

The Dividend Term Structure

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Abstract

We estimate a model for the term structure of discounted risk-adjusted dividend growth using dividend derivative prices for four major stock markets. A two-state model capturing short-term mean reversion within a year and a medium-term component which reverts at business-cycle horizon is superior over a single-state model. The dividend term structure extrapolates to an implied price-dividend ratio. This model-implied ratio, combined with current dividends, captures most of the daily stock index return variation, despite the fast mean reversion to long-run growth. Hence, investors update their valuation of dividends beyond the business cycle horizon only to a limited degree.

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"Since the level of the market index must be consistent with the prices of the future dividend flows, the relation between these will serve to reveal the implicit assumptions that the market is making in arriving at its valuation. These assumptions will then be the focus of analysis and debate.", (Brennan, 1998).

Dividends are a key ingredient for valuing stocks. Investors attach a present value to expected dividends and sum them to arrive at the value of a stock. As Campbell and Shiller (1988) have shown, stock prices thus vary because of changes in expected dividends, changes in interest rates, and changes in risk premiums. However, these elements may be horizon-dependent. For interest rates this is obvious as they can be readily observed. But also the expectations of dividends paid in the short run may at least partly be driven by other considerations than those of dividends paid in the distant future. Equally, risk premiums are likely to differ for various maturities, see for example van Binsbergen et al. (2012). Hence, investors will not only change the price of expected dividends from moment to moment, they may also change them for various maturities relative to each other, similar to a term structure of interest rates. In this paper, we focus on this term structure of the prices of expected dividends.

Given that the stock price is simply the sum of the present values of all dividends expected, Michael Brennan called in the late nineties for the development of a market for dividend derivatives. His wish came to life at the beginning of this century, with the introduction of derivatives referring to future dividend payments. These products exchange uncertain future dividends of an underlying stock or stock index for cash at the time of expiry. As such, they are forward looking in nature as they contain price information about expected dividends corrected for their risk. More precisely, the price of a single dividend future or OTC swap is the expected dividend for a given maturity discounted at the risk premium for this maturity. Finding present values of expected dividends only requires discounting these prices at the risk-free rate.

In this paper, we use data on these new dividend derivatives to study the dividend term structure for four major stock markets. A key starting point of our analysis is that we show that modelling the dynamics of a single variable is sufficient to describe the entire term

structure of discounted dividend derivative prices, and to obtain a fairly accurate total value for the stock index. This variable is equal to the difference between dividend growth and the sum of risk-free rate and a variable capturing the risk premium. We call this variable discounted risk-adjusted dividend growth. Hence, we do not need to separately assume processes for interest rates, risk premiums and dividend growth rates, the simplicity of which is a major advantage of our approach.

Inspired by the affine models often used for modelling the term structure of interest rates, we show how to set up a standard affine model for discounted risk-adjusted dividend growth. Specifically, our model resembles the interest rate model of Jegadeesh and Penacchi (1996), who use a two factor model, where the first factor reverts to a second factor, which in turn reverts to a long run constant. This model thus distinguishes a short-term component, a medium-term component and constant asymptotic growth. We cast this model in state space form and apply the Kalman filter Maximum Likelihood approach to estimate it using dividend derivative prices of one to ten years. The resulting discounted risk-adjusted dividend growth term structure describes the maturity curve of dividend present values in full, including an estimate for long-term growth.

Our key contributions are as follows. First, we find evidence that the two factor affine model describes the term structure of dividends well. It captures the dynamics of measured growth rates and it delivers an estimate for infinite growth that is economically sensible.

Second, we find that the factors driving this term structure have rather strong mean reversion. The first factor has a half-life of 6 months to one year (for reversion to the second factor) and thus captures short-term movements in expected dividends. The second factor reverts to a constant at a horizon of business-cycle proportion. Then, given the good fit to the aggregate stock market, our results show that most of the variation in stock prices is captured by short-term and business-cycle movements in discounted risk-adjusted dividends. As the estimate for infinite growth is fixed, our results suggest that investors update their day-to-day valuation of dividends beyond the business cycle horizon only to a limited degree. Apparently, depicting long term investor expectations to be fixed is not a major impediment to capturing most of the observed stock market volatility. These empirical results thus differ from the typical calibrations in long-run risk models (Bansal and Yaron (2004)). In these

calibrations shocks to expected dividend growth are very persistent. This directly implies that risk-adjusted dividend growth implied by long-run risk models is also very persistent. In contrast, our estimates of risk-adjusted dividend growth imply rather strong mean reversion for this key variable.

Third, we perform a relative pricing exercise, comparing the calibrated prices of future dividends to the observed value of the total stock index. Dividend derivatives have maturities up to ten years, but using our term structure model the extrapolated growth rates beyond that are summed to arrive at a model based estimate of the price-dividend ratio. Together with a market price for current dividends, a comparison is made to the actual stock market. This can be interpreted as an out of sample test of our model, since the model is estimated using dividend derivatives only, and not the stock index value. At an R^2 of over 50%, we find that most of the variation in the stock market is explained by current dividends and our model implied price-dividend ratio. This demonstrates that the stock market can be understood quite well in terms of the market for dividend derivatives.

We use data for four markets of dividend derivatives and contracts that extend out to horizons of up to ten years. Dividend derivative products exist in the shape of futures listed on stock exchanges and as swaps traded over the counter (OTC) between institutions. Minimum criteria for liquidity and transparency restrict the application of daily data in the estimation procedures to listed futures referring to the Eurostoxx 50 and the Nikkei 225 indices. Daily prices of OTC dividend swaps for the FTSE 100 and the S&P 500 indices are available as well, but they are illiquid and their representativeness of daily variation in dividend expectations is questionable. We therefore perform the same tests for these data using a monthly frequency.

This paper adds to a recent literature that uses dividend derivatives in asset pricing. Our work builds on and complements Binsbergen et al. (2013). They introduce the concept of equity yields, which is related to our discounted risk-adjusted growth measure. However, they do not estimate a pricing model for the term structure of discounted risk-adjusted dividend growth nor price the stock market using this model. Instead, they focus on an empirical decomposition of dividend prices into dividend growth rates and risk premiums. They do this by predicting future dividend growth from current dividend prices, and solving

for the term structure of risk premiums implied by this model. They conclude that the term structure for risk premiums is pro-cyclical, whereas expected dividend growth is countercyclical. Related to this, various authors (Binsbergen et al., 2012, Cejnek and Randl, 2014, Golez 2014) focus on realized returns of short and long-term horizon dividend derivatives or forward dividend prices derived from stock index futures and options, and find evidence for a downward sloping term structure of risk premiums. Wilkens and Wimschulte (2010) compare dividend derivative prices with dividend prices implied by index options. Suzuki (2014) assumes risk premiums are proportional to dividend volatility and then models the dividend growth curve implied by derivative prices using a Nelson-Siegel approach.

In the present value literature, expectations about the growth in dividends are often estimated by an econometric dividend model given past returns and dividend data. One of the earliest and best known examples is given by Campbell and Shiller (1988), who use vector autoregressive methods to predict returns based on past dividends, and use this to decompose returns into discount rate news and cash flow news. Many other attempts at decomposition of dividend growth and risk premiums have since followed (see Cochrane (2011) for an overview). Our approach could be a stepping stone towards a similar decomposition, but making use of forward looking information about expectations instead. These are the implicit assumptions Brennan hinted at revealing in 1998. Furthermore, the emphasis on the shape of the dividend term structure may open an approach to other finance questions. It may help to evaluate various asset pricing models such as long run risk models (Bansal and Yaron, 2004).

The remainder of this paper is organized as follows. The next section deals with the theory of dividend expectations and their fit into the present value model. It lays out the state space model which parameterizes the dividend term structure. The empirical results are discussed in the subsequent section and the paper summarizes its conclusions in section four. The data of dividend derivatives are discussed in an appendix as they are quite involved.

I. Theory

This section starts by proposing the base model for discounted risk-adjusted dividend growth, represented in terms of a stochastic discount factor. The section continues to lay out the state space approach to capturing time- and horizon-varying dividend growth.

A. The base model

We propose using a term structure model of the type familiar in the interest rate literature. We define g_{t+1} as the realized dividend growth rate for period t to $t + 1$, so that the dividend payable at maturity n is: $D_{t+n} = D_t \exp(\sum_{i=1}^n g_{t+i})$. We then apply the standard asset pricing equation to price this payoff for maturity n , where its current present value $P_{t,n}$ equals the expected product of the pricing kernel and the payoff:

$$P_{t,n} = E_t \left[D_t \exp \left(\sum_{i=1}^n m_{t+i} \right) \exp \left(\sum_{i=1}^n g_{t+i} \right) \right], \quad (1)$$

and where m_{t+1} is the log pricing kernel for period t to $t + 1$. The pricing kernel consists of the one-period risk-free rate y_t and an additional term θ_{t+1} that captures the dividend risk premium:

$$m_{t+1} = -(y_t + \theta_{t+1}), \quad (2)$$

where y_t is observed at time t and reflects the period t to $t + 1$. We aim to model a combined growth variable for the present value of future dividends and rewrite the pricing formula (1) accordingly:

$$P_{t,n} = D_t \left[E_t \exp \left(\sum_{i=1}^n \pi_{t+i} \right) \right]. \quad (3)$$

Equation (3) shows that the basic building block of the term structure model is what we denote discounted risk-adjusted dividend growth:

$$\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1} \quad (4)$$

In our data we observe dividend futures or swap prices. The relation of dividend present values to the prices of these dividend derivatives is achieved by merely discounting the futures prices at the n -period risk-free rate $y_{t,n}$:

$$P_{t,n} = F_{t,n} \exp(-ny_{t,n}), \quad (5)$$

which demonstrates that dividend present values are observable directly from market data $F_{t,n}$ and $y_{t,n}$.

If the growth rate π_t follows a lognormal distribution, equation (3) can be rewritten as:

$$\ln P_{t,n} - \ln D_t = E_t \left(\sum_{i=1}^n \pi_{t+i} \right) + \frac{1}{2} \text{Var}_t \left(\sum_{i=1}^n \pi_{t+i} \right). \quad (6)$$

The left-hand-side variable is related to the key modelling variable of Binsbergen et al. (2013). Specifically, Binsbergen et al. (2013) refer to $-(\ln P_{t,n} - \ln D_t)/n$ as the *equity yield*.

The crucial question is how to model the evolution of growth rates π_{t+1} . The approach that we advocate in the next subsection is decomposition of π_{t+1} by *horizon*. Its growth rates differ by maturity, the pattern of which is the object of this paper.

Before doing so, it is briefly explained why we choose to model π_{t+1} , rather than to assume separate models for its elements dividend growth, risk premium and risk-free discount rates. Decomposition of stock prices into dividend growth and risk premiums knows many attempts, seminal among which is the VAR based approach by Campbell and Shiller (1988). Information from dividend derivatives is also used in the VAR model of Binsbergen et al. (2013). We choose to do the exact opposite of decomposition and instead amalgamate the three variables into one; the proposed model variable is the growth rate of present values of expected dividends π_{t+1} . This amalgamation facilitates to focus on the term structure of the discounted growth trajectory alone. Connecting these growth rates via the present value identity to the stock market allows for a judgment call on the relevance of the horizon decomposition without being side tracked by additional assumptions on the constituent

variables. In fact, since we aim to value the stock market as the sum of dividend present values, a decomposition is not needed.

Furthermore, the components of π_{t+1} are likely to be correlated. For example, Bekaert & Engstrom (2010) calculate the correlation between 10 year nominal bond yields and dividend yields in the US over a 40 year period at no less than 0.77. Binsbergen et al. (2013) perform a principal components analysis of equity yields based on dividend derivatives prices. They show that the first two principal components of nominal yields explain about 30% of $g - \theta$ movements. Taken together into a single variable π_{t+1} , it should be possible to model it with a limited number of factors due to the high correlation among its components.

B. The state space model

In order to build a full term structure of discounted risk-adjusted dividend growth, we model it in state space form. We discuss the state equations and the measurement equations.

B.1. State equations

Our modelling approach to execute the decomposition by horizon closely follows Jegadeesh and Pennacchi (1996), who propose a model for estimating Libor futures with an aim to construct a term structure of interest rates based on three horizons. Their set up is a state space model in which the short-term interest rate is a latent variable. The prices of the Libor futures of different horizons are estimated by an equation consisting of the interest rates growth for the three horizons. Instantaneous growth and medium-term growth are both factors, infinite growth is a constant.

In this paper, we model discounted risk-adjusted dividend growth according to the same horizons. We specify most of the model in discrete time, following the approach in Campbell, Lo and MacKinley (1997). Specifically, we model π_{t+1} as the sum of a time-varying conditional mean p_t and a stochastic shock:

$$\pi_{t+1} = p_t + v_{t+1}, \tag{7}$$

where v_{t+1} is normal i.i.d. with zero mean. The factor p_t follows a mean reverting process to a medium-term factor \tilde{p}_t which itself is mean reverting to a long-term constant \bar{p} , where for convenience we first define their processes in continuous time:

$$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p, \quad (8)$$

$$d\tilde{p}_t = \psi(\bar{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}. \quad (9)$$

dW_p and $dW_{\tilde{p}}$ are Wiener processes, with σ_p and $\sigma_{\tilde{p}}$ scaling the instantaneous shocks to the factors. The horizon at which investors adjust their growth expectation from one state to the next is captured by mean reversion parameters φ and ψ . This two-state system results in the state equations for discrete intervals²:

$$\begin{pmatrix} p_{t+1} \\ \tilde{p}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 - e^{-\varphi} & -\frac{\varphi}{\varphi - \psi}(e^{-\psi} - e^{-\varphi}) \\ 0 & 1 - e^{-\psi} \end{pmatrix} \begin{pmatrix} \bar{p} \\ \bar{p} \end{pmatrix} + \begin{pmatrix} e^{-\varphi} & \frac{\varphi}{\varphi - \psi}(e^{-\psi} - e^{-\varphi}) \\ 0 & e^{-\psi} \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \varepsilon_{t+1}. \quad (10)$$

Finally, we model correlation between the innovation in the growth rate v_{t+1} and the errors ε_{t+1} in these state equations as $v_{t+1} = \beta' \varepsilon_{t+1}$, where $\beta = (\beta_p, \beta_{\tilde{p}})'$ is a 2-by-1 vector.³ In terms of the mathematical structure, this setup resembles the approach of Campbell, Lo and MacKinley (1997). They derive affine term structure models in discrete time by modelling the log-pricing kernel, $m_{t+1} = -(y_t + \theta_{t+1})$, in a similar way as we model the discounted risk-adjusted growth rate $\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}$. The key difference is that our growth variable depends both on the pricing kernel and the dividend growth rate. As discussed above, we only model the aggregate variable π_{t+1} and do not need to make specific assumptions on its components. This is important for the interpretation of the results. For example, when modelling interest rates, Campbell, Lo and MacKinley (1997) show that the β vector captures the risk premiums on long-term bonds. In our setup, the vector β could represent dividend risk premiums, but can also be the result of correlation of current dividend

² Refer to Appendix 2 for further details.

³ One could incorporate an independent shock to the growth rate, but this does not have an important effect on the term structure of dividend prices or the dynamics of these prices.

growth and the factors driving future dividend growth. Again, for pricing dividend derivatives there is no need to specify the source of the correlation between shocks to π_{t+1} and the factors (as determined by β).

B.2. Measurement equations

Given the dynamics of π_{t+1} , it follows that the average growth rate of dividend present values from time t to its expiry date at time n corresponds to a function of p_t and \tilde{p}_t . Specifically, as shown in Appendix B, filling in the dynamics of π_{t+1} in the pricing equation (6) and adding i.i.d. measurement error $\eta_{t,n}$ for each derivatives maturity n , the measurement equations for the state space model are:

$$\begin{aligned} \ln P_{t,n} - \ln D_t &= n\bar{p} + \varphi_n(p_t - \bar{p}) + \frac{\varphi}{\varphi - \psi}(\psi_n - \varphi_n)(\tilde{p}_t - \bar{p}) \\ &+ \frac{1}{2} \sum_{i=1}^n \left(\sigma_p^2 (\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2 \left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) \right)^2 \right) + \eta_{t,n}, \end{aligned} \quad (11)$$

in which β_p and $\beta_{\tilde{p}}$ are the covariance betas of the errors of the first and second factor and σ_p^2 and $\sigma_{\tilde{p}}^2$ are their variances. We define φ_n and ψ_n as follows:

$$\begin{aligned} \varphi_n &= \frac{(1 - e^{-n\varphi})}{(1 - e^{-\varphi})}, \\ \psi_n &= \frac{(1 - e^{-n\psi})}{(1 - e^{-\psi})}, \end{aligned}$$

with $\varphi_0 = 0$ and $\psi_0 = 0$.

C. The single-state model

We benchmark the ability of the two-state model to fit the dividend term structure by a state space model with a single factor. In essence, the medium-term factor is set to the long-term constant estimate⁴, rendering the same estimation equations as a Vasicek model:

$$dp_t = \varphi(\bar{p} - p_t)dt + \sigma_p dW_p. \quad (12)$$

Its state equation and measurement equations are:

$$p_{t+1} = \bar{p} + (p_t - \bar{p})e^{-\varphi} + \varepsilon_{t+1}, \quad (13)$$

$$\ln P_{t,n} - \ln D_t = n\bar{p} + \varphi_n(p_t - \bar{p}) + \frac{1}{2} \sum_{i=1}^n (\beta + \varphi_i)^2 \sigma^2 + \eta_{t,n}. \quad (14)$$

II. Empirical Results

A. Dividend derivatives data

The estimation methodology uses prices of dividend derivatives referring to four major stock markets: Eurostoxx 50 and Nikkei 225 dividend futures and S&P 500 and FTSE 100 OTC dividend swaps. Dividend futures were introduced in 2008 to the European market and in 2010 in Japan. Maturities extend out to ten years with annual intervals. Price data are available on a continuous basis from the relevant stock exchanges. Liquidity of European dividend futures is good with Euro billions of notional outstanding for maturities up to three years and hundreds of millions for the longest maturities. The market for Nikkei dividend futures is smaller with maturities up to two years featuring notionals of over half a billion (\$-equivalent) and tens to hundreds of millions for longer maturities (Mixon and Onur, 2014). All maturities normally trade on a daily basis and we apply the estimation procedure to daily prices in the case of dividend futures.

⁴ Alternatively, this model can be described as a nested two-state model with medium-term mean reversion parameter ψ constraint to infinity.

Liquidity and transparency of OTC dividend swaps is less developed with days passing by without a single trade taking place across all maturities more often than not (Mixon and Onur, 2014). We limit the analysis to monthly data as a consequence. The data are described in Appendix A in detail. We first discuss the results using the two dividend futures datasets.

We estimate a cross section of measurement variables for up to 8 periods⁵ on the discounted risk-adjusted dividend growth term structure. The base for the growth rates would be D_t , but current dividends are not observable. In view of the daily data that we deploy, current dividends are not well described by the realized dividend index to which the derivatives refer. These indices capture dividends paid in the 12 months preceding observation date t , whereas we are in fact looking for a value that reflects dividends as if they had to be paid on t itself. The data section in Appendix A contains more detail on this topic.

Consequently, we remove the unobserved current dividend D_t from equation (11) and instead subtract the first discounted derivatives price $P_{t,1}$ from the longer maturity prices to arrive at growth rate measurements. The term structure then consists of a present value that is observed for the first period and growth rates that are modelled for all subsequent periods. Subtracting the first period present value gives the following measurement equation for growth rates and replaces equation (11):

$$\begin{aligned} \ln P_{t,n} - \ln P_{t,1} &= (n-1)\bar{p} + \varphi_n(p_t - \bar{p}) + \frac{\varphi}{\varphi - \psi}(\psi_n - \varphi_n)(\tilde{p}_t - \bar{p}) \\ &+ \frac{1}{2} \sum_{i=1}^{n-1} \left(\sigma_p^2 (\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2 \left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) \right)^2 \right) + \eta_{t,n}. \end{aligned} \quad (15)$$

State equations (10) and measurement equations (15) together form the system of which the variables are estimated by maximum likelihood. The procedure is recursive by means of a Kalman filter (Jegadeesh and Pennacchi, 1996).

⁵ We interpolate between 10 annual derivatives expiry dates to arrive at prices of derivatives with constant maturities. These 9 prices of derivatives with a constant maturity of 1 to 9 years provide 8 growth rates. See the data section in the appendix for further details.

The error variance terms are assumed to be the same for all measurement equations (σ_η), except for the first one (which we denote σ_η^1). This is because the definition of the first derivative to expire (set to a constant maturity of one year following the observation date) differs slightly from subsequent derivative prices due to an alternative weighting scheme for finding constant maturity values as explained in Appendix A.

B. Estimation results

Tables I and II provide the results of the two-state model and a benchmark single-state model for Eurostoxx 50 and Nikkei 225 dividend markets – the two markets for which listed futures data with sufficiently long horizons of 10 years exist. Estimations are performed on daily data⁶.

B.1. Pricing errors

Before we discuss the parameters of the growth rate model, we first establish that the two-state model fits the data well⁷. To this end, we calculate mean absolute errors for the measurement equations (15). Given that they are specified for log prices of dividend futures, these mean absolute errors can be interpreted as relative pricing errors.⁸ The first measurement equation produces a mean absolute pricing error of 0.015 (1.5%) and pricing errors of subsequent expiries are between 0.0025 and 0.005 (Figures 3 and 4). The error levels are clearly small, confirming a good fit of the model to the data.

B.2. Mean reversion estimates

The mean reversion towards medium-term growth φ attains levels which translate to a half-life of less than a year. The Eurostoxx 50 mean reversion at 1.51 is twice as fast as for the Nikkei 225 ($\varphi=0.74$), which is due to the global credit crisis in 2008/09 being included in

⁶ For robustness, we perform the same tests with monthly data (not shown here). None of the parameter estimates and test coefficients change meaningfully relative to the daily dataset.

⁷ The short term beta β_p is set to zero, as discussed further below.

⁸ These errors are thus not annualized. Transformed to annual growth rates, the errors are even smaller.

the Eurostoxx 50 data period and not in the Nikkei 225 data period.⁹ Mean reversion towards the long run constant ψ is broadly measured in half-lives of 3 to 4 years¹⁰ in both markets, a space of time that comes close to that of a business cycle. All mean reversion parameters are significant at the 1% level. The estimates for φ and ψ are positive, which implies that the growth rate is stationary and thus tends to a long-term constant.

The model imposes the long-run growth rate to be constant, while the speed at which medium-term growth adjusts to it is estimated from the data. The interpretation from these results is that investors change their opinion about growth only as far ahead as the anticipated business cycle. We do not formally link an economic interpretation to the three growth stages, but given the estimates of the mean reversion parameters some intuition can be provided. Instantaneous growth can be thought of as the expectation of the immediate future. Shocks to risk aversion and to the volatility of the current business climate are likely to influence investors' valuations of dividends several months ahead, but perhaps not much further. Developments in the business cycle, on the other hand, such as credit conditions, investment growth and monetary policy set the stage for the business cycle influencing dividend expectations over a longer period ahead, measured in several years. Structural factors such as population growth and technological progress determine how investors perceive the long run, extending from the business cycle horizon into the infinite future.

Structural developments should be slow moving, if at all, and are approximated by imposing asymptotic constancy. Thus, at horizons extending well beyond business cycles, investors have opinions of economic and financial variables, but they do not change them once taken together. This means that any rise in long-maturity interest rates is exactly offset by a rise in long-term dividend growth or a fall in long-term risk premiums. Mean reversion towards such a constant implies therefore that a horizon exists at which investors never change their opinion about present value growth.

⁹ Estimating the model for the Eurostoxx 50 data over a partial data period that coincides with the Nikkei 225 data period yields mean reversion parameters that are closer to those found for the Nikkei 225: $\varphi = 0.88$ and $\psi = 0.09$.

¹⁰ Jegadeesh and Penacchi (1996) apply the two-state model to interest rates and find the opposite pattern; short mean reversion is slower than medium-term mean reversion.

B.3. Discounted risk-adjusted dividend growth rates

Given the mean reversion estimates, the instantaneous factor reflects short-term movements in risk-adjusted growth, the medium-term factor reflects an assessment of the business cycle, while \bar{p} depicts a structural level which can be linked closely to the dividend yield. Figures 5 to 8 provide estimates of expected growth rates by recalculating the factors by means of the measurement equations (15) into 1-year growth and 1-year forward 4-year growth of discounted risk-adjusted dividends. Forward growth rates imply the level of growth expected after the 1 year growth rate has materialized¹¹.

1-Year growth is mostly determined by the instantaneous factor. Figure 5 shows that it is highly volatile for the Eurostoxx 50, with the global credit crisis in 2008/09 showing a decline by nearly half and during the Eurozone sovereign debt crisis in 2011 by a quarter. Outside these periods, it moves between broadly -10 and $+5$ percent. Nikkei 225 1-year growth rates move in the same range until late 2012 (Figure 7). The period following the announcement of “Abenomics” in 2012¹² portrays high optimism with 1-year growth rates attaining 10 percent and more.

Given the values found for the mean reversion parameters, the medium-term factor largely determines 1-year forward 4-year growth depicted in Figures 6 and 8. In Europe forward growth circles around the long run constant between -2 and -6 percent. The sovereign debt crisis in 2011 shows a somewhat more negative rate than the global credit crisis. Investors apparently expected that the serious short-term blow to dividends in 2008/09 would not be corrected or reversed (by positive growth) afterwards. However, the less negative blow in 2011 would be followed by a period more negative than the long run constant (Figure 6), implying that investors expected that the European sovereign debt crisis would bear consequences for the business cycle.

The volatility of forward growth rates provides further insight into the relation between the risk and the maturity of dividends. Both the Eurostoxx 50 and the Nikkei 225 dividend markets portray declining volatility in growth rates as maturities increase (Figure 9).

¹¹ As discussed later, the short term beta is set to zero for these data, but different fixed levels do not materially change growth rates.

¹² Late 2012 the government of Shinzo Abe proclaimed a policy of monetary and fiscal expansion combined with economic reform. The two main consequences for financial markets were a substantial weakening of the Japanese Yen and a rise in the stock market.

B.4. Long-term growth and dividend yields

The economic interpretation of the long-term discounted risk-adjusted dividend growth constant is briefly recapitulated. The present value identity for stock prices S_t is recalled as:

$$S_t = \sum_{n=1}^{\infty} P_{t,n} = D_t \sum_{n=1}^{\infty} \exp(n\pi_{t,n}), \quad (16)$$

where $\pi_{t,n}$ is the observed annualized discounted risk-adjusted growth rate of dividends payable at maturity n , $\pi_{t,n} = (\ln P_{t,n} - \ln D_t)/n$, which is the negative of what Binsbergen et al. (2013) call the equity yield. The dividend-price ratio is found by rearranging identity (16) to:

$$\frac{D_t}{S_t} = \frac{1}{\sum_{n=1}^{\infty} \exp(n\pi_{t,n})}, \quad (17)$$

For the sake of interpretation, if both factors in the two-state model are equal to the mean, $\pi_{t,n}$ is a horizon invariant constant and identity (17) simplifies to:

$$\frac{D_t}{S_t} = -\bar{p}^*. \quad (18)$$

The dividend yield equals the negative of long-term growth for which the state space approach thus provides an estimate. Combined with the constant convexity term, the estimate for \bar{p} constitutes a measure of long-term growth \bar{p}^* . On an annual basis, this measure is given by:

$$\bar{p}^* = \bar{p} + \frac{1}{2} \left(\sigma_p^2 (\beta_p + \varphi_{i \rightarrow \infty})^2 + \sigma_{\bar{p}}^2 \left(\beta_{\bar{p}} + \frac{\varphi}{\varphi - \psi} (\psi_{i \rightarrow \infty} - \varphi_{i \rightarrow \infty}) \right)^2 \right), \quad (19)$$

in which the values for $\varphi_{i \rightarrow \infty}$ and $\psi_{i \rightarrow \infty}$ are set for i approaching infinity. The second factor sigma $\sigma_{\bar{p}}^2$ is small, so the term $\sigma_{\bar{p}}^2(\beta_p + \varphi_{i \rightarrow \infty})^2$ delivers most of the impact.

For longer horizons, the convexity term on the right hand side of measurement equations (15) approaches constancy. It includes the betas and the factor sigmas. While the sigmas are identified by means of the state equations, the betas are only present in the measurement equations. The long-term growth parameter \bar{p} balances out the betas under the optimization procedure. As a result, the estimates for long-term growth \bar{p} as well as of both covariance betas β_p and $\beta_{\bar{p}}$ come out unstable with sizeable standard errors. The estimation technique of the Kalman filter thus finds optimal solutions for various combinations of \bar{p} and betas, a fact which indicates multicollinearity.

Finding a relevant and well identified value for long-term growth requires fixing the short term beta β_p while solving the model for the other variables. In Figure 2, the results of this exercise is shown. There is a consistent parabolic trade-off between the long-term growth constant and the short term beta, which is as expected in view of the quadratic nature of the convexity term.

The difference between \bar{p}^* and \bar{p} is smallest for values of β_p equal to $-\varphi_{i \rightarrow \infty}$. For these values, the inverted parabola in Figure 2 reaches its maximum. The interdependence between β_p and \bar{p} in the output of the estimation procedure is such that different combinations along the parabola render very little influence on the value of \bar{p}^* .

In Tables I and II the parameters are shown of a single estimation in which β_p is set to zero. Long-term growth obtains reasonable levels, a discussion of which follows below. Standard errors show the estimates are (close to) significant at the 5% level. The mean reversion estimates and the variance estimates do not change materially when the short term beta is fixed. This is as expected since they have only a small impact in the long run, approaching zero impact at the limit. The medium term beta $\beta_{\bar{p}}$ remains large but insignificant. For the purpose of all of the subsequent discussion, the short term beta is thus set to zero.

Seen in this light, there is an economic rationale in the estimates from the state space model for the long-term growth constant \bar{p}^* . The levels found equal -2.6 percent in Japan

and Europe, which appears reasonable relative to dividend yields. Table III contains some metrics for comparison. The average dividend yield in Europe was 4.3 percent and in Japan it was 1.9 percent during the short data period. The average 1 year forward 4 year growth rate also deviates less than 1 percent from the average dividend yield, but the average short-term growth rate deviates substantially more. A tentative conclusion is that the business cycle stood close to the long-term average during the data period, but sentiment was more negative in Europe and more positive in Japan¹³. Overall, the estimates for long-term growth seem a fair assessment of the long-term cash run rate of the stock market. It is noteworthy that the estimates are produced without input from the stock market itself.

It is also important to observe that the state space model estimates discounted long-term growth to be negative as present value theory requires stock valuations to be finite. The flexibility of the model would allow for positive values, but the estimates correctly imply that dividend present values decline at a horizon that is sufficiently long.

B.5. The Dividend Term Structure

Equipped with model estimates for the growth parameters, a Dividend Term Structure (DTS) can be calibrated. The DTS depicts the present values that investors attach to expected dividends per horizon n expressed as a proportion of the total present value:

$$DTS_n = \frac{\hat{P}_{t,n}}{\sum_1^\infty \hat{P}_{t,n}}. \quad (20)$$

The value for $\hat{P}_{t,1}$ is the calibrated discounted price of the derivative expiring one year from t . The values for subsequent expiries $n \geq 2$ are calibrated from the estimated growth parameters. Figure 10 shows that the average term structure of the Nikkei 225 starts sloping upwards, and then becomes downward sloping as the horizon increases. The transition is slow given the low mean reversion and the moderate levels of the estimated averages for instantaneous and medium-term growth. The Eurostoxx 50 DTS, by contrast, is strongly negatively sloping at the outset, but adjusts to the long-term growth path rather quickly. The

¹³ In fact, in particular in Japan it turned more positive during the data period.

first dividend point on the Eurostoxx 50 DTS is therefore high, which translates into an equally high current dividend yield. The Nikkei 225 first dividend points are lower on average, which fits with the positive slope at the start of their DTS. It is also in line with the fact that the estimate for long-term growth is somewhat higher than the first dividend point.

Its DTS indicates that the fundamental value of the European stock market is more front loaded, or more heavily weighted towards the near future, than that of the Japanese stock market. The surface below the calibrated DTS equals one by definition. The relative present values of dividends of Japan cross over the European values after about thirty years into the future. Relative to the European stock market, the present value of Japanese dividends beyond the cross over makes up for their lower contribution before it.

B.6. Other long-term growth estimates

Giglio, Maggiori and Stroebel (2014) compare prices of houses of different contractual ownership to arrive at a very long-term discount rate. Leased housing reverts to the owner of the land after the lease expires, while freehold housing remains with the owner of the house indefinitely. The difference in price between the two for comparable properties equals today's present value put to ownership once the lease has expired. At lease expiries of over one hundred years, this provides an interesting comparison to the estimates for long-term discounted risk-adjusted dividend growth.

The discounts Giglio et al. (2014) find in the data equate to a value for infinite growth of around -2% for periods of 100 years and more. This level makes sense economically and is also reasonably close to the long-term discounted risk-adjusted dividend growth estimates¹⁴.

C. Reconciliation to the stock market

The second part of our research agenda is to analyze the implications of the model for the value of the stock market. Given that we estimate the model using dividend derivative data

¹⁴ It is clear that not the *level* of the rents D , but only the *growth* of rents (being part of p) matters for establishing the lease discount. We can therefore consider growth in rents with or without maintenance cost, depreciation and taxes assuming they stay constant in proportion to rents over the very long-term considered. Another aspect is the convenience provided to the occupier of a house. Growth comparisons should be made only for sufficiently remote horizons. Since the notion of convenience yield is that there is a benefit to the current user that a future user cannot currently enjoy, nearer horizon comparisons are distorted.

only, this constitutes an out-of-sample test of the model. Alternatively, if one takes the model assumptions for granted, it can be seen as a relative pricing exercise of the dividend derivative prices versus stock market levels.

C.1. The empirical approach

The present value model incorporates expected index dividends which can be extrapolated from the estimated dividend term structure. This provides the following estimate for the stock market:

$$\hat{S}_t = D_t \sum_{n=1}^{\infty} \exp(n\hat{\pi}_{t,n}) = D_t \widehat{PD}_t, \quad (21)$$

with the summation of fitted growth rates $\hat{\pi}_{t,n}$ equal to the estimated dynamic price-dividend ratio \widehat{PD}_t , and where the fitted growth rates satisfy:

$$\begin{aligned} n\hat{\pi}_{t,n} = & n\bar{p} + \varphi_n(p_t - \bar{p}) + \frac{\varphi}{\varphi - \psi} (\psi_n - \varphi_n)(\tilde{p}_t - \bar{p}) \\ & + \frac{1}{2} \sum_{i=1}^n \left(\sigma_p^2 (\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2 \left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) \right)^2 \right).^{15} \end{aligned} \quad (22)$$

It is a well-known and critical problem of the present value model that it depends on a reasonable estimate for the expected growth and the risk premium of dividends. Historical analysis of dividend growth followed by risk premium decomposition provides such estimates (Campbell and Shiller, 1988). Binsbergen et al. (2013) execute the decomposition by making use of the price data of dividend derivatives.

¹⁵ Subtracting the first growth rate $\pi_{t,1}$ from equation (22) provides an alternative representation which can be directly applied to the present value identity in equation (23);

$$\begin{aligned} n\hat{\pi}_{t,n} - \hat{\pi}_{t,1} = & n\bar{p} + \varphi_n(p_t - \bar{p}) + \frac{\varphi}{\varphi - \psi} (\psi_n - \varphi_n)(\tilde{p}_t - \bar{p}) \\ & + \frac{1}{2} \sum_{i=2}^n \left(\sigma_p^2 (\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2 \left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) \right)^2 \right). \end{aligned}$$

A key contention in this paper is that for the purpose of the present value model reconciliation, without decomposition of dividend expectations into growth and risk premiums, decomposition of risk-adjusted discounted dividend growth by horizon alone is very informative. Nonetheless, successful reconciliation of dividend derivative price information to the stock market is uncommon in the literature. For example, Suzuki (2014) builds a Nelson Siegel model of the Eurostoxx 50 dividend growth term structure and makes assumptions about the level for longer dated values. These include a fixed level imposed at 4% for discounted growth after 25 years. Under these conditions, Eurostoxx 50 dividends reconcile well with the stock market dynamically.

In contrast to Suzuki (2014), we do not impose a fixed level as the state space model itself renders an estimate for the long-term growth path of the present value of dividends independent from stock market information, while it captures the shape and the dynamics of the term structure up to the medium-term at the same time. The entirety of the present value term structure is thus described by a handful of variables from two markets¹⁶ in a single estimation procedure. The fit of the reconciliation to the observed stock market acts as a joint check on the validity and the robustness of the two-state model and the present value identity. To that end, equation (22) is used to calculate the fitted dividend growth rates and present values as implied by the estimated state space model.

All variables are taken as estimated by the state space model applied to dividend derivative data. Current dividends in (21) are approximated by the value of the first constant maturity derivative $F_{t,1}$, which is discounted at the risk-free rate. This is a better approximation for investors' estimate of current dividends than twelve month historical dividends. We thus get for the model implied stock market level:

$$\hat{S}_t = F_{t,1} \exp(-y_{t,1}) \left(1 + \sum_{n=2}^{\infty} \exp(n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}) \right) = F_{t,1} \exp(-y_{t,1}) (1 + \widehat{PD}_t^1), \quad (23)$$

¹⁶ The bond market and the dividend derivative market.

in which $n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}$ are the fitted values of the measurement variables in equations (15) and \widehat{PD}_t^1 represents the estimate for the price-dividend ratio as implied by the sum of exponential growth rates, where growth starts from the present value of the dividend derivative expiring one year following the observation date¹⁷.

C.2. Stock market level reconciliation

We now discuss the empirical results of the reconciliation with stock market levels¹⁸. In the European market, the two-state model estimates applied to equation (23) cause the stock index to be overestimated at a reasonably constant level distance to the actual stock index for most of the data period (Figure 11). There is no clear trend among the factors driving the estimated valuation away or towards the stock index. The historical dividend yield (4.3%) is somewhat higher than the negative of the long-term estimate (-2.6%) and the index is overestimated at some 20 to 30 percent except during the outbreak of the global credit crisis¹⁹. The level estimate of the stock index is highly sensitive to the long-term growth parameter. For the mean squared errors of this level comparison to be minimized, the estimate for long-term discounted growth would have to be closer to the historical dividend yield, or about 0.7% higher.

Dividend present values underestimate the Nikkei 225 index level at the beginning of the data period, but the gap closes from 2012 onwards. Short-term growth ranges between -0.20 and +0.05 percent initially, but at the onset of Abenomics in late 2012, it turns strongly positive (Figure 12). At -2.6 percent, long-term growth is more pronounced than the average Japanese dividend yield (1.9%), which contributes to the underestimation.

¹⁷ The stock index estimate is approached by numeric summation, which is approximated by:

$$\hat{S}_t \approx F_{t,1}^{CM} \exp(-y_{t,1}) \left[1 + \sum_{n=2}^{\bar{n}} \exp(n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}) + \frac{\exp(\bar{n}\hat{\pi}_{t,\bar{n}})}{-\widehat{p}^*} \right].$$

In the estimations \bar{n} is set at 50 years. Unless reduced to single digits, the number of years which \bar{n} is set to is not material to the stock index estimates.

¹⁸ The state space model estimations are produced setting the short term beta to zero.

¹⁹ Market participants consider Eurostoxx dividend derivatives prices around the turn of 2008/09 as unrepresentative of dividend expectations due to one-sided interests.

C.3. Dynamic reconciliation

Following the present value model, *stock returns* are the consequence of investors changing their valuation of future dividends. The dynamics of stock indices can be retrieved from the present value model estimate as provided in equation (23). The present value of the first dividend amount to be paid over the year to come is the starting point of the growth term structure. The first dividend is observable and the growth path of discounted risk-adjusted dividends starting after it is a model implied estimate. The dynamic fit as well as the relative importance to stock returns of the first derivative on the one hand and the growth path on the other requires testing. For this reason the estimated returns of the stock market is split into its drivers. Equation (23) is repeated with logs denoted in lower case as a regression equation:

$$\Delta \ln(S_t) = \alpha + \beta_F \Delta \ln(F_t) + \beta_y \Delta y_t + \beta_{PD} \Delta \ln(\widehat{PD}_t^1) + \varepsilon_t. \quad (24)$$

Stock index log returns are regressed by OLS on the log return of the first constant maturity derivative F_t , changes in the 1 year risk-free rate Δy_t and the log returns of the estimated price-dividend ratio \widehat{PD}_t^1 , which is the sum of the normalized dividend present values of the state space model. The betas²⁰ of the returns of the first dividend and the price-dividend ratio are predicted to be close to + 1, while the beta of the risk-free rate is expected at - 1. Data are daily.

Eurostoxx 50 and the Nikkei 225 index returns respond well to the prediction of the present value model, shown in Table IV. The model is quite capable of explaining variation in stock returns, reaching an R^2 of above 50 percent. Although we cannot benchmark this explanatory power, it appears substantial given that the model does not incorporate any direct information of the stock market. Each of the regressors add considerably to the explanatory power, while the constant is close to zero. Both stock markets appear highly sensitive to changes in the first constant maturity derivative. The daily betas are in the order of 0.85 for the Nikkei 225 to 0.90 for the Eurostoxx 50. The beta of the price-dividend ratio is close to 0.86 and 0.66 respectively. In the case of Japan, most of the explanatory power comes from

²⁰ Coefficients of \widehat{PD}_t^1 are expected below 1 due to the errors in the regressor estimates increasing their variance.

the price-dividend ratio, in Europe it is more evenly divided between short-term dividends and the price-dividend ratio.

The 1 year zero interest rate brings the price of the first derivative to its present value. Its relevance seems limited and the expected beta is -1 . It is highly significant in the estimates for the Eurostoxx 50, but reaches values of 0.15 to 0.20. In Japan, the risk-free beta is closer to zero and not significantly different from it.²¹

The interpretation of the assumption that long run discounted risk-adjusted dividend growth is constant is not that investors do not change their opinion about what value to attach to dividend present values far into the future. The value ascribed to dividends expected ten years and twenty years from today is influenced by the estimate of present values in the near term and medium-term. But the value of the twenty year dividend does not change relative to that of the ten year dividend regardless of changes in near and medium-term expectations – the relationship between them is (approximately) fixed. Therefore, long-run constancy excludes mean reversion to levels. Dividend levels attained in the past are not a target for investors to project their long-term expectations onto. Only long-term growth is.

D. Robustness

D.1. The single-state model

The two-state model distinguishes instantaneous from medium-term growth. Its ability to fit the dividend present value term structure is benchmarked by a state space model with a single factor, in which the medium-term factor is set to a long-term constant²²:

$$dp_t = \varphi(\bar{p} - p_t)dt + \sigma_p dW_p, \quad (25)$$

Figures 3 and 4 show estimation errors of the single-state model in comparison to the two-state model. While still not substantial, single-state estimation errors are larger. Table I

²¹ The impact of short-term dividends and the price dividend ratio is mitigated by negative coefficients found once lags are added to the set of regressors (not shown here). This suggests that either the stock market overreacts to shocks to dividends, which is corrected in the following day, or that dividend prices may partly follow stock prices by at least a one day lag.

²² Alternatively one can depict this model as a nested two-state model with medium-term mean reversion parameter ψ constraint to infinity.

contains the estimated parameters and Figures 5 to 8 depict the forward discounted risk-adjusted dividend growth rates as delivered by the single-state model. Overall, the parameter estimates are significant and attain reasonable levels, but the two-state model is superior.

The Eurostoxx 50 estimate for mean reversion at 1.73 is even higher than the short-term mean reversion in the two-state model, its standard error is larger. This adjustment speed implies a half-value time of instantaneous growth of only 5 months. Long-term growth is slightly lower and its standard error is smaller than in the two-state model. The 1 year growth rate is less volatile, which mirrors the quick fading of the instantaneous growth factor.

For the Nikkei 225, the picture is rather different. Mean reversion attains a value in the middle ground of the two parameters in the two-state model. Long-term growth is somewhat lower, but again economically sensible. Standard errors are smaller for both parameters. The 1 year growth rate largely overlaps with that of the two-state model.

The fit of the models measured by estimation errors is reduced in the single-state variation relative to the two-state model. The absolute measurement errors are on average always bigger in the single-state model than in the two-state model, in most cases by a factor of 2 to 3 (Figures 3 and 4). The better fit of the model is also indicated by the log likelihood statistics. Per observation the log likelihood contribution is a third higher in the two-state model than it is in the single-state variation. Benchmarking against the single-state model indicates that it appears plausible to distinguish between investors gauging the immediate future on the one hand and their considerations about the business cycle on the other, as catered for in the two-state model.

D.2. An alternative model

Our modelling approach focuses directly on discounted risk-adjusted dividend growth $\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}$. We thus incorporate discounting at the risk-free rate when valuing future dividends. An obvious alternative to this approach would be to model $z_{t+1} = g_{t+1} - \theta_{t+1}$ using a term structure model to value dividend derivatives, and subsequently discount it at observed interest rates to calculate present values. This latter step requires the assumption that interest rates and $g_{t+1} - \theta_{t+1}$ are independent. In addition, we assume the expectations hypothesis holds for bonds, so that bond risk premiums equal zero and long-term interest

rates equal expected future short rates. Given these assumptions we can rewrite equation (3) as:

$$\begin{aligned}
P_{t,n} &= D_t \left[E_t \exp \left(\sum_{i=1}^n g_{t+i} - y_t - \theta_{t+1} \right) \right] = D_t \left[E_t \exp \left(\sum_{i=1}^n -y_{t+i} \right) \right] \left[E_t \exp \left(\sum_{i=1}^n z_{t+i} \right) \right] \\
&= D_t \exp(-ny_{t,n}) \left[E_t \exp \left(\sum_{i=1}^n z_{t+i} \right) \right].
\end{aligned} \tag{26}$$

and the derivatives price equals:

$$F_{t,n} = D_t \left[E_t \exp \left(\sum_{i=1}^n z_{t+i} \right) \right]. \tag{27}$$

This shows that, to fit the futures price data, only a model for z_{t+1} is needed. To reconcile this model with the stock index level, the independence assumption and expectations hypothesis for bonds are necessary and equation (26) can be used to calculate present values of dividends and the stock index value. Using this pricing equation, one can again specify a two-state model, in this case for one period growth z_{t+1} , and estimate it using the Kalman filter in the same way as described for the base model.

As mentioned, this model assumes independence of interest rates and risk-adjusted growth rates. In the real world, however, correlation between the risk-free rate, dividend growth and the dividend risk premium is expected since often the same drivers apply: economic growth, the investment cycle, slack in the labor market and other economic variables will affect all of them. For estimating the term structure model, such correlation is not a problem if z_{t+1} is the subject of state space estimation instead of π_{t+1} , but it will cause misestimation of the implied stock market levels. It is easy to show that this separation of the two correlated variables would produce overestimation of the stock index in equation (23) if the actual correlation is positive.

Turning to the results, the long-term estimate for risk-adjusted growth \bar{z} is estimated rather high, at 0.3 percent for the Eurostoxx 50 and -1.0 percent for the Nikkei 225, which translates to 1.9 percent and 0.1 percent once \bar{z} is corrected for the convexity term (Table V). Standard errors are larger than in the case of the base model. The mean reversion parameters obtained remain reasonable and significant. However, reconciling the dividend market to the stock market based on these estimates overstates the stock market by a large margin and reduces the fit of the dynamic return reconciliation (Table VII). Compared to the base model, the coefficient of the estimated price-dividend ratio maintains its presence in the Japanese data with a coefficient of 0.78. In the data period considered, yen interest rates were close to zero and showed less variation than in the European market. The reduction to the explanatory power when modelling growth without discounting is relatively small for the Nikkei 225. In Europe, the picture is quite different due to the steep drop in interest rates in the period of 2008 to 2015. The coefficient of the estimated price-dividend ratio is almost negligibly small for the Eurostoxx 50 market. Therefore, the European data set in particular demonstrates the correlation among the three elements of π_{t+1} , which confirms the advantage of estimating risk-adjusted dividend growth *after* discounting at the risk-free rate.

D.3. OTC data

We retain price data of dividend swaps from several investment banks²³ for the dividend futures markets under investigation, and also for the S&P 500 and the FTSE 100. These data extend back to December 2005. Over-the-Counter (OTC) prices for dividend derivatives are not readily observable as are, for example, interest rate swaps, money market derivatives or foreign exchange derivatives which are posted on information systems such as Reuters. Mixon and Onur (2014) provide insight into the OTC market for dividend swaps. They investigated data from a Swap Data Repository to which participants in swap markets must report at transaction-level. It is shown that OTC swaps trade infrequently; even for the S&P 500, which is the largest OTC dividend market, they trade less than daily between dealers and once every few weeks between a dealer and a non-dealer end-user.

²³ Deutsche Bank, Goldman Sachs and Credit Suisse.

Investment banks update their pricing sheets on a daily basis, but often prices remain stale and extended periods go by without a single trade taking place. The data set of OTC prices for dividend swaps, therefore, is impacted by the model investment banks use for pricing them. We find price differences for same maturity transactions among the pricing sheets of investment banks of on average 3% with a standard deviation of 3%.

Since the OTC market does not trade regularly, it seems likely that fitting the state space model to its price data is akin to mimicking the pricing models used by the investment banks. We nonetheless perform the same set of estimations and reconciliations as above on the OTC price data of dividend swaps referring to the S&P 500 and the FTSE 100 indices. The results shown are restricted to monthly frequencies, as the daily data are stale. The results are shown in Tables VI and VIII.

The two-state model produces a high estimate for the long run growth constant \bar{p}^* of S&P 500 dividends. Indeed, at -1.3 percent for this constant, the S&P 500 present value as estimated by the model (equation (23)) overestimates its observed values by a factor of more than 2. Both mean reversion parameters attain reasonable levels, but they attract fairly large standard errors. In the case of the FTSE 100, the two-state model estimate for long run growth equals -5.3 percent, with a standard error even exceeding that level in absolute terms. At the same time, the second mean reversion parameter comes out low at 0.04, which translates into a half-value time running into decades. At such slow moving mean reversion, the role of the long run constant is essentially taken over by the medium-term factor. The single-state estimate for long run growth is more reasonable at -3.3 percent.

We then turn to the reconciliation regressions (equation (24)). The variation in the modelled price-dividend ratio produced by the estimates does not depend on the long run constant and the dynamic reconciliation to the stock indices demonstrates that it has meaningful explanatory power. Table VIII shows that the model produces a coefficient of around 0.2 for the price-dividend ratio, with reasonable significance for both the S&P 500 and the FTSE 100. Overall explanatory power is high for the S&P 500 with the adjusted R^2 reaching 0.58. However, most of it stems from the observed first dividend price F_t rather than

the modelled price-dividend ratio.²⁴ The same applies to the FTSE 100, albeit with the adjusted R^2 at a lower level of explanatory power. In both markets variation in the price-dividend ratio accounts for 8%, against 28% and 43% for the Eurostoxx 50 and Nikkei 225.

III. Conclusion

This paper proposes a method to extract information about the expectations that investors entertain of stock dividends from dividend derivatives. We show that modeling a single variable is sufficient to describe the dynamics and term of structure of dividend values. This variable is equal to the dividend growth minus the risk-free rate and a term capturing the risk premium. We propose a two-factor model for this discounted risk-adjusted growth variable, capturing the dynamics of short-term and medium-term dividend growth. The two factors shape a term structure of dividend growth which fits the data well and they determine the dynamics of the price-dividend ratio. Applied to the Eurostoxx 50 and the Nikkei 225, most of the variation of the stock market can be traced back to the model and short-term dividends together. We conclude that dividend derivatives and stock prices line up well enough to consider the information contained in one market for use of understanding the other. Several inferences from these findings can be drawn.

Short-term and medium-term dynamics

The distance into the future considered by investors affects the fit of dividend derivatives. At the extreme, a model which assumes a constant discount rate would show poor fit and explanatory power. But even a model where short-term variation in growth expectations is described by a single factor is significantly outperformed by a two-state model. The short-term factor reflects a horizon of under one year and the medium-term factor a horizon of several years. Deploying two states next to each other allows some distinction between sudden occurrences and those at business cycle proportions. Pursuing different explanations for the two states, or in other words, finding different determinants of how investors think of the short and the medium-term, seems an appropriate research avenue.

²⁴ Similar regressions based on daily estimates and stock index data produce R^2 of less than 5 percent.

Long-term growth is stable

The state space model imposes the return to a constant mean level of growth in the long run. This assumption is loosely interpreted as that investors do not change their opinion about the sequence of present values of dividends in the long run, which seems very restrictive intuitively. Nevertheless, small estimation errors, the explanatory power of the reconciliation of the model to stock returns and the near unity of the coefficients of the short-term dividend and the price-dividend ratio in the regressions of stock returns on these determinants, add credence to the imposition that long run growth is no major source of stock market variation. Interest rates are a part of discounted risk-adjusted dividend growth, and they are observable to investors. Under the assumption that they do not change their opinion about discounted risk-adjusted dividend growth \bar{p} in the long run, then our results suggest that most interest rate variation is balanced by risk-adjusted dividend growth expectations $g - \theta$ at these long horizons.

The importance of risk-free discounting of risk-adjusted dividend growth

The estimation of the dividend term structure improves when we directly discount risk-adjusted dividend growth for the time value of money. An alternative approach, which does not discount dividends at the risk-free rate and assumes independence between interest rates and risk-adjusted growth, implies estimates of dividend growth that are not economically sensible and which reconcile poorly to the stock market. Hence, jointly modeling interest rates, dividend growth and risk premium is strongly preferred.

Listed data are preferred

We perform the estimations using prices of OTC dividend swaps as well as of listed dividend futures. The prices produced by the OTC market are relevant, but generate less precise results and much lower explanatory power, while dividend futures provide intuitive and highly significant results. Not only do the long run estimates come out poorly, also the added value of the two-state model is not confirmed by OTC prices. It seems that stale prices, large price discrepancies among investment banks and infrequent trading cautions their

interpretation when applied in present value analysis. Fortunately, the set of listed data will only expand as time passes.

Shiller's "Stock Prices Move Too Much" assertion

Robert Shiller contends that realized dividends are not volatile enough to justify the observed volatility of stock markets, if discount rates are assumed to be constant over time and maturities (Shiller, 1981).

The approach we take constructs a rationally expected price for stocks in a different way. Rather than a model of future realized dividends, the term structure contains both actual expectations of future dividends and risk-adjusted discount rates. While Shiller finds observed stock return volatility to be five to thirteen times larger than modelled volatility, we find that the model produces about as much stock market volatility as is observed, regardless of the stock market that we consider.

Any model limited to using realized dividends as a proxy for expectations of dividends will fail to pick up the variation in discount rates that the market applies to those future dividends, as well as estimation error of those expectations which may well display substantial volatility of their own. The approach taken by Shiller confirms that dividends turn out much less volatile than the stock market. But it does not confirm that the volatility of the present value of dividends is too high, only because the drivers of such valuations aren't observed.

Follow up

It would be interesting to study to which fundamental variables the variation of short-term and medium-term growth in discounted risk-adjusted dividends can be ascribed. Armed with such linkages, the ability to understand stock market dynamics will improve. At the same time, Cochrane (2011) is clear in his assertion that: "We do not have to *explain* discount rates – relate expected returns to betas and understand their deep economics – in order to *use* them.". Opportunities are plentiful.

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Appendix A: Dividend derivatives data

Deriving the observed growth path of discounted risk-adjusted dividends from derivatives prices calls for several processes to make them suited for use in the state space approach. This section describes dividend derivatives, the data and these processes.

A.1. Dividend derivatives

Dividend derivatives exchange the value of a dividend index for cash at set expiry dates. The price of the derivative is set at the transaction date t and settled at the expiry date n . The difference between the transaction price and the amount of dividends actually paid is the amount settled between buyer and seller.

The transaction price reflects the growth path expected from the current level of dividends and the premium required for the risk of the actual payment differing from what is expected. It is a risk-adjusted price and equals the present value of a dividend once the time value of money is accounted for.

The dividend index measures the amount of dividends paid by the companies constituent to a stock index during a calendar year²⁵. At the end of the year, the index is the fixing at which the dividend derivative is settled. Manley and Mueller-Glissmann (2008) provide an overview of the market for dividend derivatives and its mechanisms.

A.2. Dividend derivatives data

Listed futures on dividends paid by the companies in the Eurostoxx 50 and the Nikkei 225 indices are the subject of this paper. These indices are widely accepted as representative for large cap firms based in Europe and Japan. They are chosen as dividend futures have been traded on them for a meaningful period with reasonable liquidity and good transparency.

Dividend futures are available for other markets as well. They are referenced to dividends of the FTSE 100, Hang Seng and Hang Seng China Enterprises and several other less liquid markets, although not the S&P 500²⁶. They started trading in 2008 on the Eurostoxx 50 and

²⁵ Derivatives relating to dividends paid by individual companies exist as well.

²⁶ It is not clear why no exchange has opened up to listing futures on dividends of a US stock index.

have expanded in region and horizon since then. Nonetheless, only for the Eurostoxx 50 and the Nikkei 225 dividend indices futures are traded with a maturity range of up to ten years. The other markets extend out to no longer than four years. The purpose of this paper is to estimate a term structure of dividend risk-adjusted growth, for which longer dated maturities are required. We therefore exclude the shorter maturity dividend futures markets from the dataset.

Before 2008, dividend derivatives existed as dividend swaps traded over-the-counter (OTC) only. They date back to 2002, well before the onset of listed futures. Maturities extend out to ten years and more for Eurostoxx 50, Nikkei 225, FTSE 100 and the S&P 500. We obtained dividend swap price data from several investment banks for all four stock indices mentioned²⁷, but there are problems. Before 2005, prices are stale and, throughout the data period prices, not always consistent with each other among suppliers. Moreover, turnover is very low (Mixon and Onur, 2014)²⁸.

We nonetheless perform the estimations with datasets both of listed dividend futures at daily frequency and OTC dividend swaps at monthly frequency. The main conclusion from these results is that it shows that OTC price data originate from pricing models and may or may not reflect market prices. Table AI contains a detailed description of the data.

A.3. Constant maturities

Dividend derivatives usually expire at a fixed date near the end of the calendar year²⁹ and therefore their time to maturity shortens by one day for each day that passes. For application in the state space model, growth rates of a constant horizon are required. The horizons of the measurement equations regard annual increments, the state equations regard one day increments. To obtain growth rates from prices with constant maturities, we interpolate derivatives with adjacent expiry dates. The interpolation is weighted by a scheme which reflects the uneven distribution of dividends through the year. For example, in the spring season 60% of the Eurostoxx 50 dividends of a full index year are paid in a matter of a few weeks (Figure A1).

²⁷ Deutsche Bank, Goldman Sachs and Credit Suisse.

²⁸ Dividend swaps are said not to trade daily, “sometimes not even for months”. Turnover figures are not public.

²⁹ The Nikkei 225 dividend index runs until the last trading day in March.

Derivatives prices which have a constant horizon from any observation date are constructed from observed derivatives prices. Such Constant Maturity (CM) derivative prices $F_{t,n}^{CM}$ take the following shape, attaching the seasonal pattern of the dividend index as weights to the observed derivatives prices w_i , with i standing for the day in the dividend index year, $i = 1$ being the first day of the count of the dividend index³⁰:

$$F_{t,n}^{CM} = (1 - w_i)F_{t,n} + w_iF_{t,n+1}. \quad (\text{A.1})$$

The weight w_i of the dividend index reflects the cash dividend amount paid as a proportion of the total amount during a dividend index year. The average of the years 2005 to 2013 is taken. $F_{t,n}$ is the observed price of the derivative which expires n^{th} in line into the future from the observation date onwards, $F_{t,n+1}$ expiring the following year. This weighting scheme reduces the impact of the n^{th} derivative to expire on the constant maturity derivative as time passes by the proportion w_i of dividends that have actually been declared. Its complement $(1 - w_i)$ is the proportion that remains to be declared until the expiry date and is therefore an expectation of undeclared dividends for year n at the observation date. In order to produce a derivative price with constant maturities, this undeclared amount is balanced by the proportion of the price of the derivative expiring the year after. In so doing, the constant maturity price reflects no seasonal pattern, while still accounting for the seasonal shift in impact from the n^{th} derivative to the next. For example, during the dividend season in Spring, the weight is shifted more quickly from the first to the second derivative³¹ than in other parts of the year³².

A.4. The first to expire constant maturity derivative

The weighting scheme in equation (A.1) will be applied to obtain all CM derivatives prices, except for the first CM derivative, because the proposed approach carries

³⁰ which is the first trading day following the expiry date of a dividend derivatives contract.

³¹ First and second derivatives is shorthand for the derivatives that are first and second to expire.

³² A linear weighting scheme would reflect the adjacent derivative prices unevenly. For example, half way through the dividend index year already 80% of annual dividends is declared and paid. Linear weighting would then overemphasize the information contained in the price of the derivative in equation (A.1) that is the soonest to expire.

measurement problems for it. At time t the expected dividend to be delivered at the expiration of the first derivative $E_t(D_1)$ is the sum of the dividend index DI_t as it accretes throughout the year and its unknown complement $E_t(UD_1)$:

$$E_t(D_1) = DI_t + E_t(UD_1). \quad (\text{A.2})$$

For CM derivatives with horizons longer than the first, the weight w_i is the average seasonal pattern in the preceding decade, which may not necessarily resemble that of a particular dividend index year $DI_t/E_t(D_1)$. The difference between the two is shown in Figure A2; for example in April 2013 the payments of Eurostoxx 50 dividends had already reached 33% of the annual total, while on average in the years 2005 to 2013 it stood at 20%. This advance dropped below ten percent not until a month later. In general, dividend payments in 2012 and 2013 seem to have taken place earlier in the calendar year than usual in the preceding years.

Weighting the first derivative by the average of the preceding decade when dividends realize sooner in the year than the average, as was the case in April 2013, overemphasizes the importance of that first derivative to the one year CM derivative. This first CM derivative will then contain backward looking information as well underemphasize the unrealized proportion of the contemporaneous dividend index both to the tune of the difference between the historical average and the realized dividend index. To avoid this issue, the first CM derivative is construed by defining the weight as the proportion of the dividend index that has been realized of the total expected dividend for that year only:

$$F_{t,1}^{CM} = F_{t,1} - DI_t + \frac{DI_t}{D_1} F_{t,2}. \quad (\text{A.3})$$

For building a first CM derivative with a constant one year horizon as a stochastic variable, we include unknown $E_t(UD_1)$ and exclude known DI_t . The expectation of full year dividends is proxied by the equivalent observation. Later CM derivatives do not weight variables which have already been partly realized, hence the weighting issue of the first CM

derivative does not reoccur. For $n \geq 2$, the prices of CM derivatives remain constructed as in the weighting equation (A.1).

A.5. Calculating seasonal weights for different dividend index years

Expiry years do not have the same number of trading days every year or across markets. Not only do trading holidays differ, also the expiry date is set to the third Friday in December in every expiry year. This day falls anywhere between the 15th and the 21st of December³³ and the number of trading days fluctuates accordingly.

In order to establish a seasonal pattern for w_i that is correct for the actual number of trading days in each expiry year, realized dividends are normalized and averaged. First, the amount of dividends paid on a given day is expressed as a percentage of the total dividends paid in the matching dividend index year. Next, for each expiry year these percentages are normalized to a set number of trading days. Finally, they are averaged. For calculating the values in the weighting equation, they are rescaled to the actual number of trading days in the dividend index year in question. This approach guarantees that in every expiry year, weight w_i starts at zero and ends the year at 100%, regardless of the number of trading days.

A.6. Current dividends

At the heart of the present value model are the discounted values of risk-adjusted dividends. These present values $P_{t,n}$ take current dividends D_t as the starting point from which growth is projected forward at growth rate π_{t+i} (equation (3)). It is sometimes assumed³⁴ that current dividends can be reasonably approximated by realized dividends. For daily data as applied in this paper, however, this assumption causes issues.

The asset underlying dividend derivatives is the amount of cash dividend thrown off by a stock or a stock index during the year in which the derivative expires. The index companies pay dividends throughout the calendar year³⁵ which implies that taking realized dividends as

³³ With exception of the Nikkei 225.

³⁴ E.g. Binsbergen et al. (2013), Cejnek and Randl (2014).

³⁵ In fact, the dividend index year usually runs from the first working day following the third Friday in December until and including the third Friday in December of the following year. Dividend derivatives also apply the third Friday of December as the expiry date.

current dividends at a certain day of the year would require looking back for twelve months. The dividend paying capacity of index companies are unlikely to stay constant for a year, however.

To take a strong example, around the days of the Lehman bankruptcy on the 15th of September 2008, the one year dividend history of Eurostoxx 50 companies amounted to 154. Due to the bankruptcy, investors would have changed their opinion strongly downwards about the dividend that companies would pay if they would have had to pay on these days. Even if dividends reflect the past year of earnings, company management is likely to reduce dividends if their near term outlook changes for the worse by precautionary motive. After Lehman, taking a dividend history of twelve months would then overestimate the approximation of current dividends as they stood in the fall of 2008. In the weeks following the default, the Eurostoxx 50 dividend future expiring in 2009 dropped from 140 to 100. Therefore, if twelve month realized dividends are used as current dividends, the shortest horizon observation for growth from 2008 to 2009 on the dividend curve would attain a strongly negative figure even though the actual growth expectation, starting from a level that would have been revised downwards, could be flat or even positive.

This problem rules out considering the dividend index itself, or a rolling twelve month estimate of it, as a starting point from which to calculate the growth rate until the first derivative to expire. The first derivative to expire, of course, does contain investor expectations about dividends to be paid in the remaining period until the first expiry date. But this information regards the period from the observation date until the expiry date; it is not a reflection of dividend expectations on the observation date itself.

To avoid these data difficulties, we propose an alternative base. In lieu of an estimate for current dividends, we use dividend derivatives with one year remaining life to expiry $F_{t,1}$ discounted at the one year risk-free rate $y_{t,1}$ as the base from which to calculate growth rates:

$$P_{t,1}^{CM} = F_{t,1}^{CM} \exp(-y_{t,1}), \quad (\text{A.5})$$

and the first year of growth is deducted accordingly. Discounted risk-adjusted dividend growth rates are then given by:

$$n\pi_{t,n} - \pi_{t,1} = \ln(F_{t,n}^{CM}) - ny_{t,n} - (\ln(F_{t,1}^{CM}) - y_{t,1}). \quad (\text{A.6})$$

As a consequence of estimating $n\pi_{t,n} - \pi_{t,1}$ as a single variable, we do not account for the first year of discounted dividend growth as part of the dividend term structure. At the same time, the one year dividend present value $P_{t,1}^{CM}$ includes short-term derivatives prices which encompass investor expectations extending from the observation date until a year later. Although growth for the first year is not observed, the one year discounted derivative price is included in the present value identity ensuring that no information is lost when reconciling the model estimates to the stock market as in equations (23) and (24).

Appendix B: Measurement equations

In this appendix the details of the derivation of the measurement equations are described. We rewrite the state equations (8) and (9) in vector form and then derive the discrete-time implications of the model. Denote $Q_t = \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix}$ the 2×1 vector of the factors and $\bar{Q} = \begin{pmatrix} \bar{p} \\ \bar{p} \end{pmatrix}$ as the 2×1 vector of the constant infinite growth rate. In a two equation matrix format, the system becomes:

$$dQ_t = \begin{pmatrix} dp_t \\ d\tilde{p}_t \end{pmatrix} = \begin{bmatrix} -\varphi & \varphi \\ 0 & -\psi \end{bmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \psi \bar{p} \end{pmatrix} dt + \begin{bmatrix} \sigma_p & 0 \\ 0 & \sigma_{\tilde{p}} \end{bmatrix} \begin{pmatrix} dW_p \\ dW_{\tilde{p}} \end{pmatrix}. \quad (\text{B.1})$$

This system of differential equations in matrix notation is:

$$dQ_t = C[Q_t - \bar{Q}]dt + \Sigma dW, \quad (\text{B.2})$$

which has the general solution:

$$Q_{t+1} = \bar{Q} + \Phi(Q_t - \bar{Q}) + \varepsilon_{t+1}, \quad (\text{B.3})$$

and of which the eigenmatrix solves to:

$$\Phi = \begin{pmatrix} e^{-\varphi} & \frac{\varphi}{\varphi - \psi} (e^{-\varphi} - e^{-\psi}) \\ 0 & e^{-\psi} \end{pmatrix}. \quad (\text{B.4})$$

Substituting the expression for the eigenmatrix into (B.3) delivers state equations (B.5):

$$\begin{pmatrix} p_{t+1} \\ \tilde{p}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 - e^{-\varphi} & -\frac{\varphi}{\varphi - \psi} (e^{-\psi} - e^{-\varphi}) \\ 0 & 1 - e^{-\psi} \end{pmatrix} \begin{pmatrix} \bar{p} \\ \bar{p} \end{pmatrix} + \begin{pmatrix} e^{-\varphi} & \frac{\varphi}{\varphi - \psi} (e^{-\psi} - e^{-\varphi}) \\ 0 & e^{-\psi} \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \varepsilon_{t+1}. \quad (\text{B.5})$$

We model correlation between the innovation in the growth rate v_{t+1} and the errors ε_{t+1} in these state equations as $v_{t+1} = \beta' \varepsilon_{t+1}$, where $\beta = (\beta_p, \beta_{\bar{p}})'$ is a 2-by-1 vector. Next we use this process to write the n-period ahead growth rate as a function of the factors:

$$\pi_{t+n} = \alpha'(\bar{Q} + \Phi^{n-1}(Q_t - \bar{Q})) + \alpha' \sum_{i=1}^{n-1} \Phi^{n-i} \varepsilon_{t+i} + \beta' \varepsilon_{t+n}, \quad (\text{B.6})$$

in which $\alpha' = (1 \ 0)$. This can be substituted into the pricing equation:

$$\ln P_{t,n} - \ln D_t = E_t \left(\sum_{i=1}^n \pi_{t+i} \right) + \frac{1}{2} \text{Var}_t \left(\sum_{i=1}^n \pi_{t+i} \right), \quad (\text{B.7})$$

The right hand side can be worked out as follows:

$$\ln P_{t,n} - \ln D_t = \alpha'(n\bar{Q} + B_n(Q_t - \bar{p})) + \frac{1}{2} \text{Var}_t \left(\sum_{i=1}^n \left(\alpha' \sum_{j=1}^{i-1} \Phi^{n-j} \varepsilon_{t+j} + \beta' \varepsilon_{t+i} \right) \right), \quad (\text{B.8})$$

which in turn implies:

$$\ln P_{t,n} - \ln D_t = \alpha'(n\bar{Q} + B_n(Q_t - \bar{p})) + \frac{1}{2} \sum_{i=1}^n (\beta' + \alpha' B_i) \Sigma (\beta + B_i' \alpha), \quad (\text{B.9})$$

where matrix B_i is an expression constructed from the eigenmatrix:

$$B_i = (I + \Phi + \dots + \Phi^{i-1}) = (I - \Phi)^{-1} (I - \Phi^i). \quad (\text{B.10})$$

The equations are written without vector notation. By the definition of Φ , B_n is worked out as:

$$B_n = \begin{pmatrix} \frac{(1 - e^{-n\varphi})}{(1 - e^{-\varphi})} & \frac{\varphi}{\varphi - \psi} \left(\frac{(1 - e^{-n\psi})}{(1 - e^{-\psi})} - \frac{(1 - e^{-n\varphi})}{(1 - e^{-\varphi})} \right) \\ 0 & \frac{(1 - e^{-n\psi})}{(1 - e^{-\psi})} \end{pmatrix} = \begin{pmatrix} \varphi_n & \frac{\varphi}{\varphi - \psi} (\psi_n - \varphi_n) \\ 0 & \psi_n \end{pmatrix}. \quad (\text{B.11})$$

with shorthand notation:

$$\varphi_n = \frac{(1 - e^{-n\varphi})}{(1 - e^{-\varphi})} \quad (\text{B.12})$$

$$\psi_n = \frac{(1 - e^{-n\psi})}{(1 - e^{-\psi})}. \quad (\text{B.13})$$

An expression which consists of scalars only is obtained by substituting all elements of the above in the measurement equation:

$$\begin{aligned} \ln P_{t,n} - \ln D_t &= (1 \ 0) \left(n \begin{pmatrix} \bar{p} \\ \bar{p} \end{pmatrix} + \begin{pmatrix} \varphi_n & \frac{\varphi}{\varphi - \psi} (\psi_n - \varphi_n) \\ 0 & \psi_n \end{pmatrix} \begin{pmatrix} p_t - \bar{p} \\ \tilde{p}_t - \bar{p} \end{pmatrix} \right) + \\ &\frac{1}{2} \sum_{i=1}^n \left(\begin{pmatrix} \beta_p & \beta_{\bar{p}} \end{pmatrix} + (1 \ 0) \begin{pmatrix} \varphi_i & \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) \\ 0 & \psi_i \end{pmatrix} \right) \begin{pmatrix} \sigma^2 & 0 \\ 0 & \tilde{\sigma}^2 \end{pmatrix} \begin{pmatrix} \beta_p \\ \beta_{\bar{p}} \end{pmatrix} \\ &+ \begin{pmatrix} \varphi_i & 0 \\ \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) & \psi_i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \eta_{t,n} \end{aligned} \quad (\text{B.14})$$

$$\begin{aligned} &= n\bar{p} + \varphi_n(p_t - \bar{p}) + \frac{\varphi}{\varphi - \psi} (\psi_n - \varphi_n)(\tilde{p}_t - \bar{p}) \\ &+ \frac{1}{2} \sum_{i=1}^n \left(\sigma_p^2 (\beta_p + \varphi_i)^2 + \sigma_{\bar{p}}^2 \left(\beta_{\bar{p}} + \frac{\varphi}{\varphi - \psi} (\psi_i - \varphi_i) \right)^2 \right) + \eta_{t,n}, \end{aligned} \quad (\text{B.15})$$

which is the same as equation (11) in the main text. The right hand term on the right hand side is referred to in the paper as the ‘‘convexity term’’.

Tables and Figures

Table I

Base model of Discounted Risk-adjusted Dividend Growth:

$$\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}$$

Estimates using **listed Dividend Futures of the Eurostoxx 50 Index**
Sample period: 4 August 2008 – 16 February 2015

	Two-state		Single-state	
	$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\bar{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\bar{p} - p_t)dt + \sigma_p dW_p$	
\bar{p}	-0.0586 (9.5339)	-0.0404 (0.0197)	-0.2067 (28.777)	-0.0435 (0.0144)
φ	1.5130 (0.3160)	1.5132 (0.3158)	1.7297 (0.4894)	1.7292 (0.4894)
ψ	0.2433 (0.1089)	0.2434 (0.1088)		
β_p	0.1553 (67.007)	<i>Set to 0</i>	0.6246 (69.2935)	<i>Set to 0</i>
$\beta_{\tilde{p}}$	-2.6695 (6.2539)	-2.6693 (6.2523)		
σ_p	0.5701 (0.7876)	0.5704 (0.7870)	0.7033 (1.2245)	0.7033 (1.2245)
$\sigma_{\tilde{p}}$	0.0437 (0.0947)	0.0437 (0.0946)		
σ_{η}^1	0.0219 (0.0295)	0.0219 (0.0294)	0.0177 (0.0071)	0.0177 (0.0071)
σ_{η}	0.0063 (0.0025)	0.0063 (0.0025)	0.0441 (0.0806)	0.0441 (0.0806)
Log Likelihood per contribution	24.57	24.57	18.35	18.35

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years, except for its begin until 13th May 2009 in which the number is five due to a lack of data. σ_{η} measures the standard deviations of the second until the eight measurement equations, σ_{η}^1 of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table II**Base model** of Discounted Risk-adjusted Dividend Growth:

$$\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}$$

Estimates using **listed Dividend Futures of the Nikkei 225 Index**

Sample period: 17 June 2010 – 16 February 2015

	Two-state		Single-state	
	$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\bar{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\bar{p} - p_t)dt + \sigma_p dW_p$	
\bar{p}	-0.0320 (1.3833)	-0.0371 (0.0264)	-0.0719 (3.8264)	-0.0487 (0.0304)
φ	0.7381 (0.2360)	0.7381 (0.2345)	0.2837 (0.0306)	0.2837 (0.0306)
ψ	0.1784 (0.0539)	0.1784 (0.0537)		
β_p	-1.5513 (92.822)	<i>Set to 0</i>	-7.6306 (211.137)	<i>Set to 0</i>
$\beta_{\tilde{p}}$	-3.4234 (16.191)	-3.4229 (16.1862)		
σ_p	0.1531 (0.2193)	0.1531 (0.2193)	0.0630 (0.1189)	0.0630 (0.1186)
$\sigma_{\tilde{p}}$	0.0251 (0.0731)	0.0251 (0.0730)		
σ_{η}^1	0.0147 (0.0197)	0.0147 (0.0197)	0.0137 (0.0041)	0.0137 (0.0041)
σ_{η}	0.0040 (0.0015)	0.0040 (0.0015)	0.0170 (0.0285)	0.0170 (0.0285)
Log Likelihood per contribution	29.37	29.36	22.04	22.04

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years. σ_{η} measures the standard deviations of the second until the eight measurement equations, σ_{η}^1 of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table III

Key results from two-state space model and sample period averages

Sample period		Eurostoxx 50	Nikkei 225
		4 August 2008 – 16 Feb 2015	17 June 2010 – 16 Feb 2015
Estimated LT growth ($\beta_p = 0$)	\bar{p}	– 4.0 %	– 3.7 %
Estimated convexity term for $i \rightarrow \infty$ ¹⁾		1.4%	1.1%
Estimated LT growth plus convexity term ¹⁾	\bar{p}^*	– 2.6%	– 2.6%
Average dividend yield	$\frac{D_t}{S_t}$	4.3 %	1.9 %
Average estimated 1 year growth	p_t	– 9.8 %	1.1 %
Average estimated 1 year forward 4 year growth	\tilde{p}_t	– 3.7 %	– 1.6 %
Average calibrated first dividend point	$\frac{\hat{P}_{t,1}}{\sum_1^\infty \hat{P}_{t,n}}$	3.1 %	2.1 %

¹⁾Refer to equation (19) in the main text:

$$\bar{p}^* = \bar{p} + \frac{1}{2} \left(\sigma_p^2 (\beta_p + \varphi_{i \rightarrow \infty})^2 + \sigma_{\tilde{p}}^2 \left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi} (\psi_{i \rightarrow \infty} - \varphi_{i \rightarrow \infty}) \right)^2 \right)$$

Table IV

Reconciliation of the Base Present Value Model (two-state) constituent returns to stock market returns: **listed Dividend Futures**

	Eurostoxx 50				Nikkei 225			
<i>Constant</i>	0.0005 (0.0003)	0.0002 (0.0003)	0.0006 (0.0004)	-0.0001 (0.0003)	-0.0002 (0.0003)	0.0002 (0.0004)	0.0005 (0.0004)	0.0002 (0.0003)
$\Delta \ln F_t$	0.8978 (0.0337)	1.0009 (0.0426)			0.8488 (0.0508)	0.6582 (0.0719)		
Δy_t	0.1446 (0.0127)		0.2022 (0.0178)		0.0751 (0.0619)		-0.0081 (0.0912)	
$\Delta \ln(\widehat{PD})_t$	0.6587 (0.0216)			0.6893 (0.027)	0.8619 (0.0251)			0.8156 (0.0278)
<i>Adj. R²</i>	0.540	0.248	0.071	0.280	0.540	0.068	0.000	0.429

The modelled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (24) $\Delta \ln S_t = \alpha + \beta_F \Delta \ln F_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{PD}} \Delta \ln(\widehat{PD})_t + \varepsilon_t$, in which S_t is stock index, F_t is the first constant maturity dividend derivative price, Δy_t is the change in the one year zero swap rate and \widehat{PD}_t is the sum of the normalized present value of dividends as estimated in the two-state space model. β_p is fixed at zero. Daily data for periods as in Tables I and II. Standard errors in parentheses.

Table V**Alternative Model** of Undiscounted Risk-adjusted Dividend Growth:

$$z_{t+1} = g_{t+1} - \theta_{t+1}$$

Estimates using **listed Dividend Futures**.

Sample period	Two-state		Single-state	
	$dz_t = \varphi(\tilde{z}_t - z_t)dt + \sigma_z dW_z$ $d\tilde{z}_t = \psi(\bar{z} - \tilde{z}_t)dt + \sigma_{\tilde{z}} dW_{\tilde{z}}$		$dz_t = \varphi(\bar{z} - z_t)dt + \sigma_z dW_z$	
	Eurostoxx 50	Nikkei 225	Eurostoxx 50	Nikkei 225
	4 August 2008 – 16 Feb 2015	17 June 2010 – 16 Feb 2015	4 August 2008 – 16 Feb 2015	17 June 2010 – 16 Feb 2015
\bar{z}	0.0027 (0.0241)	-0.0104 (0.0416)	-0.0147 (0.0212)	-0.0406 (0.0549)
φ	1.4108 (0.2552)	0.6547 (0.1874)	1.4142 (0.4588)	0.2944 (0.0350)
ψ	0.1860 (0.0779)	0.2006 (0.0591)		
β_z	<i>Set to 0</i>	<i>Set to 0</i>	<i>Set to 0</i>	<i>Set to 0</i>
$\beta_{\tilde{z}}$	-3.0428 (6.8937)	-3.6131 (35.6244)		
σ_z	0.5125 (0.6433)	0.1335 (0.1814)	0.4858 (1.0167)	0.0794 (0.1830)
$\sigma_{\tilde{z}}$	0.0362 (0.0687)	0.0286 (0.0810)		
σ_{η}^1	0.0233 (0.0311)	0.0138 (0.0183)	0.0259 (0.0136)	0.0144 (0.0042)
σ_{η}	0.0060 (0.0023)	0.0038 (0.0014)	0.0569 (0.1331)	0.0183 (0.0336)
Log Likelihood per contribution	24.78	29.64	15.75	21.58

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years, except for its begin until 13th May 2009 in which the number is five due to a lack of data. σ_{η} measures the standard deviations of the second until the eight measurement equations, σ_{η}^1 of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table VI

Base model of Discounted Risk-Adjusted Dividend Growth:				
$\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}$				
Estimates using OTC Dividend Swaps (monthly)				
	Two-state		Single-state	
	$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\bar{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\bar{p} - p_t)dt + \sigma_p dW_p$	
	S&P 500	FTSE 100	S&P 500	FTSE 100
Sample period	Dec 2005 – June 2014	Dec 2005 – June 2014	Dec 2005 – June 2014	Dec 2005 – June 2014
\bar{p}	-0.0188 (0.0231)	-0.0841 (0.1513)	-0.0186 (0.0108)	-0.0430 (0.0093)
φ	1.0651 (0.7296)	1.6347 (0.5865)	0.3537 (0.0583)	1.7702 (0.5828)
ψ	0.1809 (0.1431)	0.0371 (0.1422)		
β_p	<i>Set to 0</i>	<i>Set to 0</i>	<i>Set to 0</i>	<i>Set to 0</i>
$\beta_{\tilde{p}}$	-2.5935 (10.4524)	-2.1624 (8.5798)		
σ_p	0.1756 (0.2975)	0.5865 (0.9707)	0.0584 (0.0674)	0.6770 (1.1518)
$\sigma_{\tilde{p}}$	0.0293 (0.0642)	0.0173 (0.035)		
σ_{η}^1	0.0167 (0.0072)	0.0199 (0.0326)	0.0138 (0.0044)	0.0141 (0.0056)
σ_{η}	0.0078 (0.0021)	0.0054 (0.0026)	0.0261 (0.0167)	0.0298 (0.0687)
Log Likelihood per contribution	22.62	23.79	19.79	19.29

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years. σ_{η} measures the standard deviations of the second until the eight measurement equations, σ_{η}^1 of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. β_p is fixed at zero. See the Data section. Standard errors in parentheses.

Table VII

Reconciliation of the **Alternative Model** of Undiscounted Risk-Adjusted Dividend Growth (two-state) constituent returns to stock market returns:

	Eurostoxx 50				Nikkei 225			
<i>Constant</i>	0.0006 (0.0003)	0.0002 (0.0003)	0.0006 (0.0004)	0.0000 (0.0004)	-0.0002 (0.0003)	0.0002 (0.0004)	0.0005 (0.0004)	0.0002 (0.0003)
$\Delta \ln F_t$	0.9555 (0.0419)	1.0009 (0.0426)			0.8781 (0.0572)	0.6582 (0.0719)		
Δy_t	0.1693 (0.0163)		0.2022 (0.0178)		0.1448 (0.0694)		-0.0081 (0.0912)	
$\Delta \ln(\widehat{PD})_t$	0.0725 (0.0157)			-0.0113 (0.0179)	0.7800 (0.0293)			0.7121 (0.0318)
<i>Adj. R²</i>	0.293	0.248	0.071	0.000	0.424	0.068	0.000	0.305

The modelled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (24) $\Delta \ln S_t = \alpha + \beta_F \Delta \ln F_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{PD}} \Delta \ln(\widehat{PD})_t + \varepsilon_t$, in which S_t is stock index, F_t is the first constant maturity dividend derivative price, Δy_t is the change in the one year zero swap rate and \widehat{PD}_t is the sum of the normalized present value of dividends estimated in the two-state space model. β_p is fixed at zero. Daily data for periods as in Tables I/ II. Standard errors in parentheses.

Table VIII

Reconciliation of the Base Present Value Model (two-state) constituent returns to stock market returns: **OTC Dividend Swaps (monthly data)**

	S&P 500				FTSE 100			
<i>Constant</i>	-0.0019 (0.0029)	-0.0016 (0.0028)	0.0065 (0.0042)	0.0038 (0.0040)	0.0005 (0.0034)	0.0005 (0.0035)	0.0032 (0.0040)	0.0016 (0.0039)
$\Delta \ln F_t$	1.1677 (0.1118)	1.1901 (0.1073)			0.5812 (0.1111)	0.6273 (0.1033)		
Δy_t	-0.0041 (0.0158)		0.0514 (0.0221)		0.0047 (0.0163)		0.0320 (0.0173)	
$\Delta \ln(\widehat{PD})_t$	0.1869 (0.0600)			0.2561 (0.0881)	0.1190 (0.0410)			0.1428 (0.0459)
<i>Adj. R²</i>	0.582	0.552	0.051	0.078	0.307	0.269	0.033	0.088

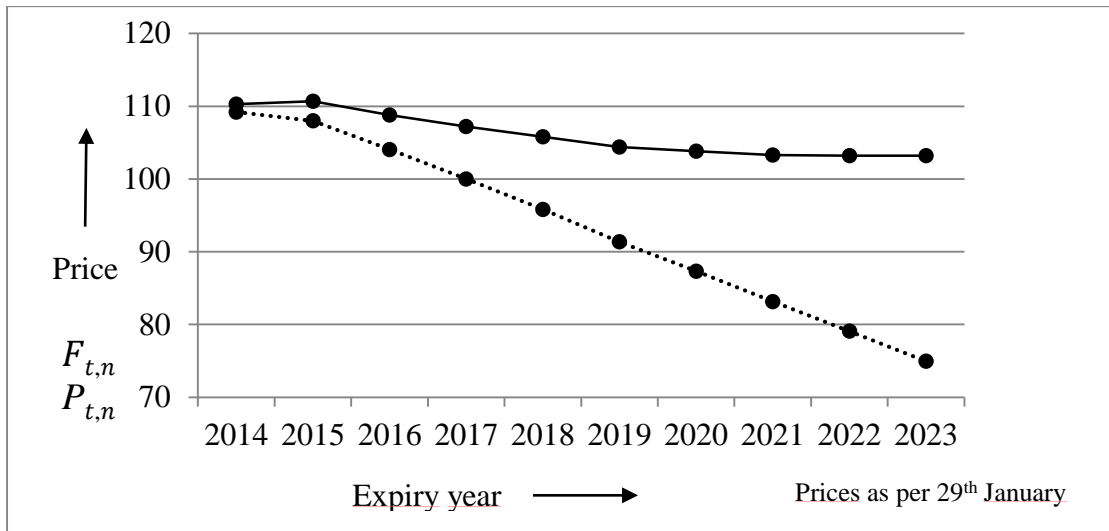
The modelled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (24) $\Delta \ln S_t = \alpha + \beta_F \Delta \ln F_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{PD}} \Delta \ln(\widehat{PD})_t + \varepsilon_t$, in which S_t is stock index, F_t is the first constant maturity dividend derivative price, Δy_t is the change in the one year zero swap rate and \widehat{PD}_t is the sum of the normalized present value of dividends estimated in the two-state space model. β_p is fixed at zero. Daily data for periods as in Table 6. Standard errors in parentheses.

Table AI

	Eurostoxx 50	S&P 500	FTSE 100	Nikkei 225
Number of companies in the index	50	500	100	225
Currency	Euro	US\$	GBP	JPY
Market capitalization in US\$ per 7 th May 2014	US\$ 3.3 trillion	US\$ 17.2 trillion	US\$ 3.1 trillion	US\$ 2.7 trillion
Data period				
Dividend swaps	N/A	19 December 2005 – 13 June 2014	19 December 2005 – 13 June 2014	N/A
Dividend futures	4 August 2008 – 16 February 2015	N/A	N/A	17 June 2010 – 16 February 2015
Source of the data				
Dividend swaps	N/A	OTC	OTC	N/A
Dividend futures	Eurex	N/A	Liffe	Singapore exchange
Average number of trading days	256	252	253	245
Liquidity	Good	Poor	Poor	Reasonable
Expiry horizon				
Dividend swaps	N/A	10 years	10 years	N/A
Dividend futures	10 years	N/A	4 years	10 years
Expiry date	3 rd Friday of December	3 rd Friday of December	3 rd Friday of December	Last trading day in March
Data frequency	Daily	Daily	Daily	Daily
Stock index ticker	SX5E	SPX	UKX	NKY
Dividend index ticker	DKESDPE	SPXDIV	F1DIVD	JPN225D

Figure 1. Eurostoxx 50 Dividend Futures Prices

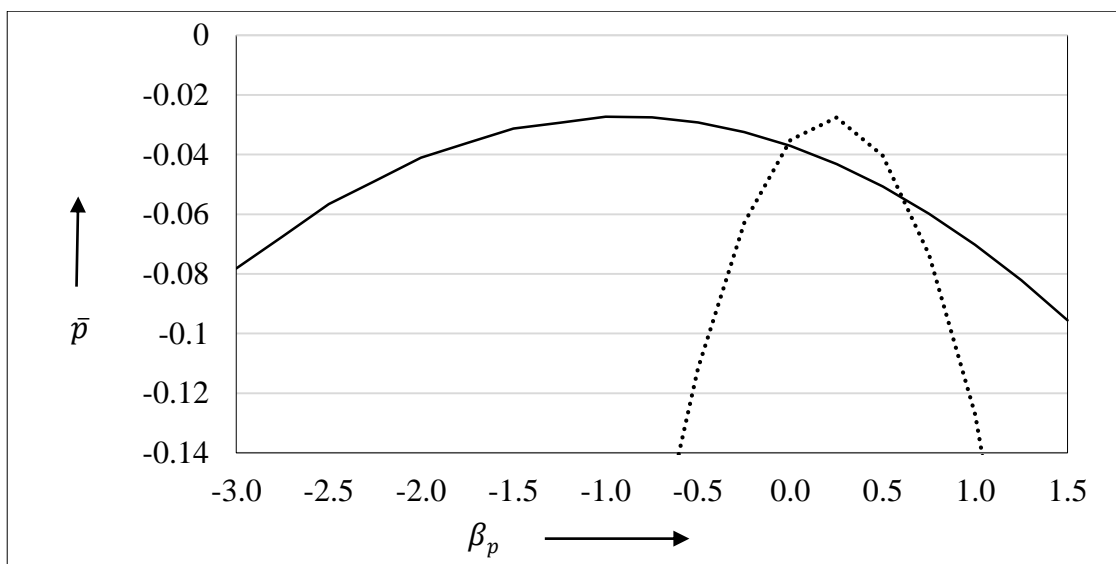
Price curve of dividend futures and of discounted dividend futures on an arbitrary day, for purpose of illustration. The discounted dividend futures equal the present value of expected dividends (equation (5)). Expiries occur on the third Friday in December of each expiry year.



Dividend futures $F_{t,n}$ ——— Discounted dividend futures $P_{t,n}$ ·····

Figure 2. Values of \bar{p} for a given value of β_p .

Values of β_p are set to 0 to arrive at values for \bar{p} . See Tables I/II for an example of estimation results. Values found for other parameters do not change materially when β_p is varied.



Nikkei 225 ——— DJ Eurostoxx 50 ·····

Mean absolute estimation errors

Figures 3 and 4 depict the average of the absolute estimation error of the two-state and the single state base model. The measurement variables are discounted risk-adjusted dividend growth rates of 1 to 8 years.

Figure 3. Eurostoxx 50

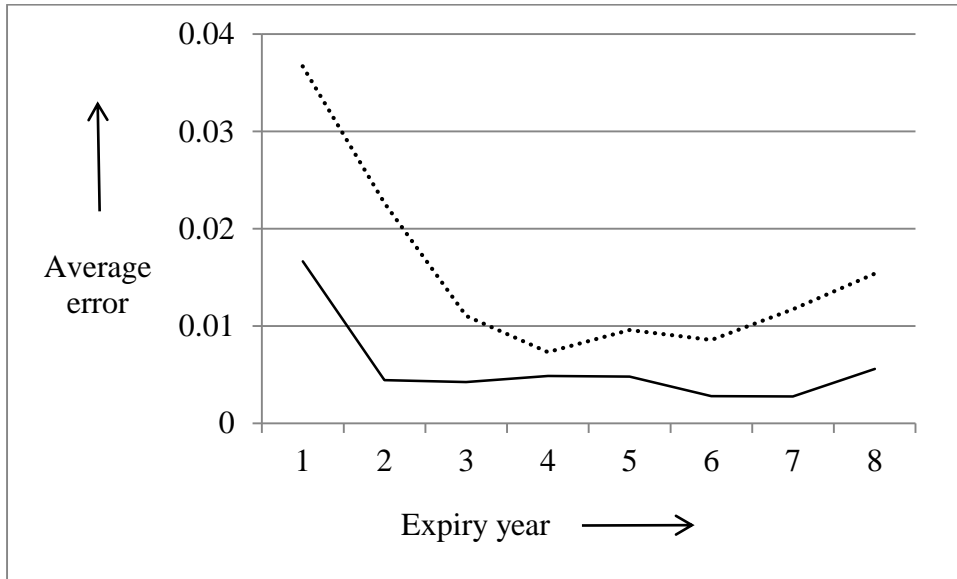


Figure 4. Nikkei 225



Two-state model ————— Single state model

Calibrated growth rates

Figures 5 to 8 show calibrated growth rates of discounted risk-adjusted dividends. Figures 5 and 7 contain the 1 year growth rate. Figures 6 and 8 contain average annual growth rates of the 4 years following the first year of growth: $p_{t,t+1 \rightarrow t+5}$.

Figure 5. Eurostoxx 50: 1 year growth growth

Figure 6. Eurostoxx 50: 1 year forward 4 year

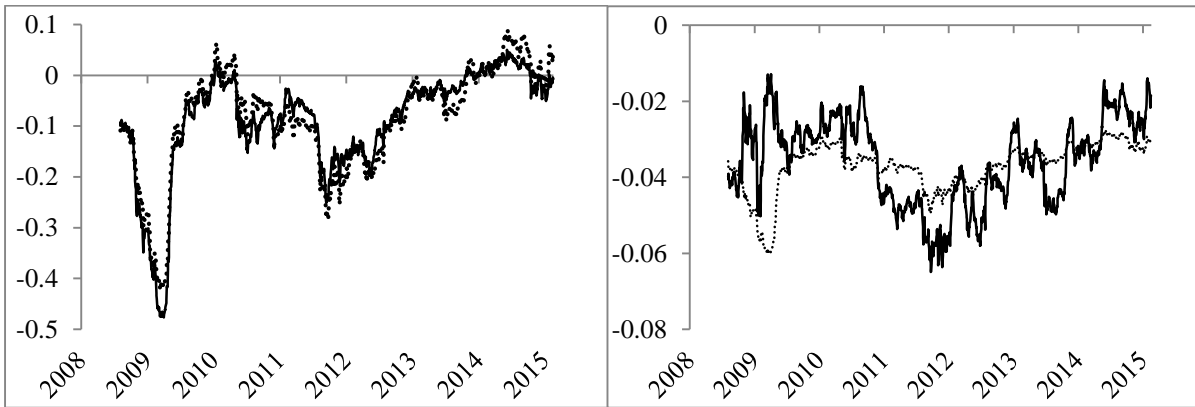
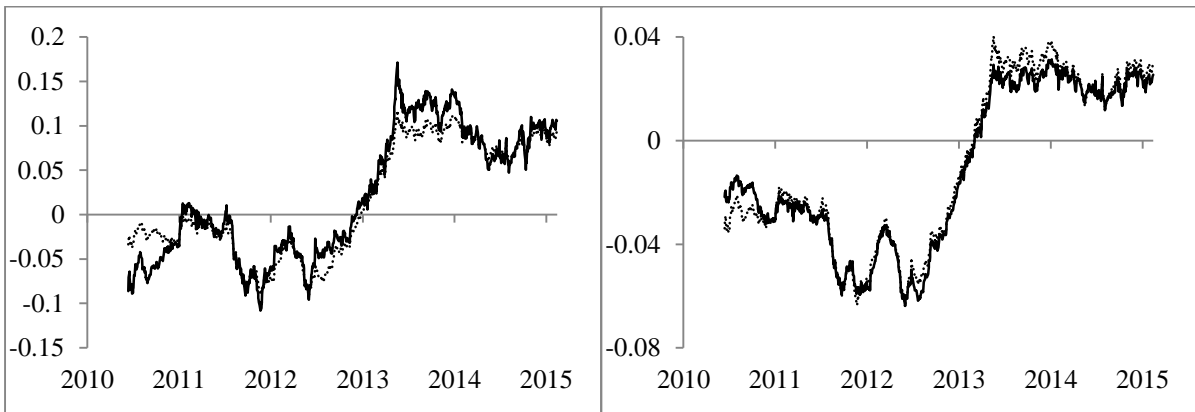


Figure 7. Nikkei 225: 1 year growth growth

Figure 8. Nikkei 225: 1 year forward 4 year



Two-state model ————— Single state model ·········

Figure 9. Discounted Risk-adjusted Dividend Growth Volatility Term Structure
 $p_{i \rightarrow i+1}$ = volatility from $n = i$ to $n = i + 1$ (= single period forward growth rates).

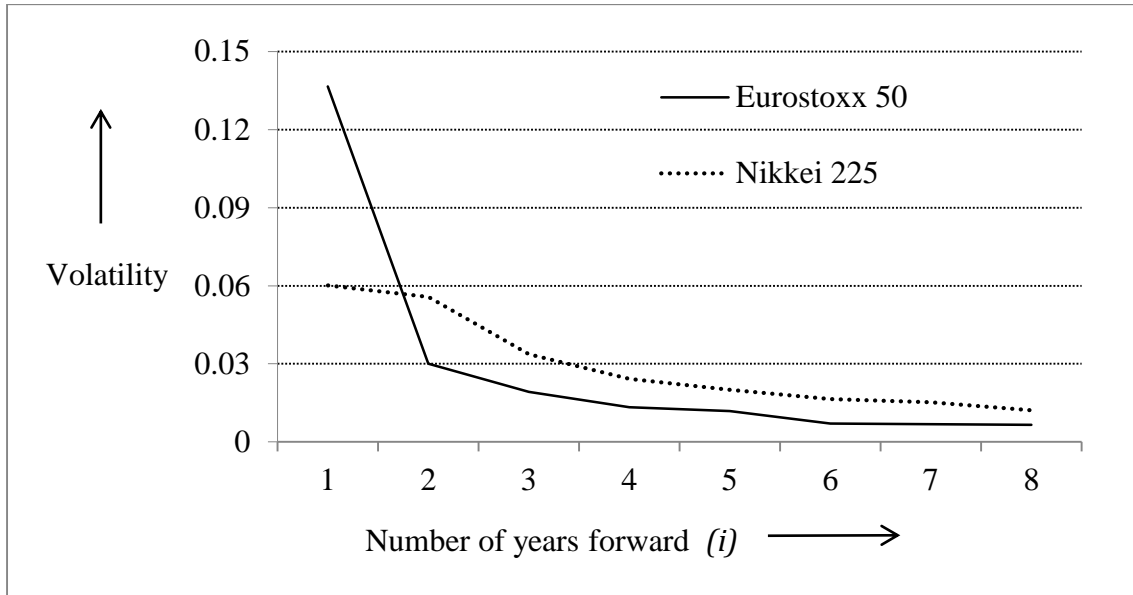
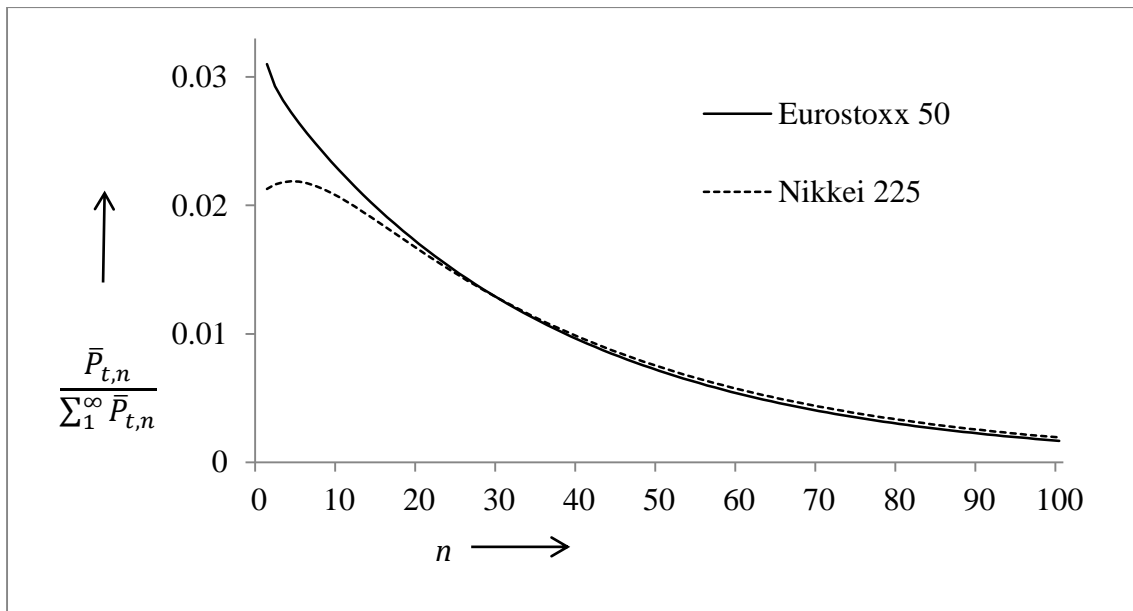


Figure 10. Calibrated average Dividend Term Structure

The average of calibrated present values of dividends per expiry year $\bar{P}_{t,n}$ is divided by the sum of the averages. This represents the average dividend yield per expiry year in present value terms.



Stock level reconciliation

Figures 11 & 12 portray the present value model estimates for the level of stock indices as described in $\hat{S}_t = F_{t,1} \exp(-y_{t,1}) (1 + \sum_{n=2}^{\infty} \exp(n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}))$ (equation (23)) in relation to stock market observations S_t .

Figure 11. Eurostoxx 50 level reconciliation

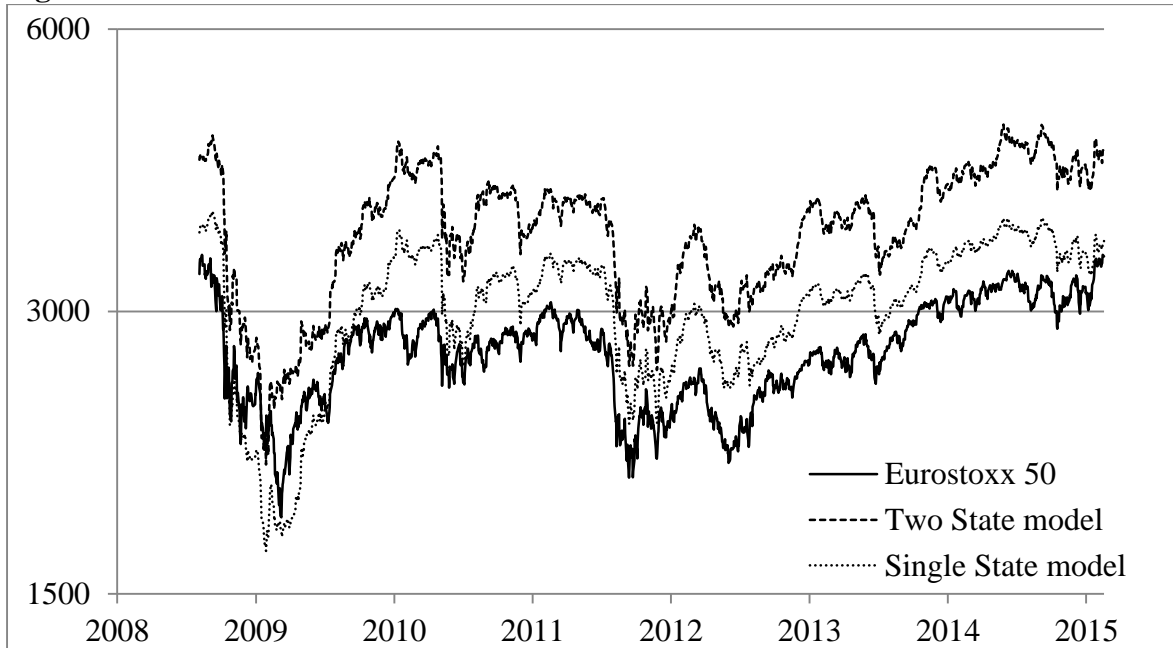


Figure 12. Nikkei 225 level reconciliation

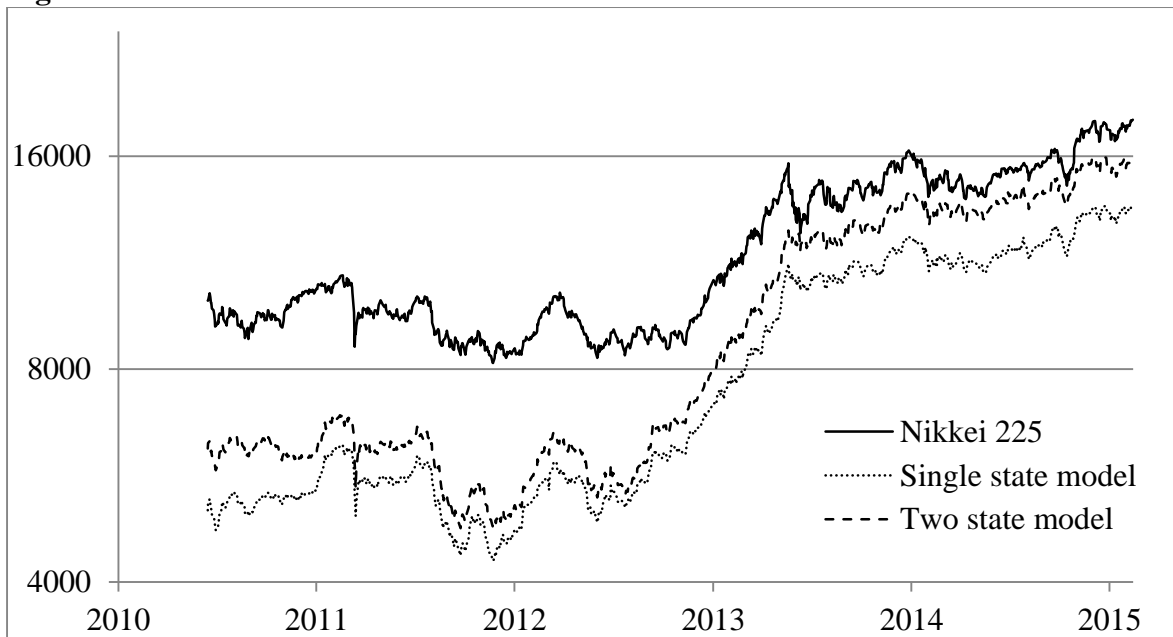


Figure A1. Proportion of dividend payments throughout the Eurostoxx 50 dividend index year. The first trading day of a dividend index year is the Monday following the third Friday of December. The chart depicts the average of the years 2005 to 2013.

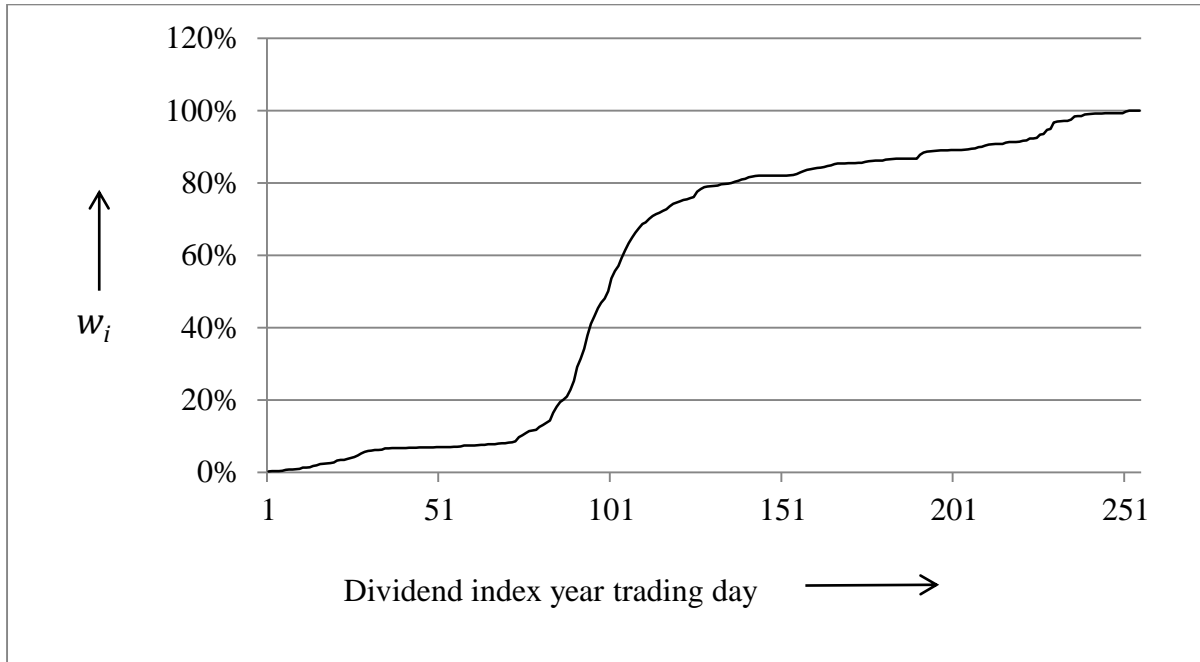


Figure A2. The difference between the proportion of annual dividends paid out at a given date and their average over the period 2005 to 2013 (Eurostoxx 50).

