Explicit vs. Statistical Targeting in Affirmative Action: Theory and Evidence from Chicago's Exam Schools*

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Abstract

In the wake of prohibitions on the use of race in affirmative action, a number of admissions systems have adopted race-neutral alternatives that encourage diversity without appearing to explicitly advantage any group. The new affirmative action system for Chicago's elite exam schools reserves a fraction of seats for students based on their neighborhood and leaves the rest to be assigned via merit. Neighborhoods are divided into four tiers based on an index of socioeconomic disadvantage, and the same fraction of seats are reserved for each tier. We show that the order in which seats are processed at schools provides an additional lever to explicitly target disadvantaged applicants beyond the size of the reserve. We next characterize tier-blind processing rules that do not explicitly discriminate between tiers. Even under tier-blind rules, it is possible to favor certain applicants by exploiting the distribution of scores across tiers, a phenomenon we call statistical targeting. We characterize the tier-blind processing order that is optimal for the most disadvantaged tier assuming that these applicants systematically have lower scores. Our main result implies that Chicago has been providing an additional boost to the disadvantaged tier beyond their reserved slots. Using data from Chicago, we show that the bias due to processing in favor of the disadvantaged tier is comparable to benefit they received from the increase in the reserve size in 2012.

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"*The way to stop discrimination on the basis of race is to stop discriminating on the basis of race.*"

— U.S. Supreme Court Justice John Roberts, 2007

1 Introduction

Affirmative action is often controversial because favoring one group inevitably involves disadvantaging another. This sentiment was behind the U.S. Supreme Court's decision to prohibit racial quotas in K-12 public school admissions in 2007, and is part of a broader movement to achieve diversity without explicitly favoring certain groups. Against this backdrop, Chicago Public Schools is undergoing one of the nation's most significant experiments in race-neutral affirmative action at the K-12 level, after abandoning their old system of racial quotas in 2009. In the new system, Chicago's neighborhoods are divided into one of four tiers based on an index of socioeconomic disadvantage. At each school, 60% of seats are reserved to be assigned based on an applicant's neighborhood tier, and the rest of the seats are assigned solely based on merit.[1] Applicants can be admitted to both types of seats.[2]

Since the reservation for the most and least advantaged neighborhoods is the same size at each school, the new Chicago system appears to be impartial because it does not favor one group of applicants over another. We show, however, that equal size reservations are not sufficient to avoid explicitly benefitting a particular tier due to the order in which school seats are processed, known as the precedence order. We characterize the optimal precedence order in the Chicago system for applicants from a given tier. Our result shows that it is possible to tweak the competition for merit seats in favor of applicants from a particular tier simply by assigning seats reserved for all other tiers before the merit seats. This precedence order handicaps applicants from those other tiers in the competition for merit seats. That is, applicant processing provides a new lever to explicitly target disadvantaged applicants beyond setting the reserve size.

It is clear that the equal size reserves in Chicago were intended to avoid the impression of impartiality with their race-neutral policy. Had the goal been to only admit the most disadvantaged, the district could have simply reserved all of the seats for applicants from the lowest tier neighborhoods. While that policy would encourage diversity, it would not reflect a major rationale for exam schools: to group together and provide a curriculum tailored to high-ability

[1] The size of reservation has increased to 70% in 2012.

[2] The fact that there are multiple categories of applicants who quality for affirmative action and these applicants can qualify for multiple types of seats is a widespread feature of affirmative action systems. Many engineering colleges in India set aside seats on the basis of gender and caste (Bertrand, Hanna, and Mullainathan 2010, Bagde, Epple, and Taylor 2016). There is widespread debate in India about processing the "creamy layer," who are high scoring or high income members of the disadvantaged caste, including an Indian Supreme Court ruling in *State of Kerala vs. NM Thomas* (1974). Finland has gender quotas which mandate at least 40% of each gender for public boards, committees, and councils (Strauss 2012). There have similar proposals with 40% male, 40% female, and 20% open on government boards in Australia (Fox 2015). The European Parliament (2008) details electoral gender quota systems used throughout Europe.

students.[3] The competition between these two objectives – grouping together the highest achievers and having diversity – is reflected in systematic reviews of the policy. For example, a report from a Blue Ribbon Commission (BRC) appointed to review Chicago's policy states:

> The BRC believes the district should strike a balance between these two extremes [100% rank order and 100% tier]. The BRC wants these programs to maintain their academic strength and excellent record of achievement, but also believes that diversity is an important part of the historical success of these programs. (page 2, BRC (2011))

To more precisely describe an impartial system, we define a tier-blind admissions policy. A tier-blind policy is one where the outcome does not depend on the labelling of tiers. We show that tier-blind precedence rules are balanced: the same number of seats from each tier are processed between any two merit seats, before the initial merit seat, and after the last merit seat. Tier-blindness therefore implies that reserve sizes must be equal as in Chicago. It imposes an even more stringent requirement since it rules out policies where the number of seats from each tier processed after merit seats differs by tier as in the optimal precedence order.

Within the set of tier-blind policies, however, it is still possible to target applicants from particular tiers by exploiting systematic differences in scores between tiers, a phenomenon we term statistical targeting. Under the assumption that applicants from the most disadvantaged tier have systematically lower scores than those from other tiers, our main result characterizes the optimal tier-blind precedence rule for the most disadvantaged applicants. Our result implies that Chicago's current tier-blind rule has been providing an additional boost to applicants from the most disadvantaged tier beyond the reserve seats.

In 2012, in an effort to target more disadvantaged applicants, Chicago increased the tier set-aside to 70% and added a sixth factor to the socioeconomic index. This change was symmetric: each tier reservation increased by 2.5% of seats. Our last result shows that this increases the assignment of applicants from the most disadvantaged neighborhoods under Chicago's affirmative action implementation. Using data from Chicago, we show that the change in assignment from the most disadvantaged tier due to the smaller merit fraction is comparable to the change in assignment from switching the processing order, holding fixed the merit fraction. Therefore, the bias in favor of the most disadvantaged due to statistical targeting is comparable to that from more explicit policy changes that alter reserve sizes.

Chicago is a compelling setting for examining affirmative action in school admissions for several reasons. Given the schools' high visibility and frequent appearance on lists of the best public U.S. high schools, it is not surprising that Chicago's reforms are being seen as a model for other cities. For instance, Kahlenberg (2014) argues that Chicago's place-based affirmative

[3]Ellison and Pathak (2016) propose a model of exam schools where a policy maker values both curriculum matching and diversity, and use it to measure whether race-neutral affirmative action systems can be an effective substitute for racial quotas.

action system is a template for other cities like New York, where there are concerns about students at their flagship exam schools not reflecting underlying district demographics.[4] Moreover, the adoption of schemes like Chicago's seem to be a likely consequence of the 2007 U.S. Supreme Court ruling in *Parents Involved in Community Schools vs. Seattle School District No. 1*, which prohibited explicit racial criteria in K-12 admissions. Indeed, the U.S. Departments of Justice and Education have held up race-neutral criteria based on geographic factors, as seen in Chicago, as a model for other districts (OCR 2011).[5] Finally, Chicago's assignment scheme is a variant of the student-proposing deferred acceptance algorithm, which is strategy-proof for participants.[6] This feature allows for the straightforward computation of the consequences of counterfactual affirmative action policies without needing to model how applicants would submit preferences under these alternatives.

The consequences of allowing applicants to be admitted to different types of seats has been studied in Dur, Kominers, Pathak, and Sönmez (2014). That paper examines comparative statics when there is only one reserve group, motivated by neighborhood priority in Boston's school choice system. The issues are more involved when there is more than one reserve group. They do not characterize optimal policies, which is the focus here. Chicago also uses scores in admissions which allows us to study statistical targeting, and there is no comparable phenomenon with lottery based tie-breaking as in Boston. Our analysis presents the first optimality results for precedence policies in the context of the framework of matching with slot-specific priorities (Kominers and Sönmez 2016).[7]

Other related studies of affirmative action include Ehlers, Hafalir, Yenmez, and Yildirim (2014), Erdil and Kumano (2012), Hafalir, Yenmez, and Yildirim (2013), Kamada and Kojima (2014), and Kojima (2012).[8] The model we study is based on a continuum model version of the matching with slot-specific priorities model introduced by Kominers and Sönmez (2016). Like Echenique and Yenmez (2015), we characterize optimal choice rules focusing on a given school. However, we take the affirmative action system as given and consider variations in implementation, while Echenique and Yenmez (2015) derive affirmative action systems from primitive axioms for diversity without considering the issues we examine here. Continuum models, like ours, are

[4]The NAACP Legal Defense Fund filed an Office for Civil Rights complaint with the US Department of Education in September 2012, asserting that the admissions process at NYC's exam schools violates the Civil Rights Act of 1964 because it uses a single admissions test. Pending New York State legislation (Senate Bill S7738) proposes to broaden the criteria for admissions.

[5]Kahlenberg (2008) reports that more than 60 public school districts use socioeconomic status as an admissions factor.

[6]The mechanism design approach to student assignment was initiated by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003). Both papers advocate the adoption of deferred acceptance (DA) in the context of priority based student assignment. Many urban school districts including Boston and New York utilize DA (Abdulkadiroğlu, Pathak, Roth, and Sönmez 2005, Abdulkadiroğlu, Pathak, and Roth 2005, Pathak and Sönmez 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Pathak and Sönmez 2013, Pathak 2016).

[7]Chicago's system motivated the model in an earlier version of that paper, Kominers and Sönmez (2013), although they do not provide any analytical results for the application at CPS.

[8]A theoretical literature examines other aspects of affirmative action policies, including Coate and Loury (1993), Sau-Chung (2000), Chan and Eyster (2003), Fryer, Loury, and Yuret (2008), and Ray and Sethi (2010).

also used in a number of other recent papers including Abdulkadiroglu, Che, and Yasuda (2015), Azevedo and Leshno (2015), and Che, Kim, and Kojima (2015).

The next section develops the model. Section 3 examines explicit targeting and characterizes the best tier-sighted choice function for a given tier. Section 4 characterizes choice functions that are tier-blind and characterizes the best and worst choice function under this constraint. Section 5 reports on data from Chicago Public Schools. The last section concludes. Proofs are given in the appendix.

2 Model

2.1 Setup

We work with a continuum model to simplify the analysis. There is a mass n of students. Let I denote the set of students. Throughout the analysis, we fix I. Each student belongs to a socioeconomic category or **tier** $t \in T = \{1, 2, .., \bar{t}\}$. In Chicago, a student's tier depends on the attributes of her census tract. When a student applies to an exam school, she must take a competitive admissions exam. The district then takes an equally-weighted combination of the admissions test score, the applicant's 7^{th} grade GPA, and a standardized test score to generate a **composite score**, which we denote by $k \in K = [\underline{k}, \bar{k}]$, where K is the continuum interval of the possible scores. For a given student i, his tier is given by $t(i)$ and his composite score is given by $k(i)$. A tier t has mass n_t of students, where $\sum_{t \in T} n_t = n$. For any subset of students $J \subseteq I$, we denote the set and mass of tier $t \in T$ students with J_t and n_t^J, respectively. For given tier t, we denote the composite score density function of students in I_t with $f_t : K \to \mathbb{R}^+$ and assume that the density function has no atom. For a given subset of students $J \subseteq I_t$, let $f_t^J : K \to \mathbb{R}^+$ be the atomless density function of tier t students in J. We represent the mass of tier t students with scores between ℓ and $\ell' < \ell$ with $\int_{\ell'}^{\ell} f_t(k)dk$.

We are interested in understanding the properties of a school's choice function. In the case of a decentralized assignment system, our analysis captures considerations relevant for a particular school. A centralized matching system based on the deferred acceptance algorithm can be interpreted as an iterated implementation of choice functions across all schools, where in the first iteration students apply to their top choices and in each subsequent iteration, students rejected in earlier iterations apply to their next choices.[9] In Section 5, we examine the extent to which our analysis of a single school's choice function captures insights relevant for Chicago's affirmative action system, which employs the deferred acceptance algorithm to assign ten schools.

[9] In centralized assignment systems, a change in one school's choice function in the deferred acceptance algorithm may generate a sequence of rejections and proposals, which might result in ambiguous overall effects across schools. Kominers and Sönmez (2016) and Dur, Kominers, Pathak, and Sönmez (2014) present examples of this phenomenon. When a matching model includes a large number of participants, there are empirical and theoretical arguments (e.g., Roth and Peranson (1999) and Kojima and Pathak (2009)), suggesting that such sequences or rejections and proposals are rare.

In our continuum model, school capacity is modeled as a set of unit capacity intervals, which we refer to as **slots**. Let S denote the finite set of slots each with a unit mass of seats to fill. Then, the school has a mass $|S|$ of seats to fill. There are $\bar{t}+1$ types of slots: tier 1 slots, tier 2 slots, ..., tier $\bar{t}$ slots, and merit slots. Function $\tau : S \to T \cup \{m\}$ specifies the type of each slot. We denote the set of tier t slots as S_t and the set of merit slots as S_m. Observe that $S_t = \{s \in S \mid \tau(s) = t\}$ and $S_m = \{s \in S \mid \tau(s) = m\}$. For each tier t, each slot $s \in S_t$ prioritizes all students in its tier I_t over all other students. For each $s \in S_t$, students in tier t are ordered by composite score. Students outside tier t are ordered by composite score, but each comes after students in tier t. Priority for each merit slot $s \in S_m$, on the other hand, is solely based on composite scores.

When a merit slot s_m considers a set of applicants J, it admits the highest-scoring unit mass subset of its applicants. Similarly, when a tier slot s_t considers a set of applicants J, it admits the highest-scoring unit mass of its applicants from tier t. The cutoff scores for both types of slots are determined by this process. Observe that for a merit slot s_m, the **cutoff score** $k_J^{s_m}$ **for** J is given by

$$\sum_{t=1}^{\bar{t}} \int_{k_J^{s_m}}^{\bar{k}} f_t^J(k)dk = 1, \tag{1}$$

and for a tier slot s_t, the **cutoff score** $k_J^{s_t}$ **for** J is given by

$$\int_{k_J^{s_t}}^{\bar{k}} f_t^J(k)dk = 1. \tag{2}$$

If J is such the expression (1) or (2) does not equal 1, we set the corresponding cutoff to $\underline{k}$.

To simplify the analysis, we rely on the following assumption throughout the paper.

Assumption 1 ***(Oversubscription):*** *For each $t \in T$,*

$$n_t \geq |S_t| + |S_m|.$$

This assumption states that for each tier t, the mass of tier t students is at least as great as the mass of slots they are competing for.

2.2 Choice Function

A choice function formally specifies the set of selected students from any given set of applicants at a given schools. To define the choice function, it is necessary to specify how slots are processed. The slots in S are processed according to a linear order $\triangleright$ on S, that we refer to as a **precedence**.

Given two slots $s, s' \in S$, the expression $s \triangleright s'$ means that slot s is to be filled before slot s' whenever possible. We say that s precedes s' or s is processed before s'. The **precedence rank** of a slot is the number of slots that precede it plus one. We say s is the ℓ^{th} (merit) slot if the number of (merit) slots preceding it is $\ell - 1$. We say a pair of merit slots $s, s' \in S_m$ are subsequent if there does not exist another merit slot $\tilde{s} \in S_m$ such that $s \triangleright \tilde{s} \triangleright s'$. Similarly, we say a pair of tier slots $s, s' \in S \setminus S_m$ are subsequent if there does not exist another tier slot $\tilde{s} \in S \setminus S_m$ such that $s \triangleright \tilde{s} \triangleright s'$.

The choice function depends on the set of slots, the types of these slots, and their precedence. Therefore, when describing a choice function, there are three inputs: the set of slots S, the τ function that specifies the types of these slots, and the linear order $\triangleright$ that specifies the precedence of these slots. For a given triple $(S, \tau, \triangleright)$, the **choice** from a set of students J is denoted by $C(S, \tau, \triangleright, J)$. Throughout the analysis, the set of slots S is fixed. Moreover, with the exception of Section 3.2, the function τ is fixed. Therefore, we drop S and τ as arguments of the choice function, referring to choice as $C(\triangleright, J)$, except in Section 3.2.

Construction of $C(\triangleright, J)$: For a given triple $(S, \tau, \triangleright)$ and set of students J, the choice $C(\triangleright, J)$ will be constructed as follows: Each slot will be filled in order of precedence $\triangleright$ given the criteria described above. That is, when it is the turn of a merit slot, it will be filled with the highest-scoring unit mass subset of applicants that are so far unchosen. When it is the turn of a tier t slot, it will be filled with the highest-scoring unit mass subset of applicants from tier t that are so far unchosen.

Under Assumption 1, for any tier t, there are more tier t applicants than the mass of slots they are competing for. Therefore, all tier t slots will be filled by tier t candidates.

3 Explicit Targeting

3.1 The Best and Worst Precedence for a Given Tier

Affirmative action schemes are designed to favor applicants from particular tiers. To have the greatest representation from a particular tier, it is of course possible to only admit applicants from that tier. However, as discussed in the introduction, this policy conflicts with the competing goals of both grouping together the highest achievers and having diversity. Moreover, a policy that only admits applicants from one tier is not tier-blind, as we more formally describe below. We therefore hold the fraction of reserved seats from each tier fixed at equal sizes and characterize the best and worst precedence policies for a given tier.

A preliminary structural result provides a convenient simplification for describing precedence orders. To determine the outcome of a given choice function, it is sufficient to specify the number of slots from each tier between any two subsequent merit slots and not their exact location relative to one another. To express this formally, we first define what it means for two precedences to be equivalent.

Definition 1 *For a given set of slots S and their types τ, the precedence $\triangleright$ is* ***equivalent*** *to precedence $\hat{\triangleright}$ if precedence $\hat{\triangleright}$ can be obtained from precedence $\triangleright$ by a sequence of swaps of the precedence ranks of any pair of tier slots $s, s' \in S \backslash S_m$ where there is no merit slot s_m such that $s \triangleright s_m \triangleright s'$.*

Equivalence of two precedences simply means that

1. they have the same number of merit slots with identical precedence ranks, and
2. for any given tier t, the number of tier t slots between any two subsequent merit slots is identical under both precedences, as is the number of tier t slots before the first merit slot and the number of tier t slots after the last merit slot.

The next lemma justifies this equivalence terminology.

Lemma 1 *Fix the set of slots S and their types τ. Let $\triangleright$ and $\hat{\triangleright}$ be two equivalent precedences. Under Assumption 1,*

$$C(\triangleright, I) = C(\hat{\triangleright}, I).$$

The proof of this and all other results in contained in the appendix.

Our first main result characterizes the precedence orders that attain the maximal and minimal mass of assignments for applicants of a given tier.[10] The statement of this result is simplified, thanks to Lemma 1.

Proposition 1 *Fix the set of slots S, their types τ, and tier $t^* \in T$.*

- *Let $\bar{\triangleright}$ be a precedence order where each slot of each tier $t \neq t^*$ precedes each merit slot and each merit slot precedes each slot of tier t^*.*
- *Let $\underline{\triangleright}$ be a precedence order where each slot of each tier t^* precedes each merit slots and each merit slot precede each slots of any tier $t \neq t^*$.*

Then under Assumption 1, among all precedence orders,

i) the maximal tier t^* assignment is attained under $\bar{\triangleright}$,

ii) the minimal tier t^* assignment is attained under $\underline{\triangleright}$.

[10] In the appendix, we show a somewhat stronger result that the set of tier t^* students chosen by the choice function induced by $\bar{\triangleright}$ includes the set of tier t^* students chosen under any other precedence $\triangleright$, and the set of tier t^* students chosen under any other precedence $\triangleright$ includes the set of tier t^* students chosen by the choice function induced by $\underline{\triangleright}$.

An important issue with the optimal precedence order $\bar{\triangleright}$ for tier t^* is that it tweaks the competition for merit seats to the benefit of applicants from tier t^* by dropping the best applicants from other tiers from competition. In contrast, the precedence order $\underline{\triangleright}$ is the other extreme that tweaks the competition for merit slots to the detriment of tier t^* applicants. Hence, this proposition raises the question of whether either of these two precedences can be considered "equitable," despite the even share of tier slots across all tiers.

3.2 Eliminating Explicit Targeting

For a given set of equally-sized reserve slots, Proposition 1 characterizes the best and worst precedence policy for a particular tier. That tier either receives favorable or unfavorable treatment for the competition at merit slots under these two policy extremes. In our view, the biased competition for merit slots due to these precedences is akin to the more visible bias associated with uneven tier reserve sizes. As such, we would like to eliminate uneven treatment of different tiers due to either visible or subtle design parameters. We therefore focus on the class of rules that do not differentiate across tiers.

Explicit targeting across tiers will be eliminated only when the label of tiers play no role in the choice function. This idea motivates the following definition of tier-blindness. A **merit-preserving bijection** $\pi : T \cup \{m\} \to T \cup \{m\}$ is a one-to-one and onto function where $\pi(m) = m$. This bijection simply relabels the tiers.

Definition 2 *A precedence $\triangleright$ is **tier-blind** if and only if for any set of slots S, for any type function τ, for any merit-preserving bijection π, and for any group of students J,*

$$C\big(S, \tau, \triangleright, J\big) = C\big(S, \pi(\tau), \triangleright, J\big).$$

In plain English, relabeling tiers does not change the outcome.

Tier-blindness is an anonymity condition across tiers, and it restricts the structure of precedence orders. We next identify the mechanical structure implied by tier-blindness.

Definition 3 *Precedence $\triangleright$ is **balanced** if for any two tiers $t, t' \in T$:*

i) there is an equal number of tier slots for tiers t and t' between any two subsequent merit slots,

ii) there is an equal number of tier slots for tiers t and t' before the first merit slot, and

iii) there is an equal number of tier slots for tiers t and t' after the last merit slot.

Due to Lemma 1, the relative position of tier slots between any pair of subsequent merit slots is immaterial.

Precedence orders characterized in Proposition 1 are clearly not balanced. To give some examples of balanced precedence orders, let us suppose that there are two tiers and denote generic slots for tier 1 as s_1 and generic slots for tier 2 as s_2. In this environment, the following three precedences are all balanced:

1) $\underbrace{s_1 \rhd s_2 \rhd s_2 \rhd s_1}_{\#\text{ tier 1}=\#\text{tier 2}} \rhd \mathbf{s_m} \rhd \mathbf{s_m}$,

2) $\mathbf{s_m} \rhd \mathbf{s_m} \rhd \mathbf{s_m} \rhd \underbrace{s_1 \rhd s_1 \rhd s_1 \rhd s_2 \rhd s_2 \rhd s_2}_{\#\text{ tier 1}=\#\text{tier 2}}$,

3) $\underbrace{s_1 \rhd s_2 \rhd s_1 \rhd s_2}_{\#\text{ tier 1}=\#\text{tier 2}} \rhd \mathbf{s_m} \rhd \underbrace{s_1 \rhd s_1 \rhd s_2 \rhd s_2 \rhd s_1 \rhd s_2}_{\#\text{ tier 1}=\#\text{tier 2}} \rhd \mathbf{s_m} \rhd \mathbf{s_m} \rhd \mathbf{s_m} \rhd \underbrace{s_2 \rhd s_2 \rhd s_1 \rhd s_1}_{\#\text{ tier 1}=\#\text{tier 2}}$.

Our next result shows that tier-blindness and balancedness are equivalent.

Proposition 2 *Fix the set of slots S and their types τ. Under Assumption 1, a precedence order is tier-blind if and only if it is balanced.*

This proposition implies that under tier-blindness, a set of slots must have an equal number of slots from each tier. Given an equal fraction of reserved seats for each tier, there are still many precedence orders that are tier-blind, but they may differ in how they distribute access to students from different tiers. How these tier-blind admission policies distribute access is a consequence of the statistical properties of score distributions across tiers, an issue we examine next.

4 Statistical Targeting

4.1 The Best and Worst Tier-Blind Precedence

In the last section, we showed that there are many possible tier-blind admissions policies, which by definition do not explicit target a given tier. Within this class, however, policies may lead to substantially different access across tiers due to the distribution of scores across tiers. **Statistical targeting**, one of the central concepts we formulate in our paper, involves choosing a policy among tier-blind precedence policies with the potential objective of optimizing the number of seats assigned to students of a specific tier, utilizing the differences between the distribution of scores across tiers.

Unlike explicit targeting, which is easier-to-understand, the implications of statistical targeting are not as straightforward. The effects of this aspect of affirmative action policies may be as large as other more explicit policy levers. It is, therefore, important to understand statistical targeting to avoid unintentionally tweaking outcomes in favor of certain groups. Some policymakers may find this more subtle policy lever easier to navigate since reaching desired outcomes

might not create a visible wedge between different groups. We caution that policies critically relying on statistical targeting may be prone to abuse without the benefit of full transparency.

Without additional structure on the problem, it is not possible to differentiate between applicants of different tiers under a tier-blind precedence by the very nature of this concept. The following empirically-motivated condition on the distribution on scores allows us to characterize the best tier-blind precedence for the lowest socioeconomic tier.

Assumption 2 $f_t(k) \geq f_1(k)$, $\forall k \in K$ *and* $\forall t \in T$.

Assumption 2 states that for any given score, tier 1 students have the lowest representation compared to all other tiers. For notational convenience, we state this assumption for all scores in K, but it only needs to hold for all "sufficiently high" scores within the relevant range where applicants may be admitted.

Theorem 1 *Fix the set of slots S and their types τ. Under Assumptions 1 and 2, among tier-blind precedence orders,*

i) the maximal tier 1 assignment is attained when all merit slots precede all tier slots, and

ii) the minimal tier 1 assignment is attained when all tier slots precede all merit slots.

Under Assumption 2, at any given score, there is lower representation of tier 1 applicants compared to other tiers. When an even share of reserve slots across tiers are filled before merit slots, the gap at the upper tail of the score distribution widens between tiers. As such, the larger the share of reserve slots that are processed prior to merit seats, the lower is the access for tier 1 applicants for merit slots. Hence, maximal access is attained when all merit slots precede tier slots, just as minimal access is attained when all tier slots precede merit slots.

How do applicants from the highest tier fare under these admissions policies? To answer this question, it is necessary to specify how scores from highest tier compares with those of other tiers. The following empirically-motivated assumption is the mirror image of Assumption 2.

Assumption 3 $f_{\bar{t}}(k) \geq f_t(k)$, $\forall k \in K$ *and* $\forall t \in T$.

Assumption 3 states that for any given score, there is higher representation of tier $\bar{t}$ students compared to all other tiers. As with Assumption 2, we state this assumption for all scores K, but it only needs to hold for all scores within the relevant range where applicants may be admitted.

Our next result shows that under Assumption 3, there is a clear conflict of interest between the highest socioeconomic tier and the lowest socioeconomic tier under our specified precedence policies.

Proposition 3 *Fix the set of slots S and their types τ. Under Assumptions 1 and 3, among tier-blind precedence orders,*

i) the maximal tier $\bar{t}$ assignment is attained when all tier slots precede all merit slots, and

ii) the minimal tier $\bar{t}$ assignment is attained when all merit slots precede all tier slots.

This result is the symmetric counterpart to Theorem 1, but for the highest socioeconomic tier. Theorem 1 and Proposition 3 show that the best tier-blind policy for the lowest tier is worst for the highest tier, and vice versa. In Section 5, we report the difference in access across tiers between the best and worst tier-blind precedence for each school. We show that the difference between the best and worst precedence policies even among tier-blind policies can be substantial.

When Chicago Public Schools launched the tier-based affirmative action system in 2009, the system adopted a precedence where all merit slots precede all tier slots. We denote this as the **CPS precedence**. Given a reservation size and tier-blindness, Theorem 1 and Proposition 3 imply that the CPS precedence is the best policy for tier 1 applicants and the worst policy for tier 4 applicants. That is, given Chicago's tier reserves, our results show that Chicago's policy is biased in favor of applicants from the most disadvantaged neighborhoods at the expense of applicants from the most advantaged neighborhoods, even though it is tier-blind.

4.2 Increasing the Size of Reservations

In the 2011-12 school year, Chicago Public Schools increased the size of tier reservations from 60% to 70%. That is, the share of tier slots increased from 15% to 17.5% for each tier. This change was made at the urging of a Blue Ribbon Commission (BRC 2011), which examined the racial makeup of schools under the 60% reservation compared to the old Chicago's old system of racial quotas. They advocated for the increase in tier reservations on the basis it would be

> improving the chances for students in neighborhoods with low performing schools, increasing diversity, and complementing the other variables.

Our next result shows that under the CPS precedence, increasing the size of reservations results in greater access for the lowest tier students, but diminishes access for the highest tier students.

Proposition 4 *Under the CPS precedence, and Assumptions 1-3, an equal sized increase in the number of tier slots,*

i) weakly increases the mass of tier 1 assignment and

ii) weakly decreases the mass of tier $\bar{t}$ assignment.

The intuition for Proposition 4 is simple. Under Assumption 2, higher tier students on average have higher scores than tier 1 students. Therefore, tier 1 students have less to lose from a reduction in the share of merit seats compared to other tiers. In contrast, under Assumption 3, tier $\bar{t}$ students have on average higher scores than other tiers, and as such, they have more to lose from a reduction in the share of merit seats.

Proposition 4 together with Theorem 1 imply that the best tier-blind rule for tier 1 students equal-size reserves with no merit seats. That is, in the case of Chicago with four tiers, the best tier-blind precedence policy is a 25% reservation for each tier and, thus, no merit slots. Indeed, in the policy discussion about modifying the plan, some advocated for the complete elimination of merit seats and equal-sized shares for each tier (see, e.g., BRC (2011)).

5 Evidence from Affirmative Action in Chicago

5.1 Examining Modeling Assumptions

In this section, we investigate the extent to which our theoretical results provide insights about Chicago's affirmative action system and quantify how the precedence affects the allocation of students from different tiers to particular schools. Our data consists of application files from Chicago Public Schools for the 2012-2013 school year, and contain student rankings, tier, and composite scores. A total of 16,818 applicants ranked schools, with 3,876 from tier 1, 4,292 from tier 2, 4,648 from tier 3, and 4,002 from tier 4. There are ten schools with a total of 4,025 seats.

In 2012-13, CPS used six factors to place neighborhoods into tiers: (1) median family income, (2) percentage of single-parent households, (3) percentage of households where English is not the first language, (4) percentage of homes occupied by the homeowner, (5) level of adult education attainment, and (6) average ISAT scores for attendance area schools. Based on these factors, each census tract was given a score, scores were ranked, and then census tracts were divided into four groups, each with approximately the same number of school-age children. Tier 1 tracts have the lowest socioeconomic index, while tier 4 tracts have the highest socioeconomic index. At each school, for 30% of the seats, the admissions criteria was merit-based using composite scores. The remaining 70% of the seats were divided into four equally-sized reserves, one for each tier. At each 17.5% reserve for a given tier, the admissions criteria was merit-based within that tier. Students could rank up to six choices, and applications were processed via the student-proposing deferred acceptance (DA) algorithm using the CPS precedence.

Chicago's affirmative action system differs from our model in one important way. We've focused on the properties of one school's choice function, but CPS uses DA to assign ten schools. This fact raises the question of how best to interpret our modelling assumptions using data from a centralized match. As we mentioned before, DA can be interpreted as the iterated application of choice functions across all schools, where in the first iteration students apply to their top choices and in each subsequent iteration, students rejected in earlier iterations apply

to their next choices. Under DA, it is sufficient to look at the cumulative set of proposals to a school during the algorithm to determine who is assigned to that school. Indeed, this property motivated Hatfield and Milgrom (2005) to define DA as the "cumulative offer" algorithm. Consequently, we investigate our three assumptions by considering the set of applicants who are subject to each school's choice function – that is, those who apply to that school during CPS's DA implementation.[11]

There is strong support for Assumption 1, which states that there are more tier t students than the number of tier t and merit slots at a school for any t. For each of the ten schools, the number of applicants from each tier is greater than the number of merit slots and slots reserved for that tier. In most cases, the number of applicants is far greater than the number of slots. For instance at Payton, the most competitive school, there are 2,091 applicants from tier 4 competing for 106 seats and the ratio of applicants to seats is similar for the other three tiers. Tier 4 applicants are less interested in schools with lower admissions cutoffs. At King, the number of tier 4 applicants is only about double the number of seats for which they compete. Moreover, at less competitive schools, the composition of applicants includes a larger share from lower tiers than higher tiers. At both King and South Shore, there are nearly three times as many applicants from tier 1 than tier 4.[12]

There is also strong support for Assumption 2, which states that tier 1 students have lower scores compared to all other tiers over a relevant range. For each tier, Figure 1 plots a smoothed estimate of the score distribution for each school ordered by merit cutoff.[13] The tier 1 line is below the corresponding lines for each other tier at nine schools, when we define the relevant range as scores above the cutoff for merit seats. Since this cutoff will likely be high when merit slots are processed first, it is also worth examining a more conservative definition of the relative range as scores greater than the minimum score needed to qualify for a merit seat. This is a conservative assumption because under the CPS precedence, the lowest scoring applicant from a given tier may have a score well below what is needed to obtain a merit seat. In such a case, applicants with scores near this threshold are unlikely to influence the competition for merit seats under different precedence orders. For the more conservative definition, the tier 1 line is below the other lines for the six most competitive schools. For these six schools, we therefore expect a close match between the best and worst tier-blind precedence computed in Theorem 1 and the Chicago data. For the other four schools, tier 1 applicants score systematically lower than applicants from other tiers, even though Assumption 2 is not exactly satisfied. Since Assumption 2 is sufficient, but not necessary, it is still possible that the optimal tier-blind precedences calculated in Theorem 1 account for empirical patterns at these schools.

[11] This set of application is identical to the "sharp sample" defined in Abdulkadiroğlu, Angrist, and Pathak (2014).

[12] South Shore is a new school that opened in 2012-13 and therefore may have experienced unusually low demand in its initial year.

[13] Since scores are discrete, we report a local linear smoother with bin size of 0.5 using STATA's lowess command. Scores range from 0 to 900, but we plot the range above 600 since no applicants below that score are admitted.

For the most competitive schools, there is strong support for Assumption 3, which states that tier 4 students have higher scores compared to the other tiers. If the relevant range starts from the minimum score needed to qualify, the tier 4 line is above the lines from other tiers for the five most competitive schools. If the relevant range starts from the minimum score needed to qualify for a merit seat, the tier 4 line is also above all the other lines at Westinghouse. Proposition 3 is, therefore, likely most relevant for these schools.

5.2 Comparing Alternative Affirmative Action Policies

The Best and Worst Precedence for Tier 1 under 30% Merit

Figure 2 reports the fraction of seats assigned to tier 1 students under the best and worst policy and the best and worst tier-blind policy for tier 1 applicants, when 30% of seats are reserved for merit. The figure tabulates the fraction of seats assigned to tier 1 students across all schools and then reports a breakdown by school, where schools are ordered from left to right by selectivity. At Payton, for instance, the cutoff for tier 1 seats is 801, while the cutoff for tier 4 seats is 892. Since merit seats precede tier seats under the CPS precedence, the merit cutoff is even higher at 898. At Northside and Young, there is also a roughly 100 point gap between the score of the last admitted tier 4 and tier 1 applicant at their respective tier seats. To obtain a merit seat at either school, an applicant must have nearly a perfect score (897 and 886, respectively). South Shore and King are the least competitive schools, both with tier 1 cutoffs of 650 and merit cutoffs of 704 and 714, respectively.

Across all schools, an additional 124 tier 1 students are assigned under the best tier 1 precedence compared to the best tier-blind precedence for tier 1. This comparison can be seen by comparing columns 1 and 2 of Figure 1 and recalling that a total of 4,025 students are assigned. It's worth noting that for a single school's choice function, Lemma 1 implies that it is not necessary to specify the ordering of the tier seats that precede the merit seats. We therefore report the allocation generated by precedence Tier2-Tier3-Tier4-Merit-Tier 1 in the first column. Under the best policy for tier 1 applicants, a total of 875 tier 1 applicants are assigned to any school, so the reduction in the number of tier 1 students assigned due to tier-blindness, the "cost of tier-blindness," of 124 students is substantial.

For particular schools, the cost of tier-blindness depends on school selectivity. At the most competitive schools, the reduction in how many tier 1 applicants are assigned in the best tier-blind policy is small. At Payton, one fewer tier 1 student is assigned in the best tier-blind policy. At Northside, four fewer tier 1 students are assigned in the best tier-blind policy. There is a substantial difference, however, at somewhat less competitive schools. At Westinghouse, 33 fewer students are assigned under the best tier-blind policy for tier 1. Figure 1 shows that there are more high-scoring applicants from higher tiers at score ranges needed to qualify for the most competitive schools. The difference in scores across tiers narrows at score ranges needed to qualify for less competitive schools. At the most competitive schools, almost all of the merit

seats are allocated to students from tiers other than tier 1, leaving little room for precedence to influence the applicant pool at merit seats. At less competitive schools, the impact of precedence is larger because the competition for merit seats across tiers is more even.

The range of outcomes from the best and worst tier-blind policy shows the potential scope for statistical targeting. This range can be seen by comparing the second and third columns of Figure 2, which are the best and worst tier-blind precedence for tier 1. A total of 39 fewer tier 1 students are assigned in worst tier-blind policy compared to the best tier-blind policy for tier 1. For particular schools, statistical targeting allows for a smaller range of outcomes at more selective schools. At Payton, there is no difference for tier 1 applicants. At Northside, the tier-blind range is 3 students, and at Young, it is 1 student. At the other extreme, the largest range is at Westinghouse and Lindblom, where 8 more tier 1 students could be assigned to each school in the best tier-blind policy for tier 1 compared to the worst one.

While there is a substantial gap between the best policy for tier 1 and the best tier-blind policy, there is almost no difference between the worst policy for tier and the worst tier-blind policy. This fact can be seen by comparing the third and fourth columns of Figure 2. Only three fewer students are assigned in the worst policy for tier 1 compared to the worst tier-blind policy for tier 1. The outcome is the same at 17.5% for all schools, except South Shore and King. This means that tier 1 students are essentially shut out entirely from merit seats under these two worst precedence policies. Appendix Figure B1 shows that tier 1 applicants obtain systematically higher choices in a way that parallels the pattern in Figure 2.

The Best and Worst Precedence for Tier 1 under 40% Merit

Our theoretical analysis studied how precedence policies influence the competition for merit seats. When the share of merit seats increases to 40%, precedence has a larger effect on the allocation of tier 1 applicants. This can be seen in Figure 3, where we tabulate the outcome of the four precedence policies in Figure 2, but for 40% merit. As described above, the initial CPS affirmative action system was launched with 40% merit, but it switched to 30% in 2010-2011. The figure shows that for all schools, the gap between the best and worst policy for tier 1 applicants is at least as large when 40% of seats are assigned via merit compared to 30%. This fact can be seen by comparing the first and fourth columns of Figures 2 and 3. At most schools, the scope for statistical targeting is also larger with more merit seats. This fact can be seen by comparing the second and third columns of Figure 3 with the corresponding columns of Figure 2.

Figure 3 shows that the King does not follow the pattern predicted by our theoretical results when 40% of seats are assigned via merit. Fewer tier 1 students assigned to King compared to the worst tier-blind policy for tier 1. This discrepancy is not inconsistent with our theoretical results given that Assumption 2 fails in a significant part of the potentially relevant score range at King.

Are the differences between precedence policies quantitatively large or small? Recall that a Blue Ribbon Commission reviewing Chicago's policy made the controversial recommendation to decrease the merit percentage from 40% to 30%. Under the CPS precedence, 63 more tier 1 students are assigned to an exam school when the merit percentage is 30%. Had CPS not decreased the merit percentage, 118 more students would be assigned in the best tier 1 precedence compared to the CPS precedence. Therefore, it would have been possible to hold the merit fraction fixed and increase access for tier 1 applicants simply by changing the precedence order. This comparison involves explicit preferential treat tier 1 applicants. When 40% of seats are merit, 34 more tier 1 students are assigned under the CPS precedence compared to the worst tier-blind policy for tier 1.[14] Even relative to the more salient tool of decreasing merit seats, the scope for statistical targeting is more than half the effect of changing the merit fraction. Given the policy review and debate leading to the adoption of the 30% merit reservation, these magnitudes suggest that precedence is far from a trivial consideration.

6 Conclusion

Chicago Public Schools has embarked on a landmark placed-based affirmative action system for assignment to its highly sought-after exam schools. Though we have focused on Chicago, many other affirmative action systems provide affirmative action to multiple categories of applicants, and allow these applicants to be assigned to multiple types of seats. This aspect of the system motivated our investigation of how applicant processing affects the implementation of affirmative action.

We have shown that it is not sufficient to specify that reserves are equally sized to eliminate explicit targeting. Moreover, even in systems that do not explicitly discriminate, there are many possible implementations of affirmative action driven by statistical differences in scores by applicant tier, due to statistical targeting. Our formal results characterize the best and worst precedence policy for a given reserve size and the best and worst tier-blind precedence policy for applicants from a given tier. In doing so, we show that CPS's current policy has been favoring the most disadvantaged applicants. We also show that the bias in favor of applicants from the most disadvantaged tier is comparable to the outright increase in the size of tier reservations in 2012.

This paper contributes to a new focus in the analysis of priority-based resource allocation problems like student assignment. A large portion of that literature has taken the social objectives embodied in the priorities as given and then examined the properties of different market-clearing mechanisms. This paper, like Echenique and Yenmez (2015), focuses on how various

[14]The range between the best and worst tier-blind precedence for tier 1 with 40% merit is smaller than the range with 30% merit, where it is 39 students, because of King. Ignoring assignments at King, the range is larger when the merit share is 40%. Excluding King, 51 more tier 1 applicants are assigned under the best tier-blind precedence for tier 1 compared to the worst tier-blind policy for tier 1 under 40% merit, and the comparable range is 35 under 30% merit.

social objectives are captured by a school's choice function. In Chicago, we've seen that the biases associated with processing applicants are present even under tier-blind admissions policies, opening the door to statistical targeting. Our findings also have implications outside of Chicago because they show that it is necessary to consider both the reserve size and processing order to determine the best way to distribute access to disadvantaged applicants.

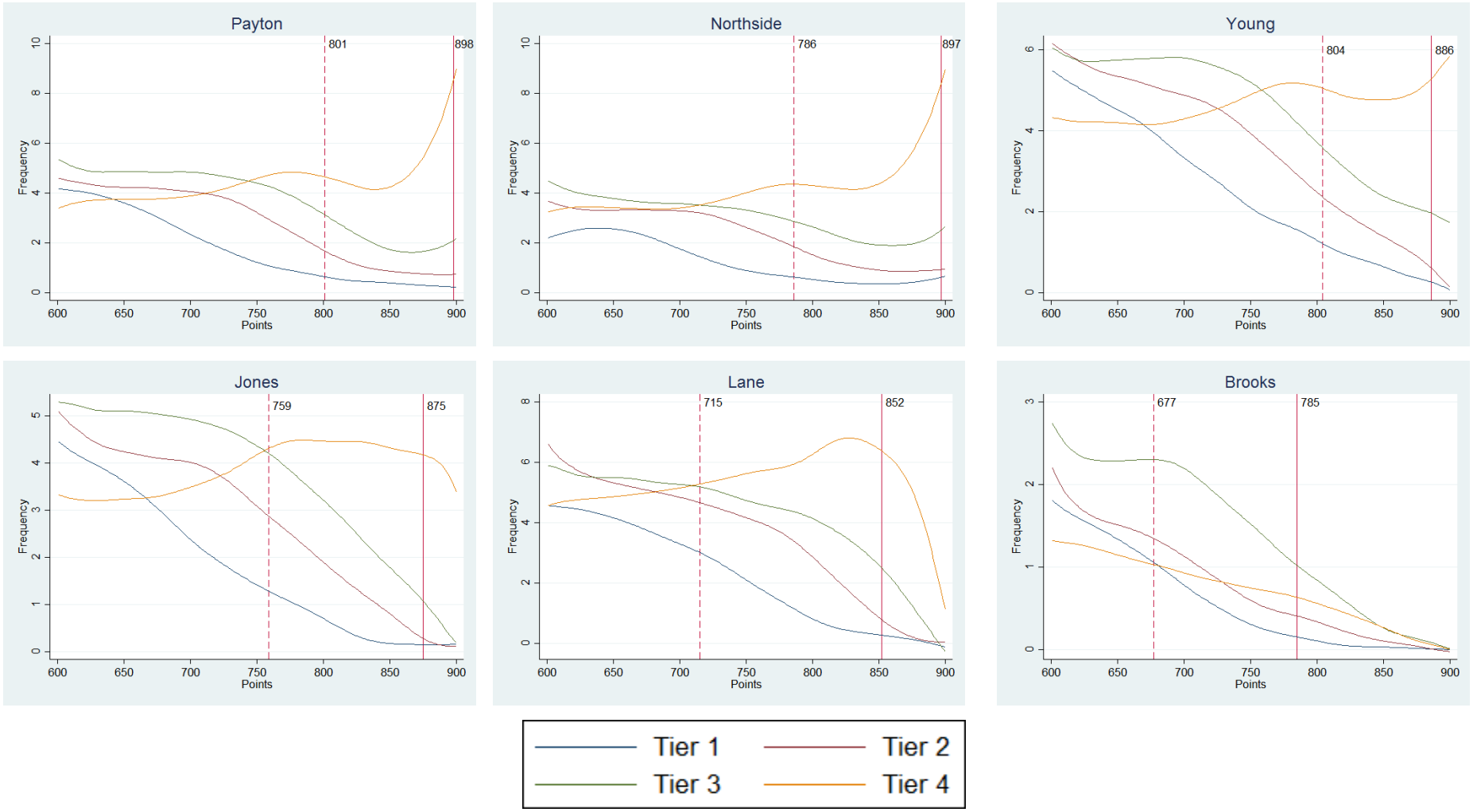


Figure 1(a): Distribution of Composite Scores for Each Tier.

Score distribution shown for applicants who apply to school during course of mechanism. Dashed vertical line indicates minimum score to be offered a seat while solid vertical line indicates minimum score to be offered a merit seat under CPS precedence. Lines from local linear smoother (lowess) with bin size of 1.

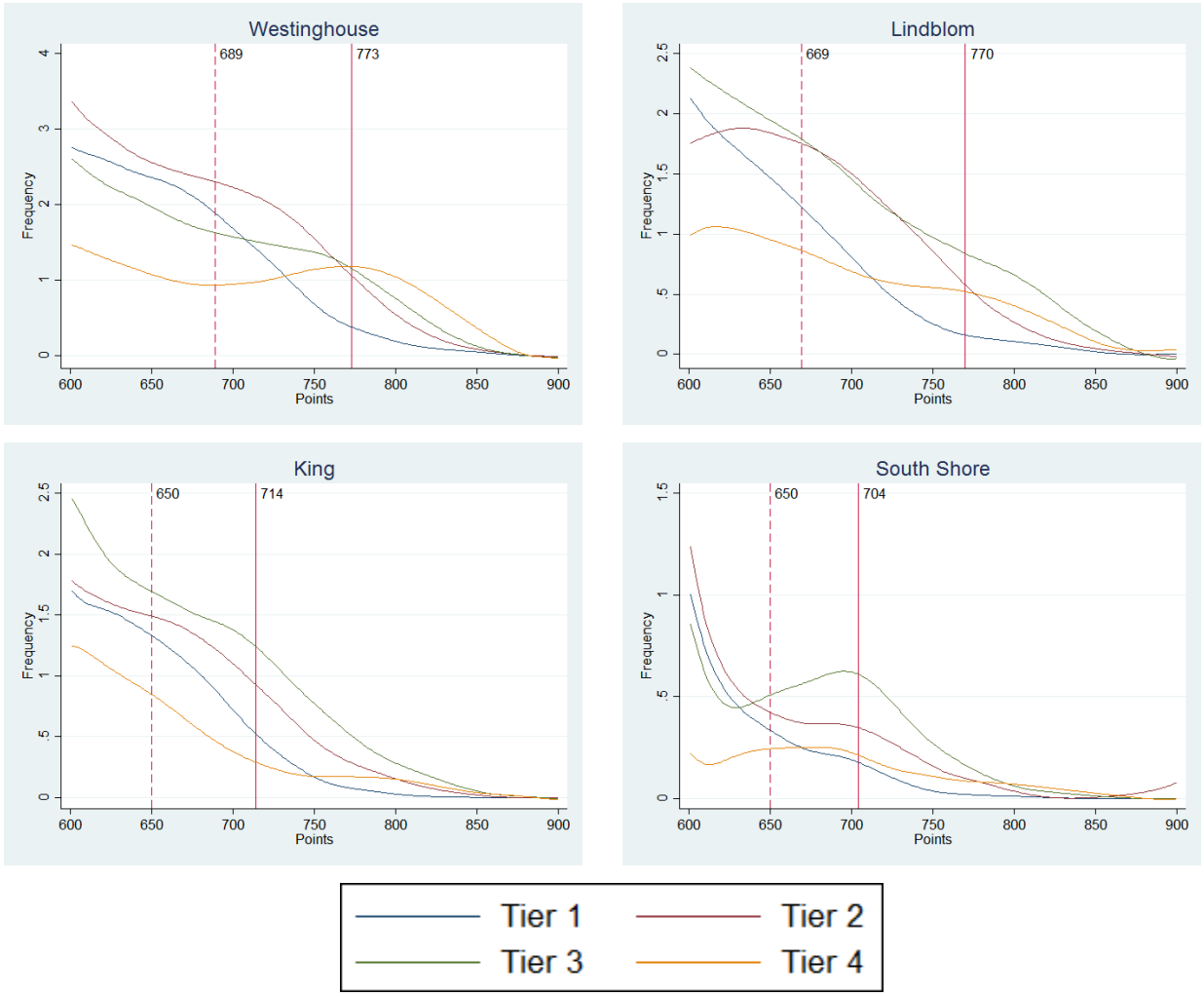


Figure 1(b): Distribution of Composite Scores for Each Tier

Score distribution shown for applicants who apply to school during course of mechanism. Dashed vertical line indicates minimum score to be offered a seat while solid vertical line indicates minimum score to be offered a merit seat under CPS precedence. Lines from local linear smoother (lowess) with bin size of 1.

Figure 2: Fraction of Seats Assigned to Tier 1 Students (30% Merit / 70% Tier)

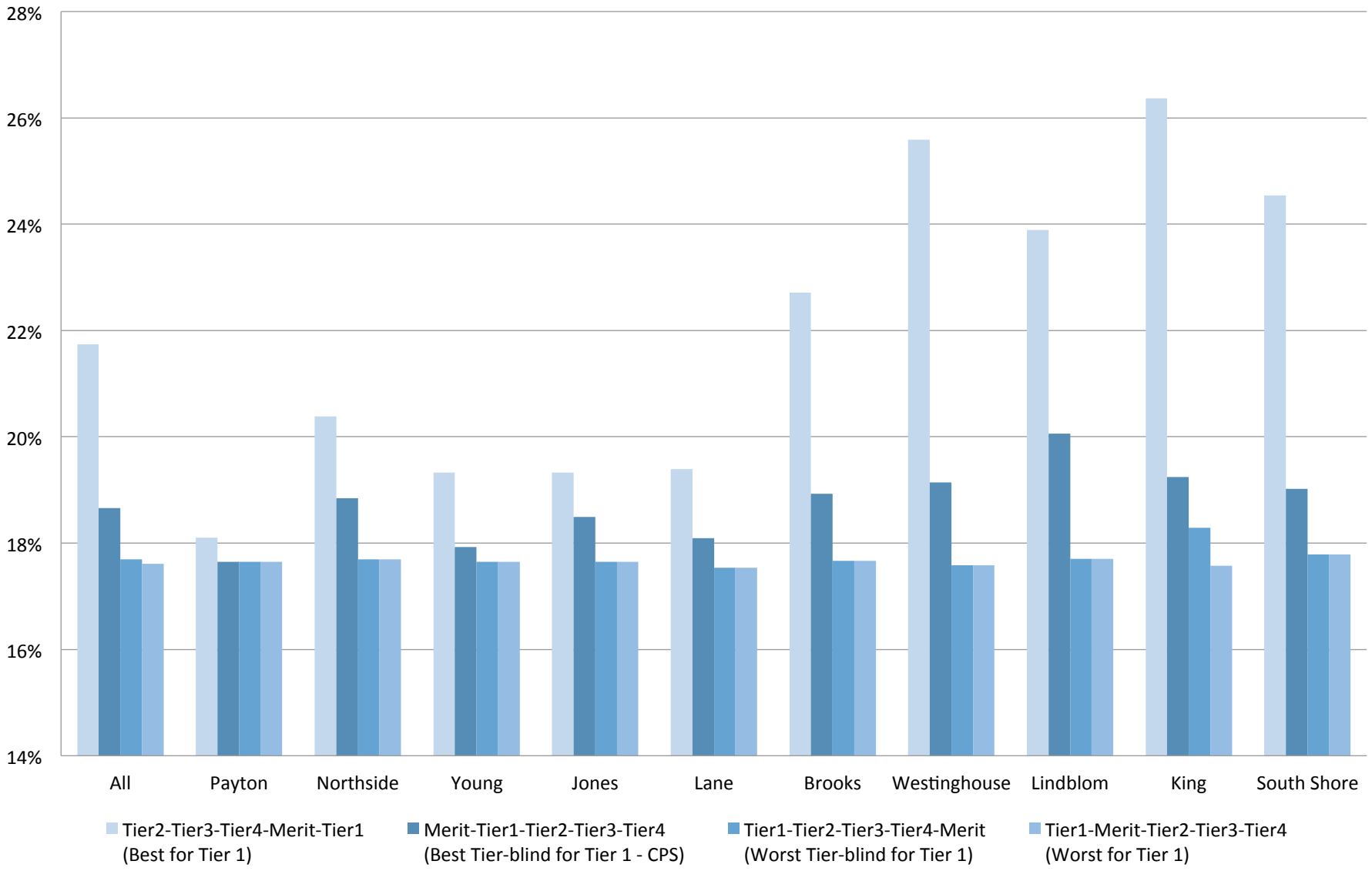
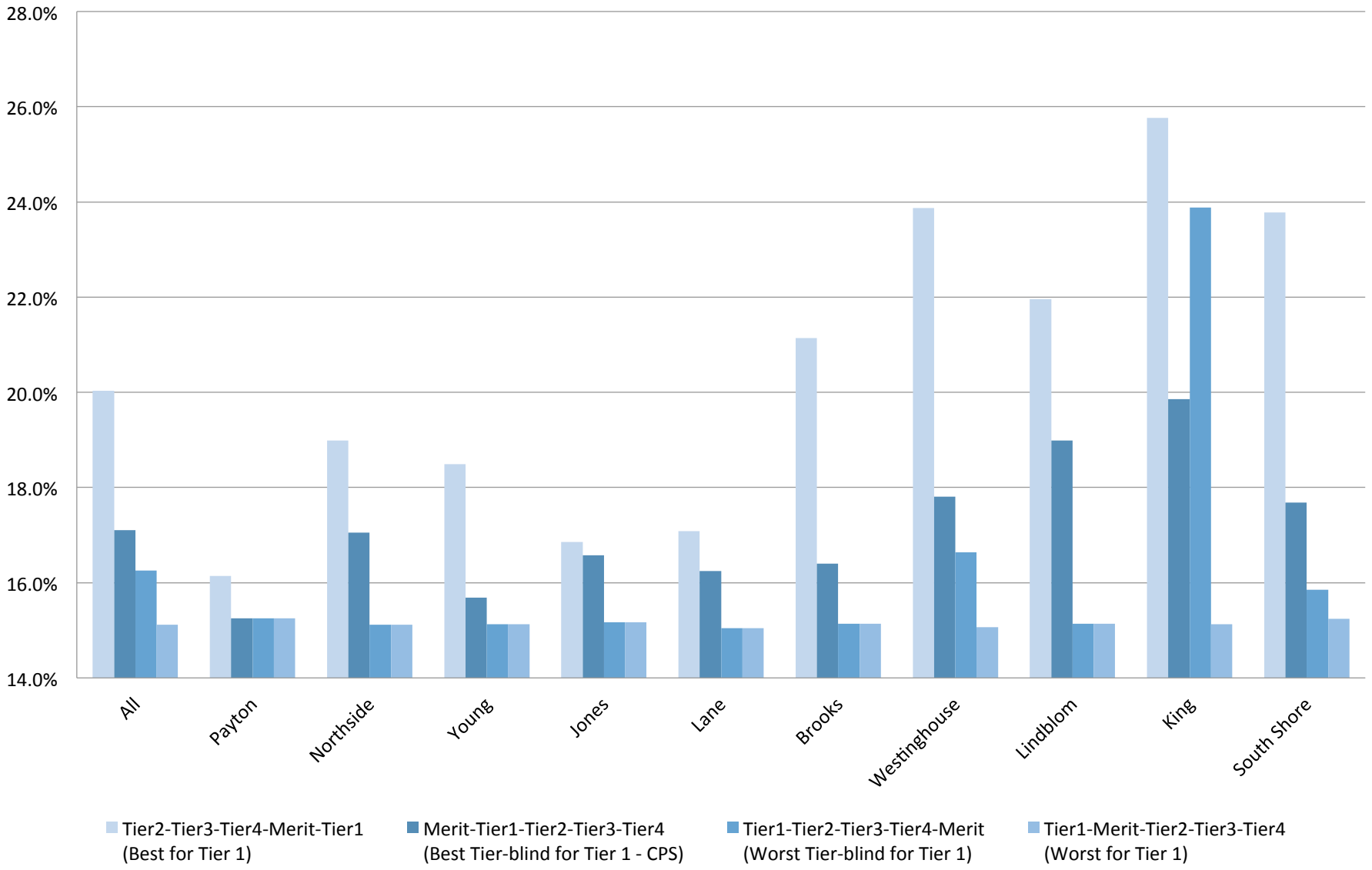


Figure 3: Fraction of Seats Assigned to Tier 1 Students (40% Merit / 60% Tier)



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A Appendix

We begin by introducing some additional notation. We fix the set of slots S and type function τ. We denote the set of students chosen for slot $s \in S$ by choice function $C(\cdot)$ from set J under precedence order $\triangleright$ with $C_s(\triangleright, J)$. Similarly, we denote the set of tier t students in $C(\triangleright, J)$ with $C^t(\triangleright, J)$ and the set of tier t students in $C_s(\triangleright, J)$ as $C^t_s(\triangleright, J)$.

For our analysis below, it is convenient to define tier and merit slot groups:

Definition 4 *Given S, τ and $\triangleright$, a subset of tier slots $G \subseteq S \setminus S_m$ is called a **tier slot group** if it consists of either*

i) all tier slots that have higher precedence than the highest precedence merit slot, or

ii) all tier slots that have lower precedence than the lowest precedence merit slot, or

iii) all tier slots between any subsequent merit slots.

Definition 5 *Given S, τ and $\triangleright$, a subset of merit slots $H \subseteq S_m$ is called an **merit slot group** if it consists of either*

i) all merit slots that have higher precedence than the highest precedence tier slot, or

ii) all merit slots that have lower precedence than the lowest precedence tier slot, or

iii) all merit slots between any subsequent tier slots.

We begin with a preliminary Lemma that shows that comparing the set of chosen students by two choice functions is equivalent to comparing the size of the two chosen sets.

Lemma 2 *Fix the set of slots S and their types τ. Let $\triangleright$ and $\triangleright'$ be two precedence orders. Then, for any $t \in T$,*

$$|C^t(\triangleright, I)| \leq |C^t(\triangleright', I)| \iff C^t(\triangleright, I) \subseteq C^t(\triangleright', I).$$

Proof. Fix a tier $t \in T$. Observe that for any precedence order, if any student i of tier t is chosen under the choice function $C(.)$ then all students of tier t with higher composite scores than student i are chosen under the choice function $C(.)$. This observation immediately implies the desired result. □

Next, we state a lemma that is used in the proofs of Theorem 1 and Propositions 1 and 3.

Lemma 3 *Fix the set of slots S and their types τ. Partition S into three sets S^1, S^2, and S^3 such that there are no merit slots in S^3. Let $\triangleright$ be a precedence order such that*

$$s^1 \triangleright s^2 \triangleright s^3 \qquad \text{for all } s^1 \in S^1,\ s^2 \in S^2, \text{ and } s^3 \in S^3.$$

Let $\triangleright'$ be another precedence order that differs from $\triangleright$ only in the precedence rankings of slots in S^2. Under Assumption 1, for all $t \in T$,

$$C^t(\triangleright, I) \subseteq C^t(\triangleright', I) \iff \bigcup_{s \in S^2} C^t_s(\triangleright, I) \subseteq \bigcup_{s \in S^2} C^t_s(\triangleright', I).$$

Proof. Each slot in S^1 not only has the same precedence ranking in $\triangleright$ and $\triangleright'$, but is also processed before slots in $S^2 \cup S^3$. Therefore, for all $t \in T$,

$$\bigcup_{s \in S^1} C^t_s(\triangleright, I) = \bigcup_{s \in S^1} C^t_s(\triangleright', I) \Longrightarrow \Big| \bigcup_{s \in S^1} C^t_s(\triangleright, I) \Big| = \Big| \bigcup_{s \in S^1} C^t_s(\triangleright', I) \Big|.$$

Moreover, since there are no merit slots in S^3, for all $t \in T$,

$$\Big| \bigcup_{s \in S^3} C^t_s(\triangleright, I) \Big| = \Big| \bigcup_{s \in S^3} C^t_s(\triangleright', I) \Big| = |S^3 \cap S_t|$$

by Assumption 1. Therefore for all $t \in T$,

$$\Big| \bigcup_{s \in S} C^t_s(\triangleright, I) \Big| \leq \Big| \bigcup_{s \in S} C^t_s(\triangleright', I) \Big| \Longleftrightarrow \Big| \bigcup_{s \in S^2} C^t_s(\triangleright, I) \Big| \leq \Big| \bigcup_{s \in S^2} C^t_s(\triangleright', I) \Big|.$$

Hence Lemma 2 implies, for all $t \in T$,

$$C^t(\triangleright, I) \subseteq C^t(\triangleright', I) \iff \bigcup_{s \in S^2} C^t_s(\triangleright, I) \subseteq \bigcup_{s \in S^2} C^t_s(\triangleright', I).$$

□

A.1 Proof of Lemma 1

We use the following result to prove Lemma 1 in the main text.

Lemma 4 *Fix the set of slots S and their types τ. Let $\hat{\triangleright}$ be a precedence order in which a tier slot $\hat{s}$ immediately precedes another tier slot s'. Let $\triangleright'$ be a precedence order obtained from $\hat{\triangleright}$ by swapping the precedence ranks of $\hat{s}$ and s' and leaving the precedence ranks of all other slots unchanged. Under Assumption 1, $C(\hat{\triangleright}, I) = C(\triangleright', I)$.*

Proof. Let $\hat{s}$ be the h^{th} slot under $\hat{\triangleright}$. Consider the outcome of choice function $C(\cdot)$ under problems $(\hat{\triangleright}, I)$ and $(\triangleright', I)$. Since the first $(h-1)$ slots are the same under both $\hat{\triangleright}$ and $\triangleright'$, $C_s(\hat{\triangleright}, I) = C_s(\triangleright', I)$ for all $s \in S$ with $s \,\hat{\triangleright}\, \hat{s}$ (and, therefore $s \;\triangleright'\; s'$). Hence, the same subset of students, denoted by I', is available to be selected for the h^{th} slot by choice function $C(\cdot)$ in both problems $(\hat{\triangleright}, I)$ and $(\triangleright', I)$. If $\hat{s}, s' \in S_t$ for some $t \in T$, then the highest-scoring two unit masses of tier t applicants in I' are selected both for $\hat{s}$ and s' by $C(\cdot)$ under both $\hat{\triangleright}$ and $\triangleright'$. If

$\hat{s} \in S_{\hat{t}}$ and $s' \in S_{t'}$ where $\hat{t} \neq t'$, then, in both problems $C(\cdot)$ selects the highest-scoring unit mass of tier $\hat{t}$ applicants in I' and the highest-scoring unit mass of tier t' applicants in I' for $\hat{s}$ and s', respectively. Hence, in both cases

$$C_{\hat{s}}(\hat{\triangleright}, I) \cup C_{s'}(\hat{\triangleright}, I) = C_{\hat{s}}(\triangleright', I) \cup C_{s'}(\triangleright', I),$$

and the same subset of students is available to be selected for the $(h+2)^{th}$ slot by $C(\cdot)$ under both $\hat{\triangleright}$ and $\triangleright'$. Since the last $(|S| - h - 1)$ slots are the same under both $\hat{\triangleright}$ and $\triangleright'$, $C_s(\hat{\triangleright}, I) = C_s(\triangleright', I)$ for all $s \in S$ with $s' \,\hat{\triangleright}\, s$ (and, therefore $\hat{s} \;\triangleright'\; s$). Hence,

$$\bigcup_{s \in S} C_s(\hat{\triangleright}, I) = \bigcup_{s \in S} C_s(\triangleright', I).$$

□

Proof of Lemma 1. Since any equivalent precedence order of $\triangleright$ can be obtained from consecutive swapping the ranks of the adjacent tier slots, Lemma 4 implies the desired result. □

A.2 Proof of Proposition 1

We use the following remark and Lemmata to prove Proposition 1. We skip the proof of the following remark for brevity.

Remark 1 *Fix the set of slots S and their types τ. Let $\triangleright$ and $\triangleright'$ be precedence orders over S such that the h^{th} slots under $\triangleright$ and $\triangleright'$ have the same type for all $h \in \{1, 2, ..., |S|\}$. Then, for any subset of students $J \subseteq I$, $C(\triangleright, J) = C(\triangleright', J)$.*

Lemma 5 *Fix the set of slots S, their types τ, and tier $t^* \in T$ such that S includes only merit and tier t^* slots under τ. Let $\triangleright$ be the precedence order over S such that merit slots precede all tier t^* slots and $\tilde{\triangleright}$ be the precedence order over S such that all tier t^* slots precede all merit slots.*[15] *Then, for any given $J \subseteq I$ with $n^J_{t^*} \geq |S_m| + |S_{t^*}| = |S|$,*

(i) $C^{t^*}(\tilde{\triangleright}, J) \subseteq C^{t^*}(\triangleright, J)$,

(ii) $C^t(\triangleright, J) \subseteq C^t(\tilde{\triangleright}, J)$ *for all* $t \in T \setminus \{t^*\}$.

Proof. With slight abuse of notation, let $C_m(\triangleright, J)$ and $C_m(\tilde{\triangleright}, J)$ be the set of students selected for the merit slots from J by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$, respectively. Since $n^J_{t^*} \geq |S|$, all slots are filled and tier $t \in T \setminus \{t^*\}$ students are only selected for merit slots by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$. Hence, it suffices to show that $C^t_m(\triangleright, J) \subseteq C^t_m(\tilde{\triangleright}, J)$ for all $t \in T \setminus \{t^*\}$.

[15] See Figure A.1 for examples of $\triangleright$ and $\tilde{\triangleright}$.

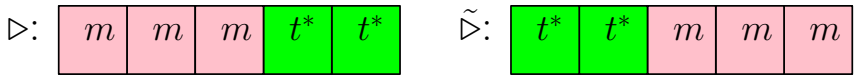

Figure A.1: Illustration of precedence orders $\triangleright$ and $\tilde{\triangleright}$

We denote the infimum scores of students in $C_m(\triangleright, J)$ and $C_m(\tilde{\triangleright}, J)$ with ℓ_m and $\tilde{\ell}_m$, respectively. Since the merit slots are filled first under $\triangleright$ and the tier slots are filled first under $\tilde{\triangleright}$, we have

$$\sum_{t=1}^{\bar{t}} \int_{\tilde{\ell}_m}^{\bar{k}} f_t^J(k)dk \geq |S_m| = \sum_{t=1}^{\bar{t}} \int_{\ell_m}^{\bar{k}} f_t^J(k)dk. \tag{3}$$

Equation (3) implies that $\ell_m \geq \tilde{\ell}_m$. For each $t \in T \setminus \{t^*\}$, if $i \in C_m^t(\triangleright, J)$, then $k(i) \geq \ell_m \geq \tilde{\ell}_m$ and hence $i \in C_m^t(\tilde{\triangleright}, J)$. That is, $C_m^t(\triangleright, J) \subseteq C_m^t(\tilde{\triangleright}, J)$ for all $t \in T \setminus \{t^*\}$, and therefore

$$C_m^{t^*}(\tilde{\triangleright}, J) \subseteq C_m^{t^*}(\triangleright, J).$$

Since all tier t^* slots are filled with tier t^* students,

$$C_m^{t^*}(\tilde{\triangleright}, J) \subseteq C_m^{t^*}(\triangleright, J) \quad \Longrightarrow \quad C^{t^*}(\tilde{\triangleright}, J) \subseteq C^{t^*}(\triangleright, J).$$

Similarly, for any $t \in T \setminus \{t^*\}$,

$$C_m^{t}(\triangleright, J) \subseteq C_m^{t}(\tilde{\triangleright}, J) \quad \Longrightarrow \quad C^{t}(\triangleright, J) \subseteq C^{t}(\tilde{\triangleright}, J).$$

□

Lemma 6 *Fix the set of slots S, their types τ, and tier $t' \in T$ such that S includes at least two merit slots under τ. Let S_m be partitioned into two non-empty subsets, S_m^1 and S_m^2. Let $\triangleright$ be a precedence order over S such that merit slots in S_m^1 precede all tier slots and all tier slots precede the merit slots in S_m^2. Let $\tilde{\triangleright}$ be a precedence order over S such that all tier $t \in T \setminus \{t'\}$ slots precede the merit slots and merit slots precede the tier t' slots. Then, for any $J \subseteq I$ with $n_t^J \geq |S_m| + |S_t|$ for all $t \in T$,*

$$C^{t'}(\triangleright, J) \subseteq C^{t'}(\tilde{\triangleright}, J).$$

Proof. We first consider precedence order $\triangleright$. When we move tier t' slots within the tier slot group such that they are preceded by all other tier slots, by Lemma 1 the mass of tier t' students selected from J by choice function $C(\cdot)$ does not change. Then, when we move tier t' slots after the merit slot group S_m^2, by Lemma 5.(i) the mass of tier t' students selected from J by choice

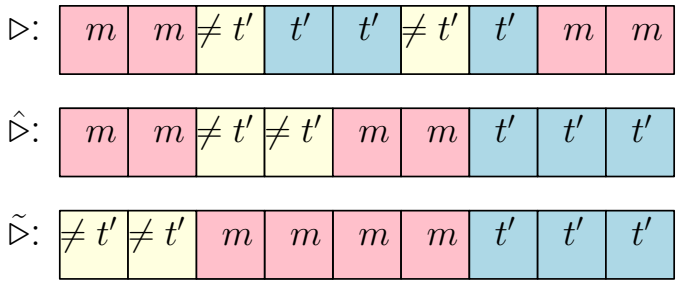

Figure A.2: Illustration of precedence orders $\triangleright$, $\hat{\triangleright}$, and $\tilde{\triangleright}$

function $C(\cdot)$ weakly increases. Let $\hat{\triangleright}$ denote the precedence order obtained from $\triangleright$ by these moves.[16] Then, $C^{t'}(\triangleright, J) \subseteq C^{t'}(\hat{\triangleright}, J)$. It therefore suffices to show that $C^{t'}(\hat{\triangleright}, J) \subseteq C^{t'}(\tilde{\triangleright}, J)$.

Under both $\tilde{\triangleright}$ and $\hat{\triangleright}$, tier t' slots have the lowest precedence and are filled with tier t' students. Hence, it is sufficient to compare the mass of tier t' students chosen for the merit slots by $C(\cdot)$ under $\tilde{\triangleright}$ and $\hat{\triangleright}$.

Denote the infimum score of tier $t \in T \setminus \{t'\}$ students assigned to tier t slots by $C(\cdot)$ under $\tilde{\triangleright}$ and $\hat{\triangleright}$ with $\tilde{k}_t$ and $\hat{k}_t$, respectively. Since some merit slots precede the tier slots under $\hat{\triangleright}$ whereas no merit slot precedes any tier slot for tier $t \neq t'$ under $\tilde{\triangleright}$, we have

$$\int_{\hat{k}_t}^{\bar{k}} f_t^J(k)dk \geq |S_t| = \int_{\tilde{k}_t}^{\bar{k}} f_t^J(k)dk \quad \text{for all } t \in T \setminus \{t'\}. \tag{4}$$

Equation (4) implies that $\tilde{k}_t \geq \hat{k}_t$ for all $t \in T \setminus \{t'\}$. Let $\tilde{\ell}_t$ and $\hat{\ell}_t$ be the infimum score of the tier $t \in T \setminus \{t'\}$ students chosen by $C(\cdot)$ under $\tilde{\triangleright}$ and $\hat{\triangleright}$, respectively. First note that, tier $t \in T \setminus \{t'\}$ students will not be selected for tier t' slots by $C(\cdot)$ under either $\tilde{\triangleright}$ or $\hat{\triangleright}$. We consider two possible cases.

Case 1 ***($\hat{\ell}_t > \tilde{\ell}_t$ for some $t \in T \setminus \{t'\}$)***: Since $\tilde{k}_t \geq \hat{k}_t$, there exist some students with score lower than $\hat{\ell}_t$ who are chosen for merit slots by $C(\cdot)$ under $\tilde{\triangleright}$, but not $\hat{\triangleright}$. In other words, the infimum score of students chosen for the merit slots by $C(\cdot)$ under $\tilde{\triangleright}$ is less than $C(\cdot)$ under $\hat{\triangleright}$. Since the merit slots precede tier t' slots under both $\tilde{\triangleright}$ and $\hat{\triangleright}$, all tier t' students chosen for the merit slots by $C(\cdot)$ under $\hat{\triangleright}$ are also chosen by merit slots by $\tilde{\triangleright}$. Hence, $C^{t'}(\hat{\triangleright}, J) \subseteq C^{t'}(\tilde{\triangleright}, J)$.

Case 2 ***($\hat{\ell}_t \leq \tilde{\ell}_t$ for each $t \in T \setminus \{t'\}$)***: Under this case, $C^t(\tilde{\triangleright}, J) \subseteq C^t(\hat{\triangleright}, J)$. Since all slots are filled, each tier t' students selected by $C(\cdot)$ under $\hat{\triangleright}$ is also selected by $C(\cdot)$ under $\tilde{\triangleright}$, i.e., $C^{t'}(\hat{\triangleright}, J) \subseteq C^{t'}(\tilde{\triangleright}, J)$. □

Remark 2 *Lemma 6 holds even if S does not include any tier t' slots.*

[16] We illustrate examples of precedence orders $\triangleright$, $\hat{\triangleright}$, and $\tilde{\triangleright}$ in Figure A.2.

Note that under any precedence order $\triangleright$, for any slot $s \in S$ when the available set of students to be admitted to s by choice function $C(\cdot)$ and the slots with weakly lower precedence than s are considered Assumption 1 holds. Hence, we can use Lemma 5 and 6 while proving Proposition 1.

Proof of Proposition 1.(i): Consider an arbitrary precedence $\triangleright'$. Without loss of generality, by Remark 1, we assume the relative precedence order of merit slots are the same under both $\bar{\triangleright}$ and $\triangleright'$. If there are no merit slots (i.e., if $S_m = \emptyset$), then $C(\triangleright', I) = C(\bar{\triangleright}, I)$ by Lemma 1. We have two cases to consider.

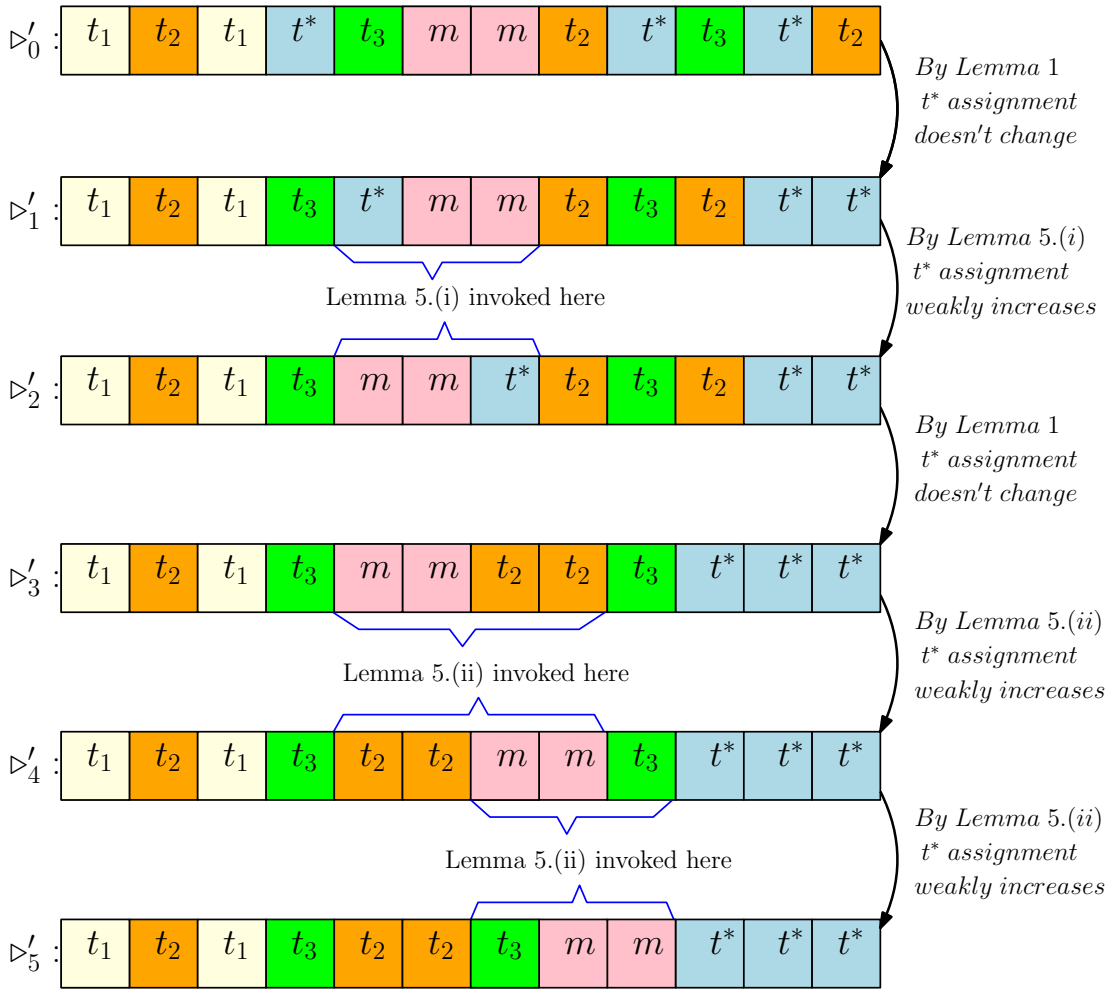

Figure A.3: Illustration of Case 1 of Proposition 1.(i)

Case 1 (There is one merit slot group under $\triangleright'$): Here is our proof strategy for Case 1. We will construct a sequence of precedences where the first element is $\triangleright'_0 \equiv \triangleright'$ and the last element is $\bar{\triangleright}$. We will show that t^* assignment weakly increases under choice function $C(\cdot)$ as we move from one element of the sequence to the next one.

Let $\rhd'_0 \equiv \rhd'$ be the first element of the sequence. Construct $\rhd'_1$ from $\rhd'_0$ by moving each slot of type t^* to the end of their tier slot group so that each tier slot of type t^* has lower precedence than each tier slot of any other type t within their tier slot group. See construction of precedence $\rhd'_1$ from $\rhd'_0$ in Figure A.3.

Claim 1 $C^{t^*}(\rhd'_1, I) = C^{t^*}(\rhd'_0, I)$.

Proof of Claim 1: Since $\rhd'_0$ and $\rhd'_1$ are equivalent, the desired result immediately follows from Lemma 1. ♢

There is potentially a tier slot of type t^* immediately before the only merit slot group under $\rhd'_1$. If such a slot does not, then $\rhd'_2 \equiv \rhd'_1$. Otherwise, construct $\rhd'_2$ from $\rhd'_1$ by moving all the adjacent tier t^* slots from immediately before the unique merit slot group to immediately after it. See construction of precedence $\rhd'_2$ from $\rhd'_1$ in Figure A.3.

Claim 2 $C^{t^*}(\rhd'_2, I) \supseteq C^{t^*}(\rhd'_1, I)$.

Proof of Claim 2: Let S' be the set of merit slots (in the unique merit slot group) together with all of the adjacent type t^* slots that are immediately before the merit slot group for the case of $\rhd'_1$ and immediately after it for the case of $\rhd'_2$. By Lemma 5.(i), we have

$$\bigcup_{s \in S'} C_s^{t^*}(\rhd'_2, I) \supseteq \bigcup_{s \in S'} C_s^{t^*}(\rhd'_1, I). \tag{5}$$

Equation (5) together with Lemma 3 complete the proof of Claim 2. ♢

Next we consider the set of tier slots after the unique merit slot group under $\rhd'_2$. Construct $\rhd'_3$ from $\rhd'_2$ by reorganizing tier slots in this set so that

1) slots of the same type are processed subsequently as a group, and

2) slots of type t^* have the lowest precedence and thus are processed at the end.

See construction of precedence $\rhd'_3$ from $\rhd'_2$ in Figure A.3.

Claim 3 $C^{t^*}(\rhd'_3, I) = C^{t^*}(\rhd'_2, I)$.

Proof of Claim 3: Since $\rhd'_2$ and $\rhd'_3$ are equivalent, the desired result immediately follows from Lemma 1. ♢

The argument for the remaining steps will be identical, and hence we only state it once.

Let t' be the tier that is processed after the unique merit slot group $\rhd'_3$. If $t' = t^*$, then $\rhd'_3 \equiv \bar{\rhd}$ and we are done. Otherwise, construct $\rhd'_4$ from $\rhd'_3$ by moving all the adjacent tier t' slots from immediately after the unique merit slot group to immediately before it. See construction of precedence $\rhd'_4$ from $\rhd'_3$ in Figure A.3.

Claim 4 $C^{t^*}(\triangleright'_4, I) \supseteq C^{t^*}(\triangleright'_3, I)$.

Proof of Claim 4: Let S' be the set of merit slots (in the unique merit slot group) together with the with all of the adjacent type t' immediately after the merit slot group for the case of $\triangleright'_3$ and immediately before it for the case of $\triangleright'_4$. By Lemma 5.(ii), we have

$$\bigcup_{s \in S'} C^{t^*}_s(\triangleright'_4, I) \supseteq \bigcup_{s \in S'} C^{t^*}_s(\triangleright'_3, I). \tag{6}$$

Equation (6) together with Lemma 3 complete the proof of Claim 4. ♢

Repeated application of the last step of the construction for each $t \neq t^*$ gives us the desired result.[17]

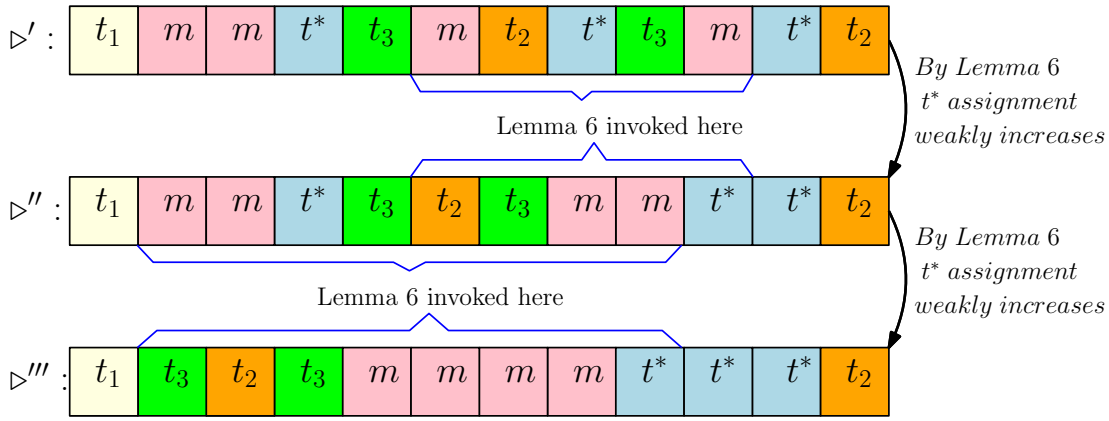

Figure A.4: Illustration of Case 2 of Proposition 1.(i)

Case 2 (There is more than one merit slot group under $\triangleright'$): Here is our proof strategy for Case 2. Given $\triangleright'$ with at least two merit slot groups, we will construct a precedence $\triangleright''$ which has one less merit slot group than under $\triangleright'$, and that weakly increases t^* assignment under choice function $C(\cdot)$. Repeated application of this construction will eventually transform Case 2 to Case 1 where we have already obtained the desired result.

Construct $\triangleright''$ from $\triangleright'$ by moving all tier t^* slots between the last two merit slot groups immediately after the last merit slot group, and moving all other tier slots immediately before the penultimate merit slot group. Observe that under the new precedence $\triangleright''$, there is one less merit slot group than under $\triangleright'$. See construction of precedence $\triangleright''$ from $\triangleright'$ in Figure A.4.

Claim 5 $C^{t^*}(\triangleright'', I) \supseteq C^{t^*}(\triangleright', I)$.

Proof of Claim 5: Let $S^{\ell}_{t^*}$ and $S^{\ell}_{-t^*}$ denote tier t^* slots and other tier slots between the last two merit slot groups under $\triangleright'$. Let S' be the last two merit slot groups together with the tier

[17] In Figure A.3 the last step is applied twice: first when $\triangleright'_4$ is constructed from from $\triangleright'_3$, and second when $\triangleright'_5$ is constructed from from $\triangleright'_4$.

slot group between them for the case of $\triangleright'$ or equivalently the last merit slot group together with the tier slots in $S^{\ell}_{t^*}$ and $S^{\ell}_{-t^*}$ for the case of $\triangleright''$. By Lemma 6, we have

$$\bigcup_{s\in S'} C^{t^*}_s(\triangleright'', I) \supseteq \bigcup_{s\in S'} C^{t^*}_s(\triangleright', I). \tag{7}$$

Equation (7) together with Lemma 3 complete the proof of Claim 5. ♢

Repeated application of this construction decreases the number of merit slot groups, and eventually gives us a precedence order with one merit slot group. Hence, application of the steps in Case 1 to this precedence order gives us the desired result.[18] □

Proof of Proposition 1.(ii): Consider an arbitrary precedence $\triangleright'$. Without loss of generality, by Remark 1, we assume the relative precedence order of merit slots are the same under both $\trianglerighteq$ and $\triangleright'$. If there are no merit slots (i.e., if $S_m = \emptyset$), then $C(\triangleright', I) = C(\trianglerighteq, I)$ by Lemma 1. We have two cases to consider.

Case 1 (There is one merit slot group under $\triangleright'$): In this case, the construction of the sequence of the precedences as well as the proof itself are completely analogous to that in Case 1 of the proof of Proposition 1.(i) with a reverse construction. Instead of repeating the entire argument, we illustrate the modified construction with Figure A.5.

Case 2 (There is more than one merit slot group under $\triangleright'$): Here is our proof strategy for Case 2. We will construct a sequence of precedences where the first element is $\triangleright'_0 \equiv \triangleright'$ and the last element is $\trianglerighteq$. We will show that t^* assignment weakly decreases under choice function $C(\cdot)$ as we move from one element of the sequence to the next one.

Let $\triangleright'_0 \equiv \triangleright'$ be the first element of the sequence. We consider the tier slot groups under $\triangleright'_0$. Construct $\triangleright'_1$ from $\triangleright'_0$ by reorganizing tier slots in each tier slot group so that

1) slots of the same type are processed subsequently as a group, and

2) slots of type t^* have the highest precedence and thus are processed at the beginning.

See construction of precedence $\triangleright'_1$ from $\triangleright'_0$ in Figure A.6.

Claim 6 $C^{t^*}(\triangleright'_1, I) = C^{t^*}(\triangleright'_0, I)$.

Proof of Claim 6: Since $\triangleright'_0$ and $\triangleright'_1$ are equivalent, the desired result immediately follows from Lemma 1. ♢

There is potentially a tier slot of type $t' \neq t^*$ immediately before the last merit slot group under $\triangleright'_1$. If such a slot does not exist, then $\triangleright'_2 \equiv \triangleright'_1$. Otherwise, construct $\triangleright'_2$ from $\triangleright'_1$ by moving all of the adjacent tier t' slots from immediately before the last merit slot group to immediately after it. See construction of precedence $\triangleright'_2$ from $\triangleright'_1$ in Figure A.6.

[18] In Figure A.4, this construction is applied twice: first when $\triangleright''$ is constructed from $\triangleright'$, and second when $\triangleright'''$ is constructed from $\triangleright''$.

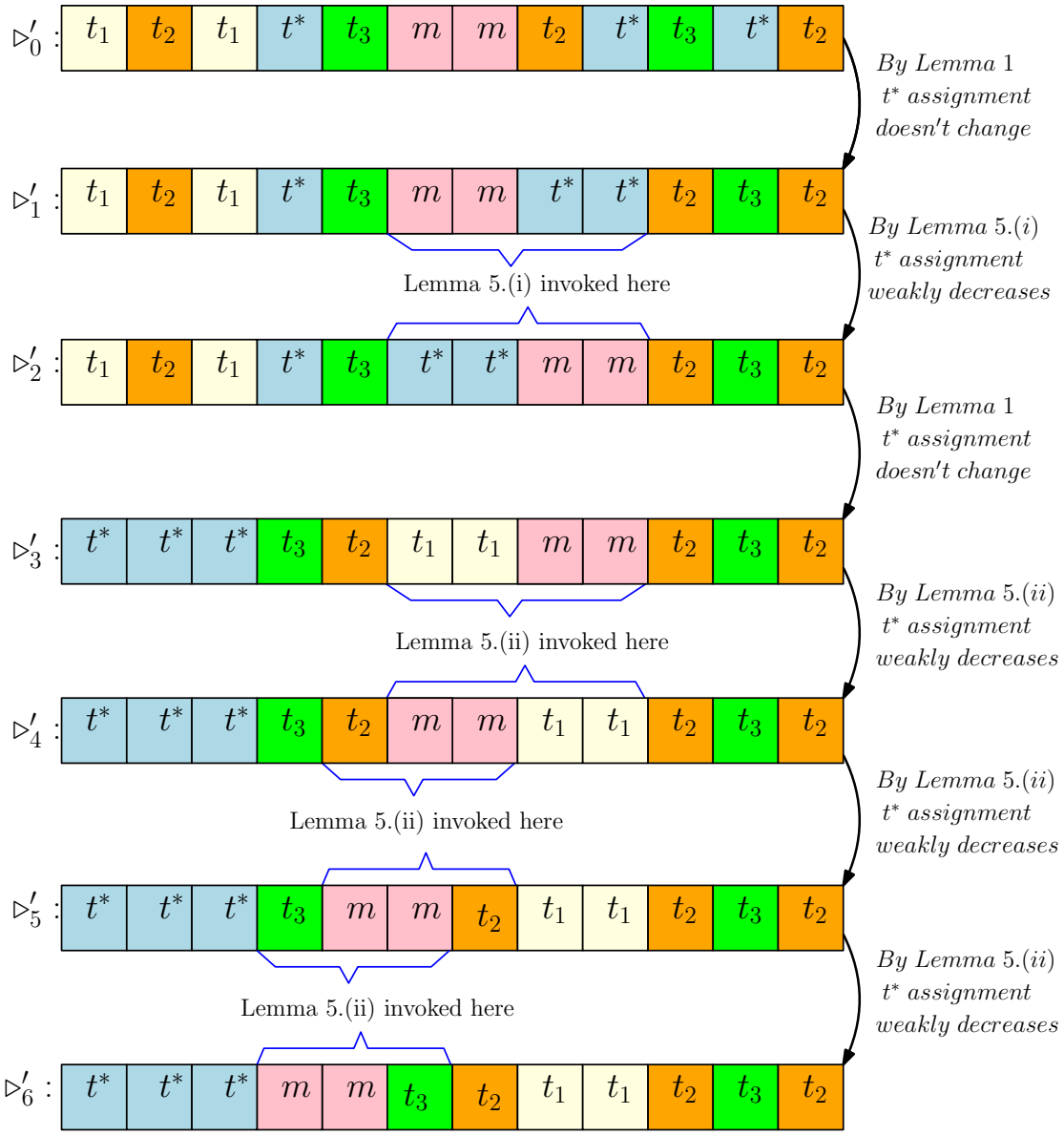

Figure A.5: Illustration of Case 1 of Proposition 1.(ii)

Claim 7 $C^{t^*}(\triangleright'_2, I) \subseteq C^{t^*}(\triangleright'_1, I)$.

Proof of Claim 7: Let S' be the last merit slot group together with all of the adjacent type t' slots immediately before the merit slot group for the case of $\triangleright'_1$ and immediately after it for the case of $\triangleright'_2$. By Lemma 5.(ii), we have

$$\bigcup_{s \in S'} C_s^{t^*}(\triangleright'_2, I) \subseteq \bigcup_{s \in S'} C_s^{t^*}(\triangleright'_1, I). \tag{8}$$

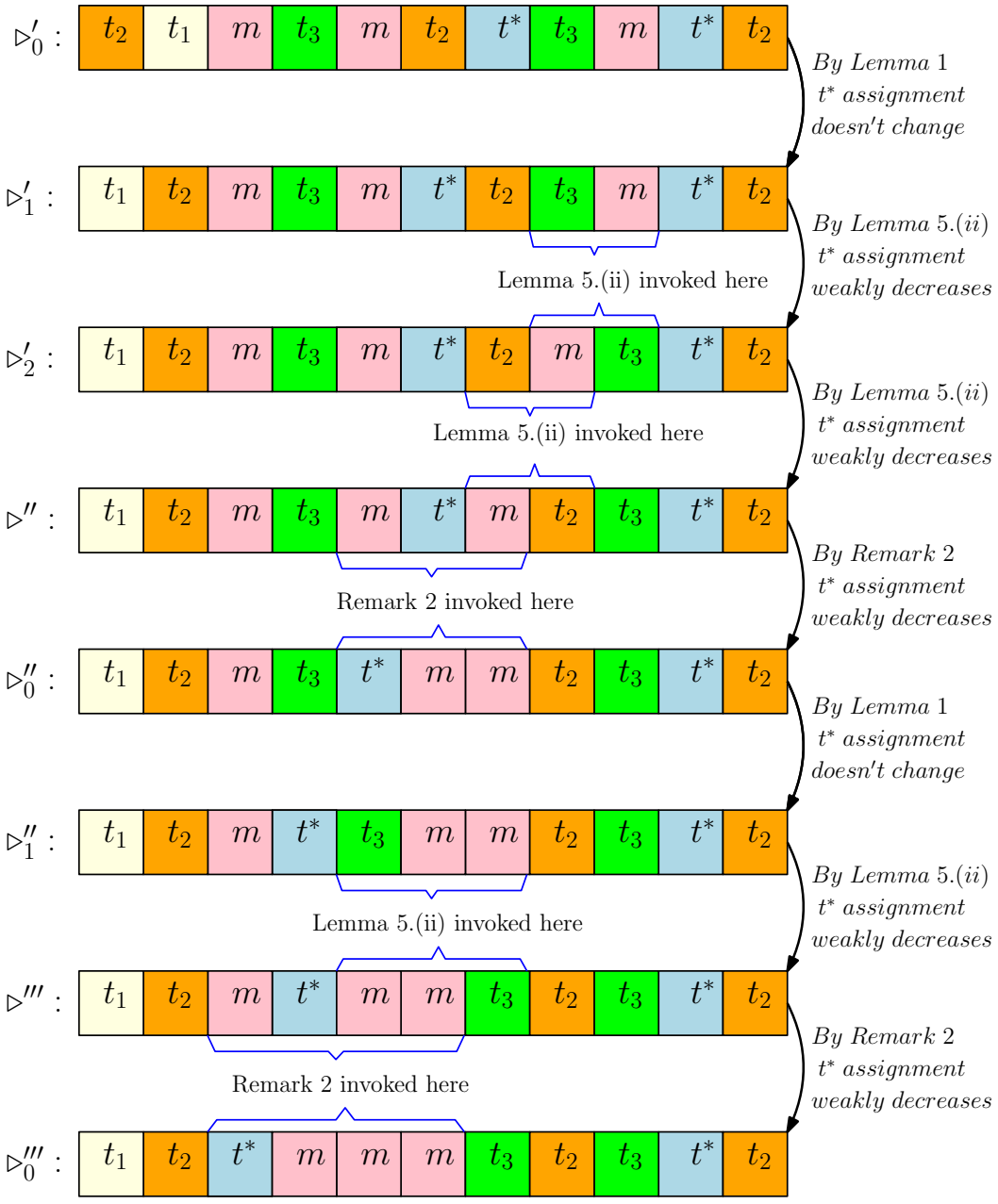

Figure A.6: Illustration of Case 2 of Proposition 1.(ii)

Equation (8) together with Lemma 3 complete the proof of Claim 7. ♢

Repeated application of this step of the construction for each $t \neq t^*$ gives us a precedence order denoted with $\triangleright''$. If there exists unique merit slot group under $\triangleright''$, then application of the steps in Case 1 to $\triangleright''$ gives us the desired result. Otherwise, under $\triangleright''$ there potentially exists a tier slot of type t^* immediately after the penultimate merit slot group. If such a slot does not exist, then $\triangleright''_0 \equiv \triangleright''$. Otherwise, construct $\triangleright''_0$ from $\triangleright''$ by moving all of the adjacent tier t^* slots from immediately after the penultimate merit slot group to immediately before it. See

construction of precedence $\triangleright_0''$ from $\triangleright''$ in Figure A.6.

Claim 8 $C^{t^*}(\triangleright_0'', I) \subseteq C^{t^*}(\triangleright'', I)$.

Proof of Claim 8: If $\triangleright_0'' \equiv \triangleright''$ then the result is immediate. Let S' be the last two merit slot groups together with type t^* slots between them for the case of $\triangleright''$ and the last merit slot group together with all of the adjacent type t^* slots immediately before it for the case of $\triangleright_0''$. By Lemma 5.(ii), we have

$$\bigcup_{s \in S'} C_s^{t^*}(\triangleright_0'', I) \subseteq \bigcup_{s \in S'} C_s^{t^*}(\triangleright'', I). \tag{9}$$

Equation (9) together with Lemma 3 complete the proof of Claim 8. ♢

Repeated application of these steps of the construction for last two merit slots decreases the number of merit slot groups and eventually gives us a precedence order with a unique merit slot group. Then application of the steps in Case 1 to this final precedence order gives us the desired result. □

A.3 Proof of Proposition 2

It is immediate from Lemma 1 that balancedness implies tier-blindness. To prove tier-blindness implies balancedness, we show that any unbalanced precedence order is not tier-blind. For a given S and τ, let $\triangleright$ be a precedence order which is not balanced. Let s_h denote the h^{th} slot under $\triangleright$. Let $S_t = \{s \in S | \tau(s) = t\}$ for each $t \in T$. Let π be a merit-preserving bijection with $\pi(\hat{t}) = \tilde{t}$, $\pi(\tilde{t}) = \hat{t}$, and $\pi(t) = t$ for all $t \in T \setminus \{\hat{t}, \tilde{t}\}$. Note that s_h is a tier slot under $(\tau, \triangleright)$ if and only if s_h is a tier slot under $(\pi(\tau), \triangleright)$ for each $h \in \{1, 2, ..., |S|\}$. We show that there exists a subset of students $J \subseteq I$ such that $C(S, \tau, \triangleright, J) \neq C(S, \pi(\tau), \triangleright, J)$ and, therefore, $\triangleright$ is not tier-blind. There are two possible cases.

Case 1 ***($|S_t| = |S_{t'}|$ for all $t, t' \in T$):*** Suppose under $(\tau, \triangleright)$ the first $b \geq 0$ slots constitute a balanced precedence order and for any $b' > b$ the first b' slots fail to constitute a balanced precedence order. We call the portion of $\triangleright$ with the lower precedence than slot s_b the *unbalanced portion* of $\triangleright$. Note that the unbalanced portion of $\triangleright$ starts with a tier slot under both τ and $\pi(\tau)$, i.e., $\tau(s_{b+1}) \neq m$ and $\pi(\tau(s_{b+1})) \neq m$.

There exists at least one merit slot in the unbalanced portion of $\triangleright$ under τ (and, therefore $\pi(\tau)$). Otherwise, $\triangleright$ would be a balanced precedence order under τ. Denote the merit slot with the highest precedence under $\triangleright$ in the unbalanced portion with $\tilde{s}$. Let u_t be the number of tier $t \in T$ slots between s_b and $\tilde{s}$ under $(\tau, \triangleright)$. Due to the unbalancedness, $u_t \neq u_{t'}$ for some $t, t' \in T$. Without loss of generality, we take $u_{\hat{t}} > u_{\tilde{t}}$.

Now consider a subset of students J with the following score distribution:

i) $\sum_{t=1}^{\bar{t}} \int_{k^*}^{\bar{k}} f_t^J(k)dk = b$, and $f_{t'}^J(\tilde{k}) = f_{t''}^J(\tilde{k})$ for all $\tilde{k} \in [k^*, \bar{k}]$ and $t', t'' \in T$,

ii) $\int_{k'}^{k^*} f_{\hat{t}}^J(k)dk = u_{\hat{t}}$ and $\int_{k'}^{k^*} f_t^J(k)dk = 0$ for all $t \in T \setminus \{\tilde{t}\}$,

iii) $\int_{k''}^{k'} f_{\hat{t}}^J(k)dk = |S|$ and $\int_{k''}^{k'} f_t^J(k)dk = 0$ for all $t \in T \setminus \{\hat{t}\}$.

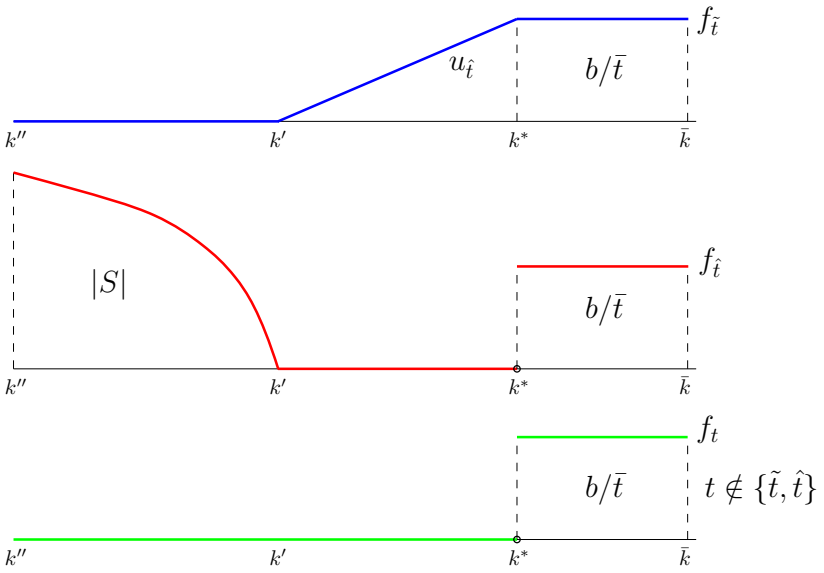

Figure A.7: Score distribution for Case 1 of Proposition 2

From J, under both τ and $\pi(\tau)$, $C(\cdot)$ selects all students with score between $[k^*, \bar{k}]$ to the first b slots, i.e., the slots in the balanced portion. Moreover, a positive mass of tier $\tilde{t}$ students with score between $[k', k^*)$ will be chosen for the merit slots in the unbalanced portion by $C(\cdot)$ under τ. However, none of the tier $\tilde{t}$ students will be selected for the merit slots in the unbalanced portion by $C(\cdot)$ under $\pi(\tau)$. Hence, the mass of tier $\tilde{t}$ students in $C(S, \tau, \triangleright, J)$ is strictly more than $(b_m/\bar{t}) + |S_{\tilde{t}}|$ and the mass of tier $\tilde{t}$ students in $C(S, \pi(\tau), \triangleright, J)$ is exactly $(b_m/\bar{t}) + |S_{\tilde{t}}|$. Therefore,

$$C(S, \tau, \triangleright, J) \neq C(S, \pi(\tau), \triangleright, J).$$

Case 2 ($|S_t| \neq |S_{t'}|$ for some $t, t' \in T$): Without loss of generality we take $|S_{\tilde{t}}| > |S_{\hat{t}}|$. Then, consider the following score distribution: $\int_{k^*}^{\bar{k}} f_{\tilde{t}}^J(k)dk = |S|$ and $\int_{k^*}^{\bar{k}} f_t^J(k)dk = 0$ for all $t \in T \setminus \{\tilde{t}\}$. Then, the mass of tier $\tilde{t}$ students in $C(S, \tau, \triangleright, J)$ is $|S_m| + |S_{\tilde{t}}|$ and the mass of tier $\tilde{t}$ students in $C(S, \pi(\tau), \triangleright, J)$ is $|S_m| + |S_{\hat{t}}|$. Since $|S_{\tilde{t}}| > |S_{\hat{t}}|$,

$$C(S, \tau, \triangleright, J) \neq C(S, \pi(\tau), \triangleright, J).$$

□

A.4 Proofs of Theorem 1 and Proposition 3

We use the following Lemmatta to prove Theorem 1 and Proposition 3.

Lemma 7 *Fix the set of slots S, type function τ, and precedence order $\triangleright$ such that $\triangleright$ is a balanced precedence. Let $\boldsymbol{G} = \{G^1, G^2, ..., G^h\}$ be the set of all tier slot groups where all slots in G^r precede the ones in G^{r+1}. For any tier $t \in T$, let ℓ_t^r and $\tilde{\ell}_t^r$ denote the supremum score of tier t students available to be admitted and the infimum score of tier t students admitted to the tier slot group G^r from I by $C(\cdot)$, respectively. If $f_{\hat{t}}(k) \geq f_{t'}(k)$ for all $k \in K$, then for any $r \in \{1, 2, ..., h\}$*

$$\ell_{\hat{t}}^r \geq \ell_{t'}^r \text{ and } \tilde{\ell}_{\hat{t}}^r \geq \tilde{\ell}_{t'}^r.$$

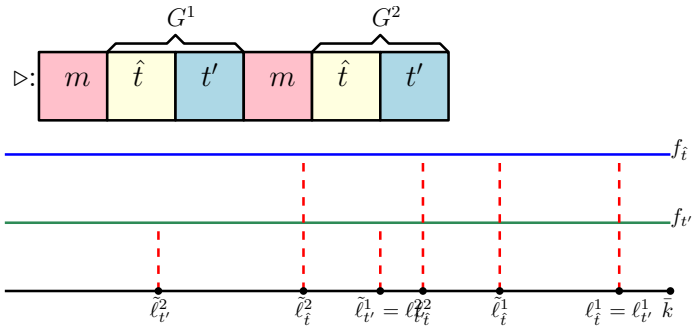

Figure A.8: Illustration of Score Distributions for $f_{\hat{t}}$ and $f_{t'}$, and the Infimum and Supremum Scores

Proof. For each $r \in \{1, 2, ..., h\}$, we denote the merit slot group between G^{r-1} and G^r by H^r where G^0 is the beginning. The proof is by induction. We start with tier slot group G^1. There exists $\ell_m^1 \in K$ such that

$$\sum_{t=1}^{\bar{t}} \int_{\ell_m^1}^{\bar{k}} f_t(k)dk = |H^1|.$$

If $H^1 = \emptyset$, then all students are available to be selected for tier slot group G^1. Otherwise, all students with score at least $\ell_m^1 < \bar{k}$ are selected for the merit slots preceding G^1. In either case, $\ell_{\hat{t}}^1 \geq \ell_{t'}^1$ since $f_{\hat{t}}(k) \geq f_{t'}(k)$ for all $k \in K$. Moreover, balancedness implies

$$\int_{\tilde{\ell}_t^1}^{\ell_t^1} f_t(k)dk = \frac{|G^1|}{\bar{t}} \quad \text{for all } t \in T. \tag{10}$$

Equation (10) and the facts that $\ell_{\hat{t}}^1 \geq \ell_{t'}^1$ and $f_{\hat{t}}(k) \geq f_{t'}(k)$ for all $k \in K$ imply that $\tilde{\ell}_{\hat{t}}^1 \geq \tilde{\ell}_{t'}^1$.

Suppose the result holds for all the tier slot groups preceding tier slot group $G^{\bar{r}}$ where $\bar{r} \leq h$. That is, $\ell_{\hat{t}}^{\bar{r}-1} \geq \ell_{t'}^{\bar{r}-1}$ and $\tilde{\ell}_{\hat{t}}^{\bar{r}-1} \geq \tilde{\ell}_{t'}^{\bar{r}-1}$. Then, there exists $\ell_m^{\bar{r}} \in K$ such that

$$\sum_{t=1}^{\bar{t}} \int_{\ell_m^{\bar{r}}}^{\tilde{\ell}_t^{\bar{r}}-1} f_t(k)dk = |H^{\bar{r}}|.$$

That is, all tier $t \in T$ students with score at least $\min\{\ell_m^{\bar{r}}, \tilde{\ell}_t^{\bar{r}}-1\}$ are selected for slots preceding $G^{\bar{r}}$. Since $\tilde{\ell}_{\hat{t}}^{\bar{r}}-1 \geq \tilde{\ell}_{t'}^{\bar{r}}-1$, $\min\{\ell_m^{\bar{r}}, \tilde{\ell}_{\hat{t}}^{\bar{r}}-1\} \geq \min\{\ell_m^{\bar{r}}, \tilde{\ell}_{t'}^{\bar{r}}-1\}$. Therefore, $f_{\hat{t}}(k) \geq f_{t'}(k)$ for all $k \in K$ implies that $\ell_{\hat{t}}^{\bar{r}} \geq \ell_{t'}^{\bar{r}}$. Balancedness implies

$$\int_{\tilde{\ell}_t^{\bar{r}}}^{\ell_t^{\bar{r}}} f_t(k)dk = \frac{|G^{\bar{r}}|}{\bar{t}} \quad \text{for all } t \in T. \tag{11}$$

Equation (11) and the facts that $\ell_{\hat{t}}^{\bar{r}} \geq \ell_{t'}^{\bar{r}}$ and $f_{\hat{t}}(k) \geq f_{t'}(k)$ for all $k \in K$ imply that $\tilde{\ell}_{\hat{t}}^{\bar{r}} \geq \tilde{\ell}_{t'}^{\bar{r}}$. This completes the proof.[19] □

Lemma 8 *Fix the set of slots S and their types τ such that $|S_m| \geq 1$ and $|S_t| = |S_{t'}| \geq 1$ for all $t, t' \in T$. Let $\triangleright$ be a precedence order over S in which all merit slots precede all tier slots and $\tilde{\triangleright}$ be a precedence order over S in which all tier slots precede all merit slots.*[20] *Let $J \subseteq I$ be a subset of students such that $n_t^J \geq |S_t| + |S_m|$ for all $t \in T$ and $k \in K$.*

(i) Under Assumption 2, i.e., $f_1^J(k) \leq f_t^J(k)$ for all $t \in T$ and $k \in K$, the mass of tier 1 students in $C(\triangleright, J)$ is weakly greater than in $C(\tilde{\triangleright}, J)$.

(ii) Under Assumption 3, i.e., $f_t^J(k) \leq f_{\bar{t}}^J(k)$ for all $t \in T$ and $k \in K$, the mass of tier $\bar{t}$ students in $C(\tilde{\triangleright}, J)$ is weakly greater than in $C(\triangleright, J)$.

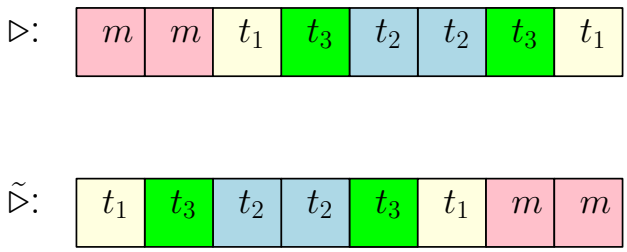

Figure A.9: Illustration of precedence orders $\triangleright$ and $\tilde{\triangleright}$

Proof. Let ℓ_m and $\tilde{\ell}_m$ denote the infimum of the scores of students selected for the merit slots from J by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$, respectively. For each $t \in T$, let ℓ_t and $\tilde{\ell}_t$ be the infimum of scores of students selected for the tier t slots from J by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$, respectively. Let g_t and

[19] See Figure A.8 for the illustration of the desired result.
[20] See Figure A.9 for examples of $\triangleright$ and $\tilde{\triangleright}$.

$\tilde{g}_t$ denote the infimum of the scores of tier $t \in T$ students in $C(\triangleright, J)$ and $C(\tilde{\triangleright}, J)$, respectively. Note that, $g_t = \ell_t$ and $\tilde{g}_t \leq \tilde{\ell}_t$ for all $t \in T$. Since merit slots are processed first under $\triangleright$, we have

$$\int_{\ell_t}^{\ell_m} f_t^J(k)dk = |S_t| \quad \text{for all } t \in T. \tag{12}$$

Similarly, since tier slots are processed before the merit slots under $\tilde{\triangleright}$, we have

$$\int_{\tilde{\ell}_t}^{\bar{k}} f_t^J(k)dk = |S_t| \quad \text{for all } t \in T. \tag{13}$$

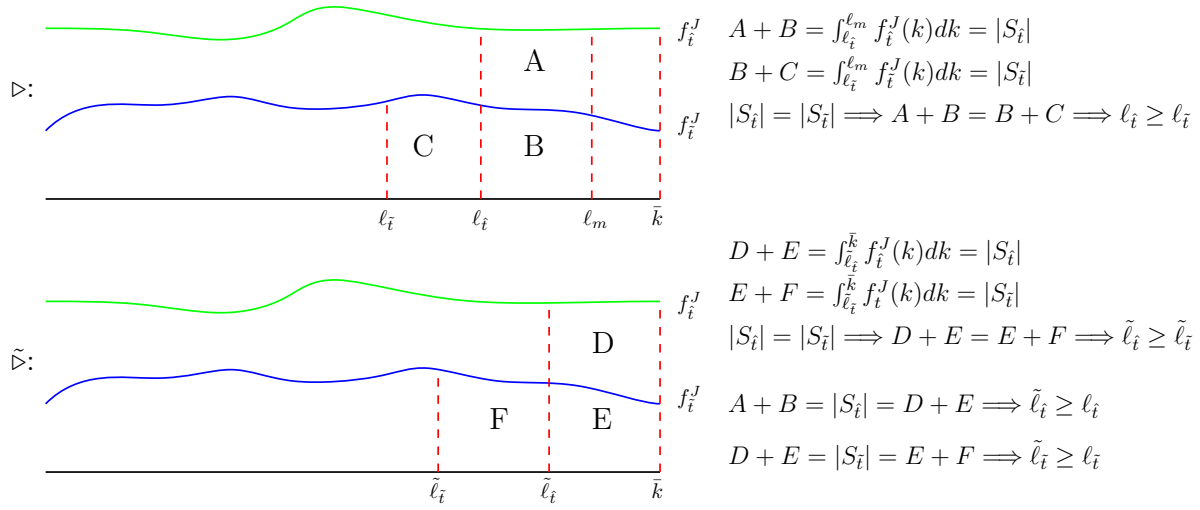

Figure A.10: Infimum scores under precedence orders $\triangleright$ and $\tilde{\triangleright}$.

Equations (12) and (13) imply that $\tilde{\ell}_t \geq \ell_t$ for all $t \in T$.[21] Next, we prove Part (i) and then Part (ii).

Part (i): Comparison of Tier 1 Assignment in $C(\triangleright, J)$ and $C(\tilde{\triangleright}, J)$

By contradiction, suppose that the mass of tier 1 students in $C(\tilde{\triangleright}, J)$ is greater than the mass in $C(\triangleright, J)$. That is,

$$\int_{g_1}^{\bar{k}} f_1^J(k)dk < \int_{\tilde{g}_1}^{\bar{k}} f_1^J(k)dk. \tag{14}$$

Equation (14) implies that $g_1 > \tilde{g}_1$. The facts that $\tilde{\ell}_1 \geq \ell_1$, $\ell_1 = g_1$, and $g_1 > \tilde{g}_1$ imply that $\tilde{\ell}_1 \geq \ell_1 > \tilde{g}_1$, and therefore, there exist tier 1 students with scores between $[\tilde{g}_1, \ell_1)$ selected for merit slots by $C(\cdot)$ under $\tilde{\triangleright}$. Hence, under $\tilde{\triangleright}$, the infimum of tier 1 students selected for the merit

[21] One can see this relation in Figure A.10.

slots by $C(\cdot)$ is $\tilde{g}_1$. Since $f_t(k) \geq f_1(k)$ for all $t \in T$ and $k \in K$, for all $t \in T$ the infimum score of tier t students in $C(\tilde{\triangleright}, J)$ is *at most* $\tilde{g}_1$. Then, we have

$$\sum_{t=1}^{\bar{t}} \int_{\tilde{g}_1}^{\bar{k}} f_t^J(k)dk \leq |S|. \tag{15}$$

Equation (14) and $g_1 > \tilde{g}_1$ imply that

$$\sum_{t=1}^{\bar{t}} \int_{\tilde{g}_1}^{\bar{k}} f_t^J(k)dk > \sum_{t=1}^{\bar{t}} \int_{g_1}^{\bar{k}} f_t^J(k)dk. \tag{16}$$

Equations (15) and (16) imply that

$$|S| > \sum_{t=1}^{\bar{t}} \int_{g_1}^{\bar{k}} f_t^J(k)dk. \tag{17}$$

For all $t \in T$, Equation (12) and the fact that $f_t(k) \geq f_1(k)$ for all $k \in K$ imply that $\ell_t \geq \ell_1$.[22] Since $g_t = \ell_t$ for all $t \in T$, $\ell_t \geq \ell_1$ implies that

$$\sum_{t=1}^{\bar{t}} \int_{g_1}^{\bar{k}} f_t^J(k)dk \geq \sum_{t=1}^{\bar{t}} \int_{g_t}^{\bar{k}} f_t^J(k)dk = |S|. \tag{18}$$

Equations (17) and (18) imply that

$$|S| > \sum_{t=1}^{\bar{t}} \int_{g_1}^{\bar{k}} f_t^J(k)dk \geq |S|.$$

This is a contradiction.

Part (ii): Comparison of Tier $\bar{t}$ Assignment in $C(\triangleright, J)$ and $C(\tilde{\triangleright}, J)$

We consider two possible cases. First, we consider the case in which for all $t \in T \setminus \{\bar{t}\}$ the mass of tier t students in $C(\tilde{\triangleright}, J)$ is weakly less than the one in $C(\triangleright, J)$. Since all slots are filled by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$, then the mass of $\bar{t}$ students in $C(\tilde{\triangleright}, J)$ is weakly greater than the mass in $C(\triangleright, J)$. Now we consider the case in which there exists some tier $t' \in T \setminus \{\bar{t}\}$ such that the

[22] Since $f_t(k) \geq f_1(k)$ for all $k \in K$ and $t \in T$, in Figure A.10, tier $\tilde{t}$ plays the role of tier 1, and tier $\hat{t}$ plays the role of tier t. Hence, Figure A.10 illustrates the relation between ℓ_t and ℓ_1.

mass of tier t' students in $C(\tilde{\triangleright}, J)$ is strictly is greater than the mass in $C(\triangleright, J)$. Then,

$$\int_{g_{t'}}^{\bar{k}} f_{t'}^J(k)dk < \int_{\tilde{g}_{t'}}^{\bar{k}} f_{t'}^J(k)dk. \tag{19}$$

Equation (19) implies that $g_{t'} > \tilde{g}_{t'}$. $\tilde{\ell}_{t'} \geq \ell_{t'}$, $\ell_{t'} = g_{t'}$, and $g_{t'} > \tilde{g}_{t'}$ imply that $\tilde{\ell}_{t'} \geq \ell_{t'} > \tilde{g}_{t'}$, and therefore there exist tier t' students with scores between $[\tilde{g}_{t'}, \ell_{t'})$ assigned to merit slots by $C(\cdot)$ under $\tilde{\triangleright}$. Since $f_{\bar{t}}(k) \geq f_{t'}(k)$ for all $k \in K$, the infimum score of tier $\bar{t}$ students assigned in $C(\tilde{\triangleright}, J)$ is *at most* $\tilde{g}_{t'}$. Equation (12) and the fact that $f_{t'}^J(k) \leq f_{\bar{t}}^J(k)$ for all $k \in K$ imply that $\ell_{t'} \leq \ell_{\bar{t}}$.[23] Since $\ell_t = g_t$ for all $t \in T$, $\ell_{t'} \leq \ell_{\bar{t}}$ implies $g_{t'} \leq g_{\bar{t}}$. Since $\tilde{g}_{t'} < g_{t'}$, $g_{t'} \leq g_{\bar{t}}$ and the infimum score of tier $\bar{t}$ students assigned in $C(\tilde{\triangleright}, J)$ is at most $\tilde{g}_{t'}$, the mass of tier $\bar{t}$ assignment in $C(\tilde{\triangleright}, J)$ is weakly is greater than the mass in $C(\triangleright, J)$. □

Lemma 9 *Fix the set of slots S and their types τ such that $|S_m| > 1$ and $|S_t| = |S_{t'}| \geq 1$ for all $t, t' \in T$ under τ. Let S_m^1 and S_m^2 be two nonempty disjoint subsets of S_m. Let $\triangleright$ be a precedence order over S in which merit slots in S_m^1 precede all tier slots and all tier slots precede the merit slots in S_m^2, and $\tilde{\triangleright}$ be a precedence order over S in which all tier slots precede the merit slots.*[24] *Let $J \subseteq I$ be a subset of students such that $n_t^J \geq |S_t| + |S_m|$ for all $t \in T$ and $k \in K$.*

(i) Under Assumption 2, i.e., $f_1^J(k) \leq f_t^J(k)$ for all $t \in T$ and $k \in K$, the mass of tier 1 students in $C(\triangleright, J)$ is weakly greater than in $C(\tilde{\triangleright}, J)$.

(ii) Under Assumption 3, i.e., $f_t^J(k) \leq f_{\bar{t}}^J(k)$ for all $t \in T$ and $k \in K$, the mass of tier $\bar{t}$ students in $C(\tilde{\triangleright}, J)$ is weakly greater than in $C(\triangleright, J)$.

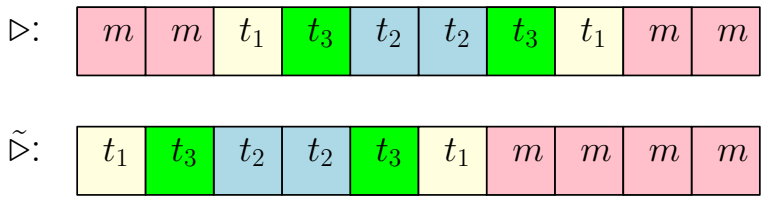

Figure A.11: Illustration of precedence orders $\triangleright$ and $\tilde{\triangleright}$

Proof. Denote the infimum score of tier t students in $C(\triangleright, J)$ and $C(\tilde{\triangleright}, J)$ with g_t and $\tilde{g}_t$, respectively. Let e_t and $\tilde{e}_t$ be the infimum score of tier t students assigned to first $\bar{t} \times |S_1| + |S_m^1|$ slots by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$, respectively. Let ℓ_t and $\tilde{\ell}_t$ denote the infimum score of tier t students assigned to tier t slots by $C(\cdot)$ under $\triangleright$ and $\tilde{\triangleright}$, respectively. Since all tier slots are processed first

[23] Since $f_{t'}^J(k) \leq f_{\bar{t}}^J(k)$ for all $k \in K$, in Figure A.10, tier $\tilde{t}$ plays the role of tier t', and tier $\hat{t}$ plays the role of tier $\bar{t}$. Hence, Figure A.10 illustrates the relation between $\ell_{t'}$ and $\ell_{\bar{t}}$.

[24] See Figure A.11 for the examples of $\triangleright$ and $\tilde{\triangleright}$.

under $\tilde{\triangleright}$ but some merit slots are processed first under $\triangleright$, $\tilde{\ell}_t \geq \ell_t$ for all $t \in T$. Next, we prove Part (i) and then Part (ii).

Part (i): Comparison of Tier 1 Assignment in $C(\triangleright, J)$ and $C(\tilde{\triangleright}, J)$

By contradiction, suppose that the mass of tier 1 students in $C(\tilde{\triangleright}, J)$ is is greater than the mass under $C(\triangleright, J)$. That is,

$$\int_{g_1}^{\bar{k}} f_1^J(k)dk < \int_{\tilde{g}_1}^{\bar{k}} f_1^J(k)dk. \tag{20}$$

Equation (20) implies that $g_1 > \tilde{g}_1$. By Lemma 8, $\tilde{e}_1 \geq e_1$. By our construction, $e_1 \geq g_1$. Then, we have $\tilde{e}_1 \geq e_1 \geq g_1 > \tilde{g}_1$. Hence, there exist tier 1 students with score between $[\tilde{g}_1, g_1)$ assigned to the last merit slot group by $C(\cdot)$ under $\tilde{\triangleright}$, and therefore the infimum of tier 1 students selected for the last merit slot group by $C(\cdot)$ under $\tilde{\triangleright}$ is $\tilde{g}_1$. Since $f_t(k) \geq f_1(k)$ for all $t \in T$ and $k \in K$, then for all $t \in T$ the infimum score of tier t students in $C(\tilde{\triangleright}, J)$ is *at most* $\tilde{g}_1$. That is,

$$\sum_{t=1}^{\bar{t}} \int_{\tilde{g}_1}^{\bar{k}} f_t^J(k)dk \leq |S|. \tag{21}$$

By Lemma 7, $e_t \geq e_1$ for all $t \in T$. Since tier 1 students with score between $[\tilde{g}_1, g_1)$ are not assigned to the last merit slot group under $\triangleright$ and $e_t \geq e_1 \geq g_1 > \tilde{g}_1$ for all $t \in T$, students with score between $[\tilde{g}_1, g_1)$ cannot be selected by $C(\cdot)$ under $\triangleright$. That is,

$$\sum_{t=1}^{\bar{t}} \int_{\tilde{g}_1}^{\bar{k}} f_t^J(k)dk > |S|. \tag{22}$$

Equations (21) and (22) imply that

$$\sum_{t=1}^{\bar{t}} \int_{\tilde{g}_1}^{\bar{k}} f_t^J(k)dk > |S| \geq \sum_{t=1}^{\bar{t}} \int_{\tilde{g}_1}^{\bar{k}} f_t^J(k)dk.$$

This is a contradiction.

Part (ii): Comparison of Tier $\bar{t}$ Assignment in $C(\triangleright, J)$ and $C(\tilde{\triangleright}, J)$

On the contrary, suppose that the mass of tier $\bar{t}$ students in $C(\triangleright, J)$ is is greater than the mass in $C(\tilde{\triangleright}, J)$. That is,

$$\int_{g_{\bar{t}}}^{\bar{k}} f_{\bar{t}}^J(k)dk > \int_{\tilde{g}_{\bar{t}}}^{\bar{k}} f_{\bar{t}}^J(k)dk. \tag{23}$$

Equation (23) implies that $g_{\bar{t}} < \tilde{g}_{\bar{t}}$. Since $f_{\bar{t}}^J(k) \geq f_t^J(k)$ for all $t \in T$ and $k \in K$, the last slots under both precedence order are merit slots, $g_{\bar{t}}$ and $\tilde{g}_{\bar{t}}$ are the infimum scores of students

assigned to the merit slots under $\triangleright$ and $\tilde{\triangleright}$, respectively. Then, the mass of tier $t \in T$ students in $C(\triangleright, J)$ is

$$\max\left\{\int_{g_{\bar{t}}}^{\bar{k}} f_t^J(k)dk, \int_{\ell_t}^{\bar{k}} f_t^J(k)dk\right\}. \tag{24}$$

Similarly, the mass of tier $t \in T$ students in $C(\tilde{\triangleright}, J)$ is

$$\max\left\{\int_{\tilde{g}_{\bar{t}}}^{\bar{k}} f_t^J(k)dk, \int_{\tilde{\ell}_t}^{\bar{k}} f_t^J(k)dk\right\}. \tag{25}$$

Since $g_{\bar{t}} < \tilde{g}_{\bar{t}}$, the first term of Equation (24) is (weakly) greater than the first term of Equation (25). Similarly, since $\tilde{\ell}_t \geq \ell_t$ the second term of Equation (24) is (weakly) greater than the second term of Equation (25). Hence, for all $t \in T$,

$$\max\left\{\int_{g_{\bar{t}}}^{\bar{k}} f_t^J(k)dk, \int_{\ell_t}^{\bar{k}} f_t^J(k)dk\right\} \geq \max\left\{\int_{\tilde{g}_{\bar{t}}}^{\bar{k}} f_t^J(k)dk, \int_{\tilde{\ell}_t}^{\bar{k}} f_t^J(k)dk\right\}. \tag{26}$$

Equation (26) implies that the mass of each tier t students in $C(\triangleright, J)$ is weakly greater than the mass in $C(\tilde{\triangleright}, J)$. However, since the same number of slots are filled under $\triangleright$ and $\tilde{\triangleright}$ and the mass of tier $\bar{t}$ students in $C(\triangleright, J)$ is strictly greater than the mass in $C(\tilde{\triangleright}, J)$, at least one tier's assignment needs to be smaller in $C(\triangleright, J)$. This is a contradiction. □

Proof of Theorem 1. Fix a set of slots S and a type function τ such that $S_m \neq \emptyset$. We first show that among the balanced precedence orders the maximal tier 1 assignment is attained when all merit slots precede the tier slots.

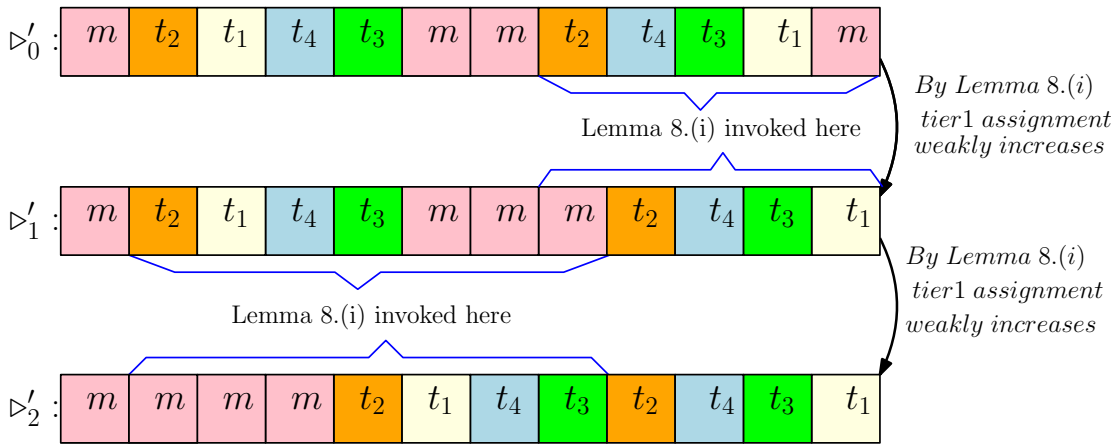

Figure A.12: Maximal Tier 1 Assignment under Balanced Precedence Orders

Maximal Tier 1 Assignment: Let $\bar{\triangleright}$ be a balanced precedence order such that all merit slots precede the tier slots. Consider an arbitrary balanced precedence order $\triangleright'$ such that at least one merit slot is preceded by a tier slot. We will construct a sequence of precedences where the first element is $\triangleright'_0 \equiv \triangleright'$ and the last element is $\bar{\triangleright}$. We will show that tier 1 assignment weakly increases under choice function $C(\cdot)$ as we move from one element of the sequence to the next one.

Let $\triangleright'_0 \equiv \triangleright'$ be the first element of the sequence. Let H and G denote the the last merit slot group and the tier slot group immediately before it under $\triangleright'_0$, respectively. Construct $\triangleright'_1$ from $\triangleright'_0$ by moving merit slot group H from immediately after the tier slot G group to immediately before it. See construction of precedence $\triangleright'_1$ from $\triangleright'_0$ in Figure A.12.

Claim 9 $C^1(\triangleright'_1, I) \supseteq C^1(\triangleright'_0, I)$.

Proof of Claim 9: First note that the score distributions of available students to be admitted by $C(\cdot)$ to tier slot group G under $\triangleright'_0$ and to merit slot group H under $\triangleright'_1$ satisfy Assumption 2. Let $S' = H \cup G$. By Lemma 8.(i) we have

$$\bigcup_{s \in S'} C^1_s(\triangleright'_1, I) \supseteq \bigcup_{s \in S'} C^1_s(\triangleright'_0, I). \tag{27}$$

Equation (27) together with Lemma 3 complete the proof of Claim 9. ♢

Repeated application of this step of the construction for the last merit slot group preceded by a tier slot group gives us a precedence order equivalent to $\bar{\triangleright}$. Hence, this fact together with Lemma 1 gives us the desired result.

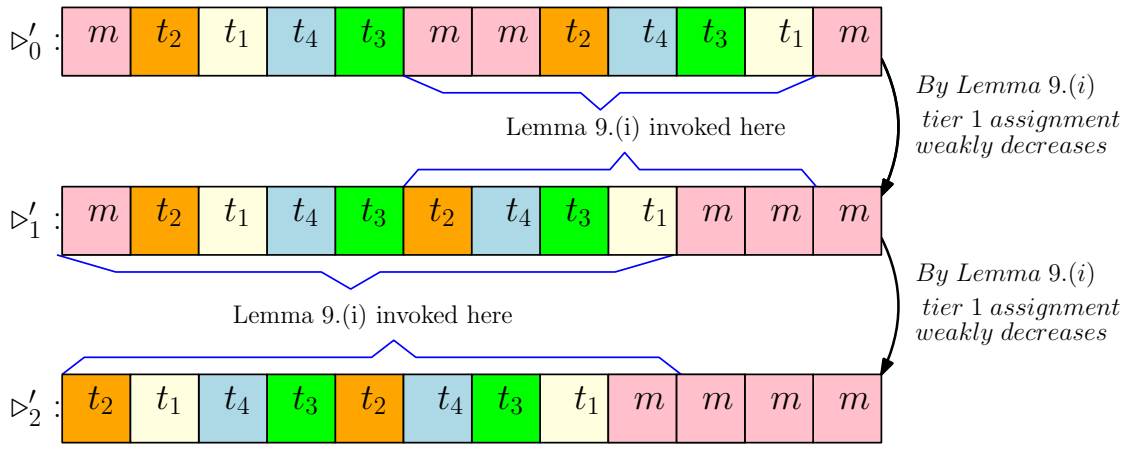

Figure A.13: Minimal Tier 1 Assignment under Balanced Precedence Orders

Minimal Tier 1 Assignment: Let $\trianglerighteq$ be a balanced precedence order such that all tier slots precede the merit slots. Consider an arbitrary balanced precedence order $\triangleright'$ such that at least one tier slot is preceded by a merit slot. We will construct a sequence of precedences where the first element is $\triangleright'_0 \equiv \triangleright'$ and the last element is $\trianglerighteq$. We will show that tier 1 assignment weakly

decreases under choice function $C(\cdot)$ as we move from one element of the sequence to the next one.

Let $\triangleright'_0 \equiv \triangleright'$ be the first element of the sequence. Let G and H denote the the last tier slot group and the merit slot group immediately before it under $\triangleright'_0$, respectively. Construct $\triangleright'_1$ from $\triangleright'_0$ by moving tier slot group G from immediately after the merit slot group H to immediately before it. See construction of precedence $\triangleright'_1$ from $\triangleright'_0$ in Figure A.13.

Claim 10 $C^1(\triangleright'_1, I) \subseteq C^1(\triangleright'_0, I)$.

Proof of Claim 10: First note that the score distributions of available students to be admitted by $C(\cdot)$ to merit slot group H under $\triangleright'_0$ and to tier slot group G under $\triangleright'_1$ satisfy Assumption 2. Let S' be the set of slots in H together with all slot groups after H for the case of $\triangleright'_0$ and be the set of slots in G together with all slot groups after G for the case of $\triangleright'_0$. If $S' = H \cup G$ by Lemma 8.(i), otherwise by Lemma 9.(i) we have

$$\bigcup_{s \in S'} C^1_s(\triangleright'_1, I) \subseteq \bigcup_{s \in S'} C^1_s(\triangleright'_0, I). \tag{28}$$

Equation (28) together with Lemma 3 complete the proof of Claim 10. ♢

Repeated application of this step of the construction for the last tier slot group preceded by a merit slot group gives us a precedence order equivalent to $\trianglerighteq$. Hence, invoking Lemma 1 to the final precedence order obtained through this step gives us the desired result. □

Proof of Proposition 3. Fix a set of slots S and a type function τ such that $S_m \neq \emptyset$. We first show that among the balanced precedence orders the minimal tier $\bar{t}$ assignment is attained when all merit slots precede the tier slots.

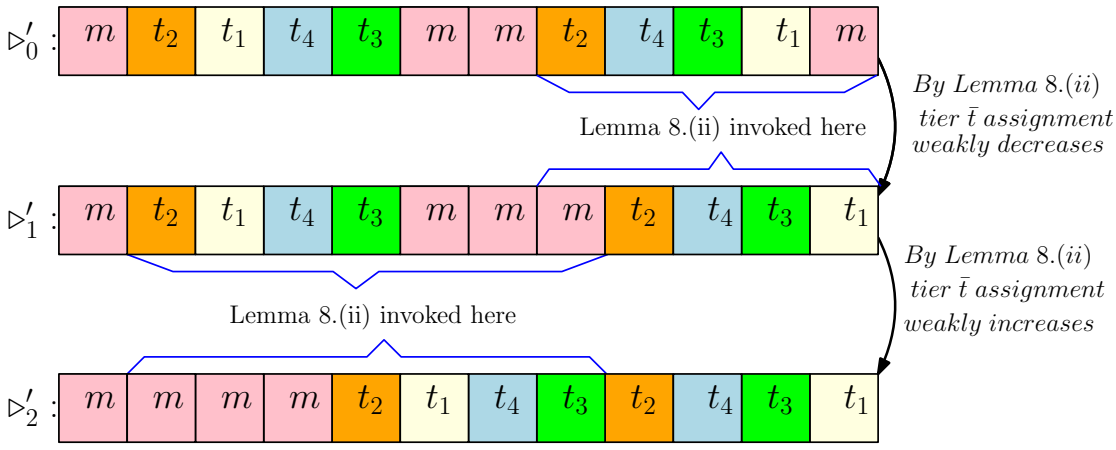

Figure A.14: Minimal Tier $\bar{t}$ Assignment under Balanced Precedence Orders

Minimal Tier $\bar{t}$ Assignment: Let $\bar{\triangleright}$ be a balanced precedence order such that all merit slots precede the tier slots. Consider an arbitrary balanced precedence order $\triangleright'$ such that at least one

merit slot is preceded by a tier slot. We will construct a sequence of precedences where the first element is $\triangleright'_0 \equiv \triangleright'$ and the last element is $\bar{\triangleright}$. We will show that tier $\bar{t}$ assignment weakly decreases under choice function $C(\cdot)$ as we move from one element of the sequence to the next one.

Let $\triangleright'_0 \equiv \triangleright'$ be the first element of the sequence. Let H and G denote the the last merit slot group and the tier slot group immediately before it under $\triangleright'_0$, respectively. Construct $\triangleright'_1$ from $\triangleright'_0$ by moving merit slot group H from immediately after the tier slot G group to immediately before it. See construction of precedence $\triangleright'_1$ from $\triangleright'_0$ in Figure A.14.

Claim 11 $C^{\bar{t}}(\triangleright'_1, I) \subseteq C^{\bar{t}}(\triangleright'_0, I)$.

Proof of Claim 11: First note that the score distributions of available students to be admitted by $C(\cdot)$ to tier slot group G under $\triangleright'_0$ and to merit slot group H under $\triangleright'_1$ satisfy Assumption 3. Let $S' = H \cup G$. By Lemma 8.(ii) we have

$$\bigcup_{s \in S'} C^{\bar{t}}_s(\triangleright'_1, I) \subseteq \bigcup_{s \in S'} C^{\bar{t}}_s(\triangleright'_0, I). \tag{29}$$

Equation (29) together with Lemma 3 complete the proof of Claim 11. ♢

Repeated application of this step of the construction for any last merit slot group preceded by a tier slot group gives us a precedence order equivalent to $\bar{\triangleright}$. Hence, this fact and Lemma 1 gives us the desired result.

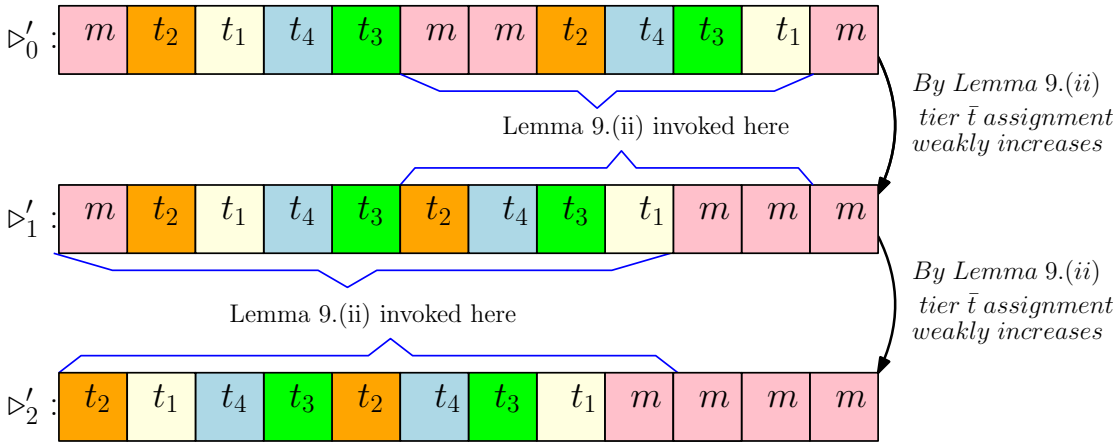

Figure A.15: Maximal Tier $\bar{t}$ Assignment under Balanced Precedence Orders

Maximal Tier $\bar{t}$ Assignment: Let $\trianglerighteq$ be a balanced precedence order such that all tier slots precede the merit slots. Consider an arbitrary balanced precedence order $\triangleright'$ such that at least one tier slot is preceded by a merit slot. We will construct a sequence of precedences where the first element is $\triangleright'_0 \equiv \triangleright'$ and the last element is $\underline{\trianglerighteq}$. We will show that tier $\bar{t}$ assignment weakly increases under choice function $C(\cdot)$ as we move from one element of the sequence to the next one.

Let $\triangleright'_0 \equiv \triangleright'$ be the first element of the sequence. Let G and H denote the the last tier slot group and the merit slot group immediately before it under $\triangleright'_0$, respectively. Construct $\triangleright'_1$ from $\triangleright'_0$ by moving tier slot group G from immediately after the merit slot group H to immediately before it. See construction of precedence $\triangleright'_1$ from $\triangleright'_0$ in Figure A.15.

Claim 12 $C^{\bar{t}}(\triangleright'_1, I) \supseteq C^{\bar{t}}(\triangleright'_0, I)$.

Proof of Claim 12: First note that the score distributions of available students to be admitted by $C(\cdot)$ to merit slot group H under $\triangleright'_0$ and to tier slot group G under $\triangleright'_1$ satisfy Assumption 3. Let S' be the set of slots in H together with all slot groups after H for the case of $\triangleright'_0$ and be the set of slots in G together with all slot groups after G for the case of $\triangleright'_0$. If $S' = H \cup G$ by Lemma 8.(ii), otherwise by Lemma 9.(ii) we have

$$\bigcup_{s \in S'} C^{\bar{t}}_s(\triangleright'_1, I) \supseteq \bigcup_{s \in S'} C^{\bar{t}}_s(\triangleright'_0, I). \tag{30}$$

Equation (30) together with Lemma 3 complete the proof of Claim 12. ♢

Repeated application of this step of the construction for any last tier slot group preceded by a merit slot group gives us a precedence order equivalent to $\trianglerighteq$. Hence, this fact and Lemma 1 gives us the desired result. □

A.5 Proof of Proposition 4

Fix the set of slots S and precedence order $\triangleright$. Let $h, e \in \mathbb{N}$ such that $h \geq e \times \bar{t}$. Let τ and $\hat{\tau}$ be two type functions such that

- The first h and $h - (e \times \bar{t})$ slots under $(\tau, \triangleright)$ and $(\hat{\tau}, \triangleright)$ are merit slots, respectively, and
- $|\{s \in S : \tau(s) = t\}| + e = |\{s \in S : \hat{\tau}(s) = t\}|$ for all $t \in T$.

Let $S_t = \{s \in S : \tau(s) = t\}$ and $\hat{S}_t = \{s \in S : \hat{\tau}(s) = t\}$ for all $t \in T$. We denote the infimum scores of students selected from I for the merit slots by $C(\cdot)$ under τ and $\hat{\tau}$ with ℓ and $\hat{\ell}$, respectively. Then,

$$\sum_{t=1}^{\bar{t}} \int_{\ell}^{\bar{k}} f_t(k)dk = h \quad \text{and} \quad \sum_{t=1}^{\bar{t}} \int_{\hat{\ell}}^{\bar{k}} f_t(k)dk = h - (e \times \bar{t}). \tag{31}$$

By Assumption 1, under both τ and $\hat{\tau}$, tier t slots are filled with only the tier t students. Hence, for each $t \in T$, the mass of tier t students in $C(S, \triangleright, \tau, I)$ is

$$\int_{\ell}^{\bar{k}} f_t(k)dk + |S_t|. \tag{32}$$

Similarly, for each $t \in T$ the mass of tier t students in $C(S, \triangleright, \hat{\tau}, I)$ is

$$\int_{\hat{\ell}}^{\bar{k}} f_t(k)dk + |\hat{S}_t|. \tag{33}$$

Equation (31) implies that $\ell < \hat{\ell}$. Hence, we can rewrite the first part of Equation (31) as

$$\sum_{t=1}^{\bar{t}} \int_{\hat{\ell}}^{\bar{k}} f_t(k)dk + \sum_{t=1}^{\bar{t}} \int_{\ell}^{\hat{\ell}} f_t(k)dk = h, \tag{34}$$

and Equation (32) as

$$\int_{\hat{\ell}}^{\bar{k}} f_t(k)dk + \int_{\ell}^{\hat{\ell}} f_t(k)dk + |S_t|. \tag{35}$$

The second part of Equation (31) and Equation (34) imply that

$$\sum_{t=1}^{\bar{t}} \int_{\ell}^{\hat{\ell}} f_t(k)dk = e \times \bar{t}. \tag{36}$$

Assumption 2 and Equation (36) imply that the mass of tier 1 students selected from I for the merit slots by $C(\cdot)$ under τ is *at most*

$$\int_{\hat{\ell}}^{\bar{k}} f_1(k)dk + e. \tag{37}$$

By Equations (35) and (37), the mass of tier 1 students in $C(S, \triangleright, \tau, I)$ is *at most*

$$\int_{\hat{\ell}}^{\bar{k}} f_1(k)dk + e + |S_1|.$$

Assumption 3 and Equation (36) imply that the mass of tier $\bar{t}$ students selected from I for the merit slots by $C(\cdot)$ under τ is *at least*

$$\int_{\hat{\ell}}^{\bar{k}} f_{\bar{t}}(k)dk + e. \tag{38}$$

By Equations (35) and (38), the mass of tier $\bar{t}$ students in $C(S, \triangleright, \tau, I)$ is *at least*

$$\int_{\hat{\ell}}^{\bar{k}} f_{\bar{t}}(k)dk + e + |S_{\bar{t}}|.$$

By construction, $|\hat{S}_t| = |S_t| + e$ for all $t \in T$. Hence, we can rewrite Equation (33) for tier 1, i.e., the mass of tier 1 students in $C(S, \triangleright, \hat{\tau}, I)$, as

$$\int_{\hat{\ell}}^{\bar{k}} f_1(k)dk + e + |S_1|,$$

which is equal to the maximal tier 1 assignment in $C(S, \triangleright, \tau, I)$. Similarly, we can rewrite Equation (33) for tier $\bar{t}$, i.e., the mass of tier $\bar{t}$ students in $C(S, \triangleright, \hat{\tau}, I)$, as

$$\int_{\hat{\ell}}^{\bar{k}} f_{\bar{t}}(k)dk + e + |S_{\bar{t}}|,$$

which is equal to the minimal tier $\bar{t}$ assignment in $C(S, \triangleright, \tau, I)$. □

Figure B1: Choice Received by Tier 1 Students (30% Merit / 70% Tier)