Pension and the family

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Abstract

The effects of pension policies on fertility have been examined in the overlapping generations (OLG) model of unitary household in which no heterogeneity exists between the wife and the husband. This paper departs from the OLG model and focuses on the marital bargaining arising from the heterogeneity in a couple in a non-unitary model. Specifically, this paper examines how the pension policy affects the endogenous fertility of a bargaining couple who have different lifespans. The analysis finds out a new channel of pension policy on fertility decisions: an increase in pension size affects fertility not only via the changes in current and future income, but through a change in marital bargaining power. This channel leads a plausible argument that an increase in a pay-as-you-go (PAYG) pension further accelerates a decline in fertility through the empowerment of women.

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1. Introduction

Acceleration in demographic aging has caused many developed countries to reform their existing pension systems. Against a background of this policy concern, the mutual dependence relationship between fertility and public pension has come under intense study (Cigno, 1993; Zhang and Zhang, 1998; Wigger, 1999; Yakita, 2001; Groezen et al., 2003; Groezen and Meijdam, 2008; Hirazawa and Yakita, 2009). Economists are intrigued with the pay-as-you-go (PAYG) pension system as an inter-generational redistribution device that involves the intra-generational redistribution effect.[1] If the PAYG pension system is generous to be equally beneficial to all individuals, it induces the redistribution among heterogeneous households. Some studies deal with heterogeneity among households, but they draw the household as a single decision unit based on unitary model, and do not consider heterogeneity within the household. This paper adopts a different approach. By focusing on the marital bargaining arising from the heterogeneity in a couple, we describe the intra-generational redistribution effect of the pension policy and how its expansion affects the balance of power between husband and wife, and the fertility of the couple having different lifespans.

Heterogeneity within a household can be factors that bring about intra-generational redistribution. For example, an expansion of a generous pension system means an implicit income transfer from the shorter longevity spouse to the spouse with longer longevity (Leroux et al., 2011).[2] Recent studies based on the non-unitary model have considered this

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[1] Sinn (2004) considers redistribution from households with children to those without, because children can be insurance devices for households who cannot have children in the PAYG pension scheme. Cremer et al. (2008) consider redistribution under both funded and unfunded pension systems in the presence of different abilities in raising children among households. Hirazawa et al. (2013) focus on redistribution among households with different contributions as a result of different childcare schedules. Heterogeneity among households is discussed in the many studies on public policies. Bommier et al. (2011a, 2011b) consider the problem of redistribution among households with different longevities. Cremer et al. (2004) examine how the redistribution effect of implicit tax imposed on postponed retirement affects households' retirement activities in the presence of different productivities and health statuses. Cremer et al. (2010) consider a trade-off between redistribution due to heterogeneous productivities and redistribution caused by heterogeneous longevity (which is positively correlated with productivities).

[2] Leroux et al. (2011) give an example of the redistribution effect of pension system, that, in

intra-redistribution effect in the retirement period.[3] Theoretically, Browning (2000) showed that a generous pension system involves redistribution from husbands to wives following the fact that women tend to live longer, and that it results in increased savings, which is a favorable household allocation for wives.[4] An empirical work by Duflo (2003) also confirms the intra-generational redistribution effect of an expansion of a generous pension in South Africa, which is more likely to be beneficial for wives because they live longer. Although they shed new light on the redistribution effect of public policies within the families in the retirement period, the long-run effects of policies on household fertility decisions are not examined. The purpose of this paper is to explore the effects of the PAYG pension system on fertility, taking into account gender differences in longevity and its effect on redistribution within the household.

Our model has three features. The first is that the household makes a decision through intra-household bargaining.[5] Homogeneous couples look like the picture of happiness, but the reality proves different. It is known that wives tend to be younger than their husbands, and that they also tend to live longer than their husbands do.[6] The difference in lifespan leads couples to bargain over the saving in the young because the wife wants to have greater wealth at the retirement stage (Lundberg and Ward-Batts, 2000; Lundberg et al.,

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France, women's life expectancy at 60 is 20% higher than that of men, which contributes to the 20％pay gap in pension benefit between men and women.

[3] The distribution effect of public policy among family members in the young period is theoretically considered by Lundberg and Pollak (1993). Komura (2013a, 2013b) theoretically examines the intra-family distribution effect of policy shifts from in-cash child support to in-kind child support, and the shift of unit of income taxation from household to individual, respectively. Lundberg et al. (1997) found a significant redistribution effect caused by the shift of the child allowance recipient from fathers to mothers in the UK.

[4] Aura (2005) also uses the American legislation change of 1984 in favor of wives, who are likely to be widowed because of their higher life expectancies, to show that this implicit income transfer leads the household wealth portfolio to reflect the wife's intentions more.

[5] In contrast to our setting of family bargaining, Glazer (2008) showed that couple's non-cooperative strategic interactions result in inefficient household savings, and that the social security system can improve welfare because it forces them to secure savings. Grossbard-Shechtman and Pereira (2013) also explores the effects of marital status on individual saving behavior, rather than household saving behavior, as responses to them in a non-cooperative game, treating the distribution problem of marriage.

[6] The United Nations (2000), based on 236 countries, reported that husbands are older than their wives in all but one country. This husband-wife age gap tends to be larger in developing countries, especially African nations, but smaller in developed countries.

2003), which may reduce the demand for private consumption and the number of children in the young period. To gain insight into the declining birthrate, we must find the channel of policy effects by accounting for the endogenous marital relationship of heterogeneous spouses. The second feature is considering the effect of the pension system on fertility in a family bargaining model in which the balance of power within the young couple is affected by social norms or peer pressure.[7] In our model, the bargaining power depends on the difference between the average lifetime income of men and women in the economy, and hence, the bargaining positions of marriage are affected by the PAYG pension system. This reflects empirical evidence that social security affects the balance of power in a couple (Duflo, 2003). The third feature of our model is that fertility is determined endogenously. Most studies based on life-cycle models of a household with multiple decision units focus on household wealth or behaviors for the retirement period, with little interest in fertility. Here, we formulate a model of family bargaining in which fertility is endogenous under the PAYG pension system.

This study reveals a new channel of pension policy on fertility decisions; an increase in pension size affects fertility not only via the changes in current and future income, but also through a change in marital bargaining power. Specifically, the study presents a plausible argument that an increase in the PAYG pension further accelerates a decline in fertility compared to the unitary model, in which the bargaining power of the couple is not of interest. Increasing a generous pension system induces intra-household redistribution between spouses with different longevities, as well as the inter-generational income redistribution between young and old generations which is discussed in the conventional unitary model. Since this redistribution from short-living husbands to long-living wives alters the balance of power of couples through the changes in their relative expected lifetime incomes, this change favors wives who expect that they will live longer than their husbands will. The change in the balance of power within the couple affects their decision on the number of

[7] This concept is based on Grossbard-Shechtman (1984) and Lundberg and Pollak (1993), both of which suggested that marriage relations are determined in a marriage market that reflects cultures or social norms.

children they have, because the wife has the longer lifespan, so that she has a larger incentive for saving by reducing expenditure in the young period.

This paper is organized as follows. Section 2 presents a model of two family members with different longevity. Section 3 explores the equilibrium of our model. Section 4 carries out our policy analysis and Section 5 gives its extended analysis. Section 6 concludes the paper.

2. Model

Consider a small open economy, comprising one representative household and a government. The household consists of two individuals $(i = f, m)$, where f and m denote the female (wife) and male (husband), respectively. Each individual lives for two periods: young and old. Following Leroux et al. (2011), we assume that everyone can live through the whole young period, but they cannot entirely in the old period.[8] The length that individual i survives in the old period is denoted by λ_i. To incorporate the gender difference in longevity, we assume that the wife has a longer lifespan than the husband: $\lambda_f > \lambda_m$. In the young period, individuals get married with the partner j ($j \neq i$), raise their children, and earn an income by supplying their time in the labor market. After paying tax, they allocate their collective earnings among their private consumptions and saving for retirement. In the retirement period, they consume by making use of the pension benefit and the return from their savings. The government employs the PAYG pension scheme for income distribution from young to old generations. It imposes a tax on each household in the young period to finance the pension benefit for the elderly living in the same period.

2.1 Household

Individual i in a household gains utility from private consumptions in the young and old

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[8] As Leroux et al. (2011) point out, although λ_i can be alternatively interpreted as uncertainty mortality rate, this interpretation makes the analyses complicated but brings no substantial difference in the main results.

periods, as well as the number of children. The utility function of individual i born at period t, who belongs to generation t, is assumed to be:

$$U_{it} = \ln n_t + \ln c_t^i + \lambda_i \ln d_{t+1}^i, \tag{1}$$

where c_t^i and d_{t+1}^i are the individual i's consumption of private goods in the young and old periods, respectively, and n_t is the number of children of each gender who belong to the couple in generation t, which means that one unit of n_t corresponds to a pair of son and daughter.[9] Even though we consider a couple whose preferences for children and consumptions are identical,[10] there exists a gender gap in longevity within their expected utilities. The household welfare function is the sum of the weighted utilities of the spouses:

$$V_t = \theta_t U_{ft} + (1-\theta_t) U_{mt}, \tag{2}$$

where $\theta_t \in [0,1]$ represents the bargaining power of the wife. Following Chiappori (1988, 1992) and Apps and Rees (1988), we assume that household members can always achieve an efficient allocation based on certain distributional rules within the household. Here, θ_t can be interpreted as the distribution rule in our model.[11]

Each individual is endowed with one unit of time in their young period, and supplies their time in market and domestic work. Childcare activities are the domestic production, so the husband and/or wife commit time to the upbringing of the children. The fixed time for parental attention per child is denoted by z and the time spent on market work by individual i is denoted by L_t^i. Based on the averages of the observations, we assume

[9] This approach was also used in the theoretical models of Galor and Weil (1996), Abio et al. (2004), Doepke and Tertilt (2009), and de la Crox and Donckt (2010). This study assumes that parents benefit from having children through changes in their utility rather than changes in their budget constraint. The interpretation of children as consumption goods are well documented in studies on social security in developed countries such as Fenge and Von Weizsäcker (2010). Other motives for having children can be children as investment goods that parents expect their children to provide some financial or physical support after their retirement (Zhang and Zhang, 1998; Cigno, 1993).

[10] The assumption of homogenous preference for children is, however, eased in Section 5, implicitly allowing the case that women may regard children as investment goods and thus, they may prefer the larger family size.

[11] The classic formulation of intra-household bargaining was studied in the Nash bargaining model (Manser and Brown, 1980; McElroy and Horney, 1981). They regarded the statuses of single or divorced as threat-points. The threat-points of the Lundberg and Pollak (1993) bargaining model are statuses of non-cooperative equilibrium. Lundberg and Pollak (1996) give an excellent survey on intra-family bargaining.

that husband's wage is higher than the wife's: $w_m > w_f$. Furthermore, we assume the parental times of the wife and the husband are completely substitutable. Thus, it is the most efficient for a couple if the wife takes care of their children, and the husband's endowed time is spent solely on market work: $L_t^f = 1 - zn_t$ and $L_t^m = 1$.[12] Note that, in this paper, we assume that there is no substitution between domestic and market childcare.

In the young period, the couple chooses the number of children they have, n_t, and allocates the disposable income among their own private goods consumptions, c_t^i, and their savings for retirement, s_t. The budget constraint of the household in the young period is given by:

$$c_t^f + c_t^m + w_f z n_t + s_t = w_f + w_m - \tau \tag{3}$$

where τ denotes the lump-sum tax imposed on the household.[13]

Here, $w_f + w_m > \tau$ is assumed in the following analysis. In the retirement period, the household members enjoy their private goods consumption, financed by the return from their savings and the pension benefit. The budget constraint of the household in the retirement period is:

$$d_{t+1}^f + d_{t+1}^m = Rs_t + (\lambda_f + \lambda_m)P_{t+1}, \tag{4}$$

where P_{t+1} is the pension benefit to each individual and

$$R \equiv \frac{1+r}{\lambda_f} \tag{5}$$

is the return rate for the savings after one period, while r is the interest rate. Here, Eq.5 implies that the savings in the young period are returned to the household, as long as one member of the couple survives. In Eq.4, the term $(\lambda_f + \lambda_m)P_{t+1}$ implies that the longer the household member i lives during the second period, the more amount of pension benefit

[12] Easing the assumption of complete substitution does not affect our main results as long as the wife is better at caring for the children.
[13] In order to make the analysis simpler and clearer, we assume here the government employs the lump-sum tax. In the case of income taxation, the model includes the substitution effect due to a change in the opportunity cost of raising children and additional redistribution effect from husbands to wives through different labor income. The extension to employ income tax is carried out in Section 5, in which our main remark on the redistribution effect caused through gender difference in longevities remains intact.

he/she can obtain.

Given the bargaining power, θ_t, the household maximizes Eq.2 subject to Eq.3 and Eq.4. Solving the optimization problem, we have the following demand functions:

$$c_t^f = \frac{\theta_t}{2+\theta_t\lambda_f + (1-\theta_t)\lambda_m} I_t, \tag{6}$$

$$c_t^m = \frac{(1-\theta_t)}{2+\theta_t\lambda_f + (1-\theta_t)\lambda_m} I_t, \tag{7}$$

$$d_{t+1}^f = \frac{R\theta_t\lambda_f}{2+\theta_t\lambda_f + (1-\theta_t)\lambda_m} I_t, \tag{8}$$

$$d_{t+1}^f = \frac{R(1-\theta_t)\lambda_m}{2+\theta_t\lambda_f + (1-\theta_t)\lambda_m} I_t, \tag{9}$$

$$n_t = \frac{1}{w_f z\left[2+\theta_t\lambda_f + (1-\theta_t)\lambda_m\right]} I_t, \tag{10}$$

$$s_t = \frac{R\left[\theta_t\lambda_f + (1-\theta_t)\lambda_m\right]\left(w_f + w_m - \tau\right) - 2\left(\lambda_f + \lambda_m\right)P_{t+1}}{R\left[2+\theta_t\lambda_f + (1-\theta_t)\lambda_m\right]}, \tag{11}$$

where $I_t \equiv w_f + w_m - \tau + (\lambda_f + \lambda_m)P_{t+1}R^{-1}$ is the net lifetime income for the couple. The demand functions show that, as the bargaining power of the wife rises, the household goods consumption in the young period and fertility fall, while the savings, and thus, the consumption in the old period, increases: $\partial(c_t^f + c_t^m)/\partial\theta_t < 0$, $\partial n_t/\partial\theta_t < 0$, $\partial s_t/\partial\theta_t > 0$ and $\partial(d_{t+1}^f + d_{t+1}^m)/\partial\theta_t > 0$. Intuitively, there is a conflict between spouses in terms of lifetime goals because the wife thinks she will outlive her husband, while the husband believes the opposite. Hence, the wife wants to save the more money for retirement than her husband. If the wife's bargaining position becomes more favorable, the household outcomes are more likely to reflect her intentions. Consequently, the private goods consumption in retirement age increases by reducing consumption in the young period, as well as the number of children they have.

2.2 PAYG Pension

The government operates the PAYG pension system. It imposes a tax on each household in the young period so that it can finance the pension benefit for people living in the old period. The government's budget constraint in per household terms is given by:

$$n_t \tau = (\lambda_f + \lambda_m) P_{t+1}. \tag{12}$$

2.3 Bargaining Power

We assume that the power balance of young couples is shaped within the marriage market and is affected by social norms or peer pressure (Grossbard-Shechtman, 1984, 1993, 2013; Lundberg and Pollak, 1993; Komura, 2013a, 2013b).[14] They anticipate the balance of power within the marriage by observing the behaviors and economic relations of their parents. Specifically, the balance of power of the couple in generation t depends on the difference in their average resources of men and women in generation $t-1$, including the income from the pension benefits of men and women in the preceding generation:

$$\theta_t = \theta\left[w_f L_{t-1}^f - w_m L_{t-1}^m + \beta(\lambda_f - \lambda_m) P_t R^{-1}\right], \tag{13}$$

where $\theta' > 0$ is assumed. For simplicity of notation, we assume $\theta'' = 0$. In Eq.11, the term $w_f L_{t-1}^f - w_m L_{t-1}^m$ represents the gap in labor income, and the term $\beta(\lambda_f - \lambda_m) P_t R^{-1}$ represents the gap in the expected pension benefits that affect the bargaining power of the couple. $\beta \in [0,1]$ captures the degree of how the pension policy (or income in the retirement period) affects the balance of power within the marital relationship. If $\beta = 1$, the gap in the expected pension benefits and the gap in labor income have the same effect on the bargaining power. Therefore, the determinant of power for the couple simply becomes the difference between the expected lifetime incomes of men and women. In contrast, $\beta = 0$ reduces our model to that of Komura (2013a, 2013b) essentially, in which the pension policy has no impact on the bargaining power.

Recalling that the husband spends his time solely in the labor market, Eq. 13 can be rewritten as:

[14] Using data of European countries and Japan, Feyrer et al. (2008) pointed out that women's status is affected not only by common economic factors but also by the longstanding cultural and social factors.

$$\theta_t = \theta\left[w_f(1 - z\bar{n}_{t-1}) - w_m + \beta(\lambda_f - \lambda_m)P_t R^{-1}\right], \qquad (14)$$

where $\bar{n}_{t-1}$ stands for the average number of children per household in generation $t-1$. Note that the wife's bargaining power decreases as the average number of children in the society increases, $\partial\theta_t / \partial\bar{n}_{t-1} = -\theta' w_f z < 0$. This implies that having children by couples in the previous generation weakens the wife's say in the next generation, because a reduction in her earning is expected from peer pressure. It is also worth mentioning that the expansion of a public pension, which mainly aims to transfer income from young to old, plays a role in the income transfer from husbands to wives in our model, and hence, it increases the wife's bargaining power, $\partial\theta_t / \partial P_t = \theta'\beta(\lambda_f - \lambda_m)R^{-1} \geq 0$.

3. Equilibrium

3.1 Dynamics

Using Eqs.10, 12 and 14, the dynamics of bargaining power can be obtained as:

$$\theta_{t+1} = \theta\left(w_f - w_m - \frac{R(w_f + w_m - \tau)A}{w_f z\left[2 + \theta_t\lambda_f + (1-\theta_t)\lambda_m\right]R - \tau} \right), \qquad (15)$$

where

$$A \equiv zw_f - \frac{\beta\tau(\lambda_f - \lambda_m)}{R(\lambda_f + \lambda_m)}. \qquad (16)$$

Differentiation gives:[15]

$$\frac{\partial\theta_{t+1}}{\partial\theta_t} = \frac{(\lambda_f - \lambda_m)z\theta' n_t^2 w_f A}{w_f + w_m - \tau}, \qquad (17)$$

$$\frac{\partial^2\theta_{t+1}}{\partial\theta_t^2} = \frac{2z^2 w_f^2(\lambda_f - \lambda_m)^2 n_t^3 \theta' A}{(w_f + w_m - \tau)^2}. \qquad (18)$$

If τ is sufficiently small, the sign of Eq.16 tends to be positive, $A > 0$. This corresponds to the case in which the pension policy is inactive, or $\tau = 0$. In contrast, when the pension policy is active and τ is sufficiently large, Eq. 16 is likely to take a negative sign, $A < 0$.

[15] See Appendix A.

Supposing that $\theta_{t+1}\big|_{\theta_t=0} > 0$, Eqs. 17 and 18 reveal that the bargaining power converges monotonically (cyclically) to the steady-state if $A > (<) 0$.[16]

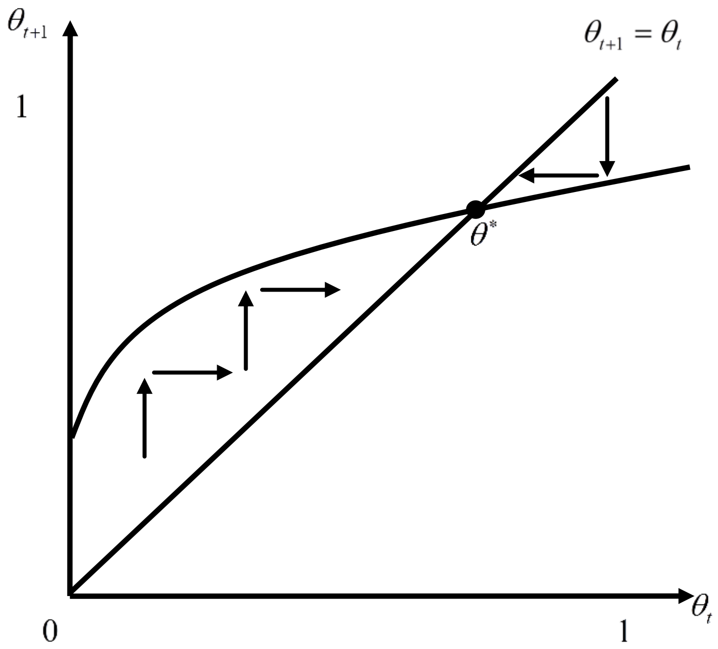

Figure 1(a). Steady-state bargaining power; $A > 0$.

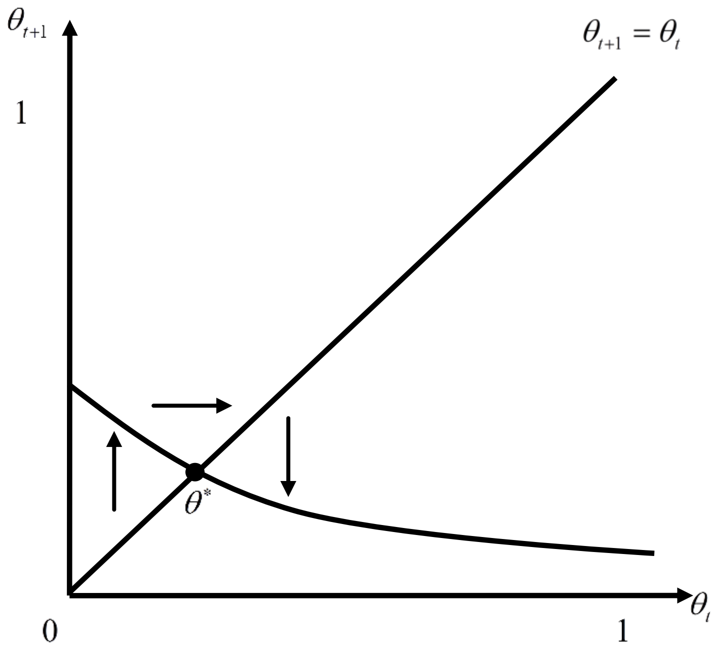

Figure 1(b). Steady-state bargaining power; $A < 0$.

[16] The stability of the steady-state is ensured by assuming $\left|\partial\theta_{t+1}/\partial\theta_t\right| < 1$.

Behind the dynamics of the bargaining power, the dynamics of fertility can be derived in a similar way to show that the fertility converges to the steady state monotonically (cyclically) if $A > (<) 0$.

3.2 Steady state

Using Eqs. 10, 12 and 14, the steady-state value of the bargaining power and the fertility satisfy:

$$\theta = \theta\left(\frac{\beta\tau n(\lambda_f - \lambda_m)}{(\lambda_f + \lambda_m)R} - w_f z n + w_f - w_m\right), \tag{19}$$

$$n = \frac{R(w_f + w_m - \tau)}{w_f z\left[2 + \theta\lambda_f + (1-\theta)\lambda_m\right]R - \tau}. \tag{20}$$

To plot combinations (θ, n) that satisfy Eqs. 19 and 20 we first reveal how the fertility affects the bargaining power in the steady state equilibrium. From Eq. 19 and $\theta'' = 0$, we have:

$$\frac{\partial\theta}{\partial n} = -\theta' A, \tag{21}$$

$$\frac{\partial^2\theta}{\partial n^2} = 0. \tag{22}$$

In Eq.21, if τ is sufficiently small, then $A > 0$, the effect of fertility on the bargaining power is negative, $\partial\theta/\partial n < 0$.This implies that having another child reduces women's bargaining position since the wife is forced into child rearing, reducing her income from the labor market. Having a child as a factor against women's bargaining power is captured by the first term in Eq. 16. If $\tau > 0$, the sign of $\partial\theta/\partial n$ depends on the relative magnitude of two terms. The second term in Eq. 16 captures the positive effect of having a child on the wife's bargaining power, owing to the PAYG pension system. The more children there are in society, the more the pension benefit increases for the elderly. Because the wife survives longer in the retirement period, her expected benefit from the PAYG pension system is higher than that of her husband's. As the weight of pension benefit in the bargaining power increases, owing to an increase in fertility, the wife becomes invulnerable, strengthening her power within the couple.

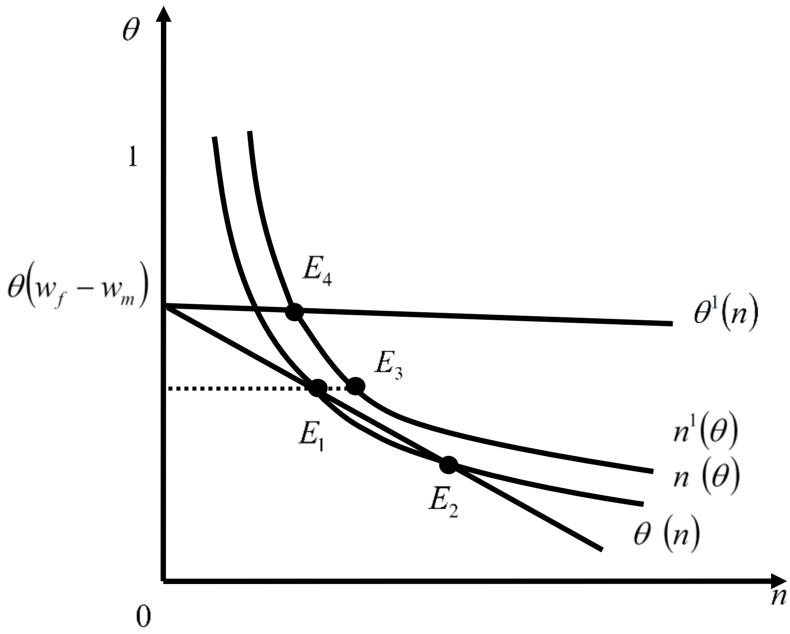

Figure 2(a). Effect of increase in τ when $R < n$ and the pension policy is inactive, $A > 0$.
Note. An increase in the size of pension policy shifts a stable equilibrium from E_1 to E_4. E_2 is an unstable equilibrium.

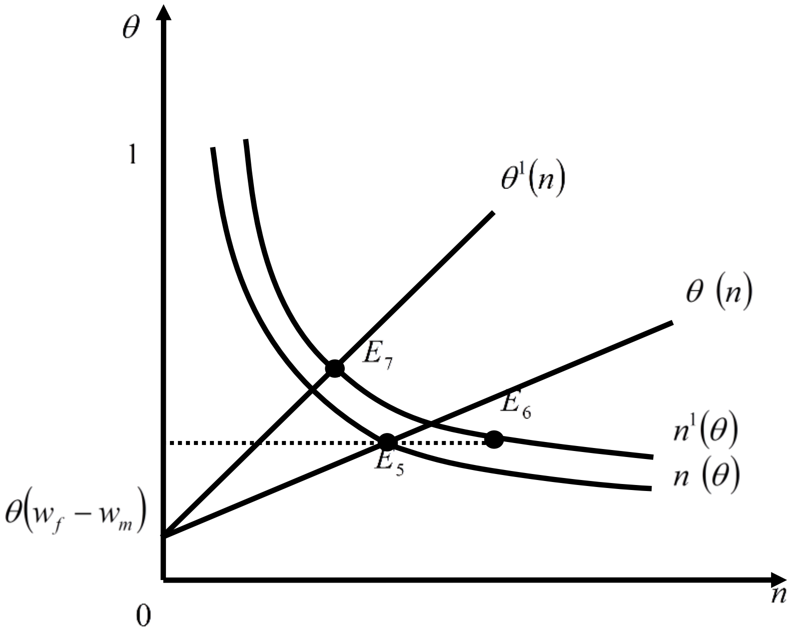

Figure 2(b). Effect of increase in τ when $R < n$ and the pension policy is active, $A < 0$.

Note. An increase in the size of pension policy shifts a stable equilibrium from E_5 to E_7.

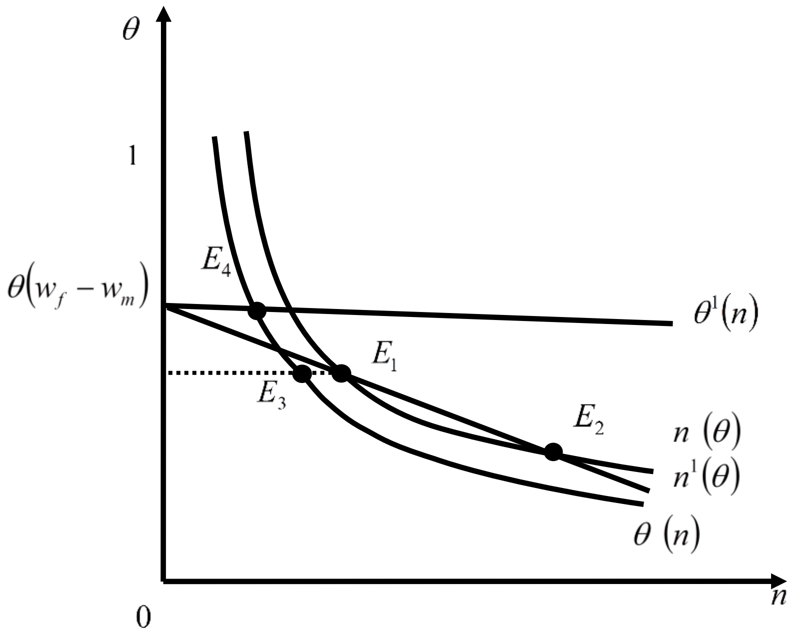

Figure 3(a). Effect of increase in τ when $R > n$ and the pension policy is inactive, $A > 0$.

Note. An increase in the size of pension policy shifts a stable equilibrium from E_1 to E_4.

E_2 is an unstable equilibrium.

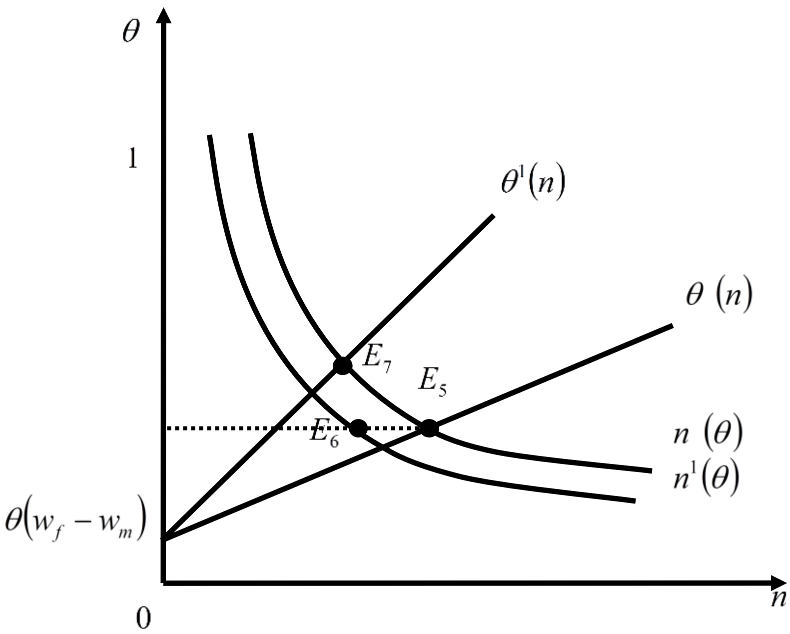

Figure 3(b). Effect of increase in τ when $R > n$ and the pension policy is active, $A < 0$.

Note. An increase in the size of pension policy shifts a stable equilibrium from E_5 to E_7.

$\theta(n)$ in Figures 2(a) and 3(a) represents Eq. 19 when the pension policy is inactive, i.e., τ is sufficiently small to cause the sign of Eq. 21 negative. In contrast, $\theta(n)$ in Figures 2(b) and 3(b) represents the alternative case, in which the pension policy is active, $\tau > 0$ leading the sign of Eq. 21 to be positive.

We next study how fertility is related to the bargaining power in the steady-state. From Eq. 20, differentiating n with respect to θ gives:[17]

$$\frac{\partial n}{\partial \theta} = -\frac{n^2 z w_f (\lambda_f - \lambda_m)}{w_f + w_m - \tau} < 0, \tag{23}$$

$$\frac{\partial^2 n}{\partial \theta^2} = 2n\left[\frac{n z w_f (\lambda_f - \lambda_m)}{w_f + w_m - \tau}\right]^2 < 0. \tag{24}$$

Eq. 20 is illustrated as $n(\theta)$ in Figures 2 and 3, showing that the number of children the

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[17] See Appendix A.

couple has decreases as the woman's bargaining power increases. This is simply because the wife lives longer than the husband does. As women live longer, they want to save more money for future consumption rather than spending it on raising children. In such situation, therefore, a rise in women's bargaining power leads to a fall in fertility rate, reflecting their intentions in household decisions.

4. Effects of Pension Policy

In this section, we examine the effects of the changes in the size of a PAYG social security system on fertility and women's bargaining power, focusing on the stable steady-state equilibrium of E_1 in Figures 2(a) and 3(a), and E_5 in Figures 2(b) and 3(b).

First, we come back to the traditional argument in which the bargaining power is fixed ($\theta = \bar{\theta}$). Using Eq. 20, this can be confirmed by differentiating n with respect to τ;[18]

$$\left.\frac{dn}{d\tau}\right|_{\bar{\theta}} = -\frac{n(R-n)}{R(w_f + w_m - \tau)}. \tag{25}$$

Here, Eq. 25 shows that, given bargaining power θ, an increase in the tax rate reduces (increases) the fertility if $R > (<) n$.

An increase in the size of the pension policy, represented by τ, affects fertility via the change in the lifetime full income in two ways: (i) the reduction in the disposable income of the working period decreases the number of children, and (ii) the increase in the pension benefit allows the household to reduce their savings for the old period and increases their fertility. In other words, the comparison between R and n means whether the present value of leaving τ as disposable income in the young period is larger or smaller than the present value of the pension benefit as a return of tax payments. The well-known Aaron-Samuelson condition states that future generations benefit from the PAYG pension system as an inter-generational transfer when the economy is dynamically inefficient, $1 + r < n$ (Samuelson, 1958; Aaron, 1966). While the interest factor is adjusted by the longevity in our model, there is strong support for the government operating the PAYG

[18] See Appendix A.

pension system, and that examining the effects of pension policy is relevant if $R < n$. When $R < n$, the positive effects of a pension expansion on lifetime income overwhelms the negative effects. As a result, an increase in lifetime income induces a rise in the number of children, as children are normal goods in our model.

On the other hand, if the economy is dynamically efficient, $1 + r > n$, the introduction of the lump-sum financed PAYG pension scheme basically loses its theoretical foundation. However, as summarized by Groezen et al. (2003), the political incentives may promote the introduction of an unfunded pension scheme or, a drastic policy reform is often difficult from a practical standpoint, even if the environment surrounding the economy becomes against a PAYG pension. If the PAYG pension system is operated under the condition that $R > 1 + r > n$, and the government increases the size of the pension policy, the negative effect on fertility of a decrease in disposable income in the young period outweighs the positive effect of an increase in pension benefit, which results in a fall in fertility rate.

In Figures 2(a) and 2(b), the policy effects on the fertility, given θ, are depicted by the shift of the $n(\theta)$ curve. When $R < n$, the sign of Eq. 25 becomes positive, which shifts the curve from $n(\theta)$ to $n^1(\theta)$. In contrast, if $R > n$, Eq. 25 becomes negative, and the curve $n(\theta)$ shifts left (see Figures 3(a) and 3(b)).

The important feature of our model is that the pension policy influences the bargaining power in the couple, which also has an impact on the fertility. From Eq. 19, given n ($n = \bar{n}$), we find that, as the pension policy increases, so does the woman's bargaining power:

$$\left.\frac{d\theta}{d\tau}\right|_{\bar{n}} = \frac{\beta n(\lambda_f - \lambda_m)}{(\lambda_f + \lambda_m)R} > 0. \qquad (26)$$

This indicates that an increase in the size of a pension policy, represented by τ, induces an upward shift of the $\theta(n)$ curve in Figures 2 and 3. This is simply because the increase in pension works to the longer-living woman's advantage, increasing her bargaining power. In this case, as β is larger, the pension benefit is appreciated in determining the balance of power between men and women, so that the change in the pension policy affects θ

significantly. Similarly, as the gender gap in life expectancy $\lambda_f - \lambda_m$ is larger, the gap in the pension benefit between men and women widens, resulting in a significant change in θ. In such situations, an increase in τ induces a relatively large upward shift in $\theta(n)$.

The steady-state equilibrium, n^* and θ^*, satisfy Eqs. 19 and 20. We first explain the effects of an increase in τ on (n^*, θ^*) when $R < n$, i.e., $\left(dn/d\tau\right)\big|_{\bar{\theta}} > 0$ in Eq. 25. Figures 2(a) and 2(b) show the cases in which, given other variables, the effect of an increase in pension size on fertility is modest, while an increase in the pension has a great impact on bargaining power. This case tends to take place when β and $\lambda_f - \lambda_m$ are large as explained above. In Figure 2(a), for instance, if we assume that the bargaining power is an exogenous parameter, then the increase in the size of the pension increases fertility along the course from E_1 to E_3. However, as the bargaining power is endogenous in our model, the woman's bargaining power is strengthened by the increase in the size of the pension, moving the stable equilibrium from E_1 to E_4. This shows that a rise in τ induces a fall in fertility. The same holds for Figure 2(b). If the bargaining power is exogenous, the increase in τ raises fertility, along the course from E_5 to E_6. However, it decreases fertility by shifting the equilibrium from E_5 to E_7 as the bargaining power of the wife increases. Consequently, the fertility rate in the economy may fall if the bargaining power is determined endogenously.

The case of $R > n$, i.e., $\left(dn/d\tau\right)\big|_{\bar{\theta}} < 0$, can be interpreted in a similar fashion, by making use of Figures 3(a) and 3(b). Both figures show that fertility decreases not only through the decrease in lifetime net income but also through the increase in the bargaining power of the wife, if the pension policy changes the balance of power significantly.

Some empirical studies with unitary models support the case in which the pension expansion by an increase in τ causes a decline in fertility (Cigno and Rosati, 1996; Boldin et al., 2005). Our model shows that the fertility rate falls not only because of the negative income effect (i.e., the leftward shift of the $n(\theta)$ curve) but also the negative bargaining effect (i.e., the upward shift of the $\theta(n)$ curve). If there is no heterogeneity within a household, $\lambda_f = \lambda_m$, the pension policy has no influence on the bargaining power (see Eq.

26), and thus the effects of the pension policy are essentially the same as that the traditional unitary model found. This is because the spouses' lifetime objects are identical, so that they do not need to negotiate household allocations. In the real economy with gender differences, if policy makers ignore the bargaining power effect, the negative effects of a pension reform on fertility could be biased or could reverse the sign of the impact estimated initially.

Using Eqs. 19 and 20, the graphical analysis mentioned above can be formally restated as follows:

$$\frac{dn^*}{d\tau} = \left(\frac{\partial n}{\partial \tau} + \frac{\partial n}{\partial \theta} \frac{\partial \theta}{\partial \tau} \right) \Delta^{-1} = \frac{n}{\tau\Delta} \left(\varepsilon_{n\tau} + \varepsilon_{n\theta} \varepsilon_{\theta\tau} \right), \tag{27}$$

$$\frac{d\theta^*}{d\tau} = \left(\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial n} \frac{\partial n}{\partial \tau} \right) \Delta^{-1} = \frac{\theta}{\tau\Delta} \left(\varepsilon_{\theta\tau} + \varepsilon_{\theta n} \varepsilon_{n\tau} \right), \tag{28}$$

where $\Delta \equiv 1 - (\partial n/\partial \theta)(\partial \theta/\partial n) > 0$ and ε_{km} $(k \neq m)$ denotes the elasticity of $k(= n, \theta)$ with respect to $m(= \tau, n, \theta)$.[19]

The first term in Eq.27 represents the effects of an increase in τ on fertility through changes in lifetime full income, which is regarded as an inter-generational distribution effect by conventional studies. The sign can be positive or negative, depending on the relative magnitude between R and n as in Eq.25. The second term in Eq.27 stands for the effect caused by a change in the power balance within a household, and can be interpreted as an intra-generational distribution effect. As other studies on intra-generational distribution effects, this effect is caused by heterogeneity among individuals in the same generation, but that within the family. Because the pension expansion by an increase in τ leads women with higher longevity to a more favorable position, the sign of $\partial \theta/\partial \tau$ is positive. On the other hand, $\partial n/\partial \tau$ is negative because women want to reduce the number of children so that they can ensure resources in the retirement period, taking into account their higher probability to survive. Thus, overall, the second term is negative. The sign of the total effect of the change in τ on n^*, therefore, is not determined a priori. If the negative effect of an increase in pension size on fertility due to a change in bargaining power exceeds the

[19] See Appendix B.

positive effect of the increase in τ, then fertility is decreased by an increase in τ.

Now, we turn to the total effects of a change in τ on θ^*. The first term in Eq.28 captures the direct positive effect of a change in τ on bargaining power by changing the gap of the pension benefit between wife and husband. The second term in Eq.28 represents the indirect effect through a change in fertility caused by an increase in τ. The sign of $\partial\theta/\partial n$ depends on the relative magnitude of the effects of n on women's bargaining power as a result of reduced labor income and an increased pension benefit, as compared to that of their husbands. Consequently, the sign of the overall effect is determined by these direct and indirect effects.

5. Discussion

5.1 Income taxation

In the previous sections, we have employed the lump-sum tax to derive the main result with clarity. However, in the real world, pension systems are generally financed by the tax revenue which depends on the level of household (individual) incomes. Moreover, considering the generous flat-benefit pension system, it may be also important to argue the possibility of its income redistribution effect not only from gender gap in life expectancy but from gender income gap. In this section, therefore, we explore the long-run effect of pension policy on fertility, employing income tax as a policy tool.

The household optimization is modified to the problem of maximizing Eq.2 subject to the budget constraint in young period,

$$c_t^f + c_t^m + (1-\tau)w_f z n_t + s_t = (1-\tau)(w_f + w_m) \tag{29}$$

and the budget constraint in the old period of Eq. 4. By solving the problem, we can have the household demand function of the number of children as:

$$n_t = \frac{1}{(1-\tau)w_f z\left[2+\theta_t\lambda_f + (1-\theta_t)\lambda_m\right]} I_t$$

where $I_t \equiv (1-\tau)(w_f + w_m) + (\lambda_f + \lambda_m)P_{t+1}/R$.

The government budget constraint is then rewritten by:

$$n_t \tau \left[w_f (1 - z n_{t+1}) + w_m \right] = (\lambda_f + \lambda_m) P_{t+1} ,$$

which means that the pension benefit for the elderly is paid by the young generations' contribution in accordance with the level of their earned income.

Finally, we can define the bargaining power of the household as:

$$\theta_t = \theta \left[(1-\tau) w_f (1 - z \bar{n}_{t-1}) - (1-\tau) w_m + \beta (\lambda_f - \lambda_m) P_t R^{-1} \right].$$

The assumption that bargaining power depends on the tax policy reflects the existing theoretical and empirical results on the relationship between tax policies and intra-household bargaining allocation (Vermeulen et al. 2006; Myck et al. 2006; Stöwhase 2011; Komura 2013b).

The combinations of (n, θ, P) in the steady-state equilibrium satisfy the following conditions:

$$n = \frac{(1-\tau)(w_f + w_m) + (\lambda_f + \lambda_m) P / R}{(1-\tau) w_f z \left[2 + \theta \lambda_f + (1-\theta) \lambda_m \right]} ,$$

$$\theta = \theta \left[(1-\tau) w_f (1 - z n) - (1-\tau) w_m + \beta (\lambda_f - \lambda_m) P R^{-1} \right], \qquad (30)$$

$$n \tau \left[w_f (1 - z n) + w_m \right] = (\lambda_f + \lambda_m) P .$$

Now, we examine the long-run effect of the pension system (an increase in τ) on the fertility rate. By calculations we have:[20]

$$\frac{dn}{d\tau} = - \left\langle \left\{ (\lambda_f + \lambda_m) \frac{\partial n}{\partial \tau} + n \left[w_f (1 - z n) + w_m \right] \frac{\partial n}{\partial P} \right\} \right.$$

$$\left. + \frac{\partial n}{\partial \theta} \left\{ (\lambda_f + \lambda_m) \frac{\partial \theta}{\partial \tau} + n \left[w_f (1 - z n) + w_m \right] \frac{\partial \theta}{\partial P} \right\} \right\rangle \Delta^{-1} \qquad (31)$$

where $\Delta \equiv -(\lambda_f + \lambda_m)(1 - \partial n / \partial \theta \cdot \partial \theta / \partial n) + \psi (\partial n / \partial P + \partial n / \partial \theta \cdot \partial \theta / \partial P) < 0$, which is the impact of an increase in P on the government's budget and is reasonable to assume negative.

The policy effect on fertility in Eq.31 is then classified into two parts: the terms in the first braces of price and income effects on fertility, and the terms in the second braces of the

[20] See Appendix C.

bargaining power effects. The first term in the first braces is composed by the two opposite effect. One is the positive price effect arising from a reduction in opportunity cost of rearing children, which is led by an increase in τ. On the other hand, an increase in τ also induces the negative income effect due to a reduction in disposal income of the household in the young period. The second term in the first braces is the positive income effect coming from the increase in disposal income in the old as a result of an increase in P. In total, the terms in the first braces are positive.

For the bargaining power effects, we have the two effects on fertility through the changes power balance within a family. The first term of the bargaining power effects is positive indicating a change in power balance as a consequence of change in the difference in the after-taxed labor incomes of men and women. The second term is positive on the wife's bargaining power because the difference in the pension benefits of men and women is widened, which has been discussed in the previous section. Consequently, the long-living wife with more say will reduce the number of children to suffice the consumption in the old period.

It is deserved to mention that an increase in pension size induces the intra-generational redistribution effects from husbands to wives in twofold: firstly it causes a redistribution through gap in labor incomes and secondly that through gap in longevities. In the existing literature, which assumes earning abilities and longevities are positively correlated in household level, found that the total direction of redistribution effect is ambiguous (Borck 2003; Cremer et al. 2010; Adema et al. 2013). This is because, the flat benefit pension system operates in the favor of the individuals with the properties of low income and high longevity, so that the positive correlation between earning and longevity implies these two distribution effects work toward opposite directions. However, if we consider the heterogeneous individuals as family members rather than households, wives earn less and live longer on average, and hence, the flat-benefit pension system is favorable for women. Our contribution to the literature on intra-generational redistribution effect of social security is further emphasized by this model specification because the positive price effect dominates the negative income effect. Thus, the sign of total effect of τ on n is simply determined by the relative magnitude of the positive price and income effects and the negative bargaining power effect.

5.2 Heterogeneous preferences

This subsection discusses the possibility of the couple's heterogeneous preferences for the number of their children. In the previous sections, we have considered their homogenous preferences except for the time preference arising from the observable gender gap in longevity. However, demographic studies on not only developing countries but developed countries have been finding the heterogeneous preferences for the number of children indicating the both cases that men prefer the larger family size than women and vice versa (Voas ,2003; Testa 2006; Hener, 2006).

To deal with the point, Eq. 1, the utility function of individual i who belongs to period t, is then rewritten as:

$$U_{it} = \phi_i \ln n_t + \ln c_t^i + \lambda_i \ln d_{t+1}^i, \tag{31}$$

where ϕ_i is the preference for the number of their children for individual i so that women prefer less children than men when $\phi_f < \phi_m$. The existing evidence supports this situation, and implies that the women's weak preference on children is attributed by physical and mental burden and the limited career options from frequent childbirth. However, we do not exclude the other empirically observed pattern of $\phi_f > \phi_m$, in which women prefer more children than men, [21] since this specialization also serves to illustrate an interesting situation from the perspective of life cycle models that women with longer life span benefit from the larger family size expecting that children may provide care and companionship after the death of their partner.

The maximization problem of household welfare function gives the demand function for children as:

[21] One possible interpretation of this case can be different religion within a couple (Lehrer, 1996).

$$n_t = \frac{\theta\phi_f + (1-\theta)\phi_m}{w_f z\left[1+\theta\phi_f + (1-\theta)\phi_m + \theta\lambda_f + (1-\theta)\lambda_m\right]} I_t ,$$

implying that the sign of the effect of bargaining power on fertility depends on ϕ_i and λ_i as:

$$\frac{\partial n_t}{\partial \theta} = \frac{\phi_f(1+\lambda_m) - \phi_m(1+\lambda_f)}{w_f z\left[1+\theta\phi_f + (1-\theta)\phi_m + \theta\lambda_f + (1-\theta)\lambda_m\right]^2} I_t . \tag{32}$$

Note that Eq. 32 is reduced to Eq. 23 if $\phi_f = \phi_m$ as we have assumed in the previous sections.

Given $\lambda_f < \lambda_m$, Eq. 32 shows that an increase in the bargaining power is likely to induce a decline in the number of children when women prefer smaller family size than their partners, $\phi_f < \phi_m$, indicating that our main result of negative effect of pension systems on fertility through a change in bargaining power can hold in the generalized model with heterogeneous preferences for the number of children.

However, our main result coming from the different life spans between men and women is weakened when $\phi_f > \phi_m$. This is because women have the strong inventive to have two goods compared to their partners at the same time: savings from different longevities, and children from the different preferences. As long as they are substitute goods, the total effect of a rise in women's bargaining power is determined by the relative magnitudes between the former and the latter. Consequently, when women's preference for children is sufficiently strong leading $\phi_f(1+\lambda_m) - \phi_m(1+\lambda_f)$ to be positive, an expansion in pension size can be likely to increase the fertility rate thorough a change in their bargaining power.

In sum, our main results may be affected when women prefer much larger family size than their partners and the effects of pension policy on fertility through a change in bargaining power are weak. Conversely, this extension have found that our results are valid as long as women prefer smaller family size than their partners,

6. Conclusion

The family is the key constitutional unit of human society. It offers comfort, security, and a place to grow. However, the role and the structure of family change according to the influence of many factors, such as changing lifestyles and increasing personal mobility. Public policy is also a significant factor affecting the family shape. The decision on having children must be affected by the system of childcare leave, educational costs, and various family policies. The systems of taxation and social security are also factors that influence the way couples work and the balance of power within the family. This paper can be placed as a variant on a line that examines the intra-familial structure, focusing on the balance of power and the number of children in the family.

There has been intense research into the effects of pension policy on the fertility. Most studies approach this matter using an overlapping generation (OLG) model with a unitary household, assuming no interaction between wife and husband. These studies are successful in analyzing long-run macroeconomic steady-state outcomes. At the same time, these standard approaches rely on some strong assumptions. In particular, they assume a unitary household with no heterogeneity in preferences or the lifespans of wives and husbands. This means they assume that a couple never bargains over household resource allocation, such as the number of children they have and/or the amount they save for the future. This paper approaches from different perspective than the orthodox homogenous couple, to explore the effects of a public pension on the household resource allocation by the bargaining couple with heterogeneity.

Following the trend of analyses on the heterogeneous couples, we also consider a setting with heterogeneity in the lifespan between husband and wife. The wife tends to live longer than her husband, causing incentives for them to bargain over the amount of saving they do and the number of children they have. The bargaining power between wife and husband is endogenously determined in the social level based on their relative average lifetime income, including pension benefit. This is affected critically by a pension reform, which may in turn influence endogenous fertility. To demonstrate our results simply, we consider a small open economy characterized by an exogenous interest rate and wages. The interest rate is the same for both the husband and wife, but the wage rate is not. Men tend to get a higher wage rate than the women, therefore the women often decide against participating in labor market.

Using this setup, we find out a new channel of pension policy on fertility decisions. An increase in pension size affects not only via the changes in current and future incomes, but it affects the fertility through the change in marital bargaining power. The conventional OLG literature has always seen marital bargaining power as fixed, which meant that the increase in the pension benefit of the old accompanied by the tax increase in the young simply changes the lifetime income. As a result, this change in lifetime income caused by an increase in the pension size affects the household fertility behaviors. In our model, however, the development of a pension alters the marital relationship defined by the gender gap in lifetime incomes, because the wife lives longer and is expected to gain higher amount of pension benefit. The change in the balance of power within the heterogeneous couple affects their saving behavior as well as their fertility. This results in the PAYG pension accelerating the falling birthrate, in contrast to the case of homogenous couples. Moreover, we explore how our main result can be influenced by introductions of income tax and heterogeneous parental preferences between men and women. The extensions show that introducing income tax helps us to have the rich and realistic argument keeping our results to hold, but the heterogeneous preferences (especially when women strongly prefer the larger family size than their partners) may induce the opposite effect from what we expected from the change in their bargaining power in the basic model.

In closing this paper, we briefly discuss the decision of the pension size in majority voting and its optimality. Suppose an economy where the size of pension system is determined by voting at every period. Every individual living in t period can vote for the decisions. It is easily expected that the individuals who enter the old period prefer to increase the tax rate as high as possible, because they realize that they surely survive the whole period once they enter the old period. On the other hand, individuals in the young period would choose the optimal level of pension size for themselves τ_i^*, which maximize their own expected indirect utility functions $V_t^i = \ln c_t(\tau) + \ln n_t(\tau) + \lambda_i \ln d_{t+1}(\tau).$ Because $\lambda_f > \lambda_m$ and the pension policy in our model is more beneficial for the longer-living individuals, women prefer the larger pension size, $\tau_f^* > \tau_m^*$. In sum, the preferred tax rate (pension size) of the elderly, the young women and the young men are the highest possible level, τ_f^* and τ_m^*, respectively. Since the relative voting power (population) is $\lambda_f + \lambda_m$, n and n, for decision branch of highest possible level, τ_f^* and τ_m^*, the political equilibrium as the Condorcet winner is τ_f^*. However, if we define the social welfare function as the sum of expected life-time utilities of a man and a woman which are weighted equally as $V_t^f + V_t^m$, it is obvious that the tax rate in the political equilibrium is larger than the social optimal level.[22]

[22] See Leroux et al. (2011) for similar analysis of political equilibrium with marital statuses and gender difference in longevity and productivity.

Appendices

Appendix A

Derivation of Eqs 17 and 18.

From Eq. 15, we have

$$\frac{\partial \theta_{t+1}}{\partial \theta_t} = \frac{\theta' R^2 (w_f + w_m - \tau) A w_f z (\lambda_f - \lambda_m)}{\left\{ w_f z \left[2 + \theta_t \lambda_f + (1-\theta_t)\lambda_m \right] R - \tau \right\}^2}. \tag{A1}$$

From Eqs. 10 and 12, we have

$$n_t = \frac{R(w_f + w_m - \tau)}{w_f z \left[2 + \theta_t \lambda_f + (1-\theta_t)\lambda_m \right] R - \tau}. \tag{A2}$$

Substituting Eq.A2 into Eq. A1, we have Eq. 17.

The differentiation of Eq. A1 gives

$$\frac{\partial^2 \theta_{t+1}}{\partial \theta_t^2} = \frac{2(\lambda_f - \lambda_m)^2 w_f^2 z^2 \theta' R^3 (w_f + w_m - \tau) A}{\left\{ w_f z \left[2 + \theta_t \lambda_f + (1-\theta_t)\lambda_m \right] R - \tau \right\}^3}. \tag{A3}$$

Substituting Eq.A2 into Eq.A3, we obtain Eq.18.

Derivation of Eqs. 23 and 24.

From Eq. 20, we have

$$\frac{dn}{d\theta} = -\frac{w_f z R^2 (\lambda_f - \lambda_m)(w_f + w_m - \tau)}{\left\{ w_f z \left[2 + \theta \lambda_f + (1-\theta)\lambda_m \right] R - \tau \right\}^2} < 0,$$

$$\frac{d^2 n}{d\theta^2} = \frac{2 w_f^2 z^2 (\lambda_f - \lambda_m)^2 R^3 (w_f + w_m - \tau)}{\left\{ w_f z \left[2 + \theta \lambda_f + (1-\theta)\lambda_m \right] R - \tau \right\}^3} > 0.$$

Eq. 20 is rewritten as

$$2 + \theta \lambda_f + (1-\theta)\lambda_m = \frac{R(w_f + w_m - \tau) + \tau n}{n w_f z R} < 0,$$

which is used to derive Eqs. 23 and 24.

Derivation of Eq. 25.

From Eq. 20, we have

$$\left.\frac{dn}{d\tau}\right|_{d\theta=0} = \frac{(w_f + w_m)R - \left[2+\theta\lambda_f + (1-\theta)\lambda_m\right]zw_f R^2}{\left\{w_f z\left[2+\theta\lambda_f + (1-\theta)\lambda_m\right]R - \tau\right\}^2}.$$

Substituting Eq. 20 into this equation, we have Eq. 25.

Appendix B

Derivation of Eqs. 27 and 28.

Fertility and bargaining power in the steady-state are given by total differentiation:

$$dn^* - \frac{\partial n}{\partial \theta} d\theta^* = \frac{\partial n}{\partial \tau} d\tau,$$

$$-\frac{\partial \theta}{\partial n} dn^* + d\theta^* = \frac{\partial \theta}{\partial \tau} d\tau.$$

Using these equations, we have

$$\begin{bmatrix} 1 & -\partial n/\partial\theta \\ -\partial\theta/\partial n & 1 \end{bmatrix} \begin{bmatrix} dn^* \\ d\theta^* \end{bmatrix} = \begin{bmatrix} \partial n/\partial\tau \\ \partial\theta/\partial\tau \end{bmatrix} d\tau,$$

where the determinant of the coefficient matrix is $1-(\partial n/\partial\theta)(\partial\theta/\partial n)$, which is reasonable to assume positive. Solving this, we have Eqs. 27 and 28.

Appendix C

Derivation of Eq. 31.

Using Eq. 30, we have

$$
\begin{bmatrix} 1 & -\dfrac{\partial n}{\partial \theta} & -\dfrac{\partial n}{\partial P} \\ -\dfrac{\partial \theta}{\partial n} & 1 & -\dfrac{\partial \theta}{\partial P} \\ \tau\left[w_f(1-zn)+w_m\right]-\tau w_f zn & 0 & -(\lambda_f+\lambda_m) \end{bmatrix} \begin{bmatrix} dn^* \\ d\theta^* \\ dP \end{bmatrix} = \begin{bmatrix} \dfrac{\partial n}{\partial \tau} \\ \dfrac{\partial \theta}{\partial \tau} \\ -n\left[w_f(1-zn)+w_m\right] \end{bmatrix} d\tau \, .
$$

Solving this allows us to obtain Eq. 31.

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