Currency Union With and Without Banking Union*

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Abstract

This paper develops a symmetric two-country model with money and banks to analyze how credit market integration affects the sustainability of a currency union. With the current Eurozone context in mind, we model limited banking integration as a higher cost for managing cross-border credit compared to domestic credit. We show that if this cost is high welfare can be higher with two currencies compared to a regime of currency union. This result derives from the interplay between the currency arrangement and endogenous credit constraints. Under limited credit integration, currency integration may magnify default incentives, leading to more stringent credit rationing. The integration of credit markets restores the optimality of the currency union.

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1 Introduction

The unification of banking markets is an overlooked issue of monetary union arrangements. In the two prominent examples of monetary unions formed during the last two centuries (the U.S. and the Euro), the original design endowed a single organization with the right of issuing currency (the Mint and the E.C.B. respectively) without taking any provision in terms of the unification of the banking markets.[1] Both ended up with the devolution of part of bank regulation to the federal authorities.[2] This paper shows that this did not occur by chance because credit market integration is a requisite to reap the gains of a common currency.[3]

A common currency is defined by common rules governing the issuance of the currency –i.e. cash and balances held at the central bank– and equal access to cash for any resident of the zone. The degree of credit market integration between countries of the zone also results from banks decisions given the technological, legal and regulatory costs of granting cross-border credit. Consequently, one can have perfect integration with respect to the currency dimension —no cost for agents to pay with currency— but imperfect integration of credit markets —i.e. a higher cost for cross-border purchases. In this paper we define imperfect credit market integration as a situation in which residents of a country pay a cross border credit premium when financing purchases in another country of the currency zone. We show why this premium may threaten the welfare gains of a common currency.

The logic of this result runs as follows. In an environment with imperfect credit market integration, the increase in transactions associated with a common currency may *worsen* default incentives on bank loans. This effect is entirely driven by the home bias triggered by the difference in funding cost created by the cross border credit premium. More specifically, banks charge a higher interest rate for cross-

[1]The U.S. constitution is the founding act of the currency union in the United States. The U.S. regime of banknote issuance varied during the 19th century but the Mint was always the authority in charge of issuing the dollar specie, see Rolnick, Smith, and Weber (2003). The Maastricht treaty creates the Euro and endows the European Central Bank with the right to issue the currency.

[2]In the Eurozone the negotiations on policies aimed at deepening credit market integration were grouped under then heading "banking union". They encompass the devolution of banks supervision from state supervisors to the E.C.B. and an agreement on common rules and funding for bank resolution, see *inter alia* Beck (2012), Nieto and White (2013).

[3]The U.S. experience exemplifies the distinction between a currency union and a fully-fledged monetary union. During the 19th century periodic localized or systemic banking problems trigger discussions on the redesign of the regime of currency issuance (Rockoff, 2003, Rousseau, 2013). For example during the National Banking Act period (Weiman and James, 2007), White (1982) argues that differences in regulatory frameworks caused distortions on the credit market that "stimulated the public to press for currency and banking reform". This political pressure for currency reform is known to us after the novel of the Wizard of Oz, see Rockoff (1990). The exact moment at which the U.S. banking market gets unified is an unsettled issue. The consensus is that the integration in terms of payment instruments (James and Weiman, 2010) and interbank funding (Davis, 1965, Sylla, 1969, James, 1976) started at the beginning of the 20$^{\text{th}}$ century. The integration was reinforced by the creation of the FED (Miron, 1986). Since then it is commonplace to pay in Boston with checks drawn for example on Chicago.

border *vis-à-vis* domestic purchases. For borrowers, this creates a wedge between the cost of foreign versus domestic purchases, which induces agents with no record of default to be biased towards domestic goods compared to what would be dictated by their preferences. Instead an agent that would default—something that does not happen on the equilibrium path—would lose access to credit and purchase goods in line with his preferences. Imposing conversion costs between currencies can thus make default less attractive, as this cost affects more severely defaulters than non-defaulters, thereby relaxing borrowing constraints. By contrast, when financing conditions are the same for domestic and cross border purchases, there is no home bias and a conversion cost between currencies does not attenuate default incentives. Thus under perfect credit integration welfare is always higher with a common currency.

The result is shown in a symmetric two-country model of fiat money and credit. Currency (fiat money) and bank credit are used in equilibrium by residents of each country to purchase domestic or foreign goods. In each period agents are subject to individual consumption and production shocks that cannot be efficiently insured by their cash holdings. As in Berentsen, Camera, and Waller (2007), banks provide insurance against those shocks: Agents with no current need for cash (producers) can deposit their currency holdings at their bank rather than keeping them as idle balances, while those with a current need for cash (buyers) can obtain credit from banks to finance additional purchases. By lending out cash received in deposits, banks effectively redistribute the money stock according to agents' current transaction needs. Banks help financing home and foreign good purchases, but at a higher price for cross-border purchases. Because agents cannot commit to repay their debt, banks resort to borrowing constraints and to the exclusion of defaulters from future access to their services to ensure debt repayment.

To evaluate the gains generated by a currency union in this environment, we compare two monetary arrangements: a common currency case, and a 'one country-one currency' case with positive conversion costs between the two currencies. The difference between the two cases lies in the conversion costs, and we ask whether the case with strictly positive conversion costs is dominated in terms of welfare by a currency union—which is equivalent to costless conversion.

Our mechanism delivers three results. First we show that the currency union is the optimal arrangement whenever the borrowing constraint is not active or when it is active and the credit markets are sufficiently integrated (i.e. under a regime of banking union). Second we show that a currency union is suboptimal *if the inflation rate is low and credit market integration is limited.* Third credit crunches —defined as a reduction in the quantity of credit caused by a substantial increase of the cross border credit premium— are sharper in a currency union with imperfectly integrated credit markets than in a regime of separate currencies.

This paper contributes to the macroeconomic literature on the benefits and costs of monetary unions by showing that their sustainability requires sufficiently integrated credit markets. Otherwise a currency union may be dominated in terms of welfare. In addition we contribute to monetary theory by suggesting a new rationale for the optimality of multiple currencies *vis-à-vis* a unique currency. In our setup a regime of separate currencies mitigates the incentive to default on credit and hence may be preferred even though it entails higher transaction costs in cross border trades.

The rest of the paper is organized as follows. The environment is laid out in section 2. The conditions for the existence of (symmetric) equilibria are presented in section 3. Section 4 presents the main results pertaining to the welfare implications of a regime of common versus multiples currencies when credit market integration varies. Section 5 illustrates the results with the varying credit market integration in the Eurozone since its inception. Section 6 discusses our contribution to the literature. Section 7 concludes. Proofs are relegated to the appendix.

2 Environment

Our model is based on Lagos and Wright (2005), Rocheteau and Wright (2005) and Berentsen et al. (2007). Time is discrete and continues forever. There are two identical countries, the home country and the foreign country, each populated by a continuum of infinitely-lived agents of unit mass. There are two perfectly divisible and non-storable goods: a home good, denoted as q_h, and a foreign good, denoted as q_f. Agents discount across periods with factor β. A period is divided in two subperiods. In each period, two competitive markets open sequentially in each country. Before the first market opens, agents receive an idiosyncratic shock that determines whether they are sellers for the current period and get no utility from consumption (with probability $(1-b)$), or buyers in which case they want to consume but cannot produce (with probability b). In the second market, all agents can produce and consume a quantity of a generic good denoted as x, and utility from consumption (or disutility of working) is linear in the quantity of good.

Buyers' preferences in the first subperiod are

$$\max\left[u\left(q_f\right)+\eta q_f, u\left(q_h\right)\right] \tag{1}$$

where η is a preference shock which can take values $\eta = 0, \eta_1, \eta_2$ with probabilities π_0, π_1 and π_2, and $0 < \eta_1 < \eta_2$.[4] The function u satisfies $u'(q), -u''(q) > 0$, $u'(0) = \infty$ and $u'(\infty) = 0$. In addition we assume that $-u''(q)\,q \leq u'(q)$. Preferences in (1) are such that in equilibrium buyers will consume the home good in periods in which their preference shock η is low and consume the foreign good in periods in which η is high. In the former case they trade in the first market of the home country. In the latter case they travel –costlessly– to trade in the foreign country and come back to participate to the home country second market. For

[4] We assume three realizations of the preference shock η to allow for domestic and foreign consumption on the equilibrium path while creating a wedge between the consumption pattern of agents who borrow and of those who do not.

sellers, producing a quantity q_j (with $j = h, f$) in the first subperiod represents a disutility equal to $c(q_j) = q_j$.

There are two storable, perfectly divisible and intrinsically useless currencies, the home currency and the foreign currency. For simplicity, the quantity of each currency at the beginning of period t is denoted as M. The money supply in each country grows at the gross rate $\gamma = M_{+1}/M$ where the subscript $+1$ indicates the following period. Agents receive monetary lump-sum transfers from the central bank equal to $T = (\gamma - 1) M_{-1}$ during the second market in period t. Consistently with the current Eurozone situation, we assume that the central bank has no power to tax agents so $\gamma \geq 1$.[5] In order to motivate a role for a medium of exchange, traders are assumed to be anonymous so that sellers require compensation at the same time as they produce. This assumption rules out bilateral credit but not banking credit.

Currencies can be exchanged before the first market opens. Exchanging currencies represents a disutility cost ε proportional to the real amount of money exchanged. Given that the money growth rate is assumed to be the same for the two countries, the case in which the cost ε is equal to zero is equivalent to a currency union.[6]

In each country there are competitive banks which take deposits and use them to grant loans. Banking activities take place before the first market opens. Deposits are taken by banks and paid back during the second subperiod with the corresponding interest. A loan is a bilateral contract between an agent and a individual bank. The loan and the interest are paid back during the second market. Hence credit is intra-period, so that loans and deposits are not rolled over.[7]

Banks have no enforcement power. However they possess a technology to keep track of agents' financial histories. This implies that they can recognize agents who have defaulted in the past and exclude them from banking activities—loans and deposits—for the rest of their lifetime. The threat of exclusion allows bank credit to be supported in equilibrium. For simplicity, we assume that defaulters are excluded from monetary transfers.

Agents contract with banks located in their country of residence since banks can only identify residents (the cost for a bank to identify a non-resident is infi-

[5]This restriction implies that the Friedman rule is not a feasible policy, so that it is optimal for agents to insure against idiosyncratic shocks using both (costly) cash holdings and banks. This assumption could be relaxed, for instance by assuming that the government can use lump-sum taxes but that agents can evade taxation by not participating in the market - see Hu, Kennan, and Wallace (2009) and Andolfatto (2010).

[6]We focus on the case in which agents only hold the currency of their country of residence. This is for simplicity and could be justified by assuming that agents face an extra cost (e.g. an accounting cost) when holding a mixed portfolio. Since the mechanism presented in the paper does not hinge on this assumption, we make this modeling choice in order to keep the model tractable. A couple of papers in international monetary economics models the choice of an asset portfolio, see Geromichalos and Simonovska (2014) and Zhang (2014)

[7]As pointed out by Berentsen et al. (2007) quasi-linear preferences in the second market imply that one-period debt contracts are optimal.

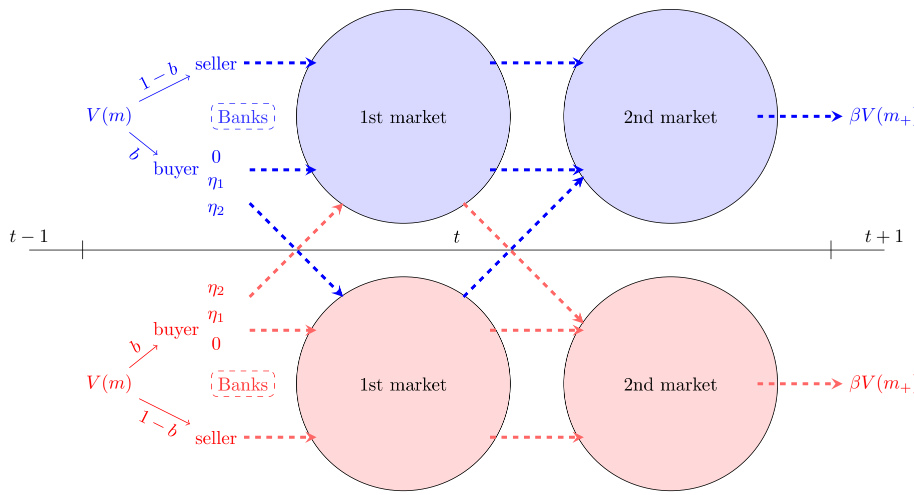

Figure 1: Sequence within a period

nite). Home (foreign) country sellers deposit in the home (foreign) country. Home (foreign) buyers borrow from home (foreign) banks. Agents can contact a bank located in their country regardless of the first market in which they trade. Consistently with Eurozone evidence Beck (2012), we assume that a home bank bears a management (regulatory) cost $c \geq 0$ when it services a client who pays his purchases abroad. In the model, since banks are competitive and make zero profit in equilibrium, this cost is shifted to borrowers.[8] We refer to this cost as the cross-border credit premium. This premium c is modeled as a disutility cost that each borrower incurs when he takes out a loan for foreign consumption. When $c = 0$ taking out a loan for foreign or home consumption is equivalent; i.e. credit market integration is perfect. When $c > 0$ financing foreign consumption is more costly than financing domestic consumption; i.e there is imperfect credit market integration.

The sequence of trades within a period is depicted in figure 1.

[8]In the Eurozone banks have information on whether transactions are carried out in another jurisdiction except for those of small amount. They incur costs when dealing with inter-jurisdiction transactions and these costs are shifted to customers. For example clearing checks payable in another jurisdiction is costly. If a buyer wants to pay a transaction of a significant amount with credit, he has to sign a specific contract at a higher interest rate, if ever a bank accepts to lend. A European Union directive forbids banks to price-discriminate payment services (e.g. ATM withdrawals) by location but withdrawals on credit card are limited to a couple of thousands euros for premium cards. Moreover customers travelling within the Eurozone with more than 10,000 euros are liable to regulatory declaration to the authorities.

3 Symmetric equilibrium

We focus on stationary equilibria in which end-of-period real money balances are constant and positive, so that

$$\gamma = M/M_{-1} = \phi_{-1}/\phi \tag{2}$$

where ϕ is the price of money in real terms during the second market. Let $V(m)$ denote the value function of an agent who holds an amount m of money at the beginning of a period, before learning the realization of the preference shock. $W(m, \ell)$ is the expected value from entering the second market with m units of money balances and an amount ℓ of loans (a negative amount ℓ denotes deposits). In what follows, we analyze a representative period t and solve the model backwards from the second to the first market. Since countries are perfectly symmetric, we only present the optimal choices by agents of the home country and we drop out the country index when this does not lead to confusion.

3.1 The second market

In the second market, agents consume (or produce), reimburse loans (or redeem deposits), and adjust their money balances. Since banks are competitive and make zero profit we already take into account that the interest rate on loans and deposits is the same and denote it by i. If an agent has taken out a loan ℓ, he pays back $\ell(1+i)$ units of money to the bank. If he has deposited an amount ℓ, he gets it back with the accrued interest, $\ell(1+i)$.

The representative agent chooses his next period monetary holdings, m_{+1}, and his consumption (or production) of the generic good, x, in order to maximize $W(m, \ell)$ subject to the budget constraint:

$$\max_{x, m_{+1}} W(m, \ell) = x + \beta V(m_{+1})$$
$$\text{s.t. } x + \phi\ell(1+i) + \phi m_{+1} = \phi m + \phi T$$

where ϕ is the price of money in terms of the second-market good and $T = (\gamma - 1) M_{-1}$ are lump-sum transfers from the central bank. The budget constraint states that the sum of an agent's current consumption, loan repayment (or deposits redemption if $\ell < 0$) and next-period money holdings equals his current money holdings plus the monetary transfers from the central bank.

Inserting the budget constraint in the objective function, the above program simplifies to

$$\max_{m_{+1}} \left[-\phi m_{+1} + \phi m - \phi\ell(1+i) + \phi T + \beta V(m_{+1})\right]$$

The first-order condition on m_{+1} is

$$\beta V'(m_{+1}) = \phi \tag{3}$$

where $V'(m_{+1})$ is the marginal value of an additional unit of money taken into period $t+1$. Notice that m_{+1} is the same for all agents, regardless of their initial money holdings m. The envelope conditions are

$$\begin{aligned} W_m &= \phi \\ W_\ell &= -\phi(1+i) \end{aligned} \tag{4}$$

3.2 The first market

3.2.1 Sellers

Since sellers do not derive utility from consumption, they choose to deposit their currency holdings at the bank instead of borrowing. Let p denote the price of first-market goods. In the first market the seller chooses how much to produce q_s, and the amount of his deposit $-\ell_s$. The program for a seller in the first market is

$$\max_{q_s,\ell_s} \left[-q_s + W\left(m_{-1} + \ell_s + pq_s, \ell_s\right)\right]$$
$$\text{s.t.} \quad -\ell_s \leq m_{-1}$$

where m_{-1} are currency holdings taken from the previous period. The first-order condition on q_s is

$$W_m p = 1$$

Using (4), it becomes

$$\phi p = 1 \tag{5}$$

Condition (5) states that sellers are indifferent between producing in the first market and producing in the second market.

The first-order condition on ℓ_s can be written as

$$\phi i = \mu_s \tag{6}$$

where μ_s is the multiplier associated with the deposit constraint. According to condition (6), if the interest rate is positive, the deposit constraint is always binding and sellers deposit their entire currency holding at a bank.

3.2.2 Buyers

At the beginning of each period, buyers learn the realized value of the preference shock η that increases the utility of consuming the foreign good. Then, buyers decide to consume in their home country or abroad during the first market. Given preferences (1), it is straightforward to see that buyers' travel decisions follow a simple cutoff rule: any buyer consumes the home good when $\eta \leq \eta^*$ and consumes the foreign good when $\eta > \eta^*$, where the threshold η^* is endogenously determined (see section 3.5).

Denote as q_h^η (q_f^η) the quantity of home (foreign) goods consumed by a buyer with preference shock η. Denote as ℓ_h^η (ℓ_f^η) the loan taken out by a buyer with preference shock η who consumes the home (foreign) good. Since banks can distinguish domestic from foreign transactions, they can potentially set different borrowing limits. Let $\bar{\ell}_f$ indicate the maximal amount that an agent traveling abroad can borrow. Similarly $\bar{\ell}_h$ indicates the borrowing limit for an agent who consumes the home good.

Since optimal quantities may differ for buyers who stay in the home country from those of buyers who travel abroad, we distinguish two cases. Consider first a buyer who consumes the home good (that is with shock $\eta \leq \eta^*$). This buyer maximizes the utility from consuming q_h^η subject to two constraints:

$$\max_{q_h^\eta, \ell_h^\eta} u\left(q_h^\eta\right) + W\left(m_{-1} + \ell_h^\eta - pq_h^\eta, \ell_h^\eta\right)$$

$$\text{s.t. } pq_h^\eta \leq m_{-1} + \ell_h^\eta \tag{7}$$

$$\ell_h^\eta \leq \bar{\ell}_h \tag{8}$$

The first constraint is the cash constraint by which the buyer cannot spend more than his initial money holdings plus his loan. The second constraint is the borrowing constraint set by banks to ensure loan repayment (see section 3.6).

The first-order condition on q_h^η is

$$u'\left(q_h^\eta\right) = 1 + \mu_h^\eta/\phi \tag{9}$$

where μ_h^η is the multiplier associated with the cash constraint (7). The first-order condition on ℓ_h^η for this buyer can be written as

$$\mu_h^\eta - \phi i = \lambda_h^\eta, \tag{10}$$

where λ_h^η is the multiplier associated with the borrowing constraint (8). Using (9) to substitute for μ_h^η, condition (10) can be rewritten as

$$u'\left(q_h^\eta\right) = 1 + i + \lambda_h^\eta/\phi. \tag{11}$$

Consider next the program for a buyer who consumes abroad (with shock $\eta > \eta^*$). His consumption solves:

$$\max_{q_f^\eta, \ell_f^\eta} u\left(q_f^\eta\right) + (\eta - \varepsilon)\, q_f^\eta - c\ell_f^\eta/p + W\left(m_{-1} + \ell_f^\eta - pq_f^\eta, \ell_f^\eta\right)$$

$$\text{s.t. } pq_f^\eta \leq m_{-1} + \ell_f^\eta, \tag{12}$$

$$\ell_f^\eta \leq \bar{\ell}_f \tag{13}$$

Compared to the buyer who consumes the home good, the buyer who consumes the foreign good incurs conversion costs on his purchase (εq_f^η) and the cross border credit premium which is proportional to the real amount of loans taken out $c\ell_f^\eta/p$.

Using (4) and (5), the first-order condition on q_f^η is

$$u'\left(q_f^\eta\right)+\eta=1+\varepsilon+\mu_f^\eta/\phi \tag{14}$$

where μ_f^η is the multiplier associated with the cash constraint (12). The first-order condition on ℓ_f^η can be written as

$$u'\left(q_f^\eta\right)+\eta-\varepsilon-c=1+i+\lambda_f^\eta/\phi \tag{15}$$

where λ_f^η is the multiplier associated with the borrowing constraint (13).

3.3 Market clearing

Market clearing in the loan market yields

$$(1-b)\,\ell_s+b\sum_{\eta\leq\eta^*}\pi_\eta\ell_h^\eta+b\sum_{\eta>\eta^*}\pi_\eta\ell_f^\eta=0 \tag{16}$$

The sum of the deposits made by sellers and the loans taken out by all buyers —i.e. those who consume the home good and those who consume the foreign good— is equal to zero. In addition, for a seller it is optimal to deposit their entire money holdings for any $\gamma\geq 1$. Thus $m_{-1}=-\ell_s$, and (16) becomes

$$(1-b)\,m_{-1}=b\sum_{\eta\leq\eta^*}\pi_\eta\ell_h^\eta+b\sum_{\eta>\eta^*}\pi_\eta\ell_f^\eta \tag{17}$$

Since countries are symmetric, market clearing in the first market for goods yields

$$b\sum_{\eta\leq\eta^*}\pi_\eta q_h^\eta+b\sum_{\eta>\eta^*}\pi_\eta q_f^\eta=(1-b)\,q_s \tag{18}$$

3.4 Marginal value of money

The expected utility for an agent who starts a period with m units of money is:

$$\begin{aligned}V(m)=&\,b\sum_{\eta\leq\eta^*}\pi_\eta\left[u\left(q_h^\eta\right)+W\left(m+\ell_h^\eta-pq_h^\eta,\ell_h^\eta\right)\right]\\&+b\sum_{\eta>\eta^*}\pi_\eta\left[u\left(q_f^\eta\right)+(\eta-\varepsilon)\,q_f^\eta-\phi\ell_f^\eta c+W\left(m+\ell_f^\eta-pq_f^\eta,\ell_f^\eta\right)\right]\\&+(1-b)\left[-q_s+W\left(m+\ell_s+pq_s,\ell_s\right)\right]\end{aligned}$$

Given (5) cash constraints (7) and (12) imply that

$$\begin{aligned}q_h^\eta&\leq\phi\left(m_{-1}+\ell_h^\eta\right)\\q_f^\eta&\leq\phi\left(m_{-1}+\ell_f^\eta\right)\end{aligned} \tag{19}$$

Using (4), (5), (6), (9) and (14), the marginal value of money is

$$\partial V/\partial m = b\phi \sum_{\eta \leq \eta^*} \pi_\eta u'\left(q_h^\eta\right) + b\phi \sum_{\eta > \eta^*} \pi_\eta \left[u'\left(q_f^\eta\right) + \eta - \varepsilon\right] + (1-b)\,\phi\,(1+i)$$

Using (2) and (3), this condition becomes

$$\gamma/\beta = b \sum_{\eta \leq \eta^*} \pi_\eta u'\left(q_h^\eta\right) + b \sum_{\eta > \eta^*} \pi_\eta \left[u'\left(q_f^\eta\right) + \eta - \varepsilon\right] + (1-b)\,(1+i) \qquad (20)$$

The left-hand side of this equation represents the marginal cost of acquiring an additional unit of money while the right-hand side represents its marginal benefit: With probability b the agent consumes the home good (for $\eta \leq \eta^*$) or the foreign good (for $\eta > \eta^*$), and with probability $(1-b)$ the agent is a seller and earns interests on his deposits.

3.5 Travel decision

As discussed above, buyers' travel equilibrium decisions can be represented by a threshold η^* such that buyers with shock $\eta \leq \eta^*$ consume at home while buyers with $\eta > \eta^*$ consume abroad. This threshold corresponds to the virtual value of the preference parameter η such that the value of staying in the home country is equal to the value of traveling to the foreign country. The threshold η^* is defined by

$$u\left(q_h^{\eta^*}\right) - \phi \ell_h^{\eta^*}(1+i) = u\left(q_f^{\eta^*}\right) + q_f^{\eta^*}(\eta^* - \varepsilon) - \phi \ell_f^{\eta^*}(1+i+c)\,. \qquad (21)$$

On the left-hand side of (21), the value of purchasing $q_h^{\eta^*}$ is equal to the utility from consumption minus the cost of reimbursing the loan for home-good consumption. On the right-hand side of (21), the value of purchasing $q_f^{\eta^*}$ is equal to the utility from consumption minus the cost of reimbursing the loan for foreign-good consumption and the conversion cost.

3.6 Borrowing constraint

Banks have no enforcement power. Therefore they must set a borrowing constraint that ensures voluntary debt repayment: They choose the amount of loans $\bar{\ell}_h$ and $\bar{\ell}_f$ such that the payoff to an agent who repays his debt is at least equal to the payoff to a defaulter.

Denote as $\hat{q}_h^\eta$ ($\hat{q}_f^\eta$) the quantity of the home (foreign) good consumed by an agent with preference shock η who has defaulted in the past. $\hat{m}_{-1}$ denotes money holdings brought by a defaulter from the previous period.

Lemma 1 *Assume $\beta \leq \left[1 + b\left(\pi_1\eta_1 + \pi_2\eta_2\right)\right]^{-1}$. Then defaulters are cash-constrained for all realizations of η.*

Lemma 1 states that defaulters are cash constrained for all realizations of η if β is low relative to the additional expected utility from consuming the foreign good as opposed to consuming the home good.[9] Since utility from consuming the foreign good is higher than utility from consuming the home good for $\eta > 0$ given (1), defaulters could be cash constrained for $\eta = \eta_1, \eta_2$ and not cash-constrained for $\eta = 0$.

Given lemma 1 we can set $\hat{q}_h^\eta = \hat{q}_f^\eta = \hat{q}$ and $\hat{m}_{-1} = p\hat{q}$ for all η since defaulters do not have access to the banking system. Let $\hat{\eta}^*$ denote the threshold describing the optimal travel decision for an agent who has defaulted in the past. In periods in which $\eta \leq \hat{\eta}^*$ the defaulter consumes the home good, whereas in periods in which $\eta > \hat{\eta}^*$ the defaulter consumes the foreign good. The threshold $\hat{\eta}^*$ is given by

$$u(\hat{q}) = u(\hat{q}) + (\hat{\eta}^* - \varepsilon)\,\hat{q}$$

According to this condition $\hat{\eta}^*$ is determined such that the utility derived from consuming the home good is equal to the utility from consuming the foreign good minus the conversion cost. Hence,

$$\hat{\eta}^* = \varepsilon \tag{22}$$

Let $\hat{V}(\hat{m})$ indicate the expected utility for a defaulter who starts a period with $\hat{m}$ units of money and $\hat{W}(\hat{m})$ indicate the expected utility for a defaulter with $\hat{m}$ units of money at the beginning of the second market. $\hat{V}(\hat{m})$ is

$$\begin{aligned}\hat{V}(\hat{m}) = {} & b\sum_{\eta\leq\hat{\eta}^*}\pi_\eta\left[u(\hat{q}) + \hat{W}(0)\right] + b\sum_{\eta>\hat{\eta}^*}\left[u(\hat{q}) + (\eta-\varepsilon)\,\hat{q} + \hat{W}(0)\right] \\ & + (1-b)\left(-q_s + \hat{W}(\hat{m} + pq_s)\right)\end{aligned}$$

where $\hat{q}$ can be determined by the optimal condition on money holdings by the defaulter:

$$\gamma/\beta = bu'(\hat{q}) + b\sum_{\eta>\hat{\eta}^*}\pi_\eta(\eta-\varepsilon) + 1 - b \tag{23}$$

Lemma 2 *An agent who borrows debt ℓ has incentives to repay his debt if and only if*

$$-\phi\ell(1+i) - \phi m_{+1} + \phi T + \beta V(m_{+1}) \geq -\phi\hat{m}_{+1} + \beta\hat{V}(\hat{m}_{+1}), \tag{24}$$

where $\hat{m}$ denotes the optimal choice of money holding following default, and $\hat{V}(\hat{m})$

[9] Notice that in this model with no preference shock ($\pi_1, \pi_2 = 0$) and $\gamma \geq 1$ the condition in lemma 1 is simply $\beta \leq 1$.

the associated continuation value. (24) can be expressed equivalently as

$$
\begin{aligned}
& -\phi\left[\ell(1+i)+m_{+1}-T\right] \\
& +\frac{\beta b}{1-\beta}\left\{\sum_{\eta \leq \eta^*} \pi_\eta\left[u\left(q_h^\eta\right)-q_h^\eta\right]+\sum_{\eta>\eta^*} \pi_\eta\left[u\left(q_f^\eta\right)+(\eta-1-\varepsilon) q_f^\eta-\phi \ell_f^\eta c\right]\right\} \\
& \geq \frac{\beta b}{1-\beta}\left[u(\hat{q})-\hat{q}+\sum_{\eta>\hat{\eta}^*} \pi_\eta(\eta-\varepsilon) \hat{q}\right]-\frac{(\gamma-\beta) \hat{q}}{1-\beta} .
\end{aligned}
\tag{25}
$$

In particular, banks set identical limits $\bar{\ell}_h = \bar{\ell}_f = \bar{\ell}$ for home-goods consumption and foreign-goods consumption loans.

The left-hand side of the borrowing constraint in equation 25 in Lemma 2 represents the pay-off to an agent who does not default. In period t, this agent works to pay his loan with the corresponding interest and to recover his money holdings. From $t+1$ onwards, his expected utility is determined by the net utility of consuming the foreign good (minus conversion costs and the cross-border credit premium) each time he turns out to be a buyer with $\eta > \eta^*$ or the home good each time he turns out to be a buyer with $\eta \leq \eta^*$.

The right-hand side of the borrowing constraint represents the pay-off to a defaulter. If an agent defaults, he does not work to repay the loan taken at the beginning of t nor the interest on it. His expected lifetime utility is given by the net utility of consuming $\hat{q}$ as a buyer from $t+1$ on, minus the cost of adjusting money holdings from t on, equal to $(\gamma-\beta)\,\hat{q}/\,(1-\beta)$.

Banks set the borrowing limits $\bar{\ell}_h$ and $\bar{\ell}_f$ at the same value $\bar{\ell}$. The reason is that since the cost c is paid at the moment at which the loan granted, it does not affect the borrowing constraint.

3.7 Unconstrained and fully-constrained equilibria

The following propositions provide conditions on parameter values for the existence of an equilibrium in which agents are not credit constrained (Proposition 1) and for the existence of a fully constrained equilibrium in which all agents are credit constrained (Proposition 2).

Definition 1 *An equilibrium is a vector of consumption quantities $\left\{q_h^\eta, q_f^\eta, \hat{q}\right\}$, traveling thresholds $\{\eta^*, \hat{\eta}^*\}$, interest rate i, price of money ϕ, money holdings m_{-1}, loans $\left\{\ell_h, \ell_f^\eta\right\}$, borrowing limit $\bar{\ell}$ and multipliers associated with the borrowing constraint $\left\{\lambda_h^\eta, \lambda_f^\eta\right\}$ for $\eta \in \{0, \eta_1, \eta_2\}$ which satisfy $m_{-1} = M_{-1}$, (11), (15), (17), (19)-(23) and (25). An equilibrium is unconstrained if the borrowing constraint (25) is slack for all values of η. An equilibrium is fully constrained if the borrowing constraint (25) binds for all values of η ($\ell_h^\eta = \ell_f^\eta = \bar{\ell}$ for all η).*

Proposition 1 *If β is sufficiently high there is $\tilde{\gamma}$ such that if $\gamma \geq \tilde{\gamma} \geq 1$, a unique unconstrained equilibrium exists.*

Proposition 1 states that if the rate of money growth γ is high enough, then an unconstrained equilibrium exists and this equilibrium is unique. For all realizations of η, agents are able to borrow as much as they desire at the prevailing interest rate. This result is usual in monetary models with limited commitment and it replicates the result of proposition 4 in Berentsen *et alii* in a two-country framework with (potentially imperfect) banking markets integration.[10]

This result comes from the impact of inflation on consumption and thus on expected utility. Agents choose the consumption quantity bought in the first market by equating the marginal utility of consumption in this market to the marginal cost of carrying money from the second market in t to the first market in $t+1$. If the rate of money growth γ is higher than the discount factor β, carrying money throughout periods is costly because agents need to acquire their money holdings before purchasing goods. The higher γ is, the higher the cost of carrying money throughout periods, and therefore the higher the marginal utility of consumption and the lower the level of consumption in the first market. The cost of carrying money is mitigated for non-defaulters by the fact that they earn interest on their idle cash balances when they turn out to be sellers. Therefore the mere existence of banks allows agents with access to the banking system to benefit from a lower level of marginal value of cash in the first market and hence from a higher level of consumption. On the contrary banks prevent defaulters from depositing their cash balances. Because they do not earn any interest, they have a higher marginal cost of carrying money and enjoy a lower level of consumption. When inflation is high enough, incentives to default are low because expected utility is too low for defaulters and the borrowing constraint is not binding. As a result there is a level of inflation above which agents borrow their desired amount at equilibrium interest rates.

Proposition 2 *If β, η_1 and η_2 are sufficiently low, there is $\left\{\gamma^1, \gamma^2\right\}$ with $1 \leq \gamma^1 < \gamma^2 < \tilde{\gamma}$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ a fully constrained equilibrium exists. In this fully constrained equilibrium the threshold η^* satisfies*

$$\eta^* = \varepsilon + (1-b)\, c \tag{26}$$

If $\eta_1 > \eta^*$ buyers consume the home good with probability π_0. If $\eta^* \geq \eta_1 > \varepsilon$ buyers consume the home good with probability $(\pi_0 + \pi_1)$.

In a fully constrained equilibrium, all buyers would like to borrow at the prevailing equilibrium interest rate more than banks allow them to. Proposition 2 states that a fully constrained equilibrium exists when the inflation rate is positive and low enough, provided that the discount factor β and the values of the

[10] See also Aiyagari and Williamson (2000), Corbae and Ritter (2004).

preference shock η_1 and η_2 are low enough.[11] When inflation is low, the marginal cost of carrying money is low and defaulters attain a relatively high level of consumption. Incentives to default are high and the borrowing constraint binds: Only a limited amount of credit can be sustained in equilibrium because the threat of being excluded from the banking system imposes too mild a cost of default.

Next we discuss how the travel decision defined in equation (21) is determined in this equilibrium. In the model agents are fully constrained when they are credit-constrained for all realizations of η in which case they borrow the same amount and consume the same quantity of goods either at home or abroad. Equation (21) boils down to equation (26): The threshold η^* defined by the right hand side of (26) depends only on the extra costs of purchasing the foreign goods composed of the conversion cost and the cross border credit premium. The conversion cost is paid on the total amount purchased whereas the cross border credit premium c is paid only on the part $(1-b)$ of the consumption financed with a bank loan.[12] The reason is that credit is used to finance the difference between the desired consumption and the cash holdings which depend positively on the probability b of becoming a buyer because agents are more inclined to carry out costly money holdings when they have a higher chance to spend them.

To decide on the country in which he trades in the first market, the buyer compares the extra utility derived from the consumption of the foreign good that depends on the realization of η with the extra cost of financing it, compared to the cost of financing the home good consumption. Given the realized preference shock there is a level of the financing cost above which an agent switches from the consumption of the foreign good towards the consumption of the home good even for a positive value of η. As stated in proposition 2 if $\eta_1 > \varepsilon + (1-b)\,c$, the cross-border credit premium is low, buyers consume the home good only when $\tilde{\eta}$ is zero —with probability π_0— and there is no home-bias. If $\eta_2 > \varepsilon + (1-b)\,c \geq \eta_1 > \varepsilon$ the cross-border credit premium is high and buyers consume the home good when $\tilde{\eta}$ is equal to zero or η_1 —i.e. with probability $(\pi_0 + \pi_1)$. This defines a home bias in consumption triggered by a high enough level of the cross border credit premium and (or) the conversion cost. When the cost of converting one currency into the other is negligible, agents' bias towards home consumption comes from the imperfect credit market integration.

4 Currency conversion cost, credit and welfare

This section presents the main results of the paper. We analyze the effect of making currency exchange costly on both credit and welfare, i.e. on the expected lifetime

[11] In the main text we do not present the case in which c is so high that buyers never consume the foreign good. The appendix presents this case with the corresponding proofs.

[12] In this equilibrium the share of consumption financed with credit l/pq is equal to $(1-b)$, see equation 41 in the proof of the proposition 2in the appendix.

utility of the representative agent. Given (1) and (18), welfare is defined as

$$\mathcal{W} = \frac{b}{1-\beta} \left\{ \sum_{\eta \leq \eta^*} \pi_\eta \left[u\left(q_h\right) - q_h \right] + \sum_{\eta > \eta^*} \pi_\eta \left[u\left(q_f^\eta\right) + \left(\eta - 1 - \varepsilon\right) q_f^\eta - \phi \ell_f^\eta c \right] \right\} \tag{27}$$

We ask when a monetary union is optimal, i.e. for which values of the parameter space welfare is maximal when $\varepsilon = 0$. We derive conditions on c and γ such that agents prefer a regime of separate currencies ($\varepsilon > 0$) instead. We then provide a comparative statics result on how credit and welfare depend on c. Finally we construct an example in which a regime of separate currencies is optimal even if inflation is optimally chosen.

4.1 When is a monetary union optimal?

Here we show that in economies with money and credit agents prefer a monetary union if for exogenous reasons the inflation rate γ is high enough or if the credit market integration between countries is deep enough (i.e. the level of cross-border credit premium c is low enough). The next proposition assesses the effect of implementing conversion costs between the two currencies when agents are not credit constrained.

Proposition 3 *In an unconstrained equilibrium, imposing positive conversion cost $\varepsilon > 0$ leaves the consumption of the home good (q_h) unchanged, decreases the consumption of the foreign good (q_f^η) for all η, increases the real quantity of credit financing home-good consumption ($\phi \ell_h^\eta$) and decreases the real quantity of credit financing foreign good consumption ($\phi \ell_f^\eta$). The overall effect is welfare worsening.*

Proposition 3 states that imposing positive conversion costs is unambiguously detrimental to welfare if agents are not borrowing constrained. There are redistributive effects across types –home or foreign– of consumption. Because a positive conversion cost increases the marginal cost of purchasing goods, buyers decrease their expected consumption so that the marginal utility from consumption matches its marginal cost. Conversion cost decreases the equilibrium quantity consumed abroad but leaves the quantity consumed domestically unchanged. Consequently, agents choose to carry a lower amount of –costly– monetary holdings from one period to the next because they are anyway able to borrow as much as they want. It follows that agents who stay in the home country choose to increase their borrowing to finance their consumption with each increase in conversion costs. Conversely, agents consuming abroad need to borrow less because the decrease in money holdings is lower than the decrease of their desired foreign consumption. Since the consumption of the foreign good decreases with conversion costs and the consumption of the home good is unaffected, it follows that the overall effect on utility is negative. Since from proposition 1 an unconstrained equilibrium exists only when inflation is high enough, proposition 3 implies that a currency is preferred for high enough of inflation.

Next we analyze the effect of conversion costs when agents are credit-constrained. The following proposition refers to the case in which agents are credit-constrained and financial integration among countries is deep enough; i.e., $c < \eta_1 / (1-b)$.

Proposition 4 *Let $c < \eta_1 / (1-b)$. In a fully constrained equilibrium imposing positive conversion cost ε triggers a reduction in the consumption of both goods (q_h, q_f^η) and in the real quantity of credit $(\phi\ell_h^\eta,\ \phi\ell_f^\eta)$ and worsens welfare.*

According to proposition 4, imposing positive conversion costs is welfare worsening when financial markets of the two countries are relatively well integrated, i.e. when the cross border credit premium c multiplied by the share $(1-b)$ of the consumption financed using credit is smaller than the preference η_1 for the foreign good.[13] The consumption of the home good decreases because agents are constrained for all realizations of η. In this equilibrium, agents who want to consume the home good are unable to borrow to exactly compensate for the decrease in monetary holdings triggered by an increase in conversion costs. Therefore, the decrease in monetary holdings implies a reduction in consumption of both the home good and the foreign good. As in the case in which agents are unconstrained, imposing conversion costs when agents are credit-constrained and financial integration is deep enough makes agents reduce their consumption and so it is unambiguously detrimental to welfare.

Therefore a monetary union is always optimal when no agent is credit constrained —i.e. if agents are patient and inflation is high enough— and when all agents are credit constrained and the cross border credit premium is low.

4.2 Monetary and non-monetary causes for monetary disunion

In this section we explain why the previous result on the optimality of a monetary union may be reversed. We depart from existing models studying the conditions for the optimality of monetary union by explicitly considering the possibility of imperfect credit market integration among countries, i.e. when the premium on granting cross-border credit c is high. We start from a situation of a monetary union between countries —agents do not pay any currency conversion cost ε— and ask whether their welfare may be improved by imposing a positive conversion cost between currencies.

Proposition 5 *Let $c > \eta_1 / (1-b)$. There are $\hat{\pi}_2 > 0$ and $\hat{\gamma}^2$ with $\gamma^1 < \hat{\gamma}^2 \leq \gamma^2$ such that for $\pi_2 \leq \hat{\pi}_2$ and $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$ in a fully constrained equilibrium imposing positive conversion cost ε increases the consumption of both goods (q_h, q_f^η) and the quantity of credit $(\phi\ell_h^\eta,\ \phi\ell_f^\eta)$ and improves welfare.*

[13] In this equilibrium the share of consumption financed using credit $\frac{l}{pq}$ is equal to $(1-b)$, see eq. (41) in the proof of proposition 2 in the appendix.

Proposition 5 states that imposing positive conversion costs is welfare improving if agents are credit-constrained –i.e. if inflation is low enough but not too low–, the cross-border credit premium c is sufficiently high and the probability π_2 of having a strong preference for the foreign good is sufficiently low. A positive conversion cost has a differential impact on the life-time utility of a defaulter on loan repayment, compared to a non defaulter. Defaulters consume abroad and hence pay the conversion cost more often than non-defaulters who are home-biased. Therefore a positive conversion cost reduces the ex ante incentives to default, which relaxes the borrowing constraint. To understand why defaulters are not home biased while non-defaulters are, let us compare their respective travel and consumption choices. A high level of c reduces the willingness of a non-defaulter to consume the foreign good. When the cost of using credit to finance purchases abroad $(1-b)\,c$ is greater than η_1 buyers choose to consume the foreign good only when the realized value of η is η_2, and consume the home good when $\eta = 0, \eta_1$. Consuming abroad then occurs with probability π_2. By contrast, a defaulter cannot borrow and hence his decision η^* is independent of c (see equation 22). When $\varepsilon = 0$, he consumes the foreign good for any η higher than 0 (for $\eta = \eta_1, \eta_2$), i.e. with probability $(\pi_1 + \pi_2)$.

Since defaulters pay the conversion cost more often than home-biased non-defaulters, a positive conversion cost makes default less attractive. In equilibrium a higher level of credit can be sustained, thereby allowing higher consumption. For this to be the case, since the conversion cost increases the marginal cost of purchasing goods also for non defaulters, it must be that the probability π_2 is sufficiently small so that the negative effect of conversion costs on consumption of foreign goods is more than compensated by the effect of conversion costs on incentives to default. Intuitively, the condition that π_2 is lower than the threshold value $\hat{\pi}_2$ states that the probability π_2 that everyone pays the conversion cost must be relatively low.[14]

This effect does not hold when the cross-border credit premium is low and agents are credit-constrained because the consumption pattern is the same for defaulters and non-defaulters. For $c < \eta_1 (1-b)$ and $\varepsilon = 0$ non-defaulters travel if their preference shock η is η_1 or η_2, since (26) implies that $\eta_1 > \eta^*$. As a result, $\eta^*, \hat{\eta}^* \leq \eta_1$; i.e., non-defaulters consume the foreign good and so pay the conversion costs as often as defaulters.

Next we discuss two potential causes for monetary disunion, first a monetary one —a variation of the level γ of monetary injections— and then a non-monetary one —an increase of the cross-border credit premium c.

[14] If c is high enough to lead buyers to consume the home good for all realizations of the preference shock η, conversion costs are only born by defaulters and hence their unique effect is to relax the borrowing constraint. Therefore an increase in conversion costs unambiguously improves welfare regardless of the probabilities associated with the different values of the preference shock. The appendix contains the proof of this result.

Monetary cause for currency disunion. We now ask whether currency disunion may follow from a variation of the growth rate of the money supply and hence of the rate of inflation. Proposition 3 shows that for any level of the cross border credit premium, if γ is high enough, agents are not credit constrained and they always prefer trading in a monetary union. Proposition 5 shows that when the inflation rate is low enough, and the cross border credit premium c is high, agents prefer a situation with positive conversion cost ε between currencies. Therefore comparison of propositions 3 and 5 suggests the following interpretation. For any sufficiently high level of the cross border credit premium, a reduction of the level of monetary injection below $\hat{\gamma}_2$ makes agents switch from a preference for the monetary union to a preference for separate monies. Implicit in this formulation is the interpretation that the authorities cannot influence the level of c. The following corollary sums up this discussion.

Corollary 6 *Comparison of propositions 3 and 4 shows that if $c < \eta_1/(1-b)$ the currency union is optimal regardless of the level of γ. Comparison of propositions 3 and 5 shows that if $c \geq \eta_1/(1-b)$ the level of γ matters for the optimality of the currency union. In particular, a decrease in the rate of inflation from a high enough level of inflation ($\gamma > \widetilde{\gamma}$) to low levels ($\gamma < \widehat{\gamma}^2$) can lead to a shift from a situation in which a currency union is optimal to one in which separate currencies are preferred.*

Non-monetary cause for monetary disunion. We now look at a potential non monetary cause for the sub-optimality of a monetary union. We follow a traditional interpretation of financial crises that sees their origin in an increase of the real cost for banks to grant credit.[15] In our model the non-monetary factor is a variation of the real cost c for banks to grant cross border loans. This interpretation is consistent with recent empirical evidence that have shown that the Japanese or the subprime crises had an asymmetric impact on lending by banks to the economy: Foreign banks cut credit more than domestic banks, something that may be interpreted as a differential cost of granting credit.[16]

In this view our model suggests that the sustainability of a monetary union is directly impacted by an increase of the cost of the non monetary factor when the inflation rate is low enough. The next corollary summarizes the effect of an increase in c in a situation of low inflation.

Corollary 7 *Comparison of propositions 4 and 5 shows that for low levels of inflation an increase in the cross border credit premium from a low level ($c < \eta_1/(1-b)$) to a high level ($c > \eta_1/(1-b)$) may lead to a shift from a situation in which a currency union is optimal to one in which separate currencies are preferred.*

[15]For example Bernanke (1983), Gertler and Kiyotaki (2007)

[16]See Peek and Rosengren (1997), De Haas and van Lelyveld (2010), Popov and Udell (2012).

Credit crunch compared across monetary regimes. We define a credit crunch as a decrease of the real amount of credit triggered by an exogenous increase in c sufficiently high to induce a home bias in consumption. We compare the size of the credit crunch when $\varepsilon = 0$ versus $\varepsilon > 0$ and show that an exogenous increase of the cross-border credit premium triggers a sharper credit crunch in a monetary union than in a regime of separate currencies. Proposition 8 states this result formally.

Proposition 8 *Let $0 < c_0 < \eta_1/(1-b) < c_1$. If a fully constrained equilibrium exists for all $c \in [c_0, c_1]$, an increase in c from c_0 to $c \leq c_1$ leads to a decrease in the real amount of total credit and worsens welfare.*

Proposition 8 establishes that any increase of the cross border credit premium c reduces the quantity of credit. Indeed an increase in c reduces the amount of credit both when it impacts the travel decision and when it does not. The dashed curve in figure 2 plots the volume of credit as a function of c in a regime of monetary union. For low level of c, credit is continuously decreasing in c. When c reaches the threshold value $\eta_1/(1-b)$, credit shrinks sharply –the credit crunch– because it makes agents less inclined to consume abroad: agents who previously consumed the foreign good with probability $(\pi_1 + \pi_2)$ now opt for consuming it with probability π_2. This switch increases the incentives to default and hence reduces the volume of credit. From $c = \eta_1/(1-b)$ onwards, the effect of c on credit is monotonously negative.

Corollary 9 *Let $0 < c_0 < \eta_1/(1-b) < c_1$. Comparison of propositions 4 and 5 shows that if c increases from c_0 to c_1 there is a range of values of γ such that the decrease in credit is greater if $\varepsilon = 0$ than if $\varepsilon > 0$.*

Corollary 9 deals with the case in which the increase in c is sufficiently high to generate a home bias in consumption. Such an increase in c generates a decrease in the quantity of credit stronger in a regime of currency union –when $\varepsilon = 0$– than in a regime of separate currency –i.e. when $\varepsilon > 0$. The solid line on figure 2 represents the evolution of credit in a regime of separate currency. Comparison with the dashed line shows that a monetary union is the regime that provides the highest possible volume of credit and consumption when $c < \eta_1/(1-b)$. However the credit crunch triggered by an increase in c above the threshold $\eta_1/(1-b)$ is lower in a regime of separate currency than in monetary union.

4.3 Conversion cost and optimal inflation

Proposition 5 shows that under appropriate conditions strictly positive conversion cost —separate currencies— may relax the borrowing constraint and improve welfare compared to the benchmark case of a currency union. This result is obtained taking as given the inflation rate (γ). However, previous studies of economies with

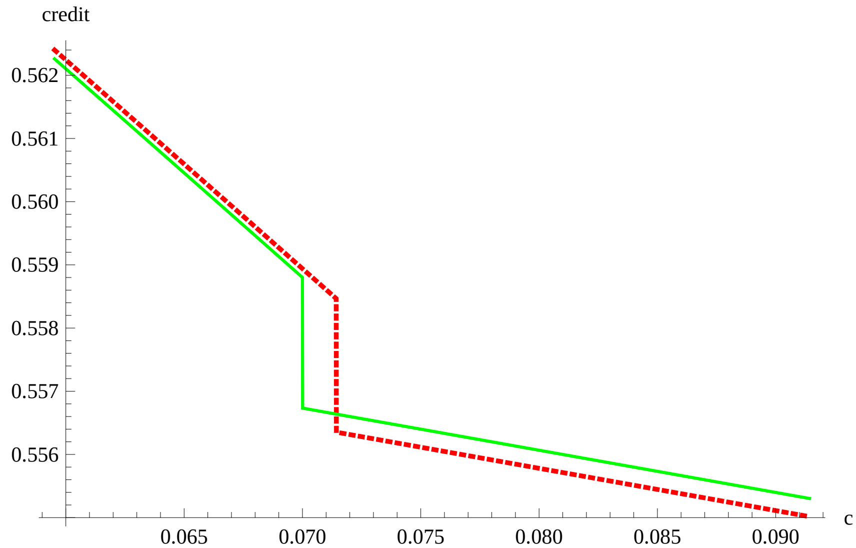

Figure 2: Quantity of credit as a function of the cross border credit premium in a monetary union (dotted line) and in a regime of separate currencies (solid line)

credit and limited commitment show that inflation can be used to curb default incentives.[17] In this section we present a parametrization of our model in which both strictly positive conversion cost and positive inflation rate allow to attain a higher level of welfare than the sole use of the optimal positive inflation rate. Figure 3 is drawn with the following specification for the utility function $u(q) = (q^{\alpha})/\alpha$ and parameters $\alpha = 0.2$, $\beta = 0.9$, $b = 0.3$, $c = 0.25$, $\eta_1 = 0.05$, $\eta_2 = 0.2$, $\pi_1 = 0.2$ and $\pi_2 = 0.01$.[18] Notice that in our example η_1 is lower than $(1 - b)\,c$.[19] For these parameter value, a fully constrained equilibrium exists up to the threshold value of γ equal to $\gamma^2 = 1.021$. The unconstrained equilibrium exists for value of γ higher than $\widetilde{\gamma} = 1.025$. For intermediate values the equilibrium is partially constrained since agents with preference shock η_0 and η_1 are credit constrained whereas agents with preference shock η_2 are not credit constrained.

The simulation reported in figure 3 shows that there exists parameter values for which welfare is higher with positive conversion costs than with an optimal positive inflation in a monetary union.[20] The dotted line corresponds to welfare

[17] In this type of environment default is a cash-intensive activity. A positive inflation rate thus acts as a tax that discourages default. In the setup we consider, default is a conversion-intensive activity.

[18] It has been checked with Mathematica that the conditions required for the existence of the fully constrained equilibrium are satisfied for the range of inflation rates that is reported on the horizontal axis.

[19] In addition, the condition on β stated in lemma 1 is verified.

[20] The figure is drawn with the specification $u(q) = (q^{\alpha})/\alpha$ and parameters $\alpha = 0.2$, $\beta = 0.9$,

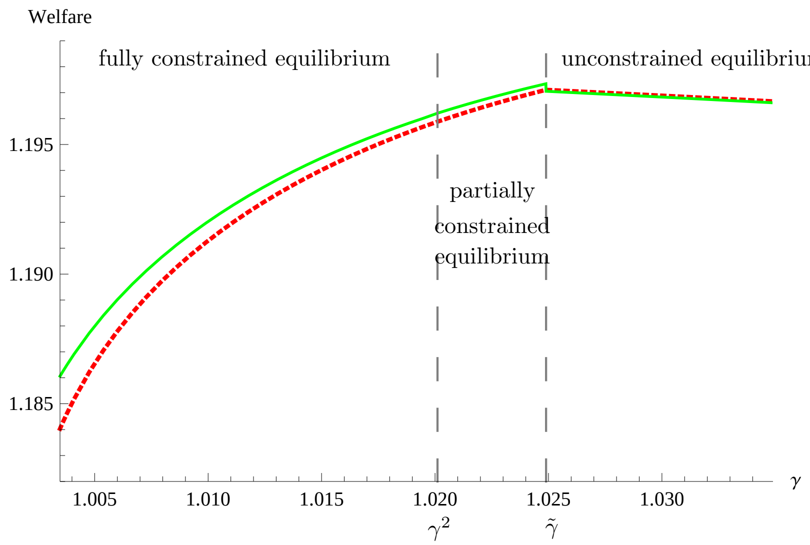

Figure 3: Welfare as a function of inflation with and without conversion costs. (dotted line: $\epsilon = 0$; solid line; $\epsilon = 0.01$)

as a function of the inflation rate when there are no conversion costs. Welfare is maximized at an inflation rate equal to 2.5% ($\gamma = 1.025$). The solid line draws welfare attained in a regime of separate currencies in which the conversion cost is set to 1%. In this example, conversion cost between currencies improves welfare in the fully constrained and partially constrained equilibria even when inflation is chosen optimally.[21] Consistently with proposition 3, in a unconstrained equilibrium conversion cost are always welfare worsening.

5 Empirical illustration with the Euro area

We believe that our model may be useful for understanding the impact of imperfect credit market integration on the efficiency of a currency union. In this section we present evidence on credit market integration in the Eurozone to infer the variations over time of the cross border credit premium. Measuring the cross border credit premium is difficult but anecdotal evidence and indirect measures abound.

Financial market integration in the Euro area increased significantly with the introduction of the Euro in 1999 and the various regulatory initiatives aimed at creating a single European financial market (Hartmann, Maddaloni, and Manganelli (2003), ECB (2007), ECB (2012, chapter 2)). The equity, bonds and money

$b = 0.3$, $c = 0.25$, $\eta_1 = 0.05$, $\eta_2 = 0.2$, $\pi_1 = 0.2$, $\pi_2 = 0.01$. It has been checked —using Mathematica— that the conditions required for the existence of the fully constrained equilibrium are satisfied for the range of inflation rates that is reported on the horizontal axis.

[21] The optimal inflation rate also depends on conversion costs.

markets became quickly integrated (Schoenmaker and Bosch, 2008).

The subprime and the Euro crises reversed the trend towards a higher financial integration.[22] The balance sheets of banks exhibited a strong increase of the home bias in asset holdings,see Jochem and Volz (2011, using IMF and Bundesbank databases) and Acharya and Steffen (2013, using data from European Banking Authority). This reversal in banking market integration was also observed in the interbank market. Manna (2011) uses BIS data on the flow of funds between Eurozone and non-Eurozone banking systems to compute a quarterly index of interbank market integration. He documents a U-shaped evolution of the home bias in bank funding of Eurozone countries with the highest integration being reached in 2007.[23] Importantly this feature is not shared by non-Eurozone countries — US, UK, Sweden, Denmark, Switzerland— where interbank relationships were not durably impacted by the subprime crisis.

Yet retail credit markets remained mostly local –before and after the crises– with few banks active in multiple jurisdictions (Sørensen and Gutiérrez, 2006, Kleimeier and Sander, 2007). The 2012 E.C.B. report on Financial Integration in Europe writes that "*Cross-border banking through branches or subsidiaries has remained limited*" (ECB, 2012, p.90-91). In the Eurozone retail banking is much less integrated than in the U.S. (Gropp and Kashyap, 2009). Foreign banking penetration in each European country —especially the largest ones— is well below the level that prevails in the U.S. (Claessens and van Horen, 2012).[24] President of the E.C.B. Draghi said that "*Integration [in the Euro area] was largely based on short term interbank debt rather than on equity or direct cross-border lending to firms and households*".[25] As a consequence a stated goal of E.C.B. is that "*a Spanish firm should be able to borrow from a Spanish bank at the same price at which it would borrow from a Dutch bank*".[26]

The evidence supports our claim that currency unification does not necessarily entail credit market integration. Moreover the evolution of the volume of credit in Eurozone countries is inversely related to the level of the cross-border credit premium. Before 2007 the Eurozone experienced a steep increase of the volume of

[22] As acknowledged repeatedly by ECB officials, see e.g. Draghi's speech "Financial integration and banking union" delivered in Brussels on the 12 February 2014. One reason mentioned by President Draghi to explain this reversal is the "hidden barriers to cross-border actiivity linked to national preferences" (p. 5). See also the speech "Banking union and European integration" delivered by E.C.B. vice president Constâncio (Vienna, 12 May 2014).

[23] This measure is computed as the average over 10 Eurozone countries.

[24] Kalemli-Ozcan, Papaioannou, and Peydró (2010) show that the adoption of the EU financial services directives aimed at fostering credit market integration varied with the preference for cross border integration. This suggests that countries that wanted to protect their banking system from outside competitors delayed the adoption of these EU directives. This may be one of the reasons for the low level of foreign banks penetration in the Eurozone.

[25] *In* "A consistent strategy for a sustained recovery", speech delivered by Draghi in Paris, 25 March 2014.

[26] *In* "Europe's pursuit of 'a more perfect union'", speech delivered by Draghi in Cambridge (MA), 9 October 2013.

credit distributed to firms and households (ECB XXX) at the moment at which cross-border credit market integration deepened. After 2008, the volume of credit granted to firms and households decreased sharply in crisis-hit countries. This is consistent with the post–2007 sharp increase in banks rejection rate of loan applications by firm, as documented by the various issues of the E.C.B. SAFE survey.[27]

6 Relation to the literature

Our paper aims at contributing to the macroeconomic literature that seeks to identify the requisites for the optimality of a monetary union and the benefits that accrue from it. We have presented a stylized framework to abstract from several dimensions that have already been singled out in previous work.

Our setup abstracts from any source of heterogeneity or asymmetric shocks so that the usual tradeoff emphasized in the literature on optimal currency areas do not arise (Mundell, 1961, Benigno, 2004, Gali and Monacelli, 2008). We assume that monetary policy is unique both in a regime of currency union and in a regime of separate currencies.[28]

The absence of a government with fiscal needs allows us to abstract from the interplay between fiscal and monetary policy. Moreover banks are modeled such that there is no room for monetary financing of fiscal deficits. Beetsma and Uhlig (1999) show that countries have incentives to commit to a balance budget *after* joining a monetary union thereby containing the free-rider problem cause by the temptation to finance fiscal deficits with other members' resources. In Alesina and Barro (2002), countries lacking internal discipline can commit to monetary stability by joining a currency union with a low-inflation anchor country. Other papers point out to the need for fiscal constraints *within a monetary union* to avoid the monetary financing of the deficit of state governments (Chari and Kehoe, 2007, Cooper, Kempf, and Peled, 2008). Cooley and Quadrini (2003) emphasize that forming a monetary union is optimal because it ensures a low inflation rate. Some papers stress that a monetary union may fail because of the incentive for individual country to tax using a higher inflation rate the domestic currency holdings of foreigners (Cooper and Kempf, 2003, Liu and Shi, 2010). Others argued that when default on public debt is not an option, a currency union may be unsustainable because it forbids overindebted governments to devalue their currency (Beetsma and Bovenberg, 1999, 2001, Chari and Kehoe, 2008). Compared to those papers, we show that a currency union may be threatened by the incomplete integration of the credit markets.

[27]See https://www.ecb.europa.eu/stats/money/surveys/sme/html/index.en.html consulted on December 23, 2014

[28]However, our results do not hinge on exogenous inflation, and the effect of conversion costs that we identify does not disappear when inflation is chosen optimally (see section 4.3).

Our work is also related to a few papers analyzing the potential benefits of multiple monies when there is a commitment issue on the side of private agents rather than public authorities. Building on Ravikumar and Wallace (2001), Kiyotaki and Moore (2003) build a model to show that multiple currencies may be preferred to a common currency when it allows agents to enjoy the benefits of a greater degree of specialization in the production of goods. Kocherlakota and Krueger (1999) provide a setup where multiple monies can be optimal because they allow agents to credibly signal private information concerning the type of goods (home vs. foreign) they prefer. We emphasize the distinct tradeoff that with imperfect credit integration a common currency can *increase* the outside option associated with default, and exacerbate agents' incentive to default on their bank loans.

7 Conclusion

Our paper analyzes when a currency union is welfare improving when banking markets are imperfectly integrated We use a two-country model with money and bank credit. The degree of the credit market integration is captured by the additional cost for banks to grant credit for purchases in the other jurisdiction. We show that when the cross-border credit premium is nil, agents always prefer using a unique currency. If countries unify their currency but are unable to reduce the cross-border credit premium, welfare is lower in a regime of currency union than with separate currencies when the cross border credit premium is high enough.

Our contribution to the literature is threefold. First we show that to be sustainable currency union has to be complement with a sufficient degree of credit market integration. Second, absent a sufficient credit market integration, a currency union may be an additional cause of credit rationing of borrowers because banks decision on the volume of credit supply in equilibrium adapt to their borrowers' incentives to default. This potential issue may be especially acute in times of crisis if impediments to cross-border credit and hence the cost to grant credit to non borrowers. The third implication is more normative. It relates to the objectives that must be assigned to a public policy aimed at strengthening the degree of credit market integration. Hurdles to integration may originate in the lower ability to seize collateral or revenue across jurisdictions when no automatic procedure for judicial cooperation among states is available.[29] They may also reflect a higher cost for banks to access the credit history of non-residents compared to residents.[30] Impediments to cross-border credit could be exacerbated by local supervisors or resolution author-

[29] Insufficient harmonization of bankruptcy procedures can result in foreign lenders being more exposed to a borrower default than domestic lenders.

[30] Jentzsch and San José Rientra (2003) reports more complex procedures for Euro-based banks from one country to access cross-border information on customers based in another country than for U.S. banks to access cross-state information on a U.S. customer. Jentzsch (2007) and Kalemli-Ozcan et al. (2010) discuss the discriminatory rules adopted by states to discourage national competition.

ities when they favor domestic credit and state collateral.[31] We leave for future research the analysis of the welfare impact of each of these primitives of the degree of credit market integration.

Appendix

Proof of Lemma 1. If defaulters are cash-constrained for all realizations of η, it must be that $u'(\hat{q}), u'(\hat{q}) + \eta_1 - \varepsilon, u'(\hat{q}) + \eta_2 - \varepsilon > 1$. Since we only consider parameter values such that $\eta_1, \eta_2 > \varepsilon$, it is sufficient to show that in the conjectured equilibria $u'(\hat{q}) > 1$ holds. From (22) and (23) we get

$$u'(\hat{q}) - 1 = (\gamma/\beta - 1)/b - [\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]$$

Thus if $\beta \leq [1 + b(\pi_1\eta_1 + \pi_2\eta_2)]^{-1}$ it follows that $u'(\hat{q}) > 1$ always holds for $\gamma \geq 1$ and $\varepsilon \geq 0$ so defaulters are cash-constrained for all realizations of η.

Proof of Lemma 2. We first show that any agent repays iff (24) holds. First, observe that (24) corresponds to the incentive constraint for any agent at the repayment date (in the second market), and as such must hold for any η. To show that condition (24) is also sufficient, it suffices to show that no type η has an incentive to deviate at the borrowing stage. Let $\Gamma = \beta\left(V(m_{+1}) - \hat{V}(\hat{m}_{+1})\right) - \phi(m_{+1} - T - \hat{m}_{+1})$, and rewrite (24) as

$$\phi\ell(1+i) \leq \Gamma. \tag{28}$$

This defines a first debt limit $\bar{\ell}^1 \equiv \frac{\Gamma}{\phi(1+i)}$ for all η. Now, consider an agent with preference shock η and debt $\ell^\eta \leq \bar{\ell}$ (with $\bar{\ell} \geq 0$ arbitrary). With no loss of generality, consider the case of local consumption. For this agent not to deviate at the borrowing stage, it must be the case that

$$u\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) - \phi m_{+1} + \phi T + \beta V(m_{+1}) \geq u\left(\frac{m+\bar{\ell}}{p}\right) - \phi\hat{m}_{+1} + \beta\hat{V}(\hat{m}_{+1}), \tag{29}$$

since an agent that will default borrows up to the limit $\bar{\ell}$. Notice that because the right-hand side is increasing in $\bar{\ell}$, (29) defines a second debt limit $\bar{\ell}^2$ to be imposed on type η. To show that (29) is redundant, we show that $\bar{\ell}^1 \leq \bar{\ell}^2$. Assume the contrary, that is $\bar{\ell}^1 > \bar{\ell}^2$. Using (29),

$$u\left(\frac{m+\bar{\ell}^2}{p}\right) = u\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) + \Gamma, \tag{30}$$

[31] See Aglietta and Scialom (2003) for a discussion of the the risks generated by the disparate views of state prudential entities. Bertay, Demirguç-Kunt, and Huizinga (2011) suggest that country-specific financial safety nets act as a barrier to cross-border banking. Gros (2012) argues that local supervision encourages the fragmentation of the banking markets.

where ℓ^η is the equilibrium borrowing for type η. Since ℓ^η is chosen optimally (and $\bar{\ell}^2$ can be chosen) we have

$$u\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta (1+i) \geq u\left(\frac{m+\bar{\ell}^2}{p}\right) - \phi\bar{\ell}^2 (1+i) \,. \tag{31}$$

From (30) and (31),

$$u\left(\frac{m+\bar{\ell}^2}{p}\right) \geq u\left(\frac{m+\bar{\ell}^2}{p}\right) - \phi\bar{\ell}^2 (1+i) + \Gamma,$$

which gives $\bar{\ell}^2 \geq \frac{\Gamma}{\phi(1+i)} = \bar{\ell}^1$, a contradiction. Hence, $\bar{\ell}^1 \leq \bar{\ell}^2$ and (24) is both sufficient and necessary for repayment incentives.

Since $\bar{\ell}^1$ does not depend on η, it also follows that $\bar{\ell}_h = \bar{\ell}_f = \bar{\ell}$.

To conclude, we check that (25) is equivalent to (24). Denote as x_j^η and x_s the amount of consumption by the buyer with preference shock η who consumes good $j = (h, f)$ and the amount of consumption by the seller, respectively, in the second market. When the settlement stage arrives, the pay-off to a buyer with preference given by η who repays his debt for consumption of good $j = (h, f)$ is:

$$x_j^\eta + \frac{\beta b}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[u\left(q_h^\eta\right) + x_h^\eta\right] + \sum_{\eta>\eta^*}\pi_\eta\left[u\left(q_f^\eta\right) + (\eta-\varepsilon)\, q_f^\eta - \phi\ell_f^\eta c + x_f^\eta\right]\right\}$$
$$- \frac{\beta(1-b)}{1-\beta}\left(q_s - x_s\right)$$

The pay-off to a defaulter with preference shock η who consumes good $j = (h, f)$ is

$$\bar{x}_j^\eta + \frac{\beta b}{1-\beta}\left\{u\left(\hat{q}\right) + \sum_{\eta\leq\hat{\eta}^*}\pi_\eta\hat{x}_h^\eta + \sum_{\eta>\hat{\eta}^*}\pi_\eta\left[(\eta-\varepsilon)\,\hat{q}_f^\eta + \hat{x}_f^\eta\right]\right\} - \frac{\beta(1-b)}{1-\beta}\left(q_s - \hat{x}_s\right)$$

where $\bar{x}_j^\eta$ is consumption by the agent in the period in which he defaults and $\hat{x}_h^\eta$, $\hat{x}_f^\eta$ and $\hat{x}_s$ are net consumption by the defaulter in subsequent periods in case he is a buyer with preference shock $\eta \leq \hat{\eta}^*$, a buyer with preference shock $\eta > \hat{\eta}^*$, or a seller.

Consumption quantities x_j^η and x_s are

$$\begin{aligned} x_j^\eta &= -\phi\ell_j^\eta (1+i) - \phi m_{+1} + \phi T \\ x_s &= -\phi\ell_s (1+i) + \phi p q_s - \phi m_{+1} + \phi T \end{aligned} \tag{32}$$

where $T = (\gamma - 1) M_{-1}$. In a symmetric equilibrium, $m_{-1} = M_{-1}$. In addition, $m_{-1} = -\ell_s$. Using (5), (16)-(19) and (32), we verify the market clearing condition in the second market:

$$b\sum_{\eta\leq\eta^*}\pi_\eta x_h^\eta + b\sum_{\eta>\eta^*}\pi_\eta x_f^\eta + (1-b)\, x_s = 0$$

Consumption quantities by the defaulter $\bar{x}_j^\eta$, $\hat{x}_h^\eta$, $\hat{x}_f^\eta$ and $\hat{x}_s$ are

$$\begin{aligned}\bar{x}_j^\eta &= \hat{x}_h^\eta = \hat{x}_f^\eta = -\phi\hat{m}_{+1} = -\gamma\hat{q} \\ \hat{x}_s &= \hat{x}_j^\eta + \phi\hat{m}_{-1} + q_s = -(\gamma-1)\hat{q} + q_s\end{aligned} \tag{33}$$

since $\phi\hat{m}_{-1} = \hat{q}$ and $\hat{m}_{+1}/\hat{m}_{-1} = \gamma$. Using (32) and (33), the borrowing constraint can be rewritten as in (25).

Proof of Proposition 1. We first rewrite the equilibrium equations that correspond to an unconstrained equilibrium and then show that the borrowing constraint is effectively slack for γ sufficiently high.

Conjecture an unconstrained equilibrium by setting $\lambda_h^\eta = 0$ and $\lambda_f^\eta = 0$ for all η. (11) and (15) become

$$\begin{aligned}u'(q_h) &= 1+i \\ u'\left(q_f^\eta\right) + \eta - \varepsilon - c &= 1+i\end{aligned} \tag{34}$$

Hence (20) can be rewritten as

$$\gamma/\beta - b\sum_{\eta>\eta^*}\pi_\eta c = 1+i \tag{35}$$

Thus, q_h^η and q_f^η are immediately pinned down for a given value of γ.

From (1), note that the consumption quantity of home goods does not depend on η so in what follows we set $q_h^\eta = q_h$ and $\ell_h^\eta = \ell_h$. From (17) and (19), we get

$$\phi\ell_h = (1-b)\,q_h - b\sum_{\eta>\eta^*}\pi_\eta\left(q_f^\eta - q_h\right) \tag{36}$$

and

$$q_h - \phi\ell_h = q_f^\eta - \phi\ell_f^\eta \tag{37}$$

for all η.

Given lemma 1 a defaulter effectively consumes $\hat{q}$ regardless of the value of the preference shock. From (20), (23), and (35), it follows that $\hat{q} < q_h$ and $\hat{q} < q_f^\eta$ for γ sufficiently high. Hence, by the mean value theorem $u(q_h) - u(\hat{q}) > u'(q_h)(q_h - \hat{q})$. Similarly, $u\left(q_f^\eta\right) - u(\hat{q}) > u'\left(q_f^\eta\right)\left(q_f^\eta - \hat{q}\right)$. Therefore, a sufficient condition for the borrowing constraint (25) to be non-binding is

$$\begin{aligned}&-\phi\left[\ell(1+i) + m_{-1}\right] + \frac{\beta b}{1-\beta}\left\{\sum_{\eta>\eta^*}\pi_\eta\left[(\eta-\varepsilon)\,q_f^\eta - \phi\ell_f^\eta c\right] - \sum_{\eta>\hat{\eta}^*}\pi_\eta(\eta-\varepsilon)\,\hat{q}\right\} \\ &+\frac{\beta b}{1-\beta}\left\{\sum_{\eta\le\eta^*}\pi_\eta\left[u'(q_h) - 1\right](q_h - \hat{q}) + \sum_{\eta>\eta^*}\pi_\eta\left[u'\left(q_f^\eta\right) - 1\right]\left(q_f^\eta - \hat{q}\right)\right\} \\ &\ge -\frac{(\gamma-\beta)\,\hat{q}}{1-\beta}\end{aligned}$$

Given (15), (11), (36) and (37), this condition can be rewritten as

$$
\begin{aligned}
&-\phi\left[\ell\left(1+i\right)+m_{-1}\right]+\frac{\beta b^{2}c}{1-\beta}\sum_{\eta>\eta^{*}}\pi_{\eta}\left[q_{h}+\sum_{\eta>\eta^{*}}\pi_{\eta}\left(q_{f}^{\eta}-q_{h}\right)\right] \\
&+\frac{\beta bi}{1-\beta}\left(\sum_{\eta\leq\eta^{*}}\pi_{\eta}q_{h}+\sum_{\eta>\eta^{*}}\pi_{\eta}q_{f}^{\eta}\right)+\frac{\beta b}{1-\beta}\left[\sum_{\eta>\eta^{*}}\pi_{\eta}\left(\eta-\varepsilon\right)-\sum_{\eta>\hat{\eta}^{*}}\pi_{\eta}\left(\eta-\varepsilon\right)\right]\hat{q} \\
&\geq-\frac{\left(\gamma-\beta\right)\hat{q}}{1-\beta}+\frac{\beta b}{1-\beta}\left(i+c\sum_{\eta>\eta^{*}}\pi_{\eta}\right)\hat{q} \qquad (38)
\end{aligned}
$$

We consider two different cases. First, consider the case of an agent who has consumed the home good in the current period. Using (19), (35) and (36), (38) becomes

$$
\begin{aligned}
&-\left(1-b\right)q_{h}i+b\sum_{\eta>\eta^{*}}\pi_{\eta}\left(q_{f}^{\eta}-q_{h}\right)i-q_{h}+\frac{\beta b^{2}c}{1-\beta}\sum_{\eta>\eta^{*}}\pi_{\eta}\left[q_{h}+\sum_{\eta>\eta^{*}}\pi_{\eta}\left(q_{f}^{\eta}-q_{h}\right)\right] \\
&+\frac{\beta bi}{1-\beta}\left(\sum_{\eta\leq\eta^{*}}\pi_{\eta}q_{h}+\sum_{\eta>\eta^{*}}\pi_{\eta}q_{f}^{\eta}\right)+\frac{\beta b}{1-\beta}\left[\sum_{\eta>\eta^{*}}\pi_{\eta}\left(\eta-\varepsilon\right)-\sum_{\eta>\hat{\eta}^{*}}\pi_{\eta}\left(\eta-\varepsilon\right)\right]\hat{q} \\
&\geq-\beta i\frac{1-b}{1-\beta}\hat{q}
\end{aligned}
$$

Since all terms with q_f^η in the above inequality are positive, one way to show that this inequality holds for γ sufficiently high is to consider the following sufficient condition

$$
\begin{aligned}
&-q_{h}i-q_{h}+\frac{\beta b^{2}cq_{h}}{1-\beta}\sum_{\eta>\eta^{*}}\pi_{\eta}\sum_{\eta\leq\eta^{*}}\pi_{\eta}+q_{h}i\frac{b}{1-\beta}\sum_{\eta\leq\eta^{*}}\pi_{\eta} \\
&\geq-\frac{\beta i\left(1-b\right)}{1-\beta}\hat{q}-\frac{\beta b}{1-\beta}\left[\sum_{\eta>\eta^{*}}\pi_{\eta}\left(\eta-\varepsilon\right)-\sum_{\eta>\hat{\eta}^{*}}\pi_{\eta}\left(\eta-\varepsilon\right)\right]\hat{q}
\end{aligned}
$$

Since $\eta^* \geq \hat{\eta}^*$, note that if γ is high enough the right-hand side in the above inequality is unambiguously negative given (35). Therefore, the right-hand side can be dismissed and it is sufficient for this inequality to hold that

$$
\left(-1+\frac{b}{1-\beta}\sum_{\eta\leq\eta^{*}}\pi_{\eta}\right)i\geq1-\frac{\beta b^{2}c}{1-\beta}\sum_{\eta>\eta^{*}}\pi_{\eta}\sum_{\eta\leq\eta^{*}}\pi_{\eta} \qquad (39)
$$

From (35), the left-hand side in the above inequality is increasing in γ, provided that β is sufficiently high (it is sufficient that $\beta > 1 - b\pi_0$ since $\sum_{\eta\leq\eta^*}\pi_\eta \geq \pi_0$).

Second, consider the case of an agent who has consumed the foreign good in the current period. Using (19), (35), (36) and (37), (38) can be written as

$$\left(-q_f^\eta + b\sum_{\eta>\eta^*}\pi_\eta q_f^\eta + \sum_{\eta\leq\eta^*}\pi_\eta b q_h^\eta\right) i - q_f^\eta + \frac{\beta bc}{1-\beta}\sum_{\eta>\eta^*} b\pi_\eta\left[q_h + \sum_{\eta>\eta^*}\pi_\eta\left(q_f^\eta - q_h\right)\right]$$
$$+\frac{\beta bi}{1-\beta}\left(\sum_{\eta\leq\eta^*}\pi_\eta q_h + \sum_{\eta>\eta^*}\pi_\eta q_f^\eta\right)$$
$$\geq -\beta i\frac{1-b}{1-\beta}\hat{q} - \frac{\beta b}{1-\beta}\left[\sum_{\eta>\eta^*}\pi_\eta(\eta-\varepsilon) - \sum_{\eta>\hat{\eta}^*}\pi_\eta(\eta-\varepsilon)\right]\hat{q}$$

In the above inequality, all terms with q_h^η are positive and the right-hand side is negative if γ is high enough (as stated in the case of the borrowing constraint for consumption of home goods). Thus, one way to show that this inequality holds for γ sufficiently high is to consider the following sufficient condition

$$\left(-q_f^\eta + \frac{b}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta q_f^\eta\right) i - q_f^\eta + \frac{\beta b^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\sum_{\eta>\eta^*}\pi_\eta q_f^\eta \geq 0$$

Since $q_f^\eta \leq q_f^{\eta_2}$ and $\sum_{\eta>\eta^*}\pi_\eta q_f^\eta \geq \pi_2 q_f^{\eta_2}$, it is sufficient that

$$\left(-q_f^{\eta_2} + \frac{b}{1-\beta}\pi_2 q_f^{\eta_2}\right) i - q_f^{\eta_2} + \frac{\beta b^2 c\pi_2 q_f^{\eta_2}}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta \geq 0$$

If $\beta > 1 - b\pi_2$, then a sufficient condition is

$$\left(-1 + \frac{b\pi_2}{1-\beta}\right) i \geq 1 - \frac{\beta b^2 c\pi_2}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta \tag{40}$$

From (35), the left-hand side in the above inequality is increasing in γ, provided that β is sufficiently high.

To sum up, (39) and (40) hold if γ is sufficiently high. In addition from (35) a high value of γ ensures $i \geq 0$. Hence an unconstrained equilibrium exists. Since (35) pins down a unique value of i and (34) pins down unique values of q_h and q_f^η for all η this equilibrium is unique.

Proof of Proposition 2. First, we derive the threshold η^* in a conjectured fully constrained equilibrium. In this equilibrium all buyers are credit-constrained. Since the multiplier associated to the borrowing constraint is positive for all realizations of η, it follows from (10), (14) and (15) that the multiplier associated to the cash constraint is also positive for all values of η and so all buyers are cash-constrained.

From Lemma 2, $\bar{\ell}_h = \bar{\ell}_f$. Thus we can write $q_h = q_f^\eta = q$ and $\ell_h = \ell_f^\eta = \ell$ for all η. Combining (17) and (19) yields

$$\begin{aligned} \phi\ell &= (1-b)\,q \\ \phi m_{-1} &= bq \end{aligned} \tag{41}$$

Therefore, from (21) the threshold η^* is equal to $\varepsilon + (1-b)\,c$.

Second, we prove the existence of a fully constrained equilibrium. We distinguish three cases depending on the value of c: $\eta_1 > \varepsilon + (1-b)\,c$, $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$ and $\varepsilon + (1-b)\,c > \eta_2$. We show for the three cases that a fully constrained equilibrium exists for $\gamma \in \left[\gamma^1, \gamma^2\right]$ where γ^1 and γ^2 depend on the value of c. The proof proceeds as follows. First, we rewrite equilibrium equations by conjecturing a fully constrained equilibrium and show that $i \geq 0$ for $\gamma \geq \gamma^1$. Then we show that there is an interval $[\gamma_1, \gamma_2]$ such that the borrowing constraint binds for all buyers; i.e., for any value of η.

For the cases $\eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$ and $\eta_2 \leq \varepsilon + (1-b)\,c$, we show that sufficiently low values of η_1 and η_2 ensure that an agent with preference shock η_1 or η_2 always prefers borrowing in order to consume the home good instead of consuming the foreign good by using only his money holdings.

Case $\eta_1 > \varepsilon + (1-b)\,c$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (22) and (26), $\eta^*, \hat{\eta}^* < \eta_1$. (20) and (23) can be rewritten as

$$\gamma/\beta - 1 = b\left[u'(q) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right] + (1-b)\,i \tag{42}$$

and

$$\gamma/\beta - 1 = b\left[u'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right] \tag{43}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$, which requires $q \geq \hat{q}$ given (42) and (43). Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ in a fully constrained equilibrium. From (42) and (43), $i = 0$ at $\gamma = \gamma^{1\prime}$. Rewrite the borrowing constraint (25) by conjecturing a fully constrained equilibrium for the case $\eta^*, \hat{\eta}^* < \eta_1$. Using (41) equation (25) becomes

$$\begin{aligned} &- i\,(1-b)\,q - q \\ &+ \frac{\beta b}{1-\beta}\left[u(q) - q + \pi_1(\eta_1-\varepsilon)\,q + \pi_2(\eta_2-\varepsilon)\,q - (\pi_1+\pi_2)(1-b)\,cq\right] \\ &= \frac{\beta b}{1-\beta}\left[u(\hat{q}) - \hat{q} + \pi_1(\eta_1-\varepsilon)\,\hat{q} + \pi_2(\eta_2-\varepsilon)\,\hat{q}\right] - \frac{(\gamma-\beta)\,\hat{q}}{1-\beta} \end{aligned} \tag{44}$$

From (44) it follows that

$$\gamma^{1\prime} = 1 + \beta b\,(\pi_1+\pi_2)(1-b)\,c$$

Define γ^1 as the minimum value of γ for which a fully constrained equilibrium exists. Since $i = 0$ at $\gamma = \gamma^{1\prime}$ it follows that $\gamma^1 = \gamma^{1\prime}$. Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^1$. Differentiate (44) with respect to γ to get

$$\begin{aligned}
&-\frac{\partial i}{\partial\gamma}(1-b)\,q - [i(1-b)+1]\frac{\partial q}{\partial\gamma} \qquad (45)\\
&+\frac{\beta b}{1-\beta}\left\{u'(q) - 1 + \pi_1(\eta_1-\varepsilon) + \pi_2(\eta_2-\varepsilon) - (\pi_1+\pi_2)(1-b)\,c\right\}\frac{\partial q}{\partial\gamma}\\
&= \frac{\beta b}{1-\beta}\left\{u'(\hat{q}) - 1 + \pi_1(\eta_1-\varepsilon) + \pi_2(\eta_2-\varepsilon)\right\}\frac{\partial\hat{q}}{\partial\gamma} - \frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\gamma} - \frac{\hat{q}}{1-\beta}
\end{aligned}$$

From (42),

$$(1-b)\frac{\partial i}{\partial\gamma} = 1/\beta - bu''(q)\frac{\partial q}{\partial\gamma} \qquad (46)$$

Use (45), (42), (43) and (46) to get

$$\frac{\partial q}{\partial\gamma} = \frac{(1-\beta)\,q/\beta - \hat{q}}{(1-\beta)\,bu''(q)\,q + \gamma - 1 - i(1-b) - \beta b(\pi_1+\pi_2)(1-b)\,c}$$

and

$$(1-b)\,\beta\frac{\partial i}{\partial\gamma} = \frac{\gamma - 1 - i(1-b) - \beta b(\pi_1+\pi_2)(1-b)\,c + \beta bu''(q)\,\hat{q}}{\gamma - 1 - i(1-b) - \beta b(\pi_1+\pi_2)(1-b)\,c + (1-\beta)\,bu''(q)\,q} \qquad (47)$$

From (44), we get

$$\begin{aligned}
&\gamma - i(1-b) - 1 - \beta b(\pi_1+\pi_2)(1-b)\,c \qquad (48)\\
&= \gamma - \beta b(\pi_1+\pi_2)(1-b)\,c + \frac{\beta b}{1-\beta}\frac{u(\hat{q}) + [-1 + \pi_1(\eta_1-\varepsilon) + \pi_2(\eta_2-\varepsilon)]\,\hat{q}}{q}\\
&- \frac{(\gamma-\beta)\,\hat{q}/q}{1-\beta} - \frac{\beta b}{1-\beta}\frac{u(q) - q + \pi_1(\eta_1-\varepsilon)\,q + \pi_2(\eta_2-\varepsilon)\,q - (\pi_1+\pi_2)(1-b)\,cq}{q}
\end{aligned}$$

By the mean value theorem, $u(q) - u(\hat{q}) > u'(q)(q-\hat{q})$ for $q > \hat{q}$. Therefore, for $q > \hat{q}$ (or $i > 0$) we verify from (48) that

$$\gamma - i(1-b) - 1 - \beta b(\pi_1+\pi_2)(1-b)\,c < -\beta(1-b)\,i\frac{\hat{q}}{q}$$

so $\gamma - i(1-b) - 1 - \beta b(\pi_1+\pi_2)(1-b)\,c$ is unambiguously negative for $i > 0$ and given γ^1 it is equal to zero for $i = 0$. Therefore, from (47) it follows that $\partial i/\partial\gamma > 0$ for $i \geq 0$ provided that the borrowing constraint binds. At $\gamma = \gamma^1$, $i = 0$ by definition of γ^1. Therefore, from (47) $\partial i/\partial\gamma > 0$ at $\gamma = \gamma^1$ and so $i > 0$ at γ slightly higher than γ^1. In turn, this implies that $\gamma - i(1-b) - 1 - \beta b(\pi_1+\pi_2)(1-b)\,c$ is negative for γ slightly higher than γ^1 and hence it follows from (47) that $\partial i/\partial\gamma > 0$ at γ slightly higher than γ^1. Therefore, $i > 0$ for higher level of γ and there is an interval of values of $\gamma \geq \gamma^1$ for which $i \geq 0$.

To conclude, we must ensure that the representative agent is credit-constrained for all values of η as we conjectured at the beginning of the proof. We show that he is credit-constrained for a range of values of $\gamma \geq \gamma^1$. Since $\eta_1 > (1-b)\,c + \varepsilon$, two subcases may exist: $\eta_1 - c - \varepsilon > 0$ and $\eta_1 - c - \varepsilon \leq 0$.

Subcase $\eta_1 - c - \varepsilon > 0$: For the agent who consumes the home good, the multiplier of the borrowing constraint (25) is positive if $u'(q) - 1 > 0$. From (42), at $\gamma = \gamma^1 = \gamma^{1\prime}$ this is the case if $\gamma^1/\beta - 1 - b\pi_1(\eta_1 - \varepsilon) - b\pi_2(\eta_2 - \varepsilon) > 0$. Since $\gamma^1 \geq 1$ and $\varepsilon \geq 0$, this inequality always holds if $1/\beta - 1 - b\pi_1\eta_1 - b\pi_2\eta_2 > 0$. Since $\eta_1 - c - \varepsilon > 0$ and $\eta_2 > \eta_1$, this condition also implies that $u'(q) - 1 + \eta_1 - \varepsilon - c > 0$ and $u'(q) - 1 + \eta_2 - \varepsilon - c > 0$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for a range of values of γ higher or equal to γ^1.

Subcase $\eta_1 - c - \varepsilon \leq 0$: For an agent with preference shock η_1 the multiplier of the borrowing constraint (25) is positive if $u'(q) + \eta_1 - 1 - c - \varepsilon > 0$. From (42), at $\gamma = \gamma^1 = \gamma^{1\prime}$ this is the case if $\left(\gamma^1/\beta - 1\right)/b - \pi_1(\eta_1 - \varepsilon) - \pi_2(\eta_2 - \varepsilon) + \eta_1 - c - \varepsilon > 0$. Since $\gamma^1 \geq 1$, $\varepsilon \geq 0$ and $\eta_1 > \varepsilon + (1-b)\,c$, this inequality always holds if $(1/\beta - 1)/b - \pi_1\eta_1 - \pi_2\eta_2 - b\eta_1/(1-b) > 0$. Since $\eta_1 - c - \varepsilon \leq 0$ and $\eta_2 > \eta_1$, this condition also implies that $u'(q) - 1 > 0$ and $u'(q) - 1 + \eta_2 - \varepsilon - c > 0$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for a range of values of γ higher or equal to γ^1.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists.

Case $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (22) and (26), $\eta^* > \eta_1$ and $\hat{\eta}^* < \eta_1$. (20) and (23) can be rewritten as

$$\gamma/\beta = bu'(q) + b\pi_2(\eta_2 - \varepsilon) + (1-b)(1+i) \tag{49}$$

and

$$\gamma/\beta - 1 = b\left[u'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right] \tag{50}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$. Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ in a fully constrained equilibrium. From (49) and (50) it follows that $(1-b)\,i = b\pi_1(\eta_1 - \varepsilon)$ at $\gamma = \gamma^{1\prime}$ so $i > 0$. Rewrite the borrowing constraint (25) by conjecturing a fully constrained equilibrium for the case $\eta_1 \leq \eta^* < \eta_2$ and $\hat{\eta}^* < \eta_1$. Using (41) equation (25) becomes

$$\begin{aligned} & -i(1-b)\,q - q + \frac{\beta b}{1-\beta}\left\{u(q) - q + \pi_2\left[\eta_2 - \varepsilon - (1-b)\,c\right]q\right\} \\ & = \frac{\beta b}{1-\beta}\left\{u(\hat{q}) + \left[-1 + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right]\hat{q}\right\} - \frac{(\gamma - \beta)\,\hat{q}}{1-\beta} \end{aligned} \tag{51}$$

From (51) it follows that

$$\gamma^{1\prime} = 1 + \beta b\pi_2(1-b)\,c + b\pi_1(\eta_1 - \varepsilon) \tag{52}$$

Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime}$. Differentiate (51) with respect to γ to get

$$-\frac{\partial i}{\partial\gamma}(1-b)\,q-\left[i(1-b)+1\right]\frac{\partial q}{\partial\gamma}+\frac{\beta b}{1-\beta}\left\{u'(q)-1+\pi_2\left[\eta_2-\varepsilon-(1-b)\,c\right]\right\}\frac{\partial q}{\partial\gamma} \tag{53}$$
$$=\frac{\beta b}{1-\beta}\left\{u'(\hat{q})-1+\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right\}\frac{\partial\hat{q}}{\partial\gamma}-\frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\gamma}-\frac{\hat{q}}{1-\beta}$$

From (49),

$$(1-b)\frac{\partial i}{\partial\gamma}=1/\beta-bu''(q)\frac{\partial q}{\partial\gamma} \tag{54}$$

Use (53), (49), (50) and (54) to get

$$\frac{\partial q}{\partial\gamma}=\frac{(1-\beta)\,q/\beta-\hat{q}}{(1-\beta)\,bqu''(q)+\gamma-1-(1-b)\,i-\beta b\pi_2(1-b)\,c}$$

and

$$(1-b)\,\beta\frac{\partial i}{\partial\gamma}=\frac{\beta bu''(q)\,\hat{q}+\gamma-1-(1-b)\,i-\beta b\pi_2(1-b)\,c}{(1-\beta)\,bu''(q)\,q+\gamma-1-(1-b)\,i-\beta b\pi_2(1-b)\,c} \tag{55}$$

From (51), we get

$$\gamma-i(1-b)-1-\beta b\pi_2(1-b)\,c \tag{56}$$
$$=\gamma-\beta b\pi_2(1-b)\,c+\frac{\beta b}{1-\beta}\frac{u(\hat{q})+\left[-1+\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right]\hat{q}}{q}-\frac{(\gamma-\beta)\,\hat{q}/q}{1-\beta}$$
$$-\frac{\beta b}{1-\beta}\frac{u(q)-q+\pi_2\left[\eta_2-\varepsilon-(1-b)\,c\right]q}{q}$$

By the mean value theorem, $u(q)-u(\hat{q})>u'(q)(q-\hat{q})$ for $q>\hat{q}$. Therefore, for $q>\hat{q}$ (or $i>b\pi_2(\eta_2-\varepsilon)/(1-b)$) we verify from (56) that

$$\gamma-i(1-b)-1-\beta b\pi_2(1-b)\,c<\beta\left[b\pi_1(\eta_1-\varepsilon)-(1-b)\,i\right]\hat{q}/q \tag{57}$$

The right-hand side in (57) is negative for $q>\hat{q}$ (or $b\pi_1(\eta_1-\varepsilon)<(1-b)\,i$). Therefore $\gamma-i(1-b)-1-\beta b\pi_2(1-b)\,c<0$ for $q>\hat{q}$ and it is equal to zero at $\gamma=\gamma^{1\prime}$. Hence from (55) $\partial i/\partial\gamma>0$ at $\gamma=\gamma^{1\prime}$ so $i>b\pi_1(\eta_1-\varepsilon)/(1-b)>0$ at γ slightly higher than $\gamma^{1\prime}$. In turn, this implies that $\gamma-i(1-b)-1-\beta b\pi_2(1-b)\,c$ is negative for γ slightly higher than $\gamma^{1\prime}$ and hence it follows from (55) that $\partial i/\partial\gamma>0$ at γ slightly higher than $\gamma^{1\prime}$. Therefore, $i>b\pi_1(\eta_1-\varepsilon)/(1-b)>0$ for higher values of γ. Define γ^1 as the minimum value of γ for which a fully constrained equilibrium exists. Since $i>0$ at $\gamma=\gamma^{1\prime}$ there is $\gamma^1<\gamma^{1\prime}$ for which $i\geq 0$.

To conclude, we must ensure that the representative agent is credit-constrained for all values of η as we conjectured at the beginning of the proof. Since $\varepsilon<\eta_1\leq\varepsilon+(1-b)\,c<\eta_2$, two subcases may exist: $\eta_2-c-\varepsilon>0$ and $\eta_2-c-\varepsilon\leq 0$.

Subcase $\eta_2 - c - \varepsilon > 0$: For the agent who consumes the home good, the multiplier of the borrowing constraint (25) is positive if $u'(q) - 1 > 0$. From (49), this is the case if $\gamma/\beta - 1 - b\pi_2(\eta_2 - \varepsilon) - (1-b)i > 0$ which always holds if $1/\beta - 1 - b\pi_1\eta_1 - b\pi_2\eta_2 > 0$ since $\gamma \geq 1$, $\varepsilon \geq 0$ and $0 \leq (1-b)i \leq b\pi_1(\eta_1 - \varepsilon)$ for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. This condition also implies that $u'(q) - 1 + \eta_2 - \varepsilon - c > 0$ since $\eta_2 - c - \varepsilon > 0$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for an interval of values of $\gamma \geq \gamma^1$.

Subcase $\eta_2 - c - \varepsilon \leq 0$: For an agent with preference shock η_2, the multiplier of the borrowing constraint (25) is positive if $u'(q) - 1 + \eta_2 - c - \varepsilon > 0$. From (49), this is the case if $\gamma/\beta - 1 - b\pi_2(\eta_2 - \varepsilon) - (1-b)i + \eta_2 - c - \varepsilon > 0$ which always holds if $(1/\beta - 1)/b - \pi_1\eta_1 - \pi_2\eta_2 - b\eta_2/(1-b) > 0$ since $\gamma \geq 1$, $\varepsilon \geq 0$, $0 \leq (1-b)i \leq b\pi_1(\eta_1 - \varepsilon)$ for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$ and $\eta_2 > (1-b)c + \varepsilon$. This condition also implies that $u'(q) - 1 > 0$ since $\eta_2 - c - \varepsilon \leq 0$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for an interval of values of $\gamma \geq \gamma^1$.

Finally, note that an agent with $\eta = \eta_1$ could prefer to consume the foreign good by using only his money holdings instead of borrowing and consuming the home good but this possibility can be dismissed. That is, the following condition is satisfied

$$u(q) - \phi\ell(1+i) \geq u(m_{-1}) + (\eta_1 - \varepsilon)m_{-1}$$

From (41), this expression can be written as

$$u(q) - (1-b)q(1+i) \geq u(bq) + (\eta_1 - \varepsilon)bq$$

Since $u(q) - u(bq) > u'(q)(1-b)q$ and in a fully constrained equilibrium $i \leq u'(q) - 1$, it follows that it is always possible to define a value $\bar{\eta}_1$ such that if $\eta_1 \leq \bar{\eta}_1$ the above inequality holds.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists and an unconstrained equilibrium does not exist.

Case $\eta_2 < \varepsilon + (1-b)c$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (22) and (26), $\eta^* > \eta_2$ and $\hat{\eta}^* < \eta_1$. (20) and (23) can be rewritten as

$$\gamma/\beta - 1 = b\left[u'(q) - 1\right] + (1-b)i \tag{58}$$

and

$$\gamma/\beta - 1 = b\left[u'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right] \tag{59}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$, which requires $q \geq \hat{q}$ given (58) and (59). Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^1 the value of γ such that $i = 0$ in a fully constrained equilibrium. From (58) and (59), $(1-b)i = b[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]$ at $\gamma = \gamma^{1\prime}$. Rewrite the borrowing constraint (25) by conjecturing a fully constrained equilibrium for the case $\eta^* > \eta_2$ and $\hat{\eta}^* < \eta_1$. Using (41) it becomes

$$
\begin{aligned}
& -i(1-b)q - q + \frac{\beta b}{1-\beta}[u(q) - q] \\
& = \frac{\beta b}{1-\beta}[u(\hat{q}) - \hat{q} + \pi_1(\eta_1 - \varepsilon)\hat{q} + \pi_2(\eta_2 - \varepsilon)\hat{q}] - \frac{(\gamma - \beta)\hat{q}}{1-\beta}
\end{aligned} \tag{60}
$$

From (60) it follows that

$$\gamma^{1\prime} = 1 + b[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]$$

Since $(1-b)i = b[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]$ at $\gamma = \gamma^{1\prime}$, next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime}$. Differentiate (60) with respect to γ to get

$$
\begin{aligned}
& -\frac{\partial i}{\partial \gamma}(1-b)q - [i(1-b)+1]\frac{\partial q}{\partial \gamma} + \frac{\beta b}{1-\beta}\left[u'(q) - 1\right]\frac{\partial q}{\partial \gamma} \\
& = \frac{\beta b}{1-\beta}\left\{u'(\hat{q}) - 1 + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right\}\frac{\partial \hat{q}}{\partial \gamma} - \frac{\gamma - \beta}{1-\beta}\frac{\partial \hat{q}}{\partial \gamma} - \frac{\hat{q}}{1-\beta}
\end{aligned} \tag{61}
$$

From (58),

$$(1-b)\frac{\partial i}{\partial \gamma} = 1/\beta - bu''(q)\frac{\partial q}{\partial \gamma} \tag{62}$$

Use (61), (58), (59) and (62) to get

$$\frac{\partial q}{\partial \gamma} = \frac{q/\beta - \hat{q}/(1-\beta)}{bqu''(q) - i(1-b) - 1 + [\gamma - \beta - \beta(1-b)i]/(1-\beta)}$$

and

$$(1-b)\beta\frac{\partial i}{\partial \gamma} = \frac{\gamma - 1 - i(1-b) + bu''(q)\hat{q}\beta}{\gamma - 1 - i(1-b) + (1-\beta)bu''(q)q} \tag{63}$$

From (60), we get

$$
\begin{aligned}
& \gamma - i(1-b) - 1 + \frac{\beta b}{1-\beta}\frac{u(q) - q}{q} \\
& = \gamma + \frac{\beta b}{1-\beta}\frac{u(\hat{q}) - \hat{q} + \pi_1(\eta_1 - \varepsilon)\hat{q} + \pi_2(\eta_2 - \varepsilon)\hat{q}}{q} - \frac{(\gamma - \beta)\hat{q}/q}{1-\beta}
\end{aligned} \tag{64}
$$

By the mean value theorem, $u(q) - u(\hat{q}) > u'(q)(q - \hat{q})$ for $q > \hat{q}$. Therefore, for $q > \hat{q}$ (or $i > b[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]/(1-b)$) we verify from (64) that

$$\gamma - i(1-b) - 1 < \beta\{b[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)] - (1-b)i\}\hat{q}/q$$

so $\gamma - i(1-b) - 1$ is unambiguously negative for $i > 0$ and given $\gamma^{1\prime}$ it is equal to zero for $i = 0$. Hence from (63) $\partial i/\partial\gamma > 0$ at $\gamma = \gamma^{1\prime}$ so $i > b[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]/(1-b) > 0$ at γ slightly higher than $\gamma^{1\prime}$. In turn, this implies that $\gamma - i(1-b) - 1$ is negative for γ slightly higher than $\gamma^{1\prime}$ and hence it follows from (63) that $\partial i/\partial\gamma > 0$ at γ slightly higher than $\gamma^{1\prime}$. Therefore, $i > b\left[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right]/(1-b) > 0$ for γ even higher. Further, from (63) it follows that $\partial i/\partial\gamma > 0$ at least for some values of $i < b\left[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right]/(1-b)$.

To conclude, we must ensure that the representative agent is credit-constrained for all values of η as we conjectured at the beginning of the proof. When $\eta_2 < \varepsilon+(1-b)\,c$, the agent consumes the home good for all realizations of the preference shock. The multiplier of the borrowing constraint (25) is positive if $u'(q)-1>0$. From (58), this is the case if $\gamma/\beta-1-(1-b)\,i>0$ which always holds if $1/\beta-1-b\pi_1\eta_1-b\pi_2\eta_2>0$ since $\gamma\geq 1$, $\varepsilon\geq 0$ and $0\leq(1-b)\,i\leq b\pi_1(\eta_1-\varepsilon)+b\pi_2(\eta_2-\varepsilon)$ for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for an interval of values of $\gamma\geq\gamma^1$.

Finally, note that an agent with $\eta=\eta_1,\eta_2$ could prefer to consume the foreign good by using only his money holdings instead of borrowing and consuming the home good but this possibility can be dismissed. That is, the following condition is satisfied

$$u(q)-\phi\ell(1+i)\geq u(m_{-1})+(\eta_2-\varepsilon)\,m_{-1}$$

From (41), this expression can be written as

$$u(q)-(1-b)\,q\,(1+i)\geq u(bq)+(\eta_2-\varepsilon)\,bq$$

Since $u(q)-u(bq)>u'(q)(1-b)\,q$ and in a fully constrained equilibrium $i\leq u'(q)-1$, it follows that it is always possible to define a value $\bar{\eta}_2$ such that if $\eta_2\leq\bar{\eta}_2$ the above inequality holds. Further, the above inequality implies that $u(q)-(1-b)\,q\,(1+i)\geq u(bq)+(\eta_1-\varepsilon)\,bq$ since $\eta_2>\eta_1$.

Therefore there is an interval $\left[\gamma^1,\gamma^2\right]$ such that if $\gamma\in\left[\gamma^1,\gamma^2\right]$ then a fully constrained equilibrium in which $i\geq 0$ exists and an unconstrained equilibrium does not exist.

Proof of Proposition 3. Given (34), (20) can be written as

$$\gamma/\beta=u'\left(q_f^\eta\right)+\eta-\varepsilon-\left(b\sum_{\eta\leq\eta^*}\pi_\eta+1-b\right)c$$

Hence

$$\frac{\partial q_f^\eta}{\partial\varepsilon}=\frac{1}{u''\left(q_f^\eta\right)}\tag{65}$$

so that $\partial q_f^\eta/\partial\varepsilon<0$. From (34) and (65), $\partial q_h/\partial\varepsilon=0$. From (36) and (37), we get

$$\frac{\partial(\phi\ell_h)}{\partial\varepsilon}=-b\sum_{\eta>\eta^*}\pi_\eta\frac{\partial q_f^\eta}{\partial\varepsilon}$$

and

$$\frac{\partial\left(\phi\ell_f^\eta\right)}{\partial\varepsilon}=\left(1-b\sum_{\eta>\eta^*}\pi_\eta\right)\frac{\partial q_f^\eta}{\partial\varepsilon}$$

Given (65), we get $\partial\left(\phi\ell_h\right)/\partial\varepsilon>0$ and $\partial\left(\phi\ell_f^\eta\right)/\partial\varepsilon<0$.

Differentiating (27) with respect to ε yields

$$\frac{\partial\mathcal{W}}{\partial\varepsilon}=\frac{b}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\left\{\left[u'\left(q_f^\eta\right)-1+\eta-\varepsilon-c+b\sum_{\eta>\eta^*}\pi_\eta c\right]\frac{\partial q_f^\eta}{\partial\varepsilon}-q_f^\eta\right\}$$

Since $u'\left(q_f^\eta\right)-1+\eta-\varepsilon-c>0$ for all $\eta>\eta^*$ from (34) and $\partial q_f^\eta/\partial\varepsilon<0$ from (65), it follows that $\partial\mathcal{W}/\partial\varepsilon<0$.

Proof of Proposition 4. Let $c<\left(\eta_1-\varepsilon\right)/\left(1-b\right)$ and consider a fully constrained equilibrium in which $\lambda_h^\eta,\lambda_f^\eta>0$ and the borrowing constraint (25) holds with equality. As in the proof of Proposition 2, we can set $\phi\ell_h=\phi\ell_f^\eta=\phi\ell$ and $q_h=q_f^\eta=q$ for all η. Given (26) and (41), welfare defines in (27) becomes

$$\mathcal{W}\left(\frac{b}{1-\beta}\right)^{-1}=u\left(q\right)+\left(-1+\pi_1\eta_1+\pi_2\eta_2\right)q-\left(\pi_1+\pi_2\right)\left[\varepsilon+\left(1-b\right)c\right]q \quad (66)$$

Differentiate the borrowing constraint for the case $c<\left(\eta_1-\varepsilon\right)/\left(1-b\right)$ stated in (44) with respect to ε to get

$$\begin{aligned}&-\left[1+\left(1-b\right)i\right]\frac{\partial q}{\partial\varepsilon}-\left(1-b\right)\frac{\partial i}{\partial\varepsilon}q-\frac{\beta b\left(\pi_1+\pi_2\right)}{1-\beta}q \quad (67)\\&+\frac{\beta b}{1-\beta}\left\{u'\left(q\right)-1+\pi_1\eta_1+\pi_2\eta_2-\left(\pi_1+\pi_2\right)\left[\varepsilon+\left(1-b\right)c\right]\right\}\frac{\partial q}{\partial\varepsilon}\\&=\frac{\beta b}{1-\beta}\left[u'\left(\hat{q}\right)-1+\pi_1\left(\eta_1-\varepsilon\right)+\pi_2\left(\eta_2-\varepsilon\right)\right]\frac{\partial\hat{q}}{\partial\varepsilon}-\frac{\beta b\left(\pi_1+\pi_2\right)}{1-\beta}\hat{q}-\frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\varepsilon}\end{aligned}$$

Differentiating (42) with respect to ε yields

$$\left(1-b\right)\frac{\partial i}{\partial\varepsilon}=-bu''\left(q\right)\frac{\partial q}{\partial\varepsilon}+b\left(\pi_1+\pi_2\right) \quad (68)$$

Rewrite (67) using (42), (43) and (68) to get

$$\frac{\partial q}{\partial\varepsilon}=\frac{b\left(\pi_1+\pi_2\right)\left[\left(1-\beta\right)q+\beta\left(q-\hat{q}\right)\right]}{\gamma-1-\left(1-b\right)i-\beta b\left(\pi_1+\pi_2\right)\left(1-b\right)c+\left(1-\beta\right)bqu''\left(q\right)} \quad (69)$$

From (42) and (43), $q=\hat{q}$ when $i=0$ and $q>\hat{q}$ when $i>0$. From the proof of Proposition 2, when $c<\left(\eta_1-\varepsilon\right)/\left(1-b\right)$ the value of γ such that $i=0$ and $q=\hat{q}$ is $\gamma^1=1+\beta b\left(\pi_1+\pi_2\right)\left(1-b\right)c$. Further, $q\geq\hat{q}$ for $\gamma\in\left[\gamma^1,\gamma^2\right]$. Therefore, the numerator at the right-hand side in (69) is positive for $\gamma\in\left[\gamma^1,\gamma^2\right]$. From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side in (69) is negative for $\gamma \in \left[\gamma^1, \gamma^2\right]$. It follows that in a fully constrained equilibrium $\partial q/\partial \varepsilon < 0$ for $\gamma \in \left[\gamma^1, \gamma^2\right]$. Since $\phi \ell_h = \phi \ell_f^\eta = \phi \ell = (1-b)\, q$ for all η from (41), $\partial \left(\phi \ell\right)/\partial \varepsilon < 0$.

Rewriting (27) differentiating with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{b}{1-\beta}\right)^{-1} = \left\{u'\left(q\right) - 1 + \pi_1 \eta_1 + \pi_2 \eta_2 - \left(\pi_1 + \pi_2\right)\left[\varepsilon + \left(1-b\right)c\right]\right\}\frac{\partial q}{\partial \varepsilon} - \left(\pi_1 + \pi_2\right) q$$

Since $\eta - \varepsilon - (1-b)\, c > 0$ for all $\eta > \eta^*$ from (26) and $\partial q/\partial \varepsilon < 0$ from (69), it follows that $\partial \mathcal{W}/\partial \varepsilon < 0$.

Proof of Proposition 5. For this proof we distinguish two cases, $\varepsilon + (1-b)\, c > \eta_2$ and $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\, c < \eta_2$. In the first case showing that $\partial \mathcal{W}/\partial \varepsilon > 0$ is straightforward since the non-defaulter consumes only the home good and hence does not incur conversion costs. For the second case it is shown that $\partial \mathcal{W}/\partial \varepsilon > 0$ holds for π_2 sufficiently low.

Consider a fully constrained equilibrium in which $\lambda_h^\eta, \lambda_f^\eta > 0$ and the borrowing constraint (25) holds with equality. As in the proof of Proposition 2, we can set $\phi \ell_h = \phi \ell_f^\eta = \phi \ell$ and $q_h = q_f^\eta = q$ for all η.

Case $\varepsilon + (1-b)\, c > \eta_2$.

Given (26), the welfare defined in (27) becomes

$$\mathcal{W}\left(\frac{b}{1-\beta}\right)^{-1} = u\left(q\right) - q \tag{70}$$

Differentiate the borrowing constraint for the case $\varepsilon + (1-b)\, c > \eta_2$ stated in (60) with respect to ε to get

$$\begin{aligned}
&-\left(1-b\right)\frac{\partial i}{\partial \varepsilon} q + \frac{\beta b\left(\pi_1 + \pi_2\right)}{1-\beta}\hat{q} - \left[1 + \left(1-b\right) i\right]\frac{\partial q}{\partial \varepsilon} \\
&+ \frac{\beta b}{1-\beta}\left[u'\left(q\right) - 1\right]\frac{\partial q}{\partial \varepsilon} \\
&= \frac{\beta b}{1-\beta}\left\{u'\left(\hat{q}\right) - 1 + \left[\pi_1\left(\eta_1 - \varepsilon\right) + \pi_2\left(\eta_2 - \varepsilon\right)\right]\right\}\frac{\partial \hat{q}}{\partial \varepsilon} - \frac{\gamma - \beta}{1-\beta}\frac{\partial \hat{q}}{\partial \varepsilon}
\end{aligned} \tag{71}$$

Differentiating (58) with respect to ε yields

$$\left(1-b\right)\frac{\partial i}{\partial \varepsilon} = -b u''\left(q\right)\frac{\partial q}{\partial \varepsilon} \tag{72}$$

Rewrite (71) using (58), (59) and (72)

$$\frac{\partial q}{\partial \varepsilon} = \frac{-\beta b\left(\pi_1 + \pi_2\right)\hat{q}/\left(1-\beta\right)}{b u''\left(q\right) q + \left[\gamma - 1 - \left(1-b\right) i\right]/\left(1-\beta\right)} \tag{73}$$

From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side of (73) is negative. Since the numerator at the right-hand side of (73) is also negative, it follows that in a fully constrained equilibrium $\partial q/\partial\varepsilon > 0$. Since $\phi\ell_h = \phi\ell_f^\eta = \phi\ell = (1-b)\,q$ for all η from (41), it follows that $\partial\,(\phi\ell)\,/\partial\varepsilon > 0$.

Differentiating (70) with respect to ε yields

$$\frac{\partial\mathcal{W}}{\partial\varepsilon}\left(\frac{b}{1-\beta}\right)^{-1} = \left[u'\,(q) - 1\right]\frac{\partial q}{\partial\varepsilon} \tag{74}$$

Since $\partial q/\partial\varepsilon > 0$ (74) implies that $\partial\mathcal{W}/\partial\varepsilon > 0$.

Case $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$.

Given (26) and (41), the welfare defined in (27) becomes

$$\mathcal{W}\left(\frac{b}{1-\beta}\right)^{-1} = u\,(q) - q + \pi_2\left[\eta_2 - \varepsilon - (1-b)\,c\right]q \tag{75}$$

Differentiate the borrowing constraint for the case $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$ stated in (51) with respect to ε to get

$$\begin{aligned}
&-(1-b)\frac{\partial i}{\partial\varepsilon}q - \frac{\beta b}{1-\beta}\left[\pi_2 q - (\pi_1+\pi_2)\,\hat{q}\right] \\
&-\left[1+(1-b)\,i\right]\frac{\partial q}{\partial\varepsilon} + \frac{\beta b}{1-\beta}\left\{u'\,(q) - 1 + \pi_2\left[\eta_2 - \varepsilon - (1-b)\,c\right]\right\}\frac{\partial q}{\partial\varepsilon} \\
&= \frac{\beta b}{1-\beta}\left\{u'\,(\hat{q}) - 1 + \left[\pi_1\,(\eta_1-\varepsilon) + \pi_2\,(\eta_2-\varepsilon)\right]\right\}\frac{\partial\hat{q}}{\partial\varepsilon} - \frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\varepsilon}
\end{aligned} \tag{76}$$

Differentiating (49) with respect to ε yields

$$(1-b)\frac{\partial i}{\partial\varepsilon} = -bu''\,(q)\frac{\partial q}{\partial\varepsilon} + b\pi_2 \tag{77}$$

Rewrite (76) using (49), (50) and (77)

$$\frac{\partial q}{\partial\varepsilon} = \frac{b\pi_2 q + \beta b\left[\pi_2 q - (\pi_1+\pi_2)\,\hat{q}\right]/\,(1-\beta)}{bu''\,(q)\,q + \left[\gamma - 1 - (1-b)\,i - \beta b\,(1-b)\,\pi_2 c\right]/\,(1-\beta)} \tag{78}$$

From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side of (78) is negative.

For $\gamma = \gamma^1$, $q = \hat{q}$. It follows that the numerator at the right-hand side of (78) is negative since $\pi_2 - \beta\pi_1/\,(1-\beta) < 0$. Therefore, $\partial q/\partial\varepsilon > 0$ at $\gamma = \gamma^1$. Since q is increasing in ε as long as the numerator at the right-hand side of (78) is negative and $\hat{q}$ is decreasing in ε given (50), the numerator at the right-hand side of (78) is increasing in ε. Define $\gamma^{2\prime}$ the value of γ such that for $\{q, \hat{q}, i\}$ which satisfy (49), (50) and (51) the numerator at the right-hand side of (78) is zero. In addition, let $\hat{\gamma}^2 = \min\left(\gamma^2, \gamma^{2\prime}\right)$. Then in a fully constrained equilibrium $\partial q/\partial\varepsilon > 0$ for $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$. Since $\phi\ell_h = \phi\ell_f^\eta = \phi\ell = (1-b)\,q$ for all η from (41), $\partial\,(\phi\ell)\,/\partial\varepsilon > 0$ for $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$.

Differentiating (75) with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{b}{1-\beta}\right)^{-1}=\left\{u'(q)-1+\pi_2\left[\eta_2-\varepsilon-(1-b)\,c\right]\right\}\frac{\partial q}{\partial \varepsilon}-\pi_2 q \tag{79}$$

Using (78) for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$ with $\gamma^{1\prime}$ stated in (52) it follows that

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{b}{1-\beta}\right)^{-1}>\frac{u'(q)-1+\pi_2\left[\eta_2-\varepsilon-(1-b)\,c\right]}{-u''(q)}\left(\frac{\beta\pi_1}{1-\beta}-\pi_2\right)-\pi_2 q$$

Since the assumed preferences satisfy $-u''(q)\,q\le u'(q)$ and $\eta_2-\varepsilon-(1-b)\,c>0$, a sufficient condition for $\partial\mathcal{W}/\partial\varepsilon>0$ for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$ is

$$\frac{u'(q)-1}{u'(q)}\left(\frac{\beta\pi_1}{1-\beta}-\pi_2\right)-\pi_2>0 \tag{80}$$

Since $0\le(1-b)\,i\le b\pi_1(\eta_1-\varepsilon)$ for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$ (49) evaluated at $\varepsilon=0$ defines lower and upper bounds for $u'(q)$ as functions of parameters. Thus the left-hand side in (80) is decreasing in π_2 and it is positive for $\pi_2=0$. Let $\bar{\pi}_2$ denote the value of π_2 such that the left-hand side in (80) is zero. Since condition (80) is sufficient (but not necessary) there is $\hat{\pi}_2>\bar{\pi}_2>0$ such that if $\pi_2\le\hat{\pi}_2$ then $\partial\mathcal{W}/\partial\varepsilon>0$ for $\gamma\in\left[\gamma^1,\hat{\gamma}^2\right]$.

Proof of Proposition 8. We proceed in three steps to show that the amount of credit is decreasing in c. Consider two cases: $\varepsilon=0$ and $\varepsilon>0$. First, we show that q is decreasing in c from some value c_0 up to some value $c<\eta_1/(1-b)$ in the case $\varepsilon=0$ and up to some value $c<(\eta_1-\varepsilon)/(1-b)$ in the case $\varepsilon>0$. To prove that an increase in c entails a decrease in credit in a fully constrained equilibrium in which $\varepsilon+(1-b)\,c\le\eta_1$ with $\varepsilon\ge 0$, differentiate the borrowing constraint stated in (44) with respect to c:

$$\begin{aligned}&-\left[(1-b)\,i+1\right]\frac{\partial q}{\partial c}-(1-b)\,q\frac{\partial i}{\partial c}-\frac{\beta b}{1-\beta}(\pi_1+\pi_2)(1-b)\,q\\&+\frac{\beta b}{1-\beta}\left\{u'(q)-1+\pi_1\eta_1+\pi_2\eta_2-(\pi_1+\pi_2)\left[\varepsilon+(1-b)\,c\right]\right\}\frac{\partial q}{\partial c}=0\end{aligned} \tag{81}$$

From (42) we get

$$(1-b)\frac{\partial i}{\partial c}=-bu''(q)\frac{\partial q}{\partial c} \tag{82}$$

Use (42) and (82) to rewrite (81) as follows

$$\frac{\partial q}{\partial c}=\frac{\beta b(\pi_1+\pi_2)(1-b)\,q/(1-\beta)}{bu''(q)\,q+\left[\gamma-1-(1-b)\,i-\beta b(\pi_1+\pi_2)(1-b)\,c\right]/(1-\beta)} \tag{83}$$

From the proof of Proposition 2, in the case $\varepsilon+(1-b)\,c<\eta_1$ of a fully constrained equilibrium the denominator at the right-hand side in (83) is negative. Since in the fully constrained equilibrium $\ell_h=\ell_f^\eta=\ell$ for all η and $\phi\ell=(1-b)\,q$

from (41), it follows that $\partial(\phi\ell)/\partial c < 0$ for $c < (\eta_1 - \varepsilon)/(1-b)$. Since $\partial q/\partial c < 0$ for all $c < (\eta_1 - \varepsilon)/(1-b)$ it follows that, in the case $\varepsilon = 0$, q and $(\phi\ell)$ are decreasing in c up to $c = \eta_1/(1-b)$ and, in the case $\varepsilon > 0$, q and $(\phi\ell)$ are decreasing in c up to $c = (\eta_1 - \varepsilon)/(1-b)$.

Second, we show that the function $q = q(c)$ is not continuous. For this, we evaluate the function $q = q(c)$ at a particular value of γ and infer that its properties hold for a range of values of γ. Consider the case $\varepsilon = 0$. The function $q = q(c)$ jumps below at $c = \eta_1/(1-b)$; i.e., $q(c^-) > q(c^+)$ with $c^- = \eta_1/(1-b) - dc$, $c^+ = \eta_1/(1-b) + dc$ and $dc \to 0$. From (42) and (49), it follows that

$$b\left[u'\left(q\left(c^-\right)\right) + \pi_1\eta_1\right] + (1-b)\,i\left(c^-\right) = bu'\left(q\left(c^+\right)\right) + (1-b)\,i\left(c^+\right) \tag{84}$$

where $q(c^-)$ and $i(c^-)$ solve (44) and (42) (with $\hat{q}$ being determined by (43)), whereas $q(c^+)$ and $i(c^+)$ solve (51) and (49) (with $\hat{q}$ being determined by (50)). At $\gamma = \gamma^1(c^+, \varepsilon = 0) = 1 + \beta b\pi_2(1-b)c^+ + b\pi_1\eta_1$, $i(c^+) = b\pi_1\eta_1/(1-b)$. Hence, at $\gamma = \gamma^1(c^+, \varepsilon = 0)$ (84) becomes

$$bu'\left(q\left(c^-\right)\right) + (1-b)\,i\left(c^-\right) = bu'\left(q\left(c^+\right)\right) \tag{85}$$

Note that $\gamma^1(c^+, \varepsilon = 0) > \gamma^1(c^-, \varepsilon = 0) = 1 + \beta b(\pi_1 + \pi_2)(1-b)c^-$ provided that $\beta < 1$. Thus, at $\gamma = \gamma^1(c^+, \varepsilon = 0)$, $i(c^-) > 0$ since $i(c^-) = 0$ at $\gamma^1(c^-, \varepsilon = 0)$ and $\partial i/\partial\gamma > 0$ in a fully constrained equilibrium with $c < \eta_1/(1-b)$ from Proposition 2. Hence, from (85) $q(c^+) < q(c^-)$. It follows that the function is discontinuous at $c = \eta_1/(1-b)$ with $q(c^+) < q(c^-)$. Since all functions in (84) ($u'(q(c^-))$ and $i(c^-)$ which solve (44) and (42), and $u'(q(c^+))$ and $i(c^+)$ which solve (51) and (49)) are continuous, we can infer that there is a range of values of γ for which the function $q = q(c)$ is not continuous at $c = \eta_1/(1-b)$ with $q(c^+) < q(c^-)$. From (41), it follows that at $c = \eta_1/(1-b)$ the function $\phi\ell$ also jumps below.

Similarly, in the case $\varepsilon > 0$, the function $q = q(c)$ jumps below at $c = (\eta_1 - \varepsilon)/(1-b)$; i.e., $q(c^-) > q(c^+)$ with $c^- = (\eta_1 - \varepsilon)/(1-b) - dc$, $c^+ = (\eta_1 - \varepsilon)/(1-b) + dc$ and $dc \to 0$. From (42) and (49), it follows that

$$b\left[u\left(q\left(c^-\right)\right) + \pi_1(\eta_1 - \varepsilon)\right] + (1-b)\,i\left(c^-\right) = bu\left(q\left(c^+\right)\right) + (1-b)\,i\left(c^+\right) \tag{86}$$

where $q(c^-)$ and $i(c^-)$ solve (44) and (42) (with $\hat{q}$ being determined by (43)), whereas $q(c^+)$ and $i(c^+)$ solve (51) and (49) (with $\hat{q}$ being determined by (50)). At $\gamma = \gamma^1(c^+, \varepsilon > 0) = 1 + \beta b\pi_2(1-b)c^+ + b\pi_1(\eta_1 - \varepsilon)$, $i(c^+) = b\pi_1(\eta_1 - \varepsilon)/(1-b)$. Hence, at $\gamma = \gamma^1(c^+, \varepsilon > 0)$ (86) becomes

$$bu\left(q\left(c^-\right)\right) + (1-b)\,i\left(c^-\right) = bu\left(q\left(c^+\right)\right) \tag{87}$$

At $\gamma = \gamma^1(c^+, \varepsilon > 0)$, $i(c^-) > 0$ since $i(c^-) = 0$ at $\gamma^1(c^-, \varepsilon > 0)$, $\partial i/\partial\gamma > 0$ in a fully constrained equilibrium with $c < (\eta_1 - \varepsilon)/(1-b)$ from Proposition 2, and $\gamma^1(c^+, \varepsilon > 0) > \gamma^1(c^-, \varepsilon > 0)$. Thus, from (87) $q(c^+) < q(c^-)$. It follows that the function is discontinuous at $c = (\eta_1 - \varepsilon)/(1-b)$ with $q(c^+) < q(c^-)$. Since all functions in (86) are continuous, we can infer that there is a range of values of γ for which the function $q = q(c)$ is not continuous at $c = (\eta_1 - \varepsilon)/(1-b)$ with $q(c^+) < q(c^-)$. From (41), it follows that the function $\phi\ell$ also jumps below at $c = (\eta_1 - \varepsilon)/(1-b)$.

Third, we show that q is decreasing in c for $c > \eta_1/(1-b)$ in the case $\varepsilon = 0$ and for $c > (\eta_1 - \varepsilon)/(1-b)$ in the case $\varepsilon > 0$. To prove that this increase in c entails a decrease in credit in a fully constrained equilibrium in which $\varepsilon + (1-b)c > \eta_1$ with $\varepsilon \geq 0$, differentiate the borrowing constraint stated in (51) with respect to c:

$$\begin{aligned}
&-[(1-b)i+1]\frac{\partial q}{\partial c} - (1-b)q\frac{\partial i}{\partial c} - \frac{\beta b}{1-\beta}\pi_2(1-b)q \\
&+\frac{\beta b}{1-\beta}\left\{u'(q) - 1 + \pi_2[\eta_2 - \varepsilon - (1-b)c]\right\}\frac{\partial q}{\partial c} = 0
\end{aligned} \tag{88}$$

From (49) we get

$$(1-b)\frac{\partial i}{\partial c} = -bu''(q)\frac{\partial q}{\partial c} \tag{89}$$

Use (49) and (89) to rewrite (88) as follows

$$\frac{\partial q}{\partial c} = \frac{\beta b\pi_2(1-b)q/(1-\beta)}{bu''(q)q + [\gamma - 1 - (1-b)i - \beta b\pi_2(1-b)c]/(1-\beta)} \tag{90}$$

As shown in the proof of Proposition 2, in a fully constrained equilibrium in the case $\varepsilon < \eta_1 \leq (1-b)c + \varepsilon < \eta_2$ the denominator at the right-hand side in (90) is negative, so $\partial q/\partial c < 0$. Since in the fully constrained equilibrium $\ell_h = \ell_f^\eta = \ell$ for all η and $\phi\ell = (1-b)q$ from (41), it follows that $\partial(\phi\ell)/\partial c < 0$.

Finally, from Proposition 4 for $\varepsilon + (1-b)c < \eta_1$, in a fully constrained equilibrium q and $\phi\ell$ are decreasing in ε. In addition, from Proposition 5 for $(1-b)c > \eta_1$, there is a range of values of γ for which q and $\phi\ell$ are increasing in ε. Then it is straightforward to verify that if c increases from c_0 to c_1 and a fully constrained equilibrium exists for c_0 and c_1 for this range of values of γ, the decrease in q and $\phi\ell$ is stronger in the case $\varepsilon = 0$ than in the case $\varepsilon > 0$.

Differentiating (66) with respect to c yields

$$\begin{aligned}
\frac{\partial\mathcal{W}}{\partial c}\left(\frac{b}{1-\beta}\right)^{-1} &= \left\{u'(q) - 1 + \pi_1\eta_1 + \pi_2\eta_2 - (\pi_1+\pi_2)[\varepsilon + (1-b)c]\right\}\frac{\partial q}{\partial c} \\
&\quad - (\pi_1+\pi_2)(1-b)q
\end{aligned}$$

Thus $\partial\mathcal{W}/\partial c > 0$ for all $(1-b)c < \eta_1 - \varepsilon$ since $\partial q/\partial c < 0$ for $(1-b)c < \eta_1 - \varepsilon$. Similarly, after differentiating (75) it is straightforward to verify that $\partial\mathcal{W}/\partial c > 0$ for all $\varepsilon < \eta_1 \leq (1-b)c + \varepsilon < \eta_2$ since $\partial q/\partial c < 0$ in this case as well. Further, since $q(c^+) < q(c^-)$ for $c^- = (\eta_1 - \varepsilon)/(1-b) - dc$, $c^+ = (\eta_1 - \varepsilon)/(1-b) + dc$ and $dc \to 0$, comparison of (66) and (75) demonstrates that welfare at c^+ is lower than welfare at c^-.

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