Rare Disasters and Exchange Rates

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Abstract

We propose a new model of exchange rates, based on the hypothesis that the possibility of rare but extreme disasters is an important determinant of risk premia in asset markets. The probability of world disasters as well as each country's exposure to these events is time-varying. This creates joint fluctuations in exchange rates, interest rates, options, and stock markets. The model accounts for a series of major puzzles in exchange rates: excess volatility and exchange rate disconnect, forward premium puzzle and large excess returns of the carry trade, and comovements between stocks and exchange rates. It also makes empirically successful signature predictions regarding the link between exchange rates and telltale signs of disaster risk in currency options.

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1 Introduction

We propose a new model of exchange rates, based on the hypothesis of Rietz (1988) and Barro (2006) that the possibility of rare but extreme disasters is an important determinant of risk premia in asset markets. The model accounts for a series of major puzzles in exchange rates. It also makes signature predictions about the link between exchange rates and currency options, which are broadly supported empirically. Overall, the model explains classic exchange rate puzzles and more novel links between options, exchange rates and stock market movements.

In the model, at any point in time, a world disaster might occur. Disasters correspond to bad times — they therefore matter disproportionately for asset prices despite the fact that they occur with a low probability. Countries differ by their riskiness, that is by how much their exchange rate would depreciate if a world disaster were to occur (something that we endogenize in the paper). Because the exchange rate is an asset price whose risk affects its value, relatively riskier countries have more depreciated exchange rates.

The probability of world disaster as well as each country's exposure to these events is time-varying. This creates large fluctuations in exchange rates, which rationalize their apparent "excess volatility". To the extent that perceptions of disaster risk are not perfectly correlated with conventional macroeconomic fundamentals, our disaster economy exhibits an "exchange rate disconnect" (Meese and Rogoff 1983).

Relatively risky countries also feature high interest rates, because investors need to be compensated for the risk of exchange rate depreciation in a potential world disaster. This allows the model to account for the forward premium puzzle.[1] Indeed, suppose that a country is temporarily risky: it has high interest rates, and its exchange rate is depreciated. As its riskiness reverts to the mean, its exchange rate appreciates. Therefore, the currencies of high interest rate countries appreciate on average.

The disaster hypothesis also makes specific predictions about option prices. This paper

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[1] According to the uncovered interest rate parity (UIP) equation, the expected depreciation of a currency should be equal to the interest rate differential between that country and the reference region. A regression of exchange rate changes on interest rate differentials should yield a coefficient of 1. However, empirical studies starting with Tryon (1979), Hansen and Hodrick (1980), Fama (1984), and those surveyed by Lewis (2011) consistently produce a regression coefficient that is less than 1, and often negative. This invalidation of UIP has been termed the *forward premium puzzle*: currencies with high interest rates tend to appreciate. In other words, currencies with high interest rates feature positive predictable excess returns.

works them out, and finds that those signature predictions are reasonably well borne out in the data. We view this as encouraging support for the disaster view.

The starting point is that, in our theory, the exchange rate of a risky country commands high put premia in option markets – as measured by high "risk reversals" (which are the difference in implied volatility between an out-of-the-money put and a symmetric out-of-the-money call). Indeed, investors are willing to pay a high premium to insure themselves against the risk that this exchange rate depreciates in the event of a world disaster. A country's risk reversal is therefore a reflection of its riskiness.

Accordingly, the model makes four predictions regarding these put premia ("risk reversals"). First, investing in countries with high risk reversals should have high returns on average. Second, countries with high risk reversals should have high interest rates. Third, when the risk reversal of a country goes up, its currency contemporaneously depreciates. These predictions, and a fourth one detailed below, are broadly consistent with the data.[2]

The model is very tractable, and we obtain simple and intuitive closed form expressions for the major objects of interest, such as exchange rates, interest rates, carry trade returns, yield curves, forward premium puzzle coefficients, option prices, and stocks.[3] To achieve this, we build on the closed-economy model with stochastic intensity of disasters proposed in Gabaix (2012) (Rietz 1988 and Barro 2006 assume a constant intensity of disasters), and use the "linearity-generating" processes developed in Gabaix (2009). Our framework is also very flexible. We show that it is easy to extend the basic model to incorporate several factors and inflation.

We calibrate a version of the model and obtain quantitatively realistic values for the quantities of interest, such as the volatility of the exchange rate, the interest rate, the forward premium puzzle, the return of the carry trade, as well as the size and volatility of risk reversals and their link with exchange rate movements and interest rates. The underlying disaster numbers largely rely on Barro and Ursua (2008)'s empirical numbers which imply that rare disasters matter five times as much as they would if agents were risk neutral. As a result, changes in beliefs about disasters translate into meaningful volatility. This is why the model yields a sizable volatility

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[2]See p. 19.

[3]Pavlova and Rigobon (2007, 2008) also provide an elegant and tractable framework for analyzing the joint behavior of bonds, stocks, and exchange rates which succeeds in accounting for comovements among international assets. However, their model is based on a traditional consumption CAPM, and therefore generates low risk premia and small departures from UIP.

which is difficult to obtain with more traditional models (e.g. Obstfeld and Rogoff 1995).

In addition, our calibration matches the somewhat puzzling link between stock returns and exchange rate returns. Empirically, there is no correlation between movements in the stock market and the currency of a country. However, the most risky currencies have a positive correlation with world stock returns, while the least risky currencies have a negative correlation. Our calibration replicates these facts. The economics is as follows: when world resilience improves, stock markets have positive returns, and the most risky currencies appreciate vis-a-vis the least risky currencies.

Finally, recent research (Lustig, Roussanov and Verdelhan (2011)) has documented a one-factor structure of currency returns (they call this new factor HML_{FX}). Our proposed calibration matches this pattern. In addition, our model delivers the new prediction that risk reversals of the most risky countries (respectively least risky) should covary negatively (respectively positively) with this common factor. This prediction holds empirically.

To sum up, our model delivers the following patterns.

Classic puzzles

1. Excess volatility of exchange rates.

2. Failure of uncovered interest rate parity. The coefficient in the Fama regression is less than 1, and sometimes negative.

   Link between options and exchange rates

3. High interest countries have high put premia (as measured by "risk reversals").

4. Investing in countries with high (respectively low) risk reversals delivers high (respectively low) returns.

5. When the risk reversal of a country's exchange rate increases (which indicates that the currency becomes riskier), the exchange rate contemporaneously depreciates.

   Link between stock markets and exchange rates

6. On average, the correlation between a country's exchange rate returns and stock market returns is zero.

7. However, high (respectively low) interest rate countries have a positive (respectively negative) correlation of their currency with world stock market: their currency appreciates (respectively depreciates) when world stock markets have high returns.

Comovement structure in exchange rates

8. There is a broad 1-factor structure in the excess currency returns (the HML_{FX} factor of Lustig, Roussanov and Verdelhan 2011): high interest rate currencies tend to comove, and comove negatively with low interest rate currencies.

9. There is a broad 1-factor structure of stock market returns: stock market returns tend to be positively correlated across countries.

10. There is a positive covariance between the above two factors.

At the same time, we match potentially challenging domestic moments, e.g.

11. High equity premium.

12. Excess volatility of stocks.

Hence, we obtain a parsimonious model of exchange rates, interest rates, options and stocks that matches the main features of the data. It delivers novel predictions borne out in the data, notably the link between movements in option prices ("risk reversals"), currency returns and stock returns.

Relation to the literature

Our paper is part of a broader research movement using modern asset pricing models to understand exchange rates, especially the aforementioned puzzles.

In the closed economy literature, there are three main paradigms for representative agent rational expectation models to explain both the level and the volatility of risk premia (something that the plain consumption CAPM with low risk aversion fails to generate): habits (Abel 1990, Campbell and Cochrane 1999), long run risks (Epstein and Zin 1989, Bansal and Yaron 2004) and rare disasters (Rietz 1988, Barro 2006).[4]

Economists have extended these closed-economy paradigms to open-economy setups to understand exchange rates. Habit models were used by Verdelhan (2010), Heyerdahl-Larsen

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[4]For the time-varying disasters, see Gabaix (2012), Gourio (2012) and Wachter (2013).

(forth.), and Stathopoulos (2012) to generate risk premia in currency markets. Long run risks models were applied by Colacito and Croce (2011) and Bansal and Shaliastovich (2013), using a two-country setting.

To the best of our knowledge, we are the first to adapt the disaster paradigm to exchange rates. After the present paper was circulated, Gourio, Siemer, and Verdelhan (2013) and Guo (2010) studied related and complementary models numerically in an RBC and a monetary context, respectively. Du (2013) explores quantitatively a related model, with a different focus: his results are mostly numerical, and do not touch on Lustig, Roussanov and Verdelhan (2011)'s HML_{FX} and the cross-moments between stocks and currencies. Martin (2013) presents a two-country model with i.i.d. shocks and characterizes the impact of deviations from lognormality using cumulants.[5]

On the empirical front, several recent papers investigate the hypothesis that disaster risk accounts for the forward premium puzzle: among these are Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Farhi et al. (2014), and Jurek (2014). Using currency options, they find some support for the disaster hypothesis for exchange rates (possibly leaving room for other determinants of the exchange rate).[6] Likewise, Brunnermeier, Nagel and Pedersen (2009) and Lustig and Verdelhan (2009) discuss evidence for crash risk in currency markets. Our paper provides a theoretical framework to understand these empirical results.

Outline. The rest of the paper is organized as follows. In Section 2, we set up the basic model and in section 3 derive its implications for the major puzzles. Section 4 shows the calibration of the model. Section 5 concludes. Most proofs are in the Appendix.

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[5] Another strand of the literature departs from the assumption of frictionless markets. Alvarez, Atkeson, and Kehoe (2002) rely on a model with endogenously segmented markets to qualitatively generate the forward premium anomaly. Pavlova and Rigobon (2012) study the importance of incomplete markets for external adjustment. Gabaix and Maggiori (2014) present a model of exchange rate determination and carry trade based on limited risk bearing capacity of the financial sector.

[6] These and our papers are also related to an older literature on so-called peso problems (Lewis 2011). Under the "pure peso" view, there are no risk premia and the forward premium puzzle is simply due to a small sample bias. By contrast, under the "rare disasters" view, there are risk premia.

2 Model Setup

2.1 Macroeconomic Environment

We consider a stochastic infinite horizon open economy model. There are N countries indexed by $i = 1, 2, ..., N$. Each country i is endowed with two goods: a traded good, called T, and a non-traded good, called NT_i. The traded good is common to all countries, the non-traded good is country-specific.

Preferences. In country i, agents value consumption streams $\left(C_{it}^T, C_{it}^{NT_i}\right)_{t \geq 0}$ according to

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \exp(-\delta t) \frac{\left(C_{it}^T\right)^{1-\gamma} + \left(C_{it}^{NT_i}\right)^{1-\gamma}/\xi}{1 - \gamma} \right], \tag{1}$$

where γ is the coefficient of relative risk aversion and $\frac{1}{\xi}$ parametrizes the expenditure share of non-traded goods.

The two goods enter the utility function separably. Together with the assumption of complete markets, this will allow us to derive a simple expression for the pricing kernel.[7]

Exchange rate. We choose the traded good as the world numéraire. We define the "absolute" exchange rate e_{it} to be the price of the non-traded good in country i in terms of the world numéraire. Hence, when e_{it} goes up, the exchange rate appreciates: one unit of the non-traded good of country i can buy more units of the world numéraire. The bilateral exchange rate between country i and country j is $\frac{e_{it}}{e_{jt}}$: an exchange rate appreciation of i with respect to

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[7]Utility function (1) could be changed to:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \exp\left(-\delta t\right) \frac{\left(C_t^T\right)^{1-\gamma}}{1 - \gamma} \right] + V(\{C_t^{NT_i}\}_{t \geq 0}), \tag{2}$$

where V is any utility function over non-traded goods consumption processes $\{C_t^{NT_i}\}_{t \geq 0}$. For instance, V could incorporate habit formation or adjustment costs. With this formulation, our formulas for the exchange rate would still hold, as the only thing that matters here is the marginal utility from one unit of tradable consumption. Though formulation (1) is, strictly speaking, subject to the Backus-Smith (1993) critique, its variant (2) can easily be made immune to it, and generate an imperfect correlation between total consumption and real exchange rates.

j corresponds to an increase of $\frac{e_{it}}{e_{jt}}$.[8,9]

Markets. Markets are complete: there is perfect risk sharing across countries in the consumption of the traded good.[10] Let C_t^{T*} be the world consumption of the traded good. The pricing kernel can therefore be expressed as

$$M_t^* = \exp(-\delta t)\left(C_t^{T*}\right)^{-\gamma}.$$

The price at time t of an asset with a stochastic stream of cash flows $\{D_{t+s}\}_{s\geq 0}$ is given by $\mathbb{E}_t\left[\sum_{s=0}^{\infty} M_{t+s}^* D_{t+s}\right]/M_t^*$.

Technology. There is a linear technology to convert the non-traded good of country i into the traded good. Investing one unit of the non-traded good at time t yields $\exp(-\lambda s)\omega_{i,t+s}$ units of the traded good in all future periods $t + s \geq t$. The interpretation is that ω_{it} is the productivity of the export technology, and the initial investment depreciates at a rate λ.

Proposition 1 (Value of the exchange rate). *The bilateral exchange rate between country i and country j is $\frac{e_{it}}{e_{jt}}$, where the absolute exchange rate e_{it} of country i is the present value of its future export productivity:*

$$e_{it} = \mathbb{E}_t\left[\sum_{s=0}^{\infty} M_{t+s}^* \exp(-\lambda s)\omega_{i,t+s}\right]/M_t^*, \tag{3}$$

with the convention that an increase in e_{it} means an appreciation of country i's currency.

Equation (3) expresses the exchange rate directly as the net present value of future fundamentals. The non-traded good is an asset that produces dividends $D_{i,t+s} = \exp(-\lambda s)\omega_{i,t+s}$,

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[8]This notion of the bilateral exchange rate differs slightly from the usual notion based on the relative price of consumption baskets across countries. However, the two notions are close for economies in which the share of non-traded goods is preponderant. In addition, there is a one-for-one correspondence between those two notions, detailed in Appendix B.

[9]Our model abstracts from interesting real-world frictions such as nominal rigidities and incomplete pass-through, which most researchers attribute to imperfect competition with non-constant demand elasticities, sticky prices, and menu costs with issues of currency denomination. Given our focus on the aggregate riskiness of the country, we believe that adding those frictions would not change the essence of the economics analyzed by the model – the impact of aggregate disaster risk on the exchange rate, option premia, and the real interest rate.

[10]Despite the completeness of markets, the consumption risk of non-traded good of country i must be borne by that country, because non-traded goods cannot be exchanged across borders.

and is priced accordingly. This is a version of the "asset view" of the exchange rate.[11,12]

A simple example. To make the model more concrete, consider the following simple example. The country produces two goods: a basket of non-traded goods and a traded good (oil). In every period t, oil can be exchanged for a basket of non-traded goods with a relative price of e_{it}. There is an inelastic supply of domestic labor. A worker can be employed in one of two activities: the worker can work in the domestic sector to produce a basket of non-traded goods, or the worker can work to expand the oil production capacity of the country (e.g., by detecting the location of an oil field and setting up the well to extract the oil in the future). These two technologies are linear. Once the oil production facility is established, the marginal cost of production is zero up to the capacity constraint. High future expected oil prices increase the profitability of expanding the oil production capacity. As a result, the domestic sector shrinks as workers move out of this sector to establish new oil production facilities. Consequently, the relative price of the basket of domestic goods in terms of the traded good (i.e., oil) increases, and the exchange rate e_{it} appreciates. A strong exchange rate therefore predicts high future commodity prices. This example is consistent with Chen, Rogoff, and Rossi (2010) who find that for commodity producing countries, high exchange rates predict high future prices of the corresponding commodities.

2.2 Disaster Risk

World consumption of the traded good. We study equilibria where the world consumption of the traded good C^{T*} follows the following stochastic process. In line with Rietz (1988) and Barro (2006), we assume that in each period $t + 1$ a disaster may happen with probability

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[11]We have made the strong assumption that there is a technology to transform non-traded goods into a flow of traded goods. We could have introduced many additional technologies without affecting our results. In particular, we could have introduced an additional reverse technology allowing to convert traded goods into a flow of non-traded goods.

[12]We could also have assumed that investment goods are a composite of traded and non-traded goods. Denote by $f(e_{it})$ the price of investment goods – corresponding to the technology for producing investment goods from traded goods and non-traded goods. Equation (3) would then become $f(e_{it}) = \mathbb{E}_t \left[\sum_{s=0}^{\infty} M^*_{t+s} \exp(-\lambda s)\omega_{i,t+s} \right] / M^*_t$. Similarly, we could let the output of the investment technology be a basket of traded and non-traded goods. The stochastic process for the exchange rate would have to be solved as the fixed point of a functional equation. The economics of the model would not be altered, but the analysis would become much more complex and closed-form solutions would be lost.

p_t. If no disaster happens, $C^{T*}_{t+1}/C^{T*}_t = \exp(g)$ where g is the normal-times growth rate of the economy. If a disaster happens, then $C^{T*}_{t+1}/C^{T*}_t = \exp(g)B_{t+1}$, with $B_{t+1} > 0$.[13] For instance, if $B_{t+1} = 0.7$, consumption falls by 30%. To sum up:

$$\frac{C^{T*}_{t+1}}{C^{T*}_t} = \exp(g) \times \begin{cases} 1 & \text{if there is no disaster at } t+1, \\ B_{t+1} & \text{if there is a disaster at } t+1. \end{cases} \tag{4}$$

Hence, the pricing kernel $M^*_t = \exp(-\delta t)\left(C^{T*}_t\right)^{-\gamma}$ evolves as:

$$\frac{M^*_{t+1}}{M^*_t} = \exp(-R) \times \begin{cases} 1 & \text{if there is no disaster at } t+1, \\ B^{-\gamma}_{t+1} & \text{if there is a disaster at } t+1, \end{cases} \tag{5}$$

where

$$R = \delta + \gamma g,$$

is the risk-free rate in an economy that has a zero probability of disasters.[14]

Productivity. We assume that productivity of country i follows:

$$\frac{\omega_{i,t+1}}{\omega_{i,t}} = \exp(g_{\omega_i}) \times \begin{cases} 1 & \text{if there is no disaster at } t+1, \\ F_{i,t+1} & \text{if there is a disaster at } t+1, \end{cases} \tag{6}$$

i.e., during a disaster, the relative productivity of the nontraded good is multiplied by $F_{i,t+1}$. For instance, if productivity falls by 20%, then $F_{i,t+1} = 0.8$.

In the model, a sufficient statistic for many quantities of interest is the "resilience" of a country i, defined as:

$$H_{it} = p_t \mathbb{E}^D_t \left[B^{-\gamma}_{t+1} F_{i,t+1} - 1 \right], \tag{7}$$

where $\mathbb{E}^D_t$ (resp. $\mathbb{E}^{ND}_t$) is the expected value conditional on a *disaster* happening at $t+1$ (resp. conditional on *no disaster* happening). A relatively safe country has a high resilience H_{it}, as it has a high recovery rate F_{it+1}. Conversely, a relatively risky country has low resilience. In equation (7), the probability p_t and the world intensity of disasters B_{t+1} are common to all

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[13] Typically, extra i.i.d. noise is added, but given that it never materially affects the asset prices, it is omitted here.

[14] In a more complex variant, disasters could be followed by partial recoveries (e.g. Gourio 2008, Nakamura et al. 2013). For a given γ, that lowers the risk premia coming from disaster risk. However, a slight increase in γ could counteract that effect. All in all, we find it simpler and more transparent to keep the simplest disaster formulation, at fairly little cost to the economics.

countries, but the recovery rate $F_{i,t+1}$ is country-specific. The changes in prospective recovery rates could be correlated across countries.

Rather than separately specifying laws of motion for its components (p_t, B_{t+1}, and F_{it+1}), we gain parsimony by directly modeling a law of motion for H_{it}. We decompose

$$H_{it} = H_{i*} + \widehat{H}_{it}, \tag{8}$$

where H_{i*} and $\widehat{H}_{it}$ are the constant and variable parts of resilience, respectively. For tractability, we posit that the law of motion for $\widehat{H}_{it}$ follows a linearity-generating process:[15]

$$\widehat{H}_{i,t+1} = \frac{1 + H_{i*}}{1 + H_{it}} \exp(-\phi_{H_i})\widehat{H}_{it} + \varepsilon_{i,t+1}^H, \tag{9}$$

where ϕ_{H_i} denotes the speed of mean reversion of resilience and the innovations $\varepsilon_{i,t+1}^H$ have mean zero, both unconditionally and conditional on a disaster ($\mathbb{E}_t \left[\varepsilon_{i,t+1}^H\right] = \mathbb{E}_t^D \left[\varepsilon_{i,t+1}^H\right] = 0$).

The economic meaning of equation (9) is that $\widehat{H}_{it}$ mean-reverts towards zero, but is subject to shocks. Because H_{it} hovers around H_{i*}, $\frac{1+H_{i*}}{1+H_{it}}$ is close to one and the process behaves like a regular AR(1) up to second-order terms in $\widehat{H}_{it}$: $\widehat{H}_{i,t+1} \simeq \exp(-\phi_{H_i})\widehat{H}_{it} + \varepsilon_{i,t+1}^H$. The "twist" term $\frac{1+H_{i*}}{1+H_{it}}$ is innocuous from an economic perspective but provides analytical tractability (see the technical appendix in Gabaix 2009 for a discussion). Linearity-generating processes allow the derivation of the equilibrium exchange rate in closed form.

3 Exchange Rates, Interest Rates, Options, and Stocks

3.1 Exchange Rates and Interest Rates

Exchange rate. We start by deriving the value of the exchange rate. We define $h_{i*} = \ln(1 + H_{i*})$ and

$$r_{ei} \equiv R + \lambda - g_{\omega_i} - h_{i*}. \tag{10}$$

As we shall see below, $r_{ei} - \lambda$ is the interest rate when the temporary component of resilience, $\widehat{H}_{it}$, is zero.

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[15]Linearity-generating (LG) processes (Gabaix 2009 and Appendix A) give rise to compact closed forms for stock and bond prices. More conventional affine processes (for instance, an AR(1) in the interest rate or the growth rate of the dividend) yield a simple closed form only for zero-coupon bonds, but yield more cumbersome infinite sums for stocks. In the present paper, the exchange rate is a stock-like asset. Hence, LG processes yield closed forms for exchange rates.

Proposition 2 (Level of the exchange rate). *The bilateral exchange rate between country i and country j is $\frac{e_{it}}{e_{jt}}$, where e_{it} is the exchange rate of country i in terms of the world numéraire and is equal to*

$$e_{it} = \frac{\omega_{it}}{r_{ei}}\left(1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}\right) \tag{11}$$

in the limit of small time intervals.[16]

Equation (11) implies that the exchange rate e_{it} increases (appreciates) with h_{i*} and $\widehat{H}_{it}$: risky (i.e., low resilience) countries have a low (depreciated) exchange rate. Safer (i.e., high resilience) currencies have a high (appreciated) exchange rate. Risky countries are those whose currency value (and more primitively, whose relative price of non-tradables) is expected to drop during disasters.[17]

The exchange rate fluctuates with the resilience $\widehat{H}_{it}$. As we shall see in the calibration, these fluctuations are plausibly large, and can therefore generate "excess volatility" of the exchange rate.[18]

To the extent that fluctuations in resilience are imperfectly correlated with traditional macroeconomic fundamentals, these fluctuations in resilience can also generate an "exchange rate disconnect".

Interest rate Consider a one-period domestic bond in country i that yields one in the numéraire of country i at time $t + 1$. It will be worth $e_{i,t+1}$ in the international numéraire. Hence, the domestic price of that bond is given by:[19]

$$\frac{1}{1 + r_{it}} = \mathbb{E}_t\left[\frac{M_{t+1}^* e_{i,t+1}}{M_t^* e_{i,t}}\right], \tag{12}$$

where r_{it} is the domestic interest rate. Recall that $r_{ei} \equiv R + \lambda - g_{\omega_i} - h_{i*}$.

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[16]See equation (42) in Appendix B for an exact expression away from the limit of small time intervals.

[17]Formula (11) implicitly exhibits a Balassa-Samuelson effect: more productive countries – countries with a higher ω_{it} – have appreciated exchange rates. Countries with high expected productivity growth also have high exchange rates. Equation (11) also implies that the exchange rate e_{it} increases (appreciates) with the growth of productivity g_ω and decreases (depreciates) with the Ramsey interest rate R.

[18]At this stage, the volatility of the exchange rate comes from the volatility of its resilience $\widehat{H}_{it}$. In the online appendix, we generalize the setup and introduce other factors.

[19]The derivation is standard. In the international currency, the payoff of the bond is e_{t+1}, so its price is $\mathbb{E}_t\left[\frac{M_{t+1}^* e_{t+1}}{M_t^*}\right]$ and its domestic price is (12).

Proposition 3 (Interest rate). *The value of the interest rate in country i is*

$$r_{it} = r_{ei} - \lambda - \frac{r_{ei}\widehat{H}_{it}}{r_{ei} + \phi_{H_i} + \widehat{H}_{it}} \tag{13}$$

in the limit of small time intervals.[20]

When a country is "risky" (low h_{i*} or $\widehat{H}_{it}$), its interest rate is high according to (13) because its currency has a high risk of depreciating in bad states of the world. Note that this risk is a real risk of depreciation, not a default risk.[21]

Safe haven countries can borrow at low interest rates and have an appreciated currency. Consider two countries, one ("Switzerland", the safe haven) with low risk / high average resilience h_{i*}, and one ("Brazil") with high risk / lower average resilience h_{i*}.[22] Equations (10)-(13) imply that, on average (i.e., when $\widehat{H}_{it} = 0$), Switzerland has low interest rates (equal to $r_{ei} - \lambda$), while Brazil has high interest rates. This is a compensation for disaster risk, not default: investors are willing to lend to Switzerland at low interest rates, because the Swiss exchange rate will appreciate relative to Brazil's in a disaster.

At the same time, the exchange rates are $e_{it} = \frac{\omega_{it}}{r_{ei}}$ when resiliences are at their central value (equation 11 with $\widehat{H}_{it} = 0$). Hence, the Swiss exchange rate is on average appreciated ("strong") compared to the Brazilian exchange rate.

Switzerland (the safe haven) therefore benefits from the "exorbitant privilege" of borrowing at low interest rates. This underlying mechanism is different from those of Gourinchas, Govillot and Rey (2010), who emphasize differences in risk aversion, and Maggiori (2013), who emphasizes differences in financial development. A distinctive feature of our model is that the exchange rate of safe haven countries appreciates in times of crises.

Existence of equilibrium. We end this section by showing sufficient conditions for the existence of an equilibrium. We choose to start with the consumption process for traded goods in equation (4), the productivity process in each country given by equation (6), and the process for the resilience in each country given by equation (9). This is enough to determine the exchange rate in each country as in equation (11) and, more generally, all the asset prices that we are interested in. Lemma 1 shows that there are endowment processes for the traded and

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[20] See equation (43) in Appendix B for an exact expression away from the limit of small time intervals.

[21] Safe countries can borrow at a lower interest rate, which may explain why historically the dollar or Swiss Franc interest rates were low (Gourinchas and Rey 2007).

[22] Hassan and Mano (2014) show that these persistent differences in riskiness are large.

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non-traded goods that can rationalize these choices as a general equilibrium outcome. This is a way of maintaining the tractability of an endowment economy in a model that features production.

Lemma 1 (Existence of equilibrium). *There exist endowment processes $\left\{\eta_{i,t}^{T}, \eta_{i,t}^{NT}\right\}_{t\geq 0, i=1\ldots n}$ for traded and non-traded goods such that in the equilibrium of the model (4), (6), and (9) hold.*

3.2 Forward Premium Puzzle and Carry Trade

We analyze the predictions of our model for Fama (1984) regressions in two different types of samples: with and without disasters. We consider countries with identical constant parameters but potentially different $\widehat{H}_{it}$, ω_{it}, and g_{ω_i}.

Consider the Fama regression of the changes in the exchange rate between countries i and j regressed on the difference in interest rates, in a sample with no disasters:

$$\text{Fama regression:} \quad \frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} = \alpha - \beta(r_{it} - r_{jt}) + \varepsilon_{ij,t+1}, \tag{14}$$

where $\varepsilon_{ij,t+1}$ is a random variable with mean zero. We will consider two possible kinds of samples for this regression: a large sample with no disasters and a full sample with a representative frequency of disasters. We denote the respective coefficients by β^{ND} and β^{Full}. As in other models with disasters, this allows us to make predictions about samples that happen not to contain disasters.

The UIP condition implies $\beta = 1$. In contrast, in our model β^{ND} and β^{Full} can be negative. For simplicity, we consider the case where the two countries i and j have the same r_e, ϕ_H.[23]

Proposition 4 (Fama coefficients). *Consider two countries i and j with the same $r_{ei} = r_{ej}$ and $\phi_{H_i} = \phi_{H_j}$, and consider the limit of small time intervals as well as small $\widehat{H}_{it}$ and $\widehat{H}_{jt}$. In the Fama regression (14), in a sample with no disasters the coefficient β is:*

$$\beta^{ND} = -\frac{\phi_{H_i}}{r_{ei}}. \tag{15}$$

If in addition B_t is constant with value B, then in a full sample the coefficient β^{Full} is:

$$\beta^{Full} = -\frac{\phi_{H_i}}{r_{ei}} + \left(1 + \frac{\phi_{H_i}}{r_{ei}}\right) B^\gamma. \tag{16}$$

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[23]In Proposition 4, as in the later Proposition 9, the expressions hold up to second-order terms in $\widehat{H}_{it}$, $\widehat{H}_{jt}$.

The intuition for the negative sign on β is as follows. Because $\widehat{H}_{it}$ is mean-reverting, risky countries are expected to be less risky in the future. As a result, the exchange rate of high interest rate countries is expected to appreciate – consistent with the "forward premium puzzle". In this simplest model with one factor (resilience), β is always negative; in richer models with more factors (resilience and inflation, see below), β can have both signs depending on the relative importance of the different factors.

To understand this proposition, it is useful to derive an expression for the appreciation of an exchange rate. For simplicity, we focus on the Fama regression in a sample without disasters. In Appendix B, we show that, in the limit of small time intervals and for small $\widehat{H}_{it}$,

$$\mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}-e_{it}}{e_{it}}\right] = -\frac{\phi_{H_i}}{r_{ei}+\phi_{H_i}}\widehat{H}_{it} + K_i \text{ and } r_{it} = -\frac{r_{ei}}{r_{ei}+\phi_{H_i}}\widehat{H}_{it} + K_i', \tag{17}$$

where K_i, K_i' (and soon K_i'') are country-specific constants. A currency with low resilience $\widehat{H}_{it}$ tends to appreciate and have a high interest rate. Eliminating the resilience term, we obtain a link between expected currency appreciation and the interest rate:

$$\mathbb{E}_t^{ND}\left[\frac{e_{i,t+1}-e_{it}}{e_{it}}\right] = -\frac{\phi_{H_i}\left(r_{ei}-\lambda\right)}{r_{ei}} + K_i'',$$

which gives the Fama coefficient on the interest rate, $\beta^{ND} = -\frac{\phi_{H_i}}{r_{ei}}$.

Here, the Fama coefficient in a sample without disasters does not depend on B (this will change when we add other factors, see Proposition 9). Even when disasters are not associated with risk premia (in other words, when $B = 1$), the Fama regression in a small sample with no disasters would indicate a violation of UIP. Time-varying risk premia are crucial to explain the forward premium puzzle in a sample with disasters: with $B = 1$, there is no disaster risk (consumption does not fall during disasters), so that $\beta^{Full} = 1$; the Fama coefficient is negative only if disaster risk is high enough. The possibility of a negative Fama coefficient β^{Full} in a full sample does not come from a peso problem.

The Carry Trade Given two currencies, the carry trade consists of borrowing one unit of the numéraire in currency j at interest rate r_{jt} and investing it in currency i at interest rate r_{it}.

Proposition 5 (Carry trade return). *The expected return of the carry trade between two countries is equal to $H_{jt} - H_{it}$, the difference in their resilience:*

$$\mathbb{E}_t^{ND} \left[\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right] + r_{it} - r_{jt} = H_{jt} - H_{it}. \tag{18}$$

Consider the particular case of two countries with identical constant parameters, but potentially different $\widehat{H}_{it}$ and ω_{it}. The idea is for an investor to borrow one unit of the world numéraire in a safe country j – the funding country – with high resilience H_{jt} and a low interest rate r_{jt}, and to invest in a risky country i with low resilience $H_{it} < H_{jt}$ and a high interest rate $r_{it} > r_{jt}$ – the investment country. If no disaster occurs, the investor pockets the interest differential. Moreover, on average, the exchange rate of country i appreciates against that of country j. However, if a disaster occurs, the exchange rate of country i depreciates against that of country j and the investor incurs a loss. Disasters correspond to bad states of the world when marginal utility (of the numéraire, i.e., the world traded good) is high. Investors are appropriately compensated for bearing this risk.

In a full sample with a representative frequency of disasters, the expected return of the carry trade is the one in (18) minus the expected loss in disasters $p\mathbb{E}_t^D [F_{jt} - F_{it}]$:

$$\mathbb{E}_t \left[\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right] + r_{it} - r_{jt} = H_{jt} - H_{it} - p\mathbb{E}_t^D [F_{jt} - F_{it}]. \tag{19}$$

3.3 Options and Risk Reversals

Disaster risk is inherently hard to measure, but options offer a powerful way to assess its importance. Here, we characterize the way disasters are incorporated into option prices. We discuss the empirical validity of the model's predictions. Consider two countries i and j. The currency j price at date 0 of a call that gives the option to buy at date 1 one unit of currency j for $K\frac{e_{j0}}{e_{i0}}$ units of currency i is $\frac{1}{e_{j0}} \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(e_{j1} - K\frac{e_{j0}}{e_{i0}} e_{i1} \right)^+ \right]$, i.e.,

$$V^C(K) = \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{j1}}{e_{j0}} - K\frac{e_{i1}}{e_{i0}} \right)^+ \right]. \tag{20}$$

Likewise, the currency j price at date 0 of a put that gives the option to sell at date 1 one unit of currency j for $K\frac{e_{j0}}{e_{i0}}$ units of currency i is $V^P(K) = \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(K\frac{e_{i1}}{e_{i0}} - \frac{e_{j1}}{e_{j0}} \right)^+ \right]$.

Option prices without disasters. The Black-Scholes formula for equity options was adapted by Garman-Kohlhagen (1983) to currency options. We call $V_{BS}^P(S, \kappa, \sigma, r, r^\star, T)$ and $V_{BS}^C(S, \kappa, \sigma, r, r^\star, T)$ the Black-Scholes prices for a put and a call, respectively, when the exchange rate is S, the strike is κ, the exchange rate volatility is σ, the home interest rate is r, the foreign interest rate is $r^\star$, and the time to maturity is T. The pricing formulas in that case are well-known.[24]

Option prices in the model. For tractability, we make two simplifying assumptions as in Gabaix (2012). First, we assume that if a disaster occurs in period $t + 1$, ε_{t+1}^H is equal to zero. Second, we assume that the distribution of $e_{j,t+1}$ conditional on date t information and no disaster occurring in period $t + 1$ is lognormal with drift μ_{it} and volatility σ_{it}, where i indexes countries: $\frac{e_{i1}}{e_{i0}} = \exp\left(\mu_i + \varepsilon_i - \sigma_i^2/2\right)$, where $\varepsilon_i \sim N(0, \sigma_i^2)$, and $\mu_i := R - \ln\left((1 + r_i)(1 + H_i)\right)$ is the expected exchange rate appreciation conditional on no disasters.[25] This enables us to derive option prices in closed form.[26] The standard deviation of the bilateral log exchange rate (conditional on no disaster) is $\sigma_{i|j} \equiv \left(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j\right)^{1/2}$, where ρ_{ij} is the correlation between ε_i and ε_j.

Proposition 6 (Option prices). *The price of a call with strike K and maturity 1 is:*

$$V^C(K) = V^{C,ND}(K) + V^{C,D}(K), \tag{21}$$

where $V^{C,ND}(K)$ and $V^{C,D}(K)$ are the part of the price corresponding respectively to the no-

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[24]Calling Φ the Gaussian cumulative distribution function, we have:

$$V_{BS}^C(S, \kappa, \sigma, r, r^\star, T) = Se^{-r^\star T}\Phi(d_1) - \kappa e^{-rT}\Phi(d_2),$$

$$V_{BS}^P(S, \kappa, \sigma, r, r^\star, T) = \kappa e^{-rT}\Phi(-d_2) - Se^{-r^\star T}\Phi(-d_1),$$

$$d_1 = \left[\ln(S/\kappa) + (r - r^\star + \sigma^2/2)T\right]/\sigma\sqrt{T}, \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

[25]This can be ensured as in Gabaix (2012). We assume that if there is no disaster, then $\omega_{t+1}/\omega_t = e^g\left(1 + \varepsilon_{t+1}^\omega\right)$, with $\mathbb{E}_t\left[\varepsilon_{t+1}^\omega\right] = \mathbb{E}_t\left[\varepsilon_{t+1}^\omega\varepsilon_{t+1}^H\right] = 0$. This does not change any of the formulas for the exchange rate and the interest rate. The disturbance term ε_{t+1}^ω can be designed to ensure that e_{i1}^{ND}/e_{i0} has the lognormal noise described above.

[26]This exact expression for μ_i comes from the Euler equation $1 = \mathbb{E}\left[(1 + r_i)e^{-R+\mu_i}(1 + H_i)\right]$.

disaster and disaster states:

$$V^{C,ND}(K) = \exp\left(-R + \mu_j\right)(1 - p_0) V^C_{BS}\left(K \exp\left(\mu_i - \mu_j\right), \sigma_{i|j}\right), \tag{22}$$

$$V^{C,D}(K) = \exp\left(-R + \mu_j\right) p_0 \mathbb{E}^D_0 \left[B_1^{-\gamma}\left(F_{j,1} - K \exp\left(\mu_i - \mu_j\right) F_{i,1}\right)^+\right], \tag{23}$$

where $V^C_{BS}(K, \sigma) := V^C_{BS}(1, K, \sigma, 0, 0, 1)$ is the Black-Scholes call value when the strike is K, the volatility σ, the interest rates 0, the maturity 1, the spot price 1.

The price of a put is given by the put-call parity equation:

$$V^P(K) = V^C(K) + \frac{K}{1 + r_i} - \frac{1}{1 + r_j}. \tag{24}$$

The option price (21) is the sum of two terms. The first one is a familiar Black-Scholes term. The second is a pure disaster term.

If the foreign currency is riskier than the home currency, then out-of-the-money put prices on the currency pair (home, foreign) should be higher than out-of-the-money call prices as the price of protection against a devaluation of the foreign currency should be high. We next present a simple metric – risk reversals – to measure the gap between out-of-the-money puts and out-of-the-money calls.

Implied volatility smile and risk reversals. Here we survey well-known notions in option theory. Given a call option with strike K and price v, the implied volatility of the option is the volatility $\widehat{\sigma}(K)$ that needs to be assumed in the Black-Scholes formula to match the price: $V^C_{BS}(K, \widehat{\sigma}(K)) = v$. Implied volatilities on puts are defined similarly. For instance, if a currency has a lot of disaster risk, its put price will be high (Proposition 6) and its implied volatility will be high.

In particular, consider the implied volatility curve (i.e., the graph of the implied volatility $\widehat{\sigma}(K)$ as a function of the strike K) of a pair of currencies: a risky currency and a safe currency. Out-of-the-money puts protect against the crash of the exchange rate of j versus i and out-of-the-money calls protect against the crash of the exchange rate of i versus j. Imagine that j is riskier than i. Then the implied volatility of deep out-of-the-money puts is higher than the implied volatility of out-of-the-money calls— a pattern referred to as a "smirk".

A popular way to quantify the smile is the "risk reversal" (RR). Intuitively, it is the difference in implied volatility of an out-of-the-money put and a symmetrically out-of-the-money call. Hence, a very risky currency will have a high RR.

To formulate a more precise definition of RR, we need to define the delta of an option. It is the derivative with respect to the time-0 currency price, in the Black-Scholes formula. Formally, if the call price (in the Black-Scholes / Garman-Kohlhagen model) is $V_{BS}^C(S, \kappa, r, r^*, \sigma, T)$, the delta is $\Delta := \frac{\partial V_{BS}^C}{\partial S}$). The delta of a call option decreases monotonically from 1 to 0 as κ/S increases. Symmetrically, the delta of a put option decreases monotonically from 0 to -1 as κ/S increases. Given a value $\Delta \in (0,1)$, we define the Δ risk reversal to be the difference in implied volatilities between an out-of-the-money put and an out-of-the-money call with the following properties. The strike of the put is chosen such that the Black-Scholes / Garman-Kohlhagen delta is $-\Delta$. Symmetrically, the strike of the call is chosen such that the Black-Scholes / Garman-Kohlhagen delta is Δ. In practice we will work with $\Delta = 0.25$, corresponding to a "25 delta" risk reversal.

We state a Lemma to better understand the risk reversal. It is drawn from Farhi et al. (2014, Proposition 5).[27]

Lemma 2 *In the limit of small time intervals, the risk reversal can be expressed as:*

$$RR_{ijt} = k_\Delta (H_{jt} - H_{it}). \tag{25}$$

where $k_\Delta := \frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))}$ is a numerical constant.

Hence, if country i has more disaster risk than country j ($H_{jt} - H_{it} > 0$), then the risk reversal is positive: put prices on currency i are very expensive and have a high implied volatility (compared to symmetric call prices).

Four signature predictions of disasters The model makes four broad predictions regarding option prices. The first three were seen above, and the fourth one will be detailed in section 4.4.

1. Countries with high risk reversals have high interest rates.

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[27]Formula (25) holds for a "one-period" option, and H_{it} is expressed per period. Suppose that "one period" is τ years (e.g. $\tau = \frac{1}{12}$ if a period is one month), and implied volatilities are expressed in annual units, and the maturity of the option is T years. Then, formula (25) becomes $\overline{RR}_{ijt} = \frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))} (\overline{H}_{jt} - \overline{H}_{it}) \sqrt{T}$, where $\overline{RR}_{ijt} = RR_{ijt}/\sqrt{\tau}$ and $\overline{H}_{it} = H_{it}/\tau$ are the RR and the resilience expressed in annual units. In addition, for 25 delta options ($\Delta = 0.25$), $\frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))} \simeq 1.57$.

2. Investing in countries with high risk reversals generates high expected returns.

3. When risk reversals go up, the exchange rate contemporaneously depreciates.

4. The risk reversal of risky (i.e., high risk reversal, high interest rate) countries should covary negatively with HML_{FX}, while the risk reversal of less risky countries should covary positively with it.[28]

Empirical support for these predictions can be found in various papers: Carr and Wu (2007, prediction 3), Brunnermeier, Nagel, and Perdersen (2009, predictions 1-3), Du (2013, prediction 1), Farhi et al. (2014, predictions 1-2).[29] Section section 4.4 finds support for prediction 4.

Those four signature predictions of the disaster hypothesis are therefore qualitatively borne out in the data. The calibration will show that the correspondence between empirics and theory can be made quantitative as well.

Illustration: impact of a change in the world disaster probability An important object is the probability of world disaster, p_t. Its movements have a number of signature effects that we now study. Consider two countries, again one safe (high F_{it}), "Switzerland", and one risky (low $F_{jt} < F_{it}$), "Brazil". The difference in their resiliences is

$$H_{it} - H_{jt} = p_t \mathbb{E}_t^D \left[B_{t+1}^{-\gamma} \left(F_{i,t+1} - F_{j,t+1} \right) \right].$$

Suppose that p_t increases, while $\mathbb{E}_t^D \left[B_{t+1}^{-\gamma} \left(F_{i,t+1} - F_{j,t+1} \right) \right]$ remains the same. Then, $H_{it} - H_{jt}$ increases: Switzerland becomes relatively more resilient than Brazil. As a result, Switzerland's exchange rate appreciates relative to Brazil's.[30]

Figure 1 illustrates this prediction of the model. We take the view that Fall 2008 was associated with an increase in the probability p_t of a disaster, rather than with the realization of a disaster. The horizontal axis is an estimate of $H_{US,t} - H_{i,t}$ during the height of the crisis (September 2008 to January 2009). The vertical axis shows the change in the exchange rate of

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[28]Recall that HML_{FX} is the payoff of a portfolio going long high interest rate currencies and short low interest rate currencies.

[29]For their empirical goal, Farhi et al. (2014) use a reduced-form version of the present model. As a result, their simple framework cannot generate some key predictions, e.g. Prediction 3.

[30]Plain CARA preferences are enough to discuss the impact of p_t. The economics would be similar with Epstein-Zin (1989) preferences.
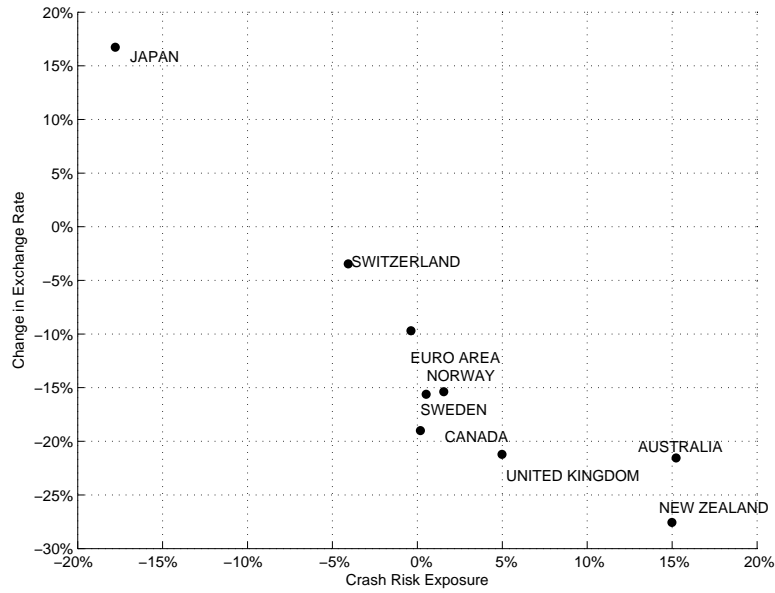
Figure 1: This Figure reports the average estimated compensation for disaster risk exposure and the cumulative percentage change in exchange rate for each country from September 2008 to January 2009. Source: Farhi et al. (2014).
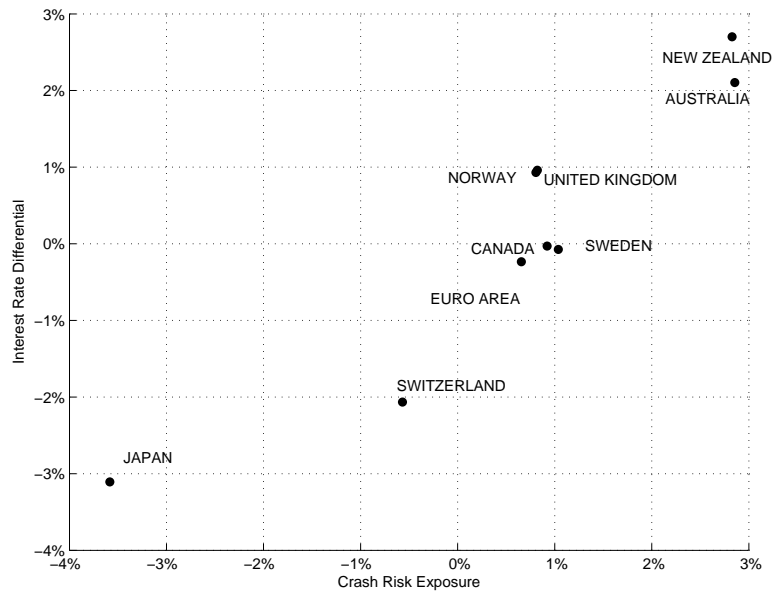

Figure 2: This Figure reports the average compensation for disaster risk exposure and the average interest rate differential (vis- a-vis the U.S.) for each country. Interest rates and risk exposures are reported in percentage points per annum. The sample period is January 1996 to May 2013. Source: Farhi et al. (2014).

various developed countries against the US dollar. According to the theory, since p_t increased during the crisis, risky countries should depreciate during the crisis. This is what Figure 1 verifies: countries with high crash risk (low resilience) depreciated a lot during the crisis.[31]

The same theory predicts that, on average, risky countries should have high interest rates (Proposition 3). This is verified in Figure 2, which shows riskiness (as measured by $H_{US,t} - H_{i,t}$) vs the currency interest rate.

3.4 Stocks

Our model allows us to think in a tractable way about the joint determination of exchange rate and equity values. We consider the case of a stock of a generic firm in country i, that produces the traded good.[32],[33]. Its dividend is D_{it} in units of the traded good, and $d_{it} = \frac{D_{it}}{e_{it}}$ when expressed in the domestic currency. It follows the following process

$$\frac{D_{i,t+1}}{D_{it}} = \exp\left(g_D\right)\left(1 + \varepsilon^D_{i,t+1}\right) \times \begin{cases} 1 & \text{if there is no disaster,} \\ F_{Di,t+1} & \text{if there is a disaster,} \end{cases}$$

where $\varepsilon^D_{i,t+1}$ is an idiosyncratic shock uncorrelated with the stochastic discount factors.

We define the resilience $H_{Di,t}$ of the dividend D_{it} of stock i as

$$H_{Di,t} = p_t\left(\mathbb{E}_t\left[B_{t+1}^{-\gamma} F_{Di,t+1}\right] - 1\right) = H_{Di*} + \widehat{H}_{Di,t}.$$

As before, we posit that the law of motion for $\widehat{H}_{D,t}$ is a LG-twisted process:

$$\widehat{H}_{Di,t+1} = \frac{1 + H_{Di*}}{1 + H_{Di,t}} \exp\left(-\phi_{H_D}\right) \widehat{H}_{Dit} + \varepsilon^{H_D}_{i,t+1},$$

where ϕ_{H_D} is the speed of mean reversion of the resilience of the stock.[34] We also define $h_{Di*} = \ln\left(1 + H_{Di*}\right),$

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[31] The loading on disaster risk is thus revealed during sharp increases in p_t: this might be one explanation for the findings of Lettau, Maggiori and Weber (2014) that downside-market risk statistically explains risk premia.

[32] The NBER working paper version of this paper also develops the case of a firm producing the nontraded good.

[33] The dividend of this firm may not be equal to total exports, as only a segment of firms are listed in the stock market.

[34] Away from the continuous time limit, the price of the stock is:

$$P_{Di,t} = d_{it} \frac{1 + \frac{\exp(-r_{Di} - h_{Di*})}{1 - \exp\left(-r_{Di} - \phi_{H_D}\right)} \widehat{H}_{Di,t}}{1 - \exp\left(-r_{Di}\right)}.$$

Proposition 7 (Price of stocks). *The domestic price of the stock $P_{Di,t}$ is, in the continuous time limit*

$$P_{Di,t} = d_{it} \frac{1 + \frac{\widehat{H}_{Di,t}}{r_{Di} + \phi_{H_{Di}}}}{r_{Di}}, \tag{26}$$

where d_{it} is the dividend expressed in the domestic currency and $r_{Di} \equiv R - g_D - h_{Di*}$.

A more resilient stock (high $\widehat{H}_{Di,t}$) has a higher price-dividend and lower future returns. The next Lemma states the equity premium.[35]

Lemma 3 (Equity premium) *The expected excess return of stocks (in the domestic currency, over the domestic risk-free rate) is, in the limit of small time intervals: $p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} \left(F_{i,t+1} - F_{Di,t+1} \right) \right]$.*

3.5 Yield Curve, Forward Rates, and Nominal Exchange Rates

Until recently, forward real interest rates were not available. Only their nominal counterparts were actively traded. Even today, most bonds are nominal bonds. To model nominal bonds, we build on the real two-factor model developed above. Let $P_t = P_0 / \prod_{s=0}^{t-1} (1 - I_s)$ be the price level, where I_t is inflation at time t (this formulation will prove tractable). The nominal exchange rate is:

$$\widetilde{e}_t = e_t / P_t, \tag{27}$$

where we denote nominal variables with a tilde. The nominal interest rate $\widetilde{r}_t$ satisfies $1 = \mathbb{E}_t \left[\frac{M_{t+1}^* \widetilde{e}_{t+1}}{M_t^* \widetilde{e}_t} \left(1 + \widetilde{r}_t \right) \right]$, so that in the continuous-time limit

$$\widetilde{r}_t = r_t + I_t, \tag{28}$$

i.e., the nominal interest rate is the real interest rate plus inflation. Fisher neutrality applies: there is no burst of inflation during disasters. With a burst of inflation, even short-term bonds would command a risk premium.

We posit that inflation hovers around I_*, roughly according to an AR(1) process. More specifically, to ensure tractability of the model, we posit the linearity-generating process:

$$I_{t+1} = I_* + \frac{1 - I_*}{1 - I_t} \exp \left(-\phi_I \right) \left(I_t - I_* \right) + \varepsilon_{t+1}^I, \tag{29}$$

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[35] This lemma neglects second-order Ito terms.

where ε_{t+1}^I has mean zero and is uncorrelated with innovations in M_{t+1}, in particular with disasters. This means, to the leading order, that $I_{t+1} - I_* \simeq \exp\left(-\phi_I\right)\left(I_t - I_*\right) + \varepsilon_{t+1}^I$, i.e. the process is a (twisted) AR (1). One could allow for non-zero correlation, but the analysis would become a bit more complicated.

Proposition 8 (Forward rates). *In the continuous-time limit, the domestic nominal forward rate is, up to second-order terms in $\widehat{H}_{it}$, and $I_t - I_*$:*

$$f_t\left(T\right) = r_{ei} - \lambda - \frac{r_{ei}}{r_{ei} + \phi_{H_i}} \exp\left(-\phi_{H_i} T\right) \widehat{H}_{it} + I_* + \exp\left(-\phi_I T\right)\left(I_t - I_*\right). \tag{30}$$

The nominal forward rate in (30) depends on real and nominal factors. The real factor is the resilience of the economy $\widehat{H}_{it}$. The nominal factor is inflation I_t.

Each term is multiplied by a term of the type $\exp\left(-\phi_{H_i} T\right)$. For small speeds of mean reversion ϕ, the forward curve is fairly flat.

We can derive the implications of our model for a Fama regression in nominal terms:

$$\frac{\tilde{e}_{i,t+1} - \tilde{e}_{it}}{\tilde{e}_{it}} - \frac{\tilde{e}_{j,t+1} - \tilde{e}_{jt}}{\tilde{e}_{jt}} = \tilde{\alpha} - \tilde{\beta}(\tilde{r}_{it} - \tilde{r}_{jt}) + \varepsilon_{ij,t+1}, \tag{31}$$

where $\tilde{r}_{it}$ and $\tilde{r}_{jt}$ are now, with some slight abuse of notation, the nominal interest rates in countries i and j. Our model's prediction is in the next proposition.

Proposition 9 (Value of the coefficient in the Fama regression in nominal terms). *In the nominal Fama regression (31) with forward rates, the coefficients are:*

$$\tilde{\beta}^{ND} = \tilde{\nu}\beta^{ND} + 1 - \tilde{\nu}, \qquad \tilde{\beta}^{Full} = \tilde{\nu}\beta^{Full} + 1 - \tilde{\nu}, \tag{32}$$

where β^{ND} and β^{Full} are the coefficients in the Fama regression defined in Proposition 4 and

$$\tilde{\nu} = \frac{1}{1 + \dfrac{\operatorname{Var}(I_{it} - I_{jt})}{\left(\frac{r_{ei}}{r_{ei} + \phi_{H_i}}\right)^2 \operatorname{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)}} \tag{33}$$

is the share of variance in the forward rate due to $\widehat{H}_t$.

In this simple model, the higher the variance of inflation, the closer $\tilde{\beta}^{ND}$ is to 1. Hence, countries with very variable inflation (typically countries with high average inflation) approximately satisfy the uncovered interest rate parity condition. Bansal and Dahlquist (2000) provide empirical support for this phenomenon. When disaster risks are very variable – and the real exchange rate is very variable – then $\tilde{\beta}^{ND}$ is more negative.

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3.6   Summing up

We gather here the key expressions we obtained. Recall $r_{ei} = R + \lambda - g_{\omega_i} - h_{i*}$, $r_{Di} = R - g_D - h_{Di*}$.

$$\text{Exchange rate: } e_{it} = \frac{\omega_{it}}{r_{ei}} \left(1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}} \right)$$

$$\text{Interest rate: } r_{it} = r_{ei} - \lambda - \frac{r_{ei} \widehat{H}_{it}}{r_{ei} + \phi_{H_i} + \widehat{H}_{it}}$$

$$\text{Option's risk reversal: } RR_{ijt} = k_\Delta \left(h_{j*} - h_{i*} + \widehat{H}_{jt} - \widehat{H}_{it} \right)$$

$$\text{Stock: } \frac{P_{Di,t}}{d_{it}} = \frac{1 + \frac{\widehat{H}_{Di,t}}{r_{Di} + \phi_{H_{Di}}}}{r_{Di}}$$

They express that asset prices are, in essence, driven by exchange rate resilience $\widehat{H}_{it}$ and stock resilience $\widehat{H}_{Di,t}$. When investors are more optimistic about country i, resilience $\widehat{H}_{it}$ is high, its exchange rate is appreciated, interest rates are low, and put premia are low. Similarly, the stock market of country i is driven by stock market resilience, $\widehat{H}_{Di,t}$. One advantage of the framework is its ability to express these intuitive dynamics in a tractable way, and to make new predictions, such as the negative correlation between exchange rates and risk-reversals. We note that other factors could be added, something we discuss in the online appendix. The next section offers a calibration of the model.

4   Calibration

4.1   Data

We use monthly data from JP Morgan presented in Farhi et al. (2014). Exchange rates are in US dollar per foreign currency. As a result, an increase in the exchange rate corresponds to an appreciation of the foreign currency and a decline of the US dollar. For each currency, our sample presents spot and forward exchange rates at the end of the month and implied volatilities from currency options for the same dates. We consider one-month forward rates and options with one-month maturity. Longer-term contracts are available but are much less traded. We construct foreign interest rates using forward currency rates and the US LIBOR. Options are quoted using their Black-Scholes implied volatilities for three different deltas: out-of-the-money
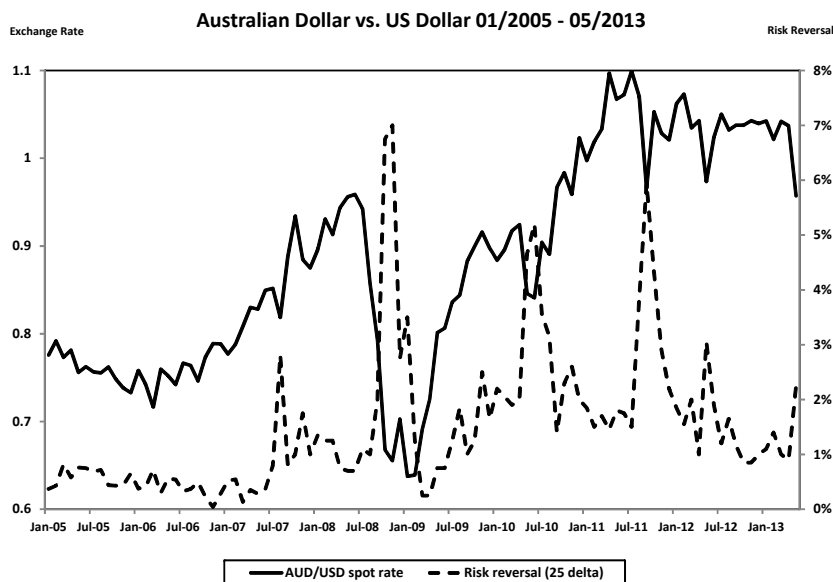
Figure 3: Australian dollar vs. US dollar exchange rate and risk reversal at 25 delta.

puts (denoted 25-delta puts), at-the-money puts and calls (50-delta), and out-of-the-money calls (denoted 25-delta calls) for the 2005-2013 period.

As will become apparent, it is easy to calibrate realistic values for the exchange rate volatility and for the interest rate level and volatility. The potential difficulty lies in calibrating option values at the same time. Using currency options, recent papers (Burnside, Eichenbaum, Kleshchelski, and Rebelo 2011, Farhi et al. 2014) have concluded that disaster risk plays an important role in currency markets. Still, to some extent, out-of-the-money put premia for risky currencies seemed somewhat low compared to those implied by a disaster model. The authors speculated that illiquidity or counterparty risk may play a role.[36]

Since the early versions of those papers, the global financial crisis happened. It turns out that it led to a large and durable increase in option prices, even after its peak – see the documentation in Farhi et al. (2014). As an illustration, Figure 3 plots the "risk reversal" of the Australian dollar, which is a major carry trade currency, since 2005. Before 2008, the risk reversals were on average 0.6%. The apex of the crisis was in the fall of 2008. But even

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[36]In the illiquidity view, deep out-of-the-money puts might be illiquid, and an agent who would like to buy them would move prices against him. In the counterparty view, insurance prices are low because in disasters the insurer (the seller of the put) will default: hence, the puts in the market are not default-free in the important (disaster) states of the world.

after spring 2009, the risk reversals were very high: they were more than three times as large as before the crisis.

This phenomenon is reminiscent of what happened with stock options before and after the 1987 crash (Jackwerth and Rubinstein 1996). Before the 1987 crash, there was no significant put premium (i.e., no skew). After the 1987 crash, a skew appeared and remained ever since. One interpretation is that options market participants became more keenly aware of crash risk: buyers of options traded them more, and option pricers incorporated crash risk into their models. Under that interpretation, options markets have become more efficient after the 1987 crash. It is conceivable that a similar phenomenon happened after the fall 2008 crisis.[37]

We show that our model can be successfully calibrated to post-crisis data, where risk reversals are higher than pre crisis, which we take to be an indication that disaster risk plays a more important role in currency markets.[38] Accordingly, for the calibration, we use data from January 2009 to May 2013.

4.2 Parameter Values

We present a calibration of the model. Our data is nominal; we therefore use the extension to a nominal setup. Up to second order terms, the differences in resiliences $H_{it} - H_{jt}$ are a sufficient statistic for the quantities of interest (which are bilateral, e.g. $\ln\left(\frac{e_{it}}{e_{jt}}\right)$, $r_{it} - r_{jt}$, etc.). Hence we specify parameters for those differences in resilience – rather than the absolute resilience H_{it} and H_{jt} and their correlation. These differences in resiliences could come from various combinations of shocks to the world disaster probability p_t, severity B_{t+1} and country-specific factors $F_{i,t+1}$. We discuss them later.

Table 1 summarizes the main inputs of the calibration. The justification is as follows.

Exchange rate and interest rate. We call Δ the time-difference operator, $\Delta x_t = x_t - x_{t-1}$, and $\sigma_x = stdev\left(\Delta x_t\right)$ the volatility of a variable x_t. For two countries, define the volatility of the bilateral exchange rate as $\sigma_e^{bil} = stdev\left(\Delta \ln \frac{e_{it}}{e_{jt}}\right)$ and the volatility of the difference in

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[37] Another possibility is that disaster-related risk simply became larger not just during the crisis but also in its aftermath. Indeed, the possibility of a second Great Depression was routinely entertained by market participants and commentators during and after the financial crisis.

[38] That same calibration would indicate that pre-crisis out-of-the-money put premia for risky currencies were somewhat low before the crisis.

interest rates $\sigma_r^{bil} = stdev\left(\Delta\left(r_{it} - r_{jt}\right)\right)$. Equations (11) and (13) give $\sigma_r^{bil} = r_e\sigma_e^{bil}$.[39] The above equation constrains our calibration. We will match $\sigma_e^{bil} \simeq 11\%$. In the sample, the volatility of the nominal interest rate is $\sigma_{\tilde{r}}^{bil} \simeq 0.7\%$. We therefore set $r_e = 6\%$.[40]

The standard deviation the innovations to relative resilience $H_{it} - H_{jt}$ are chosen to roughly match the level and volatility of the risk reversals, as well as the volatility of the exchange rate. For the speed of mean-reversion of resilience, we take $\phi_H = 18\%$, which gives a half-life of $\ln 2/\phi_H = 3.8$ years, in line with estimates from the exchange rate predictability literature (Rogoff 1996). We choose the volatilities of resilience to target the volatilities of RR and the exchange rate reported in Table 2.

<div align="center">Table 1: Key Parameter Inputs.</div>

Exchange rate discount rate	$r_e = 6\%$
Resilience: volatility and speed of mean-reversion	$\sigma_{H_{it}-H_{jt}} = 2.8\%, \phi_H = 18\%$
Inflation: volatility and speed of mean-reversion	$\sigma_I = 0.5\%, \phi_I = 30\%$
Stocks: volatility of dividends	$\sigma_D = 8.4\%$
Stocks resilience: volatility and speed of mean-reversion	$\sigma_{H_{iD}} = 3.9\%, \phi_{H_D} = 18\%$

Notes: This table reports the coefficients used in the model. σ_X is the average volatility, and ϕ_X is the speed of mean-reversion. The time unit is the year (the model is simulated at the monthly frequency, but for readability the numbers reported above are all annualized)

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[39] To keep the model parsimonious, we assume no default risk on debt. This is the cleanest assumption for developed countries. Of course, in many cases (e.g., when pricing sovereign debt), default risk can be added without changing anything about the exchange rate.

[40] The growth rate of productivity g_ω is irrelevant in practice, but for completeness we propose a specific value. We choose the growth rates so that in normal times consumption of non-tradables grows at a rate $g_c = 2\%$. We set $g_\omega = g_c$, but results are not sensitive to the choice of this parameter. We make sure that the riskless domestic short-term rate is on average around 2%, which pins down the rate of time preference δ. This parameter $r_{ei} = R + \lambda - g_{\omega_i} - h_{i*}$ is driven in the model by deeper combinations of underlying factors $p_t, B_{t+1}^{-\gamma}$, and F_{it} but mainly three parameters govern the key statics that we explore in Table 2. We take $\lambda = 4\%$ which generates a real interest rate of $r_{ei} - \lambda = 2\%$. The underlying rate of time preference δ is calibrated to match the value of r_{ei}. For simplicity, we take the recovery rate of productivity to be the average recovery rate of consumption, $F_{i*} = \mathbb{E}\left[B^{-\gamma}\right]^{1/\gamma}$. Hence we find a rate of time preference $\delta = 4.9\%$.

Inflation. Data (e.g., on currency options) are nominal, and the essence of our model is real. We pick inflation parameters that are broadly in line with averages in our sample.

Carry trade returns. We proceed as is usual in the carry trade literature, see e.g. Farhi et al. (2014). However, to better capture disaster risk, we sort on risk reversals rather than interest rates. We divide countries into two equal-sized bins of resilience; the risky countries are those in the bottom half of resilience, the less risky countries those in the top half. We define the carry trade as going long \$1 in the equal-weighted portfolio of risky countries and going short \$1 in the equal-weighted portfolio of safer countries (high $\widehat{H}_t$).

Stocks. Empirically, domestic stock returns and the exchange rate are uncorrelated on average (see Table 3). We specify the correlation between innovations to H_{Dit} and H_{it} to match this (the procedure is detailed in the online appendix). We match a volatility of dividends of 11%, as in Campbell and Cochrane (1999); this leads to $\sigma_{\varepsilon_D} = 8.4\%$. For parsimony, we take the speed of mean-reversion of dividend resilience to be that of exchange rate resilience, $\phi_{H_D} = \phi_H = 18\%$. This is in line with the range of estimates of the speed of mean reversion of the price/dividend ratio surveyed in Gabaix (2012). We choose σ_{H_D} to match the volatility of stock returns.

Interpreting resilience processes in terms of deeper disaster parameters

Resilience differentials are sufficient statistics for the calibration. We now discuss how their variations are related to deeper disaster parameters.

We take numbers from Barro and Ursua (2008).[41] The average probability of disasters is $\mathbb{E}[p] = 3.6\%$. An important parameter in the calibration is the risk-adjusted probability of disasters $\mathbb{E}[pB^{-\gamma}]$. Disasters are overweighted compared to their physical probability by a factor $\mathbb{E}[B^{-\gamma}]$. This factor is very sensitive to the severity of disasters and to the coefficient of relative risk aversion. We take $\gamma = 4$, which yields $\mathbb{E}[B^{-\gamma}]^{1/\gamma} = 0.66$. Hence, the "risk neutral" (i.e., risk-adjusted) probability of disasters equals $\mathbb{E}[pB^{-\gamma}] = 19.2\%$. Note that though $\mathbb{E}[B^{-\gamma}]^{1/\gamma} = 0.66$, which corresponds to a risk-adjusted average size of disaster of 34%, the median disaster in Barro and Ursua (2008) is much smaller: because of risk aversion, the small possibility of a large disaster matters a lot.

This calibration, strictly speaking, relies on a stark idealization in which consumption is

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[41]See also Gabaix (2012), Gourio (2012) and Wachter (2013) for related calibrations of disaster models in closed economies.

permanently affected after disasters. In practice, there is a partial recovery from disasters (Barro and Ursua 2008). For a given γ, that lowers the disaster risk premium (Gourio 2008). However, this can be remedied by increasing γ slightly. Indeed, Barro and Jin (2011) find an empirical power-law distribution of disaster sizes, so that a moderate γ can generate a very large (indeed infinite for a large enough γ) risk premium. In addition, for our purposes, the idealization of a permanent disaster seems like a good compromise between parsimony and realism.

Our calibration only requires the law of motion of the differential resilience, $H_{it} - H_{jt} = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} (F_{i,t+1} - F_{j,t+1}) \right]$. The results of the calibration do not depend on whether the shocks come from movements in p_t, $B_{t+1}^{-\gamma}$ or $F_{i,t+1} - F_{j,t+1}$.[42,43]

To interpret the volatility of $H_{it} - H_{jt} = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} (F_{i,t+1} - F_{j,t+1}) \right]$, we present the standard deviation of changes in $H_{it} - H_{jt}$ over a horizon of one year. Generally, call this object v_{X_t} for the standard deviation of a variable X_t at a one-year horizon: $v_{X_t} = stdev (X_{t+1\text{year}} - X_t)$. We take some polar cases. If the innovations come entirely from idiosyncratic movements of F_{it} (keeping p_t and $B_{t+1}^{-\gamma}$ constant at $\mathbb{E}[p]$ and $\mathbb{E}[B^{-\gamma}]$), then $v_{F_{it}} = 10.5\%$. This is broadly in line with Gabaix (2012), who argues that a one-year horizon volatility $\nu_{F_{it}} \simeq 10\%$ for the resilience of the aggregate stock market is plausible and does not violate variance bounds from historical data: hence, that calibration seems acceptable too. Conversely, suppose that innovations in differential resilience come entirely from movements in p_t (keeping $\mathbb{E}_t \left[B_{t+1}^{-\gamma} (F_{i,t+1} - F_{j,t+1}) \right]$ constant). With fixed values of F_{it}, e.g. $|F_{i,t+1} - F_{j,t+1}| = 0.4$ (similar to the numbers above), then we write $v_{p_t} = 1.3\%$. This is of the same order of magnitude as the calibration in Wachter (2013), which uses $v_{p_t} \simeq 1.1\%$.

4.3   Implications

Tables 2 and 3 present results from the calibration.

As Table 2 shows, the model hits the volatility of the bilateral exchange rate, i.e. generates the right amount of "excess volatility" in exchange rates. The model also roughly matches (and slightly undershoots) the size of disaster risk as measured by the average size of risk

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[42] For instance, movements in p_t generate a positive covariance between the innovations of H_{it} and H_{jt}, while idiosyncratic movements of $F_{i,t+1}$ and $F_{j,t+1}$ generate a 0 covariance. For the calibration, the covariance between the innovations of H_{it} and H_{jt} does not matter per se – only the variance of the innovations of $(H_{it} - H_{jt})$.

[43] This is true up to second order terms. We verify numerically that this is a good approximation.

Table 2: Moments: Empirical and in the Model

Moments	Data	Calibration
Std Dev$(\Delta \ln \tilde{e}_{ijt})$	12.35%	11.00%
Carry Trade Return	3.44%	2.74%
Mean$(|RR|)$	1.31%	0.97%
Std Dev(RR)	1.24%	1.07%
Std Dev(ΔRR_{ijt})	2.60%	0.80%
Std Dev$(\tilde{r}_{it})$	1.38%	0.94%
Std Dev$(\Delta(\tilde{r}_{it} - \tilde{r}_{jt}))$	0.71%	1.06%
Corr$(\Delta \ln \tilde{e}_{ijt}, \Delta RR_{ijt})$	-0.57	-0.67
Corr$(\ln \tilde{e}_{ij,t+1}, \ln \tilde{e}_{ijt})$	0.88	0.97
Corr$(\Delta \ln \tilde{e}_{ij,t+1}, \Delta \ln \tilde{e}_{ijt})$	-0.13	-0.011
Corr$(\tilde{r}_{it} - \tilde{r}_{jt}, RR_{ijt})$	0.55	0.69

Notes: The table reports the moments generated by the model, using the inputs from Table 1. The risk reversal RR_{ijt} is defined as the implied volatility of an out-of-the-money put minus the implied volatility of an out-of-the-money call, all at 25-delta. A high RR_{ijt} means that the price of protection from depreciation of currency i (against country j) is high. $\tilde{r}_{it}$ is the nominal interest rate. We define $\tilde{e}_{ijt} = \frac{\tilde{e}_{it}}{\tilde{e}_{jt}}$, the nominal bilateral exchange rate between countries i and j: a high $\tilde{e}_{ijt}$ means that currency i appreciates. Carry trade returns are the returns from a long-short portfolio going \$1 long (resp. short) an equal-size portfolio of high (resp. low) RR countries. The time unit is the year (the model is estimated and simulated at the monthly frequency, but for readability the numbers reported above are all annualized).

reversals.[44,45] At the same time, the model generates a moderate volatility of the interest rate, as in the data.

We showed earlier that in the model countries with high risk reversals have high interest rates and that increases in risk reversals are associated with depreciations of the exchange rate. The calibration shows that these predictions hold not just qualitatively, but also quantitatively: Table 2 reports the calibrated values of $\text{Cov}\left(\widetilde{r}_{it} - \widetilde{r}_{jt}, RR_t\right)$ and $\text{Corr}(\Delta \ln \widetilde{e}_{ijt}, \Delta RR_{ijt})$ and shows that they broadly match up with their empirical counterparts.

The carry trade generated by the model gives average returns in line with the empirical evidence (see Farhi et al. 2014 for more variants of the carry trade). Investing in countries with high risk reversals generates high expected returns. Indeed, the expected return of the carry trade (given positive RR) is about 3% per annum. Finally, the model generates Fama coefficients $\beta^{ND} = -0.66$ in line with estimates of the literature cited above.

Stocks　Table 3 shows the moments related to stocks. We call r_{it}^{stock} the return of the stock in country i, in the domestic currency. We also call $r_{it}^{\text{stock},\$}$ the return in a foreign currency, which we will take to be the dollar, and call $\widetilde{e}_{ijt} = \frac{\widetilde{e}_{it}}{\widetilde{e}_{jt}}$ the bilateral exchange rate between countries i and j.[46] The equity premium is about 6%, in line with the usual empirical estimates. We report the correlations between changes in the exchange rate of two countries, and changes in the relative stock returns: $corr\left(\Delta \ln \widetilde{e}_{ijt}, r_{it}^{\text{stock}} - r_{jt}^{\text{stock}}\right)$. Empirically, this correlation is close to 0: movements in the stock market and the exchange rate are uncorrelated on average. The model reproduces that fact. Likewise, changes in risk reversals and relative stock returns are essentially uncorrelated in the data and in the model ($corr\left(\Delta RR_{ijt}, r_{it}^{\text{stock}} - r_{jt}^{\text{stock}}\right)$ is close to 0).

We also study the correlation between stock returns in a common currency and the change in the exchange rate: $corr\left(\Delta \ln \widetilde{e}_{ijt}, r_{i,t}^{\text{stock},\$} - r_{jt}^{\text{stock},\$}\right)$. Empirically, this correlation is high, about 0.67. The model is roughly in line with this. Likewise, there is a negative correlation between

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[44]We take the mean of the absolute values of risk reversals because, by symmetry, the mean of risk reversals is 0.

[45]If we increased the value of $\mathbb{E}\left[pB^{-\gamma}\right]$, for example by slightly increasing γ, we could match better the average value of the RR and other moments, without requiring a larger volatility of the relative prospective recovery rate $F_{it} - F_{jt}$ or of the probability of disaster p_t. We thought it was more parsimonious to stick to the numbers from the previous literature for $\mathbb{E}\left[pB^{-\gamma}\right]$, e.g. Gabaix (2012).

[46]The returns are: $r_{it}^{\text{stock}} = \frac{P_{it} + D_{it}}{P_{i,t-1}} - 1$ and $r_{it}^{\text{stock},\$} = \frac{e_{it}/e_{jt}}{e_{i,t-1}/e_{j,t-1}} \frac{P_{it} + D_{it}}{P_{i,t-1}} - 1 \simeq r_{it}^{\text{stock}} + \Delta \ln \frac{e_{it}}{e_{jt}}$.

Table 3: Moments related to Stocks: Empirical and in the Model

Moments	Data	Calibration
Equity premium: Mean($r_{it}^{\text{stock}} - r_{it}$)	6%	5.9%
Volatility of stock returns: Std Dev(r_{it}^{stock})	17%	17%
Volatility of stock returns in foreign currency: Std Dev($r_{it}^{\text{stock,\$}}$)	20%	20%
Corr($\Delta \ln \tilde{e}_{ijt}, r_{it}^{\text{stock}} - r_{jt}^{\text{stock}}$)	−0.05	0.04
Corr($\Delta \ln \tilde{e}_{ijt}, r_{it}^{\text{stock,\$}} - r_{jt}^{\text{stock,\$}}$)	0.67	0.43
Corr($\Delta RR_{ijt}, r_{it}^{\text{stock}} - r_{jt}^{\text{stock}}$)	−0.09	−0.04
Corr($\Delta RR_{ijt}, r_{it}^{\text{stock,\$}} - r_{jt}^{\text{stock,\$}}$)	−0.39	−0.41

Notes. The table reports the moments generated by the model, using the inputs from Table 1. r_{it}^{stock} (resp. $r_{it}^{stock,\$}$) is the stock return of country i expressed in country i's domestic currency (resp. in US dollars). The risk reversal RR_{ijt} is defined as the implied volatility of an out-of-the-money put minus the implied volatility of an out-of-the-money call, all at 25-delta. A high RR_{ijt} means that the price of protection from depreciation of currency i (against country j) is high. We define $\tilde{e}_{ijt} = \frac{\tilde{e}_{it}}{\tilde{e}_{jt}}$, the nominal bilateral exchange rate between countries i and j: a high $\tilde{e}_{ijt}$ means that currency i appreciates. Here we take country j to be the US. The time unit is the year (the model is estimated and simulated at the monthly frequency, but for readability the numbers reported above are all annualized).

changes in risk reversals and relative stock returns: $corr\left(\Delta RR_{ijt}, r_{i,t}^{\text{stock,\$}} - r_{jt}^{\text{stock,\$}}\right) < 0$ in the data and the model. Economically, when country i becomes less risky, the risk-reversal RR_{ijt} goes down, the exchange rate $\tilde{e}_{ijt}$ appreciates, and its realized stock return differential $r_{it}^{\text{stock,\$}} - r_{jt}^{\text{stock,\$}}$ is higher.

We conclude that the disaster model can be made quantitatively broadly congruent with the salient empirical facts.

4.4 Covariance structure in currencies and HML_{FX}

In influential work, Lustig, Roussanov and Verdelhan (2011) find that a one-factor structure in currency returns. Namely, they form a portfolio, HML_{FX}, going long high interest rate currencies and short low interest rate currencies. They also find that currency excess returns are accounted for by the HML_{FX} factor. They find, in essence, that regressing currency return

$r_{it}^{Currency} = \alpha_i + \beta_i HML_{FX_t} + \varepsilon_{it}$ yields $\alpha_i = 0$. Here $r_{it}^{Currency}$ is the currency return (capital gains plus interest rate) when going long a basket i (e.g., high interest rate currencies), and short a diversified basket of currencies.

In this subsection, we keep the previous calibration, but give it the additional structure of a one-factor model in currencies and stocks, respectively. We will find that we can match the salient facts related to HML_{FX}, and make a new, successful prediction linking it to risk reversals.

Factor structure We define the normalized resiliences of stocks and exchange rates as:

$$h_{Dit} := \frac{1 + \frac{\widehat{H}_{Di,t}}{r_{Di} + \phi_{H_{Di}}}}{1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}} - 1, \qquad h_{eit} := \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}$$

Using this notation, we can re-express equations 11 and 26 as follows:

$$e_{it} = \frac{\omega_{it}}{r_{ei}}\left(1 + h_{eit}\right), \qquad P_{Di,t} = \frac{r_{ei}}{r_{Di}}\frac{D_{it}}{\omega_{it}}\left(1 + h_{Dit}\right).$$

The innovation to the exchange rate and the stock price (and return) are captured by h_{eit} and h_{Dit}, respectively. We call ε_{X_t} the innovation to a random variable X_t ($\varepsilon_{X_t} = X_t - \mathbb{E}_{t-1}\left[X_t\right]$).

We posit that the innovation to normalized resilience follows a one-factor structure: $\varepsilon_{h_{eit}} = \beta_{ei,t-1}f_{et} + \eta_{eit}$ where f_{et}, η_{eit} are mean-0 innovations. We also specify $\beta_{eit} = b\left(H_{at} - H_{it}\right)$ with $b > 0$, and H_{at} is the *average* of H_{it} over all other countries. This means that when f_{et} is positive, the spread in the resilience between risky and less risky countries shrinks, i.e. risky currencies appreciate over least risky currencies. Hence, f_{et} is proportional to $HML_{FX,t}$. So, when f_{et} is positive, $HML_{FX,t}$ is positive, and the risk reversal of risky countries goes down while their exchange rate appreciates. The key free parameter is b, which we set to 0.114.

Empirically, international stocks markets tend to covary. This naturally suggest a one-factor structure of stock resilience: $\varepsilon_{h_{Dit}} = \beta_{Di,t-1}f_{Dt} + \eta_{Dit}$. We set $\beta_{Dit} = 1$, and $corr\left(f_{et}, f_{Dt}\right) = 0.4$, which allows to match the empirical correlation between the HML_{FX} factor and average of international stock market returns.[47] The factors η_{eit} and η_{Dit} are uncorrelated with other variables.

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[47]The online appendix details the process, including linearity-generating terms.

Table 4: Moments related to HML_{FX}: Empirical and in the Model

Moments	Data	Calibration
$\text{Corr}(HML_{FX,t}, r_{at}^{\text{Stock}})$	0.39	0.28
Test of the one factor-structure: α_i	$0^{(*)}$	$0^{(*)}$
in $r_{it}^{Currency} = \alpha_i + \beta_{i,HML} HML_{FX_t} + \varepsilon_{it}$		
	(for H,M,L portfolio)	(for H,M,L portfolio)
$\text{Corr}(HML_{FX,t}, \Delta \ln \frac{\widetilde{e}_{it}}{\widetilde{e}_{at}})$	$(0.57, 0.01, -0.52)$	$(0.39, 0.02, -0.42)$
$\text{Corr}(HML_{FX,t}, \Delta RR_{it})$	$(-0.44, -0.09, 0.37)$	$(-0.38, -0.03, 0.40)$

Notes. Here $HML_{FX,t}$ is the return of a portfolio going long high interest rate currencies and short low interest rate currencies. r_{at}^{Stock} is the average of stock market returns across countries. $r_{it}^{Currency}$ is the currency return (capital gains plus interest rate) when going long a basket i (high / medium / low interest rate currencies), and short an equal-weight basket of all currencies. $0^{(*)}$ means that the value is not statistically different from 0. $\frac{\widetilde{e}_{it}}{\widetilde{e}_{at}}$ is the nominal exchange rate of country i vis-a-vis an equal-weighted basket of all currencies ($\widetilde{e}_{at}$ is the average exchange rate across countries, $\ln \widetilde{e}_{at} = \frac{1}{n} \sum_{j=1}^{n} \ln \widetilde{e}_{jt}$). Countries are sorted by interest rates, and are divided into three groups of High, Medium and Low interest rates (H,M,L).

Results Table 4 shows the results.[48] As expected, if we sort countries by interest rates (High, Medium and Low interest rates: H,M,L) (recall that in our model, risky countries have high real interest rates), we observe that risky countries have a positive correlation with HML_{FX}, and the least risky countries have a negative correlation with it (see the row $\text{Corr}(HML_{FX,t}, \Delta \ln \frac{\widetilde{e}_{it}}{\widetilde{e}_{at}})$). The model produces a good quantitative fit of this fact.

We also verify that the one-factor structure shown in Lustig, Roussanov and Verdelhan (2011) is replicated in our model (see the row on $r_{it}^{Currency} = \alpha_i + \beta_{i,HML} HML_{FX_t} + \varepsilon_{it}$).[49] In addition, the model replicates the positive correlation between HML_{FX} returns and average stock market returns.

The new prediction of the model is that risk reversals should covary with HML_{FX}: the risk

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[48] This Table is computed over the whole sample, to maximize representativeness. The numbers are broadly the same when restricting to the post-2009 sample, except $\text{Corr}(HML_{FX,t}, r_{at}^{\text{Stock}})$, which is smaller in that sample. We suspect that this number is not representative of typical samples.

[49] If we computed the returns of portfolios short a given currency (say the dollar), then we would need to add a second factor, namely the return of that currency.

reversal of risky countries should covary negatively with HML_{FX}, while the risk reversal of the less risky countries should covary positively with it. This is indeed the case in the data, as indicated in Table 4. We view this as an additional comforting, previously undocumented, disaster-like feature of the data.

In conclusion, a parsimonious calibration of the model can replicate the major moments of the link between currencies, interest rates, stocks and options, including the factor structure documented in stocks and currencies.

5 Conclusion

We have proposed a disaster-based tractable framework for exchange rates. Our framework accounts qualitatively and quantitatively for both classic exchange rate puzzles (e.g. excess volatility of exchange rates, forward premium puzzle, excess return of the carry trade) and links between currency options, exchanges rates and interest rates – signature predictions of the disaster hypothesis.

The model is fully solved in closed form. It can readily be extended in several ways. The online appendix of this paper works out various extensions, including a detailed model of the term structure and the incorporation of business cycle movement.[50]

The model offers a unified, tractable and calibrated treatment of the major assets and their links: exchange rates, bonds, stocks and options. Hence, we hope it may be a useful point of departure to think about issues in international macro-finance. In particular, given that the model calibrates correctly, studying business cycles and production in this disaster

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[50]These extensions rationalize additional empirical facts uncovered by Boudoukh, Richardson and Whitelaw (2012) on forward rates and deviations from UIP, and Lustig, Stathopoulos and Verdelhan (2014) on the term premium in the bonds and currency riskiness. Boudoukh, Richardson and Whitelaw (2012) find that the carry trades based on long-dated forward rates exhibit small deviations from UIP: in our model, this is because at long horizon, other factors (e.g. business cycle or inflation) are more important, and they tend to generate no deviation from UIP. Lustig, Stathopoulos and Verdelhan (2014) find that "risky" currencies have a lower term premium. In our model, this holds even if all countries have the same inflation dynamics, where inflation goes up in disasters (as in the historical experience on average), which creates a term premium. Because risky currencies will depreciate in a disaster, a portfolio long their short-term bond and short their long-term bond will have little value in a disaster, whereas the equivalent portfolio for safe countries will have a positive value: hence, the term premium is high in safe countries and small or zero in risky countries.

environment seems like a fruitful direction for research. Pursuing this direction could lead to a unified international macro model to think jointly about prices and quantities.

Appendix A: Results for Linearity-Generating Processes

The paper uses the linearity-generating (LG) processes defined and analyzed in Gabaix (2009). This appendix gathers the main results. LG processes are given by $M_t D_t$, a pricing kernel M_t times a dividend D_t, and X_t, an n-dimensional vector of factors (that can be thought of as stationary). For instance, for bonds, the dividend is $D_t = 1$.

By definition, a process $M_t D_t (1, X_t)$ is LG if and only if there are constant $\alpha \in \mathbb{R}$, γ, $\delta \in \mathbb{R}^n$, and $\Gamma \in \mathbb{R}^{n \times n}$ such that for all $t = 0, 1, ..., M_t D_t > 0$ and

$$\mathbb{E}_t \left[\frac{M_{t+1} D_{t+1}}{M_t D_t} \right] = \alpha + \delta' X_t, \tag{34}$$

$$\mathbb{E}_t \left[\frac{M_{t+1} D_{t+1}}{M_t D_t} X_{t+1} \right] = \gamma + \Gamma X_t. \tag{35}$$

Higher moments need not be specified. For instance, the distribution of the noise does not matter, which makes LG processes parsimonious. As a shorthand, $M_t D_t (1, X_t) =: Y_t$ is an LG process with generator $\Omega = \begin{pmatrix} \alpha & \delta' \\ \gamma & \Gamma \end{pmatrix}$. It satisfies $\mathbb{E}_t [Y_{t+1}] = \Omega Y_t$.

Stock and bond prices obtain in closed form. The price of a stock $P_t = \mathbb{E}_t \left[\sum_{s \geq t} M_s D_s \right] / M_t$ is, with I_n, the identity matrix of dimension n:

$$P_t = D_t \frac{1 + \delta' (I_n - \Gamma)^{-1} X_t}{1 - \alpha - \delta' (I_n - \Gamma)^{-1} \gamma}. \tag{36}$$

The price-dividend ratio of a "bond," or $Z_t (T) = \mathbb{E}_t [M_{t+T} D_{t+T}] / (M_t D_t)$, is:

$$Z_t (T) = \begin{pmatrix} 1 & 0_n \end{pmatrix} \Omega^T \begin{pmatrix} 1 \\ X_t \end{pmatrix} \tag{37}$$

$$= \alpha^T + \delta' \frac{\alpha^T I_n - \Gamma^T}{\alpha I_n - \Gamma} X_t \text{ when } \gamma = 0. \tag{38}$$

Hence, the "recipe" to solve a model using LG processes is very simple: First, calculate the LG moments (34)-(35), to obtain the values of α, δ, γ, and Γ. Second, use (36) and (37)-(38) to solve for stock and bond prices.

Conversely, the "recipe" to construct a model using LG processes is to force the model's primitives (e.g., twists in the AR(1) processes) to satisfy (34)-(35). Then, the model is very easy to solve by the above procedure.

To ensure that the process is well-behaved (and, hence, will prevent prices from being negative), the volatility of the process has to go to zero near some boundary. Gabaix (2009) and the online appendix to this paper detail these conditions.

Appendix B: Complements and Proofs

5.1   Different Notions of the Exchange Rate

In the paper, we define the "absolute" exchange rate e_{it} to be the price of the non-traded good in country i in terms of the world numéraire. The more traditional definition would be $\mathcal{E}_{it}$, the price of the consumption basket in country i in terms of the world numéraire.[51] Using the usual algebra of CES price indices, the link between the two is:

$$\mathcal{E}_{it} = \left(\xi^{\frac{1}{\gamma}} + e_{it}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}. \tag{39}$$

The share of traded goods in consumption is $\xi^{\frac{1}{\gamma}} \left(\xi^{\frac{1}{\gamma}} + e_{it}^{\frac{\gamma-1}{\gamma}} \right)^{-1}$. In the data, this share is small, so that ξ is close to 0. The consequence is that $\mathcal{E}_{it} \simeq e_{it}$ and $\frac{\mathcal{E}_{it}}{\mathcal{E}_{jt}} \simeq \frac{e_{it}}{e_{jt}}$, so that the two notions are quantitatively close. This approximation is exact up to a term $O\left(\xi^{\frac{1}{\gamma}}\right)$. It is analytically simpler to characterize the behavior of e_{it}. In any case, it is possible to go back and forth between the two notions using equation (39).

5.2   Proofs

We present here the proofs to the main results. Additional proofs are in the online appendix. For simplicity, we drop the country index i in most proofs.

Proof of Proposition 2.   Let $D_t = \exp\left(-\lambda t\right)\omega_t$ and $X_t = \widehat{H}_t$ (for simplicity, we drop the subscript i in this proof). By Proposition 1, we have

$$e_0 = \mathbb{E}_0 \left[\sum_{s=0}^{\infty} M_t^* D_t \right] / M_0^*. \tag{40}$$

We calculate the moments:

$$\mathbb{E}_t \left[\frac{M_{t+1}^* D_{t+1}}{M_t^* D_t} \right] = \exp\left(-R - \lambda + g_\omega\right) \left\{ (1 - p_t) + p_t \mathbb{E}_t \left[B_{t+t}^{-\gamma} F_{t+1} \right] \right\}$$

$$= \exp\left(-R - \lambda + g_\omega\right)(1 + H_t) = \exp\left(-R - \lambda + g_\omega\right)(1 + H_*) + \exp\left(-R - \lambda + g_\omega\right)\widehat{H}_t$$

$$= \exp\left(-R - \lambda + g_\omega\right)(1 + H_*) + \exp\left(-R - \lambda + g_\omega\right)X_t$$

$$= \exp\left(-r_e\right) + \exp(-r_e - h_*)X_t,$$

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[51]The bilateral exchange rate between country i and country j is then $\frac{\mathcal{E}_{it}}{\mathcal{E}_{jt}}$.

using $r_e = R + \lambda - g_\omega - h_*$. Also:

$$\mathbb{E}_t\left[\frac{M^*_{t+1}D_{t+1}}{M^*_t D_t}X_{t+1}\right] = \mathbb{E}_t\left[\frac{M^*_{t+1}D_{t+1}}{M^*_t D_t}\right]\mathbb{E}_t\left[X_{t+1}\right]$$

$$= e^{-R-\lambda+g_\omega}\left(1+H_t\right)\frac{1+H_*}{1+H_t}e^{-\phi_H}\widehat{H}_t$$

$$= e^{-R-\lambda+g_\omega-\phi_H}\left(1+H_*\right)\widehat{H}_t = e^{-r_e-\phi_H}X_t.$$

There are two ways to conclude. *The first way* uses the notations of Appendix A: the above two moment calculations show that $Y_t = M^*_t D_t\left(1, X_t\right)$ is an LG process, with generator Ω:

$$\Omega = \begin{pmatrix} \exp\left(-r_e\right) & \exp\left(-r_e - h_*\right) \\ 0 & \exp\left(-r_e - \phi_H\right) \end{pmatrix}.$$

Using equation 36, we find

$$e_{i0} = \frac{\omega_{i0}}{1-\exp\left(-r_{ei}\right)}\left(1+\frac{\exp\left(-r_{ei}-h_{i*}\right)}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)}\widehat{H}_{i0}\right). \tag{41}$$

More generally,

$$e_{it} = \frac{\omega_{it}}{1-\exp\left(-r_{ei}\right)}\left(1+\frac{\exp\left(-r_{ei}-h_{i*}\right)}{1-\exp\left(-r_{ei}-\phi_{H_i}\right)}\widehat{H}_{it}\right), \tag{42}$$

where ω_{it} is the current productivity of the country.

The second way (which is less rigorous, but does not require the results on LG processes) is to look for a solution of the type $e_t = \omega_t\left(a + b\widehat{H}_t\right)$, for some constants a and b, which satisfies: $e_t = \omega_t + \mathbb{E}_t\left[M^*_{t+1}\exp\left(-\lambda\right)e_{t+1}/M^*_t\right]$. Dividing by ω_t, this is:

$$a + b\widehat{H}_t = 1 + \mathbb{E}_t\left[\frac{M^*_{t+1}D_{t+1}}{M^*_t D_t}\left(a + b\widehat{H}_{t+1}\right)\right] = 1 + a\mathbb{E}_t\left[\frac{M^*_{t+1}D_{t+1}}{M^*_t D_t}\right] + b\mathbb{E}_t\left[\frac{M^*_{t+1}D_{t+1}}{M^*_t D_t}\widehat{H}_t\right]$$

$$= 1 + a\left[e^{-r_e} + e^{-r_e-h_*}\widehat{H}_t\right] + be^{-r_e-\phi_H}\widehat{H}_t,$$

which should hold for all $\widehat{H}_t$. Solving for a and b, we get $a = 1+e^{-r_e}a$, $b = e^{-r_e-h_*}a+be^{-r_e-\phi_H}$, and (42).

Proof of Proposition 3. In this proof, it is useful to define $x_t = e^{-h_*}\widehat{H}_t$. Then, $\mathbb{E}_t\left[\frac{M^*_{t+1}\omega_{t+1}}{M^*_t\omega_t}\right] = \exp\left(-R+g_\omega\right)\left(1+H_t\right) = \exp\left(-r_e+\lambda\right)\left(1+x_t\right)$.

Also, $\mathbb{E}_t\left[x_{t+1}\right] = \exp\left(-\phi\right)\frac{x_t}{1+x_t}$, and $e_t = \omega_t A\left(1+Bx_t\right)$ with $A = 1/\left(1-\exp\left(-r_e\right)\right)$, $B = \frac{\exp(-r_e)}{1-\exp(-r_e-\phi_H)}$. Thus:

$$1 + r_t = \frac{M_t^* e_t}{\mathbb{E}_t \left[M_{t+1}^* e_{t+1} \right]} = \frac{A(1 + Bx_t)}{\mathbb{E}_t \left[\frac{M_{t+1}^* \omega_{t+1}}{M_t^* \omega_t} A(1 + Bx_{t+1}) \right]} = \frac{1 + Bx_t}{\mathbb{E}_t \left[\frac{M_{t+1}^* \omega_{t+1}}{M_t^* \omega_t} \right] \mathbb{E}_t \left[1 + Bx_{t+1} \right]}$$

$$= \frac{1 + Bx_t}{\exp(-r_e + \lambda)(1 + x_t)\left(1 + B\frac{\exp(-\phi_H)x_t}{1 + x_t} \right)} = \exp(r_e - \lambda)\frac{1 + Bx_t}{1 + x_t(1 + B\exp(-\phi_H))}$$

$$= \exp(r_e - \lambda)\frac{1 + \frac{\exp(-r_e)}{1 - \exp(-r_e - \phi_H)}x_t}{1 + \frac{1}{1 - \exp(-r_e - \phi_H)}x_t}$$

$$= \exp(r_e - \lambda)\left[1 - \frac{(1 - \exp(-r_e))\exp(-h_*)\widehat{H}_t}{1 - \exp(-r_e - \phi_H) + \exp(-h_*)\widehat{H}_t} \right].$$

Hence,

$$r_{it} = \exp(r_{ei} - \lambda)\left[1 - \frac{(1 - \exp(-r_{ei}))\exp(-h_{i*})\widehat{H}_{it}}{1 - \exp(-r_{ei} - \phi_{H_i}) + \exp(-h_{i*})\widehat{H}_{it}} \right] - 1. \tag{43}$$

Proof of Proposition 4. *Derivation of (15).* Using (11), we calculate (up to second order terms $O(\varepsilon^2)$)

$$\mathbb{E}_t^{ND}\left[\frac{e_{i,t+1} - e_{it}}{e_{it}} \right] = \mathbb{E}_t^{ND}\left[\frac{\omega_{i,t+1}}{\omega_{it}}\frac{1 + \frac{\widehat{H}_{i,t+1}}{r_{ei} + \phi_{H_i}}}{1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}} - 1 \right]$$

$$= \mathbb{E}_t^{ND}\left[(1 + g_\omega)\frac{1 + \frac{\widehat{H}_{it+1}}{r_{ei} + \phi_{H_i}}}{1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}} - 1 \right] + O(\varepsilon^2)$$

$$= g_\omega + \mathbb{E}_t^{ND}\left[\frac{1 + \frac{\widehat{H}_{it+1}}{r_{ei} + \phi_{H_i}}}{1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}}} - 1 \right] + O(\varepsilon^2)$$

$$= g_\omega + \mathbb{E}_t^{ND}\left[\frac{\widehat{H}_{it+1} - \widehat{H}_{it}}{r_{ei} + \phi_{H_i}} \right] + O(\varepsilon^2)$$

$$= g_\omega - \frac{\phi_{H_i}\widehat{H}_{it}}{r_{ei} + \phi_{H_i}} + O(\varepsilon^2),$$

which together with (13) gives (17).

We next turn to the unconditional Fama regression. Using equation (12), we have:

$$\frac{1 + r_{jt}}{1 + r_{it}} = \frac{\mathbb{E}_t \left[\frac{M_{t+1}^* e_{i,t+1}}{M_t^* e_{it}} \right]}{\mathbb{E}_t \left[\frac{M_{t+1}^* e_{j,t+1}}{M_t^* e_{jt}} \right]}, \tag{44}$$

which in the limit of small time intervals can be expressed as:[52]

$$r_{jt} - r_{it} = \mathbb{E}_t \left[\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right] + \text{Cov}_t \left(\frac{M^*_{t+1}}{M^*_t}, \frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right),$$

i.e.,

$$\mathbb{E}_t \left[\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right] = r_{jt} - r_{it} - \text{Cov}_t \left(\frac{M^*_{t+1}}{M^*_t}, \frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right).$$

This expression highlights the role of the risk premium $\pi_t^{i,j}$:

$$\pi_t^{i,j} = - \text{Cov}_t \left(\frac{M^*_{t+1}}{M^*_t}, \frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right).$$

Consider now the Fama (1984) regression of the change in the exchange rate between countries i and j regressed on the interest rate differential in a full sample:

$$\text{Fama regression: } \mathbb{E}_t \left[\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right] = \alpha^{Full} - \beta^{Full}(r_{it} - r_{jt}). \tag{45}$$

The coefficient β^{Full} is now given by:

$$\beta^{Full} = 1 - \frac{\text{Cov}(\pi_t^{i,j}, r_{it} - r_{jt})}{\text{Var}(r_{it} - r_{jt})}.$$

Therefore, we can have $\beta^{Full} < 0$ if and only if the risk premium covaries positively enough with the interest rate differential. It is easy to compute

$$\pi_t^{i,j} = (1 - \beta^{ND})(r_{it} - r_{jt}) + p_t \mathbb{E}_t \left[F_{i,t+1} - F_{j,t+1} \right],$$

which leads to

$$\beta^{Full} = \beta^{ND} - \frac{\text{Cov}\left(p_t \mathbb{E}_t \left[F_{i,t+1} - F_{j,t+1} \right], r_{it} - r_{jt}\right)}{\text{Var}(r_{it} - r_{jt})}$$

$$\beta^{Full} = \beta^{ND} + (1 - \beta^{ND}) \frac{\text{Cov}\left(p_t \mathbb{E}_t \left[F_{i,t+1} - F_{j,t+1} \right], \widehat{H}_{it} - \widehat{H}_{jt}\right)}{\text{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)}. \tag{46}$$

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[52]To see this, express $\frac{M^*_{t+1}}{M^*_t} = 1 + m^*_{t+1}$, and $\frac{e_{i,t+1} - e_{it}}{e_{it}} = 1 + x_{it+1}$, where m^*_{t+1} and x_{it+1} are small. Then, (44) becomes, up to second order terms (denoted $O(\varepsilon^2)$):

$$1 + r_{jt} - r_{it} + O(\varepsilon^2) = \frac{\mathbb{E}_t \left[\left(1 + m^*_{t+1}\right)\left(1 + x_{it+1}\right) \right]}{\mathbb{E}_t \left[\left(1 + m^*_{t+1}\right)\left(1 + x_{jt+1}\right) \right]}$$

$$= \frac{1 + \mathbb{E}_t \left[m^*_{t+1} + x_{it+1} \right] + \text{Cov}_t \left(m^*_{t+1}, x_{it+1} \right)}{1 + \mathbb{E}_t \left[m^*_{t+1} + x_{jt+1} \right] + \text{Cov}_t \left(m^*_{t+1}, x_{jt+1} \right)}$$

$$= 1 + \mathbb{E}_t \left[m^*_{t+1} + x_{it+1} \right] - \mathbb{E}_t \left[m^*_{t+1} + x_{jt+1} \right] + \text{Cov}_t \left(m^*_{t+1}, x_{it+1} \right) - \text{Cov}_t \left(m^*_{t+1}, x_{jt+1} \right) + O(\varepsilon^2)$$

$$= 1 + \mathbb{E}_t \left[x_{it+1} - x_{jt+1} \right] + \text{Cov}_t \left(m^*_{t+1}, x_{it+1} - x_{jt+1} \right)$$

$$= \mathbb{E}_t \left[\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right] + \text{Cov}_t \left(\frac{M^*_{t+1}}{M^*_t}, \frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} \right)$$

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In the case where B_{t+1} is constant and equal to B, and $p_t \mathbb{E}_t [F_{i,t+1} - F_{j,t+1}] = \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) B^\gamma$, we have:

$$\beta^{Full} = \beta^{ND} + (1 - \beta^{ND})B^\gamma = -\frac{\phi_H}{r_e} + \left(1 + \frac{\phi_H}{r_e} \right) B^\gamma.$$

Proof of Proposition 6. *Call price.* We start with the call price:

$$V^C (K) = \mathbb{E}_0 \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}} \right)^+ \right]$$

$$= (1 - p_0) \mathbb{E}_0^{ND} \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}} \right)^+ \right] + p_0 \mathbb{E}_0^{D} \left[\frac{M_1^*}{M_0^*} \left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}} \right)^+ \right]$$

$$= (1 - p_0) e^{-R} \mathbb{E}_0^{ND} \left[\left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}} \right)^+ \right] + p_0 e^{-R} \mathbb{E}_0^{D} \left[B_1^{-\gamma} \left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}} \right)^+ \right],$$

where ND and D superscripts denote expectation conditional on no disasters and disaster, respectively. The next calculation uses the following lemma, which is standard.[53]

Lemma 4 *(Discrete-time Girsanov) Suppose that (x, y) are jointly Gaussian distributed under P. Consider the measure Q defined by $dQ/dP = \exp(x - \mathbb{E}[x] - \text{Var}(x)/2)$. Then, under Q, y is Gaussian, with distribution*

$$y \sim^Q \mathcal{N} \left(\mathbb{E}[y] + \text{Cov}(x, y), \text{Var}(y) \right), \tag{47}$$

where $\mathbb{E}[y]$, $\text{Cov}(x, y)$, and $\text{Var}(y)$ are calculated under P.

To perform the calculation, write for the ND case $\frac{e_{i,1}}{e_{i,0}} = \exp(\mu_i + \varepsilon_i - \sigma_i^2/2)$, and the analogue for j. We call $\eta = \varepsilon_j - \varepsilon_i$, and calculate:

$$V_1 = \mathbb{E}_0^{ND} \left[\left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}} \right)^+ \right] = \mathbb{E}_0^{ND} \left[(\exp(\mu_j + \varepsilon_j - \sigma_j^2/2) - K \exp(\mu_i + \varepsilon_i - \sigma_i^2/2))^+ \right]$$

$$= \exp(\mu_i) \mathbb{E}_0^{ND} \left[\exp(\varepsilon_i - \sigma_i^2/2) \left(\exp(\mu_j - \mu_i - \sigma_j^2/2 + \sigma_i^2/2 + \eta) - K \right)^+ \right].$$

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[53]To verify it, we calculate that the characteristic function of y is the characteristic function of distribution (47):

$$\mathbb{E}^Q \left[e^{ky} \right] = \mathbb{E} \left[e^{x - \mathbb{E}[x] - \sigma_x^2/2} e^{ky} \right] = \exp \left(k\mathbb{E}[y] + \frac{k^2 \sigma_y^2}{2} + k \, \text{Cov}(x, y) \right) = \exp \left[k \left(\mathbb{E}[y] + \text{Cov}(x, y) \right) + \frac{k^2 \sigma_y^2}{2} \right].$$

We define $dQ/dP = \exp\left(\varepsilon_i - \sigma_i^2/2\right)$, and use Lemma 4. Under Q, $y = \mu_j - \mu_i - \sigma_j^2/2 + \sigma_i^2/2 + \eta$ is a Gaussian variable with variance σ_η^2 and mean:

$$\mathbb{E}^Q\left[y\right] = \mu_j - \mu_i - \sigma_j^2/2 + \sigma_i^2/2 + \mathrm{Cov}\left(\varepsilon_j - \varepsilon_i, \varepsilon_i\right)$$

$$= \mu_j - \mu_i - \sigma_j^2/2 - \sigma_i^2/2 + \sigma_{i,j} = \mu_j - \mu_i - \mathrm{Var}\left(\eta\right)/2.$$

Hence,

$$V_1 = \exp\left(\mu_i\right)\mathbb{E}^Q\left[\left(e^y - K\right)^+\right] = \exp\left(\mu_i\right)\mathbb{E}^Q\left[\left(\exp\left(\mu_j - \mu_i - \mathrm{Var}\left(\eta\right)/2 + \eta\right) - K\right)^+\right]$$

$$= \exp\left(\mu_j\right)V_{BS}^C\left(K\exp\left(\mu_i - \mu_j\right), \sigma_{i/j}\right),$$

where $\sigma_{i|j} = \left(\mathrm{Var}\left(\varepsilon_j - \varepsilon_i\right)\right)^{1/2}$ and $V_{BS}^C\left(K, \sigma\right) = \mathbb{E}\left[\left(\exp\left(\sigma u - \sigma^2/2\right) - K\right)^+\right]$ (with u a standard Gaussian) is the Black-Scholes call value when the interest rate is 0, the maturity 1, the strike K, the spot price 1, and the volatility σ.

Next, we observe that:

$$\mathbb{E}_0^D\left[B_1^{-\gamma}\left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}}\right)^+\right] = \mathbb{E}_0^D\left[B_1^{-\gamma}\left(\exp\left(\mu_j\right)F_{j,1} - K\exp\left(\mu_i\right)F_{i,1}\right)^+\right].$$

We conclude that the value of the call is (21).

Put price. We use the put-call parity. Using the identity $x^+ = x + \left(-x\right)^+$ and the fact that $\mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\frac{e_{i,1}}{e_{i,0}}\right] = \exp\left(-r_i\right)$, we have:

$$V^P\left(K\right) = \mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\left(K\frac{e_{i,1}}{e_{i,0}} - \frac{e_{j,1}}{e_{j,0}}\right)^+\right] = \mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\left(K\frac{e_{i,1}}{e_{i,0}} - \frac{e_{j,1}}{e_{j,0}}\right)\right] + \mathbb{E}_0\left[\frac{M_1^*}{M_0^*}\left(\frac{e_{j,1}}{e_{j,0}} - K\frac{e_{i,1}}{e_{i,0}}\right)^+\right]$$

$$= \frac{K}{1+r_i} - \frac{1}{1+r_j} + V^C\left(K\right).$$

The following analogue of (21) also holds:

$$V^P\left(K\right) = \exp\left(-R + \mu_j\right)\left(1 - p_0\right)V_{BS}^{Put}\left(K\exp\left(\mu_i - \mu_j\right), \sigma_{i|j}\right) +$$

$$+ \exp\left(-R + \mu_j\right)p_0\mathbb{E}_0^D\left[B_1^{-\gamma}\left(-F_{j,1} + K\exp\left(\mu_i - \mu_j\right)F_{i,1}\right)^+\right]. \tag{48}$$

Proof of Proposition 7 Using the same proof as in Gabaix (2012, Theorem 1), the price of the stock in the international numéraire (the traded good) is:

$$\overline{P}_{Di,t} = D_{it}\frac{1 + \frac{\widehat{H}_{Di,t}}{r_{Di} + \phi_{HDi}}}{r_{Di}}.$$

Hence, expressed in the domestic currency, the price is: $P_{Di,t} = \frac{\overline{P}_{Di,t}}{e_{it}} = d_{it}\frac{1 + \frac{\widehat{H}_{Di,t}}{r_{Di} + \phi_{HDi}}}{r_{Di}}$.

The model with nominal prices. The inflation process is as in Gabaix (2012), so we can take results from that paper. Let $Q_t = Q_0 \prod_{s=0}^{t-1} (1 - I_s)$ be the value of money (the inverse of the price level). The expected value of one unit of currency T periods later is:

$$\mathbb{E}_t \left[\frac{Q_{t+T}}{Q_t} \right] = (1 - I_*)^T \left(1 - \frac{1 - \exp(-\phi_I T)}{1 - \exp(-\phi_I)} \frac{I_t - I_*}{1 - I_*} \right), \tag{49}$$

or $\mathbb{E}_t \left[\frac{Q_{t+T}}{Q_t} \right] = \exp(-I_* T) \left(1 - \frac{1 - \exp(-\phi_I T)}{\phi_I} (I_t - I_*) \right)$ in the continuous-time limit.

The time-t price of a nominal bond yielding one unit of currency at time $t + T$ is $\widetilde{Z}_t(T) = \mathbb{E}_t \left[\frac{M_{t+T}^* e_{t+T} Q_{t+T}}{M_t^* e_t Q_t} \right]$. Because we assume that shocks to inflation are uncorrelated with disasters, the present value of one nominal unit of the currency is:

$$\widetilde{Z}_t(T) = \mathbb{E}_t \left[\frac{M_{t+T}^* e_{t+T} Q_{t+T}}{M_t^* e_t Q_t} \right] = \mathbb{E}_t \left[\frac{M_{t+T}^* e_{t+T}}{M_t^* e_t} \right] \mathbb{E}_t \left[\frac{Q_{t+T}}{Q_t} \right].$$

Proposition 8. The derivation of the forward rate is as in Gabaix (2012, Theorem 2 and Lemma 2).

Proof of Proposition 9. We start with the case of the regression in a sample that does not contain disasters. So, up to second-order terms in $\widehat{H}_t$ and I_t,

$$\mathbb{E}_t^{ND} \left[\frac{\widetilde{e}_{i,t+1} - \widetilde{e}_{it}}{\widetilde{e}_{it}} - \frac{\widetilde{e}_{j,t+1} - \widetilde{e}_{jt}}{\widetilde{e}_{jt}} \right] = \frac{-\phi_H}{r_e + \phi_H} \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) - (I_{it} - I_{jt})$$

$$\equiv a \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) + b (I_{it} - I_{jt}) + c$$

$$\widetilde{r}_{it} - \widetilde{r}_{jt} = -\frac{r_e}{r_e + \phi_H} \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) + (I_{it} - I_{jt})$$

$$\equiv A \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) + B (I_{it} - I_{jt}) + C,$$

hence

$$\widetilde{\beta}^{ND\prime} = -\frac{\text{Cov} \left(\mathbb{E}_t^{ND} \left[\frac{\widetilde{e}_{i,t+1} - \widetilde{e}_{it}}{\widetilde{e}_{it}} - \frac{\widetilde{e}_{j,t+1} - \widetilde{e}_{jt}}{\widetilde{e}_{jt}} \right], \widetilde{r}_{it} - \widetilde{r}_{jt} \right)}{\text{Var} (\widetilde{r}_{it} - \widetilde{r}_{jt})} = -\frac{aA \, \text{Var} \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) + bB \, \text{Var} (I_{it} - I_{jt})}{A^2 \, \text{Var} \left(\widehat{H}_{it} - \widehat{H}_{jt} \right) + B^2 \, \text{Var} (I_{it} - I_{jt})}$$

$$= -\nu \frac{a}{A} - (1 - \nu) \frac{b}{B} = \nu \beta^{ND} + 1 - \nu,$$

where $\nu = \dfrac{1}{1 + \frac{B^2 \, \text{Var} \left(I_{it} - I_{jt} \right)}{A^2 \, \text{Var} \left(\widehat{H}_{it} - \widehat{H}_{jt} \right)}}$.

The case of the full sample regression is proved similarly.

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Online Appendix for "Rare Disasters and Exchange Rates"

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September 2014

This online appendix presents extensions of the model to business cycle shocks, and nominal bond risk premia. It also provides extra proofs, and technical complements on the calibration of the model.

6 A Setup with a Risk Factor and a Business Cycle Factor

6.1 Basic Theory with a Business Cycle

So far, we have only introduced one factor, so that, controlling for current productivity, exchange rates and risk premia are perfectly correlated. This is an undesirable feature. In this section, we extend our framework to a two-factor model with a risk factor and a business cycle factor (see Pavlova and Rigobon 2007, 2008 for a different framework with several factors).

We model country i's export sector productivity as follows: $\omega_{it} = \overline{\omega}_{it}(1 + y_{it})$ where $\overline{\omega}_{it}$ is the (stochastic) trend component of productivity and y_{it} is a deviation from the trend that we refer to as the business cycle factor. The trend $\overline{\omega}_{it}$ behaves according to:

$$\frac{\overline{\omega}_{it+1}}{\overline{\omega}_{it}} = \exp\left(g_\omega\right) \times \begin{cases} 1 & \text{in normal times,} \\ F_{i,t+1} & \text{if disaster.} \end{cases}$$

The business cycle factor y_{it} follows a linearity-generating process

$$\mathbb{E}_t\left[y_{i,t+1}\right] = \frac{1 + H_{i*}}{1 + H_{it}} \exp\left(-\phi_{y_i}\right) y_{it},$$

with innovation uncorrelated with those of ω_{it} and M_t^*.

Proposition 10 (Business cycle factor). *The exchange rate is given by*

$$e_{it} = \frac{\overline{\omega}_{it}}{1 - \exp\left(-r_{ei}\right)}\left(1 + \frac{\exp\left(-r_{ei} - h_*\right)}{1 - \exp\left(-r_{ei} - \phi_{H_i}\right)}\widehat{H}_{it} + \frac{1 - \exp\left(-r_{ei}\right)}{1 - \exp\left(-r_{ei} - \phi_{y_i}\right)}y_{it}\right). \qquad (50)$$

In the limit of small time intervals, the exchange rate is given by

$$e_{it} = \frac{\overline{\omega}_{it}}{r_{ei}} \left(1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}} + \frac{r_{ei} y_{it}}{r_{ei} + \phi_{y_i}} \right). \tag{51}$$

In the limit of small time intervals, the interest rate is

$$r_{it} = r_{ei} - \lambda + \frac{-\frac{r_{ei}}{r_{ei} + \phi_{H_i}} \widehat{H}_{it} + \frac{r_{ei} \phi_{y_i}}{r_{ei} + \phi_{y_i}} y_{it}}{1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}} + \frac{r_{ei} y_{it}}{r_{ei} + \phi_{y_i}}}. \tag{52}$$

The resilience $\widehat{H}_{it}$ affects the exchange rate and the interest rate in the same way as in the setup without the business cycle factor: a risky country with a low $\widehat{H}_{it}$ has a depreciated exchange rate e_{it} and a high interest rate r_{it}. As a result, the disaster factor captured by $\widehat{H}_{it}$ induces a negative correlation between e_{it} and r_{it}. In contrast, the business cycle factor induces a positive correlation between these two variables: a country with an above-trend export sector productivity y_{it} has an appreciated exchange rate e_{it} and a high interest rate r_{it}. The correlation between the exchange rate and the interest rate depends on the relative importance of the disaster factor and the business cycle factor.

Fama regressions with two factors. Denote by α' and β' respectively the constant term and the Fama coefficient for the Fama regression in the two-factor model:

$$\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} = \alpha' - \beta'(r_{it} - r_{jt}) + \varepsilon_{ij,t+1} \tag{53}$$

The next proposition relates the coefficient $\beta^{ND\prime}$ in a sample with no disasters and the coefficient β'^{Full} in a full sample to their counterparts β^{ND} and β^{Full} in the one-factor model.

Proposition 11 (Fama regression with two factors). *Consider two countries i and j with $r_{ei} = r_{ej} = r_e$, $\phi_{H_i} = \phi_{H_j} = \phi_H$, and $\phi_{y_i} = \phi_{y_j} = \phi_y$. Consider the limit of small time intervals as well as small $\widehat{H}_{it}$ and $\widehat{H}_{jt}$. Let ν be the share of the interest rate differential variance due to $\widehat{H}_{it} - \widehat{H}_{jt}$:*

$$\nu = \frac{\left(\frac{r_e}{r_e + \phi_{H_i}} \right)^2 \mathrm{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt} \right)}{\left(\frac{r_e}{r_e + \phi_{H_i}} \right)^2 \mathrm{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt} \right) + \left(\frac{r_e \phi_y}{r_e + \phi_y} \right)^2 \mathrm{Var}\left(y_{it} - y_{jt} \right)}. \tag{54}$$

The coefficient $\beta^{ND\prime}$ (respectively β'^{Full}) in the Fama regression (53) for a sample with no dis-

asters (respectively for a full sample) is given by[54]

$$\beta^{ND\prime} = \nu \beta^{ND} + 1 - \nu \tag{55}$$

$$\beta^{\prime Full} = \nu \beta^{Full} + 1 - \nu.$$

In equation (55), $\beta^{ND\prime}$ is the weighted average of two Fama coefficients: the first coefficient, β^{ND}, corresponds to variations in exchange rates and interest rate differentials driven by the disaster factor; the second coefficient, 1, corresponds to variations in exchange rates and interest rate differentials driven by the business cycle factor. The weight ν is the share of the disaster factor in the variance of interest rate differentials.

6.2 Predicting the Exchange Rate with Forwards

Nominal yield curves contain a lot of information potentially useful for predicting exchange rates. We now explain how best to extract the relevant information to compute exchange rate risk premia. As above, the expected depreciation of the nominal exchange rate is, up to second order terms, and conditional on no disasters:

$$\mathbb{E}_t^{ND}\left[\frac{d\tilde{e}_t}{\tilde{e}_t}\right]/dt = g_\omega - \frac{\phi_H \widehat{H}_t}{r_e + \phi_H} - \frac{r_e \phi_y y_t}{r_e + \phi_y} - I_t \tag{56}$$

It involves three factors that are also reflected in the nominal forward curve. Note, however, that it does not involve the inflation risk premium π_t^i. So, an optimal combination of forward rates should predict expected currency returns with more accuracy than the simple Fama regression.

Boudoukh, Richardson and Whitelaw (BRW, 2012) propose to regress the exchange rate movement on the T-period forward rate from T periods ago:

BRW regression: $\mathbb{E}_t\left[\dfrac{e_{i,t+1} - e_{it}}{e_{it}} - \dfrac{e_{j,t+1} - e_{jt}}{e_{jt}}\right] = \alpha^{Fwd}(T) - \beta^{Fwd}(T)\,(f_{t-T}^i(T+1) - f_{t-T}^j(T+1))$ (57)

Our model's prediction is in the next Proposition.

Proposition 12 (Value of the β coefficient in the Fama regression, with two factors, with forward rates). *Up to second order terms, in the BRW (57) regression with forward rates, the coefficients are:*

$$\beta^{Fwd}(T) = \nu(T)\,\beta^{ND} + 1 - \nu(T) \tag{58}$$

---

[54]The formula $\beta^{\prime Full} = \nu \beta^{Full} + 1 - \nu$ is valid even when B_t is not constant. The only difference in this case is that β^{Full} is no longer given by equation (16).

and

$$\beta^{Fwd,Full}(T) = \nu(T)\beta^{Full} + 1 - \nu(T) \tag{59}$$

where β^{ND} and β^{Full} are given in Eqs. 15 and 16, and

$$\nu(T) = \frac{\left(\frac{r_e}{r_e+\phi_H}\right)^2 \text{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)\exp(-2\phi_H T)}{\left(\frac{r_e}{r_e+\phi_H}\right)^2 \text{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)\exp(-2\phi_H T) + \left(\frac{r_e\phi_y}{r_e+\phi_y}\right)^2 \text{Var}(y_{it} - y_{jt})\exp(-2\phi_y T)} \tag{60}$$

is the share of variance in the forward rate due to $\widehat{H}_{it} - \widehat{H}_{jt}$. In particular, when $\phi_H > \phi_y$, the long horizon regression has a coefficient going to 1: $\lim_{T\to\infty}\beta^{Fwd}(T) = \lim_{T\to\infty}\beta^{Fwd,Full}(T) = 1$.

Boudoukh, Richardson and Whitelaw (2012) find that $\beta^{Fwd}(T)$ increases toward 1 with the horizon. Our theory is consistent with this empirical finding. Indeed, to interpret Proposition 12, consider the case where risk-premia are quickly mean-reverting, and the business cycle factor is slowly mean reverting, $\phi_H > \phi_y$ [55]. Then, for large T, $\nu(T)$ tends to 0, which means that, at long horizons, the forward rate is mostly determined by the level of the business cycle factor, not of the risk premium. Hence, at a long horizon the model behaves like a model without risk premia, hence generates a coefficient β close to 1.

7 Richer Nominal Model

7.1 Basic Theory

We now develop a richer model with an inflation-specific risk premium. This allows us to speak about a basic fact about the yield curve: on average, the (nominal) yield curve is upward sloping, i.e., long term interest rates are higher than short term interest rates. To do so, we extend the framework by incorporating inflation risk along the lines of Gabaix (2012): as inflation rises (on average) during disasters, long term bonds are riskier, which makes the yield curve slope upward. This will allow us to study the term premium across countries.

The variable part of inflation now follows the process:

$$\widehat{I}_{t+1} = \frac{1 - I_*}{1 - I_t} \cdot \left(\exp(-\phi_I)\widehat{I}_t + 1_{\{\text{Disaster at } t+1\}}\left(J_* + \widehat{J}_t\right)\right) + \varepsilon_{t+1}^I \tag{61}$$

---

[55] The same reasoning would hold replacing the business cycle factor by inflation

In case of a disaster, inflation jumps by an amount $J_t = J_* + \widehat{J}_t$. This jump in inflation makes long term bonds particularly risky. J_* is the baseline jump in inflation, $\widehat{J}_t$ is the mean-reverting deviation from baseline. It follows a twisted auto-regressive process, and, for simplicity, does not jump during crises:

$$\widehat{J}_{t+1} = \frac{1 - I_*}{1 - I_t} \cdot \exp\left(\phi_\pi\right) \widehat{J}_t + \varepsilon^J_{t+1}. \tag{62}$$

We define

$$\pi_t \equiv \frac{p_t \mathbb{E}_t \left[B^{-\gamma}_{t+1} F_{i,t+1}\right]}{1 + H_{i*}} \widehat{J}_t, \tag{63}$$

which is the mean-reverting part of the "risk adjusted" expected increase in inflation if there is a disaster. We parametrize the typical jump in inflation J_* in terms of a number $\kappa \leq (1 - \rho_I)/2$:

$$\frac{p_t \mathbb{E}_t \left[B^{-\gamma}_{t+1} F_{i,t+1} J_*\right]}{1 + H_{i*}} = (1 - I_*) \kappa \left(1 - \exp\left(-\phi_I\right) - \kappa\right).$$

κ represents a risk premium for the risk that inflation increases during disasters. Also, we define

$$I_{**} \equiv I_* + \kappa \tag{64}$$

and $\psi_\pi \equiv \phi_\pi - \kappa$. They represent the "risk adjusted" trend and mean-reversion parameter in the inflation process. In the continuous-time limit,

$$\kappa\left(\phi_I - \kappa\right) = p_t \mathbb{E}_t \left[B^{-\gamma}_{t+1} F_{i,t+1} J_*\right] = J_* \left(H_{it} + p_t\right) \tag{65}$$

As before, we denote nominal variables with a tilde. The price of a long term nominal bond yielding one unit of the currency at time $t + T$ is $\widetilde{Z}_t(T) = \mathbb{E}_t\left[\frac{M^*_{t+T} e_{t+T} Q_{t+T}}{M^*_t e_t Q_t}\right]$, where Q_t is the inverse of the price level.

The yield at maturity T, $\widetilde{Y}_t(T)$, and the forward rates $\widetilde{f}_t(T)$ are defined by $-\ln \widetilde{Z}_t(T) = \widetilde{Y}_t(T) T = \sum_{T'=1}^{T} \widetilde{f}_t(T')$. The forward rates can be derived in closed form. For completeness, we also import the part with business cycle risk from section 6 (the y_t term below).

Proposition 13 (Forward rates with inflation risk premia). *In the continuous time limit, in up to second order terms in $\left(\widehat{H}_t, y_t, I_t, \pi_t, \kappa\right)$:*

$$\widetilde{f}_t(T) = r_e - \lambda - \frac{r_e}{r_e + \phi_H} \exp\left(-\phi_H T\right) \widehat{H}_t + \frac{r_e \phi_y}{r_e + \phi_y} \exp\left(-\phi_y T\right) y_t + \tag{66}$$

$$+ I_{**}\left(1 - \exp\left(-\phi_I T\right)\right) + \exp\left(-\phi_I T\right) I_t + \frac{\exp\left(-\phi_I T\right) - \exp\left(-\psi_\pi T\right)}{\psi_\pi - \phi_I} \pi_t$$

where $\widehat{H}_t$ is the transitory part of the country's resilience, y_t is the state of the business cycle, I_t is inflation, π_t is the transitory part of the bond risk premium: they are all for a given country i (for simplicity, we omit here the index i).

Proof. The proof is along the lines of the Gabaix (2012, Theorem 2 and Lemma 2), and we only sketch it here as the mechanics are very similar. The first step is to calculate that $Y_t := M_t e_t Q_t \left(1, \widehat{H}_t, y_t, \widehat{I}_t, \pi_t\right)$ is LG (to the leading order), with continuous time generator:

$$
\omega = (r_e - \lambda)\,I +
\begin{pmatrix}
0 & -\frac{r_e}{r_e + \phi_H} & \frac{r_e \phi_y}{r_e + \phi_H} & 1 & 0 \\
0 & \phi_H & 0 & 0 & 0 \\
0 & 0 & \phi_y & 0 & 0 \\
-\kappa\,(\phi_I - \kappa) & 0 & 0 & \phi_I & -1 \\
0 & 0 & 0 & 0 & \phi_J
\end{pmatrix}
\tag{67}
$$

Then, we derive the bond price as $Z_t(T) = (1,0,0,0,0)'\,e^{-\omega T} Y_t$. The forward rate is then $f_t(T) = -\partial_T \ln Z_t(T)$. Here we report the limit for $\kappa \to 0$, which makes terms cleaner, and gives a sense in which the proposition is only up to second order terms. The nominal forward rate in (66) depends on real and nominal factors. The real factors are the resilience of the economy (the $\widehat{H}_t$) term, the expected growth rate of productivity $(-\phi_y y_t)$. The nominal factors are inflation I_t, and the variable component of the the risk premium for inflation jump risk, π_t. □

When a disaster occurs, inflation increases (on average). As very short term bills are essentially immune to inflation risk, while long term bonds lose value when inflation is higher, long term bonds are riskier, hence they get a higher risk premium. Hence, the yield curve slopes up on average – as implied by the term $I_{**}\,(1 - \exp(-\phi_I T)) \sim I_{**}\phi_I T$.

Each of the three terms is multiplied by a term of the type $\exp(-\phi_H T)$. For small speeds of mean reversion ϕ, it means that the forward curve is fairly flat. The last term, however, is close to T for small maturities ($\frac{\exp(-\phi_I T) - \exp(-\psi_\pi T)}{\psi_\pi - \phi_I} \sim T$). It creates a variable slope in the forward curve.

Hence, we obtain a rich forward curve. Gabaix (2012) shows that this type of yield curve generates a realistic term premium and volatility of the yield curve. Here, we have two extra terms: the country-specific resilience $\widehat{H}_t$, and the state of the business cycle y_t.

7.2 The Term Premium Across Countries

Proposition 13 implies that the short-term nominal interest rate is $\widetilde{r}_t = \widetilde{f}_t(0)$:

$$\widetilde{r}_t = r_e - \lambda - \frac{r_e}{r_e + \phi_H}\widehat{H}_t + \frac{r_e\phi_y}{r_e + \phi_y}y_t + I_t,$$

so that on average,

$$\mathbb{E}\left[\widetilde{r}_t\right] = r_e - \lambda + I_*,$$

(up to second order terms), while the long-term nominal interest rate $(\widetilde{r}_t^{LT} = \lim_{T\to\infty}\widetilde{f}_t(T))$ is equal to:

$$\widetilde{r}^{LT} := r_e - \lambda + I_* + \kappa.$$

It is independent of time, as is normal in those models.

The difference between the two rates is

$$\widetilde{r}^{LT} - \mathbb{E}\left[\widetilde{r}_t\right] = \kappa.$$

We will call this the "term premium".[56] It is also the expected excess return of long-term bonds conditional on no disasters. Below, we define p_* as the average value of the probability of disasters.

Proposition 14 (Slope of the yield curve / Term premium) *On average (and up to second order terms), the term premium (the average value of long term bond yields minus short term bond yields) in country i, κ_i, is the smaller positive root of*

$$\kappa_i\left(\phi_{I,i} - \kappa_i\right) = J_{i*}\left(H_{i*} + p_*\right) \tag{68}$$

The term premium κ_i is increasing with the expected increase of inflation in disasters (J_{i*}), and increasing *with resilience. In words, controlling for the inflation process $(\phi_{I,i}$ and $J_{i*})$, countries that are relatively riskier (lower resilience H_{i*}) have a lower term premium κ_i than countries that are safer (higher resilience H_{i*}).*

Lustig, Stathopoulos and Verdelhan (2014) find evidence for this effect (see also Ang and Chen (2010) for related work). In their sample, risky (high interest rate) countries have lower term premia than less risky (low interest rate) countries.

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[56]Empirically, the term premium is often drawn from the properties of finite-maturity bonds, e.g. 30 year bond, but for conceptual discussions very long term bonds are clearer.

The intuition is the following. To make the benchmark starker, suppose that all countries have the same inflation processes (same $\phi_{I,i}$ and J_{i*}). Because a risky currency will depreciate in a disaster, a "term premium trade" portfolio long its long-term bond and short its short-term bond will have little value in a disaster: in the limit where the currency is due to entirely collapse in a disaster ($F_{i*} = 0$, so that $H_{i*} + p_* = 0$, and $\kappa_i = 0$) this "term premium trade" will have exactly zero payoff. Hence, perhaps surprisingly at first, it should have a zero risk premium.

However, the same "term premium trade" with a safe currency will do very poorly during disasters, as the value of long term bonds falls (on average) during disasters because of the increase in inflation: this trade is risky, hence commands a risk premium, which is the term premium, κ_i.[57] In some sense, disasters "democratize" risk premia, i.e. dull the differences in riskiness between bonds (here, between short-term and long-term bonds of a risky countries), as extreme disasters just wipe out the value of all very risky bonds.[58]

8 Proofs that were omitted in the paper

Proof of Lemma 1. Call $\eta_{i,t}^T$ and $\eta_{i,t}^{NT}$ country i's endowment of the traded good and nontraded good, respectively. We work out under which conditions they generate the announced equilibrium. Say that the equilibrium is described by a social planner's maximization of $\sum_i \lambda_i^\gamma U_i$ where $U_i = \mathbb{E}_0 \left[\sum_{t=0}^\infty \exp\left(-\delta t\right) \frac{\left(C_{it}^T\right)^{1-\gamma} + \left(C_{it}^{NT_i}\right)^{1-\gamma} \frac{1}{\xi}}{1-\gamma} \right]$ is country i's utility and λ_i^γ the Negishi weight on country i. We normalize $\sum \lambda_i = 1$. Denote by q_t the Arrow-Debreu price of one unit of the traded good at date t and by Y_t^T the world production of the traded good. Among other things, the planner optimizes the consumptions of the traded good, and solves:

$$\max_{C_{it}^T} \sum_i \lambda_i^\gamma \sum_{t=0}^\infty \exp\left(-\delta t\right) \frac{\xi \left(C_{it}^T\right)^{1-\gamma} + \left(C_{it}^{NT_i}\right)^{1-\gamma}}{1-\gamma} + \sum_t q_t \left(Y_t^T - \sum_i C_{it}^T\right),$$

so that $\exp\left(-\delta t\right) \lambda_i^\gamma \xi \left(C_{it}^T\right)^{-\gamma} - q_t = 0$ and $C_{it}^T = \lambda_i \xi^{1/\gamma} q_t^{-1/\gamma} \exp\left(-\delta t/\gamma\right)$. Using $Y_t^T = \sum_i C_{it}^T$, we get: $C_{it}^T = \lambda_i Y_t^T$.

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[57] More precisely, consider small jumps and risk premia: inflation jumps by J_*, but with a speed of mean-reversion ϕ_I, so that the loss in value of long-term bonds is $\frac{J_*}{\phi_I}$. Hence, the risk premium κ satisfies: $\kappa = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} F_{i,t+1} \frac{J_*}{\phi_I} \right]$, i.e. $\kappa \phi_I = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} F_{i,t+1} J_* \right]$, which is equation (65), up to a second order term in κ.

[58] A similar intuition holds in Gabaix (2012), Proposition 4, point (i).

Let us now study country i's consumption and investment decisions. At time t, country i solves $\max_{C_{it}^T, C_{it}^{NT_i}} \frac{\xi(C_{it}^T)^{1-\gamma} + (C_{it}^{NT_i})^{1-\gamma}}{1-\gamma}$ s.t. $C_{it}^T + e_{it}C_{it}^{NT_i} = $ expenditure at time t, so $(C_{it}^{NT_i})^{-\gamma} = e_{it}\xi(C_{it}^T)^{-\gamma}$, hence $C_{it}^{NT_i} = e_{it}^{-1/\gamma}\xi^{-1/\gamma}\lambda_i Y_t^T$. The investment in the capital good (i.e., the non-traded good) is $\eta_{it}^{NT_i} - C_{it}^{NT_i} = \eta_{it}^{NT_i} - e_{it}^{-1/\gamma}\xi^{-1/\gamma}\lambda_i Y_t^T$, so that the accumulated quantity of the capital good is $K_{it} = \sum_{s=0}^{\infty} e^{-\lambda s}\left(\eta_{i,t-s}^{NT_i} - e_{i,t-s}^{-1/\gamma}\xi^{-1/\gamma}\lambda_i Y_{t-s}^T\right)$. As country i produces $K_{it}\omega_{it}$ of the world good, and also has an endowment η_{it}^T of it, the total available consumption of the world good at time t is:

$$Y_t^T = \sum_i \eta_{it}^T + \sum_i \omega_{it} \sum_{s=0}^{\infty} e^{-\lambda s}\left(\eta_{i,t-s}^{NT_i} - e_{i,t-s}^{-1/\gamma}\xi^{-1/\gamma}\lambda_i Y_{t-s}^T\right). \tag{69}$$

The first term is the endowment of the world good, and the second term is its production.

The equilibrium is described as in the paper if the endowment processes $\eta_{i,t}^T$ and $\eta_{i,t}^{NT_i}$ satisfy (69), with $Y_t^T = C_t^{T*}$. By inspection, there is an infinity of such endowment processes.

Proof of Lemma 3. The quick intuition is the following: The expected excess return of the stock claim in term of the international numéraire is $p\mathbb{E}_t\left[B_{t+1}^{-\gamma}(1 - F_{Di,t+1})\right]$, while the expected excess return on holding the currency is $p\mathbb{E}_t\left[B_{t+1}^{-\gamma}(1 - F_{i,t+1})\right]$. Hence, the excess return on holding the domestic stock is the difference between the two, namely $p_t\mathbb{E}_t\left[B_{t+1}^{-\gamma}(F_{Di,t+1} - F_{i,t+1})\right]$.

A derivation via calculations is the following: conditional on no disasters, the return on a stock, in the domestic currency, is: (we neglect the Jensen's inequality / Ito terms variance terms, which are second order).

$$\mathbb{E}r_{it}^S = \frac{dP_{Dk,t}}{P_{Dk,t}} + \frac{D_{it}/e_{it}}{P_{Dk,t}}dt$$

$$= \frac{dD_{it}}{D_{it}} - \frac{de_{it}}{e_{it}} + \frac{dh_{Dit}}{1 + h_{Dit}} + \frac{D_{it}/e_{it}}{P_{Dk,t}}dt$$

$$= g_D - g_\omega - \frac{dh_{it}}{1 + h_{it}} + \frac{dh_{Dit}}{1 + h_{Dit}} + \frac{r_{Di}}{1 + h_{Dit}}$$

$$= g_D - g_\omega + r_{Di} + \phi_H h_{it} - (\phi_H + r_{Di})h_{Dit},$$

while $r_{f,it} = r_{ei} - \lambda - r_{ei}h_{it}$, so

$$\mathbb{E}r_{it}^S - r_{f,it} = g_D - g_\omega + r_{Di} - r_{ei} + \lambda + (\phi_H + r_{ei})h_{it} - (\phi_H + r_{Di})h_{Dit}$$

$$= A + \widehat{H}_{it} - \widehat{H}_{Dit},$$

where the constant A is:

$$A = g_D - g_\omega + r_{Di} - r_{ei} + \lambda$$
$$= g_D - g_\omega + (R - g_D - h_{Di*}) - (R + \lambda - g_{\omega_i} - h_{i*}) + \lambda$$
$$= h_{i*} - h_{Di*},$$

so

$$\mathbb{E}r_{it}^S - r_{f,it} = h_{i*} - h_{Di*} + \widehat{H}_{it} - \widehat{H}_{Dit} = H_{it} - H_{Dit} = -H_{D'it}.$$

9   Allowing for Different Time Scales in Resilience

9.1   The model with two time scales for resilience

There are different time scales in most measures of risk. For instance, the VIX index (of stock market volatility) features low-frequency epochs of low vs high volatility (e.g. pre-2008 vs post-2008), and high-frequency variations (e.g. temporary rises in volatility level, e.g. after bad macroeconomic news). To capture them, we propose an extension of the model with two time scales of resilience. We decompose resilience H_{it} as:

$$H_{it} = H_{i*} + \sum_{s=1}^{2} \widehat{H}_{ist}, \tag{70}$$

where $\widehat{H}_{ist}$ for $s = 1, 2$ are the two transitory components of resilience, one slow-moving, one fast-moving (s indicates the time-$s$cale). Their laws of motion are:

$$\widehat{H}_{is,t+1} = \frac{1 + H_{i*}}{1 + H_{it}} \exp(-\phi_{H_{is}}) \widehat{H}_{ist} + \varepsilon_{is,t+1}^H, \tag{71}$$

where $\mathbb{E}_t \left[\varepsilon_{is,t+1}^H \right] = \mathbb{E}_t^D \left[\varepsilon_{i,t+1}^H \right] = 0$.

　　We assume that $\phi_{H_{i1}} < \phi_{H_{i2}}$, so that $\widehat{H}_{i1t}$ is the slow component of resilience, and $\widehat{H}_{i2t}$ is its fast component. For instance, $\widehat{H}_{i1t}$ can capture the movements of resilience happening at business cycle frequency, and $\widehat{H}_{i2t}$ the movements happening at higher (e.g. monthly) frequency.

　　All our results are easily adapted to this multi-scale setup.

Proposition 15 (Modifications when there are two time scales for resilience). *The theory's basic formulas carry over to the setup with two time scales for resilience, with minor modifications as follows. In the limit of small time intervals, the exchange rate (11), interest rate (13),*

and the Fama coefficients become:

$$e_{it} = \frac{\omega_{it}}{r_{ei}} \left(1 + \sum_{s=1}^{2} \frac{\widehat{H}_{ist}}{r_{ei} + \phi_{H_{is}}} \right), \tag{72}$$

$$r_{it} = r_{ei} - \lambda - r_{ei} \frac{\sum_{s=1}^{2} \frac{\widehat{H}_{ist}}{r_{ei} + \phi_{H_{is}}}}{1 + \sum_{s=1}^{2} \frac{\widehat{H}_{ist}}{r_{ei} + \phi_{H_{is}}}}, \tag{73}$$

$$\beta^{ND} = -\frac{\sum_{s=1}^{2} \frac{\phi_{H_{is}}}{\left(r_{ei} + \phi_{H_{is}} \right)^2} \mathrm{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst} \right)}{\sum_{s=1}^{2} \frac{r_{ei}}{\left(r_{ei} + \phi_{H_{is}} \right)^2} \mathrm{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst} \right)}, \tag{74}$$

while the expected return of the carry trade (18), and the risk-reversal (25) are unchanged.

Proof of Proposition 15. The values of e_{it} is derived as in Gabaix (2012, Proposition 12, Online Appendix). The proof for the interest rate is as above. For completeness, we state the value of the nominal Fama coefficient with two time scales.

$$\widetilde{\beta}^{ND} = \frac{-\sum_s \frac{\phi_{H_s} r_e}{\left(r_e + \phi_{H_s} \right)^2} \mathrm{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst} \right) + \mathrm{Var}\left(I_{it} - I_{jt} \right)}{\sum_s \left(\frac{r_e}{r_e + \phi_{H_s}} \right)^2 \mathrm{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst} \right) + \mathrm{Var}\left(I_{it} - I_{jt} \right)}. \tag{75}$$

Proof sketch. We proceed as in the original proof of Proposition 4. Up to second order terms,

$$\mathbb{E}_t^{ND}\left[\frac{\widetilde{e}_{i,t+1} - \widetilde{e}_{it}}{\widetilde{e}_{it}} \right] = -\sum_s \frac{\phi_{H_s}}{r_e + \phi_{H_s}} \widehat{H}_{ist} - I_{it} + K_i$$

$$\widetilde{r}_{it} = -\sum_s \frac{r_e \widehat{H}_{ist}}{r_e + \phi_{H_s}} + I_{it} + K_i',$$

so

$$\widetilde{\beta}^{ND} = \frac{-\mathrm{Cov}\left(\mathbb{E}_t^{ND}\left[\frac{\widetilde{e}_{i,t+1} - \widetilde{e}_{it}}{\widetilde{e}_{it}} \right] - \mathbb{E}_t^{ND}\left[\frac{\widetilde{e}_{j,t+1} - \widetilde{e}_{jt}}{\widetilde{e}_{jt}} \right], \widetilde{r}_{it} - \widetilde{r}_{jt} \right)}{\mathrm{Var}\left(\widetilde{r}_{it} - \widetilde{r}_{jt} \right)}$$

$$= \frac{-\sum_s \frac{\phi_{H_s} r_e}{\left(r_e + \phi_{H_s} \right)^2} \mathrm{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst} \right) + \mathrm{Var}\left(I_{it} - I_{jt} \right)}{\sum_s \left(\frac{r_e}{r_e + \phi_{H_s}} \right)^2 \mathrm{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst} \right) + \mathrm{Var}\left(I_{it} - I_{jt} \right)}.$$

9.2 Calibration with two time scales

9.2.1 Parameter Values

We present a calibration of the model. Our data is nominal; we therefore use the extension to a nominal setup. In order to match the observed autocorrelation structure of risk reversals,

we use the two time scales model.[59] Up to second order terms, the differences in resiliences $H_{it} - H_{jt}$ are a sufficient statistic for the quantities of interest (which are bilateral, e.g. $\ln\left(\frac{e_{it}}{e_{jt}}\right)$, $r_{it} - r_{jt}$, etc.). Hence we specify parameters for those differences in resilience – rather than the absolute resilience H_{it} and H_{jt} and their correlation. These differences in resiliences could come from various combinations of shocks to the world disaster probability p_t, severity B_{t+1} and country-specific factors $F_{i,t+1}$. We discuss them later.

Table 1 summarizes the main inputs of the calibration. The justification is as follows.

Exchange rate and interest rate. We call Δ the time-difference operator, $\Delta x_t = x_t - x_{t-1}$, and $\sigma_x = stdev\left(\Delta x_t\right)$ the volatility of a variable x_t. For two countries, define the volatility of the bilateral exchange rate as $\sigma_e^{bil} = stdev\left(\Delta \ln \frac{e_{it}}{e_{jt}}\right)$ and the volatility of the difference in interest rates $\sigma_r^{bil} = stdev\left(\Delta\left(r_{it} - r_{jt}\right)\right)$. Equations (11) and (13) give $\sigma_r^{bil} = r_e \sigma_e^{bil}$.[60] The above equation constrains our calibration.[61] We will match $\sigma_e^{bil} \simeq 11\%$. In the sample, the volatility of the nominal interest rate is $\sigma_{\tilde{r}}^{bil} \simeq 0.7\%$. We therefore set $r_e = 6\%$.[62]

The speeds of mean-reversion ϕ_{H_1}, ϕ_{H_2} and the variances of $H_{ist} - H_{jst}$ are chosen to roughly match the level and volatility of the risk reversals, their autocorrelations at different lags, as well as the volatility of the exchange rate. For the speed of mean-reversion of the slow component, we take $\phi_{H_1} = 0.1$, which gives a half-life of $\ln 2/\phi_H = 7$ years, in line with estimates from the

[59] We also performed a calibration with one time scale (available upon request). That calibration is essentially equally successful, except that it does not match the high-frequency movements of the risk reversals (volatility and autocorrelation), and it generates a close to perfect correlation between innovations to the exchange rate and to resilience.

[60] To keep the model parsimonious, we assume no default risk on debt. This is the cleanest assumption for developed countries. Of course, in many cases (e.g., when pricing sovereign debt), default risk can be added without changing anything about the exchange rate.

[61] This expression also holds in the two time scales model (Proposition 15).

[62] The growth rate of productivity g_ω is irrelevant in practice, but for completeness we propose a specific value. We choose the growth rates so that in normal times consumption of non-tradables grows at a rate $g_c = 2\%$. We set $g_\omega = g_c$, but results are not sensitive to the choice of this parameter. We make sure that the riskless domestic short-term rate is on average around 2%, which pins down the rate of time preference δ. This parameter $r_{ei} = R + \lambda - g_{\omega_i} - h_{i*}$ is driven in the model by deeper combinations of underlying factors $p_t, B_{t+1}^{-\gamma}$, and F_{it} but mainly three parameters govern the key statics that we explore in Table 2. We take $\lambda = 4\%$ which generates a real interest rate of $r_{ei} - \lambda = 2\%$. The underlying rate of time preference δ is calibrated to match the value of r_{ei}. For simplicity, we take the recovery rate of productivity to be the average recovery rate of consumption, $F_{i*} = \mathbb{E}\left[B^{-\gamma}\right]^{1/\gamma}$. Hence we find a rate of time preference $\delta = 4.9\%$.

exchange rate predictability literature (Rogoff 1996). For the speed of mean-reversion ϕ_{H_2} of the fast component, we target the autocorrelations of the RR (Table 6): the RR has a fast-mean reverting component, with a half-life of about 4 months. We choose the volatilities of resilience to target the volatilities of RR and the exchange rate reported in Table 6. For parsimony, we take the innovations of the fast and slow component ($H_{i1t} - H_{j1t}$ and $H_{i2t} - H_{j2t}$) to be uncorrelated.

Table 5: Key Parameter Inputs.

Exchange rate discount rate	$r_e = 6\%$
Volatility of H	$\sigma_{H_{i1t}-H_{j1t}} = 1.74\%, \sigma_{H_{i2t}-H_{j2t}} = 3.97\%$
Mean reversion of resilience ϕ	$\phi_{H_1} = 10\%, \phi_{H_2} = 210\%$
Inflation: volatility and speed of mean-reversion	$\sigma_I = 0.5\%, \phi_I = 30\%$

Notes. This table reports the coefficients used in the model. σ_X is the average volatility, and ϕ_X is the speed of mean-reversion. The time unit is the year (the model is simulated at the monthly frequency, but for readability the numbers reported above are all annualized).

Inflation. Data (e.g., on currency options) are nominal, and the essence of our model is real. We pick inflation parameters that are broadly in line with averages in our sample.

Carry trade returns. We proceed as is usual in the carry trade literature, see e.g. Farhi et al. (2014). However, to better capture disaster risk, we sort on risk reversals rather than interest rates. We divide countries into two equal-sized bins of resilience; the risky countries are those in the bottom half of resilience, the less risky countries those in the top half. We define the carry trade as going long \$1 in the equal-weighted portfolio of risky countries and going short \$1 in the equal-weighted portfolio of safer countries (high $\widehat{H}_t$).

Interpreting resilience processes in terms of deeper disaster parameters Resilience differentials are sufficient statistics for the calibration. We now discuss how their variations are related to deeper disaster parameters.

We take numbers from Barro and Ursua (2008).[63] The average probability of disasters is $E[p] = 3.6\%$. An important parameter in the calibration is the risk-adjusted probability of

[63]See also Gabaix (2012), Gourio (2012) and Wachter (2013) for related calibrations of disaster models in closed economies.

disasters $E\left[pB^{-\gamma}\right]$. Disasters are overweighted compared to their physical probability by a factor $E\left[B^{-\gamma}\right]$. This factor is very sensitive to the severity of disasters and to the coefficient of relative risk aversion. We take $\gamma = 4$, which yields $E\left[B^{-\gamma}\right]^{1/\gamma} = 0.66$. Hence, the "risk neutral" (i.e., risk-adjusted) probability of disasters equals $E\left[pB^{-\gamma}\right] = 19.2\%$. Note that though $E\left[B^{-\gamma}\right]^{1/\gamma} = 0.66$, which corresponds to a risk-adjusted average size of disaster of 34%, the median disaster in Barro and Ursua (2008) is much smaller: because of risk aversion, the small possibility of a large disaster matters a lot.

This calibration, strictly speaking, relies on a stark idealization in which consumption is permanently affected after disasters. In practice, there is a partial recovery from disasters (Barro and Ursua 2008). For a given γ, that lowers the disaster risk premium (Gourio 2008). However, this can be remedied by increasing γ slightly. Indeed, Barro and Jin (2011) find an empirical power-law distribution of disaster sizes, so that a moderate γ can generate a very large (indeed infinite for a large enough γ) risk premium. In addition, for our purposes, the idealization of a permanent disaster seems like a good compromise between parsimony and realism.

Our calibration only requires the law of motion of the differential resilience, $H_{it} - H_{jt} = p_t E_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$. The results of the calibration do not depend on whether the shocks come from movements in p_t, $B_{t+1}^{-\gamma}$ or $F_{i,t+1} - F_{j,t+1}$.[64,65]

To interpret the volatility of $H_{it} - H_{jt} = p_t E_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$, we present the standard deviation of changes in $H_{it} - H_{jt}$ over a horizon of one year. Generally, call this object v_{X_t} for the standard deviation of a variable X_t at a one-year horizon: $v_{X_t} = stdev\left(X_{t+1\text{year}} - X_t\right)$. We take some polar cases. If the innovations come entirely from idiosyncratic movements of F_{it} (keeping p_t and $B_{t+1}^{-\gamma}$ constant at $E\left[p\right]$ and $E\left[B^{-\gamma}\right]$), then $v_{F_{it}} = 12.9\%$. This is broadly in line with Gabaix (2012), who argues that a one-year horizon volatility $\nu_{F_{it}} \simeq 10\%$ for the resilience of the aggregate stock market is plausible and does not violate variance bounds from historical data: hence, that calibration seems acceptable too. Conversely, suppose that innovations in differential resilience come entirely from movements in p_t (keeping $E_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$ constant). With fixed values of F_{it}, e.g. $\left|F_{i,t+1} - F_{j,t+1}\right| = 0.4$ (similar to the numbers above),

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[64] For instance, movements in p_t generate a positive covariance between the innovations of H_{it} and H_{jt}, while idiosyncratic movements of $F_{i,t+1}$ and $F_{j,t+1}$ generate a 0 covariance. For the calibration, the covariance between the innovations of H_{it} and H_{jt} does not matter per se – only the variance of the innovations of $(H_{it} - H_{jt})$.

[65] This is true up to second order terms. We verify numerically that this is a good approximation.

then we write $v_{p_t} = 1.6\%$. This is of the same order of magnitude as the calibration in Wachter (2013), which uses $v_{p_t} \simeq 1.1\%$.

9.2.2 Implications

Table 6 presents the main results from the calibration in Table 5.

<div align="center">

Table 6: Moments: Empirical and in the Model

Moments	Data	Calibration
Std Dev($\Delta \ln \tilde{e}_{ijt}$)	12.35%	11.00%
Carry Trade Return	3.44%	2.51%
Mean($|RR|$)	1.31%	1.02%
Std Dev(RR)	1.24%	1.12%
Std Dev(ΔRR_{ijt})	2.60%	1.83%
Std Dev($\tilde{r}_{it}$)	1.38%	1.04%
Std Dev($\Delta(\tilde{r}_{it} - \tilde{r}_{jt})$)	0.71%	1.05%
Corr($\Delta \ln \tilde{e}_{ijt}, \Delta RR_{ijt}$)	−0.57	−0.32
Corr($\ln \tilde{e}_{ij,t+1}, \ln \tilde{e}_{ijt}$)	0.88	0.97
Corr($\Delta \ln \tilde{e}_{ij,t+1}, \Delta \ln \tilde{e}_{ijt}$)	-0.13	-0.012
Corr($\tilde{r}_{it} - \tilde{r}_{jt}, RR_{ijt}$)	0.55	0.43
$A(1)$	0.77	0.88
$A(6)$	0.45	0.53
$A(12)$	0.31	0.38

</div>

Notes for the Table. The table reports the moments generated by the model, using the inputs from Table 5. The risk reversal RR_{ijt} is defined as the implied volatility of an out-of-the-money put minus the implied volatility of an out-of-the-money call, all at 25-delta. A high RR_{ijt} means that the price of protection from depreciation of currency i (against country j) is high. $\tilde{r}_{it}$ is the nominal interest rate. We define $\tilde{e}_{ijt} = \frac{\tilde{e}_{it}}{\tilde{e}_{jt}}$, the nominal bilateral exchange rate between countries i and j: a high $\tilde{e}_{ijt}$ means that currency i appreciates. Carry trade returns are the returns from a long-short portfolio going $1 long (resp. short) an equal-size portfolio of high (resp. low) RR countries. $A(k)$ is the autocorrelation of RR_{ijt} at lag k. $\Delta X_t = X_t - X_{t-1}$ is the

time-difference, annualized. The time unit is the year (the model is estimated and simulated at the monthly frequency, but for readability the numbers reported above are all annualized).

The model hits the volatility of the bilateral exchange rate, i.e. generates the right amount of "excess volatility" in exchange rates. The model also roughly matches (and slightly undershoots) the size of disaster risk as measured by the average size of risk reversals.[66,67] At the same time, the model generates a moderate volatility of the interest rate, as in the data.

We showed earlier that in the model countries with high risk reversals have high interest rates and that increases in risk reversals are associated with depreciations of the exchange rate. The calibration shows that these predictions hold not just qualitatively, but also quantitatively: Table 2 reports the calibrated values of $\text{Cov}\left(\widetilde{r}_{it} - \widetilde{r}_{jt}, RR_t\right)$ and $\text{Corr}(\Delta \ln \widetilde{e}_{ijt}, \Delta RR_{ijt})$ and shows that they broadly match up with their empirical counterparts.

The carry trade generated by the model gives average returns in line with the empirical evidence (see Farhi et al. 2014 for more variants of the carry trade). Investing in countries with high risk reversals generates high expected returns. Indeed, the expected return of the carry trade (given positive RR) is about 3% per annum. Finally, the model generates Fama coefficients $\beta^{ND} = -0.66$ in line with estimates of the literature cited above.

We conclude that the disaster model can be made quantitatively broadly congruent with the empirical facts.

10   Details of the Calibration

Derivations for the main calibration. We will also consider the dispersion of variables, defined as the standard deviation of their population distributions. Recall that, for a process $X_{t+\Delta t} = (1 - \phi_X \Delta t) X_t + \sqrt{\Delta t} \sigma_\varepsilon \varepsilon_{t+1}$, the dispersion (in the limit of small time intervals) is $std(X_t) = \sigma_\varepsilon / \sqrt{2\phi_X}$. Hence, the standard deviation of the steady-state distribution of $\widehat{H}_t$ is:

$$std\left(\widehat{H}_t\right) = \frac{\sigma_H}{\sqrt{2\phi}}.$$

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[66] We take the mean of the absolute values of risk reversals because, by symmetry, the mean of risk reversals is 0.

[67] If we increased the value of $\mathbb{E}\left[pB^{-\gamma}\right]$, for example by slightly increasing γ, we could match better the average value of the RR and other moments, without requiring a larger volatility of the relative prospective recovery rate $F_{it} - F_{jt}$ or of the probability of disaster p_t. We thought it was more parsimonious to stick to the numbers from the previous literature for $\mathbb{E}\left[pB^{-\gamma}\right]$, e.g. Gabaix (2012).

Note that these results hold exactly in the limit of small shocks, i.e., $std\left(\widehat{H}_t\right) / \left(\frac{\sigma_H}{\sqrt{2\phi}}\right) \to 1$ as $\sigma_H \to 0$.

Recall that, for a Gaussian variable $X \sim N\left(0, \sigma^2\right)$, $E\left[X \mid X > 0\right] = -E\left[X \mid X < 0\right] = \sqrt{\frac{2}{\pi}}\sigma$. As $\widehat{H}_t$ is approximately Gaussian distributed (when the process goes to continuous time and $\widehat{H}_t$ is small, it is approximately an Ornstein-Uhlenbeck), we find that the carry trade return is

$$X^e = 2\sqrt{\frac{2}{\pi}}std\left(\widehat{H}_t\right) = \frac{2}{\sqrt{\pi\phi}}\sigma_H.$$

Variance processes. Consider an LG process centered at 0, $dX_t = -\left(\phi + X_t\right)X_t dt + \sigma\left(X_t\right)dW_t$ where W_t is a standard Brownian motion. Because of economic considerations, the support of X_t needs to be some $\left(X_{\min}, \infty\right)$ with $-\phi \leq X_{\min} < 0$. $X_{\min}$ cannot be less than $-\phi$ since the random variable X must always be mean reverting. For the simulation, we take $X_{\min} = -\phi$, maintaining full generality of the allowed domain. The following variance process makes this possible:

$$\sigma^2\left(X\right) = 2K\left(1 - X/X_{\min}\right)^2, \tag{76}$$

with $K > 0$. K is in units of $[\text{Time}]^{-3}$. The average variance of X is: $\overline{\sigma}_X^2 = E\left[\sigma^2\left(X_t\right)\right] = \int_{X_{\min}}^{X_{\max}} \sigma\left(X\right)^2 p\left(X\right) dX$ where $p\left(X\right)$ is the steady-state distribution of X_t. It can be calculated via the Forward Kolmogorov equation, which yields:

$$d\ln p\left(X\right)/dX = 2X\left(\phi + X\right)/\sigma^2\left(X\right) - d\ln\sigma^2\left(X\right)/dX.$$

Numerical simulations show that the process for volatility is fairly well approximated by: $\overline{\sigma}_X \simeq K^{1/2}\xi$ with $\xi = \sqrt{2}$. Also, the standard deviation of X's steady-state distribution is well approximated by $\left(K/\phi\right)^{1/2}$.

Asset prices often require to analyze the standard deviation of expressions like $\ln\left(1 + aX_t\right)$. Numerical analysis shows that the Taylor expansion approximation is a good one, yielding as the average volatility of $\ln\left(1 + aX_t\right) \simeq aK^{1/2}\xi$, which numerical simulations prove to be a good approximation, too.

When the process is not centered at 0, one simply centers the values. For instance, in our calibration, the recovery rate of the country productivity, F_t, has support $[F_{\min}, F_{\max}]$, centered around F_*. The probability and intensity of disasters (p and B) are constant. Define $H_t = p\left(B^{-\gamma}F_t - 1\right)$ and the associated $H_{\min}, H_{\max}, H_*$. The associated centered process is $X_t = \widehat{H}_t = H_t - H_*$. We use values of p and B from Gabaix (2012). Since we explicitly target

a bilateral exchange rate volatility (between two uncorrelated countries) of 11%, we use the relation $\sigma_e^{bil} = \sqrt{2}\overline{\sigma}_{\widehat{H}_t}/(r_e + \phi_H)$ to obtain a target for the resilience innovation volatility, $\overline{\sigma}_{\widehat{H}_t}$. We then take the volatility parameter to be $K = \left(\overline{\sigma}_{\widehat{H}_t}/\xi\right)^2$. The resulting volatility of F_t is equal to $\overline{\sigma}_F = \overline{\sigma}_{\widehat{H}_t}/(pB^{-\gamma})$.

Methodology for the simulations   The methodology followed in code is as follows:

1) Initialize variables in monthly units.

2) Simulate 500 shocks, obtaining monthly time series of resilience, inflation values.

3) Use resilience, inflation values and other fundamentals to obtain series of interest rates, spot rates.

4) At every date t, using the two interest rates computed at t, and the bilateral volatility, we backsolve for a pair of strikes that would give Put, Call Deltas of 0.25,-0.25. The formula used is the one in Garman-Kohlhagen (with maturity of 1 unit).

5) For the pair of strikes computed, we compute values of Calls and Puts using the formula in Farhi-Gabaix (2014); again using monthly parameters and with a time of 1 unit (which is a month in this case).

6) We use the computed prices of Puts and Calls to obtain implied volatilities using the Garman-Kohlhagen formula.

7) The risk reversal is just the difference in implied volatilities, and this is in monthly units.

8) The desired moments are obtained.

Details for the interpretation of the calibration in section 4.2.   The calculations reported in the text come from:

$$\nu_{F_{it}} = \frac{\upsilon_{H_{it}-H_{jt}}}{\mathbb{E}\left[pB^{-\gamma}\right]}\frac{1}{\sqrt{2}},$$

where the $\sqrt{2}$ come from the assumption of independent movements in H_{it} (it is easy to generalize) and

$$\upsilon_{p_t} = \frac{\upsilon_{H_{it}-H_{jt}}}{\mathbb{E}_t\left[B_{t+1}^{-\gamma}|F_{it+1} - F_{jt+1}|\right]}.$$

Stocks: details of the resilience process   The core of the methodology deals with simulating a permissible value of $\varepsilon_{t+1}^{H_D}$. On the one hand, since

$$\hat{H}_{D,t+1} = \frac{1 + H_{D*}}{1 + H_{D,t}}\exp(-\phi_{H_D})\hat{H}_{D,t} + \varepsilon_{t+1}^{H_D},$$

then, to respect the LG bound ($\hat{H}_{D,t+1} \geq -\phi_{H_D}$), we must have that $\varepsilon_{t+1}^{H_D} = 0$ whenever $\hat{H}_{D,t} = -\phi_{H_D}$. If not, there is a strictly positive probability that $\hat{H}_{D,t+1} < -\phi_{H_D}$, violating the LG bound. On the other hand, we must make sure that $\varepsilon_{t+1}^{H_D} - \varepsilon_{t+1}^{H}$ is uncorrelated with ε_{t+1}^{H}. As such, we use the following methodology.

Let $\varepsilon_{t+1}^{H_D} = \varepsilon_{t+1}^{H'_D} + \varepsilon_{t+1}^{H}$, where $\varepsilon_{t+1}^{H'_D}$ is uncorrelated with ε_{t+1}^{H}.

Let σ_H and σ_{H_D} be the standard deviations of ε_{t+1}^{H} and $\varepsilon_{t+1}^{H_D}$.

Then $Cov(\varepsilon_{t+1}^{H_D}, \varepsilon_{t+1}^{H}) = \sigma_H^2$, and $Corr(\varepsilon_{t+1}^{H_D}, \varepsilon_{t+1}^{H}) = \sigma_H/\sigma_{H_D} = \rho$ (say). Thus, it is now sufficient to simulate the $\varepsilon_{t+1}^{H_D}$ so that it has a correlation of ρ with ε_{t+1}^{H} and has the appropriate LG-implied vanishing properties.

The way to do this is by simulating

$$\varepsilon_{t+1}^{H_D} = \sqrt{2K_{H_D}}(1 + \frac{\hat{H}_{D,t}}{\phi})[\rho Z_{t+1}^{H} + \sqrt{1 - \rho^2} Z_{t+1}'],$$

where Z_{t+1}^{H} is the random number drawn to simulate ε_{t+1}^{H}, (i.e. $\varepsilon_{t+1}^{H} = \sqrt{2K_H}(1 + \frac{\hat{H}_t}{\phi})Z_{t+1}^{H}$) and $Cov(Z_{t+1}', Z_{t+1}^{H}) = 0$.

Finally, we separately simulate dividends to obtain prices and returns.

Methodology to simulate the one-factor structure in exchange rates We used the following setup:

$$\hat{H}_{i,t+1} = \frac{1 + H_{i*}}{1 + H_{it}} \exp(-\phi_H)\hat{H}_{it} + \epsilon_{ei,t+1}^{H},$$

where $\epsilon_{ei,t+1}^{H} = \sqrt{2K}(1 + \hat{H}_{it}/\phi_H)Z_{t+1}^{i}$.

Now, we introduce aggregate shocks to the resilience innovations as follows:

$$\epsilon_{ei,t+1}^{H} = \beta_{eit} f_{e,t+1} + \eta_{ei,t+1},$$

where we set $\beta_{eit} = b(H_{at} - H_{it})(1 + \hat{H}_{it}/\phi_h)$, and we set $f_{e,t+1} = Z_{t+1}$ where Z_{t+1} is a common innovation and H_{at} is the average resilience across countries. We further assume that the H_{i*} are the same across countries. The above formulation assures that $\beta_{eit} f_{e,t+1} = 0$ whenever $\hat{H}_{it} = -\phi_h$. By the Law of Large Numbers, the above reduces to

$$\beta_{eit} f_{e,t+1} = -b\hat{H}_{it}(1 + \hat{H}_{it}/\phi_h)Z_{t+1}.$$

Let $\eta_{ei,t+1} = c(1 + \hat{H}_{it}/\phi_h)Z_{t+1}^{i}$ where Z_{t+1}^{i} is a shock to country i alone and c is a constant. Therefore,

$$\epsilon_{ei,t+1}^{H} = -b\hat{H}_{it}(1 + \hat{H}_{it}/\phi_h)Z_{t+1} + c(1 + \hat{H}_{it}/\phi_h)Z_{t+1}^{i},$$

where c is such that the volatility of $\epsilon^H_{ei,t+1}$ matches with the data. The above formulation ensures that $\epsilon^H_{ei,t+1} = 0$ whenever $\hat{H}_{it} = \phi_h$ and so the LG bounds are satisfied.

To simulate the resilience innovations for stocks, we follow the below steps:

1) Recall that $\epsilon^D_i = \epsilon^{D'}_i + \epsilon^H_{ei}$, where $\epsilon^{D'}_i$ is orthogonal to the ϵ^H_{ei}. We also have common innovations to the $\epsilon^{D'}_i$ so we simulate it as follows:

$$\epsilon^{D'}_{i,t+1} = v[Z^D_{t+1} + Z^{\tilde{D}}_{i,t+1}],$$

where Z^D is a set of common innovations with a correlation of $\rho = 0.3$ to Z and with unit variance. $Z^{\tilde{D}}$ is a set of idiosyncratic innovations with unit variance. v is such that $Corr(\epsilon^D_i, \epsilon^H_{ei}) = \rho_1$, where $\rho_1 = \sigma_{He}/\sigma^D_H$, where σ_{He} is the standard deviation of $\hat{H}_i$ and σ^D_H is the standard deviation of $\hat{H}^D$. This ensures that $Corr(\epsilon^{D'}_i, \epsilon^H_{ei}) = 0$.

2) From the simulated $\epsilon^{D'}_i$, obtain $\epsilon^D_i = \epsilon^{D'}_i + \epsilon^H_{ei}$, and obtain the time series of resilience for each country.

3) Separately simulate dividends to obtain prices and returns.

To obtain correlations, we perform the following steps:

1) Using the methodology outlined above, simulate time series for resilience.

2) From the obtained time series, obtain exchange rates for both countries and the resulting (log of) bilateral exchange rate, and also obtain risk reversals.

3) Sort the real interest rate r_i into terciles, and obtain a corresponding series of "high shocks", "medium shocks" and "low shocks" of Z.

4) Look at the correlations of each tercile of Z with the corresponding bilateral exchange rate and the change in risk reversals at those indices.

Annualization Methodology This section details the techniques used to convert numbers from both the calibration and the data to annualized figures. Note that all values reported are annualized. The "annualization factor" (Ann. Factor) is the number we use to multiply the value directly obtained (from data/calibration) to annual terms.

Moments	Ann. Factor (Data)	Ann. Factor(Calibration)
$\mathbb{E}(|RR|)$	1	$\sqrt{12}$
Std.Dev(RR)	1	$\sqrt{12}$
Std.Dev($\Delta(RR)$)	$\sqrt{12}$	$\sqrt{12} \times \sqrt{12}$
Std.Dev(r_{it})(real)	1	12
Std.Dev(r_{it})(nominal)	1	12
Std.Dev($\Delta(r_{it} - r_{jt})$)(real)	$\sqrt{12}$	$12 \times \sqrt{12}$
Std.Dev($\Delta(r_{it} - r_{jt})$)(nominal)	$\sqrt{12}$	$12 \times \sqrt{12}$
Std.Dev($\Delta \ln(e_{it}/e_{jt})$)	$\sqrt{12}$	$\sqrt{12}$
Carry Trade Return	1	12

References for the online appendix

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