Convertible bonds and bank risk-taking

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Abstract

We study how contingent capital that converts in equity ahead of default affects bank risk-shifting. Going concern conversion restores equity value in highly levered states, thus reducing heightened risk incentives. In contrast, conversion at default for traditional bail-inable debt has no effect on endogenous risk. The main beneficial effect comes from reduced leverage at conversion. In contrast to traditional convertible debt, equity dilution under going concern conversion has the opposite effect. The negative effect of dilution is tempered by any value transfer at conversion. We find that CoCo capital may be less risky than bail-inable debt when lower priority is compensated by lower endogenous risk, which is beneficial as a lower bond yield improves incentives. The risk reduction effect of CoCo debt depends critically on the informativeness of the trigger, but is always inferior to pure equity.

Keywords: Banks; Contingent Capital; Risk-shifting; Financial Leverage;

JEL Classifications: G13, G21, G28.

1 Introduction

During the recent credit boom, bank capital had fallen at historical lows. In the ensuing crisis, banks could not absorb losses, leading to credit market disruption and spillovers to the real economy. The regulatory response has focused on higher core bank equity to absorb losses and protect depositors. While Basel III core capital ratios require common equity, additional buffers (required for instance for SIFIs, systematically important financial institutions) may include bail-inable and convertible debt to absorb risk. Issuance of CoCo bank debt has risen rapidly to over 200 billion dollar in the last five years. Originally proposed by Flannery (2002) and analyzed more extensively by Kashyap et al. (2008), CoCo debt is seen as a tool to reduce financial distress and deposit insurance losses (Pennacchi, 2011; Pennacchi et al., 2011; Albul et al., 2013; McDonald, 2013). It is well appreciated that bank capital does not just reduce the exogenous risk of default; by increasing "skin in the game", it attenuates incentives for endogenous risk-taking. While all authors argue that contingent capital reduces risk-shifting incentives (Madan and Schoutens, 2010; Bolton and Samama, 2012; Glasserman and Nouri, 2012; Koziol and Lawrenz, 2012; Albul et al., 2013; Hilscher and Raviv, 2014), existing models assume asset risk is exogenous and unaffected by the introduction of CoCo bonds. Our setting contributes to the literature by focusing on their risk prevention effect rather than risk absorption.[1] Our starting point is to assume that CoCos are chosen because equity is not available.[2] Risk-shifting incentives deteriorate as leverage increases. We focus on the case when the risky choice is inefficient as it involves lower return, higher risk assets.

We study how contractual features of bank debt affect the choice of inefficient "endogenous" risk by banks. In particular, we compare the impact on risk incentives of "going-concern" contingent convertible (CoCo) capital, which may convert in equity ahead of distress, with those created by debt converted (or bailed in) upon insolvency, when equity is worthless. We show how debt conversion reduce risk-taking when it forces deleveraging in highly levered states, when risk incentives are strongest. In our approach the trigger is optimally set as to avoid conversion in states when it would not contribute to reduce risk-taking.

Our basic result is that the chance of conversion in high leverage states reduces ex ante risk-shifting. The analysis reveals some surprising effects of CoCo debt's contractual features. The main contribution to endogenous risk reduction comes from debt reduction, a natural direct effect. Our surprising result is that equity dilution caused by going concern conversion does

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[1]A partial exception is Chen et al. (2013), where risk is exogenous, but strategic default shifts losses ex post, an effect reduced by conversion.

[2]Other benefits of CoCos vs equity concerns its lower after tax cost, or its effects on value creation incentives under agency conflicts (Kashyap et al., 2008).

not reduce risk incentives, unlike in Green (1984), but actually enhances them. Traditional convertible bondholders have a long option position and convert only in a favorable outcome, diluting high returns from riskier choices. In contrast, CoCo debt is in the form of reverse convertible bonds, automatically becoming equity when asset values are low and before risk is realized. This feature is essential for their preventive role, as conversion occurs at time when risk choices can still be influenced. Critically, net equity value is positive after a going-concern conversion, so dilution reduces equity returns more for a safer asset choice than an (inefficient) risky choice. Finally, we identify a third effect (extensively discussed in Berg and Kaserer (2014)) which arises because a fixed conversion ratio implies a value transfer from CoCo holders to shareholders in most states, as it is the case in all existing CoCo debt (Berg and Kaserer, 2014). In practice, it is impossible to avoid some value transfer by a contingent conversion ratio, as in some states even infinite dilution would be insufficient. In our setting, this CoCo dilution effect reduces equity dilution and thus reduces risk-shifting incentives.[3]

Another key result is that CoCo debt is better at reducing ex ante risk incentives than conventional bail-inable debt. Conversion at default has no effect, since it occurs in a state when shareholders receive nothing. As CoCo debt has on average a lower priority, it may be expected to command a higher yield. However, we show that as CoCo induces less risky choices, CoCo bonds may carry a lower yield, which also has a beneficial effect on risk incentives.

We next compare the incentive effect of standard CoCo bonds with CoCo debt leading to principal write-down. The net effect depends on the balance between two trade-offs. Write-down CoCo debt should have a higher yield (as at conversion they are worthless), which worsens risk incentives. This is to be traded off against the balance of the equity and CoCo dilution effects, both absent in write-down CoCo debt. [4]

The effectiveness of CoCo conversion critically depends on the precision of the trigger, which should deliver deleveraging whenever risk incentives deteriorate. Equity is equivalent to conversion in all states and therefore always ensures better incentives in low states. A final conclusion is that CoCo debt will fail to control risk in very highly levered banks, for which there is a clear role for direct regulatory intervention.

Finally, our approach has clear implications for pricing. CoCos are usually priced as a package of conventional bonds and a short position in a put option. This leads to underpricing as it neglects their risk-reducing effect, which reduces the value of their short option position.

Our contribution is to study how bank debt contractual features affect risk incentives for a

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[3]Berg and Kaserer (2014) illustrate how CoCo debt dilution creates poor incentives for values near the trigger, looking at the comparative statics in a model with exogenous risk.

[4]We show that if the yield were the same for both types, the principal write-down version induces safer choices.

given initial leverage. As a result, our analysis of the regulatory framework is quite limited. We take initial capital requirements and deposit insurance as given[5], and offer no explicit rationale for an optimal amount of CoCo debt. More frequent and larger CoCo conversions clearly lead to a higher equity content, and in the limit it is equivalent to equity.

In line with the literature, we consider a trigger based on observable asset value. All outstanding CoCo issues are designed to be triggered by accounting value triggers. Regulators have not encouraged market triggers, as they are feared to be manipulable, and may cause multiple pricing equilibria (Sundaresan and Wang, 2015). In practice, both market and accounting measures are imprecise and manipulable. Other approaches to conversion are offered in Duffie (2010) and McDonald (2013), where the case of conversion in a systemic crisis is examined. Glasserman and Nouri (2012) study the interesting case of a continuous conversion feature. Chan and van Wijnbergen (2014) highlight a possible systemic effect of CoCo conversion, which may trigger runs when asset risk is correlated across banks.

Our main conclusion is that CoCo debt reduces risk-taking because of its debt reduction effect, not because of its equity dilution effect. In our set up (as in all outstanding CoCo debt), conversion never results in a value transfer to CoCo holders. Hilscher and Raviv (2014) argue that a very high equity dilution rate at conversion may discourage risk-taking. In practice, converting debt into shares at below book value faces serious legal issues, and is impossible in states when net equity is negative. The introduction of CoCo debt may also be beneficial by inducing timely equity recapitalization, to avoid costly dilution of profits or loss of control, as shown in Pennacchi et al. (2011) and Calomiris and Herring (2013).

There is still limited empirical work on CoCo pricing and their effect on bank risk. Clearly, issuing CoCo debt should reduce the chance of default. Interestingly, Avdjiev et al. (2015) show how CoCo issuance results not just in lower CDS spread, but also appears to have no effect on bank equity. This suggests that issuing CoCo not only absorbs risk, but is also associated with lower expected asset risk.[6]

Section 2 presents the basic setup, and Section 3 shows how CoCo design affect the banker's risk-taking incentives. Section 4 prices different contractual forms and compares the yield among bail-inable debt, write-down CoCos and going concern CoCos. Section 5 compares the risk-reducing effect of CoCos against equity. Section 6 concludes. All proofs are in Appendix.

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[5]Deposit insurance charges would ceteris paribus increase effective leverage and thus risk incentives. Sundaresan and Wang (2014) show how even fairly priced insurance will induce banks to take risk (in their context, via higher leverage).

[6]Avdjiev et al. (2015) suggest that current CoCo prices do not reflect any expected loss at conversion, represented in our setting by the CoCo dilution effect.

2   Model Setup

All agents are risk-neutral. The bank has initial assets of value 1 at $t = 0$, financed with $1 - D$ of equity that satisfies an exogenous capital requirements. Bank debt D may include CoCo bonds with face value C and insured deposits $D - C$.[7] The interim CoCo coupon rate, the deposit rate and the deposit insurance premia are normalized to zero. CoCo debtholders are competitive, so they break-even in expectation. There is one active agent in the model: the banker/bank owner, who chooses the interim level of credit risk.[8] The banker's payoff is the value of the original bank equity at $t = 2$. Borrowers are price-takers.

At $t = 1$, the asset value v is uniformly distributed over $[1 - \delta, 1 + \delta]$, so interim leverage is $\frac{D}{v}$. This is initially observed only by the banker, who then chooses whether to exert risk control effort ($e = 1$) or not ($e = 0$). Effort is costless. We assume that no bank equity may be raised at time $t = 1$. Depending on the banker's choice, asset values at $t = 2$ may be safe or risky. If the banker exercises risk control, assets produce a safe payoff with gross return 1, and the final asset value is v. Alternatively, a risky credit strategy has payoff V_2, with distribution $F(V_2)$ and density $f(V_2)$, mean $\mathbf{E}(V_2) = v - z$, and standard deviation σ. Critically, we assume $z > 0$, so the riskier strategy is a dominated asset that yields less on average than the safer asset.[9] However, when debt exceeds asset values, a risky choice would enhance equity value.

After the risk choice is made at $t = 1$, the value v is revealed with probability φ to all investors. So φ measures the precision of public information.

CoCos are automatically converted into equity when the interim asset value v is revealed to have fallen below a pre-specified trigger level v_T at $t = 1$. The trigger value is initially set lower than the initial book value 1, else there would be immediate conversion at $t = 0$. As we study going-concern conversion, we assume that at $t = 1$ the value of equity is positive upon conversion of CoCo debt, so $v \geq D - C$ for any v. After conversion, the amount of shares is $1 + d$, where d is the amount of shares CoCo holders get upon conversion, equal to the ratio of face value of CoCos over the trigger asset value minus debt: $d = \frac{C}{v_T - D}$. This conversion ratio ensures no value transfer when asset value equals exactly v_T, but leads to a transfer from CoCo holders as soon as v is strictly below v_T.[10]

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[7]We assume that it is mandatory for the bank to issue CoCos.

[8]We assume the bank manager is the sole shareholder, to focus on risk-taking incentives rather than on other agency conflicts between managers and shareholders.

[9]As a result, the distribution of asset return in the safe outcome has second-order dominance relative to risky outcome, though not first-order dominance.

[10]Conversion at par via a contingent conversion ratio is possible only over some range, but in low states even a conversion in an infinite amount of shares would not be sufficient. Full equity dilution ahead of default would in any case be illegal.

At $t = 2$, CoCos repay $C(1 + y)$, where y is initially exogenous (we endogenize the yield in Section 4). To highlight the pure effect of going concern conversion, we assume that at $t = 2$ CoCos act as traditional junior debt, senior to equity.[11]

The current payoff structure is presented in the Figure 1. The trigger value v_T is set by a
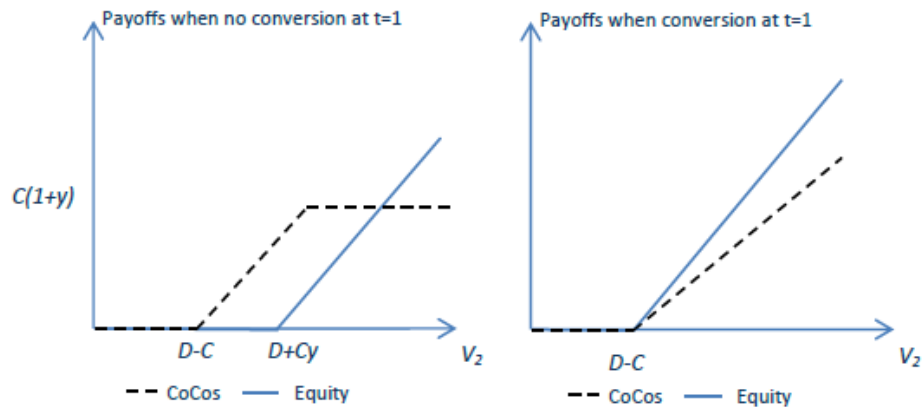
Figure 1: Payoffs in case of no conversion vs early conversion $(d < 1)$

regulator whose objective is to minimize ex ante risk (i.e. the probability that the banker chooses the inefficient risky strategy), while avoiding unnecessary conversions.
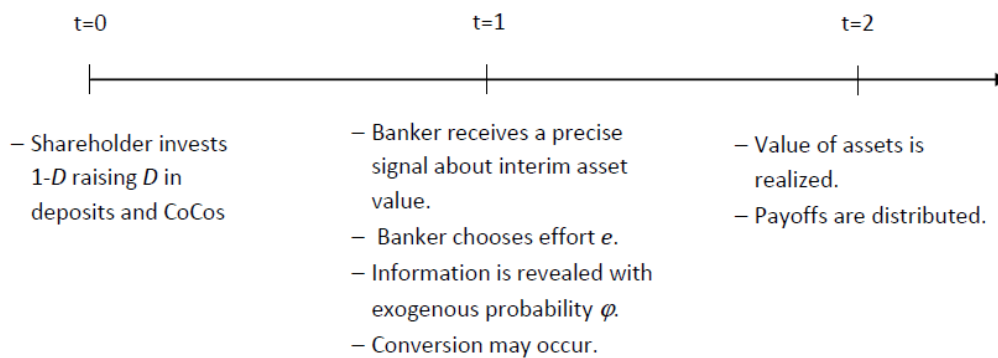
The sequence of events is presented in Figure 2.

Figure 2: The sequence of events

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3 Bank Debt Design and Risk Incentives

The banker makes her risk decision to maximize her expected payoff upon observing v. Her risk incentives can be characterized in terms of the range of values for which risk control is chosen. Under our assumption of going concern conversion $v \geq D - C$, when risk is controlled depositors are always repaid from bank assets, even in default. In contrast, if the banker chooses the risky asset, losses may fully wipe out CoCo debt and force some losses on the deposit insurance fund.

3.1 Baseline Case: Deposits

Consider the baseline case when no CoCos are issued ($C = 0$) and all debt D is represented by deposits.

The expected banker's payoff from a risky asset choice is:

$$(1 - F(D|v)) \cdot \mathbf{E}(V_2 - D|V_2 > D, v) = \int_D^{\infty} (V_2 - D)f(V_2|v)dV_2 \tag{1}$$

which is the sum of its unconditional mean $\mathbf{E}(V_2 - D) = v - z - D$ (which may be negative) and a measure of the right tail return in solvent states:

$$\int_D^{\infty} (V_2 - D)f(V_2|v)dV_2 = v - z - D - \int_{-\infty}^{D} (V_2 - D)f(V_2|v)dV_2 \tag{2}$$
$$= v - z - D + \Delta(v, D, \sigma) \tag{3}$$

where $\Delta(v, D, \sigma)$ is the value of the put option (commonly called Merton's put) enjoyed by shareholders under limited liability. It measures the temptation of the banker to shift risk, defined as the difference in her payoff between a risky and safe strategy:

$$(1 - F(D|v)) \cdot \mathbf{E}(V_2 - D|V_2 > D, v) - (v - D) = -z + \Delta(v, D, \sigma) \tag{4}$$

We henceforth refer to $\Delta(v)$ as the measure of risk-shifting incentives.

Lemma 1. *If the risky asset payoff is normally or uniformly distributed, risk-shifting incentives $\Delta(v)$ are monotonically increasing and convex in leverage $-1 \leq \Delta'(v) \leq 0, \Delta''(v) \geq 0$. Moreover, risk-shifting incentives increase with a higher volatility of risky asset σ.*

Thus, for a normal distribution, risk-shifting incentives are given by:

$$\Delta(v) = (v - D - z) \cdot \left[\Phi \left(\frac{v - D - z}{\sigma} \right) - 1 \right] + \sigma \cdot \phi \left(\frac{v - D - z}{\sigma} \right) \qquad (5)$$

Without any specific assumption on $f(V_2)$, we assume that the risk incentive function has a convex structure in leverage.

Assumption 1. *Risk-shifting incentives $\Delta(v)$ are an increasing and convex function of leverage $\frac{D}{v}$: $-1 \leq \Delta'(v) \leq 0, \Delta''(v) \geq 0$. Also $\Delta(v)$ are increasing with σ: $\Delta'(\sigma) \geq 0$.*
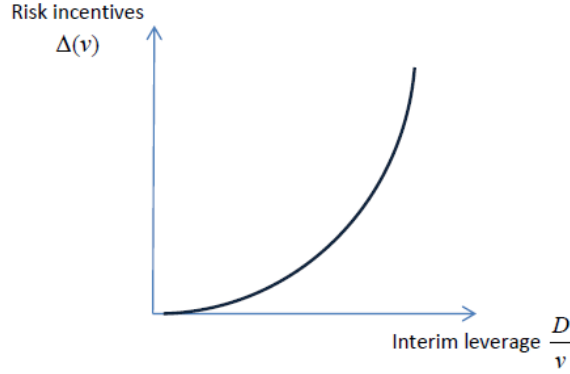

Figure 3: Risk incentives under Gaussian risk distribution

Now consider the risk choice of the banker in the absence of convertible bonds $C = 0$. The banker's program is:

$$\max_e e \cdot \underbrace{\max(v - D, 0)}_{\text{equity value if } e = 1} + (1 - e) \cdot \underbrace{(v - z - D + \Delta(v))}_{\text{equity value if } e = 0}$$

$$\text{s.t. } e \in \{0, 1\} \qquad (6)$$

At $\Delta(v) = z$ the net present value of the the banker's choice of a risky lending strategy is zero. Under the Assumption 1, the banker's choice is:

$$e = \begin{cases} 1 & \text{if } v \geq \max\left[\Delta^{-1}(z), D\right] \equiv v_D^* \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

where v_D^* is the cut-off interim asset value.

Lemma 2. *Under deposit funding, the banker controls risk if interim leverage is sufficiently low, namely if $v \geq v_D^* \equiv \max\left[\Delta^{-1}(z), D\right]$. If $\Delta^{-1}(z) > D$, the ex ante probability of risk control defined as $\frac{1+\delta-v_D^*}{2\delta}$ decreases with the volatility of risky asset σ.*

Note that the lower is the threshold asset value leading to risk control, the more frequent is risk control when incentives are poor. We now use the threshold v_D^* as the benchmark against which other forms of bank debt may be compared in terms of risk incentives.

3.2 Bail-inable debt

In our setting, CoCo debt resembles bail-inable debt at maturity date $t = 2$, unless it converts at $t = 1$. To establish the effect of its convertibility feature on risk incentives, we first derive the benchmark case of pure bail-inable debt.

Let bail-inable debt substitute deposits in the amount of C, promising a yield Cy at $t = 2$. As it is never converted, it is equivalent to CoCo debt when conversion is never triggered (such as when there is never an informative signal $\varphi = 0$).

Then the banker's program is:

$$\max_e e \cdot \max\left(v - D - Cy, 0\right) + (1 - e) \cdot (v - z - D - Cy + \Delta(v - Cy))$$
$$\text{s.t. } e \in \{0, 1\} \tag{8}$$

where $\Delta(v - Cy) = -\int_{-\infty}^{D+Cy} (V_2 - D - Cy)f(V_2|v)dV_2$ is risk-shifting incentive under bail-inable debt funding.

The optimal effort choice by the banker is:

$$e = \begin{cases} 1 & \text{if } v \geq v_D^* + Cy \equiv v_B^* \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where v_B^* is the cut-off interim asset value, above which the banker with deposits of amount $D - C$ and bail-inable debt of amount C controls risk.

Proposition 1. *(Yield effect) Under bail-inable debt funding, the banker controls risk less often than under deposits, since $v \geq v_B^* \equiv v_D^* + Cy$. The ex ante probability of risk control $\frac{1+\delta-v_B^*}{2\delta}$ declines with yield y and the amount of bail-inable debt C.*

A higher amount of bail-inable debt C increases the banker's promised payment to debtholders at $t = 2$, as the debt carries a positive yield y, higher than the zero rate on deposits. The

resulting higher leverage leads to worse risk incentives. This *yield effect* is shown in Figure 4 as the difference between v_B^* and v_D^*.
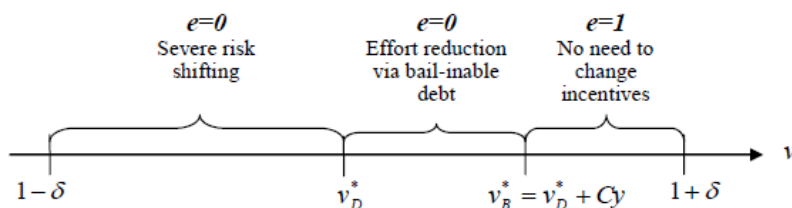

Figure 4: Risk incentives under bail-inable debt

Note that in both cases of deposit or bail-inable debt financing, an interim revelation of v has no effect on risk incentives, as this disclosure does not change leverage.[12]

3.3 CoCo Debt

This section considers the case of CoCos $C > 0$, triggered by a public signal with precision $\varphi > 0$. An efficient design of CoCo debt requires that the trigger threshold improves banker's risk incentives. Intuitively, conversion should be triggered when bank interim leverage is high enough to improve poor risk control incentives, but there should be no conversion for well capitalized banks when it would have no effect.[13]

3.3.1 Trigger value

We first consider the banker's risk decision under a given trigger v_T, and use the result to define the optimal trigger level.

From Proposition 1, setting a trigger asset value higher than v_B^* does not change risk incentives for low leverage banks (defined as those with $v \geq v_B^*$), and it would lead to unnecessary conversion. This implies that the optimal range of trigger values is the interval satisfying $v_T < \min [v_B^*, 1]$.

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[12]As depositors are insured, they would not run at $t = 1$ even if v is low.

[13]We limit attention to single thresholds, excluding the case when conversion may be triggered in a range between two values.

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Consider the banker's program:

$$\max_{e} e \cdot [\underbrace{\max\left(v - D - Cy, 0\right) \cdot \left(I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T)\right)}_{\text{equity value if no conversion and } e=1} +$$

$$\underbrace{\frac{v - D + C}{d + 1} \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion and } e=1}] +$$

$$(1 - e) \cdot [\underbrace{\left(v - z - D - Cy + \Delta(v - Cy)\right) \cdot \left(I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T)\right)}_{\text{equity value if no conversion and } e=0} +$$

$$\underbrace{\frac{v - z - D + C + \Delta(v + C)}{d + 1} \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion and } e=0}]$$

$$\text{s.t. } e \in \{0, 1\} \tag{10}$$

where $I(\cdot)$ is an indicator function, $\Delta(v + C) = -\int_{-\infty}^{D-C}(V_2 - D + C)f(V_2|v)dV_2$ is risk-shifting incentive under CoCos conversion, and d is a conversion ratio.

Figure 5 shows how the effort choice of the banker depends on the trigger, and in particular how it may not be monotonic in the interim asset value.
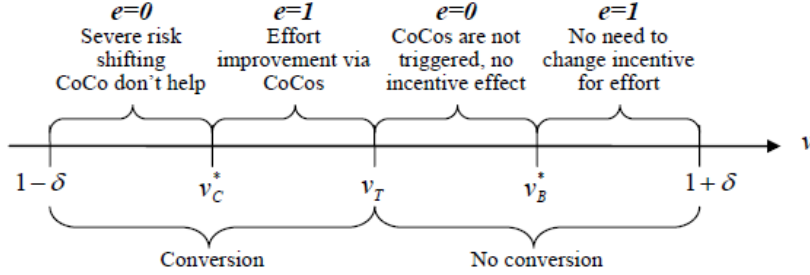
Figure 5: Risk incentives under CoCos

This identifies two critical interim asset values. First, v_B^* is the value threshold at which the banker chooses for risk control even if no conversion may take place, which is equivalent to the risk control threshold for bail-inable debt. Second, v_C^* is the risk control threshold for CoCo debt. This value depends on (11), whether the critical asset value v_C^* is above the threshold $D + Cy$, in which case bank equity is positive even without conversion:

$$\varphi \cdot \frac{z - \Delta[D + C(1 + y)]}{d + 1} + (1 - \varphi)(z - \Delta(D)) < 0 \tag{11}$$

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where v_C^* is defined implicitly in (12):

$$\begin{cases} F = \varphi \cdot \frac{z - \Delta[v+C]}{d+1} + (1-\varphi)[z - \Delta(v - Cy)] = 0 & \text{if (11) holds} \\ G = \varphi \cdot \frac{z - \Delta[v+C]}{d+1} + (1-\varphi)[z - \Delta(v - Cy) - (v - D - Cy)] = 0 & \text{otherwise} \end{cases} \quad (12)$$

Lemma 3. *The introduction of CoCos improves effort for banks relative to traditional debt, in the range of asset values $v_C^* \leq v \leq v_T$. As before, banks with extremely high leverage ($v < v_C^*$) still prefer the risk choice, while high value banks ($v > v_T$) choose to control risk.*

As with other forms of debt, banks with $v < v_C^*$ have such high leverage that they do not control risk.[14] The difference $\frac{v_T - v_C^*}{2\delta}$ measures the expected improvement in risk incentive induced by CoCos. It is easy to see that v_C^* is in the range $[v_D^* - C, v_B^*]$ and decreases with the probability of information revelation φ (see Figure 6).
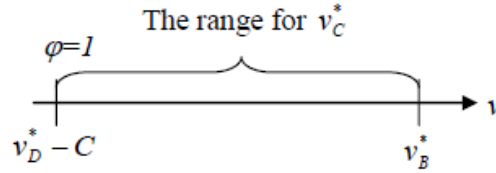

Figure 6: Cut-off value v_C^*

Lemma 4. *CoCo debt improves risk incentives more as the trigger precision φ goes up.*

The intuition is that a more informative trigger forces more frequently a recapitalization in states with high risk incentives.

Having characterized the banker's risk choice response to the trigger, we can define its optimal level.

Proposition 2. *Setting the trigger value at $v_T = v_B^*$ ensures monotonicity of risk incentives and thus maximizes expected risk reduction $\frac{(1+\delta - v_B^*) + (v_T - v_C^*)}{2\delta}$ for a given amount of CoCo debt C (and thus a given conversion ratio d).*

Figure 5 shows that unless the trigger v_T is chosen optimally, risk incentives are not necessarily monotonic in v. If the trigger is too high (above v_B^*), CoCos will not affect effort. If it

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[14]Were CoCo debt large enough ($v_C^* < 1 - \delta$), this range does not arise, and all banks with $v < v_T$ have incentives to contain asset risk.

is too low (below v_B^*), there will be no conversion for an intermediate range of highly levered banks. This is clearly inefficient, as it is easier to induce effort for higher v. As a result, setting the trigger to $v_T = v_B^*$ guarantees the monotonicity of incentives with respect to leverage, as shown in Figure 7.
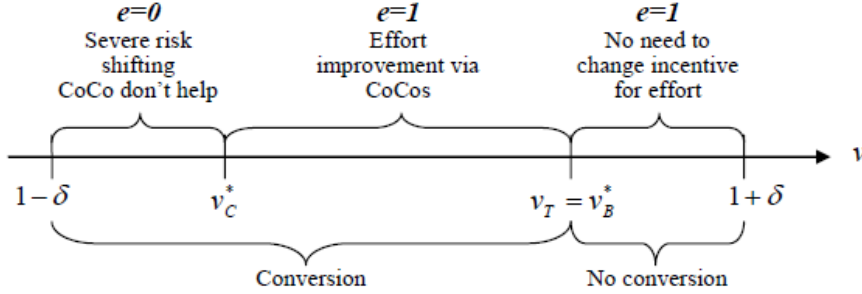

Figure 7: Risk incentives under CoCos with restricted trigger asset value $v_T = v_B^*$

Lemma 5. *Under the optimal trigger, CoCos induce safer choice than bail-inable debt, but do not necessarily have an advantage over deposit funding.*

Since under the optimal trigger, the effort improvement under CoCos against bail-inable debt ranges from 0 (if $v_C^* = v_B^*$) to $\frac{v_B^* - (v_D^* - C)}{2\delta}$ (if $v_C^* = v_D^* - C$), it is evident that CoCos ensure better risk control than bail-inable debt. However, it improves effort relative to deposit funding only if $v_C^* < v_D^*$, which is not always the case. Intuitively, the optimal trigger is affected by the basic CoCos characteristics such as their amount and promised yield.

Lemma 6. *A higher amount of CoCo debt C, or a higher yield y require that the trigger value be raised to maintain risk-shifting incentives.*

As in the case of bail-inable debt, a higher promised yield at maturity Cy increases leverage and thus willingness to take risk. An optimal trigger value here is needed to offset this effect, which leads to a higher equity content. However, it also leads to more frequent conversion and more risk bearing for CoCo debtholders. The next section endogenizes the yield for a general characterization.

Notice that when $\Delta(D) > z$, a higher asset volatility increases the return to risk-shifting, which becomes attractive for a larger range of interim values v. Intuitively, the trigger value should optimally be raised when asset values are riskier in a mean-preserving sense.

Having mapped the banker's risk decision under an optimal trigger, we turn to decompose the effect of different CoCo debt features on risk incentives.

3.4 Specific forms of CoCo debt

We here consider the different possible specifications of CoCo conversion terms.

3.4.1 Write-down CoCos

First, we consider the extreme case when conversion leads to a full write down of principal. When triggered, this type of CoCo debt "disappears". This is equivalent to the case of a zero conversion ratio $d = 0$.

With write down CoCos, the banker gets extra equity when the trigger is breached $v < v_T$, yet keeps all shares. The banker's program is similar to (10) under $d = 0$. As before, there is a critical value v_{WD}^* for the interim asset value above which the introduction of write-down CoCos improves effort (see Figure 8).

Proposition 3 (Debt reduction effect)**.** *Under write-down CoCos with conversion ratio $d = 0$, the banker controls risk if $v \geq v_{WD}^*$. This improves risk-shifting incentives relative to standard CoCo debt, since the cut-off value v_{WD}^* is lower than v_C^* for any positive d.*

Intuitively, when conversion is triggered bank leverage is immediately reduced, encouraging risk control. The risk incentive improvement relative to bail-inable debt represents a pure *debt reduction effect* represented as $\frac{v_B^* - v_{WD}^*}{2\delta}$ in Figure 8.
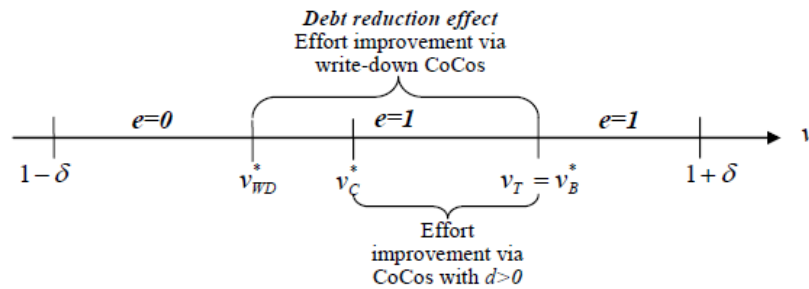
Figure 8: Risk incentives under write-down CoCos. Debt reduction effect.

This form of disappearing debt turns out to give the strongest risk reducing incentive possible associated with convertible debt. A word of caution is here necessary. Our analysis assumes that the banker cannot affect the mean value of v, while in practice a full model would recognize such a possibility. In that case, the banker may have an incentive in some range of v just above the trigger to destroy asset values, in order to trigger the debt write down (Berg and Kaserer,

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2014). This is a violation of the principle that in an optimal contract there should be no incentive for value destruction.

3.4.2 Standard CoCos

Next we establish the benchmark of CoCos converted at par, and then discuss more realistic case of CoCos with the fixed conversion ratio.

Conversion at par

We briefly consider the possibility that the CoCo conversion ratio could be set such that no value transfer would occur upon conversion, i.e $d = \frac{C}{v-D}$. In other words, the CoCo bond payoff upon conversion is exactly equal to its face value C. This feature is impossible in practice for very low v, but it is considered here as it sets a useful benchmark.

The banker's program and its solution are equivalent to (10) as well as (11) and (12) respectively under $d = \frac{C}{v-D}$. There is a critical value v_P^*, the interim asset value above which the introduction of at par CoCos improves effort.

Proposition 4 (Equity dilution effect). *Under CoCos converted at par with conversion ratio $d = \frac{C}{v-D}$, the banker controls risk if $v \geq v_P^*$. Risk control incentives are lower than for write-down CoCos, since the cut-off value v_P^* is higher than v_{WD}^*.*

Higher conversion ratio d implies higher equity dilution (the banker gets $\frac{v-D}{v-D+C} < 1$ of the total equity upon conversion). Since upon conversion net equity value is still positive $v > D - C$, a greater dilution reduces the net payoff to safe asset choice thus discouraging risk control. In other words, such a conversion increases the cut-off value v_C^* and thus enhances risk-taking incentives. We call the banker's effort reduction from at par CoCos relative to write-down Cocos an *equity dilution effect* presented as $\frac{v_P^* - v_{WD}^*}{2\delta}$ in Figure 9.

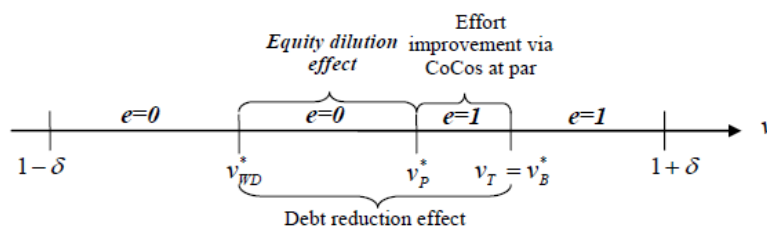Figure 9: Risk incentives under at par CoCos. Equity dilution effect.

Conversion with CoCo debt dilution

Conversion at par is not possible in practice. We consider now the realistic case of going concern CoCos with fixed conversion ratio $d = \frac{C}{v_T - D}$, a design feature common to all outstanding CoCo bonds.

The banker's program and its solution are equivalent to (10) as well as (11) and (12) respectively under $d = \frac{C}{v_T - D}$. There is a critical value v_F^* above which the introduction of standard CoCo debt improves risk control.

Proposition 5 (CoCo dilution effect). *Under CoCos with fixed conversion ratio $d = \frac{C}{v_T - D}$, the banker controls risk if $v \geq v_F^*$. Risk control incentives improve compared to CoCos converted at par, since the cut-off value v_F^* is lower than v_P^*.*

This result identifies the CoCo dilution effect, driven by the value transfer from CoCo debt to equity upon conversion. Since the fixed conversion ratio is lower than the par conversion ratio $\frac{C}{v_T - D} < \frac{C}{v - D}$, upon conversion holders of CoCo debt get less than the promised face value. As a result, the equity dilution effect is reduced, and the safe asset payoff increases relative to the risky one. Based on the result in the previous section, we can conclude that this produces higher risk control incentives. We can call the banker's effort improvement from standard CoCos relative to at par CoCos a pure *CoCo dilution effect* presented as $\frac{v_F^* - v_P^*}{2\delta}$ in Figure 10.
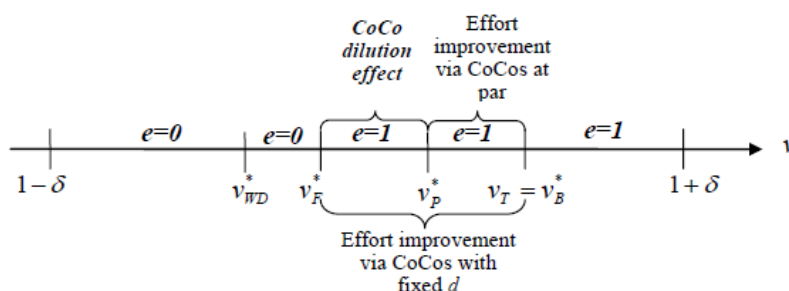
Figure 10: Risk incentives under CoCos with fixed conversion ratio. CoCo dilution effect.

Note that fixed conversion ratio depends on the trigger value.

Lemma 7. *The CoCo dilution effect increases with the trigger value v_T, so CoCo debt with a higher trigger produces (ceteris paribus) greater risk reduction incentives.*

Note that because of the CoCo dilution effect, a high trigger reduces risk-shifting incentives beyond the direct effect of their higher equity content. The key reason is that going-concern conversion ensures that equity value is positive under a safe strategy. Since risk control dominates the risky strategy, this improves the banker's incentives.

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Table 1 summarizes the effects of CoCos on the banker's risk-taking incentives under different formulations.

Effect	Bail-inable Debt	Write-down	Standard CoCos	Impact on Risk
Debt reduction	No	Yes	Yes	Risk Reduction
Equity dilution	No	No	Yes	Risk Enhancement
CoCo dilution	No	No	Yes	Risk Reduction
Yield	Yes	Yes	Yes	Risk Enhancement

Table 1: Effects of debt instruments on bank risk incentives

4   Endogenous Yield

In this section, we solve for the endogenous yield across debt types, necessary for any general calibration and ultimately for our policy conclusions. Clearly, the yield depends both on the induced endogenous risk and the relative priority of the different forms of bank debt.

Bail-inable debt

Consider possible payoffs under bail-inable debt (those are shown in the upper panels of Figure 11). Under a safe choice debtholders are repaid with $C(1+y)$ if there is enough funds $v > D + Cy$. If interim leverage is low $v < D + Cy$, then debtholders get what is left after deposits are repaid $v - (D - C)$. Under the risky choice, their payoff depends on the realization of V_2.

We showed earlier that bail-inable debt induces a safe choice when interim asset values $v \geq v_B^* = v_D^* + Cy$. Thus, debtholders expect to get safe payoff with probability $\frac{1+\delta-v_B^*}{2\delta}$, and risky payoff otherwise. Since bail-inable debt does not repay C in all states, the promised yield must be positive for debtholders to break-even.

Note that for any types of CoCos, the payoff structure of CoCo debt holders is the same as for the holders of bail-inable debt if no conversion is triggered. However, the distribution of these payoffs is different (the probability of safe return is higher).

Write-down CoCos

Next consider the case when the banker raises C of write-down CoCos and $D - C$ in deposits. If conversion is not triggered, holders of write-down CoCos get the same payoff as bail-inable debt holders under safe and risky strategies (as shown in the upper panels of Figure 11). Holders of write-down CoCo debt get nothing if conversion is triggered, independent of the strategy of the banker. Figure 8 shows that the banker chooses to control risk when $v \geq v_{WD}^*$. Two
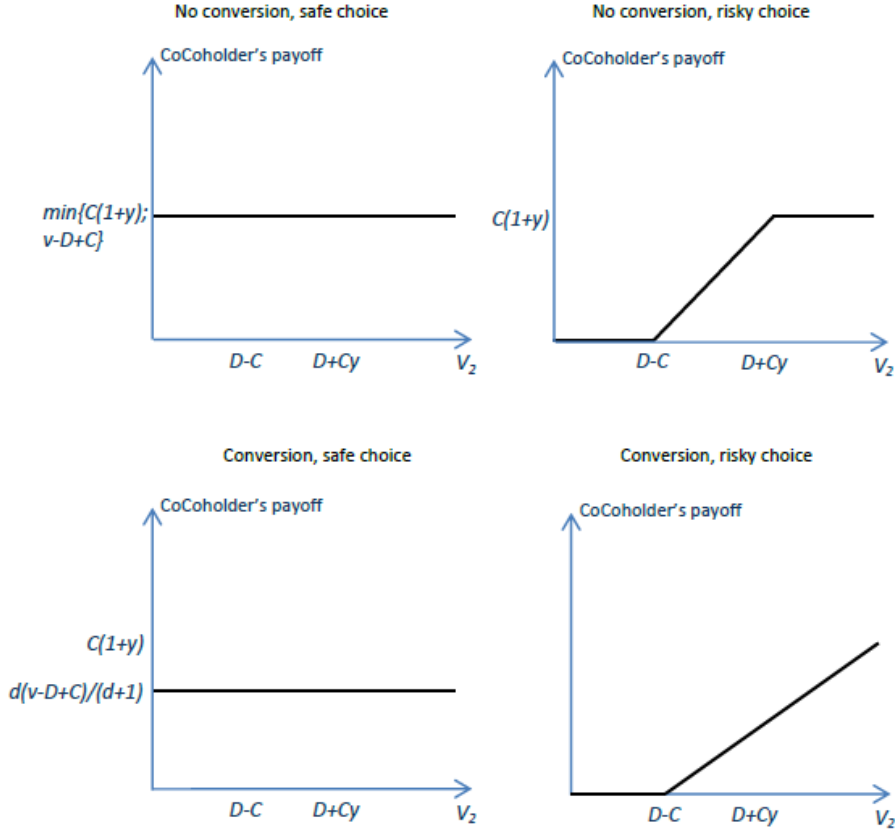
Figure 11: Payoff of CoCo debt under different banker's choice

differences between bail-inable debt and write-down CoCos affect their yields. First, in the states when conversion is triggered, write-down CoCos offer no payment even if the bank is solvent, reducing the payoff to CoCo holders. Second, write-down CoCos discourage risk, thus enhancing the probability of repayment when the interim value is not revealed. The chance of getting certain payoff $\min \{C(1 + y), v - D + C\}$ is higher in case of write-down CoCos.

Intuitively, these depend on the trigger value v_T and its precision φ. A higher trigger as well as a higher precision increases the yield to compensate for the CoCoholders' losses in conversion. At the same time, they also induce a safer banker's choice, and thus reduce the required yield.

Standard CoCo debt

We now consider the case of standard CoCos with the fixed conversion ratio.

If conversion is triggered, holders of CoCo debt receive a share of equity $\frac{d}{d+1}$. In the case of safe asset choice when $v_F^* \leq v < v_B^*$, they get a fraction of the safe payoff $\frac{d}{d+1} \cdot (v - D + C)$.

In case of a risky choice when $v < v_F^*$, their payoff depends on the asset realization at $t = 2$. If the CoCos are not converted, holders of standard CoCo debt get the same payoff as bail-inable debt holders under safe and risky strategies. Those payoffs are shown in Figure 11. Recall that risk control is chosen for $v \geq v_F^*$ (as shown in Figure 10).

There are two critical differences between bail-inable debt and standard CoCo bonds that affect their yields. First, CoCo bonds face less protection when converted than bail-inable debt. Second, CoCos induce a safer asset choice, increasing the probability of some repayment either in the form of equity or debt (for $v_F^* \leq v < v_B^*$).

Comparing risk across CoCo debt types

Finally, we compare write-down CoCo and standard CoCo debt. Again, there are two effects. Intuitively, write-down CoCos should be more expensive, since conversion leads to a complete loss of value. Higher yield itself may harm risk incentives (and will require a higher trigger). At the same time, write-down CoCo debt induces less risk than standard CoCos because of the absence of any equity dilution effect.

The comparison of the payoffs structure together with the endogenous risk choice is shown in Table 2[15]. The formulas for implicitly given yields of these three types of debt are given in the Appendix.

Debt type/ Range of v	$v < v_{WD}^*$	$v_{WD}^* \leq v < v_F^*$	$v_F^* \leq v < v_B^*$	$v \geq v_B^*$
Bail-inable debt	Risk/No Conv	Risk/No Conv	Risk/No Conv	Safe/No Conv
Write-down CoCos	Risk/Conv wp φ	Safe/Conv wp φ	Safe/Conv wp φ	Safe/No Conv
Standard CoCos	Risk/Conv wp φ	Risk/Conv wp φ	Safe/Conv wp φ	Safe/No Conv

Table 2: Payoff structure and endogenous risk of debt instruments

In conclusion, there are two key elements affecting the yield on these forms of bank debt, aside from their contractual priority. First, the induced amount of endogenous risk. Second, the resulting amount of risk bearing by the deposit insurance fund. The specific ranking of required yields depends on specific bank characteristics. Our analysis offers a framework to properly interpret and assess their relative effect on endogenous risk-shifting, although it may require a careful calibration.

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[15]"Risk" indicates that the banker makes risky choice, "Safe" indicates safe choice, "Conv wp φ" means that conversion takes place with probability φ, "No Conv" means that no conversion takes place.

5 Contingent Capital versus Equity

In this section, we compare the risk reduction effect of standard CoCo debt vs equity. This extension adopts our basic framework, assuming an exogenous yield y on standard CoCo debt.

What amount of contingent capital is required to substitute equity, to provide the same effort incentives?

Suppose the bank substitutes one unit of deposits by an extra amount of equity ϵ, or by an amount $k\epsilon$ of CoCos. We solve for the level of k which guarantees an equivalent improvement in risk incentives as with equity.[16]

The banker chooses effort according to the schedule:

$$
e = \begin{cases} 1 & \text{if } v \geq v_D^* - \epsilon \\ 0 & \text{if } v < v_D^* - \epsilon \end{cases} \tag{13}
$$

The expected improvement in effort compared to basic model with deposits as the the only source of debt funding is $\frac{\epsilon}{2\delta}$, which reflects the increased range of asset values for which there are improved risk incentives. From earlier results, the improvement in effort achieved by CoCos is $\frac{v_D^* - v_C^*}{2\delta}$, where v_C^* is the function of $k\epsilon$ instead of C.[17]

So the condition $v_D^* - v_C^* = \epsilon$ guarantees that the expected improvement in effort from introducing extra equity ϵ and CoCos $k\epsilon$ is the same.[18]

Proposition 6. *The effect of CoCos on effort is in general weaker than of equity, unless the trigger is perfectly informative ($\varphi = 1$).*

Lemma 8. *The substitution ratio k between extra equity and CoCos k decreases in a convex way with the probability of information revelation φ if promised CoCo yield is sufficiently low. Conditions for decreasing k function are (11) is satisfied and sufficiently low yield from:*

$$
\begin{aligned}
y\Delta_k'[v_D^* - \epsilon(1+ky)] \geq\ & \frac{-\varphi \cdot (v_D^* - D)}{(1-\varphi)(k\epsilon + v_D^* - D)^2} \cdot \{(k\epsilon + v_D^* - D) \cdot \Delta_k'[v_D^* + \epsilon(k-1)] \\
& +\ (z - \Delta[v_D^* + \epsilon(k-1)])\}
\end{aligned} \tag{14}
$$

---

[16]Note that after adding extra equity ϵ, the bank has debt $D - \epsilon$, so the amount of equity in the interim stage is $v - D + \epsilon$. The bank operates with lower leverage.

[17]Note that for some parameter values $v_C^* > v_D^*$ implying that equity is superior in enhancing risk control.

[18]As before, we set the trigger value to insure monotonic incentives in v, $v_T = v_B^*$.

and for its convexity - no condition (11) is satisfied and sufficiently low yield from:

$$
\begin{aligned}
y[\Delta_k'(v_D^* - \epsilon(1 + ky)) + 1] \;\leq\; & -\frac{(v_D^* - D)}{(k\epsilon + v_D^* - D)^2} \cdot \{\Delta_k'[v_D^* + \epsilon(k - 1)] \cdot (k\epsilon + v_D^* - D) \\
& + \; z - \Delta[v_D^* + \epsilon(k - 1)]\}
\end{aligned}
\tag{15}
$$

The equivalence ratio is very sensitive to φ. As φ approaches zero, there is no amount of CoCos that can substitute equity to ensure the same risk incentives. The inefficiency comes from the yield effect, i.e the banker has low incentives to control risk, i.e v_C^* can be even higher than v_D^*.

The key efficiency factor for CoCos depends on the precision of the trigger to signal a state where incentives are poor, relatively to equity which is always risk bearing. When the trigger is less precise, conversion takes less often when required. As a result, a larger amount of CoCos must be used.

However, when yield is sufficiently high, increasing the amount of CoCos becomes counter-productive, since this implies higher repayment at maturity encouraging higher leverage and thus higher risk-taking.

6 Conclusions

This paper assesses how the design of bank contingent capital affects risk incentives. The issue is extensively discussed in the literature, but in existing models the asset choice is exogenous. Many contributions have looked at the effect on risk incentives as a comparative statics, such as Pennacchi (2011), Chen et al. (2013) and Hilscher and Raviv (2014).

Our contribution is to study explicitly contingent capital's effect on bank risk choices, a necessary feature for its optimal design and pricing. The framework allows to decompose the incentive effects associated with debt characteristics. A clear result is that CoCo debt is superior to subordinated debt that may be bailed in upon default, as it actively discourages ex ante risk choices. It helps clarify the importance of going concern conversion, as it restores equity while the bank is still solvent but may choose to increase risk to favor highly leveraged equityholders.

The main beneficial effect of conversion is to contain risk-shifting by reducing leverage in states when incentives are poor. In contrast to the existing literature on conventional convertible debt (Green, 1984), we show that equity dilution has a negative effect on incentives.

The framework enables to assess the relative effectiveness of CoCos in risk reduction versus common equity, as well as other bank debt. A one for one exchange ratio of CoCos for equity is equivalent in terms of loss absorption upon default. But once the risk prevention effect is taken into account, even optimally designed contingent capital is much less efficient than equity because of limited trigger precision, which does not ensure recapitalization in all states of excessive leverage.

In current research, we plan to study how using an accounting trigger may result in regulatory forbearance, since bank reporting typically involves regulatory oversight. Future research should focus on better understanding the effect of CoCos on share pricing, which is distorted by risk-shifting. Share prices increase with bank risk when leverage is high (which may explain why Lehman shares peaked just a year before its default). As shareholders' returns drop on conversion, this creates multiple equilibria. This tendency of the share price to fall towards the trigger level once it comes in its neighborhood, is inappropriately named "death spiral". Yet it comes from the corrective effect of CoCo conversion on an underlying distortion (i.e, risk-shifting), not from a distortion it introduces.

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7   Appendix

Lemma 1

We consider two possible distribution of the asset value: normal and uniform.

In the first case let $x = V_2 - D$ be normally distributed with mean is $v - D - z$ and variance σ^2. We refer to x as the difference between the value of assets and debt.

In the second case let $x = V_2 - D$ be uniformly distributed with support $[v - D - z - \sigma\sqrt{3}, v - D - z + \sigma\sqrt{3}]$, so that mean is $v - D - z$ and variance is σ^2. We assume that the highest possible equity value when the bank takes the risky asset is positive, $v - D - z + \sigma\sqrt{3} \geq 0$. Otherwise, risky asset is never chosen. Moreover, the lowest possible capital value is negative $v - D - z - \sigma\sqrt{3} \leq 0$, else no risk-shifting takes place.

The expected value of bank equity is the expected value of assets minus debt conditional on being solvent, multiplied by the probability of being solvent.

$$(1 - F(0, v)) \cdot \mathbf{E}(x | x > 0, v)$$

For a normal distribution, it is:

$$\left(1 - \Phi\left(\frac{-(v - D - z)}{\sigma}\right)\right) \cdot \frac{\int_0^\infty x \cdot \frac{1}{\sigma} \cdot \phi(\frac{x - (v - D - z)}{\sigma}) dx}{1 - \Phi(\frac{-(v - D - z)}{\sigma})} =$$

$$\int_0^\infty x \cdot \frac{1}{\sigma} \cdot \phi(\frac{x - (v - D - z)}{\sigma}) dx =$$

$$(v - D - z) \cdot \Phi\left(\frac{v - D - z}{\sigma}\right) + \sigma \cdot \phi\left(\frac{(v - D - z)}{\sigma}\right)$$

For a uniform distribution:

$$(1 - F(0, v)) \cdot \mathbf{E}(x | x > 0, v) = \int_0^\infty x \cdot \frac{1}{2\sigma\sqrt{3}} dx = \frac{(v - D - z + \sigma\sqrt{3})^2}{4\sigma\sqrt{3}}$$

The expected value of equity in the case of risky asset is by definition the sum of unconditional mean of the value of asset minus debt $v - D - z$ and the risk-shifting incentive $\Delta(v)$ (the put option enjoyed by shareholders). Normal distribution:

$$\Delta(v) = (1 - F(0, v)) \cdot \mathbf{E}(x | x > 0, v) - (v - D - z) =$$
$$(v - D - z) \cdot \left[\Phi\left(\frac{v - D - z}{\sigma}\right) - 1\right] + \sigma \cdot \phi\left(\frac{(v - D - z)}{\sigma}\right)$$

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Uniform distribution:

$$\Delta(v) = (1 - F(0, v)) \cdot \mathbf{E}(x|x > 0, v) - (v - D - z) = \frac{(v - D - z - \sigma\sqrt{3})^2}{4\sigma\sqrt{3}}$$

Consider now how the risk-shifting incentive changes with interim asset value v. It is easy to show that under these distributions the derivative of the risk-shifting incentive function with respect to v is negative. Normal distribution:

$$\frac{\partial \Delta(v)}{\partial v} = \Phi\left(\frac{v - D - z}{\sigma}\right) - 1 \leq 0$$

Note also that for normal distribution, $\frac{\partial \Delta(v)}{\partial v} \geq -1$, since $\Phi\left(\frac{v-D-z}{\sigma}\right) \in [0, 1]$. Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial v} = \frac{2(v - D - z - \sigma\sqrt{3})}{4\sigma\sqrt{3}} \leq 0$$

Note also that for uniform distribution, $\frac{\partial \Delta(v)}{\partial v} \geq -1$, since $2(v - D - z + \sigma\sqrt{3}) > 0$. Thus, the risk-shifting incentive decrease with asset value v, or capital $v - D$.

The second derivative of function $\Delta(v)$ with respect to v is positive. Normal distribution:

$$\frac{\partial^2 \Delta(v)}{\partial v^2} = \phi\left(\frac{(v - D - z)}{\sigma}\right) \cdot \frac{1}{\sigma} \geq 0$$

Uniform distribution:

$$\frac{\partial^2 \Delta(v)}{\partial v^2} = \frac{1}{2\sigma\sqrt{3}} \geq 0$$

Thus, risk-shifting incentives fall in a convex fashion with bank capital $v - D$.

Next, we look at how risk incentives change when volatility of risky asset grows. The derivative of risk-shifting function with respect to σ is positive.

Normal distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = \phi\left(\frac{(v - D - z)}{\sigma}\right) \geq 0$$

Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = -\frac{(v - D - z - \sigma\sqrt{3}) \cdot (v - D - z + \sigma\sqrt{3})}{4\sqrt{3} \cdot \sigma^2} \geq 0$$

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Thus, the risk-shifting incentives increase with volatility of the risky asset.

And finally, we find the effect of difference in means of payoffs from safe and risky assets z on the risk-shifting incentives. The derivative of risk-shifting function with respect to z is:
Normal distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\left[\Phi\left(\frac{v - D - z}{\sigma}\right) - 1\right] \geq 0$$

Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\frac{2(v - D - z - \sigma\sqrt{3})}{4\sigma\sqrt{3}} \geq 0$$

So, higher z leads to higher risk-shifting incentives.

Lemma 2

First, we show that the banker with $v > v_D^*$ exerts effort. When solving the problem (6), there are two cases: (1) $v > D$ (positive equity upon safe choice); (2) $v \leq D$.

If $v > D$, $e = 1$ if $z > \Delta(v)$. According to the Assumption 1, $\Delta(v)$ is decreasing in v. Then $\Delta(v) \leq z$ implies that $e = 1$ if $v \geq \Delta^{-1}(z)$, which is binding if $\Delta^{-1}(z) > D$. If $v \leq D$, for any v $e = 0$.

Thus, the solution to (6) depends on the relationship between D and $\Delta^{-1}(z)$. If $\Delta(D) > z$, $\Delta^{-1}(z) > D$ and the banker exerts effort for $v > \Delta^{-1}(z)$. Otherwise, if $\Delta(D) \leq z$, $\Delta^{-1}(z) \leq D$ and the banker exerts effort for $v \geq D$. Thus, $e = 1$ if $v > \max\left[\Delta^{-1}(z), D\right] \equiv v_D^*$.

Next, we show that if $\Delta(D) > z$ (implying that $\Delta^{-1}(z) > D$), the probability of risk control ($\frac{1+\delta - \Delta^{-1}(z)}{2\delta}$) decreases with σ. Note that $\frac{1+\delta - \Delta^{-1}(z)}{2\delta}$ decreases in $\Delta^{-1}(z)$. Since $\Delta^{-1}(z)$ is given by $\Delta(v) = z$, define $H(v, z, \sigma) \equiv \Delta(v) - z = 0$. We use the implicit function theorem to compute $\frac{\partial v}{\partial \sigma}$: $\frac{\partial v}{\partial \sigma} = -\frac{\partial H/\partial \sigma}{\partial H/\partial v}$, where $\partial H/\partial \sigma = \Delta'_\sigma(v) \geq 0$, and $\partial H/\partial v = \Delta'_v(v) \leq 0$. As a result, $\frac{\partial v}{\partial \sigma} \geq 0$: v_D^* increases in σ, and thus the probability that the banker controls risk may decrease with σ.

Proposition 1

First, we show that the banker with $v > v_D^* + Cy$ exerts effort. When solving the problem (8), there are two cases: (1) $v > D + Cy$ (positive equity upon safe choice); (2) $v \leq D + Cy$.

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If $v > D + Cy$, $e = 1$ if $z > \Delta(v - Cy)$. According to the Assumption 1, $e = 1$ if $v \geq \Delta^{-1}(z) + Cy$, which is binding if $\Delta^{-1}(z) > D$. If $v \leq D + Cy$, $e = 0$ for any v from this interval. Thus, the banker exerts effort $e = 1$ if $v > \max\left[\Delta^{-1}(z), D\right] + Cy = v_D^* + Cy \equiv v_B^*$.

Next we show that the probability of risk control for $\Delta(D) > z$ (implying that $\Delta^{-1}(z) > D$) $(\frac{1+\delta-\Delta^{-1}(z)-Cy}{2\delta})$ decreases with σ. The proof of the of the derivative with respect to σ is identical to the one from Lemma 2.

Finally we show that $\frac{1+\delta-v_B^*}{2\delta}$ declines with C and y. Note that the derivatives of $\frac{1+\delta-v_D^*-Cy}{2\delta}$ with respect to C and y are equal to $-\frac{y}{2\delta}$ and $-\frac{C}{2\delta}$ accordingly and both are negative.

Lemma 3

When solving the problem (10), there are two cases: (1) $v > D + Cy$ (positive equity upon safe choice); (2) $v \leq D + Cy$.

If $v > D + Cy$, the first order condition to (10) is that $e = 1$ if:

$$F = \varphi \cdot \frac{z - \Delta(v + C)}{d + 1} + (1 - \varphi)[z - \Delta(v - Cy)] \geq 0 \tag{16}$$

Note that $F(v)$ is increasing in v:

$$\frac{\partial F(v)}{\partial v} = -(1 - \varphi)\Delta'(v - Cy) - \varphi\frac{\Delta'(v + C)}{d + 1} > 0 \tag{17}$$

Then (16) is binding if for $v = D + Cy$ it does not hold, i.e if (11) is satisfied. (11) implies that at $v = D + Cy$, no effort is exerted.

Next, consider the case when (11) is not satisfied. If $v \leq D + Cy$, the first order condition to (10) is that $e = 1$ if:

$$G = \varphi \cdot \frac{z - \Delta(v + C)}{d + 1} + (1 - \varphi)[z - \Delta(v - Cy) - (v - D - Cy)] \geq 0 \tag{18}$$

Note that it is not clear whether $G(v)$ is increasing in v:

$$\frac{\partial G(v)}{\partial v} = -(1 - \varphi)[\Delta'(v - Cy) + 1] - \varphi\frac{\Delta'(v + C)}{d + 1} \tag{19}$$

where the first item is negative, and second is positive. Also note that the second derivative of

$G(v)$ with respect to v is negative:

$$\frac{\partial^2 G(v)}{\partial v^2} = -(1-\varphi)\Delta''(v-Cy) - \varphi\frac{\Delta''(v+C)}{d+1} < 0 \tag{20}$$

Note however, that if $\frac{\partial G(v)}{\partial v}$ is non-positive for some $v \leq D + Cy$, it is not possible that for $v = D + Cy$ effort is exerted. Thus, if (11) is not satisfied, $G(v)$ is increasing in v. The banker controls risk if v is sufficiently high.

Thus, the solution to (10) depends on (11) and can be presented by v_C^* defined in (12). For $v_C^* \leq v \leq v_T$, the banker exerts effort.

Proposition 2

The trigger is set in such a way that maximizes expected effort by the banker. Recall that from Proposition 1, the banker controls risk if $v \geq v_B^*$. So there is no need to set the trigger higher than v_B^*. From Lemma 3, the banker controls risk for interim asset values $v_C^* \leq v \leq v_T$ and $v \geq v_B^*$. Thus, the ex ante probability of risk control is $\frac{(v_T - v_C^*) + (1+\delta - v_B^*)}{2\delta}$. To maximize this probability, the trigger should be set as high as possible to ensure the monotonicity of bank risk incentives for a given conversion ratio d. This results in the optimal trigger being set $v_T = v_B^*$, which increases the ex ante probability of risk control to $\frac{1+\delta-v_C^*}{2\delta}$.

Lemma 4

Using implicit function theorem for the general solution (12), we demonstrate that higher φ reduces v_C^*, or decreases equivalently risk-taking incentives:

$$\frac{\partial v_C^*}{\partial \varphi} = \begin{cases} -\frac{\partial F/\partial \varphi}{\partial F/\partial v} & \text{if (11) holds} \\ -\frac{\partial G/\partial \varphi}{\partial F/\partial v} & \text{otherwise} \end{cases} \tag{21}$$

where $\frac{\partial F}{\partial v} > 0$ and $\frac{\partial G}{\partial v} > 0$ from Lemma 3, and

$$\frac{\partial F}{\partial \varphi} = \frac{z - \Delta(v+C)}{d+1} - [z - \Delta(v - Cy)] \geq 0 \tag{22}$$

$$\frac{\partial G}{\partial \varphi} = \frac{z - \Delta(v+C)}{d+1} - [z - \Delta(v - Cy)] + (v - D - Cy) \geq 0 \tag{23}$$

the first item in (22) and (23) is non-negative, and the second is non-positive due to the fact that $\Delta(v - Cy) > \Delta(v + C)$ (recall that $\Delta'_v(v) \leq 0$). For $F(v) = 0$ and $G(v) = 0$, it must be that $z - \Delta(v - Cy) \leq 0$ and $z - \Delta(v + C) \geq 0$.

Thus, independent of condition (11), $\frac{\partial v_C^*}{\partial \varphi} < 0$, and trigger precision reduces v_C^*.

Lemma 5

Here we show that (1) $v_C^* \leq v^* B$ (CoCos induce safer choices that bail-inable debt), and (2) it can be that $v_C^* > v_D^*$ (CoCos may induce riskier choice than deposit funding).

First, note that v_C^* decreases in φ from Lemma 4. Thus, v_C^* is at maximum when $\varphi = 0$, corresponding to:

$$\begin{cases} F(v)_{\varphi=0} = z - \Delta(v - Cy) = 0 & \text{if (11) holds} \\ G(v)_{\varphi=0} = z - \Delta(v - Cy) - (v - D - Cy) = 0 & \text{otherwise} \end{cases} \tag{24}$$

If (11) holds, $v_C^* = \Delta^{-1}(z) + Cy$, otherwise $v_C^* \leq D + Cy$ (as shown in Lemma 3). Thus, for both cases, $v_C^* \leq v_B^*$.

Second, since $v_B^* \geq v_D^*$ for any positive y, for sufficiently low φ, $v_C^* > v_D^*$ (the maximum v_C^* is v_B^*). This implies that deposit funding may provide better incentives than CoCos due to the presence of yield effect.

Lemma 6

From Proposition 2, the optimal trigger is $v_T = v_B^* = v_D^* + Cy$. Note that if the amount of CoCos increases, the optimal trigger must go up:

$$\frac{\partial(v_D^* + Cy)}{\partial C} = y > 0 \tag{25}$$

The same holds for the increase in exogenous yield y:

$$\frac{\partial(v_D^* + Cy)}{\partial y} = C > 0 \tag{26}$$

Proposition 3

The banker's program is:

$$\max_e e \cdot [\underbrace{\max(v - D - Cy, 0) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=1} +$$

$$\underbrace{v - D + C \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion and } e=1}] +$$

$$(1 - e) \cdot [\underbrace{(v - z - D - Cy + \Delta(v - Cy)) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=0} +$$

$$\underbrace{v - z - D + C + \Delta(v + C) \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion and } e=0}]$$

$$\text{s.t. } e \in \{0, 1\} \tag{27}$$

The proof is similar to the one of Lemma 3. This is just a special case when CoCos have conversion ratio $d = 0$. The critical value v_{WD}^* depends on condition (28) which describes when v_{WD}^* is above $D + Cy$:

$$\varphi[z - \Delta(D + C(1 + y))] + (1 - \varphi)(z - \Delta(D)) < 0 \tag{28}$$

v_{WD}^* is defined implicitly in (29):

$$\begin{cases} \varphi[z - \Delta(v + C)] + (1 - \varphi)[z - \Delta(v - Cy)] = 0 & \text{if (28) holds} \\ \varphi[z - \Delta(v + C)] + (1 - \varphi)[z - \Delta(v - Cy) - (v - D - Cy)] = 0 & \text{otherwise} \end{cases} \tag{29}$$

This allows us to disentangle the effect of CoCos on risk incentives when there is no conversion when trigger is breached.

To show this effect, we assume that introduction of CoCos only affect leverage upon conversion. Thus, we abstract from the effect of the amount of CoCos on the yield paid upon maturity if no conversion takes place and effect of conversion ratio on risk incentives. Using implicit function theorem for the general solution (12), we demonstrate the direct debt reduction effect of CoCos introduction on v_C^*, or equivalently risk-taking incentives:

$$\frac{\partial v_C^*}{\partial C}\Big|_{d=fixed, Cy=fixed} = \begin{cases} -\frac{\partial F/\partial C}{\partial F/\partial v}\Big|_{d=fixed, Cy=fixed} & \text{if (11) holds} \\ -\frac{\partial G/\partial C}{\partial F/\partial v}\Big|_{d=fixed, Cy=fixed} & \text{otherwise} \end{cases} \tag{30}$$

where

$$\frac{\partial F}{\partial v} = -\frac{\varphi}{d+1} \cdot \Delta'_v(v+C) - (1-\varphi)\Delta'_v(v-Cy) \geq 0 \tag{31}$$

$$\frac{\partial F}{\partial C}\Big|_{d=fixed,Cy=fixed} = -\frac{\varphi}{d+1} \cdot \Delta'_C(v+C) \geq 0 \tag{32}$$

$$\frac{\partial G}{\partial v} = -\frac{\varphi}{d+1} \cdot \Delta'_v(v+C) - (1-\varphi)[\Delta'_v(v-Cy)+1] \geq 0 \tag{33}$$

$$\frac{\partial G}{\partial C}\Big|_{d=fixed,Cy=fixed} = -\frac{\varphi}{d+1} \cdot \Delta'_C(v+C) \geq 0 \tag{34}$$

$$\tag{35}$$

As a result, independent on (11), $\frac{\partial v_C^*}{\partial C}\big|_{d=fixed,Cy=fixed} \leq 0$ implying that higher amount of CoCos reduces leverage and thus decreases the threshold v_C^*. This increases the ex ante probability of bank risk control $\frac{1+\delta-v_C^*}{2\delta}$. We define debt reduction effect produced by CoCos as the effort improvement associated with the change in leverage only (but not affected by the yield or conversion ratio) compared to the baseline case of only deposit funding $\frac{v_D^*-v_{WD}^*|_{Cy=0}}{2\delta}$, where $v_{WD}^*|_{Cy=0}$ is defined by (29) for a paid yield $Cy=0$.

Proposition 4

The proof is similar to the one of Lemma 3. This is just a special case when CoCos have conversion ratio $d = \frac{C}{v-D}$. The critical value v_P^* depends on the condition (36) which describes when v_P^* is above $D+Cy$:

$$\varphi \cdot \frac{(z - \Delta[D + C(1+y)])(v-D)}{v-D+C} + (1-\varphi)(z - \Delta(D)) < 0 \tag{36}$$

v_P^* is defined implicitly in (37):

$$\begin{cases} \varphi \cdot \frac{[z-\Delta(v+C)](v-D)}{v-D+C} + (1-\varphi)[z - \Delta(v - Cy)] = 0 & \text{if (36) holds} \\ \varphi \cdot \frac{[z-\Delta(v+C)](v-D)}{v-D+C} + (1-\varphi)[z - \Delta(v - Cy) - (v - D - Cy)] = 0 & \text{otherwise} \end{cases} \tag{37}$$

Note that both functions $F(v)$ and $G(v)$ are increasing in v given conversion at par:

$$\frac{\partial F}{\partial v} = \frac{\partial F}{\partial v}\Big|_{d=fixed} + \frac{\partial F}{\partial d} \cdot \frac{\partial d}{\partial v} = \frac{\partial F}{\partial v}\Big|_{d=fixed} + \frac{\varphi(z - \Delta(v+C))}{(d+1)^2} \cdot \frac{C}{(v-D)^2} \tag{38}$$

$$\frac{\partial G}{\partial v} = \frac{\partial G}{\partial v}\Big|_{d=fixed} + \frac{\partial G}{\partial d} \cdot \frac{\partial d}{\partial v} = \frac{\partial G}{\partial v}\Big|_{d=fixed} + \frac{\varphi(z - \Delta(v+C))}{(d+1)^2} \cdot \frac{C}{(v-D)^2} \tag{39}$$

$$\tag{40}$$

where $\frac{\partial F}{\partial v}|_{d=fixed}$ and $\frac{\partial F}{\partial v}|_{d=fixed}$ are given in (17) and (19) accordingly, and $z > \Delta(v + C)$ for any $v_D^* - C \le v_C^* \le v_D^* + Cy$.

To disentangle the effect of CoCos on risk incentives when conversion leads to equity dilution, we assume that introduction of CoCos only affects conversion ratio with leverage and yield being fixed. In other words, any increase in CoCos affects only the conversion ratio and through it the incentives of the banker. Using implicit function theorem for the general solution (12), we demonstrate the equity dilution effect of CoCos introduction on v_C^*, or equivalently risk-taking incentives:

$$\frac{\partial v_C^*}{\partial d}|_{C=fixed,Cy=fixed} = \begin{cases} -\frac{\partial F/\partial d}{\partial F/\partial v}|_{C=fixed,Cy=fixed} & \text{if (11) holds} \\ -\frac{\partial G/\partial d}{\partial F/\partial v}|_{C=fixed,Cy=fixed} & \text{otherwise} \end{cases} \tag{41}$$

where $\frac{\partial F}{\partial v}|_{C=fixed,Cy=fixed}$ and $\frac{\partial G}{\partial v}|_{C=fixed,Cy=fixed}$ are given as in (31) and (33), and

$$\frac{\partial F}{\partial d}|_{C=fixed,Cy=fixed} = -\frac{\varphi(z - \Delta(v + C))}{(d + 1)^2} \le 0 \tag{42}$$

$$\frac{\partial G}{\partial d}|_{C=fixed,Cy=fixed} = -\frac{\varphi(z - \Delta(v + C))}{(d + 1)^2}) \le 0 \tag{43}$$

$$\tag{44}$$

As a result, independent on (11), $\frac{\partial v_C^*}{\partial d}|_{C=fixed,Cy=fixed} \ge 0$ implying that the increase in conversion ratio through higher amount of CoCos increases the dilution of the banker's share and thus increases the threshold v_C^*. This reduces the ex ante probability of bank risk control $\frac{1+\delta-v_C^*}{2\delta}$. We define equity dilution effect produced by CoCos as the effort reduction associated with the change in conversion ratio compared to the case of write-down CoCos with no conversion into shares $\frac{v_{WD}^*|_{Cy=0}-v_P^*}{2\delta}$, where $v_P^*|_{Cy=0}$ is defined by (37) for a paid yield $Cy = 0$.

Since higher conversion ratio implies higher risk incentives, $v_P^* > v_{WD}^*$.

Proposition 5

The proof is similar to the one of Lemma 3. This is again a special case when CoCos have conversion ratio $d = \frac{C}{v_T - D}$. Note that conversion ratio does not depend on v any more, but on the fixed trigger value. The critical value v_F^* depends on (45) which describes when v_F^* is above $D + Cy$:

$$\varphi \cdot \frac{(z - \Delta[D + C(1 + y)])(v_T - D)}{v_T - D + C} + (1 - \varphi)(z - \Delta(D)) < 0 \tag{45}$$

v_F^* is defined implicitly in (46):

$$\begin{cases} \varphi \cdot \frac{[z-\Delta(v+C)](v_T-D)}{v_T-D+C} + (1-\varphi)[z-\Delta(v-Cy)] = 0 & \text{if (45) holds} \\ \varphi \cdot \frac{[z-\Delta(v+C)](v_T-D)}{v_T-D+C} + (1-\varphi)[z-\Delta(v-Cy)-(v-D-Cy)] = 0 & \text{otherwise} \end{cases} \quad (46)$$

As before in Lemma 3, both functions $F(v)$ and $G(v)$ are increasing in v. Also note that fixed conversation $d = \frac{C}{v_T-D}$ produces lower equity dilution to the banker than conversion at par with $d = \frac{C}{v-D}$, since conversion occurs only for $v < v_T$. Recall from Proposition 4 that lower equity dilution results in better risk control. This implies that changing conversion ratio from at par to fixed one dependent on the trigger decreases $v*_C$ and thus produces better risk control incentives. As a result, $v_F^* < v_P^*$.

Changing conversion from at par to the fixed one increases the ex ante probability of bank risk control $\frac{1+\delta-v_C^*}{2\delta}$. We call this increase a CoCo dilution effect as the effort increase associated with the lower equity dilution and value transfer from CoCoholders to the equityholders (banker). We denote it by $v_F^*|_{Cy=0} - v_P^*|_{Cy=0}$, where $v_F^*|_{Cy=0}$ is defined by (46) for a paid yield $Cy = 0$.

Lemma 7

Note that fixed conversion ratio $d = \frac{C}{v_T-D}$ decreases with v_T. Thus, if v_T goes up, conversion ratio dependent on it goes down. From Proposition 5, lower conversion ratio increases banker's effort though CoCo diluting CoCoholders. As a result, higher trigger value v_T enhances CoCos' risk reduction effect.

Endogenous yield

Here we give the implicit conditions for yield of different debt types: bail-inable debt, write-down CoCos and standard CoCos.

Bail-inable debt

Given this payoff, the yield is given implicitly by break-even condition of the debtholders:

$$C = \frac{1+\delta-v_B^*}{2\delta} \cdot C(1+y) + \frac{v_B^*-1+\delta}{2\delta} \cdot \Big[Prob(V_2 \geq D + Cy|v < v_B^*) \cdot C(1+y) \quad (47)$$
$$+ Prob(D - C \leq V_2 < D + Cy|v < v_B^*) \cdot \mathbf{E}(V_2 - D + C|D - C \leq V_2 < D + Cy, v < v_B^*) \Big].$$

Write-down CoCos

The yield is given implicitly by break-even condition of holders of write-down CoCos:

$$C = \frac{1 + \delta - v_B^*}{2\delta} \cdot C(1 + y) + \frac{v_B^* - v_{WD}^*}{2\delta} \cdot (1 - \varphi)C(1 + y) +$$
$$\frac{v_{WD}^* - 1 + \delta}{2\delta} \cdot (1 - \varphi)\Big[Prob(V_2 \geq D + Cy|v < v_{WD}^*) \cdot C(1 + y) + \tag{48}$$
$$Prob(D - C \leq V_2 < D + Cy|v < v_{WD}^*) \cdot \mathbf{E}(V_2 - D + C|D - C \leq V_2 < D + Cy, v < v_{WD}^*)\Big].$$

Standard CoCos The yield of standard CoCo debt is given implicitly by the break-even condition:

$$C = \frac{1 + \delta - v_B^*}{2\delta} \cdot C(1 + y) + \frac{v_B^* - v_F^*}{2\delta} \cdot \Big((1 - \varphi)C(1 + y) + \varphi \cdot \frac{d}{d + 1} \cdot (v - D + C)\Big)$$
$$\frac{v_F^* - 1 + \delta}{2\delta} \cdot \Big[(1 - \varphi)\Big(Prob(V_2 \geq D + Cy|v < v_F^*) \cdot C(1 + y) + \tag{49}$$
$$Prob(D - C \leq V_2 < D + Cy|v < v_F^*) \cdot \mathbf{E}(V_2 - D + C|D - C \leq V_2 < D + Cy, v < v_F^*)\Big) +$$
$$\varphi \cdot Prob(V_2 > D - C|v < v_F^*) \cdot \frac{d}{d + 1} \cdot \mathbf{E}(V_2 - D + C|V_2 > D - C, v < v_F^*)\Big].$$

Proposition 6

The banker's program with extra equity is:

$$\max_e e \cdot \max[v - D + \epsilon, 0] + (1 - e) \cdot (v - D + \epsilon - z + \Delta(v + \epsilon))$$
$$\text{s.t. } e \in \{0, 1\} \tag{50}$$

which yields that the banker controls risk if $v \geq v_D^* - \epsilon$.

In order to compute the substitution ratio k, we use the condition for finding v_C^*, equivalence condition $v_C^* = v_D^* - \epsilon$ and new notation $C = k\epsilon$:

$$\begin{cases} F(v_D^* - \epsilon|k\epsilon, d) = 0 & \text{if (11) holds} \\ G(v_D^* - \epsilon|k\epsilon, d) = 0 & \text{otherwise} \end{cases} \tag{51}$$

or equivalently

$$\begin{cases} \frac{\varphi}{d+1} \cdot (z - \Delta[v_D^* + \epsilon(k - 1)]) + (1 - \varphi) \cdot (z - \Delta[v_D^* - \epsilon(1 + ky)]) = 0 \\ \frac{\varphi}{d+1} \cdot (z - \Delta[v_D^* + \epsilon(k - 1)]) + (1 - \varphi) \cdot (z - \Delta[v_D^* - \epsilon(1 + ky)] - [v_D^* - D - \epsilon(1 + ky)]) = 0 \end{cases}$$

Here we prove that $k \geq 1$. The proof is by contradiction. Assume that $k < 1$. Consider $F(v_D^* - \epsilon | k\epsilon, d) = 0$. Note that $\Delta[v_D^* - \epsilon] \geq z$, since banker with $v < v_D^*$ does not exert effort by definition of v_D^*. Since the whole expression is equal to zero, and the second term is non-negative, the first term should be non-positive. Hence,

$$\Delta[v_D^* + \epsilon(k-1)] - z \leq 0$$

The risk-shifting incentive is smaller than or equal to z only if $v \geq v_D^*$. And if $k < 1$, then $v_D^* + \epsilon(k-1) < v_D^*$. This is a contradiction. The same logic applies to $G(v_D^* - \epsilon | k\epsilon, d) = 0$, where the second item is also negative, and even more smaller than $z - \Delta[v_D^* - \epsilon]$.

Consequently, it always holds that $k \geq 1$, and higher amount of CoCos is required to provide the same effect as equity.

Note that if $\varphi = 1$, it must be the case that $z - \Delta[v_D^* + \epsilon(k-1)] = 0$, which is true only if $k = 1$.

Lemma 8

In order to show the effect of information revelation on the substitution ratio k, we compute first and second derivatives of k with respect to φ: $\frac{\partial k}{\partial \varphi}$ and $\frac{\partial^2 k}{\partial \varphi^2}$. We apply the implicit function theorem to the (52) and rewrite it using the fact that $d = \frac{k\epsilon}{v_D^* - D}$:

$$\begin{cases} \varphi \cdot \frac{(v_D^* - D)}{k\epsilon + v_D^* - D} \cdot (z - \Delta[v_D^* + \epsilon(k-1)]) + (1-\varphi) \cdot (z - \Delta[v_D^* - \epsilon(1+ky)]) = 0 \\ \varphi \cdot \frac{(v_D^* - D)}{k\epsilon + v_D^* - D} \cdot (z - \Delta[v_D^* + \epsilon(k-1)]) + (1-\varphi) \cdot (z - \Delta[v_D^* - \epsilon(1+ky)] - [v_D^* - D - \epsilon(1+ky)]) = 0 \end{cases}$$

According to the implicit function theorem:

$$\frac{\partial k}{\partial \varphi} = \begin{cases} -\dfrac{\partial F(v_D^* - \epsilon | k\epsilon)/\partial \varphi}{\partial F(v_D^* - \epsilon | k\epsilon)/\partial k} & \text{if (11) holds} \\ -\dfrac{\partial G(v_D^* - \epsilon | k\epsilon)/\partial \varphi}{\partial G(v_D^* - \epsilon | k\epsilon)/\partial k} & \text{otherwise} \end{cases}$$

$\frac{\partial F(v_D^* - \epsilon | k\epsilon)}{\partial k}$ equals to

$$\frac{\varphi \cdot (v_D^* - D) \cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot \left(\underbrace{-(k\epsilon + v_D^* - D) \cdot \Delta_k'[v_D^* + \epsilon(k-1)]}_{\geq 0} - \underbrace{(z - \Delta[v_D^* + \epsilon(k-1)])}_{\geq 0} \right)$$

$$+ (1-\varphi)\epsilon y \underbrace{\Delta_k'[v_D^* - \epsilon(1+ky)]}_{\leq 0}$$

where the item in the first line is non-negative for infinitesimal ϵ. The whole expression is non-negative if the yield y is sufficiently small.

$\frac{\partial G(v_D^* - \epsilon | k\epsilon)}{\partial k}$ equals to

$$\frac{\varphi \cdot (v_D^* - D) \cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot \left(\underbrace{-(k\epsilon + v_D^* - D) \cdot \Delta_k'[v_D^* + \epsilon(k-1)]}_{\geq 0} - \underbrace{(z - \Delta[v_D^* + \epsilon(k-1)]))}_{\geq 0} \right)$$
$$+ (1 - \varphi)\epsilon y \underbrace{[\Delta_k'[v_D^* - \epsilon(1 + ky)] + 1]}_{\geq 0}$$

where the item in the first line is non-negative for infinitesimal ϵ, and the item in the second line is also non-negative due to Assumption 1. And derivatives with respect to φ are:

$$\frac{\partial F(v_D^* - \epsilon | k\epsilon)}{\partial \varphi} = \underbrace{\frac{(v_D^* - D)}{k\epsilon + v_D^* - D} \cdot (z - \Delta[v_D^* + \epsilon(k-1)])}_{\geq 0} - \underbrace{(z - \Delta[v_D^* - \epsilon(1 + ky)])}_{\leq 0} \geq 0$$

$$\frac{\partial G(v_D^* - \epsilon | k\epsilon)}{\partial \varphi} = \frac{\partial F(v_D^* - \epsilon | k\epsilon)}{\partial \varphi} + (v_D^* - D - \epsilon(1 + ky)) \geq 0$$

Thus, the substitution ratio falls as probability of revelation rises $\frac{\partial k}{\partial \varphi} \leq 0$ if (11) is satisfied and yield y is sufficiently small, so that (14) is satisfied.

Next, consider the second derivative of substitution ratio with respect to φ:

$$\frac{\partial^2 k}{\partial \varphi^2} = \frac{\frac{\partial F}{\partial \varphi} \cdot \left(\frac{(v_D^* - D) \cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot [-\Delta_k'[v_D^* + \epsilon(k-1)] \cdot (k\epsilon + v_D^* - D) - (z - \Delta[v_D^* + \epsilon(k-1)])] \right)}{\left(\frac{\partial F}{\partial k} \right)^2}$$
$$- \frac{\frac{\partial F}{\partial \varphi} \cdot \epsilon y \Delta_k'(v_D^* - \epsilon(1 + ky))}{\left(\frac{\partial F}{\partial k} \right)^2} \geq 0 \tag{52}$$

if (11) is satisfied and otherwise:

$$\frac{\partial^2 k}{\partial \varphi^2} = \frac{\frac{\partial G}{\partial \varphi} \cdot \left(\frac{(v_D^* - D) \cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot (-\Delta_k'[v_D^* + \epsilon(k-1)] \cdot (k\epsilon + v_D^* - D) - (z - \Delta[v_D^* + \epsilon(k-1)])) \right)}{\left(\frac{\partial G}{\partial k} \right)^2}$$
$$- \frac{\frac{\partial G}{\partial \varphi} \cdot \epsilon y [\Delta_k'(v_D^* - \epsilon(1 + ky)) + 1]}{\left(\frac{\partial G}{\partial k} \right)^2} \geq 0$$

which is non-negative for sufficiently low yield y.

This result implies that the substitution ratio k is a convex function of the probability of

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information revelation φ if the promised yield y is sufficiently low, i.e when (15) is satisfied.