A Robust Redesign of High School Match

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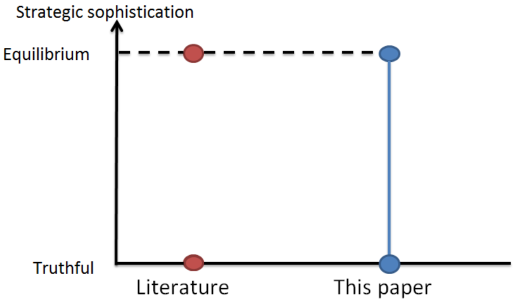
Introduction

- Parents report rankings of schools to get their children assigned to schools
- Assignment policy often not incentive compatible
- Debate: should we switch to incentive compatible policy?
  1. Estimate the distribution of parents' cardinal utilities for schools
  2. Simulate the benefit (efficiency) / cost (inequity) of the policy
- Contribution: relax the assumption about how strategic/unstrategic parents may be



Incentive-incompatible Boston Mechanism (BM) and Deferred Acceptance (DA)

- Algorithm
  - Round 1. Assign as many students as possible to their first choices
  - Round k. Assign as many remaining students as possible to their k^{th} choices
- Why not IC?
  - By the end of Rd. 1, all the good schools will be already filled up.
  - You want to be assigned to some school in round 1
  - Avoid $1^{st}-$ranking low-probability schools; $1^{st}-$rank high-probability, good-enough schools
- I-C alternative: DA

Benefit and Cost of BM

- BM is more efficient than DA when everyone plays equilibrium / has same ordinal preferences
  - Experimetal evidence suggests that 14 to 40% report truthfully
  - Students might have different ordinal preferences
- Truthfully reporting students may be penalized under BM under complete information about other's preferences/lottery number
  - Students are not likely to have complete information
- Is the BM more efficient than DA with heterogeneity in strategic sophistication/ordinal preferences and by how much?
  - Yes, by 0.6 to 3.2 min. of per-capita daily commuting
- Is naivété in BM penalized without complete information?
  - Yes, more likely to be assigned to lesser favorite schools

Literature Review

1. Structural Estimation: Hastings et al. (2009), He (2012), Agarwal and Somaini (2014), Calsamiglia et al. (2014)
   - Strong behavioral assumptions: correctly predict assignment probabilities, fully optimize
2. Theory: Ergin and Sönmez (2006), Kojima (2008), Miralles (2008), Pathak and Sönmez (2008), Haeringer and Klijn (2009), Abdulkadiroğlu et al. (2011), Troyan (2012), Akyol (2013)
3. Experiment: Chen and Sönmez (2006), Pais and Pintér (2008), Calsamiglia et al. (2010)
4. Similar in spirit: Haile and Tamer (2003), Hortaçsu and McAdams (2010)
5. Partially identified model: Romano et al. (2014)

Model

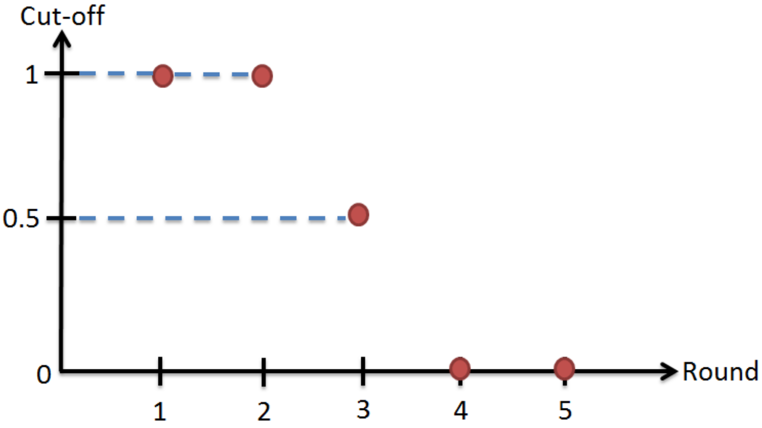
- Measure 1 of students, i
- S: a set of finite number of schools, s
  - $S = \{1, 2, 3\}$
- q_s: capacity of s
  - $q_1 = 0.3$, $q_2 = 0.4$, $q_3 = 0.5$
- $R = \{r_1, r_2, \ldots, r_m\}$: set of all rankings parents can report, r
  - $R = \{(1, 2, 3), (1, 3, 2), (2, 1, 3), \ldots\}$
- B: a set of beliefs about the distribution of rankings reported by parents
  - $b = (0.1, 0.15, 0.2, \ldots)$

The Boston Mechanism

- Equal priority, lottery is drawn from Unif[0,1], lower number is better
- A cut-off for a school in a round: highest (worst) lottery number that guarantees assignment to a school in the round
- Example 1: $q_s = 0.3$, measure of applicants in round $1 = 0.2$
  - Cut-off: 1
- Example 2: $q_s = 0.3$, measure of applicants in round $1 = 0.6$
  - Cut-off: $\frac{0.3}{0.6} = 0.5$

Cut-off Plot

- Critical rounds: the round at which cut-off $\in (0, 1)$
- Critical cutoffs: cut-offs at critical rounds



Cut-off Table

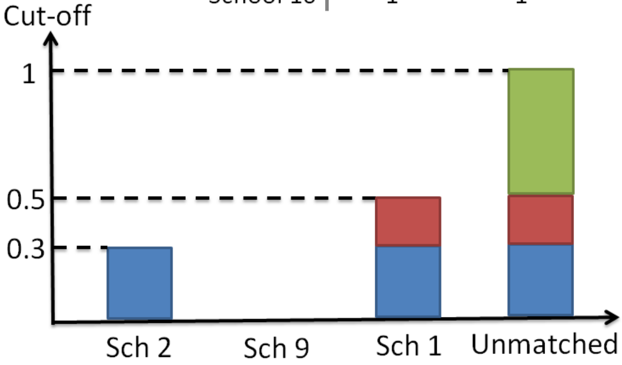
	Round 1	Round 2	Round 3	Round 4	Round 5	...
School 1	1	1	0.5	0	0	...
School 2	0.3	0	0	0	0	...
...	...	...	...	...	...	...
School 9	1	0.2	0	0	0	...
School 10	1	1	1	1	0.4	...

- Previous literature assumes that strategic parents correctly predict the cut-off table

Calculation of Assignment Probabilities

1st	School 2
2nd	School 9
3rd	School 1

	Round 1	Round 2	Round 3
School 1	1	1	0.5
School 2	0.3	0	0
...	...	...	...
School 9	1	0.2	0
School 10	1	1	1



Model of Parents' Decision Process

1. i's vNM utility for school s: $u_{is} = u(x_{is}, \epsilon_{is}; \theta)$
   - Observable on school-student pair x_{is}
   - Unobservable ϵ_{is}
2. Belief about the distribution of ranking reported by all parents $b_i \in B$
3. A reporting strategy σ_i maps utility and belief to a probability distribution over R
   - $\sigma_i^r(\mathbf{u}_i, b_i) > 0$: σ_i "recommends" r to i
4. Draws $(x_{is}, \epsilon_{is}, b_i, \sigma_i)$ from μ

Reporting Strategies

- Truth-telling strategy, fully-optimizing strategy
- Simple strategy: never recommends rankings that violate the simple rule
  - Simple rule: do not rank a school if you do not prefer it to higher-probability schools

	Rd 1	Rd 2	Rd 3
Sch 1	0.3	0	0
Sch 2	1	0.5	0
Sch 3	1	1	1
Sch 4	1	0.7	0

	Sch 1	Sch 2	Sch 3	Sch 4
u_{is}	20	15	10	18

		u_{is}	Cut-offs
1st	School 1	20	0.3
2nd	School 2	15	0.5
3rd	School 3	10	1

NO!

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		u_{is}	Cut-offs
1st	School 1	20	0.3
2nd	School 4	18	0.7
3rd	School 3	10	1

Proposition

Truth-telling/fully-optimizing strategies are simple strategies

Assumptions about Reporting Behavior

1. Everyone correctly predicts the ex-post critical round and the ranking of critical cut-offs

	Rd. 1	Rd. 2	Rd. 3	Rd. 4	Rd. 5
Sch. 1	1	1	0.7	0	0
Sch. 2	1	0.3	0	0	0
Sch. 3	1	1	1	1	1
Sch. 4	1	1	0.4	0	0
Sch. 5	0.2	0	0	0	0

Cut-off table

	Critical Round	Cut-off ranking
Sch. 1	3	4
Sch. 2	2	2
Sch. 3	5	5
Sch. 4	3	3
Sch. 5	1	1

Reduced Cut-off table

2. Everyone uses a simple strategy

Identification

	Critical Round	Ranking of cut-offs	Example
School 1	1	1	0.3
School 2	2	2	0.5
School 3	4	4	1
School 4	2	3	0.7

		Ranking of cut-offs
1st	School 1	1
2nd	School 2	2
3rd	School 3	4

r_1

		Ranking of cut-offs
1st	School 1	1
2nd	School 4	3
3rd	School 3	4

r_2

- If $u_{i2} < u_{i4}$, then $\sigma^{r_1}(\mathbf{u}_i, b_i) = 0$
- If $u_{i2} \geq u_{i4}$, then $\sigma^{r_1}(\mathbf{u}_i, b_i) \in [0, 1]$
- $\sigma^{r_1}(\mathbf{u}_i, b_i) \leq \mathbb{1}\{u_{i2} \geq u_{i4}\}$
- $$\underbrace{\int \sigma^{r_1}(\mathbf{u}_i, b_i) d\mu}_{\text{Prob. parents submit } r_1} \leq \underbrace{\int \mathbb{1}\{u(x_{i2}, \epsilon_{i2}; \theta) \geq u(x_{i4}, \epsilon_{i4}; \theta)\} d\mu}_{\text{Prob. } u_{i2} \geq u_{i4}}$$
- Identified $\theta = \{\theta' : \text{All moment inequalities hold at } \theta'\}$

Parameters To Be Estimated

- $u(x_{is}, \epsilon_{is}; \theta) = x_{is}\beta_i - d_{is} + \epsilon_{is}$
- Observables x_{is}
  1. Quality index, measured by the % of students scoring on or above average at a standardized test
  2. $\mathbb{1}\{\text{science magnet}\}$
  3. $\mathbb{1}\{\text{charter}\}$
  4. $\mathbb{1}\{\text{private}\}$
  5. $\mathbb{1}\{\text{i boy} \cap \text{s boys only}\}$
  6. $\mathbb{1}\{\text{i girl} \cap \text{s girls only}\}$
- Distance d_{is}: minutes spent in commuting from i to s
- Assume $\beta_i \sim N(\mu, \Sigma)$, $\epsilon_{is} \sim N(0, \sigma_\epsilon)$
- Estimate the confidence region of the true $\mu, \Sigma, \sigma_\epsilon$

Computation of Confidence Region

- Romano et al. (2014): Test each $\theta \in \Theta$ whether they should be in the confidence region
- Get two point-estimates under the assumptions that
  1. everyone is truth-telling
  2. everyone plays equilibrium
- Θ : a 28-dimensional interval that contains the two point estimates in its interior
- Draw 4 million points; 9 points pass the test
- Confidence region: smallest 28-dimensional interval that contains the 9 points

95% Confidence Region of the True Parameter

	$\mathbb{E}[\beta_i]$	$\text{Std}[\beta_i]$
Quality Index	$[12.3, 14.0]$	$[9.9, 14.2]$
Science magnet	$[-60.0, -54.1]$	$[3.3, 17.9]$
Charter	$[-9.8, -4.7]$	$[39.3, 44.7]$
Private	$[3.2, 7.0]$	$[29.0, 35.5]$
Boys-only	$[7.0, 9.9]$	$[15.3, 21.4]$
Girls-only	$[-2.9, 1.0]$	$[8.2, 13.3]$
ϵ_{is}	0	$[0.1, 0.14]$

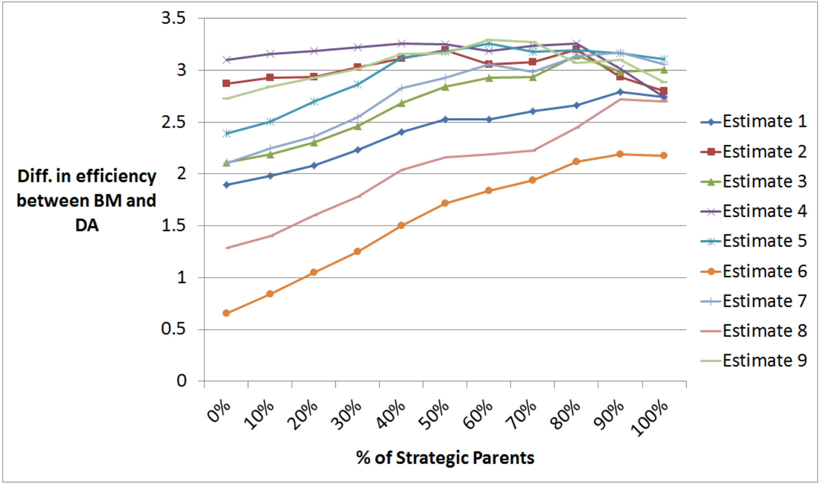
	Science	Charter	Private	Boys	Girls
Quality	[-0.99,-0.6]	[0.4,0.5]	[-0.8,-0.6]	[0.4,0.6]	[-0.3,0.3]
Science		[-0.5,-0.3]	[0.4,0.7]	[-0.6,-0.2]	[-0.2,0.4]
Charter			[-0.7,-0.5]	[0.6,0.7]	[0.3,0.5]
Private				[-0.98,-0.9]	[-0.8,-0.4]
Boys					[0.7,0.9]

Simulation Procedure

- 9 parameter estimates in the confidence region
- For each of the 9 parameter estimates, fix the fraction of naive parents at X%
  1. Draw 80,000 $\mathbf{u_i}$
  2. Randomly choose naive parents from the population
  3. Deferred Acceptance: let everyone report truthfully
  4. The Boston Mechanism
     - Naive parents: report truthfully
     - Strategic parents: best respond to the rest of the parents
  5. Run each mechanism 100 times with different lottery numbers
- Repeat 1 to 5 10 times
- Vary X from 0 to 100%, 10% increment

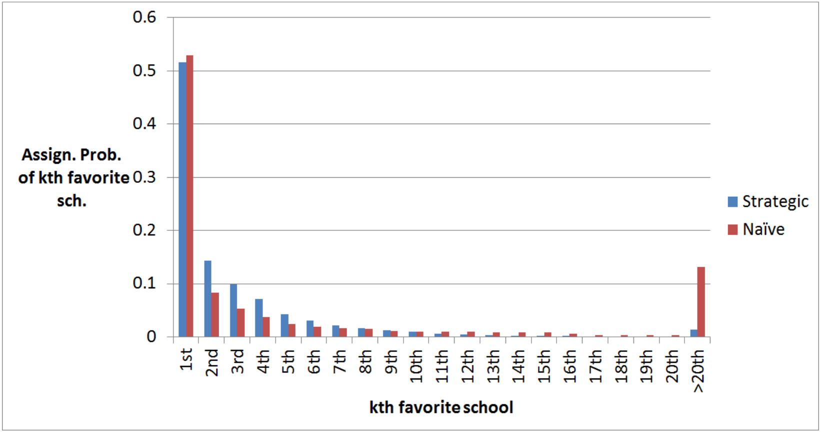
Simulation Results: Efficiency

- Measure of efficiency: $\sum_{i \in I} \frac{1}{|I|}(\text{Expected utility})_i$
- Families under the Boston Mechanism are better off by 0.6 to 3.2 minutes on average than under Deferred Acceptance



Simulation Results: Inequity

- Naive families are more likely to be assigned to lower ranked schools
- Strategic families are better off by 16 to 32 minutes in daily commuting time on average



Conclusion

- Can learn enough about preferences without strong assumptions on the behavior
  - Bounds are "tight", i.e. we got the answer we wanted
- Many decisions to make in designing the mechanism
  - Who gets what priority to which school
  - Tie-breaking scheme
  - The size of the matching market
- Bounds might not be tight enough to be informative for these decisions

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