Information and Competition with Speculation and Hedging*

Jeongmin Lee† Albert S. Kyle‡

October 9, 2017

Abstract

Can financial markets aggregate information dispersed among traders and allow traders to achieve their target inventories? To examine this question, we study a model of a double auction among finitely many traders who are all rational, strategic, risk averse, and informed about the value of a risky asset. Traders trade both to speculate on their private information and to hedge their endowments. Using concrete measures of competition and informational efficiency, we show that the strategic incentives of traders prevent financial markets from achieving both full informational efficiency and perfect competition, even with infinitely many traders.

Keywords: competition, informational efficiency, strategic trading, private information, vanishing noise equilibrium, rational expectations equilibrium.

*This paper supersedes the earlier versions, titled "Information and Competition with Symmetry" and "Information Aggregation with Symmetry." We thank Leland Crane, Julien Cujean, Jason Donaldson, Phil Dybvig, Maryam Farboodi, Jungsuk Han, Tibor Heumann, Xing Huang, Ohad Kadan, Mark Loewenstein, Rich Mathews, Thomas Maurer, Stephen Morris, Anna Obizhaeva, Giorgia Piacentino, Marzena Rostek, Shri Santosh, Gyuri Venter, Yajun Wang, and the co-editor and anonymous referees for helpful comments.

†Olin Business School, Washington University, St. Louis, MO 63130, USA; jlee89@wustl.edu.

‡Robert H. Smith School of Business, University of Maryland, College Park, MD 20742, USA; akyle@rhsmith.umd.edu. Kyle has worked as a consultant for various companies, exchanges, and government agencies. He is a non-executive director of a U.S.-based asset management company.

1 Introduction

Can financial markets aggregate information dispersed among traders? Are financial markets perfectly competitive so that traders can achieve their target inventories? Is there any relationship between information aggregation and competition? To address these questions, we study a model of a double auction among finitely many traders who are all rational, strategic, risk averse, and informed about the value of a risky asset. Traders trade both to speculate on their private information and to hedge their endowments.

To provide insights into the questions above, we develop concrete measures of competition and informational efficiency. We measure *competition* by the ratio between the quantity a trader optimally trades and the hypothetical quantity the trader would have traded if he were a price taker. We measure *informational efficiency* by the precision of information a trader learns from the price as a fraction of the precision of all information available in the market. We find that the strategic incentives of traders, who trade off speculating on their information against hedging their endowments, prevent financial markets from achieving both full informational efficiency and perfect competition simultaneously, even with infinitely many traders.

Our model builds on Kyle (1989), in which traders with constant absolute risk aversion (CARA) preferences compete in demand schedules. We remove noise traders and instead assume traders receive both deterministic and random endowments, following the competitive model of Diamond and Verrecchia (1981). We allow the asset value to contain residual uncertainty, about which no trader receives any information.

The private information and endowments may be conditionally correlated provided that their pairwise correlations are the same on average, similar to the concept of "equicommonality" that Rostek and Weretka (2012) use to generalize the model of Vives (2011). Traders then can be considered to be divided into groups so that the private signals are identical within each group and conditionally independent across different groups, since the distribution of the correlations does not affect equilibrium; we refer to the number of traders in each group and the number of groups as the *industrial organization* parameters.

There are six main results in the paper. Our first main result is that competition in equilibrium depends only on the two industrial organization parameters and on informational efficiency. This implies informational efficiency fully captures the effects that the other exogenous parameters, private information, endowments, and residual uncertainty, have on competition.

Our second main result is that competition and informational efficiency are inversely related. As intuition suggests, competition increases when there are more traders in each group and when there are more groups. More surprisingly, holding the two industrial organization parameters constant, competition decreases in informational efficiency; any changes in private information, endowments, and residual uncertainty that increase informational efficiency would decrease competition, and vice versa.

This inverse relationship between competition and informational efficiency results from the traders' strategic incentives: (1) Prices are made informative by traders incorporating their private information into the price. (2) From the traders' perspective, incorporating their information into the price simply means incurring trading costs. (3) To avoid incurring large trading costs, traders restrict the quantities they trade, sacrificing the opportunity to hedge endowments.

Furthermore, increasing the industrial organization parameters—which always increases competition—does not make the price more informationally efficient. Intuitively, informational efficiency is determined by the importance of the speculative motive relative to the hedging motive. While more competition makes traders trade more aggressively overall, it does not strengthen the traders' motive to speculative on private information relative to their motive to hedge risky endowments. Thus, the inverse relationship between competition and informational efficiency remains unchanged when the industrial organization parameters are also allowed to vary.

Our third main result, which follows from this inverse relationship, is that obtaining perfect competition with infinitely many traders depends on the industrial organization parameters and informational efficiency. If the number of traders in each group sharing identical signals approaches infinity, perfect competition obtains if and only if the price is *not* fully informationally efficient. If the number of traders in each group remains finite as the number of groups approaches infinity, perfect competition obtains if and only if the price reveals to traders a *zero* fraction of the available information, with informational efficiency approaching zero.

Therefore, the market may remain imperfectly competitive with infinitely many traders even when random endowments create gains from risk sharing. Moreover, if the market is perfectly competitive, the price cannot be fully informationally efficient. Perfect competition and efficient information aggregation cannot be achieved at the same time.

All three main results are possible because our new measure of competition allows a complete and clean characterization of competition. After describing the main results, we validate our measure by comparing it with other measures.

The next three results concern the case when strategic incentives prevent equilibrium from existing because the price would aggregate information *too* efficiently. Our fourth main result is introducing the new concept of a vanishing noise equilibrium. Taking advantage of the property that noise traders are willing to incur whatever losses are necessary to support equilibrium, we add noise trading to the environment and take a limit as the variance of noise trading vanishes; we call this a *vanishing noise equilibrium*. In the spirit of the trembling hand perfect equilibrium, we interpret vanishing noise as small perturbations to the trading environment.[1] Although the variance of noise trading and so the expected losses of noise traders vanish, a vanishing noise equilibrium always exist.

The intuition is that adding vanishing noise when equilibrium does not exist without noise trading makes the price sufficiently inefficient in aggregating information. Our fifth main result is that in this equilibrium there is no trade, with the market being *noncompetitive*, and each trader moves the price *halfway* toward his valuation, with the price incorporating half of traders' marginal information. This contrasts with the model of Milgrom and Stokey (1982), in which the no trade equilibrium price reveals information that swamps all private signals.

A vanishing noise equilibrium allows us to examine the well-known paradox in the model of Grossman and Stiglitz (1980), in which the market is exogenously assumed perfectly competitive. The paradox is that traders have no incentive to acquire private information if the price would reveal their information. But if traders do not acquire information, the price would not reveal that information, which then would make traders want to acquire information. Traders, therefore, can neither acquire nor not acquire private information.

Our sixth, and final, main result is that in our model the price becomes fully informationally efficient if and only if there is no trade and the number of traders in each group sharing identical signals approaches infinity. Importantly, informational efficiency continues to increase with more traders *because* the market remains noncom-

[1] In a competitive rational expectations equilibrium model, Anderson and Sonnenschein (1985) add random variations in demand to allow linear least squares estimation of the parameters.

petitive and so each trader continues to move the price, incorporating his marginal information into the price.

Thus, when the price is fully informationally efficient, the market is noncompetitive and there is no trade. Traders would not acquire information simply because they cannot trade on it. If they do not acquire information, there is no adverse selection, which will allow traders to hedge their endowments and share risk better. There is no paradox. To sum up, the Grossman-Stiglitz paradox arises from assuming perfect competition when it is inappropriate.

All six main results highlight the importance of measuring and understanding competition and information correctly. We next compare our measures of competition and informational efficiency with the existing measures in the literature to show that our measures are appropriate for studying information aggregation and competition in financial markets.

Our measure of competition is different from the price impact parameter from Kyle (1985), which measures the change in the asset price in response to an informed trader's buying one more share. Our measure of competition appropriately weighs the importance of price impact relative to the disutility associated with the risk aversion and riskiness of the asset. This reflects the way real-world asset managers deal with trading costs in financial markets. By measuring how closely traders can reach their target inventories, our measure of competition quantifies the concept of *liquidity*.

Traditional measures of competition from the industrial organization literature are not applicable to our double auction setting. For example, the Lerner index, which is the difference between the price and the marginal cost divided by the price, is not applicable because there is no concept of the marginal cost in our model. To corroborate our measure of competition, we apply our measure to Cournot competition with symmetric information. We then show our measure of competition is consistent with the Lerner index and with the Herfindahl index normalized by the price elasticity of the demand.

Informational efficiency is different from the concept of a fully revealing price and from the concept of a privately revealing price. A fully revealing price refers to the price from which traders can learn the value perfectly, extracting infinite precision. We choose informational efficiency for our measure for three reasons. First, a fully revealing price is restrictive in that it requires infinite precision to be available in the market. Traders in financial markets may not produce infinite precision, even in aggregate. Second, even when infinite precision is available in the limit, a fully revealing price does not measure how *efficiently* the price aggregates available information. Understanding efficiency in information aggregation is useful because information is a scarce good in financial markets. Lastly, as discussed above, informational efficiency fully captures the effects of private information, endowment shocks, and residual uncertainty on competition.

A privately revealing price is the price from which traders infer all that they could hypothetically learn from observing all signals about correlated private values. In models with one signal, Vives (2011) and Rostek and Weretka (2012) show that the price is privately revealing if and only if the correlations between two traders' values are the same across all pairs.

Informational efficiency exclusively concerns information about the liquidation value of a risky asset, which is common across all traders, while the concept of a privately revealing price concerns information about traders' correlated private values. We think that the informational role of financial markets in the broader economy operates through providing information about a common value, which benefits both participants and nonparticipants in trading. Furthermore, there is a long tradition in the finance literature of emphasizing speculation and hedging as two distinct motives for trading. It is then reasonable to imagine that traders receive their information about the asset value separately from endowment shocks. Therefore, focusing on the information about a common value is appropriate in our model of speculation and hedging.

In models with each trader receiving one signal, Vives (2011) and Rostek and Weretka (2012) show the price is privately revealing if and only if the correlations are identical across all pairs.

In our model traders receive two signals about the liquidation value and endowments. Our model does not prevent the price from being privately revealing. In any given equilibrium we can construct a composite signal, which is a linear combination of the private information and endowment shocks. The optimal demand schedule and the equilibrium price can be expressed in terms of the composite signals. Equilibrium would remain the same if we replace two signals with a composite signal. Then the price always reveals the average of the composite signals, like in models with one signal. The price, therefore, would be privately revealing of composite signals if and only if the correlations between two composite signals are the same across all pairs.

The key difference between our model with two signals and models with one signal is, therefore, not whether the price is privately revealing but what the signals that the price reveals are. Unlike in models with one signal, the composite signals in our model are endogenous. The relative weights between private information and endowment shocks in composite signals are determined by informational efficiency, reflecting the importance of the traders' speculative motive relative to the hedging motive.

In this sense, our composite signals have an element of "confounding." In the model of Bergemann, Heumann and Morris (2015), the signal is a combination of the aggregate and the idiosyncratic components. They show that confouding between the two components given by their relative weights in the signal are important for the equilibrium price impact, their measure of market power. In their model, confounding is exogenous and affects how similar the signals are to one another. In our model, the relative weights are determined endogenously and can vary independently of the similarity of the signals.

Vives (2014) shows that traders have incentives to acquire private information even when the price is privately revealing. Although a trader can perfectly learn about the other signals from the price, he cannot perfectly learn his private value because his value is imperfectly correlated with the other traders' values. Thus, he shows that there is no Grossman and Stilglitz paradox when traders' values are imperfectly correlated. We show that the paradox does not arise when traders' values are perfectly correlated and equilibrium is sustained by vanishing noise trading.

Our result on perfect competition is different from the models that assume quadratic storage costs. Vives (2011) shows that the market becomes perfectly competitive in the limit as the number of traders approaches infinity if the correlation between traders' values is less than one. Rostek and Weretka (2015) show that the market becomes perfectly competitive in the limit as the number of traders approaches infinity if the average of the correlations is bounded away from one.

Their results on perfect competition are consistent with the result from our model with exponential utility *and* residual uncertainty. The market becomes perfectly competitive in the limit as the number of traders approaches infinity if there is any endowment shock that keeps the correlation between traders' values strictly below one. Adding residual uncertainty or using quadratic preferences makes the hedging motive dominate the speculative motive when infinitely many traders bring new information. This drives informational efficiency to zero and results in perfect competition.

Without residual uncertainty, however, the market may remain imperfectly competitive with infinitely many traders and endowment shocks. This result is unique to models with exponential utility and does not arise in models with quadratic storage costs.

Whether financial markets can aggregate dispersed information has been studied by many economists. The rational expectations equilibrium (REE), in which the market is perfectly competitive and the price is fully revealing, has had a profound impact on economic theory. Widely used REE models, however, exogenously assume perfect competition, lacking a strategic foundation. Reny and Perry (2006) and references therein provided a strategic foundation for REE in various market settings. Ostrovsky (2012) provided a sufficient condition for information aggregation in a dynamic trading model building on Kyle (1985). We contribute to this literature by clearly characterizing the trade-off between competition and informational efficiency. Our result, however, does not preclude REE. REE with the less than perfectly informationally efficient price may be strategically founded in our model.

Palfrey and Srivastava (1986), Blume and Easley (1990), McLean and Postlewaite (2002), and McLean, Peck and Postlewaite (2005) study the effect of information asymmetry on competition in general equilibrium models. They emphasize the effect of the exogenous distribution of signals ("informational smallness" or "informational irrelevance") on competition. In our paper competition is determined not only by the exogenous parameters M and N but also by endogenous informational efficiency.

The plan for this paper is as follows. Section 2 describes the setup of the model and defines an equilibrium. Section 3 characterizes an equilibrium. Section 4 presents measures of information and competition. Section 5 analyzes how information and competition vary in equilibrium. Section 6 provides a necessary and sufficient condition for perfect competition, which allows a mapping between strategic equilibrium and rational expectations equilibrium. Section 7 introduces a vanishing noise equilibrium and discusses its implications for the Grossman-Stiglitz paradox and no trade theorem. Section 8 concludes.

2 Setup

There is one round of trading in which traders exchange a risky asset against a safe asset whose return is normalized to one. There are L informed traders with $L > 1$. Each trader has exponential utility with constant risk aversion parameter A.

The liquidation value of the risky asset is

$$v + \sigma_V y, \qquad \text{where} \qquad v \sim \mathrm{N}\left(0, \sigma_V^2\right) \qquad \text{and} \qquad y \sim \mathrm{N}\left(0, \sigma_Y^2\right). \tag{1}$$

Each trader receives two private signals before trading. First, trader l receives private information about v given by

$$i_l = \tau_I^{1/2}\left(\frac{v}{\sigma_V}\right) + e_l, \qquad \text{where} \qquad e_l \sim \mathrm{N}(0,1). \tag{2}$$

No trader receives any information about y. If $\sigma_Y^2 > 0$, the liquidation value contains residual uncertainty. The precision parameter τ_I is a ratio of the variance of the signal to the variance of the noise. The noise variables $e_1, \dots, e_L$ are independently distributed from v and y.

Second, trader l receives an endowment of the risky asset. The endowment is a sum of deterministic and random endowments

$$\bar{s}_l + s_l \sim \mathrm{N}\left(\bar{s}_l, \sigma_S^2\right) \qquad \text{and} \qquad \frac{\sum_{l=1}^{L} \bar{s}_l}{L} = \bar{s} \tag{3}$$

The random endowments $s_1, \dots, s_L$ are independently distributed from v, y, and $e_1, \dots, e_L$.

Adopting the concept of "equicommonality" that Rostek and Weretka (2012) define in the model with one signal to the model with two signals, we assume that the errors in private information and endowment shocks have the same correlation *on average*:[2]

$$\rho := \frac{\sum_{l' \neq l} \operatorname{corr}(e_l, e_{l'})}{L-1} = \frac{\sum_{l' \neq l} \operatorname{corr}(s_l, s_{l'})}{L-1}, \qquad \text{where} \qquad \frac{-1}{L-1} < \rho \leq 1 \text{ and for all } l. \tag{4}$$

As we shall see, each trader trades against an anonymous residual supply schedule. Thus, with a given average of deterministic endowments $\bar{s}$ and a given average correlation ρ, equilibrium does not depend on how the deterministic endowments $\bar{s}_1$, $\dots$, $\bar{s}_L$ are distributed, how the pairwise correlations of the errors in private information $\operatorname{corr}(e_l, e_{l'})$ are distributed, or how the pairwise correlations of the random endowments $\operatorname{corr}(s_l, s_{l'})$ are distributed. We assume that the deterministic endowments

[2] The assumption that the average correlations are the same for the errors in private information and endowment shocks helps us clearly show the relationship between competition and the similarity of signals. Allowing the average correlations to be different for the errors in private information and endowment shocks is left for future study.

and the pairwise correlations are arbitrarily given.

We assume that traders know their own deterministic endowment $\bar{s}_l$, the average of deterministic endowments $\bar{s}$, and the average correlation ρ, but do not know how deterministic endowments or the pairwise correlations are distributed. Except for deterministic endowments and the pairwise correlations, the model is *symmetric* in that the model looks the same from the perspective of every trader.

There are two dimensions of measurement: dollars and shares. The parameter σ_V has a dimension of dollars-per-share, the parameters σ_S and $\bar{s}_1, \ldots, \bar{s}_L$ have dimensions of shares, the parameter A has dimensions of per-dollar. We assume $A > 0$ and $\sigma_V > 0$ and use A and σ_V as units to scale variables in dollars-per-share by σ_V and variables in shares by $(A\sigma_V)^{-1}$. This scaling convention is used throughout the paper.

Trading. After observing his own private information i_l and random endowment s_l, each trader l submits a demand schedule $X_l\left(p \mid i_l, s_l\right)$. This notation means that X_l is a function of the price p, and the function is measurable with respect to s_l and i_l.

Let X denote the L vector of submitted demand functions whose lth element corresponds to X_l. An auctioneer aggregates all L functions to calculate a market clearing price, denoted $p(X)$, which satisfies the market clearing condition

$$\sum_{l=1}^{L} X_l\left(p\right) = 0. \tag{5}$$

If there is no market clearing price, then nobody trades ($x_l = 0$ for all l). If there are many market clearing prices, then the auctioneer chooses the smallest price which minimizes trading volume, with possible ties resolved by flipping a coin. Given the matrix of submitted demand schedules, trader l realizes wealth

$$w_l(X) := \left(v + \sigma_V y\right)(s_l + \bar{s}_l) + \left(v + \sigma_V y - p(X)\right) X_l\left(p(X)\right) \tag{6}$$

and achieves expected utility

$$u_l(X) := \mathrm{E}\left\{-\exp\left(-A w_l(X)\right)\right\}. \tag{7}$$

The equilibrium concept is a Bayesian Nash equilibrium. An *equilibrium* is a vector of demand schedules X such that (1) a market clearing price $p(X)$ is always well defined and (2) for all $l = 1, \ldots, L$, trader l chooses his demand schedule X_l to maximize his expected utility $u_l(X)$, taking as given the demand schedules of the other $L-1$ traders.

A *symmetric linear equilibrium* is an equilibrium in which all traders choose the same linear demand schedule

$$A\sigma_V X_l\left(p \mid i_l, s_l\right) = \pi_C + \pi_0 A\sigma_V \bar{s} - \pi_1 A\sigma_V \bar{s}_l - \pi_S A\sigma_V s_l + \pi_I i_l - \pi_P \frac{p}{\sigma_V}, \tag{8}$$

where the six endogenous parameters π_C, π_0, π_1, π_S, π_I, and π_P define the same linear function X_l for all $l = 1,\ldots,L$. The constant π_C can be shown to be zero in equilibrium regardless of the values of other parameters. Without loss of generality, we assume $\pi_C = 0$.

If $\pi_P = 0$, then every trader submits a totally inelastic demand schedule, and the resulting aggregate demand is either identically zero or some random quantity which is non-zero with probability one. Market clearing requires this aggregate demand to be identically zero; this further requires each trader's demand to be identically zero ($X_l \equiv 0$, for all l). In such a no-trade equilibrium, the market clearing price is not uniquely determined since any price can support the allocation. Such an equilibrium always exists. We call this a *trivial no-trade equilibrium* and exclude it from the following analysis.

Our goal is to characterize existence and uniqueness of symmetric linear equilibria. Discussing asymmetric equilibria or equilibria with non-linear strategies takes us beyond the scope of this paper.

3 Equilibrium

The equilibrium solution proceeds in five steps using the no-regret pricing approach. A trader (1) observes his residual supply schedule, (2) learns about other traders' private information from the intercept of this schedule, (3) finds the optimal quantity on his residual supply schedule, (4) and implements this optimal quantity by submitting a demand schedule, which (5) is the same as the demand schedules conjectured for other traders.

Trader l conjectures and takes as given symmetric linear demand schedules for the other traders, described by the six endogenous parameters π_C, π_0, π_1, π_I, π_S and π_P as in (8). Having ruled out trivial no-trade equilibria by assuming $\pi_P \neq 0$ as above, the market clearing condition (5) implies that trader l has a well-defined residual supply schedule given by

$$\frac{p}{\sigma_V} = \frac{p_l}{\sigma_V} + \frac{1}{(L-1)\,\pi_P} A\sigma_V x_l. \tag{9}$$

The price p_l, which would prevail if he did not trade ($x_l = 0$), is defined by

$$\frac{p_l}{\sigma_V} = \frac{\sum_{l' \neq l} (\pi_0 A\sigma_V \bar{s} - \pi_1 A\sigma_V \bar{s}_{l'} - \pi_S A\sigma_V s_{l'} + \pi_I i_{l'})}{(L-1)\,\pi_P}. \tag{10}$$

With each trader trading against an anonymous residual schedule (9), the prevailing price p_l depends on the average of the endowments and the average of private information of the other traders, but does not depend on how the deterministic endowments or the pairwise correlations are distributed.

To describe how traders learn about the other traders' private information from the price, define $\hat{p}_l$ as

$$\frac{\hat{p}_l}{\sigma_V} := \frac{p_l}{\sigma_V} - \frac{\pi_0}{\pi_P} A\sigma_V \bar{s} + \frac{\pi_1 (LA\sigma_V \bar{s} - A\sigma_V \bar{s}_l)}{(L-1)\,\pi_P} + \rho \frac{\pi_S}{\pi_P} A\sigma_V s_l. \tag{11}$$

Then

$$\frac{\hat{p}_l}{\sigma_V} = \frac{\pi_I}{\pi_P} \frac{\sum_{l' \neq l} i_{l'}}{L-1} - \frac{\pi_S}{\pi_P} A\sigma_V \left(\frac{\sum_{l' \neq l} s_{l'}}{L-1} - \rho s_l \right), \tag{12}$$

which makes $\hat{p}_l$ a signal of the average private information of the other traders ($\frac{\sum_{l' \neq l} i_{l'}}{L-1}$) with noise from random endowments. Subtracting the predicted endowment ρs_l makes the signal more accurate. Trader l's information about v contained in $\{i_l, s_l, p_l\}$ is summarized by $\{i_l, \hat{p}_l\}$.

Define τ^*, the ratio of prior variance to posterior variances of v, as

$$\tau^* := \frac{\sigma_V^2}{\mathrm{var}\{v \mid i_l, \hat{p}_l\}} = \frac{\sigma_V^2}{\mathrm{var}\{v \mid i_l, s_l, p_l\}}. \tag{13}$$

The symmetry assumption makes τ^* common across all traders. Since the posterior variance is at least as high as the prior variance and each trader observes his own signal perfectly, the inequality $\tau^* \geq 1 + \tau_I$ holds by definition.

Since traders do not have information about y, the posterior variance of the liquidation value $v + \sigma_V y$ is given by

$$\mathrm{var}\{v + \sigma_V y \mid i_l, \hat{p}_l\} = \frac{\sigma_V^2}{\tau^*} + \sigma_Y^2. \tag{14}$$

Trader l's learning from the price is described by the following lemma. All proofs are in the Appendix.

Lemma 1 (Learning From Prices.). *Assume* $(1-\rho)\tau_I\pi_I \neq 0$. *Then* τ^* *can be written*

$$\tau^* = 1 + \tau_I + \frac{(1-\rho)(L-1)\tau_I}{1+(L-1)\rho}\varphi, \tag{15}$$

where the endogenous parameter φ *is given by*

$$\frac{1}{\varphi} = 1 + \left(\frac{A\sigma_V\sigma_S\pi_S}{\pi_I}\right)^2. \tag{16}$$

The dimensionless endogenous parameter φ is important. We postpone discussing economic interpretations of φ until Section 4.1. When all private information is identical ($\rho = 1$), no trader has private information ($\tau_I = 0$), or traders do not trade on their private information ($\pi_I = 0$) so that $(1-\rho)\tau_I\pi_I = 0$, then there is no learning from the price; we set $\varphi = 0$ by continuity in these cases.

Since all random variables are jointly normally distributed and trading strategies are linear, the optimal trading strategy solves the quadratic maximization problem[3]

$$\max_{x_l}\left\{\mathrm{E}\left\{w_l(x_l) \mid p_l, s_l, i_l\right\} - \frac{A}{2}\operatorname{var}\left\{w_l(x_l) \mid p_l, s_l, i_l\right\}\right\}. \tag{17}$$

Using (9), this is in turn equivalent to

$$\max_{x_l}\left(E\left\{v+\sigma_V y \mid i_l, \hat{p}_l\right\} - p_l - \frac{A\sigma_V^2}{\pi_P(L-1)}x_l\right)x_l - \frac{A}{2}(s_l+\bar{s}_l+x_l)^2\operatorname{var}\left\{v+\sigma_V y \mid i_l, \hat{p}_l\right\}. \tag{18}$$

This implies that, with τ^* given by (13), the first-order condition is

$$\left(\frac{2}{\pi_P(L-1)} + \frac{1}{\tau^*} + \sigma_Y^2\right)A\sigma_V x_l = E\left\{\frac{v}{\sigma_V} + y \,\middle|\, i_l, \hat{p}_l\right\} - \frac{p_l}{\sigma_V} - \left(\frac{1}{\tau^*} + \sigma_Y^2\right)A\sigma_V(s_l+\bar{s}_l), \tag{19}$$

and the second-order condition is equivalent to

$$\frac{2}{\pi_P(L-1)} + \frac{1}{\tau^*} + \sigma_Y^2 > 0. \tag{20}$$

[3]With a slight abuse of notation, $w_l(x_l)$ means $w_l(X)$ defined in (6) where $X_l(P) = x_l$ and $X_{l'}(P)$ is fixed for all $l' \neq l$.

The best response demand schedule—which follows from the first order condition and traders' learning from the price—depends on the exogenous parameters L, A, σ_V, σ_Y, ρ, τ_I, and σ_S; the endogenous parameters $\pi_0, \pi_1, \pi_I, \pi_P, \pi_S$; and τ^* (or equivalently φ using (15)). A linear symmetric equilibrium is found when the trader's best response can be implemented using the same demand schedule that the trader conjectures that the other traders submit:

Theorem 1 (Characterization of Symmetric Linear Equilibrium)**.** *Suppose $A > 0$, $\sigma_V > 0$, and $L > 2$. If $(1-\rho)\,\tau_I \neq 0$, then the set of symmetric linear equilibria, excluding trivial no-trade equilibria, is characterized by the set of all endogenous parameters φ such that (1) φ solves the cubic polynomial*

$$\frac{1}{\varphi} - 1 = \frac{(A\sigma_V\sigma_S)^2}{\tau_I}\left(1 + \sigma_Y^2 + \sigma_Y^2\tau_I\left(1 + \left(\frac{1-\rho}{\rho + \frac{1}{L-1}}\right)\varphi\right)\right)^2, \tag{21}$$

and (2) φ satisfies

$$\varphi < \varphi_{soc} := \frac{\rho + \frac{1}{L-1}}{\rho + \frac{2}{L-2}}. \tag{22}$$

If $(1-\rho)\,\tau_I = 0$, an equilibrium is characterized by $\varphi = 0$.

With τ^* given by (15), the equilibrium demand schedule of trader l is given by

$$\begin{aligned} &A\sigma_V X_l\left(p \mid i_l, s_l\right) \\ &= \left(\frac{L-2}{L-1}\right)\left(1 - \frac{\varphi}{\varphi_{soc}}\right)\left(-\frac{A\sigma_V\bar{s}}{1 + \frac{(1-\rho)(L-1)}{1+(L-1)\rho}\varphi} - \frac{A\sigma_V\left(\bar{s}_l - \bar{s}\right)}{1-\varphi} - A\sigma_V s_l\right) \\ &+ \left(\frac{L-2}{L-1}\right)\left(1 - \frac{\varphi}{\varphi_{soc}}\right)\left(1 + \tau^*\sigma_Y^2\right)^{-1}\left(\tau_I^{1/2} i_l - \frac{\tau^*}{1 + \frac{(1-\rho)(L-1)}{1+(L-1)\rho}\varphi}\frac{p}{\sigma_V}\right), \end{aligned} \tag{23}$$

and the market clearing price is given by

$$\begin{aligned} \frac{p}{\sigma_V} &= -\frac{A\sigma_V}{\tau^*}\left(1 + \tau^*\sigma_Y^2\right)\bar{s} \\ &+ \frac{1}{\tau^*}\left(1 + \frac{(1-\rho)\,(L-1)}{1 + (L-1)\,\rho}\varphi\right)\left(\tau_I^{1/2}\frac{\sum_{l=1}^L i_l}{L} - \frac{A\sigma_V}{\tau^*}\left(1 + \tau^*\sigma_Y^2\right)\frac{\sum_{l=1}^L s_l}{L}\right). \end{aligned} \tag{24}$$

Equilibrium demand schedules (23) and the resulting market clearing price (24) are derived as functions of exogenous parameters and one endogenous variable φ, which in turn is determined by (21) and (22), fully characterizing the set of symmetric linear equilibria. The condition (22) follows from applying the second order condition (20) and ruling out a trivial no-trade equilibrium ($\pi_P \neq 0$).

Substituting (22) into (21) yields the following necessary and sufficient conditions for the existence and uniqueness of an equilibrium:

Theorem 2 (Existence and Uniqueness)**.** *Assume $A > 0$, $\sigma_V > 0$. Then there exists a symmetric linear equilibrium, excluding trivial no-trade equilibria, if and only if $L > 2$ and at least one of the following two conditions holds: Either $\left(1-\rho_I\right)\tau_I = 0$ or*

$$\frac{L\tau_I}{1+(L-1)\rho} < (L-2)\left(A\sigma_V\sigma_S\right)^2\left(1+\sigma_Y^2+\sigma_Y^2\left(\frac{L\tau_I}{2+(L-2)\rho}\right)\right)^2. \tag{25}$$

If a symmetric linear equilibrium exists, it is unique.

Three observations can be made from (25). First, without residual uncertainty ($\sigma_Y = 0$), (25) simplifies to

$$\frac{L}{1+(L-1)\rho}\tau_I < (L-2)\left(A\sigma_V\sigma_S\right)^2, \tag{26}$$

where the left side measures the amount of information and the right side measures scaled endowment shocks. There are two motives for trade, hedging random and deterministic endowments and speculating on private information. Equation (26) implies that for an equilibrium to exist, the hedging motive from random endowments—but not deterministic endowments—must be sufficiently strong to overcome adverse selection from the speculative motive related to economy-wide private information.

Second, increasing σ_Y increases the right side of (25), increasing the set of other parameters that support existence of equilibrium. Intuitively, more residual uncertainty *both* increases the hedging motive, because given endowments become riskier, *and* decreases speculative motive, because it becomes riskier to speculate on a given private signal.

Lastly, the effect of residual uncertainty interacts with private information. When traders learn more information about v, residual uncertainty becomes relatively more important. So the effect of residual uncertainty becomes stronger when the precision a trader extracts from the price τ^* increases. In contrast with the case without residual uncertainty, the existence of equilibrium may require traders to have more private information.[4]

[4]Mathematically, this implies that with $\sigma_Y > 0$, (25) becomes equivalent to $\tau_I > \alpha$ or $\tau_I < \beta$ where α

In (21), (22), (23), and (24), changing units of measurement has no real effect. This implies if the dimensional exogenous variables A, σ_V, σ_S, and $\bar{s}_1, \ldots, \bar{s}_L$, change in such a way that the dimensionless products $A\sigma_V\sigma_S$ and $A\sigma_V\bar{s}_1, \ldots, A\sigma_V\bar{s}_L$ do not change, then φ and equilibrium properties described by φ do not change.[5]

4 Measuring Information and Competition

This section introduces our measures of informational efficiency and competition.

4.1 Measuring Information

In a symmetric linear equilibrium, the market clearing condition (5) implies

$$\frac{p}{\sigma_V} = \left(\frac{\pi_0 - \pi_1}{\pi_P}\right) A\sigma_V \bar{s} - \frac{\pi_S}{\pi_P} \frac{\sum_{l=1}^{L} A\sigma_V s_l}{L} + \frac{\pi_I}{\pi_P} \frac{\sum_{l=1}^{L} i_l}{L}. \tag{27}$$

The information that a trader can learn from the price is at most the average of private information $\frac{1}{L}\sum_{l=1}^{L} i_l$.

Let τ_E denote the precision obtained by observing average information $\frac{1}{L}\sum_{l=1}^{L} i_l$ (the right side of (26)):

$$\tau_E := \frac{\sigma_V^2}{\text{var}\left\{v \mid i_l, \frac{1}{L}\sum_{l=1}^{L} i_l\right\}} - 1 = \frac{L}{1+(L-1)\rho}\tau_I. \tag{28}$$

Our measure of information is φ, given by (15), which we call informational efficiency. From (16), φ lies between zero and one. As φ varies from zero to one, τ^* monotonically increases from $1+\tau_I$ to $1+\tau_E$. Informational efficiency φ is the precision of information a trader learns from the price $(\tau^* - 1)$ as a fraction of the precision of the available information (τ_E).

Informational efficiency φ is determined by the strength of the traders' motive to speculate on their private information (π_I) relative to their motive to hedge their risky

and β $(\alpha \geq \beta)$ are the solutions of the quadratic equation which results from replacing the inequality in (25) with an equality if $\beta > 0$. If $\beta \leq 0$, (25) becomes equivalent to $\tau_I > \alpha$.

[5] This property is shared by many finance models. Fundamental model properties depend on the ratio of the risks to be borne—$\sigma_V\sigma_S$—to dollar risk-bearing capacity A^{-1}. For example, $A\sigma_V\sigma_S$ can become small either because risk bearing capacity increases (A becomes small) or because the risks to be borne $\sigma_V\sigma_S$ become small. Either way, the effect on equilibrium is the same.

endowments ($A\sigma_V\sigma_S\pi_S$). The price becomes more informationally efficient when the speculative motive becomes more important relative to the hedging motive.

Full informational efficiency ($\varphi \to 1$) is different from the concept of a fully revealing price ($\tau^* \to \infty$). A fully revealing price ($\tau^* \to \infty$) does not imply a full informational efficiency ($\varphi \to 1$), and vice versa. We choose φ for our measure of information for three reasons. First, a fully revealing price is restrictive in that it requires infinite precision to be available in the market ($\tau_E \to \infty$). Traders in financial markets may not produce infinite precision, even in aggregate. Second, even when infinite precision is available in the limit, a fully revealing price does not measure how *efficiently* the price aggregates available information. Understanding efficiency in information aggregation is useful because information is a scarce good in financial markets. Lastly, as we shall see, informational efficiency fully captures the effects of private information, endowment shocks, and residual uncertainty on competition.

In models with one signal, Vives (2011) and Rostek and Weretka (2012) show that the price is privately revealing if and only if the correlations between two traders' values are the same across all pairs. A privately revealing price is the price from which traders infer all that they could hypothetically learn from observing all signals about correlated private values.

In our model, each trader receives two signals. This does not prevent the price from being privately revealing. In any given equilibrium we can construct a composite signal y_l defined by

$$y_l := i_l - \frac{\pi_S}{\pi_I} A\sigma_V s_l, \qquad \text{implying} \qquad y_l = i_l - \left(\frac{1-\varphi}{\varphi}\right)^{1/2} \frac{s_l}{\sigma_S}. \tag{29}$$

The optimal demand schedule (23) and the equilibrium price (24) can be expressed in terms of the composite signals. Equilibrium would remain the same if we replace two signals with a composite signal because receiving additional information about i_l or s_l would not change traders' optimal demand schedule. Then the price always reveals the average of the composite signals $\frac{1}{L}\sum_{l=1}^{L} y_l$, like in models with one signal. The price, thus, would be privately revealing of composite signals if and only if the correlations between two composite signals are the same across all pairs.

The key difference between our model with two signals and models with one signal is, therefore, not whether the price is privately revealing but what the signals that the price reveals are. Unlike in models with one signal, the composite signals in our model are endogenous. The relative weights between private information and endowment shocks in composite signals are determined by informational efficiency φ, reflecting the importance of the traders' speculative motive relative to the hedging motive.

Informational efficiency exclusively concerns information about the liquidation value of a risky asset, which is common across all traders, while the concept of a privately revealing price concerns information about traders' correlated private values. We think that the informational role of financial markets in the broader economy operates through providing information about a common value, which benefits both participants and nonparticipants in trading. Furthermore, there is a long tradition in the finance literature of emphasizing speculation and hedging as two distinct motives for trading. It is then reasonable to imagine that traders receive their information about the asset value separately from endowment shocks. Therefore, focusing on the information about a common value is appropriate in our model of speculation and hedging.

Furthermore, the composite signal y_l has an element of "confounding." In the model of Bergemann, Heumann and Morris (2015), the signal is a combination of the aggregate and the idiosyncratic components. They show that confouding between the two components given by their relative weights in the signal are important for the equilibrium price impact, their measure of market power. In their model, confounding is exogenous and affects how similar the signals are to one another. In our model, the relative weights are determined endogenously and can vary independently of the similarity of the signals.

4.2 Measuring Competition

We start by comparing the quantity a trader chooses strategically in our Bayesian Nash equilibrium with the hypothetical competitive quantity a trader would choose if he ignored his own price impact while taking the strategies of other traders as given. If trader l were a price taker, the first order condition (19) would change to

$$\left(\frac{1}{\tau^*}+\sigma_Y^2\right)A\sigma_V x_l^{PT} = E\left\{\frac{v}{\sigma_V}+y \,\middle|\, i_l,\hat{p}_l\right\} - \frac{p_l}{\sigma_V} - \left(\frac{1}{\tau^*}+\sigma_Y^2\right)A\sigma_V\,(s_l+\bar{s}_l)\,. \tag{30}$$

Comparing the two first order conditions (19) and (30) reveals that the ratio of the optimal strategic demand x_l to the hypothetical price-taking demand x_l^{PT} is a constant that does not depend on realizations of private information or random endowments.

Symmetry implies this constant is the same for all traders. We let χ denote this dimensionless endogenous ratio, our measure of competition:

$$\chi := \frac{x_l}{x_l^{PT}} \qquad \text{for all } l = 1, \ldots, L. \tag{31}$$

As we shall see, χ lies between zero and one in equilibrium. The market is perfectly competitive if and only if $\chi \to 1$. As χ decreases, traders optimally reduce their quantities traded (or shade their bids), reflecting a less competitive market. In the limit $\chi \to 0$, there is no trade.[6]

To see how χ is related to traditional measures of competition, consider Cournot competition among n identical firms. Each firm can produce q units at a cost αq^2 for $\alpha > 0$. There is an industry demand curve with constant elasticity $Q = (P/P_0)^{-e}$ for $P_0 > 0$ and $e > 0$. Then χ, defined as the ratio the optimal quantity produced by each firm to the hypothetical price-taking quantity, is given by

$$\chi := \frac{q^*}{q_{PT}} = \left(1 - \frac{1}{en}\right)\left(1 - \frac{1}{n}\right)^{\frac{1}{e}} = (1 - LI)\,(1 - HI)^{\frac{1}{e}}, \tag{32}$$

where LI denotes the Lerner index defined as $\left(P - \text{Marginal cost}\right)/P$, HI denotes the Herfindal index defined as $\left(\sum_{n=1}^{N} q_n^2\right)/\left(\sum_{n=1}^{N} q_n\right)^2$, and e denotes the absolute value of the price elasticity of demand defined as $-(dQ/Q)/(dP/P)$. The detailed derivations are in the Appendix A.

Competition χ decreases in market power measured by the Lerner index and in market power measured by the Herfindahl index normalized by the absolute value of the price elasticity of demand. In the limit $n \to \infty$, the market becomes perfectly competitive ($\chi \to 1$) with $LI \to 0$, and $HI \to 0$.

Competition χ is also a measure of liquidity, or the demand for immediacy. From (19), trader l would not want to trade any more ($x_l = 0$) if and only if he reached his target inventory s_l^{TI} given by

$$s_l^{TI} := \frac{\mathrm{E}\left\{v + \sigma_V y \,\middle|\, p_l, i_l, s_l\right\} - p_l}{A\sigma_V^2\left(1/\tau^* + \sigma_Y^2\right)}, \qquad \text{implying} \qquad x_l^{PT} = s_l^{TI} - (s_l + \bar{s}_l)\,. \tag{33}$$

So χ, defined by (31), measures how closely traders reach their target inventories after

[6] If traders were risk neutral, there could be trades when $\chi \to 0$ since $x^{PT} \to \infty$. This does not happen when A is bounded away from zero.

trading. Unless $\chi \to 1$, traders choose not to reach their target inventories, retaining further need to trade. In contrast to the view of Grossman and Miller (1988) that liquidity is determined by the supply and demand for immediacy, where customers are willing to pay any price that market makers charge for immediate execution of their desired quantities, traders in our model demand immediacy if and only if the market is perfectly competitive. Understanding this relationship between competition and liquidity, therefore, has a practical implication for better market designs.

Another measure of market power is the price impact parameter λ in Kyle (1985), or the analogous parameter λ_I in Kyle (1989). The price impact parameter λ is defined as the changes in the price of the risky asset in response to an informed trader's buying one more share. Then χ can be expressed in terms of λ:

$$\chi = \frac{A\sigma_V^2\left(\frac{1}{\tau^*}+\sigma_Y^2\right)}{2\lambda + A\sigma_V^2\left(\frac{1}{\tau^*}+\sigma_Y^2\right)}. \tag{34}$$

Competition χ appropriately weighs the importance of price impact relative to the disutility associated with risk aversion and riskiness of the asset. This captures the way in which real-world asset managers deal with trading costs. Traders trading less risky assets are more constrained by price impact than traders trading riskier assets. Thinking of risk tolerance $1/A$ as assets under management, large asset managers are intuitively more constrained by price impact than small asset managers.[7]

5 Information and Competition in Equilibrium

This section studies how the two endogenous parameters, informational efficiency φ and competition χ, are related when the exogenous parameters vary.

The number of traders L and the average correlation parameter ρ defined by (4) affect both the similarity among the signals and the amount of available information (τ_E) at the same time. If $\rho = 0$, each private information is unique. As ρ increases from zero to one, private information become more repetitive across traders and identical when $\rho \to 1$. As ρ decreases from zero to $-\frac{1}{L-1}$, private information become more different

[7]It is easy to see how risk tolerance maps into assets under management. For a small mean μ and small variance σ^2, the competitive demand function for a log-utility investor with wealth W is approximately $W\mu/\sigma^2$. When this is compared to the CARA-normal competitive demand $\mu/\left(A\sigma^2\right)$, the demands are the same when $W = 1/A$.

across traders.

To clarify the economic effects of these different channels, we define M and N by

$$M := 1 + (L-1)\,\rho \in (0, L] \qquad \text{and} \qquad N := \frac{L}{M} \in [1, \infty)\,. \tag{35}$$

Then M captures the effect of varying L and ρ on the similarity of the signals and N captures the effect on the amount of information, with (28) implying $\tau_E = N\tau_I$.

Since equilibrium outcomes do not depend on the distribution of pairwise correlations but only on the average of the correlations, the traders can be considered to be divided into N groups with M traders in each group, with private information perfectly correlated within groups but conditionally independently distributed across groups. We thus informally refer to N as the number of groups with uniquely different information and M as the number of traders with the same information in each group; of course, this intuition is strictly valid only when M and N happen to be positive integers. We call M and N industrial organization parameters since they reflect the exogenous competitiveness of the environment.

Traders can be also considered to be divided into groups according to endowment shocks. The assumption that private information and random endowments have the same average conditional correlation (4) implies the information groups and the endowments groups may be different provided that the number of traders in each information and endowments group is same.

The characterization of equilibrium using (36) and (37) in Theorem 1 is easily expressed in terms of M, N, and φ:

Theorem 3. *Suppose $N \neq 1$. An equilibrium is characterized by φ that uniquely solves*

$$\frac{1}{\varphi} - 1 = \frac{(A\sigma_V\sigma_S)^2}{\tau_I}\left(1 + \sigma_Y^2 + \sigma_Y^2\left(1 + (N-1)\,\varphi\right)\tau_I\right)^2 \tag{36}$$

and satisfies the inequality

$$\varphi < \varphi_{soc} = \frac{MN-2}{MN-2+N}. \tag{37}$$

Competition χ, defined by (31), is given by

$$\chi = \left(1 + 2\left(\frac{1+(N-1)\,\varphi}{MN-2-(MN-2+N)\,\varphi}\right)\right)^{-1} \in (0,1)\,. \tag{38}$$

Competition χ is a function of the industrial organization parameters M and N and informational efficiency φ. This implies that φ fully captures the effects of the other exogenous parameters (τ_I, $A\sigma_V\sigma_S$, and σ_Y) on competition.

As a function of M, N, and φ, the value of χ in (38) satisfies

$$\frac{\partial \chi}{\partial M} > 0, \qquad \frac{\partial \chi}{\partial N} > 0, \qquad \text{and} \qquad \frac{\partial \chi}{\partial \varphi} < 0. \tag{39}$$

Intuitively, holding φ constant, increasing the number of traders competing with the same information M or the number of groups competing with different information N makes the market more competitive.

Surprisingly, holding M and N constant, competition decreases in informational efficiency. This implies that any changes in the parameters τ_I, $A\sigma_V\sigma_S$, and σ_Y that increase φ must decrease χ.

The economic intuition for this result is as follows: (1) Prices are made informative by traders incorporating their private information into the price. (2) From a traders' perspective, making prices more informative means incurring greater trading costs. (3) To avoid incurring large trading costs, a traders restrict the quantities they trade, just like firms exercising market power restrict the quantity they produce to raise prices in a product market.

Rewriting the residual supply curve in (9) in terms of χ illustrates this intuition:

$$p - p_l = \frac{1-\chi}{2}\left(E\left\{v \mid i_l, p_l, s_l\right\} - \left(\frac{1}{\tau^*} + \sigma_Y^2\right) A\sigma_V^2 \left(s_l + \bar{s}_l\right) - p_l\right). \tag{40}$$

The per-share trading cost on the left hand side is proportional to the difference between the trader's valuation of the asset based on his private information and endowments and the prevailing price p_l. The extent to which each trader incorporates his information into the price, by moving the price from p_l toward his valuation, decreases in competition.

We now allow the industrial organization parameters M and N to vary. We consider two separate cases for the comparative statics analysis of M and N.

Constant Individual Characteristics First, we assume the individual trader's risk aversion A, private information τ_I, and endowment shocks σ_S^2 are constant as M, N, or both vary. This assumption is relevant for studying the implications of new traders bringing their information, endowment shocks, and risk bearing capacity to the market.

From (36), we have

$$\frac{d\varphi}{dM} = 0 \qquad \text{and} \qquad \frac{d\varphi}{dN} \leq 0 \text{ with equality if and only if } \sigma_Y = 0. \tag{41}$$

Increasing M or N, while always increasing competition, does not make prices more efficient in aggregating private information.

Without residual uncertainty ($\sigma_Y = 0$), informational efficiency is the ratio between private information and endowment shocks ($\frac{\tau_I}{(A\sigma_V\sigma_S)^2}$). The industrial organization parameters have no effect on φ. While increasing M or N makes traders trade more aggressively, the proportion by which traders shade their trading, as a function of M and N, is the same for private information and endowment shocks because they are governed by the same optimal exercise of market power.

With residual uncertainty ($\sigma_Y > 0$), informational efficiency decreases N. Residual uncertainty strengthens the hedging motive and weakens the speculative motive. As the amount of available information (τ_E) increases with more traders bringing new information, the total precision traders extract from the price (τ^*) also increases. As traders learn more about the non-residual component of the liquidation value v, residual uncertainty becomes more important. Informational efficiency, which is determined by the ratio between the speculative and the hedging motives, thus decreases.

As N increases, the amount of available information τ_E automatically increases. Even when informational efficiency decreases in N with residual uncertainty, the total precision traders extract from the price τ^* increases in N because traders' improved learning about v is what causes informational efficiency to decrease in the first place.[8] This implies that, regardless of residual uncertainty, the price becomes fully revealing in the limit as N approaches infinity:

$$\tau^* \to \infty \qquad \text{as} \qquad N \to \infty. \tag{42}$$

In this limit, the price may aggregate information inefficiently. With residual uncertainty, the price reveals a zero fraction of the available information ($\varphi \to 0$).

[8]Equation (36) is equivalent to $\frac{1}{\varphi} - 1 = \frac{(A\sigma_V\sigma_S)^2}{\tau_I}\left(1 + \sigma_Y^2\tau^*\right)^2$, where τ^* decreases in φ with $\sigma_Y^2 > 0$.

Constant Market Characteristics Second, we assume the market risk aversion $A_E = \frac{A}{MN}$, market private information $\tau_E = N\tau_I$, and market endowment shock $\Sigma_S^2 = M^2 N\sigma_S^2$ are constant as M, N, or both vary. The choice for A_E follows from assuming exponential utility.[9] The choice for Σ_S^2 follows from assuming that traders can be divided into N groups with M traders within each group according to their endowment shocks. This assumption is relevant for studying the implications of industrial organization in a given market, such as coalitions of traders forming or alliances breaking down.

In terms of A_E, Σ_S^2, and τ_E, (36) can be expressed as

$$\frac{1}{\varphi} - 1 = \frac{N^2 (A_E \sigma_V \Sigma_S)^2}{\tau_E} \left(1 + \sigma_Y^2 + \sigma_Y^2 \left(\frac{1 + (N-1)\varphi}{N}\right) \tau_E\right)^2. \tag{43}$$

Again, increasing M or N, while always increasing competition, does not make prices more efficient in aggregating private information.

From (43), informational efficiency does not depend on M and decreases in N regardless of residual uncertainty. Increasing M makes each trader more risk averse proportionally to his decreasing endowment shocks, while private information is unaffected. This keeps constant the ratio between the hedging and the speculative motives. Increasing N, on the other hand, reduces each trader's private information and increases his risk aversion more than proportionally to his decreasing endowment shocks. The hedging motive, therefore, becomes more important relative to the speculative motive, and so informational efficiency decreases.

This implies, regardless of the amount of available information τ_E, traders learn nothing from the price in the limit as N approaches infinity:

$$\varphi \to 0 \qquad \text{and} \qquad \tau^* \to 1 \qquad \text{as} \qquad N \to \infty. \tag{44}$$

When a given stock of information is divided across many traders, the private information dispersed in the economy completely evaporates.

Summarizing, competition, as a function of M, N, and φ decreases in φ. Increasing M or N, while always increasing competition, does not increase φ. In this sense, the two endogenous variables φ and χ are inversely related.

[9] With exponential utility, or more generally with hyperbolic absolute risk aversion (HARA) preferences, the inverse of risk aversion is linear in wealth. This means when you combine two agents with the same risk aversion into one, the risk aversion should exactly halve.

6 Perfect Competition.

This section provides necessary and sufficient conditions under which the market becomes perfectly competitive. The results provide strategic foundations for the competitive model of Diamond and Verrecchia (1981).

From (38) the market is not perfectly competitive ($\chi < 1$) if M and N are both finite. Thus, perfect competition ($\chi \to 1$) implies either $M \to \infty$ or $N \to \infty$. In the limit $M \to \infty$, we obtain perfect competition as long as equilibrium exists with $\varphi < 1$, since φ_{soc} in (37) approaches one. As N approaches infinity while M remains finite, we have

$$\chi \to \frac{M\left(1-\varphi\right)-\varphi}{M\left(1-\varphi\right)+\varphi}, \qquad \text{implying} \qquad \chi \to 1 \text{ if and only if } \varphi \to 0. \tag{45}$$

Whether markets become perfectly competitive as more traders compete with one another depends on how similar the signals are to one another and how informationally efficient the price is. If M remains finite because the signals are sufficiently different from one another, the rather restrictive condition $\varphi \to 0$, meaning the speculative motive must become unimportant relative to the hedging motive, is necessary to achieve perfect competition. Information asymmetry affects competition in this nontrivial manner.

From the comparative statics analysis in Section 5, we know informational efficiency depends on N if and only if there is residual uncertainty or a constant stock of risk aversion, private information, and endowment shocks is divided among many traders, in which case $\varphi \to 0$ as N approaches infinity. If there is no residual uncertainty, the individual trader characteristics are constant, and the endowment shocks satisfy $\sigma_S^2 > \frac{\tau_I}{MA^2\sigma_V^2}$ so that an equilibrium exists, then as N approaches infinity,

$$\chi \to \frac{\frac{(A\sigma_V\sigma_S)^2}{\tau_I} - \frac{1}{M}}{\frac{(A\sigma_V\sigma_S)^2}{\tau_I} + \frac{1}{M}} \leq 1 \qquad \text{with equality if and only if} \qquad M \to \infty. \tag{46}$$

The market remains imperfectly competitive if M is finite. The finitely many traders in each group maintain their market power even though their private information becomes small compared to the available information with $\frac{\tau_I}{\tau_E} \to 0$.

Theorem 4. *The market becomes perfectly competitive as infinitely many traders with risk aversion A, private information τ_I, and endowment shock σ_S enter the market if* *and only if at least one of the following conditions holds:*

(i) the variance of endowment shock σ_S^2 is nonzero and the number of traders in each group $M = 1 + (L-1)\rho$ goes to infinity.

(ii) the variance of residual risk σ_Y^2 is nonzero.

The market becomes perfectly competitive as the given stock of market-wide risk aversion A_E, private information τ_E, and endowment shock Σ_S is divided among infinitely many traders if and only if the variance of the market endowment shock Σ_S^2 is nonzero.

In a model with quadratic storage costs, Vives (2011) shows that the market becomes perfectly competitive in the limit as the number of traders approaches infinity if the correlation between traders' values is less than one. In a model with quadratic storage costs *and* equicommonality, Rostek and Weretka (2015) show that the market becomes perfectly competitive in the limit as the number of traders approaches infinity if the average correlation is bounded away from one.

Their results on perfect competition are consistent with the result from our model with exponential utility *and* residual uncertainty. The market becomes perfectly competitive in the limit as the number of traders approaches infinity if there is any endowment shock that keeps the correlation between traders' values strictly below one.

To compare the implications of quadratic storage costs with that of exponential utility, we solve our model with exponential utility replaced with quadratic storage costs but everything else remaining the same. The utility maximization problem (17) becomes

$$\max_{x_l} \left\{ \mathrm{E}\left\{ \left(v + \sigma_V y - p\right) x_l - \frac{\mu}{2}\left(s_l + \bar{s}_l + x_l\right)^2 \,\middle|\, p_l, s_l, i_l \right\} \right\}, \tag{47}$$

where the constant $\mu > 0$ is the marginal cost of holding inventory. Then the first order condition becomes equivalent to that with exponential utility in (19) when $A = \sigma_V = 1$ and $\frac{1}{\tau^*} + \sigma_Y^2$ are replaced by μ.

Then equilibrium with the quadratic storage costs can be simply characterized by informational efficiency φ that solves

$$\frac{1}{\varphi} - 1 = \frac{\mu^2 \sigma_S^2}{\tau_I}\left(1 + \tau_I + \left(1 + (N-1)\varphi\right)\tau_I\right)^2 \tag{48}$$

and satisfies the existence condition (37). Equation (48) follows from substituting $A = \sigma_V = 1$ and $\frac{1}{\tau^*} + \sigma_Y^2 = \mu$ into (36).

Assuming quadratic storage costs, therefore, affects how informational efficiency depends on the exogenous parameters. Comparing (48) with (36) shows that informational efficiency with quadratic storage costs is, in fact, qualitatively identical to informational efficiency with exponential utility *and* residual uncertainty.

Assuming quadratic storage costs, however, does not affect the relationship between competition and informational efficiency. Our measure of competition is still the same function of M, N, and φ. So our measures of competition and informational efficiency capture the relationship between the two concepts in models with quadratic storage costs as equally well as in models with exponential utility. This explains why the results from models with quadratic storage costs on perfect competition correspond to the result from our model with exponential utility *and* residual uncertainty.

Perfect Risk Sharing Traders hedge their deterministic and random endowments. From (56), the coefficients π_1 for the deterministic endowments $(-A\sigma_V \bar{s}_l)$ and π_S for random endowments $(-A\sigma_V s_l)$ in the equilibrium demand schedule are given by

$$\pi_1 = \frac{\pi_S}{1-\varphi} \qquad \text{and} \qquad \pi_S = \left(\frac{MN-2}{MN-1}\right)\left(1-\frac{\varphi}{\varphi_{\text{soc}}}\right). \tag{49}$$

Informational efficiency restricts the extent to which traders can hedge deterministic endowments as well as random endowments. Traders typically hedge their deterministic endowments more than they hedge their random endowments ($\pi_1 \geq \pi_s$). The extent to which they can hedge their endowments decreases in informational efficiency. Traders perfectly hedge their endowment ($\pi_1 = \pi_S \to 1$) if and only if informational efficiency φ approaches zero and the number of total traders MN approaches infinity. Therefore, perfect risk sharing is a sufficient condition, but not a necessary condition, for perfect competition.

Mapping to Rational Expectations Equilibrium Models. In the rational expectations equilibrium models of Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981), traders rationally learn other traders' information from prices but they do not trade strategically. They take prices as given, even though their trading affects the price. This "schizophrenia" problem, articulated by Hellwig (1980), is addressed in our model in which all traders trade strategically.

In the model of Diamond and Verrecchia (1981) price-taking traders trade with different private information ($\tau_I > 0$) and different endowment shocks ($\sigma_S^2 > 0$). Each trader has unique information and endowment shocks are independent across traders. There is no residual uncertainty ($\sigma_Y^2 = 0$) and no deterministic endowments (with $\bar{s}_l = 0$ for all l). Replacing each trader in their model with a group of M traders in our model and taking the limit $M \to \infty$ in (36) and (38) result in

$$\varphi = \left(1 + \frac{A\sigma_V \sigma_S^2}{\tau_I}\right)^{-1} < 1 \qquad \text{and} \qquad \chi \to 1. \tag{50}$$

The market becomes perfectly competitive, consistently with their assumption that traders are price takers. With φ and τ^* being independent of M, this means the price in the model of Diamond and Verrecchia (1981) would be the same as that in our strategic model with arbitrary M in (24).[10] This means the competitiveness of the market is not reflected in the price. M, and thus competition affects the quantities traded in (23). In this setting, information is in the price; competition is in the quantity.

While φ remains strictly below one, the price can be made fully revealing in the limit as N also approaches infinity. As new traders bring new information to the market, the total precision a trader extracts from the price τ^* approaches infinity. This implies REE with perfect competition and fully revealing price, but not with full informational efficiency, may be a limit of an equilibrium in our strategic model as both M and N approach infinity.

In the models of Grossman and Stiglitz (1980) and Hellwig (1980), traders do not have the hedging motive but trade with noise traders who trade exogenously. We discuss the models in the next section where we introduce noise trading to the model.

7 Equilibrium with Vanishing Noise

A symmetric linear equilibrium does not exist when $(1-\rho)\tau_I = 0$ or (25) fails to be satisfied in Theorem 2. While informal intuition may suggest that there is no trade when equilibrium fails to exist, nonexistence does not imply no trade but instead represents failure of the model to make a prediction about the equilibrium outcome. We address this modeling issue by introducing an equilibrium with vanishing noise and using it to provide precise intuition for the paradox of Grossman and Stiglitz (1980) and for the no-trade theorem of Milgrom and Stokey (1982).

[10]Note that in (24) the price does not directly depend on M since $\frac{(1-\rho)(L-1)}{1+(L-1)\rho}$ simplifies to $N-1$.

7.1 Exogenous Noise Trading

We first generalize the model by adding exogenous noise traders who demand a completely inelastic random quantity z distributed $\mathrm{N}\left(0, \Sigma_Z^2\right)$ with $\Sigma_Z^2 > 0$. An *equilibrium with exogenous noise trading* is defined in the same way as an equilibrium without exogenous noise trading. The only change is that the exogenous noise trading is added to the market clearing condition. The equilibrium is specified by the original exogenous parameters A, σ_V, σ_S, τ_I, σ_Y, M, N, and $\bar{s}_1, \ldots, \bar{s}_{MN}$ (with $\bar{s}$ given by $\frac{1}{MN}\sum_{l=1}^{MN}\bar{s}_l$) plus an additional exogenous parameter Σ_Z^2 defining the variance of zero-mean exogenous noise trading.

Here we have replaced the original exogenous parameters L and ρ with the equivalent exogenous parameters $M = 1 + (L-1)\rho$ and $N = L/M$ from (35). In terms of M and N, the values of φ_{soc} is given by (37) and τ^* from (15) is given by $\tau^* = 1 + \tau_I + (N-1)\tau_I\varphi$.

Lemma 2 (Equilibrium with Exogenous Noise Trading)**.** *Suppose $A > 0$, $\sigma_V > 0$, $\tau_I > 0$, and $MN > 2$. A symmetric linear equilibrium with non-zero exogenous noise trading, $\Sigma_Z^2 > 0$, always exists and is unique. The unique equilibrium is characterized by the endogenous parameter φ that is the unique solution to*

$$\begin{aligned}\frac{1}{\varphi} - 1 = {} & \frac{(A\sigma_V\sigma_S)^2}{\tau_I}\left(1+\sigma_Y^2+\sigma_Y^2\tau_I\left(1+(N-1)\varphi\right)\right)^2 \\ & + \frac{(A\sigma_V\Sigma_Z)^2}{\tau_I M^2 (N-1)}\left(\frac{MN-1}{MN-2}\right)^2\left(\frac{\varphi_{soc}}{\varphi_{soc}-\varphi}\right)^2\left(1+\sigma_Y^2+\sigma_Y^2\tau_I\left(1+(N-1)\varphi\right)\right)^2.\end{aligned} \tag{51}$$

The market clearing price is given by

$$\begin{aligned}\frac{p}{\sigma_V} = {} & -\frac{\left(1+\tau^*\sigma_Y^2\right)}{\tau^*}A\sigma_V\bar{s} + \frac{\left(1+(N-1)\varphi\right)}{\tau^*}\frac{\tau_I^{1/2}\sum_{l=1}^{MN} i_l}{MN} \\ & - \frac{\left(1+(N-1)\varphi\right)\left(1+\tau^*\sigma_Y^2\right)}{\tau^*}\left(\frac{\sum_{l=1}^{MN} A\sigma_V s_l}{MN} - \left(\frac{MN-1}{MN-2}\right)\left(\frac{\varphi_{soc}}{\varphi_{soc}-\varphi}\right)\frac{A\sigma_V z}{MN}\right).\end{aligned} \tag{52}$$

As functions of equilibrium φ, the demand schedule for any trader l and competition χ are given by (23) *and* (38)*, respectively.*

Exogenous noise trading, widely used as a modeling device in the finance literature, is sometimes justified as a shortcut that proxies for various trading needs such as hedging endowments. Exogenous noise trading and endowment shocks are similar in that they are independent of the liquidation value of the asset, mitigating adverse selection created by information asymmetry.

They are, however, different in that endogenous hedging is sensitive to its effects on the price while noise trading is completely inelastic. Unlike endowment shocks, with which equilibrium may not exist because φ does not stay below φ_{soc}, any positive amount of noise trading ($\Sigma_Z^2 > 0$) guarantees the existence of equilibrium since the coefficient of Σ_Z^2 in the second line of (51) explodes to infinity as φ increases to φ_{soc}.

Intuitively, noise traders are willing to incur whatever losses are necessary so that they can trade the exogenous quantity z. In this sense, noise traders are similar to the "schizophrenic" price takers in the model of Grossman and Stiglitz (1980) and customers in the model of Grossman and Miller (1988). They are willing to pay any cost to trade their desired quantities immediately. This inability or unwillingness to adjust quantities to limit price impact distinguishes noise traders from strategic traders with hedging demands. How strategic traders adjust the quantities they trade is how we measure competition with χ. If the market is not perfectly competitive ($\chi < 1$), hedgers never trade the entire quantities they desire while noise traders pay whatever trading losses to do so.

Instead of relying on the losses of exogenous noise trading to support equilibrium, we propose a "vanishing noise equilibrium," which takes a limit as exogenous noise trading vanishes. In the spirit of trembling hand perfect equilibrium, we interpret vanishing noise as small perturbations to the trading environment. Our approach differs from trembling hand perfection by using exogenous perturbations rather than perturbations of the players' actions.

Let $\left(\Sigma_{Z,k}^2\right)_{k=1}^{\infty}$ denote a sequence of positive real numbers with $\lim_{k\to\infty} \Sigma_{Z,k}^2 = 0$. Then there is a sequence of unique linear symmetric equilibrium for models specified by fixed exogenous parameters A, σ_V, σ_S, τ_I, σ_Y, M, N, and $\bar{s}_1, \ldots, \bar{s}_{MN}$ and changing exogenous noise trading $z_k = \Sigma_{Z,k} u$ with $u \sim \mathrm{N}(0,1)$. For all k, let X_k, $p(X_k)$, and φ_k denote the equilibrium demand schedules, prices, and informational efficiency characterized by Lemma 2.

Theorem 5. *Suppose $A > 0$, $\sigma_V > 0$, $\tau_I > 0$, and $MN > 2$. In a model with exogenous noise trading z_k, the equilibrium informational efficiency, price, and demand schedules of all traders satisfy the following well-defined limits when $\Sigma_{Z,k}^2 \to 0$ as $k \to \infty$. Define*

φ_{soc} by (37) and define φ_0 as the unique solution to

$$\frac{1}{\varphi_0} - 1 = \frac{(A\sigma_V\sigma_S)^2}{\tau_I}\left(1 + \sigma_Y^2 + \sigma_Y^2\tau_I\left(1 + (N-1)\,\varphi_0\right)\right)^2. \tag{53}$$

Then informational efficiency φ_k satisfies the limit

$$\varphi_k \to \varphi = \min\left\{\varphi_0, \varphi_{soc} = \frac{MN-2}{MN-2+N}\right\}. \tag{54}$$

The price satisfies the limit

$$\begin{aligned}\frac{p_k}{\sigma_V} \to &-\left(\frac{1}{\tau^*} + \sigma_Y^2\right)A\sigma_V\bar{s} + \frac{\left(1+(N-1)\,\varphi\right)}{\tau^*}\left(\frac{\tau_I^{1/2}\sum_{l=1}^{MN} i_l}{MN} - \left(1+\tau^*\sigma_Y^2\right)\frac{\sum_{l=1}^{MN} A\sigma_V s_l}{MN}\right)\\ &-\frac{\left(1+(N-1)\varphi\right)}{\tau^*}\frac{(N-1)^{1/2}}{N}\left(\frac{1-\varphi}{\varphi}\tau_I - A^2\sigma_V^2\sigma_S^2\left(1+\tau^*\sigma_Y^2\right)^2\right)^{1/2} u.\end{aligned} \tag{55}$$

Quantities traded satisfy the limit

$$\begin{aligned}&\left(\frac{MN-1}{MN-2}\right)\left(\frac{\varphi_{soc}}{\varphi_{soc}-\varphi_k}\right)A\sigma_V X_{l,k}\left(p \mid i_l, s_l\right)\\ &\to -\frac{A\sigma_V\bar{s}}{1+(N-1)\,\varphi} - \frac{A\sigma_V\left(\bar{s}_l - \bar{s}\right)}{1-\varphi} - A\sigma_V s_l + \frac{1}{1+\tau^*\sigma_Y^2}\left(\tau_I^{1/2} i_l - \frac{\tau^*}{1+(N-1)\,\varphi}\frac{p}{\sigma_V}\right).\end{aligned} \tag{56}$$

A *vanishing noise equilibrium* is defined as strategies $\lim_{k\to\infty} X_{l,k}$ for all l given by (56) and prices $\lim_{k\to\infty} p_k$ given by (55), with informational efficiency $\varphi = \lim_{k\to\infty}\varphi_k$ characterized by (54). Competition χ in a vanishing noise equilibrium is given by (38).

Theorem 5 covers two distinct cases, depending on whether an equilibrium would exist without any exogenous noise trading. First, when $\varphi = \varphi_0 < \varphi_{\text{soc}}$ holds, an equilibrium already exists without exogenous noise trading (Theorem 1), in which case the limit with vanishing exogenous noise trading is this equilibrium. There is always finite trading volume, and the vanishing noise trading has no effect on prices. In this case, the square root expression in second line of (55) is exactly zero because it corresponds to φ_0 solving (53).

We next discuss the second case when $\varphi = \varphi_{\text{soc}} \le \varphi_0$ holds, and so an equilibrium does not exist without exogenous noise trading.

7.2 No-Trade Theorem.

When $\varphi = \varphi_{\text{soc}} \leq \varphi_0$ holds, there is no trade since $X_l = \lim_{k\to\infty} X_{l,k} = 0$ for all l from (56). While the at-the-limit-strategies $X_l(p) \equiv 0$ themselves define a trivial no-trade equilibrium in which there is no well-defined price, there is trading "along the way to the limit", and such trading is proportional to the difference $\varphi_k - \varphi_{\text{soc}}$. This allows the price to be well-defined in the limit.

Informational Efficiency in No-Trade Equilibrium. Vanishing noise trading supports an equilibrium by keeping the price sufficiently uninformative with $\varphi = \varphi_{\text{soc}}$. The price has "noise" in the second line in (55). When the inequality is strict ($\varphi_{\text{soc}} < \varphi_0$), this noise is nonzero even in the limit as exogenous noise trading vanishes.

The intuition is that the price impact of noise trading goes to infinity at a specific speed as noise trading Σ_Z^2 vanishes. According to (52), the price impact of noise trading λ_Z—the per-share price change in response to per-share noise trading—is defined by

$$\lambda_Z := \left(\frac{MN-1}{MN-2}\right)\left(\frac{\varphi_{\text{soc}}}{\varphi_{\text{soc}}-\varphi}\right)\frac{\left(1+(N-1)\,\varphi\right)\left(1+\tau^*\sigma_Y^2\right)A\sigma_V}{MN\tau^*}. \tag{57}$$

This per-share trading cost of noise trading λ_Z explodes to infinity as $\varphi_k \to \varphi_{\text{soc}}$, consistent with the idea that noise traders incur whatever trading losses are necessary to support equilibrium.

As noise trading vanishes ($\Sigma_Z \to 0$), informational efficiency φ approaches φ_{soc} from (54). From (51) the noise in the price created by noise trading has variance

$$\operatorname{var}\{\lambda_Z z\} = \lambda_Z^2\Sigma_Z^2 \to \frac{\tau_I\,(N-1)}{\left(N\tau_I + \frac{MN-2+N}{MN-1}\right)^2}\left(\frac{1}{\varphi_{\text{soc}}} - \frac{1}{\varphi_0}\right). \tag{58}$$

The noise created by noise trading $\lambda_Z^2\Sigma_Z^2$ to converge to a constant that is nonzero if and only if the inequality ($\varphi_{\text{soc}} \leq \varphi_0$) is strict because the price impact λ_Z goes to infinity at the same rate as the noise trading Σ_Z vanishes.

This also implies that the expected dollar losses of noise traders $E\{z\cdot\lambda_Z z\} = \lambda_Z\Sigma_Z^2$ vanish as noise trading vanishes. Thus, the vanishing noise equilibrium introduced in Theorem 5 does not rely on noise traders' suffering nonzero trading losses.

Milgrom and Stokey (1982) show that there may be no trade when traders have private information in environments with more general preferences and distributions of random variables.[11] They also argue that in such a no-trade equilibrium, the price nevertheless fully aggregates information so that each trader's private signal is "swamped" by information contained in the price. They do not provide a specific mechanism for determining equilibrium prices. Which price should we choose when any price would clear the market in a trivial no-trade equilibrium?

Our model requires the market-clearing equilibrium price to be uniquely defined by the well-defined mechanism of aggregating demand schedules. The vanishing noise equilibrium uniquely pins down the price even when there is no trade in the limit. This approach is essential for understanding the somewhat counterintuitive result that the price can be noisy even when there is no noise in (58). Contrary to Milgrom and Stokey (1982), traders' private signals may not be "swamped" by information contained in the price when traders exercise market power strategically. With informational efficiency $\varphi = \varphi_{\text{soc}}$ being less than one, a fraction $1 - \varphi$ of their private information is not incorporated in the price.

Combining (38) with (54), there is no trade in equilibrium if and only if the value of φ_0 that uniquely solves (53) satisfies

$$\varphi_0 \geq \varphi = \varphi_{\text{soc}} = \frac{MN - 2}{MN - 2 + N}. \tag{59}$$

Informational efficiency increases in both M and N and the market is noncompetitive ($\chi = 0$). Some readers might find it counterintuitive that prices can reveal any information when there is no trade. The intuition for this is that traders choose not to trade because prices incorporate so much of their information.

Recall that in (40), the extent to which traders trade toward their target inventory is inversely related to the extent to which traders move the price toward their valuation. Traders choose to not trade if and only if they move the price halfway toward their valuations. If each trader has unique information with $M = 1$, this intuition implies that informational efficiency φ satisfies $\varphi \approx 1/2$ for large enough N (or, more precisely, $\varphi = \frac{1}{2}\left(1 - \frac{1}{N-1}\right)$). If more traders share the same information with M increasing to 2, 3, ..., each trader still incorporates half of his marginal information. Since the trader's marginal information becomes smaller as M increases, informational efficiency φ increases to approximately 2/3, 3/4, 4/5, ...

[11] Tirole (1982) considers both static and dynamic settings. Dow, Madrigal and da Costa Werlang (1990) emphasize market completeness and common knowledge. Morris (1994) shows there may be no trade with heterogeneous priors.

The outcome of this oligopolistic competition among M traders resembles quantity competition in a Cournot equilibrium in which each firm tries to maximize its profit by supplying only the half of its residual demand. As the number of firms increases in quantity Cournot competition, each firm becomes a smaller fraction of the market, and the total quantity produced increases to fractions $2/3$, $3/4$,..., of the quantity with perfect competition. Of course, the important distinction here is that informational efficiency φ continues to increase in M *because* the market remains perfectly noncompetitive. If the market were becoming perfectly competitive, the marginal trader would stop incorporating his information into the price and so φ would be independent of M as shown in Section 5.

One example of (59) being satisfied occurs when M is finite and there is no residual uncertainty $\sigma_Y^2 = 0$. In this case, even with some endowment shock $\sigma_S^2 \in \left(0, \frac{\tau_I}{MA^2\sigma_V^2}\right)$, there is no trade even as the number of groups N approaches infinity. Positive endowment shocks and infinitely many traders are not only insufficient for achieving perfect competition but also insufficient for creating any trade.

Ex-ante gains from trade. Another difference between our model and that of Milgrom and Stokey (1982) is that we allow initial allocations to be not Pareto optimal. Recall the initial endowments have deterministic components $\bar{s}_l$ as well as random components s_l for all l in (3). If trader l's mean endowment $\bar{s}_l$ is not equal to the aggregate mean endowment $\bar{s}$, he knows that the allocations are not Pareto optimal because all traders are risk averse. It is common knowledge among all traders whose deterministic endowments are not the same as $\bar{s}$ that there are ex-ante gains from trade.

The deterministic endowments $\bar{s}_1, \ldots, \bar{s}_{MN}$ affect the levels of the price and the demand schedules in Theorem 5. First, the average deterministic endowment $\bar{s}$ lowers the level of the price from (55) in Theorem 5. The average deterministic endowments cannot be hedged away and, thus, risk averse traders need to be compensated for holding the asset. Second, both the individual deterministic endowments $\bar{s}_l$ and the average deterministic endowments $\bar{s}$ affect the level of the trader's demand schedule. From (56) a trader's demand schedule shifts down with $\bar{s}_l$ since traders hedge them at least partly. It shifts up with $\bar{s}$ since the aggregate risk cannot be hedged.

The deterministic endowments have no effect on equilibrium informational efficiency φ or competition χ. Since deterministic endowments do not affect whether there is no trade, traders may not participate in any trade at all despite large potential gains from trade to equalize inventories across traders. The economic intuition for the failure of the market to realize ex-ante gains from trade is reminiscent of the lemons problem in the model of Akerlof (1970), in which the seller has private information and the buyer does not. We show that the lemons problem can remain when all traders have private information.

The equilibrium in demand schedules is attractive because all traders are treated symmetrically and limit orders are protected. These are properties of well-functioning markets which organized exchanges and their regulators strive to implement. From the perspective of welfare economics, the main weakness of the equilibrium in demand schedules is that modest adverse selection can make trade break down even when there are large gains from trade due to large non-stochastic initial endowments. Whether there are better trading mechanisms for internalizing gains from trade is an interesting issue and left for future study.[12]

7.3 The Grossman–Stiglitz Paradox

Our model assumes the quality of private information and the variance of endowment shocks are the same across all traders, while the model of Grossman and Stiglitz (1980) assumes informed and uninformed traders differ in the quality of their information. To compare our model with theirs, we replace the informed and uninformed in their model with two groups of symmetrically informed traders ($N = 2$) with different private signals to capture their strategic interactions. Similarly, the model of Hellwig (1980) can be approximated by our model with N groups of traders.

In their models, there is exogenous noise trading ($\Sigma_Z^2 > 0$) but no endowment shocks ($\sigma_S^2 = 0$) and no residual uncertainty ($\sigma_Y^2 = 0$). With given market-wide exogenous noise trading $\Sigma_Z^2 > 0$ and $\sigma_S^2 = \sigma_Y^2 = 0$ in our model, informational efficiency φ solves (51),

[12] Liu and Wang (2016) examine a model in which dealers make profits by buying at the bid and selling at the offer while customers are not allowed to trade with one another. The monopolistic spread profits earned by dealers may allow trade to occur. Duffie and Zhu (Forthcoming) study a workup process that allows traders to trade at fixed prices which do not necessarily clear the market. Glode and Opp (2016) study the welfare effects of trading with intermediation chains. Malamud and Rostek (2016) study the welfare effects of decentralized exchanges when traders have heterogeneous risk aversion. Also, Bond and Eraslan (2010) show that allowing the liquidation value of the asset to depend on the action of the final owner of the asset can generate trades.

which simplifies to

$$\frac{1}{\varphi} - 1 = \frac{N^3 (A_E \sigma_V \Sigma_Z)^2}{\tau_E (N-1)} \left(\frac{MN-1}{MN-2}\right)^2 \left(\frac{\varphi_{\text{soc}}}{\varphi_{\text{soc}} - \varphi}\right)^2 \tag{60}$$

using the market-wide risk aversion $A_E = \frac{A}{MN}$ and information $\tau_E = N\tau_I$.

With given $\Sigma_Z^2 > 0$, replacing each trader in their models with a group of M traders with the same information and taking the limit $M \to \infty$ results in

$$\frac{(1-\varphi)^3}{\varphi} = \frac{N^3 (A_E \sigma_V \Sigma_Z)^2}{\tau_E (N-1)}, \qquad \text{implying} \qquad \lim_{M\to\infty} \varphi < 1. \tag{61}$$

Since $\varphi < 1$ is sufficient for achieving perfect competition as M approaches infinity in (38) from Theorem 3, the market becomes perfectly competitive with $\chi \to 1$, consistent with the price-taking assumption in the competitive models of Grossman and Stiglitz (1980) and Hellwig (1980). Importantly, perfect competition is obtained *because* the marginal information of each trader is not being incorporated into the price, which also explains why informational efficiency remains strictly below one.

The paradox of Grossman and Stiglitz (1980) involves taking a limit as the exogenous noise trading vanishes. Taking the limit $\Sigma_Z^2 \to 0$ in (61), the prices become fully informationally efficient with $\varphi \to 1$. Then traders have no incentive to acquire private information since their information is completely incorporated into the price. If traders do not acquire private information, the information is not revealed in the price. Then traders have an incentive to acquire information. Traders, therefore, can neither acquire nor not acquire private information; this is the paradox.

In our strategic trading model, this paradox does not arise. Taking the limit $\Sigma_Z^2 \to 0$ in (60) results in a no trade equilibrium with $\chi \to 0$ and informational efficiency φ is given by (59). Regardless of informational efficiency, traders do not have any incentive to acquire costly information because they will not be able to trade on it. This does not create a paradox. In fact, if traders do not acquire information, there is no information asymmetry and $\tau_I = \varphi = 0$. This will allow traders to hedge their deterministic endowments. If the number of traders approaches infinity, the market becomes perfectly competitive with $\chi \to 1$ from (38). Traders can share their risk by equalizing their endowments since $\pi_0 = \pi_1 \to 1$ in (49).

The two different results can be summarized as taking limits in different orders. Grossman and Stiglitz (1980), by assuming perfect competition exogenously, implicitly take the limit $M \to \infty$, while our strategic trading model takes the limit $\Sigma_Z^2 \to 0$ first:

$$\lim_{M\to\infty} \chi = 1 \qquad \text{for all } \Sigma_Z^2 > 0 \qquad \text{and} \qquad \lim_{\Sigma_Z^2\to 0} \chi = 0 \qquad \text{for all } M. \tag{62}$$

By first assuming M to be infinity, their approach neglects the delicate interaction between information and competition. With vanishing noise and no endowment shock, the market does not become perfectly competitive. In fact, the market remaining noncompetitive is why the price becomes fully informationally efficient as M approaches infinity. Therefore the perfect competition assumption is not strategically appropriate in this setting when noise trading vanishes.

The fact that different results are obtained when limits are taken in different orders implies that different results are also possible when a double limit is taken with both $\Sigma_Z^2 \to 0$ and $M \to \infty$. This result that competition in the limiting economy crucially depends on the order in which limits are taken is not unique to rational expectations equilibrium models. Below we show a similar result is also obtained in Cournot competition with a linear demand curve. This example highlights the importance of understanding and measuring competition correctly.

Consider Cournot competition among n identical firms.[13] There is a linear industry demand curve with $Q = \delta - P/P_0$ for constants $P_0 > 0$ and $\delta > 0$. Each firm produces q units at a cost $\frac{\alpha}{2}q^2$ for $\alpha > 0$. Then it is well known that both the equilibrium price and the marginal cost approach zero in the limit $n \to \infty$. The details are derived in the Appendix A. The conventional wisdom is that, therefore, the market becomes perfectly competitive in the limit $n \to \infty$.

Our measure of competition χ, defined as the ratio the optimal quantity produced by each firm to the hypothetical price-taking quantity produced, is given by

$$\chi := \frac{q^*}{q_{PT}} = \frac{\alpha}{\alpha + 2P_0} = \frac{n\beta}{n\beta + 2P_0}, \tag{63}$$

where the industry production cost is given by $\frac{\beta}{2}q^2$ with $\beta = \frac{\alpha}{n}$ so that $\frac{\beta}{2}q^2 = n\frac{\alpha}{2}\left(\frac{q}{n}\right)^2$.

If the firm's production cost parameter α stays constant as the number of firms n varies, competition χ is independent of the number of firms. χ in (63) can be expressed in terms of the Lerner index (LI), the Herfindal index (HI), and the absolute value of

[13] We thank an anonymous referee for suggesting the Cournot competition example.

the price elasticity of demand (e)[14]:

$$\frac{1-\chi}{1+\chi} = LI = \frac{1}{e}HI = \frac{P_0}{\alpha + P_0} = \frac{P_0}{n\beta + P_0}. \tag{64}$$

The market becomes perfectly competitive with $\chi \to 1$ if and only if the market power measured by the Lerner index, which is the same as the Herfindahl index normalized by the absolute value of the elasticity, approaches zero. With the linear demand curve, unlike the constant elasticity demand, the demand becomes completely inelastic as the price approaches zero. This enables firms to limit their quantities and charge higher prices, keeping the relative markup constant. Because of the elasticity, in contrast to the conventional wisdom, the market does not become perfectly competitive in the limit $n \to \infty$.

If, instead, the industry's production cost parameter β stays constant as the number of firms n varies, competition now depends on n in (63) and in (64). With the constant industry production cost $\beta > 0$, the equilibrium price does not approach zero in the limit $n \to \infty$. Similar to the Grossman–Stiglitz case, the order in which limits are taken is crucial in determining competition χ. From (63), we have

$$\lim_{n\to\infty} \chi = 1 \qquad \text{for all } \beta > 0 \qquad \text{and} \qquad \lim_{\beta\to 0} \chi = 0 \qquad \text{for all } n \geq 1. \tag{65}$$

With a linear demand curve, vanishing marginal cost with $\beta \to 0$ implies $e \to \frac{1}{n}$. As the number of firms approaches infinity $n \to \infty$, the demand becomes infinitely inelastic, keeping the market from becoming competitive. As in the Grossman–Stiglitz example, this intuition is lost if the limit $n \to \infty$ is taken first.[15] These examples demonstrate that measuring and understanding competition correctly is an essential issue in economics.

[14] Recall the measures are defined as $LI := (P - MC)/P$, $HI := \left(\sum_{n=1}^{N} q_n^2\right)/\left(\sum_{n=1}^{N} q_n\right)^2$, and $e := -(dQ/Q)/(dP/P)$.

[15] We can interpret the paradox of Diamond (1971) in a sequential search model as taking limits in two different orders. His model assumes firms post wages, which is equivalent to implicitly taking the limit as firms have all the bargaining power and workers none. Then taking the limit as the search cost of workers vanish implies that the firms still extract all the rents. This is considered a paradox because without any search cost, firms would have Bertrand competition. As is well known, this result is overturned when the wage is determined by bargaining.

8 Conclusion

By examining the properties of an equilibrium in a model of speculation and hedging, we have found that financial markets cannot achieve both full informational efficiency and perfect competition simultaneously, even with infinitely many traders. Our measure of competition, defined by the ratio between the quantity a trader optimally trades and the hypothetical quantity the trader would have traded if he were a price taker, has been shown useful for providing clear intuitions for the relationship between information and competition. The result that traders choose not to trade even when there are large gains from trade by equalizing their deterministic endowments indicates that an equilibrium in demand schedules does not efficiently internalize gains from trade. It is an interesting question for future research whether other trading mechanisms can achieve greater gains from trade than the single-price double auction analyzed in this paper.

References

Akerlof, George A. 1970. “The Market for ‘Lemons’: Quality Uncertainty and the Market Mechanism.” *Quarterly Journal of Economics*, 84(3): 488–500. 7.2

Anderson, Robert M., and Hugo Sonnenschein. 1985. “Rational Expectations Equilibrium with Econometric Models.” *The Review of Economic Studies*, 52(3): 359–369. 1

Bergemann, Dirk, Tibor Heumann, and Stephen Morris. 2015. “Information and Market Power.” Cowles Foundation Discussion Paper No. 2017, available at `https://ssrn.com/abstract=2639692` or `http://dx.doi.org/10.2139/ssrn.2639692`. 1, 4.1

Blume, Lawrence, and David Easley. 1990. “Implementation of Walrasian expectations equilibria.” *Journal of Economic Theory*, 51(1): 207–227. 1

Bond, Philip, and Hülya Eraslan. 2010. “Information-based trade.” *Journal of Economic Theory*, 145(5): 1675–1703. 12

Diamond, Douglas W, and Robert E Verrecchia. 1981. “Information Aggregation in a Noisy Rational Expectations Economy.” *Journal of Financial Economics*, 9(3): 221–235. 1, 6, 6, 6

Diamond, Peter A. 1971. “A model of price adjustment.” *Journal of economic theory*, 3(2): 156–168. 15

Dow, James, Vicente Madrigal, and Sergio Ribeira da Costa Werlang. 1990. “Preferences, Common Knowledge, and Speculative Trade.” London Business School, available at `http://bibliotecadigital.fgv.br/dspace/bitstream/handle/10438/417/000075109.pdf?sequence=1`. 11

Duffie, Darrell, and Haoxiang Zhu. Forthcoming. “Size Discovery.” *Review of Financial Studies*, Stanford University Graduate School of Business Research Paper No. 15–56, available at `https://ssrn.com/abstract=2682668` or `http://dx.doi.org/10.2139/ssrn.2682668`. 12

Glode, Vincent, and Christian Opp. 2016. “Asymmetric Information and Intermediation Chains.” *American Economic Review*, 106(9): 2699–2721. 12

Grossman, Sanford J., and Joseph E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review*, 70(3): 393–408. 1, 6, 6, 7, 7.1, 7.3, 7.3

Grossman, Sanford J., and Merton H. Miller. 1988. "Liquidity and Market Structure." *Journal of Finance*, 43(3): 617–633. 4.2, 7.1

Hellwig, Martin F. 1980. "On the Aggregation of Information in Competitive Markets." *Journal of Economic Theory*, 22(3): 393–408. 6, 6, 7.3, 7.3

Kyle, Albert S. 1985. "Continuous Auctions and Insider Trading." *Econometrica*, 53(6): 1315–1335. 1, 4.2

Kyle, Albert S. 1989. "Informed Speculation with Imperfect Competition." *Review of Economic Studies*, 56(3): 317–355. 1, 4.2

Liu, Hong, and Yajun Wang. 2016. "Market Making With Asymmetric Information and Inventory Risk." *Journal of Economic Theory*, 163: 73–109. 12

Malamud, Semyon, and Marzena Rostek. 2016. "Decentralized Exchange." Working Paper, Swiss Finance Institute, available at http://sfi.epfl.ch/files/content/sites/sfi/files/users/192820/public/DE.pdf. 12

McLean, Richard, and Andrew Postlewaite. 2002. "Informational size and incentive compatibility." *Econometrica*, 70(6): 2421–2453. 1

McLean, Richard, James Peck, and Andrew Postlewaite. 2005. "On price-taking behavior in asymmetric information economies." In *Essays in Dynamic General Equilibrium Theory*. 129–142. Springer. 1

Milgrom, Paul, and Nancy Stokey. 1982. "Information, Trade and Common Knowledge." *Journal of Economic Theory*, 26(1): 17–27. 1, 7, 7.2, 7.2

Morris, Stephen. 1994. "Trade with heterogeneous prior beliefs and asymmetric information." *Econometrica: Journal of the Econometric Society*, 1327–1347. 11

Ostrovsky, Michael. 2012. "Information Aggregation in Dynamic Markets with Strategic Traders." *Econometrica*, 80(6): 2595–2647. 1

Palfrey, Thomas R, and Sanjay Srivastava. 1986. "Private information in large economies." *Journal of Economic Theory*, 39(1): 34–58. 1

Reny, Philip J, and Motty Perry. 2006. "Toward a Strategic Foundation for Rational Expectations Equilibrium." *Econometrica*, 74(5): 1231–1269. 1

Rostek, Marzena, and Marek Weretka. 2012. "Price Inference in Small Markets." *Econometrica*, 80(2): 687–711. 1, 2, 4.1

Rostek, Marzena, and Marek Weretka. 2015. "Information and Strategic Behavior." *Journal of Economic Theory*, 158(Part B): 536 – 557. Symposium on Information, Coordination, and Market Frictions. 1, 6

Tirole, Jean. 1982. "On the Possibility of Speculation Under Rational Expectations." *Econometrica*, 50(5): 1163–1181. 11

Vives, Xavier. 2011. "Strategic Supply Function Competition With Private Information." *Econometrica*, 79(6): 1919–1966. 1, 4.1, 6

Vives, Xavier. 2014. "On the possibility of informationally efficient markets." *Journal of the European Economic Association*, 12(5): 1200–1239. 1

A Cournot Competition Examples

Cournot Competition with Constant Elasticity Demand Curve. Consider the following example of Cournot competition. There are n identical firms that can each produce q units at a cost αq^{δ} for $\alpha > 0$ and $\delta > 1$. There is an industry demand curve with constant elasticity $Q = (P/P_0)^{-e}$ for constants $P_0 > 0$ and $e > 0$. Then taking as given the quantity produced by the $n-1$ other firms Q_0, firm n chooses to produce the optimal quantity q^* that solves

$$\max_q P_0 \left[\left(q + Q_0\right)^{-1/e} \cdot q - \alpha q^{\delta} \right]. \tag{66}$$

In a symmetric equilibrium, $Q_0 = (n-1)\, q^*$ can be substituted into the first order condition to obtain the equilibrium price P^*:

$$P^* \left(1 - \frac{1}{en}\right) = \alpha\delta \left(q^*\right)^{\delta-1}. \tag{67}$$

In industrial organization, the degree of monopoly power is often measured by the Lerner index, given by

$$LI := \frac{P - MC}{P} = 1 - \frac{\alpha\delta \left(q^*\right)^{\delta-1}}{P^*} = \frac{1}{en}, \tag{68}$$

or the Herfindal-Hirschman index, given by symmetry by

$$HI := \sum_{n=1}^{N} \left(\frac{q_n}{\sum_{n=1}^{N} q_n} \right)^2 = \frac{1}{n}. \tag{69}$$

To find χ, defined by (31), first find the price P_n that would prevail if firm n's produced quantity were zero. The constant elasticity demand schedule implies that $P_n = P^* \left(1 - \frac{1}{n}\right)^{-\frac{1}{e}}$. A price taker would choose to produce q^{PT} so that the marginal cost equals P_n:

$$P^* \left(1 - \frac{1}{n}\right)^{-\frac{1}{e}} = \alpha\delta q_{PT}^{\delta-1}. \tag{70}$$

Comparing (67) and (70) yields

$$\chi := \frac{q^*}{q_{PT}} = \left(\left(1 - \frac{1}{en}\right) \left(1 - \frac{1}{n}\right)^{\frac{1}{e}} \right)^{\frac{1}{\delta-1}} = \left((1 - LI)\,(1 - HI)^{\frac{1}{e}} \right)^{\frac{1}{\delta-1}}. \tag{71}$$

As $n \to \infty$, the market becomes perfectly competitive with $\chi \to 1$, $LI \to 0$, and $HI \to 0$.

Cournot Competition with Linear Demand Curve. There are n identical firms, which can each produce q units at a cost $\frac{\alpha}{2}q^2$ for $\alpha > 0$. The industry production cost is given by $\frac{\beta}{2}q^2$ with $\beta = \frac{\alpha}{n}$. There is a linear industry demand curve with $Q = \delta - P/P_0$ for constants $P_0 > 0$ and $\delta > 0$. Taking as given the quantity produced by $n-1$ firms Q_0, firm n chooses to produce the optimal quantity q^* that solves

$$\max_q \left[P_0 \left(\delta - Q_0 - q \right) \cdot q - \frac{\alpha}{2} q^2 \right]. \tag{72}$$

In a symmetric equilibrium, $Q_0 = (n-1)\, q^*$ can be substituted into the first order condition to obtain the equilibrium quantity Q^* and price P^*:

$$q^* = \frac{P_0 \delta}{\alpha + P_0 (n+1)} \qquad \text{and} \qquad P^* = \frac{P_0 \delta (\alpha + P_0)}{\alpha + P_0 (n+1)}. \tag{73}$$

The Lerner index LI and the Herfindal-Hirschman index HI are given by

$$LI := \frac{P - MC}{P} = \frac{P_0}{\alpha + P_0} \qquad \text{and} \qquad HI = \frac{1}{n}. \tag{74}$$

The price elasticity of demand is $-e$, with $e > 0$ given by

$$e = -\frac{dQ/Q}{dP/P} = \frac{\alpha + P_0}{n P_0}. \tag{75}$$

To find χ, defined by (31), first find the price P_n, that would prevail if firm n's produced quantity were zero. The linear demand schedule implies that P_n and the price taker's quantity q^{PT} are given by

$$P_n = \frac{P_0 \delta (\alpha + 2P_0)}{\alpha + P_0 (n+1)} \qquad \text{and} \qquad q^{PT} = \frac{P_0 \delta (\alpha + 2P_0)}{\alpha (\alpha + P_0 (n+1))}, \tag{76}$$

which yields

$$\chi := \frac{q^*}{q_{PT}} = \frac{\alpha}{\alpha + 2P_0} = \frac{n\beta}{n\beta + 2P_0}. \tag{77}$$

This implies our measure of competition χ satisfies

$$\frac{1-\chi}{1+\chi} = LI = \frac{1}{e} HI = \frac{P_0}{\alpha + P_0} = \frac{P_0}{n\beta + P_0}. \tag{78}$$

B Proofs

Proof of Lemma 1. Information that trader l can learn from $\{p_l, i_l, s_l\}$ is equivalent to information that he can learn from $\{\hat{p}_l, i_l\}$. The variables $\frac{v}{\sigma_V}$, i_l, and $\frac{\hat{p}_l}{\sigma_V}$ are jointly normally distributed as

$$\begin{pmatrix} \frac{v}{\sigma_V} \\ i_l \\ \frac{\hat{p}_l}{\sigma_V} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \tau_I^{1/2} & \frac{\pi_I}{\pi_P}\tau_I^{1/2} \\ \tau_I^{1/2} & 1+\tau_I & \frac{\pi_I}{\pi_P}(\tau_I+\rho) \\ \frac{\pi_I}{\pi_P}\tau_I^{1/2} & \frac{\pi_I}{\pi_P}(\tau_I+\rho) & \operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} \end{pmatrix}\right), \tag{79}$$

where $\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\}$ is given by

$$\begin{aligned} \operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} &= \left(\frac{\pi_I}{\pi_P}\right)^2 \left(\tau_I + \operatorname{var}\left\{\frac{\sum_{l'\neq l} e_{l'}}{L-1}\right\}\right) + \left(\frac{A\sigma_V \pi_S}{\pi_P}\right)^2 \operatorname{var}\left\{\frac{\sum_{l'\neq l} s_{l'}}{L-1} - \rho s_l\right\} \\ &= \left(\frac{\pi_I}{\pi_P}\right)^2 \left(\tau_I + \frac{1+(L-2)\rho}{L-1}\right) + \left(\frac{\pi_S}{\pi_P} A\sigma_V\sigma_S\right)^2 \frac{(1-\rho)\left(1+(L-1)\rho\right)}{L-1}. \end{aligned} \tag{80}$$

By the projection theorem we have

$$\begin{aligned} E\left\{\frac{v}{\sigma_V} \,\middle|\, i_l, \frac{\hat{p}_l}{\sigma_V}\right\} &= \frac{\tau_I^{1/2} \begin{pmatrix} 1 \\ \frac{\pi_I}{\pi_P} \end{pmatrix}^T \cdot \begin{pmatrix} \operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} & -\frac{\pi_I}{\pi_P}(\tau_I+\rho) \\ -\frac{\pi_I}{\pi_P}(\tau_I+\rho) & 1+\tau_I \end{pmatrix} \cdot \begin{pmatrix} i_l \\ \frac{\hat{p}_l}{\sigma_V} \end{pmatrix}}{(1+\tau_I)\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I+\rho)^2} \\ &= \frac{\tau_I^{1/2}\left(\left(\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I+\rho)\right) i_l + \frac{\pi_I}{\pi_P}(1-\rho)\frac{\hat{p}_l}{\sigma_V}\right)}{(1+\tau_I)\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I+\rho)^2}, \end{aligned} \tag{81}$$

and

$$\begin{aligned} \operatorname{var}\left\{\frac{v}{\sigma_V} \,\middle|\, i_l, \frac{\hat{p}_l}{\sigma_V}\right\} &= 1 - \frac{\tau_I \begin{pmatrix} 1 \\ \frac{\pi_I}{\pi_P} \end{pmatrix}^T \cdot \begin{pmatrix} \operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} & -\frac{\pi_I}{\pi_P}(\tau_I+\rho) \\ -\frac{\pi_I}{\pi_P}(\tau_I+\rho) & 1+\tau_I \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \frac{\pi_I}{\pi_P} \end{pmatrix}}{(1+\tau_I)\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I+\rho)^2} \\ &= \frac{\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I+\rho^2)}{(1+\tau_I)\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I+\rho)^2}. \end{aligned} \tag{82}$$

Substituting (80) and (82) into (13) yields

$$\tau^* = 1 + \tau_I + \frac{\left(\frac{\pi_I}{\pi_P}\right)^2 (1-\rho)^2}{\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} - \left(\frac{\pi_I}{\pi_P}\right)^2 (\tau_I + \rho^2)} \tau_I. \tag{83}$$

Then substitute (15) and (80) into (83) to obtain (16).

Using (83), we can express $\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\}$ in terms of τ^* as

$$\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} = \left(\frac{\pi_I}{\pi_P}\right)^2 \left(\tau_I + \rho^2 + \frac{\tau_I (1-\rho)^2}{\tau^* - 1 - \tau_I}\right), \tag{84}$$

which can be substituted into (81) to obtain

$$\mathrm{E}\left\{\frac{v + \sigma_V y}{\sigma_V} \,\middle|\, i_l, \hat{p}_l\right\} = \frac{\tau_I - \rho(\tau^* - 1)}{\tau^* \tau_I^{1/2} (1-\rho)} i_l + \frac{\frac{\pi_P}{\pi_I}(\tau^* - 1 - \tau_I)}{\tau^* \tau_I^{1/2} (1-\rho)} \frac{\hat{p}_l}{\sigma_V}. \tag{85}$$

since $E\{y\} = E\{y \mid \hat{p}_l, i_l\} = 0$. □

Proof of Theorem 1. Substituting the conditional expectation (85) into the first order condition (19) allows the optimal quantity demand x_l to be written as

$$\begin{aligned}\left(\frac{2}{\pi_P (L-1)} + \frac{1}{\tau^*} + \sigma_Y^2\right) A \sigma_V X_l = {} & \frac{\tau_I - \rho(\tau^* - 1)}{\tau^* \tau_I^{1/2} (1-\rho)} i_l - \left(\frac{1}{\tau^*} + \sigma_Y^2\right) A \sigma_V (s_l + \bar{s}_l) \\ & + \frac{\frac{\pi_P}{\pi_I}(\tau^* - 1 - \tau_I)}{\tau^* \tau_I^{1/2} (1-\rho)} \frac{\hat{p}_l}{\sigma_V} - \frac{p_l}{\sigma_V}.\end{aligned} \tag{86}$$

Substituting the residual supply curve (9) and the definition of $\hat{p}_l$ (11) allows the optimal quantity demanded $x_{m,n}$ in (86) to be implemented with a demand schedule

X_l given by

$$\left(1+\sigma_Y^2\tau^*+\frac{\tau^*}{\pi_P(L-1)}+\frac{(\tau^*-1-\tau_I)}{(L-1)\,\tau_I^{1/2}(1-\rho)\,\pi_I}\right)A\sigma_V X_l$$
$$=\frac{(\tau^*-1-\tau_I)}{\tau_I^{1/2}(1-\rho_I)}\left(-\frac{\pi_0}{\pi_I}+\left(\frac{L}{L-1}\right)\frac{\pi_1}{\pi_I}\right)A\sigma_V\bar{s}-\left(1+\sigma_Y^2\tau^*+\frac{(\tau^*-1-\tau_I)}{(L-1)\,\tau_I^{1/2}(1-\rho_I)}\frac{\pi_1}{\pi_I}\right)A\sigma_V\bar{s}_l$$
$$+\frac{\tau_I-\rho(\tau^*-1)}{\tau_I^{1/2}(1-\rho)}i_l-\left(\tau^*-\frac{(\tau^*-1-\tau_I)\,\pi_P}{\tau_I^{1/2}(1-\rho_I)\,\pi_I}\right)\frac{p}{\sigma_V}-\left(1+\sigma_Y^2\tau^*-\frac{\rho\frac{\pi_S}{\pi_I}(\tau^*-1-\tau_I)}{\tau_I^{1/2}(1-\rho)}\right)A\sigma_V s_l. \tag{87}$$

A linear symmetric equilibrium is found by equating a trader's best response (87) to the strategy the trader conjectures (π_0, π_1, π_S, π_I, and π_P) that others are playing. This implies that

$$\frac{\pi_P}{\pi_I}=\frac{\tau^*-\frac{\frac{\pi_P}{\pi_I}(\tau^*-1-\tau_I)}{\tau_I^{1/2}(1-\rho)}}{\frac{\tau_I-\rho_I(\tau^*-1)}{\tau_I^{1/2}(1-\rho)}}=\frac{\tau^*\tau_I^{-1/2}}{1+\frac{(1-\rho)(L-1)}{1+(L-1)\rho}\varphi}, \tag{88}$$

$$\frac{\pi_S}{\pi_I}=\frac{1+\sigma_Y^2\tau^*-\frac{\rho\frac{\pi_S}{\pi_I}(\tau^*-1-\tau_I)}{\tau_I^{1/2}(1-\rho)}}{\frac{\tau_I-\rho(\tau^*-1)}{\tau_I^{1/2}(1-\rho)}}=\left(1+\sigma_Y^2\tau^*\right)\tau_I^{-1/2}, \tag{89}$$

$$\frac{\pi_1}{\pi_I}=\frac{1+\sigma_Y^2\tau^*+\frac{(\tau^*-1-\tau_I)}{(L-1)\tau_I^{1/2}(1-\rho)}\frac{\pi_1}{\pi_I}}{\frac{\tau_I-\rho(\tau^*-1)}{\tau_I^{1/2}(1-\rho)}}=\frac{\left(1+\sigma_Y^2\tau^*\right)\tau_I^{-1/2}}{1-\varphi}, \tag{90}$$

$$\frac{\pi_0}{\pi_I}=\frac{\frac{(\tau^*-1-\tau_I)}{\tau_I^{1/2}(1-\rho)}\left(-\frac{\pi_0}{\pi_I}+\left(\frac{L}{L-1}\right)\frac{\pi_1}{\pi_I}\right)}{\frac{\tau_I-\rho(\tau^*-1)}{\tau_I^{1/2}(1-\rho)}}=\left(1-\frac{1-\varphi}{1+\frac{(L-1)(1-\rho)}{1+(L-1)\rho}\varphi}\right)\frac{\pi_1}{\pi_I}, \tag{91}$$

and

$$\pi_I=\frac{\frac{\tau_I-\rho(\tau^*-1)}{\tau_I^{1/2}(1-\rho)}}{1+\sigma_Y^2\tau^*+\frac{\tau^*}{\pi_P(L-1)}+\frac{(\tau^*-1-\tau_I)}{(L-1)\tau_I^{1/2}(1-\rho)\pi_I}}=\frac{\frac{L-2}{L-1}\left(1-\frac{\rho+\frac{2}{L-2}}{\rho+\frac{1}{L-1}}\varphi\right)}{\left(1+\sigma_Y^2\tau^*\right)\tau_I^{-1/2}}, \tag{92}$$

where the second equality in (92) follows (88).

Substituting (88) to (92) into (8) yields (23). Substituting (23) into the market clearing condition (5) and rearranging yields (24).

To exclude trivial no-trade equilibrium, we require $L>1$ and $(L-1)\pi_P\neq 0$. Using

(88) and (92), this implies

$$L > 2 \qquad \text{and} \qquad \varphi \neq \frac{\rho + \frac{1}{L-1}}{\rho + \frac{2}{L-2}}. \tag{93}$$

Substituting (88) and (92) into the second order condition (20) yields

$$\left(\frac{1}{L-2}\right)\left(\frac{1 + \frac{1-\rho}{\rho + \frac{1}{L-1}}\varphi}{1 - \frac{\rho + \frac{2}{L-2}}{\rho + \frac{1}{L-1}}\varphi}\right) > -\frac{1}{2}, \tag{94}$$

which, combined with (93), is equivalent to (22). To see this, first note that (94) implies that if (22) doesn't hold, (94) and (93) imply that $1 < \left(\frac{\rho}{\rho + \frac{1}{L-1}}\right)\varphi$, which cannot hold because $\varphi \leq 1$.

Finally, (22) implies that $\pi_I = 0$ if and only if $\tau_I = 0$. Then assuming $(1-\rho)\tau_I \neq 0$, substituting (89) and (92) into (16) yields (21). If $(1-\rho)\tau_I = 0$, φ is set to zero and this satisfies (22). □

Proof of Theorem 2. Proving this corollary is accomplished by analyzing the two equations (21) and (22) determining the endogenous parameter φ. If $(1-\rho)\tau_I = 0$, φ is set to zero, which satisfies (22).

Suppose $(1-\rho)\tau_I \neq 0$. The left hand side of (21) is monotonically deceasing for all $\varphi \in [0,1]$ with it approaching to infinity as φ approaches zero. The right hand side of (21) is monotonically increasing for all $\varphi \in [0,1]$. Therefore, a symmetric linear equilibrium is unique, if it exists. The unique solution to (21) exists and satisfies (22) if and only if the left hand side of (21) is strictly less than the right hand side of (21) when φ equals φ_{soc}, which can be written as (25) using (28). □

Proof of Theorem 3. From the two first order conditions (19) and (30), χ, defined by (31), is given by

$$\chi = \left(1 + \frac{2\tau^*}{\pi_P (L-1)\left(1 + \sigma_Y^2 \tau^*\right)}\right)^{-1}. \tag{95}$$

Substituting equilibrium values of τ^* and π_P from (15) and (23) respectively into (95) and rewriting in terms of M and N using (35) yield (38). □

Proof of Theorem 4. From the discussion in the main text of the paper, perfect competition is achieved if and only if $\varphi < 1$ if $M \to \infty$ and $\varphi \to 0$ if $M < \infty$ and $N \to \infty$ in (45). With $\sigma_S^2 > 0$, $\varphi < 1$ from . In Section (5), we have shown that $\varphi \to 0$ in the limit $N \to \infty$ if and only if residual uncertainty $\sigma_Y^2 > 0$ or the market characteristics, A_E, Σ_S^2, and τ_E are constant in (44). □

Proof of Lemma 2. Proving this lemma is similar to proving Theorem 1 with two exceptions. First, the market clearing condition (5) should be replaced with

$$\sum_{l=1}^{L} X_l\left(p\right) + z = 0. \tag{96}$$

Second, learning from prices in (16) should have additional noise from Σ_Z^2.

Equation (96) implies the price p_l, that would prevail if his traded quantity were zero ($x_l = 0$) in (10) is now replaced by

$$\frac{p_l}{\sigma_V} = \frac{\sum_{l' \neq l} \left(\pi_C + \pi_0 A \sigma_V \bar{s} - \pi_1 A \sigma_V \bar{s}_{l'} - \pi_S A \sigma_V s_{l'} + \pi_I i_{l'}\right) + A \sigma_V z}{\left(L-1\right) \pi_P}, \tag{97}$$

which means $\hat{p}_l$, defined by (11), now has an additional term as

$$\frac{\hat{p}_l}{\sigma_V} = \frac{\pi_I}{\pi_P} \frac{\sum_{l' \neq l} i_{l'}}{L-1} - \frac{\pi_S}{\pi_P} A \sigma_V \left(\frac{\sum_{l' \neq l} s_{l'}}{L-1} - \rho_S s_l\right) + \frac{A \sigma_V z}{\left(L-1\right) \pi_P}. \tag{98}$$

Then $\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\}$ also increases by new noise trading as

$$\operatorname{var}\left\{\frac{\hat{p}_l}{\sigma_V}\right\} = \left(\frac{\pi_I}{\pi_P}\right)^2 \left(\tau_I + \frac{1 + \left(L-2\right) \rho}{L-1}\right) + \left(\frac{\pi_S}{\pi_P} A \sigma_V \sigma_S\right)^2 \frac{\left(1-\rho\right)\left(1 + \left(L-1\right)\rho\right)}{L-1} + \left(\frac{A \sigma_V \Sigma_Z}{\left(L-1\right) \pi_P}\right)^2. \tag{99}$$

Substituting and (99) into (83) to obtain

$$\frac{1}{\varphi} - 1 = \left(\frac{\pi_S}{\pi_I} A \sigma_V \sigma_S\right)^2 + \frac{1}{\left(1-\rho\right)\left(\rho + \frac{1}{L-1}\right)} \left(\frac{A \sigma_V \Sigma_Z}{\left(L-1\right) \pi_I}\right)^2. \tag{100}$$

Exogenous noise trading only affects the optimal strategy and equilibrium demand schedules only through φ. This means the optimal demand schedules are given by (23).

Substituting $\frac{\pi_S}{\pi_I}$ and π_I from (23) into (100) yields

$$\frac{1}{\varphi}-1=\left(1+\tau^{*}\sigma_Y^2\right)^2\left(\frac{(A\sigma_V\sigma_S)^2}{\tau_I}+\frac{(A\sigma_V\Sigma_Z)^2}{(1-\rho)\left(\rho+\frac{1}{L-1}\right)\left(1-\frac{\varphi}{\varphi_{\text{soc}}}\right)^2(L-2)^2\tau_I}\right). \tag{101}$$

Substituting τ^* from (15) and M and N, given by (35) into (101) yields (51).

With $\Sigma_Z^2>0$, the right hand side of (51) approaches infinity as φ approaches φ_{soc} and the left hand side of (51) approaches infinity as φ approaches zero. By the Intermediate Value Theorem, there is a $\varphi\in\left(0,\varphi_{\text{soc}}\right)$ that solves (51). Since the left hand side of (51) is strictly decreasing in φ and the right hand side of (51) is increasing in φ for all $\varphi\in\left[0,\varphi_{\text{soc}}\right]$, the solution to (51) is unique. Therefore, there exists a unique symmetric linear equilibrium characterized by φ.

Substituting (23) into the market clearing condition (96) yields

$$\begin{aligned}\frac{p}{\sigma_V}=&-\frac{\left(1+\tau^{*}\sigma_Y^2\right)A\sigma_V\bar{s}}{\tau^{*}}+\frac{\left(1+\tau^{*}\sigma_Y^2\right)\left(1+\frac{(1-\rho_I)(L-1)}{1+(L-1)\rho_I}\varphi\right)}{\tau^{*}\left(\frac{L-2}{L-1}\right)\left(1-\frac{\varphi}{\varphi_{\text{soc}}}\right)}A\sigma_V z\\&+\frac{1}{\tau^{*}}\left(1+\frac{\left(1-\rho_I\right)(L-1)}{1+(L-1)\rho_I}\varphi\right)\left(\tau_I^{1/2}\frac{\sum i_l}{L}-\left(1+\tau^{*}\sigma_Y^2\right)\frac{\sum A\sigma_V s_l}{L}\right).\end{aligned} \tag{102}$$

Again, substituting τ^* from (15) and M and N, given by (35) into (101) yields (51). □

Proof of Theorem 5. If $\varphi<\varphi_{\text{soc}}$, then the coefficient of Σ_Z^2 in (51) is finite and (51) is continuous in σ_Z^2 and φ. The limit of (51) as Σ_Z^2 approaches zero is (53) and $\varphi=\varphi_0$ solves (53). Since (51) is continuous in φ and φ_k is a unique solution to (51) in $\left(0,\varphi_{\text{soc}}\right)$ for all k, by the Implicit Function Theorem, $\varphi_k\to\varphi_0$ as $k\to\infty$ if $\varphi_0<\varphi_{\text{soc}}$.

If $\varphi_{\text{soc}}<\varphi_0$, then φ_k does not approach φ_0 since $\varphi_k<\varphi_{\text{soc}}$ for all k. This means

$$\lim_{k\to\infty}\frac{1}{\varphi_k}-1-\frac{(A\sigma_V\sigma_S)^2}{\tau_I}\left(1+\sigma_Y^2+\sigma_Y^2\tau_I\left(1+(N-1)\varphi_k\right)\right)^2>0. \tag{103}$$

Substituting (103) into (51) implies

$$\lim_{k\to\infty}\frac{(A\sigma_V\Sigma_Z)^2}{M^2(N-1)\tau_I}\left(\frac{MN-1}{MN-2}\right)^2\left(\frac{\varphi_{\text{soc}}}{\varphi_{\text{soc}}-\varphi_k}\right)^2>0. \tag{104}$$

Therefore, as $k \to \infty$, $\varphi_k \to \varphi_{\text{soc}}$ and

$$\frac{(A\sigma_V \Sigma_Z)^2}{M^2 (N-1)\tau_I} \left(\frac{MN-1}{MN-2}\right)^2 \left(\frac{\varphi_{\text{soc}}}{\varphi_{\text{soc}} - \varphi_k}\right)^2 \to \left(\frac{1}{\varphi} - 1\right)\left(1 + \sigma_Y^2 \tau^*\right)^{-2} - \frac{(A\sigma_V \sigma_S)^2}{\tau_I} > 0. \quad (105)$$

where the inequality follows $\varphi_0 > \varphi_{\text{soc}}$.

Finally, if $\varphi_0 = \varphi_{\text{soc}}$, (105) approaches zero. So $\varphi = \varphi_0 = \varphi_{\text{soc}}$ solves the limit of (51) as σ_Z^2 approaches zero, which is (53). By the Implicit Function Theorem, $\varphi_k \to \varphi_0 = \varphi_{\text{soc}}$ as $k \to \infty$. This proves (54).

Using (54), taking a limit to (52) as $k \to \infty$ and substituting (105) yields $p_k \to p$ in (55). Taking a limit to (23) and substituting (35) yields $X_{l,k} \to X_l$ in (56). $\square$