The paradox of financial fire sales and the role of arbitrage capital

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August, 2015

Abstract

Why do fire sales happen for financial assets, even when there are well-capitalized investors somewhere in the economy? We propose a theory of financial fire sales based on a noisy rational expectations equilibrium framework with endogenous adverse selection. When informed market participants are liquidity-constrained due to market-wide shocks, prices become less informative. This creates an adverse selection problem, decreasing the supply of high-quality assets. This "lemons" problem makes well-capitalized uninformed market participants unwilling to absorb the supply, thereby freezing the market. Our results shed light on the paradoxical nature of fire sales in which capital seems to move out of the market precisely when it appears likely to earn higher returns.

Keywords: fire sales, adverse selection, market freeze, illiquidity, flight-to-quality, informed trading

JEL Classification: G14, G21, D82, D83, D84

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‡We thank Julian Franks, Simon Gervais, Pete Kyle, Paolo Sodini, Per Strömberg, and Jörgen Weibull for helpful comments and suggestions. We also thank seminar participants at BI Norwegian Business School, Korea University, London Business School, SKKU, Stockholm School of Economics, University of Gothenburg, University of Mannheim, and Yonsei University.

1 Introduction

In a fire sale, sellers are forced to sell assets at deep discounts because no one is willing to buy them at fair prices. Sellers could be forced to sell because of financial distress, credit market frictions, regulation, margin calls, etc. Why do fire sales happen? What makes investors avoid buying assets that are apparently cheap? In the previous literature, Shleifer and Vishny (1992) argue that fire sales can happen when industry experts with higher private valuations do not have enough liquidity. Therefore, those assets are bought at a discount by non-experts who cannot use these assets efficiently. This argument naturally applies to real assets rather than financial securities because private valuations of real assets, unlike those of financial securities, can differ significantly among investors. Since financial securities typically require the holder only to collect cash flows, and not to take any actions, it cannot explain significant differences in private valuations.[1] Shleifer and Vishny (1997), Gromb and Vayanos (2002), and Brunnermeier and Pedersen (2009) argue that fire sales may occur even for financial securities due to limits to arbitrage. If all investors are constrained in their buying capacities for various reasons, then they are not able to buy apparently undervalued assets. However, many episodes (such as the LTCM crisis and the collapse of the mortgage-backed securities (MBS) markets during the 2007-8 financial crisis) described in the literature as fire sales occurred even though there were plenty of well-capitalized investors somewhere in the world economy (e.g., Warren Buffett, or sovereign wealth funds). Why wouldn't well-capitalized investors want to step in and buy undervalued assets whenever a fire sale starts, thereby preventing significant price drops? This seems paradoxical.

A possible resolution of the paradox is based on "lemons" problems as described by Akerlof (1970). If the well-capitalized investors are not specialists in valuing the assets then, by buying, they could be exposing themselves to adverse selection perpetrated by industry insiders. This could happen when insiders choose to hold back high-quality assets, and sell only low-quality assets, causing a classic lemons problem in which anybody who is forced to sell high-quality assets would suffer a loss. But this explanation has a major flaw because it does not explain why the lemons problem would suddenly increase during a crisis. In a crisis, asset sellers would have less discretion on which assets to sell, thus, liquidity shocks should even mitigate the lemons problem. It seems that the

[1]Some differences in valuation for financial securities may exist, for example if some holders are able to repo the security and others are not, but these differences should be minor.

paradox remains.

We give an intuitive answer to this question by examining the role of informed trading in preventing adverse selection. We argue that informed traders who can *buy* assets help to make market prices informative. They are willing to buy the asset if underpriced, and sell short if overpriced. This prevents high-quality assets from being underpriced and trading at the same prices as low-quality assets. Hence, informed buyers remove the incentive for sellers to favor selling low-quality assets. But in case of a large shock to the market, this mechanism breaks down. A severe need for liquidity affecting traders who specialize in an asset prevents them from using their private information to bid up undervalued assets: there are no buyers to keep prices informative. This can make prices uninformative, leading to adverse selection in which sellers supply only overvalued assets to the market. This in turn leads uninformed agents, who are potential buyers for those assets, to withdraw from the market even though they are not wealth-constrained. This causes a market freeze for high-quality assets because no one is willing to buy or sell them, while agents holding these assets but subject to a severe liquidity shortage are forced to sell at fire sale prices.

To formalize this argument, we develop an information-based theory of fire sales using a noisy rational expectations equilibrium (REE) framework in which adverse selection arises endogenously. We aim to achieve the following goals in our paper. First, we explain the role of informed trading in fire sales and market freeze. Unlike the traditional literature on adverse selection that only features informed sellers, we highlight the role of informed buyers who compete to exploit mispricing, and thereby make the price informative. Second, we answer the question of why fire sales occur even when, somewhere in the economy, there is enough capital to correct prices. In particular, we show that liquidity shocks to informed traders can make them stay away from buying, thereby allowing asset prices to fall. This also sheds light on the paradoxical nature of fire sales in which capital moves out of the market when it is needed most and would seemingly earn higher returns. Third, we explain why a market freeze happens at the same time as fire sales. That is, our paper explains the "double whammy" situation where fire sales and low trading volume occur together (see, for example, Tirole, 2011 for a discussion of the double whammy during the recent financial crisis).

Consider a two-period model with informed, but financially constrained, intermediaries and unconstrained, but uninformed, investors. There exists a marketable asset with risky payoffs whose value is only known to the informed intermediaries.[2] There are two types of informed intermediaries: there is a distressed seller who needs to raise liquidity, and there are arbitrageurs who aim to make trading profits. The seller is forced to meet liquidity needs, which he can do either by selling his holdings of the marketable asset, or alternatively liquidate another asset which is non-marketable. It is this choice that leads to the lemons problem; the seller only sells either when the marketable asset is overvalued (i.e., it is a lemon), or when liquidating the non-marketable asset is very costly.

The arbitrageurs compete to exploit arbitrage opportunities, thereby driving price close to the fundamental value. In normal market conditions where the arbitrageurs have enough liquidity, the price reveals the fundamental value of the asset because informed trading volume overwhelms the impact of noise in the supply. This allows an intermediary in liquidity shortage (the seller) to fund itself by selling assets on its balance sheet at intrinsic value regardless of the quality of those assets. This in turn makes uninformed investors willing to absorb the supply of assets without worrying about adverse selection. On the other hand, in a crisis situation where the arbitrageurs are liquidity-constrained due to a market-wide liquidity shock, prices become less informative. This makes the seller only willing to supply low-quality assets to the market unless it is effectively forced to sell because the alternative of liquidating the non-marketable asset is very costly. This adverse selection problem in turn makes uninformed investors unwilling to absorb the supply of assets unless there is a drop in price to reflect a lemons discount. It also creates a market freeze because the supply of high-quality assets decreases. We also show that, in our model, price falls further because risk averse investors require a further risk premium to compensate for the risk of buying an asset whose price is uninformative.

Figure 1 illustrates this idea. An initial liquidity shock to the arbitrageurs reduces their capacity to trade, so prices become less informative. This discourages the seller from selling high-quality assets. This lemons problem causes prices to fall on average, further reinforcing the seller's reluctance to sell high-quality assets.

We further show that economic efficiency is maximized when prices are close to the fundamental value for any realizations of the random variables because mispricing leads to misallocation of resources. Mispricing causes inefficient liquidation of otherwise valuable assets. In case traded assets are underpriced, intermediaries raise funds to meet

[2] We call these agents "intermediaries" because in practice they are likely to be financial intermediaries such as banks, hedge funds.

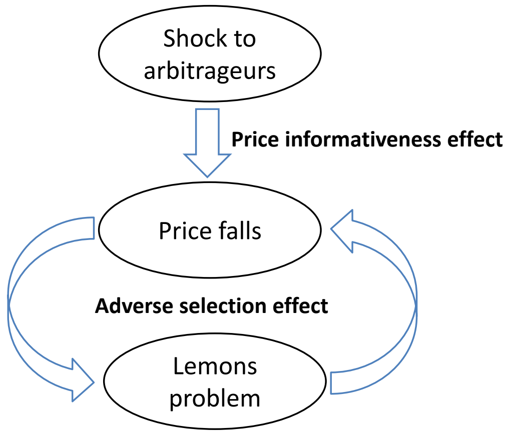


Figure 1: Price informativeness effect vs. adverse selection effect

liquidity shortages by inefficiently liquidating non-marketable assets, rather than selling traded assets.

It is worth noting that, in our model, the majority of the supply from the seller is absorbed by the uninformed investors rather than the informed arbitrageurs. Even a small amount of informed capital is able to promote liquidity in the market as long as it can make prices sufficiently informative. Therefore, informed capital facilitates the movement of uninformed capital from markets with excess liquidity to those with low liquidity. This potency of arbitrage capital, while apparently attractive, is however a double-edged sword. Small capital shocks to arbitrageurs in the market can trigger fire sales and market freezes by driving away the demand of uninformed investors. Our theory implies that this "multiplier" effect of arbitrage capital can actually serve as the source of financial *in*stability rather than financial stability.

Our results also contribute to the debate on flight-to-quality by suggesting an alternative mechanism for flight-to-quality. When prices are uninformative, uninformed investors lower their portfolio weights on the marketable asset because its expected payoffs are perceived to be lower. At the same time, they increase their portfolio weights on the risk-free asset. Notice that this flight-to-quality mechanism does not involve any change in preferences such as risk-aversion. Rather, it is a consequence of the lemons problem caused by the change in the amount of information about the fundamental value that is endogenously determined by constrained informed trading. Furthermore, illiquidity does happen together with flight-to-quality, but they are both consequences of a liquidity crisis, rather than illiquidity being the cause for flight-to-quality.

Our model provides a useful tool for understanding fire-sale episodes, such as the financial crisis of 2007-2009 during which many financial institutions were forced to unwind their positions as well as to reduce their leverage (See, for example, Brunnermeier, 2009; Shleifer and Vishny, 2011). Our theory suggests how reductions in informed trading can act as a transmission mechanism of liquidity shocks to fire sales in unrelated asset classes such as the repo market (e.g., Gorton and Metrick, 2012) and the residential mortgage-backed securities (RMBS) market (e.g., Merrill, Nadauld, Stulz, and Sherlund, 2014); it also sheds light on market freezes such as the collapse of the non-agency RMBS market (e.g., Vickery and Wright, 2013).

The organization of the paper is as follows. Section 2 relates our paper to the literature. Section 3 describes the basic model. Section 4 solves for the equilibrium. Section 5 describes our main results about fire sales and market freezes. Section 6 studies implications of our paper about financial stability (including policy implication), flight-to-quality, and the financial crisis of 2007-2009. Section 7 concludes.

2 Literature review

There is a large literature on both the theory and the empirics of fire sales.[3] On the empirical side, there is evidence of fire sales across various classes of assets and securities: (i) real assets (e.g., Pulvino, 1998; Schlingemann, Stulz, and Walkling, 2002), (ii) equities (e.g., Coval and Stafford, 2007; Jotikasthira, Lundblad, and Ramadorai, 2012), (iii) bonds (e.g., Ellul, Jotikasthira, and Lundblad, 2011; Jotikasthira, Lundblad, and Ramadorai, 2012), (iv) structured products (e.g., Merrill, Nadauld, Stulz, and Sherlund, 2014), and (v) repos (e.g., Duarte and Eisenbach, 2014).

As discussed in the introduction, the previous theoretical literature suggests that fire sales occur because of liquidity shocks to industry experts (e.g., Shleifer and Vishny, 1992), and limits to arbitrage (e.g., Shleifer and Vishny, 1997; Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009). Several papers also focus on the amplification mechanism of fire sales as in Gromb and Vayanos (2002), Geanakoplos (2003), Brunnermeier and Pedersen (2009), Krishnamurthy (2010), Greenwood, Landier, and Thesmar (2015), and Kuong (2015). For example, Krishnamurthy (2010) shows that fire sales can

[3]Shleifer and Vishny (2011) and Tirole (2011) provide excellent surveys on the literature.

occur due to feedback effects between asset prices and balance sheets. Counterparty risk can also contribute to the amplification mechanism of fire sales (e.g., Krishnamurthy, 2010; Caballero and Simsek, 2013). For example, Caballero and Simsek (2013) study a model where fire sales occur because of counterparty risk in a complex network.

In his seminal paper, Akerlof (1970) shows that a market collapse (or freeze) can happen due to adverse selection. If the quality of assets is only known to sellers, buyers' valuation that depends on the average quality cannot satisfy sellers who own assets with the highest quality. This makes them withdraw from the market, and subsequently lowers buyers' expectation about the quality, and this in turn makes more sellers withdraw from the market if they own assets with the highest quality among the remaining sellers. This process can continue until there is no seller left, thus the market collapses. This intuition has been extended and applied in the finance literature. In particular, several papers have emphasized the role of fire sales where sellers are forced to sell due to distress. However, buyers cannot tell whether the supply is coming from liquidity-driven sales or information-driven sales, if other sellers try to sell when they get bad signals. This type of adverse selection can be source of market freeze. For example, Eisfeldt (2004) shows that market illiquidity endogenously arises due to lemons problem in a dynamic consumption economy where agents trade for both informational reasons and liquidity needs.[4] Bolton, Santos, and Scheinkman (2009) show that distressed sellers may choose to sell earlier at fire sale prices to avoid potential adverse selection problems in the future. Dang, Gorton, and Holmstrom (2012) suggest that a market freeze can occur in the debt market by extending the argument of Myers and Majluf (1984). Although debt securities are information-insensitive relative to equity, they become information sensitive when approaching a default state. This causes increased information acquisition, but such information asymmetries may lead to adverse selection and a market freeze. Malherbe (2014) focuses on the self-fulfilling nature of liquidity dry-up. If sellers are forced to sell their assets due to liquidity needs, and do not choose to sell for informational reasons, adverse selection problems do not arise. Because prices will not reflect adverse selection, sellers do not need to hoard liquidity. On the other hand, if asset sales are driven by information, prices will reflect adverse selection. Anticipating this, sellers will need to hoard liquidity. This creates multiple equilibria.

[4] Other authors also show that market illiquidity arises in dynamic models: Daley and Green (2012) show that information release can create a market freeze by creating delayed sales of high-quality assets. Guerrieri and Shimer (2014) show that high-quality assets may be traded at low frequencies to signal their types.

Our theory is in stark contrast with the existing theories of fire sales or market freeze. Akerlof (1970) shows how a market freeze can happen in the presence of information asymmetries. But, there is no informed trader who can correct market prices in that argument. On the other hand, Grossman and Stiglitz (1980) show that the existence of informed traders can improve the informativeness of prices, but the supply of assets is inelastic to market prices. In our paper, Akerlof (1970) meets Grossman and Stiglitz (1980), and this gives a mechanism that creates fire sales. This idea is illustrated in Figure 2.

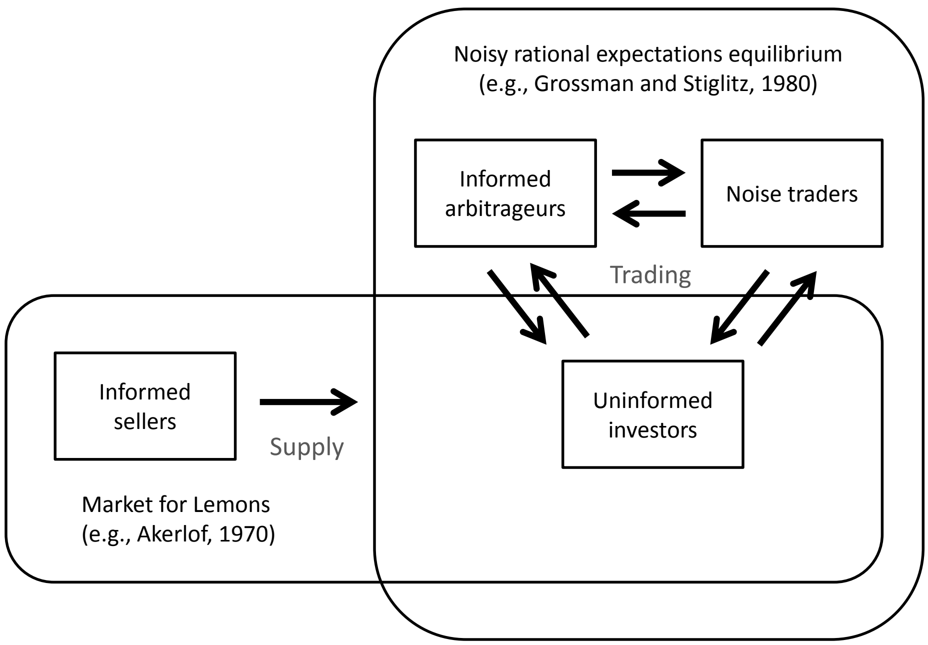


Figure 2: An illustration of our model: Akerlof (1970) meets Grossman and Stiglitz (1980)

Our paper combines limits-to-arbitrage and adverse selection to provide a plausible mechanism for fire sales. We argue that informed experts set asset prices to be fair, and, thus uninformed deep-pocketed buyers can supply liquidity to potential sellers. During crisis episodes, experts are constrained, thereby making price uninformative. Consequently, uninformed buyers are not willing to supply liquidity, and this leads to fire sales because there are no willing buyers unless prices fall sharply. Our theory is different from Shleifer and Vishny (1992) because it does not require private valuations.

Our theory differs from Shleifer and Vishny (1997), Gromb and Vayanos (2002), and Brunnermeier and Pedersen (2009) because it does not rely on exogenous exclusion of deep-pocketed investors (also, our model generates adverse selection). Our theory differs from Malherbe (2014) because a market freeze (or fire sale) occurs when informed traders are liquidity constrained. Specifically, Malherbe (2014) argues that fire sales occur because of increased adverse selection. However, for this argument to hold, traders should be only moderately distressed (i.e., more funding liquidity for sellers) during fire sales so that they can choose to sell lemons instead of good assets. Yet fire sales typically occur when traders are highly distressed.

Fire sales and market freezes create negative externalities to the economy because they distort resource allocations. In the literature, it has been argued that those externalities will not disappear by themselves in the absence of publicly coordinated efforts. For example, Diamond and Rajan (2011) show that distressed financial institutions do not have enough incentives to prevent their own fire sales at a socially optimal level because of risk-shifting incentives. Many papers in the literature suggest various policy implications to mitigate their adverse effects on the economy (e.g., Bolton, Santos, and Scheinkman, 2009; Krishnamurthy, 2010; Shleifer and Vishny, 2010; Guerrieri and Shimer, 2014). Our paper also contributes to the discussion of policy implications by looking at the problem from a new perspective that highlights the value of arbitrage capital in facilitating the efficient functioning of markets.

Recent papers have extended the limits-to-arbitrage argument, and suggest that "Slow-moving capital" could be a reasons why fire sales still occur when there is enough capital in the economy (e.g., Mitchell, Pedersen, and Pulvino, 2007; Duffie, 2010). However, it is hard to see why uninformed investors would be slow to buy undervalued assets.[5] What stops capital flowing to correct mispricing? Our paper contributes to the discussion of slow-moving capital by suggesting that the presence of sufficient informed capital can facilitate movement of uninformed capital. However, the market can quickly become illiquid if informed capital providers are subject to liquidity shocks, as the resulting lemons problem prevents entry of uninformed capital.

Finally, our paper is also related to the literature of financial markets with intermediaries. The usual setup is that only intermediaries participate in asset markets, thus, they trade assets on behalf of consumers or outside investors (e.g., Shleifer and Vishny, 1997;

[5] We accept that there are a small number of financial markets which are prohibited to unregulated outside investors. For examples, only registered insurers can provide insurance.

Gromb and Vayanos, 2002; Allen and Gale, 2004; Brunnermeier and Pedersen, 2009; He and Krishnamurthy, 2011; He and Krishnamurthy, 2013; Vayanos and Woolley, 2013; Dow and Han, 2015). The difference in the objectives of intermediaries and delegating investors, which is often endogenously determined in the model, is the key to generate mispricing or shock amplification in asset prices. For example, He and Krishnamurthy (2011) and He and Krishnamurthy (2013) study dynamic general equilibrium economies in which households, who do not have access to a long-lived risky asset, invest in the equity of capital-constrained financial intermediaries (or specialists), who have access to the risky asset. They show that shocks are amplified through the channel of financial intermediation; as intermediary capital shrinks up to the point where their capital constraint binds, risk premia of the risky asset rise. More generally in this line of literature (e.g., Shleifer and Vishny, 1997; Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009), a negative shock to intermediary capital restricts investment by outside investors and hence lowers asset prices. Our paper differs from this line of literature in several respects. First, all agents in our model can participate in the market. This allows us to study why and how uninformed traders endogenously inject or withdraw capital given shocks to informed capital. Second, the main mechanism of shock amplifications involves endogenous lemons problem resulting from varying degrees of informed trading activities. This creates a market freeze, which is typically absent in this line of literature, even though investors have enough capital to absorb asset supply in the market.

3 Model

3.1 Basic setup

Consider a two-period economy ($t = 1, 2$) with a risk-free asset in infinitely elastic supply with an exogenously-given return r_f.[6] There is also a risky asset that is tradable by all the participants in the financial market (henceforth, "the marketable asset"). The marketable asset is illiquid in the sense that it cannot be liquidated at $t = 1$, but pays a random liquidation value v at $t = 2$ with

$$v = \begin{cases} v_H & \text{with probability } \rho; \\ v_L & \text{with probability } 1 - \rho; \end{cases} \tag{1}$$

[6]See Appendix A for a list of symbols used in this paper.

where $v_H > v_L > 0$ and $0 < \rho < 1$. We call the marketable asset a "high-quality" asset in the event $v = v_H$, and "low-quality" otherwise. Examples of the marketable asset include most tradable financial securities such as equities, corporate bonds, MBS, or CDO. For example, in the case of a fixed income security such as corporate bond, one can interpret v_H as the promised payoff (face value plus coupon) of the security, v_L as its recovery value, and $1 - \rho$ as the default probability.

There are four classes of participants: (i) a seller, (ii) arbitrageurs, (iii) investors, and (iv) noise traders.[7] There is a unit mass of arbitrageurs and investors in the economy. We denote $\mathcal{A}$ and $\mathcal{I}$ to be the set of arbitrageurs and investors, respectively. The seller and arbitrageurs are risk-neutral, capital-constrained, and informed (the risk-neutrality assumption for these agents is for simplicity). On the other hand, the investors are risk-averse, unconstrained, and uninformed.

The seller is short of liquidity (or distressed), thus, he participates to sell his endowment of the marketable asset to meet liquidity needs. One can consider the seller to be a representative bank in the economy which makes loans and sells securities made out of them. The seller is endowed with $\bar{x}$ unit of the marketable asset ($\bar{x} > 0$). Besides the marketable asset, he is also endowed with a "non-marketable asset" which cannot be traded or transferred to other participants. The non-marketable asset can be considered as the profit-generating operations of the firm. The non-marketable asset generates a return r_I per unit of investment with probability density function $f_I(\cdot)$ and cumulative distribution function $F_I(\cdot)$ with support $[\underline{r}, \infty)$ where $\underline{r} > r_f$. The interpretation is that without any liquidity shock, the seller would have exploited the investment opportunity in the non-marketable asset until the return is reduced to the risk-free rate r_f. In the presence of the liquidity shock, however, the non-marketable asset is not fully utilized. Thus, the return remains higher than the risk-free rate. The seller's liquidity shortage is given by l, and it can be met by either selling the marketable asset or liquidating the non-marketable asset. For simplicity, we further assume that the seller decides to sell either none or the entire holdings of the marketable asset. Any remaining proceeds from selling the marketable asset can be reinvested in the non-marketable asset. The seller

[7]Notice that there is only a single seller. The reason is as follows: In Akerlof's lemons market for used cars, each car could be of different quality, and hence in principle is a different good with its own market and a single seller (i.e., the existing owner). Because there is no private information on the buy-side of the market, all cars will trade at the same price, so it is possible to treat these different markets as a single representative market. With private information among buyers, this is not possible. Hence, in our model, we have a single market with a single seller.

can observe the realization of v and r_I.

The arbitrageurs participate to generate profits by exploiting any mispricing by trading the asset. They are informed about the value of the marketable asset. Each arbitrageur $a \in \mathcal{A}$ has liquidity position w_A which is strictly positive (w_A is endogenized later in the paper). w_A can be thought of as the arbitrageurs' cash position and available credit (e.g., cash from pledging their inventory of other assets). As in the case of the sellers, the arbitrageurs observe the realization of v. We assume that the arbitrageurs are subject to margin requirements. In a similar fashion to Brunnermeier and Pedersen (2009), we assume that arbitrageur a's total margin on his position x^a cannot exceed his available capital w_A:

$$|mx^a| \leq w_A, \tag{2}$$

where m is the dollar margin on the position.

The investors participate to transfer their liquidity to the future, and also potentially to make trading profits. They do not know the value of the marketable asset. Each investor $i \in \mathcal{I}$ is endowed with identical initial wealth w_I at $t = 1$, and has an identical constant absolute risk aversion (CARA) utility $U(w_2^i) = -\frac{1}{\gamma}e^{-\gamma w_2^i}$ where w_2^i is his wealth at $t = 2$. We assume that the investors are unconstrained in their capacity to invest in available investment opportunities, i.e., they can borrow at r_f if their initial funding w_I is not large enough to cover investment expenses, and they can sell short. The investors are uninformed about the realization of v, but in equilibrium they will learn about it from the price of the marketable asset. For notational convenience, we define $q(p) \equiv Pr(v = v_H|p)$ to be their updated belief the marketable asset is high quality.

Intuitively, the seller can be considered to be a firm that invests in projects and creates financial securities. A bank that originates and distributes loans is an example. If the bank writes down some assets in its balance sheet, capital requirements force it to either shrink its balance sheet (by selling assets) or raise capital. The arbitrageurs are firms that specialize in securities trading and have particular expertise in trading the marketable asset but have limited capital. In practice such investors are typically firms rather than individuals. Examples include investment banks that specialize in market-making, hedge funds that specialize in investing in the marketable asset and similar assets, etc. Such arbitrageurs can be subject to liquidity constraints in the form of regulatory requirements, margin calls, client capital withdrawals, etc. Elsewhere in the economy, capital is not limited but the investors deploying this capital have no expertise in the tradable asset. Such investors could be pension funds, insurance companies, financial institutions in other countries, sovereign wealth funds, wealthy investors like Warren Buffett, etc—in other words, any investors who do not have particular expertise in valuing and trading the marketable asset. A more conceptual interpretation is that the investors are the "representative consumer" of the economy who, although he may choose to hold assets via intermediaries, has the option of investing directly.

The noise traders participate in the market for exogenous reasons. We denote ϵ to be the demand of the noise traders, and assume that ϵ follows a probability distribution with probability density function $f_\epsilon(\cdot)$ with support on the real line. We assume that ϵ is independent of asset value v. Although we have modeled noise by introducing a separate class of traders, it is possible to add noise in other ways via utility maximizing agents. For example, this could be done by making the return on the non-marketable asset correlated across sellers, or giving investors a random endowment of the marketable asset as in Diamond and Verrecchia (1981) (See Dow and Gorton, 2008 for survey). The exogenous noise trade we have specified is simpler.

We assume that $f_\epsilon(\cdot)$ satisfies the following conditions for all ϵ:

$$f''_\epsilon(\epsilon) f_\epsilon(\epsilon) \leq f'_\epsilon(\epsilon)^2; \tag{3}$$

$$\frac{f'_\epsilon(\epsilon)}{f_\epsilon(\epsilon)} < \frac{\gamma(v_H - v_L)}{2}. \tag{4}$$

The condition in Eq. (3) alone is sufficient for us to show existence of equilibrium (see Proposition 1). It is a weak condition; for example, it is satisfied by many unimodal distribution. The pair of conditions in Eqs. (3) and (4) are sufficient for one of our results, Proposition 2, although they are much stronger than necessary. The pair of conditions are satisfied by a variety of distributions such as the (generalized) logistic distribution, which we use for our numerical calculations.[8]

The market opens if the seller decides to sell his holdings of the marketable asset. If there is no supply of the marketable asset from the seller, the market does not open. Once the market is open, arbitrageurs and investors then condition their demands on

[8] A normal distribution satisfies condition in Eq. (3), but does not satisfies condition in Eq. (4). However, we have performed computations of the equilibrium for several examples using the normal distribution, and found that Proposition 2 holds. As well as the logistic distribution, the pair of conditions in Eqs. (3) and (4) holds for the hyperbolic distribution and the hyperbolic secant distribution for some parameter values.

the price. The market clears by equating the supply of the seller to the demand from the other traders (arbitrageurs, investors, and noise traders).

3.2 Optimization problems

The seller maximizes future expected profit by deciding whether to sell his endowment of marketable asset or liquidate the non-marketable asset. We denote x^s to be the units of marketable asset sold by the seller. Then, the seller solves an optimal trading problem given v and r_I:

$$\max_{x^s \in \{0,\bar{x}\}} \; E\big[v(\bar{x} - x^s) + (1 + r_I)(px^s - l)\big|v, r_I\big], \tag{5}$$

where l is the size of liquidity shortage.[9]

Each arbitrageur $a \in \mathcal{A}$ maximizes profit by choosing a portfolio of the marketable asset and the risk-free asset under the liquidity constraint. Arbitrageur a solves the following constrained optimization problem given p and v:

$$\max_{x^a} \; vx^a + (1 + r_f)(w_A - px^a), \tag{6}$$

subject to

$$|mx^a| \leq w_A, \tag{7}$$

where Eq. (7) is the margin constraint.

Each investor $i \in \mathcal{I}$ maximizes expected utility of future wealth by choosing a portfolio that consists of the marketable asset and the risk-free asset. Investor i solves the following optimization problem given the price p:

$$\max_{x^i} \; E\left[-\frac{1}{\gamma}\exp\Big(-\gamma\big[w_I(1 + r_f) + (v - p(1 + r_f))x^i\big]\Big)\Big|p\right]. \tag{8}$$

[9]Notice that the seller's objective function is independent of l because the non-marketable asset has constant returns to scale.

4 Equilibrium

As in standard REE models, investors infer the fundamental value from the price because the equilibrium price is a function of the fundamental value and noise.

4.1 Definition of equilibrium

Equilibrium is defined in the standard manner except that we augment the equilibrium concept with learning from net supply. The reason is as follows.

Standard models using the CARA-normal framework are typically solved by assuming a linear solution. Unlike such models, however, the equilibrium price function in our model is nonlinear not only because of the distributions of the fundamental value and noise trading but, more importantly, because of the financial constraints of the arbitrageurs. Instead of requiring the solution to be linear, we require that the investors in our model first infer net supply of the asset from the price, then use it to infer the fundamental value.[10] The net supply is defined to be the sum of an informed trading component, which is the supply from the seller minus demand from the arbitrageurs, and a noise component, which is the demand from the noise traders.

Definition 1. *A noisy rational expectation equilibrium is a price p and an allocation $\left(x^s, (x^a)_{a\in\mathcal{A}}, (x^i)_{i\in\mathcal{I}}\right)$ such that (i) x^s solves the seller's problem, (ii) x^a solves each arbitrageur a's problem, (iii) x^i solves each investor i's problem, (iv) in case the market opens, p clears the market under (i.e., supply from the seller equals demand from the other agents):*

$$\bar{x} = \int_{i\in\mathcal{I}} x^i di + \int_{a\in\mathcal{A}} x^a da + \epsilon, \tag{9}$$

and (v) p is a sufficient statistic for ξ_p, i.e.,

$$q(p) \equiv Pr(v_H|p) = Pr(v_H|p, \xi_p), \tag{10}$$

[10] Investors' belief update based on net supply is parallel with Kyle (1985) in which the uninformed market makers use order flows to infer the fundamental value. Unlike the market makers in a Kyle model, however, the investors in our model do not directly observe net supply, but can infer it from the price.

where ξ_p denote the net supply of the marketable asset to the investors such that

$$\xi_p \equiv \bar{x} - \int_{a \in \mathcal{A}} x^a da - \epsilon. \tag{11}$$

In solving, we first conjecture that Eq. (10) holds, then, later, we verify that it is indeed true in equilibrium.

We define a market freeze to be a situation in which the market fails to open with a positive probability because the seller may sometimes prefer not to sell the marketable asset. Define

$$\begin{aligned} \mu_H &\equiv Pr\Big(x^s > 0 \Big| v_H\Big); \\ \mu_L &\equiv Pr\Big(x^s > 0 \Big| v_L\Big). \end{aligned} \tag{12}$$

As we will see, in equilibrium, the seller always sells the low-quality asset whereas he may not always want to sell the high-quality asset. This is because the low-quality assets may be overvalued but the high-quality may be undervalued. Therefore, we can measure the degree of market freeze by the probability that the market opens for the high-quality asset, μ_H. Because the low-quality asset is circulated in the market with probability one, μ_H can be also interpreted as the relative circulation rate of the high-quality asset to that of the low-quality asset.

Definition 2. *There is a market freeze if the high-quality asset fails to fully circulate in the market, i.e., $\mu_H < 1$.*

We define a fire sale to be an event in which the seller sells his holdings at a price so that he would never want to sell unless he is forced to sell for non-informational reasons (such as liquidity shortage).

Definition 3. *The seller engages in a fire sale if he sells his holdings of the high-quality asset at a discount rate greater than the minimum cost of capital, i.e., $x^s > 0$ when $E[p|v_H] < \frac{v_H}{1+\underline{r}}$.*

The intuition behind these definitions is as follows. Since sellers are informed, one could conjecture that they will normally sell overvalued assets and not sell undervalued assets. Hence, a seller who sells an undervalued asset must be trying to raise cash, not selling for informational reasons. We call this a fire sale. When prices are lower than

$\frac{v_H}{1+\underline{r}}$, the seller would not want to sell unless the realization of r_I is high enough that he wants to sell the apparently-undervalued high-quality asset, so as not to give up more profitable non-marketable asset.

Turning to the volume of sales, it may seem natural in this model for the seller to always sell his holdings because the returns on the non-marketable asset dominate the returns required by the investors (i.e., r_I is always greater than r_f). So, if the seller is keeping back his holdings from the market, this suggests that there is some kind of malfunction in the market. As we will show, it is impossible for the marketable asset to trade in equilibrium at less than the present value of the low-quality asset. This implies that the seller will always sell the low-quality asset. But if the price is not revealing, the seller may hold back the high-quality asset for classic "lemons" motives. This is what we describe as a market freeze.

4.2 Solving for equilibrium

4.2.1 Demand and supply

We first describe the seller's supply of the marketable asset given the value of the marketable asset and the return on the non-marketable asset. The following is immediate from the seller's problem in Eq. (5):

Lemma 1. *Given v and r_I, the seller's supply is as follows:*[11]

$$x^s \in \begin{cases} 0 & \text{if } E[p|v] < \frac{v}{1+r_I}; \\ \bar{x} & \text{if } E[p|v] \geq \frac{v}{1+r_I}. \end{cases} \tag{13}$$

The seller sells his holdings of the marketable asset if the expected price exceeds the value of the asset, discounted at the opportunity cost of capital. (This is the seller's return on the non-marketable asset, since we have assumed that is higher than the risk-free rate.) In other words, when the non-marketable asset has a high return, the seller will sell his endowment of the marketable asset to meet liquidity needs. When the non-marketable asset has a low return, the seller will instead meet his liquidity needs by

[11] We assume a tie-breaking rule that the seller prefer selling in case the seller is indifferent between choices. This assumption is purely to simplify exposition, and does not affect the result. The other possible tie-breaking rule (i.e., not selling his holdings when indifferent) results in an identical outcome because it is a measure-zero event.

liquidating the non-marketable asset.

Next, given the price, arbitrageurs buy the marketable asset if undervalued, and sell if overvalued. Because the arbitrageurs are identical, their aggregate demand $X_A(p,v) \equiv \int_{a\in\mathcal{A}} x^a da$ is equivalent to each arbitrageur's optimal demand. The following is immediate from the arbitrageur's problem in Eq. (6):

Lemma 2. *Given p and v, the arbitrageurs' aggregate demand is as follows:*

$$X_A(p,v) \in \begin{cases} \frac{w_A}{m} & \text{if } p < \frac{v}{1+r_f}; \\ [-\frac{w_A}{m}, \frac{w_A}{m}] & \text{if } p = \frac{v}{1+r_f}; \\ -\frac{w_A}{m} & \text{if } p > \frac{v}{1+r_f}. \end{cases} \tag{14}$$

The arbitrageurs have perfectly elastic demands when the asset is correctly priced (i.e. it is priced at discount rate r_f), subject to a minimum and a maximum. The maximum they can demand is the quantity of the asset that exhausts their wealth, $\frac{w_A}{m}$, according to the margin constraint in Eq. (2), and this is their demand if the asset is underpriced. Likewise, the minimum they can demand is $-\frac{w_A}{m}$, which is also their demand if the asset is overpriced.

Once the market opens the investors trade with the arbitrageurs and the sellers who are privately informed about the true value of v. Each investor attempt to infer v from the market clearing price p, thus, the demand of the asset is based on their posterior belief $q(p)$. Let $X_I(p) \equiv \int_{i\in\mathcal{I}} x^i di$ denote the aggregate demand of the investors given p.

Lemma 3. *Given p, the investors's aggregate demand is given by*

$$X_I(p) = \frac{1}{\gamma(v_H - v_L)}\left[\log\Big(\frac{q(p)}{1-q(p)}\Big) + \log\Big(\frac{v_H - (1+r_f)p}{(1+r_f)p - v_L}\Big)\right]. \tag{15}$$

Proof. See Appendix B. □

This is a standard CARA demand function for the case where there only two possible outcomes for the terminal value of the asset. Because they are risk-averse, the investors require a larger risk premium when they are less confident about the expected payoff of the asset.[12]

[12]This is a standard feature of any noisy REE models such as Grossman and Stiglitz (1980).

Notice that Lemma 3 implies that the equilibrium price p will be between $\frac{v_L}{1+r_f}$ and $\frac{v_H}{1+r_f}$. This is because the investors are unconstrained, thus, they are the marginal buyers (or sellers) who set the price unlike the seller or the arbitrageurs who are constrained. In equilibrium, the arbitrageurs' demand only depends on v. In other words, the arbitrageurs buy whenever the asset is of high quality, and sell short otherwise. Therefore, we replace $X_A(p, v)$ with $X_A(v)$ for notational convenience.

4.2.2 Learning

There are two groups of informed market participants: the seller and the arbitrageurs.

Recall that the market does not open in case there is no supply of the asset from the seller, i.e., $x^s = 0$. Lemma 1 shows that the market may or may not be open sometimes for the high-quality asset whereas the market is always open for the low-quality asset. The lemma further implies that the probability of the market opening for the high- and the low- quality asset, respectively, is given by:

$$\mu_H = 1 - F_I\Big(\frac{v_H}{E[p|v_H]} - 1\Big), \tag{16}$$

$$\mu_L = 1. \tag{17}$$

As mentioned earlier in Section 4.1, μ_H and μ_L are also interpreted as the circulation rates of the high- and the low- quality asset, respectively.

Because the market is less likely to be open for the high-quality asset relative to the low-quality asset, the fact that the investors are participating in the market delivers some information about the quality of the traded asset. Conditional on the market being open, the investors' probability assessment that the asset is of high quality is given by[13]

$$\hat{\rho} \equiv Pr(v = v_H | x^s > 0) = \frac{\rho \mu_H}{\rho \mu_H + (1-\rho)}. \tag{19}$$

[13] Using Bayes' rule, we have

$$Pr(v = v_H | x^s > 0) = \frac{\rho Pr(x^s > 0 | v = v_H)}{\rho Pr(x^s > 0 | v = v_H) + (1-\rho) Pr(x^s > 0 | v = v_L)}. \tag{18}$$

Then, Eq. (19) is immediate from Eq. (18) because $Pr(x^s > 0 | v = v_H) = 1 - F_I\big(\frac{v_H}{E[p|v_H]} - 1\big)$ and $Pr(x^s > 0 | v = v_L) = 1$.

The investors assess that the quality of the traded asset is likely to be poorer when they expect a greater undervaluation for the high-quality asset (i.e., $E[p|v_H]$ is lower). This reflects the classic lemons intuition that good assets are in smaller supply than lemons. This adverse selection problem becomes more severe as prices become more dislocated. Therefore, the investors' prior belief of the quality of the asset being high is adjusted to be $\hat{\rho}$ rather than ρ.

Unlike the seller who influences the average quality of the traded asset, arbitrageurs influence the informativeness of prices. That is, their arbitrage activities make the price partially reveal the quality of the traded asset. Therefore, uninformed investors update their beliefs about the asset value from the price. Given the condition that price is a sufficient statistic for ξ_p (the net supply trading of the asset to the investors as defined in Eq. (11)), they update their belief conditional on ξ_p.

Lemma 4. *Given the prior belief $\hat{\rho}$, the investors' posterior belief conditional on p is given by*

$$q(p) = \frac{\hat{\rho} f_\epsilon(\bar{x} - X_A(v_H) - \xi_p)}{\hat{\rho} f_\epsilon(\bar{x} - X_A(v_H) - \xi_p) + (1-\hat{\rho}) f_\epsilon(\bar{x} - X_A(v_L) - \xi_p)}. \tag{20}$$

Proof. See Appendix B. □

Finally, we can verify that p is indeed a sufficient statistic for ξ_p in equilibrium by finding a mapping ξ from prices to net supply that gives equivalent information to directly observing ξ_p. That is, given p, the investors' posterior belief is identical whether it is conditioned on ξ_p or $\xi(p)$, confirming the initial conjecture in Eq. (10). The result is not obvious: in a noisy REE framework, updating beliefs based on price for general distributions of asset value is potentially complex. The canonical formulation is a CARA-Gaussian model where one can infer the updating rule based on the conjecture that price is linear in the state variables (e.g. noise and asset value). Grossman and Stiglitz (1980), Diamond and Verrecchia (1981) and Hellwig (1980) are examples. Linearity is a special property which does not hold in general.[14] In our model, price is non-linear because of the Bernoulli distribution of asset value, the liquidity and margin constraints, and the general distribution of noise trading. Updating beliefs based on net supply as well as price is straightforward, by Bayes' rule, but in principle price might

[14]For example, the standard CARA-Gaussian models also feature unconstrained traders to maintain the linearity. Constraining traders by borrowing constraints (e.g., Yuan, 2005) or a short-sale constraint (e.g., Bai, Chang, and Wang, 2006) results in a non-linearity.

not reveal net supply: a single price might correspond to two or more possible values for net supply that lead to different posterior beliefs.[15] Our assumption in Eq. (3) on the distribution of noise trade, however, rules this out. Furthermore Eq. (3) is quite a weak condition that is satisfied by a wide-variety of unimodal distributions that nest the commonly-used distributions (e.g., normal distribution; logistic distribution). The details are in appendix.

4.2.3 Price informativeness and arbitrage trading variation

When the arbitrageurs have more liquidity, prices reveal fundamental value better. Let ΔX be the maximum difference in the information component of net supply for the high- and low- quality assets:

$$\Delta X \equiv X_A(v_H) - X_A(v_L) = \frac{2w_A}{m}. \tag{21}$$

Because ΔX measures the variability in the net supply due to arbitrage activities, we call it the "arbitrage trading variation." We will show that price informativeness increases in arbitrage trading variation.

Using Lemma 1, Lemma 2 and the definition of arbitrage trading variation, Eq. (21), we can derive an equilibrium relationship between the investors' posterior belief $q(p)$ and the realizations of v and ϵ as follows:

Corollary 1. *Given v and ϵ, the investors' equilibrium posterior belief is equal to*

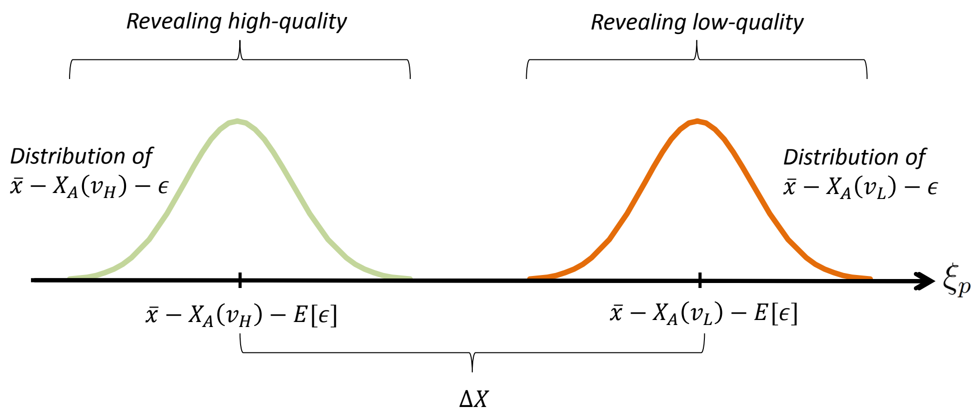
$$q(p) = \begin{cases} \frac{\hat{\rho} f_\epsilon(\epsilon)}{\hat{\rho} f_\epsilon(\epsilon)+(1-\hat{\rho}) f_\epsilon(\epsilon+\Delta X)}; & \textit{if } v = v_H; \\ \frac{\hat{\rho} f_\epsilon(\epsilon-\Delta X)}{\hat{\rho} f_\epsilon(\epsilon-\Delta X)+(1-\hat{\rho}) f_\epsilon(\epsilon)}; & \textit{if } v = v_L; \end{cases} \tag{22}$$

Proof. See Appendix B. □

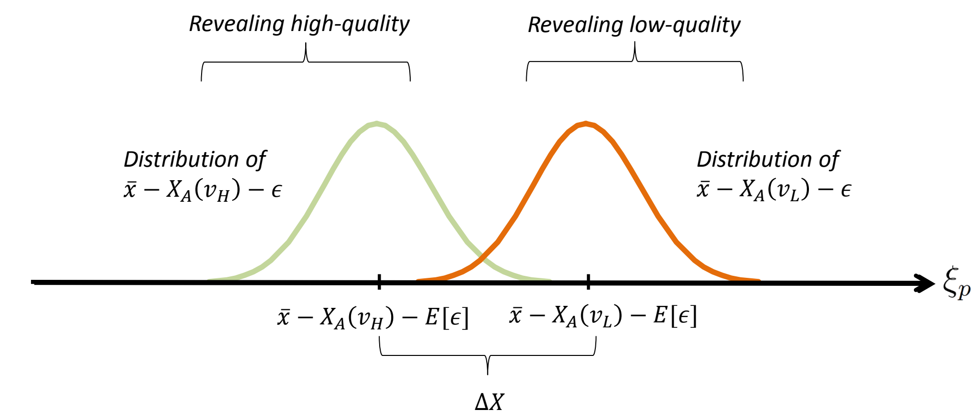
From Eq. (22), one can observe that higher arbitrage trading variation in general increases the chance of revealing the fundamental value of the marketable asset. As ΔX increases, $q(p)$ gets larger on average if $v = v_H$, and gets smaller on average if $v = v_L$.

We have assumed that the noise in the supply has unbounded support. Therefore, prices will not be fully revealing. However, when ΔX is sufficiently large, arbitrage

[15] It is fine for a price to correspond to two or more values for net supply as long as those values lead to a single posterior belief.



(i) large arbitrage trading variation ΔX (prices are almost fully-revealing)



(ii) small arbitrage trading variation ΔX (prices are less revealing)

Figure 3: An illustration of price informativeness under large and small arbitrage trading variation

activities push prices arbitrarily close to the fundamental value of the asset, thereby making prices almost fully-revealing. Large enough arbitrage trading variation will reveal the fundamental value of the asset with arbitrarily high precision.

Figure 3 illustrates this. When arbitrage trading variation, ΔX, is large as in Figure 3.(i), most realizations of ξ_p will not overlap between the two cases with $v = v_H$ and $v = v_L$. This makes v almost fully revealed, so the price is very close to either the fundamental value of the high-quality asset ($\frac{v_H}{1+r_f}$) or that of the low-quality asset ($\frac{v_L}{1+r_f}$). On the other hand, when arbitrage trading variation is small as in Figure 3.(ii), a large portion of realizations of ξ_p are likely to overlap between the two cases with $v = v_H$ and $v = v_L$. This results in a noisier price.

4.2.4 Equilibrium

The model always has an equilibrium that can be characterized as follows:

Proposition 1. *(Existence of equilibrium) There always exists an equilibrium, i.e., there exists an asset quality $\hat{\rho} \in (0, \rho]$ that satisfies*

$$\hat{\rho} = \frac{\rho\big(1 - F_I\big(\frac{v_H}{E[p(v,\epsilon;\hat{\rho},w_A)|v_H]} - 1\big)\big)}{\rho\big(1 - F_I\big(\frac{v_H}{E[p(v,\epsilon;\hat{\rho},w_A)|v_H]} - 1\big)\big) + (1-\rho)}, \tag{23}$$

where the equilibrium price p given $v, \epsilon, \hat{\rho}$ and w_A is uniquely given by

$$p(v, \epsilon; \hat{\rho}, w_A) = \frac{1}{1+r_f}\big(\theta(v, \epsilon; \hat{\rho}, w_A)v_H + (1 - \theta(v, \epsilon; \hat{\rho}, w_A))v_L\big), \tag{24}$$

with the weight $\theta(v, \epsilon; \hat{\rho}, w_A)$

$$\theta(v_H, \epsilon; \hat{\rho}, w_A) = \frac{\hat{\rho} f_\epsilon(\epsilon)}{\hat{\rho} f_\epsilon(\epsilon) + (1-\hat{\rho}) f_\epsilon(\epsilon + \Delta X) \exp(\gamma(v_H - v_L)(\bar{x} - \frac{w_A}{m} - \epsilon))}; \tag{25}$$

$$\theta(v_L, \epsilon; \hat{\rho}, w_A) = \frac{\hat{\rho} f_\epsilon(\epsilon - \Delta X)}{\hat{\rho} f_\epsilon(\epsilon - \Delta X) + (1-\hat{\rho}) f_\epsilon(\epsilon) \exp(\gamma(v_H - v_L)(\bar{x} + \frac{w_A}{m} - \epsilon))}. \tag{26}$$

Proof. See Appendix B. □

Proposition 1 shows existence of equilibrium by proving that there exists a fixed point for $\hat{\rho}$ that solves Eq. (23). The weights $(\theta(v, \epsilon; \hat{\rho}, w_A), 1 - \theta(v, \epsilon; \hat{\rho}, w_A))$ in the equilibrium price in Eq. (24) reflect the investors' pricing kernels given their posterior beliefs and risk aversion. As the investors become more risk tolerant (i.e., γ goes to zero), the weight approaches the posterior probability $q(p)$ (i.e., the price approaches the risk-neutral value).

5 Main results

5.1 Fire sales and market freezes

In this subsection, we show that fire sales and market freezes can occur due to reductions in the arbitrageurs' capital. When the arbitrageurs have enough capital, we find that there is no price dislocation (i.e., prices are close to the fundamental value of the marketable asset). Because prices are sufficiently informative, uninformed investors provide liquidity to the market by absorbing the entire supply of the asset. In that case, the seller always sells his holdings of the marketable asset rather than giving up on their profitable non-marketable assets. Therefore, the supply of the marketable asset is insensitive to the quality of the asset, and is driven only by the liquidity needs of the seller.

On the other hand, in case arbitrageurs' capital is scarce, prices are dislocated. Because prices are not informative enough, uninformed investors cannot provide much liquidity. In that case, seller's decisions depend on the asset quality. If the marketable asset is of low quality, he sells it. However, if the marketable asset is of high quality, he only sells it when divesting from the non-marketable asset is very costly. Therefore, average asset quality per trading volume goes down. This deterioration of average asset quality further depresses prices, thereby creating fire sales in case seller has to sell. To summarize, the reduction in arbitrageurs' capital leads to uninformative prices, which in turn cause further price drops through adverse selection. This idea is illustrated in Figure 4.

We state our main results about fire sales and market freezes in the following proposition.

Proposition 2. *There exists a constant $\bar{w}_A$ such that (i) (Fire sales) fire sales occur when arbitrage capital is below $\bar{w}_A$, i.e.,*

$$E[p|v_H] \in \begin{cases} [\frac{v_H}{1+\underline{r}}, \frac{v_H}{1+r_f}) & \textit{if } w_A \geq \bar{w}_A; \\ (\frac{v_L}{1+r_f}, \frac{v_H}{1+\underline{r}}) & \textit{if } w_A < \bar{w}_A, \end{cases} \tag{27}$$

(ii) (Lemons problem) the average quality of the traded asset becomes poorer when arbi-

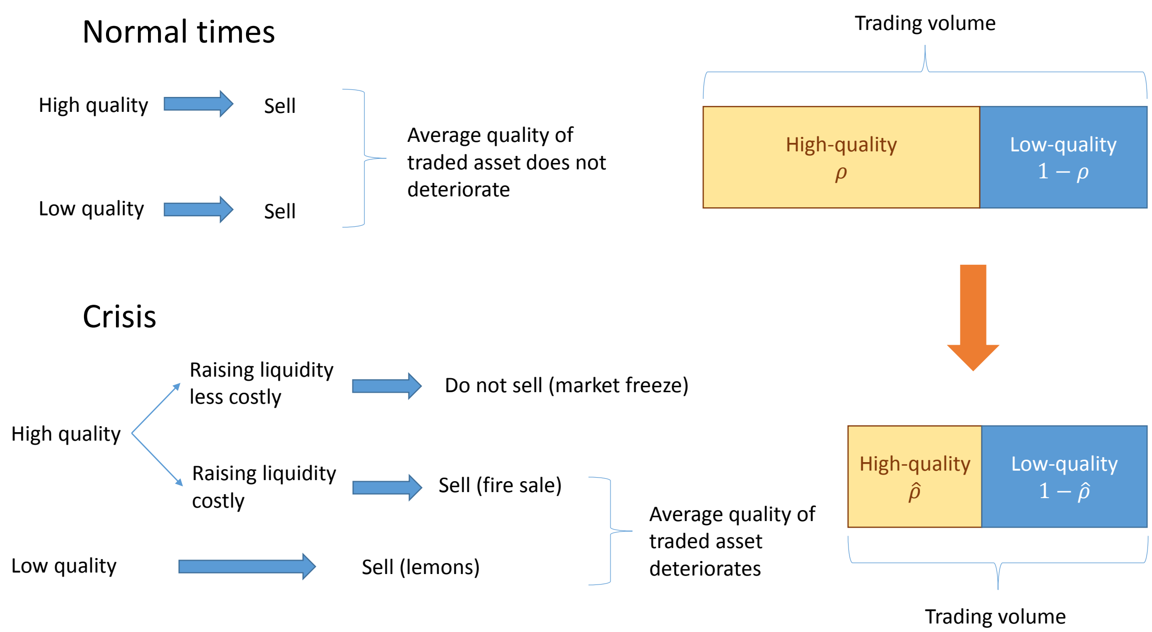


Figure 4: How a crisis causes a lemons problem

trage capital is below $\bar{w}_A$, i.e.,

$$\hat{\rho} \in \begin{cases} \{\rho\} & \text{if } w_A \geq \bar{w}_A; \\ [0, \rho) & \text{if } w_A < \bar{w}_A, \end{cases} \tag{28}$$

Proof. See Appendix B. □

Proposition 2 shows that there is neither a fire sale nor a market freeze when there is enough capital for the arbitrageurs. $\bar{w}_A$ is the level of arbitrage capital that eliminates the possibility of any fire sale or market freeze. We call this level of arbitrage capital the "fire-sale-free" level.[16] On the other hand, insufficient arbitrageur capital (or liquidity shocks to arbitrageurs) can create a double whammy of fire sales and market freeze. In case of the high-quality asset, a reduction in arbitrage capital creates a large price reduction because it affects both supply and demand. That is, the investors are less willing to absorb the net supply at any given level of prices.

Why does demand for high-quality assets shrink so much when there is a reduction

[16] Note that prices are not fully revealing, but because there is a gap between the r_f and $\underline{r}$, the seller will be always willing to sell the high-quality asset if prices are close enough to the true value.

in arbitrage capital? Answering this question is the main purpose of our analysis in this subsection. We argue that a reduction in arbitrage capital has both a direct effect and an indirect effect that reduce demand, thereby precipitating price falls.

The direct effect, which we call the "price informativeness effect" (or "noisy rational expectations equilibrium effect"), is related to price dislocation. This effect is captured by our noisy REE framework that connects price informativeness to informed trading. The reduction of arbitrage capital creates an initial price fall by lowering the investors' demand. This is because the investors cannot infer the quality of the traded asset as much as before.

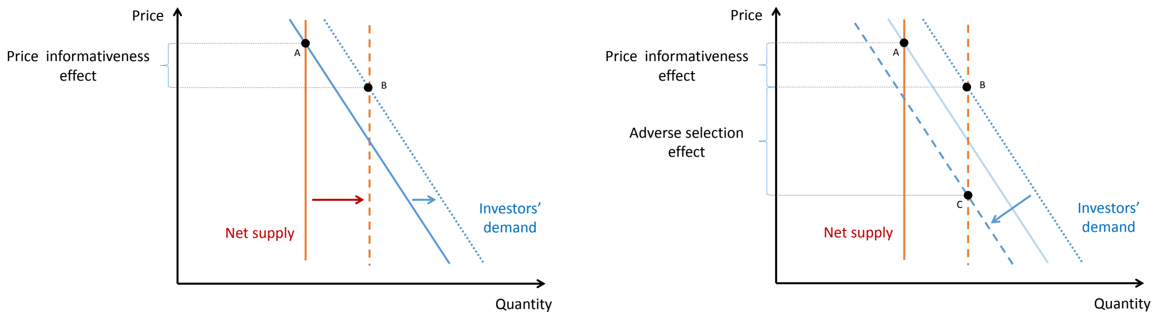
The indirect effect, which we call the "adverse selection effect" (or "lemons effect") of a liquidity shock, is related to feedback effects between price dislocation and the lemons problem. When prices are noisy, the seller will not sell the high-quality asset unless they are effectively forced by very pressing liquidity needs (i.e., high liquidation costs for the non-marketable asset). That is, the initial price dislocation due to uninformative prices (i.e., the direct effect) worsens adverse selection of the seller, thereby lowering the average quality of the traded asset poorer. Because the average quality gets poorer, prices fall further, but this in turn lowers the quality further by worsening the adverse selection problem, and so on. Therefore, price becomes more dislocated as the overall quality of traded assets becomes poorer. This feedback mechanism causes the "double whammy" of a large fire sale discount and a market freeze for the high-quality asset.

When arbitrage capital is reduced from $\bar{w}_A$ to w'_A, the fire sale discount (or the change in prices) can be decomposed into two components as follows:

$$\begin{aligned}\underbrace{\frac{v_H}{1+\underline{r}} - E[p'|v_H]}_{\text{fire sale discount}} &= \underbrace{E\Big[\frac{(v_H - v_L)}{1+r_f}\Big(\theta(v_H, \epsilon; \rho, \bar{w}_A) - \theta(v_H, \epsilon; \rho, w'_A)\Big)\Big]}_{\text{price informativeness effect}} \\ &\quad + \underbrace{E\Big[\frac{(v_H - v_L)}{1+r_f}\Big(\theta(v_H, \epsilon; \rho, w'_A) - \theta(v_H, \epsilon; \hat{\rho}', w'_A)\Big)\Big]}_{\text{adverse selection effect}},\end{aligned} \tag{29}$$

where $E[p'|v_H]$ and $\hat{\rho}'$ are the expected price of the high-quality asset and the average quality of the traded assets given w'_A, respectively.

This idea is illustrated in Figure 5. First consider an equilibrium prior to the reduction in arbitrage capital. The equilibrium is determined at point A where demand and



(i) Price informativeness effect (ii) Adverse selection effect

Figure 5: Fire sales of the high-quality asset: price informativeness effect vs. adverse selection effect

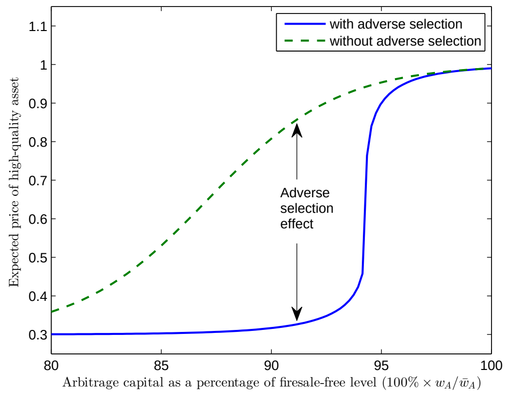
supply are matched given the initial size of arbitrage capital. Now, suppose that there is a reduction in arbitrage capital, and this shifts the supply curve outward. Because the shock to arbitrage capital is common knowledge among all participants, the investors incorporate that information, thus, their demand shifts outward to accommodate the change. However, the demand will not shift enough to support the same price as at point A because prices are now less revealing. Therefore, the price will be lowered to point B; because the uninformed investors are risk-averse, they demand a risk premium for buying an asset whose quality is uncertain conditional on the price. This in turn results in a further decrease in prices through the lemons problem. That is, the reduction in arbitrage capital lowers price informativeness, thus, this increases adverse selection. Therefore, demand shrinks as a result of the increased lemons problem. As a result of the decrease in demand, the equilibrium is determined at point C rather than B.

For numerical examples, we choose the following parameter values unless stated otherwise: $\gamma = 1, v_H = 1, v_L = 0.3, \rho = 0.8, \bar{x} = 10, r_f = 0\%, \underline{r} = 1\%, w_I = 15$. We also assume that ϵ follows a logistic distribution with mean zero and scale parameter 0.2, and $\log(r_I - \underline{r})$ follows a normal distribution with mean $\log(0.2)$ and standard deviation 1.8 (i.e., $r_I - \underline{r}$ follows a lognormal distribution).[17]

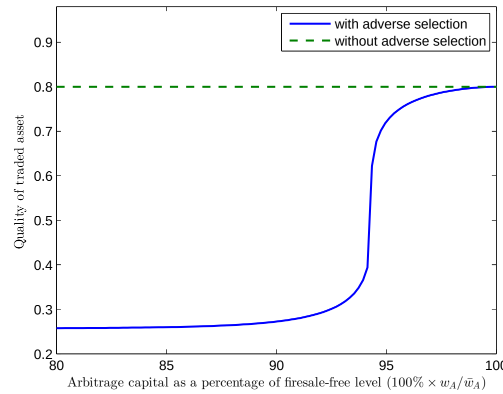
Figure 6 illustrates the impact of liquidity shocks on the expected price and the

[17] The parametric assumptions imply the probability density function of ϵ is given by

$$f_\epsilon(\epsilon) = \frac{e^{-\frac{\epsilon}{s}}}{s\left(1 + e^{-\frac{\epsilon}{s}}\right)^2}, \tag{30}$$



(i) Expected price of high-quality asset ($E[p|v_H]$)



(ii) Average quality of traded asset ($\hat{\rho}$)

Figure 6: The impact of reductions in arbitrage capital on the expected price of the high-quality asset and the average quality of the traded asset (the x-axis is scaled as the percentage of $\bar{w}_A$)

average quality of the traded asset in case of the high-quality asset.[18] When there is enough arbitrage capital, the marketable asset is in full supply and price approaches fundamental value. As arbitrage capital falls, however, the fire sale discount increases (see the left panel of Figure 6). Also, the average quality of the traded asset falls because the supply of the high-quality asset decreases (see the right panel of Figure 6). That is, market freezes and fire sales occur together due to adverse selection. Furthermore, notice that the fire sale discount can increase sharply due to the adverse selection effect even for a relatively small reduction in arbitrage capital (i.e., where $\frac{w_A}{\bar{w}_A} \approx 94\%$).

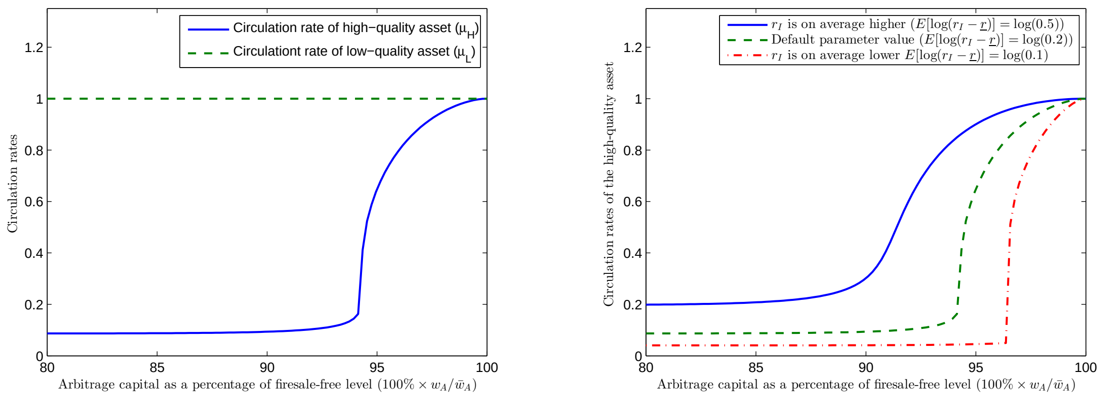
The left panel of Figure 7 illustrates the impact of liquidity shocks on the circulation rates of the high- and low- quality asset, respectively. When there is enough arbitrage capital, the seller sells the marketable asset regardless of its quality (i.e., $\mu_H = \mu_L = 1$). When there is a reduction in arbitrage capital, however, the seller is more likely to sell lemons rather than the high-quality asset (i.e., $\mu_H < 1$ and $\mu_L = 1$). The right panel of Figure 7 illustrates the impact of liquidity shocks on the circulation rate of the

and the probability density function of r_I is given by

$$f_I(r_I) = \begin{cases} \frac{1}{(r_I - \underline{r})\sigma\sqrt{2\pi}} e^{-\frac{(\log(r_I - \underline{r}) - \mu_I)^2}{2\sigma^2}} & \text{if } r_I \geq \underline{r}; \\ 0 & \text{otherwise,} \end{cases} \tag{31}$$

where $s = 0.2, \mu_I = \log(0.2)$ and $\sigma = 1.8$.

[18]Notice that $\bar{w}_A$ is endogenously derived given other parameter values.



(i) Circulation rates (μ_H and μ_L)

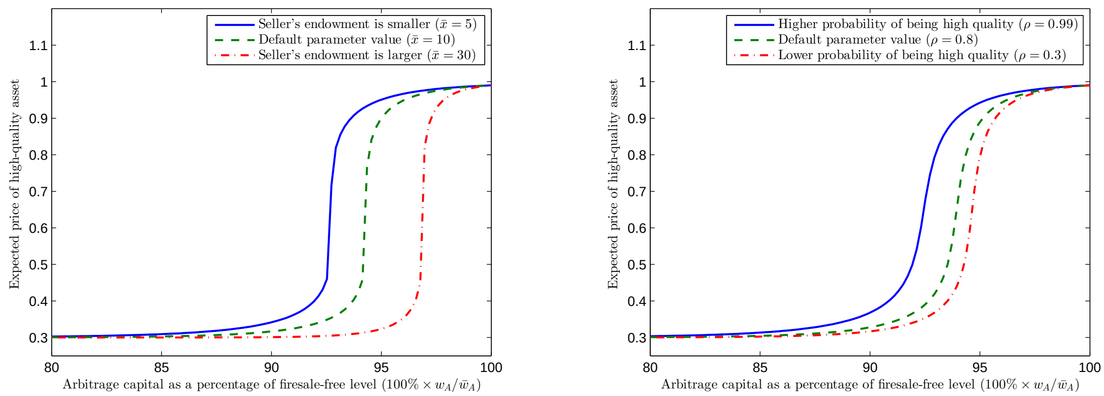
(ii) μ_H under different distributions of r_I

Figure 7: The impact of reductions in arbitrage capital on the circulation rate of the marketable asset (the x-axis is scaled as the percentage of $\bar{w}_A$)

high-quality asset given different distributions of r_I (the return on the non-marketable asset).[19] When the non-marketable asset is less profitable (r_I is on average lower), the seller is more likely to liquidate the non-marketable asset instead of selling the high-quality asset. Therefore, there is more adverse selection with lower r_I (notice that the circulation rate falls down much faster when r_I is on average lower). The adverse selection problem is actually costly for the seller with the high-quality asset because it causes inefficient liquidation of the otherwise valuable non-marketable asset. Therefore, the circulation rate of the high-quality, μ_H, reflects how easily the seller can raise liquidity by selling the asset in the market. That is, μ_H proxies for the degree of liquidity in the market. By comparison to the classical informed trading literature (such as Grossman and Stiglitz (1980) and Kyle (1985)), our model contributes to the literature by suggesting an alternative mechanism that can create market illiquidity. It suggests that reductions in funding liquidity decreases market liquidity through the lemons problem.

Figure 8 illustrates the impact of liquidity shocks on the expected price of the high-quality asset under different parameter values of the seller's endowment, $\bar{x}$, and the probability of being high quality, ρ. In the left panel of Figure 8, the fire sale discount increases faster when the seller's endowment is larger (i.e., $\bar{x}$ is higher). As arbitrage

[19]In the right panel of Figure 7, we first calculate $\bar{w}_A$ for each case of the three distributions of r_I, then scale the x-axis as the percentage of each $\bar{w}_A$.



(i) Expected price under different $\bar{x}$ (ii) Expected price under different ρ

Figure 8: The impact of reductions in arbitrage capital on the expected price of the high-quality asset $E[p|v_H]$ under various levels of the seller's endowment $\bar{x}$ and the probability of being high quality ρ (the x-axis is scaled as the percentage of $\bar{w}_A$)

capital decreases, the investors become less certain about the quality of the traded asset (due to both the price informativeness effect and the adverse selection effect). Because the investors are risk averse, they require greater risk premium for the increased uncertainty. Thus, a greater supply shock translates into a greater fall in prices. We can interpret a greater value of $\bar{x}$ as representing that distress occurs in a larger market. The accelerated price falls in case of higher $\bar{x}$ are also related to increased market illiquidity. In the right panel of Figure 8, the fire sale discount increases faster when the marketable asset is more likely to be of low quality (i.e., ρ is lower). In case of fixed income securities, we can interpret that speculative-grade securities are more vulnerable to fire sales under small reductions in arbitrage capital. Notice, however, that fire sales eventually occur even for those assets with higher ρ (or investment-grade securities) as arbitrage capital decreases further.

5.2 Economic efficiency

We can measure the productoin efficiency of the economy using the expected total payoffs of all the existing assets. These are the marketable asset, the non-marketable assets, and the risk-free asset. Because the marketable asset merely changes hands through trades, any change in efficiency arises from the allocations between the non-marketable assets and the risk-free asset. Recall that the non-marketable assets are ex-ante operating at an optimal level in the absence of liquidity constraints, so liquidation destroys value. Therefore, efficiency is maximized when liquidation of the non-marketable assets is minimized. In the previous subsections, we have shown that the arbitrageurs will trade the marketable asset according to their liquidity needs when the price is equal to the fundamental value. Then, more price informativeness improves efficiency by helping efficient allocations of resources.[20] Conversely, common liquidity shocks to arbitrageurs will make price uninformative, resulting in reduced efficiency.

This reduction in efficiency due to inefficient liquidation of the non-marketable assets is a negative spillover effect generated by fire sales. Mispricing caused by fire sales creates incentives to sacrifice the efficiency of investment in two ways. First, those in liquidity shortage are forced to liquidate their non-marketable assets. Second, even those in liquidity surplus choose to liquidate their non-marketable assets to raise funds for speculating on the marketable asset. For example, Ivashina and Scharfstein (2010) find that bank lending decreased significantly after the onset of the financial crisis.

We can measure the efficiency of the economy by the seller's expected payoff from the non-marketable asset, $\Pi \equiv E\big[(1+r_I)(px^s - l)\big]$, as follows:

$$\Pi = \rho \int_{\underline{r}}^{\bar{r}} \Big[(1+r_I)(E[p|v_H]x^s - l)\Big] f_I(r_I)dr_I + (1-\rho)(1+E[r_I])(E[p|v_L]\bar{x} - l). \quad (32)$$

It is immediate from Proposition 2 that the economic efficiency is maximized when there is enough arbitrage capital.

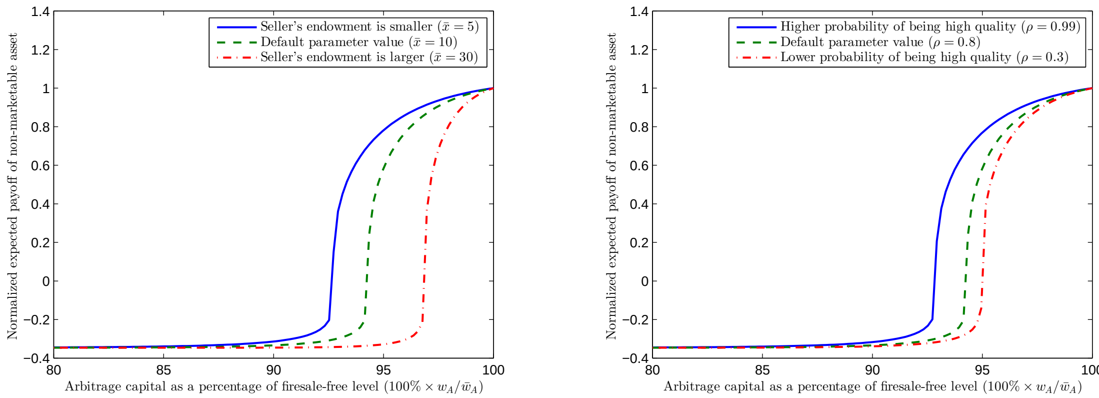
Corollary 2. Π *is maximized if* $w_A \geq \bar{w}_A$.

Proof. See Appendix B. □

As Proposition 2 implies efficiency decreases with more mispricing. That is, liquidity shocks reduce efficiency by misallocating resources. Figure 9 illustrates this with numerical examples. The liquidity shortage parameter is assumed to be $l = \left(\frac{v_L}{1+r_f}\right)\bar{x}$ in the numerical examples in Figure 9. The results show that negative spillover effects

[20]Our analysis considers production efficiency. In line with Grossman and Stiglitz (1980) and most of the literature on noisy REE, we do not perform a full welfare analysis that also considers changes in risk allocation. The reasons why the literature does not consider the allocation of risk as an indicator of welfare include the presence of noise traders and the exogenous risk-free rate.

are larger (i) when the seller's endowment is larger ($\bar{x}$ is higher), and (ii) when the marketable asset is more likely to be of low quality (ρ is lower).



(i) Efficiency under different $\bar{x}$ (ii) Efficiency under different ρ

Figure 9: The impact of reductions in arbitrage capital on the expected payoff of the non-marketable asset under various levels of the seller's endowment $\bar{x}$ and the probability of being high quality ρ (the x-axis is scaled as the percentage of $\bar{w}_A$, and the y-axis is scaled as the percentage of the maximum level of efficiency)

5.3 Ex-ante arbitrage capital allocations

In this subsection, we examine the ex-ante capital allocations of the arbitrageurs. We do this by adding an earlier stage ($t = 0$) to our model, in which the arbitrageurs decide their liquidity position.

We assume that there are two states of the economy; it is in a good state with probability ψ, and in a bad state with probability $1 - \psi$. The realization of the state is initially unknown, but it is observed by all the participants at $t = 1$. At $t = 0$, each arbitrageur is endowed with initial cash w_0 and a risky asset in place that gives a payoff u at $t = 1$ where $u = u_G$ in the good state, and $u = u_B$ in the bad state with $u_G > 0 > u_B$. The risky asset can be interpreted as financial institutions' operational profits that have general exposure to macroeconomic conditions, while their arbitrage activities can be understood as proprietary trading and market making. Therefore, the bad realization of u can be interpreted as a liquidity shock to the arbitrageurs.

The arbitrageurs can invest in a long-term investment opportunity that yields a random return y at $t = 2$. This investment opportunity is illiquid in the sense that arbitrageurs cannot liquidate it at $t = 1$ to raise extra funds.[21] Each arbitrageur allocates his initial cash between the investment opportunity and the risk-free asset. We assume that the risk-free rate at $t = 0$ is normalized to zero without loss of generality. Finally, we assume that all random variables v, r_I, ϵ, y and u are jointly independent.

Will arbitrageurs have enough liquid capital to eliminate any possibility of a fire sale and market freeze? We find that there will always be some fire sales in equilibrium if illiquid assets are attractive enough because the arbitrageurs do not internalize the spillover effects of a fire sale. We can prove this by contradiction. Suppose that there is no fire sale in equilibrium. Then, the arbitrageurs will not want to hold enough liquid arbitrage capital because the return will be too low. Instead, they will hold illiquid assets. Consequently, the arbitrageurs will have insufficient liquidity to prevent fire sales, thereby making the equilibrium unsustainable. Therefore, it is not possible to have an equilibrium without fire sales. This argument is parallel with the argument in Grossman and Stiglitz (1980) who find that informationally-efficient markets are not possible in equilibrium if information acquisition is costly.[22]

Proposition 2 states that liquid arbitrage capital w_A at $t = 1$ should be greater than some threshold $\bar{w}_A$ to prevent fire sales. In particular, the arbitrageurs should hold enough liquidity to offset the potential liquidity shock in the bad state. The following proposition states that it is not possible to eliminate the possibility of fire sales if the investment opportunity offers sufficiently high returns:

Proposition 3. *There is a constant $\bar{y}$ such that there does not exist a fire-sale-free equilibrium whenever $E[y] > \bar{y}$.*

Proof. See Appendix B. □

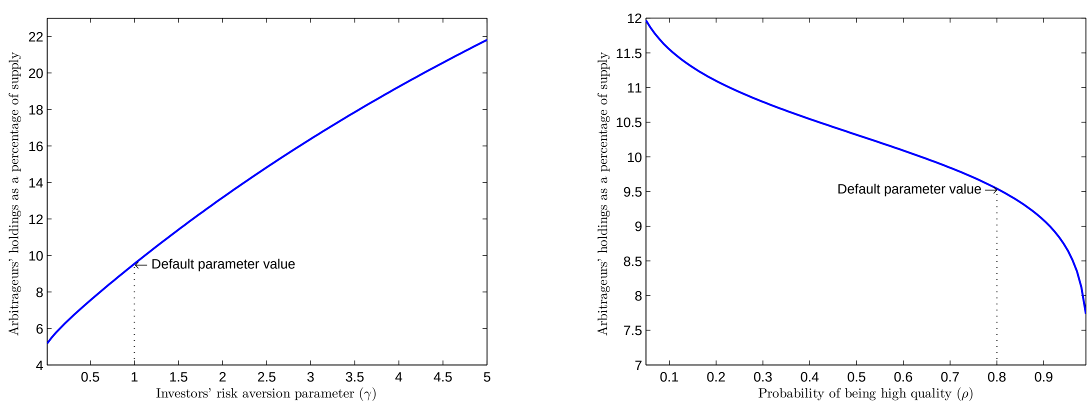
[21] We can alternatively assume that it can be partially liquidated at a loss.

[22] Dow and Han (2015) also features a similar argument that proves the impossibility of equilibrium without a bubble in the presence of contractual incompleteness and limited liability.

6 Discussion

6.1 Financial stability vs. financial fragility

We have shown that arbitrage capital can minimize mispricing and promote liquidity. It is worth stressing that arbitrageurs do not have to absorb the majority of the supply from the seller in our model. On the contrary, the uninformed investors absorb the major share of the supply of the risky asset whereas the arbitrageurs simply play a supporting role of setting the price close to the fundamental value. The role of arbitrage capital is, however, crucial because the investors absorb the supply due to the presence of arbitrage capital. A moderate amount of arbitrage capital is enough to make the market efficient and stable. We draw the conclusion that informed capital facilitates the movement of uninformed capital from the markets with excessive liquidity to those with lack of liquidity. This is the mechanism of market stability in our model.



(i) Arbitrageurs' share under different γ (ii) Arbitrageurs' share under different ρ

Figure 10: Average arbitrageurs' holdings as a percentage of total supply at the fire-sale-free level across various levels of investors' risk aversion parameter γ and probability of high quality ρ: The figure illustrates that the majority of the supply from the seller is absorbed by the investors rather than the arbitrageurs around the default parameter values

Figure 10 illustrates how many percentage of the supply is on average absorbed by the arbitrageurs assuming that the arbitrageurs have the fire-sale-free level capital $\bar{w}$. With the same parameter values used in the previous numerical example (i.e., $\gamma = 1, \rho = 0.8$), we find that about only about ten percent of the supply needs to be absorbed to make the market fully functional. The investors on average absorb about 90% of the supply, and yet there is no fire sale. In other words even a small amount of informed capital can make the price efficient. Therefore, arbitrage capital creates information spillover effects in the market that multiplies its price-setting ability; the revelation of private information through prices makes otherwise-uninformed capital work like informed capital.

This efficiency of arbitrage capital may seem advantageous for financial stability, but it is in fact a double-edge sword. The potency of a small amount of arbitrage capital may actually be the reason for financial fragility because it means prices will be sensitive to a reduction of arbitrage capital. That is, small capital shocks to arbitrageurs in the market can trigger fire sales and market freezes by driving away the demand of uninformed investors. Indeed, as we have seen in Figure 6, small shocks to arbitrage capital create fire sales by exacerbating lemons problems in the market. Notice that such large drops in prices are triggered by small changes in arbitrage holdings (relative to total holdings) as shown in Figure 10. The "multiplier" effects of arbitrage capital can actually serve as the source of financial *in*stability rather than financial stability.

Fire sales and market freezes create negative externalities and distort resource allocations. Our paper highlights the value of arbitrage capital in facilitating the efficient functioning of markets. This has policy implications for capital adequacy regulation. On the other hand, our analysis suggests that asset purchases will be ineffective in preventing fire sales if they are caused by a lemons problem,

Recent papers such as Mitchell, Pedersen, and Pulvino (2007) and Duffie (2010) suggest that institutional impediments such as such as search frictions, taxes, regulations, and market segmentation can slow down the speed of arbitrage capital, thereby creating fire sales and market freezes during market stress. But isn't there anybody who can try to arbitrage away obvious mispricing? A very few markets are truly closed to outsiders by regulatory fiat (writing insurance is an example) but most markets are open to investors who want to participate. Our results suggest an explanation for the underlying assumption of "slow-moving capital"; the presence of sufficient informed capital can facilitate movement of uninformed capital during normal times, but the market can quickly become illiquid if informed capital providers are subject to liquidity shocks, as the resulting lemons problem prevents entry of uninformed capital.

Since the financial crisis, there has been an extensive debate about how to prevent fire sales because they can lead to negative welfare consequences in the economy (e.g., Shleifer and Vishny, 2011; Tirole, 2011). The suggested remedies for fire sales can be usually categorized into two types. One is an ex-ante approach that reduces the possibility of fire sales (e.g., Krishnamurthy, 2010; Diamond and Rajan, 2011; Perotti and Suarez, 2011), and the other is an ex-post approach that mitigates the magnitude of fire sales and the following adverse effects (e.g., Shleifer and Vishny, 2010; Diamond and Rajan, 2011; Tirole, 2012; Guerrieri and Shimer, 2014). Our theory has implications for both ex-ante and ex-post measures against fire sales.

On the ex-post side, our results imply that asset purchase programs may not be effective if asset purchase programs merely aim to reduce the net supply of assets sold in fire sales. Asset purchase programs might be effective if fire sales were caused simply by cash-in-the-market pricing that does not involve information asymmetries. However, they will not be effective if fire sales are caused by lemons problems in the market because reducing asset supply in itself does not improve price informativeness.[23] Therefore, uninformed capital would not move into the market in fire sales simply because of an asset purchase program. On the other hand, extending liquidity to arbitrageurs can improve price informativeness, thereby restoring the price mechanism that allows uninformed capital to participate in the market. Of course, incentive problems should be properly addressed because financial institutions that receive liquidity support may have very different objectives from the government agency that provides liquidity to them.

On the ex-ante side, our results imply that regulations on arbitrageurs such as capital requirements may or may not achieve desired effects of stabilizing the market. We have argued that lemons problems are endogenously determined by the availability arbitrage capital, and that they have an intrinsic vulnerability to shocks. In Section 5, we showed that enough arbitrage capital will prevent fire sales, but that arbitrageurs do not have incentives to provide enough capital because they do not internalize negative spillover effects. In that sense, requiring financial institutions to keep enough capital for potential crisis could have a direct effect of lowering the frequency of fire sales. However, our model shows such tightening of capital requirements would lower the return on arbitrage capital. One might consider that in practice, this could have negative indirect effects, for example by inducing arbitrageurs to exit entirely.[24] In Proposition 3, the very reason for the impossibility of fire-sale-free equilibrium is the low profitability of arbitrage capital

[23]In Tirole (2012), asset purchase programs can still be effective in the presence of a lemons problem if the government is able to "clean up" the market by removing low-quality assets.

[24]For example, capital may flow to unregulated sectors such as non-bank financial intermediaries rather than to regulated sectors.

in the absence of fire sales.

6.2 Flight to quality

During financial crises (or episodes of market stress), investors tend to migrate their portfolios to safer and more liquid asset classes. Consequently, prices of low-quality assets naturally fall while those of high-quality assets rise. This flight-to-quality (and flight-to-liquidity) is well documented in various financial markets such as the Euro-area government bond market (e.g., Beber, Brandt, and Kavajecz, 2009), the corporate bond market and the stock market (e.g., Acharya, Amihud, and Bharath, 2013). What are the driving forces behind those flight-to-quality or flight-to-liquidity phenomena? Potential explanations include time-varying risk or liquidity preference, time-varying return correlations among assets and time-varying liquidity factors. However, these explanations are rather complex. They require an elaborate mechanism of dynamic changes in preferences or correlations. In contrast our model suggests a comparatively simpler explanation based on lemons problems in financial markets.

In our model, a shock to financial institutions (which causes reductions in arbitrage capital) triggers a flight-to-quality among the uninformed investors. Because the investors are unwilling to absorb the supply of assets at the previous price, they require a lower price and in equilibrium they end up holding more of their wealth in the risk-free asset. The new, lower price is a consequence of both the lemons problem and the extra risk they bear because the asset value is not revealed by the price. Figure 11 illustrates how the investors change their portfolio as arbitrage capital decreases. Notice that this change in portfolios occurs together with significant price falls in the marketable asset as shown in Figure 6.[25] Indeed, Acharya, Amihud, and Bharath (2013) find that there is a flight-to-quality regime in which prices of speculative bonds fall substantially whereas those of investment grade bonds rise. Furthermore, they argue that the flight-to-quality regime can be predicted by economic "stress".

It seems that flight-to-quality and flight-to-liquidity are interconnected phenomena which cannot be easily disentangled. In Section 5, we have shown that illiquidity also increases for the marketable asset as flight-to-quality occurs. Therefore, we find that the two phenomena happen together. However, in our interpretation illiquidity is an

[25]Notice that the portfolio weight on the marketable asset and the risk-free asset is given by $\frac{px_I}{w_I}$ and $1 - \frac{px_I}{w_I}$, respectively.

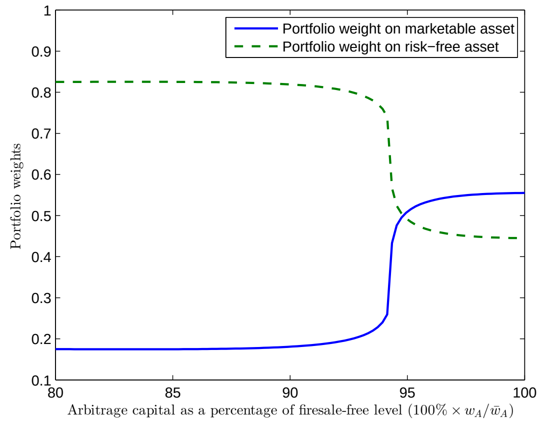


Figure 11: The impact of reductions in arbitrage capital on the average portfolio weights of investors on the marketable asset and the risk-free asset (the x-axis is scaled as the percentage of $\bar{w}_A$): The figure illustrates that with low levels of arbitrage capital, the uninformed investors hold less of the marketable asset and more of the risk-free asset

indication of the uninformed investors' uncertainty about other investors' intention of trading, rather than the reason they want to avoid the marketable asset.

6.3 Interpretation of the 2007-2009 financial crisis

In this section, we discuss the recent financial crisis in the light of our results in this paper, and find some implications on the policy or regulations. The recent financial crisis is often viewed as a consequence of the collapse of the housing bubble that grew through the 2000s. Along with real estate, other assets and securities were also overvalued. The aftermath of the collapse during 2007-2009 induced many financial institutions to unwind their positions as well as to reduce their leverage. This process created liquidity shortage among leveraged institutions, thus created further illiquidity spillover effects in other markets. There was market freeze across a large class of assets in particular for structured financial products.[26]

Gorton and Metrick (2012) document that banks suffered liquidity shortage during the financial crisis. They further find that relatively small amount of subprime risk lead to spread rises of unrelated asset classes. Our theory suggests a reduction in informed trading can act as a transmission mechanism of liquidity shocks to unrelated asset classes.

[26]See, for example, Brunnermeier (2009).

During the onset of the recent financial crisis, there was a liquidity crunch among the financial institutions as a result of downturn in housing markets (and subsequent price falls in structured products on housing mortgages). The liquidity constraints affected the arbitrage trading variation of financial institutions which are likely to be more informed about many securities they were trading. Merrill, Nadauld, Stulz, and Sherlund (2014) find evidence about fire sales of RMBS in the aftermath of the subprime crisis.

In our model, the seller decides whether to bring their endowment of the marketable asset to the market or not. Bringing the asset to the market can be interpreted as trading the asset in the market, but it can be also alternatively interpreted as creating the asset. Vickery and Wright (2013) document that issuance of non-agency RMBS (or private-label RMBS) decreased significantly relative to agency RMBS since mid-2007 and, during this period, secondary markets for trading non-agency RMBS were extremely illiquid. Non-agency RMBS are less regulated and tend to include riskier underlying loans (i.e., subprime mortgages) relative to agency RMBS, thus, they are more subject to information asymmetries than agency RMBS. Our theory can explain why there was a collapse of non-agency RMBS market during the recent crisis.

7 Conclusions

In our paper, we have developed an information-based theory of fire sales using a noisy REE framework with endogenous lemons problem. Our model combines limits to arbitrage and adverse selection to provide a plausible mechanism of financial fire sales and market freezes. In a situation when informed market participants are not highly liquidity constrained, arbitrage activity is high and prices are informative. This allows uninformed investors to absorb the supply of assets without worrying about adverse selection. But a market wide shock can cause liquidity constraints to arbitrageurs. When this happens, arbitrage activity is reduced and prices become less informative. This creates an adverse selection problem, increasing the supply of low-quality assets. This lemons problem makes well-capitalized uninformed market participants unwilling to absorb the supply, thereby freezing the market. This can explain the “double whammy” in which fire sales and market freezes occur together. Our results shed light on the paradoxical nature of fire sales in which capital moves out of the market when it is needed most and apparently would earn higher returns. Furthermore, our results show how a financial fire sale can reduce economic efficiency. This may also be accompanied by a flight-to-quality.

The model has implications for financial fragility. Only a small amount of arbitrage capital is required to make the market work efficiently because, so long as it ensures assets are priced efficiently, this can facilitate the movement of uninformed capital. However, this is a double-edge sword because a small reduction in arbitrage capital can create a lemons problem and slow down the movement of uninformed capital, causing a fire sale.

Appendix A. List of symbols

v	payoff of marketable asset
ρ	probability that marketable asset pays v_H
$\bar{x}$	seller's endowment of marketable asset
l	seller's liquidity shortage
r_f	risk-free rate
r_I	return on non-marketable asset
$\underline{r}$	lower bound of r_I
w_A	liquidity position of arbitrageur
m	dollar margin on arbitrageur's position
γ	risk aversion parameter of investor
w_I	initial wealth of investor
ϵ	demand of noise traders
x^s	seller's supply of marketable asset
$X_A(v)$	arbitrageurs' demand for marketable asset given v
$X_I(p)$	investors' demand for marketable asset given p
μ_H	circulation rate of high-quality asset
μ_L	circulation rate of low-quality asset
$\hat{\rho}$	average quality of traded asset
ξ_p	net supply of marketable asset
ΔX	arbitrage trading variation
θ	adjusted weight on v_H in price function
u	payoff of risky asset
ψ	probability that risky asset pays u_G
y	return on long-term investment opportunity

Appendix B.

Proof of Lemma 3: An investor $i \in \mathcal{I}$ maximizes his expected utility:

$$EU(w_2^i) = -\frac{1}{\gamma}\Big[q(p)e^{-\gamma[w_I(1+r_f)+(v_H-p(1+r_f))x^i]} + (1-q(p))e^{-\gamma[w_I(1+r_f)+(v_L-p(1+r_f))x^i]}\Big], \tag{B.1}$$

where $q(p)$ is the posterior belief of the investor conditional on the price. Because p is a sufficient statistic for ξ_p, we have

$$q(p) = Pr(v = v_H|p, \xi_p). \tag{B.2}$$

Then, the first order condition is equal to

$$\begin{aligned}\frac{\partial EU(w_2^i)}{\partial x^i} = e^{-\gamma w_I(1+r_f)}\Big[&q(p)(v_H - p(1+r_f))e^{-\gamma(v_H-p(1+r_f))x^i} \\ &+ (1-q(p))(v_L - p(1+r_f))e^{-\gamma(v_L-p(1+r_f))x^i}\Big] = 0.\end{aligned} \tag{B.3}$$

Notice that the second order condition is always satisfied because

$$\begin{aligned}\frac{\partial^2 EU(w_2^i)}{(\partial x^i)^2} = -\gamma e^{-\gamma w_I(1+r_f)}\Big[&q(p)(v_H - p(1+r_f))^2e^{-\gamma(v_H-p(1+r_f))x^i} \\ &+ (1-q(p))(v_L - p(1+r_f))^2e^{-\gamma(v_L-p(1+r_f))x^i}\Big] < 0.\end{aligned} \tag{B.4}$$

Solving Eq. (B.3) for x^i gives the optimal portfolio given p as follows:

$$x^i(p) = \frac{1}{\gamma(v_H - v_L)}\Big[\log\Big(\frac{q(p)}{1-q(p)}\Big) + \log\Big(\frac{v_H - (1+r_f)p}{(1+r_f)p - v_L}\Big)\Big]. \tag{B.5}$$

By aggregating each individual demand x^i across all investors in $\mathcal{I}$, we can obtain the aggregate demand of investors. □

Proof of Lemma 4: According to the initial conjecture that p is a sufficient statistic for ξ_p, the investors can infer ξ_p from p to update their beliefs about v. We define $q^*(p, \xi_p) \equiv Pr(v = v_H|p, \xi_p)$ to be the investors' posterior belief conditional on p and ξ_p.

Using Bayes' rule, the investors' posterior beliefs that the marketable asset is of high quality can be derived as follows:

$$
\begin{aligned}
q^*(p,\xi_p) &= \frac{\hat{\rho}L(\xi_p;v_H,p)}{\hat{\rho}L(\xi_p;v_H,p)+(1-\hat{\rho})L(\xi_p;v_L,p)} \\
&= \frac{\hat{\rho}f_\epsilon(\bar{x}-X_A(v_H)-\xi_p)}{\hat{\rho}f_\epsilon(\bar{x}-X_A(v_H)-\xi_p)+(1-\hat{\rho})f_\epsilon(\bar{x}-X_A(v_L)-\xi_p)},
\end{aligned} \tag{B.6}
$$

where $L(\cdot;v,p)$ is the likelihood function of ξ_p given v and p.[27]

Now, we turn to the second step in which we prove that p is indeed a sufficient statistic for ξ_p in equilibrium. Market clearing (see Eq. (9)) together with the definition of ξ_p implies that for any value of ξ_p and p,

$$
\xi_p = X_I(p). \tag{B.7}
$$

Whenever there exists a unique solution for ξ_p that solves Eq. (B.7), there exists an injective mapping from p to ξ_p. Therefore, the investors can infer ξ_p correctly.

Eqs. (15) and (B.7) imply that

$$
\xi_p = \frac{1}{\gamma(v_H-v_L)}\Big[\log\Big(\frac{q^*(p,\xi_p)}{1-q^*(p,\xi_p)}\Big)+\log\Big(\frac{v_H-(1+r_f)p}{(1+r_f)p-v_L}\Big)\Big]. \tag{B.8}
$$

To prove that there exists a unique solution that solves Eq. (B.7), we first claim that $q^*(p,\xi_p)$ is non-increasing in ξ_p for any given p, i.e., $\frac{\partial q^*(p,\xi_p)}{\partial \xi_p}\le 0$ for all ξ_p. To see this, we first obtain the first-order derivative of $q^*(p,\xi_p)$ with respect to ξ_p from Eq. (B.6):

$$
\begin{aligned}
\frac{\partial q^*(p,\xi_p)}{\partial \xi_p} = &-\frac{\hat{\rho}(1-\hat{\rho})f_\epsilon(\bar{x}-X_A(v_H)-\xi_p)f_\epsilon(\bar{x}-X_A(v_L)-\xi_p)}{\big(\hat{\rho}f_\epsilon(\bar{x}-X_A(v_H)-\xi_p)+(1-\hat{\rho})f_\epsilon(\bar{x}-X_A(v_L)-\xi_p)\big)^2} \\
&\times\Big(\frac{f'_\epsilon(\bar{x}-X_A(v_H)-\xi_p)}{f_\epsilon(\bar{x}-X_A(v_H)-\xi_p)}-\frac{f'_\epsilon(\bar{x}-X_A(v_L)-\xi_p)}{f_\epsilon(\bar{x}-X_A(v_L)-\xi_p)}\Big).
\end{aligned} \tag{B.9}
$$

[27] The numerator of the second expression in Eq. (B.6) is the likelihood of the quantity ξ_p and price p if the asset is high quality, since $\hat{\rho}$ is the probability the asset is high quality and $f_\epsilon(\bar{x}-X_A(v_H)-\xi_p)$ is the likelihood of the demand of the noise traders taking exactly the value that offsets a net supply of the asset to the investors of ξ_p, when informed demand takes the value $X_A(v_H)$ that corresponds to informed traders knowing the asset is of high quality. Similarly the denominator has two terms, one of which is the same as the numerator and the other being the similar term for the case of low asset quality.

Because $f''_\epsilon f_\epsilon \leq f'^2_\epsilon$ (see Eq. (3)), $\frac{f'_\epsilon}{f_\epsilon}$ is non-increasing (this can be verified by differentiating $\frac{f'_\epsilon}{f_\epsilon}$). Because net supply is larger when the asset is of low quality, $\bar{x} - X_A(v_L) > \bar{x} - X_A(v_H)$, and we have

$$\frac{f'_\epsilon(\bar{x} - X_A(v_H) - \xi_p)}{f_\epsilon(\bar{x} - X_A(v_H) - \xi_p)} \geq \frac{f'_\epsilon(\bar{x} - X_A(v_L) - \xi_p)}{f_\epsilon(\bar{x} - X_A(v_L) - \xi_p)}. \tag{B.10}$$

From Eqs. (B.9) and (B.10), it is immediate that $\frac{\partial q^*(p,\xi_p)}{\partial \xi_p} \leq 0$.

Therefore, the rhs of Eq. (B.8) is continuous and non-increasing in ξ_p.[28] Because, trivially, the lhs of Eq. (B.8) is continuous and increasing in ξ_p, there exists a unique solution that solves Eq. (B.8) for any given p. That is, there exists a unique injective function that maps p to ξ_p. We denote $\xi(p)$ to be the unique mapping that maps p to ξ_p. Then, we have $q(p) = q^*(p, \xi(p))$. Using Eq. (B.6), we can finally represent the investors' posterior belief as a function of p as in Eq. (20). $\square$

Proof of Proposition 1: We prove existence of equilibrium by construction in two steps. First, we fix the quality of the traded asset $\hat{\rho}$ as given, allowing us to solve for a unique price p that clears the market at any given level of v and ϵ. Second, we show that there exists a solution for $\hat{\rho}$ given the expected price $E[p|v]$ using the price function derived in the first step.

We start with the first step by fixing $\hat{\rho}$. For any given v and ϵ, the investors' demand has to be equal to the net supply of the asset due to the market clearing condition in Eq. (9). Then, we get the following equation that should be satisfied by the equilibrium price p:

$$\frac{1}{\gamma(v_H - v_L)}\left[\log\left(\frac{q(p)}{1-q(p)}\right) + \log\left(\frac{v_H - (1+r_f)p}{(1+r_f)p - v_L}\right)\right] = \bar{x} - X_A(v) - \epsilon. \tag{B.11}$$

Substituting the expression for investors' beliefs from Eq. (20) into Eq. (B.11) and solving for p, we can derive the equilibrium price as follows:

$$p = \frac{1}{1+r_f}\big(\theta(v,\epsilon;\hat{\rho},w_A)v_H + (1-\theta(v,\epsilon;\hat{\rho},w_A))v_L\big), \tag{B.12}$$

[28] It can be easily verified that the rhs of Eq. (B.8) is non-increasing in ξ_p given p because it is increasing in q^*, and $q^*(\cdot)$ is non-increasing in ξ_p at any given level of p.

where the weight $\theta(v, \epsilon; \hat{\rho}, w_A)$ is defined by

$$\theta(v, \epsilon; \hat{\rho}, w_A) \equiv \frac{\hat{\rho} f_\epsilon(\epsilon - X_A(v_H) + X_A(v))}{D(v, \epsilon; w_A)}, \tag{B.13}$$

and

$$\begin{aligned} D(v, \epsilon; w_A) \equiv & \hat{\rho} f_\epsilon(\epsilon - X_A(v_H) + X_A(v)) + (1 - \hat{\rho}) f_\epsilon(\epsilon - X_A(v_L) + X_A(v)) \\ & \times \exp(\gamma(v_H - v_L)(\bar{x} - X_A(v) - \epsilon)). \end{aligned} \tag{B.14}$$

Using the fact that $X_A(v_H) = \frac{w_A}{m}$ and $X_A(v_L) = -\frac{w_A}{m}$ (see Lemma 2), we can obtain Eqs. (25) and (26) from Eqs. (B.13) and (B.14). The equilibrium price function in Eq. (24) is immediate from Eq. (B.12).

Now, we turn to the second step that proves existence of the equilibrium supply from the sellers. From Eqs. (16) and (19), in equilibrium the quality of the traded asset $\hat{\rho}$ should satisfy:

$$\hat{\rho} = H(\hat{\rho}), \tag{B.15}$$

where

$$H(\hat{\rho}) \equiv \frac{\rho\left(1 - F_I\left(\frac{v_H}{E[p|v_H]} - 1\right)\right)}{\rho\left(1 - F_I\left(\frac{v_H}{E[p|v_H]} - 1\right)\right) + (1 - \rho)}. \tag{B.16}$$

Notice that Eq. (B.13) implies that $E[p|v_H] = \frac{v_L}{1+r_f}$ when $\hat{\rho} = 0$. Thus,

$$H(0) = \frac{\rho\left(1 - F_I\left(r_f + \frac{(1+r_f)(v_H - v_L)}{v_L}\right)\right)}{\rho\left(1 - F_I\left(r_f + \frac{(1+r_f)(v_H - v_L)}{v_L}\right)\right) + (1 - \rho)} > 0. \tag{B.17}$$

Furthermore, Eq. (B.15) implies that $\hat{\rho}$ is always smaller than or equal to ρ. This implies that $H(\rho) \leq \rho$. Because $H(\cdot)$ is continuous in $\hat{\rho}$, there must exist a fixed point solving Eq. (B.15) on the set $(0, \rho]$. The equilibrium condition in Eq. (23) is immediate from Eqs. (B.15) and (B.16). $\square$

Proof of Proposition 2: From Eq. (24), we can represent the expected price of the high-quality asset given w_A to be a function $\hat{P}(w_A)$ as follows:

$$\begin{aligned}\hat{P}(w_A) &\equiv \int_{-\infty}^{\infty}\Big[\frac{1}{1+r_f}\big(\theta(v_H,\epsilon;\hat{\rho},w_A)v_H + (1-\theta(v_H,\epsilon;\hat{\rho},w_A))v_L\big)\Big]f_\epsilon(\epsilon)d\epsilon \\ &= \frac{v_L}{1+r_f} + \frac{v_H-v_L}{1+r_f}\int_{-\infty}^{\infty}\theta(v_H,\epsilon;\hat{\rho},w_A)f_\epsilon(\epsilon)d\epsilon.\end{aligned} \tag{B.18}$$

Notice that $\hat{P}$ is equal to $E[p|v_H]$ by definition. If there is no lemons problem, $\hat{\rho}$ is equal to ρ. Therefore, we can represent the expected price of the high-quality asset given w_A in the absence of lemons problem as follows:

$$\begin{aligned}P(w_A) &\equiv \int_{-\infty}^{\infty}\Big[\frac{1}{1+r_f}\big(\theta(v_H,\epsilon;\rho,w_A)v_H + (1-\theta(v_H,\epsilon;\rho,w_A))v_L\big)\Big]f_\epsilon(\epsilon)d\epsilon \\ &= \frac{v_L}{1+r_f} + \frac{v_H-v_L}{1+r_f}\int_{-\infty}^{\infty}\theta(v_H,\epsilon;\rho,w_A)f_\epsilon(\epsilon)d\epsilon.\end{aligned} \tag{B.19}$$

We let $\bar{p} \equiv \frac{v_H}{1+\underline{r}}$, above which price level there is no lemons problem (i.e., $\hat{\rho}$ is equal to ρ for any $E[p|v_H] \geq \bar{p}$). By definition, $P(w_A)$ and $\hat{P}(w_A)$ are identical whenever $E[p|v_H] \geq \bar{p}$. On the other hand, $P(w_A)$ is smaller than or equal to $\hat{P}(w_A)$ whenever $E[p|v_H] < \bar{p}$ because $\hat{\rho}$ is smaller than or equal to ρ (or equivalently, $\mu_H < 1$). This implies that $E[p|v_H]$ will be smaller than $\bar{p}$ if and only if $P(w_A)$ is smaller than $\bar{p}$.

Differentiating $P(w_A)$ with respect to w_A yields

$$\frac{dP}{dw_A} = \frac{v_H-v_L}{1+r_f}\int_{-\infty}^{\infty}\frac{d\theta(v_H,\epsilon;\rho,w_A)}{dw_A}f_\epsilon(\epsilon)d\epsilon. \tag{B.20}$$

and differentiating $\theta(v_H,\epsilon;\rho,w_A)$ with respect to w_A (using Eq. (25)) yields

$$\begin{aligned}&\frac{d\theta(v_H,\epsilon;\rho,w_A)}{dw_A} \\ &\quad= \frac{2\theta(v_H,\epsilon;\rho,w_A)(1-\theta(v_H,\epsilon;\rho,w_A))}{m}\Big(\frac{\gamma(v_H-v_L)}{2} - \frac{f'_\epsilon(\epsilon+\Delta X)}{f_\epsilon(\epsilon+\Delta X)}\Big).\end{aligned} \tag{B.21}$$

Because $\theta(v_H,\epsilon;\rho,w_A) \in (0,1)$, Eq. (B.21) implies that $\frac{d\theta_{ex}(\epsilon;w_A)}{dw_A}$ is positive under the condition in Eq. (4). Therefore, $P(w_A)$ is strictly increasing in w_A, i.e., $\frac{dP}{dw_A} > 0$ for all $w_A \in [0,\infty)$. Because $P(w_A)$ approaches $\frac{v_H}{1+r_f} > \bar{p}$ as w_A approaches infinity, there exists a unique threshold $\bar{w}_A \in [0,\infty)$ above which $P(w_A)$ is greater than $\bar{p}$, and below which $P(w_A)$ is smaller than $\bar{p}$. Notice that $\bar{w}_A$ is equal to zero if $P(0) > \bar{p}$.

Recall that $\hat{P}(w_A)$ is smaller than $\bar{p}$ if and only if $P(w_A)$ is smaller than $\bar{p}$. Therefore, we conclude that $\hat{P}(w_A)$ is smaller than $\bar{p}$ if and only if w_A is smaller than $\bar{w}_A$. $\square$

Proof of Proposition 3: We prove this proposition by showing that it is not possible to have w_A greater than $\bar{w}_A$ at $t = 1$ whenever $E[y] > \bar{y}$ for some constant $\bar{y}$.[29] We first take I_A^* as the aggregate investment in the investment opportunity. Given I_A^*, we obtain the equilibrium prices, then solve each arbitrageur's ex-ante optimization problem. Then, we show that the arbitrageurs always invest more than I_A^* whenever $E[y]$ is high enough. Therefore, there does not exist an equilibrium where w_A exceeds $\bar{w}_A$.

Given I_A^*, the aggregate arbitrage capital at $t = 1$ is given by $w_A^* = w_0 - I_A^* + u$ (recall that u is the risky component of the arbitrageur's initial endowment, interpreted as the payoff on a risky asset). Then, the equilibrium price of the marketable asset is determined by Eq. (24) in Proposition 1, and we denote this $p(w_A^*)$. Then, $E[p(w_A^*)|v_H]$ and $E[p(w_A^*)|v_L]$ are the expected price of the high- and low- quality asset given w_A^*, respectively.

At $t = 1$, the market for the marketable asset may or may not be open depending on the seller's choice of $x^s \in \{0, \bar{x}\}$. If the market is open (i.e., $x^s = \bar{x}$), the value function of arbitrageur $a \in \mathcal{A}$ given p, v, w_A^a and I^a is

$$V^a(p, v; w_A^a, I^a) \equiv \max_{x^a} \; vx^a + (1 + r_f)(w_A^a - p(w_A^*)x^a) + (1 + E[y])I^a, \tag{B.22}$$

subject to

$$|mx^a| \leq w_A^a. \tag{B.23}$$

Notice that this is equivalent to the arbitrageur's optimization problem in the original setup in Eq. (6). The extended setup in Section 5.3 adds the extra term $(1 + E[y])I^a$ to the objective function in Eq. (B.22), but only the return from the investment opportunity is relevant in the arbitrageur's ex-ante decision making.

[29] Recall that $\bar{w}_A$ is the threshold above which there is no fire sale. See Proposition 2 for details.

If the market is not open (i.e., $x^s = 0$), the value function of arbitrageur $a \in \mathcal{A}$ given p, v, w_A^a and I^a is

$$V^a(p, v; w_A^a, I^a) = (1 + r_f)w_A^a + (1 + E[y])I^a. \tag{B.24}$$

Using Lemma 2 and the fact that $w_A^a = w_0 - I^a + u$, we can obtain the expected utility of arbitrageur a conditional on I^a and u from the value functions in Eqs. (B.22) and (B.24):

$$E[V^a(p, v; w_A)|I^a, u] = (w_0 + u)\big[1 + r_f + R(w_A^*)\big] + I^a\big[E[y] - r_f - R(w_A^*)\big], \tag{B.25}$$

where

$$\begin{aligned} R(w_A^*) \equiv &\rho\mu_H\left(\frac{v_H - E[p(w_A^*)|v_H](1 + r_f)}{m}\right) \\ &+ (1 - \rho)\left(\frac{E[p(w_A^*)|v_L](1 + r_f) - v_L}{m}\right). \end{aligned} \tag{B.26}$$

At $t = 0$, arbitrageur a maximizes $E[V^a(p, v; w_A)|I^a]$ by optimally choosing I^a (before u realizes at $t = 1$). Using Eq. (B.25), we obtain the ex-ante optimization problem as follows:

$$\max_{I^a}\ (w_0 + E[u])\big[1 + r_f + E[R(w_A^*)]\big] + I^a\big[E[y] - r_f - E[R(w_A^*)]\big]. \tag{B.27}$$

We now prove that there cannot exist a fire-sale-free equilibrium by contradiction. Suppose that there exists a fire-sale-free equilibrium. Then, available arbitrage capital in the bad state should be greater than the minimum threshold, i.e., $w_A^* = w_0 - I_A^* + u_B \geq \bar{w}_A$. That is, $I_A^* \leq \bar{I} \equiv w_0 - \bar{w}_A + u_B$.[30] For this to be an equilibrium, each arbitrageur should be indifferent about the choice of I^a due to the risk-neutrality. That is, the following equation should be true:

$$E[y] - r_f - E[R(w_A^*)] = 0, \tag{B.28}$$

[30]This implies that arbitrage capital in the good state will also be greater than $\bar{w}_A$ because $u_G > 0 > u_B$.

or equivalently (recall that ψ is the probability of the high state),

$$E[y] - r_f = \psi R(w_0 - I_A^* + u_G) + (1-\psi) R(w_0 - I_A^* + u_B). \tag{B.29}$$

Notice that there is no lemons problem when $I_A^* \leq \bar{I}$ (or equivalently, $w_A^* > \bar{w}_A$ for any realization of u). Then, Proposition 2 implies that $\hat{\rho} = \rho$, which in turn implies that the expected price is monotone in w_A^*. We show this in the following:

From Eq. (25) under the condition $\hat{\rho} = \rho$, differentiating $\omega(v_H, \epsilon; \rho, w_A^*)$ with respect to w_A^* yields

$$\begin{aligned} &\frac{d\theta(v_H, \epsilon; \rho, w_A)}{dw_A} \\ &\quad = \frac{2\theta(v_H, \epsilon; \rho, w_A)(1 - \theta(v_H, \epsilon; \rho, w_A))}{m} \Big(\frac{\gamma(v_H - v_L)}{2} - \frac{f_\epsilon'(\epsilon + \Delta X)}{f_\epsilon(\epsilon + \Delta X)} \Big), \end{aligned} \tag{B.30}$$

which is strictly positive because of Eq. (4). Therefore, we can show from Eq. (24) that the expected price of the high-quality asset is monotone increasing in w_A^*:

$$\frac{dE[p(w_A^*)|v_H]}{dw_A^*} = \frac{v_H - v_L}{1 + r_f} \int_{-\infty}^{\infty} \frac{d\theta(v_H, \epsilon; \rho, w_A^*)}{dw_A^*} f_\epsilon(\epsilon) d\epsilon > 0.$$

Similarly, from Eq. (26), differentiating $\theta(v_L, \epsilon; \rho, w_A^*)$ with respect to w_A^* yields

$$\begin{aligned} &\frac{d\theta(v_L, \epsilon; \rho, w_A)}{dw_A} \\ &\quad = -\frac{2\theta(v_L, \epsilon; \rho, w_A)(1 - \theta(v_L, \epsilon; \rho, w_A))}{m} \Big(\frac{\gamma(v_H - v_L)}{2} - \frac{f_\epsilon'(\epsilon - \Delta X)}{f_\epsilon(\epsilon - \Delta X)} \Big), \end{aligned} \tag{B.31}$$

which is strictly negative. Therefore, the expected price of the low-quality asset is monotone decreasing in w_A^*:

$$\frac{dE[p(w_A^*)|v_L]}{dw_A^*} = \frac{v_H - v_L}{1 + r_f} \int_{-\infty}^{\infty} \frac{d\theta(v_L, \epsilon; w_A^*)}{dw_A^*} f_\epsilon(\epsilon) d\epsilon < 0. \tag{B.32}$$

From Eq. (B.26) under the condition that $\mu_H = 1$, differentiating $R(w_A^*)$ with respect to w_A^* yields

$$\frac{dR(w_A^*)}{dw_A^*} = \frac{1+r_f}{m}\left[-\rho\left(\frac{dE[p|v_H, w_A^*]}{dw_A^*}\right) + (1-\rho)\left(\frac{dE[p|v_L, w_A^*]}{dw_A^*}\right)\right] < 0. \tag{B.33}$$

Therefore, $R(w_A^*)$ is strictly decreasing in the absence of lemons problem, i.e., $\frac{dR(w_A^*)}{dw_A^*}$ is strictly negative if $w_A^* \geq \bar{w}_A$. This implies that there exists a constant $\bar{R} \equiv \psi R(w_0 - \bar{I} + u_G) + (1-\psi)R(w_0 - \bar{I} + u_B)$ which is an upper bound of $R(w_A^*)$, i.e.,

$$\bar{R} \geq \psi R(w_0 - I_A^* + u_G) + (1-\psi)R(w_0 - I_A^* + u_B), \text{ for any } I_A^* \leq \bar{I}. \tag{B.34}$$

Arbitrageur a's ex-ante optimization problem in Eq. (B.27) implies that the arbitrageur will increase I^a whenever $E[y] - r_f > \psi R(w_0 - I_A^* + u_G) + (1-\psi)R(w_0 - I_A^* + u_B)$. Therefore, the arbitrageurs allocate more capital in the illiquid long-term investment opportunity whenever $E[y] > r_f + \bar{R}$, thus, I_A^* cannot be an equilibrium aggregate investment (or equivalently, w_A^* goes below $\bar{w}_A$ for the bad realization of u). Therefore, there cannot be a fire-sale-free equilibrium if $E[y] > r_f + \bar{R}$. $\square$

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