General Equilibrium Dampened

(i) from Micro to Macro (ii) Forward Guidance

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Motivation

- **GE effects key to macroeconomics** (and elsewhere)
  - upend partial-equilibrium (PE) intuitions
  - limit usefulness of micro-based evidence a la Mian-Sufi
  - drive interpretations of phenomena + policy implications

- But: GE effects hinge on
  - common knowledge (CK) of structure and state of economy
  - immense coordination in beliefs and behavior
    - ⋆ all hardwired in solution concept + info assumptions

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This talk, Part I: Dampening GE

- Formalize notion

  "GE Adjustment Takes Times"

- Framework: abstract but flexible "supply and demand"

- Main result: Equivalence between
  - relax solution concept $\rightarrow$ Tattonment ("off equilibrium")
  - relax info / CK $\rightarrow$ imperfect coordination ("on equilibrium")

- Broader lessons/implications:
  - lack of CK = relaxation of RE solution concept = dampen GE
  - resuscitate PE intuitions in GE settings
  - enhance value of empirical work a la Mian-Sufi

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This talk, Part II: Application to Forward Guidance

- Context: A New-Keynesian economy during a liquidity trap
- Question: Ability to stimulate economy my promising low interest rates after ZLB has ceased to bind
- Puzzling prediction: Ability is large and increases with horizon at which forward guidance operates
- Our contribution:
  - puzzle driven solely by GE effects
  - lack of CK $\rightarrow$ anchors expectations of income and inflation
    $\rightarrow$ attenuates relevant GE effects $\rightarrow$ reduces power of forward guidance
  - additional results: paradox of flexibility, discounted Euler/NKPC...

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Related Literature

- Part I: Higher-order uncertainty in macroeconomics
  - Morris and Shin (1998, 2000, 2002), Woodford (2003), Angeletos and Pavan (2007), Angeletos and La'O (2009), Nimark (2011), etc
  - Angeletos and Lian (2016): chapter in *Handbook of Macroeconomics*

- Part II: Forward guidance
  - Wiederholt (2016), Farhi and Werning (2016)
  - McKay, Nakamura and Steinsson (2016a,b), Gabaix (2016)
  - Garcia-Schmidt and Woodford (2015)

First paper: Angeletos and Lian (2016a)

Dampening General Equilibrium: Macro is Micro in the Short Run

Framework

- Minimal framework for studying PE vs GE, and micro vs macro
  - many locations, competitive firms and households
  - idiosyncratic and aggregate shocks
  - two (relative) prices $\rightarrow$ three goods

    = numeraire + another tradable + one non-tradable per location
- What's next?
  - micro-foundations, demand and supply
  - review standard predictions
  - two variants: (i) Tatonnment and (ii) Incomplete Info/Lack of CK

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Supply

- Representative competitive firm at each location $i \in [0,1]$.
- Technology (production possibilities set):

$$F(q_i, q_i^*, q_i^Z; a_i) \leq 0$$

  - q_i, q_i^*, q_i^Z = production of non-tradables, tradables, numeraire
  - a_i = technology shock = "supply shock"

Supply

- Firm's problem

$$\max \left\{ p_i q_i + p^* q_i^* + q_i^Z \right\}$$

$$\text{s.t. } F(q_i, q_i^*, q_i^Z; a_i) \leq 0$$

  - $p_i, p^* =$ price of non-tradables, tradables

- Supply of local non-tradable

$$q_i = S(p_i, p^*, a_i)$$

- Supply of tradable and numeraire:

$$q_i^* = S^*(...) \qquad q_i^Z = S^Z(...)$$

Demand

- Representative competitive household at each location $i \in [0,1]$.
- Preferences:

$$u_i = U(c_i, c_i^*, c_i^Z; \xi_i)$$

  - ξ_i = preference shock = "demand shock"

- Budget constraint:

$$p_i c_i + p^* c_i^* + c_i^Z = y_i$$

  - y_i = income $= p_i q_i + p^* q_i^* + q_i^Z$

Demand

- Demand for local non-tradable:

$$c_i = D\left(p_i, p^*, \xi_i, y_i\right)$$

- Demand for tradable and numeraire

$$c_i^* = D^*\left(...\right) \qquad c_i^Z = D^Z\left(...\right)$$

Partial Equilibrium

- Partial equilibrium $\equiv$
  - market clearing for non-tradable, but arbitrary p^*
  - i.e., momentarily allow market for tradable to be off equilibrium
- Let $\theta_i = (a_i, \xi_i)$. Quantity and price of non-tradable

$$c_i = q_i = Q(p^*, \theta_i) \qquad \text{and} \qquad p_i = P(p^*, \theta_i)$$

- Net (excess) demand for tradable

$$n_i \equiv c_i^* - q_i^* = N^*(p^*, \theta_i)$$

General Equilibrium

- Market clearing for tradable: p^* solves

$$\int N^*\left(p^*, \theta_i\right) di = 0$$

- GE imposes

$$p^* = P^*\left(\bar{\theta}\right)$$

- Assumptions
  - $\frac{\partial}{\partial p^*} \int N^* di < 0$ (stable equilibrium)
  - $\frac{\partial}{\partial \bar{\theta}} \int N^* di \neq 0$, or equivalently $\frac{\partial}{\partial \bar{\theta}} P^* \neq 0$ (non-zero GE effects)

Macro Effect of an Aggregate Shock

- How does the economy respond to a shock that moves θ_i for all i?
  - demand shock: housing wealth, consumer deleveraging...
  - supply shock: productivity, payroll taxes...
- To simplify, work with log-linearized conditions
  - all variables in log-deviations from "steady state"
- To be concrete, focus on expenditure on non-tradable
  - local expenditure on non-tradable:

$$x_i \equiv q_i + p_i = X(p^*, \theta_i)$$

  - corresponding aggregate:

$$\bar{x} \equiv \int x_i di = X(p^*, \bar{\theta})$$

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Micro vs Macro

- **Micro elasticity, or PE effect**
  - at local level

$$\varepsilon_i^{micro} \equiv \left. \frac{dx_i}{d\theta_i} \right|_{p^* \text{ constant}}$$

  - aggregate counterpart

$$\varepsilon^{micro} \equiv \int \varepsilon_i^{micro} di = \left. \frac{d\bar{x}}{d\bar{\theta}} \right|_{p^* \text{ constant}} = \frac{\partial X}{\partial \theta}$$

- Macro effect

$$\varepsilon^{Macro} \equiv \left. \frac{d\bar{x}}{d\bar{\theta}} \right|_{p^* \text{adjusts in GE}} = \underbrace{\frac{\partial X}{\partial \theta}}_{PE} + \underbrace{\frac{\partial X}{\partial p^*} \frac{\partial P^*}{\partial \theta}}_{GE} \neq \varepsilon^{micro}$$

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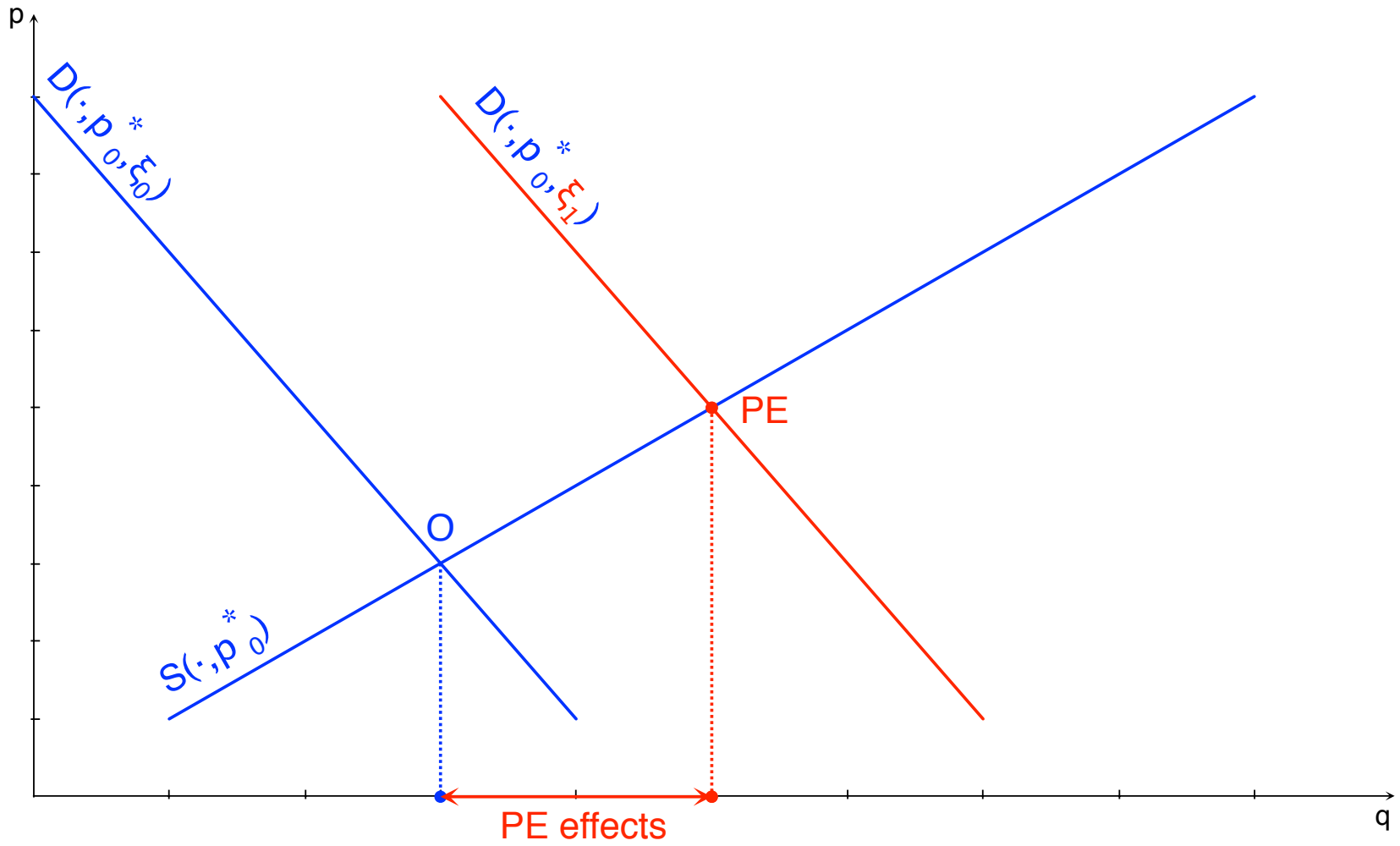
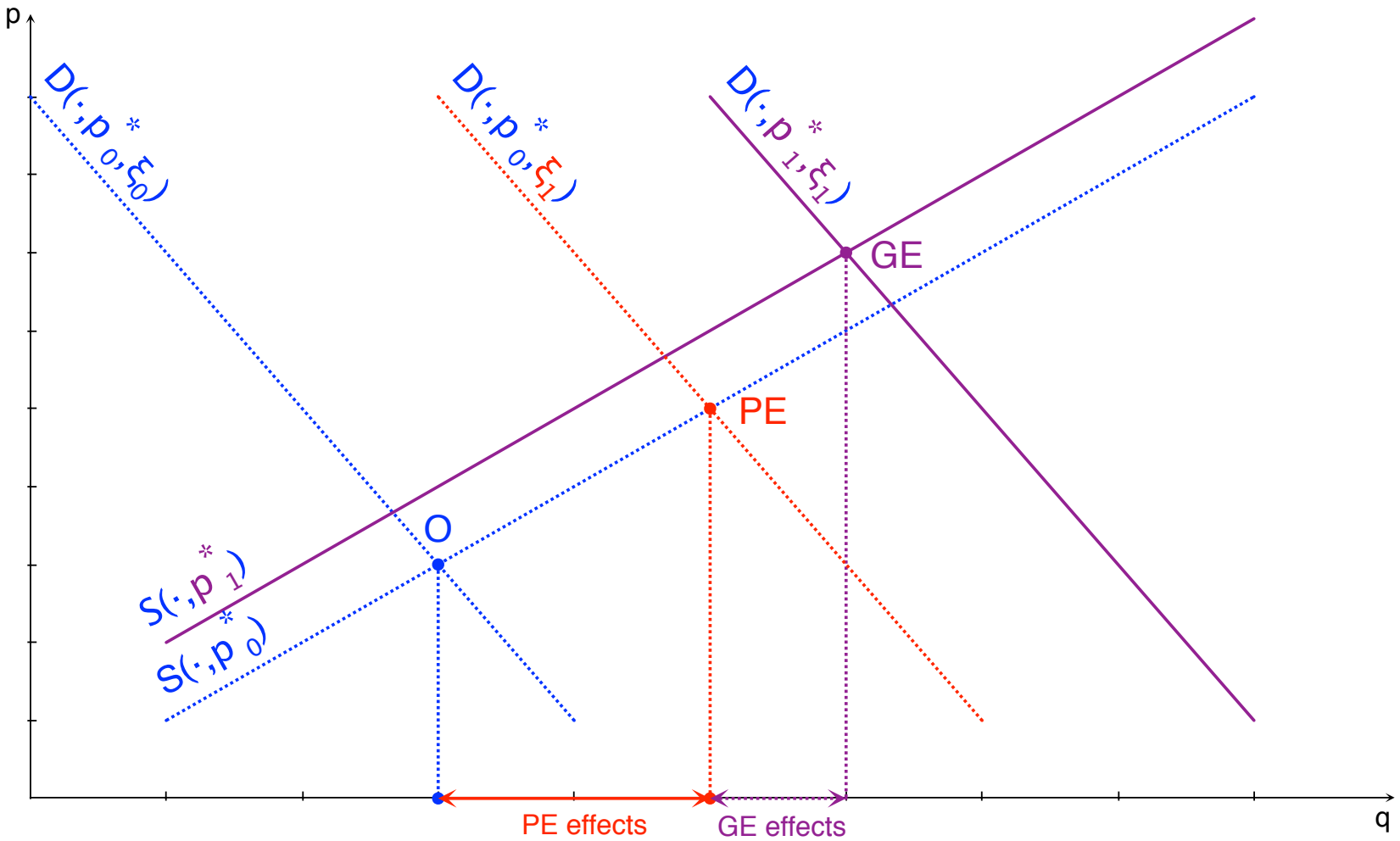
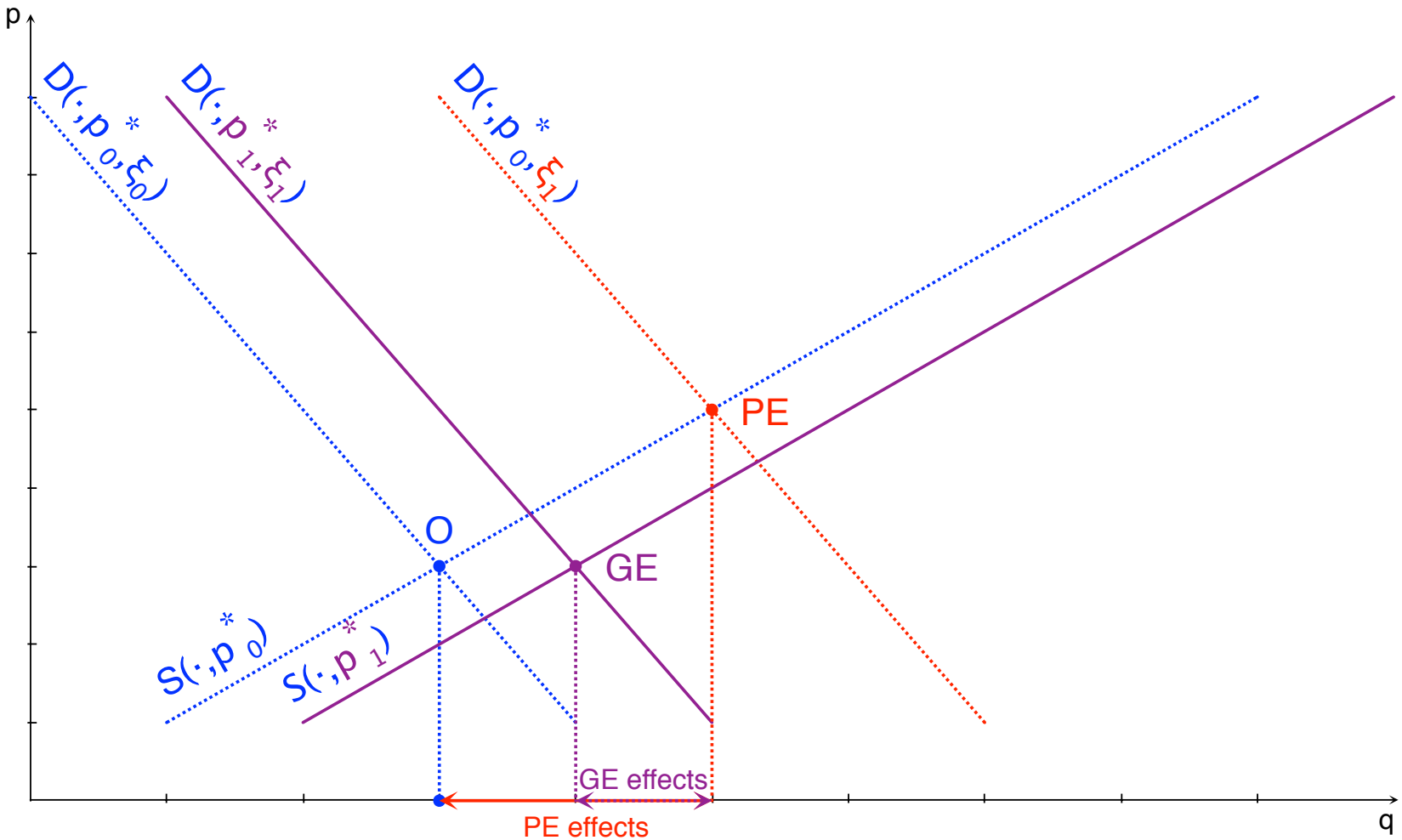
PE effect



GE amplifies PE



GE attenuates PE



Connection to Empirical Work

- Recent empirical macro:
  - exploits cross-sectional variation
  - provides estimate of ε^{micro}
  - Mian-Sufi, Nakamura-Steinsson, etc
- Tension between
  - what is estimated (ε^{micro})
  - what is of interest (ε^{Macro})
- Key problem:
  - GE effect partialed out as time fixed effect in regressions
- Our contribution: lessens the problem (at least in the short run)

Preview

- Standard paradigm:
  - adjustment in p^* is instantaneous
  - perfect coordination
- What we are after:
  - slow adjustment in p^*
- How?
  - relax solution concept: Tâtonnement
  - relax info assumption: remove common knowledge

Tâtonnement

- Let t index round of iteration in Tatonnement process
  - soon to reinterpret t as time
- $\forall i, t$, local market for non-tradables clears with given perception $\hat{p}_t^*$
  - gives PE outcomes with $p^* = \hat{p}_t^*$ to reinterpret t as time
- "Walrasian auctioneer" adjusts $\hat{p}^*$ slowly from old GE level to new one
  - $\hat{p}^*$ solves the following ODE

$$\frac{d\hat{p}_t^*}{dt} = -b_t \cdot \left[N^* \left(\hat{p}_t^*, \bar{\theta}_{new} \right) \right]$$

  - with initial condition

$$\hat{p}_0^* = P^*(\bar{\theta}_{old})$$

  - and for some exogenous $\{b_t\}$ with $b_t \geq \underline{b} > 0 \ \forall t$.

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Tâtonnement: micro vs macro

- **Macro effect at t:**

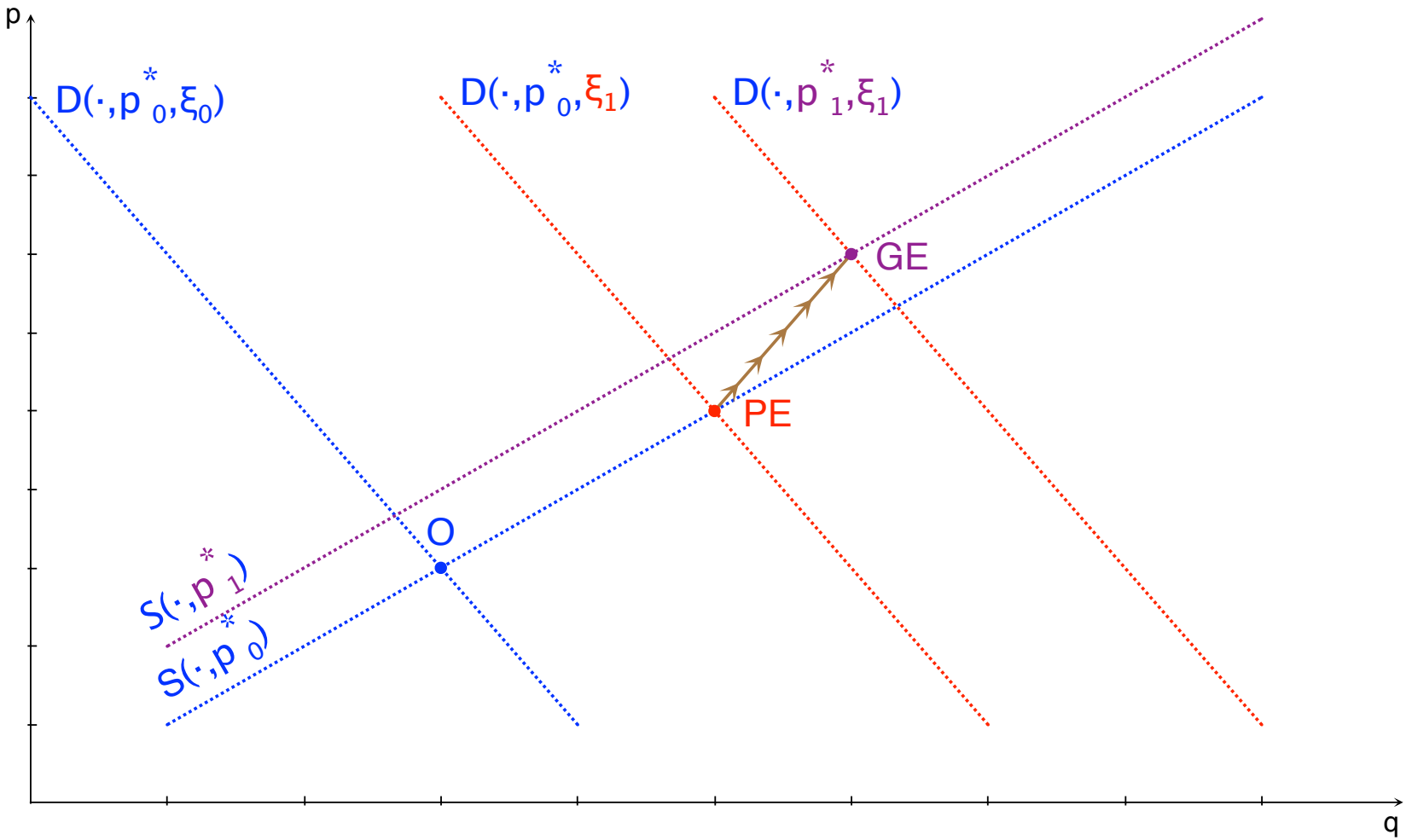
$$\varepsilon_{T\hat{a}t}(t) \;=\; \varepsilon^{micro} \;+\; \underbrace{w(t)\cdot\left(\varepsilon^{Macro}-\varepsilon^{micro}\right)}_{\text{GE at } t}$$

  where $w(t)$ is increasing in t, with $w(0)=0$ and $w(\infty)=1$

- PE same as in benchmark
  - because of local market clearing

- GE is dampened by factor w
  - because of erroneous perceptions of p^*

Tâtonnement: micro vs macro



Tâtonnement: micro vs macro

Corollary. $\varepsilon_{T\hat{a}t}$ is monotone and continuous in t, with

$$\varepsilon_{T\hat{a}t}(0) = \varepsilon^{micro} \qquad \text{and} \qquad \varepsilon_{T\hat{a}t}(\infty) = \varepsilon^{Macro}$$

- That is, we can span the gap between the micro and the macro by varying the round t in Tâtonnement

Incomplete Information

- Goal: translate from "off equilibrium" to "on equilibrium"
- Same payoff environment
- Non-tradable decisions in the "morning" under incomplete information
  - perfect knowledge of local conditions (θ_i, q_i, p_i)
  - lack common knowledge (CK) of global conditions $(\bar{\theta}, p^*)$
  - private signal about the latter: $s_i = \bar{\theta} + v_i$
- Tradable decisions in the "afternoon"
  - global conditions $(\bar{\theta}, p^*)$ become common knowledge

Equilibrium

- **Rational-Expectations Equil** with inco info (similar to PBE)
- Morning: local markets for non-tradable clear, giving

$$q_i = Q(E_i[p^*], \theta_i) \qquad p_i = P(E_i[p^*], \theta_i)$$

  where $E_i[p^*]$ is the rational expectation of p^* conditional on s_i

- Afternoon: p^* clears global market for tradable, giving

$$p^* = P^q(\bar{q}, \bar{\theta})$$

  where $\bar{q}$ is the realized agg quantity of non-tradable

Characterization

- **Lemma.** Equilibrium outcomes satisfy

$$q_i = \alpha E_i[\bar{q}] + \eta E_i[\bar{\theta}] + \zeta \theta_i$$

- Isomorphic to a "beauty contest"
  - GE effect akin to strategic interaction in games
  - $\alpha < 1$: degree of strategic complementarity/substitutability

- **Corollary.** Rational expectation of p^* = hierarchy of beliefs about $\bar{\theta}$

$$\bar{E}[p^*] = \Omega \cdot \sum_{h=1}^{\infty} \alpha^{h-1} \bar{E}^h[\bar{\theta}]$$

  - GE effects = HOB

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Lack of CK = Anchored Expectations

- Beliefs
  - first-order beliefs:

$$\bar{E}^1 \bar{\theta} \equiv \int E_i \bar{\theta} di = \lambda \bar{\theta}$$

    where $\lambda \equiv \frac{\kappa}{\kappa + \sigma_\theta^{-2}} \in (0,1)$ captures deviation of common knowledge

  - higher-order beliefs (HOB):

$$\bar{E}^h \bar{\theta} \equiv \bar{E}\left[\bar{E}^{h-1}[\bar{\theta}]\right] = \lambda^h \bar{\theta}$$

- HOB vary less than lower-order beliefs

  $\Rightarrow$ expectations of p^* are anchored

- GE is stronger $\Rightarrow$ HOB more important

  $\Rightarrow$ the stronger the GE effect, the stronger its own attenuation

Micro vs Macro

- PE as in benchmark
  - due to perfect knowledge of local conditions
- GE dampened
  - due to lack of common knowledge of global conditions
- **Macro effect revisited:**

$$\varepsilon_{Inc}(\lambda) \equiv \frac{d\bar{x}}{d\bar{\theta}} = \varepsilon^{micro} + \underbrace{g(\lambda) \cdot \left(\varepsilon^{Macro} - \varepsilon^{micro}\right)}_{\text{GE effect parameterized by } \lambda}$$

  - $g(\lambda)$ is monotone in λ, with $g(0) = 0$ and $g(1) = 1$
  - $g(\lambda)$ decreases with α
    - ★ tends to be closer to zero when GE effect is larger

Equivalence Result

Proposition. For any $\{b_t\}$ and any t, there exists a λ such that

1. rational expectations $\bar{E}[p^*]$ in inco-info economy same as ad hoc perceptions $\hat{p}_t^*$ in Tâtonnement economy
2. outcomes in inco-info economy same as in Tâtonnement economy
3. equal GE attenuation

$$\varepsilon_{T\hat{a}t}(t) = \varepsilon_{Inc}(\lambda)$$

The converse is also true.

Complementary Results and Take-home Lesson

- Similar equivalence results for
  - adaptive expectations
  - reflective equilibrium (Garcia-Schmidt & Woodford, 2015)
  - limited-depth reasoning

- Take-home lesson:

lack of CK = relaxation of solution concept = GE dampened

Extension: GE Takes Time

- A dynamic extension
- Essentially repeated version of static economy

$$F(q_{i,t}, q^*_{i,t}, q^Z_{i,t}; a_i) \leq 0$$

$$u_i = \sum_{t=0}^{\infty} e^{-\rho t} U(c_{i,t}, c^*_{i,t}, z_{i,t}; \xi_i)$$

- Slow learning about $bar\theta$ (or, equivalently, about global response)

Learning and GE Adjustment

- To avoid perfect aggregation of information:
  - idiosyncratic "iceberg costs" for tradable good
  - noisy private learning through realized prices
- **Lemma.** There exists an increasing sequence $\{\lambda_t\}$ such that, $\forall t$,

$$\bar{E}_t^h\left[\bar{\theta}\right] = \lambda_t^h \bar{\theta}$$

$$\bar{E}_t\left[p^*\right] = \Omega \sum_{h=1}^{\infty} \alpha^{h-1} \bar{E}_t^h\left[\bar{\theta}\right] = f(\lambda_t)\bar{\theta}$$

- **Proposition.** Macro elasticity at t

$$\varepsilon_t \;=\; \varepsilon_{Inc}(\lambda_t) \;=\; \varepsilon^{micro} + \underbrace{w_t \cdot \left(\varepsilon^{Macro} - \varepsilon^{micro}\right)}_{GE_t}$$

  where $w_t = g(\lambda_t)$ is increasing in t, with $w_0 < 1 = w_\infty$

- Similar to static model, except that now λ_t increases with time

Slow GE Adjustment

- Formalization of notion that GE adjustment takes time
  - **in short run, macro effect is close to micro/PE effect**
  - but as time passes, it converges to what predicted by standard model
- Speed of convergence?
  - not surprisingly, it depends on quality of learning
  - more interestingly, it depends on magnitude of GE effect
- **Prop.** For any given $\{\lambda_t\}$, the sequence $\{w_t\}$ converges to 1 at a rate that is decreasing in α. In this sense,

stronger GE effect $\rightarrow$ slower GE adjustment !

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Second paper: Angeletos and Lian (2016b)

Forward Guidance without Common Knowledge

Forward Guidance Puzzle

- Context: an economy during a liquidity trap
  - zero-lower bound (ZLB) binds for $t \leq T-1$
- Forward Guidance = promise at t to keep interest rates low at $t \geq T$
- The Puzzle: standard NK model predicts that the stimulating effect is quantitatively large, increases with T, and explodes as $T \to \infty$

Our contribution

- Prelim result: beauty-contest representation of NK model
  - disentangle PE and GE effects
  - recast GE effects as HOB
- Main result: remove CK $\Rightarrow$
  $\Rightarrow$ anchor expectations of income and inflation
  $\Rightarrow$ attenuate relevant GE effects
  $\Rightarrow$ reduce power of forward guidance
- Complementary results: paradox of flexibility, discounted Euler/NKPC

Differences from Earlier Abstract Framework

- Concrete context, precise micro-foundations, policy focus
- Truly dynamic environment
  - forward-looking expectations
  - dynamic beauty contest
- Specific novel insights
  - GE effects tied to expectations of future income and future inflation
  - longer horizons map to beliefs of higher order
  - dampening increases with $T \rightarrow$ lessen forward-guidance puzzle
  - dampening increases with price flexibility $\rightarrow$ lessen paradox of flexibility

Textbook NK Model

- demand block: Euler condition (aka IS curve)

$$y_t = \mathbb{E}_t \left[y_{t+1} - (R_t - \pi_{t+1}) \right]$$

- supply block: NK Philips Curve

$$\pi_t = \beta \mathbb{E}_t \left[\pi_{t+1} \right] + \kappa y_t$$

- Monetary Policy: ZLB and forward guidance
  - $R_t = 0 \ \forall t \leq T - 1$
  - R_T free $\rightarrow$ forward guidance moves $\mathbb{E}_0[R_T]$
  - $y_t = \pi_t = 0 \ \forall t \geq T + 1 \rightarrow$ ex post optimal

- The puzzle: $\left| \frac{\partial y_0}{\partial \mathbb{E}_0[R_T]} \right|$ increases with T and explodes as $T \rightarrow \infty$

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NK as Multi-Layer Beauty Contest

- Remove CK (of policy and/or of responses of others)
- Euler condition $\rightarrow$ dynamic BC among consumers
  - feedback from future spending to future income to current spending
- NKPC $\rightarrow$ dynamic BC among firms
  - feedback from future inflation to future MCs to current inflation
- Equilibrium: higher-layer BC between consumers and firms
  - feedback from future inflation to current spending

NK as Multi-Layer Beauty Contest

- Euler condition $\rightarrow$ dynamic BC among consumers

$$\left.\begin{array}{l} y_t = \int c_{it}\, di \\ c_{it} = f(\text{expected PV of income}) = f\left(\mathbb{E}_{it}[y_{t+k}]\right) \end{array}\right\} \Longrightarrow$$

$$y_t = -\sum_{k=1}^{+\infty} \beta^{k-1} \left\{ \bar{E}_t[R_{t+k-1}] - \bar{E}_t[\pi_{t+k}] \right\} + (1-\beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[y_{t+k}] \right\}$$

- NKPC $\rightarrow$ dynamic BC among firms

$$\left.\begin{array}{l} p_t = \theta\, p_{t-1} + (1-\theta) \int p_{it}^*\, di \\ p_{it}^* = f(\text{expected PV of nominal MC}) = f\left(\mathbb{E}_{it}[p_{t+k}]\right) \end{array}\right\} \Longrightarrow$$

$$\pi_t^* = \kappa y_t + \kappa \sum_{k=1}^{+\infty} (\beta\theta)^k \bar{E}_t^f[y_{t+k}] + \frac{1-\theta}{\theta} \sum_{k=1}^{+\infty} (\beta\theta)^k \bar{E}_t^f[\pi_{t+k}] + \kappa\mu_t$$

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GE Attenuation

- three GE mechanisms = three types of strategic complementary
  - within demand block: income multiplier
  - within supply block: pricing complementarity
  - between two blocks: inflationary/deflationary spiral

- key insight: **lack of CK attenuates all three at once!**

Forward Guidance Dampened

Proposition. With non-vanishing lack of CK,

$$\frac{\left.\frac{\partial y_0}{\partial \bar{\mathbb{E}}_0 R_T}\right|_{variant}}{\left.\frac{\partial y_0}{\partial \mathbb{E}_0 R_T}\right|_{standard}} \to 0 \quad \text{as } T \to \infty$$

Proposition. When lack of CK is sufficiently large,

$$\left.\frac{\partial y_0}{\partial \bar{\mathbb{E}}_0 R_T}\right|_{variant} \to 0 \quad \text{as } T \to \infty,$$

whereas $\left.\frac{\partial y_0}{\partial \mathbb{E}_0 R_T}\right|_{standard} \to \infty.$

Forward Guidance Dampened

Proposition. With non-vanishing lack of CK,

$$\frac{\left.\frac{\partial y_0}{\partial \bar{\mathbb{E}}_0 R_T}\right|_{variant}}{\left.\frac{\partial y_0}{\partial \mathbb{E}_0 R_T}\right|_{standard}} \to 0 \quad \text{as } T \to \infty$$

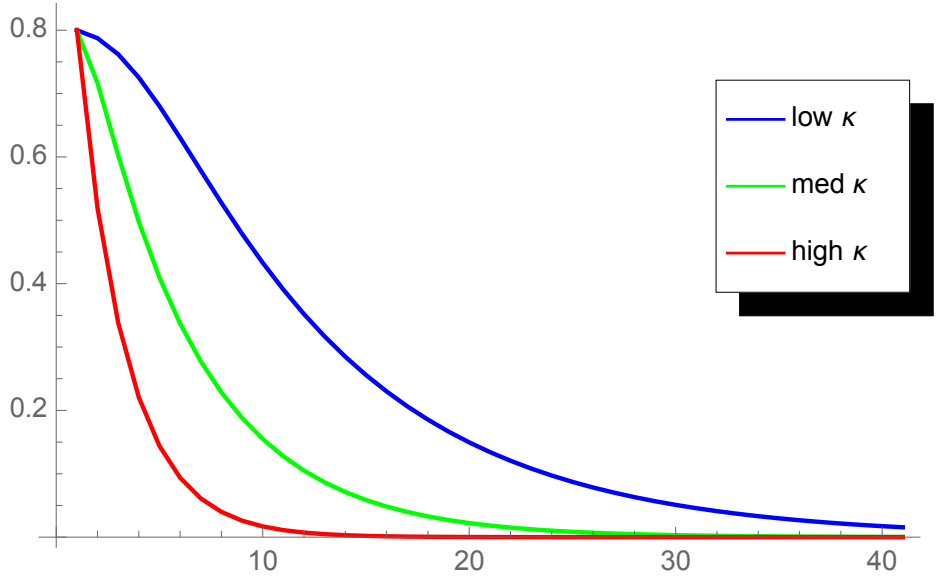
Proposition. When lack of CK is sufficiently large,

$$\left.\frac{\partial y_0}{\partial \bar{\mathbb{E}}_0 R_T}\right|_{variant} \to 0 \quad \text{as } T \to \infty,$$

whereas $\left.\frac{\partial y_0}{\partial \mathbb{E}_0 R_T}\right|_{standard} \to \infty.$

Paradox of Flexibility, Discounting, and More

- Dampening is stronger when prices are more flexible



- Lack of CK manifests as discounting of future expectations in Euler and NKPC of isomporphic representative-agent model

- Insights relevant also for
  - shocks at ZLB, deflationary spirals, paradox of flexibility, eq. selection, neo-Fisherian predictions...

Conclusion

- Worth revisiting solution concept and GE effects in macro
  - even if we maintain individual rationality and PE effects
- Lack of CK = relaxation of solution concept = GE dampened
  - formalization of "GE takes time"
  - in short run, "Macro is (close) to Micro"
- Topical application: Forward Guidance
- Other applications...
  - aggregate demand and Keynesian multipliers
  - Ricardian equivalence, fiscal stimuli