Financial Crises and Lending of Last Resort in Open Economies*

Luigi Bocola[†] **Guido Lorenzoni**[‡]

October 2017

Abstract

We study financial panics in a small open economy with floating exchange rates. In our model, bank runs trigger a decline in domestic wealth and a currency depreciation. Runs are more likely when banks have dollar debt. Dollar debt emerges endogenously in response to the precautionary motive of domestic savers: dollar savings provide insurance against crises; so when crises are possible it becomes relatively more expensive for banks to borrow in local currency, which gives them an incentive to issue dollar debt. This feedback between aggregate risk and savers' behavior can generate multiple equilibria, with the bad equilibrium characterized by financial dollarization and the possibility of bank runs. A domestic lender of last resort can eliminate the bad equilibrium, but interventions need to be fiscally credible. Holding foreign currency reserves hedges the fiscal position of the government and enhances its credibility, thus improving financial stability.

Keywords: Financial crises, Dollarization, Lending of Last Resort, Foreign Reserves.

JEL codes: F34, E44, G11, G15

*First draft: August 29, 2016. We thank Mark Aguiar, Fernando Alvarez, Javier Bianchi, Charles Brendon, Fernando Broner, Alessandro Dovis, Pierre-Olivier Gourinchas, Matteo Maggiori, Fabrizio Perri, and participants at EIEF, CSEF-IGIER 2016, EEA-ESEM 2016, Cambridge-INET 2016, ASSA 2017, CREI, PSE, Boston College, Cornell, Carnegie Mellon, Wharton, University of Chicago, MFS 2017 spring meeting, IMF, Federal Reserve Bank of Richmond, SED 2017, NBER SI 2017, ITAM-PIER 2017, Stanford SITE 2017, Federal Reserve Board, Columbia, NYU, Duke, UNC, MIT, and Harvard. Jane Olmstead-Rumsey provided excellent research assistance. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve System.

[†]Northwestern University, Federal Reserve Bank of Minneapolis and NBER

[‡]Northwestern University and NBER

1 Introduction

After the financial crisis of 2007-2008, there has been a renewed interest in understanding financial panics and in designing appropriate policy responses. Financial panics are situations in which banks suddenly lose access to short-term funding, leading to asset sales, a sharp downward adjustment in asset prices, and a contraction in credit. The standard recipe for dealing with financial panics is relatively well understood, going back to Bagehot (1873). The central bank can stop a panic by providing ample access to emergency funding to distressed banks, that is, by acting as a lender of last resort.

For emerging economies with an open capital account, financial panics tend to go together with other sources of stress. A domestic financial crisis is often associated with an international flight from domestic assets that leads to a depreciation of the domestic currency (Kaminsky and Reinhart, 1999). A depreciation in turn can further exacerbate the crisis if, as is often the case, domestic banks or firms are indebted in foreign currency (Krugman, 1999). This combination of tensions makes it especially challenging for domestic authorities to effectively act as lenders of last resort. Financing the intervention by expanding the domestic money supply can lead to inflationary concerns and further exacerbate the currency depreciation. Financing it by issuing government bonds may be limited by investors' concerns about public debt sustainability.

Several economists and policymakers have suggested that the buildup of foreign currency reserves in emerging markets over the past twenty years is a response to the challenges just described. A large stock of foreign reserves, the argument goes, helps domestic authorities intervene in a financial panic, acting—in Mervyn King's language—as do-it-yourself lenders of last resort in US dollars to their own financial system.[1] The idea that reserves are needed to fight the combination of an "internal drain" (a domestic bank run) with an "external drain" (a capital flight) also goes back to Bagehot (1873), and it has been recently articulated by Obstfeld, Shambaugh, and Taylor (2010), who provide cross-country empirical evidence in its support.[2] Complementary evidence by Gourinchas and Obstfeld (2012) shows that foreign currency reserves are indeed effective at reducing the probability of financial crises.

There are, however, several open questions regarding lending of last resort in emerging market economies. The theoretical argument for why reserves help lending of last resort

[1] From a speech given as governor of the Bank of England (King, 2006).

[2] They show that the size of the banking sector, measured by bank deposits, plays a crucial role in explaining variation in reserves holdings across countries and across time. In related work, Aizenman and Lee (2007) document that foreign reserve stocks are related to financial openness and to measures of exposure to financial crises.

has been developed in the context of pegged exchange rate regimes.[3] It is thus puzzling that the acceleration in reserve accumulation by emerging markets occurred over a period during which many of these economies abandoned hard pegs and opted for more flexible exchange rate arrangements. What is the argument for reserve accumulation in a floating exchange rate regime? Furthermore, a common concern with ex post financial interventions is that they can distort incentives to take risk ex ante. In our context, the concern is that the accumulation of reserves to support the financial sector during a crisis might backfire and induce domestic agents to borrow more in foreign currency, through a mechanism similar to the one in the literature on bailout guarantees (Burnside, Eichenbaum, and Rebelo, 2001a; Schneider and Tornell, 2004; Farhi and Tirole, 2012). This means that we need to address two questions: What are the fundamental forces that give domestic agents incentives to borrow in foreign currency? Do these incentives get worse when government intervention is expected?

In this paper we formulate a model that can help us understand how panics play out in open economies and shed some light on the questions above. Our model features a fully flexible exchange rate regime and it captures explicitly the decisions of the private sector regarding the currency composition of its assets and liabilities. First, we show that our environment can generate multiple equilibria with a bad equilibrium in which domestic financial institutions primarily issue dollar debt and are prone to runs. Differently from existing theories, dollar debt arises in equilibrium because domestic savers want to save in dollars as a way of insuring against financial panics. We then provide an argument for why foreign currency reserves improve financial stability in a floating exchange rate regime. The main idea is that reserves are a good hedge against the pessimistic expectations of the private sector, and they can thus help official authorities to credibly intervene to eliminate bad equilibria. Surprisingly, in our framework these interventions can have a stabilizing effect ex ante, leading to less dollar debt.

Our model combines ingredients from the recent macro-financial literature (Gertler and Kiyotaki, 2010; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2015) with a standard open economy framework. There are two domestic agents, households and bankers, and risk-neutral international investors. Households work for domestic firms and save in domestic and foreign currency. Bankers borrow in domestic and foreign currency, and use these resources along with their accumulated net worth to purchase domestic assets, which are used as inputs in production. The model features two sources of financial frictions: banks face a potentially binding financial constraint, and foreign investors only

[3] The argument in Bagehot (1873) was made in the context of the gold standard. Modern versions of this argument— for example, the model in Section 3 of Obstfeld, Shambaugh, and Taylor (2010)— build on models of currency crises originating from unsustainable pegs.

borrow and lend in foreign currency.

We first show that a financial panic triggers a depreciation through its effects on domestic wealth and domestic demand. In particular, when banks lose access to funding, asset prices and credit fall, and so does in equilibrium the wealth of domestic households. The associated decline in the demand for domestic goods requires the relative price of non-tradables to fall. This Balassa-Samuelson channel leads to a pattern of comovement that is important for the analysis to follow: in a crisis, domestic asset prices, banks' net worth, the real exchange rate, households' incomes, and the fiscal capacity of the government all fall at the same time.

We then show that the possibility of financial panics in the future might lead the banks to issue more dollar debt ex ante. When panics are possible, households have an incentive to insure by saving more in foreign currency and less in domestic currency. This happens because of the comovement properties highlighted above: households' income goes down in a panic and foreign currency assets are a good hedge against such event. In general equilibrium, the households' behavior puts upward pressure on the interest rate in domestic currency relative to the one in foreign currency. As a result, borrowing in foreign currency becomes relatively cheaper for banks, and this can dominate their motive to insure from the risk of a run, leading them to issue more dollar debt.

The feedback between the precautionary motives of households and the risk of financial panics can be so strong as to produce multiple equilibria. In a low-risk equilibrium, households are happy to save in domestic currency, banks mostly borrow in domestic currency, balance sheet effects of currency depreciation are weak, and panics cannot arise in equilibrium. This confirms households' expectations. In a high-risk equilibrium, domestic households are worried about future panics and save in dollars. Domestic currency funding is more expensive, so banks borrow in dollars, making the financial sector more fragile and opening the door to the possibility of panics. Again, households' expectations are confirmed. This form of multiplicity emphasizes the importance of allowing for endogenous risk premia as determinants of the currency denomination of debt.

The presence of these high-risk equilibria motivates our analysis of lending of last resort. We consider a benevolent government that can extend a liquidity facility to banks. This intervention can stimulate credit when banks are financially constrained and it can potentially eliminate panics. Thus, the government has an incentive to promise aggressive interventions when savers have pessimistic expectations. These promises, however, are not necessarily optimal ex post. In a crisis, households also hold pessimistic expectations about future tax revenues, limiting the ability of the government to finance its interventions by issuing debt. The government then faces a trade-off between stabilizing the financial sector and increasing distortionary taxes, leading to less aggressive interventions. We show that dollar reserves hedge the fiscal position of the government because reserves appreciate precisely when households hold pessimistic expectations. The ex ante accumulation of dollar reserves thus allows the government to credibly operate as a lender of last resort and eliminates the possibility of panics.

Therefore our model offers a novel argument for why reserves promote financial stability. The government in our framework does not use reserves to sustain the currency.[4] However, foreign currency reserves help because they have good hedging properties against bad equilibria. In other words, a desirable feature of reserves is that if private sector beliefs deteriorate, pushing the economy in the direction of a panic, the value of reserves increases, giving the government more resources to intervene.

Finally, we address the question of whether reserves accumulation leads the private sector to take on more risk ex ante. In our model, the accumulation of reserves by the official sector does not necessarily induce the banking sector to increase dollar borrowing. When the government can credibly rule out financial panics, it reduces the incentives of domestic savers to hold dollar assets for precautionary reasons, which in turn reduces the costs of borrowing in domestic currency. This makes it cheaper for the banks to borrow in domestic currency, and it can reduce the dollarization of their liabilities. In this sense, official holdings of foreign currency reserves play a catalytic role by encouraging virtuous behavior of local borrowers and by promoting financial stability also from an ex ante perspective.

Literature. Our research is related to several strands of literature. Following the crises of the 1990s, several authors have developed equilibrium models to explain the joint occurrence of financial and currency crises. Burnside, Eichenbaum, and Rebelo (2001b) and Corsetti, Pesenti, and Roubini (1999) emphasize the role of prospective deficits due to bailout guarantees. Chang and Velasco (2000, 2001) point to the role of maturity mismatches and illiquidity in the banking sector. Aghion, Bacchetta, and Banerjee (2001, 2004) focus on the interactions between the nominal exchange rate and firms' balance sheet in a model with price rigidities. We share with all these papers the emphasis on the self-fulfilling nature of these crises, although we focus on different economic mechanisms.

Closest to our work is the seminal paper by Krugman (1999), who emphasizes how the feedback between investment demand and real exchange rates can lead to multiple

[4] This does not mean that we think reserves can't be used in reality to dampen a depreciation or that we ignore that most emerging economies choose to intervene either by monetary policy or by currency interventions to limit exchange rate flexibility. Ilzetzki, Reinhart, and Rogoff (2017) argue convincingly that most emerging economies do. It just means that our hedging argument for reserve accumulation can be made independently of those ingredients.

equilibria when firms have dollar debt.[5] Dollar debt is not crucial to produce multiple equilibria in our environment, but it plays an important amplifying effect by making banks' balance sheets further exposed to pessimistic expectations.

A key innovation in our paper relative to this literature is that we endogenize debt denomination and show how risk premia can lead banks to endogenously choose currency positions that make multiple equilibria possible. The economic mechanisms that produce dollar-denominated liabilities in our setting are distinct from other explanations offered in the literature and, in particular, from Schneider and Tornell (2004), Burnside, Eichenbaum, and Rebelo (2001a) and Farhi and Tirole (2012), who emphasize the role of bailout guarantees.[6] In contrast, we emphasize the portfolio choices of domestic savers and how their demand for safety in equilibrium translates into deviations from uncovered interest rate parity, which incentivizes financial institutions to issue dollar debt. The view that dollarized liabilities are a response to risk premia is consistent with recent empirical work by Dalgic (2017), Salomao and Varela (2017), and Wiriadinata (2017).[7]

The feedback between risk and portfolio choices as a source of equilibrium multiplicity is shared by other papers, although in different contexts. Bacchetta, Tille, and Van Wincoop (2012) show that volatility in asset prices can be self-fulfilling when investors are risk averse. Heathcote and Perri (2015) and Ravn and Sterk (2017) study the feedback between unemployment risk and self-insurance motives of savers in models with nominal rigidities. See also Broner and Ventura (2016) for an application to cross-borders capital flows. To the best of our knowledge, we are the first to identify a mechanism of this sort in a macroeconomic model with a financial sector.

Our treatment of lending of last resort builds on Gertler and Kiyotaki (2015). In their environment, providing liquidity to the financial sector during a panic has ex ante benefits because it reduces the probability of future runs, and it is always optimal ex post because the government does not face borrowing constraints. Apart from working in a small open economy framework, the main innovation in our paper relative to their approach is that we explicitly formulate a game between the government and private investors, which embeds equilibrium in goods and asset markets. This allows us to analyze whether off-the-equilibrium-path promises to intervene in a "bad" equilibrium are credible and to discuss

[5] A recent paper by Céspedes, Chang, and Velasco (2017) uses similar ingredients to discuss non-conventional monetary policy in emerging economies.

[6] On the normative side, Caballero and Krishnamurthy (2003) suggest that dollar debt might be excessive relative to the social optimum because of pecuniary externalities.

[7] An earlier empirical literature provides additional evidence supporting this portfolio view of dollar debt in emerging markets. For instance, Ize and Levy-Yeyati (2003) and Levy-Yeyati (2006) show that indicators of inflation volatility have historically been an important predictor of dollar-denominated deposits in emerging markets. See also the evidence in Du, Pflueger, and Schreger (2017).

how limited fiscal capacity can interfere with lending of last resort policies. The only previous work we know of that discusses credibility issues in lending of last resort policies is Ennis and Keister (2009), who analyze deposit freezes in the Diamond and Dybvig (1983) model.

An important literature studies the role of reserves as insurance against various types of shocks (Caballero and Panageas, 2008; Durdu, Mendoza, and Terrones, 2009; Jeanne and Rancière, 2011; Bianchi, Hatchondo, and Martinez, 2012). We share with these papers a precautionary view on the accumulation of foreign reserves. But our focus on reserves as a means of help in fighting financial panics and our approach to modeling the official and financial sector lead to a distinct set of predictions.[8] In particular, our model can rationalize why reserves across countries are well explained by the size of the financial sector's total liabilities, as shown by Obstfeld, Shambaugh, and Taylor (2010). Moreover, our framework can explain why reserves seem to be underutilized by domestic authorities, as documented by Aizenman and Sun (2012). In our framework, a government that accumulates enough reserves can rule out financial panics, so reserves never need to be used in equilibrium.

Finally, our paper relates to recent research aimed at understanding the patterns of global capital flows and low interest rates in the world economy (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Jeanne, 2013; Mendoza, Quadrini, and Rios-Rull, 2009; Maggiori, 2017; Fahri and Maggiori, 2017). Our paper offers a fully fledged model of financial instability as a cause for increased accumulation of reserves by emerging economies and it identifies important differences between the private and the official sector demand for dollars (see Section 5.4).

Layout. Section 2 presents the model. We then move on to characterize the equilibria of the model, proceeding backward in time. Section 3 describes the continuation equilibria of the model from period 1 onward, taking the currency denomination of assets and liabilities as given. Section 4 studies the optimal portfolio choices of households and banks in the initial period. In Section 5 we introduce a government and study lending of last resort and the role of foreign currency reserves. Section 6 concludes.

2 Model

Consider a small open economy that lasts three periods, $t = 0, 1, 2$, populated by two groups of domestic agents, households and bankers, who trade with a large number of

[8] Models that focus more on equilibrium multiplicity are Hur and Kondo (2016) and Hernandez (2017), although in different contexts.

foreign investors.

There are two goods: a tradable and a nontradable good. There are two units of account: the domestic one and the foreign one. We will refer to the first as "pesos" and to the second as "dollars". The price of tradables in dollars is exogenously given by foreign monetary policy. To focus on a purely floating exchange rate regime, we assume that domestic monetary policy pursues a strict inflation target, keeping the domestic price index constant. This means that adjustments in the relative price of tradables versus nontradables lead to fluctuations in the nominal exchange rate. The model features flexible prices, but movements in the nominal exchange rate matter because agents trade financial claims in different currencies.

The bankers act as intermediaries: they hold all capital goods in the economy and issue liabilities denominated in pesos and dollars. Therefore, the price of capital goods and the exchange rate affect bankers' net worth and, due to collateral constraints, bankers' net worth affect investment in the economy. To allow for the endogenous determination of the price of capital goods, we assume an upward-sloping supply of new capital coming from firms producing capital goods subject to convex costs.

We now turn to a detailed description of the environment and to the definition of an equilibrium. Along the way, we make a number of simplifying assumptions. Their role is discussed in detail at the end of the section.

2.1 Agents and their decision problems

Households. Household preferences are represented by the utility function

$$\mathbb{E}\left[\sum_{t=0}^{2} \beta^t U(c_t)\right],$$

where c_t is the consumption aggregator

$$c_t = (c_t^T)^{\omega}(c_t^N)^{1-\omega},$$

and c_t^T and c_t^N are consumption of tradable and nontradable goods. The prices of tradable and nontradable goods, in pesos, are p_t^T and p_t^N.

Each period t, households supply a unit of labor inelastically at the wage w_t (in pesos), receive an endowment of non-tradable goods, e^N, and receive the profits of the firms producing capital goods, Π_t (also in pesos), which are described below. Households also trade risk-free one-period claims denominated in pesos and dollars, denoted by a_t and a_t^*.

The interest rates in pesos and dollars are i_t and i_t^*. The nominal exchange rate (pesos per dollar) is s_t. Accordingly, the household period t budget constraint is

$$p_t^T c_t^T + p_t^N c_t^N + \frac{1}{1+i_t} a_{t+1} + s_t \frac{1}{1+i_t^*} a_{t+1}^* \leq w_t + p_t^N e^N + \Pi_t + a_t + s_t a_t^*. \tag{1}$$

The households choose consumption and asset positions in order to maximize expected utility subject to the budget constraints (1) and the terminal conditions $a_3 = a_3^* = 0$.

Bankers. Bankers are agents who own and run banks. They are risk neutral and only consume tradable goods at date 2. On the asset side, banks hold capital k_t, which is used as input in the production of tradable goods,

$$y_t^T = k_t^\alpha l_t^{1-\alpha}, \tag{2}$$

and earns the rental rate

$$r_t = p_t^T \alpha k_t^{\alpha-1}, \tag{3}$$

since labor supply is 1 and it is only employed in the production of tradables. The peso price of capital is Q_t. Capital does not depreciate in periods 0 and 1 and fully depreciates after production at $t = 2$.

On the liability side, banks enter period t with debt in pesos and in dollars, respectively, b_t and b_t^*. The banks' net worth in pesos is then

$$n_t = (Q_t + r_t) k_t - b_t - s_t b_t^*, \tag{4}$$

and the banks' budget constraint is

$$Q_t k_{t+1} = n_t + \frac{1}{1+i_t} b_{t+1} + s_t \frac{1}{1+i_t^*} b_{t+1}^*, \tag{5}$$

in $t = 0, 1$, as banks use their net worth and new borrowing to purchase the capital good.

We assume that banks face limits in their ability to raise external finance. Namely, banks have to satisfy the following collateral constraint, which requires total end-of-period liabilities to be bounded by a fraction of the capital held by the bank:

$$\frac{1}{1+i_t} b_{t+1} + s_t \frac{1}{1+i_t^*} b_{t+1}^* \leq \theta Q_t k_{t+1}, \tag{6}$$

where θ is a parameter in $[0, 1]$.

The bankers' problem is to choose $\{k_{t+1}, b_{t+1}, b^*_{t+1}\}$ to maximize the expected value of n_2/p^T_2, subject to the law of motion for net worth (4), the budget constraint (5), the collateral constraint (6), and the terminal condition $b_3 = b^*_3 = 0$.

Capital goods production. Competitive firms owned by the households transform tradable goods into capital at date 0 and 1. In order to produce $\iota_t \geq 0$ units of capital, these firms require $G(\iota_t)$ units of tradable goods. The function $G(\iota_t)$ takes the form

$$G(\iota_t) = \phi_0 \iota_t + \frac{\phi_1}{1+\eta} \iota_t^{1+\eta}.$$

The profits of the capital producing firms are

$$\Pi_t = \max_{\iota_t \geq 0} Q_t \iota_t - p^T_t G(\iota_t). \tag{7}$$

Market clearing in the capital goods market in periods $t = 0, 1$ is given by

$$k_{t+1} = k_t + \iota_t, \tag{8}$$

as the capital inherited from past periods plus the newly produced capital are accumulated by banks for future production. The capital goods market is not active at date 2 because all capital fully depreciates.

Foreign investors. Foreign investors are risk neutral, and consume only tradable goods. Their discount factor is denoted by β. We assume that foreign investors can only buy claims in dollars. Therefore, equilibrium in the market for domestic claims requires $a_t = b_t$.

Let p^{T*}_t denote the price of tradable goods in dollars, which is exogenous to the small open economy. The law of one price implies that

$$p^T_t = s_t p^{T*}_t. \tag{9}$$

This price p^{T*}_t is normalized to 1 at date 0, and it is subject to random shocks at date 1 and 2. Specifically, at $t = 1$ the permanent shock ε is realized and

$$p^{T*}_1 = p^{T*}_2 = \varepsilon.$$

The variable ε satisfies $\mathbb{E}[1/\varepsilon] = 1$.

The interest rate in dollars is pinned down by the Euler equation of foreign investors

$$1 = (1 + i_t^*)\beta \mathbb{E}_t \left[\frac{p_t^{T*}}{p_{t+1}^{T*}} \right],$$

which yields $1 + i_t^* = 1/\beta$ given the assumed properties of p_t^{T*}.

Monetary regime and the exchange rate. Our economy features flexible prices, so the only role of monetary policy is to determine nominal prices and the nominal exchange rate. These prices matter for the real allocation because assets and liabilities are denominated in different currencies, so fluctuations in the nominal exchange rate reallocate wealth across agents.

We assume that the monetary authority is only concerned with price stability. Given consumers' preferences, the price index is

$$p_t = \omega^{-\omega} (1 - \omega)^{-(1-\omega)} (p_t^T)^{\omega} (p_t^N)^{1-\omega}. \tag{10}$$

We assume that the monetary authority successfully targets a constant price index

$$p_t = \bar{p} = \omega^{-\omega}(1 - \omega)^{-(1-\omega)}. \tag{11}$$

Combining this rule with the consumer price index (CPI) definition (10) and the law of one price (9), we obtain the nominal exchange rate

$$s_t = \frac{1}{p_t^{T*}} \left(\frac{p_t^T}{p_t^N} \right)^{1-\omega}. \tag{12}$$

Two forces drive the nominal exchange rate: nominal fluctuations in the price level in the rest of the world and movements in the relative price of tradables and nontradables at home. Both forces are relevant for our analysis.

2.2 Equilibrium

There are two sources of uncertainty in this economy, both realized at date 1. The nominal shock ε, and a sunspot variable ζ uniformly distributed in $[0, 1]$. The sunspot determines which equilibrium is in play at date 1 when multiple equilibria are possible. We are leaving implicit in our notation that all variables dated 1 and 2 are functions of the state of the world $(\zeta, \varepsilon, k_1, b_1, b_1^*, a_1, a_1^*)$.

A competitive equilibrium is a vector of prices $\{Q_t, i_t, i_t^*, r_t, w_t, p_t^T, p_t^N, s_t\}$, households' choices $\{a_{t+1}, a_{t+1}^*, c_t^T, c_t^N\}$, bankers' portfolio choices $\{k_{t+1}, b_{t+1}, b_{t+1}^*\}$, and choices of capital good producers $\{\iota_t\}$, such that: **(i)** the choices of households, bankers, capital good producers, and foreign investors are individually optimal; **(ii)** markets clear; and **(iii)** law of one price holds; **(iv)** and the price index p_t is constant.

2.3 Discussion of assumptions

Let us briefly discuss the main simplifying assumptions made in the model.

First, we are assuming that tradables are produced with capital and labor while nontradables are in fixed endowment. This assumption simplifies the analysis because we don't have to determine how labor is allocated among the two sectors, and we only need to keep track of capital accumulation in one sector. Moreover, it captures in a stylized way the fact that the tradable sector is typically more capital intensive than the nontradable sector in emerging markets.

Second, foreign investors in the model cannot lend to domestic agents in pesos, an assumption that plays an important role in our analysis, as we will discuss in Section 4.3. Our results, however, do not require this stark form of segmentation, and they would go through as long as the supply of peso claims by foreigners is not infinitely elastic. Ruling out foreign investors' participation in the peso debt market is a useful simplification.

Third, we are representing monetary policy purely as a choice of numeraire, and we are assuming the monetary authority can commit to perfect price stability. This is a simple way to model a floating exchange rate regime, where nominal exchange rate volatility is not driven by the inflationary choices of the central bank. As we shall see, our main mechanism is based on the relation between the country's real wealth and the real exchange rate, so it is useful to mute other policy-driven channels of exchange rate instability. This assumption allows us to leave aside connections between banking crises and currency crises driven by inflationary concerns as in, for example, Burnside, Eichenbaum, and Rebelo (2001b), and focus on a mechanism based on real depreciations.

2.4 Road map

In the following two sections, we analyze the model in two steps, moving backward in time. First, we analyze how the equilibrium in the last two periods is determined, taking as given $(\varepsilon, k_1, b_1, b_1^*, a_1, a_1^*)$. We call this a *continuation equilibrium*, and we show that for a subset of initial conditions, there can be multiple continuation equilibria in the model.

This part of the analysis shows that our model captures many insights of so-called third-generation models of currency crises previously studied in the literature. The second, and most novel, step of our analysis consists of going back to date 0 and studying the currency denomination of assets and liabilities in the economy. Here we will characterize the feedback loop between the precautionary motives of domestic savers and the risk of future financial crises, which, through general equilibrium forces, can lead to a dollarization of the banks' liabilities at date 0.

3 Continuation equilibria

In this section we characterize the behavior of the economy from date 1 onward. Our approach consists of using a subset of equilibrium conditions to express all the endogenous variables of the model as a function of the price of capital in terms of tradables

$$q_1 \equiv \frac{Q_1}{p_1^T}.$$

This allows us to characterize the continuation equilibria of the model using a simple diagram that plots the demand and supply of capital against q_1.

3.1 The supply of capital goods

From the optimization problem of capital-producing firms (7) we obtain the supply of capital goods at date 2, which takes the form

$$K_S(q_1) = k_1 + \left(\frac{q_1 - \phi_0}{\phi_1}\right)^{1/\eta}, \tag{13}$$

if $q_1 \geq \phi_0$. If $q_1 < \phi_0$, the solution is $\iota_1 = 0$ and the supply of capital goods is just k_1.

3.2 The exchange rate

Before deriving the demand for capital, we need to obtain a relation between q_1 and the equilibrium exchange rate.

The following lemma derives useful properties of a continuation equilibrium.

Lemma 1. *Any continuation equilibrium must satisfy the following conditions:*

i. Consumption is constant over time, $c_1 = c_2$;

ii. The relative price of tradable and nontradable goods is constant over time: $p_1^N/p_1^T = p_2^N/p_2^T$;

iii. The domestic real interest rate is: $(1+i_1)p_1/p_2 = 1/\beta$.

The logic of this lemma is simple. At date 1, all uncertainty is revealed, and households want to smooth consumption over time. Tradable consumption is perfectly smoothed by trading with foreign investors. Nontradable consumption is constant because the nontradable endowment is constant. So the relative price of tradables and non-tradables must be constant. The expression for the domestic real interest rate comes from the intertemporal Euler equation of households for local currency bonds and constant consumption.

Using the previous lemma and the household intertemporal budget constraint, we can write consumption expenditure at date 1 as

$$p_1 c_1 = \frac{1}{1+\beta}\left(w_1 + \beta w_2 + p_1^N e^N + \beta p_2^N e^N + \Pi_1 + a_1 + s_1 a_1^*\right).$$

Since consumers spend a fraction $1-\omega$ of their total expenditure on nontradables, the market clearing condition for nontradable goods is

$$(1-\omega)\frac{p_1 c_1}{p_1^N} = e^N.$$

Combining the last two conditions, after some manipulation, gives[9]

$$\frac{1-\omega}{1+\beta}\left\{\frac{p_1^T}{p_1^N}\left[(1-\alpha)k_1^\alpha + \beta(1-\alpha)k_2^\alpha + \pi(q_1) + \frac{1}{\varepsilon}a_1^*\right] + \left(\frac{p_1^T}{p_1^N}\right)^\omega a_1\right\} = \omega e^N, \tag{14}$$

where the function $\pi(q_1)$ gives the profits of the capital producers (in tradables) as a function of q_1 and is obtained from their profit maximization problem.

Equation (14) defines an implicit relation between p_1^N/p_2^T and (k_2, q_1). More capital invested in the tradable sector or a higher price of capital leads to higher wages and profits for households. This shifts up the demand for non-tradables and leads to a real appreciation (a lower value of p_1^T/p_1^N). This is just a version of the Balassa-Samuelson effect. Equation (12) translates this result in terms of the nominal exchange rate.

Using the supply of capital (13), we can further express k_2 as a function of q_1. This allows us to express all the variables in a continuation equilibrium, and, especially, the exchange rate, as a function of the price of capital q_1.

[9] Equate real wages to the marginal product of labor and use the law of one price to substitute $s_1 = (1/\varepsilon)p_1^T$. To substitute for $1/p_1^N$ in front of a_1, use $(p_1^T)^\omega(p_1^N)^{1-\omega} = 1$, which gives $1/p_1^N = (p_1^T/p_1^N)^\omega$.

Lemma 2. *Given the initial conditions* $(a_1, a_1^*, b_1, b_1^*, k_1)$, *with* $a_1^* \geq 0$, *the shock* ε, *and a candidate value of the capital price* q_1, *there exists a unique vector of prices and quantities*

$$(i_1, i_1^*, s_1, s_2, p_1^T, p_2^T, p_1^N, p_2^N, c_1, c_2)$$

consistent with a continuation equilibrium. Let

$$s_1 = \mathcal{S}(q_1, \varepsilon) \tag{15}$$

denote the relation between the capital price q_1 *and the nominal exchange rate. The function* $\mathcal{S}$ *is decreasing in* q_1.

3.3 The demand for capital goods

The demand for capital goods can be obtained using the optimality conditions of the bankers. The rate of return to tradable capital is r_2/Q_1 because buying capital costs Q_1 at $t = 1$, earns the dividend r_2 at $t = 2$, and then capital fully depreciates. Comparing this rate of return to the interest rate, two cases are possible in equilibrium:

1. *Unconstrained banks.* The marginal gain from borrowing an extra peso and investing it in capital is zero, and the collateral constraint is slack:[10]

$$\frac{r_2}{Q_1} = 1 + i_1, \quad Q_1 k_2 - n_1 \leq \theta Q_1 k_2.$$

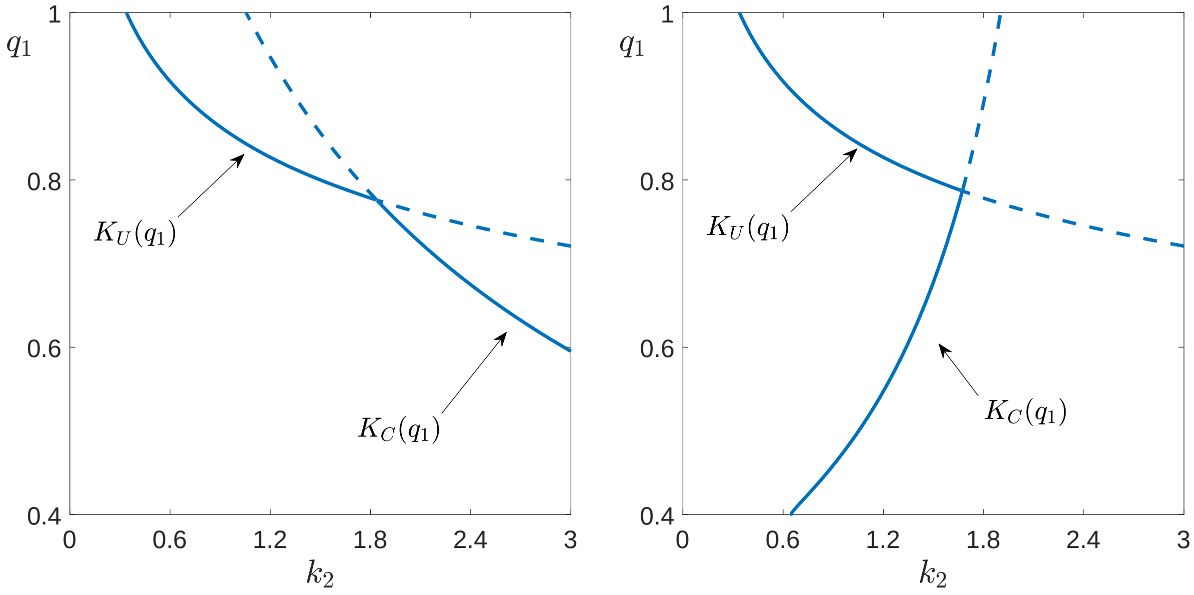
2. *Constrained banks.* The marginal gain from borrowing an extra peso and investing it in capital is positive, and the collateral constraint is binding:

$$\frac{r_2}{Q_1} > 1 + i_1, \quad Q_1 k_2 - n_1 = \theta Q_1 k_2.$$

The conditions above can be used to derive the demand schedule for capital goods. Substituting the rental rate from (3) and $1 + i_1 = 1/\beta$, we get the unconstrained demand for capital:

$$K_U(q_1) = \left(\frac{\alpha\beta}{q_1}\right)^{\frac{1}{1-\alpha}}. \tag{16}$$

[10]Here we use the budget constraint (5) to substitute $\frac{b_1}{1+i_1} + s_1 \frac{b_1^*}{1+i_1^*}$ on the left-hand side of the collateral constraint (6). Moreover, there is no residual uncertainty from $t = 1$ onward, which implies that banks are indifferent between borrowing in pesos or in dollars.



(a) Downward-sloping demand

(b) Nonmonotone demand

Notes: The parameters used in the example are: $\omega = 0.50$, $\alpha = 0.85$, $\beta = 1.00$, $e^N = 0.70$, $\theta = 0.90$, $\phi_0 = 0.40$, $\phi_1 = 0.20$, $\eta = 3.00$. *We set* $\varepsilon = 1.00$ *and* $k_1 = 0.30$ *in both examples. In the left panel the initial conditions are* $b_1 = a_1 = 0.05$, $a_1^* = 0.71$, *and* $b_1^* = 0.36$. *In the right panel they are* $b_1 = a_1 = 0.05$, $a_1^* = 0.71$, *and* $b_1^* = 0.36$.

Figure 1: The demand for capital goods

In the constrained case, we can rewrite the binding collateral constraint in terms of tradables and obtain the constrained demand for capital:

$$K_C(q_1) = \frac{1}{(1-\theta)\,q_1}\left[q_1 k_1 + \alpha k_1^{\alpha} - \frac{b_1}{\varepsilon \mathcal{S}(q_1,\varepsilon)} - \frac{b_1^*}{\varepsilon}\right], \tag{17}$$

where debt in pesos b_1 is converted into tradables by dividing by the peso price of tradables $p_1^T = \varepsilon \mathcal{S}(q_1, \varepsilon)$ (from the law of one price and $p_1^{T*} = \varepsilon$) and debt in dollars is converted into tradables by dividing by $p_1^{T*} = \varepsilon$. The demand for capital is given by the lower value between the constrained and the unconstrained demand at each q_1:

$$K_D(q_1) = \min\{K_U(q_1), K_C(q_1)\}. \tag{18}$$

The unconstrained portion of the demand curve is always downward sloping because of substitution effects. When banks are constrained, however, changes in q_1 might also have income effects, and the demand curve might have upward-sloping regions. We will return shortly to the determinants of this slope. For now we just show two numerical examples in Figure 1, one in which the demand curve is downward sloping everywhere, in panel (a), and one in which the constrained portion of the curve is upward sloping for some values of q_1, in panel (b).

3.4 Equilibrium in the capital goods market

We can now combine the supply and demand curves just derived to find the equilibrium price q_1. First, we establish a sufficient condition for the existence of a continuation equilibrium.

Proposition 1. *Given ε, if the following inequalities are satisfied:*

$$\alpha k_1^{\alpha-1} > \phi_0, \quad \alpha k_1^{\alpha} + \theta\phi_0 k_1 > \frac{1}{\varepsilon S(\phi_0, \varepsilon)} b_1 + \frac{1}{\varepsilon} b_1^*, \tag{A1}$$

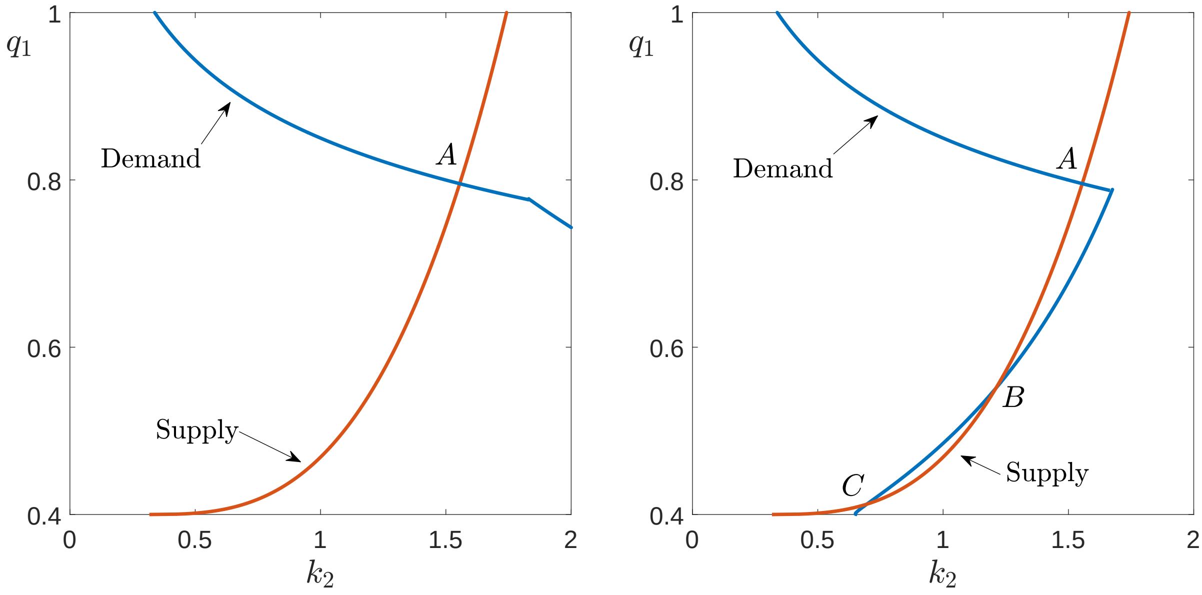
there exists a continuation equilibrium with $q_1 > \phi_0$. There is at most one equilibrium in which banks are unconstrained.

From now on, we focus on economies that satisfy (A1) and restrict attention to continuation equilibria with $q_1 > \phi_0$. The main advantage of these restrictions is that we do not need to worry about the possibility that banks have negative net worth, and so we don't need to specify how banks' bankruptcy is resolved for bondholders.[11]

In Figure 2 we plot the demand and supply for two numerical examples. In panel (a) the equilibrium is unique. In panel (b), instead, there are three equilibria, given by points A, B, and C. At equilibrium A, banks are unconstrained. Because the unconstrained demand curve is downward sloping, there can be at most one equilibrium of this type. At equilibria B and C, instead, the collateral constraint is binding. Equilibrium B can be ruled out on stability grounds, so we focus on the two stable equilibria A and C.

The equilibrium multiplicity here can be interpreted in terms of a standard coordination problem between atomistic lenders (both domestic households and foreign investors). In the good equilibrium, each lender expects the others to extend credit to the banks. As a result, the lender anticipates that the banks will be able to invest, and the price of capital will be high. The resulting high valuation of the banks' collateral induces the lender to extend credit to the banks. In the bad equilibrium, instead, each lender expects the others to extend little credit to banks, which induces expectations of low investment, low asset prices, and low collateral values. Given these expectations, it is then rational for an individual lender to offer little credit to the banks.

[11] Of course, individual banks' bankruptcies are commonplace in financial crises. However, our stylized model captures the entire financial system with a single representative bank, and it thus seems reasonable to model a crisis as a severe reduction in the total net worth of the financial sector rather than a complete depletion of its equity.



(a) Unique continuation equilibrium (b) Multiple continuation equilibria

Notes: Parameters and initial conditions are the same as in Figure 1.

Figure 2: Capital market equilibrium

If we interpret a financial crisis as a continuation equilibrium with constrained banks, such as point C in panel (b) of Figure 2, we obtain a number of predictions about the behavior of consumption, investment, the exchange rate and the current account.

Proposition 2. *If multiple continuation equilibria are possible, and we compare a low q_1 to a high q_1 equilibrium, we obtain the following predictions for the former:*

i. *Investment and consumption are lower;*

ii. *The current account balance is higher;*

iii. *The real and nominal exchange rates are more depreciated.*

The improvement in the current account shows that the domestic financial crisis is associated with a capital flight from the entire country. The capital flight has a double nature: the contraction in investment is driven by the binding collateral constraints of the banks, while the contraction in consumption is driven by the reduction in the country's wealth due to lower future wages. The recent literature includes papers that emphasize both uncertainty about future income growth (Aguiar and Gopinath, 2007) and binding financial constraints (Mendoza, 2010) as sources of fluctuations in emerging markets. Here both channels are operative because a crisis in our three-period model acts like a permanent shock to households' future income.[12] An interesting observation here is that even though some agents in

[12]As documented by Cerra and Saxena (2008), financial crises are historically associated with permanent

the economy are not forced to borrow less from the rest of the world, spillovers from the financially constrained agents induce them to move in the same direction.[13]

3.5 Sources of financial instability

The fact that the demand curve is locally upward sloping is crucial for the possibility of obtaining multiple equilibria. So let us now go back to the slope of the demand curve. Differentiating the constrained demand curve (17) yields

$$K_C'(q_1) = \frac{b_1/(\varepsilon\mathcal{S}(q_1,\varepsilon)) + b_1^*/\varepsilon - \alpha k_1^\alpha}{(1-\theta)q_1^2} + \frac{b_1 \partial \mathcal{S}(q_1,\varepsilon)/\partial q_1}{(1-\theta)q_1\varepsilon\mathcal{S}(q_1,\varepsilon)^2}. \tag{19}$$

The first term on the right-hand side of equation (19) shows that leverage makes the curve upward sloping. Namely, when total debt is large enough, the expression at the numerator is positive.[14] The second term shows that borrowing in domestic currency can mitigate the effects of leverage because the value of peso obligations depreciates exactly when the price of capital q_1 goes down (recall that $\partial\mathcal{S}(q_1,\varepsilon)/\partial q_1 < 0$ from Lemma 2), thus providing hedging against a reduction in asset prices. Note that this is somewhat distinct from the *currency mismatch* channel emphasized, for instance, by Krugman (1999) and Aghion, Bacchetta, and Banerjee (2004). Our banks have revenues in tradable goods, so matching the balance sheet would require the issuance of dollar liabilities. Yet, peso debt is a hedge for the banks because it requires lower payments when the market value of their assets falls.

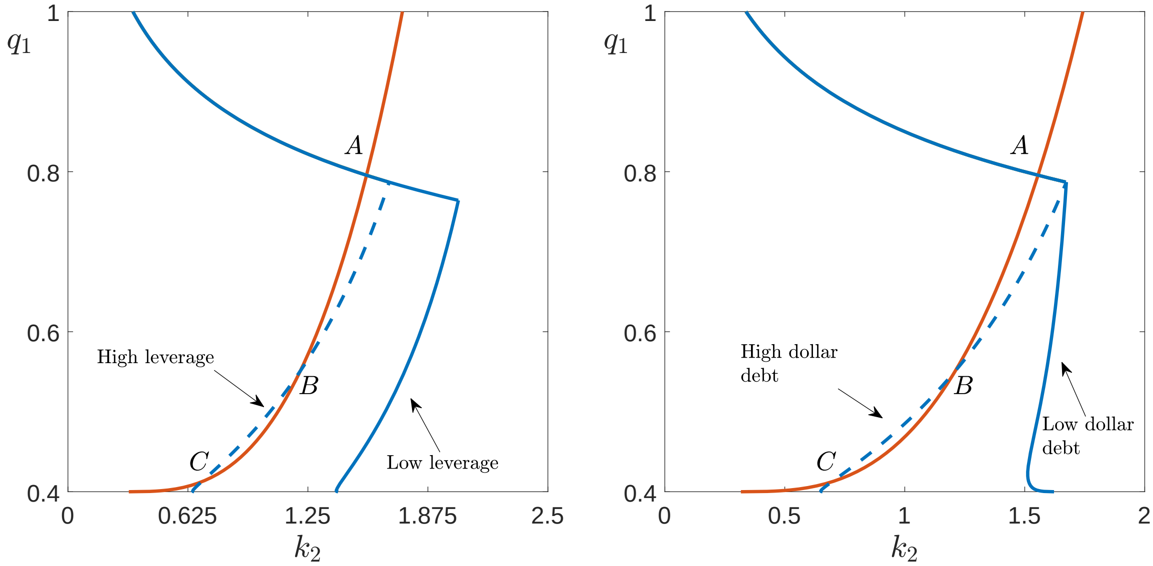
Figure 3 shows by numerical examples the role of the banks' balance sheet in determining the slope of the demand curve and the possibility of multiple equilibria. In panel (a) we consider an increase in banks' leverage, holding constant their currency exposure. This is achieved by increasing b_1^*. Ceteris paribus, an increase in leverage raises the sensitivity of net worth to asset prices, increasing the elasticity of the demand function in the constrained region. Comparing the solid and dotted line, we can see that an increase in banks' leverage makes the economy more prone to multiple equilibria.

A similar result is obtained when considering a change in the currency composition

reductions in income. An infinite horizon extension of our model would miss this feature, thus muting the response of consumption and the exchange rate to a tightening of the collateral constraint. However, it would not be complicated to further extend that framework to incorporate these empirically relevant effects. See Queralto (2014) for such an example.

[13]The fact that the unconstrained agents here are identified with the household sector is just because of specific modeling assumptions. It would be easy to extend the model to a case where constrained and unconstrained agents are present both in the household and in the business sector.

[14]The effect of leverage on the slope of the capital demand curve has been remarked in closed economy financial accelerator models (e.g., Lorenzoni (2008)), and has been used to generate equilibrium multiplicity in Gai, Kapadia, Millard, and Perez (2008) and Gertler and Kiyotaki (2015).



(a) Leverage

(b) Currency composition

Notes: Parameters and initial conditions are the same as in the right panel of Figure 1. For the "Low leverage" schedule, we set $b_1^* = 0.33$, and for the "Low dollar debt" schedule, we set $b_1 = 0.23$ and $b_1^* = 0.18$.

Figure 3: The role of leverage and the currency composition of debt

of debt, holding total debt constant. Panel (b) of Figure 3 shows how the demand for capital changes when we increase dollar debt b_1^* and offset such increase by a corresponding reduction in peso debt b_1. We can verify from equation (19) that such a change makes the constrained portion of the demand schedule steeper, raising in this fashion the potential for multiple equilibria.

The economy can thus exhibit financial crises characterized by low asset prices, a depreciated exchange rate, and capital flights, and these are more likely to arise when the financial sector is more levered and has more dollar debt. These debt positions, however, are determined endogenously at date 0. So we next turn to study their determination. Before continuing, though, we must adopt a rule for selecting among continuation equilibria when we have more than one, as agents at date 0 need to form expectations over future outcomes. First, we restrict attention to stable continuation equilibria in the tâtonnement sense. Moreover, we focus on economies with at most two stable continuation equilibria. As the equilibria are ranked in terms of welfare, we refer to the one with high asset prices as the "good" equilibrium and to the other as the "bad" equilibrium—see, for example, points A and C in panel (b) of Figure 2. When multiple equilibria are possible, the sunspot ζ selects the bad equilibrium with probability μ.

4 Endogenous dollarization

We now go back to date 0 and describe two classes of equilibria that can arise. These equilibria differ in the currency denomination of households' savings and banks' liabilities, and in the risk of financial panics. We show, by numerical examples, that these two types of equilibria can coexist for the same initial conditions.

We start by characterizing equilibria in which the banks' collateral constraint is always slack and the economy is not exposed to financial panics at date 1. These equilibria are supported by the households' incentive to save in pesos. Because households don't expect a crisis in the future, they prefer peso bonds as they insure them against fluctuations in the price of foreign tradables. The households' demand for peso assets allows the banks to borrow in pesos, thus making their balance sheet safer. Hence, the financial sector is not exposed to runs at $t = 1$. We call this a "nondollarized" equilibrium.

We then describe equilibria in which financial panics can arise with positive probability at date 1. These equilibria are supported by the households' incentive to save in dollars. When households are afraid of future banking panics, they prefer dollar bonds because they provide insurance against a crisis at date 1. When sufficiently strong, this precautionary motive is met in equilibrium with a dollarization of the banks' liabilities, which is what exposes the financial sector to runs at date 1. We call this a "dollarized" equilibrium.

4.1 Nondollarized equilibrium

We simplify the analysis further by assuming that at date 0, capital good producing firms are not operative, so market clearing requires $k_1 = k_0$. We can then use equations (4) and (5), along with market clearing, to write the banks' budget constraint as

$$\frac{b_1}{1+i_0} + s_0 \frac{b_1^*}{1+i_0^*} = b_0 + s_0 b_0^* - r_0 k_0. \tag{20}$$

The total liabilities of the financial sector are given, and the only choice of the bankers regards the currency composition of their debt. The households at $t = 0$ decide how much to consume and save, and in which currency to denominate their savings.

We now state a result that characterizes our first class of equilibria.

Proposition 3 (Nondollarized equilibrium)**.** *Suppose that there is an equilibrium in which the collateral constraint of the banks is slack in period* 0 *and is slack almost surely in period* 1*. This equilibrium has the following properties:*

i. *There is a unique continuation equilibrium from $t = 1$ onward, with (k_2, q_1) solving*

$$k_2 = \left(\frac{\alpha\beta}{q_1}\right)^{\frac{1}{1-\alpha}} \qquad k_2 = k_1 + \left(\frac{q_1 - \phi_0}{\phi_1}\right)^{\frac{1}{\eta}}; \tag{21}$$

ii. *The prices of tradables and nontradables are constant over time. The domestic real interest rate is constant over time and equal to $1 + i = 1/\beta$;*

iii. *Household consumption is constant over time and equal to*

$$\bar{c} = \frac{1}{1+\beta+\beta^2}\left[p^T\left((1+\beta)(1-\alpha)k_0^\alpha + \beta^2(1-\alpha)k_2^\alpha + \beta\pi(q_1) + a_0^*\right) + a_0\right] + p^N e^N.$$

At $t = 0$, households set $a_1^* = 0$, and save only in pesos;

iv. *The banks' debt levels in pesos and dollars at $t = 1$ are*

$$b_1 = (1+i)\left(p^T(1-\alpha)k_0^\alpha + p^N e^N + a_0 + p^T a_0^* - \bar{c}\right),$$
$$b_1^* = (1+i)\left((b_0 - \beta b_1)/p^T + b_0^* - \alpha k_0^\alpha\right).$$

Because the banks are unconstrained at date 1, the equilibrium in the capital market is unique, with the quantity and the price of capital independent of ε. As a result, the wages and profits that households obtain in periods 1 and 2 are nonstochastic. The households can then achieve perfect consumption smoothing by setting $a_1^* = 0$: in this way, their lifetime wealth is nonstochastic, and they can consume $\bar{c}$ in every period. Because consumption is constant over time, we obtain that the domestic real interest rate and the relative prices of tradables and nontradables are also constant. Banks' debt in pesos and dollars in (*iv*) is then obtained from the market clearing condition $a_1 = b_1$ and from the banks' budget constraint (20).

In a nondollarized equilibrium, households do not save or borrow in dollars at date 0. To understand this property, let us characterize their portfolio problem in general. In any equilibrium, the households' optimality conditions for dollar and peso bonds give rise to the standard asset pricing relation

$$\mathbb{E}_0\left[(1+i_0^*)\frac{s_1}{s_0}\right] = 1 + i_0 - \mathrm{Cov}_0\left[(1+i_0^*)\frac{s_1}{s_0}, \frac{U'(c_1)}{U'(c_0)}\right]. \tag{22}$$

The return on peso bonds is always safe for domestic households because of the assumption of a stable price index in pesos. The return on dollar bonds, instead, is stochastic and equal to $(1+i_0^*)(s_1/s_0)$. Equation (22) says that households require a positive or negative premium on dollar bonds depending on whether their return covaries negatively or positively with the marginal utility at $t = 1$.

In the nondollarized equilibrium the dollar return at date 1 is $1/(\varepsilon\beta)$ and the households' nonfinancial income is constant and independent of ε.[15] Setting $a_1^* > 0$ would then be a pure bet for households, because it would expose their consumption to the ε shock. Equation (22) then implies that households would save in dollars only if they expected an excess return, $\mathbb{E}[(1+i_0^*)(s_1/s_0)] > (1+i_0)$. This, however, is not possible in the nondollarized equilibrium. To understand why, note that an asset pricing condition similar to (22) can be derived from the banks' optimality conditions, giving

$$\mathbb{E}_0\left[(1+i_0^*)\frac{s_1}{s_0}\right] = 1 + i_0 - \mathrm{Cov}_0\left[(1+i_1^*)\frac{s_1}{s_0}, \frac{\lambda_1}{\mathbb{E}_0[\lambda_1]}\right], \tag{23}$$

where

$$\lambda_1 = \frac{1}{p_2^T}\frac{r_2 - \theta Q_1/\beta}{(1-\theta)Q_1} \tag{24}$$

is the banks' marginal value of wealth at date $t = 1$.[16] In any equilibrium, equations (22) and (23) must both hold. Because the collateral constraint of the banks does not bind at $t = 1$, their marginal value of wealth is constant, implying that $\mathbb{E}[(1+i_0^*)(s_1/s_0)] = (1+i_0)$ in equilibrium. But at those prices, households' optimal choice is to set $a_1^* = 0$.

Importantly, the households' choice to save in pesos is a stabilizing force. From the analysis of the continuation equilibrium, we know that crises at date 1 are more likely when banks have dollar debt. Because the leverage of banks is fixed by equation (20), the fact that households are willing to save in pesos minimizes the dollar debt of the financial sector and the risk of a bank panic at date 1.

4.2 Dollarized equilibria

We now ask whether the model can feature equilibria in which banking panics are possible at date 1, as in the example of Figure 2, panel (b).

We start by studying the portfolio choices of the households at $t = 0$, which are still determined by the asset pricing condition (22). The main difference with the case analyzed earlier is that the possibility of financial panics at date 1 influences the joint distribution

[15] Wages and profits are functions of k_2 and q_1, which are independent of ε.

[16] A unit of net worth in pesos can be levered to purchase $k_2 = 1/((1-\theta)Q_1)$ units of capital at $t = 1$, as θQ_1 can be borrowed per unit of capital. After paying the interest $1/\beta$ on the borrowed amount $\theta Q_1 k_2$, the return obtained at $t = 2$ is $r_2 k_2 - \theta Q_1 k_2 = (r_2 - \theta Q_1/\beta)/((1-\theta)Q_1)$, which, converted in tradables, yields the expression above.

of realized returns and households' consumption. Indeed, the $t = 1$ returns on bonds denominated in dollars are now

$$(1+i_0^*)\frac{s_1}{s_0} = \frac{\left(\frac{p_1^T}{p_1^N}\right)^{1-\omega}}{\varepsilon\beta\left(\frac{p_0^T}{p_0^N}\right)^{1-\omega}}. \tag{25}$$

Differently from before, there is now an endogenous component to these returns, which is due to changes in the relative price of tradables and nontradables between date 0 and 1.

To understand how this component influences the portfolio choices of the households, consider the case in which $\sigma_\varepsilon \to 0$, so that uncertainty arises only because of the sunspot: with probability $(1-\mu)$ the economy will be in the good continuation equilibrium, and with probability μ in the bad continuation equilibrium. From Proposition 2, we know that the exchange rate is more appreciated in the good than in the bad equilibrium. Therefore, by equation (25) we can see that bonds denominated in dollars have higher returns in the bad continuation equilibrium. This property makes these assets a good hedge for the households at date 0 because they pay more in the low consumption state.

Thus, the risk of a future crisis generates a motive for the households to save in dollars. When sufficiently strong, this incentive might lead to little peso savings in the economy and might effectively push banks to issue dollar debt in order to finance their operations. Whether this occurs in equilibrium, however, also depends on the behavior of the banks.

Differently from the case analyzed in the previous section, banks no longer act as risk-neutral because their marginal value of wealth at date 1 depends on the sunspot shock.[17] A unit of net worth at date 1 is worth more when the collateral constraint binds than when it is slack because in the former case it allows the banks to relax their financial constraint. Because of this property, banks now have an incentive to issue peso debt because it requires lower payments in states of the world in which they are constrained (see equation (23)).

In equilibrium, the currency denomination of assets and liabilities balances the precautionary motive of the households to save in dollars with the one of the banks to borrow in pesos. We now present a numerical example in which the households' precautionary motive dominates, and the economy has a dollarized financial sector. Importantly, as the example shows, a dollarized equilibrium can coexist with a nondollarized one and thus be self-fulfilling. Appendix B reports the details of the computation.

Table 1 reports several statistics of interest across the two equilibria. In the table, $\tilde{c}_1$ and

[17]See Aiyagari and Gertler (1999), Mendoza and Smith (2006) and Rampini and Viswanathan (2010) for a discussion of the asset pricing and risk management implications of models with collateral constraints.

$\tilde{s}_1$ are the consumption and the nominal exchange rate in logs at date 1, and $\tilde{w}_1$ is the nonfinancial present value of households' income at date 1, also in logs. First, we can verify that the nondollarized equilibrium conforms with Proposition 3. In the nondollarized equilibrium, households save only in pesos ($a_1^* = 0.00, a_1 = 0.70$). Banks absorb all the desired pesos savings of the households and issue dollar bonds to finance the rest of their debt. In this example, households' savings are more than enough to cover banks' borrowing needs, and as a result, the domestic financial sector accumulates foreign reserves ($b_1^* = -0.28$). Given this balance sheet structure, the banks are not exposed to runs at date 1, households' income and consumption have zero variance, and the nominal exchange rate fluctuates in period 1 only because of variation in the price of foreign tradables. The uncovered interest parity condition holds.

Table 1: Nondollarized and dollarized equilibria: a numerical illustration

	Nondollarized	Dollarized
a_1, b_1	0.703	0.050
a_1^*	0.000	0.710
b_1^*	-0.280	0.360
Stdev$(\tilde{w}_1)$	0.000	0.043
Stdev$(\tilde{s}_1)$	0.005	0.016
Corr$(\tilde{w}_1, \tilde{s}_1)$	0.000	-0.969
Stdev$(\tilde{c}_1)$	0.000	0.015
$\mathbb{E}[(1+i_0^*)(s_1/s_0)]$	1.000	0.941
$(1+i_0)$	1.000	1.034

Notes: The parameters used in the example are: $\omega = 0.50$, $\alpha = 0.85$, $\beta = 1.00$, $e^N = 0.70$, $\theta = 0.90$, $\phi_0 = 0.40$, $\phi_1 = 0.20$, $\eta = 3.00$, $\sigma_\varepsilon = 0.005$, $\mu = 0.10$. *The initial conditions are* $k_0 = 0.30$, $a_0 = b_0 = 0.00$, $a_0^* = 1.29$, $b_0^* = 0.71$. *See Appendix B for additional details.*

In the dollarized equilibrium, households save mostly in dollars ($a_1^* = 0.71, a_1 = 0.05$). Banks have little access to pesos, and they need to finance their operations at date 0 by issuing dollar debt. This structure of the banks' balance sheet exposes them to runs at date 1, and it endogenously generates risk in the economy. At date 1, households' nonfinancial income is exposed to the realization of the sunspot and it is negatively correlated with the exchange rate, because the exchange rate depreciates in a crisis. These hedging properties justify the households' choice to save in dollars at date 0, and it provides some insurance, as households' consumption at $t = 1$ is less volatile than their nonfinancial income. However, the precautionary motive of the households is met in equilibrium by a riskier balance sheet of the banks, which is what exposes the economy to financial instability.

Why are banks happy to borrow in dollars and be exposed to exchange rate risk? The answer is that dollar borrowing is effectively "cheaper" than peso borrowing. This can be seen by comparing the interest rates of peso and dollar borrowing expressed in the same currency. In Table 1, we can see that in the dollarized equilibrium the rate of return on peso borrowing is higher than the rate of return on dollar borrowing converted in the same currency. This deviation from the uncovered interest rate parity is effectively a result of households' demand for dollars, which in equilibrium bids up the peso interest rate. Paradoxically, this behavior generates in equilibrium the very risk against which households are trying to insure.

4.3 Discussion

Before continuing, it is useful to discuss in more detail some properties of dollarized equilibria. First, the presence of segmentation in financial markets is critical in generating these equilibria. The segmentation has both an international and a domestic dimension.

At the international level, it is important that some mechanism prevents foreign investors from lending to domestic banks in local currency. To understand why, consider the dollarized equilibrium in Table 1 and suppose that we allow foreign investors to trade in peso denominated claims. Because foreign investors are risk neutral, they have an incentive at date 0 to lend in pesos to the banks because of the positive excess returns. Eventually, this force would eliminate the dollarized equilibrium. Importantly, this argument works only because foreign investors are risk neutral and have deep pockets— assumptions that generate an infinitely elastic supply of funds. There is ample empirical evidence that the supply of foreign capital to emerging market economies is not perfectly elastic. For example, Borri and Verdhelan (2013) and Tourre (2017) document large risk premia in emerging markets' sovereign bonds, while Du and Schreger (2016) and Maggiori, Nieman, and Schreger (2017) recently document the dominance of dollar denomination in cross-border flows. In our paper, we consider the extreme view in which the supply of foreign funds in local currency is perfectly inelastic, but this is just a useful simplification. Our results would not be qualitatively different if we were to relax this assumption, and instead model foreign lenders as risk-averse specialists that are not diversified toward the small open economy.

At the domestic level, we have assumed that the bankers and the households are distinct agents. This assumption, shared by recent papers such as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012), allows us to consider parametrizations in which the households are relatively more risk averse than the bankers, which is important in obtaining the dollarized equilibrium. More primitive frictions such as limited asset market participation on the households' side, or limited liability on the banks' side, would offer a justification for our assumption.

A second aspect that we wish to emphasize is that the model does not have predictions on whether domestic agents save in dollars by opening a foreign currency deposit with domestic banks, or whether they do so by opening it abroad. After the crises of the 1990s, several emerging markets imposed restrictions on households' ability to open dollar deposits in domestic banks. To the extent that these regulations do not interfere with an open capital account, they would not discourage domestic agents in our model from saving in dollars.

Finally, our current analysis mutes an interesting aspect of the dollarized equilibrium: the leverage choice of the banks at date 0. This was done mostly for tractability, as we wanted to isolate how expectations of future crises affect the currency positions of households and banks. Introducing a leverage choice would add an interesting layer, as the expectations of binding financial constraints in the dollarized equilibrium might lead banks to scale down production at date 0 and induce a recession. See Bocola (2016) for an analysis of these effects in a related framework.

5 Lending of last resort and foreign currency reserves

After having characterized the sources of financial instability in this economy, we turn to government interventions. We introduce a benevolent government that intervenes in financial markets at $t = 1$ and ask what is the optimal intervention and whether it can eliminate the multiplicity identified in the previous sections. Although lending of last resort is often done by the central bank, here we do not distinguish the role of the central bank from the role of the treasury and just call "government" the consolidated public sector.

We first describe how we model government interventions at date 1, introducing distortionary taxation and government lending to banks. We then study the continuation equilibria in this version of the model. Finally, we move back to date 0 and analyze how reserve accumulation affects the ex ante choices of the private sector. Appendix C contains a more extensive description of the model with government interventions, along with numerical examples and the proofs of the results presented in this section.

5.1 Modeling ex post interventions

The role of a lender of last resort is to provide emergency liquidity to banks and to restore the confidence of private savers in the liabilities of the financial sector. We model this class of interventions by assuming that at date 1 the government extends a liquidity facility to the banks and guarantees private sector loans to the banks. The liquidity facility takes the form of a credit line from which the banks can draw resources, and the guarantees cover the private sector loans up to the pleadgeable value of the banks' collateral.

The government is financed with distortionary taxes on labor income, so we extend the model by introducing a labor supply decision for households. The government enters period 1 with net positions in pesos and dollars, denoted by A_1 and A_1^*. Each period the government chooses a proportional tax rate τ_t on labor income and nonnegative lump-sum transfers to households T_t. The budget constraints of the government are

$$s_1 \frac{1}{1+i_1^*}(b_2^{*g} + A_2^*) + T_1 = \tau_1 w_1 l_1 + A_1 + s_1 A_1^*,$$

and

$$T_2 = \tau_2 w_2 l_2 + s_2(\hat{b}_2^{*g} + A_2^*), \tag{26}$$

where b_2^{*g} denotes the loans extended to banks at date 1, and $\hat{b}_2^{*g}$ are the repayments made by the banks at date 2. Here we are assuming that all borrowing at date 1, by banks and the government, is in dollars and that it takes place at the world interest rate $(1 + i_1^*) = \beta^{-1}$.[18]

The government is benevolent and sets policy in order to maximize the present value of domestic tradable GDP net of labor disutility

$$k_1^{\alpha} l_1^{1-\alpha} - v\left(l_1\right) - G\left(k_2 - k_1\right) + \beta\left(k_2^{\alpha} l_2^{1-\alpha} - v\left(l_2\right)\right). \tag{27}$$

In order to introduce a notion of credibility for government interventions we need to spell out the timing of the game between the government and the private sector. We split period 1 into three subperiods and assume the following timing:

1.*i* Private savers (domestic households and foreign investors) choose the maximum amount of bank and government claims they are willing to buy, denoted respectively by $\bar{b}_2^{*p}$ and $-\bar{A}_2^*$, and they bid an exchange rate $\tilde{s}_1$ at which they are willing to exchange pesos for dollars.

[18]Because there is no further uncertainty from date 1 onward, agents will be indifferent in equilibrium on whether they borrow/save in dollars or pesos. Having financial transactions in dollars simplifies some expressions in the analysis.

1.*ii* The government sets the credit line to the banks $\bar{b}_2^{*g}$, and decides how to finance it by choosing the vector $(\tau_1, \tau_2, T_1, T_2, A_2^*)$. The banks decide how much to borrow from the private sector and the government. The capital market, the factor markets, and the goods markets clear.

1.*iii* If banks' total liabilities exceed $\theta q_1 k_2$, banks make a take-it-or-leave-it offer to the investors (both private and public), and they renegotiate their debt down to $\theta q_1 k_2$. Investors accept the offer because in case of default they can only recover a fraction θ of the assets.

Under this timing, the government acts after the private sector has set the debt limits $\bar{b}_2^{*p}$ and $-\bar{A}_2^*$ and the exchange rate $\tilde{s}_1$. These debt limits and this exchange rate will have to be consistent with rational expectations, as we shall see shortly. The key assumption here is that the government takes them as given when deciding the optimal intervention. Further details on the timing are provided in Appendix C, along with a characterization of prices and quantities that arise in subperiod 1.*ii* for a given policy of the government.

The modeling of the debt renegotiation stage in subperiod 1.*iii* provides a justification for the collateral constraint used in previous sections. This way of modeling the credit friction implies that the government does not have superior enforcement powers relative to the private sector.

We can then define an equilibrium for the model with government interventions.

Definition 1. *An equilibrium of the model with government interventions is given by debt limits set by the private sector, a policy of the government, and prices and quantities such that:*

i. Taking $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$ as given, the government chooses the policy $(\bar{b}_2^{*g}, \tau_1, \tau_2, T_1, T_2, A_2^*)$ to solve its decision problem, subject to the optimal behavior of the private sector;

ii. Prices and quantities constitute a competitive equilibrium given the debt limits set by investors, the exchange rate $\tilde{s}_1$ in subperiod 1.i, and the policy of the government;

iii. Debt limits and the exchange rate in subperiod 1.i satisfy rational expectations.

It is useful to discuss the rational expectation requirement in the above definition. When private savers set the limits $\bar{b}_2^{*p}$ and $-\bar{A}_2^*$, they form expectations about the repayment capacity of the banks and the government.

Because of debt renegotiation and the government guarantees, savers expect to receive at most $\theta q_1 k_2$ from the banks. Thus, in equilibrium we must have

$$\bar{b}_2^{*p} = \theta q_1 k_2 / \beta. \tag{28}$$

The amount that savers expect to receive from the government depends on the repayments of the loans that the government has issued to the banks, and on its ability to tax in period 2. The banks' repayment to the government in period 2 is

$$\hat{b}_2^{*g} = \min\{\theta q_1 k_2/\beta - b_2^{*p}, b_2^{*g}\},$$

and the government makes a loss on its portfolio when $\theta q_1 k_2/\beta < b_2^{*p} + b_2^{*g}$. In Appendix C we show that the maximum tax revenue that the government can extract in period 2 equals $\Xi k_2^{\alpha(1+\phi)/(\alpha+\phi)}$, with ϕ being the inverse of the Frisch elasticity of labor supply and Ξ a function of structural parameters. Therefore, rational expectation implies that in equilibrium we must have

$$-\bar{A}_2^* = \hat{b}_2^{*g} + \Xi k_2^{\alpha\frac{1+\phi}{\alpha+\phi}}. \tag{29}$$

Before moving forward, we can draw a parallel with the continuation equilibria studied in Section 3. For this purpose, suppose that we restrict the government from intervening in financial markets, $\bar{b}_2^{*g} = 0$. It is then straightforward to show that the equilibria for the model described in this section correspond to the continuation equilibria of Section 3, with two differences. First, labor is endogenous. Second, the government uses taxes and transfers to satisfy its budget constraint.

In what follows, we restrict attention to economies in which the parameters and initial conditions guarantee the existence of two stable continuation equilibria without government interventions, with banks being unconstrained in one of them (as in the case analyzed in panel (b) of Figure 2). We refer to the unconstrained equilibrium as the "good" equilibrium and use the superscript *Good* to denote its prices and quantities. The superscript *Bad* denotes prices and quantities in the constrained equilibrium.

The government has an incentive to support banks only when the bad equilibrium is played. To see that, we can verify that the objective function of the government (27) is maximized when (k_2, l_1, l_2) satisfy

$$\begin{aligned} \beta\alpha k_2^{\alpha-1} l_2^{1-\alpha} &= G'(k_2 - k_1) \\ (1-\alpha) k_t^{\alpha} l_t^{-\alpha} &= v'(l_t), \end{aligned}$$

which are also the optimality conditions of a private sector equilibrium when banks are not constrained and the government sets zero labor taxes.[19] This is a useful property that

[19]The first condition is equivalent to the optimality of bankers and capital good producers in an unconstrained equilibrium. The second condition combines optimal labor choices of the households with the competitive wage and $\tau_t = 0$.

allows us to isolate the role of the government as a lender of last resort.

5.2 Government liquidity and government interventions

We now study the equilibria of the model with government interventions. To do so, we first derive properties of the optimal policy of the government for a given triple $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$. Our first result shows that if the government has access to enough liquidity, then its best response implements the good equilibrium.

Proposition 4 (Abundant government liquidity)**.** *Suppose that the banks and the government are net debtors in pesos, $b_1 \geq 0$ and $A_1 \leq 0$. Moreover, assume that absent government interventions in financial markets, there are three continuation equilibria, one of which is unconstrained, with $A_1 + s_1^{Good} A_1^* \geq 0$. The best response of the government to any triple $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$ that satisfies $\tilde{s}_1 \geq s_1^{Good}$ and*

$$q_1^{Good}(k_2^{Good} - k_1) - \alpha k_1^\alpha \left(l_1^{Good}\right)^{1-\alpha} + b_1^* + \frac{b_1}{\tilde{s}_1} \leq +A_1^* + \frac{A_1}{\tilde{s}_1} - \beta\left(\bar{A}_2^* - \bar{b}_2^{*p}\right) \tag{30}$$

is to set zero labor income taxes in both periods, to extend a credit line

$$\beta\bar{b}_2^{*g} = q_1^{Good}(k_2^{Good} - k_1) - \alpha k_1^\alpha \left(l_1^{Good}\right)^{1-\alpha} + b_1^* + \frac{b_1}{\tilde{s}_1} - \beta\bar{b}_2^{*p}$$

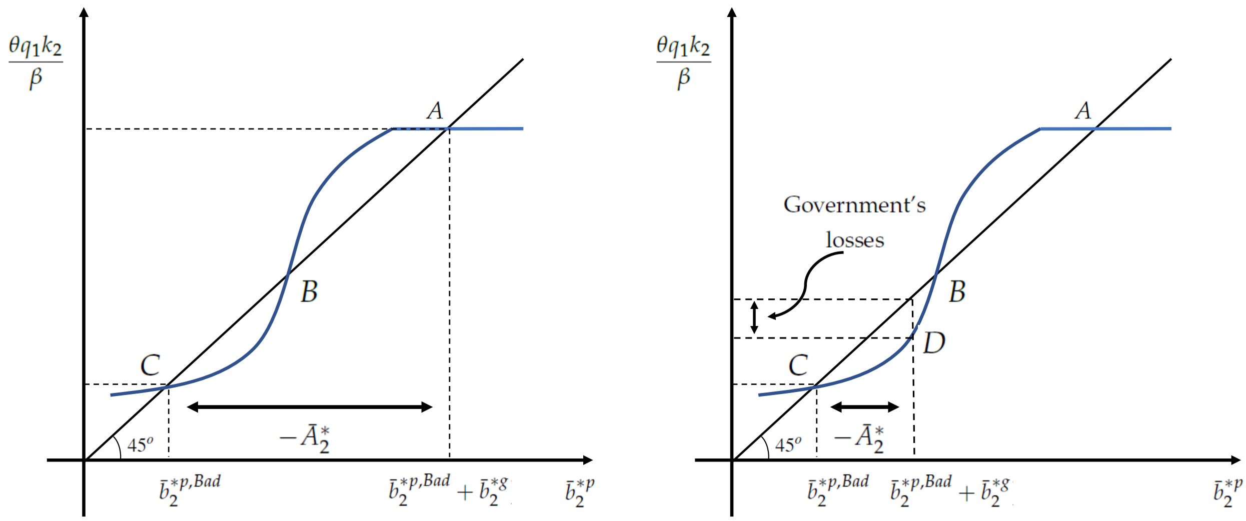
to the banks, and to borrow

$$-\beta A_2^* = \beta\bar{b}_2^{*g} - \left(A_1^* + \frac{A_1}{\tilde{s}_1}\right)$$

from private investors. With this policy, the government implements the allocation and prices of the good equilibrium.

The proof of the proposition is in Appendix C. To understand the logic behind this result, suppose that the government enters period 1 with zero net financial positions ($A_1 = A_1^* = 0$). This implies that it would be optimal to set zero taxes and transfers when $\bar{b}_2^{*g} = 0$. Let us now look at competitive equilibria without government interventions, that is restricting $\bar{b}_2^{*g}$ to be equal to zero. For each value of the credit limit $\bar{b}_2^{*p}$ set by private savers, we can compute prices and allocations that clear the capital, goods, and factors markets, and the implied value for the bank collateral $\theta q_1 k_2/\beta$. This defines a mapping between $\bar{b}_2^{*p}$ and $\theta q_1 k_2/\beta$. An equilibrium without government interventions is then a fixed point of this mapping.

Panel (a) of Figure 4 shows an example. The mapping from $\bar{b}_2^{*p}$ to $\theta q_1 k_2/\beta$ is first increasing, then flat. For low levels of the credit limit $\bar{b}_2^{*p}$, the banks are constrained: an



(a) No liquidity constraints for the government

(b) Liquidity constrained government

Figure 4: Optimality of government interventions

increase in $\bar{b}_2^{*p}$ raises their demand for capital and leads to a higher price and quantity of capital. For higher levels of $\bar{b}_2^{*p}$, banks are unconstrained, and a higher credit limit does not result in more investment, which implies that $\theta q_1 k_2/\beta$ does not respond to $\bar{b}_2^{*p}$. In Figure 4 the mapping crosses the 45^o line three times, so we have three equilibria without government interventions, which are analogous to the three equilibria identified in the price space in Figure 2.

Suppose now that households have pessimistic expectations about the banks' collateral values, and they lend at most $\bar{b}_2^{*p,Bad}$. If condition (30) is satisfied, the government can set zero labor taxes at date 1 and borrow from private investors in order to extend a liquidity facility to the banks. When condition (30) is satisfied, the intervention can be made sufficiently large to finance the unconstrained demand of capital. Banks use these funds up to the point at which they are unconstrained, and collateral values move from point C to point A. At point A banks' collateral is

$$\theta q_1^{Good} k_2^{Good} = \beta(\bar{b}_2^{*p,Bad} + \bar{b}_2^{*g}),$$

so banks repay both private investors and the government. The government can then use these proceeds to settle its debt. This policy implements an allocation with undistorted labor and unconstrained investment, which, as seen earlier, maximizes the objective function (27). Hence, it is the best response of the government.

This argument shows that lending of last resort can always be successfully conducted if the government faces no liquidity constraints. Being a large player, the government understands that by lending enough resources to the banks, it can increase the value of their collateral to the one prevailing in the good equilibrium. Intervening has zero costs because in the good equilibrium the banks can meet their debt payments, and the government can use the proceeds from these loans to repay its debt. This seems to be a general argument that extends beyond the specific model presented here.

So far we have seen that a government that does not face liquidity constraints is always in a position to eliminate the bad equilibrium. But is it possible to have equilibria in which the government is unable or unwilling to do so? To answer this question, we can study the trade-offs faced by a government with limited liquidity. Let us go back to the example in Figure 4. Suppose again that private savers start with pessimistic expectations at $\bar{b}_2^{*p,Bad}$, but now the loans they extend to the government are low enough that condition (30) is violated. An example of this case is represented in panel (b) of the figure.

We can verify that the policy described in Proposition 4 is no longer feasible. If the government keeps labor taxes at zero in period 1 and finances its interventions only by issuing debt, the economy reaches point D. At point D the collateral value of banks is smaller than $\bar{b}_2^{*p,Bad} + \bar{b}_2^{*g}$, so the government incurs losses on the loans extended to the banks. Because of that, it will need to raise taxes at date 2 to repay private savers.

Therefore, a liquidity-constrained government faces a trade-off because financial market interventions can be sustained only at the cost of introducing distortions in the labor market. This trade-off can be analyzed by inspecting the objective function of the government (27). On the one hand, an increase in $\bar{b}_2^{*g}$ when the banks are constrained leads to more investment and a higher k_2. This raises the payoff of the government because in the constrained region we have that $\beta k_2^{\alpha-1} l_2^{1-\alpha} > q_1 = G'(k_2 - k_1)$.[20] On the other hand, this increase in $\bar{b}_2^{*g}$ can be achieved only by raising taxes at date 1. This induces a distortion in the labor market, and it reduces the payoff of the government. While the best response in this case depends on the model's parameters, it will not typically involve a move from point D to the unconstrained allocation.

The fact that the government might decide not intervene aggressively for some triples $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$ implies that the bad equilibrium can survive even when government interventions are allowed.

Proposition 5. *The model with government interventions can feature multiple equilibria.*

In Appendix C we provide numerical examples that prove this proposition and we also provide a characterization of the government's best response that helps to interpret these

[20] Indirect effects are also associated with such an increase in investment, which we analyze in Appendix C. An increase in k_2 increases the tax base in period 2 and alters the total portfolio losses made by the government. These effects can relax or tighten the budget constraint of the government.

equilibria. In these situations, the private sector holds pessimistic expectations about the ability to repay of both banks *and* the government, and it extends little credit to both. Facing tight credit limits, the best response of the government is a partial intervention that does not bring the economy back to the good equilibrium allocation. The implied low capital accumulation leads to low collateral values and low future tax revenues, validating the private sector expectations (see equations (28) and (29)).

5.3 The role of reserves

We now build on Proposition 4 and study the role of foreign currency reserves in sustaining effective lending of last resort. In particular, we derive a sufficient condition on A_1 and A_1^* that ensures that the government uniquely implements the good equilibrium. To state this condition, we first define the function $\mathcal{S}(k_2)$. For each k_2, the function $\mathcal{S}(k_2)$ gives the exchange rate that clears the nontradable goods market under no government interventions.[21] The definition of $\mathcal{S}(.)$ is in Appendix C.

Proposition 6 (Reserves and lending of last resort)**.** *Suppose that the banks and the government are net debtors in pesos, $b_1 \geq 0$ and $A_1 \leq 0$. Suppose also that with no government interventions there are three continuation equilibria, one of which is unconstrained, with $A_1 + s_1^{Good} A_1^* \geq 0$. If the following condition*

$$q_1^{Good}\left(k_2^{Good} - k_1\right) - \alpha k_1^{\alpha}(l_1^{Good})^{1-\alpha} + b_1^* + \frac{b_1}{\mathcal{S}\left(k_2\right)} \leq \beta \Xi k_2^{\alpha\frac{1+\phi}{\alpha+\phi}} + \theta\left[\phi_0 + \phi_1\left(k_2 - k_1\right)\right] k_2 + A_1^* + \frac{A_1}{\mathcal{S}\left(k_2\right)} \tag{31}$$

is satisfied for all $k_2 \leq k_2^{Good}$, then the model with government interventions has a unique equilibrium which yields the unconstrained allocation.

When reserves satisfy condition (31), the government is always in a position to intervene as in Proposition 4 and uniquely implement the unconstrained equilibrium. Let us provide some intuition for condition (31). As discussed above, the borrowing capacity of the banks and the government depends on the private sector expectations about k_2. If the private sector has pessimistic expectations about k_2, it is willing to lend little to banks and the government because both the value of banks' collateral and tax revenues are expected to be low. However, pessimistic expectations about k_2, in equilibrium, go together with pessimistic expectation about the exchange rate because when k_2 is low households are

[21] Here we are abusing notation by using $\mathcal{S}$ to denote a function different from the one defined in Lemma 2. That function is not used here, so no confusion should arise.

poorer and nontradable demand is depressed. The function $\mathcal{S}(.)$ captures this relation.

If the government holds more dollars and borrows more pesos, its net financial position $A_1^* + A_1/s_1$ increases exactly when the private sector holds pessimistic expectations and helps the government to intervene. In other words, a position long in dollars and short in pesos is a good "hedge" against the pessimistic expectations of the private sector.[22]

Condition (31) can also be used to interpret patterns in the data that have been documented in the empirical literature. Obstfeld, Shambaugh, and Taylor (2010) show that the size of the banking sector liabilities is an important predictor in explaining the accumulation of foreign currency reserves by emerging markets. Our model offers a rationale for this relation. To see why, suppose we compare two economies that are identical in all respects except for the balance sheet of the financial sector at date 1, that is, for the debt levels b_1 and b_1^*. Inspecting (31), we can then state the following.

Remark 1 (Reserves and banks' balance sheet)**.** *All else equal, an economy with a more levered financial sector or with relatively more dollar debt, requires a higher value of A_1^* for a given net financial position $A_1^* + A_1/s_1^{Good}$ to satisfy condition (31).*

It is useful to clarify that condition (31) is sufficient but not necessary. A government not satisfying (31) may still have sufficient incentives to use distortionary taxation when facing adverse expectations, so as to rule out multiplicity. Unfortunately, it's harder analytically to derive comparative statics showing how the *minimal* reserve level needed to eliminate a financial panic responds to changes in b_1 and b_2^*. Numerical examples suggest that minimal reserves go in the same direction as the reserves that satisfy (31).

A second remark comes out of our analysis.

Remark 2 (Unused reserves)**.** *Reserves can play a useful role in credibly ruling out financial panics and yet never be used in equilibrium.*

When condition (31) is satisfied, the government doesn't intervene in equilibrium and rebates the reserves back to the households. However, the presence of reserves is important to rule out the bad equilibrium. This remark connects our analysis to another empirical observation: emerging markets seem to hold on to their stocks of reserves, even in times of distress. Aizenman and Sun (2012) provide evidence of this type of behavior—which they dub "fear of losing international reserves"—looking at emerging economies' response to the global downturn of 2009. Jeanne and Sandri (2016) make a related observation in the context of a model of precautionary reserve accumulation. They calibrate their model

[22]We put the word "hedge" in quotes because if condition (31) is satisfied, bad expectations never materialize. So the government is really hedging against an event that never takes place.

to fit the average reserve-to-GDP ratio observed in emerging markets and show that the country in their model depletes reserves fast whenever a negative shock hits. So a standard precautionary model (without a financial sector and multiple equilibria) cannot account for the observed stability of reserve stocks.

5.4 The ex ante effects of reserve accumulation

A possible concern is that the beneficial effects of foreign currency reserves might be undone if interventions in financial markets are anticipated by the private sector. Banks may choose a riskier balance sheets ex ante if they anticipate a government rescue and domestic savers may offset an increase in A_1^* by borrowing more in dollars. The surprising result in our environment is that it is possible that the accumulation of foreign currency assets by the government actually promotes financial stability from an ex ante perspective.

We make this point using an example, which we report in Table 2, based on the model with optimal intervetnions of the government at $t = 1$ described above. The first two columns consider the case in which the government sets $A_1 = A_1^* = 0$ at $t = 0$. In this case, two equilibria are possible, analogous to the two equilibria analyzed in Section 4. The last column considers a case in which the government borrows in pesos at date 0 to finance dollar reserves, so $A_1^* > 0 > A_1$ and

$$\frac{1}{1+i_0}A_1 + \frac{s_0}{1+i_0^*}A_1^* = 0.$$

In this case, the government has sufficient resources to eliminate the bad equilibrium at $t = 1$, so only the nondollarized equilibrium is possible at $t = 0$.

Let us compare the nondollarized equilibrium without government reserves to the unique equilibrium that arises with reserves. Table 2 reports the consolidated positions of the government and the banking sector, in dollars and in pesos. Both the consolidated dollar position $A_1^* - b_1^*$ and the consolidated peso debt $b_1 - A_1$ are higher in the case in which the government accumulates reserves. In other words, increased dollar holdings by the government are not undone by increased dollar borrowing by the banks. We summarize this finding in the next remark.

Remark 3 (Catalytic reserves)**.** *When reserves are large enough to eliminate a bad equilibrium, their presence can lead to a higher net consolidated dollar position of banks and the government.*

In this example, stabilizing interventions ex post instead of encouraging more risk taking ex ante encourage banks to take safer positions. To understand the logic behind this exam-

Table 2: The ex-ante effects of reserve accumulation

	No Reserves		Reserves
	Nondollarized	Dollarized	Nondollarized
a_1	0.633	0.045	0.822
a_1^*	0.000	0.710	-0.200
b_1	0.633	0.045	0.633
b_1^*	-0.187	0.375	-0.187
A_1	0.000	0.000	-0.189
A_1^*	0.000	0.000	0.200
$b_1 - A_1$	0.633	0.045	0.822
$A_1^* - b_1^*$	0.187	-0.375	0.387
$\mathbb{E}[(1+i_0^*)(s_1/s_0)]$	1.000	0.909	1.000
$(1+i_0)$	1.000	1.042	1.000

Notes: The parameters used in the example are: $\omega = 0.50$, $\alpha = 0.85$, $\beta = 1.00$, $e^N = 0.70$, $\theta = 0.90$, $\phi_0 = 0.40$, $\phi_1 = 0.20$, $\eta = 3.00$, $\phi = 0.33$, $\chi = 0.05$, $\sigma_\varepsilon = 0.005$, $\mu = 0.10$. *The initial conditions are* $k_0 = 0.30$, $a_0 = b_0 = 0.00$, $a_0^* = 1.22$, $b_0^* = 0.72$. *See Appendix C for additional details.*

ple, it is useful to identify two opposing channels through which government intervention affects banks' behavior ex ante.

First, if we fix the interest rates in pesos and dollars at date 0, there is a direct effect of intervention that leads banks to issue more dollar debt. The argument is as follows. As argued in Section 4, the presence of the bad equilibrium gives banks an incentive to borrow less in dollars, as the marginal value of net worth is higher in that state. Therefore, when the bad equilibrium is removed, the incentive to borrow in dollars goes up. This is a traditional moral hazard argument, where reducing the risk to which banks are exposed (by eliminating the bad equilibrium) leads to increased risk taking.

Second, there is a general equilibrium effect that works in the opposite direction. When government interventions remove the bad equilibrium, domestic savers are no longer concerned about a large peso depreciation correlated to a contraction in consumption. Hence, savers will demand more peso assets and fewer dollar assets. This force pushes down the interest rate differential between dollar and peso debt and induces banks to borrow less in dollars.

In the example of Table 2, the general equilibrium effect dominates, so banks end up with less dollar and more peso debt.

It is also useful to look at the consolidated position of the government plus the household sector. Comparing now the two nondollarized equilibria (under no reserves and under reserves), we see that Ricardian equivalence is fully at play. In the case with reserves, the government does not use its dollar assets on the equilibrium path and rebates them back to the households in period 1. Since households fully anticipate that, they offset the actions of the government by borrowing more in dollars. However, this does not mean that official purchases of foreign reserves are ineffective. Differently from private savers, the government is a large player, and can use dollar reserves more effectively by deploying them in a coordinated way to fight financial panics off the equilibrium path. Thus, the accumulation of dollar reserves by official authorities has beneficial effects for financial stability even if domestic households perfectly offset it.

An additional implication of these forces, which can be seen from the example in Table 2, is that the presence of reserves induces the country as a whole (aggregating households, banks, and the government) to accumulate a smaller net position in dollars. Since foreign investors only trade dollar assets and the initial net dollar position of the country is given, this also means that the trade balance is smaller under intervention. The intuition here has to do with the precautionary behavior of domestic savers: households, facing less volatile incomes, choose a higher level of tradable consumption and so run a smaller trade surplus. We then get the counterintuitive result that reserve accumulation by the government leads to less total accumulation of dollar assets by the economy. The idea is that if dollars are accumulated in a way that makes the domestic financial system more stable, the private incentive to accumulate dollars for precautionary reasons may be reduced.

Here we have not analyzed the decision of an optimizing government to accumulate reserves at date 0. In the example presented, reserve accumulation entails no costs for the government because, by ruling out the bad continuation equilibrium, it equalizes the interest rate on peso and dollar debt at date 0. It would be interesting to extend the model to allow for a country or a currency premium that cannot be completely eliminated by ex ante policy.[23] In that case, the government would face a genuine trade-off at date 0 as reserves are costly—because they pay a relatively low rate of return—but would help provide a credible backstop to the financial system. We leave the analysis of this trade-off to future work.

6 Conclusion

We presented a model of a small open economy with a financial sector. This framework provides a novel perspective on liability dollarization in emerging markets, pointing out the

[23]For example, by adding other sources of real uncertainty for the domestic economy at date 1.

interaction between financial instability and the precautionary motive of domestic savers. We have used the model to study the view that the accumulation of official foreign currency reserves in emerging markets has an underlying financial stability motive. Our formal analysis is consistent with this view, as foreign currency holdings allow domestic authorities to deal more effectively with financial panics. When large enough, foreign reserves can promote the development of local currency asset markets and protect an economy from liability dollarization pressures.

In the paper, we assumed that domestic authorities do not interfere with the openness of their capital accounts, and that the economy has fully flexible exchange rates under a strict inflation targeting regime. The framework can be enriched to study the role of capital controls and alternative monetary policies. Moreover, we have conducted our analysis in a three-period environment in order to clearly isolate the rich general equilibrium linkages between the financial side and the real side of the economy. We believe that considering a more quantitative version of the model represents a fruitful avenue for future research.

References

Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee. 2001. "Currency crises and monetary policy in an economy with credit constraints." *European economic review* 45 (7):1121–1150.

———. 2004. "A corporate balance-sheet approach to currency crises." *Journal of Economic theory* 119 (1):6–30.

Aguiar, Mark and Gita Gopinath. 2007. "Emerging market business cycles: The cycle is the trend." *Journal of Political Economy* 115 (1):69–102.

Aiyagari, Rao S. and Mark Gertler. 1999. ""Overreaction" of Asset Prices in General Equilibrium." *Review of Economic Dynamics* 2 (1):3–35.

Aizenman, Joshua and Jaewoo Lee. 2007. "International reserves: precautionary versus mercantilist views, theory and evidence." *Open Economies Review* 18 (2):191–214.

Aizenman, Joshua and Yi Sun. 2012. "The financial crisis and sizable international reserves depletion: From "fear of floating" to the "fear of losing international reserves"?" *International Review of Economics & Finance* 24:250–269.

Bacchetta, Philippe, Cédric Tille, and Eric Van Wincoop. 2012. "Self-fulfilling risk panics." *The American Economic Review* 102 (7):3674–3700.

Bagehot, Walter. 1873. *Lombard Street: A description of the money market*. Scribner, Armstrong & Company.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez. 2012. "International reserves and rollover risk." Tech. rep., National Bureau of Economic Research.

Bocola, Luigi. 2016. "The Pass-Through of Sovereign Risk." *Journal of Political Economy* 124 (4):879–926.

Borri, Nicola and Adrien Verdhelan. 2013. "Sovereign Risk Premia." Manuscript, MIT Sloan.

Broner, Fernando and Jaume Ventura. 2016. "Rethinking the effects of financial globalization." *The Quarterly Journal of Economics* 131 (3):1497–1542.

Brunnermeier, Markus K and Yuliy Sannikov. 2014. "A macroeconomic model with a financial sector." *The American Economic Review* 104 (2):379–421.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 2001a. "Hedging and financial fragility in fixed exchange rate regimes." *European Economic Review* 45 (7):1151–1193.

———. 2001b. “Prospective deficits and the Asian currency crisis.” *Journal of political Economy* 109 (6):1155–1197.

Caballero, Ricardo J, Emmanuel Farhi, and Pierre-Olivier Gourinchas. 2008. “An Equilibrium Model of" Global Imbalances" and Low Interest Rates.” *The American Economic Review* 98 (1):358.

Caballero, Ricardo J and Arvind Krishnamurthy. 2003. “Excessive dollar debt: Financial development and underinsurance.” *The Journal of Finance* 58 (2):867–894.

Caballero, Ricardo J and Stavros Panageas. 2008. “Hedging sudden stops and precautionary contractions.” *Journal of Development Economics* 85 (1):28–57.

Cerra, Valerie and Sweta Chaman Saxena. 2008. “Growth Dynamics: The Myth of Economic Recovery.” *American Economic Review* 98 (1):439–57.

Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco. 2017. “Financial Intermediation, Real Exchange Rates, and Unconventional Policies in an Open Economy.” *Journal of International Economics* .

Chang, Roberto and Andres Velasco. 2000. “Financial fragility and the exchange rate regime.” *Journal of economic theory* 92 (1):1–34.

———. 2001. “A model of financial crises in emerging markets.” *Quarterly Journal of Economics* :489–517.

Corsetti, Giancarlo, Paolo Pesenti, and Nouriel Roubini. 1999. “Paper tigers?: A model of the Asian crisis.” *European Economic Review* 43 (7):1211–1236.

Dalgic, Husnu. 2017. “Financial Dollarization in Emerging Markets as an Insurance Mechanism.” Manuscript, Northwestern University.

Diamond, Douglas W and Philip H Dybvig. 1983. “Bank runs, deposit insurance, and liquidity.” *Journal of Political Economy* :401–419.

Du, Wenxin, Carolin E. Pflueger, and Jesse Schreger. 2017. “Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy.” Tech. rep., National Bureau of Economic Research.

Du, Wenxin and Jesse Schreger. 2016. “Sovereign Risk, Currency Risk, and Corporate Balance Sheets.” Manuscript, Columbia Business School.

Durdu, Ceyhun Bora, Enrique G Mendoza, and Marco E Terrones. 2009. “Precautionary demand for foreign assets in Sudden Stop economies: An assessment of the New Mercantilism." *Journal of development Economics* 89 (2):194–209.

Ennis, Huberto M and Todd Keister. 2009. "Bank runs and institutions: The perils of intervention." *The American Economic Review* 99 (4):1588–1607.

Fahri, Emmanuel and Matteo Maggiori. 2017. "A Model of the International Monetary System." Forthcoming, Quarterly Journal of Economics.

Farhi, Emmanuel and Jean Tirole. 2012. "Collective moral hazard, maturity mismatch, and systemic bailouts." *The American Economic Review* 102 (1):60–93.

Gai, Prasanna, Sujit Kapadia, Stephen Millard, and Ander Perez. 2008. "Financial innovation, macroeconomic stability and systemic crises." *The Economic Journal* 118 (527):401–426.

Gertler, Mark and Nobuhiro Kiyotaki. 2010. "Financial intermediation and credit policy in business cycle analysis." *Handbook of Monetary Economics* 3 (3):547–599.

———. 2015. "Banking, liquidity, and bank runs in an infinite horizon economy." *The American Economic Review* 105 (7):2011–2043.

Gourinchas, Pierre-Olivier and Olivier Jeanne. 2013. "Capital flows to developing countries: The allocation puzzle." *Review of Economic Studies* 80 (4):1484–1515.

Gourinchas, Pierre-Olivier and Maurice Obstfeld. 2012. "Stories of the twentieth century for the twenty-first." *American Economic Journal: Macroeconomics* 4 (1):226–265.

He, Zhiguo and Arvind Krishnamurthy. 2012. "Intermediary Asset Pricing." *The American Economic Review* 20:1–42.

———. 2015. "A Macroeconomic Framework for Quantifying Systemic Risk." Manuscript, Stanford GSB.

Heathcote, Jonathan and Fabrizio Perri. 2015. "Wealth and volatility." Tech. rep., National Bureau of Economic Research.

Hernandez, Juan. 2017. "How International Reserves reduce the Probability of Debt Crises." Manuscript, University of Pennsylvania.

Hur, Sewon and Illenin O Kondo. 2016. "A theory of rollover risk, sudden stops, and foreign reserves." *Journal of International Economics* 103:44–63.

Ilzetzki, Ethan, Carmen M. Reinhart, and Kenneth S. Rogoff. 2017. "Exchange arrangements entering the 21st century: Which anchor will hold?" NBER working paper No 23134.

Ize, Alain and Eduardo Levy-Yeyati. 2003. “Financial dollarization.” *Journal of International Economics* 59 (2):323–347.

Jeanne, Olivier and Romain Rancière. 2011. “The optimal level of international reserves for emerging market countries: a new formula and some applications.” *The Economic Journal* 121 (555):905–930.

Jeanne, Olivier and Damiano Sandri. 2016. “Optimal Reserves in Financially Closed Economies.” Tech. rep., National Bureau of Economic Research.

Kaminsky, Graciela L and Carmen M Reinhart. 1999. “The twin crises: the causes of banking and balance-of-payments problems.” *American economic review* :473–500.

King, Mervyn. 2006. “Reform of the International Monetary Fund.” Speech at the Indian Council for Research on International Economic Relations.

Krugman, Paul. 1999. “Balance sheets, the transfer problem, and financial crises.” In *International finance and financial crises: Essays in Honor of Robert P Flood*, edited by A. Razin P. Isard and A. Rose. Springer, 31–55.

Levy-Yeyati, Eduardo. 2006. “Financial dollarization: evaluating the consequences.” *Economic Policy* 21 (45):62–118.

Lorenzoni, Guido. 2008. “Inefficient credit booms.” *The Review of Economic Studies* 75 (3):809–833.

Maggiori, Matteo. 2017. “Financial intermediation, international risk sharing, and reserve currencies.” Forthcoming, American Economic Review.

Maggiori, Matteo, Brent Nieman, and Jesse Schreger. 2017. “Unpacking Global Capital Flows.” Manuscript, Harvard.

Mendoza, Enrique G. 2010. “Sudden stops, financial crises, and leverage.” *The American Economic Review* 100 (5):1941–1966.

Mendoza, Enrique G, Vincenzo Quadrini, and Jose-Victor Rios-Rull. 2009. “Financial integration, financial development, and global imbalances.” *Journal of Political economy* 117 (3):371–416.

Mendoza, Enrique G and Katherine A Smith. 2006. “Quantitative implications of a debt-deflation theory of Sudden Stops and asset prices.” *Journal of International Economics* 70 (1):82–114.

Obstfeld, Maurice, Jay C Shambaugh, and Alan M Taylor. 2010. “Financial stability, the trilemma, and international reserves." *American Economic Journal: Macroeconomics* 2 (2):57–94.

Queralto, Pablo. 2014. "A model of slow recoveries from financial crises." Tech. rep., National Bureau of Economic Research.

Rampini, Adriano A and S Viswanathan. 2010. "Collateral, risk management, and the distribution of debt capacity." *The Journal of Finance* 65 (6):2293–2322.

Ravn, Moreton O. and Vincent Sterk. 2017. "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach." Unpublished manuscript, UCL.

Salomao, Juliana and Liniana Varela. 2017. "Exchange Rate Exposure and Firm Dynamics." Manuscript, Carlson School of Management.

Schneider, Martin and Aaron Tornell. 2004. "Balance sheet effects, bailout guarantees and financial crises." *The Review of Economic Studies* 71 (3):883–913.

Tourre, Fabrice. 2017. "A Macro-Finance Approach to Sovereign Debt Spreads and Returns." Manuscript, University of Chicago.

Wiriadinata, Ursula. 2017. "External Debt, Currency Risk, and International Monetary Policy Transmission." Manuscript, University of Chicago.

Appendix to "Financial Crises and Lending of Last Resort in Open Economies"

by Luigi Bocola and Guido Lorenzoni

A Proofs of results in Section 3 and Section 4

Proof of Lemma 1

Since there is no uncertainty left after period 1, the Euler equations for domestic and foreign bonds are

$$U'(c_1) = \frac{p_1}{p_2}\beta(1+i_1)U'(c_2), \tag{A.1}$$

$$U'(c_1) = \frac{s_2}{s_1}\frac{p_1}{p_2}\beta(1+i_1^*)U'(c_2). \tag{A.2}$$

Using market clearing in nontradables, the definition of the price index (10), and the constant endowment of nontradables, we get

$$(1-\omega)\left(\frac{p_1^T}{p_1^N}\right)^{\omega} c_1 = (1-\omega)\left(\frac{p_2^T}{p_2^N}\right)^{\omega} c_2. \tag{A.3}$$

Using $(1+i_1^*) = 1/\beta$, the law of one price (9), the assumption of constant foreign prices, and the definition of CPI (10), we can write the Euler equation for foreign bonds (A.2) as

$$U'(c_1) = \frac{\left(p_1^N/p_1^T\right)^{1-\omega}}{\left(p_2^N/p_2^T\right)^{1-\omega}} U'(c_2).$$

Combining these two conditions yields

$$\left(U'(c_1)\right)^{-\frac{\omega}{1-\omega}} c_1 = \left(U'(c_2)\right)^{-\frac{\omega}{1-\omega}} c_2,$$

which, given the concavity of $U(.)$, implies $c_1 = c_2$. Equation (A.3) then implies a constant relative price of nontradables. The domestic interest rate is then obtained from (A.1). □

Proof of Lemma 2

Substituting the supply of capital (13) in condition (14), we have, for any given q_1, an equation in p_1^N/p_1^T. When $a_1^* \geq 0$ the left-hand side of (14) is monotonically increasing in p_1^T/p_1^N and ranges from 0 to ∞, so this equation has a unique solution. Moreover, this solution is monotonically decreasing in q_1, since both profits π and the capital stock k_2 are increasing in q_1. All other variables are easily derived. In particular, equation (12) transforms the relative price p_1^N/p_1^T into the nominal exchange rate s_1. $\square$

Proof of Proposition 1

As $q_1 \to \infty$, the demand of capital (18) goes to 0 and the supply of capital (13) goes to ∞. It is easy to check that under conditions (A1) demand exceeds supply at $q_1 = \phi$. Thus, by continuity, we have existence. The second statement follows because the unconstrained demand of capital is decreasing in q_1 while the supply is increasing. $\square$

Proof of Proposition 2

The low q_1 equilibrium is associated with low investment, as we move along an upward-sloping supply schedule. The fact that consumption is lower and the exchange rate is more depreciated follows from Lemma 2. The result on the current account follows from the fact that y_1 is the same across the continuation equilibria (as capital is predetermined and labor is inelastically supplied), while investment and consumption of tradables are lower in the low q_1 equilibrium. $\square$

Proof of Proposition 3

We first describe the equilibrium conditions of the model and then verify the statement of the proposition.

The equilibrium conditions from date 1 onward are discussed in the main text. At date 0 we need to determine the choices of households $\{c_0, a_1, a_1^*\}$, the choices of bankers, $\{b_1, b_1^*\}$, and the prices $\{p_0^T, p_0^N, p_0, s_0, i_0\}$. The monetary rule pins down p_0, while we have $s_0 = p_0^T$ by the law of one price and the normalization of the foreign price of tradables. The remaining variables are determined through households' optimality, bankers' optimality, and market clearing.

Households' optimality at date 0 requires

$$
\begin{aligned}
U(c_0) &= \beta(1+i_0)\mathbb{E}_0[U'(c_1)], &\text{(A.4)}\\
s_0 U(c_0) &= \beta(1+i_0^*)\mathbb{E}_0[U'(c_1)s_1], &\text{(A.5)}\\
\frac{1}{1+i_0}a_1 + s_0\frac{1}{1+i_0^*}a_1^* + c_0 &= p_0^T(1-\alpha)k_0^\alpha + p_0^N e^N + a_0 + s_0 a_0^*, &\text{(A.6)}
\end{aligned}
$$

where we have used $p_0 = p_1$.

The bankers' optimality at date 0 requires that

$$
\begin{aligned}
(\lambda_0+\mu_0) &= \beta(1+i_0)\mathbb{E}_0[\lambda_1], &\text{(A.7)}\\
s_0(\lambda_0+\mu_0) &= \beta(1+i_0^*)\mathbb{E}_0[\lambda_1 s_1], &\text{(A.8)}\\
\frac{1}{1+i_0}b_1 + s_0\frac{1}{1+i_0^*}b_1^* &= a_0 + s_0 a_0^* - p_0^T \alpha k_0^\alpha, &\text{(A.9)}
\end{aligned}
$$

where λ_t is the bankers' marginal value of wealth at date t, and μ_0 is the Lagrange multiplier on the date 0 collateral constraint.

Finally, there are two market clearing conditions:

$$
\begin{aligned}
a_1 &= b_1, &\text{(A.10)}\\
(1-\omega)c_0 &= e^N p_0^N. &\text{(A.11)}
\end{aligned}
$$

We can then use Lemma 2 to determine a continuation equilibrium for an arbitrary asset position (a_1, a_1^*, b_1, b_1^*). Because the collateral constraint does not bind at date 1 by assumption, the price and quantity of capital are uniquely pinned down by (21), which implies that w_2/p_2^T and $\pi(q_1)$ are unique and independent of ε. From equation (14) and the monetary rule, we can then see that $\{c_t, p_t^T, p_t^N\}$ are also independent of ε if and only if $a_1^* = 0$.

Consider now the bankers' optimality conditions at date 0. Because the collateral constraint does not bind at date 1, we know by equation (24) and equations (A.7)-(A.8) that

$$(1+i_0) = (1+i_0^*)\mathbb{E}_0\left[\frac{s_1}{s_0}\right]$$

must hold in equilibrium.

Using the above results and equations (A.4) and (A.5), it then follows that a_1^* must be equal to 0 in equilibrium.[24] This implies that households' future consumption is non-

[24] See the main text for a discussion of this result.

stochastic. Using the same logic of Lemma 1 and the monetary rule at date 0, we can then show that an equilibrium must feature $c_0 = c_1 = \bar{c}$, $p_0^N = p_1^N = p^N$, $p_0^T = p_1^T = p^T$, and $(1 + i_0) = (1 + i) = (1/\beta)$.

From equation (A.6) and the law of one price, we then obtain

$$a_1 = (1 + i) \left[p^T (1 - \alpha) k_0^\alpha + p^N e^N + a_0 + p^T a_0^* - \bar{c} \right].$$

The positions of the bankers in point (iv) of the proposition are obtained by using the bankers' budget constraint (A.9) and the market clearing condition (A.10).

Note that a nondollarized equilibrium exists if initial conditions and model parameters are such that the asset positions $\{a_1, a_1^*, b_1, b_1^*\}$ derived here imply a slack collateral constraint in period 1 (almost surely) and the existence of an unconstrained continuation equilibrium in the capital market. □

B Constructing dollarized equilibria

In this section we present an algorithm to "reverse engineer" a dollarized equilibrium and to verify whether it coexists with a nondollarized equilibrium. In what follows, we take as given the structural parameters of the model with the exception of those governing the households' utility function,

$$U(c_t) = \frac{(c_t - \bar{c})^{1-\sigma}}{1 - \sigma}.$$

This utility function allows us to flexibly parametrize households' precautionary behavior.

The algorithm is composed of three main steps. First, we start with a guess for the asset position $[a_1, a_1^*, b_1^*]$ that guarantees that the model features multiple continuation equilibria from date 1. Second, given the continuation equilibria, we solve for $[\sigma, \bar{c}]$ and for the initial conditions $[a_0, a_0^*, b_0^*]$ that guarantee that $[a_1, a_1^*, b_1^*]$ are optimally chosen at date 0 by households and bankers, and that all markets clear. Third, given $[\sigma, \bar{c}]$ and $[a_0, a_0^*, b_0^*]$, we verify whether a nondollarized equilibrium exists. We now describe these steps in details.

> **Step 1**: Fixing $[a_1, a_1^*, b_1^*]$ and a realization of ε, we use the demand and supply of capital derived in Section 3 in order to compute the (q_1, k_2) that clear the capital market. We focus on the case in which there are two stable solutions in the capital market, which we label "Good" and "Bad". We next use the results in Lemma 1 and 2 to compute $\{c_1^{\text{Good}}(\varepsilon), s_1^{\text{Good}}(\varepsilon), \lambda_1^{\text{Good}}(\varepsilon)\}$ and $\{c_1^{\text{Bad}}(\varepsilon), s_1^{\text{Bad}}(\varepsilon), \lambda_1^{\text{Bad}}(\varepsilon)\}$, with λ_1 as the marginal value of net worth of the bankers, defined in equation (24). We denote

the implied distribution of these variables by $\{c_1(\varepsilon,\xi), s_1(\varepsilon,\xi), \lambda_1(\varepsilon,\xi)\}$, where ξ is the sunspot selecting between the two equilibria.

Step 2: Given $\{c_1(\varepsilon,\xi), s_1(\varepsilon,\xi), \lambda_1(\varepsilon,\xi)\}$, we compute

$$A = \frac{\mathbb{E}_0[(c_1(\varepsilon,\xi)-\bar{c})^{-\sigma}s_1(\varepsilon,\xi)]}{\mathbb{E}_0[(c_1(\varepsilon,\xi)-\bar{c})^{-\sigma}]}, \qquad B = \frac{\mathbb{E}_0[\lambda_1(\varepsilon,\xi)s_1(\varepsilon,\xi)]}{\mathbb{E}_0[\lambda_1(\varepsilon,\xi)]},$$

on a grid from $(\sigma, \bar{c})$.[25] We select the $(\sigma, \bar{c})$ that makes sure that $A = B$. In this fashion, we are guaranteed that the Euler equations of bankers and households, equations (22) and (23), hold at date 0. We can then solve for $\{c_0, p_0^N/p_0^T, i_0\}$ using the following equations:

$$(c_0-\bar{c})^{-\sigma} = \beta\frac{(1+i_0^*)}{(p_0^N/p_0^T)^{-(1-\omega)}}\mathbb{E}_0[(c_1(\varepsilon,\xi)-\bar{c})^{-\sigma}s_1(\varepsilon,\xi)],$$

$$(c_0-\bar{c})^{-\sigma} = \beta(1+i_0^*)\mathbb{E}_0[(c_1(\varepsilon,\xi)-\bar{c})^{-\sigma}],$$

$$(1-\omega)c_0 = \left(\frac{p_0^N}{p_0^T}\right)^{\omega} e^N.$$

Finally, we select initial conditions $\{a_0, a_0^*, b_0^*\}$ such that the budget constraints of the bankers and the households is satisfied.

Step 3: To verify whether a nondollarized equilibrium exists at $\{a_0, a_0^*, b_0^*\}$, we can use Proposition 3 to compute the asset positions $\{a_1, a_1^*, b_1^*\}$. The nondollarized equilibrium exists if at $\{a_1, a_1^*, b_1^*\}$ networth at date 0 is positive and the collateral constraint at date 1 is slack for all the realizations of ε in our grid.

C The model with government interventions

This section presents the model with government interventions. Section C.1 gives some preliminaries and describes in detail the timing of events taking place at date 1. In Section C.2 we define how the private sector achieves an equilibrium for a given policy of the government, and we derive the conditions characterizing an equilibrium. We then present the decision problem of the government in Section C.3 and formally define an equilibrium in Section C.4. Section C.5 presents some numerical examples of equilibria in the model

[25] At this stage, we apply Gauss-Hermite quadrature to integrate over the ε shock.

with government interventions, and Section C.6 reports the proofs of the results that appear in the main body of the paper.

C.1 Preliminaries and timing

For the ex post analysis, we consider our economy from date 1 onward. The government enters date 1 with net positions in pesos and dollars, A_1 and A_1^*. In operating as a lender of last resort, the government can extend a credit line to the banks up to a limit $\bar{b}_2^{*g}$. These operations at date 1 are financed with its net assets, with tax revenues, and with debt issued to private savers. In period 2, the government collects the proceeds of the loans it extended to banks, repays the debt contracted in period 1, and taxes labor income. We assume that the government can transfer resources to households in a lump-sum fashion.

We make a number of assumptions in our analysis. First, we assume that the objective of the government is to maximize the present value of tradable production net of labor disutility,

$$[k_1^\alpha l_1^{1-\alpha} - v(l_1)] - G(k_2 - k_1) + \beta[k_2^\alpha l_2^{1-\alpha} - v(l_2)].$$

Second, we assume that the initial financial positions of the private sector, (a_1, a_1^*, b_1, b_1^*), are such that the economy would feature two stable continuation equilibria in absence of a financial intervention by the government, an equilibrium in which banks are unconstrained and one in which banks are constrained.

Third, and most importantly, we consider a specific timing of actions of the government and the private sector. Specifically, we divide period 1 into three subperiods.

In subperiod 1.i, only the market for financial claims is open. Banks and the government repay the claims issued in period 0 by issuing new claims. Trading of claims takes place as follows. First, investors (domestic and foreign) choose the maximum amount of period 2 claims they are willing to buy from the banks and the government. The limits are denoted, respectively, by $\bar{b}_2^{*p}$ and $-\bar{A}_2^*$. At this stage, investors also make a bid to exchange any amount of peso claims for dollar claims at the exchange rate $\tilde{s}_1$. Next, the government chooses the amount of period 2 claims to issue, $-A_2^* \leq -\bar{A}_2^*$, and the maximum amount of banks' period 2 claims it is willing to acquire, $\bar{b}_2^{*g}$. It also chooses a path for labor tax rates, τ_1, τ_2, and for nonnegative lump-sum dollar transfers to households, T_1^*, T_2^*. Banks choose how much to borrow from the government and the private sector:

$$b_2^{*p} \leq \bar{b}_2^{*p}, \quad b_2^{*g} \leq \bar{b}_2^{*g}.$$

The government chooses a vector $(\bar{b}_2^{*g}, \tau_1, \tau_2, T_1^*, T_2^*, A_2^*)$ such that, given the banks' choice of b_2^{*g}, the budget constraint of the government is satisfied:

$$T_1^* + \frac{1}{1+i_1^*} b_2^{*g} \leq \tau_1 \frac{w_1}{p_1^T} l_1 + A_1^* + \frac{A_1}{\tilde{s}_1} - \frac{1}{1+i_1^*} A_2^*.$$

The implicit assumption here is that the government can issue in subperiod 1.*i* claims due in subperiod 1.*ii* against the tax receipts $\tau_1(w_1/p_1^T)l_1$. Therefore, the government's financial resources are given by claims on taxes to be received in period 1.*ii* plus claims due in period 2, $-[1/(1+i_1^*)]A_2^*$. The government uses these resources to settle its claims from period 0 and to finance transfers to households and lending to banks. The peso claims inherited from period 0 are converted into dollars at the rate $\tilde{s}_1$ because that is the exchange rate at which agents are willing to trade pesos for dollars in subperiod 1.*i*.

In subperiod 1.*ii*, the capital market, factor markets, and good markets all clear, and the government uses labor tax receipts to repay its claims due in 1.*ii*.

In subperiod 1.*iii*, banks make a take-it-or-leave-it offer to the investors to renegotiate their debt down to $\theta q_1 k_2$. Investors accept the offer, since in case of default they can only recover a fraction θ of the assets. Creditors expect to recover only a fraction θ of the assets if debt renegotiation fails and they can perfectly coordinate during a renegotiation, so they will accept any offer greater than or equal to $\theta q_1 k_2$.

C.2 Private equilibrium

We now characterize an equilibrium of the private sector given the credit limit $\bar{b}_2^{*p}$, a policy of the government $(\bar{b}_2^{*g}, \tau_1, \tau_2, T_1^*, T_2^*, \bar{A}_2^*)$, and the exchange rate $\tilde{s}_1$ prevailing in subperiod 1.*i*. We first describe the decision problem of households and bankers.[26] We next define a private equilibrium and discuss the conditions characterizing it.

Households. In subperiod 1.*i*, the households receive payments of their domestic and foreign bond holdings. In subperiod 1.*ii*, they choose consumption of tradable and non-tradable goods, labor effort, and savings in foreign bonds in order to maximize

$$\mathbb{E}\left[\sum_{t=1}^{2} \beta^t U\left(\left(c_t^T - \chi \frac{l_t^{1+\phi}}{1+\phi}\right)^{\omega} \left(c_t^N\right)^{1-\omega}\right)\right],$$

[26]The decision problems of the remaining agents are identical to those in Section 2.

subject to the budget constraints

$$
\begin{aligned}
c_1^T + \frac{p_1^N}{p_1^T} c_1^N + \frac{1}{1+i_1^*} a_2^* &\leq (1-\tau_1)\frac{w_1}{p_1^T} l_1 + \frac{p_1^N}{p_1^T} e^N + \pi_1 + \frac{a_1}{\tilde{s}_1} + a_1^* + T_1^*, \\
c_2^T + \frac{p_2^N}{p_2^T} c_2^N &\leq (1-\tau_2)\frac{w_2}{p_2^T} l_2 + \frac{p_2^N}{p_2^T} e^N + a_2^* + T_2^*.
\end{aligned}
$$

Note that a_2^* here represents purchases of bonds from foreign investors that take place in subperiod 1.*ii*, gross of the credit that households extend to the banks and the government in subperiod 1.*i*.

The first order conditions characterizing $\{c_t^T, c_t^N, l_t\}_{t=1,2}$ and $\{a_2^*\}$ are

$$
\begin{aligned}
\left(\frac{c_1^N}{(c_1^T - v(l_1))}\right)^{1-\omega} U'(\tilde{c}_1) &= \beta(1+i_1^*)\left(\frac{c_2^N}{(c_2^T - v(l_2))}\right)^{1-\omega} U'(\tilde{c}_2), \\
\omega c_t^N &= (1-\omega)\frac{p_t^T}{p_t^N}(c_t^T - v(l_t)) \quad \text{for } t = 1,2, \\
\chi l_t^{\phi} &= (1-\tau_t)\frac{w_t}{p_t^T} \qquad\qquad\qquad \text{for } t = 1,2,
\end{aligned}
$$

where $\tilde{c}_t = \left(c_t^T - v(l_t)\right)^{\omega}\left(c_t^N\right)^{1-\omega}$ and $w_t/p_t^T = (1-\alpha)k_t^{\alpha} l_t^{-\alpha}$.

Bankers. In subperiod 1.*i*, the bankers pay back their debt b_1^* and b_1 and borrow from private savers b_2^{*p} and the government b_2^{*g} in order to finance purchases of capital in subperiod 1.*ii*. The budget constraint of the bankers is

$$
q_1(k_2 - k_1) = \alpha k_1^{\alpha} l_1^{1-\alpha} + \frac{1}{1+i_1^*}(b_2^{*p} + b_2^{*g}) - b_1^* - \frac{b_1}{\tilde{s}_1},
$$

while the borrowing constraints are

$$
b_2^{*p} \leq \bar{b}_2^{*p}, b_2^{*g} \leq \bar{b}_2^{*g}.
$$

Bankers' demand for capital goods satisfies the asset pricing conditions

$$
q_1 \leq \frac{1}{1+i_1^*}\alpha k_2^{\alpha-1} l_2^{1-\alpha},
$$

with equality if the borrowing constraints are slack.

In subperiod 1.*iii*, bankers renegotiate their debt with the government and private savers. If $\theta q_1 k_2 < [1/(1+i_1^*)](b_2^{*p} + b_2^{*g})$, bankers renegotiate their obligations. Because the government provides a guarantee of private deposits up to $\theta q_1 k_2$, we have that private savers maintain their claims b_2^{*p} after the renegotiation stage, while the claims of the government become[27]

$$\hat{b}_2^{*g} = \min\left\{\theta q_1 k_2(1+i_1^*) - b_2^{*p}, b_2^{*g}\right\}.$$

Private equilibrium. A *private equilibrium* given the credit limit $\bar{b}_2^{*p}$, the exchange rate $\tilde{s}_1$, and a government's policy $(\bar{b}_2^{*g}, \tau_1, \tau_2, T_1^*, T_2^*, A_2^*)$, is a vector of prices $(Q_t, i_t^*, r_t, w_t, p_t^T, p_t^N, s_t)$, households' choices $\left(\{c_t^T, c_t^N, l_t\}_{t=1,2}, a_2^*\right)$, bankers' portfolio choices $(k_2, b_2^{*p}, b_2^{*g})$, and choices of capital good producers ι_1 such that: **(i)** the choices of households, banks, capital good producers, and foreign investors are individually optimal; **(ii)** markets clear; **(iii)** the law of one price holds; **(iv)** the price index is constant.

Equilibrium conditions. We can verify by inspecting the households' decision problem that Lemma 1 extends to this setting, with the exception that households now equalize $\tilde{c}_t$ between date 1 and 2 rather than $(c_t^T)^\omega (c_t^N)^{1-\omega}$. Moreover, because in equilibrium $c_t^N = e^N$, we must have that $\left(c_t^T - v(l_t)\right)$ is constant between date 1 and 2. Coupled with the monetary rule, this result also implies that (p_t^N, p_t^T, s_t) are time invariant. Also, the optimality of foreign investors implies that $1/(1+i_1^*) = \beta$. Hence, we can collapse the problem of determining a private equilibrium into that of finding prices $(w_1/p_1^T, w_1/p_2^T, s_1, p_1^T/p_1^N, q_1)$, households' choices (c_1^T, l_1, l_2), and bankers' choices $(k_2, b_2^{*p}, b_2^{*g})$ such that

$$\omega e^N = (1-\omega)[c_1^T - v(l_1)]\frac{p_1^T}{p_1^N} \tag{A.12}$$

$$\frac{w_t}{p_t^T} = (1-\alpha)k_t^\alpha l_t^{-\alpha} \tag{A.13}$$

$$s_1 = \left(\frac{p_1^T}{p_1^N}\right)^{1-\omega} \tag{A.14}$$

$$q_1 = \phi_0 + \phi_1(k_2 - k_1)^\eta \tag{A.15}$$

$$(1-\tau_t)\frac{w_t}{p_t^T} = \chi l_t^\phi \tag{A.16}$$

$$\begin{aligned} c_1^T - v(l_1) &= \frac{1}{1+\beta}\left\{\sum_{t=1}^{2}\beta^{t-1}\left[(1-\tau_t)\frac{w_t}{p_t^T}l_t - v(l_t)\right] + \right. \\ &\quad + \left. \pi(q_1) + a_1^* + T_1^* + \beta T_2^* + \frac{a_1}{\tilde{s}_1}\right\} \end{aligned} \tag{A.17}$$

[27] For ease of notation, we rule out the possibility that $\theta q_1 k_2 < [1/(1+i_1^*)]b_2^{*p}$.

$$q_1(k_2 - k_1) = \left[\alpha k_1^{\alpha} l_1^{1-\alpha} + \beta(b_2^{*p} + b_2^{*g}) - b_1^* - \frac{b_1}{\tilde{s}_1}\right] \tag{A.18}$$

$$q_1 \leq \beta\alpha k_2^{\alpha-1} l_2^{1-\alpha}, \tag{A.19}$$

where the last expression holds with equality if $b_2^{*p} < \bar{b}_2^{*p}$ and $b_2^{*g} < \bar{b}_2^{*g}$.[28]

For future reference, we now detail how the equilibrium quantity and price of capital are determined in this version of the model. Suppose first that equation (A.19) holds with strict inequality in a private equilibrium. Then, the equilibrium quantity of capital is such that equations (A.15) and (A.18) hold with $\bar{b}_2^* = \bar{b}_2^{*p} + \bar{b}_2^{*g}$, that is, k_2 solves

$$[\phi_0 + \phi_1(k_2 - k_1)^{\eta}](k_2 - k_1) = \left[\alpha k_1^{\alpha} l_1^{1-\alpha} + \beta\bar{b}_2^* - b_1^* - \frac{b_1}{\tilde{s}_1}\right].$$

Note that this equation has a unique solution, as long as the right-hand side is positive.

If, instead, equation (A.19) holds with equality in a private equilibrium, then the optimal quantity of capital is such that equations (A.15) and (A.19) hold, that is, it satisfies

$$[\phi_0 + \phi_1(k_2 - k_1)^{\eta}] = \beta\alpha k_2^{\alpha-1} l_2^{1-\alpha},$$

which again admits a unique solution under standard assumptions.

As we shall see, the absence of equilibrium multiplicity in the capital market conditional on $\bar{b}_2^*$ does not rule out the type of multiplicity analyzed in Section 3, once the credit limits are endogeneized in a rational expectations equilibrium.

C.3 The decision problem of the government

The government enters date 1 with net positions in pesos and dollars, $A_1 and A_1^*$, and chooses $\bar{b}_2^{*g}$ along with $(\tau_1, \tau_2, T_1^*, T_2^*, A_2^*)$ in order to maximize

$$[k_1^{\alpha} l_1^{1-\alpha} - v(l_1)] - G(k_2 - k_1) + \beta[k_2^{\alpha} l_2^{1-\alpha} - v(l_2)].$$

[28] There is an element of indeterminacy when the borrowing constraints are slack because the system of equations only pins down $b_2^* = b_2^{*p} + b_2^{*g}$ in those cases. To resolve this indeterminacy, we assume that $b_2^{*g} = \max\{b_2^* - \bar{b}_2^{*p}, 0\}$, that is, the bankers preferably borrow from private savers.

The budget constraints for the government are

$$\begin{aligned} T_1^* + \beta b_2^{*g} &\leq \tau_1 \frac{w_1}{p_1^T} l_1 + A_1^* + \frac{A_1}{\tilde{s}_1} - \beta A_2^* \\ T_2^* &\leq \tau_2 \frac{w_2}{p_2^T} l_2 + \hat{b}_2^{*g} + A_2^*, \end{aligned}$$

with $\hat{b}_2^{*g}$ given by

$$\hat{b}_2^{*g} = \min \left\{ \theta q_1 k_2 / \beta - b_2^{*p}, b_2^{*g} \right\}.$$

When choosing the optimal policy, the government faces the debt limit $A_2^* \geq \bar{A}_2^*$, and it takes as given the optimal behavior of the private sector in periods 1 and 2. This latter requirement means that $(w_1/p_1^T, w_2/p_2^T, q_1, k_2, l_1, l_2, b_2^{*p}, b_2^{*g})$ must be part of a private equilibrium defined in the previous section.

For our numerical examples, it will be useful to assume that there is a maximum labor tax rate $\bar{\tau}$ that the government is willing to impose in period 2.[29] Given the optimality conditions of the private sector, this implies that the maximum tax revenue that the government can obtain in period 2 takes the form $\Xi k_2^{\alpha(1+\phi)/(\alpha+\phi)}$, with

$$\Xi = (1-\alpha)^{\frac{1+\phi}{\alpha+\phi}} \chi^{-\frac{1-\alpha}{\alpha+\phi}} \bar{\tau}(1-\bar{\tau})^{\frac{1-\alpha}{\alpha+\phi}}.$$

By varying the parameter $\bar{\tau}$, we will be able to study how the optimal policy of the government changes with its fiscal capacity.

It is more convenient to work with a primal formulation of the problem in which the government directly chooses allocations. For that purpose, we can use equations (A.13) and (A.16) to express the tax revenues at date t as

$$\mathcal{L}(l_t, k_t) = (1-\alpha) k_t^\alpha l_t^{1-\alpha} - \chi l_t^{1+\phi}. \tag{A.20}$$

Then, we can rewrite the decision problem of the government as choosing the borrowing limit for banks $\bar{b}_2^*$ and the labor effort of the households to solve the following problem:

$$\max_{\bar{b}_2^*, l_1, l_2} [k_1^\alpha l_1^{1-\alpha} - v(l_1)] - G(k_2 - k_1) + \beta[k_2^\alpha l_2^{1-\alpha} - v(l_2)],$$

[29] A maximum tax rate can be microfounded by assuming that the government has the option to default on its claims in period 2, that all debt is held by domestic agents, and that the default penalty takes the form of a wedge $\bar{\tau}$ in the labor supply decision of households.

subject to

$$
\begin{aligned}
\beta \bar{b}_2^* &\leq \mathcal{L}(l_1, k_1) + A_1^* + \frac{A_1}{\tilde{s}_1} + \beta(\bar{b}_2^{*p} - \bar{A}_2^*) \\
\min\{\theta q_1 k_2 - \beta b_2^*, 0\} &\leq \sum_{t=1}^{2} \beta^{t-1} \mathcal{L}(l_t, k_t) + \left(A_1^* + \frac{A_1}{\tilde{s}_1}\right) \\
\mathcal{L}(l_t, k_t) &= (1-\alpha) k_t^\alpha l_t^{1-\alpha} - \chi l_t^{1+\phi} \\
\mathcal{L}(l_2, k_2) &\leq (1-\alpha)^{\frac{1+\phi}{\alpha+\phi}} \chi^{-\frac{1-\alpha}{\alpha+\phi}} \bar{\tau} (1-\bar{\tau})^{\frac{1-\alpha}{\alpha+\phi}} k_2^{\alpha \frac{1+\phi}{\alpha+\phi}} \\
b_2^* &\leq \bar{b}_2^* \\
q_1(k_2 - k_1) &= \left[\alpha k_1^\alpha l_1^{1-\alpha} + \beta b_2^* - b_1^* - \frac{b_1}{\tilde{s}_1}\right] \\
q_1 &= \phi_0 + \phi_1 (k_2 - k_1)^\eta \\
q_1 &\leq \beta \alpha k_2^{\alpha-1} l_2^{1-\alpha}.
\end{aligned}
$$

The first constraint is a liquidity constraint, stating that the maximum credit that can go to banks in period 1 cannot exceed the total credit that private savers extend to banks and the government, the government net asset positions at the beginning of the period, and the tax revenues that the government collects at date 1. The second constraint is the intertemporal budget constraint of the government, stating that any loss that the government makes with financial interventions must be backed by its net assets at date 1 and the present value of its tax revenues. The remaining expressions define tax revenues and determine (b_2^*, q_1, k_2) that are consistent with a private equilibrium for given $(\bar{b}_2^*, l_1, l_2)$. Given the allocations chosen by the government, it is then straightforward to derive the policies of the government implementing them.

First order conditions. It is useful to derive the first order conditions of this problem in order to understand the optimal behavior of the government. For this purpose, we let μ and ψ be the Lagrange multipliers associated, respectively, with the liquidity constraint and the intertemporal budget constraint. Then, maximizing the objective function with respect to $(\bar{b}_2^*, l_1, l_2)$ subject to these two constraints and to the optimizing behavior of the private sector leads to the following conditions:[30]

[30] These conditions assume that the constraint on second-period tax revenues doesn't bind.

$$[\beta\alpha k_2^{\alpha-1}l_2^{1-\alpha} - G'(k_2-k_1)]\frac{\partial k_2}{\partial \bar{b}_2^*} = \beta\mu - \psi\left\{\beta\frac{\partial \mathcal{L}(l_2,k_2)}{\partial k_2}\frac{\partial k_2}{\partial \bar{b}_2^*} - \right.$$
$$\left. - \left[\frac{\partial \theta q_1 k_2}{\partial b_2^*} - \beta\right]\frac{\partial b_2^*}{\partial \bar{b}_2^*}\mathbf{1}\left(\theta q_1 k_2/\beta < b_2^*\right)\right\} \quad \text{(A.21)}$$

$$[(1-\alpha)k_1^{\alpha}l_1^{\alpha-1} - v'(l_1)] = -[\beta\alpha k_2^{\alpha-1}l_2^{1-\alpha} - G'(k_2-k_1)]\frac{\partial k_2}{\partial l_1} - \mu\frac{\partial \mathcal{L}(l_1,k_1)}{\partial l_1} - \psi\left[\frac{\partial \mathcal{L}(l_1,k_1)}{\partial l_1} + \right.$$
$$\left. + \beta\frac{\partial \mathcal{L}(l_2,k_2)}{\partial k_2}\frac{\partial k_2}{\partial l_1} - \frac{\partial \theta q_1 k_2}{\partial l_1}\mathbf{1}\left(\theta q_1 k_2/\beta < b_2^*\right)\right] \quad \text{(A.22)}$$

$$\beta[(1-\alpha)k_2^{\alpha}l_2^{\alpha-1} - v'(l_2)] = -[\beta\alpha k_2^{\alpha-1}l_2^{1-\alpha} - G'(k_2-k_1)]\frac{\partial k_2}{\partial l_2} - \psi\left\{\beta\left[\frac{\partial \mathcal{L}(l_2,k_2)}{\partial l_2} + \right.\right.$$
$$\left.\left. + \frac{\partial \mathcal{L}(l_2,k_2)}{\partial k_2}\frac{\partial k_2}{\partial l_2}\right] - \frac{\partial \theta q_1 k_2}{\partial l_2}\mathbf{1}\left(\theta q_1 k_2/\beta < b_2^*\right)\right\}, \quad \text{(A.23)}$$

where the derivatives of $(k_2, q_1, b_2, \mathcal{L}_t)$ are constructed using the remaining constraints in the problem of the government.

Equation (A.21) represents the trade-off for the government when changing $\bar{b}_2^*$. When bankers are constrained, an increase in $\bar{b}_2^*$ increases banks' investment, and a higher k_2 raises the objective function of the government because in the constrained region, $\alpha k_2^{\alpha-1}l_2^{1-\alpha} \geq G'(k_2-k_1) = q_1$. A higher credit limit, however, potentially has some fiscal costs for the government, and these are represented on the right-hand side of (A.21). The first term is unambiguously positive, stating that a higher $\bar{b}_2^*$ tightens the liquidity constraint. The second term tells us that an increase in $\bar{b}_2^*$ implies a change in fiscal resources at date 2, which might tighten or relax its intertemporal budget constraint.[31]

Equations (A.22) and (A.23) determine the optimal tax policy of the government. To understand the logic of these two equations, consider first the case in which the government is not liquidity constrained ($\bar{b}_2^{*p}$ or $-\bar{A}_2^*$ large enough). In that case, we know that $\mu = 0$, and the government implements the unconstrained capital allocation. In this scenario, the

[31]The increase in investment associated with higher $\bar{b}_2^*$ raises future tax revenues and unambiguously relaxes the intertemporal budget constraint. However, a higher $\bar{b}_2^*$ potentially magnifies the losses that the government makes in its loans to the banks, which would tighten it.

optimal tax policy of the government satisfies

$$
\begin{aligned}
(1-\alpha)k_1^{\alpha}l_1^{\alpha-1} - v'(l_1) &= -\psi\frac{\partial\mathcal{L}(l_1,k_1)}{\partial l_1} \\
(1-\alpha)k_2^{\alpha}l_2^{\alpha-1} - v'(l_2) &= -\psi\left[\frac{\partial\mathcal{L}(l_2,k_2)}{\partial l_2} + \frac{\partial\mathcal{L}(l_2,k_2)}{\partial k_2}\frac{\partial k_2}{\partial l_2}\right].
\end{aligned}
$$

If the government is a net debtor, the intertemporal constraint binds ($\psi > 0$), and the optimal policy consists of choosing taxes in periods 1 and 2 in order to smooth labor supply distortions over time. In the more general case in which the government is liquidity constrained ($\mu > 0$), the government has an incentive to tax more in period 1 because this relaxes its liquidity constraint and allows more credit to be extended to the banks.

C.4 Equilibrium

An equilibrium of the model with government interventions is given by debt limits set by the private sector, a government policy, and prices and quantities such that:

i. Taking $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$ as given, the government chooses the policy $(\bar{b}_2^{*g}, \tau_1, \tau_2, T_1, T_2, A_2^*)$ to solve its decision problem, subject to the optimal behavior of the private sector;

ii. Prices and quantities constitute a competitive equilibrium given the debt limits set by investors, the exchange rate $\tilde{s}_1$ in subperiod $1.i$, and the government policy;

iii. Debt limits and the exchange rate in subperiod $1.i$ satisfy the rational expectations.

C.5 Numerical Examples

We now present some numerical examples of the equilibria in the model with government interventions. The section has two main objectives. First, we provide an illustration of Proposition 5 in the main text and show that the model with government interventions admits multiple equilibria. Second, we study the optimal policy of the government and how it is affected by some key model parameters.

Our first step consists of parametrizing the model. We borrow most of the parameters from the numerical example of Section 4. Specifically, we set $\omega = 0.50$, $\alpha = 0.85$, $\beta = 1.00$, $e^N = 0.70$, $\theta = 0.90$, $\phi_0 = 0.40$, $\phi_1 = 0.20$, $\eta = 3.00$. In the model with government interventions, we also need to specify the preference parameters for the households and the maximum tax rate that the government can impose at date 2. We consider three different specifications. As a benchmark, we set $\phi = 0.33$, consistent with a Frisch elasticity of 3, and $\bar{\tau} = 0.05$. We consider also two alternative specifications: one with less elastic labor

Table A-1: Equilibria with and without government interventions

	No interventions	Benchmark	Less elastic labor	High fiscal capacity
		Good equilibrium		
k_2	1.698	1.698	1.680	1.698
q_1	0.946	0.946	0.925	0.946
$\bar{b}_2^{*p}$	1.446	1.446	1.399	1.446
$-\bar{A}_2^*$	0.014	0.014	0.014	0.123
$\bar{b}_2^{*g}$		0.000	0.000	0.000
$-A_2^*$		0.000	0.000	0.000
$\mathcal{L}_1$		0.000	0.000	0.000
$\mathcal{L}_2$		0.000	0.000	0.000
$\hat{b}_2^{*g}$		0.000	0.000	0.000
		Bad Equilibrium		
k_2	0.427	0.659	0.821	
q_1	0.400	0.409	0.428	
$\bar{b}_2^{*p}$	0.154	0.243	0.316	
$-\bar{A}_2^*$	0.004	0.006	0.007	
$\bar{b}_2^{*g}$		0.014	0.021	
$-A_2^*$		0.006	0.007	
$\mathcal{L}_1$		0.009	0.014	
$\mathcal{L}_2$		0.006	0.007	
$\hat{b}_2^{*g}$		0.000	0.000	

($\phi = 0.50$) and one with more fiscal capacity for the government ($\bar{\tau} = 0.50$). In all of these three specifications, we set χ so that $l_1 = 1.00$ when labor taxes equal zero.

The initial conditions for assets and liabilities mimic those of the dollarized equilibrium in Table 1, with $k_1 = 0.30$, $a_1 = b_1 = 0.05$, $a_1^* = 0.71$, and $b_1^* = 0.37$. In these examples, we also set $A_1 = A_1^* = 0$. These initial conditions guarantee that the economy features two stable equilibria in absence of government interventions, with the banks being unconstrained in the good equilibrium.

Table A-1 presents features of the stable equilibria for these three different parametrizations. We report the price and quantity of capital along with the policy sets by the government. As a benchmark, we also report the price and quantity of capital in the version of the model where the government does not intervene.

Starting from the benchmark parametrization, we can first verify that the model with government interventions features two stable equilibria. In the good equilibrium, bankers are unconstrained, and the government has no incentives to intervene. Moreover, labor taxes are optimally set to zero because the government does not have inherited debt to repay. Hence, the price and quantity of capital in this scenario are equal to the ones that arise in the model without government interventions.

In the bad equilibrium, instead, the government extends a credit line to the banks. The credit line allows the banks to invest more, which leads to a higher price and quantity of capital relative to what would happen without the intervention. This policy spurs collateral values, and it induces the private sector to lend more to the banks. However, it also entails costs for the government. Indeed, we can see from Table A-1 that the government is liquidity constrained, as it borrows up to the credit limit $-\bar{A}_2^*$. So, part of these interventions are necessarily financed by introducing distortionary taxes in period 1. Moreover, the table shows that the government makes losses on these bank loans ($\hat{b}_2^{*g} = 0.00$), and it needs to repay its debt by also taxing labor at date 2.[32]

This is an equilibrium because, with low capital accumulation, savers extend little credit to the government as tax revenues at date 2 are small. But for a liquidity-constrained government, it is costly to act as a lender of last resort. Thus, the government cannot sustain the banks, and the associated financial crisis results in low capital accumulation.

The remaining columns report the results for the two alternative parametrizations. A lower elasticity of labor supply reduces the distortionary costs of taxation. Holding everything else constant, this means that it is less costly for the government to act as a lender of last resort. Hence, the price and quantity of capital improve relative to the benchmark specification.

A similar comparative static result occurs when we consider an increase in $\bar{\tau}$. To understand why, consider the bad equilibrium under the benchmark specification, and assume that $\bar{\tau}$ increases marginally. The government can borrow more from savers. This means that the government could implement the same allocation by reducing labor taxes in period 1, increasing borrowing, and repaying the higher debt with an increase in the labor tax at date 2. This reduces the costs of financial market interventions because it allows the government to better smooth tax distortions over time. Moreover, as we have seen in the discussion of Proposition 4, an increased ability to borrow moves the government closer to the region in which interventions become self-financed. Both of these forces push the government to increase the size of its interventions. In our numerical example, this force is so strong that the bad equilibrium is eliminated.

[32]This is actually a property of the bad equilibrium as we demonstrate in the proof of Proposition 6.

C.6 Proofs of results in Section 5

Proof of Proposition 4

We have already shown in the main text that the allocation of a competitive equilibrium in which bankers are unconstrained and the government sets zero labor taxes is the solution to the following problem:

$$\max_{l_1,l_2,k_2} [k_1^{\alpha} l_1^{1-\alpha} - v(l_1)] - G(k_2 - k_1) + \beta[k_2^{\alpha} l_2^{1-\alpha} - v(l_2)],$$

and it thus provides an upper bound to the objective function of the government. The proof of the result consists of showing that the policy described in Proposition 4 implements this allocation and that it is feasible for the government.

To prove that the policy implements this allocation, we can verify from equation (A.18) that the credit limit offered by the government allows the bankers to finance their unconstrained demand for capital for every $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$. Hence, under the policy of the government, the private sector achieves an unconstrained equilibrium with zero labor income taxes.

To prove feasibility, equation (30) in the statement of the proposition guarantees that the liquidity constraint in the decision problem of the government is satisfied for every $(\bar{b}_2^{*p}, -\bar{A}_2^*, \tilde{s}_1)$. To show that the intertemporal budget constraint also holds with $\mathcal{L}(l_t, k_t) = 0$, we can first see that $A_1^* + \frac{A_1}{\tilde{s}_1} \geq 0$ for all $\tilde{s}_1 \geq s_1^{Good}$ because of the assumption that this inequality holds at $\tilde{s}_1 = s_1^{Good}$ and that $A_1 < 0$. Moreover, we can show that

$$\theta q_1 k_2 \geq \beta b_2^*$$

under the policy of the government. This inequality is satisfied at $\tilde{s}_1 = s_1^{Good}$ because in the unconstrained equilibrium, the collateral constraint is slack by definition. Moreover, it also holds for $\tilde{s}_1 > s_1^{Good}$ because $b_1 \geq 0$ by assumption. It follows that the intertemporal budget constraint of the government is also satisfied.

Thus, the policy of the government is feasible and achieves the maximum of its objective function. As a result, it is the best response of the government. □

Proof of Proposition 6

As a preliminary step, we first define the function $\mathcal{S}(k_2)$. Given k_2, $\mathcal{S}(k_2)$ returns the exchange rate consistent with an equilibrium with no government intervention. To obtain it, we can rearrange the equations defining a private equilibrium in the absence of government intervention. First, we can rewrite equation (A.17) as

$$c_1^T - v(l_1) = \frac{1}{1+\beta}\left\{\sum_{t=1}^{2}\beta^{t-1}\left[\frac{w_t}{p_t^T}l_t - v(l_t)\right] + \pi(q_1) + (a_1^* + A_1^*) + \frac{a_1 + A_1}{\tilde{s}_1}\right\},$$

where we have used $\tau_t = 0$ and substituted for T_1 and T_2 using the government budget constraints. Next, we can use equation (A.16) in the households' problem and the functional form for $v(.)$ to obtain

$$\frac{w_t}{p_t^T}l_t - v(l_t) = \frac{\phi\chi}{1+\phi}l_t^{1+\phi}.$$

The function $\mathcal{S}$ then solves

$$\frac{\mathcal{S}(k_2)^{\frac{1}{(1-\omega)}}}{1+\beta}\left\{\frac{\phi\chi}{1+\phi}\sum_{t=1}^{2}\beta^{t-1}l_t^{1+\phi} + \pi(q_1) + (a_1^* + A_1^*) + \frac{a_1 + A_1}{\mathcal{S}(k_2)}\right\} = \frac{\omega}{1-\omega}e^N, \tag{A.24}$$

where q_1 and l_2 are increasing functions of k_2 given, respectively, by equation (A.15) and

$$l_2 = \left[\frac{(1-\alpha)k_2^\alpha}{\chi}\right]^{\frac{1}{\alpha+\phi}}.$$

The function $\mathcal{S}(.)$ is decreasing in k_2 to the extent that $a_1 + A_1 \geq 0$, a maintained assumption in our analysis.

To prove the proposition, we first note that in any equilibrium, we must have $k_2 \leq k_2^{Good}$ and $s_1 \geq \mathcal{S}(k_2)$. The first inequality arises because banks' investment is maximized when $\tau_2 = 0$ and the banks are unconstrained. The proof of the second inequality is more involved. By construction of $\mathcal{S}(.)$, we must have $s_1 = \mathcal{S}(k_2)$ in any equilibrium in which the government does not intervene. Consider now an equilibrium in which the government intervenes and sets strictly positive taxes at some t. In such an equilibrium, the collateral constraint of the bankers binds. Moreover, the government does not receive any payments from the bankers at date 2, $\hat{b}_2^{*g} = 0$.[33] Therefore, government reserves and the collected tax revenues are transferred to the bankers, and we must have $T_1 = T_2 = 0$. Thus, the exchange rate in this candidate equilibrium must satisfy

$$\frac{s_1^{\frac{1}{(1-\omega)}}}{1+\beta}\left\{\frac{\phi\chi}{1+\phi}\sum_{t=1}^{2}\beta^{t-1}l_t^{1+\phi} + \pi(q_1) + a_1^* + \frac{a_1}{s_1}\right\} = \frac{\omega}{1-\omega}e^N, \tag{A.25}$$

[33] Rational expectations, in fact, require that $\bar{b}_2^{*p}$ equals the pledgeable value of the bank collateral. Moreover, because the bankers are constrained, they borrow up to $\bar{b}_2^{*p} + \bar{b}_2^{*g}$. Hence, in the renegotiation stage, they will repay only private savers and set $\hat{b}_2^{*g} = 0$.

for each k_2, with l_t being given by

$$l_t = \left[\frac{(1-\tau_t)k_t^{\alpha}}{\chi}\right]^{\frac{1}{\phi+\alpha}}.$$

Comparing equations (A.24) and (A.25), we can verify that $s_1 < \mathcal{S}(k_2)$ when $\tau_t > 0$ because labor is strictly decreasing in τ_t and $A_1^* + A_1/\mathcal{S}(k_2) \geq 0$ for all $k_2 \leq k_2^{Good}$ by assumption.

We now show that if condition (31) is satisfied, then there is no equilibrium with $k_2 \leq k_2^{Good}$ and $s_1 \geq \mathcal{S}(k_2)$, unless $k_2 = k_2^{Good}$ and $s_1 = \mathcal{S}(k_2^{Good})$. Suppose, by contradiction, that such (k_2, s_1) are part of an equilibrium. Then, by rational expectations, we would have

$$\begin{aligned} \beta\bar{b}_2^{*p} &= \theta[\phi_1 + \phi_1(k_2 - k_1)^{\eta}]k_2 \\ -\beta\bar{A}_2^* &\geq \beta\Xi k_2^{\alpha\frac{1+\phi}{\alpha+\phi}}. \end{aligned}$$

These two conditions, coupled with (31), imply that condition (30) in Proposition 4 is satisfied at $\tilde{s}_1 = s_1$. Hence, the best response of the government is to implement the unconstrained equilibrium allocation and prices. This contradicts the assumption that $k_2 \leq k_2^{Good}$ and $s_1 \geq s_1^{Good}$, unless these two hold with equality. □