Global Imbalances and Currency Wars at the ZLB

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Abstract

This paper explores the consequences of extremely low equilibrium real interest rates in a world with integrated but heterogenous capital markets, and nominal rigidities. In this context, we establish five main results: (i) Economies experiencing liquidity traps pull others into a similar situation by running current account surpluses; (ii) Reserve currencies have a tendency to bear a disproportionate share of the global liquidity trap—a phenomenon we dub the "reserve currency paradox;" (iii) Beggar-thy-neighbor exchange rate devaluations provide powerful stimulus to the domestic economy at the expense of other economies; (iv) While more price and wage flexibility exacerbates the risk of a deflationary global liquidity trap, it is the more rigid economies that bear the brunt of the recession; (v) (Safe) Public debt issuances and increases in government spending anywhere are expansionary everywhere. We use these results to shed light on the evolution of global imbalances, interest rates, and exchange rates since the beginning of the global financial crisis.

JEL Codes: E0, F3, F4, G1,

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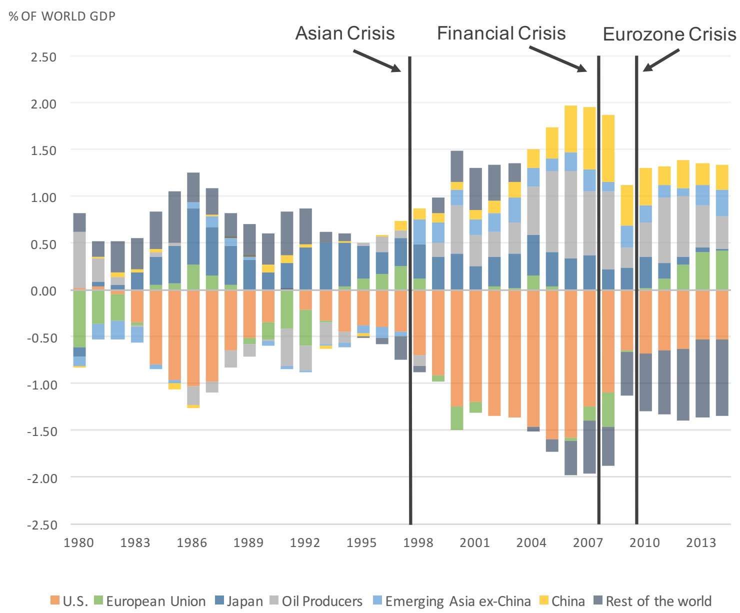
1 Introduction

In Caballero, Farhi and Gourinchas (2008a), (2008b), we argued that the (so called) "global imbalances" of the late 1990's and early 2000's (cf. Figure 1) and the low and declining world real interest rates (cf. Figure 2) were primarily the result of the great diversity in the ability to produce (safe) stores of value around the world, and of the mismatch between this ability and the local demands for these assets—countries with a low ability to produce safe assets and a high demand for safe assets run current account surpluses and put downward pressure on world real interest rates.

Much has happened since then. Following the Subprime and European Sovereign Debt crises, we entered a world of unprecedented low natural interest rates across the developed world and in many emerging market economies. Figure 2 shows that global nominal interest rates have remained at or close to the Zero Lower Bound (ZLB) since 2009. With nominal rates so low, the equilibrating mechanism we highlighted in our previous work has little space to operate. Yet the global mismatch between local demand and local supply of stores of value remains. The goal of this paper is to understand how this global mismatch plays out and shapes global economic outcomes, in an environment of extremely low global equilibrium real interest rates. We address questions such as: How do liquidity traps spread across the world? What is the role played by capital flows and exchange rates in this process? What are the costs of being a reserve currency in a global liquidity trap? How do differential inflation targets and degree of price rigidity influence the distribution of the impact of a global liquidity trap? What is the role of (safe) public debt and government spending?

Building on our previous work, we provide a stylized model to answer these questions. The main mechanism in this model is that once real interest rates cannot play their equilibrium role any longer, global output becomes the active margin: lower global output, by reducing income and therefore asset demand, rebalances global asset markets. In this world, liquidity traps emerge naturally and countries drag each other into them by running current account surpluses. Indeed, Figure 3 shows that, following the financial crisis, unemployment rates have increased persistently across most regions.

Our basic framework is a perpetual-youth overlapping generations model with nominal

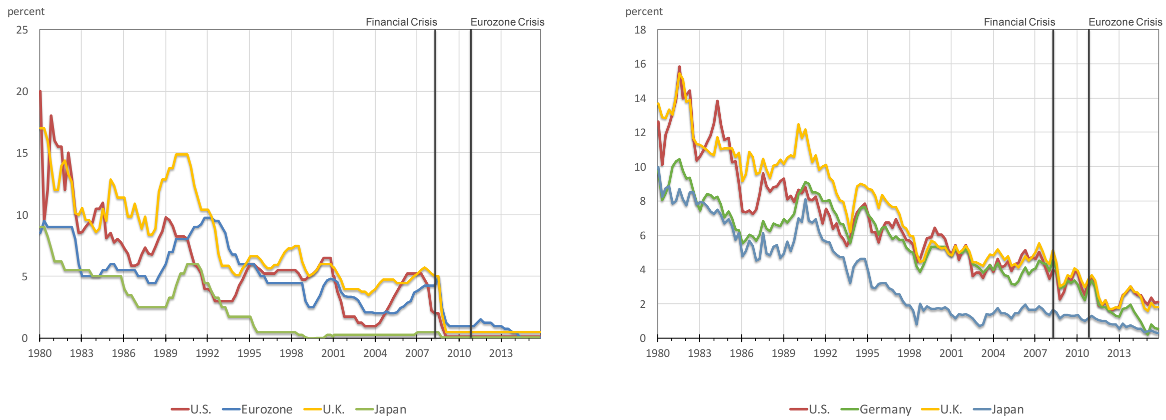


Note: The graph shows Current Account balances as a fraction of world GDP. We observe the build-up of global imbalances in the early 2000s, until the financial crisis of 2008. Since then, global imbalances have receded but not disappeared. Notably, deficits subsided in the U.S., and surpluses emerged in Europe. Source: World Economic Outlook Database, April 2015 and Authors' calculations. Oil Producers: Bahrain, Canada, Iran, Iraq, Kuwait, Lybia, Mexico, Nigeria, Norway, Oman, Russia, Saudi Arabia, United Arab Emirates, Venezuela; Emerging Asia ex-China: India, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, Vietnam.

Figure 1: Global Imbalances

rigidities designed to highlight the heterogeneous relative demand for and supply of financial assets across different regions of the world. Given the nominal rigidities, output is aggregate-demand determined as soon as the global demand for financial assets exceeds their supply at the ZLB. We study a stationary world in which all regions of the world share the same preferences for home and foreign goods (i.e. there is no home bias) and financial markets are fully integrated. This is an *all-or-none world*: Either all regions experience a permanent liquidity trap, or none. However the relative severity of these traps varies depending on a region's capacity to produce financial assets and on the level of the exchange rate.

We characterize global imbalances in terms of a *Metzler diagram in quantities,* that connects the size of the global liquidity trap and net foreign assets (and current accounts) positions to the size of the liquidity traps that would prevail in each region under financial autarky. This is analogous to the analysis outside of a liquidity trap, where the world



(a) Policy Rates

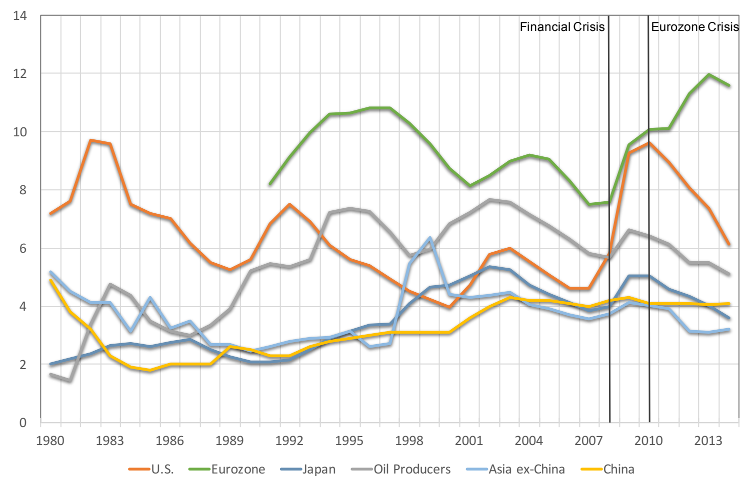
(b) Long Rates

Note: Panel (a) reports policy rates while panel (b) reports nominal yields on 10-years government securities. Both panels show the large decline in global interest rates. Following the financial crisis, the developed world remained at the Zero Lower Bound. Source: Global Financial Database.

Figure 2: Short and Long Nominal Interest Rates

equilibrium real interest rates and net foreign assets (and current accounts) positions are connected to the equilibrium real interest rate that would prevail in each region under autarky. This analysis shows that when a region's autarky liquidity trap is more (less) severe than the global liquidity trap, that country is also a net creditor (debtor) and runs current account surpluses (deficits) in the financially integrated environment, effectively exporting its liquidity trap abroad. Other things equal, a country experiences a more severe liquidity trap than average when its ability to produce financial assets is low relative to its own demand for these assets. For the same reason, in this environment, a large country with a severe asset shortage can pull the world economy into a global liquidity trap through its downward pressure on world equilibrium interest rates.

But other things need not be equal. In particular, the benchmark model has a critical degree of indeterminacy when at the ZLB (and not when away from it). This indeterminacy is related to the seminal result by Kareken and Wallace (1981) that the nominal exchange rate is indeterminate in a world with pure interest rate targets. This is *de facto* the case when the global economy is in a liquidity trap, since both countries are at the ZLB. However, in our framework this indeterminacy has substantive implications since money is not neutral. Different values of the nominal exchange rate correspond to different values of the



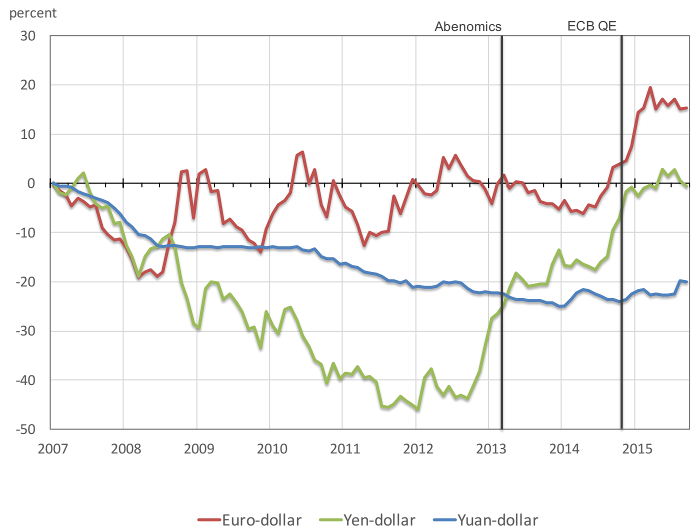
Note: The graph shows the persistent increase in unemployment rates following the global financial crisis and European sovereign debt crises. Source: World Economic Outlook, April 2015. The regional unemployment rate is defined as the median unemployment rate across the countries of the region, defined in Figure 1.

Figure 3: Global Unemployment Rates (percent).

real exchange rate and therefore different levels of output and the current account across countries. This means that, via expenditure switching effects, the exchange rate affects the *distribution* of a global liquidity trap across countries. This creates fertile grounds for "beggar-thy-neighbor"devaluations achieved by direct interventions in exchange rate, stimulating output and improving the current account in one country at the expense of the others. Thus, our model speaks to the debates surrounding "currency wars".

By the same token, the indeterminacy implies that if agents coordinate on an appreciated home exchange rate, as could be the case, for example, for a reserve currency, then this economy would experience a disproportionate share of the global liquidity trap. That is, while outside of a global liquidity trap a reserve currency status is mostly a blessing as it buys additional purchasing power, *in* a liquidity trap the reserve currency status exacerbates the domestic liquidity trap.

Section 2 contains our baseline model in which prices are fully rigid. In Section 3 we allow for milder forms of nominal rigidities by introducing Phillips curves, which can differ across countries. As usual, inflation is important because higher expected inflation reduces the impact of the (nominal) ZLB constraint. Our interest here is to study the interaction



Note: The graph shows the cumulated depreciation (+) or appreciation (-) of the euro, the yen, and the yuan against the dollar since January 2007. The 40% yen appreciation against the dollar between 2007 and 2012 was entirely reversed following the implementation of Abenomics (April 2013). The euro remained mostly stable against the dollar, until the second half of 2014, with increased expectations of Quantitative Easing by the European Central Bank (January 2015). Throughout the period the yuan appreciated agains the dollar, until the August 2015 yuan devaluation. Source: Global Financial Database. The figure reports $\ln(E/E_{2007m1})$ where E denotes foreign currency value of the dollar.

Figure 4: Global Exchanges Rates

between this mechanism and a *global* liquidity trap. In this setting, we show that if inflation targets in all countries are high enough, then there exists an equilibrium with no liquidity trap. But there is also an equilibrium with a global liquidity trap. In that equilibrium, wage and price flexibility plays out differently across countries and at the global level: countries with more price or wage flexibility bear a smaller share of the global recession than countries with less price or wage flexibility; but at the global level, more downward price or wage flexibility exacerbates the global recession. And finally, there is an asymmetric liquidity trap equilibrium where only one country experiences a liquidity trap and a larger recession than in the global liquidity trap equilibrium.

In Section 4, we consider the role of public debt and fiscal policy. Our model is non-Ricardian, which gives a role to debt policy. Additional debt issuance in one country can potentially address the net shortage of assets and stimulate the economy in *all* countries, alleviating a global liquidity trap. This also worsens the current account and the net foreign asset position of the country issuing additional debt.

The effect of a balanced-budget increase in domestic government spending in one country depends on the severity of nominal rigidities. When prices are perfectly rigid, it stimulates domestic output more than one-for-one and stimulates foreign output, albeit less, and it worsens the domestic current account. When some price adjustment is possible, the short-run increase in domestic and foreign output is even larger as increased government spending raises inflation and reduces real interest rates, further stimulating output. Over time, however, increased government spending at home appreciates the domestic terms of trade, which rebalances spending away from the domestic goods and toward foreign goods. The appreciation of the domestic terms of trade reduces the effect on domestic output and increases the effect on foreign output, but the overall effect on world output remains more than one-for-one and further worsens the domestic current account and its net foreign asset position.

We also present several important extensions in an Appendix, which we briefly summarize in Section 5. There we introduce home bias, relax some elasticity assumptions, and introduce risk-aversion and hence the concept of safe assets.

A brief model-based tour of the world. We wrap up this introduction by providing a brief narrative of the evolution of global imbalances and global interest rates since the early 1990s through the lens of our model (cf. Figures 1 and 2), and of the role played by exchange rates in these dynamics. We divide the period into two sub-periods, before and after the onset of the 2008 Subprime crisis, when the ZLB starts binding in the U.S.

The first sub-period, from 1990 to 2008, was the focus of our earlier papers (Caballero et al. (2008a), (2008b)). We refer the reader to these papers for a detailed account and only provide here a quick summary. This period saw the emergence of large current account deficits in the U.S., offset by current accounts surpluses in Japan throughout the period, and, starting at the end of the 1990s, by large surpluses in emerging Asia (in particular China) and commodity producers

The second sub-period, 2008-2015, is the focus of this paper. During that period, the U.S. current account deficit was halved, Japan's current account surplus disappeared, Europe's current account surplus increased substantially, and China's current account deficit was considerable reduced (see Figure 1). Global interest rates accelerated their decline and eventually hit the ZLB in the developed world. The U.S. and Europe experienced the largest recessions since the Great Depression (cf. Figure 3). In our framework, these phenomena can be understood as the consequence of a combination of severe shocks and large exchange rate swings.

The Subprime crisis and European Sovereign Debt crisis shocks triggered a sharp contraction in the supply of (safe) assets—U.S. “private label ”safe assets as well as European Sovereign assets from crisis countries. They also triggered a surge in demand for safe assets, as households and the financial sector in both regions attempted to de-leverage. Taken together, these shocks exacerbated the global shortage of safe assets, pushing interest rates to the ZLB throughout the developed world, where they have remained since (Figure 2). They also increased domestic net asset scarcity in the U.S. and Europe, resulting in the sharp reduction in the U.S. current account deficit and the increase in European current account surpluses in the wake of both crises.[1]

In this new environment, the ultra-accommodating monetary policy of the U.S. achieved initially a substantial depreciation of the dollar, especially against the yuan throughout the period and against the yen until 2014. This depreciation contributed further to the reduction of the current account surpluses of China and Japan. After this initial phase, the Bank of Japan in 2013 and the European Central Bank in 2014 started to implement aggressive expansionary monetary policies, leading to a sharp depreciation of the yen and the euro against the dollar. The depreciation of these two currencies offset and began to shift back onto the U.S. a significant share of the global adjustment burden, slowing down the prospects of a normalization of U.S. monetary policy. In turn, the appreciation of the dollar, combined with domestic developments, forced China in August 2015 to de-peg its currency in order to mitigate the additional slowdown due to the imported appreciation. See Figure 4 for a graphical illustration of these exchange rate swings . Although the expression “currency wars ”was originally coined by emerging market policymakers, we use it in this paper to capture the just described exchange rate trade-offs faced by economies like the U.S., Japan, or the Eurozone at the ZLB, as well those of countries, like China, who effectively peg their

[1]Some of the reduction in the U.S. current account deficits can also be attributed to the improvement in its petroleum trade balance caused by the expansion of U.S. shale oil production and lower oil prices.

currency to the dollar.

Finally, our framework helps us to understand the constraints faced by a regional reserve currency issuer such as Switzerland, illustrating our "paradox of the reserve currency". Confronted with a surge in the demand for its currency and deposits in the wake of the European Sovereign Debt crisis, Switzerland either had to allow its currency to appreciate at the risk of a recession, or to prevent its currency from appreciating. It chose the latter by imposing a floor in the value of the Euro in terms of Swiss francs, a highly contentious policy (inside and outside of Switzerland), until it was abandoned in early 2015.

Related literature. Our paper is related to several strands of literature. First and most closely related is the literature that identifies the shortage of assets, and especially of safe assets, as a key macroeconomic driver of global interest rates and capital flows (see e.g. Bernanke (2005), Caballero (2006), Caballero et al. (2008a) and (2008b), Caballero and Krishnamurthy (2009), Mendoza, Quadrini and Ríos-Rull (2009), Caballero (2010), Bernanke, Bertaut, DeMarco and Kamin (2011), and Barclays (2012)). In particular, Caballero et al. (2008a) developed the idea that global imbalances originated in the superior development of financial markets in developed economies, and in particular the U.S. This paper analyzes how the same forces play out at when the world economy experiences ultra-low natural real interest rates and is constrained by the Zero Lower Bound. In particular, it articulates how adjustment now occurs through quantities (output) rather than prices (interest rates) and the role of exchange rates in allocating a global slump across countries.

Second, there is by now an abundant literature on liquidity traps (see e.g. Keynes (1936), Krugman (1998), Eggertsson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2011), Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Werning (2012), and Correia, Farhi, Nicolini and Teles (2013)). This literature emphasizes that the binding Zero Lower Bound on nominal interest rates presents an important challenge for macroeconomic stabilization. A subset of that literature considers the implications of a liquidity trap in the open economy (see e.g. Svensson (2003), Jeanne (2009), Farhi and Werning (2012), Cook and Devereux (2013a), (2013b) and (2014), Devereux and Yetman (2014), Benigno and Romei (2014) and Erceg and Lindé (2014)). Our paper shares with that literature the result that global liquidity traps can propagate across countries and have significant international side effects. Cook and Devereux (2014) argue that the exchange rate may exacerbate the impact of adverse shock when a country hits the liquidity trap, making a fixed exchange rate more desirable. In their paper, flexible exchange rates remain desirable if the monetary authorities can implement credible forward guidance, a result reminiscent of Svensson (2003) who argues that forward guidance allows for a faster exit from a liquidity trap. Similarly, Benigno and Romei (2014) argues that movements in exchange rates in a global liquidity trap may be inefficient from the perspective of the global planner. Jeanne (2009), like us, finds that a negative shock in one country may be sufficient to push the world economy into a global liquidity trap. In his model, fiscal policy and raising inflation targets can help restore full employment. While many of these papers share similar themes and mechanisms, our paper also elucidates the link between the size of the global liquidity trap and Net Foreign Assets positions, with our *Metzler diagram in quantities*.

Third, there is an emerging literature on secular stagnation: the possibility of a permanent zero lower bound situation (see e.g. Kocherlakota (2014), Eggertsson and Mehrotra (2014), Caballero and Farhi (2015)). Like us, these papers use an OLG structure with a zero lower bound and nominal rigidities, but in a closed economy. Our contribution is to explore the open economy dimension of the secular stagnation hypothesis and the important question of whether some but not all countries can be in a permanent liquidity equilibrium. In our base model all countries are either in or out of a permanent liquidity trap since they face the same real (and nominal) interest rate. By contrast, the model with inflation of Section 3 features asymmetric equilibria where some countries are in a permanent liquidity trap but not others. While real interest rates are equated, countries that avoid the liquidity trap manage to maintain a positive nominal interest rate.[2] Overall, our model elucidates the conditions under which it is possible for some but not all countries to experience a secular stagnation equilibrium. From this perspective, the paper closest to ours is Eggertsson, Mehrotra, Singh and Summers (2015) which finds that exchange rates have powerful effects when the economy is in a global liquidity trap. Complementary to ours, their paper explores

[2] Appendix A.5 shows how, in presence of home bias in preferences, real interest rates can differ permanently across countries, allowing for some, but not all countries to be in a permanent liquidity trap.

the role of varying degrees of capital market integration and capital controls. Our paper emphasizes other methodological and substantive dimensions, such as the Metzler diagram in quantities, the reserve currency paradox, the spillovers of safe public debt issuance, and the role of capital flows in spreading liquidity traps and macroeconomic policies.

2 A Model of the Diffusion of Liquidity Traps

In this section we introduce our baseline model and main analytical tool (which we label the *Metzler Diagram in Quantities*). We use these to illustrate how countries are pulled into and out of liquidity traps by capital flows, and to show how a depreciation shifts the burden of absorbing a global liquidity trap onto others.

2.1 Model

Time is continuous. There are two countries, Home and Foreign. Foreign variables are denoted with stars. We first describe Home, and then move on to Foreign.

Demographics. Population is constant and normalized to one. Agents are born and die at hazard rate θ, independent across agents. Each dying agent is instantaneously replaced by a newborn. Therefore, in an interval dt, θdt agents die and θdt agents are born, leaving total population unchanged.

Preferences. We assume that agents only have an opportunity to consume when they die and we denote their consumption by c_t. We denote by τ_θ the stopping time for the idiosyncratic Poisson process controlling death for the agent under consideration.

Agents value home and foreign goods according to a Cobb Douglas aggregate, are risk neutral over short time intervals, and do not discount the future. More precisely, for a given stochastic consumption process of home and foreign goods $\{c_{H,t}, c_{F,t}\}$ which is measurable with respect to the information available at date t, we define the utility U_t of a an agent alive at date with the following stochastic differential equation:

$$U_t = 1_{\{t-dt \leq \tau_\theta < t\}} c_{H,t}^{\gamma} c_{F,t}^{1-\gamma} + 1_{\{t \leq \tau_\theta\}} \mathbb{E}_t[U_{t+dt}],$$

where we use the notation $\mathbb{E}_t[U_{t+dt}]$ to denote the expectation of U_{t+dt} conditional on the information available at date t.

Note that the information at date t contains the information about the realization of the idiosyncratic Poisson shocks up to t, implying that $1_{\{t-dt \leq \tau_\theta < t\}}$ and c_{Ht} and c_{Ft} are known at date t. Similarly, the conditional expectation $\mathbb{E}_t$ is an expectation over idiosyncratic Poisson death shocks.

Nominal rigidities, potential output and actual output. In an interval dt, potential output of the home good is given by $X_t dt$, where X_t grows at rate g.

The prices of home goods are fully rigid in the home currency and we normalize them to one: $P_{H,t} = 1$. For this reason, actual output $\xi_t X_t dt$ is demand determined and might be lower than potential output. $0 \leq \xi_t \leq 1$ denotes the output gap, i.e. the ratio of output to potential output, so that a lower value of ξ_t indicates more slack in the economy. For short, and with some abuse of notation, we refer to ξ_t as output from now on.

Private incomes, assets, and financial development. Output has two components: income of newborns and a financial income. In the interval dt, newly born agents receive income $(1-\delta)\xi_t X_t dt$. The remainder, $\delta \xi_t X_t dt$, is distributed as financial income. This financial income can be capitalized into home Lucas trees that can be traded between agents. Thus, $\delta \xi_t X_t dt$ represents the dividends on the stock of these home Lucas trees.

Each tree capitalizes a constant stream of dividends. With independent and instantaneous probability ρ each tree dies and the corresponding stream of dividends is transferred to a new tree, endowed to newborns. We assume that the stock of trees grows at rate g to accommodate growth in potential output. All new trees are also endowed to newborns.

The assumption that trees die, and that they only capitalize a constant (and not growing) dividend before they die, can be interpreted as a consequence of a process of creative-destruction, whereby technological innovations render older technologies and the associated dividend claims obsolete, or as a form of weak property rights that transfers without compensation claims on future output from old generations to new generations. While this creative destruction induces a risk premium on the return on each tree, in equilibrium we shall see that it depresses the risk-free rate.

In practice, δ captures many factors behind the pledgeability of income into tradable financial assets. At the most basic level, one can think of δ as the share of capital in production. But in reality only a fraction of this share can be committed to asset holders, as the government, managers, and other insiders can dilute and divert much of profits. For this reason, we refer to δ as an index of financial capacity of the home country, by which we mean an index of the extent to which property rights over earning are well defined and tradable in financial markets.[3]

Public debt. There is a home government that issues short-term public debt D_t, which it services by levying taxes τ_t on the income $(1-\delta)\xi_t X_t$ of newborns. We denote by $d_t = D_t/X_t$ Home's debt to output ratio.

The assumption that taxes are levied on the income of newborns is important. It ensures that public debt reduces the excess demand for assets. This occurs both by increasing the net supply of assets since public debt does not crowd out private assets, and by decreasing the demand for assets, since taxes reduce the income of newborns.[4]

Monetary policy. We assume that home monetary policy follows a truncated Taylor rule

$$i_t = \max\{r_t^n - \psi(1-\xi_t), 0\}.$$

In this equation, i_t is the home nominal interest rates which, with rigid prices, is equal to the real interest rate r_t. r_t^n is the relevant natural interest rate at Home, which depends on

[3]Overall, denoting by r_t the real interest rate, the present value of the economy's future output is $PV_t = \int_t^\infty X_s e^{-\int_t^s r_u du} ds$ while the current value of tradable private financial assets is $V_t = \delta \int_t^\infty X_t e^{-\int_t^s (r_u+\rho) du} ds$. Two factors lower the ratio V_t/PV_t: financial development (captured by the pledgeability parameter δ), and the process of creative destruction (captured by the death rate of trees ρ and the fact that a given tree capitalizes a constant dividend before its death while global output increases over time with new trees).

[4]These effects remain as long as taxes do not fall entirely on financial income. If taxes were levied entirely on financial income, then the environment would become Ricardian with respect to debt policy, despite the fact that there are overlapping generations: public debt would crowd out private assets one-for-one.

whether we are analyzing the equilibrium with financial integration or with financial autarky. For simplicity, we place ourselves in the limit of very reactive Taylor rules $\psi \to \infty$. This specification of monetary policy guarantees that we either have $\xi_t = 1$ and $i_t > 0$, or $\xi_t \leq 1$ if $i_t = 0$.

Foreign. Foreign is identical to Home except in three aspects. First, potential output of the foreign good is given by X_t^*, which also grows at rate g. Second, the financial capacity of the foreign country is given by δ^*.[5] Third, public debt in the foreign country is given by D_t^*, the debt to output ratio by d_t^*, and taxes by τ_t^*. Fourth, Foreign has its own currency and the prices of foreign goods are sticky in this currency. We normalize the price of the Foreign good to one in the foreign currency: $P_{F,t}^* = 1$.

We assume that there is no home bias and that the share γ of home consumption expenditure in total expenditures in both countries is equal to the share of potential output of home goods in total output: $\gamma = x$, where $x \equiv X_t/(X_t + X_t^*)$.

Wealth, asset values, interest rates, exchange rates, and output gaps. We denote by E_t the exchange rate between the home and the foreign currency, defined as the home price of the foreign currency, so that an increase in E represents a depreciation of the home currency; W_t and W_t^* are the total wealth of home and foreign households in their respective currencies; and V_t and V_t^* are the total value of home and foreign private assets (Lucas trees) in their respective currencies, so that the total value of home and foreign private and public assets in their respective currencies are $V_t + D_t$ and $V_t^* + D_t^*$.

[5] To a large extent, differences in propensity to consume θ play a similar role as differences in financial development δ in determining capital flows and interest rates, but the expressions of the model become more cumbersome when we introduce heterogeneity in θ. For this reason, we capture differences in $\theta\delta$ (an inverse index of country-specific asset shortage) only through differences in δ in the benchmark model. We refer the reader to Appendix A.3, where we introduce an alternative model featuring within country heterogeneity between borrowers and savers, which generates differences in propensity to save and consume across countries, driven by demographics (identifying borrowers with the young and savers with the middle-aged and the old) or credit markets (captured by the tightness of borrowing constraints). This model yields similar qualitative insights to the simpler baseline model.

Roadmap. We start with the simple observation that under financial integration, Uncovered Interest Parity (UIP) holds between Home and Foreign:

$$i_t = i_t^* + \frac{\dot{E}_t}{E_t}.$$

We focus on steady state balanced growth paths in the benchmark model and with some abuse of notation, we drop the time subscripts. In a steady state, the exchange rate is constant at E, and the home and foreign interest rates are necessarily equal to each other: $i = i^* = i^w$ and $r = r^* = r^w$ with $i^w = r^w$ since prices (and wages) are constant. This implies that under the maintained assumption of financial integration, either no country is in a liquidity trap $i^w = r^w > 0$, or all countries are in a liquidity trap $i^w = r^w = 0$, although, as we shall see, the severity of each country's liquidity trap depends on the exchange rate E.

2.2 No Liquidity Trap

Outside of a liquidity trap, we have $r = r^* = r^w > 0$ and $\xi = \xi^* = 1$. We take the latter as given and solve for the equilibrium r^w and E. This section illustrates in detail the steps we follow in finding equilibrium in this class of models, which are then repeated more succinctly in the more complex extensions found later in the paper.

Equilibrium equations. The equilibrium equations are as follows: First, there are the asset pricing equations for home and foreign Lucas' trees, taking into account depreciation (creative destruction):

$$r^w V = -\rho V + \delta X, \tag{1a}$$

$$r^w V^* = -\rho V + \delta^* X^*, \tag{1b}$$

Consider equation (1a). The return on home Lucas trees has two components: a dividend yield $\delta X/V$, and a capital loss associated with the depreciation of existing trees, $-\rho$. By arbitrage, this return should be equal to the global interest rate r^w, and equation (1a) follows. A similar argument yields equation (1b).

The second set of equations characterizes the evolution of home and foreign financial wealth:

$$\dot{W} = gW = -\theta W + (1-\tau)(1-\delta)X + r^w W + (\rho + g)V, \tag{2a}$$

$$\dot{W}^* = gW^* = -\theta W^* + (1-\tau^*)(1-\delta^*)X^* + r^w W^* + (\rho + g)V^*. \tag{2b}$$

Along the balanced growth path, home and foreign wealth grow at rate g. This change in wealth is composed of three terms. First, newborn's net of tax income $(1-\tau)(1-\delta)X$ is earned and consumption θW from dying agents is subtracted; Second, wealth earns a risk-free return r^w; Third, new trees with an aggregate value $(\rho + g)V$, accounting both for creative destruction and growth of potential output, are endowed to newborns.

The third set of equations characterizes the government budget constraints:

$$(r^w - g)D = \tau (1-\delta) X, \tag{3a}$$

$$(r^w - g)D^* = \tau^* (1-\delta^*) X^*. \tag{3b}$$

Note that positive taxes are required to sustain positive debt when the economy is dynamically efficient with $r^w > g$ but that when the economy is dynamically inefficient with $r^w < g$, positive debt is associated with tax rebates. We will return to this observation when we consider the use of public debt as a stimulus.

Lastly, the home and foreign goods market clearing conditions are:

$$\gamma\theta(W + EW^*) = X, \tag{4a}$$

$$(1-\gamma)\theta(W + EW^*) = EX^*. \tag{4b}$$

The asset market clearing condition

$$(V + D) + E(V^* + D^*) = W + EW^*$$

can be omitted since it is redundant by Walras' law.

Solving for the equilibrium. To solve for the equilibrium, we first note that since there is no home bias and $\gamma = x$, the home and foreign goods market clearing conditions (4a) and (4b) imply that the equilibrium exchange rate is:

$$E = 1.$$

Using the linearity of the equilibrium equations, we can combine the asset market clearing condition (1a) and (1b) with the wealth accumulation equations (2a) and (2b), and the government budget constraints (3a) and (3b) so as to characterize the asset pricing equation for world private assets $V^w = V + V^*$, and the accumulation equation for world wealth $W^w = W + W^*$ as a function of world output $X^w = X + X^*$ and world public debt $D^w = D + D^*$. Thus, world equilibrium is characterized by:

$$\begin{aligned}
r^w V^w &= -\rho V^w + \bar{\delta} X^w, \\
g W^w &= -\theta W^w + (1 - \bar{\delta}) X^w - (r^w - g) \bar{d} X^w + r^w W^w + (\rho + g) V^w, \\
\theta W^w &= X^w,
\end{aligned}$$

where $\bar{\delta} = x\delta + (1-x)\delta^*$ is the world's financial capacity, and $\bar{d} = xd + (1-x)d^*$ is the world's public debt to output ratio. Substituting $W^w = V^w + D^w$ into these equations and solving for the world interest rate r^w, yields:

$$r^w = r^{w,n} \equiv -\rho + \frac{\bar{\delta}\theta}{1 - \theta \bar{d}},$$

where $r^{w,n}$ is the *Wicksellian natural interest rate* consistent with full employment. This equilibrium is valid as long as $r^{w,n} \geq 0$. The natural interest rate decreases when global asset demand is high (low θ corresponding to a low propensity to consume), or global private asset supply is low (low $\bar{\delta}$ or high ρ, corresponding respectively to a low capacity to securitize financial claims, or a high rate of creative destruction), or global public asset supply is low (low $\bar{d}$ corresponding to a low public supply of assets).

Financial autarky. We can follow an identical set of steps to find the natural interest rate for Home and Foreign that would prevail in *financial autarky* (that is, in the absence of *intertemporal* trade). These *financial autarky natural interest rates,* denoted by $r^{a,n}$ and $r^{a,n*}$ respectively, will play a useful role in the characterization of capital flows and net external liabilities and are given by:

$$r^{a,n} \equiv -\rho + \frac{\delta\theta}{1-\theta d},$$
$$r^{a,n*} \equiv -\rho + \frac{\delta^*\theta}{1-\theta d^*}.$$

Under financial autarky, Home and Foreign equilibrium real interest rates satisfy:

$$r^a = \max\{r^{a,n}, 0\} \quad ; \quad r^{a*} = \max\{r^{a,n*}, 0\},$$

that is, they equate their financial autarky *natural* counterpart as long as the latter is positive.

Net Foreign Assets, Current Accounts, and (conventional) Metzler diagram. We can now characterize Net Foreign Asset positions and Current Accounts. For a given world interest rate r^w, we first return to Home's asset pricing equation (1a), Home's wealth accumulation equation (2a), and Home's government budget constraint (3a) to find:

$$V = \frac{\delta}{r^w + \rho} X,$$

$$W = \frac{(1-\delta) - (r^w - g)d + (\rho + g)\frac{\delta}{r^w+\rho}}{g + \theta - r^w} X,$$

from which, using the fact that $NFA = W - (V + D)$ and $CA = N\dot{F}A = gNFA$, we obtain:

$$\frac{NFA}{X} = \frac{(1-\theta d)(r^w - r^{a,n})}{(g+\theta - r^w)(\rho + r^w)}, \tag{6a}$$
$$\frac{CA}{X} = g\frac{NFA}{X}. \tag{6b}$$

Equation (6a) tells us that Home's Net Foreign Asset position increases with global interest rates r^w. Moreover, Home is a net creditor (resp. net debtor) if the world interest rate is larger (resp. smaller) than the financial autarky natural rate $r^{a,n}$.

Similar equations hold for Foreign, which together with equilibrium in the world asset market

$$x\frac{NFA}{X} + (1-x)\frac{NFA^*}{X^*} = 0, \tag{7}$$

allow us to see the derivation of the financial integration world interest rate r^w in a friendly Metzler diagram (Figure 5).

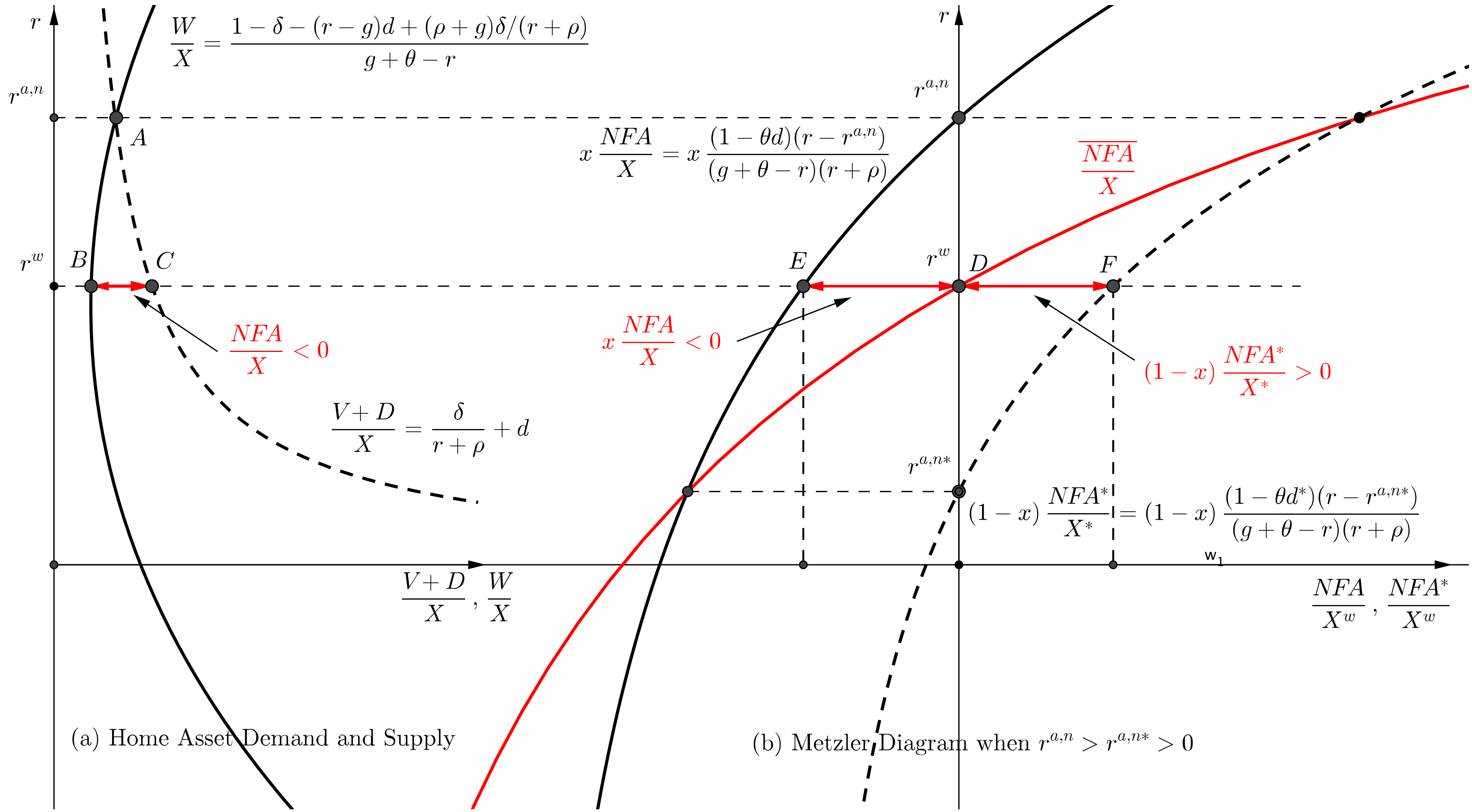
Panel (a) of Figure 5 reports Home asset demand W (solid line) and Home's asset supply $V + D$ (dashed line) scaled by domestic output X, as functions of the world interest rate r^w.[6] The two curves intersect at the financial autarky natural interest rate $r^{a,n}$,—assumed positive in the figure—where the country is neither a debtor nor a creditor (point A). For lower values of the world interest rate, Home is a net debtor: $NFA/X < 0$. For higher values, it is a net creditor. Panel (b) reports the Net Foreign Asset positions of Home $xNFA/X$, Foreign $(1-x)NFA^*/X^*$, and the world $\overline{NFA/X} = xNFA/X + (1-x)NFA^*/X^*$, as a function of the world interest rate r^w. The figure assumes that $0 < r^{a,n*} < r^{a,n}$, so that both countries would escape the liquidity trap under financial autarky. The equilibrium world interest rate r^w has to be such that global asset markets are in equilibrium, i.e. equation (7) holds or equivalently $\overline{NFA/X} = 0$ (point D).

Under financial integration, the world interest rate is a weighted average of the home and foreign financial autarky natural interest rates, with:

$$r^{w,n} = x\frac{1-\theta d}{1-\theta\bar{d}}r^{a,n} + (1-x)\frac{1-\theta d^*}{1-\theta\bar{d}}r^{a,n*}.$$

To summarize, Home runs a Current Account deficit if and only if its financial autarky

[6] Asset supply $(V+D)/X$ is monotonically decreasing in the world interest rate r^w. Asset demand W/X is non-monotonous because of two competing effects. First, higher interest rates imply that wealth accumulates faster. But higher interest rates also reduce the value of the new trees endowed to the newborns, and increase the tax burden required to pay the higher interests on public debt. For high levels of the interest rate and low levels of debt, the former effect dominates and asset demand increases with r^w. For low levels of the interest rate, the latter effect dominates and asset demand decreases with r^w. Regardless of the shape of W/X, one can easily verify that NFA/X is always increasing in the interest rate.



Panel (a) reports asset demand W/X (solid line) and asset supply $(V+D)/X$ (dashed line). The two lines intersect at the autarky natural interest rate $r^{a,n}$ (point A). Panel (b) reports home (solid line) and foreign (dashed line) net foreign assets scaled by world output, $xNFA/X$ and $(1-x)NFA^*/X^*$ when $r^{a,n} > 0$ and $r^{a,n*} > 0$. The world natural interest rate r^w is such that net foreign asset positions are equilibrated, or equivalently, when the world's NFA (red line) $\overline{NFA/X} = xNFA/X + (1-x)NFA^*/X^*$ is equal to zero (point D). If $r^{a,n} > r^w$, Home is a net debtor ($NFA/X < 0$) and runs a current account deficit ($CA/X < 0$).

Figure 5: World Interest Rates and Net Foreign Asset Positions: the Metzler Diagram

natural interest rate is above the financial autarky natural interest rate of Foreign, i.e. when Home's net asset scarcity is smaller than Foreign's. This ocurs occurs when financial capacity in Home is higher than that in Foreign, $\delta > \delta^*$, or when Home sustains a higher public debt ratio than Foreign $d > d^*$. Foreign's Current Account surplus helps propagate its asset shortage, increasing its interest rate and reducing Home's interest rate all the way to the point at which they are equal to each other.

In the integrated equilibrium with $r^{a,n} > r^{a,n*}$, it is possible for Home to pull Foreign out of a liquidity trap, in the sense that $r^{w,n} > 0$ while $r^{a,n*} < 0$. In that case, financial integration helps prevent the occurrence of a liquidity trap in Foreign. It is also possible for Foreign to pull Home into a liquidity trap, in the sense that $r^{w,n} < 0$ while $r^{a,n} > 0$. We turn next to the global liquidity trap equilibrium.

2.3 Liquidity Trap

When $r^{w,n} < 0$, the global economy is in a liquidity trap. The economy is at the ZLB with $r = r^* = r^w = 0$. At this interest rate, and if output is at potential, there is a global asset shortage, which cannot be resolved by a reduction in world interest rates.

Instead, an alternative (perverse) equilibrating mechanism arises in the form of a recession with $\min\{\xi, \xi^*\} < 1$, where it is important to bear in mind that the quantities ξ and ξ^* are endogenous equilibrium variables in a rigid-price equilibrium.

At a fixed zero interest rate, the recession *endogenously reduces asset demand more than asset supply* which restores equilibrium in the global asset market. It is important to note that this key property is about *endogenous* changes in asset supply and asset demand brought about by a reduction in output, and not about the exogenous movements in asset supply and asset demand directly caused by any exogenous shock that may trigger a recession. In other words, this property is entirely consistent with large drops in asset supply and large increases in asset demand during recessions. In fact, these shocks, which exacerbate asset shortages, are one of the main reasons why the economy might end up in a liquidity trap.

This logic rests on the following assumptions, which we maintain throughout: $\delta\theta/\rho < 1$, $\delta^*\theta/\rho < 1$,$1/\theta > d > 0$ and $1/\theta > d^* > 0$. To understand the role of these, note that a recession endogenously reduces asset demand because it reduces the income of newborns.

Further, the recession reduces the dividend paid out on Lucas trees, reducing the value of the trees. This reduces both the asset demand of the newborns (who receive these new trees) and private asset supply (the value of the trees themselves). The assumptions that $\delta\theta/\rho < 1$ and $\delta^*\theta/\rho < 1$ guarantee that a recession endogenously reduces net private asset demand.

The assumptions that $d > 0$ and $d^* > 0$ ensure that part of the asset supply (public asset supply) is *not* endogenously affected by the recession (a sort of "safe debt"). The recession resolves the asset shortage because equilibrium in the global asset market requires net private asset demand for public assets to equal public asset supply. The assumptions that $d < 1/\theta$ and $d^* < 1/\theta$ ensure that debt can be sustained in both countries even under financial autarky.

Equilibrium equations. The equilibrium equations are similar to those in the previous case except for the zero interest rates and endogenous output at Home and Foreign (ξ, ξ^*), where we have already replaced taxes at Home and Foreign using the government budget constraints $\tau\,(1-\delta)\,\xi X = -gD$ and $\tau^*\,(1-\delta)\,\xi^* X^* = -gD^*$:

$$0 = -\rho V + \delta\xi X, \tag{8a}$$

$$0 = -\rho V^* + \delta^*\xi^* X^*, \tag{8b}$$

$$gW = -\theta W + (1-\delta)\xi X + gD + (\rho+g)V, \tag{8c}$$

$$gW^* = -\theta W^* + (1-\delta^*)\xi^* X^* + gD^* + (\rho+g)V^*, \tag{8d}$$

$$x\theta(W + EW^*) = \xi X, \tag{8e}$$

$$(1-x)\theta(W + EW^*) = E\xi^* X^*. \tag{8f}$$

As before, the first two lines correspond to the asset pricing equations (for Home and Foreign), the next two lines characterize wealth dynamics, and the last two lines represent the market clearing conditions. The global asset market clearing condition $(V+D)+E(V^*+D^*) = W + EW^*$ can be omitted by Walras' Law.

Solving for the equilibrium. To solve for the equilibrium, we proceed as in the case with no liquidity trap. We first note that the home and foreign goods market clearing conditions

(8e) and (8f) imply that the equilibrium nominal exchange rate is:

$$E = \frac{\xi}{\xi^*}. \tag{9}$$

The nominal exchange rate is the *ratio of Home and Foreign's outputs* (each normalized by its corresponding potential output). This simple expression is a direct consequence of the assumption of a unit elasticity of substitution between Home and Foreign goods, and the fact that there is no home bias in preferences (we relax these assumptions in appendices A.1 and A.2). Given that Home and Foreign prices are constant (in their own currency), a more depreciated nominal exchange rate implies a more depreciated real exchange rate, which shifts relative demand toward the Home good. Since output is demand determined in the global liquidity trap, this expenditure switching effect requires a smaller Home recession relative to Foreign.

We can combine Home and Foreign equations (8a)-(8d) to derive the asset pricing equation for global private assets (in Home currency) $V^w = V + EV^*$ and the evolution of global wealth (in Home currency) $W^w = W + EW^*$:

$$0 = -\rho V^w + \delta \xi X + \delta^* E \xi^* X^*, \tag{10a}$$

$$gW^w = -\theta W^w + (1-\delta)\xi X + (1-\delta^*)E\xi^* X^* + gD^w + (\rho + g)V^w, \tag{10b}$$

where $D^w = D + ED^*$ is global public debt in Home currency. In addition, equilibrium in goods markets then requires:

$$\theta W^w = \xi X + E \xi^* X^*. \tag{11}$$

This is a system of *four* equations (9)-(11), in *five* unknowns V^w, W^w, ξ, ξ^*, and E. That is, there is a *degree of indeterminacy.* This indeterminacy is related to the seminal result by Kareken and Wallace (1981) that the exchange rate is indeterminate with pure interest rate targets, which is *de facto* the case when both countries are at the ZLB. An important difference here is that money is *not* neutral, and therefore different values of the exchange rate correspond to different levels of output at Home and in Foreign, as prescribed by equation (9).

From now on, we index the solution by the exchange rate E, and solve for the remaining equations.gaps:

$$\xi = 1 + \frac{1-\theta\bar{d}}{1-\frac{\bar{\delta}\theta}{\rho}}\frac{r^{w,n}}{\rho} + \frac{(1-x)d^*(E-1)}{1-\frac{\bar{\delta}\theta}{\rho}}, \tag{12a}$$

$$\xi^* = 1 + \frac{1-\theta\bar{d}}{1-\frac{\bar{\delta}\theta}{\rho}}\frac{r^{w,n}}{\rho} + \frac{xd(\frac{1}{E}-1)}{1-\frac{\bar{\delta}\theta}{\rho}}. \tag{12b}$$

It follows that both Home and Foreign outputs ξ and ξ^* are increasing in the world natural interest rate $r^{w,n}$, and Home (Foreign) output is increasing (decreasing) in the exchange rate E. The former is natural: the more acute the global (safe) asset shortage, the lower the world natural interest rate $r^{w,n}$, and the larger the required endogenous reduction in output (the lower ξ and ξ^*) to restore asset market equilibrium. The latter is intuitive since, as explained above, a depreciation in the home exchange rate stimulates the home economy to the detriment of the foreign economy through an expenditure switching effect. Finally, note that the indeterminacy is bounded as it only applies while *both* countries are in a liquidity trap. Concretely, E must be within a range $[\underline{E}, \overline{E}]$, with $\xi = 1$ for $E = \overline{E}$, and $\xi^* = 1$ for $E = \underline{E}$, where $\bar{E} = 1 - (1-\theta\bar{d})r^{w,n}/(1-x)d^*\rho > 1$ and $\underline{E} = 1/[1-(1-\theta\bar{d})r^{w,n}/xd\rho] < 1$. By depreciating (appreciating) the exchange rate sufficiently, Home (Foreign) can avoid the global liquidity trap altogether.

Financial autarky. Following similar steps, we can define the *financial autarky output gaps*, denoted by $\xi^{a,l}$ and $\xi^{a,l*}$, as the outputs that obtain when intertemporal trade is not allowed, with

$$\xi^{a,l} = 1 + \frac{1-\theta d}{1-\frac{\delta\theta}{\rho}}\frac{r^{a,n}}{\rho}, \tag{13a}$$

$$\xi^{a,l*} = 1 + \frac{1-\theta d^*}{1-\frac{\delta^*\theta}{\rho}}\frac{r^{a,n*}}{\rho}. \tag{13b}$$

Under financial autarky, Home and Foreign equilibrium outputs satisfy:

$$\xi^a = \min\left\{\xi^{a,l}, 1\right\} \quad ; \quad \xi^{a*} = \min\left\{\xi^{a,l*}, 1\right\}.$$

In particular, there is a liquidity trap at Home with $\xi^a < 1$ if and only if $r^{a,n} < 0$, and a liquidity trap in Foreign with $\xi^{a*} < 1$ if and only if $r^{a,n} < 0$, and the corresponding liquidity trap are deeper the more negative are the corresponding natural interest interest rates, i.e. the lower is financial capacity. These equations make clear that both at Home and in Foreign, the interest rate and outputs are entirely determined by domestic factors. Under financial autarky, and in contrast with the case of financial integration, each country's interest rate reflects its own asset scarcity. An asset shortage in one country cannot propagate to the another country when the capital account is closed.

Using the goods market clearing conditions, it follows that the *unique* equilibrium exchange rate under financial autarky satisfies:

$$E^a = \frac{\xi^a}{\xi^{a*}}, \tag{14}$$

which has the intuitive implication that if Home experiences a more severe liquidity trap (a lower ξ^a), it also has a more appreciated exchange rate. Unlike the financially integrated case, this is purely a goods market linkage, reflecting the relative scarcity of home goods.

In the special case where both countries experience a liquidity trap under financial autarky (that is, when $r^{a,n} < 0$ and $r^{a,n*} < 0$), the equilibrium exchange rate simplifies to:

$$E^a = \frac{d}{d^*} \frac{1 - \frac{\delta^* \theta}{\rho}}{1 - \frac{\delta \theta}{\rho}}.$$

The country with larger asset scarcity (low d or low δ) has a lower output level and a stronger currency under financial autarky.

Net Foreign Assets, Current Accounts, and "Metzler diagram in quantities". We now have all the ingredients to characterize Net Foreign Asset positions and Current Accounts, and to introduce one of our main analytical innovation, *the Metzler diagram in* *quantities.*

Let's start by fixing the nominal exchange rate E. We can rewrite the home asset pricing and wealth accumulation equations (8a) and (8c) as:

$$V = \frac{\delta \xi X}{\rho}, \tag{15a}$$

$$W = \frac{\xi X + gD + g\frac{\delta \xi X}{\rho}}{g+\theta}, \tag{15b}$$

which implies:

$$\frac{NFA}{X} = \frac{W-(V+D)}{X} = \frac{\xi(1-\frac{\delta\theta}{\rho}) - \theta d}{g+\theta} = \frac{(1-\frac{\delta\theta}{\rho})(\xi - \xi^{a,l})}{g+\theta}, \tag{16a}$$

$$\frac{CA}{X} = g\frac{NFA}{X}. \tag{16b}$$

Equation (16a) has a similar interpretation as equation (6a): Home's Net Foreign Asset and Current Account positions increase with Home output ξ. Home is a net creditor (resp. debtor) if its output exceeds its financial autarky level: $\xi > \xi^{a,l}$ (resp. $\xi < \xi^{a,l}$).[7]

Similar equations hold for Foreign. In particular, we have:

$$\frac{NFA^*}{X^*} = \frac{\xi^*(1-\frac{\delta^*\theta}{\rho}) - \theta d^*}{g+\theta} = \frac{(1-\frac{\delta^*\theta}{\rho})(\xi^* - \xi^{a,l*})}{g+\theta},$$

which can be rewritten (replacing the exchange rate equation (9) into it) as:

$$\frac{NFA^*}{X^*} = \frac{\frac{\xi}{E}(1-\frac{\delta^*\theta}{\rho}) - \theta d^*}{g+\theta}. \tag{17}$$

Combining these equations with equilibrium in the world asset market expressed in the home currency

$$x\frac{NFA}{X} + (1-x)E\frac{NFA^*}{X^*} = 0, \tag{18}$$

yields a simple re-derivation of the home liquidity trap as a function of the exchange rate E, equation (12a).

[7] Observe that since $\xi \leq 1$, Home always runs a current account deficit when $\xi^{a,l} > 1$, i.e. when Home would escape the liquidity trap under financial autarky.

Given E, both the home and foreign Net Foreign Asset Positions are increasing in ξ. And given ξ, the foreign Net Foreign Asset position is decreasing in E. Both effects are intuitive.

Indeed, given E, expressing all values in the home currency, an increase in ξ raises both home and foreign asset demands as well as the home and foreign supply of private assets, but leaves the home and foreign public supply of assets (public debt) unchanged.

Similarly, given ξ, an increase in E increases foreign asset supply (foreign asset values in home currency) more than foreign asset demand (foreign wealth in home currency) because of the increased value of foreign public debt (in home currency).

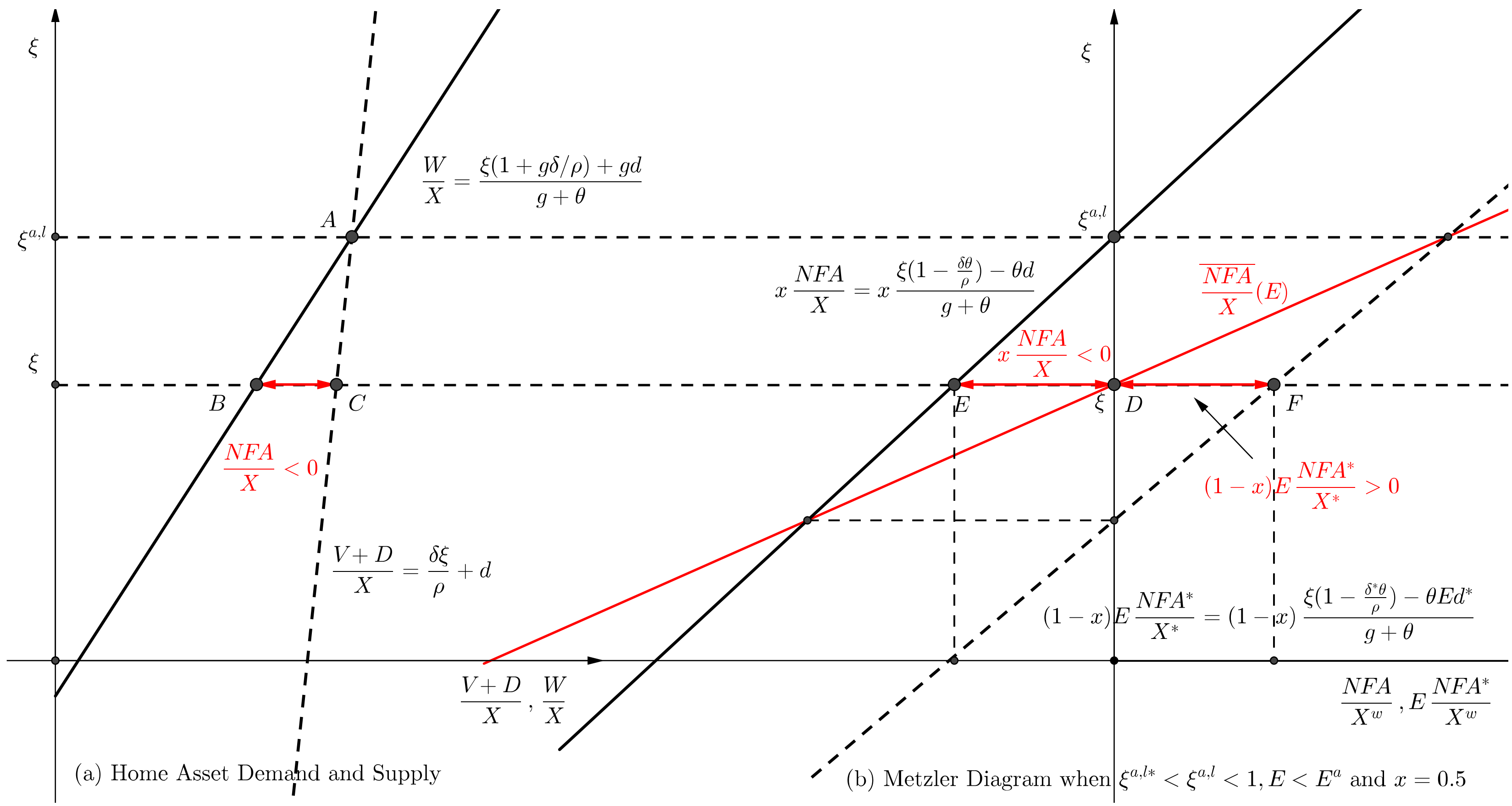
Taken together, this "*Metzler diagram in quantities*" immediately implies that ξ is increasing in E.

Panel (a) of Figure 6 reports Home asset demand W (solid line) and Home asset supply $V + D$ (dashed line) scaled by Home's potential output X, as a function of the domestic output ξ (equations (15a) and (15b)). Both asset demand and asset supply are increasing in output, but the former increases faster than the latter. The two curves intersect at the financial autarky output $\xi^{a,l}$ (point A). For lower values of output, Home is a net debtor: $NFA/X < 0$. For higher values, it is a net creditor: $NFA/X > 0$. Panel (b) reports scaled Home and Foreign Net Foreign Asset position $NFA/(X + X^*)$ and $E\,NFA^*/(X + X^*)$, as a function of Home's output ξ (equations (16a) and (17)). The figure assumes that $\xi^{a,l*} < \xi^{a,l} < 1$ so that both countries are in a liquidity trap under financial autarky. The equilibrium liquidity trap ξ has to be such that global asset markets are in equilibrium, i.e. equation (18) holds (point D).

The Metzler diagram in quantities indicates that, under financial integration, domestic output is an average of the Home and exchange-rate-weighted Foreign financial autarky outputs. From equations (12a), (13a) and (13b), we obtain:

$$\xi = \frac{\theta \bar{d}(E)}{1 - \frac{\bar{\delta}\theta}{\rho}} = x \frac{1 - \frac{\delta\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} \xi^{a,l} + (1 - x) \frac{1 - \frac{\delta^*\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} E \xi^{a,l*}, \tag{19}$$

where $\bar{d}(E) = xd + (1 - x)Ed^*$ denotes the exchange rate-adjusted world public debt ratio, which increases with E since a depreciation of the home currency increases the value of foreign public debt (expressed in the home currency).



Panel (a) reports asset demand W/X (solid line) and asset supply $(V+D)/X$ (dashed line) as a function of Home output ξ. The two lines intersect at the autarky level of output $\xi^{a,l}$ (point A). Panel (b) reports home (solid line) and foreign (dashed line) net foreign assets scaled by world potential output, $xNFA/X$ and $(1-x)ENFA^*/X^*$, in the Home currency, as a function of home's output ξ, for a given E, when $\xi a, l < 1$ and $\xi^{a,l*} < 1$. Given an exchange rate $E \in [\underline{E}, \bar{E}]$, Home output is such that net foreign asset positions are equilibrated, or equivalently such that the world's NFA (red line) $\overline{NFA/X}(E) = xNFA/X + (1-x)ENFA^*/X^*$ is zero (point D). If $\xi < \xi^{a,l}$, Home is a net debtor ($NFA/X < 0$), and runs a current account deficit ($CA/X < 0$).

Figure 6: Recessions and Net Foreign Asset Positions in a Global Liquidity Trap: the Metzler Diagram in Quantities

We can use (19) to rewrite the home Net Foreign Asset position and Current Account as:

$$\frac{NFA}{X} = \frac{(1-\frac{\delta\theta}{\rho})}{g+\theta}\left[\frac{\theta\bar{d}(E)}{1-\frac{\bar{\delta}\theta}{\rho}} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}}\right],$$

$$\frac{CA}{X} = g\frac{NFA}{X}.$$

Clearly, the more depreciated is the exchange rate, the greater is the home Net Foreign Asset position and so is its Current Account, allowing Home to export more of its recession abroad. It is apparent in these expressions that the sign and size of the home Net Foreign Asset position and its Current Account is determined by the gap between the exchange rate-adjusted world index of world asset supply $\theta\bar{d}(E)/(1-\bar{\delta}\theta/\rho)$, and the home index of asset supply $\theta d/(1-\delta\theta/\rho)$. Depending on the value of the exchange rate E, Home can be a surplus country, or a deficit country.[8]

When both countries are in a liquidity trap under financial autarky, we can express the Net Foreign Asset position and Current Account directly as a function of the exchange rate E, relative to the autarky exchange rate E^a. Substituting the expression for the exchange rate-adjusted financial capacity, we obtain:

$$\frac{NFA}{X} = \frac{1-\frac{\delta\theta}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}}\frac{(1-x)d^*(E-E^a)}{g+\theta},$$

$$\frac{CA}{X} = g\frac{NFA}{X}.$$

We can now connect back with the financial autarky results. In the latter, the exchange rate is determinate precisely because the capital account is closed. In the case where both countries are in a liquidity trap under financial autarky, $\xi^a = \xi^{a,l} < 1$, $\xi^{a*} = \xi^{a,l*} < 1$ and $E^a = \xi^{a,l}/\xi^{a,l*}$. Then for $E = E^a$, the financial integration equilibrium coincides with the financial autarky equilibrium. For $E > E^a$, we have $\xi > \xi^a$, $\xi^* < \xi^{a*}$, and $NFA/X > 0$, and vice versa for $E < E^a$. It follows that, by manipulating its exchange rate, a country can

[8]This shows that when there in a global liquidity trap, there can be global imbalances even though the two countries are identical, which could never happen outside of a global liquidity trap. In this case, when $E > 1$, Home has a lower recession than Foreign and runs a Current Account Surplus, and vice versa if $E < 1$.

influence the relative size of its liquidity trap.

In order for a global liquidity trap to emerge under financial integration, it must be the case that at least one of the two regions is in a liquidity trap under financial autarky, in the sense that $r^{a,n} < 0$ or $r^{a,n*} < 0$. This does not, however, require *both* countries to experience a liquidity trap under financial autarky. Hence, it is perfectly possible for financial integration *per se* to drag a country into a global liquidity trap. This happens for Home if $r^{a,n} > 0$ but $r^{a,n*} < 0$ and $r^{w,n} < 0$. In that case, the preceding discussion indicates that Home must run a Current Account deficit and have a negative Net Foreign Asset position.

"Secular Stagnation Hypothesis". Our model is consistent with the Secular Stagnation Hypothesis, put forward by Hansen (1939) and recently revived by Summers (2014) and Summers (2015a). Each economy could find itself in a permanent liquidity trap, with a deficient aggregate demand. The model offers a natural way of connecting the active debate, most recently illustrated by the exchange between Bernanke (2015) and Summers (2015b), surrounding secular stagnation and global imbalances. In particular, our model highlights that secular stagnation can be exported from one country to the other through Current Account surpluses in the origin country and Current Account deficits in the destination country. In other words, a "global savings glut" or a "global asset shortage", as discussed by Bernanke (2005) and Caballero et al. (2008a), can contribute to pushing the world economy into a secular stagnation equilibrium. Unlike the analysis in the benchmark case, where global imbalances affect the equilibrium real interest rate but are otherwise relatively benign, in an environment with very low autarky real interest rates, global imbalances may now contribute toward pushing the world economy into a global liquidity trap. See also Eggertsson et al. (2015) for a related analysis.

"Currency wars" and "reserve currency paradox". Outside the liquidity trap, the exchange rate is pinned down ($E = 1$), output in each country is at its potential ($\xi = \xi^* = 1$) and the real interest rate is equal to its Wicksellian natural counterpart ($r = r^* = r^{w,n}$). It follows that nothing can be gained by a country's attempt to manipulate its exchange rate.

In the global liquidity trap equilibrium, the global asset shortage cannot be offset by lower world interest rates and a world recession occurs. This global recession is propagated by global imbalances, with surplus countries pushing world output down and exerting a strong negative externality on the world economy.

Even though in this global liquidity trap regime the exchange rate is indeterminate, it is in principle possible for the home monetary authority to peg the exchange rate at any level E (within the indeterminacy region $[\underline{E}, \bar{E}]$) it sees fit by simply standing ready to buy and sell the home currency for the foreign currency at the exchange rate E. By choosing a sufficiently depreciated exchange rate, Home is able to partly export its recession abroad by running a current account surplus. That is, once interest rates are at the ZLB, our model indicates that exchange rate policies generate powerful beggar-thy-neighbor effects. This zero-sum logic resonates with current concerns regarding currency wars: in the global stagnation equilibrium, attempts to depreciate one's currency affect one for one relative outputs, according to equation (9).

Of course, if both countries attempt to simultaneously depreciate their currency, these efforts cancel out, and the exchange rate remains a pure matter of coordination. Moreover, if agents coordinate on an equilibrium where the home exchange rate is appreciated, as could be the case if the home currency were perceived to be a "reserve currency," then this would worsen the recession at Home. In other words, while the reserve currency status may be beneficial outside a liquidity trap as it increases purchasing power and lowers funding costs, it exacerbates the domestic recession in a global liquidity trap. We dub this effect the *paradox of the reserve currency.* This mechanism may capture a dimension of the recent appreciation struggles of Switzerland during the recent European turmoil, and of Japan before the implementation of Abenomics. Similarly, it helps us understand some of the difficulties faced by the U.S. in normalizing its monetary policy.

3 Inflation

So far, we have assumed that prices are fully rigid. In this section, we relax this assumption and allow for *some* price adjustment through a Phillips curve.

This extension gives us the opportunity to reiterate some well-known insights about the economics of liquidity traps, and to obtain a few new ones. The former are that credibly higher inflation targets reduce the severity of a liquidity trap, that more (downward) price flexibility can exacerbate the severity of the trap as the economy may fall into a deflationary spiral. The less known one is that in a *global* liquidity trap, it is the *more rigid* country that experiences the worst trap (note the contrast between this relative rigidity and the aggregate rigidity implication). Moreover, it is now possible for some regions of the world to escape the liquidity trap if their inflation expectations are sufficiently high.

3.1 Extending the Model

Phillips curve. We wish to capture the idea that wages, or prices, are rigid downwards, but not upwards. We follow the literature and assume that prices and wages cannot fall faster than a certain limit pace, and that this limit pace is faster if there is more slack in the economy:[9]

$$\pi_{H,t} \geq -\kappa_0 - \kappa_1(1 - \xi_t),$$
$$\pi^*_{F,t} \geq -\kappa^*_0 - \kappa^*_1(1 - \xi^*_t),$$

where $\pi_{H,t} = \dot{P}_{H,t}/P_{H,t}$ (resp. $\pi^*_{F,t} = \dot{P}^*_{F,t}/P^*_{F,t}$) denotes the *domestic* (resp. foreign) inflation rate, and where $\kappa_0 \geq 0$, $\kappa_1 \geq 0$, $\kappa^*_0 \geq 0$, and $\kappa^*_1 \geq 0$. Moreover, we assume that if there is slack in the economy, prices or wages fall as fast as they can: $\xi_t < 1$ implies that $\pi_{H,t} = -\kappa_0 - \kappa_1(1 - \xi_t)$ and $\xi^*_t < 1$ implies that $\pi^*_{F,t} = -\kappa^*_0 - \kappa^*_1(1 - \xi^*_t)$. We capture this requirement with the complementary slackness conditions:

$$[\pi_{H,t} + \kappa_0 + \kappa_1(1 - \xi_t)](1 - \xi_t) = 0,$$
$$[\pi^*_{F,t} + \kappa^*_0 + \kappa^*_1(1 - \xi^*_t)](1 - \xi^*_t) = 0.$$

To summarize, there are two Phillips curves, one for Home and one for Foreign. The home Phillips curve traces out an increasing curve in the $(\pi_{H,t}, \xi_t)$ space, which becomes vertical

[9] The introduction of this kind of Phillips curves borrows heavily from Eggertsson and Mehrotra (2014) and Caballero and Farhi (2015).

at $\xi_t = 1$. The foreign Phillips curve is similar.

Monetary policy. We assume that monetary policy is conducted according to simple truncated Taylor rules, where the nominal interest rate responds to domestic inflation:

$$i_t = \max\{r_t^n + \bar{\pi} + \phi(\pi_{H,t} - \bar{\pi}), 0\},$$
$$i_t^* = \max\{r_t^{n*} + \bar{\pi}^* + \phi^*(\pi_{F,t}^* - \bar{\pi}^*), 0\}.$$

In these equations r_t^n and r_t^{n*} are the relevant natural interest rates at Home and in Foreign, which depend on whether we analyze the financial integration equilibrium or the financial autarky equilibrium. We denote by $\bar{\pi} > 0$ and $\bar{\pi}^* > 0$ the home and foreign inflation targets, and $\phi > 1$ and $\phi^* > 1$ are the Taylor rule coefficients.

For simplicity, we take the limit of large Taylor rule coefficients $\phi \to \infty$ and $\phi^* \to \infty$. This specification of monetary policy implies that inflation in any given country is equal to its target and that there is no recession as long as the country's interest rate is positive. For example, for Home, either $\pi_{H,t} = \bar{\pi}$, $\xi_t = 1$, and $i_t = r_t^n + \bar{\pi} > 0$ or $\pi_{H,t} \leq \bar{\pi}$, $\xi_t \leq 1$, and $i_t = 0$. The same holds for Foreign.

3.2 Equilibria

Let us focus on the case $r^{w,n} < 0$, which yields a *global* liquidity trap in the absence of inflation. We show that even in this case, there are several possible equilibrium configurations once inflation considerations are added. First, there can be equilibria with no liquidity traps either at Home or in Foreign. Second, there can be equilibria with a global liquidity trap both at Home and in Foreign. Third, there can be asymmetric liquidity trap equilibria with a liquidity trap only in one country. We treat each in turn.

No liquidity traps equilibrium. We solve for the no-liquidity traps case: $i > 0$ and $i^* > 0$. This equilibrium is such that $\pi_H = \bar{\pi}$, $\pi_F^* = \bar{\pi}^*$, $i = r^{w,n} + \bar{\pi}$, and $i^* = r^{w,n} + \bar{\pi}^*$, where $r^{w,n} = -\rho + \bar{\delta}\theta/(1 - \bar{d}\theta) < 0$ is the world natural real interest rate, as before.

It is straightforward to show (by analogy with the derivation of the equilibrium nominal exchange rate in the benchmark model) that the terms of trade (defined as the relative price of foreign and domestic goods) satisfy:

$$S_t = \frac{E_t P^*_{F,t}}{P_{H,t}} = 1,$$

which implies that:

$$\frac{\dot{E}_t}{E_t} = \pi_H - \pi^*_F = \bar{\pi} - \bar{\pi}^*.$$

The condition for this equilibrium to exist is that $i > 0$ and $i^* > 0$, i.e. $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} > 0$. This condition shows that when the world natural interest rate is negative $r^{w,n} < 0$, *the no-liquidity traps equilibrium exists if and only if the inflation targets $\bar{\pi}$ and $\bar{\pi}^*$ in both countries are high enough.*

Note, however, that this is an existence, not a uniqueness result. In fact, as we shall see next, other equilibria exist even if inflation targets are high enough to make the no-liquidity traps equilibrium feasible.

Global liquidity trap equilibrium. Let us now focus on the other extreme and solve for the global liquidity trap case $i = i^* = 0$.

Observe that in a stationary equilibrium the terms of trade $S_t = E_t P^*_{F,t}/P_{H,t}$ must be constant so that

$$\frac{\dot{E}_t}{E_t} = \pi_H - \pi^*_F.$$

Uncovered Interest Parity then requires that $i = i^* + \dot{E}_t/E_t$, which combined with $i = i^* = 0$ implies that $\dot{E}_t/E_t = 0$ and hence $\pi^*_F = \pi_H = \pi^w$. That is, *in a global liquidity trap, inflation rates are equal across countries,* hence real interest rates are equalized, $r = r^* = -\pi^w$.

The equilibrium values of $V^w = V + SV^*$, $W^w = W + SW^*$ (expressed in terms of the home good numeraire) and π_H, π_F^*, S, ξ, and ξ^* solve the following system of equations

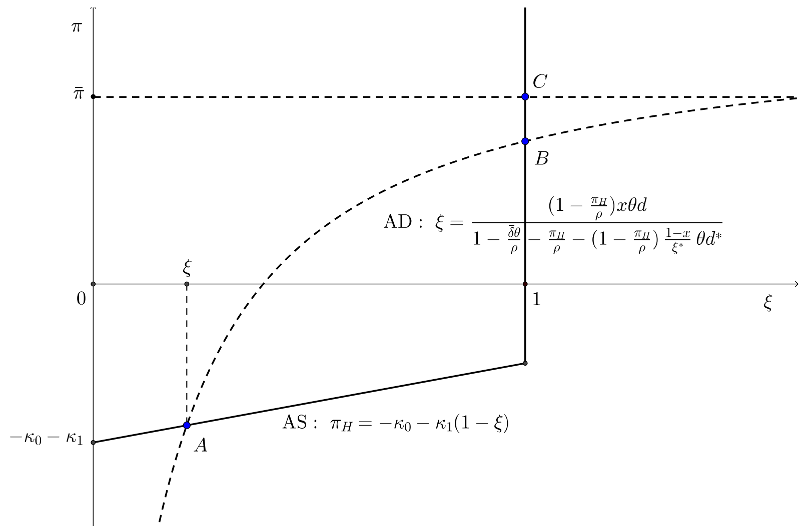
$$
\begin{aligned}
S &= \frac{\xi}{\xi^*}, \\
\theta W^w &= \xi X + S\xi^* X^* \\
-\pi_H V^w &= -\rho V^w + \delta \xi X + \delta^* S \xi^* X^*, \\
g W^w &= -\theta W^w + (1-\delta)\xi X + (1-\delta^*) S \xi^* X^* + g D^w - \pi_H W^w + (\rho + g) V^w, \\
\pi_H &= -\kappa_0 - \kappa_1 (1-\xi), && \text{(22a)} \\
\pi_F^* &= -\kappa_0^* - \kappa_1^* (1-\xi^*) && \text{(22b)} \\
\pi_F^* &= \pi_H, && \text{(22c)}
\end{aligned}
$$

where $D^w = D + SD^*$. The first equation is the equation for the terms of trade. The second equation is the equation for total world wealth. Both result directly from combining the home and foreign goods market clearing conditions. The third equation is the asset pricing equation for world private assets. The fourth equation is the accumulation equation for world wealth, where we have used the government budget constraints to replace taxes as a function of public debt $\tau(1-\delta)\xi X = -gD$ and $\tau^*(1-\delta)\xi^* X^* = -gD^*$. The fifth and sixth equations are the home and foreign Phillips curves. The seventh equation represents the requirement derived above that the terms of trade be constant.

We can represent the equilibrium as an Aggregate Demand (AD)-Aggregate Supply (AS) diagram which constitutes a system of four equations in four unknowns π_H, π_F^*, ξ, and ξ^*. The home and foreign AD curves are given by:

$$
\xi = \frac{\frac{1-\frac{\pi_H}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_H}{\rho}} x\theta d}{1 - \frac{1-\frac{\pi_H}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_H}{\rho}} \frac{1-x}{\xi^*}\theta d^*},
$$

$$
\xi^* = \frac{\frac{1-\frac{\pi_F^*}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_F^*}{\rho}} (1-x)\theta d^*}{1 - \frac{1-\frac{\pi_F^*}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_F^*}{\rho}} \frac{x}{\xi}\theta d}.
$$



The figure reports Aggregate Supply (solid line) and Aggregate Demand (dashed line), for a given value of Foreign output, ξ^*.

Figure 7: Aggregate Demand and Aggregate Supply.

The home and foreign AS curves are given by:

$$\pi_H = -\kappa_0 - \kappa_1(1 - \xi),$$
$$\pi_F^* = -\kappa_0^* - \kappa_1^*(1 - \xi^*).$$

The AD-AS diagram for the Home economy is reported on Figure 7, for a given value of ξ^*. The AS curve (solid line) slopes upwards, then becomes vertical at $\xi = 1$: a smaller recession is associated with less deflation, until full employment is achieved. At the ZLB, the AD curve (dashed line) also slopes upwards since an increase in inflation reduces the real interest rate, which increases output. Away from the ZLB, the AD curve becomes horizontal at $\bar{\pi}$.

We always assume that the upward sloping part of the AD curve is steeper than the non-vertical part of the AS curve and that they intersect at one point, A. The AD and AS schedules intersect at either exactly point A, or at three points, A, B, and C. Point A is the liquidity trap equilibrium: $i = 0$, $\pi_H = -\kappa_0 - \kappa_1(1 - \xi) < \bar{\pi}$, and $\xi < 1$. Point C, if it exists, corresponds to an asymmetric liquidity trap equilibrium with $i > 0$, $\pi_H = \bar{\pi}$, and $\xi = 1$. Point B, if it exists, is an equilibrium with $i = 0$, $-\kappa_0 < \pi_H \leq \bar{\pi}$ and $\xi = 1$. Points

B exists if the inflation target $\bar{\pi}$ is high enough. It is unstable, so we ignore it. Point C, if it exists, is treated below. Therefore, for the time being, we focus on point A.

It can be verified that the home and foreign AD equations imply $\pi_H = \pi_F^* = \pi^w$. If $\kappa_0 = \kappa_0^*$, this implies that

$$\frac{1-\xi^*}{1-\xi} = \frac{\kappa_1}{\kappa_1^*},$$

so that *Home has a smaller recession than Foreign, $\xi > \xi^*$, if and only if home prices or wages are more flexible than foreign prices or wages: $\kappa_1 > \kappa_1^*$.* More (downward) wage flexibility reduces the size of the recession at Home relative to Foreign because it depreciates the domestic terms of trade. In a stationary equilibrium, deflation rates are equalized across countries, and because deflation in any given country increases with this country's wage flexibility and with this country's recession.

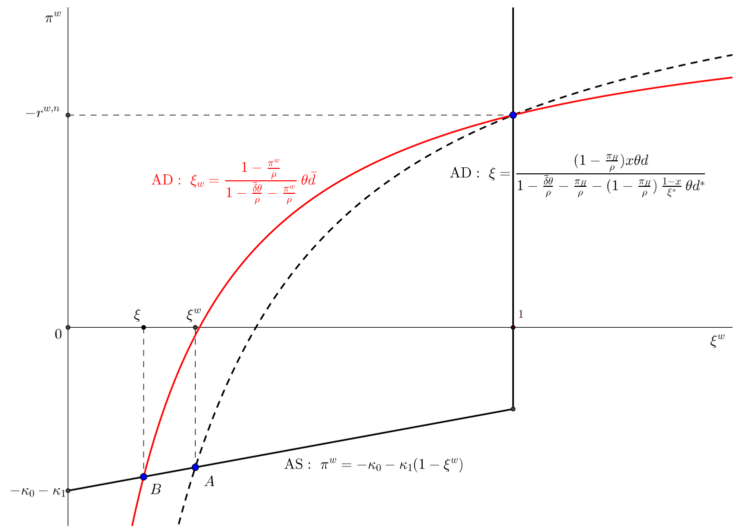
The rest of the equilibrium simplifies greatly when the Phillips curves are identical in both countries so that $\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$. Indeed, this requires that the recession is identical at Home and in Foreign: $\xi = \xi^* = \xi^w$. This implies $S = 1$. Moreover, in this case, we have the following simpler *global* AD-AS representation:

$$\xi^w = \frac{1 - \frac{\pi^w}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho} - \frac{\pi^w}{\rho}} \theta \bar{d},$$

$$\pi^w = -\kappa_0 - \kappa_1(1 - \xi^w),$$

where the AD curve implicitly defines an increasing relationship between ξ^w and π^w. Figure 8 reports this global AD-AS diagram.

This representation makes clear that compared with the case with no inflation, *there is now a negative feedback loop between the global recession and inflation.* A larger recession reduces inflation, which in turn raises the real interest rate, causing a further recession etc. *ad infinitum.* This *feedback loop is stronger, the more flexible prices and wages are*, as captured by the slope of the Phillips curve κ_1. That is, wage flexibility plays out differently across countries and at the global level: countries with more price flexibility bear a smaller share of the global recession than countries with less wage flexibility; but at the global level, more wage flexibility exacerbates the global recession.



The figure reports Aggregate Supply (solid line) and Aggregate Demand (dashed line) in a symmetric equilibrium, when $\kappa_0 = \kappa_0^*$ and $\kappa_1 = \kappa_1^*$. The red solid line represents the home AD curve in the asymmetric equilibrium with $\xi^* = 1$.

Figure 8: Aggregate Demand and Aggregate Supply in a symmetric and asymmetric equilibrium.

The equilibrium is guaranteed to exist under some technical conditions on the Phillips curves parameters κ_0 and κ_1, which ensure that the feedback loop is not so powerful to lead to a total collapse of the economy.[10]

Asymmetric liquidity trap equilibria. Can we have an asymmetric equilibrium where one country is in a liquidity trap but not the other? As we shall see, it is *always possible*. These asymmetric liquidity trap equilibria are associated with different values of the real exchange rate, and are a manifestation of the same fundamental indeterminacy that we identified in the case with no inflation.

Suppose that one country is in a liquidity trap (say Home) but not the other (say Foreign). Then we must have $i = 0$, $i^* = i - \dot{E}_t/E_t = \pi_F^* - \pi_H > 0$, $\xi < 1$, $\xi^* = 1$, $\pi_F^* = \bar{\pi}^*$, and

[10] For $\xi^w = 1$, the AD curve has $\pi^w = -r^{w,n} > 0$, while the AS curve has $\pi^w = -\kappa_0 < 0$. For $\xi^w = 0$, the AD curve has $\pi^w = \rho$, while the AS curve has $\pi^w = -(\kappa_0 + \kappa_1)$. A sufficient condition for a unique intersection is that $\kappa_0 + \kappa_1 \leq -\rho$.

$\pi_H + \kappa_0 + \kappa_1(1-\xi) = 0$. Going through the same steps as above, we find:

$$\xi = \frac{x\frac{1-\frac{\pi_H}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_H}{\rho}}\theta d}{1-(1-x)\frac{1-\frac{\pi_H}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi_H}{\rho}}\theta d^*},$$

$$\pi_H = -\kappa_0 - \kappa_1(1-\xi),$$

$$S = \xi,$$

$$i^* = \bar{\pi}^* - \pi_H.$$

The equilibrium is guaranteed to exist under the same technical conditions on Phillips curves that the ones derived above.

It is easy to see that *the home recession is larger and home inflation is lower in this asymmetric liquidity trap equilibrium where only Home is in a liquidity trap than in the symmetric equilibrium where both Home and Foreign are in a liquidity trap.* On Figure 8, the red solid line reports the Home AD curve when Foreign is not in a liquidity trap and point B the corresponding equilibrium. We can verify immediately that $\xi < \xi^w$, that is: the recession is more severe for the country that remains in the trap. This follows directly from two observations: First, the home AD curve in the symmetric liquidity trap equilibrium is decreasing in ξ^*, since higher foreign output depreciates the foreign real exchange rate and stimulates demand for the foreign good. Second, the home AD curve in the asymmetric liquidity trap equilibrium is obtained when $\xi^* = 1$.

Inflation, exchange rates, and the structure of equilibria. Let us take stock and summarize the structure of equilibria when $r^{w,n} < 0$. There may exist an equilibrium with no liquidity traps, which occurs if and only if $\min\{r^{w,n}+\bar{\pi}, r^{w,n}+\bar{\pi}^*\} > 0$. But there always exists a symmetric liquidity trap equilibrium, as well as two asymmetric liquidity trap equilibria where only one country is in a liquidity trap. These symmetric and asymmetric liquidity trap equilibria are associated with different values of the real exchange rate and this multiplicity is a manifestation of the same fundamental indeterminacy that we identified in the case with no inflation. Indeed, it is immediate to see that terms of trade S are the most depreciated in the asymmetric liquidity trap equilibrium where Home is not in a liquidity trap but Foreign is, the most appreciated in the asymmetric liquidity trap equilibrium where Home is in a liquidity trap but Foreign is not, and intermediate between these two values in the symmetric liquidity trap equilibrium where both Home and Foreign are in a liquidity trap. The severity of the recession at Home is directly commensurate with the degree of appreciation of the terms of trade S.

Currency wars. Suppose that $\min\{r^{w,n} + \bar{\pi}, r^{w,n} + \bar{\pi}^*\} < 0$. A country (say Home) can target its exchange rate by standing ready to exchange unlimited quantities of home currency for foreign currency at a given crawling exchange rate. Doing so can in effect rule out both the symmetric liquidity trap equilibrium and the asymmetric liquidity trap equilibrium where it is in a liquidity trap. Home can therefore always guarantee that it will not be in a liquidity trap, and avoid a recession by shifting it entirely to Foreign. This is to the benefit of Home which avoids a recession altogether, but to the detriment of Foreign which experiences a deeper recession.

3.3 Net Foreign Assets, Current Accounts, and Metzler Diagram in Quantities

As before, we conclude by characterizing Net Foreign Asset positions and Current Accounts in all the equilibria described above. We express these quantities in real terms in the home good numeraire. In the no liquidity trap equilibrium, these quantities are given by exactly the same formula as in the case with no inflation. In a symmetric liquidity trap equilibrium, or in an asymmetric liquidity trap equilibrium, we have

$$\frac{NFA}{X} = \frac{W - (V + D)}{X} = \frac{\xi(1 - \frac{\delta\theta}{r+\rho}) - \theta d}{g + \theta - r},$$
$$\frac{CA}{X} = g\frac{NFA}{X},$$

where $\xi < 1$ and $r = -\pi_H$ if Home is in a liquidity trap and $\xi = 1$ and $r = -\pi_F^*$ if Home is not in a liquidity trap (but Foreign is).

The same forces that we identified in the model with no inflation are at play. For example, in a symmetric liquidity trap equilibrium when the Phillips curves are identical across countries (so that $\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$),

$$\begin{aligned}
\frac{NFA}{X} &= \frac{W-(V+D)}{X} = \frac{(1-\frac{\delta\theta}{\rho}-\frac{\pi^w}{\rho})[\frac{\theta\bar{d}}{1-\frac{\delta\theta}{\rho}-\pi^w} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}-\frac{\pi^w}{\rho}}]}{g+\theta+\pi^w}, \\
\frac{CA}{X} &= g\frac{NFA}{X}.
\end{aligned}$$

Hence to the extent that Home has a higher financial capacity than Foreign $\delta > \delta^*$, or a higher public debt ratio than Foreign $d > d^*$, then Home runs a negative Net Foreign Asset position and a Current Account deficit. We can also represent the equilibrium with a Metzler diagram in quantities augmented with a global AS curve. Indeed we have

$$\begin{aligned}
\frac{NFA}{X} &= \frac{\xi^w(1-\frac{\frac{\delta\theta}{\rho}}{1-\frac{\pi^w}{\rho}}) - \theta d}{g+\theta+\pi^w}, \\
S\frac{NFA^*}{X^*} &= \frac{\xi^w(1-\frac{\frac{\delta^*\theta}{\rho}}{1-\frac{\pi^w}{\rho}}) - \theta d^*}{g+\theta+\pi^w},
\end{aligned}$$

and we must have

$$\begin{aligned}
x\frac{NFA}{X} + (1-x)S\frac{NFA^*}{X^*} &= 0, \\
\pi^w + \kappa_0 + \kappa_1(1-\xi^w) &= 0,
\end{aligned}$$

with $S = 1$.

4 Public Debt and Fiscal Policy

Liquidity traps occur in environments with a scarcity of assets. In a liquidity trap, both public debt and fiscal policy have special roles to play. They both have the potential to address the financial markets (stock) insufficiency, and the corresponding goods demand (flow) insufficiency. We illustrate these roles next.

4.1 Public Debt

In the interest of space, we only consider the case of global liquidity trap. In the base model with rigid prices, we have

$$
\begin{aligned}
\xi &= \frac{\theta \bar{d}(E)}{1-\frac{\bar{\delta}\theta}{\rho}}, \\
\xi^* &= \frac{1}{E}\frac{\theta \bar{d}(E)}{1-\frac{\bar{\delta}\theta}{\rho}}, \\
\frac{NFA}{X} &= \frac{(1-\frac{\delta\theta}{\rho})}{g+\theta}\left[\frac{\theta \bar{d}(E)}{1-\frac{\bar{\delta}\theta}{\rho}} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}}\right], \\
\frac{CA}{X} &= g\frac{NFA}{X}.
\end{aligned}
$$

These equations make apparent that for a given exchange rate E, an increase in debt at Home D or in Foreign D^* increases world net asset supply and reduces the world asset shortage. As a result home and foreign outputs ξ and ξ^* increase. An increase in debt at Home decreases the home Net Foreign Asset position and pushes the home Current Account toward a deficit.

Similar conclusions emerge in the extended model with inflation of Section 3, where we have

$$
\begin{aligned}
\xi^w &= \frac{1-\frac{\pi^w}{\rho}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi^w}{\rho}}\theta\bar{d}, \\
\pi^w &= -\kappa_0-\kappa_1(1-\xi^w), \\
\frac{NFA}{X} &= \frac{(1-\frac{\delta\theta}{\rho}-\frac{\pi^w}{\rho})}{g+\theta+\pi^w}\left[\frac{\theta\bar{d}}{1-\frac{\bar{\delta}\theta}{\rho}-\frac{\pi^w}{\rho}} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}-\frac{\pi^w}{\rho}}\right], \\
\frac{CA}{X} &= g\frac{NFA}{X}.
\end{aligned}
$$

The effect of public debt on output gains an extra kick through the feedback loop between output and inflation. The increase in output increases inflation, reducing real interest rates, further increasing output etc. ad infinitum. This boosts the Keynesian output multiplier associated with public debt issuance.

It is important to note that in a liquidity trap, we necessarily have $r^w \leq g$ since $r^w = 0$ and $g \geq 0$. This implies that the government does not need to levy taxes to sustain debt, and in fact can afford to rebate some tax revenues to households. Fiscal capacity is therefore not a constraint on the use of debt as an instrument to stimulate the economy. This stark conclusion rests on the assumption that the trap is permanent. If the trap were only temporary (as in the model with exit we discuss in Appendix A.4), then the results could be different, depending on whether the post-exit economy is dynamically efficient or not. If the post-exit economy is dynamically inefficient, then the conclusion above still holds. But if the post-exit economy is dynamically efficient, then fiscal capacity is eventually required to service debt after exit, constraining the use of debt issuance as a stimulus tool during the liquidity trap.[11]

4.2 Fiscal Policy

We now introduce balanced budget government spending. At Home, government spending on home goods $\gamma_G X$ is financed by increasing the tax τ on the income of newborns $(1-\delta)\,\xi X$, for a constant level of public debt D/X. The same applies to Foreign where government spending $\gamma_G^* X^*$ is financed by increasing the tax τ^* on the income of newborns $(1-\delta^*)\,\xi^* X^*$, for a constant level of public debt D^*/X^*.

In the interest of space, we only consider the case of a global liquidity trap. In the base

[11] When the economy is dynamically inefficient with $r^w < g$, it is possible to reinterpret government debt as a pure bubble. The only difference is that the lump sum rebate to newborns takes the form of new bubbles, with the same value. For example, at Home, we would replace debt D by a bubble B, and rebate to newborns $-\tau(1-\delta)\,\xi X = -(r^w - g)B$ by new bubbles. Crucially, the requirement that the value of new bubbles be positive requires that the economy be dynamically inefficient with $r^w < g$.

This reinterpretation offers the following insights. Outside of a liquidity trap, world interest rates are increasing in home and foreign bubbles. In a liquidity trap, home and foreign outpu gaps are both increasing in home and foreign bubbles. In both cases, an increase in the bubble at Home decreases the home Net Foreign Asset position and pushes its Current Account toward a deficit.

model with perfectly rigid prices, we have

$$
\begin{aligned}
E &= \frac{\xi - \gamma_G}{\xi - \gamma_G^*}, \\
\xi - \gamma_G &= \frac{\theta \bar{d}(E) + \frac{\overline{\delta \gamma_G}(E)\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}}, \\
\xi - \gamma_G^* &= \frac{1}{E} \frac{\theta \bar{d}(E) + \frac{\overline{\delta \gamma_G}(E)\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}}, \\
\frac{NFA}{X} &= \frac{\xi(1 - \frac{\delta\theta}{\rho}) - \gamma_G - \theta \frac{D}{X}}{g + \theta} = \frac{(1 - \frac{\delta\theta}{\rho})[\frac{\theta \bar{d}(E) + \frac{\overline{\delta \gamma_G}(E)\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} - \frac{\frac{\delta \gamma_G \theta}{\rho} + \theta d}{1 - \frac{\delta\theta}{\rho}}]}{g + \theta}, \\
\frac{CA}{X} &= g \frac{NFA}{X}.
\end{aligned}
$$

These equations make apparent that given the exchange rate E, home government spending stimulates home output more than one-for-one i.e. with a Keynesian government spending multliplier

$$
1 + \frac{\frac{x\delta\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} > 1,
$$

and stimulates foreign output but less with a Keynesian government spending multiplier of

$$
\frac{1}{E} \frac{\frac{x\delta\theta}{\rho}}{1 - \frac{\bar{\delta}\theta}{\rho}} > 0.
$$

These two effects are intuitive given that government spending not only increases the demand for home goods, reduces the asset demand arising from home agents who must now pay extra taxes to finance the increase in government spending, but also indirectly increases asset supply by stimulating home output. This explains why the domestic government spending multiplier is greater than one, and why the effect on foreign output is positive. Moreover, and for the same reason, home government spending reduces the home Net Foreign Asset position and pushes the home Current Account toward a deficit. Similar effects apply for foreign government spending.

We stress that these results rely on the assumption of perfectly rigid prices. Important differences arise when nominal rigidities are less extreme and some price adjustment is possible, as in the model of Section 3. Focusing on the global liquidity trap equilibrium with identical Phillips curves ($\kappa_0^* = \kappa_0$ and $\kappa_1^* = \kappa_1$) so that $\xi = \xi^* = \xi^w$, we have

$$\begin{aligned}
S &= \frac{\xi^w - \gamma_G}{\xi^w - \gamma_G^*} \\
\xi^w - \gamma_G &= \frac{x\theta d + (1-x)\frac{\xi^w-\gamma_G}{\xi^w-\gamma_G^*}\theta d^* + \frac{x\theta\delta\gamma_G + (1-x)\theta\delta^*\gamma_G^*\frac{\xi^w-\gamma_G}{\xi^w-\gamma_G^*}}{\rho-\pi_H}}{1-\frac{\theta\bar{\delta}}{\rho-\pi_H}}, \\
\pi^w &= -\kappa_0 - \kappa_1(1-\xi^w), \\
\frac{NFA}{X} &= \frac{\xi^w(1-\frac{\delta\theta}{\rho-\pi^w}) - \gamma_G - \theta d}{g+\theta+\pi^w}, \\
\frac{CA}{X} &= g\frac{NFA}{X}.
\end{aligned}$$

For a given E, in response to an increase in home government spending, a transition to a new steady state where the home terms of trade is more appreciated takes place over time, during which home inflation runs higher than foreign inflation. In the short run, home output increases more than one-for-one i.e. with a Keynesian government spending multiplier greater than one, and foreign output increases, for the same reasons as above and for the additional reason that inflation increases, reducing the real interest rate, further stimulating output, further increasing inflation etc... ad infinitum. The home Net Foreign Asset position decreases, and the home Current Account is pushed toward a deficit.

In the long run home and foreign output increase by the same amount, because the appreciation of the home terms of trade rebalances demand toward foreign goods through an expenditure switching effect, and global output increases more than one-for-one (i.e. with a multiplier greater than one). The appreciation of the home terms of trade results in a further deterioration of the home Net Foreign Asset Position and a further increase in the home Current Account deficit.[12]

5 Extensions

We present several extensions in the appendix, which we briefly summarize here.

[12]These effects are similar to the ones derived in Farhi and Werning (2012) in the context of a standard open-economy New-Keynesian model of a currency union confronted with a temporary liquidity trap.

Our baseline model features no home bias, a unitary trade elasticity (the elasticity of substitution σ between home and foreign goods), and only differences in financial development (asset supply) across countries. We relax these assumptions and analyze the effect of home bias (Appendix A.1), a higher trade elasticities (Appendix A.2), and differences across countries in their propensities to save (asset demand), which we capture with a model featuring within-country heterogeneity between borrowing-constrained borrowers and savers (Appendix A.3).[13] A rough characterization of our results is that in a global liquidity trap, a high degree of home bias mitigates the impact of the exchange rate on the allocation of the global liquidity trap, Net Foreign Asset positions, and Current Accounts, while large trade elasticities exacerbate these impacts. Tighter credit constraints (because of lower financial development or an asymmetric deleveraging shock) or a smaller fraction of income accruing to borrowers (because of aging) in one country depress world interest rates, push toward current account surpluses in that country, and can send the global economy into a liquidity trap.

Our baseline model studies a stationary environment where the global economy can experience a permanent liquidity trap. We consider the effect of expected transitions, either out of the current liquidity trap into a good state (Appendix A.4), or into a bad state (Appendix A.5).

Appendix A.4 allows us to discuss the role of *expected exchange rate movements* in creating *real interest rate differentials*. In the presence of home bias, this raises the possibility of a more asymmetric situation than the one encountered in the base model, where one country, the currency of which is expected to appreciate, is in a liquidity trap and has zero real interest rates, while the other is not in a liquidity trap and has positive real interest rates.

Appendix A.5 allows us to introduce a distinction between *safe assets* and *risky assets*, and focuses on the implications of safe asset shortages rather than asset shortages in general.

[13] As mentioned earlier, it is also possible to model differences in propensities to save across countries as differences in θ. Appendix A.3 instead introduces within country heterogeneity between borrowers and savers. The tightness of credit constraints reflects a country's financial development, and can be affected by deleveraging shocks, as in Eggertsson et al. (2015). Identifying borrowers with the young and savers with the middle-aged and the old, the relative importance of borrowers and savers can be used to capture a country's demographics. These features can generate differences in each country's propensity to save and in asset demand across countries, yet the model remains tractable.

We do so by relaxing the risk neutrality assumption, which allows us to refine our view along three dimensions. First, asset shortages are concentrated in safe assets, giving a prominent role a country's securitization and safe-asset tranching capacity. It also requires taking a more granular view of external accounts by disaggregating them by asset class, with the Net Foreign Asset positions and Current Accounts in safe assets playing the center stage in propagating recessions. Second, it leads to a possible rationalization of the *exorbitant privilege*, whereby a country with a large securitization capacity runs a permanent negative Net Foreign Asset Position and a Current Account deficit. Third, it gives rise to a *risk premium in UIP*, leading to the possibility of asymmetric safety traps equilibrium with *real interest rate differentials* and a new version of the *reserve currency paradox*.

6 Final Remarks

World interest rates and global imbalances go hand in hand: countries with large (small) safe asset shortages run Current Account surpluses (deficits) and push the world interest rate down (up). At the ZLB, the global asset market is in disequilibrium: there is a global safe asset shortage which cannot be resolved by lower world interest rates. It is instead dissipated by a world recession, which is propagated by global imbalances: Current Account surplus (deficit) countries push world output down (up), exerting a negative (positive) externality on the world economy. Economic policy enters a regime of increased interdependence across the world, with either negative or positive spillovers depending on the policy instrument. Exchange rate policy becomes a zero-sum game of currency wars where each country seeks to depreciate its exchange rate to stimulate its economy, at the expense of other countries. In contrast, safe public debt issuances and increases in government spending are positive-sum and benefit other countries beyond the frontier of the issuer. Our Metzler Diagram in Quantities is a powerful new tool to crystallize the economics of global imbalances and currency wars at the ZLB.

Unfortunately, this state of affairs is not likely to go away any time soon. In particular, there are no good substitutes in sight for the role played by US Treasuries in satisfying global safe asset demand. With mature US growing at rates lower than those of safe asset demander countries (as highlighted by The Economist, October 1, 2015), its debt and currency are likely to remain under upward pressure, dragging down (safe) interest rates and inflation, and therefore keeping the world economy (too) near the dangerous ZLB zone.

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A Appendix

A.1 Home Bias

We assume that the spending share on home goods of home agents is $x + (1-x)\beta$, and similarly that the spending share on foreign goods of foreign agents is $x^* + (1-x^*)\beta$, where $x^* = 1-x$ and $\beta \in [0,1]$ indexes the degree of home bias. Full home bias corresponds to $\beta = 1$. The case of no home bias analyzed previously corresponds to $\beta = 0$.

With home bias in preferences, the good market clearing conditions (4a) and (4b) become:

$$[x + (1-x)\beta]\theta W + (1-x^*)(1-\beta)\theta W^* E = \xi X, \tag{A.1a}$$

$$(1-x)(1-\beta)\theta W + [x^* + (1-x^*)\beta]\theta W^* E = E\xi^* X^*. \tag{A.1b}$$

This applies both under financial integration and under financial autarky, whether Home is in a liquidity trap ($\xi < 1$) or not ($\xi = 1$), and similarly whether Foreign is in a liquidity trap ($\xi^* < 1$) or not ($\xi^* = 1$).

For conciseness, we only consider the case where there is a global liquidity trap under financial integration. In that case, just like in the case of no home bias, there is a degree of indeterminacy indexed by the exchange rate E.

The asset and wealth dynamic equations (8a)-(8d) are unchanged. After simple manipulations, we can express all equilibrium variables as a function of the nominal exchange rate:

$$\xi = \frac{\theta \bar{d}^\beta(E)}{1 - \frac{\bar{\delta}^\beta \theta}{\rho}}, \tag{A.2a}$$

$$\frac{NFA}{X} = \frac{(1-\frac{\delta\theta}{\rho})[\frac{\theta\bar{d}^\beta(E)}{1-\frac{\bar{\delta}^\beta\theta}{\rho}} - \frac{\theta d}{1-\frac{\delta\theta}{\rho}}]}{g+\theta} = \frac{(1-\frac{\delta\theta}{\rho})(\xi - \xi^{a,l})}{g+\theta},$$

$$\frac{CA}{X} = g\frac{NFA}{X},$$

where we have defined the averages modified by home bias β as

$$\bar{d}^\beta(E) = \frac{[\beta + (1-\beta)x\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)]\theta d + (1-x)(1-\beta)\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)E\theta d^*}{[\beta + (1-\beta)x\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)] + (1-x)(1-\beta)\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)},$$

$$\bar{\delta}^\beta = \frac{[\beta + (1-\beta)x\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)]\left(1 - \frac{\delta\theta}{\rho}\right) + (1-x)(1-\beta)\frac{1+\frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\left(1 - \frac{\delta^*\theta}{\rho}\right)}{[\beta + (1-\beta)x\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)] + (1-x)(1-\beta)\left(1 + \frac{\frac{\delta^*\theta}{\rho} + \frac{\theta}{g}}{1-\frac{\delta^*\theta}{\rho}}\right)}.$$

and where $\xi^{a,l}$ is defined exactly as in the case with no home bias, and given by the same formula, equation (13a). Equations (A.2a) and its equivalent for the foreign country show that, as before, home output ξ is increasing in the exchange rate E while foreign output ξ^* is decreasing in E. Finally, as before, the home Net Foreign Asset Position and Current Account are increasing in the gap between the domestic recession and the home financial autarky recession $\xi^{a,l}$. *The key difference introduced by home bias β is that the home and foreign outputs ξ and ξ^* become less responsive to the exchange rate E.* This can be seen directly by examining (A.2a) in the case of home bias ($\beta > 0$) and comparing to (12a) in the case with no home bias ($\beta = 0$). This effect is seen most transparently in the limit with full home bias ($\beta \to 1$) in which case the outputs ξ and ξ^* become completely insensitive to the exchange rate E.

Assume further that both countries are in a liquidity trap under financial autarky. Then, just like in the case with no home bias, the integrated equilibrium coincides with financial autarky when $E = E^a$. For

$E > E^a$, we have $\xi > \xi^a$ and $\xi^* < \xi^a$ and vice versa for $E < E^a$.[14]

A.2 Trade Elasticities

We now assume away home bias and investigate instead the role of the elasticity of substitution σ between home and foreign goods. The main difference in the system of equilibrium equations is once again the goods market clearing conditions, which become

$$\frac{x}{x + E^{1-\sigma}(1-x)}(W + EW^*) = \frac{\xi X}{\theta},$$
$$\frac{(1-x)E^{1-\sigma}}{x + E^{1-\sigma}(1-x)}(W + EW^*) = E\frac{\xi^* X^*}{\theta}.$$

This applies both under financial integration and under financial autarky, whether Home is in a liquidity trap ($\xi < 1$) or not ($\xi = 1$), and similarly whether Foreign is in a liquidity trap ($\xi^* < 1$) or not ($\xi^* = 1$). This implies that we now have

$$E = \hat{E}^{\frac{1}{\sigma}}$$

where $\hat{E}$ is a renormalized exchange rate given by

$$\hat{E} = \frac{\xi}{\xi^*}.$$

The analysis under financial autarky is identical to the case $\sigma = 1$ except for the value of the financial autarky exchange rate. Unfortunately, the analysis under financial integration, although conceptually straightforward, leads to a nonlinear system of equations which is not amenable to a closed form solution.

Things simplify in the limit $\sigma \to \infty$, where the goods become perfect substitutes, to which we now turn. For conciseness, we only treat the case where there is a global liquidity trap under financial integration. As we take the limit $\sigma \to \infty$, we have $E = 1$, but there is still a degree of indeterminacy indexed by the renormalized exchange rate $\hat{E}$. Indeed, we can compute all the equilibrium variables as a function of $\hat{E}$:

$$\xi = \frac{x\theta d + (1-x)\theta d^*}{x(1 - \frac{\delta\theta}{\rho}) + \frac{1}{\hat{E}}(1-x)(1 - \frac{\delta^*\theta}{\rho})},$$
$$\frac{NFA}{X} = \frac{(1 - \frac{\delta\theta}{\rho})\xi - \theta d}{g + \theta} = \frac{(1 - \frac{\delta\theta}{\rho})(\xi - \xi^{a,l})}{g + \theta},$$
$$\frac{CA}{X} = g\frac{NFA}{X},$$

where $r^{w,n}$ and $\xi^{a,l}$ are defined exactly as in the unitary elasticity case, and are given by the same formulas. Home output ξ is increasing in the renormalized exchange rate $\hat{E}$, and foreign output is decreasing in the renormalized exchange rate $\hat{E}$. Finally, the home Net Foreign Asset Position is increasing in the gap between home output and home financial autarky output $\xi^{a,l}$ under zero home nominal interest rates. *The key difference introduced by $\sigma > 1$ over $\sigma = 1$ is that home and foreign outputs ξ and ξ^* become more responsive to the exchange rate E.* Indeed in the limit $\sigma \to \infty$, ξ and ξ^* become infinitely sensitive to the exchange rate E. In other words, larger trade elasticities magnify the stimulative effect of an exchange rate depreciation on the home recession.

Assume further that both countries are in a liquidity trap under financial autarky. Then, just like in the case with $\sigma = 1$, the financially integrated equilibrium coincides with financial autarky when $\hat{E} = \hat{E}^a$. For $\hat{E} > \hat{E}^a$, we have $\xi > \xi^a$ and $\xi^* < \xi^a$ and vice versa for $\hat{E} < \hat{E}^a$.

[14] One can readily check that the range $\left[\underline{E}, \bar{E}\right]$ increases with β, so that the model with home bias admits a larger range of indeterminacy. In the limit of full home bias, any value of the exchange rate is admissible.

A.3 Within Country Heterogeneity: Borrowers and Savers

Here we work out a version of our model incorporating within country heterogeneity between borrowers and savers. For simplicity, we abstract away from public debt by setting $D/X = D^*/X^* = 0$.

We add a mass of borrowing constrained impatient borrowers (B) agents. The rest of the agents are savers (S) and are modeled as before. Borrowers consume as much as possible when they are born, and the rest when they die. They only get an endowment when they die, and they can only pledge a part of it. They must therefore borrow in order to consume when born. They then roll over their debt until they die, at which point they use their income to repay their debt and consume the remainder. In a small interval dt, a part $\eta\xi_t X_t dt$ of total income accrues to dying borrowers in the form of labor income. Because of the borrowing constraint, borrowers born in the interval dt can only consume $\chi X_t dt$, where we imagine that χ is small compared to η.[15] We assume that the new trees accrue to savers.

Note that there is now a distinction between financial wealth and human wealth for borrowers. Indeed a borrower receives income when he dies. This future income is a form a human wealth and is not part of his financial wealth. When the borrower dies, this human wealth allows him to repay the debt that he has incurred to borrow when he was born and rolled over until his death (his financial wealth), and to consume the residual.

The evolution equations for the financial wealth of borrowers and savers are given by:[16,17]

$$\begin{aligned}
gW^B &= -\theta W^B - \chi X + r^w W^B, \\
gW^S &= -\theta W^S + (1-\delta-\eta)\xi X + r^w W^S + (\rho+g)V, \\
gW^{B*} &= -\theta W^{B*} - \chi^* X^* + r^w W^{B*}, \\
gW^{S*} &= -\theta W^{S*} + (1-\delta-\eta)\xi^* X^* + r^w W^{S*} + (\rho+g)V^*.
\end{aligned}$$

We continue to denote by $W = W^B + W^S$ total home wealth and by $W^* = W^{B*} + W^{S*}$ total foreign wealth and obtain the evolution equations for total wealth by aggregating the evolution equations for wealth by borrowers and savers in both countries:

$$\begin{aligned}
gW &= -\theta W + (1-\delta-\eta)\xi X - \chi X + r^w W + (\rho+g)V, \\
gW^* &= -\theta W^* + (1-\delta^*-\eta^*)\xi^* X^* - \chi^* X + r^w W + (\rho+g)V^*.
\end{aligned}$$

[15] Note that, as in Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012), the borrowing limit χX does not depend on whether the economy is in recession. This assumption is crucial to generate a liquidity trap, as it implies that the debt issued by borrowers does not scale with output, so asset demand declines faster than asset supply in the recession. While the assumption that the credit constraint is invariant to the recession is perhaps extreme, all that is needed for our result to go through is that the borrowing limit does not scale one for one with output.

[16] Let us for example explain in details the wealth evolution equation for borrowers at Home. The wealth of borrowers is negative $W^B < 0$, it represents their debt. In an interval dt, the wealth of borrowers W^B changes because of because of dying borrowers repaying their debt ($-\theta W^B dt$), because of newborn borrowers taking on new debt ($-\chi X dt$), and because of the accumulation of interest ($r^w W^B$). In a steady state, the wealth of borrowers W^B must also change by $gW^B dt$. This gives the wealth evolution equation for borrowers.

[17] Note that the wealth of borrowers does not take into account the income of borrowers when they die, because it is not part of their financial wealth. But of course, the income of borrowers influences their consumption when they die. It therefore appears in the goods market clearing conditions. This explains the terms $\eta\xi X$ and $E\eta^*\xi^* X^*$ in the market clearing conditions at Home and in Foreign.

The good market clearing conditions are now given by:[18]

$$\begin{aligned}\xi X &= x(\theta(W+EW^*)+\eta\xi X+E\eta^*\xi^*X^*+\chi X+E\chi^*X^*)\\ E\xi^*X^* &= (1-x)(\theta(W+EW^*)+\eta\xi X+E\eta^*\xi^*X^*+\chi X+E\chi^*X^*).\end{aligned}$$

The asset pricing equations are unchanged:

$$\begin{aligned}r^wV &= -\rho V+\delta\xi X,\\ r^wV^* &= -\rho V^*+\delta^*\xi^*X^*.\end{aligned}$$

And we must still impose $r^w \geq 0$, $0 \leq \xi \leq 1$, $0 \leq \xi^* \leq 1$, and the complementary slackness conditions $r^w(1-\xi)=0$ and $r^w(1-\xi^*)=0$.

In the interest of space, we only treat the liquidity trap case. We get:

$$\begin{aligned}E &= \frac{\xi}{\xi^*},\\ \xi &= \frac{\bar{\chi}(E)}{1-\bar{\eta}-\frac{\bar{\delta}\theta}{\rho}},\\ \frac{NFA}{X} &= \frac{\frac{1-\eta-\frac{\delta\theta}{\rho}}{1-\bar{\eta}-\frac{\bar{\delta}\theta}{\rho}}\bar{\chi}(E)-\chi}{g+\theta},\\ \frac{CA}{X} &= g\frac{NFA}{X}.\end{aligned}$$

where for any variable z, we use the notation $\bar{z}(E)=xz(E)+(1-x)z^*(E)$.

The variable χ (χ^*) increases with home (foreign) financial development, and decreases with a home (foreign) deleveraging shock. Identifying the borrowers with the young and the savers with the middle-aged and the old, proportional decreases in the variables η and χ (η^* and χ^*) capture home (foreign) population aging.

These equations indicate that a deleveraging shock at Home (a decrease in χ) or in Foreign (a decrease in χ^*) can push the global economy into a liquidity trap. For a given exchange rate E, the larger the world deleveraging shock, the larger the recession in any given country. For a given exchange rate E and world deleveraging shock $\bar{\chi}(E)$, a larger home deleveraging shock (a lower χ) pushes the home Current Account towards a surplus.

Similarly, aging at Home (a proportional decrease in χ and η) or in Foreign (a proportional decrease in χ^* and η^*) can push the global economy in a liquidity trap. For a given exchange rate E, the larger the shock, the larger the recession in any given country. For a given exchange rate E, more aging at Home pushes the Home Current Account towards a surplus.

This analysis also shows how countries with tighter credit constraints or lower fraction of income accruing to borrowers act as if they had a larger asset demand (lower θ).

A.4 Recovery: Exchange Rates Movements and Interest Rates Differentials

We start with the model with permanently rigid prices, and consumption home bias of section A.1. For simplicity, we assume that there is no public debt so that $D/X = D^*/X^* = 0$.

[18]For example, the home market clearing condition can be understood as follows. The demand arising from dying savers is given by $x\theta(W-W^B+EW^*-EW^{B*})$. The demand arising from newborn borrowers is given by $x(\chi X+E\chi^*X^*)$. And the demand arising from dying borrowers is given by $x(\eta\xi X+\theta W^B+E\eta^*\xi^*X^*+E\theta W^{B*})$.

We then assume that a Poisson shock occurs with instantaneous probability $\lambda > 0$. When the Poisson shock occurs, the fraction of output δ that accrues in the form of dividends jumps instantaneously and permanently by a factor $\nu > 1$ in both countries. This alleviates the asset shortage and increases the world natural interest rate. We assume that ν is large enough that upon the realization of the Poisson shock the world natural interest rate rises above zero: $-\rho + \nu\bar{\delta}\theta > 0$. This implies that the economy may experience a liquidity trap *before* the Poisson shock, but never *after* it.

The steady state of the post-Poisson shock economy is uniquely determined, and so are its dynamics from any initial position.[19] By backward induction, this means that the exchange rate during the liquidity trap phase is also pinned down, conditional on the exchange rate E_τ that occurs at the time τ of the realization of the Poisson shock. This removes the indeterminacy in the nominal exchange rate a la Kareken and Wallace (1981) that we found in the previous sections.

But another form of indeterminacy appears which we can index by the exchange rate E_τ. This is because, in our model, agents are risk neutral, so that international portfolios are indeterminate. Yet, a given portfolio allocation will determine relative wealths immediately after the Poisson shock. In the presence of home bias in consumption, this pins down relative demands for Home and Foreign goods and therefore the nominal exchange rate E_τ. Conversely, for a given value E_τ, one can construct international portfolios that are consistent with this value of the exchange rate at the time of the Poisson shock. We summarize by writing domestic and foreign wealth and asset values at the time of the shock as $W_\tau = w_\tau X/\theta$, $W_\tau^* = w_\tau^* X^*/\theta$, $V_\tau = v_\tau X/\theta$, and $V_\tau^* = v_\tau^* X^*/\theta$ where it is understood that the coefficients w_τ,w_τ^*, v_τ and v_τ^* are functions of the exchange rate E_τ at the time of the shock.[20]

We focus on the stochastic steady state before the Poisson shock. Because of the jump in the exchange rate at the time of the Poisson shock, Home and Foreign typically experience different real interest rates. This is in contrast to our baseline model where real interest rates are always equalized across countries. To see this most clearly, note that financial integration imposes that in the stochastic steady state prior to the Poisson shock, we have the following UIP equation:

$$r = r^* + \lambda(\frac{E_\tau}{E} - 1). \tag{A.5}$$

This implies that the home interest rate $r < r^*$ if the home currency is expected to appreciate after the Poisson shock $E_\tau/E < 1$.

The asset pricing equations include news terms accounting for capital gains and losses triggered by the realization of the Poisson shock:

$$rV = -\rho V + \delta\xi X + \lambda\left(V_\tau - V\right),$$

$$r^*V^* = -\rho V^* + \delta^*\xi^* X^* + \lambda\left(V_\tau^* - V^*\right).$$

For example, a higher value of home assets V_τ after the Poisson shock increases the value of home assets V in the stochastic steady state before the Poisson shock.

The wealth accumulation equations include new terms accounting for the risk and return of each country's portfolio:

$$gW = -\theta W + (1-\delta)\,\xi X + rW + \lambda\left(W - W_\tau\right) + (g+\rho)\,V,$$

$$gW^* = -\theta W^* + (1-\delta)\,\xi^* X^* + r^*W^* + \lambda\left(W^* - W_\tau^*\right) + (g+\rho)\,V^*.$$

For example, a lower value of home wealth W_τ after the Poisson shock means that home agents have a riskier portfolio, and therefore collect higher returns as long as the Poisson shock does not materialize. This in turn increases home wealth W in the stochastic steady state before the Poisson shock.

The goods market clearing equations (A.1a) and (A.1b) are unchanged, and we must still impose $r \geq 0$, $r^* \geq 0$, $0 \leq \xi \leq 1$, $0 \leq \xi^* \leq 1$, and the complementary slackness conditions $r(1-\xi) = 0$ and $r^*(1-\xi^*) = 0$.

The jump in the exchange rate at the time of the Poisson shock opens the door to the possibility that

[19] One can verify that the dynamics of the economy are saddle-path stable.

[20] While this is outside the model, we can think of the indeterminacy in E_τ as being resolved by a form of forward guidance which effectively coordinates exchange rate expectations and the portfolios consistent with this exchange rate.

Home and Foreign may not experience a liquidity trap simultaneously prior to the shock. Real interest rates can differ across countries, resulting in the possibility of more strongly asymmetric liquidity trap equilibria than those we have encountered so far, where one country has zero nominal interest rates, zero real interest rates and a recession, while the other country has positive nominal interest rates, positive real interest rates, and no recession.

Going back to the UIP equation (A.5), we see that for Home to be the only country in a liquidity trap, we need $r = 0$, $\xi < 1$, $r^* > 0$, $\xi^* = 1$ and $\lambda\left(E_\tau/E - 1\right) = -r^* < 0$. This requires that the home exchange rate appreciate at the time of the shock, $E_\tau < E$. We focus on this configuration from here onwards.

Home output ξ is then given by

$$\xi = \frac{\beta\frac{g-\lambda}{g-\lambda+\theta}\frac{\lambda v_\tau - \frac{(\rho+\lambda)\lambda}{g-\lambda}(w_\tau - v_\tau)}{\rho+\lambda} + x^*(1-\beta)E}{[1-\frac{\beta\frac{g-\lambda}{g-\lambda+\theta}}{\beta\frac{g-\lambda}{g-\lambda+\theta}+x^*(1-\beta)}\frac{\delta\theta}{\rho+\lambda}][\beta\frac{g-\lambda}{g-\lambda+\theta}+x^*\left(1-\beta\right)]}. \tag{A.6a}$$

This equation shows that everything else equal, as long as there is home bias $\beta > 0$, a higher value v_τ/θ of the home asset after the Poisson shock, and a lower value of the home Net Foreign Asset position after the Poisson shock $(w_\tau^* - v_\tau^*)/\theta$, contribute to a lower home output. Both increase the value of home wealth before the Poisson shock and, because of home bias, of the demand for home goods. The reason is that a higher value of v_τ/θ increases the value of new trees, and that a lower value of $(w_\tau^* - v_\tau^*)/\theta$ indicates that home agents take more risk, and are hence rewarded by a higher return before the Poisson shock.

The foreign interest rate r^* and the exchange rate are then given by the following system of nonlinear equations

$$0 = r^* + \lambda(\frac{E_\tau}{E} - 1),$$

$$1-(\frac{\xi}{E}-1)x\frac{1-\beta}{\beta} = \frac{\theta + \frac{g-\lambda-r^*}{\rho+\lambda+r^*}(\delta^*\theta + \lambda v_\tau^*) - \lambda\left(w_\tau^* - v_\tau^*\right)}{g-\lambda+\theta-r^*},$$

where we use the equation above to express ξ as a function of E. This is an equilibrium as long as $\xi < 1$ and $r^* \geq 0$.

We can also compute

$$\frac{NFA}{X} = \frac{\xi(1-\frac{\delta\theta}{\rho+\lambda}) - \frac{\lambda v_\tau}{\rho+\lambda} - \lambda\frac{\theta}{g-\lambda}(w_\tau - v_\tau)}{g-\lambda+\theta} = \frac{1-\frac{\delta\theta}{\rho+\lambda}}{g-\lambda+\theta}(\xi - \hat{\xi}^{a,l}),$$

$$\frac{CA}{X} = g\frac{NFA}{X}.$$

where $\hat{\xi}^{a,l}$ is home output in the equilibrium where Home is in financial autarky before the Poisson shock, but not after the Poisson shock (and where the equilibrium coincides with that under consideration after the Poisson shock).[21] Financial integration before the Poisson shock increases output $\xi \geq \hat{\xi}^{a,l}$ if and only if the exchange rate is more depreciated $E \geq \hat{E}^{a,l}$.[22]

A.5 Fear: Safe Assets and Risk Premia

As in Section A.4, we assume that a Poisson shock occurs with instantaneous probability $\lambda > 0$ and we assume away public debt $D/X = D^*/X^* = 0$. However the nature of the shock is different. When the

[21] By financial autarky we mean that Net Foreign Asset positions at Home and in Foreign are equal to 0. We allow countries to trade actuarially fair insurance contracts on the realization of the Poisson shock. These contracts have zero ex-ante value for both home and foreign agents.

[22] The analysis simplifies drastically in the limit of full home bias ($\beta \to 1$). In this case, we have $\xi^a = \xi^{a,l}$, $w_\tau = v_\tau = w_\tau^* = v_\tau^* = 1$. This implies that ξ and r^* are given by their financial autarky values $\xi = \xi^{a,l}$, $r^* = r^{*a,n} = \delta^*\theta - \rho$, and Net Foreign Asset Positions and Current Accounts are zero $\frac{NFA}{X} = \frac{CA}{X} = 0$. This is an equilibrium if and only if $r^{*a,n} \geq 0$ and $r^{a,n} \leq 0$ (which is equivalent to $\xi^{a,l} \leq 1$).

Poisson shock occurs, output in both countries drops instantaneously and permanently by a factor $\mu < 1$. The possibility of an adverse future shock reduces the supply of assets, depressing the world natural interest rate before the Poisson shock below its value after the Poisson shock. We assume that $\delta = \delta^*$ so that there are no differences between Home and Foreign in their ability to pledge future output into current assets. Instead we focus on a different dimension of financial development, the capacity to securitize assets into safe and risky tranches. We assume that this securitization capacities, denoted ϕ and ϕ^* and defined formally below, differs across countries.

Neutrals and Knightians. Following Caballero and Farhi (2015), we allow for a fraction α of agents in each country to be Knightian (infinitely risk averse over short time intervals). The remaining fraction $1-\alpha$ of agents is Neutral (risk neutral over short time intervals) as in the benchmark model. We assume that Knightians have full home bias: they only consume the goods of their own country. This implies that domestic Knightians only value financial assets whose payoffs are constant in the Home good numeraire. By contrast, Neutrals have an interior degree of home bias indexed by β as in Section A.1. The preferences of Knightians and Neutrals at Home are given by the following stochastic differential equations

$$\begin{aligned} U_t^K &= 1_{\{t-dt \le \tau_\theta < t\}} c_{H,t} + 1_{\{t \le \tau_\theta\}} \min{}_t \{U_{t+dt}^K\}. \\ U_t^N &= 1_{\{t-dt \le \tau_\theta < t\}} c_{H,t}^{\gamma} c_{F,t}^{1-\gamma} + 1_{\{t \le \tau_\theta\}} \mathbb{E}_t[U_{t+dt}^N], \end{aligned}$$

where $\gamma = x + \beta(1-x)$. Similar equations hold for Foreign with $1-\gamma^* = x^* + \beta(1-x^*)$.

Tranching and securitization. In the presence of Knightians, equilibrium portfolios are no longer indeterminate. Knightians shield themselves from aggregate risk, which is instead borne by Neutrals. Neutrals are of course compensated for the risk that they are bearing in the form of a positive risk premium. This allocation is achieved through an imperfect process of securitization of Lucas trees, which takes the following form.

At any point in time, in each country, a fraction of the existing Lucas trees can be arbitrarily tranched into Arrow-Debreu securities which can then be arbitrarily traded and recombined. These Arrow-Debreu securities cannot be sold short. The remaining fraction of the trees cannot be tranched, and can only be traded as a whole. Countries differ in their ability to generate tranched trees, and this is the only difference between the two counties. The fraction of tranched trees at Home is ϕ, and it is ϕ^* in Foreign.

At any point in time, Knightians invest all their wealth in safe assets, which they synthesize by constructing a portfolio with the right mix of Arrow-Debreu securities originating from the securitization of home and foreign tranched trees. Home (resp. foreign) Knightians synthesize home (resp. foreign) safe assets, i.e. assets that are riskless in the home (resp. foreign) good numeraire. At any point in time, Neutrals invest their wealth in portfolios of untranched trees and Arrow-Debreu securities originating in the securitization process of tranched trees.

It will prove convenient for our analysis to classify combinations of Arrow-Debreu securities into two categories. We use the term Poisson puts to denote combinations of Arrow-Debreu securities which pay off zero dividends until the Poisson shock realizes, and positive dividends after the Poisson shock realizes. Similarly, we use the term Poisson calls to denote Arrow-Debreu securities which only pay off positive dividends before the Poisson shock realizes, but zero dividends after the Poisson shock realizes. Poisson puts can be combined with Poisson calls in different proportions to create home safe assets demanded by home Knightians and foreign safe assets demanded by foreign Knightians.

The limit $\lambda \to 0$. For simplicity, we study the limit $\lambda \to 0$. Because of the presence of Knightians, the Poisson shock matters even in this limit where its intensity becomes vanishingly small. We also assume that new trees accrue only to Neutrals, an assumption which simplifies the analysis but does not matter for our substantive results.

Equilibrium equations. We focus on stochastic steady states throughout. The equilibrium variables are as follows. First, there are the risk free interest rates r^K and r^{K*} in the home and foreign numeraires. Second, there is the risky interest rate r^w which is the same in the home and foreign numeraires (since we are working in the limit $\lambda \to 0$). Third, there are the values of home and foreign Poisson puts V^S and V^{S*}, and the values of home and foreign assets V and V^*. Fifth, there are the values of home and foreign Knightian wealth W^K and W^{K*}, as well as the value of home and foreign Neutrals wealth W^N and W^{N*}. Sixth, there are the home and foreign outputs ξ and ξ^*. We treat the value of the exchange rate E_τ after the Poisson shock as a parameter exactly as in Section A.4: different values can be rationalized by different portfolios for Neutrals at Home and in Foreign.

The equilibrium equations are as follows. First, there are the asset pricing equations for home and foreign assets

$$V = V^S + \frac{\delta \xi X}{r^w + \rho}, \tag{A.7a}$$

$$V^* = V^{S*} + \frac{\delta \xi^* X^*}{r^w + \rho}, \tag{A.7b}$$

and the asset pricing equations of the Poisson puts

$$V^S = \frac{r^w - r^K}{r^w} \mu \phi v_\tau \frac{X}{\theta}, \tag{A.8a}$$

$$V^{S*} = \frac{r^w - r^{K*}}{r^w} \mu \phi^* v_\tau^* \frac{X^*}{\theta}, \tag{A.8b}$$

where the value of home and foreign assets after the Poisson shock is $v_\tau \frac{\mu X}{\theta}$ and $v_\tau^* \frac{\mu X^*}{\theta}$ (where both v_τ and v_τ^* can be written as functions of E_τ as explained in Section A.4).

The logic behind the first two equations is as follows. Consider a particular date t. Home assets are composed of home Poisson puts, home Poisson calls, and untranched home trees. Home Poisson puts are worth V^S. Poisson calls constitute claims to a future stream of dividends $\phi \delta X e^{-\rho(s-t)} ds$ in any small interval ds until the Poisson shock realizes, and 0 after the Poisson shock realizes. They are worth $\phi \frac{\delta X}{r^w + \rho}$. Home trees that cannot be tranched constitute claims to a future stream of dividends $(1-\phi)\delta X e^{-\rho(s-t)} ds$ in any small interval ds until the Poisson shock realizes, and $(1-\phi)\mu\delta X e^{-\rho(s-t)} ds$ after the Poisson shock realizes. They are worth $(1-\phi)\frac{\delta X}{r^w+\rho}$.[23] The same applies to Foreign. Equations (A.7a) and (A.7b) follow.

The logic behind the last two equations is as follows. Consider a particular date t. Home Poisson puts can be combined with Poisson calls to create safe assets from the perspective of home Knightians worth $\mu \phi v_\tau \frac{X}{\theta}$—a fraction ϕ of the value $\mu v_\tau \frac{X}{\theta}$ of home assets after the Poisson shock. The required Poisson calls represent a constant future stream of dividends $r^K \mu \phi v_\tau \frac{X}{\theta} ds$ in any small time interval ds until the Poisson shock realizes. They are worth $\frac{r^K}{r^w} \mu \phi v_\tau \frac{X}{\theta}$. The value of home Poisson puts is the residual $\mu \phi v_\tau \frac{X}{\theta} - \frac{r^K}{r^w} \mu \phi v_\tau \frac{X}{\theta}$. The same applies to Foreign. Equations (A.8a) and (A.8b) follow.

Moreover, home Poisson puts can also be combined with Poisson calls to create safe assets from the perspective of foreign Knightians worth $\frac{E}{E_\tau} \mu \phi v_\tau \frac{X}{\theta}$. The required Poisson calls are worth $\frac{r^{K*}}{r^w} \frac{E}{E_\tau} \mu \phi v_\tau \frac{X}{\theta}$. The value of home Poisson puts is therefore also given by $\mu \phi v_\tau \frac{X}{\theta} - \frac{r^{K*}}{r^w} \frac{E}{E_\tau} \mu \phi v_\tau \frac{X}{\theta}$. This implies the following no-arbitrage condition

$$\frac{r^w - r^K}{r^w - r^{K*}} = \frac{E}{E_\tau}. \tag{A.9}$$

[23] The reason for this last statement is the following. Because these trees cannot be tranched, they are held by Neutrals, who do not value the associated dividends after the Poisson shock realizes because the intensity of the Poisson process λ is vanishingly small

Second, there are the wealth accumulation equations

$$
\begin{aligned}
gW^K &= -\theta W^K + \alpha(1-\delta)\xi X + r^K W^K, \\
gW^{K*} &= -\theta W^{K*} + \alpha(1-\delta)\xi^* X^* + r^{K*} W^{K*}, \\
gW^N &= -\theta W^N + (1-\alpha)(1-\delta)\xi X + r^w W^N + (\rho+g)V, \\
gW^{N*} &= -\theta W^{N*} + (1-\alpha)(1-\delta)\xi^* X^* + r^w W^N + (\rho+g)V^*.
\end{aligned}
$$

Third, there are the market clearing conditions

$$
\begin{aligned}
W^K + [x+\beta(1-x)]W^N + (1-x^*)(1-\beta)EW^{N*} &= \xi\frac{X}{\theta}, \\
EW^{K*} + (1-x)(1-\beta)W^N + [x^* + (1-x^*)\beta]EW^{N*} &= E\xi^*\frac{X^*}{\theta},
\end{aligned}
$$

Fourth, there is the condition that total Knightian wealth after the Poisson shock be less or equal than the total value of Poisson puts

$$
W^K + E_\tau W^{K*} \leq [x\phi v_\tau + x^*\phi^* E_\tau v_\tau^*]\mu\frac{X+X^*}{\theta}.
$$

Finally, we also have inequality requirements $r^w \geq \max\{r^K, r^{K*}\}$, $r^K \geq 0$, $r^{K*} \geq 0$, $0 \leq \xi \leq 1$, and $0 \leq \xi^* \leq 1$ together with the associated complementary slackness conditions.[24]

Unconstrained and constrained regimes. As explained in Caballero and Farhi (2015), there are different regimes in this model depending on whether the marginal holder of a Poisson put is a Neutral (unconstrained regime) or a Knightian (constrained regime). If it is a Neutral, then there are no risk premia $r^w = r^K = r^{K^*}$. If it is a Knightian, then there are risk premia $r^w > r^K$ and $r^w > r^{K*}$ and we have

$$
W^K + E_\tau W^{K*} = [x\phi v_\tau + x^*\phi^* E_\tau v_\tau^*]\mu\frac{X+X^*}{\theta}.
$$

We assume throughout that we are in the latter case, which always occurs if α is large enough, if μ is small enough or if ϕ and ϕ^* are small enough.[25]

A.5.1 Risk Premium in UIP and Reserve Currency Paradox

In this section, we zoom in on the exchange rate and characterize the deviation from UIP as a risk premium. We show that in a world of safe asset shortages, this can result in a reserve currency paradox, whereby a country whose currency is expected to appreciate in bad times is more at risk of experiencing a safety trap.

Risk Premium in UIP. We call the no-arbitrage equation (A.9) the "modified UIP" equation. It illustrates that the country with *the lowest safe interest rate* has a depreciated real exchange rate. The exchange rate is the ratio of the risk premium at Home to the risk premium abroad: a country with a high risk premium has a depreciated exchange rate. Another way to understand this equation is as follows. An

[24]The complementary slackness conditions are

$$
[[x\phi v_\tau + x^*\phi^* E_\tau v_\tau^*]\mu\frac{X+X^*}{\theta} - W^K - E_\tau W^{K*}][r^w - \max\{r^K, r^{K*}\}] = 0,
$$

$$
(1-\xi)r^K = 0, \quad (1-\xi^*)r^{K*} = 0.
$$

[25]This is a valid equilibrium if untranched trees are worth more to Neutrals than to Knightians in either country (which is always verified if μ is small enough).

investor can borrow one at r^K and invest at r^w, but that requires buying one unit worth of Poisson put as "collateral". Similarly, an investor can borrow one at r^{K*} and invest at r^w, but that requires $\frac{E_\tau}{E}$ units worth of Poisson put as collateral. Or in other words, an investor can borrow $\frac{E}{E_\tau}$ at r^{K*} and invest it at r^w, with one unit worth of Poisson put. Both strategies require one unit worth of Poisson put as collateral. They must net out the same return, which can be expressed as $\frac{E}{E_\tau}(r^w - r^{K*}) = r^w - r^K$.

Yet another way to interpret this expression is that the modified UIP equation relates the steady state deviation from UIP in our model to the risk premium. Indeed, we have an endogenous risk premium ψ in the UIP equation so that

$$r^K = r^{K*} + \psi + \frac{\dot{E}}{E}$$

where (in the stochastic steady state prior to the shock)

$$\frac{\dot{E}}{E} = 0,$$

$$\psi = (1 - \frac{E}{E_\tau})(r^w - r^{K*}).$$

We have $\psi < 0$ if an only if $E > E_\tau$, i.e. if and only if the exchange rate of Home is expected to appreciate after the bad Poisson shock.

Reserve currency paradox and asymmetric liquidity traps. In general we can have a safety trap in one country but not in the other. For example, Home can be in a safety trap with $r^K = 0$ and $\xi < 1$ while Foreign is not: $r^{K*} > 0$ and $\xi^* = 1$. In this case, going back to the modified UIP equation, the risk premium in UIP is negative $\psi < 0$ and the exchange rate appreciates when the bad Poisson shock occurs $E > E_\tau$. This is another version of the reserve currency paradox. If the home currency is a reserve currency, which is expected to appreciate in bad times, then the home risk free rate is lower, and Home can be in a safety trap even if Foreign is not.

A.5.2 Solving the Model with No Home Bias

The equilibrium equations can be boiled down to a system of nonlinear system of six equations in six unknowns E, r, r^K, r^{K*}, ξ, and ξ^*, which cannot be solved in closed form in general. This system is derived in the appendix. Things drastically simplify when there is no home bias, i.e. $\beta = 0$. We therefore maintain the assumption that there is no home bias from now on, unless explicitly stated otherwise.

Solving the equilibrium with no home bias. In the absence of home bias, we have $E_\tau = 1$, since the two economies are identical after the Poisson shock apart from their sizes (there is no more risk, and so the differences in securitization capacities become irrelevant). In that case, the economy jumps immediately to its long run steady state. We then have $E = 1$, $r^K = r^{K*} = r^{K,w}$, $\xi = \xi^* = \xi^w$, $v_\tau = v_\tau^* = 1$.

Outside of a safety trap, we have $r^{K,w} = r^{K,w,n}$, $r^w = r^{w,n}$, and $\xi^w = 1$, where

$$r^{K,w,n} = g + \delta\theta - (1-\delta)\theta\frac{\alpha - \bar{\phi}\mu}{\bar{\phi}\mu},$$

$$\frac{r^{w,n} - r^{K,w,n}}{r^{w,n}} = \frac{1}{\mu\bar{\phi}}\frac{g - r^{w,n}}{g - r^{w,n} + \rho}(1 - \frac{\delta\theta}{r^{w,n} + \rho}),$$

where $\bar{\phi}$ is the world average securitization capacity

$$\bar{\phi} = x\phi + (1-x)\phi^*.$$

The requirement that we are in the constrained regime is equivalent to $r^{w,n} > r^{K,w,n}$. And we are outside a safety trap if and only if $r^{K,w,n} > 0$. This happens when α, which indexes the demand for safe assets, is small enough compared to $\bar{\phi}\mu$, which indexes the supply for safe assets.

If we are in a safety trap, we have $r^{K,w} = 0$ and $\xi^w < 1$. This happens when $r^{K,w,n} < 0$, i.e. when α, which indexes the demand for safe assets, is large enough compared to $\bar{\phi}\mu$, which indexes the supply for safe assets. We then have

$$\xi^w = 1 + r^{K,w,n} \frac{\bar{\phi}\mu}{(1-\delta)\,\alpha\theta},$$

$$1 = \frac{\xi^w}{\mu\bar{\phi}} \frac{r^w - g}{r^w - g - \rho}(1 - \frac{\delta\theta}{r^w + \rho}).$$

The requirement that we are in the constrained regime is equivalent to $r^w > 0$. Importantly, we see that everything else equal, r^w is decreasing in ξ^w. In particular, an increase in the demand for safe assets, captured by an increase in α, results in a decrease in ξ^w and an increase in risky interest rates r^w.

Net Foreign Assets, Current Accounts, and the "exorbitant privilege". We can compute the home Net Foreign Asset Position and Current Account independently of wether Home, Foreign, or both are in a safety trap. Indeed, we show in the appendix that both in a safety trap and outside of a safety trap, we have

$$\begin{aligned} \frac{NFA}{X} &= \frac{(r^w + \rho - \theta)\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}}{\theta + g - r^w}(\phi - \bar{\phi}), \\ \frac{CA}{X} &= g\frac{NFA}{X}. \end{aligned}$$

Suppose that Home has a larger securitization capacity than Foreign $\phi > \phi^*$. In this case, we see that Home runs a negative Net Foreign Asset position and a Current Account deficit if $r^w + \rho - \theta < 0$, and a positive Net Foreign Asset position and a Current Account surplus otherwise. This is because there are two opposing effects of a larger securitization capacity. First, it increases the value of home assets. Second, it means that the wealth of home agents accumulates faster because of the larger value of the sales of new securitized trees. The first effect worsens the Net Foreign Asset position and Current Account, while the second effect improves them.

Since the strength of the second effect depends negatively on the propensity to consume θ and positively on the rate of depreciation ρ, the condition $r^w < \theta - \rho$ essentially bounds the strength of the second effect, and guarantees that the first effect dominates. It turns out that this condition is automatically verified in equilibrium.[26] Therefore, Home experiences a version of the "exorbitant privilege" documented by Gourinchas and Rey (2007): it is able to run a permanent negative Net Foreign Asset position and Current Account deficit because it pays a lower interest rates on its liabilities than on its assets.

Metzler diagram in safe assets. We can represent the equilibrium determination of the safe interest rate $r^{K,w}$ and of the recession ξ^w in a safety trap, through a *Metzler diagram in safe assets.* The key is to focus on a component CA^K of the Current Account, with corresponding Net Foreign Asset position NFA^K, in safe assets:

$$\begin{aligned} \frac{NFA^K}{X} &= \frac{W^K - \frac{\phi\mu X}{\theta}}{X} = \frac{\alpha(1-\delta)\xi^w}{\theta + g - r^{K,w}} - \frac{\phi\mu}{\theta}, \\ \frac{CA^K}{X} &= g\frac{NFA^K}{X}. \end{aligned}$$

Similar equations hold for Foreign

$$\frac{NFA^{K*}}{X} = \frac{W^{K*} - \frac{\phi^*\mu X^*}{\theta}}{X^*} = \frac{\alpha(1-\delta)\xi^w}{\theta + g - r^{K,w}} - \frac{\phi^*\mu}{\theta},$$

[26]Indeed, if it were violated, we would have $W^N < 0$ and $W^{N*} < 0$, which is impossible.

$$\frac{CA^{K*}}{X} = g\frac{NFA^{K*}}{X}.$$

We then have the following Metzler diagram representation

$$x\frac{NFA^{K}}{X} + (1-x)\frac{NFA^{K*}}{X} = 0.$$

If we are not in a safety trap with $\xi^w = 1$, this becomes a Metzler diagram for safe assets, which determines $r^{K,w} = r^{K,w,n} > 0$. If we are in a safety trap with $r^{K,w} = 0$, this becomes a Metzler diagram for safe assets in quantities, which determines output ξ^w.[27]

A.5.3 Derivations

We can rewrite the system equilibrium equations as

$$x\frac{\alpha(1-\delta)\xi}{\theta+g-r^K} + (1-x)E^\mu\frac{\alpha(1-\delta)\xi^*}{\theta+g-r^{K*}} = [x\phi v(E^\mu) + (1-x)\phi^* E^\mu v^*(E^\mu)]\frac{\mu}{\theta},$$

$$\frac{r^w - r^K}{r^w - r^{K*}} = \frac{E}{E^\mu},$$

$$\begin{aligned}\frac{\alpha(1-\delta)\xi}{\theta+g-r^K} &+ [x+\beta(1-x)]\frac{(1-\alpha)(1-\delta)\xi + (g+\rho)[\frac{r^w-r^K}{r^w}\frac{\mu}{\theta}\phi v(E^\mu) + \frac{\delta\xi}{r^w+\rho}]}{\theta+g-r^w}\\ &+ (1-x)(1-\beta)E\frac{(1-\alpha)(1-\delta)\xi^* + (g+\rho)[\frac{r^w-r^{K*}}{r^w}\frac{\mu}{\theta}\phi^* v(E^\mu) + \frac{\delta\xi^*}{r^w+\rho}]}{\theta+g-r^w} = \frac{\xi}{\theta},\end{aligned}$$

$$\begin{aligned}E\frac{\alpha(1-\delta)\xi^*}{\theta+g-r^{K*}} &+ x(1-\beta)\frac{(1-\alpha)(1-\delta)\xi + (g+\rho)[\frac{r^w-r^K}{r^w}\frac{\mu}{\theta}\phi v(E^\mu) + \frac{\delta\xi}{r^w+\rho}]}{\theta+g-r^w}\\ &+ [x^* + (1-x^*)\beta]E\frac{(1-\alpha)(1-\delta)\xi^* + (g+\rho)[\frac{r^w-r^{K*}}{r^w}\frac{\mu}{\theta}\phi^* v(E^\mu) + \frac{\delta\xi^*}{r^w+\rho}]}{\theta+g-r^w} = E\frac{\xi^*}{\theta},\end{aligned}$$

$$r^K \geq 0, \quad 0 \leq \xi \leq 1, \quad (1-\xi)r^K = 0,$$
$$r^{K*} \geq 0, \quad 0 \leq \xi^* \leq 1, \quad (1-\xi^*)r^{K*} = 0.$$

With no home bias, we have $E = E^\mu = 1$, $\xi = \xi^* = \xi^w$ and $r^K = r^{K*} = r^{K,w}$. We can solve the system and get

$$V^w = \frac{\xi^w X^w}{\theta},$$

$$W^{K,w} = \bar{\phi}\mu\frac{X^w}{\theta},$$

$$W^{N,w} = [\xi^w - \bar{\phi}\mu]\frac{X^w}{\theta},$$

[27] Our model with safe assets is consistent with the Secular Stagnation Hypothesis. The safety trap can be very persistent, and in fact permanent, because the mere possibility of the bad Poisson event casts a long shadow. Compared to our simple baseline model, it highlights an additional factor contributing to secular stagnation: a global savings glut or global asset shortage of a particular form, concentrated along the safe assets dimension. Secular stagnation can be exported from one country to the other through a surplus of the safe asset component of the Current Account and a corresponding deficit in the destination country.

$$r^{K,w} = \delta\theta + g - (1-\delta)\theta\frac{\alpha\xi^w - \bar{\phi}\mu}{\bar{\phi}\mu},$$

$$\frac{g - r^w + \rho}{g - r^w}\frac{r^w - r^{K,w}}{r^w}\mu\bar{\phi} = \xi^w(1 - \frac{\delta\theta}{r^w + \rho}),$$

$$r^{K,w} \geq 0, \quad 0 \leq \xi^w \leq 1, \quad (1 - \xi^w)r^{K,w} = 0.$$

In addition, we have

$$\frac{NFA}{X} = \frac{\bar{\phi}\mu}{\theta} + \frac{(1-\alpha)(1-\delta)\xi^w + (g+\rho)[\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}\phi + \frac{\delta\xi^w}{r^w+\rho}]}{\theta + g - r^w} - [\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}\phi + \frac{\delta\xi^w}{r^w + \rho}],$$

$$\frac{NFA^*}{X^*} = \frac{\bar{\phi}\mu}{\theta} + \frac{(1-\alpha)(1-\delta)\xi^w + (g+\rho)[\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}\phi^* + \frac{\delta\xi^w}{r^w+\rho}]}{\theta + g - r^w} - [\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}\phi^* + \frac{\delta\xi^w}{r^w + \rho}].$$

The requirement that

$$x\frac{NFA}{X} + (1-x)\frac{NFA^*}{X^*} = 0$$

boils down to

$$0 = \frac{\bar{\phi}\mu}{\theta} + \frac{(1-\alpha)(1-\delta)\xi^w + (g+\rho)[\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}\bar{\phi} + \frac{\delta\xi^w}{r^w+\rho}]}{\theta + g - r^w} - [\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}\bar{\phi} + \frac{\delta\xi^w}{r^w + \rho}].$$

This can be combined with the expression for $\frac{NFA}{X}$ above to yield

$$\frac{NFA}{X} = \frac{(r^w + \rho - \theta)\frac{r^w - r^{K,w}}{r^w}\frac{\mu}{\theta}}{\theta + g - r^w}(\phi - \bar{\phi}).$$