t, is conditioned on both the entire path of past news and on the fact that the seller has rejected all past offers. It will be convenient to separate these two sources of information. Let f_t^θ denote the density of X_t conditional on θ, which for $t > 0$ is normally distributed with mean $\mu_\theta t$ and variance $\sigma^2 t$.[10] Let $S_{t^-}^\theta \equiv \lim_{s \uparrow t} S_s^\theta$ (which is well defined for $t > 0$ given that S^θ is bounded and non-decreasing), and specify that $S_{0^-}^\theta = 0$. Belief "at time t" should be interpreted to mean *before* observing the seller's decision at time t, which is why left limits are appropriate. If $S_{t^-}^L \cdot S_{t^-}^H < 1$ (i.e., given the sequence of offers up to t, there is positive probability that the seller will not have accepted yet in equilibrium), then the probability the buyer assigns to $\theta = H$ is defined by Bayes Rule as

$$\frac{P_0 f_t^H(X_t)(1 - S_{t^-}^H)}{P_0 f_t^H(X_t)(1 - S_{t^-}^H) + (1 - P_0) f_t^L(X_t)(1 - S_{t^-}^L)}. \tag{2}$$

Taking the log-likelihood ratio of (2) results in

$$Z_t = \underbrace{\ln\left(\frac{P_0}{1 - P_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)}_{\hat{Z}_t} + \underbrace{\ln\left(\frac{1 - S_{t^-}^H}{1 - S_{t^-}^L}\right)}_{Q_t} \tag{3}$$

By working in log-likelihood space we are able to represent Bayesian updating as a linear process, and the buyer's belief as the sum of two components, $Z = \hat{Z} + Q$, as seen in (3). Notice that the two component processes separate the two sources of information to the buyer. $\hat{Z}$ is the belief process for a Bayesian who updates *only based on news* starting from $\hat{Z}_0 = Z_0 = \ln\left(\frac{P_0}{1-P_0}\right)$. Q is the stochastic process that keeps track of the information conveyed in equilibrium by the fact that the seller has rejected all past offers.

Condition 3 (Belief Consistency). *For all t such that $S_{t^-}^L \cdot S_{t^-}^H < 1$, Z_t is given by (3).*

Option for Immediate Trade The next condition is simple: if the buyer offers K_H, then both types accept with probability one. Since the buyer has all of the offering power, this feature is easy to establish in any discrete-time analog.[11]

Condition 4 (Option for Immediate Trade). *If $W(Z_t) = K_H$, then $S_t^L = S_t^H = 1$.*

[10] Let $f_0^H = f_0^L$ be the Dirac delta function.

[11] See Fudenberg and Tirole (1991, pp. 409). Ortner (2017) imposes a similar condition in a continuous-time bargaining model.

The Buyer's Problem Given *Stationarity*, the value functions for each player depend only on the current state. Let $F_\theta(z)$ denote the expected payoff for the type-θ seller given state z. That is, for any $\tau \in \mathcal{S}^\theta$

$$F_\theta(z) \equiv E_z^\theta \left[e^{-r\tau}(W(Z_\tau) - K_\theta) \right],$$

where $\mathbb{E}_z^\theta$ is the expectation with respect to the probability law of the process $\{Z_t, 0 \leq t \leq \infty\}$ conditional on θ and $Z_0 = z$. Similarly, let $F_B(z)$ denote the expected payoff to the seller in any given state z:

$$F_B(z) \equiv (1-p(z))E_z^L \left[\int_0^\infty (V_L - W(Z_t)) dS_{t^-}^L \right] + p(z)E_z^H \left[\int_0^\infty e^{-rt}(V_H - W(Z_t)) dS_{t^-}^H \right], \quad (4)$$

where $p(z) \equiv \frac{e^z}{1+e^z}$.

Taking the reservation values of each type seller as given, the buyer has essentially three options in any state z. She can make an offer of K_H and trade immediately. She can make a non-serious offer that both types will reject and wait for more news. Or, she can make an intermediate offer that will be rejected by the high type, but has positive probability of acceptance by the low type.

Rather than write the buyer's problem in terms of offers, it will be more convenient to do so in terms of "quantities" (i.e., the probability of trade).[12] Thus, the buyer's problem is to choose a stopping time, denoted by T, at which she trades for sure at price K_H and a process, denoted by Q, representing the intensity of trade with the low type prior to T. The intensity of trade at time $t < T$, dQ_t, determines the belief conditional on rejection (in accordance with (3) above), and therefore the price at time t must be the low type's expected payoff conditional on rejecting the offer (i.e., $W(Z_t) = F_L(Z_t) = F_L(\hat{Z}_t + Q_t)$). We refer to the pair (T, Q) as a *policy*. A policy is *feasible* if T is an $\mathcal{F}_t$-measurable stopping rule and Q is non-negative, non-decreasing process, $\mathcal{F}_t$-measurable process. Let Φ denote the set of feasible policies.

Condition 5 (Buyer Optimality)**.** *For any z, F_B as defined by* (4) *satisfies:*

$$F_B(z) = \sup_{(Q,T)\in\Phi} \Bigg\{ (1-p(z))E_z^L \left[\int_0^T e^{-rt}(V_L - F_L(\hat{Z}_t + Q_t))e^{-Q_{t^-}} dQ_t + e^{-(rT+Q_T)}(V_L - K_H) \right] \\ + p(z)E_z^H \left[e^{-rT}(V_H - K_H) \right] \Bigg\} \quad (5)$$

[12]Formally dealing with continuation play following deviations from W posses well-known existence problems in a continuous-time setting (Simon and Stinchcombe, 1989) and would require a substantially more complicated set of available strategies for the seller.

Definition 2. *An* ***equilibrium*** *of the model is a quadruple (W, S^L, S^H, Z) that satisfies Conditions 1-5.*

3 Equilibrium

The equilibrium of the game is characterized by a belief threshold, β, and, for all $z < \beta$, a rate of trade with the low type. Specifically, when $z \geq \beta$, the buyer offers $W(z) = K_H$, which is accepted with probability one and hence trade is immediate. When $z < \beta$, the buyer offers some $W(z) < K_H$, which the high type rejects. The low type accepts at a state-specific flow rate (i.e., proportional to time), meaning rejection is a (weakly) positive signal that $\theta = H$. Therefore, the buyer's belief conditional on rejection, Z, has additional upward drift, denoted $\dot{q}(z) \geq 0$.

The next definition gives a formal description of the equilibrium candidate parameterized by $(\beta, \dot{q})$.

Definition 3. *For $\beta \in \overline{\mathbb{R}}$ and measurable function $\dot{q} : (-\infty, \beta) \to \mathbb{R}_+$, let $T(\beta) \equiv \inf\{t : Z_t \geq \beta\}$ and $\Sigma(\beta, \dot{q})$ be the strategy profile and belief process:*

$$Z_t = \begin{cases} \hat{Z}_t + \int_0^t \dot{q}(Z_s)ds & \text{if } t \leq T(\beta) \\ \text{arbitrary} & \text{otherwise}^{13} \end{cases} \tag{6}$$

$$S_t^H = \begin{cases} 0 & \text{if } t < T(\beta) \\ 1 & \text{otherwise} \end{cases} \tag{7}$$

$$S_t^L = \begin{cases} 1 - e^{-\int_0^t \dot{q}(Z_s)ds} & \text{if } t < T(\beta) \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

$$W(z) = \begin{cases} K_H & \text{if } z \geq \beta \\ \mathbb{E}_z^L[e^{-rT(\beta)}]K_H & \text{if } z < \beta \end{cases} \tag{9}$$

In a candidate $\Sigma(\beta, \dot{q})$ equilibrium, the high type seller plays a pure strategy $\tau^H = T(\beta)$ whereas the low-type mixes over $\mathcal{H}_t$-adapted stopping times.[14] The offer in each state $z < \beta$ equals the low-type seller's continuation value. If $\dot{q}(z) > 0$, the equivalency is necessary, as

[13]According to $\Sigma(\beta, \dot{q})$, if $t > T(\beta)$, trade should commence by time t with probability one. Hence, the evolution of Z—the belief conditional on rejection—in this event is the specification of the buyer's off-path beliefs. Because the buyer never offers more than K_H, no matter how high Z becomes, the specification of these off-path belief has no bearing on our results.

[14]While this mixing may appear rather involved, it can be accomplished by drawing single random variable at $t = 0$. For instance, let $\nu \sim$ exponential(1), independent from (B, θ). Let $\hat{\tau} = \inf\{t \geq 0 : \nu \leq \int_0^t \dot{q}(Z_s)ds\}$. Then $\tau^L = \hat{\tau} \wedge T(\beta)$ is distributed according to S^L.

the low type is mixing and must be indifferent. If $\dot{q}(z) = 0$, the low type weakly prefers to reject, so our specification of offers is without loss for this case.

Theorem 1. *There exists a unique pair $(\beta, \dot{q})$ such that $\Sigma(\beta, \dot{q})$ is an equilibrium.*

Theorem 1 is established by construction. In the next subsections, we derive necessary conditions of any Σ-equilibrium to identify a unique candidate $(\beta, \dot{q})$-pair (verifying that candidate is indeed an equilibrium is straightforward and left for the appendix). Before doing so, we state our second main result.

Theorem 2. *The equilibrium in Theorem 1 is unique.*

The two key features of a $\Sigma(\beta, \dot{q})$ profile are (1) a threshold β above which trade takes place immediately at a price of K_H, and (2) for $z < \beta$, trade takes place at a rate proportional to time. It is not hard to prove that (1) must be true of any equilibrium, but proving that (2) must hold in any equilibrium requires more work. We do so by employing Lesbesgue's Decomposition Theorem: since Q must be monotonic, it can be decomposed into an absolutely continuous component and a singular component. Any singular component corresponds to trade with the low type at a rate "faster" than dt, which can take the form of an atom (i.e., a jumps in Z) or local time (e.g., a reflecting boundary). We then argue that a singular component cannot be sustained in equilibrium. Appendix A.2 contains the formal proof.

Although some of the details are technical, the intuition for the argument is actually quite simple. If the equilibrium Q-process were to involve a singular component, then the low type's value function must have a right derivative of zero at the state where it ends (either the "jump-to" point or the reflecting boundary). Denote this state by α. Note that if the low type's value function has slope zero at α, then the low type is no more expensive to trade with just above α. But if a singular component is optimal at $z = \alpha$ and the low type is no more expensive to trade with just above α, then the buyer must prefer trading at an intensity greater than dt just above α. Hence, α cannot be the endpoint of the singular component.

3.1 Necessary Conditions: Determining β and F_B

Let $V(z) \equiv \mathbb{E}_z[V_\theta]$. For any state $z \geq \beta$, the buyer's value is $F_B(z) = V(z) - K_H$. For $z < \beta$, given the buyer's information, Z evolves according to

$$dZ_t = \left(\frac{\phi^2}{2} \left(2p(z) - 1\right) + \dot{q}(Z_t) \right) dt + \frac{\phi^2}{2} dB_t,$$

and the buyer's value function satisfies

$$
\begin{aligned}
rF_B(z) = \dot{q}(z)(1-p(z))(V_L - F_L(z) - F_B(z)) \\
+ \left(\frac{\phi^2}{2}(2p(z)-1) + \dot{q}(z)\right) F_B'(z) + \frac{\phi^2}{2} F_B''(z). \quad (10)
\end{aligned}
$$

Collecting the $\dot{q}$ terms gives

$$
\begin{aligned}
rF_B(z) = \underbrace{\frac{\phi^2}{2}(2p(z)-1)\,F_B'(z) + \frac{\phi^2}{2}F_B''(z)}_{\text{Evolution due to news}} \\
+ \dot{q}(z)\underbrace{\Big((1-p(z))\big(V_L - F_L(z) - F_B(z)\big) + F_B'(z)\Big)}_{\Gamma(z)\equiv\text{Net benefit of screening at } z}. \quad (11)
\end{aligned}
$$

The first term on the right-hand side of (11) is the evolution of the buyer's value arising from the news. To interpret the second term, let

$$
J(z,z') \equiv \frac{p(z') - p(z)}{p(z')}(V_L - F_L(z')) + \frac{p(z)}{p(z')}F_B(z'),
$$

which represents the buyer's payoff from moving the (post-rejection) belief to z' starting from some state $z \geq z'$ and notice that $\Gamma(z) = \frac{\partial}{\partial z'}J(z,z')\big|_{z'=z}$.

In a Σ-equilibrium the belief does not jump, meaning $z' = z$ must be weakly optimal. The necessary first-order condition is

$$
\Gamma(z) \leq 0. \quad (12)
$$

So either, $\Gamma(z) = 0$ or $\Gamma(z) < 0$. But if $\Gamma(z) < 0$ then, the buyer strictly prefers $\dot{q}(z) = 0$. In either case,

$$
\dot{q}(z)\Gamma(z) = 0. \quad (13)
$$

Therefore, (11) simplifies to

$$
rF_B(z) = \frac{\phi^2}{2}(2p(z)-1)\,F_B'(z) + \frac{\phi^2}{2}F_B''(z). \quad (14)
$$

The ODE has unique closed-form solution

$$F_B(z) = \frac{1}{1+e^z} C_1 e^{u_1 z} + \frac{1}{1+e^z} C_2 e^{u_2 z}, \tag{15}$$

where $(u_1, u_2) = \frac{1}{2}(1 \pm \sqrt{1 + 8r/\phi^2})$ and C_1, C_2 are constants yet to be determined. The boundary conditions are:

$$\lim_{z \to -\infty} |F_B(z)| < \infty \tag{16}$$

$$F_B(\beta) = V(\beta) - K_H. \tag{17}$$

Because the buyer's payoff is uniformly bounded between 0 and V_H, (16) must hold (which implies $C_2 = 0$). When Z_t hits β, trade is immediate regardless of θ. Hence, (17) is the required *value-matching* condition. Finally, *smooth pasting* of F_B is required at β:

$$F_B'(\beta) = V'(\beta). \tag{18}$$

To see why smooth pasting is required, consider the buyer at $z = \beta$. Given (17), if $F_B'(\beta^-) < V'(\beta)$, then a convex combination of $F_B(\beta - \epsilon)$ and $V(\beta + \epsilon) - K_H$ is strictly greater then $F_B(\beta) = V(\beta) - K_H$. This implies that the buyer can improve his payoff by simply waiting (i.e., make non-serious offers) for all $z \in [\beta, \beta + \delta)$ for sufficiently small δ. On the other hand, if $F_B'(\beta^-) > V'(\beta)$, then there exists an ϵ such that $F_B(\beta - \epsilon) < V(\beta - \epsilon) - K_H$, meaning the buyer would do better to trade at K_H immediately, in violation of Conditions 4-5.[15]

These necessary conditions yield a unique solution for β and F_B, as we characterize in Lemma 1. To do so, let $\underline{z} \equiv \ln\left(\frac{K_H - V_L}{V_H - K_H}\right)$ (i.e., $V(\underline{z}) = K_H$).

Lemma 1. *If $\Sigma(\beta, \dot{q})$ is an equilibrium, then*

(i) $\beta = \beta^* \equiv \underline{z} + \ln\left(\frac{u_1}{u_1 - 1}\right)$,

(ii) For all $z \geq \beta$, $F_B(z) = V(z) - K_H$, and

(iii) For all $z < \beta$, $F_B(z)$ is given by (15)*, with $C_1 = C_1^* \equiv \frac{K_H - V_L}{u_1 - 1}\left(\frac{u_1}{u_1 - 1}\frac{K_H - V_L}{V_H - K_H}\right)^{-u_1}$ and $C_2 = C_2^* \equiv 0$.*

3.2 Necessary Conditions: Determining $\dot{q}$ and F_L

In the candidate equilibrium, the low type weakly prefers to reject $W(z)$ when $z < \beta$. Hence, his equilibrium payoff must be equal to the payoff he would obtain by always rejecting in

[15]See Shiryaev (1978, Sect. 3.8) for a more formal treatment of the necessity of smooth-pasting conditions or Dixit (1993, Sect. 4.1) for a more intuitive exposition.

these states, and waiting for K_H to be offered: $F_L(z) = \mathbb{E}_z^L[e^{-rT(\beta)}]K_H$.[16] So, for $z \geq \beta$, $F_L(z) = K_H$. From the low type's perspective, for $z < \beta$, Z evolves according to

$$dZ_t = \left(\dot{q}(Z_t) - \frac{\phi^2}{2}\right) dt + \frac{\phi^2}{2} dB_t$$

and therefore F_L satisfies

$$rF_L(z) = \left(\dot{q}(z) - \frac{\phi^2}{2}\right) F_L'(z) + \frac{\phi^2}{2} F_L''(z). \tag{19}$$

Solving for $\dot{q}(z)$ gives that

$$\dot{q}(z) = \frac{rF_L(z) + \frac{\phi^2}{2} F_L'(z) - \frac{\phi^2}{2} F_L''(z)}{F_L'(z)}. \tag{20}$$

Now, recall from (12) that $\Gamma(z) \leq 0$, meaning the buyer weakly prefers not to trade "faster" with the low type. The next lemma states that the buyer is actually indifferent over all rates of trade.

Lemma 2. *If $\Sigma(\beta, \dot{q})$ is an equilibrium, then for all $z < \beta$*

$$\Gamma(z) = 0. \tag{21}$$

To understand why, notice that if $\Gamma(z) < 0$, then by (13), $\dot{q}(z) = 0$. Without any additional drift, Z_t takes longer to reach β, reducing the low type's continuation value, which (we just argued) coincides with F_L. This, in turn, raises $\Gamma(z)$ and leads to a violation of (12). The interpretation is that, if trade ever came to a halt, the low type's continuation value would become so low that he would be too cheap for the buyer resist trading faster.

Solving (21) for F_L and using Lemma 1's characterization of F_B, gives that, for all $z < \beta$,

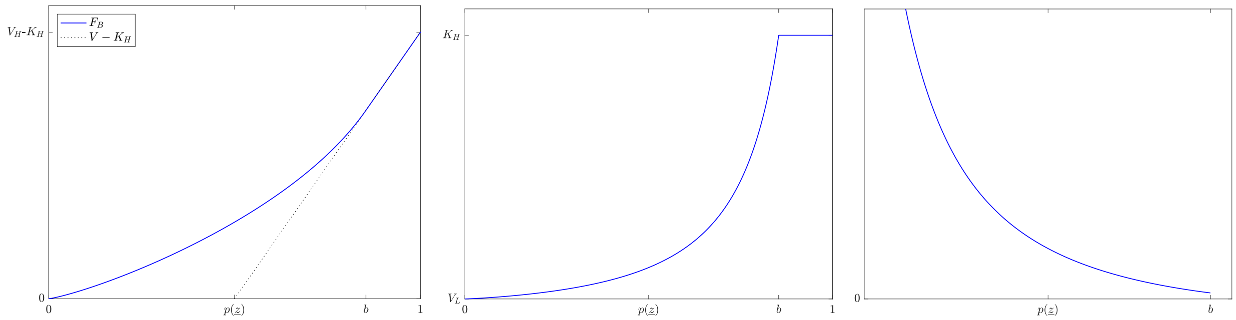
$$F_L(z) = (1 - p(z))^{-1} F_B'(z) + V_L - F_B(z). \tag{22}$$

$$= V_L + C_1^*(u_1 - 1)e^{u_1 z} \tag{23}$$

Substituting (23) into (20) gives

$$\dot{q}(z) = \frac{rV_L e^{-u_1 z}}{C_1^* u_1(u_1 - 1)} = \frac{\phi^2 V_L}{2C_1^*} e^{-u_1 z} > 0 \tag{24}$$

[16]$\mathbb{E}_z^\theta$ denotes the expectation with respect to the law of the process Z starting from $Z_0 = z$ and conditional on θ, which we denote by $\mathcal{Q}_z^\theta$.



(a) Buyer Payoff (F_B) (b) Low Type Payoff and Buyer's Offer ($F_L = W$) (c) Rate of Trade ($\dot{q}$)

FIGURE 2: Illustation of equilibrium payoffs and the rate of trade as a function of the state variable, $p(z)$.

Lemma 3. *If $\Sigma(\beta^*, \dot{q})$ is an equilibrium, then for all $z < \beta^*$, $F_L(z)$ is given by* (23) *and $\dot{q}(z)$ is given by* (24).

Henceforth, we use $(\beta, \dot{q})$ in reference to the pair that characterize the unique equilibrium of the game. Figure 2 depicts the equilibrium buyer's value function, low-type seller's value function (which is equal to the buyer's offer), and rate of trade for beliefs below β.[17]

4 Bargaining Dynamics

Having constructed the equilibrium, we now examine several of the novel implications.

4.1 Who Benefits from the Negotiation?

One interesting feature of the equilibrium is that, although the buyer engages in a negotiation, the she does not actually benefit from her ability to do so. To formalize this finding, consider an alternative version of the model in which the buyer cannot negotiate the price. Rather, the price is exogenously fixed at K_H (the lowest price that a seller would surely accept). The buyer still observes $\hat{Z}$, but the only decision that the buyer makes is when (if ever) to complete the transaction. We refer to this auxiliary model as the due diligence problem.[18]

[17] In all figures, beliefs are measured as probabilities (for example, $b = p(\beta)$, etc.).

[18] It is not hard to provide conditions under which K_H is the optimal first-stage offer in an extension of the due diligence problem where the buyer first makes a take-it-or-leave-it offer, which, if accepted, endows the buyer with the right to conduct due diligence and a perpetual option to purchase at the accepted offer price. In particular, the optimal offer in the first-stage is K_H provided that $F_B(Z_0) > (1 - P_0)(V_L - K_L)$

The due diligence problem reduces to a standard optimal stopping problem for the buyer. Her belief updates only based on news, $Z = \hat{Z}$, and stopping corresponds to a payoff of $V(z) - K_H$. Hence, she chooses a stopping time, T, to maximize $\mathbb{E}_z[e^{-rT}(V(\hat{Z}_T) - K_H)]$.

It is not difficult to establish that the unique solution of the due diligence problem is a threshold policy: $T_d = \inf\{t : Z_t \geq \beta_d\}$. Further, below β_d, the buyer's value function satisfies the ODE in (14). Finally, the value-matching and smooth-pasting conditions (16)-(18) are also required. Therefore, $\beta_d = \beta$ and we have the following result.

Proposition 1. *In the unique equilibrium of the (true) bargaining game:*

1. *The buyer's payoff is identical to her payoff in the due diligence problem.*

2. *The L-seller has a higher payoff than he would under the buyer's optimal policy in the due diligence problem.*

Intuition might have suggested that the buyer will make use of the news in two ways: (i) to ensure she is sufficiently confident that $\theta = H$, before offering the price needed to compensate a H-type seller, and (ii) to extract value out of the L-type seller with low prices if she becomes sufficiently confident that $\theta = L$. Our result is consistent with (i), but not (ii). Even though the buyer does negotiate with the seller by making offers below K_H and there is probability that such a price will be accepted, the buyer's equilibrium payoff, F_B is identical to what she would garner if she had no ability to screen using prices. This can be viewed as the manifestation of the "Coasian" force in our model.

Starting from a low belief, the buyer would like to be able commit to a low offer for at least some discrete interval of time. The rejection of this offer would, however, increase the buyer's belief at which point she would again be tempted to "experiment" by offering a price that may be accepted by the low type as described above. Without any ability to commit, she will indeed make this offer, which the low type foresees. This raises low-type continuation value, which coincides with price. See Section 7 for further discussion on the relation to the Coase Conjecture.

An immediate corollary of Proposition 1 is that total surplus is higher when the buyer can negotiate the price. However, the additional surplus is captured entirely by the seller despite the fact that the buyer makes all the offers.

(which holds provided that P_0 is not too small and/or the gains from trade with the low type are not too large).

4.2 Experimentation

Another feature of the equilibrium is that for all $z < \beta$, the buyer makes offers that are both strictly greater than V_L and only accepted by the low type. Therefore, the buyer's realized payoff is *negative* whenever such an offer is accepted.[19]

Property 1. *For all $z < \beta$, $W(z) > V_L$ and $\dot{q}(z) > 0$.*

Making these relatively high offers can be rationalized as a form of costly experimentation. The buyer's value function is strictly increasing, and therefore she values pushing the belief up. Making an offer that the low type may accept, generates a potential benefit (rejection raises the belief and, therefore, the buyer's expected payoff), but also a potential cost (acceptance means the buyer overpays, and earns a negative payoff). As shown in Proposition 1, these costs and benefits perfectly cancel each other out as the buyer exhausts all of the potential gains from experimentation leaving her with precisely the same payoff she would obtain if she were unable to experiment through price offers.

As the buyer becomes certain she is facing a low type, the implications of the buyer's willingness to engage in costly experimentation are even more extreme.

Property 2. *As $z \to -\infty$ (i.e., $p \to 0$):*

1. $F_B(z) \to 0$,

2. $F_L(z), W(z) \to V_L$,

3. $\dot{q}(z) \to +\infty$.

The buyer's value goes to zero as the probability that $\theta = L$ tends to 1. However, this is *not* due to destruction of total surplus through inefficient delay. In fact, the rate of rate with the low type grows arbitrarily large, and the low-type seller's value tends to V_L as $z \to -\infty$. Hence, trade is fully efficient in this limit (see Property 3 below), but the *entire* surplus is captured by the low type.

4.3 Efficiency

In the absence of trade, each player gets a payoff of zero. The (expected) *surplus* obtained by the seller's side of the game in state z is given by

$$\Pi^S(z) \equiv (1 - p(z))(F_L(z) - K_L) + p(z)(F_H(z) - K_H).$$

[19]Unlike in DL06 and FS10.

The buyer's suplus in state z is simply $F_B(z)$. So, total surplus realized in state z is then given by $\Pi(z) \equiv \Pi^S(z) + F_B(z)$. Due to common knowledge of gains from trade, the efficient outcome is to trade immediately, resulting in a total potential (or first-best) surplus of

$$\Pi^{FB}(z) \equiv (1 - p(z))(V_L - K_L) + p(z)(V_H - K_H).$$

Hence, $\Pi^{FB}(z) - \Pi(z) \geq 0$ and any strictly positive difference is the efficiency loss due to expected delays in trade. We define the normalized loss in efficiency as a function of z by

$$\mathcal{L}(z) \equiv \frac{\Pi^{FB}(z) - \Pi(z)}{\Pi^{FB}(z)}.$$

Property 3. *$\mathcal{L}(z) = 0$ if and only if $z \geq \beta$, but $\mathcal{L}(z) \to 0$ as $z \to -\infty$.*

5 Buyer Competition

In this section, we explore the effect of competition among buyers by contrasting our findings with DG12, which analyzes an analogous setting except with perfectly competitive buyers.[20] In most economic settings, one expects a more competitive market to lead to more efficient outcomes. However, when the uninformed side of the market can learn from news, we will see that introducing competition can have exactly the opposite effect.

By way of terminology, we refer to the *competitive outcome* as the equilibrium with multiple competing buyers from DG12, and the *bilateral outcome* as the unique Σ-equilibrium with only a single buyer. Notionally, we use a subscript $s \in \{b(ilateral), c(competetive)\}$ on objects when referencing the respective outcomes.

When buyers are competitive (and the SLC holds), DG12 show that the unique equilibrium is characterized by a pair of beliefs $\alpha_c < \beta_c$ and the following three regions. For $z > \beta_c$, trade takes place immediately at a price $V(z)$. For $z < \alpha_c$, buyers offer V_L, the high type rejects and the low type mixes. Conditional on a rejection at some $z < \alpha_c$, buyer's belief jumps to α_c. For all $z \in (\alpha_c, \beta_c)$ trade occurs with probability zero and the buyers' beliefs evolve solely due to news. Finally, at $z = \alpha_c$, the low type trades at an intensity proportional to the local time of the belief process.

Both equilibria involve a threshold belief above which trade is fully efficient and below

[20] Specifically, they replace the *Option for Immediate Trade* and *Buyer Optimality* equilibrium conditions with *Zero Profit* and *No Deals* conditions. The first ensures that any trade that takes place earns zero expected profit for a buyer. The second ensures that when trade does not take place, there does not exist an offer that a buyer could make and earn positive profit. They also impose a modest refinement on off-path beliefs.

which there is positive probability of delay. However, unlike the smooth and strictly positive trading intensity in the bilateral outcome, the trading intensity below the threshold in the competitive outcome is "lumpy" (i.e., either zero or singular). Given the upper threshold determines the set of states in which the outcome is fully efficient, the following proposition has important efficiency implications.

Proposition 2. *$\beta_c > \beta_b$.*

The intuition behind this result is the following. In the competitive outcome, buyers are willing to offer $V(z)$ at any z such that the high-type seller is willing to accept. Thus, it is the high-type seller that decides when to "stop," which nets him $V(z) - K_H$. In the bilateral outcome, it is the buyer who decides when to "stop" (i.e., offer K_H) which nets her $V(z) - K_H$.[21] While the net payoff to player who determines when to stop in the respective settings is the same, they have different expectations about the evolution of $\hat{Z}$. In particular, the drift of $\hat{Z}$ under the high-type seller's filtration is strictly greater than under the buyer's filtration. Hence, the solution to the high-type's stopping problem involves waiting longer (i.e., a higher threshold). The intuition is further strengthened by the lower boundary, α_c, in the competitive outcome where the low-type seller "pushes" the belief process upward, making the high-type even more willing to wait.

Clearly, Proposition 2 implies there exists a set of states (i.e., $z \in (\beta_b, \beta_c)$) such that the bilateral outcome is fully efficient and the competitive outcome is not. By continuity, the bilateral outcome remains more efficient just below β_b. However, for low z, the ranking reverses and the competitive outcome is more efficient as can be seen in Figure 3(a) and in the following proposition.

Proposition 3. *There exist a $z_1 \le z_2$, both in $(-\infty, \beta_b)$, such that,*

- *$\mathcal{L}_c(z) \ge \mathcal{L}_b(z)$ for all $z > z_2$ where the inequality is strict for all $z \in (z_2, \beta_c)$, and*
- *$\mathcal{L}_c(z) < \mathcal{L}_b(z)$ for all $z < z_1$.*

Intuitively, when the belief is low, trade is more efficient in the competitive outcome because the low type is trading more rapidly (i.e., with an atom compared to with a rate in the bilateral outcome), and when z is low it is the low type's trading behavior that determines efficiency.[22]

[21] Recall that the stopping threshold, β_b, is the same as the solution to the buyer's stopping problem in which she is unable to screen (i.e., the due diligence problem).

[22] In Figure 3(a), $z_1 = z_2$. This feature appears to be general, but we have not attempted to prove it formally.

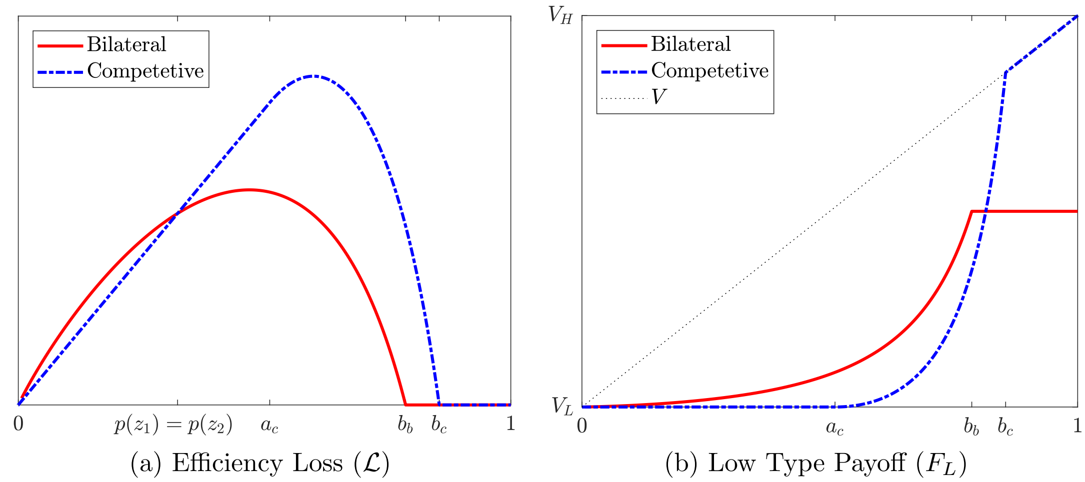


FIGURE 3: Comparison of loss in efficiency and low-type payoff across the bilateral and competitive outcomes

In terms of player welfare, the comparison for the both the buyer and the high-type seller is trivial. The buyer earns positive surplus (for all z) in the bilateral outcome, and zero in the competitive one. Conversely, the high-type seller earns zero surplus in the bilateral outcome (since the price never exceeds K_H), but earns positive surplus in the competitive outcome.

The comparison for the low-type seller is more nuanced. When the belief is low, he is better off in the bilateral outcome than in the competitive, but the reverse when the belief is high, as seen in Figure 3(b). As discussed in Section 4, in the bilateral setting, the buyer offers prices above V_L as form of experimentation, which benefits the low-type seller. There is no scope for costly experimentation in the competitive setting, as buyer-profits are driven to zero. In contrast, when the belief is high, the low-type seller enjoys buyer competition since it raises the price to V instead of only K_H.

6 News Quality

In this section we investigate the effect of news quality. First, we explore how an increase in news quality affects equilibrium play and payoffs. Then we take the limit as news becomes arbitrarily informative (i.e., $\phi \to \infty$) and arbitrarily noisy (i.e., $\phi \to 0$). Finally, we compare the $\phi \to 0$ limit to a model with no news analyzed by DL06.

6.1 An Increase in News Quality

We first state the result and then provide intuition.

Proposition 4. *As the quality of news, ϕ, increases:*

(i) *β increases.*

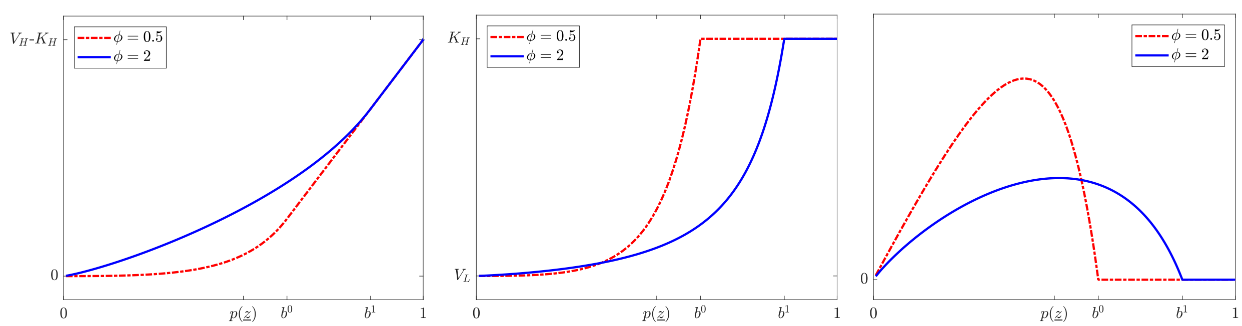
(ii) *The rate of trade, $\dot{q}$, decreases for $z < \beta - \frac{2u_1 - 1}{u_1(u_1-1)}$ but increases for $z \in (\beta - \frac{2u_1-1}{u_1(u_1-1)}, \beta)$.*

(iii) *The buyer's payoff increases for all $z < \beta$.*

(iv) *The low-type seller's payoff increases for $z < \beta - \frac{1}{u_1 - 1}$ but decreases for $z \in (\beta - \frac{1}{u_1-1}, \beta)$.*

(v) *Total surplus increases for $z < \beta - \frac{1}{u_1}$ but decreases for $z \in (\beta - \frac{1}{u_1}, \beta)$.*

Intuitively, as the quality of news increases, the buyer learns about the seller's type faster, and therefore finds it optimal to choose a higher belief threshold before exercising the option for immediate trade. Thus, both β and F_B increase with ϕ. These findings are illustrated in Figure 4(a).



(a) Buyer Payoff (F_B) (b) Low Type Payoff (F_L) (c) Efficiency loss ($\mathcal{L}$)

FIGURE 4: The effect of news quality on equilibrium payoffs and efficiency.

The effect of news quality on F_L is more subtle because there are several opposing forces. To understand them, recall that the low type's equilibrium payoff is equal to the expected discounted value of waiting until $z = \beta$, when K_H is offered. Now, holding β and $\dot{q}$ fixed, a higher ϕ means an increase in the volatility of $\hat{Z}$ which reduces the expected waiting cost and therefore increases F_L. On the other hand, a higher β (or lower $\dot{q}$) increases the waiting costs, thereby decreasing F_L. For intuition about *(iii)*, consider a discrete increase in news quality from ϕ^0 to ϕ^1 and therefore by *(i)*, $\beta^0 < \beta^1$. Clearly, the low type must be worse off with the higher news quality for $z \in (\beta^0, \beta^1)$. Continuity implies this ranking must persist for z just below β^0. However, for low enough z, the volatility effect dominates as illustrated in Figure 4(b).

These same opposing forces also affect the overall efficiency as illustrated in Figure 4(c). On the one hand, a higher ϕ "speeds things up" and reduces $\mathcal{L}$. On the other hand, because β increases, there are states in which trade would be fully efficient under ϕ^0, but is delayed with positive probability under ϕ^1. Thus, a higher ϕ leads to less efficient outcomes for z near the upper threshold, while the first effect dominates and $\mathcal{L}$ decreases for low z.

6.2 Arbitrarily Informative News ($\phi \to \infty$)

The following proposition characterizes the limit properties of the equilibrium as news quality becomes arbitrarily high. Let $\stackrel{pw}{\to}$ and $\stackrel{u}{\to}$ denote pointwise and uniform convergence, respectively.

Proposition 5. *As $\phi \to \infty$:*

(i) $\beta \to \infty$.

(ii) $\dot{q} \stackrel{pw}{\to} \infty$, *but for any* $x > 0$, $\dot{q}(\beta - x) \to \frac{rV_L}{K_H - V_L} e^x$.

(iii) $F_B \stackrel{u}{\to} p(z)(V_H - K_H)$.

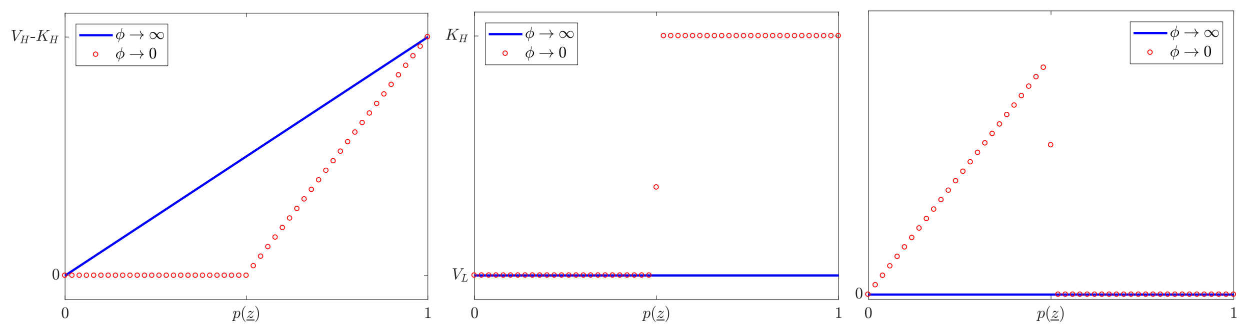
(iv) $F_L \stackrel{pw}{\to} V_L$.

(v) $\mathcal{L} \stackrel{u}{\to} 0$.

Property (i) says that the buyer waits until she is virtually sure that the seller is a high type before offering K_H. As $\phi \to \infty$, this learning happens so quickly that the delay becomes trivial and the buyer captures all of the surplus from trading with the high type seller (i.e., $V_H - K_H$). Intuition might suggest that a similar type of pattern should obtain when trading with a low type. That is, one might have expected the buyer would wait until she is virtually sure that the seller is of the low type before offering K_L; this learning would happen arbitrarily quickly as $\phi \to \infty$; and thereby the buyer would also extract all the surplus from trading the low-type seller.

Recall from Section 4.1, however, that this intuition is incorrect. For *any* ϕ, as $z \to -\infty$: $\dot{q}(z) \to \infty$, $F_B(z) \to 0$, $F_L(z) \to V_L$ and $\mathcal{L}(z) \to 0$, due to the Coasian force. Properties (ii)-(v) demonstrate that this temptation to speed up trade with the low type overwhelms the motivation to learn about the seller's type, even when this learning takes place arbitrarily quickly.[23]

Properties (*iii*)-(*v*) are illustrated in Figure 5. The disparity between the strength of convergence for F_L and F_B is due to the fact that, even for large ϕ, $F_L(z) = K_H$ for all $z \geq \beta$, meaning the convergence of F_L to V_L is only pointwise.



(a) Buyer Payoff (F_B) (b) Low Type Payoff (F_L) (c) Efficiency loss ($\mathcal{L}$)

FIGURE 5: Limiting payoffs and efficiency loss as $\phi \to \infty$ and $\phi \to 0$.

6.3 Arbitrarily Uninformative News ($\phi \to 0$)

We now turn to the other extreme in which news tends to pure noise.

Proposition 6. *As $\phi \to 0$:*

(i) $\beta \to \underline{z}$.

(ii) For all $z < \underline{z}$, $\dot{q}(z) \to \infty$, *but* $\dot{q}(\underline{z}) \to 0$.

(iii) $F_B \xrightarrow{u} \begin{cases} 0 & \text{if } z < \underline{z} \\ V(z) - K_H & \text{if } z \geq \underline{z}. \end{cases}$

(iv) $F_L \xrightarrow{pw} \begin{cases} V_L & \text{if } z < \underline{z} \\ (1 - e^{-1})V_L + e^{-1}K_H & \text{if } z = \underline{z} \\ K_H & \text{if } z > \underline{z}. \end{cases}$

[23] This fact may partially be attributed to the order of limits. By analyzing a continuous-time model directly, we have implicitly taken the period length to zero first (i.e., before taking $\phi \to \infty$). If we were to interchange the order of limits (i.e., consider a discrete-time model with news and take the limit as $\phi \to \infty$ *before* taking the period length to zero), then it is plausible that the intuition given above would prove correct.

$$(v)\ \mathcal{L} \xrightarrow{pw} \begin{cases} \frac{p(z)(V_H - K_H)}{\Pi^{FB}(z)} & \text{if } z < \underline{z} \\ \frac{p(z)(V_H - K_H) - (1-p(z))e^{-1}(K_H - V_L)}{\Pi^{FB}(z)} & \text{if } z = \underline{z} \\ 0 & \text{if } z > \underline{z}. \end{cases}$$

To interpret these results, it is useful to draw a comparison to DL06. For convenience, we restate their result below using our notation.

Result (DL06, Proposition 2)**.** *Consider a two-type, discrete-time model with no news (i.e., $\phi = 0$), and suppose that SLC holds. In equilibrium, as the period length between offers goes to zero,*

(a) For all $z > \underline{z}$, the buyer offers K_H and the seller accepts w.p.1.

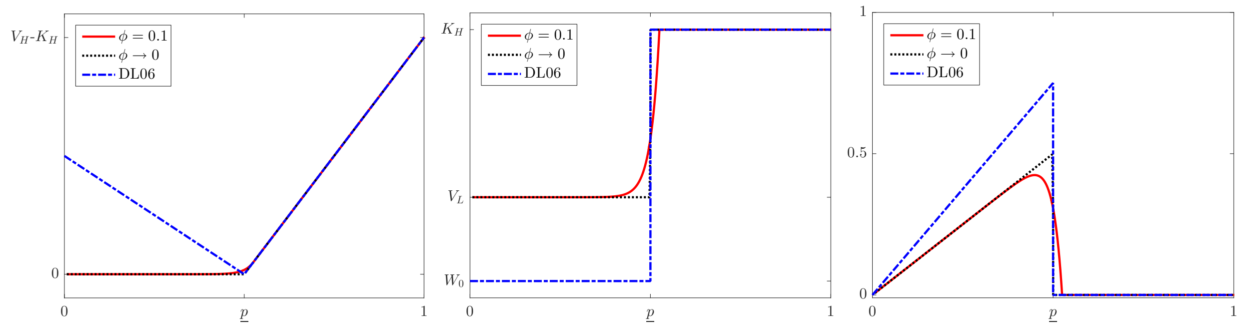
(b) For $z < \underline{z}$, the buyer makes an offer of $w_0 = \frac{V_L^2}{C_H}$. The high type rejects and the low type mixes such that the belief is $\underline{z}$ following a rejection.

(c) For $z = \underline{z}$, there is delay of length 2τ, where τ satisfies $V_L = e^{-r\tau}K_H$, after which the buyer offers K_H and the seller accepts w.p.1.

There are notable similarities between the result above and our findings in Proposition 6. For $z > \underline{z}$, the predictions are perfectly aligned; trade takes place immediately at a price equal to the high-type's cost. In addition, for $z < \underline{z}$, in both settings there is a "burst" of trade with the low type and delay ensues conditional on a rejection. The key differences are the buyer's offer when $z < \underline{z}$ and the amount of ensuing delay. In our case, the offer is V_L and the amount of ensuing delay is τ, whereas in DL06 the offer is $w_0 < V_L$ and the amount of ensuing delay is exactly twice as along.

A perhaps surprising implication is that the buyer is strictly worse off for all $z < \underline{z}$ in our limit (continuous time, $\phi \to 0$) than in that of DL06 (discrete time, $\phi = 0$, period length $\to 0$). An intuition for this result is as follows. In DL06, if the buyer delays trade (by making unacceptable offers), the belief remains constant and when the buyer's belief is $\underline{z}$, the temptation to speed up trade (i.e., the Coasian force) is absent because the buyer's continuation value from this state is zero. Hence, in DL06, the buyer can leverage an endogenous form of commitment power: it is both feasible and sequentially rational for the buyer to delay trade at $\underline{z}$ and for her belief to remain constant during such a delay. This allows her to extract more surplus from the low type in states $z < \underline{z}$.

In contrast, with even an arbitrarily small amount of Brownian news, the buyer's belief will instantaneously diverge from $\underline{z}$ almost surely. That is, the buyer cannot just "sit" at $\underline{z}$, and make non-serious offers for any amount of time, because she observes news and updates her belief arbitrarily quickly, which strengthens the Coasian force and reduces her ability to extract surplus in all states $z < \underline{z}$.



(a) Buyer Payoff (F_B) (b) Low Type Payoff (F_L) (c) Efficiency loss ($\mathcal{L}$)

FIGURE 6: Payoffs and efficiency comparison to DL06 with $\phi > 0$ and as $\phi \to 0$.

Another implication is that even a small amount of news can lead to a discontinuous improvement in efficiency. Without news, in order to extract the extra surplus, the buyer uses her (endogenous) commitment power at $\underline{z}$, which implies more delay and hence more inefficiency. These findings are illustrated in Figure 6.

7 When the SLC Fails and the Coase Conjecture

We now turn to equilibrium when the SLC fails. In this case, the unique equilibrium outcome involves no delay.

Theorem 3. *When the SLC fails, there is a unique equilibrium. In it, $W(z) = K_H$ and trade is immediate for all z.*

One intuition for the result comes via the connection to the due diligence problem from Section 4. Recall that the buyer's equilibrium payoff (in the true game) coincides with her payoff in the due diligence problem. Without the SLC, however, the solution to the due diligence problem is to "stop" (i.e., trade at price K_H) immediately. Why? The buyer's reward from stopping in the due diligence problem is strictly positive and linear in her belief $p \in (0, 1)$, which is a martingale. Since she discounts future payoffs, she can do no better than stopping immediately.

Strikingly, Theorem 3 holds regardless of the quality of the news process, ϕ. This can be viewed as an extension of the Coase conjecture. Interpreted within our setting, Coase (1972) conjectured that the buyer's competition with her future self would lead to immediate trade at a price K_H when there is no news, $\phi = 0$, and independent values, $V_H = V_L > K_H$ (which implies the SLC fails). Our results show that, without the SLC, the Coasian force swamps the incentive to delay and learn from Brownian news.[24]

However, this result also brings a subtlety to the interpretation of the Coasian force. Often, the force is interpreted as: competition with the future self simulates competition from other buyers, leading to efficient trade. With news however, DG12 shows that the outcome with competitive buyers features periods of delay, and therefore is *not* efficient, even when the SLC fails (Proposition 5.3 therein). Moreover, as Section 5 makes clear, competition with the future self does not simulate intra-temporal competition in the presence of news.

We believe this suggests a different interpretation of the Coasian force. Namely, the inability to commit to prices means that the buyer (i.e., uninformed party) gains nothing from the ability to screen using prices. In Coase's setting (independent values, no news), it then *follows* that trade will be immediate and efficient, just as it would be if competitive buyers were introduced. In general however, the inability to profit by screening through prices need not lead to a pattern of trade resembling the pattern from the competitive-buyer environment. In fact, with news the bargaining outcome is *more* efficient than the competitive outcome if *i*) the SLC fails, or *ii*) the SLC holds and the belief is sufficiently optimistic (Proposition 3).

8 Extensions

In this section we consider two extensions of the model: costly information acquisition and lumpy information arrival. We view these extensions as serving multiple purposes. First, to illustrate how our interpretation of the Coasian force (described above) can be used for constructing equilibrium. Second, to demonstrate robustness of our main results and provide several additional insights.

8.1 Costly Investigation

In many applications, information is not freely generated. Rather the buyer must "investigate" by actively engaging in activities to unearth information. For example, during due diligence, acquiring firms hire auditors, lawyers, and other consultants to investigate the financial soundness of the target. Such activities require resources, which we now model explicitly by introducing a flow cost, $m > 0$, incurred by the buyer while still engaged in the negotiation with the seller. Costly investigation introduces the possibility that the buyer

[24] DL06 show that the Coase conjecture holds for the interdependent case (again, without news) if the Static Incentive Constraint is satisfied (i.e., $K_H \le E[V_\theta|P_0]$). FS10 show delay can arise if the news instead has the potential to perfectly reveal θ in finite time.

may prefer to terminate the negotiation, if she anticipates that it will take too long to reach an agreement. We therefore endow the buyer with this strategic option, which if exercised, generates a payoff of zero for both players.[25]

The Due Diligence Problem with Costly Investigation. To construct the equilibrium, we start by using our conjecture that the buyer will be unable to profit from the ability to negotiate the price. Hence, we first solve for the buyer's value function in the analog of the due diligence problem. In the original due diligence problem (Section 4.1), the buyer chooses a stopping time τ to maximize $\mathbb{E}_z[e^{-r\tau}(V(\hat{Z}_\tau) - K_H)]$. With the addition of the flow cost, the buyer's problem becomes:

$$\sup_{\tau} \ \mathbb{E}_z \left[-\int_0^\tau e^{-rt} m dt + e^{-r\tau} \max \left\{ V(\hat{Z}_\tau) - K_H, 0 \right\} \right]. \tag{25}$$

The integral term captures the cumulative investigation costs incurred, and the max operator incorporates the idea that when the buyer "stops" she may be exercising the option to trade at price K_H or terminating the negotiation.

Lemma 4. *The unique solution to* (25) *is of the form* $\tau = \inf \left\{ t : \hat{Z} \notin (\alpha_m, \beta_m) \right\}$, *with* $-\infty < \alpha_m < \underline{z} < \beta_m < \infty$. *For* $z \in (\alpha_m, \beta_m)$ *the buyer's value function satisfies*

$$rF_B(z) = -m + \frac{\phi^2}{2}\big((2p(z) - 1)F_B'(z) + F_B''(z)\big),$$

where (α_m, β_m) *and the constants in the buyer's value function are characterized by the boundary conditions*

$$F_B(\alpha_m) = 0 \tag{26}$$

$$F_B'(\alpha_m) = 0 \tag{27}$$

$$F_B(\beta_m) = V(\beta_m) - K_H \tag{28}$$

$$F_B'(\beta_m) = V'(\beta_m). \tag{29}$$

As before, the buyer exercises the option to trade when her beliefs are sufficiently optimistic ($z \geq \beta_m$), but with the investigation now being costly, the buyer chooses to terminate the negotiation if her beliefs are sufficiently pessimistic ($z \leq \alpha_m$).[26]

[25] Notice that the buyer would never exercise the option to terminate the bargaining in the model of Section 2 (i.e., with $m = 0$) as she can always guarantee herself a positive payoff by playing the optimal strategy from the due diligence problem (Section 4.1).

[26] Note, as $m \to 0$, $\alpha_m \to -\infty$ and $\beta_m \to \beta_d$, in line with Section 4.1.

Equilibrium with Costly Investigation. Characterizing the equilibrium offers and acceptance rates that garner the buyer her due diligence payoff for $z > \alpha_m$ is analogous to the construction in the model with $m = 0$ (Sections 3.1-3.2). For $z \geq \beta_m$, trade is immediate at a price K_H. For $z \in (\alpha_m, \beta_m)$, there is zero net benefit to screening ($\Gamma(z) = 0$), implying the offer and low-type continuation value is as in (22), which is accepted at the smooth rate characterized by (20). For these beliefs, $F_B(z) > 0$, so the buyer never walks away.

The new piece of the equilibrium construction is determining the behavior and low-type payoffs for $z \leq \alpha_m$ (i.e., when $F_B(z) = 0$). As before, $F_L(z) \geq V_L$ for any z, otherwise the buyer would seek to trade with the low type at a higher intensity than the equilibrium called for, generating a contradiction. Therefore, set $W(z) = V_L = F_L(z)$ for all $z \leq \alpha_m$, where $W(z)$ should be interpreted as the offer in state z *conditional* on the buyer *not* terminating the negotiation. Given that the seller continuation payoff is constant below α_m, the belief must exit the region in zero time conditional on rejection. Hence, for $z < \alpha_m$ the low type accepts with probability $\frac{p(\alpha_m)-p(z)}{p(\alpha_m)(1-p(z))}$, so that z jumps to α_m conditional on rejection.

The last part of the construction is to characterize the behavior precisely at $z = \alpha_m$. We first argue that the buyer must sometimes terminate the negotiation. If not, then (conditional on rejection) the belief process would have a reflecting boundary at $z = \alpha_m$, and the implied boundary condition is $F_L'(\alpha_m^+) = 0$. However, differentiating (22) gives that F_L must satisfy

$$F_L'(\alpha_m^+) = (1 + e^{\alpha_m})F_B''(\alpha_m^+) + (e^{\alpha_m} - 1)\underbrace{F_B'(\alpha_m^+)}_{=0}. \tag{30}$$

This implies $F_L'(\alpha_m^+) > 0$ by the convexity of the buyer's value function in the "continuation" region (α_m, β_m), which obviously contradicts the boundary condition implied by reflection. Hence, the buyer must sometimes terminate the negotiation at $z = \alpha_m$. Let ζ denote the (random) date of termination and denote the termination rate by $\kappa \geq 0$, which is sometimes referred to as a "killing rate."[27] Because the buyer earns $F_B(\alpha) = 0$ by continuing the negotiation, she is indifferent between remaining in the negotiation and exiting, so is willing to mix. The implied boundary condition for the low type is now $F_L'(\alpha_m^+) = \kappa(F_L(\alpha_m) - 0) = \kappa V_L$.[28] To satisfy (30), set $\kappa = \frac{(1+e^{\alpha_m})F_B''(\alpha_m^+)}{V_L}$, which completes the equilibrium construction.

Proposition 7. *There exists an equilibrium of the bargaining game with costly investigation (as characterized above) in which the buyer's value function is equal to her value function in the due diligence problem with costly investigation (as characterized in Lemma 4).*

[27]Formally, $\zeta = \inf\{t \geq 0 : \kappa L_t = \xi\}$, where L_t is the local time of Z_t at α_m and $\xi \sim$ exponential(1) and independently distributed. See Harrison (2013, Section 9.3), for details on the construction of this process.

[28]See Harrison (2013, p.160). The buyer's Robin condition is $F_B'(\alpha_m^+) = \kappa F_B(\alpha_m) = 0$, which is redundant given (26) and (27).

One interesting implication of this extension is its effect on *seller* welfare. The threshold at which the buyer offers K_H is strictly decreasing in m. Hence, there exists a cutoff belief above which the low-type seller benefits from higher buyer investigation costs. However, below that cutoff the low type seller is worse off when the buyer must pay more to investigate. Intuitively, the higher investigation cost prompts the buyer to end the game more quickly—be it by offering K_H (which benefits the seller) or by walking away (which harms the seller). When the belief is high, the former effect dominates and F_L increases with m; when the belief is low, the latter effect dominates and F_L decreases with m.

8.2 Lumpy Information Arrivals

We now consider an extension where in addition to learning gradually from the news process X_t, the buyer may also learn from "lumpy" information arrivals. Specifically, there is a Poisson process with intensity $\lambda > 0$, and at its first arrival time, ν, the buyer (publicly) learns θ, at which point trades occurs immediately at price K_θ.[29]

The Due Diligence Problem with Lumpy Information. To construct the equilibrium, we again start with our conjecture that the buyer will receive the same payoff as she would in the analog of the due diligence problem. Hence, we first solve an updated version of that game. In the original due diligence problem (Section 4.1), the buyer chooses a stopping time τ to maximize $\mathbb{E}_z[e^{-r\tau}(V(\hat{Z}_\tau) - K_H)]$. With the addition of the perfectly revealing arrival, the buyer's problem becomes:

$$\sup_\tau \mathbb{E}_z \left[e^{-r(\tau\wedge\nu)} \left(V(\hat{Z}_{\tau\wedge\nu}) - \left[K_H \mathbb{1}_{\{\tau<\nu\}} + K(\hat{Z}_\nu)\mathbb{1}_{\{\nu\le\tau\}} \right] \right) \right] \tag{31}$$

where $K(z) = \mathbb{E}_z[K_\theta]$. To understand (31), notice that trade occurs at $\tau \wedge \nu$ regardless of θ. The only difference is that if $\tau < \nu$, then the buyer pays K_H for both types whereas if $\nu \le \tau$ then she pays K_θ (since θ is revealed at ν, $p(\hat{Z}_\nu) \in \{0, 1\}$).

Lemma 5. *The unique solution to* (31) *is of the form* $\tau = T(\beta_\lambda) = \inf\{t : \hat{Z} \ge \beta_\lambda)\}$, *with* $\underline{z} < \beta_\lambda < \infty$. *For* $z < \beta_\lambda$ *the buyer's value function satisfies*

$$(r+\lambda)F_B(z) = \lambda(V(z) - K(z)) + \frac{\phi^2}{2}\left((2p(z)-1)F_B'(z) + F_B''(z)\right), \tag{32}$$

where β_λ *and the constants in the buyer's value function are characterized by the boundary conditions* (16)-(18) *(with* β *replaced by* β_λ*).*

[29] The case in which $\lambda = 0$ is the model from Section 2. A type-dependent arrival rate would simply add a drift of $(\lambda_L - \lambda_H)$ to $d\hat{Z}$ prior to an arrival.

Not surprisingly, lumpy information arrivals benefit the buyer in the due diligence problem and induce her to wait longer before offering K_H. That is, it is straightforward to show that both β_λ and F_B are increasing in λ.

Equilibrium with Lumpy Information. Characterizing the equilibrium offers and acceptance rates that garner the buyer her due diligence payoff is analogous to the construction in the model with $\lambda = 0$ (Sections 3.1-3.2). For $z \geq \beta_\lambda$, trade is immediate at a price K_H. For $z < \beta_\lambda$, there is zero net benefit to screening ($\Gamma(z) = 0$), implying the offer and low-type continuation value is as in (22). The low type's acceptance rate is given by the analog of (20):

$$\dot{q}(z) = \frac{(r+\lambda)F_L(z) + \frac{\phi^2}{2}F_L'(z) - \frac{\phi^2}{2}F_L''(z)}{F_L'(z)}, \tag{33}$$

which reflects that, because he earns nothing if his type is revealed, his discount rate effectively increases to $r + \lambda$.

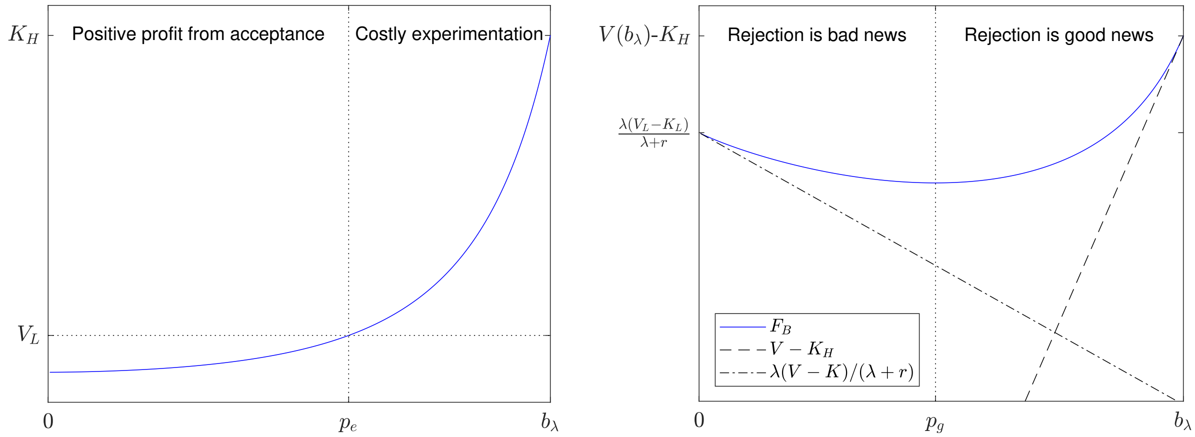
Proposition 8. *There exists an equilibrium of the bargaining game with lumpy information arrivals (as characterized above) in which the buyer's value function is equal to her value function in the due diligence problem with lumpy information arrivals (as characterized in Lemma 5).*

Lumpy information arrivals alter the price dynamics when the buyer is trading only with the low type (i.e., for $z < \beta_\lambda$). First, because the buyer has the option of waiting for θ to be perfectly revealed, she is able to earn price concessions from the low-type seller in proportion to the value of this option. For example,

$$\lim_{z \to -\infty} F_B(z) = \frac{\lambda}{r+\lambda}V_L > 0 \qquad \text{and} \qquad \lim_{z \to -\infty} F_L(z) = \frac{r}{r+\lambda}V_L < V_L.$$

Hence, the buyer earns a positive profit if the low type accepts (i.e., $V_L - F_L(z) > 0$) when the belief is low (below p_e in Figure 7(a)), unlike in the $\lambda = 0$ case. This finding illustrates a fundamental difference between the Brownian news and perfectly revealing arrivals. That is, information that changes the support of the buyer's beliefs allows her to extract concessions from the low type whereas the Coasian force overwhelms her ability to do so with Brownian news. Nevertheless, even with lumpy arrivals, a region of costly experimentation (see Section 4.2) always persists (above p_e in Figure 7(a)).

One manifestation of the buyer's ability to extract concessions from the low type is that her value function F_B may be non-monotone in z (first decreasing then increasing). This



(a) Buyer's Offer ($W(z) = F_L(z)$) (b) Buyer's Payoff (F_B)

FIGURE 7: Lumpy information arrival allows the buyer to extract surplus from trading with the low-type seller for low beliefs (i.e., below p_e in panel (a)) and can lead to an equilibrium buyer value function that is decreasing for low beliefs (i.e., below p_g in panel (b)).

occurs when the gains from trade with the low type are larger than the gains from trade with the high type ($V_L - K_L > V_H - K_H$) and λ is sufficiently large. When F_B is decreasing (below p_g in Figure 7(b)), a rejection, which moves her belief upward, is "bad news" from the buyer's perspective. To see this, recall that in equilibrium, the net benefit of screening, Γ, is zero, which implies that

$$F_B(z) < V_L - W(z) \iff F_B'(z) < 0.$$

Hence, the buyer's value function is decreasing at z if and only if the buyer's payoff from an acceptance ($V_L - W(z)$) is strictly higher than her expected payoff prior to making the offer ($F_B(z)$). Because F_B is always positive, the region over which it is decreasing is a subset of the region over which $V_L - W(z) > 0$. That is, $p_g < p_e$.

Notice the contrast to the model with $\lambda = 0$ in which F_B is everywhere increasing. Intuitively, without lumpy arrivals the buyer loses money on all trades with the low type, and hence the total surplus generated from such trades is irrelevant for her payoff because she is not able to extract any of it. Because the buyer only profits on trades with the high type, a rejection is always good news for $z < \beta$.

9 Concluding Remarks

We have investigated a bilateral-bargaining model in which the seller's private information is gradually revealed to the buyer until agreement is reached. In equilibrium, the buyer's ability to leverage her access to information in order to extract more surplus from the seller is remarkably limited. In particular, the buyer's payoff is identical to what she would achieve if she were unable to renegotiate the price based on new information. Both the trading dynamics and efficiency differ from the competitive-buyer analog. Hence, insofar as the buyer "competes with her future self," this inter-temporal competition is *not* a perfect proxy for intra-temporal competition.

Rather, the robust implication of the Coasian force is that competition with future self renders the ability to screen through prices useless. We adopt this heuristic to solve several extensions of the model including costly investigation and lumpy information arrival. In both cases, the equilibrium can be constructed in a straightforward and "stepwise" fashion by first solving a simple stopping problem for the uninformed player, which is independent of the informed player's value function. Our methodology appears to be useful for constructing equilibria in bargaining models with frequent offers.

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A Appendix

A.1 Proofs for Theorem 1

Proof of Lemma 1. From Section 3.1, if $\beta \in \mathbb{R}$, then F_B, β, C_1, C_2 must satisfy (15)-(18). First, from (15),

$$\lim_{z \to -\infty} F_B(z) = \lim_{z \to -\infty} \frac{1}{1+e^z} C_1 e^{u_1 z} + \frac{1}{1+e^z} C_2 e^{u_2 z} = \begin{cases} -\infty & \text{if } C_2 < 0 \\ 0 & \text{if } C_2 = 0 \\ \infty & \text{if } C_2 > 0. \end{cases}$$

To satisfy (16), therefore, in any solution $C_2 = 0$. This simplifies the remaining two equations, (17) and (18):

$$F_B(\beta) = \frac{C_1 e^{u_1 \beta}}{1+e^\beta} = V(\beta) - K_H = \frac{e^\beta}{1+e^\beta}(V_H - V_L) + V_L - K_H$$
$$F_B'(\beta) = \frac{C_1 e^{u_1 \beta}\left((u_1 - 1)e^\beta + u_1\right)}{(1+e^\beta)^2} = V'(\beta) = \frac{e^\beta}{(1+e^\beta)^2}(V_H - V_L).$$

The unique solution to the two equations above is

$$\beta = \beta^* \equiv \underline{z} + \ln\left(\frac{u_1}{u_1 - 1}\right)$$
$$C_1 = C_1^* \equiv \frac{K_H - V_L}{u_1 - 1}\left(\frac{u_1}{u_1 - 1}\frac{K_H - V_L}{V_H - K_H}\right)^{-u_1}.$$

If $\beta = \infty$, then $F_B(z) = 0$ for all $z \in \mathbb{R}$. But this violates *Buyer Optimality* (Condition 5) since the buyer could improve his payoff by offering K_H (leading to payoff $V(z) - K_H$ by Condition 4) for any $z > \underline{z}$. Finally, if $\beta = -\infty$, then $F_B(z) = V(z) - K_H$ for all $z \in \mathbb{R}$. But this also violates *Buyer Optimality* as the buyer's payoff is negative all $z < \underline{z}$, and he would improve his payoff by making a non-serious offers in these states. □

Proof of Lemma 2. Fix $\beta = \beta^*$ and F_B as given by Lemma 1. Given an arbitrary $\dot{q}$ on $z < \beta$, let $G_L^{\dot{q}}(z)$ be the expected payoff of a low type who rejects all offers until $Z_t \geq \beta$ (i.e., $E_z^L[e^{-rT(\beta)}]K_H$). Let $\dot{q}^*$ denote expression for $\dot{q}$ given in (24). Therefore, for all $z < \beta$,

$$\frac{1}{1+e^z}\left(V_L - G_L^{\dot{q}^*}(z) - F_B(z)\right) + F_B'(z) = 0.$$

From (12), $\Gamma(z) \leq 0$ for all $z < \beta$. For the purpose of contradiction, suppose there exists $z_0 < z_1 < \beta$ such that $\Gamma(z) < 0$ for all $z \in (z_0, z_1)$. To satisfy (13), then $\dot{q}(z) = 0$ for all $z \in (z_0, z_1)$. Hence, $G_L^{\dot{q}}(z) < G_L^{\dot{q}^*}(z)$ for all $z \in (z_0, z_1)$, which implies that

$$\frac{1}{1+e^z}\left(V_L - G_L^{\dot{q}}(z) - F_B(z)\right) + F_B'(z) > 0, \quad \forall z \in (z_0, z_1).$$

Finally, recall that in equilibrium, the low type always weakly prefers rejection in state $z < \beta$, so $F_L(z) = G_L^{\dot{q}}(z)$. Hence, for all $z \in (z_0, z_1)$,

$$\Gamma(z) = \frac{1}{1+e^z}\left(V_L - F_L(z) - F_B(z)\right) + F_B'(z) > 0,$$

producing a contradiction. Finally, since for any absolutely continuous Q, $G_L^{\dot{q}}$ and F_B are continuous on $(-\infty, \beta)$, and in equilibrium $F_L(z) = G_L^{\dot{q}}(z)$, $\Gamma(z)$ is continuous. Hence, if $\Sigma(\beta, \dot{q})$ is an equilibrium, $\Gamma(z) = 0$ for all $z < \beta$. $\square$

Proof of Lemma 3. Immediate from Lemmas 1 and 2, and analysis in Section 3.2. $\square$

Proof of Theorem 1. Lemmas 1 and 3 show that there exists a unique candidate $\Sigma(\beta, \dot{q})$. Thus, to prove the theorem, we need only verify that this candidate satisfies the equilibrium conditions. Conditions 1, 3, and 4 are satisfied by construction for any $(\beta, \dot{q})$: 1 follows immediately from (6), 3 can be verified by inserting (7) and (8) into (3), 4 is also immediate from (7)-(9) since $S_t^\theta = 1$ for all $t \geq T(\beta) = \inf\{s : Z_s \geq \beta\} = \inf\{s : W(Z_s) = K_H\}$.

Next we verify *Seller Optimality* (Condition 2). Consider first the high type and note from (7) that $\mathcal{S}^H = \{T(\beta)\}$ and from (9) that $W(z) \leq K_H$. Therefore,

$$\sup_{\tau \in \mathcal{T}} E^H\left[e^{-r\tau}(W(Z_\tau) - K_H)\right] \leq 0 = F_H(z),$$

where $F_H(z)$ is equal to the high-type's payoff under the candidate equilibrium strategy, $T(\beta)$, which verifies that S^H solves (SP_H).

For the low type, recall that, by construction, $F_L(z) = \mathbb{E}_z^L[e^{-rT(\beta)}]K_H$. Let $\mathcal{T}(\beta) \equiv \mathcal{T} \cap \{\tau : \tau \leq T(\beta), \forall\, \omega\}$, i.e., the set of all $\mathcal{H}$-adapted stopping times such that $\tau \leq T(\beta)$ for all ω. Observe that $\mathbb{E}_z^L[e^{-r\tau}W(Z_\tau)] \leq F_L(z)$ for any $\tau \in \mathcal{T} \setminus T(\beta)$ since W is bounded above by K_H and delay is costly. That is, since K_H is the largest possible offer, it is optimal for the low type to accept it as soon as it is offered. Note further that $\mathcal{S}^L \subseteq \mathcal{T}(\beta)$. To prove S^L solves (SP_L), we show that, in fact, for *any* $\tau \in \mathcal{T}(\beta)$, $E_z^L\left[e^{-r\tau}W(Z_\tau)\right] = F_L(z)$, which verifies that S^L solves (SP_L).

Let $f_L(t, z) \equiv e^{-rt}W(z)$ and note that f_L is C^2 for all $z \neq \beta$. Conditional on $\theta = L$ and $t < T(\beta)$, Z evolves according to

$$dZ_t = \left(\dot{q}(Z_t) - \frac{\phi^2}{2}\right)dt + \phi dB_t.$$

By Dynkin's formula, for any $\tau \in \mathcal{T}(\beta)$,

$$E_z^L[f_L(\tau, Z_\tau)] = f_L(0, z) + E_z^L\left[\int_0^\tau \mathcal{A}^L f_L(s, Z_s)ds\right],$$

where $\mathcal{A}^L$ is the characteristic operator for the process $Y_t = (t, Z_t)$ under $\mathcal{Q}^L$, i.e.,

$$\mathcal{A}^L f(t, z) = \frac{\partial f}{\partial t} + \left(\dot{q}(z) - \frac{\phi^2}{2}\right)\frac{\partial f}{\partial z} + \frac{1}{2}\phi^2\frac{\partial^2 f}{\partial z^2}. \tag{A.1}$$

Applying $\mathcal{A}^L$ to f_L, we get that

$$\begin{aligned}\mathcal{A}^L f_L(t,z) &= e^{-rt}\left[-rW(z) + \left(\dot{q}(z) - \frac{\phi^2}{2}\right)W'(z) + \frac{\phi^2}{2}W''(z)\right]\\ &= e^{-rt}\left[-rF_L(z) + \left(\dot{q}(z) - \frac{\phi^2}{2}\right)F_L'(z) + \frac{\phi^2}{2}F_L''(z)\right]\\ &= 0,\end{aligned}$$

where the first equality follows from the fact the $W(z) = F_L(z)$ (by construction, see (9)) and the second equality from the fact that $\dot{q}$ satisfies (20). Hence, for any $\tau \in \mathcal{T}(\beta)$, $E_z^L[f_L(\tau, Z_\tau)] = F_L(z)$, as desired.

The last step in the proof is to verify *Buyer Optimality* (Condition 5). In order to do so, we first characterize an upper bound on the buyer's payoff in Lemma A.1 (below) and then verify that F_B achieves this bound. An immediate corollary of Lemma A.1 is that if there exists a feasible (Q,T) under which the buyer's expected payoff satisfies the hypothesis of the Lemma, then the policy is optimal. By construction, F_B is the buyer's payoff under the policy $Q_t = \int_0^t \dot{q}(Z_s)ds$, $T = T(\beta)$. Observe that $F_B \in C^1$ and is C^2 for all $z \neq \beta$, therefore, it suffices to verify that F_B satisfies (A.3)-(A.5).

Verification that F_B satisfies (A.3)-(A.5)*:*

- For $z \leq \beta$. First, note that (A.5) holds with equality for all $z < \beta$ by construction. Hence, we need only verify (A.3) and (A.4). For (A.4), recall that

$$J(z,z') \equiv \frac{p(z') - p(z)}{p(z')}(V_L - F_L(z')) + \frac{p(z)}{p(z')}F_B(z').$$

To see that $F_B(z) \geq \sup_{z' \geq z}\{J(z,z')\}$, note that

$$\frac{d}{dz'}J(z,z') = \frac{C_1 e^{(u_1-1)z'}\left(e^z - e^{z'}\right)(-1+u_1)u_1}{1+e^z} < 0,\ \forall z' \in (z,\beta). \tag{A.2}$$

Since $J(z,z')$ is decreasing in z', we have that $F_B(z) = J(z,z) = \sup_{z'\in(z,\beta)} J(z,z')$. Furthermore, $J(z,z') = V(z) - K_H \leq F_B(z)$ for $z' \geq \beta$ (the latter inequality is shown below), which verifies that the first term is non-positive for all $z' > z$.

To see that $F_B \geq V - K_H$, as required by (A.3), note that

$$F_B(\beta - x) - (V(\beta - x) - K_H) = \frac{e^{-u_1 x}\left(e^x + e^{x(1+u_1)}(u_1 - 1) - u_1 e^{u_1 x}\right)(V_H - K_H)(K_H - V_L)}{e^x(u_1 - 1)(V_H - K_H) + u_1(K_H - V_L)}$$

The denominator on the RHS is positive since $V_H > K_H > V_L$. The numerator is positive provided that for all $x > 0$, $e^x + e^{x(1+u_1)}(u_1 - 1) - u_1 e^{u_1 x} \geq 0$, which can be shown to hold for all $u_1 \geq 1$ (i.e., over the entire relevant parameter space).

- For $z > \beta$. First, note that $F_B = V - K_H$ by construction so (A.3) holds with equality. Hence, it remains to verify (A.4) and (A.5). Since $F_L(z) = K_H$ for all $z \geq \beta$, we get that

$$
\begin{aligned}
&\frac{p(z') - p(z)}{p(z')}(V_L - F_L(z')) + \frac{p(z)}{p(z')}F_B(z') \\
&\qquad\qquad\qquad\qquad = \frac{p(z') - p(z)}{p(z')}\big(V_L - K_H\big) + \frac{p(z)}{p(z')}\big(V(z') - K_H\big) \\
&\qquad\qquad\qquad\qquad = \frac{p(z') - p(z)}{p(z')}\big(V_L - K_H\big) + \frac{p(z)}{p(z')}\big(p(z')V_H + (1 - p(z'))V_L - K_H\big) \\
&\qquad\qquad\qquad\qquad = p(z)V_H + (1 - p(z)V_L - K_H \\
&\qquad\qquad\qquad\qquad = V(z) - K_H,
\end{aligned}
$$

and therefore (A.4) holds with equality *for all* $z' \geq z$. Verifying (A.5) is equivalent to showing that for all $z > \beta$,

$$
\frac{\phi^2}{2}\bigg((2p(z) - 1)V'(z) + V''(z)\bigg) - r(V(z) - K_H) \leq 0.
$$

Noting that $(2p(z) - 1)V'(z) + V''(z) = 0$ and $\beta > \underline{z} \Rightarrow V(z) - K_H > 0$ for all $z > \beta$ implies the above inequality and completes the proof.

$\square$

Lemma A.1. *Let $F^{Q,T}(z)$ denote the buyer's payoff under an arbitrary feasible policy $(Q, T) \in \Gamma$ starting from $Z_0 = z$. Let $\mathcal{A}$ denote the characteristic operator of $\hat{Z}_t$ under $\mathcal{Q}^B$. Suppose that $f \in C^1$, $f \in C^2$ almost everywhere and satisfies*

$$
\begin{aligned}
f(z) &\geq V(z) - K_H && \textit{for all } z \in \mathbb{R}, && \text{(A.3)} \\
f(z) &\geq J(z, z') && \textit{for all } z' \geq z \in \mathbb{R}; && \text{(A.4)} \\
0 &\geq (\mathcal{A} - r)f(z) && \textit{for almost all } z \in \mathbb{R}; && \text{(A.5)}
\end{aligned}
$$

then $f \geq F^{Q,T}$.

Proof. If f is the buyer's value function, (A.3) says that the buyer cannot benefit by stopping immediately (i.e., offering K_H). (A.4) says that the buyer cannot benefit by enforcing a jump from z to z' and is a standard optimality condition in impulse control. The inequality in (A.5) says that the buyer cannot benefit by making a non-serious offer and "wait for news" and is a standard optimality condition in optimal stopping. That (A.3)-(A.5) combined with the smoothness properties are sufficient for an upper bound on the buyer's payoff follows closely standard arguments (see e.g., Harrison (2013), Corollary 5.2, Proposition 7.2) and is therefore omitted. $\square$

A.2 Proofs for Theorem 2

Proof of Theorem 2. In Lemma A.4, we show that in any equilibrium there exists a β such that the buyer offers K_H (and the seller accepts w.p.1.) if and only if $z \geq \beta$. Consider equilibrium play for $t < T(\beta)$, by Lesbesgue's decomposition for monotonic functions (cf. Proposition 5.4.5, Bogachev, 2013), we can decompose Q into two processes

$$Q = Q^{abs} + Q^{sing}$$

where Q^{abs} is an absolutely continuous process and Q^{sing} is non-decreasing process with $dQ_t^{sing} = 0$ almost everywhere. We have already demonstrated that the equilibrium is unique among those in which Q is absolutely continuous, therefore it is sufficient to rule out equilibria with singular intervention.

To do so, first note that Q^{sing} can further be decomposed into a continuous nondecreasing process and a nondecreasing jump process. Thus, a singular intervention can take one of two possible forms. Either, (i) a jump from some z_0 to some $z_1 > z_0$ or (ii) intervening on order greater than dt at some isolated z_0. In Lemma A.8, we show that (i) cannot be part of an equilibrium. Lemmas A.9 eliminates the possibility of (ii). $\square$

In order to prove Lemmas A.4, A.8, and A.9, (and thus Theorem 2), we will use the following preliminary lemmas.

Lemma A.2. *For all z, (i) $F_L(z) \leq K_H$, and (iii) $F_L(z) = K_H \implies F_B(z) = V(z) - K_H$.*

Proof. Since the buyer can ensure trade w.p.1. at a price of K_H, any offer higher than K_H is suboptimal, which implies (i). For (ii), if $F_L(z) = K_H$ then $w(z) = K_H$, which from the *Option for Immediate Trade*, implies $F_B(z) = V(z) - K_H$. $\square$

Lemma A.3. *In any equilibrium, the buyer's value function must satisfy*

$$F_B(z) \geq V(z) - K_H \tag{A.6}$$

$$F_B(z) \geq \max_{z' \geq z} J(z, z'). \tag{A.7}$$

Further, if F_B is $\mathcal{C}^2$ on any interval (z_1, z_2), then for all $z \in (z_1, z_2)$

$$(\mathcal{A} - r)F_B(z) \leq 0, \tag{A.8}$$

where $\mathcal{A}$ is the characteristic operator of $\hat{Z}$ under $\mathcal{Q}_z^B$.

Proof. The buyer always has the option to offer K_H and trade immediately implying (A.6). If (A.7) is violated at z, then the buyer can profitably deviate by enforcing a jump to some $z' \geq z$. Finally, if (A.8) is violated at such a $z \in (z_1, z_2)$, then since F_B is $\mathcal{C}^2$ on the interval, there exits $\epsilon > 0$ such that (A.8) is violated over the interval $(z-\epsilon, z+\epsilon)$. But then, starting from any $z \in (z-\epsilon, z+\epsilon)$, the buyer can profitably deviate by adopting a policy such that $Q_\tau = 0$ for $\tau = \inf\{t : Z_t \notin (z-\epsilon, z+\epsilon)\}$ and then resuming the original policy. $\square$

Lemma A.4. *In any equilibrium, there exists $\beta < \infty$ such that $F_L(z) = w(z) = K_H$ if and only if $z \geq \beta$.*

Proof. First, note that for any z, there must exist some $z' > z$ such that $F_L(z') = w(z') = K_H$ and $F_B(z') = V(z') - K_H$. If not, then the high type never trades in states above z, the probability of trade goes to zero as $z \to \infty$, and thus $F_B(z) \to 0$, which violates (A.6).

Hence, there exists $z_1 < \infty$ such that $F_L(z_1) = K_H$ and $F_B(z_1) = V(z_1) - K_H$. To prove the lemma (by contradiction), suppose that there is some $z_2 > z_1$ such that $F_L(z_2) < K_H$. Consider the policy which, starting from $Z_t = z_1$, the buyer chooses $Q_t = z_2 - z_1$ (by offering $F_L(z_2)$) and then resumes the original policy. The buyer's payoff under this policy is

$$\begin{aligned} J(z_1, z_2) &\equiv \frac{p(z_2) - p(z_1)}{p(z_2)} \left(V_L - F_L(z_2)\right) + \frac{p(z_1)}{p(z_2)} F_B(z_2) \\ &\geq \frac{p(z_2) - p(z_1)}{p(z_2)} \left(V_L - F_L(z_2)\right) + \frac{p(z_1)}{p(z_2)} \left(V(z_2) - K_H\right) \\ &= V(z_1) - \left(\frac{p(z_2) - p(z_1)}{p(z_2)} F_L(z_2) + \frac{p(z_1)}{p(z_2)} K_H\right) \\ &> V(z_1) - K_H = F_B(z_1), \end{aligned}$$

where the first inequality follows from (A.6) and the second by our hypothesis that $F_L(z_2) < K_H$. Notice that $J(z_1, z_2) > F_B(z_1)$ violates (A.7), which yields the contradiction. □

Three additional lemmas will be used in the proofs of Lemmas A.8 and A.9.

Lemma A.5. *In any equilibrium, $\beta > \underline{z}$ and $F_B(z) \geq \mathbb{E}_z\left[e^{-rT(\beta)}(V(\beta) - K_H)\right] > 0$.*

Proof. For any $z_1 > \underline{z}$, the policy of not trading for $z < z_1$ and immediately trading at price K_H for all $z \geq z_1$ is feasible for the buyer and, starting from any z, generates a payoff of $\mathbb{E}_z\left[e^{-rT(z_1)}(V(z_1) - K_H)\right] > 0$. Hence, the buyer's equilibrium payoff must be at least as large. Finally, if $\beta < \underline{z}$, then $F_B(\beta) = V(\beta) - K_H < 0$ by definition of $\underline{z}$, which we just established cannot be true. □

Lemma A.6. *In any equilibrium, $F_L(z) = \mathbb{E}_z^L\left[e^{-rT(\beta)}K_H\right]$.*

Proof. From Lemma A.4, we know that any equilibrium must feature a threshold $\beta < \infty$, above which trade takes place immediately at a price of K_H and below which trade only occurs with the low type. For all $z \geq \beta$, the lemma is immediate. Starting from $z < \beta$, since $\beta < \infty$, there is positive probability that the low-type rejects all offers until the state reaches β. Therefore she must be weakly willing to reject the equilibrium offer in any state $z < \beta$, meaning her equilibrium payoff in any state $z < \beta$ must equal her payoff from playing $T(\beta)$. □

Lemma A.7. *In any equilibrium: (i) F_L is non-decreasing, (ii) F_L is continuous, and (iii) F_B is continuous.*

Proof. For (i), first suppose that Q (and therefore Z) has continuous sample paths. By Lemma A.6 then, for any $z_1 < z_2 < \beta$,

$$\begin{aligned}F_L(z_1) &= \mathbb{E}_{z_1}^L\left[e^{-rT(\beta)}K_H\right] \\ &= \mathbb{E}_{z_1}^L\left[e^{-rT(z_2)}\left(\mathbb{E}_{z_2}^L\left[e^{-rT(\beta)}K_H\right]\right)\right] \\ &= \mathbb{E}_{z_1}^L\left[e^{-rT(z_2)}F_L(z_2)\right] \\ &\leq F_L(z_2).\end{aligned}$$

Thus, if $F_L(z_2) < F_L(z_1)$, there must exist a $z_0 < z_2$ such that the buyer enforces a jump from z_0 to some $z_3 > z_2$, with $F_L(z_0) = F_L(z_3) > F_L(z_2)$. By an argument similar to the one used in Lemma A.4, such a policy violates *Buyer Optimality* (i.e., the policy could be improved upon by first enforcing a jump from z_0 to z_2 and then enforcing a jump to z_3).

For (ii), suppose that F_L is discontinuous at $z_1 \leq \beta$. Then by Lemma A.6, Z must also be discontinuous at z_1. The monotonicity of Q implies that Z can only have upward jumps, so $F_L(z_1^-) = F_L(z_2)$ for some "jump-to" point $z_2 > z_1$. By (i), F_L is non-decreasing, so

$$F_L(z_2) \geq F_L(z_1^+) \geq F_L(z_1^-) = F_L(z_2),$$

contradicting a discontinuity of F_L at z_1.

For (iii), $F_B(z_0^-) < F_B(z_0^+)$ violates (A.7): starting from $z_0 - \epsilon$, the buyer can enforce a jump to $z_0 + \epsilon$ (i.e., trade with arbitrarily small probability at price which is bounded above by K_H), and therefore achieve a payoff arbitrarily close to $F_B(z_0^+)$. Since F_L is continuous, if $F_B(z_0^-) > F_B(z_0^+)$, then $F_B(z_0^-) = J(z_0, z_1)$ for some $z_1 > z_0$ (i.e., Z must jump upward as it approaches z_0 from the left). But J is continuous in its first argument and therefore $F_B(z_0^+) < J(z_0, z_1)$ violating (A.7). □

Lemma A.8. *In any equilibrium, Q has continuous sample paths (i.e., there cannot exist an atom of trade with only the low type).*

Proof. Suppose that starting from $Z_t = z_0$, the buyer enforces a jump such that $Z_{t^+} = \alpha > z_0$. By Lemma A.6, it must be that $F_L(z_0) = F_L(\alpha)$ and F_L non-decreasing (Lemma A.7) then implies that $F_L(z) = F_L(z_0)$ for all $z \in (z_0, \alpha)$. Thus, conditional on rejection, the belief jumps immediately to α starting from any $z \in (z_0, \alpha)$. Moreover, there must exist a $z_1 > \alpha$ such that Z evolves continuously in the interval (α, z_1) (otherwise $Z_{t^+} \neq \alpha$). *Stationarity* also requires that α be a reflecting barrier for the belief process conditional on rejection starting from any $Z_t \geq \alpha$. We claim that these equilibrium dynamics require the following properties.

(i) $(\mathcal{A} - r)F_B(z) = 0$ and $\Gamma(z) \leq 0$ for all $z \in (\alpha, z_1)$

(ii) $\Gamma(z) = 0$ for all $z \in (z_0, \alpha)$

(iii) $F_L'(\alpha) = 0$

(iv) F_B is $\mathcal{C}^2$ at α.

The properties in (i) follow from the arguments in Section 3.1. For the second, note that the buyer's payoff starting from any $z \in (z_0, \alpha)$ is given by

$$F_B(z) = J(z,\alpha) \equiv \frac{p(\alpha) - p(z)}{p(\alpha)}(V_L - F_L(\alpha)) + \frac{p(z)}{p(\alpha)}F_B(\alpha). \tag{A.9}$$

Since, $\alpha \in \sup_{z' \geq z} J(z, \alpha)$, the envelope theorem yields

$$F_B'(z) = J_1(z, \alpha). \tag{A.10}$$

Solving (A.9) for $F_B(\alpha)$ and plugging into (A.10) gives

$$F_B'(z) = \frac{p'(z)}{p(z)}(F_B(z) - (V_L - F_L(\alpha)) = \frac{p'(z)}{p(z)}(F_B(z) - (V_L - F_L(z)),$$

which implies (ii). For (iii), note that $F_L'(\alpha^-) = 0$ is implied by $F_L(z) = F_L(\alpha)$ for all $z \in (z_0, \alpha)$ and $F_L(a^+) = 0$ is implied by the reflecting barrier. For (iv), note that $\mathcal{C}^1$ at α follows from physical conditions. Namely, the *Robin* condition

$$F_B'(\alpha^+) = \frac{p'(\alpha)}{p(\alpha)}\big(F_B(\alpha) - (V_L - F_L(\alpha))\big),$$

where $\frac{p'(\alpha)}{p(\alpha)}$ is the (unconditional) rate at which the seller accepts at α and the second term on the right hand side is the difference between the buyer's payoff following rejection versus acceptance. Differentiating (A.9) and taking the limit as $z \uparrow \alpha$ shows that the left and right derivatives at α must be equal. For $\mathcal{C}^2$, if $F_B''(\alpha^+) < F_B''(\alpha^-)$ then $(\mathcal{A} - r)F_B(z) > 0$ in a neighborhood just below α, which violates (A.8). On the other hand, if $F_B''(\alpha^+) > F_B''(\alpha^-)$ then

$$\begin{aligned}
\Gamma'(\alpha^+) &= \frac{p''(\alpha)}{p(\alpha)}\big(F_B(\alpha) - (V_L - F_L(\alpha))\big) - \frac{p'(\alpha)}{p(\alpha)}\big(F_L'(\alpha^+) + F_B'(\alpha^+)\big) + F_B''(\alpha^+) \\
&= \frac{p''(\alpha)}{p(\alpha)}\big(F_B(\alpha) - (V_L - F_L(\alpha)\big) - \frac{p'(\alpha)}{p(\alpha)}\big(F_L'(\alpha^-) + F_B'(\alpha^-)\big) + F_B''(\alpha^+) \\
&= \Gamma'(\alpha^-) + F_B''(\alpha^+) - F_B''(\alpha^-), \\
&> 0,
\end{aligned}$$

where the second equality uses (iii) and the final inequality contradicts that $\Gamma(z) \leq 0$ established in (i). Thus, we have established (i)-(iv).

We now claim that (i)-(iv) requires $F_B(\alpha) \leq 0$, which contradicts Lemma A.5. First, (ii)-(iv) imply $\Gamma(\alpha) = 0$. Therefore to satisfy $\Gamma(z) \leq 0$ for the neighborhood above α requires

$\Gamma'(\alpha) \leq 0$. But,

$$\begin{aligned}\Gamma'(\alpha) \leq 0 \iff & \frac{-e^{\alpha}}{(1+e^{\alpha})^2}(V_L - F_L(\alpha) - F_B(\alpha)) - \frac{1}{1+e^{\alpha}}F_B'(\alpha) + F_B''(\alpha) \leq 0 \\ \iff & (2p(\alpha)-1)F_B'(\alpha) + F_B''(\alpha) \leq \frac{e^{\alpha}}{1+e^{\alpha}}\Gamma(\alpha) \\ \iff & \mathcal{A}F_B(\alpha) \leq 0 \\ \iff & F_B(\alpha) \leq 0,\end{aligned}$$

where the first $\iff$ follows by differentiating Γ, the second is simple algebra, the third follows from multiplying both sides of the second by $\phi^2/2$ and using $\Gamma(\alpha) = 0$, and the fourth from the fact that (A.8) holds at α. □

Lemma A.9. *There cannot exist an isolated point, $\alpha < \beta$, at which singular intervention occurs.*

Proof. We first prove the F_B must be $\mathcal{C}^2$ at any such α. Since there are no jumps and α is an isolated singular point of intervention, the buyer's policy is absolutely continuous in a neighborhood of α. Hence, there exists a $\epsilon > 0$ such that

$$(\mathcal{A} - r)F_B(z) = 0, \quad \forall z \in N_\epsilon(\alpha) \setminus \alpha. \tag{A.11}$$

By Lemma A.7, F_L and F_B are continuous. Therefore, if intervention at α is optimal, it must be that

$$\frac{1}{1+e^{\alpha}}(V_L - F_L(\alpha) - F_B(\alpha)) + F_B'(\alpha^+) = 0. \tag{A.12}$$

To prove that F_B must be C^1 at α, suppose that $F_B'(\alpha^-) < F_B'(\alpha^+)$ (i.e., F_B has an upward kink at α). Starting from $Z_t = \alpha$, consider an alternative policy that involves no intervention until $\tau_\epsilon = \inf\{s \geq t : Z_s \notin N_\epsilon(\alpha)\}$. Let $f(\alpha)$ denote the payoff under this alternative policy and let $\Delta \equiv F_B'(\alpha^+) - F_B'(\alpha^-) > 0$. An extension of Ito's formula (see Harrison, 2013, Proposition 4.12) gives

$$\begin{aligned}e^{-r\tau_\epsilon}F_L(Z_{\tau_\epsilon}) = F_B(\alpha) + & \int_0^{\tau_\epsilon} e^{-rs}(\mathcal{A} - r)F_B(Z_s)I(Z_s \in U)ds \\ & + \int_0^{\tau_\epsilon} e^{-rs}\phi F_B'(Z_s)dB_s + \frac{1}{2}\phi^2\Delta l(\tau_\epsilon, \alpha).\end{aligned}$$

Taking the expectation over sample paths, we get that

$$\begin{aligned}f(\alpha) = F_B(\alpha) + \frac{1}{2}\sigma^2\Delta\mathbb{E}\left[l(\tau_\epsilon, \alpha)\right] = & F_B(\alpha) + \frac{1}{2}\sigma^2\Delta\int_0^{\tau_\epsilon} p_0(s, \alpha)ds \\ > & F_B(\alpha),\end{aligned}$$

where $p_0(t, \cdot)$ is the density of Z_t starting from $Z_0 = \alpha$. Thus, we have found an alternative policy that generates a higher payoff for the buyer. Therefore, an upward kink in F_B violates buyer optimality.

Next, suppose that $F_B'(\alpha^-) > F_B'(\alpha^+)$ (i.e., F_B has a downward kink at α). Then,

$$\begin{aligned}\Gamma(\alpha^-) &= \frac{1}{1+e^\alpha}(V_L - F_L(\alpha) - F_B(\alpha)) + F_B'(\alpha^-)\\ &> \frac{1}{1+e^\alpha}(V_L - F_L(\alpha) - F_B(\alpha)) + F_B'(\alpha^+)\\ &= \Gamma(\alpha^+) = 0,\end{aligned}$$

which violates (12) in a neighborhood below α. Intuitively, if the buyer can benefit from pushing at α and there is a downward kink in the value function, then he can benefit from pushing just below α. Thus, we have established that F_B must be C^1 at α.

For C^2, since (A.8) holds with equality at z_0^+ and F_B is C^1 at α, if $F_B''(z_0^-) > F_B''(z_0^+)$ then (A.8) is violated in a neighborhood below z_0. Next suppose that $F_B''(z_0^+) > F_B''(z_0^-)$. Then it must be that (A.8) holds strictly in a neighborhood below α, which violates (A.11). We have thus established the smoothness of F_B at α.

Now, recall that a singular intervention at α means that for $t \le \tau_\epsilon$, Q_t^{sing} increases only at times t such that $Z_t = \alpha$. Thus, Q_t^{sing} is proportional to the *local time* of Z_t at α (see Harrison, 2013, Section 1.2), which we denote by $l_\alpha^Z(t)$. And, for $t \le \tau_\epsilon$, Z evolves according to

$$Z_t = \hat{Z}_t + Q_t^{abs} + \delta l_\alpha^Z(t). \tag{A.13}$$

Harrison and Shepp (1981) show that (A.13) has a (unique) solution if and only if $|\delta| \le 1$, in which case Z is distributed as skew brownian motion (SBM) with δ capturing the degree of skewness. If $\delta = 1$, then Z has a reflecting boundary at α, whereas for $\delta = 0$ there is no singular intervention at α and Z is a standard Ito diffusion. By Lemma A.6, SBM involves a kink in the low type's value function at α, namely

$$\gamma F_L'(\alpha^+) = (1-\gamma) F_L'(\alpha^-), \tag{A.14}$$

where $\gamma = \frac{1+\delta}{2}$ (see Kolb, 2016). There are three (exhaustive) cases to rule out.

First, suppose $F_L'(\alpha^+) = F_L'(\alpha^-) = 0$. Then we have $\Gamma(\alpha) = 0$, $F_L'(\alpha) = 0$, and (A.8) holds in a neighborhood around α. Using an argument virtually identical to the one used in the Proof of Lemma A.8 leads to the conclusion that $F_B(\alpha) \le 0$, which yields a contradiction. Second, suppose $F_L'(\alpha^+) = F_L'(\alpha^-) \ne 0$. Then (A.14) requires $\gamma = \frac{1}{2}$. But then $\delta = 0$, contradicting that α is a point of singular intervention. Third, and finally, suppose $F_L'(\alpha^+) \ne F_L'(\alpha^-)$. By F_L nondecreasing (Lemma A.7), $F_L'(\alpha^+), F_L'(\alpha^-) \ge 0$. Further, (A.14) and $\gamma \ge \frac{1}{2}$ then imply that $F_L'(\alpha^-) > F_L'(\alpha^+) > 0$. In addition, we know that $\Gamma(\alpha) = 0$, and therefore $\Gamma'(\alpha^-) \ge 0$ in order to maintain (12) in the neighborhood just below α. Next, immediate calculation yields that Γ' is strictly decreasing in F_L'. Therefore, if $F_L'(\alpha^+) < F_L'(\alpha^-)$ implies that $\Gamma'(\alpha^+) > \Gamma'(\alpha^-) \ge 0$. Since $\Gamma(\alpha) = 0$, this implies $\Gamma(z) > 0$ for z in the neighborhood just above α, in violation of (12). Hence a contradiction arises in all cases, and there cannot exist an isolated point of singular intervention. □

A.3 Remaining Proofs

Proof of Proposition 1. The first statement is immediate from the analysis in Sections 3.1 and 4.1; the buyer's value function in both cases satisfy the same ODE and boundary conditions. For the second statement, notice that the low types' payoff in the due diligence problem is $\mathbb{E}_z^L[e^{-r\hat{T}(\beta)}K_H]$, where $\hat{T}(\beta) = \inf\{t \geq 0 : \hat{Z}_t \geq \beta\} \geq T(\beta) = \inf\{t \geq 0 : Z_t \geq \beta\}$ and hence $\mathbb{E}_z^L[e^{-r\hat{T}(\beta)}K_H] \leq \mathbb{E}_z^L[e^{-rT(\beta)}K_H] = F_L(z)$. □

Proof of Proposition 2. As shown in DG12 (see the proof of Lemma B.3 therein), $\beta_c > z_H^*$, where z_H^* is the threshold belief at which a high-type seller would stop in a game where $V(z)$ is always offered and beliefs evolve only according to news. Using the closed form expressions for z_H^* (see (41) in DG12) and β_b (see Lemma 1), it is straightforward to check that $z_H^* > \beta_b$, which proves the lemma. □

Proof of Proposition 3. First, $\mathcal{L}_b, \mathcal{L}_c \geq 0$, $\mathcal{L}_b(z) > 0$ if and only $z < \beta_b$, and $\mathcal{L}_c(z) > 0$ if and only $z < \beta_c$. By Proposition 2, $\beta_b < \beta_c$. Hence, by continuity of $\mathcal{L}_c$ and $\mathcal{L}_b$, there exists $z_2 < \beta_b$ such that $\mathcal{L}_b(z) < \mathcal{L}_c(z)$ for all $z \in (z_2, \beta_c)$.

In the bilateral outcome, $F_H^b = 0$, so $\Pi_b(z) = F_B^b(z) + (1 - p(z))\, F_L^b(z)$. In the competitive outcome, $F_B^c = 0$, so $\Pi_c(z) = p(z)F_H^c(z) + (1 - p(z))F_L^c(z)$. Further, in the competitive outcome, for all $z < \alpha_c$, both seller payoffs are constant: $F_L^c(z) = V_L$ and $F_H^c(z) = A \in (0, V_H - K_H)$. Direct calculations then show:

$$\lim_{z \to -\infty} \mathcal{L}_b(z) = \lim_{z \to -\infty} \mathcal{L}_c(z) = 0.$$

Therefore, by L'Hospital's rule:

$$\lim_{z \to -\infty} \left(\frac{\mathcal{L}_b(z)}{\mathcal{L}_c(z)} \right) = \lim_{z \to -\infty} \left(\frac{\mathcal{L}_b'(z)}{\mathcal{L}_c'(z)} \right) = \frac{V_H - K_H}{V_H - K_H - A} > 1.$$

Hence, there exists $z_1 > -\infty$ such that $\mathcal{L}_b(z) > \mathcal{L}_c(z)$ for all $z < z_1$. □

Proof of Proposition 4. From the expression in Lemma 1, β is decreasing in u_1, which recall is defined as $u_1 \equiv \frac{1}{2}\left(1 + \sqrt{1 + 8r/\phi}\right)$. Clearly u_1 decreases with ϕ, which implies (i). For (ii), using the expression in (24) we have that

$$\frac{d}{du_1}\dot{q}(z) = \frac{rV_L}{e^{u_1 z}(u_1 - 1)^2 u_1^2 (K_H - V_L)} \zeta^{u_1} \left(1 + u_1(z - 2) - u_1^2 z + (u_1 - 1)u_1 \ln(\zeta)\right)$$

where $\zeta \equiv \frac{u_1(K_H - V_L)}{(u_1 - 1)(V_H - K_H)} = e^{\beta} > 0$. The expression above is strictly positive (negative) for $z > (<) \beta - \frac{2u_1 - 1}{u_1(u_1 - 1)}$, which implies (ii). For (iii), it is sufficient to show that F_B is decreasing in u_1 below β. To do so, plug in the expression for $C_1 = C_1^*$ into F_B and differentiate with respect to u_1 to get that

$$
\begin{aligned}
\frac{d}{du_1}F_B(z) &= \frac{1}{1+e^z}e^{u_1 z}\left(\frac{\partial C_1^*}{\partial u_1} + zC_1^*\right) \\
&= \frac{1}{1+e^z}e^{u_1 z}\left(\frac{K_H - V_L}{u_1 - 1}\right)\zeta^{-u_1}(z - \ln(\zeta)) \\
&< 0,
\end{aligned}
$$

where the inequality follows from noting that $\ln(\zeta) = \beta$. For (iv), note that for $z < \beta$,

$$
\begin{aligned}
\frac{d}{du_1}F_L(z) &= e^{u_1 z}\left((1 + (u_1 - 1)z)C_1^* + (u_1 - 1)\frac{\partial C_1^*}{\partial u_1}\right) \\
&= e^{u_1 z}\left(\frac{K_H - V_L}{u_1 - 1}\right)\zeta^{-u_1}\left(1 + (u_1 - 1)(z - \ln(\zeta)\right).
\end{aligned}
$$

Noting that $e^{u_1 z}\left(\frac{K_H - V_L}{u_1 - 1}\right)\zeta^{-u_1} > 0$, we have that $F_L(z)$ increases with u_1 (decreases with ϕ) for $z \in (\beta - \frac{1}{u_1 - 1}, \beta)$ and decreases in u_1 (increasing in ϕ) for $z < \beta - \frac{1}{u_1 - 1}$, which proves (iv). For (v), note that $\Pi(z) = F_B(z) + (1 - p(z))F_L(z)$ and therefore

$$
\begin{aligned}
\frac{d}{du_1}\Pi(z) &= \frac{d}{du_1}F_B(z) + (1 - p(z))\frac{d}{du_1}F_L(z) \\
&= \frac{1}{1+e^z}e^{u_1 z}\left(\frac{K_H - V_L}{u_1 - 1}\right)\zeta^{-u_1}\left(1 + u_1(z - \ln(\zeta))\right)
\end{aligned}
$$

Thus, Π increases with u_1 (decreases with ϕ) for $z \in (\beta - \frac{1}{u_1}, \beta)$ and decreases with u_1 (increases with ϕ) for $z < \beta - \frac{1}{u_1}$. As a result, (v) immediately follows. □

Proof of Proposition 5. First, note that taking the limit as $\phi \to \infty$ is equivalent to taking the limit as $u_1 \to 1$ from above. For (i), using the expression for β in Lemma 1, we have that

$$
\lim_{u_1 \to 1} \beta = \underline{z} + \lim_{u_1 \to 1} \ln\left(\frac{u_1}{u_1 - 1}\right) = \infty.
$$

For (ii), using the expressions for $\dot{q}$ and C_1^* from Lemmas 3 and 1,

$$
\dot{q}(z) = \frac{rV_L e^{-u_1 z}}{C_1^* u_1(u_1 - 1)} = \frac{rV_L e^{-u_1 z}\left(\frac{u_1(K_H - V_L)}{(u_1 - 1)(V_H - K_H)}\right)^{u_1}}{u_1(K_H - V_L)},
$$

which, for all $z < \beta$, tends to ∞ as $u_1 \to 1$ from above. Incorporating the expression for β yields:

$$
\dot{q}(\beta - x) = \frac{rV_L e^{u_1 x}}{u_1(K_H - V_L)} \to \frac{rV_L e^x}{K_H - V_L}
$$

as u_1 goes to 1. For (iii), from Lemma 1,

$$F_B(z) = \begin{cases} V(z) - K_H & \textit{if } z \geq \beta \\ \frac{e^{u_1 z}(V_H - K_H)\left(\frac{u_1(K_H - V_L)}{(u_1-1)(V_H-K_H)}\right)^{1-u_1}}{(1+e^z)u_1} & \textit{if } z < \beta \end{cases}$$

As $u_1 \to 1$, $\beta \to \infty$, meaning for any $z \in \mathbb{R}$,

$$\lim_{u_1 \to 1} F_B(z) = \lim_{u_1 \to 1} \frac{e^{u_1 z}(V_H - K_H)\left(\frac{u_1(K_H - V_L)}{(u_1-1)(V_H-K_H)}\right)^{1-u_1}}{(1+e^z)u_1} = \frac{e^z}{1+e^z}(V_H - K_H) = p(z)(V_H - K_H).$$

Further, since $F_B(z)$ is continuous in z and non-decreasing in ϕ (Proposition 4), the convergence is uniform by Dini's Theorem.[30] For (iv), from Lemma 3,

$$F_L(z) = \begin{cases} K_H & \textit{if } z \geq \beta \\ V_L + e^{u_1 z}(V_H - K_H)^{u_1}(K_H - V_L)\left(\frac{u_1(K_H - V_L)}{u_1 - 1}\right)^{-u_1} & \textit{if } z < \beta \end{cases}$$

As $u_1 \to 1$, $\beta \to \infty$, meaning for any $z \in \mathbb{R}$,

$$\lim_{u_1 \to 1} F_L(z) = V_L + \lim_{u_1 \to 1} e^{u_1 z}(V_H - K_H)^{u_1}(K_H - V_L)\left(\frac{u_1(K_H - V_L)}{u_1 - 1}\right)^{-u_1} = V_L.$$

Finally, for (v),

$$\begin{aligned} 0 \leq \mathcal{L}(z) = \frac{\Pi^{FB}(z) - \Pi(z)}{\Pi^{FB}(z)} &= \frac{p(z)(V_H - K_H) - F_B(z) + (1-p(z))(V_L - F_L(z))}{\Pi^{FB}(z)} \\ &\leq \frac{p(z)(V_H - K_H) - F_B(z)}{\Pi^{FB}(z)}, \end{aligned} \tag{A.15}$$

where the last inequality follows from $F_L(z) \geq V_L$ for all z (regardless of ϕ). By (iii), the term in (A.15) uniformly converges to 0 as $u_1 \to 1$, implying $\mathcal{L}$ does as well. □

Proof of Proposition 6. First, note that taking the limit as $\phi \to 0$ is equivalent to taking the limit as $u_1 \to \infty$. For (i), using the expression for β in Lemma 1, we have that

$$\lim_{u_1 \to \infty} \beta = \underline{z} + \ln\left(\lim_{u_1 \to \infty} \frac{u_1}{u_1 - 1}\right) = \underline{z} + \ln(1) = \underline{z}.$$

From (24), we have that $\dot{q}(z) = \frac{rV_L}{C_1^* u_1(u_1-1)e^{u_1 z}}$. Therefore, to prove (ii) it suffices to show that $\lim_{u_1 \to \infty} C_1^* u_1(u_1 - 1)e^{u_1 z} = 0$ for $z < \underline{z}$ and $\lim_{u_1 \to \infty} C_1^* u_1(u_1 - 1)e^{u_1 \underline{z}} = \infty$. Using the

[30] To apply Dini's Theorem, the function's domain must be compact. However, simply transform log-likelihood states, z, back into probability states, $p \in [0, 1]$, and, for all ϕ-values, extend the function to $p = 0, 1$ to preserve continuity.

closed form expression for C_1^* in Lemma 1, we have that

$$C_1^* u_1(u_1-1)e^{u_1 z} = (K_H - V_L)\left(\frac{u_1-1}{u_1}\right)^{u_1}\left(\frac{V_H-K_H}{K_H-V_L}e^z\right)^{u_1} u_1$$

The first term on the right hand side is a constant. The second term limits to e^{-1} as $u_1 \to \infty$. Thus, the remaining terms determine the limiting properties. They can be written as $u_1 y^{u_1}$, where $y \equiv \frac{V_H-K_H}{K_H-V_L}e^z$. Notice that $z < \underline{z} \Rightarrow y < 1 \Rightarrow u_1 y^{u_1} \to 0$, whereas $z = \underline{z} \Rightarrow y = 1 \Rightarrow u_1 y^{u_1} = u_1 \to \infty$. This completes the proof of (ii).

For (iii), note that for all $z \le \underline{z}$, $0 \le F_B(z) \le C_1^* e^{u_1 z} \le C_1^* e^{u_1 \underline{z}}$. And further, $C_1^* e^{u_1 \underline{z}} = (K_H - V_L)\left(\frac{u_1-1}{u_1}\right)^{u_1}\frac{1}{u_1-1} \to 0$ as $u_1 \to \infty$. Thus, we have obtained uniform bound on $F_B(z)$ below $\underline{z}$, which converges to zero implying the first part of (iii). That $F_B(z) \xrightarrow{u} V(z) - K_H$ for $z \ge \underline{z}$ follows from continuity of F_B, $F_B(z) = V(z) - K_H$ for $z \ge \beta$, and $\beta \to \underline{z}$.

For (iv), the pointwise convergence above $\underline{z}$ is immediate. For $z \le \underline{z}$,

$$\begin{aligned}
0 \le F_L(z) - V_L &= C_1^*(u_1-1)e^{u_1 z} \\
&= (K_H - V_L)\left(\frac{u_1-1}{u_1}\right)^{u_1}\left(\frac{V_H-K_H}{K_H-V_L}e^z\right)^{u_1} \\
&\to (K_H - V_L)e^{-1}\lim_{u_1\to\infty} y^{u_1}.
\end{aligned}$$

The remainder of (iv) follows from $z < \underline{z} \Rightarrow y < 1 \Rightarrow y^{u_1} \to 0$ and $z = \underline{z} \Rightarrow y = 1 \Rightarrow y^{u_1} \to 1$. Finally, (v) is immediately implied by (iii) and (iv). □

Proof of Theorem 3. In the proposed equilibrium candidate, for all $z \in \mathbb{R}$, trade is immediate, $W(z) = F_L(z) = K_H$, and $F_B(z) = V(z) - K_H$. Hence, the equilibrium candidate is of $\Sigma(\beta, \dot{q})$ form in which $\beta = -\infty$. As in the proof of Theorem 1, Conditions 1, 3, and 4 are by construction of the Σ-profile. In the candidate, $\beta = -\infty$, so verification of *Seller Optimality* (Condition 2) is trivial: for all z, $W(z) \le K_H$, so for $\theta \in \{L, H\}$:

$$\sup_{\tau \in \mathcal{T}} E^\theta\left[e^{-r\tau}(W(Z_\tau) - K_\theta)\right] \le K_H - K_\theta = F_\theta(z).$$

Finally, the verification of *Buyer Optimality* (Condition 5) is identical to the one given for the case of $z > \beta^*$ in the proof of Theorem 1.

To see that no other Σ-equilibrium exists, suppose first that $\Sigma(\beta, \dot{q})$ was an equilibrium with $\beta \in \mathbb{R}$. The analysis from Section 3.1 again applies, and therefore F_B, β, C_1, C_2 must satisfy (15)-(18). Solving the system, as in Lemma 1, gives the unique solutions as

$$\beta = \ln\left(\frac{K_H - V_L}{V_H - K_H}\right) + \ln\left(\frac{u_1}{u_1-1}\right),$$

which is not in $\mathbb{R}$ when the SLC fails, contradicting the supposition. Finally, if $\beta = \infty$, then $F_B(z) = 0$ for all $z \in \mathbb{R}$. But then the buyer would improve her payoff by offering K_H (leading to payoff $V(z) - K_H > 0$) for any z. Hence, no other Σ-equilibrium exists.

The argument for why that there does not exist an equilibrium not of the Σ form follows closely the proof of Theorem (2) with two minor modifications. First, since $\underline{z}$ does not exist when the Static Lemons Condition does not hold, the first statement in Lemma A.5 (i.e., that $\beta > \underline{z}$) is vacuous and no longer required. Second, the proof of Lemmas A.6 and A.7 are immediate if $\beta = -\infty$ and follow the same argument for any $\beta > \infty$. $\square$

Proof of Lemma 4. We first construct the buyer's value function under the candidate policy and show there is a unique (α_m, β_m) satisfying (26)-(29). We then apply a standard verification argument to demonstrate the policy is indeed optimal.

For $z \in (\alpha_m, \beta_m)$, the buyer's value under the candidate policy satisfies

$$(\mathcal{A} - r)F_B(z) = m,$$

which has a solution of the form

$$F_B(z) = -\frac{m}{r} + \frac{1}{1+e^z}\left(C_1 e^{u_1 z} + C_2 e^{u_2 z}\right). \tag{A.16}$$

For an arbitrary β, using the functional form of F_B in (A.16), solve (28) and (29) for C_1 and C_2. These equations are linear so the solution is unique, denote it by $C_1(\beta)$ and $C_2(\beta)$. Plugging the solution into (A.16), the resulting function, which is given by

$$f_B(z;\beta) \equiv -m/r + (1+e^z)^{-1}\left(C_1(\beta)e^{u_1 z} + C_2(\beta)e^{u_2 z}\right),$$

has the following properties for arbitrary β (which are straightforward to verify).

(i) $f_B(\cdot;\beta)$ is continuously differentiable, strictly convex, and has a unique global minimum.

(ii) $f_B(z;\beta)$ is continuous and increasing in β for all $z < \beta$.

(iii) $\frac{\partial}{\partial z} f_B(z;\beta) > 0$ for z close enough to β.

(iv) $f_B(z;\beta) > V(z) - K_H$ for all $z \neq \beta$.

An immediate implication of (i) is that (for an arbitrary β) the unique candidate α such that $\frac{\partial}{\partial z} f_B(\alpha;\beta) = 0$ (i.e., such that (27) is satisfied) is $\alpha_{sp}(\beta) \equiv \arg\min_z f_B(z;\beta)$. Note that $\alpha_{sp}(\beta) < \beta$ by (i) and (iii). Further, (ii) implies that $f_B(\alpha_{sp}(\beta);\beta)$ is strictly increasing in β. Hence, there is at most one value for β_m satisfying $f_B(\alpha_{sp}(\beta_m);\beta_m) = 0$ (i.e., such that (26) is also satisfied).

To see that such a β_m in fact exists, note that $f_B(\underline{z};\underline{z}) = 0$ (and hence $f_B(\alpha_{sp}(\underline{z}),\underline{z}) < 0$), while $\alpha_{sp}(\beta) \to \beta$ as $\beta \to \infty$ and hence $\lim_{\beta\to\infty} f_B(\alpha_{sp}(\beta),\beta) = V_H - K_H > 0$. Thus, we have shown there is a unique candidate pair (α_m, β_m), which satisfies (26)-(29). Further, note that because $f_B(\alpha_m;\beta_m) = 0$ and $f_B(\alpha_m;\beta_m) > V(\alpha_m) - K_H$, we have that $\alpha_m < \underline{z}$. And since $f_B(\beta_m;\beta_m) > f_B(\alpha_m;\beta_m)$ (since α_m is a global minimum), we have that $\beta_m > \underline{z}$.

We next verify that the policy $\tau = \inf\left\{t : \hat{Z} \notin (\alpha_m, \beta_m)\right\}$ is indeed optimal. To do so, note that by construction, the buyer's value function under the candidate policy is $\mathcal{C}^1$ and satisfies:

$$F_B(z) = \begin{cases} 0 & z \leq \alpha_m \\ f_B(z;\beta_m) & z \in (\alpha_m, \beta_m) \\ V(z) - K_H & z \geq \beta_m \end{cases}$$

Using a standard verification theorem (e.g., Oksendal, 2007, Theorem 10.4.1) to verify the policy is optimal, it suffices to check that (1) $F_B(z) \geq g(z) \equiv \max\{V(z) - K_H, 0\}$ for all $z \in (\alpha_m, \beta_m)$, and (2) that $(\mathcal{A} - r)F_B - m \leq 0$ for all $z \notin (\alpha_m, \beta_m)$. That (1) holds follow immediately from (iv) above. For (2), first note that $(\mathcal{A} - r)F_B = (\mathcal{A} - r)g$ for all $z \notin (\alpha_m, \beta_m)$. Next, recall that $\alpha_m < \underline{z}$ and therefore $g(z) = 0$ for all $z \leq \alpha_m$. Thus, $(\mathcal{A} - r)F_B - m = (\mathcal{A} - r)g - m = -m$ for all $z \leq \alpha_m$. For $z \geq \beta_m$, $(\mathcal{A} - r)F_B = \frac{\phi^2}{2}\left((2p(z) - 1)V'(z) + V''(z)\right) - r(V(z) - K_H)$. Noting that $(2p(z) - 1)V'(z) + V''(z) = 0$ and $\beta_m > \underline{z}$ implies that $(\mathcal{A} - r)F_B < 0$, which is clearly sufficient for (2). □

Proof of Proposition 7. That the buyer's value function is equal to the one from the due diligence problem in Lemma 4 follows the same logic as given in the proof of Proposition 1. Verifying that the proposed candidate is an equilibrium then follows closely the proof of Theorem 1. Conditions 1, 3, and 4 are again by construction. *Seller Optimality* (Condition 2) for $\theta = H$ is immediate. For $\theta = L$, it is again by construction that $F_L(z) = \mathbb{E}^L_z[e^{-rT(\beta)}]K_H$ and therefore any $\tau \in \mathcal{T}(\beta)$ achieves the same payoff (the only difference is the law of motion of Z). To verify *Buyer Optimality* (Condition 5), we must first incorporate the option to terminate into the buyer's policy and modify conditions (A.3) and (A.5) of Lemma A.1 to account for the cost of investigation as follows.

$$f(z) \geq \max\{V(z) - K_H, 0\} \quad \text{for all } z \in \mathbb{R}, \tag{A.3'}$$

$$m \geq (\mathcal{A} - r)f(z) \quad \text{for almost all } z \in \mathbb{R}; \tag{A.5'}$$

With these modifications, any smooth function satisfying (A.3'), (A.4), and (A.5') provides an upper bound on F_B (analogous to Lemma A.1). The proof of Lemma 4, demonstrates that the buyer's value function satisfies (A.3') and (A.5'). Thus, all that remains is to check (A.4). Following a similar argument to the one used in the proof of Theorem 1, recall that $J(z, z') \equiv \frac{p(z') - p(z)}{p(z')}(V_L - F_L(z')) + \frac{p(z)}{p(z')}F_B(z')$ for any $z \leq z'$.

If $z, z' \leq \alpha_m$ then $J(z, z') = F_B(z) = 0$. If $z, z' \in [\alpha_m, \beta_m)$, then using the functional form for F_B (from Lemma 4) and F_L (implied by (22)) we get that

$$\frac{d}{dz'}J(z, z') = \underbrace{-\frac{e^{-z'}(e^{z'} - e^{z})}{1 + e^{z}}}_{(-)} \times \left(C_1 \underbrace{e^{u_1 z'}(u_1 - 1)u_1}_{(+)} + C_2 \underbrace{e^{u_2 z'}(u_2 - 1)u_2}_{(+)} \right),$$

where the (+) signs come from the fact that $u_1 > 1$ and $u_2 < 0$. Thus, to verify that $J(z, z')$ is decreasing in z', it is sufficient to show that $C_1 > 0$ and $C_2 > 0$. From the two boundary conditions at α, we have that

$$\begin{aligned} C_1 &= -\frac{e^{-\alpha u_1} m \left(e^{\alpha}(u_2 - 1) + u_2\right)}{r(u_1 - u_2)} > 0 \\ C_2 &= \frac{e^{-\alpha u_2} m \left(e^{\alpha}(u_1 - 1) + u_1\right)}{r(u_1 - u_2)} > 0, \end{aligned}$$

which verifies that $J(z, z')$ is decreasing in z' for $z, z' \in [\alpha_m, \beta_m)$. If $z < \alpha_m < z' < \beta_m$, then

$$\begin{aligned}
J(z,z') &\equiv \frac{p(z')-p(z)}{p(z')}\left(V_L - F_L(z')\right) + \frac{p(z)}{p(z')}F_B(z') \\
&\leq \frac{p(z')-p(z)}{p(z')}\left(V_L - F_L(z')\right) + \frac{p(z)}{p(z')}F_B(z') + \frac{p(\alpha_m)-p(z)}{p(\alpha_m)}\left(F_L(z') - F_L(\alpha_m)\right) \\
&= \frac{p(\alpha_m)-p(z)}{p(\alpha_m)}\left(V_L - F_L(\alpha_m)\right) + \frac{p(z)}{p(\alpha_m)}\left(\frac{p(z')-p(\alpha_m)}{p(z')}\left(V_L - F_L(z')\right) + \frac{p(\alpha_m)}{p(z')}F_B(z')\right) \\
&= \frac{p(\alpha_m)-p(z)}{p(\alpha_m)}\left(V_L - F_L(\alpha_m)\right) + \frac{p(z)}{p(\alpha_m)}J(\alpha_m, z') \\
&\leq \frac{p(\alpha_m)-p(z)}{p(\alpha_m)}\left(V_L - F_L(\alpha_m)\right) + \frac{p(z)}{p(\alpha_m)}F_B(\alpha_m) = J(z,\alpha_m) = F_B(z) = 0,
\end{aligned}$$

where the first inequality comes from $z' > \alpha_m$ and $F_L(z') \geq F_L(\alpha_m)$, the subsequent equality is from algebra, and the remaining statements follow from the definition of J and established properties of F_B in the candidate equilibrium. Thus, we have shown that $J(z, z') \leq F_B(z)$ for all $z \leq z' < \beta$. If $z' \geq \beta_m$, then $J(z, z') = V(z) - K_H \leq F_B(z)$ (from Lemma 4), which completes the verification of (A.4) □

Proof of Lemma 5. As in the proof of Lemma 4, we proceed by constructing the candidate value function, demonstrate there is a unique β_λ satisfying the boundary conditions, and then verify the candidate policy is indeed optimal.

For $z < \beta_\lambda$, the buyer's value function satisfies (32), which has solution of the form

$$F_B(z) = \frac{\lambda}{r+\lambda}(V(z) - K(z)) + \frac{1}{1+e^z}\left(C_1 e^{\hat{u}_1 z} + C_2 e^{\hat{u}_2 z}\right)$$

where $(\hat{u}_1, \hat{u}_2) = \frac{1}{2}\left(1 \pm \sqrt{1 + \frac{8(\lambda+r)}{\phi^2}}\right)$. The boundary condition (16) requires $C_2 = 0$, and jointly solving (17)-(18) for C_1 and β_λ yields:

$$\begin{aligned}
\beta_\lambda^* &= \ln\left(\frac{\hat{u}_1}{\hat{u}_1 - 1}\frac{(\lambda+r)K_H - rV_L}{r(V_H - K_H)}\right) \\
C_1^* &= \frac{(\lambda+r)K_H - rV_L}{(r+\lambda)(\hat{u}_1 - 1)}e^{-\hat{u}_1\beta_\lambda}.
\end{aligned}$$

Thus, there is a unique candidate solution. To verify that the policy $\tau = \inf\left\{t : \hat{Z} \geq \beta_\lambda\right\}$ is optimal, note that by construction, the buyer's value function under the candidate policy is $\mathcal{C}^1$ and satisfies:

$$F_B(z) = \begin{cases} \frac{\lambda}{r+\lambda}(V(z) - K(z)) + \frac{1}{1+e^z}C_1^* e^{\hat{u}_1 z} & z \leq \beta_\lambda^* \\ V(z) - K_H & z \geq \beta_\lambda^* \end{cases}$$

Analogous to the proof of Lemma 4, it suffices to check that (1) $F_B(z) \geq V(z) - K_H$ for all $z \leq \beta_\lambda$, and (2) that $(\mathcal{A} - (r+\lambda))F_B(z) + \lambda(V(z) - K(z)) \leq 0$ for all $z \geq \beta_\lambda$. To verify

(1), make a change of variables from z to p (i.e., substitute $\ln\left(\frac{p}{1-p}\right)$ for z into both F_B and V). Note that F_B is convex in p, while V is linear. Given that both the slopes and values match at $p(\beta_\lambda)$, F_B must lie everywhere above to the left. For (2), since $\mathcal{A}F_B = 0$ for $z > \beta_\lambda$, it suffices to show that $V(z) - K_H \geq \frac{\lambda}{\lambda+r}(V(z) - K(z))$ for all $z \geq \beta_\lambda$. Making the same change of variables from z to p, observe that both $V - K_H$ and $\frac{\lambda}{\lambda+r}(V - K)$ are linear in p and that $V - K_H > \frac{\lambda}{\lambda+r}(V - K)$ for all $p > \hat{p} \equiv \frac{(r+\lambda)K_H - rV_L}{r(V_H - V_L)+\lambda K_H}$. The final step is to observe that $\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = \beta_\lambda - \ln\left(\frac{\hat{u}_1}{\hat{u}_1-1}\right) < \beta_\lambda$. $\square$

Proof of Proposition 8. The proof follows the same steps as Proposition 7 with the exception of verifying *Buyer Optimality* (Condition 5). In order to do so, we must modify condition A.5 of Lemma A.1 to account for the possibility of the fully revealing information arrival as follows:

$$0 \geq (\mathcal{A} - (r+\lambda))f(z) + \frac{\lambda}{\lambda + r}(V(z) - K(z)) \quad \text{for almost all } z \in \mathbb{R}. \tag{A.5"}$$

With this modification, any smooth function satisfying (A.3), (A.4), and (A.5") provides an upper bound on F_B (analogous to Lemma A.1). By construction (A.5") holds with equality for $z < \beta_\lambda$. The proof of Lemma 5 (and the fact that the buyer's value function in equilibrium is the same as in the due diligence problem) shows that F_B satisfies (A.5") for $z > \beta_\lambda$ and (A.3) for all z. Thus, all that remains is to check (A.4) and for this, the same argument as given in the proof of Theorem 1 applies. In particular, $\frac{d}{dz'}J(z, z')$ has the same form as given in (A.2) where u_1 is replaced by $\hat{u}_1$ and therefore is strictly negative for all $z' \in (z, \beta_\lambda)$. $\square$