

Optimal Reserves in Financially Closed Economies*

Olivier Jeanne
Johns Hopkins University

Damiano Sandri
International Monetary Fund**

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Abstract

Financially closed economies insure themselves against current-account shocks with international reserves. We characterize the optimal level of reserves in a simple dynamic welfare-based model that captures this motive. We compare the welfare-based opportunity cost of reserves with the measures that have been used by practitioners. The model is calibrated to a sample of countries with a low level of international financial integration. Under plausible calibrations the model is consistent with the rule of thumb that the average level of reserves should be close to three months of imports. We discuss robust linear rules that capture most of the welfare gains from optimal reserves management.

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**Olivier Jeanne: Department of Economics, Johns Hopkins University, 3400 N.Charles Street, Baltimore MD 21218; email: ojeanne@jhu.edu. Damiano Sandri: International Monetary Fund, 700 19th street NW, Washington DC; email: dsandri@imf.org

1 Introduction

One question of considerable practical relevance for international economists is the appropriate level of international reserves for a small open economy (see IMF, 2011, 2013). In this paper we study this question in the context of a model where reserves play a very simple and basic role of precautionary savings against current account shocks. We study the pure case of an economy that insures against current account shocks with international reserves only. This case is applicable, at least as an approximation, to countries in which the private sector cannot insure itself directly because it has little access to foreign assets or external borrowing. This situation was more prevalent under the Bretton Woods system than today, but it retains some relevance for developing countries with the lowest levels of international financial integration, which are often low-income countries.

We study the question of the optimal level of reserves using an intertemporal optimizing model of an open economy populated by an infinitely-lived representative consumer. The consumer consumes nontradable goods as well as imported goods. The economy is hit by shocks to the value of exports in terms of imports (which might come from shocks to the quantity of exports or to the terms of trade). The consumer does not have access to international financial markets and holds claims against the domestic government. The government holds bonds denominated in foreign currency (reserves). Reserves are foreign assets that are held by the government on behalf of the domestic consumer.

We assume that the government is benevolent and manages the reserves so as to maximize domestic welfare. Reserves allow the country to smooth imports in response to shocks to the value of its exports in terms of imports (for example shocks to the terms of trade). The optimal level of reserves is the optimal level of precautionary savings in response of shocks to export income. This is the type of thinking that underpinned old rules of thumb such as "reserves should cover three months of imports".

We show that in the model the optimal level of reserves tends to converge toward a target that depends on the parameter values. The influence of several parameters is summarized in a key variable, which we call the "carry cost" of reserves. The carry cost is the difference between the real interest rate at which the domestic consumer would be willing to borrow abroad in the deterministic autarkic steady growth path and the imports own-rate of interest. Measuring the carry cost as a spread between two interest rates is

reminiscent of the way that the cost of holding reserves is measured in the non-welfare-based literature on international reserves but there are important differences. In particular, the carry cost is not the same as two measures that are often used by practitioners, the quasi-fiscal cost of holding reserves for the central bank, and the spread between the cost of external borrowing and the return on reserves. The carry cost is a theoretical construct that can be calibrated based on the model and involves a counterfactual interest rate that is not observed in the data.

We then calibrate our model using data on a sample of 21 developing countries from 1960 to 2014. We select the countries that were the least internationally financially integrated and so for which the current account was arguably the main source of shocks. For these countries the average level of reserves was close to 4.5 months of imports. Under our benchmark calibration this average level of reserves is the optimal one if the carry cost is equal to 6.7 percent, which is well in the range of plausible values for this parameter. The model predictions are however sensitive to the carry cost. In particular, the optimal level of reserves goes to infinity as the carry cost goes to zero. This result is not surprising from the point of view of the literature on precautionary savings, but it implies that it may be difficult to determine that an observed level of reserves is "excessive" based on a precautionary savings model. If one assumes that the consumer's discount factor is lower than 1 it is however difficult to rationalize a carry cost under 5 percent for the countries in our sample.

We also investigate the extent to which the gains from optimal reserves management can be reaped using simple linear rules. We show that in our model the optimal policy is well approximated by a linear rule that combines certainty equivalence with a convergence policy whose sole role is to bring reserves back to target. The optimal weight put on convergence is not negligible since it implies that a deviation from the target has a half-life of about three years. The best linear rules are relatively robust to errors in the reserves target and yield more than 90 percent of the welfare gains from optimal reserves management.

Literature. The paper is related to the literature on the optimal level of reserves for an open economy. The idea of a cost-benefit approach to the optimal level of reserves has inspired a long line of literature that goes back at least to Heller (1966). In Heller's analysis the optimal level of reserves was determined in the context of a trade-off between their opportunity cost

and the risk of an external disequilibrium leading to a costly adjustment—a contraction in domestic absorption.¹ More recently calibrated models include Jeanne (2007) or Jeanne and Rancière (2011). These contributions are not based however on precautionary savings models however. Here we compare the implications of a precautionary savings model to the heuristic measures of the benefits and costs of reserves that have been used in the empirical or policy literature (IMF, 2011, 2013).

The paper is also related to the question of how to make foreign assets stationary in stochastic models of the current account. Linearizing a stochastic model of an open economy leads to nonstationarity of foreign assets, in contradiction with the maintained assumption that the economy should stay close to a steady state. Many authors solve this problem by using the assumptions proposed by Schmitt-Grohé and Uribe (2003) to make foreign assets stationary. As noted by Coeurdacier, Rey and Winant (2011) another way of making foreign assets stationary is to solve for the equilibrium with precautionary savings, as we do here. We show that the approximate rules combining certainty equivalence and convergence are similar to the equilibrium policies derived from linearizing models a la Schmitt-Grohé and Uribe (2003), but that the weight that should be put on convergence is significantly higher than in the typical calibrations.

This paper contributes to the literature on precautionary saving in the open economy.² This literature has developed in the recent period to address the challenge of explaining global imbalances and upstream capital flows from developing to advanced economies. Most of this new literature looks at precautionary savings in response to idiosyncratic shocks (Sandri, 2014; Carroll and Jeanne, 2009). Some papers, like this one, look at aggregate shocks (Fogli and Perri, 2015; Kent, 2015). Ghosh and Ostry (1997) show with a CARA-Gauss specification that the greater the uncertainty in national cash flow, defined as output less investment less government expenditure, the greater is the precautionary demand for savings and, other things equal, the larger is the current account surplus.

¹The dynamic aspect of the authorities' optimization problem was treated more rigorously in the buffer stock models of international reserves of Hamada and Ueda (1977) and Frenkel and Jovanovic (1981).

²There is a long line of literature on precautionary savings against income shocks that we draw on (this literature is too vast to be reviewed here, see Heathcote, Storesletten and Violante, 2009 for a review).

2 Model

2.1 Assumptions

The economy is populated by a unitary mass of identical atomistic consumers and a government. The representative consumer maximizes,

$$U_t = E_t \left[\sum \beta^t u(C_t) \right],$$

with

$$u(C) = \frac{C^{1-\gamma} - 1}{1-\gamma},$$

$$C = [\alpha^{1/\eta} M^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} N^{(\eta-1)/\eta}]^{\eta/(\eta-1)}, \quad (1)$$

where M is the consumption of imported good and N is the consumption of nontraded good.

We write the budget constraints in terms of a foreign currency, which for the sake of concreteness we call the dollar. The representative consumer's budget constraint is,

$$\frac{B_t^g}{\mathcal{E}_t} + P_{Mt} M_t = P_{Xt} X_t + \frac{(1+i_{t-1}^g) B_{t-1}^g}{\mathcal{E}_t} + T_t,$$

where X_t and M_t are respectively the quantities of exports and imports, and P_{Xt} and P_{Mt} are dollar prices, B_t^g is government debt expressed in terms of domestic currency and \mathcal{E}_t is the domestic currency per dollar exchange rate, i^g is the domestic currency interest rate and T_t is a transfer from the government. The representative consumer does not have access to foreign assets and can invest only in the liabilities of the domestic government.

The government budget constraint is,

$$\frac{(1+i_{t-1}^g) B_{t-1}^g}{\mathcal{E}_t} + B_t + T_t = \frac{B_t^g}{\mathcal{E}_t} + (1+i_{t-1}) B_{t-1},$$

where B is the amount of dollar reserves, and i is the dollar nominal interest rate.

One should think of the government as including the central bank. The government can engage in "open market" operations in which for example it increases its holdings of reserves and sells domestic bonds to the private

sector. The words "open market" are in quotes because the market is not really open: the domestic bonds must be purchased by residents who do not have access to foreign assets. A sterilized foreign exchange intervention in which the central bank sells government debt (or sterilization bonds) to buy reserves corresponds, in our model, to a simultaneous increase in B_t and B_t^g .

The two budget constraint can be consolidated into the current account balance identity,

$$B_t - B_{t-1} = P_{Xt}X_t - P_{Mt}M_t + i_{t-1}B_{t-1}.$$

There are exogenous processes for the value of exports $P_X X$, the price of imports P_M , the output of nontradable good N , and the dollar nominal interest rate i . The imports M and reserves B are endogenous.

We assume that there is a trend growth factor G in income. We denote in lower case the detrended variables expressed in terms of imports (except for the consumption of nontradable good and total consumption, which are simply detrended):

$$\begin{aligned} m_t &= G^{-t}M_t, \\ x_t &= G^{-t}P_{Xt}X_t/P_{Mt}, \\ n_t &= G^{-t}N_t, \\ b_t &= G^{-t}B_t/P_{Mt}, \\ c_t &= G^{-t}C_t. \end{aligned}$$

The country's aggregate budget constraint can then be written in normalized form,

$$b_t + m_t = w_t + x_t,$$

where

$$w_t = (1 + r_t) \frac{b_{t-1}}{G},$$

is the beginning-of-period stock of reserves, and r_t denotes the imported goods own real rate of interest between period $t - 1$ and period t ,

$$1 + r_t = (1 + i_{t-1}) \frac{P_{Mt-1}}{P_{Mt}}.$$

We assume that the level of reserves must be positive,

$$b_t \geq 0.$$

If the constraint $b_t \geq 0$ is not binding the first-order condition for consumption is,

$$u'(C_t) \frac{\partial C_t}{\partial M_t} = \beta E_t \left[(1 + r_{t+1}) u'(C_{t+1}) \frac{\partial C_{t+1}}{\partial M_{t+1}} \right],$$

or, after detrending,

$$c_t^{1/\eta - \gamma} m_t^{-1/\eta} = \beta G^{-\gamma} E_t \left[(1 + r_{t+1}) c_{t+1}^{1/\eta - \gamma} m_{t+1}^{-1/\eta} \right]. \quad (2)$$

The equilibrium is driven by exogenous stochastic processes for x_t , n_t , and r_t , which are assumed to be stationary and Markov. The state at time t , thus, is summarized by the current values of these variables plus current tradable wealth, $s_t = (x_t, n_t, r_t, w_t)$. The equilibrium is characterized by endogenous policy functions for total consumption, imports and reserves, $c(s_t)$, $m(s_t)$ and $b(s_t)$. The policy functions satisfy (2) and

$$c(s_t) = \left[\alpha^{1/\eta} m(s_t)^{(\eta-1)/\eta} + (1 - \alpha)^{1/\eta} n_t^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

2.2 Target level of reserves

In the long run b_t might tend to fluctuate around a level that could be defined as a “target” for the level of reserves. One way of defining the target level of reserves more formally is to look at the deterministic dynamics of the system that is not perturbed by shocks, i.e., assuming that x_t , n_t , and r_t remain equal to their average levels \bar{x} , \bar{n} , and \bar{r} in the stochastic model. Then one could define the target level of reserves as the level b^* towards which b_t converges. The following proposition gives a condition that is both necessary and sufficient for this target to exist.

Proposition 1 *In the absence of shocks the detrended level of reserves converges towards a finite target b^* if and only if,*

$$1 + \bar{r} < \frac{G^\gamma}{\beta}. \quad (3)$$

If this condition is not satisfied the level of reserves grows without bound.

Proof. See appendix. ■

Condition (3) is the analog of the “impatience condition” in Carroll (2009). To understand this condition it is useful to define the “carry cost” of reserves as,

$$\delta = \frac{G^\gamma}{\beta(1+\bar{r})} - 1. \quad (4)$$

G^γ/β is the real gross interest rate at which the domestic consumer would be willing to borrow abroad at the margin in the steady growth path with no shock. The carry cost of reserves reflects the amount by which this interest rate exceeds the average real return on reserves in terms of imports. The proposition says that for the country to accumulate a bounded level of reserves the carry cost must be positive.

The carry cost is a key variable that summarizes the combined effects of the discount factor, the growth rate and the elasticity of intertemporal substitution of consumption on the consumer’s willingness to borrow against future income. The first-order condition (2) can be rewritten,

$$c_t^{1/\eta-\gamma} m_t^{-1/\eta} = \frac{1}{1+\delta} E_t \left[\frac{1+r_{t+1}}{1+\bar{r}} c_{t+1}^{1/\eta-\gamma} m_{t+1}^{-1/\eta} \right].$$

Thus δ summarizes all we need to know about β and G in the Euler equation. Because G appears in the budget constraint we keep it as a separate parameter but we can treat δ instead of β as an exogenous parameter.

Our definition of the carry cost is reminiscent of how the opportunity cost of reserves is defined and measured in the literature, but there are interesting differences. First, the opportunity cost of reserves is often measured as the quasi-fiscal cost of accumulating reserves for the central bank, measured as the spread between the cost of issuing sterilization bonds (generally denominated in domestic currency) and the return on reserves (Frenkel and Jovanovic, 1981; Flood and Marion, 2001). In the model this quasi-fiscal cost can be written as the difference between the interest rate paid on the domestic debt securities and the return on the reserves,

$$(1+i_t^g) \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - (1+i_t). \quad (5)$$

Note that for consistency the two interest rates must be expressed in dollar which implies that the local currency interest rate must be adjusted for depreciation. To put it differently the valuation gains or losses on the reserves must be taken into account in the quasi-fiscal cost of reserves.

The first-order condition for the consumer's holding of government debt is,

$$c_t^{1/\eta-\gamma} m_t^{-1/\eta} = \beta G^{-\gamma} (1 + i_t^g) E_t \left[\frac{c_{t+1}^{1/\eta-\gamma} m_{t+1}^{-1/\eta} \mathcal{E}_t}{1 + \pi_{t+1} \mathcal{E}_{t+1}} \right]. \quad (6)$$

Assume for a moment that there is no exchange rate uncertainty one period ahead, i.e., \mathcal{E}_{t+1} is known in period t . Then comparing (2) and (6) shows that uncovered interest parity applies,

$$1 + i_t = (1 + i_t^g) \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}. \quad (7)$$

This implies that the quasi-fiscal cost given by (5) is equal to zero. Observe that uncovered interest parity applies not because of arbitrage by private agents (since the market for domestic bonds is completely insulated from the market for foreign bonds) but because of the optimizing behavior of the government. Since the government invests the reserves on behalf of the consumers, it should not distort the intertemporal allocation of consumption relative to the equilibrium with perfect financial integration. The government must manage reserves in such a way that the private sector faces the same interest rate on domestic securities as it would on foreign securities, which implies that there is no quasi-fiscal cost of holding reserves. A positive quasi-fiscal cost arises only if the government accumulates more reserves than the private sector would under perfect financial integration.

This argument does not apply however if there is exchange rate risk. In this case there will in general be a wedge in condition (5) that comes from the exchange rate risk premium. In general the domestic interest rate could be lower or higher than the level implied by uncovered interest parity, and if it is higher there is a quasi-fiscal cost of holding the reserves. There might also be other reasons outside of the model for which the government might be willing to incur a quasi-fiscal cost for holding reserves, for example if depreciating the domestic currency has welfare benefits.³

Irrespective of whether or not the government incurs a quasi-fiscal cost for holding the reserves, this cost is not an appropriate measure of the welfare cost of reserves for the country as a whole. In general the quasi-fiscal cost given by (5) has no reason to be equal to the carry cost δ . The main reason is that the quasi-fiscal cost is a cost for the government but a gain for the private

³See Korinek and Serven (2010), Rabe (2013), Michaud and Rothert (2014).

sector. The logical counterpart of the fact that the government receives a lower return on the reserves than the interest rate it pays on its debt is obviously that the private sector receives a higher return on its assets than if it directly held the reserves. The quasi-fiscal cost, thus, can be viewed as a transfer to the private sector that the government would not have to pay if it simply forced the private sector to hold the reserves. It is not a welfare cost for the country as a whole.

Another measure of the cost of reserves that is often used in the literature is the difference between the interest rate on external debt and the return on reserves.⁴ This does not in general coincide with the carry cost as defined in equation (4) because there is no presumption in general that a credit-constrained borrower pays the interest rate that makes him willing to borrow the constrained amount. For example in many open-economy models a credit-constrained open economy pays the riskless interest rate on its external borrowing because of perfect competition between lenders and the absence of default risk. In this case the difference between the external cost of borrowing and the return on reserves underestimates the true cost of holding reserves. However, the external cost of borrowing may include a default risk premium which in principle should not be included in the carry cost of reserves Jeanne (2007).

To summarize, the carry cost given by (4) is a theoretical construct that is not directly observable using market data because it involves a counterfactual interest rate, the interest rate that would be observed in the autarkic steady-growth path. Our approach in the calibration will be to look for the carry cost that one needs to assume in order to rationalize the observed level of reserves.

A final note of caution is that one should not confuse the *target* level of reserves with the *average* level of reserves. The unconditional average level of reserves based on stochastic simulations, $E(b)$, is in general higher than the target b^* . First, the concavity in the policy function implies that reserves converge towards the target at a faster pace when they are below target than when they are above target. The zero-bound on reserves is another reason for this result. For example, if the target level of reserves is equal to zero the average level will be strictly higher than zero simply because reserves are always at or above target, and never below.

⁴See for example Edwards (1985), Rodrik (2006), Hauner (2006).

3 Calibration

The model is calibrated to a group of developing countries that are affected primarily by current account shocks on the external side because they receive relatively little private capital flows. Our country sample has all the countries in the World Bank’s World Development Indicators (WDI) database such that (i) long-term Public or Publicly-Guaranteed (PPG) debt represents at least 75 percent of their total external debt; (ii) they are not classified as fragile states in IMF (2014); and (iii) they have at least 15 consecutive years of data. The first condition ensures that the countries in our sample have a relatively low exposure to financial account shocks. The second condition was imposed because we found the quality of the data to be suspicious in fragile states, and the last condition ensures that we have enough data to estimate the exogenous stochastic processes. The resulting sample includes 21 countries: Bangladesh, Benin, Botswana, Burkina Faso, Egypt, Ethiopia, Gabon, The Gambia, Kenya, Lesotho, Mauritania, Morocco, Mozambique, Namibia, Nigeria, Pakistan, Panama, Rwanda, Senegal, Sri Lanka, Uganda. The data are annual from 1960 to 2014.

The WDI database provides volume and value indexes for both exports and imports. These indexes give us respectively $\{X_t, M_t\}$ and $\{P_{X_t}X_t, P_{M_t}M_t\}$, conditional on initial values. By dividing import values by import volumes we can infer a price index P_{M_t} that we use to express export values in units of imports, $P_{X_t}X_t/P_{M_t}$. The WDI also provides series for exports and GDP in constant local currency units. Using the identity $GDP_t^r = P_X X_t + P_N N_t$, where GDP_t^r is real GDP in year t and the prices P_X and P_N are constant since real GDP is expressed in constant local currency, we construct an index for nontradable output N_t by subtracting gross real exports from real GDP. We then detrend exports (expressed in units of imports) and nontradable output to obtain indexes that measure the time variations in x_t and n_t .

In order to calibrate the model we need levels (and not only first-differences) for the series x_t and n_t . The model being homogeneous of degree 1 in x_t and n_t we can normalize n_t so that its average value is equal to 1 for each country. The only piece of information that we then need to derive the whole path x_t is the ratio n/x in a given year. For this we divide the identity for nominal GDP, $GDP_t = P_{X_t}X_t + P_{N_t}N_t$ by the nominal value of exports $P_{X_t}X_t$ to obtain,

$$\frac{GDP_t}{P_{X_t}X_t} = 1 + \frac{n_t P_{N_t}}{x_t P_{M_t}}.$$

We assume that the quantity of imported goods is expressed in a unit such that, in a given base year t^* , the prices of the nontradable good and the imported good are equal to each other, $P_{Nt^*} = P_{Mt^*}$. This is without loss of generality since the unit of nontradable good being given, it is always possible to define the unit for the imported good so that this condition is satisfied. We thus obtain the ratio n_{t^*}/x_{t^*} from

$$\frac{n_{t^*}}{x_{t^*}} = \frac{GDP_{t^*}}{P_{Xt^*} X_{t^*}} - 1.$$

Note also that the restriction $P_{Nt^*} = P_{Mt^*}$ can be used to determine the value of α . Since the CES index (1) implies

$$\frac{N_t}{M_t} = \frac{1 - \alpha}{\alpha} \left(\frac{P_{Nt}}{P_{Mt}} \right)^{-\eta},$$

we can express α as a function of the ratio of nominal expenditures on nontradable good to nominal expenditures on imports in the base year,

$$\frac{1 - \alpha}{\alpha} = \frac{N_{t^*}}{M_{t^*}} = \frac{P_{Nt^*} N_{t^*}}{P_{Mt^*} M_{t^*}}.$$

We use the year $t^* = 2005$ for the normalization.

We then estimate AR(1) processes for the series $\{x_t, n_t\}$ for each country in our sample. The table below reports the average autocorrelation coefficient and standard deviation in our country sample. More precisely, the table reports the values of ρ and σ of the AR(1) regression $y_t - \bar{y} = \rho(y_{t-1} - \bar{y}) + \varepsilon_t$ where σ^2 is the variance of ε and y is the column variable. We find that the value of exports in terms of imports is significantly more volatile but slightly less persistent than nontradable output. We also estimate an AR(1) process for the imports real rate of interest, $1 + r_t = (1 + i_t)P_{Mt}/P_{Mt+1}$ where i_t is the one-year eurobond interest rate in U.S. dollars. The imports own rate of interest is equal to 3.6% on average. This interest rate exhibits little persistence and significant volatility because the price of imports is volatile.

Table 1. Time series properties of state variables

	x_t	n_t	r_t
\bar{y}	0.676	1	3.56%
ρ	0.778	0.877	0.186
σ	0.161	0.107	12.9%

The estimated autoregressive processes reported in Table 1 are used to calibrate the model. We approximate each process using the method of Tauchen and Hussey (1991) with five gridpoints.⁵ The other parameters in our benchmark calibration are reported in Table 2. The value for risk aversion ($\gamma = 2$) is standard in the literature. The value of α is the cross-country average of the ratio of nominal expenditures on nontradable good to nominal expenditures on imports in 2005, as explained above. The elasticity of substitution between tradable goods and nontradable goods is set to 1, a value that is standard in the literature. The growth factor is calibrated to the average growth in nontradable output and in the value of exports in terms of imports that we found in the data for our country sample.⁶ The value for the discount factor is within the range considered in the literature for models with growth. The global interest rate were determined by the steady growth path of advanced economies growing at 2% per year, it would be equal to $1.02^2/\beta - 1 = 5.1\%$. The sensitivity of the results to these parameter values will be discussed in the next section. The benchmark calibration implies that the carry cost of reserves is equal to $\delta = 6.72\%$.

Table 2. Benchmark calibration

γ	α	η	G	β
2	0.36	1	1.046	0.99

4 Quantitative Results

We focus first on the target level of reserves and then on the impulse response functions of reserves to shocks in the exogenous variables. We report our results using the reserves-to-imports ratio in months, $\rho_t = 12 * b_t/m_t$, as this

⁵The number of gridpoints is not indifferent because the model predictions depend on the lowest value for x in the grid. The Tauchen-Hussey method implies that the lowest point in the grid decreases with the number of gridpoints, and even becomes negative if the number of gridpoints is too large. With five gridpoints the lowest value of x is 70% below the average level of x under our benchmark calibration. In the data the lowest value of x is 48% below the average level of x on average in our country sample.

⁶The average growth rates in n and x are respectively 4.5% and 5.5% in our sample. In the model these growth rates are assumed to be the same and we set its value to 5%.

One issue is that the growth rate is higher than the interest rate. This cannot be true forever otherwise the country would not have a well-defined intertemporal budget constraint. The implicit assumption, in the rest of the paper, is that the growth rate will fall at a very distant point in the future.

is a standard way of measuring reserves. The average reserves-to-imports ratio, $E(\rho)$, is the average value of ρ taken over five thousand 200-period simulations of the model. The target reserves-to-imports ratio, ρ^* , is the ratio b^*/m^* where b^* and m^* are the long-run levels of reserves and imports respectively in the model without shocks.

4.1 Target and average levels of reserves

Under our benchmark calibration the average level of reserves $E(\rho)$ is equal to 4.6 months of imports and the target level of reserves ρ^* is equal to 3.3 months of imports. The average amount of reserves predicted by the model is quite close to average level of reserves in the data (4.5 months of imports) and the target is close to the 3-months of import rule of thumb. This coincidence is remarkable as we did not use any data about reserves to calibrate the model. Is this a fluke, or do plausible calibrations tend to deliver similar results?

Figure 1 shows how the target and average levels of reserves vary with the carry cost δ , risk aversion γ , the elasticity of substitution η , and the share of imports in consumption α . The average level of reserves is always higher than the target (for the reasons explained at the end of section 2.2).

The most important parameter is the carry cost δ . In Figure 1 we change the carry cost, given the other parameters, by adjusting the discount factor β . The target and average levels of reserves both decreases with the carry cost. The figure shows the variation of reserves when the carry cost remains above 2 percent. The target level of reserves diverges to infinity as the carry cost goes to zero, and already exceeds 15 months of imports when the carry cost is equal to 2 percent. However under our benchmark calibration the carry cost cannot fall below 5.6 percent if the discount factor β stays below 1. For this level of the carry cost the average level of reserves amounts to 6.1 months of imports, and the reserves target to 4.6 months of imports.

We also consider the sensitivity of the target to the risk aversion parameter γ that we vary between 1 and 5, a range of values often considered in the literature. The results differ depending on whether we hold constant the carry-cost δ or the intertemporal discount β . If we keep δ constant by adjusting β , changes in γ capture purely the effect of risk aversion. In this case, higher values of γ make the consumer more willing to smooth consumption against shocks and so increase the desired level of reserves. If instead we did not adjust β , increasing γ would also make the consumer less willing to

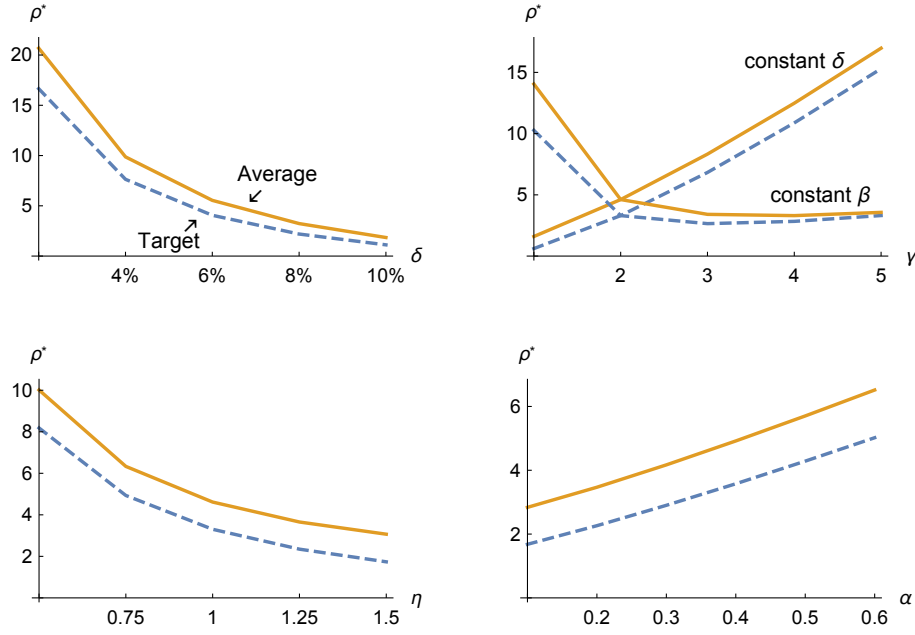


Figure 1: Sensitivity of reserves target ρ^* and average reserves $E[\rho]$.

change consumption intertemporally, which would increase the carry cost of foreign assets and could decrease the target. On the one hand, increasing γ above 2 does not have a significant impact on the optimal level of reserves because the impact of higher risk aversion is more or less offset by the impact of a lower elasticity of intertemporal elasticity. When γ increases the consumer is more willing to hold reserves for insurance but at the same time more willing to borrow against future income and these two effects broadly cancel out. On the other hand, reducing risk aversion below 2 significantly increases the optimal level of reserves: the average level of reserves increases to almost 15 months of imports if γ is equal to 1.

The reserves target decreases with the elasticity of substitution between tradable and nontradable goods η . When the two goods are more substitutable the marginal utility of consumption is less sensitive to the consumption of imported goods, which reduces the benefits of precautionary savings. Finally, raising the share of imports in consumption increases the country's exposure to external shocks and so the desirable level of reserves.

To conclude, the results from our benchmark calibration are robust in the sense that they are not very sensitive to increasing risk aversion above

2 or changing the consumer’s discount rate, as long as it remains positive. However, reducing risk aversion (i.e., increasing the elasticity of intertemporal substitution) significantly increases the optimal level of reserves. If β and γ are both equal to 1 (the combination of values in the plausible set that maximizes the optimal level of reserves), the optimal level of reserves amounts to about 30 months of imports on average.

4.2 Robust rules [sketch]

The policy function for reserve management predicted by the model is non-linear and therefore cannot easily guide the actions of policy makers. In this section, we are interested in solving for simple policy rules that can be more easily followed by practitioners and still deliver most of the welfare gains provided by official reserves. Note first that, as shown in the previous section, the model predicts that reserves should be mostly used in response to shocks x_t . Therefore, we consider the following simple linear rule according to which reserves have to converge towards a target \widehat{b} while buffering export shocks x_t

$$b_t = \frac{1 + r_t}{1 + \bar{r}} b_{t-1} + \lambda (x_t - \bar{x}) + \mu (\widehat{b} - b_{t-1}) \quad (8)$$

Note that this rule is a combination of certainty equivalence and convergence toward a target. If $\mu = 0$ this rule corresponds to certainty equivalence: reserves are increased by a fraction of the deviation of income from its average. In a certainty equivalence model the value of λ would be determined by the parameters and the process for x_t but we leave it as a free parameter. If $\lambda = 0$ this rule stipulates that reserves converge to the target \widehat{b} , where parameter μ captures the speed of convergence.

We then look for the values of λ , μ and \widehat{b} that maximize average welfare based on a large number of simulations.⁷ Under our benchmark calibration we find

$$\widehat{b} = 0.22, \quad \lambda = 0.35, \quad \mu = 0.2.$$

⁷More specifically, we run Montecarlo simulations starting from the stochastic distribution of reserves under the non-linear policy functions. These simulations are conducted assuming that countries follow the linear rule (4.2) for various combinations of the parameters $\{\lambda, \mu, \widehat{b}\}$. Finally, we select the parameters that return the highest average welfare across countries.

The optimal target is a little larger than in the nonlinear policy (0.22 instead of 0.18). Furthermore, the value for λ implies that the country should accumulate 35% of its export income in excess of the average in reserves. Finally, reserves converge to the target relatively quickly: deviations from the target have a half-life of about three years. This is much quicker than the speed of convergence to the target in calibrated version of models à la Schmitt-Grohé and Uribe (2003).

The linear rule performs reasonably well in welfare terms, especially when compared with the solution under no reserves. In particular, we assess the welfare properties of the linear rule as a share of the welfare gain from optimal reserves management. Let us denote by U_{\max} the welfare level when reserves are used optimally, by U_{lin} the average welfare under the linear rule, and by U_{\min} the average level of welfare when reserves are simply set to zero. The welfare loss from the linear rule is computed as the ratio $(U_{\max} - U_{lin}) / (U_{\max} - U_{\min})$ in percentage points.

Under the optimal parameter values for $\{\lambda, \mu, \hat{b}\}$, the loss from using the linear rule is 8.8%, meaning that the best linear rule provides 91.2% of the welfare gains from the best nonlinear rule. Nonetheless, these welfare gains are not negligible. The no-reserves rule is equivalent to a permanent reduction in consumption by 56% relative to the best nonlinear rule. This is much larger than typical estimates of the welfare cost from the business cycle because export income is quite volatile in the countries under consideration.

Figure 2 shows how the welfare loss from the linear rule varies with the parameters. It appears that it is important to set the values of λ and μ at the appropriate levels, and especially not to set them too low. By contrast, the level of the target \hat{b} does not seem to be very important. The linear rule should become more responsive to shocks as the target increases (see Figure 3) but again, λ and μ are not very sensitive to \hat{b} .

Overall, this suggests that the level of the target is much less consequential for welfare than how the government accumulates and decumulates reserves in response to shocks.

5 Conclusion [sketch]

We presented an optimization model to study the optimal level of foreign reserves for countries that use only reserves to smooth export income shocks. The fact that the model is micro-founded has allowed to clarify some points

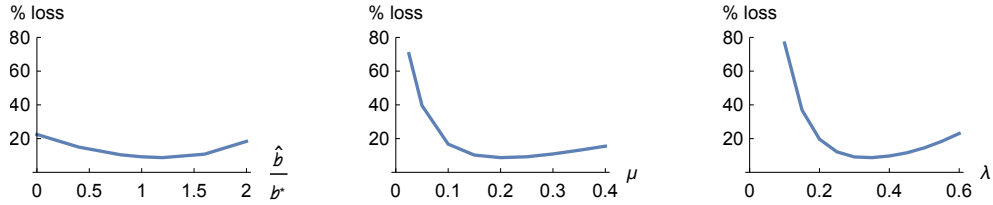


Figure 2: Variation of welfare loss from linear rule with rule parameters.

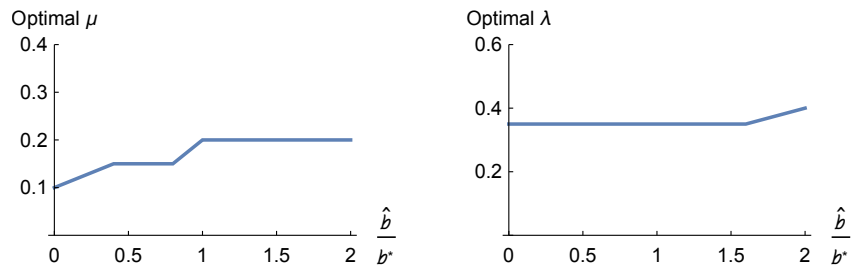


Figure 3: Variation of λ and μ with \hat{b} .

about the welfare cost of reserves. On the quantitative side, we find that plausible calibrations give results for the average level of reserves that are remarkably close to both the average level of reserves in the data and the 3-months-of-imports rule of thumb. However, real-world governments seem to be excessively cautious in their use of reserves. The model suggests that they should use reserves more actively in response to shocks. Furthermore the welfare gains from reserves management come from using the reserves well rather than keeping them at the appropriate level on average.

Several directions of research could be pursued in future work. First, it would be interesting to separate risk aversion from the elasticity of intertemporal substitution of consumption by using Epstein-Zin preferences. Second, one could introduce other shocks such as demand shocks in the discount factor. Finally it would be interesting to explore how the model can be applied to the optimal management of commodity funds and sovereign wealth funds.

6 Appendix

Proof of Proposition 1

TBA [The marginal utility of consumption is bounded so basic contraction theorems should apply.]

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