Asset Encumbrance, Bank Funding and Fragility*

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Abstract

We offer a model of asset encumbrance by banks subject to rollover risk and study how secured debt issuance influences fragility, funding costs, and welfare. A banker encumbers assets to trade off expanding profitable investment funded with secured debt with greater fragility due to unsecured debt runs. We derive several testable implications about the privately optimal level of encumbrance. A constrained planner encumbers *more* assets, since the bankruptcy-remote pool of assets satisfies a demand for safety. We evaluate the efficacy of policy tools aimed at boosting private encumbrance levels, including interest rate cuts, capital injections, guarantees, lender of last resort, and stable funding ratios.

Keywords: asset encumbrance, wholesale funding, rollover risk, fragility, secured debt, unsecured debt, prudential policy.

JEL classifications: D82, G01, G21, G28.

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1 Introduction

Bank funding structures matter for financial stability. The reliance of many banks on short-term unsecured wholesale funding was a key factor contributing to the global financial crisis (IMF, 2013). Since then, bank funding structures – particularly in the euro area and the United States – have shifted towards secured debt instruments such as covered bonds and repurchase agreements (repos). These trends have been driven by market forces and reinforced by regulatory developments. These include reforms emphasizing statutory 'bail-in' tools for bank resolution and Basel III liquidity regulations stipulating that long-term assets be financed by stable funding.

The increasing emphasis on collateral to secure wholesale funding has led banks to encumber assets into bankruptcy-remote entities on their balance sheets. The implications of rising asset encumbrance levels, however, are poorly understood. On one hand, longer-term secured debt might seem appropriate since it is a cheap source of funding for banks and a safe asset for investors. On the other hand, encumbrance reduces assets available to meet the claims of unsecured debt holders, thus potentially exacerbating funding liquidity risk. Regulators in a number of countries have, therefore, sought to restrict the encumbrance of bank balance sheets on stability grounds.[1]

We offer a theory of asset encumbrance by banks subject to rollover risk. It contains a positive analysis of how encumbrance affects bank fragility and funding costs and contributes to the policy debate on collateral use in financial transactions. A novel aspect of the analysis is a positive externality associated with asset encumbrance that is not considered by the bank. Following bankruptcy, bankers do not value the encumbered assets that remain bankruptcy-remote. These safe assets, how-

[1]For example, in the United States, there are ceilings on the amount of secured funding by banks. In Canada, regulators include encumbrance levels in deposit insurance premia alongside ceilings. In Australia and New Zealand, explicit limits are imposed on total asset encumbrance.

ever, have a social value that must be accounted for by a planner when designing optimal regulation. While rising asset encumbrance can exacerbate financial fragility, a planner may be prepared to tolerate this to satisfy a societal demand for safe assets. Our work highlights a hitherto neglected benefit of asset encumbrance that, arguably, warrants greater attention in the policy debate.

Our approach sidesteps the Modigliani-Miller theorem by presuming the existence of segmented funding markets, each with its own distinct investor clientele.[2] In the model, a banker seeks funding for profitable long-term investment. Debt is issued in two markets. The banker attracts unsecured funding from risk-neutral investors by offering demandable debt and procures secured funding from infinitely risk-averse investors, such as pension funds, by issuing debt backed by the encumbered assets on balance sheet. Since encumbered assets are protected, shocks to a bank's balance sheet affect unencumbered assets and are thus borne by unsecured debt holders.

An unsecured debt run is modeled as a coordination failure and we use the global games approach to pin down the unique equilibrium of the model (Morris and Shin, 2003; Rochet and Vives, 2004; Goldstein and Pauzner, 2005). The decision to roll over unsecured debt is based on a private signal about the balance sheet shock. A critical run threshold ensures that a run occurs if, and only if, the shock is sufficiently large in relation to the value of unencumbered assets. We link the incidence of ex post runs to the ex ante issuance of secured debt and the proportion of assets encumbered, and solve for the face values of secured and unsecured debt.

Asset encumbrance alters the run dynamics by driving a wedge between illiquidity and insolvency conditions of a bank. If the banker has to prematurely liquidate assets to satisfy withdrawals by unsecured debt holders, encumbered assets are re-

[2]This segmentation of markets according to risk aversion breaks the irrelevance between secured and unsecured debt. Moreover, liquidation costs break the irrelevance between debt and equity.

served for secured debt holders and only unencumbered assets can be used. However, if unsecured debt is rolled over, it can also be repaid with residual encumbered assets once secured debt holders have been paid. Such additional assets are available because of over-collateralization of the pool of assets that back secured debt. We show that the illiquidity condition is more binding than the insolvency condition if unsecured debt is cheap. In other words, asset encumbrance makes solvent banks illiquid and prone to unsecured debt runs. This result sharply contrasts with our benchmark of Rochet and Vives (2004), in which any illiquid bank is also insolvent.

Our analysis shows that greater asset encumbrance heightens fragility. Although more secured funding supports greater investment and lowers run risk, the quantum of unencumbered assets is lower for any given level of investment. This exacerbates bank fragility. And since infinitely risk-averse investors value collateral at its liquidation value, the overall effect of rising asset encumbrance is greater financial fragility.

The privately optimal level of encumbrance chosen by the banker balances the marginal cost – greater fragility and a lower probability of surviving a run – against the marginal benefit of more secured funding to finance profitable investment. As a result, the model yields positive results that offer testable implications about the relationship between asset encumbrance and (i) monetary conditions; (ii) market stress; and (iii) bank profitability. Our results also point to a non-monotonic relationship between asset encumbrance and bank capital. Under mild conditions on the shock distribution, a positive relationship between encumbrance and capital arises.

The banker's choice of asset encumbrance is constrained inefficient. The demand for safe assets by infinitely risk-averse investors, coupled with the bankruptcy remoteness of the encumbered assets, generates a positive externality that is not accounted for by a banker subject to limited liability. The banker encumbers fewer assets than a planner who also cares about the expected utility of investors in the funding markets, taking as given the rollover risk of unsecured debt and equilibrium conditions in secured and unsecured debt markets.

The model has several normative implications. Financial policies that seek to reduce the impact and incidence of bank fragility can encourage the banker to choose levels of encumbrance that are closer to the constrained efficient level. We study the role played by (i) interest rate cuts; (ii) capital injections; (iii) a credible lender of last resort; (iv) a net stable funding ratio; and (v) guarantees on unsecured debt and evaluate their efficacy in facilitating the socially desired level of asset encumbrance.

Although our analysis focuses on secured lending that is backed by assets that remain on the issuing bank's balance sheet, including covered bonds, our results are also relevant for off-balance sheet instruments (e.g., residential mortgage-backed securities), particularly to the extent that the issuing bank makes implicit or explicit guarantees to support a special-purpose vehicle. Similar interactions may also be found for term repos, where safe harbor arrangements ensure the bankruptcy remoteness of collateral (Goralnik, 2012). Credit card asset-backed securities also feature bankruptcy remoteness (Furletti, 2002).

The literature on bank funding structures and its implications for financial stability is sparse. An early contribution is Greenbaum and Thakor (1987). They study the choice between deposit funding (on-balance sheet) and securitized funding (off-balance sheet). Borrowers effectively choose the funding mode by signaling private information about the quality of their projects. Higher quality projects are securitized, while lower quality projects remain on the bank's balance sheet and are funded by deposits. Prudential regulations, such as the deposit insurance and capital requirements, impact the relative appeal of deposit funding in their model.

Our contribution builds on Rochet and Vives (2004) and the literature on bank runs, coordination failure, and global games pioneered by Carlsson and van Damme (1993). Our modeling of unsecured debt follows Rochet and Vives (2004) and we extend their work to allow for asset encumbrance and the pricing of unsecured debt. In doing so, we argue that asset encumbrance can render solvent banks illiquid. And in emphasizing the externalities that arise from safe asset demand, our work has points of contact with the emerging literature on safe assets pioneered by Dang et al. (2016), Gorton and Ordonez (2014), and Caballero et al. (2016), and on the role of sovereign debt in bank balance sheets (Brunnermeier et al., 2016).

Recent analytical work by policymakers has also attempted to examine the interplay between secured and unsecured funding. Gai et al. (2013) and Eisenbach et al. (2014) adopt a balance sheet approach to examine the financial stability implications of alternative funding structures. Eisenbach et al. (2014) highlight some of the ex-post balance sheet dynamics associated with asset encumbrance and collateralized funding in the context of exogenous creditor behavior. Using global games techniques to describe endogenous creditor behavior, Gai et al. (2013) study how bank liquidity and solvency risks change with the composition of funding in partial equilibrium and show how 'dashes for collateral' by short-term secured creditors can occur.

The paper proceeds as follows. Section 2 sets out the model. Section 3 studies the rollover decision of unsecured debt claims, and solves for the equilibrium in the secured and unsecured funding markets. Section 4 highlights implications of our model for the empirical analysis of asset encumbrance. It also studies the welfare implications of the private allocation and evaluates the efficacy of several financial stability policies. Section 5 concludes.

2 Model

Our analytical framework builds on Rochet and Vives (2004). There are three dates, $t = 0, 1, 2$, and a single good for consumption and investment. The economy is populated by a large mass of investors, who are each endowed with a unit at $t = 0$ and are indifferent between consuming at $t = 1$ and $t = 2$. Investors differ in their risk preferences: a first investor clientele is risk-neutral, while a second is infinitely risk-averse. The latter group may be thought of as pension funds or large institutional investors mandated to hold high-quality and safe assets in their portfolios (IMF, 2012). All investors have access to a storage technology at $t = 0$, which yields a gross return, $r > 0$, at $t = 2$.

The economy also comprises a unit mass of identical risk-neutral bankers with access to profitable and high-quality investment opportunities at $t = 0$. Bankers can invest their own funds, $E \geq 0$, at $t = 0$ in order to consume at $t = 2$. But they can also obtain funding by issuing unsecured and secured debt to investors at $t = 0$. Each investment matures at $t = 2$ with a gross return, $R > r$. Premature liquidation of the investment at $t = 1$ yields a fraction ψ of the return at maturity, where $0 < \psi R < r$.

Given the segmented investor base, bankers issue unsecured demandable debt to risk-neutral investors and asset-backed secured debt to risk-averse investors. In the spirit of Rochet and Vives (2004), an exogenous quantum of unsecured debt, $U \equiv 1$, can be withdrawn at $t = 1$ or rolled over until $t = 2$. The rollover decision is made by a group of professional fund managers, indexed by $i \in [0, 1]$, who typically prefer to roll over funds but are penalized by investors if the bank fails. Fund managers face strategic complementarity in their decisions in the sense that individual manager's incentive to roll over increases in the proportion of managers who roll over. A manager's conservatism, $0 < \gamma < 1$, is crucial to this decision.[3] The more conservative the manager, the higher is γ and the less likely that unsecured debt is rolled over.[4] The face value of unsecured debt, D_U, is independent of the withdrawal date.

A banker attracts secured funding from risk-averse investors by encumbering – or ring-fencing – a proportion $\alpha \in [0, 1]$ of assets into a bankruptcy-remote entity on its balance sheet.[5] We denote by $S \geq 0$ the total amount of secured funding raised, and by D_S the face value of secured debt at $t = 2$. Table 1 illustrates the balance sheet of the representative bank at $t = 0$ once funding is raised, investment $I = E + S + U$ is made, and assets are encumbered.

Assets		Liabilities
(encumbered assets)	αI	S
(unencumbered assets)	$(1-\alpha)I$	U
		E

Table 1: Balance sheet at $t = 0$ after funding, investment, and asset encumbrance.

We suppose that a bank's balance sheet is subject to shock, A, at $t = 2$. The shock may enhance the value of assets, in which case $A < 0$. But the crystallization of operational, market, credit or legal risks may require write downs, corresponding to $A > 0$. The shock has a continuous probability density function $f(A)$ and cumulative distribution function $F(A)$, with decreasing reverse hazard rate, $\frac{d}{dA}\frac{f(A)}{F(A)} < 0$. This common condition supports the concavity of the expected equity value in the level of

[3]The conservatism ratio, $\gamma \equiv \frac{c}{b+c} \in (0, 1)$, derives from managerial compensation in Rochet and Vives (2004). In bankruptcy, a manager's relative compensation from rolling over is negative, $-c < 0$. Otherwise, the relative compensation is positive, $b > 0$.

[4]Reviewing debt markets during the financial crisis, Krishnamurthy (2010) argues that investor conservatism was an important determinant of short-term lending behavior. See also Vives (2014).

[5]Secured funding typically takes the form of covered bonds, which are backed by assets that remain on the issuing bank's balance sheet, or RMBS (residential mortgage-backed securities), which are generally off-balance sheet instruments. To the extent that the issuing bank offers implicit or explicit guarantees, RMBS also affect encumbrance. Term repos (repurchase agreements) also contribute to asset encumbrance levels.

asset encumbrance and equilibrium uniqueness.

Bankers and investors are protected by limited liability. The value of bank equity at $t = 2$ is $E_2(A) \equiv \max\{0, RI - A - UD_U - SD_S\}$. Table 2 shows the balance sheet at $t = 2$ for a small shock and when all unsecured debt is rolled over. Since encumbered assets are ring-fenced, the shocks affects only unencumbered assets.[6]

Assets		Liabilities
(encumbered assets)	$R\alpha I$	SD_S
(unencumbered assets)	$R(1-\alpha)I - A$	UD_U
		$E_2(A)$

Table 2: Balance sheet at $t = 2$ after a small shock and unsecured debt is rolled over.

If a proportion $\ell \in [0, 1]$ of unsecured debt is not rolled over at $t = 1$, the banker liquidates an amount $\frac{\ell UD_U}{\psi R}$ to meet withdrawals. A bank is **illiquid** at $t = 1$ and closed early if the liquidation value of unencumbered assets is insufficient to meet withdrawals,

$$R(1-\alpha)I - A < \frac{\ell UD_U}{\psi}. \quad (1)$$

The assumption of bankruptcy-remoteness means that the pool of encumbered assets is reserved for secured debt holders who each receive an equal share of the liquidation value $R\psi\alpha I$. In stark contrast, bankruptcy costs are assumed to be so high that unsecured debt holders recover zero in bankruptcy. The illiquidity threshold for the shock is $A_{IL}(\ell) \equiv R(1-\alpha)I - \frac{\ell UD_U}{\psi}$.

If the bank is liquid at $t = 1$, then the total value of bank assets is $RI - \frac{\ell UD_U}{\psi} - A$ at $t = 2$. The bank is **insolvent** at $t = 2$ if it is unable to repay its secured debt holders and the proportion $1 - \ell$ of unsecured debt holders, that is

[6] Our modeling of the shock is also consistent with the notion of 'replenishment,' whereby credit and market risks are expunged from the encumbered assets and concentrated onto the unencumbered part of the bank's balance sheet. Such mechanisms are present for covered bonds and repos.

$$RI - A - \frac{\ell\, UD_U}{\psi} < SD_S + (1-\ell)\, UD_U. \tag{2}$$

Upon repaying secured debt holders at $t = 2$, the banker uses any residual encumbered assets (due to over-collateralization) to repay remaining unsecured debt. The insolvency threshold of the shock is thus $A_{IS}(\ell) \equiv RI - SD_S - UD_U \left[1 + \ell \left(\frac{1}{\psi} - 1\right)\right]$.

At $t = 1$, fund managers receive a noisy private signal about the shock upon which they base their rollover decisions. Specifically, they receive the signal

$$x_i \equiv A + \epsilon_i, \tag{3}$$

where ϵ_i is idiosyncratic noise drawn from a continuous distribution G with support $[-\epsilon, \epsilon]$ for $\epsilon > 0$. The idiosyncratic noise is independent of the shock, and is independently and identically distributed across fund managers.

Table 3 summarizes the timeline of events in the model.

$t = 0$	$t = 1$	$t = 2$
1. Issuance of secured and unsecured debt	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
3. Asset encumbrance	3. Unsecured debt withdrawals	3. Debt repayments
	4. Consumption	4. Consumption

Table 3: Timeline of events.

3 Equilibrium

Our focus is on the symmetric, pure-strategy, perfect Bayesian equilibrium of the model. Without loss of generality, we study threshold strategies for the rollover of unsecured debt (Morris and Shin, 2003). Thus, fund managers roll over unsecured debt if, and only if, their private signals indicate a healthy balance sheet, $x_i \leq x^*$.

Definition 1. *The symmetric, pure-strategy, perfect Bayesian equilibrium comprises a proportion of encumbered assets* (α^*)*, an amount of secured debt for each bank* (S^*)*, face values of unsecured and secured debt* (D_U^*, D_S^*)*, and critical thresholds for the private signal* (x^*) *and balance sheet shock* (A^*) *such that:*

a. At $t = 1$*, the rollover decision of fund managers* (x^*, A^*) *is optimal, given the level of asset encumbrance and secured debt* (α^*, S^*) *and face values of debt* (D_U^*, D_S^*)*;*

b. At $t = 0$*, each banker optimally chooses* (α^*, S^*) *given the face values of debt* (D_U^*, D_S^*)*, the participation of secured debt holders, and the rollover thresholds of fund managers* (x^*, A^*)*; and*

c. At $t = 0$*, secured and unsecured debt are priced by binding participation constraints, given the bankers' choices* (α^*, S^*) *and rollover thresholds* (x^*, A^*)*.*

We proceed to construct the equilibrium in four steps. First, we price secured debt. Second, we derive the optimal rollover decision of fund managers. Third, we characterize the optimal asset encumbrance choice of the banker and, in doing so, obtain the endogenous level of secured debt issuance. And, in a final step, we price unsecured debt.

3.1 Pricing secured debt

A secured debt holder either receives the face value D_S or an equal share of the value of encumbered assets. Since early closure at $t = 1$ occurs with positive probability for a large balance sheet shock, competitive pricing of secured debt by infinitely risk-averse investors implies a binding participation constraint, $r = \min\left\{D_S, \psi\, \frac{R\alpha I}{S}\right\}$.

Lemma 1. Asset encumbrance and cheap secured debt. *Secured debt is cheap, $D_S^* = r$, and the maximum issuance of secured debt tolerated by risk-averse investors is $S \leq S^*(\alpha) = \alpha\psi z I^*(\alpha)$, where $z \equiv R/r$ is the relative return and $I^*(\alpha) = \frac{U+E}{1-\alpha\psi z}$ is total investment. Greater encumbrance increases secured debt issuance and investment, $\frac{dS^*}{d\alpha} = \frac{dI^*}{d\alpha} = \frac{\psi z I^*(\alpha)}{1-\alpha\psi z} > 0$.*

Note that the bounds on the liquidation value can be expressed as $0 < \psi z < 1$. Competitive pressures push the face value of secured debt to the outside option of investors. Since infinitely risk-averse investors evaluate the secured debt claim at the worst outcome (that is, the liquidation value in early closure), the maximum level of secured debt increases in the level of asset encumbrance. As we make clear below, a banker always chooses this maximum level of secured debt for a given level of asset encumbrance, since it reduces fragility and enhances expected equity value.

3.2 Rollover risk of unsecured debt

Asset encumbrance and secured debt issuance fundamentally alter the dynamics of rollover risk, as shown in Figure 1. Panel A shows the illiquidity and insolvency thresholds, $A_{IL}(\ell)$ and $A_{IS}(\ell)$, that obtain without any asset encumbrance and secured debt issuance, that is $\alpha = 0 = S$. It thus recovers the dynamics in Rochet and Vives (2004) for the case where liquid cash reserves are set at zero. An illiquid bank at $t = 1$ is always insolvent at $t = 2$. And so, in this case, the insolvency threshold is the relevant condition for analysis. Panel B shows the illiquidity and insolvency thresholds in the case of asset encumbrance and secured debt issuance. Over-collateralization means that the thresholds do not coincide at $\ell = 1$. Additional assets worth $R\alpha(1-\psi)I^*(\alpha)$ become available to service unsecured debt withdrawals at $t = 2$, which are not available at $t = 1$ because of encumbrance. As a result, a bank that is illiquid at $t = 1$ can, nevertheless, be solvent at $t = 2$, that is $R\alpha\psi I^*(\alpha) \geq (1 - \ell)\, UD_U$. Thus, a sufficient condition for the illiquidity threshold to be the relevant condition for analysis is a requirement for an upper bound on the face value of unsecured debt, namely $D_U \leq \hat{D}_U \equiv (1 - \psi)R\alpha I$. In what follows, we suppose this condition holds and later verify that it holds in equilibrium.

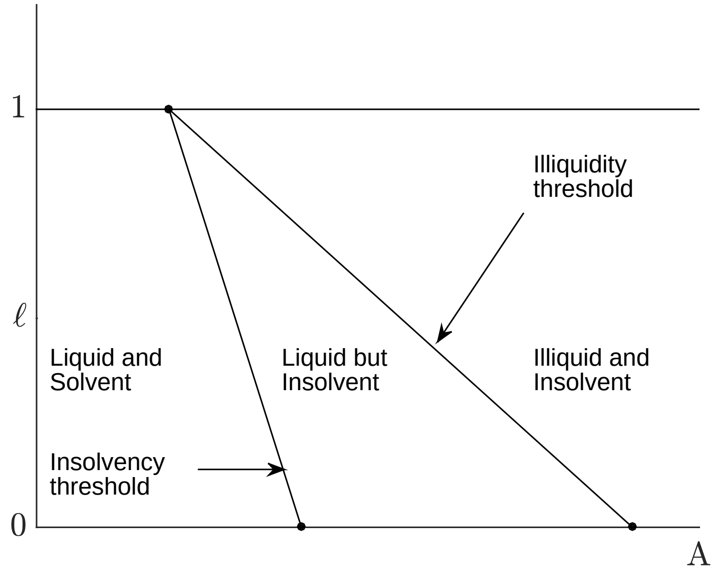
Proposition 1. Run threshold. *There exist unique signal and run thresholds such that unsecured debt is not rolled over and a run occurs if, and only if,*

$$A > A^* \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma\, UD_U}{\psi} \leftarrow x^*, \tag{4}$$

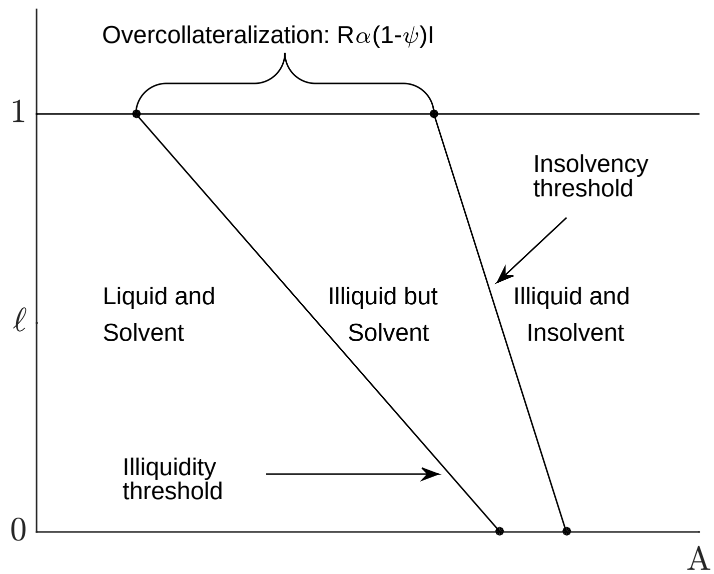
where we consider the limit of vanishing private noise, $\epsilon \to 0$.

Proof. See Appendix A. ■

Proposition 1 employs global games techniques to pin down the unique incidence of an unsecured debt run by fund managers and relates it to model parameters. A higher conservatism of fund managers decreases the threshold above which a run occurs, $\frac{\partial A^*}{\partial \gamma} < 0$. A higher return on investment increases the value of unencumbered assets as well as the amount of secured debt raised for a given level of asset encumbrance. Both effects act to reduce run risk, so $\frac{\partial A^*}{\partial R} > 0$. A higher liquidation value of



(a) Without asset encumbrance



(b) With asset encumbrance

Figure 1: Asset encumbrance and secured debt issuance alters the run dynamics. The figure depicts the illiquidity and insolvency thresholds, A_{IL} and A_{IS}, as a function of the proportion of withdrawing investors ℓ for two cases: without asset encumbrance in panel A and with asset encumbrance in panel B. Panel A replicates the results of Rochet and Vives (2004) without liquid asset holdings and with a balance sheet shock, where the relevant condition is the insolvency threshold. As depicted in panel B, over-collateralization shifts the insolvency threshold to the right. Therefore, the upper bound $\hat{D}_U$ ensures that the relevant condition is the illiquidity threshold.

investment decreases the extent of strategic complementarity among fund managers and increases the amount of secured debt issued, given encumbrance. Since both these effects also lower run risk, it follows that $\frac{\partial A^*}{\partial \psi} > 0$.

The comparative statics for an increase in r and E are also intuitive. A higher cost of funding decreases the amount of secured debt raised for a given level of encumbrance. This reduces the value of unencumbered assets, so $\frac{\partial A^*}{\partial r} < 0$. And a better capital of banks reduces run risk through its effect on increased investment and unencumbered asset values, implying $\frac{\partial A^*}{\partial E} > 0$.

Lemma 2. Asset encumbrance and fragility. *Greater asset encumbrance heightens bank fragility,* $\frac{dA^*}{d\alpha} < 0$.

Proof. See Appendix A. ■

Our model shows that greater asset encumbrance influences bank fragility in two important ways. First, for a given level of investment, the amount of unencumbered assets is reduced, exacerbating the fragility of the balance sheet at $t = 1$. Second, greater encumbrance allows the banker to raise more secured funding. This increases investment and reduces fragility. But since infinitely-risk averse investors value the collateral backing secured debt claims at the liquidation value, the overall effect of greater asset encumbrance is to unambiguously heighten fragility.

3.3 Optimal asset encumbrance and secured debt issuance

The representative banker chooses a level of asset encumbrance to maximize the expected equity value of the bank, taking as given the face value of unsecured debt D_U, and subject to the run threshold, $A = A^*(\alpha)$, and the (maximum) amount of secured debt that can be raised, $S \leq S^*(\alpha)$. Since a higher level of secure debt for a given level of asset encumbrance both increases the expected equity value of the bank and lowers fragility, we obtain $S = S^*(\alpha)$. So the problem of the banker is:

$$\max_{\alpha} \pi \equiv \int E_2(A) dF(A) = \int_{-\infty}^{A^*(\alpha)} \left[RI^*(\alpha) - UD_U - S^*(\alpha)\, r - A\right] dF(A). \tag{5}$$

Figure 2 shows the relationship between encumbrance and expected equity value. It illustrates the existence of a unique interior solution for asset encumbrance.

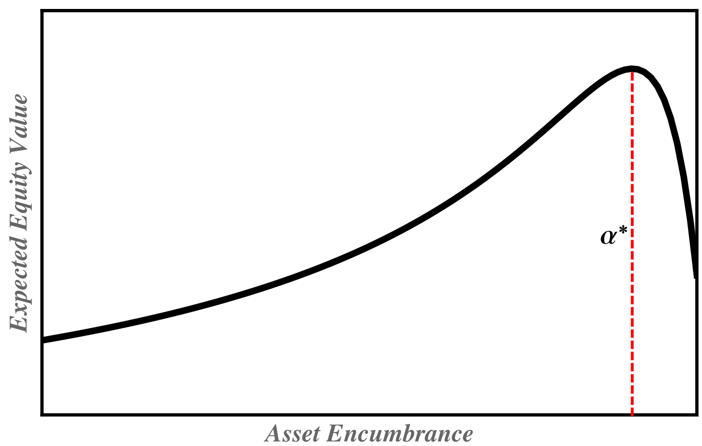


Figure 2: A unique interior privately optimal level of asset encumbrance: an example with $R = 1.5$, $r = 1.2$, $E = 0.5$, $\psi = 0.6$, $\gamma = 0.8$, $D_U = 3.3$, and the balance sheet shock follows the Logistic distribution with mean -3 and scaling parameter 1.

Proposition 2. Asset encumbrance schedule. *There is a unique asset encumbrance schedule, $\alpha^*(D_U)$. If $\gamma > \psi$, that is fund managers are sufficiently conservative, the schedule decreases in the face value of unsecured debt, $\frac{d\alpha^*}{dD_U} \leq 0$, and an interior solution for $\underline{D}_U < D_U < \overline{D}_U$ is implicitly given by:*

$$\frac{F(A^*)}{f(A^*)} = \frac{(1-\psi z)}{\psi\,(z-1)} \left[\alpha^*(1-\psi) RI^*(\alpha^*) + UD_U \left(\frac{\gamma}{\psi} - 1\right)\right]. \tag{6}$$

Proof. See Appendix B. ■

The banker balances the marginal benefits and costs of asset encumbrance when choosing the privately optimal level. The marginal benefit of encumbrance is an increase in collateral and hence more secured funding. Since secured debt is cheap and investments are profitable, the equity value of the bank – conditional on it surviving an unsecured debt run – is increased. But the marginal cost of encumbrance is an increase in bank fragility and, therefore, a lower probability of surviving an unsecured debt run. Consequently, a higher face value of unsecured debt exacerbates rollover risk and lowers the run threshold, inducing the banker to encumber fewer assets.

Figure 3 shows the relationship between the asset encumbrance schedule and the face value of unsecured debt. The bounds on the face value of unsecured debt ensure a zero marginal profit at the boundaries, $\frac{d\pi}{d\alpha}\Big|_{\alpha=1,\underline{D}_U} \equiv 0$ and $\frac{d\pi}{d\alpha}\Big|_{\alpha=0,\overline{D}_U} \equiv 0$.

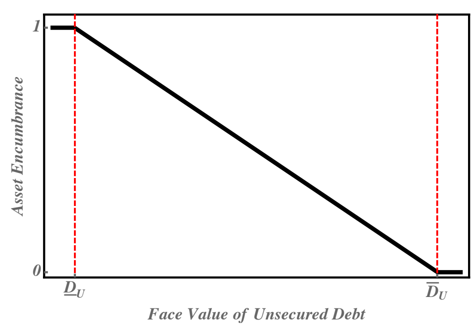


Figure 3: Asset encumbrance schedule as a function of the face value of unsecured debt with $R = 1.5$, $r = 1.2$, $E = 0.5$, $\psi = 0.6$, $\gamma = 0.8$, and the balance sheet shock follows the Logistic distribution with mean -3 and scaling parameter 1.

3.4 Pricing of unsecured debt

The repayment of unsecured debt depends on the size of the balance sheet shock. Without a run, $A \leq A^*$, unsecured debt holders receive the promised payment D_U, while for larger shocks, $A > A^*$, bankruptcy occurs and they receive zero. Thus, the value of an unsecured debt claim is $V(D_U, \alpha) \equiv D_U\, F(A^*(\alpha, D_U))$, and competitive pricing implies that it equals the cost of funding for any given level of encumbrance:

$$r = D_U^* F(A^*(\alpha, D_U^*)). \tag{7}$$

Proposition 3. Private optimum. *If the shock distribution is truncated from below, bank capital is bounded from above, $E < \bar{E}$, the conservatism satisfies $\gamma \geq \underset{\sim}{\gamma}$, there exists a unique face value of unsecured debt, $D_U^* > r$. If funding is sufficiently costly, $r > \underset{\sim}{r}$, the implied proportion of asset encumbrance is interior, $\alpha^{**} \equiv \alpha^*(D_U^*) \in (0, 1)$.*

Proof. See Appendix C. ■

Figure 4 shows the privately optimal allocation and its construction. The constraint $\gamma \geq \underset{\sim}{\gamma}$ ensures that the schedule $r = V(D_U^*, \alpha)$ is upward-sloping at α^{**}, ensuring uniqueness. Since $V(D_U = r) < r$, we obtain that $D_U^* > r$. The sufficient condition $E < \bar{E}$ ensures that the supposition always holds, $D_U^* < \hat{D}_U(\alpha^*)$. The lower bound on the cost of funding ensures that the equilibrium face value of unsecured debt is sufficiently high such that the proportion of asset encumbrance is interior.

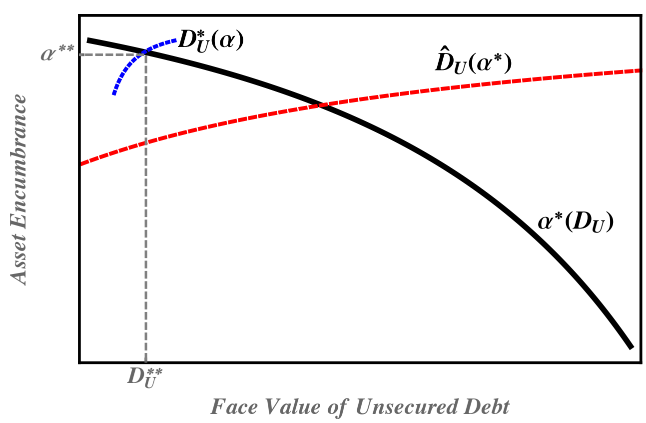


Figure 4: Privately optimum of asset encumbrance and face value of unsecured debt: an example with $R = 1.5$, $r = 1.2$, $E = 0.5$, $\psi = 0.6$, $\gamma = 0.8$, and the shock follows the Logistic distribution with mean -3 and scaling parameter 1.

4 Implications for policy and empirical work

We state the main comparative static results of the model and contrast the privately optimal level of asset encumbrance with the socially optimal level chosen by a planner. Our results have implications for both financial stability policy and empirical analysis.

Proposition 4. *Privately optimal asset encumbrance, α^{**}, decreases in the cost of funding, r, and in the conservatism of fund managers, γ. It increases in the profitability of investment, R, improvements in the shock distribution, and the liquidation value, ψ. Greater bank capital, E, has an ambiguous effect on asset encumbrance.*

Proof. See Appendix D. ■

Parameter changes affect the unique, interior equilibrium in two ways. First, for a given face value of unsecured debt, the banker trades off heightened fragility against the funding of investments with positive net present value. Second, the equilibrium face value of unsecured debt changes with underlying parameter values, influencing the incentives of investors to participate in the debt contract.

The social planner maximizes the expected payoffs of all agents in the economy, taking as given the incomplete information and the associated run threshold. As such, our notion of welfare is constrained efficiency and the planner's problem is:

$$\max_{\alpha} W(\alpha, A^*) \quad \equiv \quad \pi(\alpha, A^*) + r\left[S^*(\alpha) + U\right] \tag{8}$$

$$\text{s.t.}$$

$$A^* \quad = \quad R(1-\alpha)I^*(\alpha) - \gamma\frac{rU}{\psi F(A^*)}. \tag{9}$$

Equation (8) states that the planner cares about the expected value of bank equity and the value of payments to debt holders, where expected equity value is evaluated at the equilibrium in unsecured debt market, namely $\pi(\alpha, A^*) \equiv \pi|_{r=V(D_U^*,\alpha)}$. Equation (9) also uses the unsecured debt market equilibrium to implicitly define the run threshold A^*. We focus on parameter values than uniquely pin down the run threshold.

The segmented nature of the investor base drives a wedge between the privately optimal allocation and that chosen by the social planner. The demand for safe assets by infinitely risk-averse investors, combined with the bankruptcy remoteness of encumbered assets, generates a positive externality from asset encumbrance that is not accounted for by the banker. The banker maximizes expected equity value subject to limited liability. But the pricing of secured debt does not fully induce the banker to take the social value of safe assets into account – while equity is zero in bankruptcy, the bankruptcy remoteness of the ring fence ensures that encumbered assets are safe.

In our model, the banker does not want to increase asset encumbrance as it would exacerbate financial fragility, increasing the range of shocks over which bank equity gets wiped out. Moreover, while the social planner values ring-fenced assets, the bankrupt banker does not. Note that the run threshold is the same for both the planner and the banker. The objective functions differ, however, and this difference is increasing in the level of asset encumbrance as Figure 5 suggests. Thus, too few assets are encumbered from a societal perspective. More formally,

Proposition 5. Constrained inefficiency. *The privately optimal level of encumbrance is below the socially optimal level, $\alpha_P^{**} \equiv \alpha^{**} < \alpha_S^{**}$.*

Proof. See Appendix F. ■

Propositions 4 and 5 allow us to consider several policy measures aimed at reducing fragility. By reducing the incidence or impact of fragility, these policies raise the privately optimal level of encumbrance and bring it closer to the social optimum.

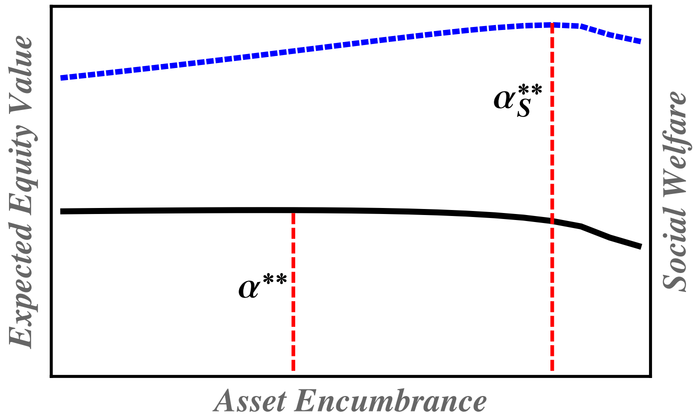


Figure 5: Constrained inefficiency: the privately optimal level of encumbrance, α_P^{**}, is below the socially optimal level, α_S^{**}. The expected equity value (black) and the planner's objective function (blue, dashed) are plotted on log scale for an example with $R = 1.5$, $r = 1.2$, $E = 0.5$, $\psi = 0.6$, $\gamma = 0.8$, and the shock follows the Logistic distribution with mean -3 and scaling parameter 1.

Monetary policy. Bebchuk and Goldstein (2011) highlight how an interest rate cut helps avoid credit freezes by inducing banks to lend more. On the funding side, our model suggests that a lower cost of funding, r, increases the amount of secured funding that the banker can raise for a given level of asset encumbrance, increasing investment and the value of unencumbered assets. At the same time, the face value of unsecured debt is also lowered. The two effects combine to lower bank fragility and increase the privately optimal level of asset encumbrance, that is $\frac{d\alpha^{**}}{dr} < 0$. Figure 6 illustrates how a reduction in interest rates increases the level of encumbrance chosen by both the banker and the planner. The gap between the privately and socially optimal level declines as interest rates decrease, reflecting the role that easier monetary conditions play in satisfying the demand for safe assets.

Empirically, our model suggests that less restrictive monetary conditions lead to a shift towards safety, that is toward secured lending. This implication appears

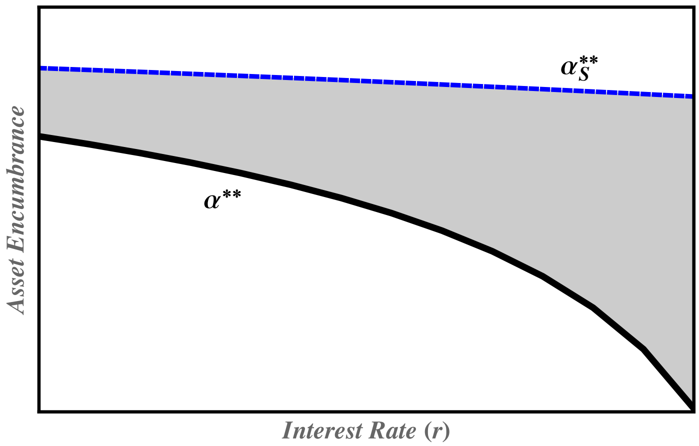


Figure 6: The effect of an interest rate cut on the privately and socially optimal levels of asset encumbrance. An example with $R = 1.5$, $E = 0.5$, $\psi = 0.6$, $\gamma = 0.8$, and the shock follows the Logistic distribution with mean -3 and scaling parameter 1.

consistent with the stylized evidence in the wake of the global financial crisis. Since 2007/8, central banks in advanced countries have run expansionary monetary policy and there has been an increased appetite for safe assets among investors (IMF, 2013; Caballero et al., 2016). Juks (2012) and Bank of England (2012) document a clear, increasing trend in the encumbrance ratios of Swedish and UK banks following the implementation of extraordinary monetary policy measures in response to the crisis.

Capital buffers. Proposition 4 states that the effect of an increase in bank capital on asset encumbrance is ambiguous. There are two opposing effects. On the one hand, more capital enables the bank to withstand larger balance sheet shocks and so lowers fragility. While this "loss absorption" effect induces greater encumbrance, the bank risks losing more of its own funds in bankruptcy. The result of such "greater skin in the game" is to lower encumbrance. Figure 7 illustrates how these two opposing effects induce a non-monotonic relationship between bank capital and asset encumbrance. For a uniformly distributed shock, the loss absorption effect dominates.

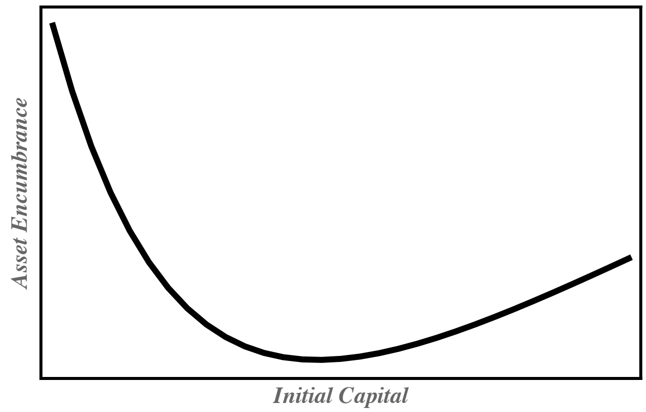


Figure 7: Non-monotonic relationship between bank capital and encumbrance: an example with $R = 1.5$, $r = 1.2$, $\psi = 0.6$, $\gamma = 0.8$, and the shock follows the Logistic distribution with mean -3 and scaling parameter 1.

Lemma 3. *If the balance sheet shock is uniformly distributed over $[-A_L, A_H]$, then the privately optimal level of asset encumbrance increases in bank capital.*

Proof. See Appendix E. ■

Suppose the planner injects Δ_E into the bank at $t = 0$, increasing its equity capital to $\tilde{E} \equiv E + \Delta_E$. There is a social cost to injecting such capital, reflecting resource costs (including distortionary taxation) as well as political costs. As a result, the planner's welfare is $W - \lambda\Delta_E$, where $\lambda \geq 1$ reflects the costs of the injection. Under the conditions described in Lemma 3, the improved capital buffer increases the optimal level of encumbrance chosen by the banker. But the effect on the socially optimal level is less clear-cut. In Figure 8, we consider an alternative case in which the shock follows a logistic distribution. In this instance, the privately and socially optimal levels of encumbrance converge as the capital injection increases. But once the costs of providing the injection become excessive, the socially optimal level falls below the privately optimal level of encumbrance and a gap re-emerges.

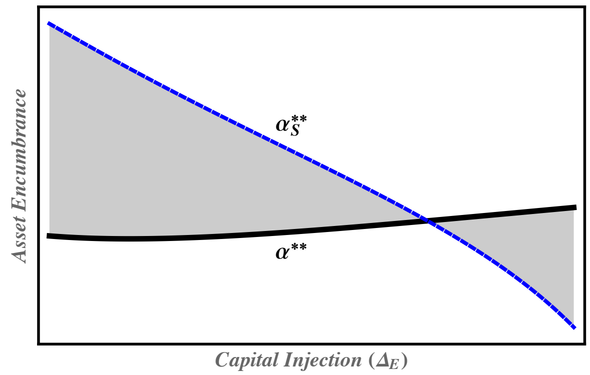


Figure 8: Capital injections alter the privately and socially optimal levels of encumbrance. An example with $R = 1.5$, $r = 1.2$, $E = 0.5$, $\lambda = 0.2$, $\psi = 0.6$, $\gamma = 0.8$, and the shock follows the Logistic distribution with mean -3 and scaling parameter 1.

Guarantees. A similar dynamic – an increase in the private encumbrance level and a decrease in the social encumbrance level – arises for guarantees of unsecured debt. In many jurisdictions, unsecured debt holders enjoy the benefits of explicit or implicit public guarantee schemes. Such schemes usually apply to retail deposits, but were also extended to unsecured wholesale debt during the global financial crisis.[7] If a proportion of unsecured debt is guaranteed, a banker faces lower rollover risk and cheaper unsecured debt, since the guarantor pays in bankruptcy. Both effects increase the privately optimal encumbrance level. In contrast, the welfare needs to reflect the cost of the guarantor, which tends to reduce the socially optimal encumbrance.

Lender of last resort (LOLR). Bagehot (1873) dictum holds that a central bank should lend freely to a solvent but illiquid bank at penalty rates. In line with Rochet and Vives (2004), the LOLR function in our model can be viewed as a commitment by the central bank to purchase assets from illiquid but solvent banks at $t = 1$ at a

[7] Between 2007 and 2011, many countries enacted special arrangements for banks to have new and existing wholesale bank funding guaranteed by the government until market conditions normalized. Recent analyses of the interplay between government guarantees and financial stability include König et al. (2014), Allen et al. (2015), and Leonello (2016).

value between ψ and 1. The possibility of central bank action affects both the fragility of the bank at $t = 1$ and its incentives to raise secured funding ex ante at $t = 0$.

A LOLR intervention can be viewed as equivalent to an increase in the liquidation value of bank assets at $t = 1$. As Proposition 4 shows, a higher liquidation value decreases the degree of strategic complementarity among fund managers for any given level of asset encumbrance. This reduces illiquidity at $t = 1$ and, hence, bank fragility. The banker, therefore, encumbers more assets and increases investment with positive NPV. Lower fragility, in turn, reduces the face value of unsecured debt required for investors to participate, increasing encumbrance still further. Accordingly, $\frac{d\alpha^{**}}{d\psi} > 0$.[8]

As Figure 9 shows, both the privately and socially optimal levels of encumbrance increase in the liquidation value of bank assets. Although gap between these encumbrance levels converges, it does not close. Even with a credible LOLR regime, fewer assets are encumbered in equilibrium than is socially desirable.

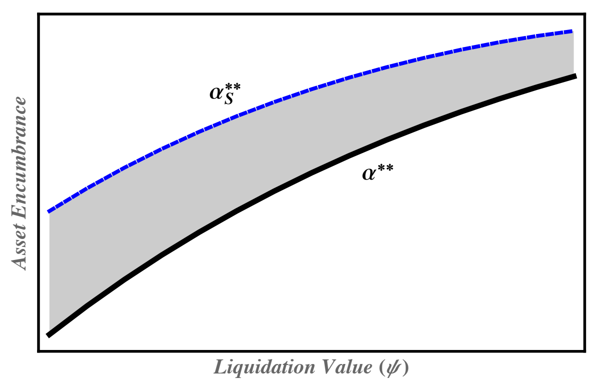


Figure 9: The effect of a lender of last resort policy on the privately and socially optimal levels of asset encumbrance. An example with $R = 1.5$, $r = 1.2$, $E = 0.5$, $\gamma = 0.8$, and the shock follows the Logistic distribution with mean -3 and scaling parameter 1.

[8]Chen et al. (2010) identify strategic complementarities in mutual fund investment and show that illiquid funds result in greater strategic complementarities than liquid funds because of the higher costs associated with redemptions.

Net Stable Funding Ratio (NSFR). Our model suggests that if the planner can directly influence asset encumbrance levels by imposing a floor at the socially optimal level, then constrained efficiency can be achieved. The banker's problem would include the additional constraint, $\alpha \geq \alpha_S^{**}$. Since the banker's expected equity value is concave, the smallest feasible level of asset encumbrance is optimal, $\alpha^{**} = \alpha_S^{**}$.

The Basel III regulations propose a NSFR that seeks to match the maturity of the assets and liabilities on a bank's balance sheet in order to avoid runs by short-term creditors (BIS, 2014). In our model, investment is long term, secured debt is a stable source of funding, and unsecured debt is fickle. The NSFR in our context is

$$NSFR(\alpha) \equiv \frac{S^*(\alpha) + E}{I^*(\alpha)}, \tag{10}$$

which increases in the level of encumbrance. Thus, a sufficiently high NSFR can implement the constrained efficient allocation. Imposing a minimum ratio of $NSFR(\alpha_S^{**})$ ensures that the banker encumbers assets at the socially desired level.

In addition to these normative implications, our comparative static results also provide testable implications about the positive relationship between bank funding and market stress and the link between asset returns and encumbrance.

Market stress. The conservatism parameter γ can be broadly interpreted as a measure of market stress. Krishnamurthy (2010) documents how, during the global financial crisis, fund managers turned conservative and became less inclined to roll over unsecured debt. Proposition 4 implies that, faced with a deterioration in investor sentiment, the bank is more fragile for any given level of encumbrance. The banker responds to the heightened fragility in a precautionary fashion, lowering the extent of encumbrance and forgoing profitable investment from the issuance of secured debt, in order to induce rollovers by fund managers. The combined effects of increased fragility and the greater face value of unsecured debt reduce encumbrance, so $\frac{d\alpha^{**}}{d\gamma} < 0$.

Increased market stress thus induces a reduction in the share of secured debt (as a proportion of total debt). A joint analysis of banks' borrowing and lending behavior during both normal and crisis episodes would shed light on this implication.

Profitability. Higher returns on investment or a more favorable distribution of the balance sheet shock, in the sense of a first-order stochastic dominance shift according to the reverse hazard rate, reduces fragility and induces the banker to encumber more. The model implies, therefore, that banks with less risky balance sheets or more profitable assets increase the share of secured debt on their balance sheet. Consistent with these implications, DiFilippo et al. (2016) document that higher risks reduce the share of secured debt held by banks.

The channel identified in our model is quite distinct from theories based on asymmetric information (e.g., Stiglitz and Weiss 1981; Freixas and Jorge 2008). An asymmetric information approach might suggest that banks with low risk have less incentive to borrow in unsecured markets, since lenders do not know the borrower's quality and therefore over-charge low-risk borrowers. Empirical work in the spirit of DiFilippo et al. (2016) may clarify the nature and relative importance of the mechanism proposed here.

5 Conclusion

This paper examines how bank funding structures affect financial fragility, funding costs, and welfare. We offer a model in which a banker issues unsecured demandable debt and secured long-term debt that is backed by collateral assets on its balance sheet. We use this model to explore how asset encumbrance affects bank fragility, given by the incidence of runs by unsecured debt holders. Critically, encumbered assets are bankruptcy-remote and wholesale investors in funding markets are segmented. Our model clarifies why greater encumbrance heightens financial instability and sheds light on the cost-benefit analysis facing the banker when choosing the optimal level of encumbrance. Greater encumbrance raises funding for investment with positive NPV, while increasing fragility.

A novel feature of our framework is the presence of a positive externality – ignored by the banker – stemming from the segmentation of the investor base. The existence of risk-averse investors, combined with the bankruptcy remoteness of encumbered assets, drives a wedge between the privately and socially optimal allocations. We explore a rich set of comparative static results that has both positive and normative implications and discuss the efficacy of several policy tools.

Several assumptions in the model merit comment. First, the source of the externality in our model derives from the risk aversion of some wholesale investors. While the assumption of infinitely risk-averse investors is made for analytical tractability, our results continue to hold for finite risk aversion. Clearly, assuming all investors in funding markets are risk-neutral would eliminate the externality created by the demand for safety and overturn the normative results. But the idea that wholesale investors differ according to their risk preferences seems plausible, particularly given the strict mandates imposed upon institutional investors including global reserve managers and pension, mutual, and sovereign debt funds (IMF, 2012).

Second, the balance sheet shock is scale-invariant. This simplifying assumption implies an upward bias to the level of asset encumbrance since larger investment funded by secured debt reduces the average shock to the bank's balance sheet. Despite this bias, and the assumption of a constant returns to scale technology, an interior solution still obtains – due to an increase in bank fragility as asset encumbrance rises.

Third, our model implicitly assumes that the banker has bargaining power in the secured funding market. Again, relaxing this assumption does not qualitatively alter the results. In other market structures, the amount of secured funding would be lower, reducing investment and equity value. And, as a result, the bank encumbers fewer assets in equilibrium.

Finally, our model shares with Rochet and Vives (2004) the assumption that the face value of unsecured debt is independent of the withdrawal date. The model could, in principle, be extended along the lines of Freixas and Ma (2015) to allow for a maturity premium as long as early withdrawals are still costly for a fund manager who opts to roll over.

Our analysis has highlighted some benefits of asset encumbrance and the quest for safety by investors. Many policymakers (e.g., CGFS (2013) and Haldane, 2012) have highlighted the systemic instability that can arise from an excessive desire for financial instruments secured on high-quality collateral. In our model, the social planner is utilitarian and maximizes the expected payoffs of bankers and investors. But the planner does not take any social costs of systemic bank failure into account when selecting the socially optimal level of encumbrance. Allowing for real costs of systemic crisis (fiscal, reputational etc.) in the social welfare function would temper the positive externality and reduce the gap between the privately and socially desired levels of encumbrance. We plan to address this extension of the model to nest both the positive and negative social aspects of asset encumbrance in future work.

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A Proof of Proposition 1

This proof is in three steps. First, we show that the dominance regions at the rollover stage based on the illiquidity threshold are well defined for any choice of bankers and face values of debt. If the balance sheet shock were common knowledge, the rollover behavior of fund managers would be characterized by multiple equilibria, as shown in Figure 10. If no unsecured debt is rolled over, $\ell = 1$, the bank is liquid at $t = 1$ whenever the shock below a lower dominance bound $\underline{A} \equiv R(1-\alpha)I - \frac{UD_U}{\psi}$. For $A < \underline{A}$, it is a dominant strategy for fund managers to roll over. If $\ell = 0$, the bank is illiquid whenever the shock is above an upper dominance bound $\overline{A} \equiv R(1-\alpha)I$. For $A > \overline{A}$, it is a dominant strategy for managers not to roll over.

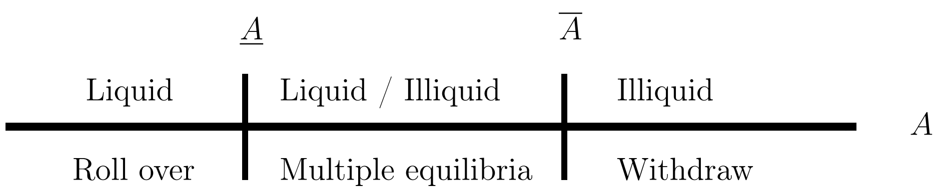

Figure 10: Tripartite classification of the balance sheet shock

Second, in any rollover stage, it suffices to establish the optimality of threshold strategies for sufficiently precise private information. Morris and Shin (2003) and Frankel et al. (2003) show that only threshold strategies survive the iterated deletion of strictly dominated strategies. Specifically, we consider the case of vanishing private noise, $\epsilon \to 0$. Therefore, each fund manager i uses a threshold strategy, whereby unsecured debt is rolled over if and only if the private signal about the balance sheet shock is below some signal threshold, $x_i < x^*$.

Third, we characterize this threshold equilibrium. For a given realization $A \in [\underline{A}, \overline{A}]$, the proportion of fund managers who do not roll over unsecured debt is:

$$\ell\Big(A, x^*\Big) = \text{Prob}\left(x_i > x^* \middle| A\right) = \text{Prob}\left(\epsilon_i > x^* - A\right) = 1 - G\Big(x^* - A\Big). \tag{11}$$

A **critical mass condition** states that illiquidity occurs when the shock equals A^*, where the proportion of managers not rolling over is evaluated at A^*:

$$R(1-\alpha)I - A^* = \ell\big(A^*, x^*\big)\frac{UD_U}{\psi}. \tag{12}$$

The posterior distribution of the shock conditional on the private signal is derived using Bayes' rule. An **indifference condition** states that the manager who receives the threshold signal $x_i = x^*$ is indifferent between rolling and not rolling over:

$$\gamma = \Pr\left(A < A^* | x_i = x^*\right). \tag{13}$$

Using the definition of the private signal $x_j = A + \epsilon_j$, the conditional probability is

$$\begin{aligned} 1-\gamma &= \Pr\left(A \geq A^* | x_i = x^*\right) = \Pr\left(A \geq A^* | x_i = x^* = A + \epsilon_j\right), && (14) \\ &= \Pr\left(x^* - \epsilon_j \geq A^*\right) = \Pr\left(\epsilon_j \leq x^* - A^*\right) = G\Big(x^* - A^*\Big). && (15) \end{aligned}$$

The indifference condition implies that $x^* - A^* = G^{-1}\big(1-\gamma\big)$. Inserting this expression into $\ell\big(A^*, x^*\big)$, the proportion of managers who do not roll over when the shock equals the threshold level A^* is perceived by the indifferent manager to be:

$$\ell\Big(A^*, x_i = x^*\Big) = 1 - G\Big(x^* - A^*\Big) = 1 - G\Big(G^{-1}\Big(1-\gamma\Big)\Big) = \gamma. \tag{16}$$

The run threshold A^* stated in Proposition 1 follows. For vanishing private noise, the signal threshold also converges to this value, $x^* \to A^*$. The run threshold varies with asset encumbrance and the face value of unsecured debt according to:

$$\frac{dA^*}{d\alpha} = -R(1-\psi z)\frac{I^*(\alpha)}{1-\alpha\psi z} < 0, \tag{17}$$

$$\frac{dA^*}{dD_U} = -\frac{\gamma U}{\psi} < 0. \tag{18}$$

B Proof of Propositions 2

The banker's problem is given in (5). Calculating the total derivative $\frac{d\pi}{d\alpha}$, which takes indirect effects via $A^*(\alpha)$ and $S^*(\alpha)$ into account, we obtain an implicit function $G(\alpha) \equiv \frac{1-\alpha\psi z}{RI^*(\alpha)} f(A^*)\frac{d\pi}{d\alpha}$:

$$G(\alpha) = \frac{F(A^*)}{f(A^*)}\psi(z-1) - (1-\psi z)\left[R(1-\psi)\alpha I^*(\alpha) + UD_U\left(\frac{\gamma}{\psi}-1\right)\right], \tag{19}$$

which is the difference between two terms where the second term is positive because of the bound $D_U \leq \hat{D}_U$. If an interior solution $0 < \alpha^* < 1$ exists, it is given by $G(\alpha^*) = 0$. This solution is a local maximum, since

$$\frac{dG}{d\alpha} = \underbrace{\frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}}_{+}\underbrace{\frac{dA^*}{d\alpha}}_{-}\psi(z-1) - (1-\psi z)\frac{R(1-\psi)I^*(\alpha)}{1-\alpha\psi z} < 0, \tag{20}$$

because the first term is positive because of decreasing reverse hazard rate of $f(A)$. Using the implicit function theorem, we obtain $\frac{d\alpha^*}{dD_U} < 0$ for the interior solution since

$$\frac{dG}{dD_U} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dD_U}\psi(z-1) - (1-\psi z)U\left(\frac{\gamma}{\psi}-1\right) < 0 \tag{21}$$

where we used $\gamma > \psi$. Hence, the weak inequality $\frac{d\alpha^*}{dD_U} \leq 0$ follows for the entire schedule of asset encumbrance.

We next study whether the solution to $G(\alpha^*) = 0$ is interior. An interior solution requires two conditions, $G(\alpha = 0) > 0$ and $G(\alpha = 1) < 0$, that we consider in turn. First, evaluating the implicit function at no encumbrance $\alpha = 0$, we obtain

$$\frac{F(A^*(0))}{f(A^*(0))}\psi(z-1) - (1-\psi z)\left(\frac{\gamma}{\psi} - 1\right) U D_U \,. \tag{22}$$

Since this expression decreases in D_U, $\alpha^* > 0$ requires $D_U < \bar{D}_U$, where this bound $\bar{D}_U$ is implicitly defined by

$$\frac{F(R(1+E) - \frac{\gamma}{\psi} U \bar{D}_U)}{f(R(1+E) - \frac{\gamma}{\psi} U \bar{D}_U)}\psi(z-1) - (1-\psi z)\left(\frac{\gamma}{\psi} - 1\right) U \bar{D}_U = 0. \tag{23}$$

Strong monotonicity of the left-hand side of equation (23) in D_U ensures the uniqueness of $\bar{D}_U$. Second, evaluating the implicit function at $\alpha = 1$, we obtain

$$\frac{F(A^*(1))}{f(A^*(1))}\psi(z-1) - (1-\psi z)\left[\frac{R(1-\psi)(1+E)}{1-\psi z} + \left(\frac{\gamma}{\psi} - 1\right) U D_U\right]. \tag{24}$$

Since this expression again decreases in D_U, $\alpha^* < 1$ requires $D_U > \underline{D}_U$, where this bound $\underline{D}_U$ is implicitly defined by

$$\frac{F(-U\frac{\gamma}{\psi}\underline{D}_U)}{f(-U\frac{\gamma}{\psi}\underline{D}_U)}\psi(z-1) - (1-\psi z)\left[\frac{R(1-\psi)(1+E)}{1-\psi z} + \left(\frac{\gamma}{\psi} - 1\right) U \underline{D}_U\right] = 0\,. \tag{25}$$

Strong monotonicity of the left-hand side of equation (25) in D_U again ensures the uniqueness of $\underline{D}_U$. It readily follows that $\underline{D}_U < \bar{D}_U$.

C Proof of Proposition 3

The proof is in five steps. First, we insure that the equilibrium level of asset encumbrance is interior, $\alpha^{**} \in (0,1)$. Since $\alpha = 0$ implies $\hat{D}_U = 0$, there is no equilibrium consistent with our supposition $D_U \leq \hat{D}_U$, so $\alpha^{**} > 0$. (We verify the supposition below.) Moreover, if $\alpha = 1$, then $A^*(1) = -\frac{\gamma D_U}{\psi}$ and $V(1, D_U) = D_U F(A^*(1))$, which reaches its maximum value at D_{max} that is uniquely and implicitly defined by $\frac{\gamma D_{max}}{\psi} \equiv H\left(\frac{\gamma D_{max}}{\psi}\right)^{-1}$. Thus, the maximum value of the unsecured debt claim is $V(1, D_{max})$. For $r > \underset{\sim}{r} \equiv V(1, D_{max})$, any solution is interior, $0 < \alpha^{**} < 1$, if it exists.

Second, we consider changes in the proportion of asset encumbrance and the face value of unsecured debt on the value of the unsecured debt claim. Greater asset encumbrance reduces the value of unencumbered assets and therefore reduces the range of balance sheet shocks for which unsecured debt holders are repaid:

$$\frac{\partial V}{\partial \alpha} = D_U f(A^*) \frac{dA^*}{d\alpha} < 0. \tag{26}$$

In general, the effect of a higher face value of unsecured debt is ambiguous:

$$\begin{aligned} \frac{\partial V}{\partial D_U} &= F(A^*) - f(A^*) D_U \frac{\gamma}{\psi} && (27) \\ \left.\frac{\partial V}{\partial D_U}\right|_{\alpha^*(D_U)} &= \frac{1-\psi z}{\psi(z-1)} R(1-\psi)\alpha I + \beta_0 U D_U, && (28) \end{aligned}$$

which is non-negative whenever $\beta_0 \equiv \frac{1-\psi z}{\psi(z-1)}\left(\frac{\gamma}{\psi} - 1\right) - \frac{\gamma}{\psi} \geq 0$ that is equivalent to $\gamma \geq \underset{\sim}{\gamma} \equiv \frac{1-\psi z}{1-\psi z - \psi(z-1)}\psi$. An unsecured debt claim with face value $D_U = r$ always violates the participation constraint, $V(D_U = r) = rF(A^*(D_U = r)) < r$, so $D_U^* > r$.

Third, uniqueness of equilibrium follows from the fact that $\alpha^*(D_U)$ is strictly downward-sloping in (α, D_U) - space for any interior solution (see Proposition 2), while the schedule $r = V(\alpha, D_U^*)$ is non-decreasing whenever it hits the schedule $\alpha^*(D_U)$ (see step 2). Hence, there can be at most one intersection.

Fourth, we need to demonstrate that the equilibrium specified above exists. To this end, define $T(D_U) = r/F(A^*(\alpha^*(D_U), D_U))$ as a mapping from the set $\mathcal{U}$ of face values of unsecured debt into itself. If $\mathcal{U}$ is a closed an compact set, then by Brouwer's fixed-point theorem there exists at least one fixed-point for the mapping. The lower bound on D_U is, by definition, r. For the upper bound, note that if the banker could pledge all assets to unsecured creditors, then $D_U \leq RI^*(\alpha) - A$. If we truncate the shock distribution at some $-A_L$, then we have a well defined upper bound on D_U.

Fifth, we verify the supposition $D_U \leq \hat{D}_U = R(1-\psi)\alpha I^*(\alpha)$. Evaluating the run threshold and the asset encumbrance schedule at $\hat{D}_U$ yields

$$\hat{A}^* \equiv A^*(\hat{D}_U) = RI^*(\alpha)\left[1 - \alpha\left(1 + \frac{\gamma(1-\psi)}{\psi}\right)\right] \tag{29}$$

$$\hat{\alpha}^* \quad : \quad H(\hat{A}^*)^{-1} = \frac{1-\psi z}{\psi(z-1)}\frac{\gamma(1-\psi)}{\psi}R\hat{\alpha}^* I^*(\hat{\alpha}^*), \tag{30}$$

where the right-hand side increases in α and the left-hand side decreases in α since

$$\frac{d\hat{A}^*}{d\alpha} = -\frac{RI^*(\alpha)}{1-\alpha\psi z}\left[1 - \psi z + \frac{\gamma(1-\psi)}{\psi}\right] < 0. \tag{31}$$

As a result, $\hat{\alpha}^*$ is unique. Since $\frac{\partial V}{\partial D_U}$ at α^* increases in D_U, it suffices to show that

$$r \leq V(\hat{\alpha}^*, \hat{D}_U(\hat{\alpha}^*)). \tag{32}$$

A sufficient condition is $r \leq V(1, \hat{D}_U(1))$, which yields an upper bound on bank capital, $E < \bar{E}$:

$$\frac{\psi^2(z-1)}{\gamma(1-\psi z)} \frac{F\left(-R\frac{1+\bar{E}}{1-\psi z}\frac{\gamma(1-\psi)}{\psi}\right)^2}{f\left(-R\frac{1+\bar{E}}{1-\psi z}\frac{\gamma(1-\psi)}{\psi}\right)} - r = 0. \tag{33}$$

D Proof of Proposition 4

This proofs is in two steps. First, we show the effect of a parameter on the schedule of asset encumbrance $\alpha^*(D_U)$. Second, we show that this direct effect is reinforced by the indirect effect via the equilibrium cost of unsecured debt D_U^*.

Direct effects Regarding the direct effect via $\alpha^*(D_U)$, we take D_U as given and use the implicit function theorem, whereby $\frac{d\alpha^*}{dy} = -\frac{\frac{dG}{dy}}{\frac{dG}{d\alpha}}$ for $y \in \{\gamma, r, R, \psi, E, F(\cdot)\}$:

$$\begin{aligned}
\frac{dG}{d\gamma} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\gamma}\psi(z-1) - (1-\psi z)U\frac{D_U}{\psi} < 0, && (34)\\
\frac{dG}{dr} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dr}\psi(z-1) - \frac{F(A^*)}{f(A^*)}\psi\frac{z}{r} && (35)\\
&- \psi\frac{z}{r}\left[\frac{R(1-\psi)\alpha I^*(\alpha)}{1-\alpha\psi z} + D_U\left(\frac{\gamma}{\psi}-1\right)\right] < 0 \\
\frac{dG}{dR} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dR}\psi(z-1) + \frac{F(A^*)}{f(A^*)}\frac{\psi}{r} && (36)\\
&+ \frac{\psi}{r}D_U\left(\frac{\gamma}{\psi}-1\right) + (1-\psi)\alpha I^*(\alpha)\left\{\psi z - \frac{(1-\psi z)}{1-\alpha\psi z}\right\}.
\end{aligned}$$

The expression in Equation (36) has an ambiguous sign in general. Therefore, we evaluate this derivative at α^* by substituting $F(A^*)/f(A^*)$ from the first-order condition in Equation (6). Grouping terms, we obtain that $dG/dR > 0$ as long as

$$(1-\psi z)\left[\frac{z}{1-z} - \frac{1}{1-\alpha^*\psi z}\right] + \psi z > 0\,. \tag{37}$$

The condition in Equation (37) is toughest to satisfy for $\alpha^* = 1$. Evaluating it at this bound, we get the sufficient condition $(1-\psi z)\left[\frac{z}{z-1} - 1\right] > 0$, which requires $z > z-1$, which is always true.

For the derivative with respect to ψ, we obtain

$$\begin{aligned} \frac{dG}{d\psi} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\psi}\psi(z-1) + \frac{F(A^*)}{f(A^*)}(z-1) \\ &+ z\left[R(1-\psi)\alpha I^*(\alpha) + UD_U\left(\frac{\gamma}{\psi} - 1\right)\right] \\ &- (1-\psi z)\left[R\alpha I^*(\alpha)\frac{\alpha z - 1}{1-\alpha\psi z} - UD_U\frac{\gamma}{\psi^2}\right]. \end{aligned} \tag{38}$$

Grouping the terms involving $R\alpha I^*(\alpha)$, a sufficient condition for $dG/d\psi > 0$ is $z(1-\psi) - (1-\psi z)(\alpha z - 1)/(1-\alpha\psi z) > 0$. This condition is most binding for $\alpha = 1$, for which we require that $1 - \psi z > 0$, which is always true.

For the derivative with respect to E, we obtain

$$\frac{dG}{dE} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dE}\psi(z-1) - \frac{1-\psi z}{1-\alpha\psi z}R(1-\psi)\alpha \lesseqgtr 0. \tag{39}$$

Finally, suppose that the balance sheet shock distribution $\widetilde{F}$ stochastically dominates the distribution F according to the reverse hazard rate. This implies that

$$\frac{\widetilde{f}}{\widetilde{F}} \geq \frac{f}{F}, \tag{40}$$

or, equivalently, $F/f \geq \widetilde{F}/\widetilde{f}$. Let $\widetilde{G}(\widetilde{\alpha}^*) = 0$ denote the implicit function defining the privately optimal level of asset encumbrance, $\widetilde{\alpha}^*$, under the balance sheet shock distribution $\widetilde{F}$. Therefore, we have that $\widetilde{G}(\alpha) \leq G(\alpha)$ for all levels of encumbrance. Since $d\widetilde{G}/d\alpha < 0$ and $dG/d\alpha < 0$, it follows that the privately optimal levels of encumbrance satisfy $\widetilde{\alpha}^* \geq \alpha^*$.

Indirect effects The indirect effects arise from the equilibrium face value of unsecured debt. For any given level of asset encumbrance, it is given by the implicit function $J(\alpha, D_U^*) = 0$ where:

$$J \equiv -r + D_U F(A^*(\alpha, D_U)) \tag{41}$$

Using the implicit function again, and noting that $\frac{\partial J}{\partial D_U}\Big|_{\alpha^*} > 0$, we obtain the desired reinforcing effects since:

$$\frac{\partial J}{\partial R} = D_U f(A^*) \frac{dA^*}{dR} > 0 \tag{42}$$

$$\frac{\partial J}{\partial \psi} = D_U f(A^*) \frac{dA^*}{d\psi} > 0 \tag{43}$$

$$\frac{\partial J}{\partial \gamma} = D_U f(A^*) \frac{dA^*}{d\gamma} < 0 \tag{44}$$

$$\frac{\partial J}{\partial E} = D_U f(A^*) \frac{dA^*}{dE} > 0 \tag{45}$$

$$\frac{\partial J}{\partial r} = -1 + D_U f(A^*) \frac{dA^*}{dr} < 0. \tag{46}$$

Finally, note that an improvement in the distribution of the balance sheet shock also increases J. Taking the direct and indirect effects together, we arrive at the total effects reported in Proposition 4.

E Proof of Lemma 3

If the balance sheet shock is uniformly distributed in the interval $[-A_L, A_H]$, then the equilibrium level of asset encumbrance, for a given D_U is

$$RI^*(\alpha^*)\left[1-\alpha^*\left\{1+\frac{(1-\psi)(1-\psi z)}{\psi(z-1)}\right\}\right]=UD_U\left\{\frac{\gamma}{\psi}+\left(\frac{\gamma}{\psi}-1\right)\frac{(1-\psi z)}{\psi\,(z-1)}\right\}-A_L\,, \tag{47}$$

which uniquely defines α^*.

It readily follows that $d\alpha^*/dE > 0$ by the implicit function theorem, since the left-hand side of Equation (47) decreases in the level of asset encumbrance and increases in bank capital.

F Proof of Proposition 5

The planner's welfare function implies that $W_\alpha = \pi_\alpha + rS^*_\alpha > \pi_\alpha$, where the derivative of the banker's expected equity value with respect to the level of encumbrance is

$$\pi_\alpha = F(A^*)\psi(z-1)I^*(\alpha)1-\alpha\psi z+\pi_{A^*}\frac{dA^*}{d\alpha}\,, \tag{48}$$

where A^* is implicitly defined by Equation (9). Furthermore, one can show that $dA^*/d\alpha < 0$ whenever $\frac{F(A^*)}{f(A^*)} > \frac{\gamma r U}{\psi F(A^*)}$ holds in any interior equilibrium. The banker's private choice of asset encumbrance is given by $\pi_\alpha(\alpha^{**}_P) = 0$. Insofar $\alpha^{**}_P \in (0,1)$, we have that $W_\alpha(\alpha^{**}_P) = rS^*_\alpha(\alpha^{**}_P) > 0$. The planner's choice of asset encumbrance is given by $W_\alpha(\alpha^{**}_S) = \pi_\alpha(\alpha^{**}_S) + rS^*_\alpha(\alpha^{**}_S) = 0$. If we plug the planner's choice into the banker's first-order condition, we have $\pi_\alpha(\alpha^{**}_S) = -rS^*_\alpha(\alpha^{**}_S) < 0$. Since the banker's equity value is (locally) concave, it follows that $\alpha^{**}_P < \alpha^{**}_S$.